Scalar Triple Product of Vectors

SUBJECT : MATHEMATICS
CHAPTER NUMBER:10
CHAPTER NAME :VECTOR ALGEBRA

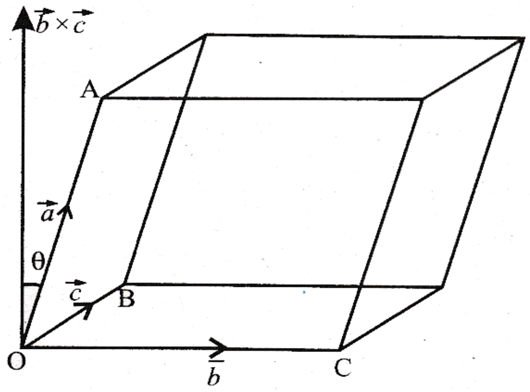
Scalar Triple Product

The scalar triple product of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is defined as the number $\vec{a}.(\vec{b} \times \vec{c})$ and is denoted by $[\vec{a} \quad \vec{b} \quad \vec{c}]$.

Hence $[\vec{a} \quad \vec{b} \quad \vec{c}] = \vec{a}.(\vec{b} \times \vec{c})$

Geometrical Interpretation Scalar Triple Product

The scalar triple product $[\vec{a} \quad \vec{b} \quad \vec{c}]$ represents the volume of the parallelepiped whose coterminous edges $\vec{a}, \vec{b}$ and $\vec{c}$.



Properties of Scalar Triple Product

- If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, then

  $$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

- If three vectors are cyclically permitted then the scalar triple product remain the same

  i.e. $[\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{a}] = [\vec{c} \quad \vec{a} \quad \vec{b}]$.

- If $\vec{a}, \quad \vec{b}, \quad \vec{c}$ be any three vectors then $[\vec{a} \quad \vec{b} \quad \vec{c}] = -[\vec{a} \quad \vec{c} \quad \vec{b}]$

Properties of Scalar Triple Product

- In scalar triple product the position of dot and cross can be interchanged provided cyclic order of vector remains the same. i.e. $\vec{a}.(\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$
- The scalar triple product of three vectors is zero, If any two of the given vector are equal i.e. for any three vectors $\vec{a} \quad \vec{b} \quad \vec{c}$ with $\vec{a} = \vec{b}$ or $\vec{b} = \vec{c}$ or $\vec{c} = \vec{a}$ then $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$.
- The scalar triple product of three vectors is zero if any two of them are parallel (or collinear)
- If $\vec{a} \quad \vec{b} \quad \vec{c}$ are coplanar then $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$. As the volume of parallelopiped vanishes.

EXAMPLE

If $\vec{a} = 4\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} - 10\hat{\jmath} - 7\hat{k}$ and $\vec{c} = 5\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$ then find $[\vec{a} \quad \vec{b} \quad \vec{c}]$.

EXAMPLE

Show that the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ are coplanar?

EXAMPLE

Find the volume of the parallelepiped where three coterminous edges are represented by vectors $\hat{\imath} + \hat{\jmath} + \hat{k}, -\hat{\imath} + \hat{\jmath} - \hat{k}$ and $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$.

EXAMPLE

Prove that $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$.

Assignments

1. Find λ if the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} - 3\hat{k}$ are coplanar.
2. Show that the four points $A(4,5,1), B(0,-1,-1), C(3,9,4)$ and $D(-4,4,4)$ are coplanar.
3. Prove that $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$. If $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, then show that $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar.
4. Find the value of λ, if four points with position vectors $3\hat{\imath} + 6\hat{\jmath} + \lambda\hat{k}$, $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + 6\hat{\jmath} + \lambda\hat{k}$ are coplanar.
5. Prove that $\vec{a}.(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})$= $[\vec{a}\ \vec{b}\ \vec{c}]$.

THANKING YOU

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