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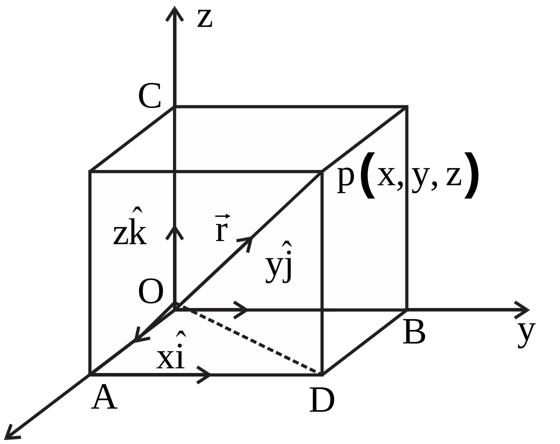
COMPONENTS OF A VECTOR IN A PLANE

SUBJECT : MATHEMATICS
CHAPTER NUMBER:10
CHAPTER NAME :VECTORS

Components of a Vector

Let us consider a point $P(x, y, z)$ in space with position vector OP as shown in the figure. $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors along the $x-$axis, y-axis, and z-axis respectively. Let D be the foot of the perpendicular from P on the XOY plane. Thus PD is parallel to the z-axis.



So $\overrightarrow{DP} = \overrightarrow{OC} = z\hat{k}$ similarly $\overrightarrow{AD} = \overrightarrow{OB} = y\hat{\jmath}$ and $\overrightarrow{OA} = x\hat{\imath}$.

Now $\overrightarrow{OP} = \overrightarrow{OD} + \overrightarrow{DP}$ $\Rightarrow \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AD} + \overrightarrow{DP} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

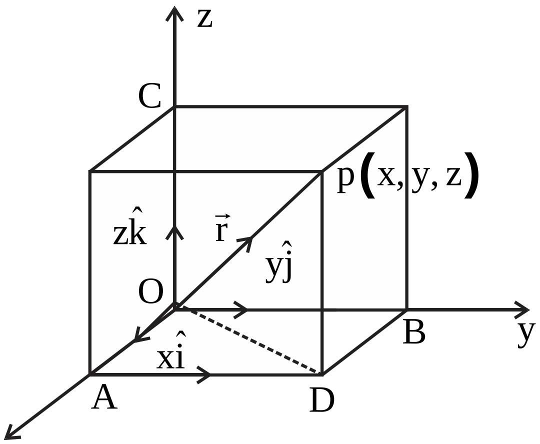
This representation of any vector is called the component form.

Thus position vector of the point $P(x, y, z)$ is $\vec{r} = O\vec{P} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

Components of a Vector

Position vector of the point $P(x, y, z)$ is $\vec{r} = \overrightarrow{OP} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

- x, y, and z are called **scalar components**
- $x\hat{\imath}, y\hat{\jmath}$ and $z\hat{k}$ are called **vector components** of $\vec{r}$.
- The **magnitude** of the vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

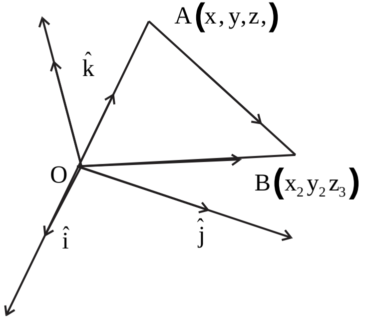


Vector Joining two Points

If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are any two points then vector joining A and B is

the vector $\overrightarrow{AB} = P.V\ of\ B - P.V.of\ A = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

And, $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$



Some Useful Results on DCS and DRS

Let $P(x, y, z)$ be a point in space such that $\overrightarrow{OP} = \vec{r}$ has dcs l, m, n .Then

- $x = \ell|\vec{r}|,\ y = m|\vec{r}|,\ z = n|\vec{r}|$
- Unit vector along $\vec{r} is \hat{r} = \ell\hat{\imath} + mj + n\hat{k}$. This scalar components of a unit vector give the dcs of that vector.
- If a, b, c be the drs of the vector whose dcs as l, m, n then $\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c}$. Here $\ell^2 + m^2 + n^2 = 1$ but, $a^2 + b^2 + c^2 \neq 1$ and $\ell = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$ & $n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$
- The dcs of $\vec{r}$ are unique but drs of a vector a not unique. If a, b, c are drs of a vector then ka, kb, kc are also drs.
- If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, Here x, y, z can be taken as the drs of $\vec{r}$. They are its scalar components.

EXAMPLE

Find the unit vector in the directing $\overrightarrow{AB}$ when A and B are points $(-2,4,3)$ and $(-1,-4,6)$ respectively.

Also find a vector of magnitude 5 units in the direction of $\overrightarrow{AB}$.

EXAMPLE

If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$, then find a vector of magnitude 6 units in the direction of $\vec{a} - \vec{b}$.

EXAMPLE

Find the value of x, y, z so that the vector $3\hat{\imath} + y\hat{\jmath} - 2\hat{k}$ and $x\hat{\imath} + 5\hat{\jmath} + z\hat{k}$ are equal.

Hints: If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Then, $\vec{a} = \vec{b}$ iff $a_1 = b_1, a_2 = b_2$ & $a_3 = b_3$

EXAMPLE

Show that the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear.

EXAMPLE

Find the direction cosines and direction ratios of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Assignments

1. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ

2. Find the diction cosines of the vector joining the points $A(1,2,-3)$ and $B(-1,-2,1)$ directed from A to B.

THANKING YOU

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