Chapter- 5
Continuity and Differentiability

**Very Short Type[ 1 mark Questions]**

1. If $f\left(x\right)=2x $and$g\left(x\right)=\frac{x^{2}}{2}+1$, then which of the following may not be a continuous function?

a) $f\left(x\right)+g(x)$ b) $f\left(x\right)-g(x)$ c)$f\left(x\right).g(x)$ d) $\frac{g(x)}{f(x)}$

2. The function $f\left(x\right)=cotx$ is discontinuous on the set

a) $\left\{x=nπ :n\in Z\right\}$ b) $\left\{x=2nπ :n\in Z\right\}$

c) $\left\{x=\left(2n+1\right)\frac{π}{2}:n\in Z \right\}$ d) $\left\{x=\frac{nπ}{2} :n\in Z\right\}$

3. The function $f\left(x\right)=\frac{4-x^{2}}{4x-x^{3}}$

a) Discontinuous at only one point b) discontinuous at exactly two points

c) Discontinuous at exactly three points d) None of these

4. Determine the value of $'k'$ for which the following function is continuous at $x=3.$

$f\left(x\right)= \left\{\begin{array}{c}\frac{(x+3)^{2}-36}{x-3} , x\ne 3\\k , x=3\end{array}\right. $

5. Determine the value of the constant $' k '$ so that the function $f\left(x\right)= \left\{\begin{array}{c}\frac{kx}{\left|x\right|} if x<0\\3 if x\geq 0\end{array}\right.$ is continuous at $x=0$.

6. The function $f\left(x\right)= e^{\left|x\right|}$is

a) Continuous everywhere but not differentiable at $x=0$

b) Continuous and differentiable everywhere

c) Not continuous at $x=0$

d) None of these

7. The set of points where the function $f$ given by $f\left(x\right)= \left|2x-1\right| sinx$ is differentiable is

a) $R$ b) $R- \left\{\frac{1}{2}\right\}$ c)$(0, \infty )$ d) None of these

8. Fill in the blank: The greatest integer function defined by $f\left(x\right)=\left[x\right], 0<x<2$ is not differentiable at $x= $­­­­\_\_\_\_\_\_\_\_ .

**Long Type – I [ 4 Marks Questions]**

9. For what value of k is the function continuous at x=2? *f(x)*=

10. Discuss the continuity of the function x=0: *f(x)=*

11. If the function *f(x)* given by *f(x)=*  is continuous at x=1, find the value of a & b.

12. Find the relationship between a and b so that the function *f* defined by *f(x)= *is continuous at x=3.

13. Find the value of k, for which f(x)=is continuous at x=0.

14. Find the value of k so that the function f, defined by *f(x)* = is continuous at x=

15. Find the value of a for which the function f defined as *f(x)=* is continuous at x=0.

16. Find all points of discontinuity of f, where f is defined as follows f(x)=

17. For what value of $k$, the following function is continuous at $x=0. $

$$f\left(x\right)= \left\{\begin{array}{c}\frac{1-cos4x}{8x^{2}} if x \ne 0 \\k if x=0\end{array}\right.$$

18. Find the value of $a$ such that the function $f$ defined by $f\left(x\right)= \left\{\begin{array}{c}\frac{x^{2}-3x+2}{x^{2}-1} when x\ne 1\\4a when x=1\end{array}\right.$ is continuous at $x=1$.

19.The function $f\left(x\right)$ is defined as $f\left(x\right)= \left\{\begin{array}{c}x^{2}+ax+b, 0\leq x<2\\3x+2, 2\leq x\leq 4\\2ax+5b, 4<x\leq 8\end{array}\right. .$ If $f(x)$ is continuous in $[0, 8]$, find the values of $a$ and $b$.

20. If the function $f(x)$ is differentiable at $x=2$, then find the value of $a$ and $b$. $f\left(x\right)= \left\{\begin{array}{c}x^{2} if x\leq 2\\ax+b if x>2\end{array}\right.$

21. Find the values of $a$ and $b$ so that the function $f\left(x\right)= \left\{\begin{array}{c}x^{2}+3x+a, x\leq 1\\bx+2 , x>1\end{array}\right.$ is differentiable for$ x\in R$.

22. Show that the function f*(x)* = ,xR is continuous but not differentiable at x=3.

23. Show that the function $f\left(x\right)= \left|2x+1\right|$ is not differentiable at $x= -\frac{1}{2}$.

24. If function $f\left(x\right)= \left|x-3\right|+\left|x-4\right|,$ show that $f$ is not differentiable at $x=3 $and $x=4$.

25. Discuss the differentiability of the function $f\left(x\right)=x\left|x\right|$at $x=0$.

26. Show that the function defined as follows is continuous at $x=1, x=2$ but not differentiable at $x=2.f\left(x\right)= \left\{\begin{array}{c}3x-2, 0< x\leq 1\\2x^{2}-x, 1<x\leq 2\\5x-4, x>2\end{array}\right.$

27. If $y=log⁡(\cos(e^{x}))$, then find $\frac{dy}{dx}$.

28. If $y=cosec(cot\sqrt{x})$, then find $\frac{dy}{dx}$.

29. If f(x)= then find f’(x).

30. Differentiate with respect to x:

31. If, find.

32. If, find 

33. Differentiate the function w.r.t x:

34. Differentiate  w.r.t x.

35. Differentiate  w.r.t x

36. Find ,if y= (cosx)x+(sinx)1/x

37. Differentiate the function w.r.t x: (x)cosx+(sinx)tanx

38. If (cosx)y=(siny)x,find .

39. Find ,if (x2+y2)2=xy

40. If siny=x sin(a+y),Prove that 

41. If log(x2+y2)=2 tan-1(y/x),then show that 

42. If , then show that 

43.If xy=ex-y then show that

44. If  then show that .

45. If xmym=(x+y)m+n find 

46. If x16y9=(x2+y)17 then find 

47. If  prove that

48. If y=,then prove that 

49. If y=ex(sinx+cosx),then show that 

50. If x=a(cost+tsint) and y=a(sint-tcost),then find .

51. If, then show that 

52. If y=cosec-1x,x>1 then show that 

53. If y=a cos(logx)+b sin(logx) then show that 

54. Prove that 

55. If x=a(t-sint),y=a(1+cost),then find 

56. If  then show that 

57. If y=sin-1x, then show that 

58. If x=acos3t and y=asin3t then find the value of  at t=

59. Differentiate  with respect to 

60. Differentiate  w.r.t 

61. Verify Rolle’s theorem for $f\left(x\right)=sin2x$ in $\left[0, \frac{π}{2}\right]$ and find the value of $c \in \left[0,\frac{π}{2}\right]$.

62. Using Rolle’s theorem find a point on the curve $y=sinx+cosx-1, x\in \left[0, \frac{π}{2}\right]$, where the tangent is parallel to $x-axis$.

63. Verify Mean value Theorem for $f\left(x\right)=(x-1)(x-2)(x-3)$in $\left[1, 4\right]$.

64. Verify Lagrange’s mean value theorem for the function f(x) =x2+2x+3, for [4,6].

65. For what value of $c$, Mean value theorem is applicable for the function $f\left(x\right)=x+\frac{1}{x}$ on $[1, 3]$ ?

66. Find  if 

67. Find  if and 