ODM EDUCATIONAL GROUP
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Product of Vectors

SUBJECT : MATHEMATICS
CHAPTER NUMBER:10
CHAPTER NAME :VECTOR ALGEBRA

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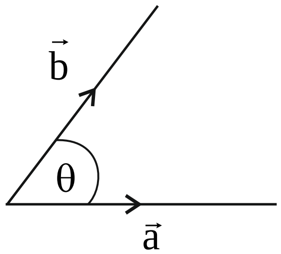
Product of Vectors

Multiplication of two vectors is defined in two ways, namely scalar (or dot) product where the result is a scalar and vector (or cross) product where the result is a vector.

Scalar (or dot) product of two vectors

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors. Then, the scalar product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}.\vec{b}$ (read as $\vec{a}$ dot $\vec{b}$) and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\, cos\, \theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$.



Some Properties and Observations

(i) As $\vec{a}.\vec{b}$ is a scalar that is why dot product is called scalar product.

(ii) $\vec{a}.\vec{b}$ can be positive, negative or zero according to as $cos\,\theta$ is positive, negative or zero

(iii) If $\vec{a}$ & $\vec{b}$ are like vectors $(i.e. \theta = 0)$ then $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\, cos\,\theta = |\vec{a}||\vec{b}|$

(iv) If $\vec{a}$ & $\vec{b}$ are unlike vectors $(i.e. \theta = \pi)$ then $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\, cos\,\pi = -|\vec{a}||\vec{b}|$

(v) $(\vec{a})^2 = \vec{a}.\vec{a} = |\vec{a}||\vec{a}|\, cos\, 0 = |\vec{a}|^2$

(vi) If $\vec{a}$ and $\vec{b}$ are perpendicular or orthogonal vectors then $\vec{a}.\vec{b} = 0$

Some properties and observations

(vii) The dot product of vectors commutative i.e. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

(viii) For unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$

$\hat{\imath}.\hat{\imath} = 1$	$\hat{\jmath}.\hat{\jmath} = 1$	$\hat{k}.\hat{k} = 1$
$\hat{\imath}.\hat{\jmath} = 0$	$\hat{\jmath}.\hat{k} = 0$	$\hat{k}.\hat{\imath} = 0$

(xi) For $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors $\vec{a}.\left(\vec{b} + \vec{c}\right) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$.

(x) let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

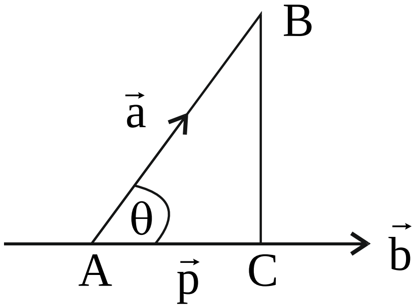
(xi) The angle between the two vectors $\vec{a}$ and $\vec{b}$ is $cos\,\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$.

(xii) If $\vec{a}.\vec{b} = 0$ then either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a} \perp \vec{b}$.

Projection of a Vector on Another Vector

The projection of any object can be obtained by focusing a source of light on the object and obtaining its shadow on a surface. Light is focused on a vector to obtain its shadow on a plane. The shadow so formed is the projection of the vector on the plane.

Let $\vec{a}$ and $\vec{b}$ be any two vectors with $\vec{b} \neq \vec{0}$. Let $\vec{a} = \overrightarrow{AB}$ and θ be the angle between $\vec{a}$ & $\vec{b}$. Draw $\overrightarrow{BC} \perp \vec{b}$ as shown in fig.



Then, projection of $\vec{a}$ on $\vec{b}$ (also known as the scalar projection of $\vec{a}$ on $\vec{b}$)

$$|\vec{p}| = AC = AB\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$$

Note:- Vector projection $\vec{a}$ on $\vec{b}$ is given by $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

EXAMPLE

If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{c} = 5\hat{\imath} - 4\hat{\jmath} + 3\hat{k}$, then find $(\vec{a} + \vec{b}).\vec{c}$

EXAMPLE

Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ are at right angles.

EXAMPLE

Find the angle between the vectors $4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$.

EXAMPLE

Find the value of μ so that the two vectors $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $4\hat{\imath} - 6\hat{\jmath} - \mu\hat{k}$ are

(i) Parallel (ii) perpendicular to each other.

EXAMPLE

Find the projection of $2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\hat{\imath} - 2\hat{\jmath} - \hat{k}$ are

Cauchy Schwartz Inequality

For any two vectors $\vec{a}$ and $\vec{b}$; $\left|\vec{a}.\vec{b}\right| \leq |\vec{a}|\left|\vec{b}\right|$

Triangle Inequality

For any two vectors $\vec{a}$ & $\vec{b}$ Prove that $\left|\vec{a}+\vec{b}\right| \leq |\vec{a}| + \left|\vec{b}\right|$

Assignments

1. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ

2. Find the diction cosines of the vector joining the points $A(1,2,-3)$ and $B(-1,-2,1)$ directed from A to B.

If two vectors $\vec{a}$ *and* $\vec{b}$ *are such that* $|\vec{a}| = 2$, $|\vec{b}| = 3$ *and* $\vec{a} \cdot \vec{b} = 4$, *then find*

(i) $|\vec{a} + \vec{b}|$. (ii) $|\vec{a} - \vec{b}|$.

If $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ *are three vectors such that* $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ *and* $|\vec{a}| = 5$, $|\vec{b}| = 12$, $|\vec{c}| = 13$, *then find* $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

THANKING YOU

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