(CHAPTER -13)

NUCLEI

From Rutherford's model and Scattering experiment,

(a) The entire positive charge and most of the mass of the atom is concentrated in a small volume called Nucleus and electrons revolve around it.

(b) The nuclear size is found to be of the order of $10^{-14}\,\text{m}$ and the diameter of an atom is of the order of $10^{-10}\,\text{m}$

(c) Protons and neutrons are the main building blocks of the nuclei of all atoms

(d) $_{Z}X^{A} \Rightarrow$ Atomic number = z, No of protons = Z, No. of neutrons = $(A - Z)$, A = Mass number.

Nucleons:-

The Protons and neutrons which are present in the nuclei of atoms are called Nucleons.

Proton :- (a) Charge = $+1.6\times10^{-19}\,\text{C}$ (b) Rest mass = $1.672\times10^{-27}\,\text{kg}$

(c) Mass of proton = $1836\times\text{mass of electron}$ (d) Spin angular momentum = ½

Neutron:-

(a) Charge = Neutral (b) Rest mass = $1.6749\times10^{-27}\,\text{kg}$ (c) Magnetic moment = very small.

Mass Number:-

The total number of protons and neutrons present in a nucleus. It is denoted by A.

$A = N + Z$

Nuclear mass:-

The total mass of protons and neutrons present in a nucleus.

Isotopes:-

(The atoms of an element having same atomic number but a different mass number)

Example:-

Protium $\left({}_1H^1\right)$, Deuterium $\left({}_1H^2\right)$ and Tritium $\left({}_1H^3\right)$ are isotopes of hydrogen

$\left({}_6C^{12},{}_6C^{13}\right)$ and $\left({}_3Li^6,{}_3Li^7\right)$ are isotopes of carbon and lithium respectively.

Isobars:-

(The atoms having different atomic number but having the same mass number

Example:-

(a) ${}_1H^3 \ \&_2 H^3$ (b) ${}_{17}C^{37},{}_{16}S^{37}$ (c) ${}_{20}Ca^{40},{}_{18}Ar^{40}$

Isotones:-

Nuclides having same number of neutrons

Example:-

(a) ${}_{17}Cl^{37},{}_{19}K^{39}$ (b) ${}_{80}Hg^{198}$ and ${}_{79}Pu^{197}$

Atomic mass unit:-

1/12 th of the actual mass of carbon – 12 atoms is called a.m.u.

$1\,\text{a.m.u} = 1.66 \times 10^{-27}\,\text{kg}$

Nuclear Size:- If R = Radius of a nucleus having mass number A, then

$$\frac{4}{3}\pi R^3 \propto A$$

$$\Rightarrow R \propto A^{1/3}$$

$\Rightarrow R = R_0 A^{1/3}$ R_0 is constant = $1.2 \times 10^{-15}\,\text{m} = 1.2\,\text{fm}$

Nuclear Density:-

Different nuclei are like drops of liquid, of different sizes but same density,

If m = Average mass of a nucleon

A = Mass number, Mass of nucleus = mA

The volume of nucleus = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3 = \frac{4}{3}\pi R_0^3 A$

$\Rightarrow$ Nuclear density = $\frac{\text{mass of nuclus}}{\text{volume of nucleus}}$

$\Rightarrow \rho_{nu} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$

$\Rightarrow$ Nuclear density is independent of mass number A or size of the nucleus.

Putting $m = 1.67\times10^{-27}\,\text{kg}, \quad R_0 = 1.2\times10^{-15}\,\text{m}$

We get

$\Rightarrow \rho_{nu} = 2.3\ x\ 10^{17}\ kg/m^3$

The density of nuclei of all elements is the same

Example - 01:- What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 3.6 fermi.

Solution:- $\frac{R_{Fe}}{R_{Al}} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3} \quad \Rightarrow R_{Fe} = \frac{5}{3}\times R_{Al} = \frac{5}{3}\times 3.6 = 6\,\text{fermi}$

Example –0 2:-

Two nuclei have mass numbers in the ratio 27:125. What is the ratio of their nuclear radii?

Solution:- $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$

Example – 03:- Assuming the nuclei to be spherical, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2?

Solution:- $\frac{S_1}{S_2}=\left(\frac{R_1}{R_2}\right)^2=\left[\left(\frac{A_1}{A_2}\right)^{1/3}\right]^2=\left(\frac{A_1}{A_2}\right)^{2/3}$

Example – 04:- Boron has two stable isotopes, ${}_5B^{10}$ and ${}_5B^{11}$. Their respective masses are 10.01294 am and 11.00931 am and the atomic weight of boron is 10.811 amu. Find the abundances of ${}_5B^{10}$ and ${}_5B^{11}$.

Solution:-

Let natural boron contains $x\%$ ${}_5B^{10}$ isotope and $(100-x\%)$ ${}_5B^{11}$ isotope, then

The atomic mass of natural Borons = Weighted average of the masses of two isotopes

$$\Rightarrow 10.811=\frac{x\times10.01294+(100-x)\times11.00931}{100}$$

$\Rightarrow 1081.1=-0.99637x+1100.931$ $\qquad\Rightarrow x=\frac{19.831}{0.99637}=19.9$

$\therefore$ The relative abundance of ${}_5B^{10}$ isotope = 19.9 %

$\therefore$ The relative abundance of ${}_5B^{11}$ isotope = 80.1 %

Example – 05:- The nuclear radius of ${}_8O^{16}$ is 3×10^{-15}m. Find the density of nuclear matter

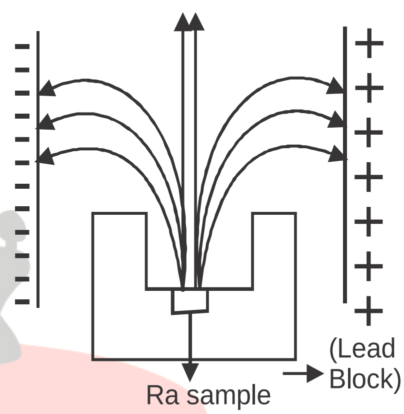
Solution:- Here $R=3\times10^{-15}$m $\qquad$ Nuclear mass = 16 amu = $16\times1.66\times10^{-27}$kg

$$\Rightarrow \rho_{nu}=\frac{Nuclear\ mass}{Nuclear\ volume}=\frac{16\times1.66\times10^{-27}}{\frac{4}{3}\pi(3\times10^{-15})^3}=2.359\times10^{17}\,kg/m^3$$

Radioactivity:-

The phenomenon of spontaneous disintegration of the nucleus of an atom with the emission of radiations like α particles, $\beta-$ particles or $\gamma-$ rays is called radioactivity. The phenomenon of radioactivity was discovered by French scientist Henry Becquerel in 1896. The substances which spontaneously emit radiations are called radioactive substances.

Example:- Uranium, Polonium, Radium, thorium, actinium etc.



Electrical nature of Radioactive Radiation:-

In a thick lead block with a small hole in its centre, we place a radioactive material. The ejected radiations are subjected to an electric field by two oppositely charged plates.

Observations:- The beam is divided into three parts.

(a) $\alpha-$decay in which ${}_2He^4$ is emitted (moves towards –ve plate)

(b) $\beta-$ decay in which electrons or positions are emitted (moves towards +ve plate)

(c) $\gamma-$decay in which high energy photons are emitted (moves straight)

Properties of $\alpha-$ray:-

(a) +vely charged particles identified as ${}_2He^4$ (charge = 2e)

(b) Deflected by electric and magnetic field

(c) They affect photographic plate

(d) Ionising power is high

(e) Small penetrating power

(f) They produce fluorescence in zinc solphide etc.

(g) ${}_2X^A \rightarrow {}_{z-2}Y^{A-4} + {}_2He^4\,(\alpha-\text{particle})$

Properties of $\beta-$ rays:-

(a) This consists of electrons or positrons

(b) Deflected by an electric and magnetic field

(c) Can affect photographic plate

(d) Less ionizing power than $\alpha-$the ray

(e) Large penetrating power that $\alpha-$ray

(f) Less effect on ZnS plate than $\alpha-$the ray

(g) ${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0}\left(\beta^{-}\text{ decay}\right)$, ${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{+1}e^{0}\left(\beta^{+}\text{ decay}\right)$

Properties of $\gamma-$ rays :-

(a) These are e.m waves consisting of photons

(b) Since photons have rest mass zero and charge zero so they are not deflected by electric or magnetic fields

(c) No ionizing power

(d) High penetrating power than $\alpha\,\&\,\beta-$ rays

(e) They travel with the velocity of light

(f) Less effect on photographic plate

(g) ${}_{Z}X^{A} \rightarrow {}_{Z}X^{A} + h\gamma(\gamma - rays)$

(h) $\gamma \rightarrow {}_{-1}e^{0} + {}_{+1}e^{0}$

Displacement Laws:- Soddy and Fajon in 1913 states that law as

(a) When a radioactive nucleus emits $\alpha-$particle, its atomic number decreases by 2 and the mass number decreases by 4.

(b) When a radioactive nucleus emits β^{-} particle, its atomic number increases by 1 and no change in mass number. For β^{+} decay, atomic number decreases by 1 and no change in mass number)

(c) There is no change in atomic number and mass number for the emission of a $\gamma-$ particle.

Alpha Decay ($\alpha-$decay):-

In $\alpha-$decay, the mass number of product nucleus (daughter nucleus) is 4 less than that of the decaying nucleus (parent nucleus) while the atomic number decrease by 2.

Example:-

${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}H^{4}$

(a) ${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4} (\alpha - decay) + Q$

(b) ${}_{84}Po^{208} \rightarrow {}_{82}Pb^{204} + {}_{2}He^{4} + Q$

Mechanism:-

$\alpha -$ decay is possible spontaneously if the mass of the initial nucleus is more than the total mass of the decay products. The difference in mass appears as the kinetic energy of the products.

Q Value disintegration

Energy:- The difference between the initial mass-energy and the total mass-energy of the decay products

For $\alpha -$ decay $Q = [m_x - m_y - m_{He}]C^2$

$\Rightarrow$ Q is called K.E gained in the process or K.E of products

$\Rightarrow$ Q > 0 for exothermic process

$\Rightarrow K_{\alpha(He)} \text{Particle} = \left(\frac{A-4}{A}\right)Q$ and V_α or $V_{He} = \sqrt{\frac{2(A-4)Q}{Am_{He}}}$

Example:- Find the velocity of $\alpha -$the particle in the disintegration of ${}_{86}Rn^{222}$ if Q value = 5.587 MeV.

Solution:- $K_{He} = \left(\frac{222-4}{222}\right) \times 5.587 = 5.486 \text{Mev} = 5.486 \times 1.6 \times 10^{-19} \text{J}$

$m_{He} = 4 \times 1.66 \times 10^{-27} \text{Kg}$

$v_{He} = \sqrt{\frac{2 \times 5.486 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27}}} \text{m/s} = 1.62 \times 10^{7} \text{m/s}$

Beta Decay:-

The process of spontaneous emission of an electron $\left({}_{-1}e^0\right)$ or a positron $\left({}_{+1}e^0\right)$ from a nucleus is called $\beta-$decay. In both β^- decay $\left({}_{-1}e^0\right)$ or β^+ decay $\left({}_{+1}e^0\right)$, the mass number remains constant.

In β^- decay atomic number (Z) increases by 1 and in β^+ decay, the atomic number (Z) decreases by one.

β^- decay:- ${}_Z X^A \rightarrow {}_{Z+1} Y^A + {}_{-1} e^0 + \bar{\gamma}$ (antineutrino)

Example:- ${}_{15}P^{32} \rightarrow {}_{16} S^{32} + {}_{-1}e^0 + \bar{\gamma}$

$(or)\ {}_{15}P^{32} \rightarrow {}_{16} S^{32} + \beta^{-1} + \bar{\gamma}$ $\quad (T_{1/2} = 2.6\,\text{days})$

Mechanism:-

$n \rightarrow P + {}_{-1} e^0 + \bar{\gamma}$ (antineutrino), if the unstable nucleus has excess neutron than needed for stability, it undergoes β^- decay by converting its neutron into a proton, an electron and an antineutrino.

[Neutrons and antineutrons are massless and charge less]

β^+ Decay:- ${}_Z X^A \rightarrow {}_{Z-1} Y^A + {}_{+1} e^0 + \gamma$ (Neutrino)

Example:- ${}_{11}Na^{22} \rightarrow {}_{10} Ne^{22} + {}_{+1} e^0 + \gamma$ or ${}_{11}Na^{22} \rightarrow {}_{10} Ne^{22} + \beta^+ + \gamma$ $(T_{1/2} = 2.6\ \text{yrs})$

Mechanism:- $P \rightarrow n + {}_{+1} e^0 + \gamma\ (Neutrino)$

If the unstable nucleus has excess proton than needed for stability, it undergoes β^+ decay by converting its proton into a neutron, positron and a neutrino.

Gamma Decay:- The process of emission of γ ray photon during the radioactive disintegration of the nucleus.

$${}_Z X^A \rightarrow {}_Z X^A + \gamma$$

(Excited State) (Ground state)

When a nucleus is in the excited state, spontaneously decays to its ground state and a photon is emitted with energy equal to the difference in two energy levels of the nucleus. This is called γ decay. γ-decay is emitted when α or β decay results in a daughter nucleus in an excited state.

Example:- ${}_{27}CO^{60} \xrightarrow{\beta^-(\text{excited})} {}_{28}Ni^{60} \xrightarrow{\gamma(\text{decay})(E\gamma=1.17\text{mev})} {}_{28}Ni^{60} \xrightarrow{\gamma-\text{de}(E\gamma=1.33\text{mev})} {}_{28}Ni^{60}$ (Stable)

Question:- Write nuclear reaction equations for

(a) α the decay of ${}_{88}Ra^{226}$ and ${}_{94}PO^{242}$

(b) β^- decay of ${}_{15}P^{32}$ and ${}_{83}Bi^{210}$

(c) β^+ the decay of ${}_{6}C^{11}$ and ${}_{43}TC^{97}$

(d) Electron culture of ${}_{54}Xe^{120}$

Solution:-

(a) ${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_{2}He^{4},\ {}_{94}PO^{242} \rightarrow {}_{92}U^{238} + {}_{2}He^{4}$

(b) ${}_{15}P^{32} \rightarrow {}_{16}S^{32} + {}_{-1}e^{0} + \bar{\gamma},\ {}_{83}Bi^{210} \rightarrow {}_{84}PO^{210} + {}_{-1}e^{0} + \bar{\gamma}$

(c) ${}_{6}C^{11} \rightarrow {}_{5}B^{11} + {}_{+1}e^{0} + \gamma,\ {}_{43}Tc^{97} \rightarrow {}_{42}Mo^{97} + {}_{+1}e^{0} + \gamma$

(d) ${}_{54}Xe^{120} + {}_{-1}e^{0} \rightarrow {}_{53}I^{120} + \gamma$

Question:- Give the following atomic mass

${}_{92}U^{238} = 238.05079U,\ {}_{2}He^{4} = 4.00260U,\ {}_{90}Th^{234} = 234.04363U,\ {}_{1}H^{1} = 1.00783U,\ {}_{91}Pa^{237} = 237.05121U$

(a) Calculate the energy released during $\alpha-$the decay of ${}_{92}U^{238}$

(b) Show that ${}_{92}U^{238}$ can't spontaneously emit a proton.

Solution:-

(a) ${}_{92}U^{238} \rightarrow {}_{92}Th^{234} + {}_{2}He^{4}$

$$Q = (M_U - M_{Th} - M_{He})c^2$$

putting the atomic masses

Q= 238.05079-(234.04363+4.00260)

$$Q = (0.004560)\,c^2 = (0.00456\,U) \times 931.\,5\frac{Mev}{U} = 4.25\,mev$$

(b) If ${}_{92}U^{238}$ spontaneously emits a proton, ${}_{92}U^{238} \rightarrow {}_{91}Pa^{237} + {}_{1}H^{1}$

$$\Rightarrow Q = (Mu - M_{Pa} - M_H)c^2$$

Putting the atomic masses,

$$\Rightarrow Q = [-0.00825u]c^2$$

$$= 0.00825 \times 931.5\,mev$$

$$= -7.68\,\text{mev}$$

Since Q < 0 it cannot proceed spontaneously. We have to supply energy of 7.68 MeV to ${}_{92}U^{238}$ the nucleus to emit a proton.

Radioactive Decay Law:-

The number of nuclei disintegrating per second of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that instant.

If No = No of radioactive nuclei present at time t = 0 (initially)

N = No. of radioactive nuclei present at any instant t.

dN = No. of radioactive nuclei disintegrates in small time dt.

Now $-\frac{dN}{dt} \propto N \Rightarrow -\frac{dN}{dt} = \lambda N$ .... (1)

Where λ is a proportionality constant called decay constant or disintegration constant.

-ve sign shows that the number of un-decayed nuclei N decreases with time

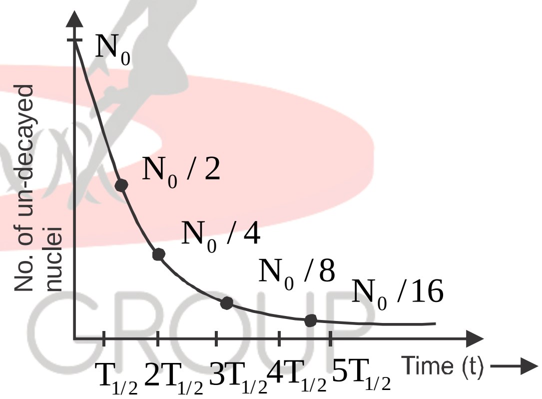
Now $\frac{dN}{N} = -\lambda dt$

Integrating $\int_{M_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$

$\ln N - \ln N_0 = -\lambda(t-0)$

$\Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$

$\Rightarrow N(t) = N_0 e^{-\lambda t}$ .... (2)



Equation (2) is a radioactive decay law. It shows the number of active nuclei in a radioactive sample decreases exponentially with time. The graph between the number (N) of undecayed nuclei and time (t) is shown below.

<u>Decay Constant & Disintegration constant (λ) :-</u>

If $t = \frac{1}{\lambda} \Rightarrow N = \frac{N_0}{e} = 36.8\%$ of N_0. The reciprocal of time during which the number of active nuclei in a radioactive sample reduces to 1/e times of its initial value is called decay constant.

λ depends on the nature of radioactive substance...

Half-Life $\left(T_{1/2}\right)$**:-**

The time interval in which one-half of the radioactive nuclei originally present in radioactive sample disintegrates is called the half-life of the radioactive substance. It is denoted by $T_{1/2}$.

At $t = T_{1/2} \Rightarrow N = \frac{N_0}{2}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda} T_{1/2} \Rightarrow e^{-\lambda T_{1/2}} = \frac{1}{2} \Rightarrow \lambda T_{1/2} = 2$$

$$T_{1/2} = \frac{2.303 \log 2}{\lambda} \Rightarrow T_{1/2} = \frac{0.693}{\lambda} \qquad T_{1/2} \propto \frac{1}{\lambda}$$

Number of radioactive nuclei left un-decayed after n-half lives = $N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \left[N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}\right]$

Mean Life (T_{av} **or** τ**):-**

The ratio of the total age of all the nuclei to the total number of nuclei present in the sample is called average life.

$$\Rightarrow \tau \text{ or } T_{av} = \frac{1}{N_0} \int_0^{N_0} t dN$$

But $N = N_0 e^{-\lambda t}$

$$\Rightarrow dN = -\lambda N_0 e^{-\lambda t}.dt$$

If $N = N_0 \Rightarrow t = 0 \; \& \; N = 0 \Rightarrow t = \infty$

$$\Rightarrow \tau = \frac{1}{N_0} \int_\infty^0 -\lambda N_0 e^{-\lambda t} t dt = \frac{1}{N_0} \int_0^\infty \lambda N_0 e^{-\lambda t}.tdt$$

$$\Rightarrow \tau \text{ or } T_{av} = \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}$$

$$\Rightarrow T_{av} = \frac{1}{\lambda} \; again \; T_{1/2} = \frac{0.693}{\lambda} = 0.693\tau$$

The relation between half-life and average life:-

$T_{1/2} = 0.693\tau$

The activity of Radioactive substance:-

The number of radioactive disintegrations per second in the sample is called activity. Also called decay rate.

$$R = \frac{-dN}{dt}$$

Putting $N = N_0 e^{-\lambda t} \Rightarrow R = \lambda N_0 . e^{-\lambda t}$

$\Rightarrow R = R_0 e^{-\lambda t}$

Where $R_0 = \lambda N_0$ is the decay rate at t = 0

Also $R = \lambda N$

R also decreases exponentially with time.

Units of radioactivity:-

S.I unit:- Becquerel (Bq) = 1 decay per second

Other units:- Curie (Cu), $1Ci = 3.7 \times 10^{10} Bq$

Ruther ford (rd) = $\mathbf{1\ ci = 3.7 \times 10^4 rd}$

Example:- Tritium has a half-life of 12.5 yr undergoing $\beta-$ decay. What fraction of sample of pure tritium will remain undecayed after 25yr?

Solution:- $n = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \Rightarrow \frac{N}{No} = \left(\frac{1}{2}\right)^{\frac{25}{12.5}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow N = No/4 \Rightarrow$ ¼ that of the sample of initial pure tritium remains undecayed.

Example:- The half-life of ${}_{92}U^{238}$ undergoing α decay is $4.5\times10^{9}\,yr$. What is the activity of 1g sample of ${}_{92}U^{238}$?

Solution:- $T_{1/2} = 4.5\times10^{9}\,yr = 1.42\times10^{17}\,sec$

No of atoms in 1gm of uranium = $\frac{6.023\times10^{23}}{238}\,atoms$

$$R = \lambda N = \frac{0.693}{T_{1/2}}.N$$

$$\Rightarrow R = \frac{0.693}{1.42\times10^{17}}\times\frac{6.023\times10^{23}}{238}\,sec^{-1} = 1.235\times10^{4}\,Bq$$

Example:- The half-life of the radioactive sample is 20 sec. calculate

(a) The decay constant and (b) Time is taken for the sample to decay by (7/8)th of initial value?

Solution:-

(a) $T_{1/2} = 20\,sec, \lambda = \frac{0.693}{20} = 0.0346\,sec^{-1}$

(b) Fraction un decayed = $1 - 7/8 = \frac{1}{8}$ of original value

$$\frac{N}{No} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{t}{20}} \Rightarrow \left(\frac{1}{2}\right)^{3} \Rightarrow \left(\frac{1}{2}\right)^{\frac{t}{20}} \Rightarrow t = 60\,sec$$

Numerical:-

A radioactive sample has a half-life of 5 years. How long will it take the activity to reduce to 3.125%

Solution:-

$T_{1/2} = 5yrs$, activity is proportional to the number of radioactivity atoms

$$\Rightarrow \frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Now $\frac{R}{R_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$

$$\Rightarrow \left(\frac{1}{2}\right)^{\frac{t}{5}} = \left(\frac{1}{2}\right)^5 \Rightarrow t = 25\,\text{years}$$

Numerical:-

The half-life of a radioactive substance is 50 sec. Find (a) the decay constant and (b) time was taken of the sample to decay by (3/4)th of its initial value?

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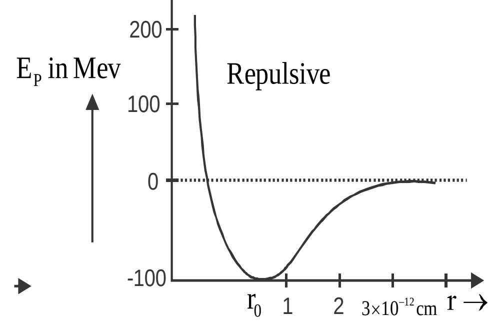
Nuclear Force:- It is a strong attractive force that binds the nucleons together inside the nucleus.



Properties:-

(a) Strong interaction (b) Short-range fore

(c) Charge independent (d) Saturation effect

(e) Spin-dependent (f) Non central force

Variation with distance:- For $r < r_0$ = strong repulsive nuclear force

For $r > r_0 \Rightarrow E_P$ decreases to zero with the increase in r.

$\Rightarrow$ at $r = r_0 \Rightarrow$ a maximum attractive force

$\Rightarrow$ For $r = 4\text{fm} \Rightarrow$ nuclear force is zero (short round)

Einstein's Mass-Energy Equivalence:-

Einstein's mass-energy equivalence relation is

$E = mc^2$, where $m = \text{mass}, c = \text{speed of light}$

Einstein's showed that mass is another form of energy

This relation shows that the energy content of an object is equal to its mass times the source of the speed of light.

Example:- Find an energy equivalent of one a.m.u?

Solution:- $\Rightarrow m = 1\text{amu} = 1.66\times10^{-27}\text{kg},\ C = 3\times10^{8}\text{m/s}$

$$\Rightarrow E = mc^2 = 1.66\times10^{-27}\times\left(3\times10^{8}\right)^2 \text{J} \qquad = \frac{1.66\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}}\text{ev} = 931.5\text{mev}$$

Example:- Find the energy equivalent of 1g of substance.

Solution:- $E = 10^{-3} \times (3\times10^{8})^{2}\,J \qquad = 10^{-3}\times 9\times10^{16} = 9\times10^{13}\,J$

Mass Defect:-

It has been experimentally observed that the rest mass of the nucleus of a stable atom is always less than the mass of the constituent nucleons in the free state.

The difference between the actual mass of the nucleus and the sum of masses of the constituent nucleons is called the mass defect.

If M = mass of the nucleus, then mass defect of the nucleus of an atom is

$$\Delta m = \left[Zm_P + (A-Z)m_N\right] - M$$

$m_P =$ mass of protons $\qquad m_N =$ mass of the neutron

Binding Energy- (E_b)

The energy with which nucleons are bound in the nucleus or amount of energy required to separate nucleons from the nucleus is called Binding energy.

Binding energy $E_b = \Delta mc^2$

Where Δm is the mass defect? E_b is the binding energy

And c= velocity of light

$$E_b = \left\{\left[ZM_P + (A-Z)M_N\right] - M\right\}\times C^2$$

The energy required to split up a nucleus into its constituent nucleons in a free state is called binding energy.

Question:- Find mass defect of ${}_8O^{16}$ also find binding energy per nucleon of ${}_8O^{16}$ the nucleus. Given $M_P = 1.00727$ and $M_N = 1.00866\,amu$ and mass of ${}_8O^{16} = 15.99053\,amu$.

Solution:- $\Delta m = \left[ZM_P + (A-Z)M_N\right] - M$

$= [8\times1.00727 + 8\times1.00866] - 15.99053 \qquad = 16.12744 - 15.99053 = 0.13691\,amu$

$E_b = \Delta m \times C^2 = 0.13691 \times 931.5\,mev \qquad = 127.5\,mev$

$$\frac{BE}{Nucleon}\left(\frac{E_b}{A}\right) = \frac{127.5}{16} = 7.96\,mev$$

Question:- Calculate the binding energy per nucleon of ${}_{20}Ca^{40}$ the nucleus. Given $m\left({}_{20}C^{40}\right) = 39.962589\,a.m.u,\ m_n = 1.005665\,amu,\ m_p = 1.007825\,amu$

Solution:- Δm (mass defect) = $\left[ZM_P + (A-Z)M_N\right] - M$

$\Rightarrow \Delta M = \left[20(1.007825) + 20(1.008665)\right] - 39.962589$

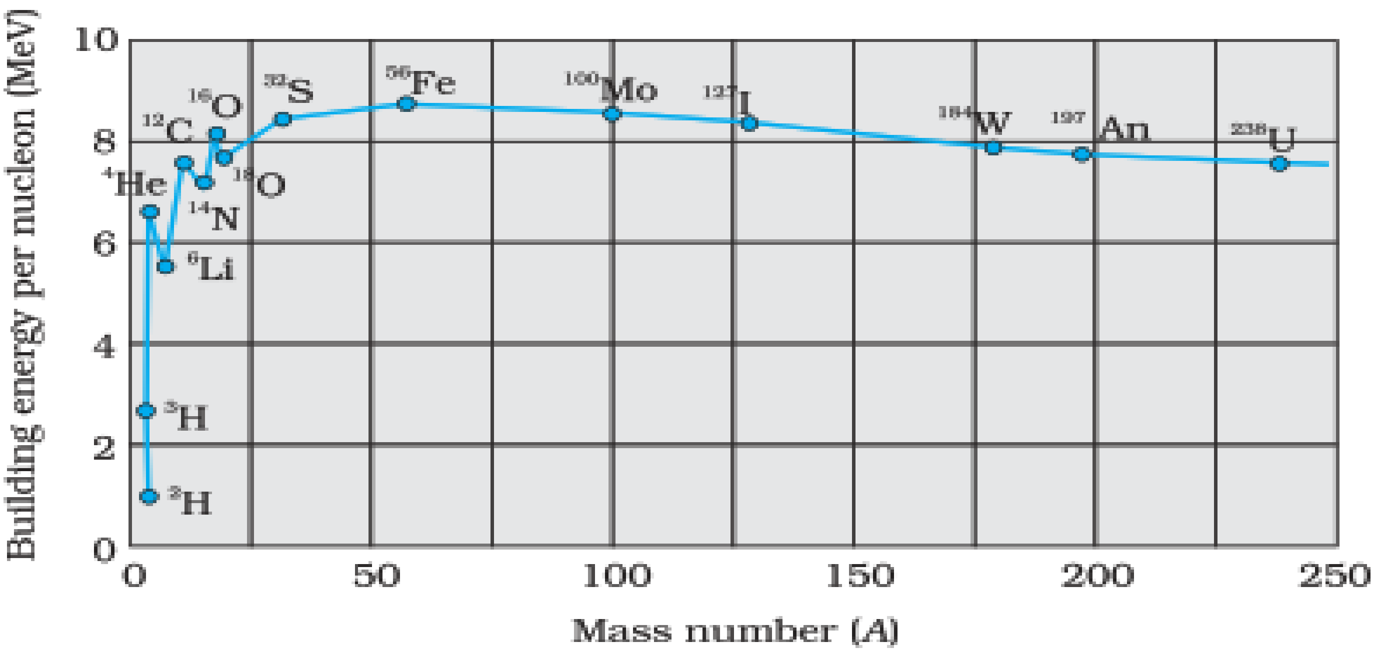
$\Rightarrow \Delta m = 0.367211\ a.m.u$

$\Rightarrow B.E\ (E_b) = \Delta m \times C^2 = 0.367211 \times 931.5 = 342.05705\,mev$

BE Per nucleon $\left(\frac{E_b}{A}\right) = \frac{342.05705}{40} = 8.55143\,mev$

<u>Binding Energy Curve</u>:-

This is the graph between B.E. per nucleon $(E_{bn})\,mev$ and mass number (A) for a large number of nuclei.



Features of graph:-

(a) It gives the measure of the stability of that nucleus.

(b) E_{bA} lies on or near a smooth curve except for lighter nuclei like $_2He^4$, $_6C^{12}$ and $_8O^{16}$. This indicates higher stability of these three nuclei then neighbours.

(c) E_{ven} is independent of atomic number for nuclei between mass number 30 to 170.

(d) The curve has maximum value of 8.75 mev for A = 56 and has a value of 7.6 mev or A = 238

(e) E_{ven} is lower of both lighter nuclei (A < 30) and heavier nuclei (A > 170).

(f) The constancy of B.E. in the range 30 < A < 170 is a consequence of fact that nuclear force is short-range

(g) If we increase A by adding nucleons they will not change E_{bA} since nucleons reside inside the nucleus. A given nucleon influences only nucleons close to it is referred to as saturation property of nuclear force.

(h) When a heavy nucleus splits into lighter nuclei, the E_{bn} increases. This shows the liberation of energy in nuclear fission

(i) When 2 light nuclei join to form a heavy nucleus, the E_{bn} of heavy nuclei increases then lighter nuclei. Also, energy is released in a nuclear fusion reaction.

Nuclear Fission:-

The phenomenon in which a heavy nucleus breaks into two light nuclei of almost equal masses with the release of a huge amount of energy is called fission reaction. When a neutron strikes $_{92}U^{235}$ nucleus, it produces an unstable $_{92}U^{235}$ nucleus which splits into $_{56}Ba^{114}$and $_{36}Kr^{99}$ three neutrons and the huge amount of energy.

Example:-

(a) $_0n^1 +_{92} U^{235} \rightarrow_{92} U^{236} \rightarrow_{56} Ba^{144} +_{36} Kr^{89} + 3_0n^1 + Q$

(b) $_0n^1 +_{92} U^{235} \rightarrow_{92} U^{236} \rightarrow_{54} Xe^{140} +_{38} Sr^{94} + 2_0n^1$

Also, nuclear fission occurs in

(a) An atom bomb (uncontrolled fission)

(b) In Nuclear reactor (controlled fission reason)

Source of Energy:-

The sum of the masses of the final products is less than the sum of masses of the reactant. The difference in masses is converted into energy according to relation $E = mc^2$.

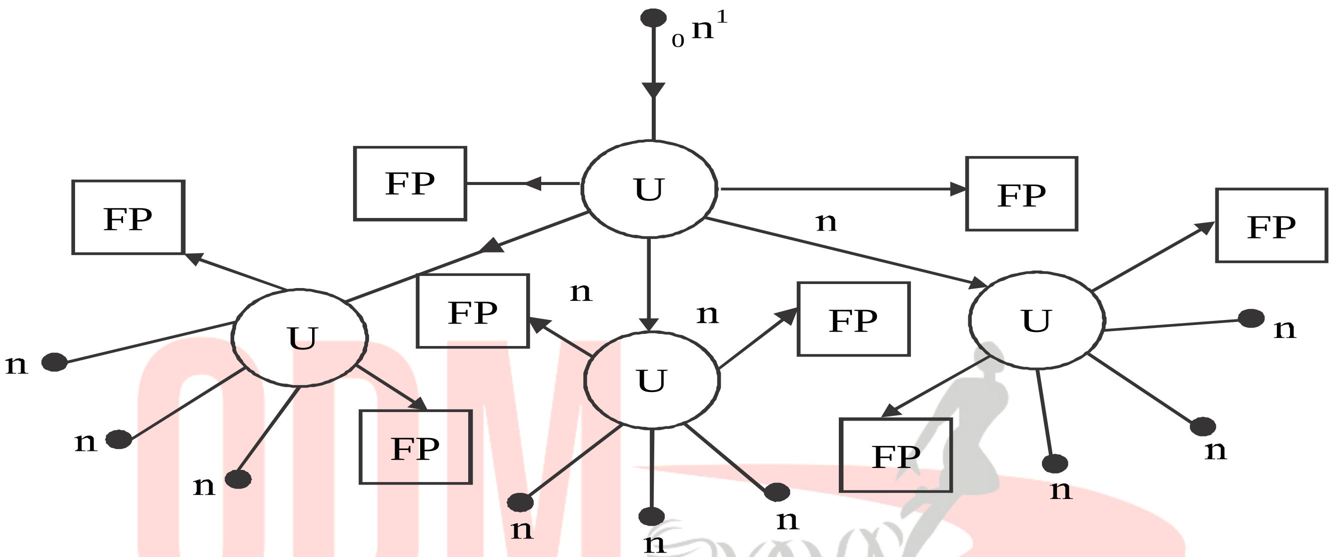
⇒This energy appears as the kinetic energy of fission products and as γ rays and heat energy.

⇒ In single fission of $_{92}U^{235}$, about 216 MeV energy is released which is equivalent to 0.9 MeV/ nucleon.

⇒ Fission of 1kg of uranium generates $10^{14}J$ of energy compared to burning of 1kg of coal that gives $10^7 J$.

Chain Reaction:-

The three neutrons emitted in fission reaction are fast enough to cause a further reaction. This leads to a chain of nuclear fission which continues till the whole of uranium is consumed. The energy produced goes on increasing. This is known as a chain reaction.



Types of Chain Reaction:-

(a) **Uncontrolled chain reaction**:- If the fissionable material is more than critical mass then the reaction will accelerate at a rapid rate that whole of material is consumed within a fraction of second. The huge amount of energy is released in a very short interval of time and is released as an explosion. Example- Atom Bomb.

(b) **Controlled chain reaction:-** If only one of the neutrons produced in each fission can cause further fission then the process is slow and the energy is released steadily. Such a chain reaction is called a controlled chain reaction that occurs in a nuclear reactor. The energy released is utilized peacefully.

Nuclear Reactor:- The device in which controlled chain reaction is initiated and it supplies energy at a constant rate is called Nuclear reactor.

Main Parts:-

(a) Nuclear Fuel:- Isotopes like U-235, Th-232 and Pu-239

(b) Moderator:- Heavy water, graphite and beryllium oxide are called as the moderator to slow down the fast neutrons.

(c) Control Rods:- Cadmium or boron rods are used to absorb fast neutrons inside the reactor

(d) Coolant:- heavy water and liquid sodium are used as a coolant to decrease the temperature inside the reactor.

(e) Shielding:- It is a thick concrete wall to prevent radiations produced in the nuclear reactor.

Multiplication factor and critical size:- It is denoted by K.

K = No. of neutrons present at the beginning of one generation / No. of neutrons present at the beginning of the previous generation

If $K > 1 \Rightarrow$ chain reaction on grows (K measures the growth rate of neutron and in a reactor)

If $K = 1 \Rightarrow$ chain reaction remains steady

If $K < 1 \Rightarrow$ chain reaction on gradually dies out.

Critical mass:-

The size of fissionable material for which M.F (K) = 1 is called critical size and mass is called critical mass.

If $K > 1 \Rightarrow$ The size of a material is called supercritical (very fast reaction results explosion)

If $K < 1 \Rightarrow$ The size of a material is called subcritical where chain reaction gradually comes to an end

Nuclear Fusion:-

The process in which two light nuclei combine at high temperature to form a single heavier nucleus is called nuclear fusion. The mass of the heavier nucleus formed is less than the sum of the masses of combining nuclei. The mass defect is released as energy under Einstein's relation $E = mc^2$.

Example:-

${}_1H^1 +_1 H^1 \rightarrow_1 H^2 +_{+1} e^0 + \gamma + 0.42\,\text{mev}$ .... (1)

${}_1H^2 +_1 H^2 \rightarrow_2 He^3 +_0 n^1 + 3.27\,\text{mev}$ ....(2)

${}_1H^2 +_1 H^2 \rightarrow_1 H^3 +_1 H^1 + 4.03\,\text{mev}$ ....(3)

(1) For fusion reaction, the two nuclei must close enough so that they come under attractive short-range nuclear force.

(2) Two nuclei experience repulsion force when they close each other due to their +ve charge (protons)

(3)The height of coulomb's barrier depends on charges and radii of two nuclei coming each other.

(4) For two protons, to overcome coulomb's barrier of 400 Kev, the temperature of the order of $3\times10^9\,K$ is required.

(5)The K.E of a particle increases with an increase in temperature to nullify the repulsive force between them. That is why nuclear fusion is called **thermonuclear reaction.**

6)The source of energy in sun and stars is due to the fusion reaction.

Fusion Reaction in sun and Stars:-

Protons are abundant in the body of sun and star. At high temperature, the hydrogen is burned into helium and releasing a huge amount of energy.

The **proton-proton (P – P) cycle** that occurs is shown below.

$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_{+1}e^0 + \gamma + 0.42\,\text{mev}$ .... (1)

$_{+1}e^0 + {}_{-1}e^0 \rightarrow \gamma + \gamma + 1.02\,\text{mev}$ .... (2)

$_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + \gamma + 5.49\,\text{mev}$ ....(3)

$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1 + 12.86\,\text{mev}$ .... (4)

By multiplying (2) (in equation (1), equation (2) and equation (3)] and adding with equation (4) the net effect is

$$4\,{}_1H^1 + 2\,{}_{-1}e^0 \rightarrow {}_2He^4 + 2\gamma + 6\gamma + 26.7\,\text{mev}$$

When hydrogen converted to He, the core starts to cool. Star begins to collapse under gravity which increases the temperature of the core. Age of sun is about $5 \times 10^9\,y$. If hydrogen-burning will stop after some years, the sun will collapse under gravity. It will raise the core temperature. The outer envelope of the sun will expand, turning it into a red giant.

Controlled Thermo Nuclear Fusion:-

In a controlled fusion reactor to generate steady power, high temperature of range 10^8 K is needed. Since No. container can sustain high temperature at a plasma state. Many countries are developing techniques in this connection for the fusion reactor to supply power to humanity.

Numerical:- A neutron is absorbed by a Li^6 nucleus with subsequent emission of $\alpha-$ particle. Write nuclear reaction find the energy released in this reaction.

Given

$m\left({}_3Li^6\right) = 6.015126\,amu, m\left({}_2He^4\right) = 4.0026044\,amu, m\left({}_0n^1\right) = 1.0086654\,amu, m\left({}_1H^3\right) = 3.016049\,amu$

Solution:- ${}_3Li^6 +_0 n^1 \rightarrow_2 He^4 +_1 H^3 + Q$

$\Rightarrow \Delta m = 7.0237914 - 7.0186534 = 0.005138\,amu$

$\Rightarrow Q = \Delta m \times c^2 = 0.005138 \times 931\,mev = 4.78\,mev$

Question:- Find the disintegration energy (Q) for the fission of ${}_{42}Mo^{98}$ into two equal fragments, ${}_{21}Sc^{49}$. If Q turns out to be positive, explain why this process does not occur spontaneously. Giventhat

$m\left({}_{42}Mo^{98}\right) = 97.90541\,amu, m\left({}_{21}Sc^{49}\right) = 48.95002\,amu, M_n = 1.00867\,amu$

Solution:- ${}_{42}Mo^{98} +_0 n^1 \rightarrow 2\,{}_{21}Sc^{49} + Q$

Q value in the fission of ${}_{42}Mo^{98}$ is given by

$$Q = [m(\,{}_{42}Mo^{98}) + M_n - 2m(\,{}_{21}Sc^{49})] \times c^2$$

$$= [97.90541 + 1.00867 - 2 \times 48.95002] \times c^2$$

$$= 1.01404 \times 931.5 = 944.6\,mev$$

The nucleus must overcome the energy barrier which is larger the 944 Mev. The +ve sign of Q does not necessarily mean to find plenty of molybdenum nuclei spontaneously fissioning.