DIRECTION RATIOS AND DIRECTION COSINES OF A VECTOR

SUBJECT : MATHEMATICS
CHAPTER NUMBER:10
CHAPTER NAME :VECTORS

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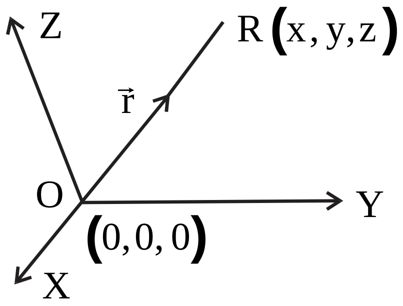
Position Vector

Definition:

Let R be any point in space having co-ordinates (x, y, z) with reference to the origin $(0,0,0,)$. Then vector $\overrightarrow{OR}$ is called position vector of the point R, with reference to the origin.

The position vector of R is denoted by $\vec{r}$.

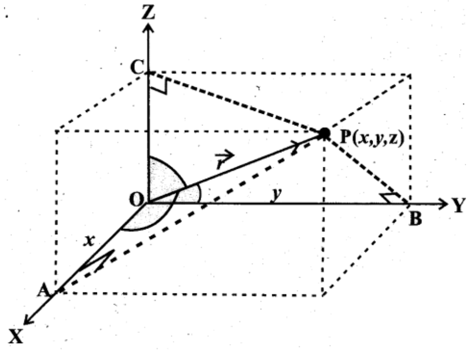


Here $\overrightarrow{OR} = \vec{r}$.

- The magnitude of $\overrightarrow{OP}$ is $\left|\overrightarrow{OP}\right| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- in practice, the position vectors of the points A, B, C, etc with respect to origin O one denoted by $\vec{a}, \vec{b}, \vec{c}$, etc respectively.

Direction Cosines

Let us consider a point $R(x, y, z)$ whose position vector as $\overrightarrow{OR}$ or $\vec{r}$. The angle α, β, γ made by the vector $\vec{r}$ with the positive directions of x, y and z-axes respectively are called its direction angles. The cosine values of these angles, i.e. $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines (dcs) of $\vec{r}$.



- Direction cosines are denoted by l, m, n
- $\cos\alpha = \frac{x}{|r|} \Rightarrow \ell = \frac{x}{r}$, $\cos\beta = \frac{y}{|r|} \Rightarrow m = \frac{y}{r}$, $\cos\gamma = \frac{z}{|r|} \Rightarrow n = \frac{z}{r}$

Direction Ratios

The number $\ell r, mr$ and nr proportional to the direction cosines are called direction ratios (drs) of $\vec{r}$. They are denoted by a, b and c respectively.

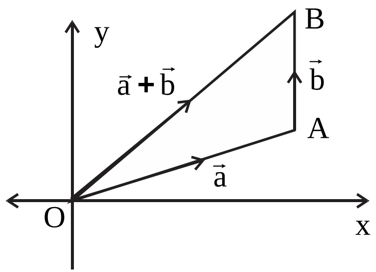
Note :

(1) $\ell^2 + m^2 + n^2 = 1$ But $a^2 + b^2 + c^2 \neq 1$

(2) $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$ But $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

Addition of Vectors

The sum or resultant of more than two vectors is called the **composition** of vectors.



Triangle law of addition If $\vec{a}$ & $\vec{b}$ are two vectors represented by directed line segments $\overrightarrow{OA}$ and $\overrightarrow{AB}$, i.e., $\overrightarrow{OA} = \vec{a}\ and\ \overrightarrow{AB} = \vec{b}$.

Then the sum or resultant of $\vec{a}$ and $\vec{b}$ is defined as the vector represented by the line segment $\overrightarrow{OB}$. i.e. $\overrightarrow{OB} = \vec{a} + \vec{b}$ or $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$.

Parallelogram Law of Addition :

If two vector $\vec{a}\ and\ \vec{b}$ is represented by the two adjacent sides of a parallelogram in magnitude and direction, then the sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point.

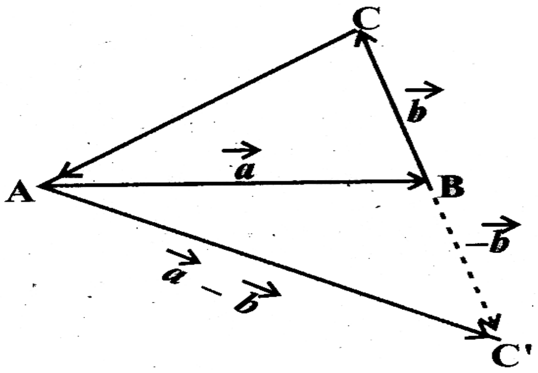
Properties of Addition

- For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative)
- For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (Associative)
- For any vector $\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$
- For any vector, $\vec{a}$ there exists another vector - $\vec{a}$ such that $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$.

Subtraction of two vectors :

If $\vec{a}$ and $\vec{b}$ are any two vectors then subtraction of $\vec{b}$ from $\vec{a}$ is defined as the sum of $\overrightarrow{a}$ and $-\vec{b}$. It is written as $\vec{a} - \vec{b}$.

In process of subtracting $\vec{b}$ from $\vec{a}$, we find $-\vec{b}$ (by reversing the direction) and add to $\vec{a}$.



Multiplication of a vector by a scalar:

Let $\vec{a}$ be a given vector and k be a scalar. Then multiplication of the vector $\vec{a}$ by a scalar, k is defined as a vector denoted by $k\vec{a}$. Such that.

- The magnitude of $k\vec{a}$ is $|k|$ times of the magnitude of $\vec{a}$. i.e., $|k\vec{a}| = |k||\vec{a}|$
- Direction of $k\vec{a}$ is the same as that of $\vec{a}$ if k is positive and direction of $k\vec{a}$ is opposite to that of $\vec{a}$ if k is negative.

EXAMPLE:

If $|\vec{a}| = 3$ and $-4 \leq k \leq 1$, then what can you say about $|k\vec{a}|$?

EXAMPLE:

Can $\frac{-1}{2\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{3}}$ be the direction cosines of any vector ? Justify your answer.

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EXAMPLE:

If a vector makes angles $\alpha, \beta,$ with the positive directions of x, y and z-axes respectively. then prove that

$sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

EXAMPLE:

What are the direction cosines of a line which is equally inclined to coordinate axes.

Assignments

1. If a vector makes angles 60^0 and 45^0 to positive direction of x-axis and y-axis respectively.Find angle at which it inclined to z-axis.

2. What is the direction cosines of a line whose direction ratios are $3, 4, 12$.

THANKING YOU

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