Chapter- 2

Electric Potential and Capacitance

ELECTROSTATIC POTENTIAL –

An electric field can also be represented in terms of a scalar quantity called Electrostatic Potential.

IMPORTANCE OF ELECTROSTATIC POTENTIAL –

Electric Potential represents:

(i) The idea of potential energy possessed by a unit charge at that point.

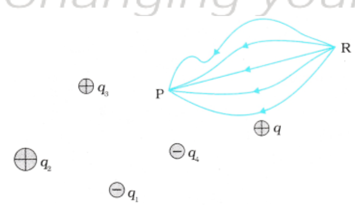
(ii) The degree of Electrification of a body.

(iii) The direction of flow of charge between two bodies in contact.

Note:

- The actual value of potential energy is not physically significant, it is only the difference of potential i.e significant.

ELECTROSTATIC POTENTIAL DIFFERENCE



Definition

The potential difference between two points in an electric field is defined as :

The amount of work done by an external force in carrying a unit +ve charge (test charge) from one point to another along any path (without acceleration)

MATHEMATICALLY; $V_P - V_R = \frac{W_{RP}}{q_0}$

Points to ponder:

- ✓ $\left(W_{P\rightarrow R}\right)$ electric force = $q_0\left(V_p - V_R\right)$
- ✓ $\left(W_{P\rightarrow R}\right)_{elec} + \left(W_{P\rightarrow R}\right)_{external} = (K)_R - (K)_P$
- ✓ $\left(W_{P\rightarrow R}\right)_{ext} = (K_R - K_p) + q_0(V_R - V_P)$
- ✓ The work done by an electrostatic field in moving a charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to another. Thus, the electric field is usually conservative.

ELECTROSTATIC POTENTIAL AT A POINT:

- The infinity is taken as zero potential

  Thus at infinity $V_R = 0$

  Therefore

  $$\Rightarrow V_P - 0 = \frac{W_{\infty P}}{q_o}$$

  $$\Rightarrow \boxed{V_P = \frac{W_{\infty P}}{q_o}}$$

Thus, Electrostatic potential at any point in the electric field is defined as the work done in carrying a unit +ve charge from infinity to that point along any path without acceleration against the field.

SI unit $\rightarrow$ Volt = Joule/Columb

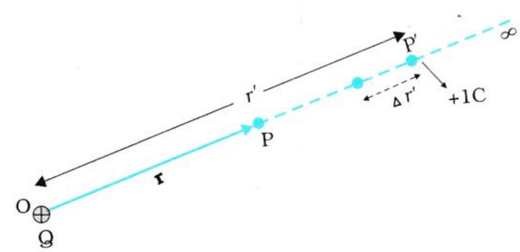
Define 1 volt?

Ans.: Electrostatic Potential at a point is said to be one volt when one joule of work is done in moving one coulomb of positive charge from infinity to that point against the electrostatic force of the field without acceleration.

3. Dimensional formula: $V_P = \frac{W}{q_0} = \frac{\left[ML^2T^{-2}\right]}{AT} = \left[A^{-1}ML^2T^{-3}\right]$

POTENTIAL DUE TO A POINT CHARGE:

Let 'P' be any point in the field of a single point charge at 'O'



The electrostatic force on the unit positive charge at a distance 'x' at some intermediate point 'A' on this path:

$F = \frac{1}{4\pi \in_O} \frac{Q}{x^2} \times q_0$ .... (Along OA)-------------------(1)

$\therefore$ A small amount of work done in moving a unit +ve charge from A to B through distance 'dx' is given by :

$$dW = \vec{F} \cdot \vec{dx}$$
$$= F(-dx)\cos 0^0$$ .... (2) *(as x is decreasing –dx is taken)*
$$= -Fdx$$

Total work done in moving unit +ve charge from ∞ to the point P is :

$$W = \int_{\infty}^{r} -Fdx$$
$$= \frac{-Qq_0}{4\pi \in_0} \int_{\infty}^{r} x^{-2}dx$$
$$= \frac{Qq_0}{4\pi \in_0 r}$$

By Definition; $V = \frac{W}{q_0} = \frac{KQ}{r}$

$$\boxed{V = \frac{Q}{4\pi \in_0 r}}$$

<u>GRAPH:</u> For the variation of V and E with r due to a point charge

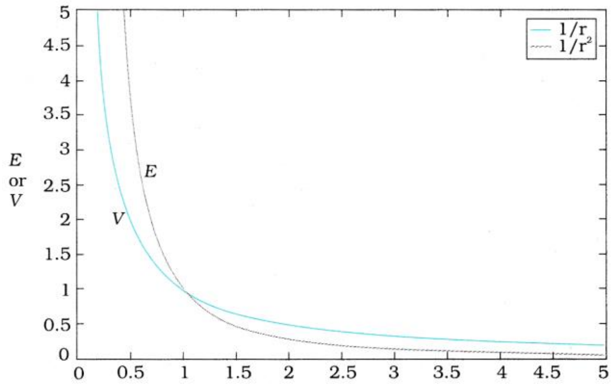


Fig shows the variation of electrostatic potential with distance i.e. $V \propto \frac{1}{r}$: and also the variation of the electrostatic field with distance i.e. $E \propto \frac{1}{r^2}$

Q. Can a metal sphere of radius 1centimeter hold a charge of 1C? Given that the ionizing potential of air is 3×10^4 volts.

Ans – We know, $V = \frac{Q}{4\pi \in_0 r}$ or $\frac{KQ}{r}$

Now, $V = \frac{KQ}{r}$

$$\Rightarrow 9\times10^9 \times \frac{1}{1\times10^{-2}}$$
$$= \frac{9\times10^9}{1\times10^{-2}}$$
$$= 9\times10^{11} V$$

(The Potential is very much greater than the required to ionize the air $(3\times10^4 V)$

Q . **(a) Calculate the potential at a point P' due to a charge $4\times10^{-7}C$ located 9 cm away**

(b) Hence obtain the work done in bringing a charge of $2\times10^{-9}C$ from infinity to the point 'P'. Does the answer depend on the path along which the charge is brought?

Ans(a) $V=\dfrac{KQ}{r}=9\times10^{9}\times\dfrac{4\times10^{-7}}{9\times10^{-2}}$

$=\dfrac{36\times10^{2}}{9\times10^{-2}}$

$=4\times10^{4}V$

(b) $W=qV=2\times10^{-9}\times4\times10^{4}V=8\times10^{-5}J$

No, work done is path independent. Since the electric field is conservative.

Q 3. **An isolated small spherical body is given a charge 'q' in the air. What will be its potential (i) in the air?**

(ii) in a medium of a dielectric constant (ϵ_r) ?

Ans – (i) Potential in air

$$V=\frac{KQ}{r}=\frac{Q}{4\pi\epsilon_0 r} \quad -(1)$$

(ii) In a medium of di-electric : $V_m=\dfrac{KQ}{r}$

$$=\frac{1}{4\pi\epsilon}\frac{Q}{r}\left[\because \epsilon_r=\frac{\epsilon}{\epsilon_0}\Rightarrow \epsilon=\epsilon_r\cdot\epsilon\right]$$

$$\Rightarrow V_m=\frac{Q}{4\pi(\epsilon_r\epsilon_0)r} \quad -(2)$$

Comparing equations (1) and (2)

$$\Rightarrow \boxed{V_m=\frac{1}{\epsilon_r}(V)}$$

Q. A charge of 1mC is displaced from point A of potential 25V to another point B of potential 5V.

(i) Find the work done by the electrostatic force on the charge for displacement $A \rightarrow B$.

(ii) If K.E. of the particle increases by 2mJ during displacement $A \rightarrow B$, then calculate the work done by external force on the charge.

(iii) What would be the work done by the external force on the charge during the motion, if K.E of the charged particle remains constant?

Ans: (i) $(W_{el})_{A\rightarrow B} = q_0(V_A - V_B)$

$= 1(25-5)V$ = 20 m J

(ii) $(W_{ext})_{A\rightarrow B} = \{(K.E.)_B - (K.E.)_A\} - (W_{el})_{A\rightarrow B}$

$= 2mJ - 20mJ$ = -18 m J

(iii) As $(K.E.)_A = (K.E.)_B$

$\therefore (W_{ext})_{A\rightarrow B} = 0 - (W_{el})_{A\rightarrow B}$ = -20 mJ.

Q. Electric field intensity and electric potential at a point due to a point charge are 10 N/C aid 100 V respectively.

(a) What is the magnitude of the charge?

(b) What is the distance of the point from the charge?

Ans.: (a) $E = \frac{kq}{r^2} = 10\, N/C$

$$V = \frac{kq}{r} = 100V$$

$$\therefore \frac{kq/r}{kq/r^2} = \frac{100}{10}$$

$$\Rightarrow r = 10m$$

(b) As $\frac{kq}{r} = 100$

$$\Rightarrow \frac{9\times10^9 \times q}{10} = 100$$

$$\Rightarrow q = \frac{1000}{9\times10^9} C$$

$$\Rightarrow q = \frac{1}{9}\times10^{-6} C$$

Points to ponder: Let charge q_0 moves from $A \rightarrow B$

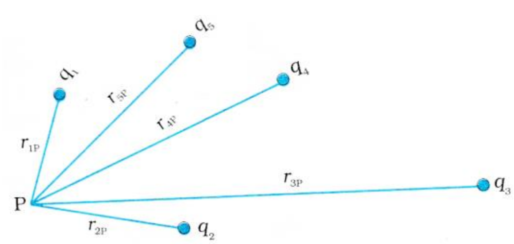
- ✓ $W_{el} = q_0(V_A - V_B)$
- ✓ $W_{el} + W_{ext} = k_B - k_A.$

$$\Rightarrow W_{ext} = (k_B - k_A) + q_0(V_B + V_A)$$

- ✓ If the charge is not accelerated $k_B - k_A = 0$

$$\therefore W_{ext} = q_0(V_B - V_A).$$

POTENTIAL DUE TO SYSTEM OF CHARGES :



The potential at P due to charge 'q_1' :

$$V_1 = \frac{1}{4\pi \in_0} \frac{q_1}{r_{1P}}$$

Similarly values of potential due to other charges

$$V_2 = \frac{1}{4\pi \in_0} \frac{q_2}{r_{2P}}$$

$$V_3 = \frac{1}{4\pi \in_0} \frac{q_2}{r_{3P}} \quad ....$$

Similarly, $V_n = \frac{1}{4\pi \in_0} \frac{q_n}{r_{nP}}$

Using Superposition Principle:

$$V = V_1 + V_2 + V_3 + ... + V_n$$

$$= \frac{1}{4\pi \in_0} \left[\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} .... + \frac{q_n}{r_{nP}} \right]$$

Therefore,

$$\boxed{V = \frac{1}{4\pi \in_0} \sum_{i=1}^{n} \frac{q_i}{r_{iP}}}$$

Electric potential due to continuous charge distribution:

- If we have to calculate electric potential due to a continuous charge distribution, characterized by volume charge density ρ, we divide the entire volume into a large, number of small volume elements, each of size dV.

  Hence now, charge on each element; $dq = \rho \mathrm{dV}$

  Find the potential at the point due to the element, i.e. $dV = \frac{kdq}{r} = \frac{k\rho \mathrm{dV}}{r}$

  $\therefore$ Total potential due to the body, $V = \int \frac{kdq}{r} = \int \frac{k\rho \mathrm{dV}}{r}$

- For the linear charge distribution; $V = \int \frac{k\lambda dl}{r}$

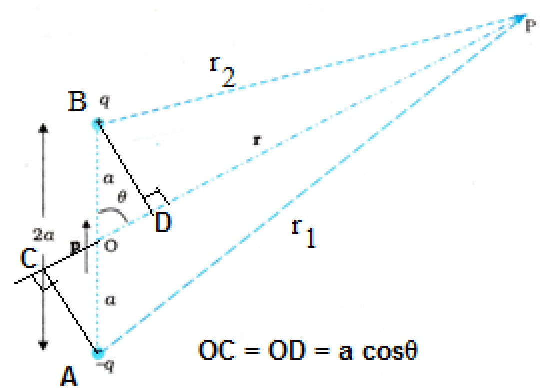
- For the surface charge distribution; $V = \int \frac{k\sigma dA}{r}$

ELECTROSTATIC POTENTIAL AT A POINT DUE TO AN ELECTRIC DIPOLE

Potential At 'P' due to Q charge : $V_1 = \frac{1}{4\pi \in_0} \frac{(-q)}{r_1} = \frac{-1}{4\pi \in_0} \frac{q}{r_1}$

The potential at 'P' due to +Q Charge

$$V_2 = \frac{1}{4\pi \in_0} \frac{q}{r_2}$$



Potential At 'P' due to the dipole is given by

$$V = V_1 + V_2$$

$$= \frac{-1}{4\pi \in_0} \frac{q}{r_1} + \frac{1}{4\pi \in_0} \frac{q}{r^2}$$

$$= \frac{q}{4\pi\in_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \qquad .....(1)$$

Now by geometry:

$$r_1 = AP{\sim}CP = CO + OP = r + a\cos\theta$$

$$r_2 = BP{\sim}DP = OP - OD = r - a\cos\theta$$

Thus,

$$V = \frac{q}{4\pi\in_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{r + a\cos\theta - r - a\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$$

$$= \frac{q}{4\pi \in_0} \frac{2a\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

$$= \frac{P\cos\theta}{4\pi \in_0 (r^2 - a^2\cos^2\theta)} (\because p = q \times 2a)$$

If $r \gg a, a^2\cos^2 Q$ will be neglected in comparison to r^2

Hence, $\boxed{V = \frac{P\cos\theta}{4\pi\in_0 r^2}}$

In vector notation $V = \frac{\vec{P}\cdot\hat{r}}{4\pi\epsilon_0 r^2}\{P\cos\theta = \vec{P}\cdot\hat{r}\}$

Special Cases:

1. If the point 'P' lies on the axial line of the dipole. i.e $\theta = 0^0$ or 180^0

   Then, $V = \pm\frac{P}{4\pi\epsilon_0 r^2}$

2. If the point 'P' lies on the equatorial line of the dipole i.e $\theta = 90^0$

   Then, $V = 0$

Points to ponder :

- ✓ $V \propto \frac{1}{r}$ due to point charge
- ✓ $V \propto \frac{1}{r^2}$ due to an electric dipole
- ✓ Potential due to dipole depends upon :

  (i) Displacement

  (ii) The angle between position vector and displacement vector
- ✓ The potential due to a dipole is axially symmetric about $\vec{P}$. i.e if we rotate the position vector $\vec{r}$ and $\vec{P}$ keeping θ fixed, the points corresponding to P on the cone so generated will have the Same potential.

Q. Two charges $3\times10^{-8}C$ and $2\times10^{-8}C$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential 0? Take the potential at infinity to be zero.

Ans: The potential at 'P' is on the axis at which V = 0, i.e. $V_A + V_B = 0$

Now,

$$V_A = \frac{K(3\times10^{-8})}{x} \text{ AND } V_B = \frac{-K(2\times10^{-8})}{(15-x)} = 0$$

$$\Rightarrow \frac{K\left(3\times10^{-8}\right)}{x} - \frac{K\left(2\times10^{-8}\right)}{15-x} = 0$$

$$\Rightarrow \frac{K(3\times10^{-8})}{x} = \frac{K(2\times10^{-8})}{15-x}$$

$$\Rightarrow \frac{3}{x} = \frac{2}{15-x}$$

$$\Rightarrow 45 - 3x = 2x$$

$$\Rightarrow 5x = 45$$

$$\Rightarrow x = 9cm \text{ (from charge A)}$$

Now if x lies in the extended line OA the required condition is :

The potential at 'P' on the extended line 'BP' where V = 0

i.e $V_A + V_B = 0$

$$V_A = \frac{K\left(3\times10^{-8}\right)}{x} \text{ AND } V_B = \frac{K\left(2\times10^{-8}\right)}{x-15} = 0$$

$$\Rightarrow \frac{K\left(3\times10^{-8}\right)}{x} - \frac{K\left(2\times10^{-8}\right)}{15-x} = 0$$

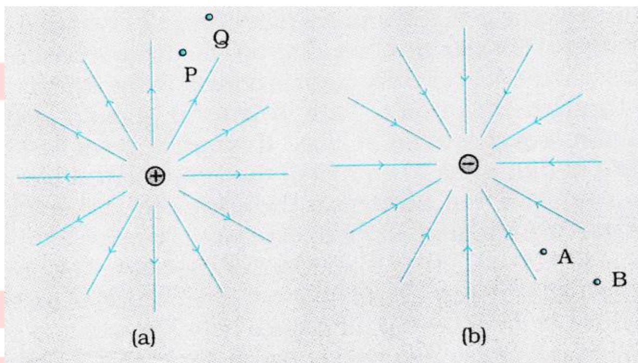
$$\Rightarrow K\frac{3\times10^{-8}}{x} = K\frac{3\times10^{-8}}{x-15}$$

$$\Rightarrow \frac{3}{x} = \frac{2}{x-15}$$

$$\Rightarrow 3x - 45 = 2x$$

$$\Rightarrow \boxed{x = 45cm}$$

Q. The figures (a) and (b) show the field lines of a positive and negative point charge respectively



(a) Give the signs of the potential difference $V_P - V_Q$; $V_B - V_A$

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Ans :

(a) $V \propto \frac{1}{r}, V_P > V_Q$. Thus $(V_P - V_Q)$ is positive. Also V_B is less negative than V_A. Thus $V_B > V_A$ or $(V_B - V_A)$ is positive.

(b) A small negative charge will be attracted to a positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore the sign of potential energy difference of small negative charge between Q and P is positive.

Similarly $(\mathrm{P.E})_A > (\mathrm{P.E})_B$ and hence the sign of potential energy difference is positive

(c) In moving a small positive charge from Q to P work has to be done by an external agency against the electric field. Therefore, the work done by the field is negative.

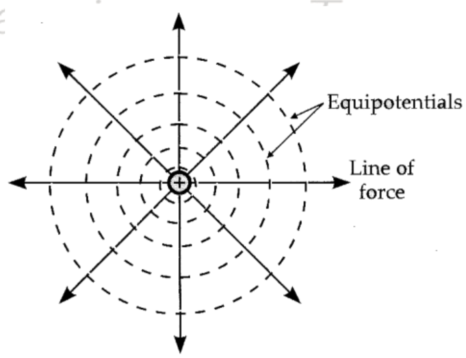
(d) In moving a small negative charge from B to A work has to be done by the external agency. It is positive.

(e) Due to the force of repulsion on the negative charge velocity decreases and hence kinetic energy decreases in going from B to A.

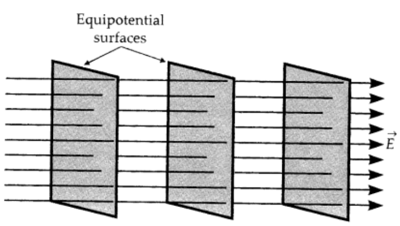
EQUIPOTENTIAL SURFACES

An equipotential is that surface at every point of which electric potential is the same.

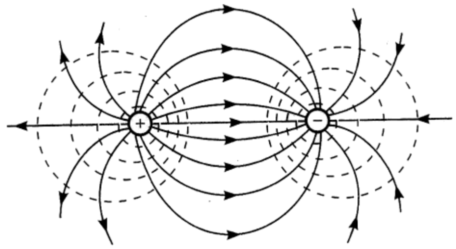
(i) For a single charge 'q':



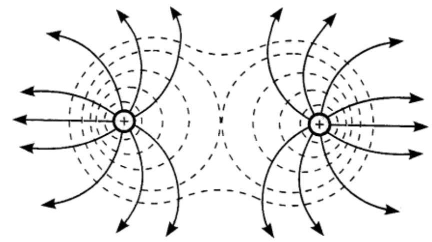
(ii) For uniform Electric Field:



(iii) For Dipole:



(iv) Two Identical Positive Charges:



PROPERTIES OF EQUIPOTENTIAL SURFACES:

(1) NO WORK IS DONE IN MOVING THE TEST CHARGE OVER AN EQUI-POTENTIAL SURFACE:

By definition, the potential difference between two points B and A = Work done in carrying a unit positive, test charge from A to B.

i.e $V_B - V_A = W_{AB}$

For Equipotential Surface:

$V_B = V_A$ Therefore, $\boxed{W_{AB} = V_B - V_A = 0}$

(2) FOR ANY CHARGE CONFIGURATION, EQUIPOTENTIAL SURFACE THROUGH A POINT IS NORMAL TO THE ELECTRIC FIELD AT THAT POINT:

$d\ell$ is the small distance over the equipotential surface through which unit positive charge is carried:

Then $dW = \vec{E}.d\vec{\ell}$

$= Ed\ell\cos\theta$

$= 0$

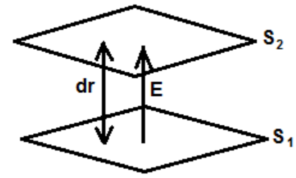
Therefore, $\cos\theta = 0 \text{ or } \theta = 90^0$

i.e $\vec{E} \perp d\vec{\ell}$

(3) EQUI-POTENTIAL SURFACE HELPS TO DISTINGUISH REGION OF STRONG FIELD FROM THOSE OF WEAK FIELD:

The equipotential surfaces are close to each other in the region of a stronger field.

As; $\left|\vec{E}\right| = \frac{|dV|}{dr}$



$\Rightarrow \left|\vec{E}\right| \alpha \frac{1}{dr}$ (For same potential difference)

This shows that if the electric field is stronger, then the separation between the equipotential surfaces is small.

(4) NO TWO EQUIPOTENTIAL SURFACES CAN INTEREST EACH OTHER:

In case, if they intersect there, will be two values of potential at a single point in the field which is impossible.

(5) EQUI-POTENTIAL SURFACES OFFER AN ALTERNATIVE, VISUAL PICTURE also OF FIELD LINES AROUND A CHARGE FIELD.

ILLUSTRATION: What is the work done in moving a test charge 'q' through a distance of 1cm along the equatorial axis of an electric dipole?

Solution:

On the equatorial line of a dipole,

V = 0

Then, $W = Q \times V = 0$

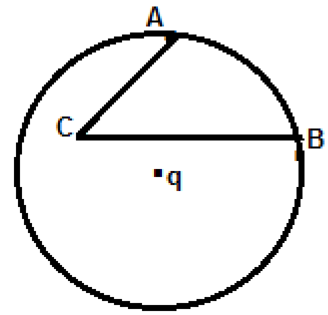
ILLUSTRATION: What would be the work done if a point charge +Q is taken from a point A to B

(a) On the circumference of the circle with another point charge $\pm q$ at the center.

(b) Via C.

Solution:

(a) As $V_A = V_B \Rightarrow W = Q(V_B - V_A) = 0$



(b) As $V_A = V_B \Rightarrow W = Q(V_B - V_A) = 0$ (As electrostatic force is conservative, its work done is path independent.)

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL:

Let us consider two equipotential surfaces A and B spaced closely as shown. Let the potential of A be 'V' and B be V + dV.

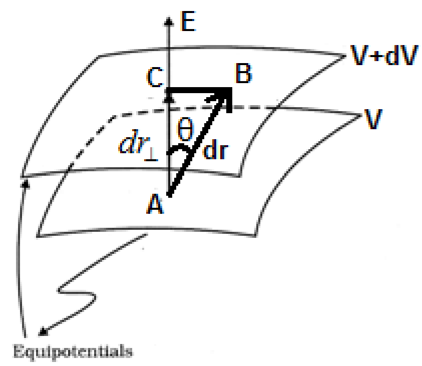
$dV \rightarrow$ The change in potential between A and B.

The work done in moving a unit charge against the electric field from A to B is;

$$dW = -\vec{F}.d\vec{r} = -1\vec{E}.d\vec{r} = -\vec{E}.d\vec{r} \quad ......(i)$$

By Definition of potential difference;

$$dW = 1dV = dV \quad ....(ii)$$



From equations (i) and (ii) we have

$$dV = -\vec{E}.d\vec{r} = -Edr\cos\theta = -Edr_\perp$$

$$\Rightarrow E = \frac{-dV}{dr_\perp}$$

In magnitude, $|\vec{E}| = \frac{|dV|}{dr_\perp}$

Two important conclusions from the relation;

(i) The electric field at any point is directed along the direction in which the potential decreases steepest.

(ii) The magnitude of the electric field intensity at any point is equal to the potential difference per unit length along a perpendicular direction to an equipotential point considered there.

Points to Ponder:

- ✓ In general; $\vec{E} = -\vec{\nabla} V$
- ✓ In Cartesian co-ordinate system; $\vec{E} = \frac{-\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$
- ✓ In 2D polar coordinate system (r,θ); $\vec{E} = \frac{-\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}$
- ✓ In one dimensional Cartesian coordinate system; $\vec{E} = \frac{-dV}{dx}\hat{i}$
- ✓ In one dimensional polar coordinate system; $\vec{E} = \frac{-dV}{dr}\hat{r}$
- ✓ Now potential at a point; $V = -\int_{\infty}^{r} Edr$

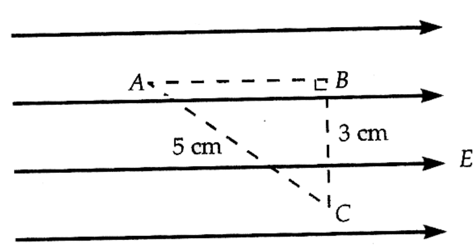
  In 1D Cartesian coordinate system; $V = -\int_{\infty}^{x} Edx$

  In 3D; $V = -\int_{\infty}^{\vec{r}} \vec{E}.d\vec{r}$
- ✓ Now the potential difference between two points; $V_B - V_A = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E}.d\vec{r}$

ILLUSTRATION: Three points A, B, and C lie in a uniform electric field $E = 5\times10^{-3}\,N/c$ **as shown in the figure. Find out the potential difference between A and C.**

Solution:

The electric field in a region is given by

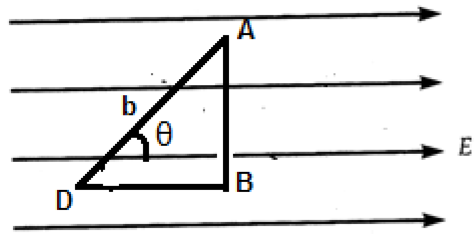


$$E = \frac{-dV}{dr} \Rightarrow dV = -Edr$$

ATQ,

$$V_A - V_C = V_A - V_B = +E(AB) \; = +5\times10^3\times4\times10^{-2} = +20\times10^1 \; = +200V$$

ILLUSTRATION: -A test charge 'q_0' is moved from A to D along the path ABD as shown in the figure. Find the P.D between points D and A.

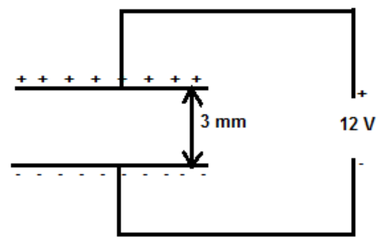


Solution:

We know, $V_A = V_B$

And $V_D - V_A = V_D - V_B = -E(BD) = -E(b\cos\theta) = Eb\cos\theta$

ILLUSTRATION: Find the electric field between two metal plates 3mm apart connected to a 12V battery



Solution:

$$E = \frac{V}{d} = \frac{12}{3\times10^{-3}} = 4\times10^{+3}\,\mathrm{N/C\ or\ volt/m}$$

ILLUSTRATION -4: Given $V = x^2y + yz$, calculate the magnitude of $\vec{E}$ at (1, 3, 1)

Solution:

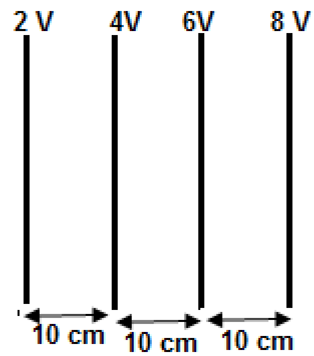
$$E_x = -\frac{\partial V}{\partial x} = -2xy = 2(1).(3) = -6\,\text{unit}$$

$$E_y = -\frac{\partial V}{\partial y} = -x^2 - z = \left((1)^2 + (1)\right) = -2\,\text{unit}$$

$$E_z = -\frac{\partial V}{\partial z} = -y = -3\,\text{unit}$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(-6)^2 + (-2)^2 + (-3^2)} = \sqrt{36+4+9} = \sqrt{49} = 7\,\text{unit}$$

ILLUSTRATION: Equipotential surfaces, with potential 2V, 4V, 6V, and 8V parallel to the y-axis as shown. Calculate the electric field intensity



Solution:

We know, $|\vec{E}| = \frac{-dV}{dx}$

$$|\vec{E}| = \frac{dV}{dx}$$

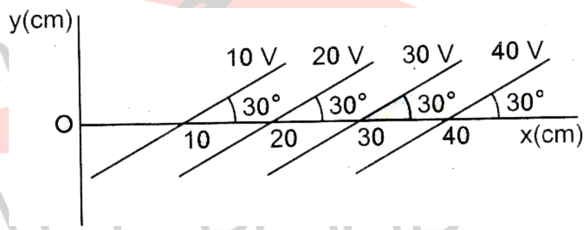
Now, $dV = 4 - 2 = 2$

$dx = 10cm = 0.1m$

$|\vec{E}| = \frac{2}{0.1} = 20\,V/m$

We know the electric field is along the direction of decreasing potential.

ILLUSTRATION - 6:



In the above equipotential surface. What can you say about the magnitude and direction of E?

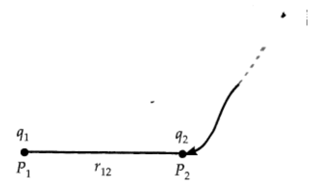
Solution: Left to the students.

Electrostatic Potential Energy for a System of Charges:

(Definition)

Electrostatic Potential energy of a system of point charges is defined as the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF TWO-POINT CHARGES



Suppose a point charge q_1 is held at the point with position vector $\vec{r}_1$ in space. Another point charge q_2 is at infinite distance from q_1. This is to be brought to the position $P_2(\vec{r}_2)$.

Where $P_1P_2 = (\vec{r}_{12})$

Now electrostatic potential at P_2 due to charge q_1 at P_1 is

$$V= \frac{k}{r_{12}} q_1$$

By definition work done in carrying charge q_2 from infinity to P_2

W = (Potential due to q_1) × charge (q_2)

$$W = \frac{kq_1q_2}{r_{12}}$$

This is stored in the system of two point charges q1 and q2 in the form of electrostatic potential energy U. Thus

$$\boxed{U=W=\frac{kq_1q_2}{r_{12}}}$$

FOR A THREE-POINT CHARGE SYSTEM

Suppose a point charge $+q_1$ is at a point P in space.

NO WORK IS DONE since another charge is at ∞

the charge $+q_2$ is brought from ∞ to P_2 at a distance r_{12}.

W_2= (potential due to q_1) × q_2

$$=\frac{kq_1}{r_{12}}(q_2) =\frac{kq_1q_2}{r_{12}}$$

When a charge $+q_3$ is brought from infinity to P_3 at a distance of $\vec{r}_{13}$

Work has to be done against q_1 and q_2.

W_3= (potential due to q_1 and q_2) × (charge (q_3))

$$=k \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)q_3$$

$=k(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}})$

POTENTIAL ENERGY FOR THE SYSTEM

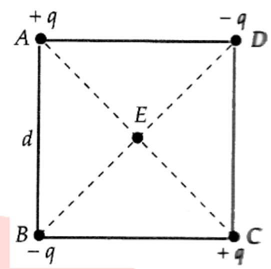
U= W= $W_1 + W_2 + W_3$

$= k(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_2}{r_{12}})$

Points to ponder :

For n charges, $\boxed{U = \frac{1}{2}k\sum_{\substack{i=1\\k=1}}^{n}\frac{q_iq_k}{r_{ik}}}$

ILLUSTRATION: Four charges are arranged at the corners of a square ABCD of side "d"



(a) Find the work required to put together this arrangement.

(b) A charge q_0 is brought to the center "E" of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?

Solution:

(a) (i) work needed to bring charge +q to A when no charge is present elsewhere=0

(ii) work needed to bring the charge –q to B when +q is at A

$W= -q \times (\frac{q}{4\pi\varepsilon_0 d}) = -\frac{q^2}{4\pi\varepsilon_0}$

(iii) work needed to bring charge ''+q'' to C when +q is at A and ''–q ''is at B

$W = +q\ (\frac{+q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{-q}{4\pi\varepsilon_0 d})$

$= -\frac{q^2}{4\pi\varepsilon_0 d}(1 - \frac{1}{\sqrt{2}})$

(iv) work needed to bring –q to D when +q is at A, -q is at B , and +q is at C

$W = -q\ (\frac{q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{q}{4\pi\varepsilon_0 d})$

$= -\frac{q^2}{4\pi\varepsilon_0 d}(2 - \frac{1}{\sqrt{2}})$

Net work done;

$W = \frac{-q^2}{4\pi\varepsilon_0 d}(0 + 1 + (1 - \frac{1}{\sqrt{2}}) + (2 - \frac{1}{\sqrt{2}}) = \frac{-q^2}{4\pi\varepsilon_0 d}(4 - \sqrt{2})$

(b) The electrostatic potential at center "E" is 0 since potential due to A and C is canceled by that due to B and D. Hence no work is required to bring any charge to point E.

Because $W = V(q_0) = 0(q_0) = 0$

Potential Energy in an External Field:

1. **For Single charge:**

Electric potential is different at different points of an external field. Let 'V' be the potential at any point 'P'.

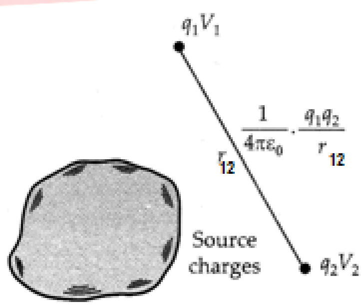
$\therefore$ the potential energy of the charge 'q' = work done in bringing the charge from ∞ to that point.

i.e. $U = qV$

Potential Energy in terms of position vector:

Therefore, $U = qV(\vec{r})$

2. Two Charges:



Suppose q_1 and q_2 are two-point charges at position vector r_1 and r_2 respectively in a uniform field E.

Work done in bringing charge q_1 from ∞ to position $\vec{r_1}$

$$W_1 = q_1.V(\vec{r}_1)$$

Again work done in charge q_2 from ∞ to the position $\vec{r}_2$ against the external field.

$$W_2 = q_2.V(\vec{r}_2)$$

Which q_2 is brought from ∞ to position $\vec{r}_2$. Work has also been done against the field due to q_1

Thus, $W_3 = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$

By the superposition principle:

P.E of the system = Total work done in assembling the charge configuration

Thus, $U = W_1 + W_2 + W_3 = q_1.V(\vec{r}_1) + q_2.V(\vec{r}_2) + \frac{q_1.q_2}{4\pi\varepsilon_0 r_{12}}$

The above equation represents Potential energy in terms of a position vector.

ILLUSTRATION:

(a) Determine the electrostatic potential energy of a system consisting of two charges $7\mu C$ and $-2\mu C$ (and with no external field) placed at($-9cm, 0, 0$) and (9cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external field $E = \frac{A}{r^2}$, $A = 9\times10^5 cm^2$. What would be the electrostatic energy of the configuration be?

Solution:

(a) We know for a two-point charge system

The potential energy is

$$U = K\frac{q_1 q_2}{r} = \frac{9\times10^9\times7\times10^{-6}\times(-2)\times10^{-6}}{0.18} = -0.7J$$

(b) $W = U_2 - U_1 = 0 - U = 0 - (-0,7) = 0.7J$

(c) In the given field, the potential at any point is;

$$V = -\int_{\infty}^{r} E dr = \frac{A}{r}$$

The mutual interaction energy of the two charges remains unchanged. Also, there, is the energy of interaction of the two charges with the external electric field. We find,

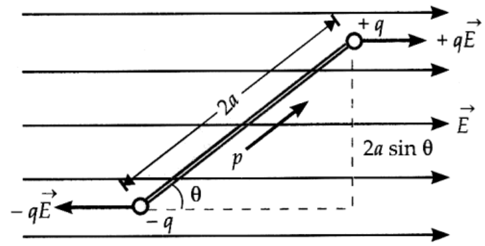
$$q_1 V(r_1) + q_2 V(r_2) = A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m}$$

And the net electrostatic energy is:

$$U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m} - 0.7J$$

$\Rightarrow U = 70J - 20J - 0.7J = 49.3J$

WORK DONE IN ROTATING A DIPOLE IN AN EXTERNAL FIELD:



Let a dipole of dipole moment $\vec{P}$ be placed in an electric field making an angle θ with the direction of $\vec{E}$.

The magnitude of Torque acting on dipole:

$$\tau = PE\sin\theta$$

Work done in rotating the dipole in a field through $d\theta$ is given by.

$$dW = \tau d\theta = PE\sin\theta d\theta$$

Work done in rotating the dipole from $\theta_1 \text{to} \theta_2$

$$W = \int dW = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta$$

$$= PE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= -PE(\cos\theta_2 - \cos\theta_1)$$

Therefore $\boxed{W = PE(\cos\theta_1 - \cos\theta_2)}$

ILLUSTRATION: If a dipole is rotated from field direction to any position θ. i.e $\theta_1 = 0, \text{then}\, \theta_2 = \theta$

Show that $W = PE(1 - \cos\theta)$

Solution: Left to the students

ILLUSTRATION: Calculate the amount of work done in rotating the dipole from the field direction to position normal to the field direction.

Solution:

In this case; $\theta_1 = 0^0$ and $\theta_2 = 90^0$

Now, $W = PE\left[1-\cos\theta\right] = PE\left[1-\cos 90^0\right] = PE\left[1-0\right] = PE$

ILLUSTRATION: Calculate the amount of work done in deflecting the dipole through an angle of 180^0, if it was placed normal to the field.

Solution:

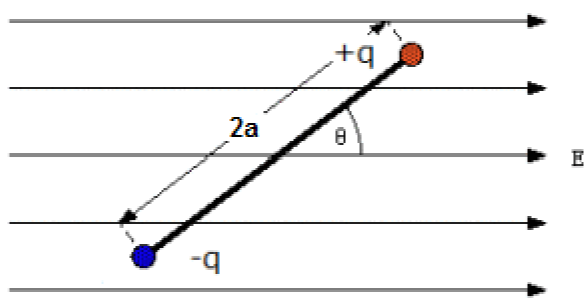
$$W = PE\left[\cos\theta_1 - \cos\theta_2\right]$$

$$= PE\left[\cos 90^0 - \cos\left(90+180\right)\right]$$

$$= PE\left[0-0\right] = 0$$

The potential energy of a dipole in an electric field:

It is defined as the work done in rotating a dipole from a direction perpendicular to the field to a given direction



Expression for potential energy of a dipole:

Mathematically,

$$U = W_{90^0 \to \theta} = \int_{90^0}^{\theta} pE\sin\theta d\theta$$

$$\Rightarrow U = pE\left(\cos 90^0 - \cos\theta\right)$$

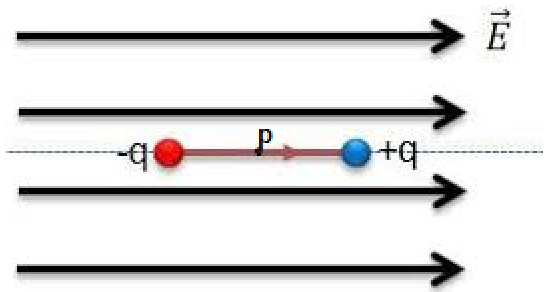
$$\Rightarrow U = -pE\cos\theta$$

In vector form: $\boxed{U = -\vec{P}.\vec{E}}$

ILLUSTRATION:

(a) When the dipole is said to be in stable equilibrium in an electric field?

Solution:



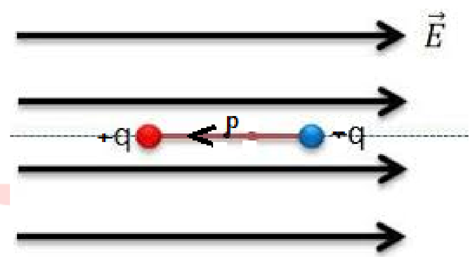
When, $\theta = 0^0$

$$U = -\vec{P}.\vec{E} \;= -PE\cos\theta \;= -PE\,\text{(minimum)}$$

Here, $\vec{F}_{net} = 0, \vec{\tau} = 0$

(b) When the dipole is said to be in unstable equilibrium in an electric field?

Solution:

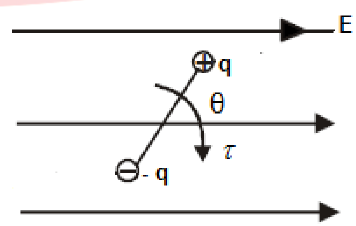


When, $\theta = 180^0$

$$U = \vec{P}.\vec{E} \;= -PE\cos\theta \;= -PE(-1) = PE$$

Here U = maximum, $\vec{F}_{ext} = 0, \tau = 0$

ILLUSTRATION: An electric dipole of dipole moment $(\vec{P})$ is placed in a uniform electric field $(\vec{E})$ in a stable equilibrium position. Its moment of inertia about the central axis is I. It is displaced slightly from its mean position. Find the period of small oscillation.



Solution:

When displaced at an angle θ from its mean position. The magnitude of restoring force.

We know, $\tau = -PE\sin\theta$

For slight displacement: $\sin\theta \to \theta$

Therefore, $\tau = -PE\theta$

We know, $\tau = I\propto$

$$\therefore \propto = \frac{\tau}{I} = \frac{-PE\theta}{I}$$

Now, $\alpha = -\omega^2\theta\left[\text{for SHM} \to \alpha = -\omega^2\theta\right]$

Substituting the values:

$$\alpha = \frac{-PE\theta}{I}$$

$$\Rightarrow -\omega^2\theta = \frac{-PE\theta}{I}$$

$$\Rightarrow \omega^2 = \frac{PE}{I} \Rightarrow \omega = \sqrt{\frac{PE}{I}}$$

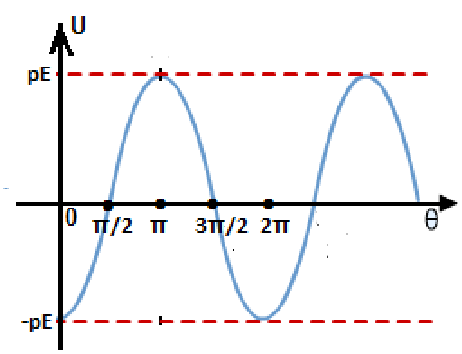
Also $\omega = \dfrac{2\pi}{T} \Rightarrow \dfrac{2\pi}{T} = \sqrt{\dfrac{PE}{I}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{PE}}$$

ILLUSTRATION:

Draw the variation of the potential energy of an electric dipole in the electric field with θ.

Solution:



Unit of Electrostatic Potential Energy:

SI unit – Joule

Q. Define 1J Potential Energy?

Solution:

The energy which is required to move 1c of charge through a p.d of 1V is called 1J

Other common units of potential energy:

1eV – It is the energy gained by electron by moving through a field of p.d of V

Conversion:

1eV = charge of 1e x 1 volt $=\left(1.6\times10^{-19}C\right)\times1volt=1.6\times10^{-19}C.V$

Therefore $1eV=1.6\times10^{-19}J$

Now, $1meV=10^{-3}eV=1.6\times10^{-19}\times10^{-3}=1.6\times10^{-22}J$

Kilo electron volt : $1keV=10^{3}eV=1.6\times10^{-19}\times10^{3}=1.6\times10^{-16}J$

Mega electron volt: $1MeV=10^{6}eV=1.6\times10^{-19}\times10^{6}=1.6\times10^{-13}J$

Giga electron volt: $1GeV=10^{9}eV=1.6\times10^{-10}J$

Tera electron volt: $1TeV=10^{12}eV=1.6\times10^{-7}J$

ILLUSTRATION: A molecule of a substance has a permanent electric dipole moment of magnitude $10^{-29}cm$. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude $10^{6}Vm^{-1}$. The direction of the field is suddenly changed by an angle 60^{0}. Estimate the heat released by the substance, in aligning its dipoles along the new direction of the field. For simplicity assume 100% polarization of the sample.

Solution:

Here, the dipole moment of each molecule = $10^{-29}Cm$.

As 1 mole of the substance contains $6\times10^{23}mol$

The total dipole moment of all the molecules, $P=6\times10^{23}\times10^{-29}Cm=6\times10^{-6}Cm$

Initial potential energy $U_{t}=-PE\cos\theta=-6\times10^{-6}\times10^{6}\cos\theta=-6J$

Final potential energy (when $\theta = 60^0$)

$$U_f = -6\times10^{-6}\times10^{6}\cos 60^{o} = -6\times\frac{1}{2} = -3J$$

Change in Potential Energy = $-3J - (-6J) = 3J$

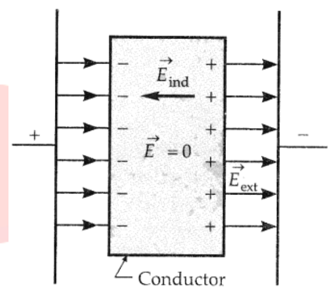
So, there is a loss in potential energy. This must be the energy released by the substance in the form of heat in aligning its dipoles.

ELECTROSTATICS OF A CONDUCTOR

The behavior of metal conductors in the electrostatic field.

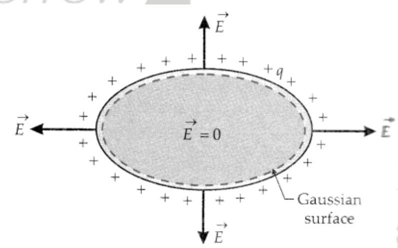
Some of the important results regarding the electrostatics of conductors are discussed below:

(1) Inside a conductor, the electric field is zero



When a conductor is held in an external electric field of intensity $\vec{E}_0$, then free electrons in the conductor move in the opposite direction of the field and gather at one end leaving the +ve charges at the other end. Hence an electric field of intensity $\vec{E}_P$ is induced opposite to the external field. Hence net field is 0.

(2) The interior of a conductor can have no excess charge in a static situation:



Let us consider any arbitrary volume element 'V'

A Gaussian surface is imagined just inside the element

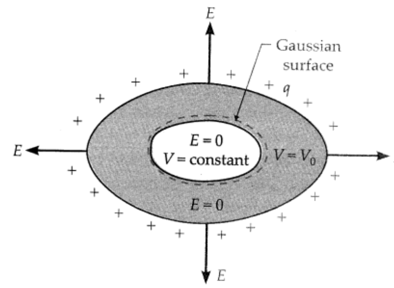
Then according to Gauss Law: $\oint_s \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$

Inside the conductor E = 0

Therefore $q_{in} = 0$

Thus if an excess charge is placed on an isolated conductor the charge move quickly spreads over the surface because like charges repel each other.

(3) Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point:



Under electrostatic conditions, once the charges on a conductor are re-arranged, the flow of charges stops. Therefore components of the electric field along the tangent to the surface of the conductor must be zero. Hence the electric field must be normal to the surface of the conductor in the electrostatic conditions.

(4) Electrostatic Potential is constant through the volume of the conductor and has the same value as on its surface:

We know; $\vec{E} = -\frac{dV}{dr}$

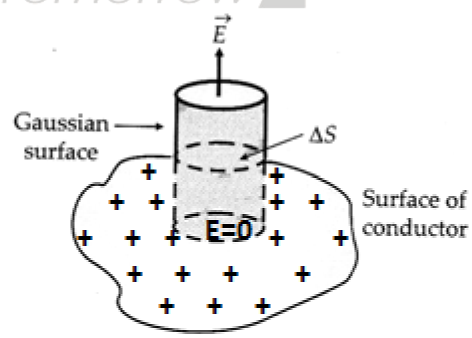
Inside the conductor $E = 0$

Therefore $\frac{dV}{dr} = 0 \Rightarrow V = \text{constant}$

Thus the interior of a charged conductor is an equipotential region

<u>Therefore the surface of the conductor is equipotential.</u>

(5) The electric field at any point close to the charged conductor is σ / ε_0



Let us consider a short cylinder of the small area of cross-section ds and negligible height partly inside and partly outside the surface of a conductor of surface charge density 'σ'.

Just inside the surface, E = 0

Just outside, the field $\vec{E}$ is normal to the surface.

By Gauss theorem, $\oint_s \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$

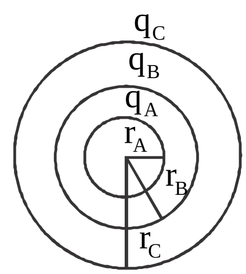
$$\Rightarrow E_{Plane} ds \cos 0 + E_{curved} ds \cos 90^0 + 0.ds = \frac{\sigma ds}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Vectorially; $\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}}$

ILLUSTRATION: In the figure shown, find out the electric potential at A, B, and C

Solution: Potential at A $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A} + \frac{q_B}{r_B} + \frac{q_C}{r_c}\right]$



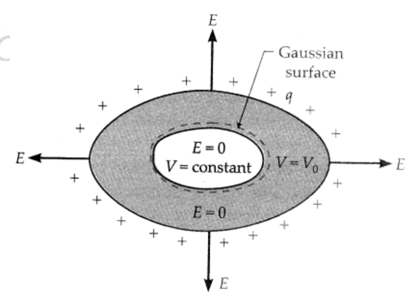
Similarly $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B} + \frac{q_B}{r_B} + \frac{q_C}{r_C}\right]$ and $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_C} + \frac{q_B}{r_C} + \frac{q_C}{r_C}\right]$

Electrostatic Shielding:

Definition: The phenomenon of making a region free from any electric field is called electrostatic shielding. it is based on the fact that the electric field varnishes inside the charity of a hollow conductor.

Proof:

For the Gaussian Surface inside the conductor



$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

We know, E = 0 (inside the conductor)

Therefore $q_{in} = 0$

Furthermore, if we consider the surface of the cavity as a Gaussian surface

By Gaussis Theorem,

$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0}$$

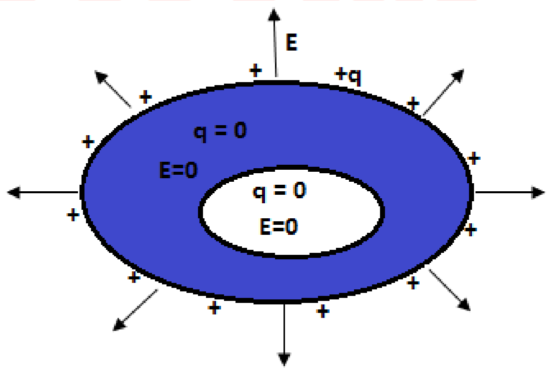
E = 0 (inside the cavity)

Applications of Electrostatic Shielding:

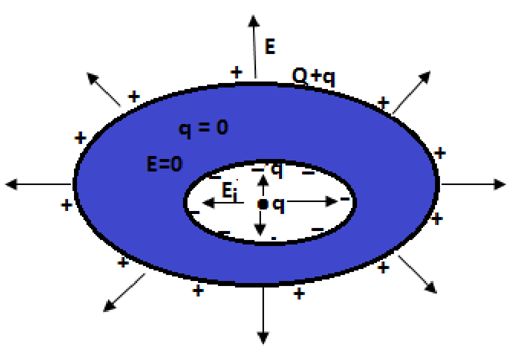
- In a thunderstorm accompanied by lightning, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lightning.
- Sensitive components of electronic devices are protected or shielded from external electric disturbances by placing metal shields around them.
- In a coaxial cable, the outer conductor connected to the ground provides an electrical shield to the signals carried by the central conductor.

Point to ponder:

✓ **If there is a cavity in a conductor and there is no charge in the cavity, then**



✓ **If there is a cavity in a conductor and a point charge q is in the cavity but insulated from the conductor and conductor is given with a charge Q, then**



DIELECTRICS AND THEIR POLARIZATION:

Dielectric: Dielectrics are insulating materials which transmit electric effect without actually conducting electricity.

Classification: (1) Polar

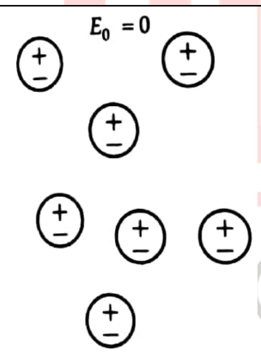
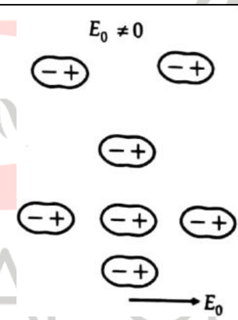
(2) Non-Polar

Non-Polar Dielectric

In such dielectrics, the center of mass of +ve charge coincides with C.M of –ve charge in the molecule.

Example: Hydrogen nitrogen, oxygen, CO_2, Benzene, methane

(Note: Such molecules are symmetric)

 Each molecule has zero dipole moment in its normal state	 In the presence of E, the C.M of +ve charges is displaced in the direction of the external field while C.M of –ve charges are displaced in the opposite direction. These induced dipole moments add up to give a net dipole moment. This is the polarization of non-polar dielectrics.

Polar Dielectric:

Such dielectric is made up of polar molecules

Example: Water HCl NH_3, Alcohol, etc

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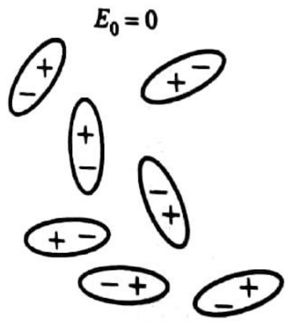

Under normal condition the cm of +ve and C.M of –ve charges do not coincide because of the asymmetric shape of the molecules
Each molecule has a spontaneous or permanent electric dipole moment

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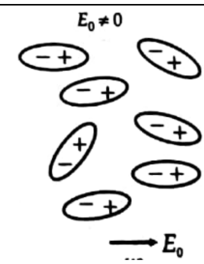

In the presence of an external electric field, the cm of +ve charges and cm of –ve charges are displaced.
The induced dipole moments of different molecules are added up giving a net dipole-moment to the dielectric in the direction of the field. This is the polarization of polar dielectrics.

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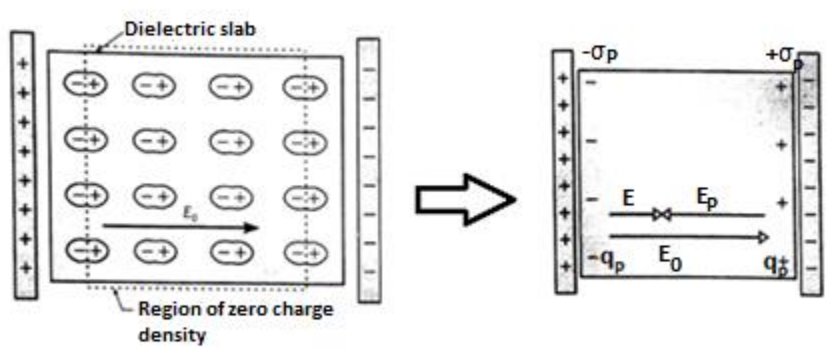
</table>

ILLUSTRATION: What is electric polarisation? Explain why the polarization of the dielectric reduces the electric field inside the dielectric. Hence define dielectric constant.

Answer:

- When a dielectric slab is placed in an electric field, then a net electric dipole moment is induced in the direction of the applied field. This is called electric polarisation.
- Due to polarisation bound charges ($-q_P$ and q_P) are developed at the edges of the slab.
- These charges induce a field $\vec{E}_P$ in the opposite of the applied field $\vec{E}_0$.



So the net field in the dielectric; $E = E_0 - E_P$

$\therefore\ E < E_0$

- The dielectric constant is defined as the ratio between the electric field in the vacuum and the electric field in the medium.

  i.e. $K = \dfrac{E_0}{E} = \dfrac{E_0}{E_0 - E_P}$

Polarisation vector or polarisation density:

- Polarisation vector $\vec{P}$ is defined as the total dipole moment induced per unit volume.

  i.e. $\vec{P} = \dfrac{\vec{p}_{net}}{V}$

- The direction of 'P' is the same as that of the external field.
- Its S.I. unit is Cm^{-2}.

Electric susceptibility:

- The ratio of the polarization to ε_0 times the electric field is called the electric susceptibility of the dielectric.

  i.e. $\chi_e = \dfrac{P}{\varepsilon_0 E}$

- It is a unitless and dimensionless quantity.
- **Physical Significance:**

  It describes the electrical behavior of dielectric. The dielectrics with constant χ_e are called linear dielectric.

For vacuum:

- ✓ Polarisation = 0
- ✓ Thus $\chi = 0$

The relation between polarization density and induced surface charge density:

Suppose a dielectric slab of surface area 'A' and thickness 'd' acquires a surface charge density $\pm\sigma_P$ due to polarization in the electric field and its two faces acquire charges $\pm Q_P$. Then

$$\sigma_P = \frac{Q_P}{A}$$

We can consider the whole dielectric slab as a large dipole having a dipole moment $Q_P d$. The dipole moment per unit volume or the polarization density will be:

$$P = \frac{\text{dipole moment of dielectric}}{\text{volume of dielectric}}$$

$$= \frac{Q_P d}{Ad} = \frac{Q_P}{A} = \sigma_P$$

Dielectric Strength:

The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called the dielectric strength of the material.

Unit: Vm^{-1}. But the more common practical unit is $(kV)(mm^{-1})$

Note: For air it is about $3\times10^6\,Vm^{-1}$

The relation between dielectric constant and electric susceptibility of the material:

$E = E_0 - E_P$ ....(1)

But $E_P = \frac{\sigma_P}{\varepsilon_0} = \frac{P}{\varepsilon_0}$

Thus from (1)

$$E = E_0 - \frac{P}{\varepsilon_0} = E_0 - \frac{\varepsilon_0 \chi_e E}{\varepsilon_0} = E_0 - \chi_e E$$

$$\Rightarrow E_0 = E + \chi_e E$$

$$\Rightarrow E_0 = E(1+\chi_e)$$

$$\Rightarrow \frac{E_0}{E} = 1+\chi_e$$

$$\Rightarrow K = 1+\chi_e$$

CAPACITOR AND CAPACITANCE:

It is an arrangement, which can store more electric charge or potential energy in a small space compared to an isolated conductor.

The capacitance of an isolated conductor: -

When a conductor is given some q charge, it spreads over its outer surface. Hence its potential increases.

Thus; $q \alpha V$

$\Rightarrow q = CV$ ……………………..(1)

Here constant C is called capacitance of the conductor

Definition of capacitance:

From equation (1) when V = 1 volt, then C = q

Thus capacitance of a conductor is the charge required to increase its potential by unity.

Units of capacitance:

- S.I Unit is Farad (F)
- If 1C of charge is required to increase the potential by 1 volt the capacitance of the conductor is said to be 1 Farad.

  Thus, 1 Farad = $\frac{1\text{coulomb}}{1\text{volt}}$

Note: -

- ✓ 1 F is a very large quantity. Generally smaller units $\mu F, nF, PF$ are used.

  Micro Farad $1\mu F = 10^{-6} F$

  Nano Farad $1nF = 10^{-9} F$

  Pico Farad $1PF = 10^{-12} F$

- ✓ Dimensional formula; $C = \frac{Q}{V} = \frac{Q}{W/Q} = \frac{Q^2}{W} = \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$

The capacitance of the spherical capacitor: -

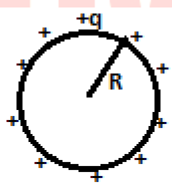
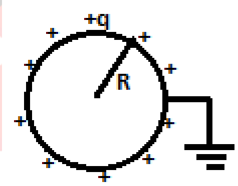
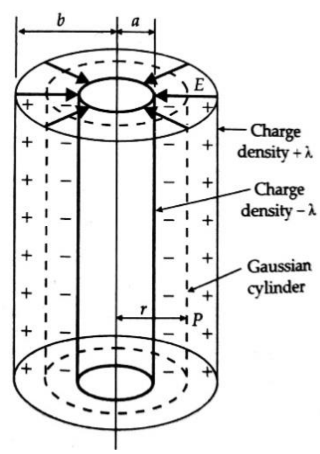
Isolated sphere	Isolated earthed sphere
$V = \frac{kq}{R}$, $C = \frac{q}{V} = 4\pi\varepsilon_0 R$ 	$V = 0$, $C = \frac{q}{0} = \infty$ 

ILLUSTRATION: A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of $3.5\mu C$. Determine the capacitance of the system.



Solution:

Applying Gauss's law. The electric field in between plates

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$\therefore$ P.D between plates are

$$V = -\int_b^a E.dr$$

$$= -\int_b^a \frac{\lambda}{2\pi\varepsilon_0 r}.dr$$

$$= -\int_b^a \frac{Q}{2\pi\varepsilon_0 \ell}\frac{dr}{r}$$

$$= -\frac{Q}{2\pi\varepsilon_0 \ell}[\log r]_b^a$$

$$= -\frac{Q}{2\pi\varepsilon_0 \ell}(\log a - \log b) = \frac{Q}{2\pi\varepsilon_0 \ell}(\log b - \log a)$$

$$\boxed{V = \frac{Q}{2\pi\varepsilon_0 \ell}\ln\left(\frac{b}{a}\right)} \quad ....(2)$$

Thus, $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi\varepsilon_0 \ell}\ln\left(\frac{b}{a}\right)}$

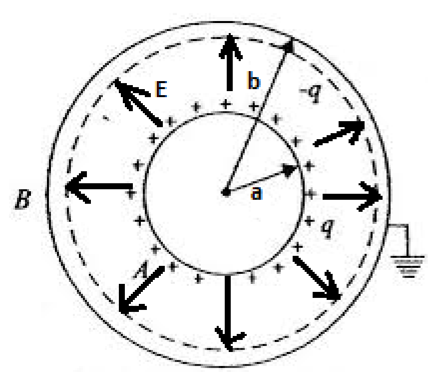
$$\boxed{C = \frac{2\pi\varepsilon_0 \ell}{\ln\left(\frac{b}{a}\right)}} \quad ....(3)$$

ILLUSTRATION: Find out the capacitance of the spherical concentric capacitor

(a) With earthed outer sphere

(b) With earthed inner sphere

Solution: -



(a) By using Gauss law it can be shown that;

$$E = \frac{q}{4\pi\varepsilon_0 r^2} \qquad (a < r < b)$$

$$\therefore V = V_A - V_B = -\int_b^a E dr = -\int_b^a \frac{q}{4\pi\varepsilon_0 r^2} dr$$

$$\Rightarrow V = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_b^a = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\Rightarrow C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)}$$

$$\Rightarrow C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

(b) Left to the students

ILLUSTRATION: Find the radius of an isolated spherical capacitor to achieve the capacitance of 1 microfarad.

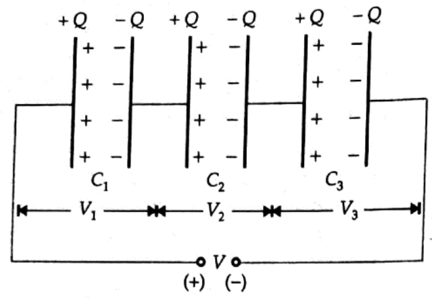
Solution:

$$C = 4\pi\varepsilon_0 r$$

$$\Rightarrow r = \frac{C}{4\pi\varepsilon_0} = 1\times10^{-6}\text{F}\times9\times10^9$$

$$= 9\times10^3\text{m} = 9\text{km}$$

COMBINATIONS OF CAPACITORS



In series:

Here the magnitude of the charge on all the plates same but the potential is distributed in the inverse ratio of the capacity.

$$\therefore V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \text{.................... (1)}$$

If we regard the combination as an effective capacitor with charge A and P.D 'V' then the effective capacitance of the combination.

$$C_s = \frac{Q}{V} \Rightarrow V = \frac{Q}{C_S} \text{................................(2)}$$

Comparing equations (1) and (2)

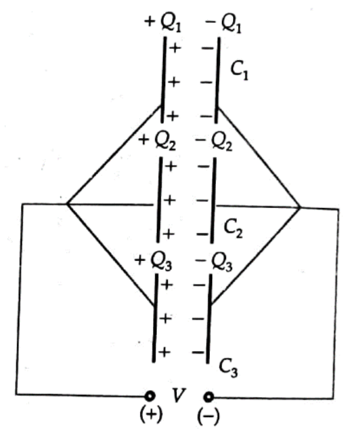
$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For n no of capacitors, $\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \text{.......} + \frac{1}{C_n}}$

For n identical capacitors, $\boxed{C_S = \frac{C}{n}}$

Capacitors in Parallel:



Here the P.D for all individual capacitors is the same but the total charge 'Q' is distributed in the ratio of their capacitance.

$$\therefore Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V$$

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V \text{ ....(1)}$$

If C_P is the equivalent capacitance of the parallel combination

then $Q = C_PV$ ....(2)

Comparing equation (1) and (2)

$$C_PV = C_1V + C_2V + C_3V$$

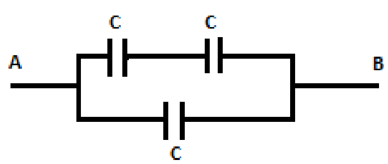
$$\Rightarrow \boxed{C_P = C_1 + C_2 + C_3}$$

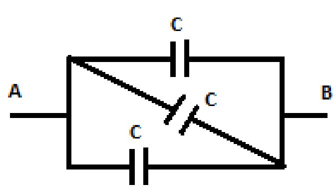
For n no of capacitors

$$\boxed{C_P = C_1 + C_2 + C_3 + ... + C_n}$$

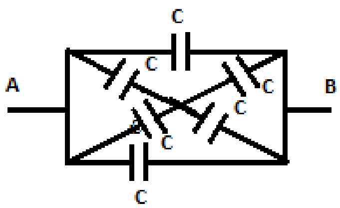
For n identical capacitors $\boxed{C_P = nC}$

ILLUSTRATION: Find the equivalent capacitance between points A and B in the following figures.

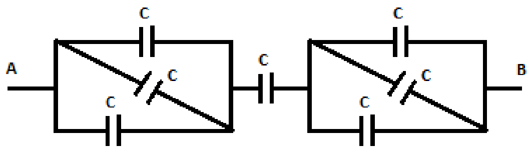
a) 

b) 

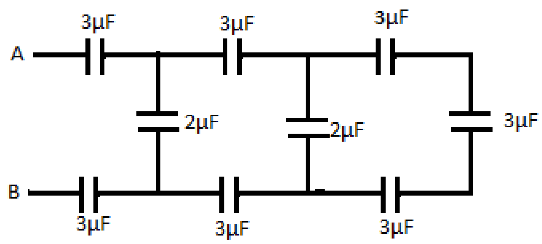
c)


C
C
C
A
B
C
C
C

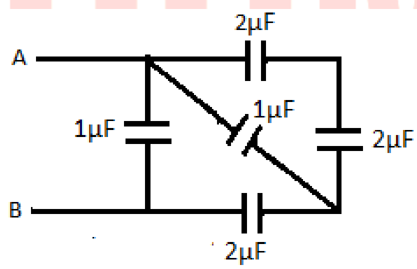
d)


C
A
C
C
C
C
C
B
C

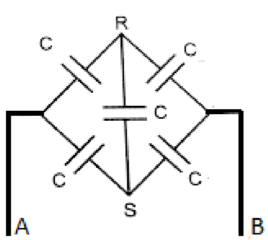
e)


3μF
3μF
3μF
A
2μF
2μF
3μF
B
3μF
3μF
3μF

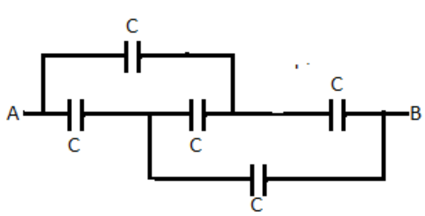
f)

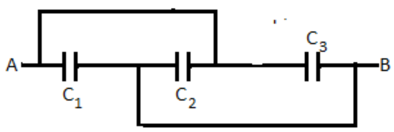

2μF
A
1μF
1μF
2μF
B
2μF

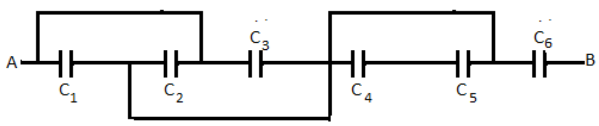
g)


R
C
C
C
C
C
S
A
B

h)


C
C
A
C
C
B
C

i) 

j) 

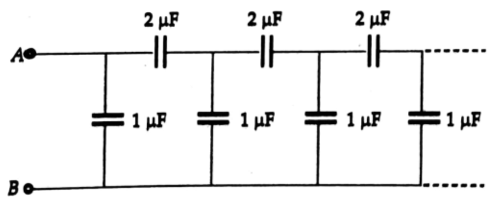
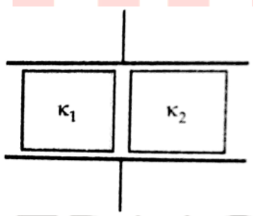
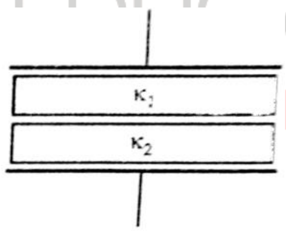
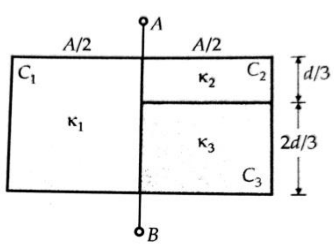
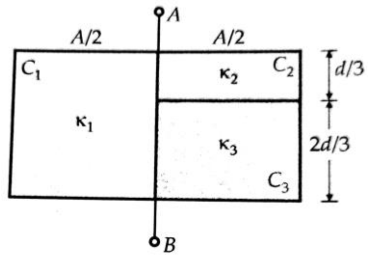
k) 

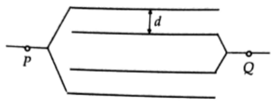
ILLUSTRATION: Find the equivalent capacitance between the terminals as shown in the figures.

a) 

b) 

c) 

d) 

e) 

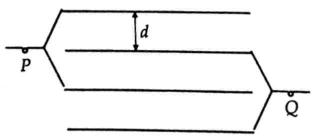
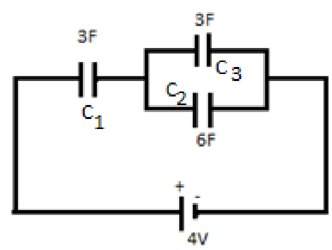
f) 

ILLUSTRATION: In the circuit diagram find out the charge of the capacitors C_1, C_2, and C_3.



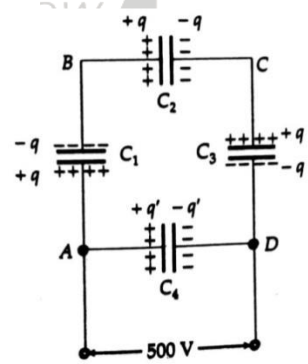
Solution:

$$C_{eq} = \frac{9}{4}F$$

$$\Rightarrow Q = C_{eq}V = \frac{9}{4} \times 4C = 9C$$

Thus charges on the capacitors, $C_1 = 9C, C_2 = 3C, C_3 = 6C$

ILLUSTRATION: Determine (a) the equivalent capacitance of the network (b) Charge on each capacitor of the network.



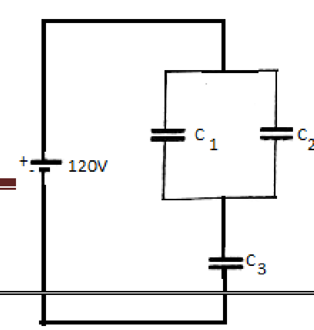
Solution: -

(a) $C_{eq} = 13.3\mu F$

(b) $Q = V \times C_{eq} = 500 \times 13.3\mu F$

ILLUSTRATION: In the figure $C_1 = 10\mu F, C_2 = 20\mu F, C_3 = 15\mu F$. Find out the P.D across the capacitor

Solution: -



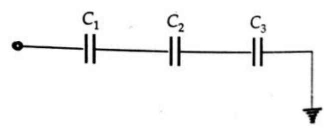
$V_1 + V_2 = 120$

$2x + x = 120$

$x = 40$

P.D across $C_3 = 40 \times 2 = 80$ volt

ILLUSTRATIONs-6: In the diagram find P.D between the plate C_2.



Solution: - $Q = C_{eq} \times V_{diff}$

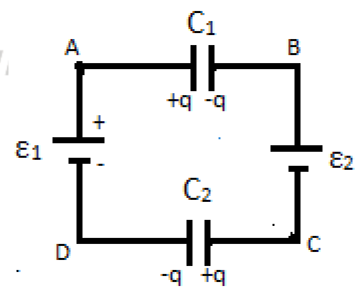
$$= \frac{20\mu F}{3} \times (90 - 0) = \frac{20}{3} \times 10^{-6} \times 90 = 600 \times 10^{-6} C$$

P.D across $C_2 = \dfrac{600 \times 10^{-6}}{30 \times 10^{-6}} = 20V$

ILLUSTRATIONs-7: In the given figure if $\varepsilon_1 > \varepsilon_2$ finding potential difference across $C_1 and C_2$.

Solution:

In DABCD Loop



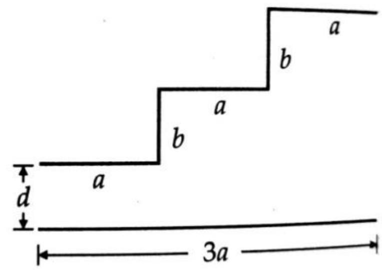
$$-\varepsilon_1 + \frac{q}{C_1} + \varepsilon_2 + \frac{q}{C_2} \Rightarrow q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \varepsilon_1 - \varepsilon_2$$

$$\Rightarrow q = \left(\frac{C_1 C_2}{C_1 + C_2}\right)(\varepsilon_1 - \varepsilon_2)$$

$$\therefore V_1 = \frac{q}{C_1} = \frac{C_2(\varepsilon_2 - \varepsilon_1)}{C_1 + C_2}$$

$$\therefore V_2 = \frac{q}{C_2} = \frac{C_1(\varepsilon_2 - \varepsilon_1)}{C_1 + C_2}$$

ILLUSTRATIONs-8: Find the capacitance of the capacitor shown in the figure?



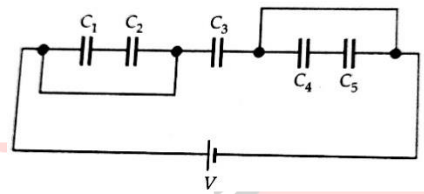
Solution:

$$C = \frac{\varepsilon_0 A/3}{d} + \frac{\varepsilon_0 A/3}{2d} + \frac{\varepsilon_0 A/3}{3d}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9}\right)$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{18+9+6}{54}\right) = \frac{11\varepsilon_0 A}{18d}$$

ILLUSTRATIONs-9: In the given figure. Find C_{eq} and q and energy stored.



Solution:

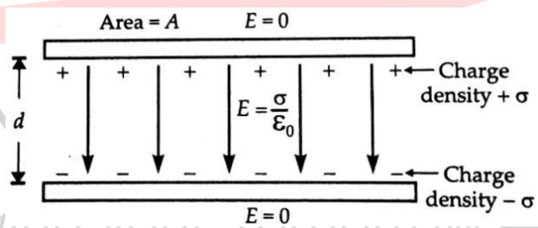
$$C_{eq} = C_3 = 2\mu F$$

As others are shorted

$$q = 5 \times 2 = 10\mu C$$

$$U = \frac{1}{2} \times 2\mu F \times (5V)^2 = 25\mu J$$

PARALLEL PLATE CAPACITOR:



A – Plate area

d – the distance between plates

The outer face of the 2nd plate is earthed because if the charged conductor is placed near an earthed conductor its capacity increases.

Case – I: When only air is between the Electric field in the inner region between plates :

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

But the $V = Ed = \frac{Q}{A\varepsilon_0}d$

$$\therefore C = \frac{Q}{V} = \frac{Q}{Qd/A\varepsilon_0} \Rightarrow C_{air} = \frac{A\varepsilon_0}{d}$$

Case – II: When there is Di-electric in between the plates

$C_{med} = \frac{A\varepsilon}{d} = \frac{A\varepsilon_0\varepsilon_r}{d} = \frac{A\varepsilon_0 k}{d}$

$\boxed{\frac{C_{med}}{C_{air}} = k}$

ILLUSTRATION: Justify that 1F too big unit in practice

OR

Calculate the area of the plates needed to have a capacitance of 1F for separation of 1cm.

Solution: Given that

C=1F

d = 1cm=10^2m

C = $\frac{A\varepsilon_0}{d}$

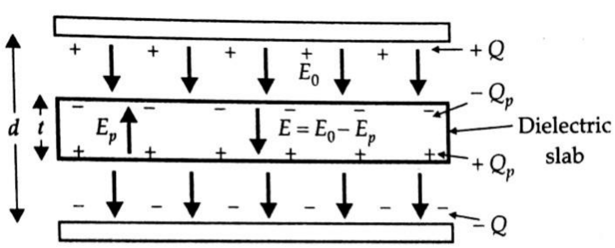
A= $\frac{Cd}{\varepsilon_0} = \frac{1\times10^{-2}}{8.8\times10^{-12}}$

A = $\frac{1}{8.85\times10^{10}} \simeq 10^9 m^2$

Which is a plate about 30km in length and breadth.

The capacitance of a parallel plate capacitor with a dielectric slab:

Let us consider a parallel plate capacitor of plates A and B, each of area A and distance of separation d between the plates. Let charge on plate A be +q and that of plate B be -q Let a dielectric slab of thickness t and dielectric constant K be introduced in the space between the plates (Let t < d). So the region between the plates with vacuum has a width (d - t).



Now the electric field at any point in the vacuum region between the plates has a magnitude

$E_0 = \frac{\sigma}{\varepsilon_0}$ .... (i) ($\sigma = \frac{q}{A}$ = surface charge density)

Now the dielectric slab will be polarized. So, a bound charge q_p is gathered at the surfaces of the slab.

The bound charge density has a magnitude $\sigma_P = \frac{q_P}{A}$

So an electric field is induced within the dielectric slab opposite to the field E_0.

This is given by, $E_P = \frac{\sigma_P}{\varepsilon_0}$ ....(ii)

So-net field inside the dielectric slab is $E = \frac{E_0}{K} = E_0 - E_P$...........(iii) (By definition $K = \frac{E_0}{E}$)

The potential difference between the plates i.e. potential of the capacitor is

$$V = E\,t + E_0(d - t) = \frac{E_0}{K}t + E_0(d - t) = E_0\left(d - t + \frac{t}{K}\right) = \left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

The capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{\frac{q}{A\varepsilon_0}(d - t + \frac{t}{K})} = \frac{A\varepsilon_0}{d - t + \frac{t}{K}}$$ ....(iv) (this is the expression for capacitance)

$$\Rightarrow C = \frac{\varepsilon_0 A/d}{1 - t/d + t/Kd} = \frac{C_0}{1 - t/d + t/Kd}$$

Where $C_0 = \frac{\varepsilon_0 A}{d}$ = capacitance of the capacitor when space between the plates is vacuum

If space between the plates is filled with dielectrics i.e. t = d, then eq. (iv)

gives, $C_m = \frac{KA\varepsilon_0}{d}$

$$\frac{C_m}{C_0} = \frac{\frac{KA\varepsilon_0}{d}}{\frac{A\varepsilon_0}{d}}$$

$$\frac{C_m}{C_0} = K$$

Since $K\varepsilon_r \geq 1$

$$\Rightarrow \frac{C_m}{C_0} \geq 1$$

Thus, the capacitance of the parallel plate capacitor increases due to the introduction of a dielectric slab between its plates (keeping the charge to be constant).

Points to ponder:

- ✓ Here electric field hence P.D decreases by a factor k (dielectric constant)

  $\therefore E = \frac{E_0}{K}$ and $V = \frac{V_0}{K}$ [Here E = Reduced Field = E_0 - E_P]

- ✓ Induced charge in the dielectric is given by

  $q_i = q[1 - \frac{1}{K}]$ [for metallic K = infinity, $q_i = q$]

- ✓ If a conducting slab ($K = \infty$) partially fills between plates, then

  $C = \frac{A\varepsilon_0}{d-t}$

- ✓ If the metal slab fills the space between the plates i.e. t = d then

  $C = \frac{A\varepsilon_0}{0} = \infty$

ILLUSTRATION: A dielectric slab (dielectric constant =k) is introduced between the plates of a charged air capacitor when the battery remains connected what happens to

i. **P.D between plates**
ii. **Electric field**
iii. **Capacitance**
iv. **Charge**
v. **Electrostatic potential energy**

Solution

i. V becomes constant
ii. $E = \frac{V}{d} \Rightarrow E \propto \frac{1}{d}$, since d=constant so E=constant
iii. $\frac{C_m}{C_0} = K \Rightarrow C_m = KC_0$, capacitance becomes K times
iv. $q = CV \Rightarrow q \propto C$, thus charges will increase to K times.
v. $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$, thus U will increase to K times.

ILLUSTRATION: A dielectric slab of dielectric constant k is introduced between the plates of a charged air capacitor when the battery is disconnected, what happens to its

i. **Electric charge**
ii. **P.D**
iii. **Capacitance**
iv. **Field**
v. **Electrostatic potential energy**

Solution:

i. q = constant

ii. $V = \frac{q}{C} \Rightarrow V \propto \frac{1}{C}$ therefore, V becomes 1/K times its initial value

iii. $C = \frac{KA\varepsilon_0}{d} \Rightarrow C \propto \frac{K}{d}$

iv. If q=constant, $E = \frac{E_0}{K} \Rightarrow E \propto \frac{1}{K}$

v. If q=constant $U = \frac{1}{2}\frac{q^2}{C} \Rightarrow U \propto \frac{1}{C}$ i.e. decreases by K times

ILLUSTRATION: A slab of material of dielectric constant 'K' has the same area as the plates of parallel plate capacitor, but has thickness 3d/4, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates.

Solution: When there is no dielectric

$$V_0 = E_0 d$$

But in the presence of dielectric

$$V = Ed = E_0(d-t) + Et$$

$$\Rightarrow V = E_0(\frac{d}{4}) + E(\frac{3d}{4}) = E_0(\frac{d}{4}) + \frac{E_0}{K}(\frac{3d}{4})$$

$$\Rightarrow V = V_0(\frac{K+3}{4K})$$

$$\therefore C = \frac{Q_0}{V} = \frac{4K}{K+3}(\frac{Q_0}{V_0}) = \frac{4K}{K+3}C_0$$

ENERGY STORED IN A CAPACITOR:

Work has to be done in charging the conductor against the force of repulsion by the already existing charge on it.

This work is stored as potential energy in the electric field of the conductor.

Suppose a conductor of capacity C is charged to a potential V and the charge at that instant be q. Therefore the potential of the conductor V= $\frac{q}{C}$

Now the work done in bringing small charge dq at this potential is

dW=V dq=$(\frac{q}{C})$ dq

therefore total work done in charging it from 0 to q is given by

$$W = \int_0^W dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

This work is stored as potential energy.

Thus $U = \frac{1}{2}\frac{Q^2}{C}$ ....(1)

Further by using Q = CV in eqn (1) we have

$$U = \frac{1}{2}\frac{(CV)^2}{C}$$

$$\boxed{U = \frac{1}{2} CV^2} \quad ....(2)$$

Again $C = \frac{Q}{V}$

From equation (2) $U = \frac{1}{2}\left(\frac{Q}{V}\right)V^2$

$\boxed{U = \frac{1}{2} QV}$ ....(3)

Again, for a parallel plate capacitor

$E = \frac{\sigma}{\varepsilon_0}$

$\sigma = \varepsilon_0 E$

and $Q = \sigma A = \varepsilon_0 E A$

Capacitance $C = \frac{A\varepsilon_0}{d}$

Putting the volume of Q and C in equation (1)

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\varepsilon_0 EA)^2}{\frac{A\varepsilon_0}{d}} = \frac{1}{2}\varepsilon_0 E^2 Ad$$

Thus, $\boxed{U = \frac{1}{2}\varepsilon_0 E^2 Ad}$ ....(4)

In general, if a conductor of capacity C is charged to a potential V by giving it a charge Q then.

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\varepsilon_0 E^2 Ad}$$

ILLUSTRATION: In which form energy is stored inside a capacitor?

Answer: In the form of the electric field

ILLUSTRATION: Where is the energy stored in a parallel plate capacitor?

Answer: In between the plates (space)

ILLUSTRATION: A dielectric is introduced between plates of parallel plate capacitors. Has it any effect on the force?

Answer: No. Because induced charges on opposite faces of dielectric are equal and opposite and the electric field in the vicinity of the plates depends only on the net charge.

Note

The total energy stored in a series combination or parallel combination of capacitors is equal to the sum of the energies stored in the individual capacitors.

i.e. $U = U_1 + U_2 + U_3 + \ldots$

ILLUSTRATION: An unknown capacitor is connected to the battery. Show that half of its energy supplied by the battery is lost as heat while charging the capacitor.

Solution: The work done by the battery in charging a capacitor

$$W = QV$$

But energy stored in the capacitor

$$U = \frac{1}{2}QV$$

The remaining energy = $QV - \frac{1}{2}QV = \frac{1}{2}QV$ is lost as heat radiation.

Thus, $\boxed{W_{ExternalSource} = 2U}$

ILLUSTRATION: Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}QE$ .where Q is the charge in the capacitor and E is the magnitude of the electric field between the plates.

Solution: Force between the plates of the capacitor

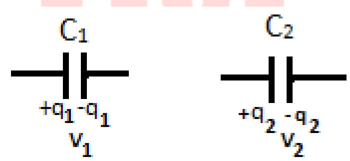
$$F = -\frac{du}{dx} = -\frac{d}{dx}\left(\frac{\varepsilon_0 E^2 Ad}{2}\right) = -\frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}(\varepsilon_0 EA)E$$

$\Rightarrow F = \frac{1}{2}QE$ (as $Q = \varepsilon_0 EA$)

Thus, $\boxed{F = \frac{1}{2}QE = \frac{1}{2}\varepsilon_0 E^2 A = \frac{Q^2}{2\varepsilon_0 EA}}$

ILLUSTRATION: When two charged conductors having different capacities and different potentials are joined together, show that there is always a loss of energy.

Solution: Sharing of charges

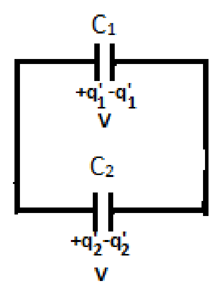


If two capacitors C_1 and C_2 at potential differences V_1 and V_2 respectively.

Charges on capacitors before sharing are $q_1 = C_1 V_1$ and $q_2 = C_2 V_2$

Now they are connected in parallel, then they share charge till both attain equal potential V.

Charges on capacitors after sharing are $q'_1 = C_1 V$ and $q'_2 = C_2 V$ law of conservation of charge $q_1 + q_2 = q'_1 + q'_2$



$$\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

This is the expression of common potential.

Loss of energy during the sharing of charge

$\Delta U = U_i - U_f =$ total potential energy before sharing - total potential energy after sharing

$$= \frac{1}{2}\left[\left(C_1V_1^2 + C_2V_2^2\right) - \left(C_1 + C_2\right)V^2\right]$$

$$=\frac{1}{2}\left[\left(C_1V_1^2+C_2V_2^2\right)-\left(C_1+C_2\right)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2\right]$$

$$=\frac{1}{2}\left[\frac{C_1^2V_1^2+C_1C_2V_2^2+C_2C_1V_1^2+C_2^2V_2^2-C_1^2V_1^2-C_2^2V_2^2-2C_1C_2V_1V_2}{C_1+C_2}\right]$$

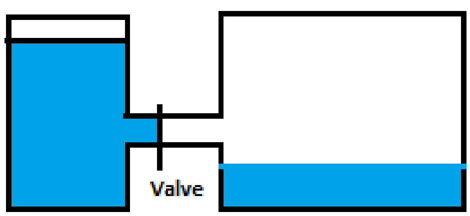
$$=\frac{1}{2}\left[\frac{C_1C_2V_2^2+C_2C_1V_1^2-2C_1C_2V_1V_2}{C_1+C_2}\right]$$

$$=\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)\left(V_2^2+V_1^2-2V_1V_2\right)$$

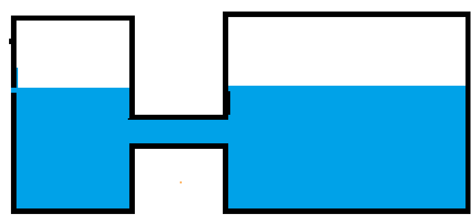
$$=\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)\left(V_1-V_2\right)^2$$

Now as C_1C_2 and $\left(V_1-V_2\right)^2$ are always positive, $U_i > U_f$ i.e. there is a decrease in energy. Hence, energy is always lost in the redistribution of charge.

Note: The redistribution of charge is analogous to the following example



(Before opening the valve)



(After opening the valve)

When the valve is open, the level in both the vessels becomes equal but the volume of liquid in the right vessel is more than the left vessel.

ILLUSTRATION:

a) A 900 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor?

b) The capacitor is disconnected from the battery and connected to another uncharged 900 pF capacitor. What is the electrostatic energy stored now?

c) Where has the remained energy gone?

Solution:

a) $U_i=(1/2)CV^2$

$=\frac{1}{2}\left(900x10^{-12}\right)(100)^2 = 4.5x10^{-6}\,J$

b) After connection, the common potential will be

$$V_{common} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{CV+0}{900+900} = \frac{1}{2}x10^2\,volts$$

Now the final energy stored $\mathrm{U_f} = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) = 2.25x10^{-6}\,J$

c) Loss of energy = $\Delta\mathrm{U}=\mathrm{U_i}\text{-}\mathrm{U_f} = (2.25x10^{-6} - 0.5x10^{-6}) = 1.75x10^{-6}\,J$

This energy is lost in form of heat and electromagnetic radiation.

ILLUSTRATION: A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supplier and is connected to another 600 pF capacitor. How much electrostatic energy is lost in the process?

Solution: -C_1 = C_2 = 600 pF, $V_1 = 200V, V_2 = 0$

Putting the formula for energy loss $= \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2 = 6x10^{-6}\,J$

ILLUSTRATION: Two parallel plate condensers A and B having capacitances of $1\mu F$ and $5\mu F$ are charged separately to the same potential of 100 V. Now the positive plate of A is connected to the negative plate of B. And the negative plate of A is connected to the positive plate of B. Find the final charge on each condenser and total loss of electric energy in the condenser?

Solution: Before connection;

$Q_1 = C_1V = 100x10^{-6}C$

$Q_2 = C_2V = 500x10^{-6}C$

The initial energy of the capacitor = $\mathrm{U_i} = \frac{1}{2}\left(C_1V_1^2 + C_2V_2^2\right) = 0.03J$

After connection;

$$V_{Commom} = \frac{500\times10^{-6} - 100\times10^{-6}}{(1+5)\times10^{-6}} = \frac{200}{3}V$$

Therefore, the final energy of capacitance = $\frac{1}{2}(C_1+C_2)V_{common}^2 = \frac{0.04}{3}\,J$

Therefore, loss in energy = $0.03-\frac{0.04}{3}=\frac{0.05}{3}J$