ODM EDUCATIONAL GROUP
Changing your Tomorrow

Power of lens , combination of thin lenses in contact

XII- SCIENCE

SUBJECT : PHYSICS
CHAPTER NUMBER: 9
CHAPTER NAME : RAY OPTICS

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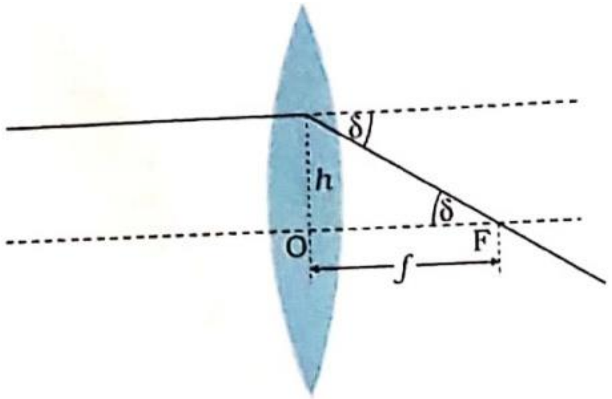
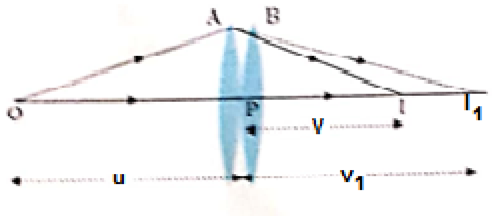
NUMERICAL

- **Power of lens:**
- Power of a lens is a measure of the convergence or divergence , which a lens introduces to the light falling on it.
- Power of a lens is numerically equal to the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from optical centre.
- In the figure a beam parallel to principal axis strikes the lens at a height h.
- From figure , δ = deviation angle
- As tan δ = h/f. If h = unity , then tan δ = 1/f
- So power of lens is ; **P = 1/f**
- S.I. unit of power is dioptre (D) and **1 D = 1m^{-1}**.



Two thin lenses in contact:

In figure two thin lenses A and B of focal lengths f_1 and f_2 with powers P_1 and P_2 are in contact.
O is the object. I_1 is the image for A and object for B. I is the image of B and image of the combination.



Now for lens A using lens formula , $\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$ ....(i)

Now for lens B using lens formula , $\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$ ....(ii)

Adding equations (i) and (ii) we get , $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$ ..(iii)

Since u = object distance for the combination and v = image distance of combination ,

Hence for the combination ; $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ...(iv)

(Where f = equivalent focal length of the combination)

Comparing equations (iii) and (iv) we get , $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \Rightarrow f = \frac{f_1 f_2}{f_1 + f_2}$

Power of the combination is ; $P = P_1 + P_2$

- For large number of lenses in contact , $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ...$ OR $P = P_1 + P_2 + ...$

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NUMERICAL

Numerical: A combination of two lenses form two times magnified real image at a distance of 40 cm from the combination. If one of the lenses is convex with focal length 10 cm , then what is the nature and focal length of the second lens ?

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Solution: For combination ; v = 40 cm (positive as real image) and m = -2 (as inverted)

Since , $m = \frac{f-v}{f} \Rightarrow -2 = \frac{f-40}{f} \Rightarrow -2f = f - 40 \Rightarrow -3f = -40cm \Rightarrow f = \frac{40}{3}cm$

Now $f_1 = 10cm$.

As , $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \Rightarrow \frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{3}{40} - \frac{1}{10} = -\frac{1}{40cm} \Rightarrow f_2 = -40cm$

So second lens is divergent lens with focal length 40 cm.

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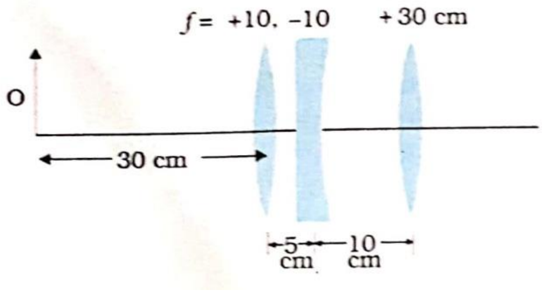
NUMERICAL

Numerical: In the given combination of lenses find the final image position.



NUMERICAL

Numerical: In the given combination of lenses find the final image position.

Solution: For 1st lens ; u_1 =- 30 cm and f_1 = 10 cm

By lens formula ; $\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{1}{15cm}$

$\Rightarrow v_1 = 15cm$

Now for combination cases , image of first lens is the object of the second lens ‘

So for second lens ; $u_2 = v_1 - d_{12}$ = 15 cm – 5 cm = 10 cm (+ve because virtual object)

f_2 = -10cm

By lens formula ; $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0$

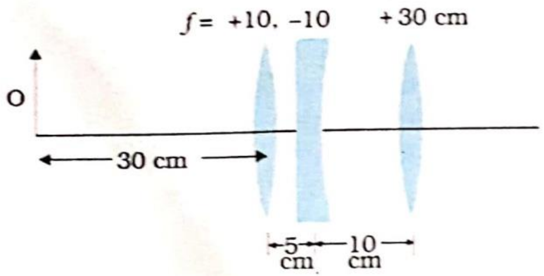
$\Rightarrow v_1 = \infty$

So for third lens ; $u_3 = v_2 - d_{23} = \infty - 10cm = \infty$

F_3 = 30cm

By lens formula ; $\frac{1}{v_3} = \frac{1}{f_3} + \frac{1}{u_3} = \frac{1}{30} + \frac{1}{\infty} = \frac{1}{30cm}$

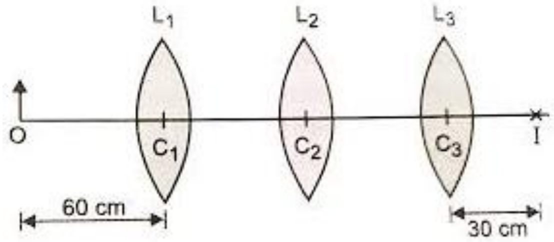
$\Rightarrow v_3 = 30cm$



So final image is produced 30 cm right to the third lens.

NUMERICAL

Numerical: In the given combination of lenses , all lenses are identical with focal length 30 cm. I is the final image position of the object O. Find the distances between the lenses.



NUMERICAL

Numerical: In the given combination of lenses , all lenses are identical with focal length 30 cm. I is the final image position of the object O. Find the distances between the lenses.

Solution: For 3rd lens ; v_3 = 30 cm and f_3 = 30 cm

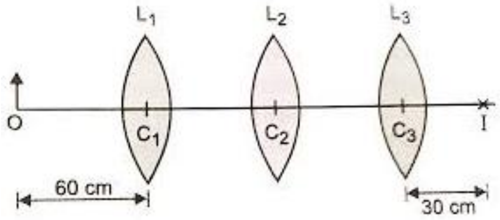
i.e. image is at the secod focus of the third lens. So the incident ray for thid lens must be parallel to principal axis. So $u_3 = \infty$.

So what ever the separation between L_2 and L_3 may be for 2nd lens $v_2 = \infty$. Again f_2 = 30 cm.

So object for 2nd lens must be at its 1st focus.

So u_2 = - 30 cm.

For 1st lens ; u_1 =- 60 cm and f_1 = 30 cm



By lens formula ; $\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{30} - \frac{1}{60} = \frac{2-1}{60} = \frac{1}{60cm}$

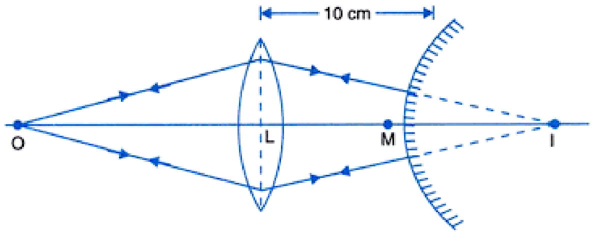
$$\Rightarrow v_1 = 60cm$$

So for second lens ; $u_2 = v_1 - d_{12} \Rightarrow d_{12} = v_1 - u_2$ = 60 cm – (-30cm) = 90 cm

So separation between L_1 and L_2 = 90 cm and separtion betwee L_2 and L_3 can be of any value.

NUMERICAL

Numerical: A convex lens of focal length 30 cm and a convex mirror of radius of curvature 40 cm are kept coaxially at a separation 10 cm. A pont object O is kept on the principal axis such that image coincides the object. Draw the ray diagram and find the position of object.



NUMERICAL

Numerical: A convex lens of focal length 30 cm and a convex mirror of radius of curvature 40 cm are kept coaxially at a separation 10 cm. A pont object O is kept on the principal axis such that image coincides the object. Draw the ray diagram and find the position of object.

Solution: The ray diagram is shown below.

As the refracted rays from lens are returning along same line that means the rays must be normal to the mirror i.e. approaching towards the centre of curvature.

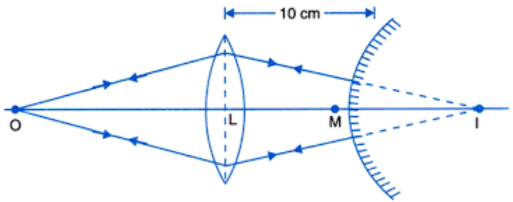
Hence image of lens is the centre of curvature of the mirror.

Hence for the lens ; v = 10 cm + R of the mirror = 10 cm + 40 cm = 50 cm

And f = 30 cm

By lens formula ; $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{50} - \frac{1}{30} = \frac{3-5}{150} = \frac{-2}{150}$

$\Rightarrow u = -75cm$

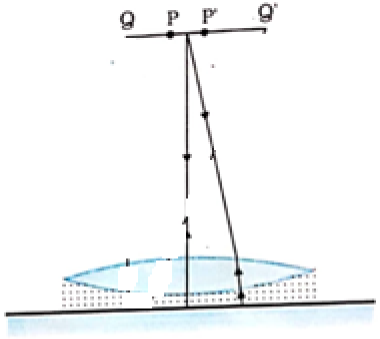


Hence object is at 75 cm left of the convex lens.

NUMERICAL

Numerical: In the given figure an equi-convex lens of refractive index 1.5 is kept on a liquid of unknown refractive index on a plane mirror. In this situation a very small pin PQ is moved along the axis till its image coincide with it. This happens at a distance x from the combination. Now the same experiment is repeated after removing the liquid and the distance of pin is found to be y. Find the refractive index of the unknown liquid.

Solution: When a lens system is kept on a mirror and the image coincides with the object , then the refracted rays from the lens system must be normal to the mirror. As here mirror is plane mirror , hence normal to this means parallel to the principal axis of the lens system. So refracted rays of the lens system are parallel to the principal axis.
Hence object is at the 1st focus of the lens.



NUMERICAL

$$\therefore f = x$$

Here f is the focal length of the lens system i.e. the equi-convex lens(focal length f_1) in contact with the plano-concave lens (focal length f_2) formed by the liquid and lens.

Hence $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{x} = \frac{1}{f_1} + \frac{1}{f_2}$ ....(i)

When the liquid is removed then only convex lens remains. Here he observed distance is y. Hence focal length of the lens is y

$\therefore f_1 = y$ ....(ii)

Using equation (ii) in (i) we have ; $\frac{1}{x} = \frac{1}{y} + \frac{1}{f_2} \Rightarrow \frac{1}{f_2} = \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$ ....(iii)

Using lens maker's formula for equi-convex lens ;

$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow \frac{1}{y} = 0.5\left(\frac{2}{R}\right) = \frac{1}{R} \Rightarrow R = y$ .... (iv)

For the liquid lens , R_1 = - R = -y and $R_2 = \infty$. Let the refractive index of the liuid = n.

Using Lens maker's formula for the liquid lens:

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{f_2} = -\left(\frac{n-1}{R}\right)$$

$\Rightarrow \frac{y-x}{xy} = -\left(\frac{n-1}{R}\right) = -\left(\frac{n-1}{y}\right)$ (using equation (iii) and (iv)

$$\Rightarrow \frac{x-y}{x} = n-1 \Rightarrow n = \frac{x-y}{x} + 1 = \frac{2x-y}{x}$$

THANKING YOU

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