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Coplanarity of two lines and System of Planes

SUBJECT : MATHEMATICS
CHAPTER NUMBER:11
CHAPTER NAME :THREE DIMENTIONAL GEOMETRY

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Co-planarity of Two lines

Two given lines be $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if $\vec{a}_2 - \vec{a}_1$ is perpendicular to $\vec{b}_1 \times \vec{b}_2$

i.e. when $(\vec{a}_2 - \vec{a}_1).\left(\vec{b}_1 \times \vec{b}_2\right) = 0$

And the equation of the plane containing them is $(\vec{r} - \vec{a}_1).\left(\vec{b}_1 \times \vec{b}_2\right) = 0$

In Cartesian form for two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ & $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar if

$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ and equation of the plane containing them is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

Note:- If the two lines are parallel then the two lines will be coplanar.

EXAMPLE

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-5}{8}$ are coplanar.

Plane Passing Through the Intersection of two Planes

The **vector equation** of the plane passing through the intersection of the planes $\vec{r}.\vec{N_1} = d_1$ and $\vec{r}.\vec{N_2} = d_2$ is $\vec{r}.(\vec{N_1} + \lambda\vec{N_2}) = d_1 + \lambda d_2$ for the different real value of λ representing a family (system) of planes through the line of intersection of planes $\vec{r}.\vec{N_1} = d_1$ and $\vec{r}.\vec{N_2} = d_2$.

The **Cartesian equation** of the plane through the line of intersection of planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2 = d_2$ is $(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2 - d_2) = 0.$

EXAMPLE

Find the equation of the plane through the intersection of the planes $x + y + z + 1 = 0$ and $2x - 3y + 5z - 2 = 0$ and the point $(-1,2,1)$.

EXAMPLE

Find the equation of the plane through the line of intersection of the planes $\vec{r}.(2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r}.(\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$

EXAMPLE

Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

Assignments

1. Find the vector and Cartesian equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 6$ and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = -5$ and the point $(1, 1, 1)$.

2. Find the equation of the plane passing the line of intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$, and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

3. Show that the line of intersection of the planes $x + 2y + 3z = 8$ and $2x + 3y + 4z = 11$ is coplanar with the line $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$. Also, find the equation of the plane containing them.

4. Question no 9,10, 11 from Exercise 11.3 from NCERT book.

THANKING YOU

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