16

PLAYING WITH NUMBERS

INTRODUCTION

We know various numbers such as natural numbers, whole numbers, integers and rational numbers. They have various interesting properties that can create a magical mathematics.

NUMBERS IN GENERAL FORM

In general, any two digit number ab made of digits a and b can be written as

$ab = 10 \times a + b = 10a + b$

$ba = 10 \times b + a = 10b + a$

For example, $37 = 10 \times 3 + 7$

In general, a 3-digit number abc made up of digits a, b and c is written as

$abc = 100 \times a + 10 \times b + 1 \times c = 100a + 10b + c$

In the same way, $cab = 100c + 10a + b$

$bca = 100b + 10c + a$ and so on.

For example, $351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$

Similarly $497 = 100 \times 4 + 10 \times 9 + 1 \times 7$

GAMES WITH NUMBERS

1. If you multiply the number 142,857 by anything between 1 and 6 the answer contains the same digits.
e.g. $142857 \times 3 = 428571$ and $142857 \times 6 = 857142$
If you multiply the same figure by 7 the answer is 999,999.

2. Ignore your area code and use only your seven digit phone number
(i) What are the first 3 digits of your phone number? (do not include the area code)
(ii) Multiply by 80 (iii) Add 1 (iv) Multiply by 250 (v) Add in the last four digits of phone number
(vi) Add in the last four digits of phone number again (vii) Subtract 250 (viii) Divide by 2.
The result is your phone number. Surprised. It's a simple algebra.

3. Game to find out a person's age and how many coins they are carrying.

* Ask someone to double their age (either mentally or on paper) but, obviously, they must not tell you.
* Then to add 5 and multiply by 50.
* Finally, they must add the number of coins they have on them. If they do not have any, nothing is added.
* They then tell you the final answer.
* Now all you have to do is take away 250. The first two digits of the answer you are left with are the persons age and the last two the number of coins in their possession.

This works unless they are carrying over 100 loose coins.
Example : $25 \times 2 = 50 + 5 = 55 \times 50 = 2750 + 6 = 2756$; $2756 - 250 = 2506$

4. Take any three digit number in which the first and last number differ by more than one i.e. 335 would be O.K. but not 333, or 332.
Reverse this number
Subtract the smaller number from the larger.
Add this answer to the same number reversed and the answer is always 1089.
Example : 335 Reversed = 533 ; $533 - 335 = 198$; $198 + 891$(198 reversed) $= 1089$
Example : 932 reversed = 239 ; $932 - 239 = 693$; $693 + 396 = 1089$

5. Get a friend to throw a dice three times and you will be able to tell them which numbers came up and in which order. This is what they have to do -

* Throw dice
* Multiply number by 2
* Add 5
* Multiply by 5 (Remember this total)
* Throw dice second time
* Add this second number to the previous total.
* Multiply by 10 (Remember total)
* Throw the dice for the third time and add the number to the last total.

Ask for the final total. Subtract 250 and you will be left with three figures. These figures represent the numbers thrown and the in which they appeared.

Example: First throw = 4 ; $4 \times 2 = 8$; $8 + 5 = 13$; $13 \times 5 = 65$
Second throw = 2 ; $65 + 2 = 67$; $67 \times 10 = 670$
Third throw = 6 ; $670 + 6 = 676$; $676 - 250 = 426 = 4\ 2\ 6$

Example 1 :

Insert the symbols +, –, ×, ÷ and parenthesis in the following sequence of nos. so that the expression equals 100.
1 2 3 4 5 6 7 8 9

Sol. The desired expression is $1 + (2 \times 3) - 4 + (56 \div 7) + 89$. Clearly, the value of the above expression is 100.

Example 2 :

x stands for one digit, y stands for another digit and z stands for another third digit. Find out three digits if

$$\begin{array}{r} x\,x\,x\,x \\ \times\, x\,x \\ \hline y\,y\,y\,y \\ y\,y\,y\,y\,y \\ \hline y\,z\,z\,z\,y \\ \hline \end{array}$$

Sol. Suppose x = 1, then we have, $\begin{array}{r} 1111 \\ \times 11 \\ \hline 1111 \end{array}$

x = 1 is not possible because 1111 × 1111 i.e., x x x x but we need y y y y.

Suppose x = 2, then we have, $\begin{array}{r} 2\,2\,2\,2 \\ \times\, 2\,2 \\ \hline 4\,4\,4\,4 \\ 4\,4\,4\,4 \\ \hline 4\,8\,8\,8\,4 \end{array}$. Hence, x = 2, y = 4 and z = 8.

FIBONACCI SERIES

A group of numbers in ascending order is called Fibonacci series if each number in the series is equal to the sum of two numbers just left to it.

Example 3 :

Write a Fibonacci series upto six numbers taking 3 and 4 as its first two members.

Sol. First number = 3 ; Second number = 4
∴ Third number = First number + second number = 3 + 4 = 7
Fourth number = Third number + Second number = 7 + 4 = 11
Fifth number = Fourth number + Third number = 11 + 7 = 18
Sixth number = Fifth number + Fourth number = 18 + 11 = 29
Hence, the required Fibonacci series is: 3, 4, 7, 11, 18, 29.

LETTERS FOR DIGITS

We can have various puzzles in which letters take the place of digits in an arithmetic 'sum', and the problem is to find out which letter represents which digit; so it is like cracking a code. Here we stick to problems of addition and multiplication.

Two rules we follow while doing such puzzles.

1. Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter.

2. The first digit of a number cannot be zero. Thus, we write the number "sixty three" as 63, and not as 063, or 0063.

Example 4 :

Find the value of a, b and c such that

$$\begin{array}{ccccc} & 2 & a & 6 & 2 \\ + & a & 2 & 4 & a \\ \hline b & c & c & b & 1 \\ \hline \end{array}$$

Sol. 'a' may be any number from 0 to 9. But it cannot be zero as 2 + a is given to be 1. Also, 'a' cannot be any number upto 8 as the sum of 2 and a must have 1 at the units place. This can happen only when a = 9 because then 2 + 9 = 11 in which 1 is in the units place and 10 is carried to the next digit.

Moving on to the next column on the left, we find that 6 tens + 4 tens = 1 ten (which has been carried over the from previous column) gives 11 tens. Hence b has to be 1 and one hundred is carried to the next column.

In the hundreds place, a = 9. So, 900 + 200 + 100 (carried) = 12 hundreds

So, 2 from 12 comes in place of c. Thus, c has to be 2.

Since, we know that a is 9, so in the first column, 9000 + 2000 + 1000 (carried) = 12000.

Hence, we again get b = 1 and c = 2. $\quad\therefore$ a = 9, b = 1, c = 2

Example 5 :

Find out the correct digit for each letter:

$$\begin{array}{ccc} A & B & C \\ -3 & 6 & 9 \\ \hline C & A & B \end{array}$$

Sol. Putting A = 6, B = 3 and C = 2, we get

$$\begin{array}{ccc} 6 & 3 & 2 \\ -3 & 6 & 9 \\ \hline 2 & 6 & 3 \end{array}$$

Example 6 :

In the following problem, replace the letters of the English alphabet by digits (two or more letters may have the same value) to complete the procedure of division.

$$\begin{array}{r} 5\,C \\ 9\overline{)\;4\,A\,B} \\ -D\,E \\ \hline 3\,F \\ -G\,H \\ \hline 0 \\ \hline \end{array}$$

Sol. In the quotient, the first number is 5 and we know that $9 \times 5 = 45$

$\therefore$ D = 4 and E = 5

Now, 48 – 45 = 3. Therefore, A = 8. Also, to make the number 3F to be divisible by 9 we must have F = 6.

And so, C = 4 and B = 6. Also, G = 3, H = 6

Thus, the division works out as shown :

$$\begin{array}{r} 54 \\ 9\overline{)\,486} \\ -45 \\ \hline 36 \\ -36 \\ \hline 0 \\ \hline \end{array}$$

TEST OF DIVISIBILITY FOR NUMBERS EXPRESSED IN THE GENERALIZED FORM

A number is divisible by another number if after dividing, the remainder is zero.

For example, 18 is divisible by 3 because $18 \div 3 = 6$ with 0 remainder. However, 25 is not divisible by 4 because $25 \div 4 = 6$ with a remainder of 1. There are several mental math tricks that can be used to find the remainder after division without actually having to do the division.

The test of divisibility by a number 'x' is a short-cut method to detect whether a particular number 'y' is divisible by the number 'x' or not.

1. **Test of divisibility by 2 :**

A number is divisible by 2, if its units digit is even, i.e., if its units digit is any of the digits 0, 2, 4, 6 or 8.

For a number in the generalized form:

(i) A general two-digit number 10a + b is divisible by 2 if 'b' is any of the digits 0, 2, 4, 6 or 8.

(ii) A general three-digit number 100a + 10b + c is divisible by 2 if 'c' is any of the digits 0, 2, 4, 6 or 8.

For example, numbers 12, 42, 62, 70, 88, 96, 342, 406, 964, 730. etc., are divisible by 2.

2. **Test of divisibility by 3 :** A number is divisible by 3, if the sum of its digits is divisible by 3.

For a number in the generalized form:

(i) A general two-digit number l0a + b is divisible by 3 if (a + b) is divisible by 3.

(ii) A general three-digit number 100a + 10b + c is divisible by 3 if (a + b + c) is divisible by 3.

For example, each of the numbers 42, 12, 24,36, 123, 456,789.972, etc., is divisible by 3.

Also, numbers 71, 53, 94, 26, 134, 361, 985, etc., are not divisible by 3.

3. **Test of divisibility by 5 :** A number is divisible by 5, if its units digit is either 0 or 5.

For a number in the generalized form:

(i) A general two-digit number 10a + b is divisible by 5 if 'b' is either 0 or 5.

(ii) A general three-digit number 100a + 10b + c is divisible by 5 if 'c' is either 0 or 5.

For example, each of the numbers 70, 35, 15, 90, 340, 265, 805, etc., is divisible by 5.

4. **Test of divisibility by 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

For a number in the generalized form:

(i) A general two-digit number 10a + b is divisible by 9 if (a + b) is divisible by 9.

(ii) A general three-digit number 100a + 10b + c is divisible by 9 if (a +b + c) is divisible by 9.

For example, each of the numbers 45, 63, 72, 18, 324, 459, 792, 387, etc., is divisible by 9.

5. **Test of divisibility by 10 :** A number is divisible by 10, if its unit's digit is 0.

For a number in the generalized form:

(i) A general two-digit number 10a + b is divisible by 10, if 'b' is equal to 0.

(ii) A general three-digit number 100a + 10b + c is divisible by 10 if 'c' is equal to 0.

For example, each of the numbers 20, 70, 40, 10, 300, 530, 690, 180, etc., is divisible by 10.

Example 7 :

Test the divisibility of the following numbers by (i) 2 (ii) 5 (iii) 10

26, 240, 153, 6032, 7035, 18290

Sol. (i) We know that a number is divisible by 2 if its units digit is one of 0, 2, 4, 6, 8
Thus, 26, 240, 6032, 18290 are divisible by 2.

(ii) We know that a number is divisible by 5 if its units digit is either 0 or 5.
Hence, 240, 7,035, and 18,290 are divisible by 5.

(iii) We know that a number is divisible by 10 if its units digit is 0.
Hence, 240 and 18,290 are divisible by 10.

SELF CHECK

Q.1 Which of the following numbers are divisible by 2 ? 57, 34, 60, 93, 126, 365

Q.2 Which of the following numbers are divisible by 3 ? 42, 73, 84, 105, 314, 726

Q.3 Which of the following numbers are divisible by 5 ? 30, 49, 75, 210, 305, 640

Q.4 Insert '+' and '–' symbols between the numbers so that the following equation becomes correct.
1 2 3 4 5 8 = 7

Q.5 If we delete the numeral occurring in both the numerator and the denominator of the fraction $\frac{26}{25}$, we don't change its value because $\frac{26}{25} = \frac{2}{5}$. What other fractions consisting of two digits figures in the numerator and denominator can be similarly reduced?

Q.6 Suppose you ask a friend to choose a number. Then, if your directions are followed correctly, you will be able to state the answer which your friend has obtained without knowing the original number. Develop the set of directions.

Q.7 There are 365 days in a year. This number is the sum of the squares of 2 consecutive number.... $365 = 13^2 + 14^2$. But 365 is also the sum of the squares of three consecutive numbers. Can you find them ?

Q.8 Each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 is –

A B C
D
E F G
H
I

(a) Represented by a different letter in the figure above.

(b) Positioned in the figure above so that each A + B + C, C + D + E, E + F + G and G + H + I is equal to 13. Which digit does E represent ?

Q.9 Veena told Rita that "every number can be written as a difference of two squares" was discovered during Vedic times. For example, $24 = 7^2 - 5^2$. Do you know the reason? Write down 25, 31 & 6 as difference of two square.

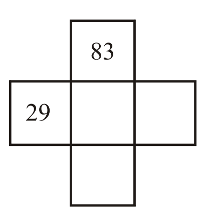
Q.10 My house has a number.

(1) If my house number is a multiple of 3 (0 × 3, 1 × 3, 2 × 3, etc..), then it is a number from 50 through 59.

(2) If my house number is not a multiple of 4, then it is a number from 60 through 69.

(3) If my house number is not a multiple of 6, then it is a number from 70 through 79. What is my house number?

Q.11 Fill in the blank squares by 2-digit prime numbers so that the sum of the numbers in both the horizontal and vertical squares is 123.



USEFUL TIPS

* **Dividing By 2:** A number is divisible by 2 if the last digit is even.
* **Dividing By 3 :** A number is divisible by 3 if the sum of all the digits is divisible by 3.
Ex 34,164 is divisible by 3 because 3+4+1+6+4 = 18 which is divisible by 3.
To find the remainder of a number divided by 3, add the digits and find that remainder. So if the digits added together equal 13 then the number has a remainder of 1 since 13 divided by 3 has a remainder of 1.
* **Dividing By 4:** A number is divisible by 4 if the last 2-digits are divisible by 4.
Ex 34,164 is divisible by 4 because 64 is divisible by 4.
To find the remainder of a number divided by 4 take the remainder of the last 2 digits. So if the last 2-digits are 13 then the number has a remainder of 1 since 13 divided by 4 has a remainder of 1.
* **Dividing By 5:** A number is divisible by 5 if the last digit is a 5 or a 0.
To find the remainder of a number divided by 5 simply use the last digit. If it is greater than 5, subtract 5 for the remainder.
* **Dividing By 6:** A number is divisible by 6 if it is divisible by 2 and by 3.
Ex 34,164 is divisible by 6 because it is divisible by 2 and 3.
* **Dividing By 7:** A number is divisible by 7 if the following is true:
1. Multiply the ones digit by 2.
2. Subtract this value from the rest of the number.
3. Continue this pattern until you find a number you know is or is not divisible by 7.
Ex :7203 is divisible by 7 because (a) $2 \times 3 = 6$. (b) $720 - 6 = 714$ which is divisible by 7.
Ex :14443 is not divisible by 7 because (a) $3 \times 2 = 6$. (b) $1444 - 6 = 1438$. (c) $8 \times 2 = 16$.
(d) $143 - 16 = 127$ which is not divisible by 7.
* **Dividing By 8:** A number is divisible by 8 if the last 3-digits are divisible by 8.
Ex :34,168 is divisible by 8 because 168 is divisible by 8.
To find the remainder of a number divided by 8 take the remainder of the last 3-digits. So if the last 3-digits are 013 then the number has a remainder of 5.
* **Dividing By 9:** A number is divisible by 9 if the sum of the digits is divisible by 9.
Ex :34,164 is divisible by 9 because $3 + 4 + 1 + 6 + 4 = 18$ which is divisible by 9.
To find the remainder of a number divided by 9, add the digits and find that remainder. So if the digits added together equal 13 then the number has a remainder of 4 since 13 divided by 9 has a remainder of 4.
* **Dividing By 10:** A number is divisible by 10 if the last digit is a 0.
To find the remainder of a number divided by 10 simply use the last digit.
* **Dividing By 11:** A number is divisible by 11 if this is true:
1st Step: Starting from the one's digit add every other digit
2nd Step: Add the remaining digits together
3rd Step: Subtract 1st Step from the 2nd Step
If this value is 0 then the number is divisible by 11. If it is not 0 then this is the remainder after dividing by 11 if it is positive. If the number is negative add 11 to it to get the remainder.
Ex :6613585 is divisible by 11 since $(5 + 5 + 1 + 6) - (8 + 3 + 6) = 0$.
* **Dividing By 12:** A number is divisible by 12 if it is divisible by 3 and by 4.
Ex : 34,164 is divisible by 12 because it is divisible by 3 and 4.