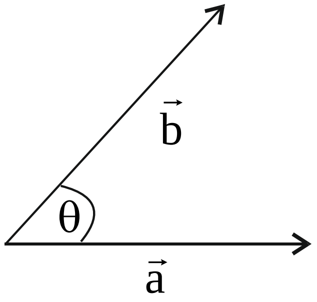
Cross Product of Vectors

SUBJECT : MATHEMATICS
CHAPTER NUMBER:10
CHAPTER NAME :VECTORS

Cross Product of Vectors

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors. Then, vector product (or Cross product) of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$ (read as $\vec{a}$ cross $\vec{b}$) and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \, sin \, \theta \, \hat{n}$

Here θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \leq \theta \leq \pi$ and $\hat{n}$ is a unit vector perpendicular to the plane containing the vectors $\vec{a}$ and $\vec{b}$.



Some Properties and Observations

- $\vec{a} \times \vec{b}$ is always a vector. That is why the cross product is called a vector product.
- If $\vec{a}$ and $\vec{b}$ are like vectors (i.e. $\theta = 0$) then $\vec{a} \times \vec{b} = \vec{0}$.
- If $\vec{a}$ and $\vec{b}$ are unlike vectors (i.e. $\theta = \pi$) then $\vec{a} \times \vec{b} = \vec{0}$.
- $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$. But $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|$ and $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.
- If $\vec{a} \times \vec{b} = \vec{0}$ then either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a}$ & $\vec{b}$ is collinear.
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

Some Properties and Observations

- $\vec{a} \times \vec{a} = \vec{0}$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$$

$$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}\,,\ \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

- Let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}.\ then \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

- If θ be the angle between $\vec{a}$ and $\vec{b}$, then$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\, sin\,\theta \Leftrightarrow sin\,\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

- The unit vector perpendicular to both vectors $\vec{a}$ and $\vec{b}$ is $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Area of Triangle and Parallelogram

If $\vec{a}$ and $\vec{b}$ be the adjacent sides of a parallelogram then the area of the parallelogram is

$|\vec{a} \times \vec{b}|$.

- If $\vec{a}$ and $\vec{b}$ be the adjacent sides of a triangle then area of the triangle is $\frac{1}{2}|\vec{a} \times \vec{b}|$.
- If $\vec{d}_1$and $\vec{d}_2$ be the two diagonals of a parallelogram then the area of the parallelogram is

  $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$.

EXAMPLE

If $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$ then find $\vec{a} \times \vec{b}$ & $|\vec{a} \times \vec{b}|$

EXAMPLE

Find the *sine* of the angle between the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.

EXAMPLE

Find a vector of magnitude 15 units, which is perpendicular to both the vectors $4\hat{\imath} - \hat{\jmath} + 8\hat{k}$ and $-\hat{\jmath} + \hat{k}$.

EXAMPLE

Find the area of the parallelogram whose adjacent sides are $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

EXAMPLE

Using vector method, find the area of the triangle whose vertices are $A(1,1,1), B(1,2,3), C(2,3,1)$.

Assignments

1. Find the value of $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{k} \times \hat{\imath}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$
2. Prove that $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a}.\vec{b})^2$.
3. Find the value of λ for which the vectors $\vec{a} = 3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel.
4. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of ΔABC, show that the area of ΔABC is

   $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$ sq units.

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