Chapter-

Dual Nature of Radiation and Matter

STUDY NOTE

Introduction:-

Classical physics treats particles and waves as separate components.

But in relativistic mechanics particles and waves are complementary to each other.

Under suitable conditions radiations exhibits wave character and under other it exhibits particle character

Example:- Wave nature of radiation was suggested in different phenomena like interference, diffraction polarisation et.

Some phenomena like photoelectric effect, Crompton effect can be explained by considering the radiation as particle. Hence radiation (light) has dual nature. The exchange of energy between radiation and matter is governed by its particle properties where as the propagation of radiation is governed by its wave properties.

Particle nature of light:-

1st suggested by Einstein. According to him when radiation interacts with matter, radiation behaves as if it is made of packet of energy or quanta. It is latter named as Photon.

Characteristics of Photons:-

Energy of Photon $E = h\upsilon = \frac{hc}{\lambda}$

Where h is Plank's constant $\left(6.63\times10^{-34}\ Js\right)$

υ is the frequency of radiation

λ is the wavelength associated with the photon.

C is the velocity of light $\left(=3\times10^{8} m/s\right)$

Mass of Photon:-

Photon is the energy particle and so has no meaning at rest. Its rest mass is zero. The kinetic mass of the photon can be obtained from $E = mc^2 \Rightarrow m = \frac{E}{c^2}$

Momentum of photon:-

It is defined as $p = mc$, $= \frac{E}{c^2}.c$, $= \frac{E}{C} = \frac{hc}{\lambda C}$ $\Rightarrow P = \frac{h}{\lambda}$

Intensity of light:-

It is defined as energy crossing per unit area per unit time.

$$I = \frac{E}{At} = \frac{Power}{Area}$$

Note:- For a point source at a distance r

$$I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

For a line source at a distance r $I = \frac{P}{2\pi rL} \Rightarrow I \propto \frac{l}{r}$

Definition of intensity of radiation in terms of photon picture of light:-

The number of photons incident normally per unit area per unit time is determined the intensity of radiation.

With increase in intensity of radiation only the number of photons crossing a given area per sec will increase but energy of photon will not change.

Number of Photons: - We known energy transmitted in a radiation per sec is given by

$(P) =$ [No. Of photons emitted per sec] [Energy of each photon]

$$\Rightarrow P = NE \qquad \Rightarrow N = \frac{P}{E}$$

Radiation force:-

Radiation possesses momentum and so exerts force on the object on which it strikes. (It can be easily demonstrated by placing a charge in the plane perpendicular to the direction of propagation of radiation. The charge will be set and sustained in motion). $F = \frac{\Delta P}{\Delta t}$

(a) If the radiation is absorbed by the object $P = \frac{E}{c}$ and $E = IA\Delta t \quad \therefore P = \frac{IA\Delta t}{C}$

$$\text{Thus } F = \left(\frac{IA\Delta t}{c}\right) \Big/ \Delta t = \frac{IA}{C} = \frac{power}{C}$$

(b) If the surface of the body is perfectly reflecting (normally) $\quad F = \frac{2IA}{c} = 2\left(\frac{power}{c}\right)$

(c) If r is the coefficient of reflection, then $F = (1+r)\frac{Power}{c}$

(d) When Emw is completely reflected at an angle of incident i $F = \frac{2\rho \cos i}{c}$

Photons are electrically neutral:-

Being neutron photon is not affected by electric and magnetic field.

Question:- Calculate the energy associated with the photon of light of $\lambda = 500nm$.

Solution:- $E = h\upsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times(3\times10^{8})}{500\times10^{-9}} = 3.97\times10^{-19}J$

Short cut way: - $E = \frac{hc}{\lambda} = \frac{1242evnm}{500nm} = 2.484ev$

Or $E = \frac{hc}{\lambda} = \frac{1242evnm}{500nm} = 2.484ev$ $\left[\therefore 1ev = 1.6\times10^{-19}J\right]$

Question:-

Monochromatic light of frequency $6\times10^{14}Hz$ is produced by Laser. The power emitted is $2\times10^{-3}W$.

(a) What is the energy of each photon?

(b) How many photons parsec in the average are emitted by the source?

Solution:-

(i) $E = h\upsilon = 6.63\times10^{-34}\times(6\times10^{14}) = 3.98\times10^{-19}J$

(ii) $N = \frac{\rho}{E} = \frac{2\times10^{-3}}{3.98\times10^{-19}} = 5\times10^{15}$

Electron Emission:-

The phenomenon of emission of electrons from the surface of metal with supply of energy is known as electron emission.

Different types of emission are

(a) Thermionic emission (ii) Photo ionic emission

(c) Field emission (d) Secondary emission

Thermionic Emission:-

The phenomenon of emission of electron from a metal surface on heating is known as thermoionic emission

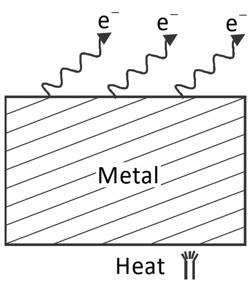
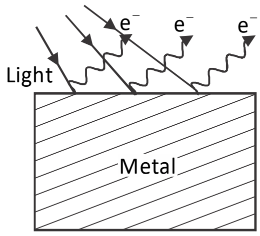


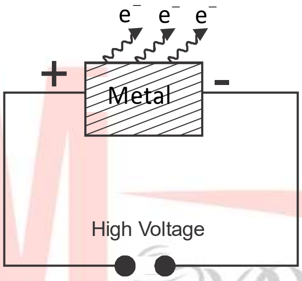
Photo ionic Emission:-

The phenomenon of emission on electron from metal surface when radiation of suitable frequency fall on it.



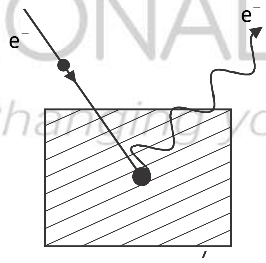
Field Emission:-

The phenomenon of emission of light on the metal surface under the application of strong electric field.



Secondary Emission:-

The phenomenon of emission of ions in metal surface on large no. When 1st moving electron called primary electron strikes the metal surface.



Work Function:-

Question:- Define work function of a metal. Define its unit. On what factor does it depend?

Solution:- When an electron attempts come out of metal, the metal surface acquires +ve charge and pulls the electron back. The free electron this are held by metal by a attraction free. The minimum amount of energy required by the free electron to just leave the metal surface without imparting any kinetic energy to the electron is known as work function.

Work function depend upon (i) property of material (ii) Nature of surface (iii) Nature of impurity.

Practical unit of work function $\rightarrow$ ev

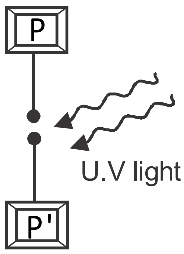
Note:- Max K.E of electrons emitted decrease with increase in work function.

Photoelectric Emission:- The phenomenon of emission of electrons from metal surface, when radiation of suitable frequency fall on it.

Photsensitive substance:- Metals like Zinc, cadmium respond U.V . Alkali metal respond visible radiation.

Hertz's Observation:-

Sparks are produced in the air gap of his transmitter when u.v light was directed at one of the metal sphere s,s^1 attached to the plate p,p^1. Thus in the process of verifying Electro-magnetic wave theory of light Hertz had discovered the photoelectric effect.



Question on work function:-

The work functions of two metals A and B are 1 ev and 4 ev respectively. Which of these two would be suitable for use in a photocell where visible light is used?

Solution:- Since work function of metal A is less than the energy of the photon of visible light therefore metal A would be suitable in the given photocell.

Hallwachs' and Lenarol observation:-

Lenard observation:- When u.v allowed to fall on emitter of an evacuated glass tube current flows in the circuit. As soon as the u.v were stopped, the flow of current is stopped.

This indicate that electrons are ejected due to action of u.v radiation of metal. Which is the photo electric effect.

Hallwaches observation:- When u.v radians falls on a –ve charged zinc plate which is connected to GLE, the collapsing of leafs is observed. Which indicates that e^- are emitted from the surface of zinc plate when u.v radiation falls on it. Which is photoelectric effect.

Experimental study of photo electric effect:-

(a) Set up:-

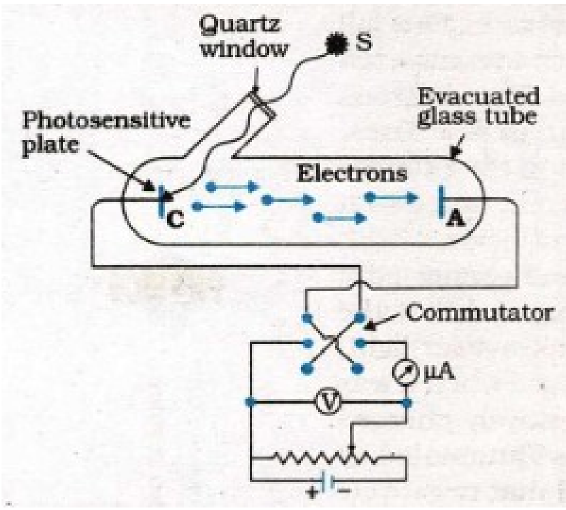
An evacuated glass tube having photosensitive plate C and a metal plate A.

S is the monochromatic light source

W Quartz window to facilitate the incident radiation to fall on c.

Commutator – to reverse the potential on A

Voltmeter (v) – To measure p.d between A and C



Micro Ammeter – To measure the photocurrent

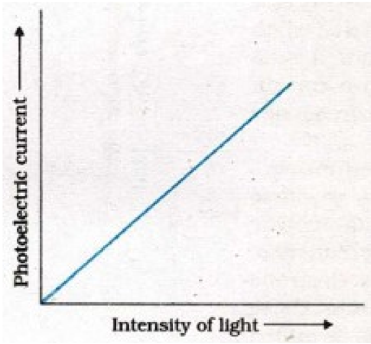
Battery – To measure the electric field between A and C

Effect of intensity of light on photo current:-

Collector A = +ve potential (fixed)

Emitter C = Illuminated with radiation of fixed frequency

Intensity of (s) = Varies slowly (By changing the distance of source)



Observation:-

Conclusion:- Photo electric current is directly proposal to intensity of radiation.

Experiment:- Due to change in intensity, no. Of photo emitted per second from a given surface increases. But the energy of each photon remain same.

Effect of potential on photo current:-

(a) Plate A - +ve potential (varing slowly)

Plat C – Illuminated with radiation of fixed υ and intensity.

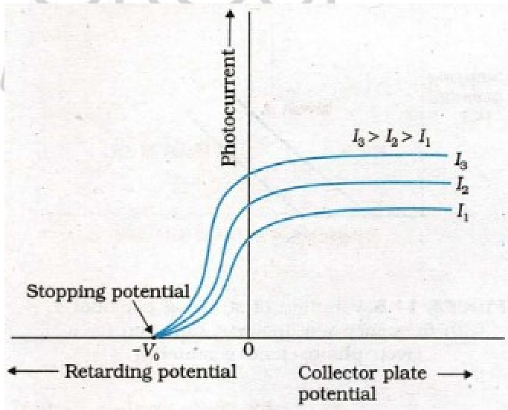
Observation – photo current increases till saturation (current Yz)

(b) Plate A - -ve potential (varing slowly)

Plate C – Illuminated with that radiation

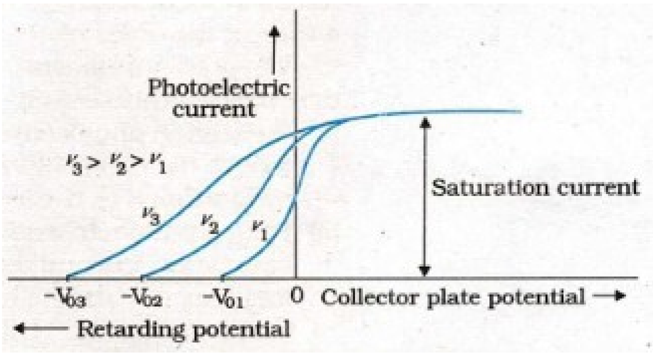
Observation – Photo current decreases and becomes zero for a minimum –ve potential given to A called (stopping potential/ cut off potential) current XY.

(c) If we increase the intensity for different values the saturation current increases but the cut off potential remain same.



Conclusion:- For a given frequency of incident radiation stopping potential is independent of intensity. Stopping potential is the measure of K.E of photo electron. Hence K.E is independent of intensity.

Explanation:- Due to increase of intensity only the number of photon or the no of electrons per sec from a given surface increase but K.E of electrons will not be hampered.



(d) Variation of stopping potential on frequency:-

Plate A – Potential varied

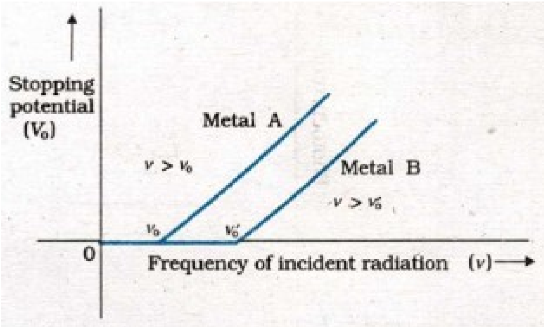
Plate C – Intensity same but frequency varies

Observation:-

Conclusion: - For different frequency, the stopping potential is different.

Explanation: - On increasing υ, energy of each photon increases, hence K.E of electrons increase. (i.e V_0 increase with frequency)

Threshold Frequency: - If a graph is drawn between frequency and stopping potential for different material



Conclusion:-

(i) Stopping potential $\propto$ Frequency

(i.e greater the frequency, greater is the maximum K.E of photo electrons)

(ii) There exist a minimum cut off frequency (called threshold frequency υ) for which V_0 = 0 (i.e emission is not possible)

Laws of Photoelectric emission:-

(i) For a given photo sensitive material and frequency (above υ_0).

Photo current $\propto$ Intensity

(ii) For a given photo sensitive material and for a given frequency

Intensity $\propto$ Saturation current

But independent of stopping potential

(iii) For a given photosensitive material, there exist a certain minimum frequency (called threshold frequency) below which no emission of photo electron is possible.

(iv) The photo electric emission is an instantaneous process (i.e it takes 10^{-9}sec or less)

Einstein's Photo electric effect:-

To Einstein it appears if the radiation from blackbody is in form of quanta, why not the absorption of radiation by emitter is in form of quanta.

Therefore, he explained the quantum theory to explain the photoelectric effect. According to Einstein – Each photon interacts with one electron. The energy of incident photon is used up in two parts.

(i) Part of energy is used up in liberating the electrons from the metal surface. (work function ϕ_0)

(ii) Remaining part is used up in imparting K.E of ejecting electrons. Very few (< 1%) photons, whose energies are greater than ϕ_0 are capable of ejecting the photo electrons. Thus by conservation of energy. Energy of incident photon = maximum K.E of photo electron + Work function

i.e. $h\upsilon = \phi_0 + \frac{1}{2}mV_{max}^2$ .... (1)

If the incident photon is of the threshold frequency, the energy $h\upsilon_0$ is just sufficient to free the electrons from metal surface and does not gives its kinetic energy.

i.e. At $\upsilon = \upsilon_0$

$(KE)_{max} = 0$

Thus, from equation (1) $h\upsilon_0 = \phi_0 + 0$

$$\Rightarrow \phi_0 = h\upsilon_0$$

Substituting for ϕ_0 in equation (1) we have $h\upsilon = h\upsilon_0 + \frac{1}{2}mV_{max}^2$

$\Rightarrow h(\upsilon - \upsilon_0) = \frac{1}{2}mV_{max}^2$ .... (2)

Equation (1) and (2) are called Einstein's photoelectric equation.

Failure of wave theory to explain photo electric effect:-

(a) Acc. To wave theory, when wave front of light strikes a metal surface, the free electrons at the surface absorb the radiate energy continuously.

Greater the intensity of incident radiation greater the amplitude of electric and magnetic field and greater is the energy density of the wave. This higher intensity should liberate photo electron with greater K.E

But this is control to the experimental result that max K.E of photo electron is independent of intensity.

(b) Wave theory fails to explain the existence of threshold frequency:-

No matter what the frequency of incident radiation a light wave of sufficient intensity over a sufficient time should be able to impact enough energy required to eject electrons from the metal surface. This wave they fall to explain the existence of threshold frequency.

(c) Failed to explain the photo electric emission is an instantaneous process.

Wave nature of matter:-

The idea of dual behaviour of radiation inspired de-Braglie to put forward a revolutionary hypothesis. He suggested that nature loves symmetry. Universe is composed of radiation and matter. If radiation behaves as s particle on same occasion it is reasonable to suppose a particle may behave like a wave.

For photon: - $\lambda = \frac{h}{p} = \frac{h}{mc}$ .... (1)

The above equation is equally valid for other material particles.

If a body of mass m, moving with speed V (may be charged or uncharged) its momentum is given by P = mV

Thus $\lambda = \frac{h}{mv}$ .... (2)

Equation (2) is called de-Broglie's wave equation

$\lambda \rightarrow$ de-Broglie's wave length

Wave length of debroglie's matter wave in terms of K.E:-

We know that $\frac{P^2}{2m} = E_K \Rightarrow P = \sqrt{2mE_K}$

Thus $\lambda = \frac{h}{p}$

$\Rightarrow \lambda = \frac{h}{\sqrt{2mE_K}}$ .... (3)

Wave length of de-Broglie wave in terms of electric potential:-

When an electron 9mass m, charge = e) is accelerated from rest through a potential V

Then $E_K =$ Charge of electron X potential

$\Rightarrow E_K = eV$

Substituting this value in equation (3)

$\lambda = \frac{h}{\sqrt{2meV}}$ .... (4)

Again substituting $n = 6.63 \times 10^{-34}$ Js $m_e = 9.11 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C

We have from equation (4)

$\lambda = \frac{1.227}{\sqrt{V}}$ nm .... (5)

Thus $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_K}} = \frac{1.227}{\sqrt{V}} nm$

Or $\lambda = \sqrt{\frac{150}{V}} A^{o}$

Note:- The LHS of the above equation – 2 attributes the waves, while the RHS attributes the particle. Plank's constant (h) relates the two attributes.

Conceptual Question:-

Question:- Why the macroscopic objects in our daily life do not show wave like property. On the other hand in the sub atomic domains the wave property is significant.

Solution:-

This can be made clear by the following example.

NCERT; Ex = 11.4:-

What is the de-Broglie wave length associated with (a) an electron moving with a speed of $5.4 \times 10^{6} m/s$. (b) A ball of mass 150 g moving with speed of 30 m/s?

Solution:-

For electron mass (m) = $9.11 \times 10^{-31} kg$

Speed (v) = $5.4 \times 10^{6} m/s$

Its momentum $P = mv$

$= 9.11 \times 10^{-31} \times 5.4 \times 10^{6} = 4.92 \times 10^{-24} kgm/s$

$\therefore$ de-Broglie wave length $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.92 \times 10^{-24}} = 0.135 nm$

(b) For ball mass (m) = 0.15 kg

Speed (v) = 30 m/s

Its momentum = $0.15 \times 30 = 4.5 kgm/s$

$\therefore$ de-Broglie wave length $\lambda' = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.5} = 1.47 \times 10^{-34} m$

Conclusion: - The de-Broglie wave length of electron is comparable with X-ray wave lengths. For the ball is about 10^{-19} times the size of the proton, quite beyond experiment measurement.

NCERT. Example – 11.5:-

An electron, an α-particle and a proton. Which of these particles have shortest de-Broglie wave length?

Solution: - $\lambda = \frac{h}{\sqrt{2mE}}$

When E is same $\lambda \propto \frac{1}{\sqrt{m}}$

We know that $m_\alpha < m_p < m_e$

Thus $\lambda_\alpha < \lambda_p < \lambda_e$

Thus, the α-particle has shortest wave length.

NCERT, Example – 11.7:-

What is the de-Broglie's wave length associated with an electron, accelerated through a p.d of 100 volt?

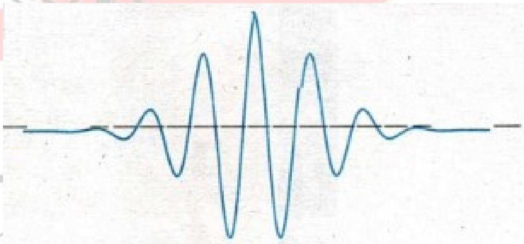
Solution: - $\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{V}} nm$

$= \frac{1.227}{\sqrt{100}} nm = 0.123 nm$

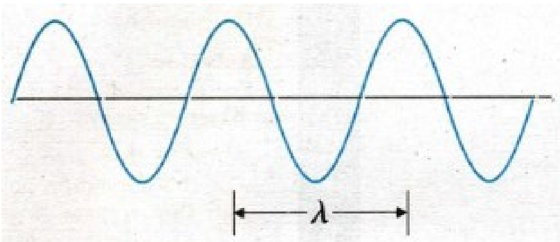
Which is order of X-ray wave length of electromagnetic spectrum.

Heisen Berg's uncertainty principle:-

Introduction: - In general the matter wave associated with moving particle should be regarded as a wave packet, instead of continuous wave. The wave packet moves with speed V.



If the wave packet is long, it contains a large no. Of waves. Thus, the chance of determining its λ hence momentum $(p) = \frac{h}{\lambda}$ are better. But in this case, the particle may be ay where within the long packet. Thus position of particle is not precise.



If the wave packet is short:- Δx is small i.e position of the particle is determined more precisely. But it is not easy to find its wave length.

Conclusion:- Therefore, there is a reciprocal relationship between uncertainty in position (Δx) of the particle and its momentum ΔP.

Heisen Berg's uncertainty principle:-

$\Delta x \propto \frac{1}{\Delta P}$

$\Rightarrow \Delta x\ \Delta P \le \tau$

Statement:- It is not possible to measure both the position and momentum of an electron (or any other sub atomic particle) at the same time exactly.

Case – 1, If a electron has definite momentum $\Delta p = 0$

$$\Rightarrow \Delta x = \frac{h}{\Delta p} \Rightarrow \infty$$

Case – 2, If the electron has definite position then $\Delta x = 0$

$$\Rightarrow \Delta P = \frac{h}{\Delta x} = \frac{h}{0} \Rightarrow \infty$$

Experimental Demonstration of wave nature of electron:-

Division and Germer's Expt.:-

Aim: - To verify the wave nature of matter (slow moving electron)

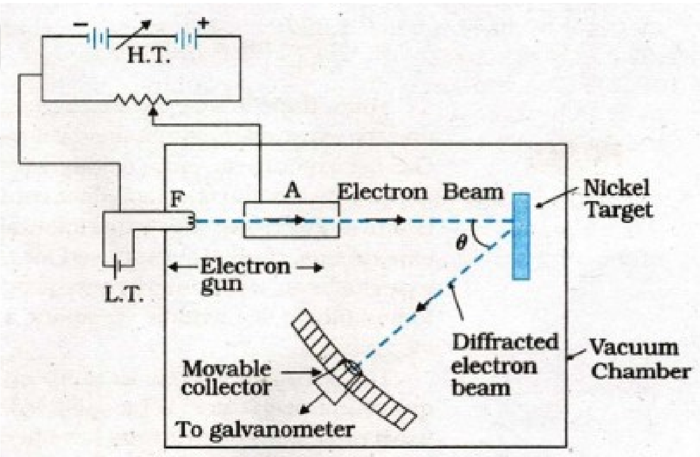
Experimental setup:-

Main components are:-

(i) Electron gun (ii) Nil target/ Ni-ergstal

(iii) Electron detector (iv) Housing (Evacuated chamber)

(1) Electron Gun:- Cathode- Tungsten cooled with BaO_2 act as a cathode which is heated up by a low tension. Battery (LTB). This heating gives enough energy to electron to surpass the work function and collected around the cathode.



(b) Anode: - It consist of a cylinder with a fine hole along its axis and kept +ve potential w.r.t cathode. Anode has two function (a) to pull the e^- away from cathode (b) To calumniate the electrons. The p.d between cathode and anode is variable in order to change the velocity of e^- as per requirement. The anode and cathode collectively form an electron gun.

(2) Ni-Crystal: - It is capable of rotating about an axis 1r to the plane of the paper. The ni-crystal whose lattice spacing is $2.15A^0$ is chosen for this purpose.

(3) Electron detector: - The intensity of scattered beam of electron in a given direction is measured by electron detector, which can be rotated on a circular scale. It is connected to a sensitive galvanometer which records the current.

(4) Housing: - The apparatus is enclosed in an evacuated chamber.

Experimental Result: - The intensity of scattered e^- was recorded at different diffracting angle and at different velocity of electron. It was observed that at 54V and at diffracting angle 50^0, the intensity of e^- was max.

Theory or Explanation:-

Appearance of peak at 50^0 is due to the constructive interference of electrons from different layer of regularly special atom of the crystal. The interference pattern itself prove that electrons were travelling as a wave.

Confirmation of wave nature of e^- by the expt.:-

According to Bragg's law.

For the 1st order diffraction max

$2^{nd}\ \sin\theta = \lambda$

$\rightarrow 2\left(2.15\times10^{-10}\right)\sin50^0 = 0.165\text{nm}$

According to de-Broglie $\lambda = \dfrac{1.227}{\sqrt{V}}\text{nm} = \dfrac{1.227}{\sqrt{54}} = 0.167\text{nm}$

There is a close agreement between experimental value (i.e 0.165 nm) given by Division and Germer and the estimated value by de-Broglie.

Conclusion:- This proves the existence of de-Broglie wave for slow moving electron.