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MOTION

INTRODUCTION

Motion is everywhere : friendly and threatening, terrible and beautiful. It is fundamental to our human existence. We need motion for growing, for learning, for thinking and for enjoying life. We use motion for walking through a forest, for listening to its noises and for talking about all this.

Like all animals, we rely on motion to get food and to survive dangers. Plants by contrast cannot move (much); for their self-defence, they developed poisons. Examples of such plants are the stinging nettle, the tobacco plant, digitalis, belladonna and poppy; poisons include cateine, nicotine and many others. Poisons such as these are at the basis of most medicines therefore, most medicines exist essentially because plants have no legs. Like all living beings, we need motion to reproduce, to breathe and to digest; like all objects, motion keeps us warm.

Motion is the most fundamental observation about nature at large. It turns out that everything that happens in the world is some type of motion. Motion is also important to the human condition. Who are we ? Where do we come from ? What will we do ? What should we do ? What will the future bring ? Where do people come from ? Where do they go ? What is death ? Where does the world come from ? Where does life lead? All these questions are about motion, therefore study of motion provides answers that are both deep and surprising.

To study the motion branch of physics called **Mechanics** is defined. To simplify study it is further divided into two sections, Kinematics and Dynamics. **Kinematic** deals with the study of motion of objects without considering the cause of motion, here measurement of time is essential

$$\left(\text{Kinematics} \xrightarrow[\text{Word}]{\text{Greek}} \text{Kinema} \rightarrow \text{motion}\right).$$

Dynamics deals with the study of objects taking into consideration and cause of their motion

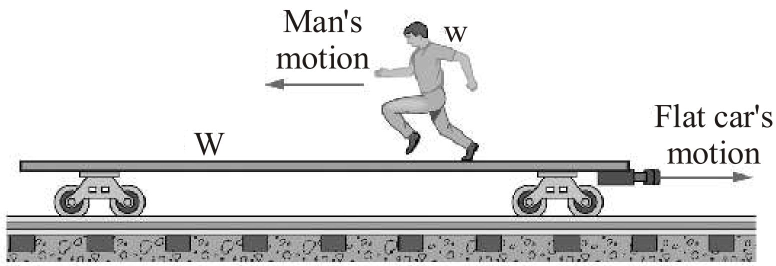
$$\left(\text{Dynamics} \xrightarrow[\text{Word}]{\text{Greek}} \text{Dynamis} \rightarrow \text{power}\right)$$

REST V/S MOTION

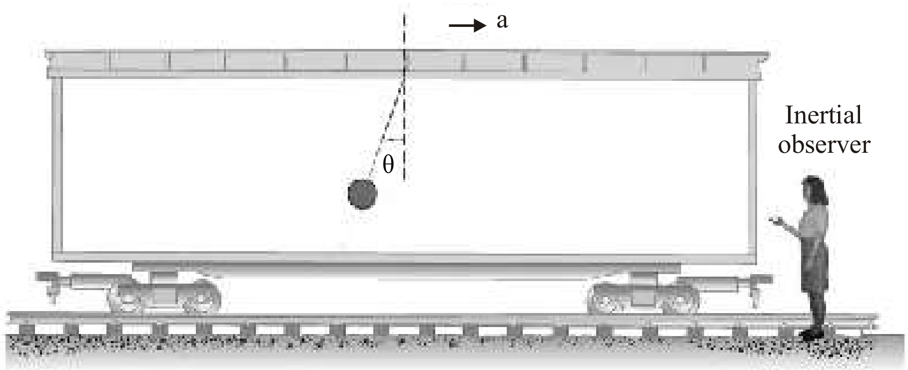
Rest : An object is said to be at rest if it does not change its position with respect to its surroundings with the passage of time.

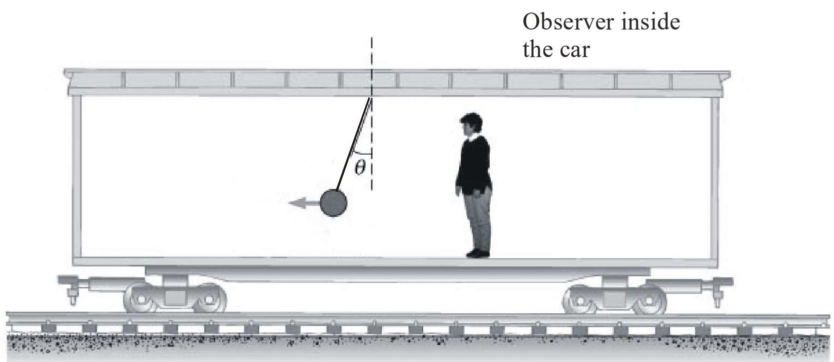
Motion : A body is said to be in motion if its position changes continuously with respect to the surroundings (or with respect to an observer) with the passage of time.

We know that earth is rotating about its axis and revolving around the sun. The stationary objects like your class-room, a tree and the lamp posts etc. do not change their position with respect to each other i.e. they are at rest. Although earth is in motion. To an observer situated outside the earth say in a space ship, your classroom, trees etc. would appear to be in motion. Therefore, all motions are relative. There is nothing like absolute motion. If you move with book in your hand, book is not moving with respect to you.



Observe like a science student : To the passengers in a moving bus or train, trees, buildings and people on the roadsides observe that the bus or the train and its passengers are moving in the forward direction. At the same time, each passenger in a moving bus or train finds that fellow passengers are not moving, as the distance between them is not changing. These observations tell us that the motion is relative. If you will observer the man moving on moving flat car from ground your observation will be different from what man himself will observe. Similarly, if you will observe pendulum in moving car from ground your observation will be different from what person inside car will observe.





BASIC TERMS

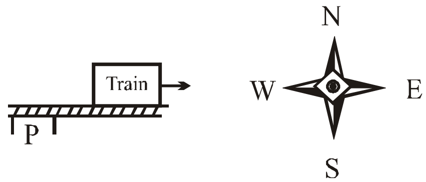
(i) **Frame of Reference :** To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axes and name them x-y-z axes. The co-ordinates (x, y, z) of the particle then specify the position of object w.r.t. that frame. If any one or more co-ordinates change with time, then we say that the object is moving w.r.t. this frame.

(ii) **Motion in one, two and three dimensions (Types of motion) :** As position of object may change with time due to change in one or two or all the three co-ordinates so we have classified motion.

Accordingly :

(a) Motion in 1-D : If only one of the three co-ordinates specifying the position of object changes w.r.t. time. In such a case the objects moves along a straight line and the motion therefore is also known as rectilinear or linear motion.

Simply, rectilinear motion means motion along a straight line. This is a useful topic to study for learning how to describe the movement of cars along a straight road or of trains along straight railway tracks. In this section you have only 2 directions to worry about: (1) along the direction of motion, and (2) opposite to the direction of motion. To illustrate this imagine a train heading east.



If it is accelerating away from the station platform (P), the direction of acceleration is the same as the direction of the train's velocity - east. If it is braking the direction of acceleration is opposite to the direction of its motion, i.e. west.

(b) Motion in 2-D : If two of the three co-ordinates specifying the position of object change w.r.t. time, then the motion of object is called two dimensional. In such a motion the object moves in a plane like motion of queen on carom board.

(c) Motion in 3-D : If all the three co-ordinates specifying the position of object change w.r.t. time, then the motion of object is called 3-D. In such a motion the object moves in a space.

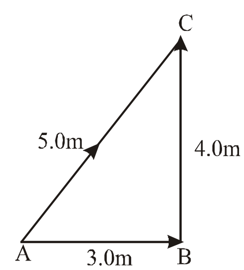
(1) A bird flying in a sky. (also kite)

(2) Motion of an aeroplane in space.

(iii) Distance and Speed, Displacement and Velocity

Motion is related to change of position. The length traveled in changing position may be expressed in terms of distance, the actual path length between two points. Distance is a scalar quantity,

which only a magnitude with no direction.



The direct straight line pointing from the initial point to the final point is called displacement (change in position). Displacement only measures the change in position, not the details involved in the change in position. Displacement is a vector quantity, which has both magnitude and direction. In the figure shown, an object goes from point A to point C by following paths AB and BC.

The distance traced is 3.0m + 4.0m = 7.0m, and the displacement is 5.0m in the direction of the arrow.

If one states 'the car has travelled 200m', it means that is the distance travelled by the car. But if one states 'the car has travelled 200m due east' it means that the displacement of the car 200m towards east.

The displacement can be zero even if the distance is not zero. For example when a body is thrown vertically upwards from a point on the ground, after sometime it returns back to the same point, then the displacement of the body is zero but the distance travelled by the body is not zero, it is 2h if h is the maximum height attained by the body.

Similarly, if a body is moving in a circular or closed path and reaches its original position after one rotation, then the displacement in one rotation is zero, but the distance travelled is equal to the circumference of the circular path = $2\pi r$ if r is the radius of the circular path.

In describing motion, the rate of change of position may be expressed in terms of speed and velocity.

Average speed is defined as the distance traveled divided by the time interval to travel that distance.

Average speed = d/t, d is distance traveled, and t is time interval (change in time).

Instantaneous speed is the speed at a particular time instant (t is infinitesimal small or close to zero).

Since distance is a scalar quantity with no direction, so are average speed and instantaneous speed. Both tell us only how fast objects are moving.

A body is said to be moving with uniform speed if it covers equal distances in equal time intervals and with non-uniform or variable speed if covers unequal distances in the same time intervals.

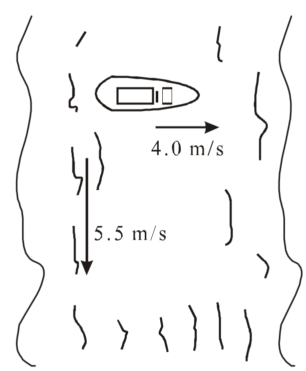
Average velocity is defined as displacement divided by the time interval, $\vec{v} = \frac{\Delta x}{\Delta t}$, where $\vec{v}$ is average velocity, Δx is displacement (change in position), and Δt is time interval. (Direction of displacement indicated by sign, + or – for one-dimensional motion) It is this velocity that is Instantaneous velocity, v is the velocity (magnitude and direction) at a particular instant of time (Δt is close to zero). Since displacement is a vector quantity, so are average velocity and instantaneous velocity. Both tell us not only how fast, but in which directions objects are moving. (Direction of velocity indicated by sign, + or – for one-dimensional motion.)

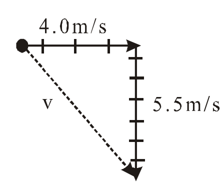
The SI units of speed v and velocity $(\vec{v})$ are m/s.

If a body travels equal distances in equal intervals of time along a particular direction, the body is said to be moving with a uniform velocity and if a body moves unequal distances in a particular direction in equal intervals of time or it moves equal distances in equal intervals of time but its direction of motion changes, such as in circular motion, the velocity of the body is said to be variable or non-uniform. In such a case we specify the instantaneous velocity and the average velocity of the body.

(iv) Velocity vector : Sometimes object have two different velocities at the same time. Object total velocity is then the vector sum of the separate velocities. It is found by vector adding, using the same rules as in adding displacement vectors.

Suppose, for example, a boat can travel 4.0 meters per second in still water. It is in a river that flows southward at 5.5 meters per second, as shown in figure. If the boat heads eastward, directly across the river, what are the direction and magnitude of its total velocity ?





The vector diagram in figure shows how the two velocity vectors can be added.

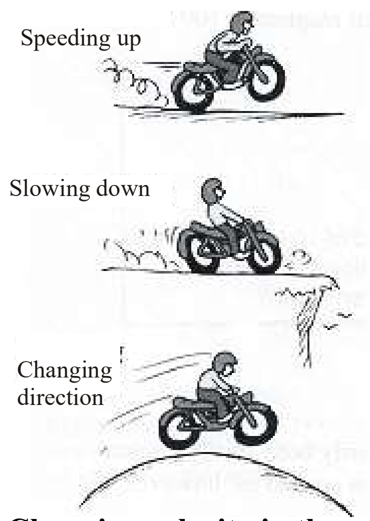
boat's velocity + river's velocity = total velocity

Since these velocities are vectors, they must be added by the rules of vector mathematics, that is, as shown in the vector diagram. We get the answer by solving the triangle. The magnitude of the total velocity of the boat is the hypotenuse of the triangle. Using the theorem of Pythagoras, we obtain

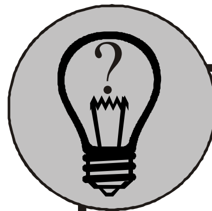
$$v_{total}^2 = (4.0\ m/s)^2 + (5.5\ m/s)^2 \; ; \qquad \text{from which} \quad v_{total} = 6.8\ m/s$$

The boat will obviously not travel directly eastward, but at some angle south of east. To find the angle, use the tangent function: $\tan\theta = \frac{5.5\ m/s}{4.0\ m/s}$, which gives $\theta = 54°$ (from trigonometry table).

The total velocity of the boat is therefore v = 6.8 meters per second 54° south of east.



Changing velocity in three ways



Q.1 Which of the following could not be a unit of speed ? km/h, s/m, mph, m/s, m s.

Q.2 Mental arithmetic : A runner travels 400m in 50s. What is her average speed ?

Q.3 Mental arithmetic : How far will a bus travel in 30s at a speed of 15 m/s ?

(v) **Acceleration :** Acceleration is the rate of change of velocity of time. It is velocity (a vector), not speed (a scalar). Hence acceleration is also a vector.

Average acceleration is defined as the change in velocity divided by the time interval to make the change,

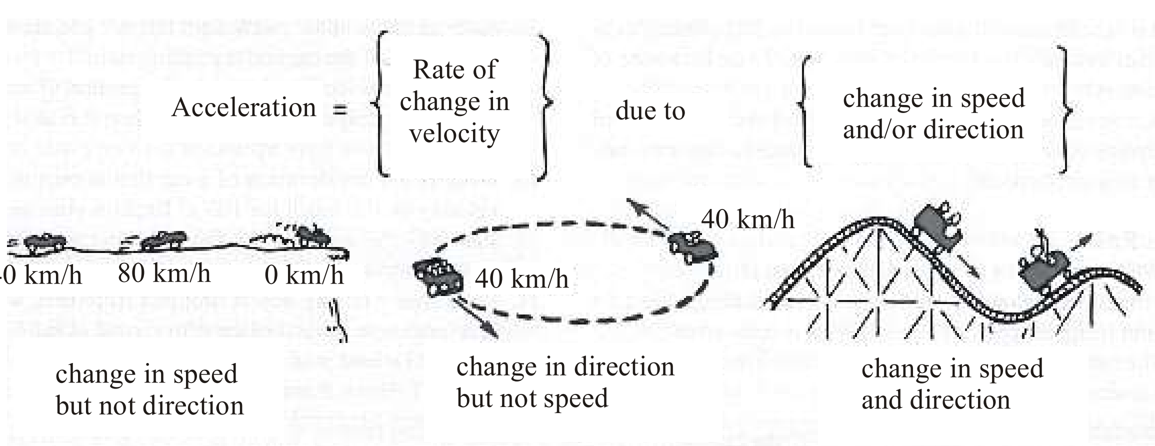
$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$, where $\vec{a}$ is average acceleration, Δv is change in velocity, and Δt is time interval.

Instantaneous acceleration is the acceleration at a particular instant of time (Δt is close to zero). As noted, velocity is a vector quantity, so are average acceleration and instantaneous acceleration. The SI units of acceleration are m/s/s or m/s^2.

Types of Acceleration :

(i) **Positive acceleration :** If the velocity of an object increases in the same direction, the object has a positive acceleration.

(ii) **Negative acceleration (Retardation) :** If the velocity of a body decreases in the same direction, the body has a negative acceleration or it is said to be retarding eg : A train slows down.



Three ways to accelerate

Misconception : A common misconception about velocity and acceleration has to do with their directions. Since velocity has both magnitude and direction, a change in either magnitude (speed) and/or direction will result in a change in velocity, therefore an acceleration. We can accelerate objects either by speeding them up or down (change magnitude) and/or by changing their directions of travel.

For motion in one-dimension, when the velocity and acceleration of an object are in the same direction (they have the same directional signs), the velocity increases and the object speeds up (acceleration). When the velocity and acceleration are in opposite direction, the velocity decreases and the object slows down deceleration.

CHARACTERIZE THE MOTION OF A PARTICLE

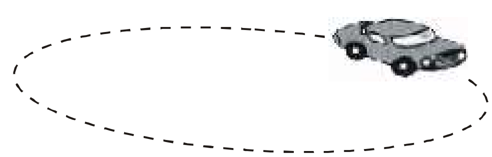
Let us first learn some variables:

Time (t) : How much time t has elapsed since some initial time. The initial time is often referred to as "the start of observations" and even more often assigned the value 0. We will refer to the amount of time t that has elapsed since time zero as the stopwatch reading. A time interval Δt (to be read "delta t") can be referred to as the difference between two stopwatch readings.

Position (x) : Where the object is along the straight line. To specify the position of an object on a line, one has to define a reference position (the start line) and a forward direction. Having defined a forward direction, the backward direction is understood to be the opposite direction. It is conventional to use the symbol x to represent the position of a particle.

The values that x have units of length. The SI unit of length is the meter. The symbol for the meter is m. The physical quantity x can be positive or negative where it is understood that a particle which is said to be minus five meters forward of the start line (more concisely stated as $x = -5$ m) is actually five meters behind the start line.

Velocity (v) : How fast and which way the particle is going. We use the symbol v for this and call it the velocity of the object. Because we are considering an object that is moving only along a line, the "which way" part is either forward or backward. Since there are only two choices, we can use an algebraic sign ("+" or "–") to characterize the direction of the velocity.



Speed is constant. Direction is changing.
Hence a velocity is not constant.

By convention, a positive value of velocity is used for an object that is moving forward, and, a negative value is used for an object that is moving backward.

Velocity has both magnitude and direction. The magnitude of a physical quantity that has direction is how big that quantity is, regardless of its direction. So the magnitude of the velocity of an object is how fast that object is going, regardless of which way it is going. Consider an object that has a velocity of 5 m/s. The magnitude of the velocity of that object is 5 m/s. Now consider an other object that has a velocity of -5 m/s. (It is going backward at 5 m/s.) The magnitude of its velocity is also 5 m/s. Another the name for the magnitude of the velocity is the speed. In both of the cases just considered, the speed of the object is 5 m/s despite the fact that in one case the velocity was –5 m/s. To understand the "how fast" part, just imagine that the object whose motion is under study has a built-in speedometer. The magnitude of the velocity, i.e. the speed of the object, is simply the speedometer reading.

Acceleration (a) : Next we have the question of how fast and which way the velocity of the object is changing. We call this the acceleration of the object. Instrumentally, the acceleration of a car is indicated by how fast and which way the tip of the speedometer needle is moving. In a car, it is determined by how far down the acceleration pedal is pressed, or, in the case of car that is slowing down, how hard the driver is pressing on the brake pedal. In the case of an object that is moving along a straight line, if the object has some acceleration, then the speed of the object is changing.

Average velocity : Consider a moving particle that is at position x_1 when the clock reads t_1 and at position x_2 when the clock reads t_2 .

The displacement of the particle is, by definition, the change in position $\Delta x = x_2 - x_1$ of the particle.

The average velocity (symbolically $\bar{v}$ or v_{avg} or $\langle v \rangle$) is, by definition, $\bar{v} = \dfrac{\Delta x}{\Delta t}$; where $\Delta t = t_2 - t_1$ is the change in clock reading. In the case of a constant velocity, to calculate the instantaneous velocity, all we have to do is calculate the average velocity, using any displacement with its corresponding time interval, that we want. Suppose we have position vs. time data, for instance, a car traveling a straight path at 24m/s. Some idealized fictitious data are given :

Data Reading Number	Time [seconds]	Position [meters]
0	0	0
1	0.100	2.30
2	1.00	23.0
3	10.0	230
4	100.0	2300

Note that for this special case of constant velocity, you get the same average velocity, the known value of constant speed, no matter what time interval we choose. For instance, if we choose the time interval from 1.00 seconds to 10.0 seconds:

$$\bar{v} = \frac{\Delta x}{\Delta t} \;;\; \bar{v} = \frac{x_3 - x_2}{t_3 - t_2} \;;\; \bar{v} = \frac{230\text{m} - 23.0\text{m}}{10.0\text{s} - 1.00\text{s}} = 23.0 \text{ m/s}$$

and, if we choose the time interval 0.100 seconds to 100.0 seconds:

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \text{or} \quad \bar{v} = \frac{x_4 - x_1}{t_4 - t_1} \quad \text{or} \quad \bar{v} = \frac{2300\text{m} - 2.30\text{m}}{100.0\text{s} - 0.100\text{s}} \quad \text{or} \quad \bar{v} = 23.0 \text{ m/s}$$

The points that need emphasizing here is, if the velocity is constant then the calculation of the average speed yields the instantaneous speed (the speedometer reading, the speed we have an intuitive feel for), and, when the velocity is constant, it doesn't matter what time interval's used to calculate the average velocity; in particular, a small time interval works just as well as a big time interval.

So how do we calculate the instantaneous velocity of an object at some instant when the instantaneous velocity is continually changing? Let's consider a case in which the velocity is continually increasing. Here we show some idealized fictitious data (consistent with the way an object really moves) for just such a case.

Data Reading Number	Time since object was at start line. [s]	Position (distance ahead of start line) [m]	Velocity (This is what we are trying to calculate.Here are the correct answers) [m/s]
0	0	0	10
1	1	14	18
2	1.01	14.1804	18.08
3	1.1	15.84	18.8
4	2	36	26
5	5	150	50

What we want to do with this fictitious data is to calculate an average velocity during a time interval that begins with t = 1 s and compare the result with the actual velocity at time t = 1 s. The plan is to do this repeatedly, with each time interval used being smaller than the previous one.

Average velocity from t = 1 s to t = 5 s:

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \text{or} \quad \bar{v} = \frac{x_5 - x_1}{t_5 - t_1} \quad \text{or} \quad \bar{v} = \frac{150\text{m} - 14\text{m}}{5\text{s} - 1\text{s}} \quad \text{or} \quad \bar{v} = 34 \text{ m/s}$$

Note that this value is quite a bit larger than the correct value of the instantaneous velocity at t = 1s (namely 18 m/s). It does fall between the instantaneous velocity of 18 m/s at t = 1s and the instantaneous velocity of 50 m/s at t = 5 seconds. That makes sense since, during the time interval, the velocity takes on various values which for 1s < t < 5 s are all greater than 18 m/s but less than 50 m/s.

For the next two time intervals in decreasing time interval order (calculations not shown):

Average velocity from t = 1 to t = 2 s: 22 m/s, Average velocity from t = 1 to t = 1.1 s: 18.4 m/s

And for the last time interval, we do show the calculation:

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \text{or} \quad \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} \quad \text{or} \quad \bar{v} = \frac{14.1804\text{m} - 14\text{m}}{1.01\text{s} - 1\text{s}} \quad \text{or} \quad \bar{v} = 18.04 \text{ m/s}$$

Average velocity from t = 1 to t = 5 s: 34 m/s, Average velocity from t = 1 to t = 2 s: 22 m/s

Average velocity from t = 1 to t = 1.1 s: 18.4 m/s, Average velocity from t = 1 to t = 1.01 s: 18.04 m/s

Every answer is bigger than the instantaneous velocity at t = 1s (namely 18 m/s). Why?

Because the distance traveled in the time interval under consideration is greater than it would have been if the object moved with a constant velocity of 18 m/s. Why? Because the object is speeding up, so, for most of the time interval the object is moving faster than 18 m/s, so, the average value during the time interval must be greater than 18 m/s. But notice that as the time interval (that starts at t = 1s) gets smaller and smaller, the average velocity over the time interval gets closer and closer to the actual instantaneous velocity at t = 1s. By induction, we conclude that if we were to use even smaller time intervals, as the time interval we chose to use was made smaller and smaller, the average velocity over that tiny time interval would get closer and closer to the instantaneous velocity, so that when the time interval got to be so small as to be virtually indistinguishable from zero, the value of the average velocity would get to be indistinguishable from the value of the instantaneous velocity.

Note : (i) In mechanics while studying the motion of an object, sometimes its dimensions are of no importance and the object may be treated as a point object without much error.

(ii) A mistake that is often made in linear motion problems involving acceleration, is using the velocity at the end of a time interval as if it was valid for the entire time interval. The mistake crops up in constant acceleration problems when students try to use the definition of average velocity $\overline{v} = \frac{\Delta x}{\Delta t}$ in the solution.

Unless we are asked specifically about average velocity, you will never need to use this equation to solve a physics problem. Avoid using this equation it will only get you into trouble. For constant acceleration problems, use the set of constant acceleration equations.

ACTIVITY -1

Purpose : To compute the average speed of at least three different and to participate in at least one race.

Discussion : In this activity, you will need to think about what measurements are necessary to make in order to compute the average speed of an object. How does the average speed you compute compare with the maximum speed ? How could you find the maximum speed of a runner or a car between stoplights ?

Procedure : Work in groups of about three students. Select instruments to measure distance and time. Develop a plan that will enable you to determine speed. Two students race each other in races such as hopping on one foot, rolling on the lawn, or walking backward. The third student collects and organizes data to determine the average speed of each racer. Repeat this process until each member of your group has a chance to be the timer. For the race in which you are the timer, record your plan and the type of race. When measurements are to be made in an experiment, a good experimenter organizes a table showing all data, not just the data that "seem to be right." Record your data in Data Table. Show the units you used as well as the quantities. For each measurement, record as many digits as you can read directly from the measuring instrument, plus one estimated digit. Then calculate the average speed for each student.

Activity	Distance	Time	Speed

Analysis :

1. How does average speed relate to the distance covered and the time taken for travel ?
2. Should the recorded average speed represent the maximum speed for each event ? Explain.
3. Does your measurement technique for speed enable you to measure the fastest speed attained during an event

ACTIVITY -2

Take a plastic bag. Make a pinhole near its bottom and fill it with water. Hold the bag in your hand and observe water dripping from the pinhole. You may have to adjust the quantity of water in the bag or the size of the pinhole to make sure that water drips from it drop by drop. Holding this bag in one hand, walk as fast as you can along a straight path on a concrete floor. Make sure that the hand holding the bag remains steady while you are walking. The water drops that fall on the surface will make their mark with a patch.

Each patch or dot of water on the floor will mark your position at a given time with respect to the release of a water drop. The time interval between two consecutive drops of water can be considered to be nearly the same. Therefore, the distance between two consecutive patches or dots of water on the floor would give the distance moved by you in nearly equal intervals of time.

The pattern of dots obtained in the above activity can be utilized to find out the distance moved in a fixed interval of time. Measure the distance between the consecutive dots in this pattern and note your observations. The pattern of dots obtained in a similar activity is shown in Fig., the corresponding measurements are given in Table.

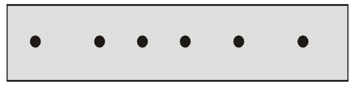


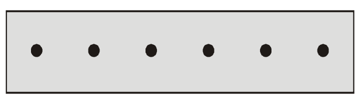
Fig. : Dots showing position of water drop at different instants of time

Table : Positions of water drops at equal intervals of time and distance between two consecutive dots

S.N.	Distance of the dot from starting point dots	Distance between two consecutive
1	0m	
2	1.5 m	1.5 m
3	2.7m	1.2 m
4	3.9m	1.2 m
5	5.2m	1.3 m
6	6.7m	1.5m

You can obtain a few more sets of pattern of dots by repeating the above activity. You may increase or slow down your pace of walking or you may even use a bicycle to move. If you study the pattern of dots obtained on different occasion, you will find that the distance moved in equal intervals of time is not always the same. Your motion in such a case is an example of a non-uniform motion. A body is said to be in non-uniform motion if the distance moved by it in equal intervals of time is not the same. Most of the motions, which we observe around us, are non-uniform in nature.

On the other hand, if a body covers equal distances in equal intervals of time it is said to be in uniform motion. Figure shows the pattern of dots in the case of a uniform motion.



In uniform motion, the distance covered in equal intervals of time should be same whether the duration of time is small or large. For example, if a body in uniform motion covers 100 m in 50 s then it should move 10 m in every 5 s, 1.0 m in every 0.5 s, 0.1 m in every 0.05 s and so on. We rarely observe uniform motion in everyday life. Thus, to describe motion we need to classify it as uniform or non uniform motion.

UNIFORM AND NON-UNIFORM MOTION

Uniform rectilinear motion : It is a motion in which a material point moves in a straight line and covers equal distances in equal intervals of time. The path length of a body in a uniform rectilinear motion is equal to the magnitude of the displacement. Consequently, the path length in the motion is equal to the magnitude of the velocity multiplied by the time : $s = vt$.

for the x-coordinate of the body at instant t : $x = x_0 + s = x_0 + vt$

Nonuniform motion : It is a motion in which the velocity varies with time. The change in the velocity of a material point in nonuniform motion is characterized by acceleration. Uniformly variable motion is a motion with a constant acceleration. Uniformly variable motion can be curvilinear like circular motion. If a uniformly variable motion is rectilinear, i.e., the velocity v changes only in magnitude, it is convenient to take the straight line in which a material point moves as one of the coordinate axes (say, the x-axis).

Non-uniform acceleration : If during motion of a body its velocity increases by unequal amounts in equal intervals of time.

Solving a Numerical : General method of approaching numerical problems.

1. Draw a 'sketch' diagram wherever possible.
2. Copy down the numerical information given in the question.
3. Write down the relevant formula.
4. Substitute the given values into the formula.
5. Calculate the answer, remembering to show all steps in the working out and giving the correct units for our final answer.

Example 1 :

An object moving to the right has a decrease in velocity from 5.0 m/s to 1.0 m/s in 2.0 s. What is the average acceleration ? What does your result mean ?

Sol. Given : $v_0 = +5.0$ m/s, $v = +1.0$ m/s, $t = 2.0$s. Find $\vec{a}$

According to the definition of average acceleration,

$$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = \frac{+1.0\ m/s - (+5.0\ m/s)}{2.0s} = \frac{-4.0\ m/s}{2.0\ s} = -2.0\ m/s^2$$

The negative sign means the acceleration is opposite to velocity (deceleration). The result means that the object decreases its velocity by 2.0 m/s every s or 2.0 m/s^2.

Example 2 :

A car covers the 1st half of the distance between two places at a speed of 40 km/hr and the 2nd half at 60km/hr what is the average speed of the car ?

Sol. Suppose the total distance covered is 25. Then time taken to cover first's distance with speed 40 km/hr.

$t_1 = S/40$ hrs.

Time taken to cover second S distance with speed 60 km/hrs., $t_2 = S/60$ hrs.

$$V_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{S+S}{\left(\frac{S}{40}+\frac{S}{60}\right)} \quad \text{or} \quad V_{av} = \frac{2S}{\left(\frac{3S+2S}{120}\right)} = \frac{2S}{5S}\times 120 \quad \text{or} \quad V_{av} = 48\ \text{km/hrs.}$$

Example 3 :

The table below shows the distance in cm, travelled by the objects A, B and C during each second.

Time	Distance (in cm) covered in each second by A, B and C		
	Object A	Object B	Object C
1st second	20	20	20
2nd second	20	36	60
3rd second	20	24	100
4th second	20	40	140
5th second	20	48	180

(i) Which object is moving with constant speed ? Give a reason for your answer.
(ii) Which object is moving with a constant acceleration ? Give a reason.
(iii) Which object is moving with irregular acceleration ?

Sol. The object A is moving with constant speed. The reason is that it covers equal distance = 20 cm. in each second.

(i) The object A is moving with constant speed. The reason is that it covers equal distance = 20cm. in each sound.

(ii) The object C is moving with a constant acceleration. The reason is that for the object C, the distance covered increases by the same amount in each second. It can further be verified by drawing graph between S (total distance covered) and t^2 (square of time taken). The graph will be a straight line.

t^2	1	4	9	16	25
S	20	80	180	320	500

(iii) The object B is moving with irregular acceleration.

SELF CHECK

Q.1 A cheetah is the fastest land animal and can achieve a peak velocity of 100 km/h up to distances less than 500m. If a cheetah spots his prey at a distance of 100m, what is the minimum time it will take to get its prey, if the average velocity attained by it is 90km/h.

Q.2 A sprinter in a 100m race, covers 4m in first second, 30m in next 4s, 52m in another 4s and finishes the race in 10s.

(a) Calculate the average velocity of the sprinter.

(b)What is the peak velocity attained by the sprinter ?

(c) During which time-interval is the acceleration highest ?

Q.3 An athlete runs along a circular tracks of diameter 28m. The displacement of the athlete after he completes more one circle is –

(1) 28m (2) 88m (3) 44m (4) zero

Q.4 A body is moving with uniform velocity of 10 m/s. The velocity of the body after 10s is –

(1) 100 m/s (2) 50 m/s (3) 10 m/s (4) 5 m/s

Q.5 A particle covers equal distance in equal intervals of time, it is said to be moving with uniform –

(1) speed (2) velocity (3) acceleration (4) force

ANSWERS

(1) 4 sec (2) (a) 10 m/s (b) 14 m/s (c) second and third time interval

(3) 4 (4) 3 (5) 1

KINEMATIC EQUATIONS

Kinematic equations can be used to describe the motion with constant acceleration.

The symbols used in the kinematic are : v_0 or u initial velocity; v, final velocity; a, acceleration; x, displacement; t, time interval. Be aware that the terms initial and final are relative. The end of one event is always the beginning of another. There are three general equations and two algebraic combinations of these equations that provide calculation convenience.

$x = \bar{v}t$, displacement = average velocity × times interval

$\bar{v} = \dfrac{v+u}{2}$, average velocity = (final velocity + initial velocity) / 2

First equation : $v = u + at$, final velocity = initial velocity + acceleration × time interval, By definition, Acceleration $= \frac{\text{change in velocity}}{\text{time taken}} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$

or $a = \frac{v-u}{t}$ or $at = v - u$ or $v = u + at$ .... (1)

Second eq. : $x = ut + \frac{1}{2}at^2$, Displacement = initial velocity × time interval + $\frac{1}{2}$ × acceleration × time interval squared

Distance travelled = Average velocity × time $= \left(\frac{\text{Initial velocity+final velocity}}{2}\right) \times \text{time}$ or $S = \frac{u+v}{2} \times t$

But from eq. (1), $v = u + at$ $\therefore$ $x = \frac{u + (u + at)}{2} \times t$ or $x = \frac{2u + at}{2} \times t$ or $x = ut + \frac{1}{2}at^2$ .... (2)

Third eq. : $v^2 = u_0^2 + 2ax$, final velocity squared = initial velocity squared + 2 × acceleration × displacement

Distance travelled = Average velocity × time

$S = \frac{u+v}{2} \times t$ But from eq. (1), $v = u + at$ or $t = \frac{v-u}{a}$ $\therefore$ $S = \frac{u+v}{2} \times \frac{v-u}{a}$

or $S = \frac{v^2 - u^2}{2a}$ or $v^2 - u^2 = 2aS$ or $v^2 = u^2 + 2ax$ .... (3)

Three kinematic equations can be used to solve the majority of kinematic problems.
Which equation should you select for a particular problem ? The equation you select must have the unknown quantity in it and everything else must be given, because we can only solve for one unknown in one equation.

Distance covered by body in n^{th} second :

$S = ut + \frac{1}{2}at^2$, is the distance covered by a body in t sec. or $S_n = un + \frac{1}{2}an^2$............ (i) distance covered by a body along straight line in n sec.

$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$............ (ii) distance covered by a body along straight line in sec. in (n–1) sec.

$\therefore$ the distance covered by the body in n^{th} second will be $s_{nth} = s_n - s_{n-1}$

$\therefore$ $S_n th = un + \frac{1}{2}an^2 - \{u(n-1) + \frac{1}{2}a(n-1)^2\}$

$S_n th = un + \frac{1}{2}an^2 - \{nu - u + \frac{1}{2}a(n^2 + 1 - 2n)\} \Rightarrow S_n th = u + \frac{a}{2}(2n-1)$

Tips to solve problem on kinematic equations :

1. Make a drawing to represent the situation being studied.

2. Decide which directions are to be called positive (+) and negative (–) relative to a conveniently chosen coordinate origin. Do not change your decision during the course of a calculation.

3. In an organized way, write down the values (with appropriate plus and minus signs) that are given for any of the five kinematic variables (x, a, v, v_0 and t).
4. Before attempting to solve a problem, verify that the given information contains values for at least three of the five kinematics variables.
5. When the motion of an object is divided into segments, remember that the final velocity of one segment is the initial velocity for the next segment.

Example 4 :

An automobile accelerates uniformly from rest to 25 m/s while traveling 100m. What is the acceleration of the automobile ?

Sol. Given : $v_0 = 0$ (rest), $v = 25$ m/s, $x = 100$m. Find a.

Since $v^2 = v_0^2 + 2ax$, $a = \frac{v^2 - v_0^2}{2x} = \frac{(25\ m/s)^2 - (0)^2}{2\ (100m)} = 3.1\ m/s^2$

Since a is positive, it is in the direction of the velocity or motion.

Example 5 :

A car is moving at a speed 50 km/h. Two seconds there after it is moving at 60 km/h. Calculate the acceleration of the car.

Sol. Here, $v_0 = 50$ km/h $= 50 \times \frac{5}{18}\ m/s = \frac{250}{18}\ m/s$ and $v = 60$ km/h $= 60 \times \frac{5}{18} = \frac{300}{18}\ m/s$

Since $a = \frac{v - v_0}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{\frac{50}{18}}{2} = \frac{50}{36} = 1.39\ m/s^2$

Example 6 :

A bus moving with a velocity of 60 km/h is brought to rest in 20 seconds by applying brakes. Find its acceleration.

Sol. $v_0 = 60$ km/h $= 60 \times \frac{5}{18} = \frac{50}{3}\ m/s$; $v = 0$ (as bus comes to rest)

as $a = \frac{v - v_0}{t}$ $\therefore$ $a = \frac{0 - \frac{50}{3}}{20} = \frac{-50}{3 \times 20} = \frac{-5}{6} = -0.83\ m/s^2$

Example 7 :

A bullet moving with 10 m/s hits the wooden plank the bullet is stopped when it penetrates the plank 20 cm. deep calculate retardation of the bullet.

Sol. $v_0 = 10$ m/s, $v = 0$ & $s = 20$ cm. $= \frac{2}{100} = 0.02$m

Using $v^2 - v_0^2 = 2ax$; $0 - (10)^2 = 2a\ (0.2) \Rightarrow \frac{-100}{2 \times 0.02} = a$ or $a = -2500\ m/s^2$

Retardation = $2500\ m/s^2$

Example 8 :

A body covers a distance of 20m in the 7th second and 24m in the 9th second. How much distance shall it cover in 15th sec.

Sol. $S_7th = u + \frac{a}{2}(2 \times 7 - 1)$ but $S_7th = 20m$

$\therefore \; 20 = u + \frac{a}{2} \times 13 \Rightarrow 20 = u + \frac{13a}{2}$ .... (1) also $s_9th = 24m \;\therefore\; 24 = u + \frac{17a}{2}$ .... (2)

from (1) equation $u = 20 - \frac{13a}{2}$ .... (3)

Substitute this value in (2)

$24 = 20 - \frac{13a}{2} + \frac{17a}{2}$; $24 - 20 = \frac{17a}{2} - \frac{13a}{2}$; $4 = \frac{4a}{2} \Rightarrow 4 = 2a \Rightarrow a = \frac{4}{2} = 2\ m/s^2$

Use this value in (3) $u = 20 - \frac{13a}{2}$ $\therefore$ $u = 20 - \frac{13 \times 2}{2} \Rightarrow u = 20 - 13$ $\therefore\; u = 7\ m/s$

Now, $s_{15}th = u + \frac{a}{2}(2 \times 15 - 1) = 7 + \frac{2}{2}(29) = 7 + 29 = 36m$

Example 9 :

A body with an initial velocity of 18 km/hr accelerates uniformly at the rate of $9cms^{-2}$ over a distance of 200m. Calculate :

(i) the acceleration in ms^{-2}. (ii) its final velocity in ms^{-1}

Sol. (i) Acceleration $= 9\ cm\ s^{-2} = \frac{9}{100} ms^{-2} = 0.09\ ms^{-2}$

(ii) Initial velocity $u = 18\ km\ h^{-1} = \frac{18000m}{60 \times 60s} = 5\ ms^{-1}$; Acceleration, $a = 0.09\ ms^{-2}$ and distance $S = 200m$

From equation of motion $v^2 = u^2 + 2aS$; $v^2 = (5)^2 + 2 \times 0.09 \times 200$ or $v^2 = 25 + 36 = 61$

$\therefore\; v = \sqrt{61} = 7.81\ ms^{-1}$. Thus, final velocity $= 7.81\ ms^{-1}$.

SELF CHECK

Q.1 In outer space a rocket ship is traveling at the enormous speed of 2800 meters per second. What is its acceleration if it increases its speed uniformly and is going 2840 meters per second after 25 seconds ?

Q.2 What is the acceleration of a rocket-driven sled that travels 360 meters in 8.3 seconds, starting at 22 meters per second ?

Q.3 In each of the following cases, state the direction of the acceleration :

(a) A rising elevator is coming to rest (b) A car going west speeds up

(c) A car going east slows down (d) A bicycle going southward starts to turn left.

Q.4 The average time taken by a normal person to react to an emergency is one fifteenth of a second and is called the 'reaction time'. If a bus is moving with a velocity of 60 km/h and its driver sees a child running across the road, how much distance would the bus had moved before he could press the brakes?

The reaction time of the people increases when they are intoxicated. How much distance has the bus moved if the reaction time of the driver were ½s under the influence of alcohol ?

Q.5 A car increase its speed from 20 km/hr. to 50 km/hr. in 10 sec, its acceleration is –
(1) 30 m/s^2 (2) 3 m/s^2 (3) 18 m/s^2 (4) 0.83 m/s^2

Q.6 A car accelerates uniformly from 18 km/h to 36 km/h in 5s. The distance travelled by the car is –
(1) 25.5 m (2) 27.5m (3) 32.5 m (4) 37.5 m

Q.7 A body is moving along a straight line at 20 m/s under goes an acceleration of 4 m/s^2. After 2s, its speed will be –
(1) 8 m/s (2) 12 m/s (3) 16 m/s (4) 28 m/s

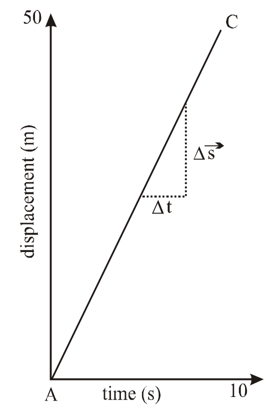
ANSWERS

(1) 1.6 m/s^2 **(2)** a = 5.2 m/s^2 **(3)** (a) Down, when slowing down, a is opposite v.
(b) West, when speeding up, a and v are in the same direction. (c) West
(d) East, in the direction of the turn. **(4)** 1.1m, 8.33m **(5)** 4 **(6)** 4 **(7)** 4

GRAPHICAL INTERPRETATION

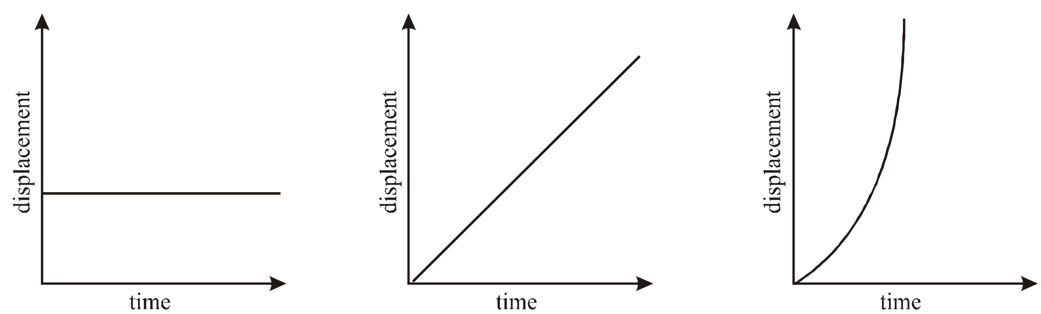
In physics we often use graphs as important tools for picturing certain concepts. Below are some graphs that help us picture the concepts of displacement, velocity and acceleration.

Displacement-Time Graphs : Below is a graph showing the displacement of the cyclist from A to C:



This graphs shows us how, in 10 seconds time, the cyclist has moved from A to C. We know the gradient (slope) of a graph is defined as the change in y divided by the change in x, i.e Δy/Δx. In this graph the gradient of the graph is just $\frac{\Delta\vec{s}}{\Delta t}$ and this is just the expression for velocity.

The slope of a displacement-time graph gives the velocity.
The slope is the same all the way from A to C, so the cyclist's velocity is constant over the entire displacement he travels.
Observe the following displacement-time graphs.



(a) (b) (b)

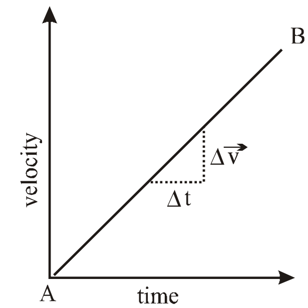
(a) Shows the graph for an object stationary over a period of time. The gradient is zero, so the object has zero velocity.

(b) Shows the graph for an object moving at a constant velocity. You can see that the displacement is increasing as time goes on. The gradient, however, stays constant (remember is the slope of straight line) so the velocity is constant. Here the gradient is positive, so the object is moving in the direction we have defined as positive.

(c) Shows the graph for an object moving at a constant acceleration. You can see that both the displacement and the velocity (gradient of the graph) increase with time. The gradient is increasing with time, thus the velocity is increasing with time and the object is accelerating.

Velocity-Time Graphs : Look at the velocity-time graph :

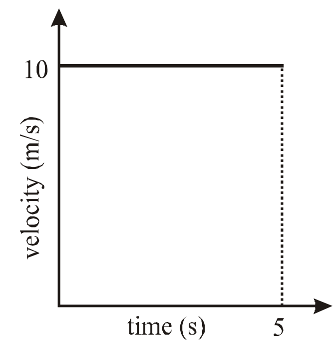
This is the velocity-time graph of a cyclist traveling from A to B at a constant acceleration, i.e. with steadily increasing velocity. The gradient of this graph is just $\frac{\Delta\vec{v}}{\Delta t}$ and this is just the expression for acceleration. Because the slope is the same at all points on this graph, the acceleration of the cyclist is constant.



The slope of a velocity-time graph gives the acceleration.

We can also calculate displacement traveled from velocity-time graph.

This graph shows an object moving at a constant velocity of 10m/s for a duration of 5s. The area between the graph and the time axis of the above plot will give us the displacement of the object during this time. In this case we just need to calculate the area of a rectangle with width 5s and height 10m/s



Area of rectangle = height × width

$= \vec{v} \times t$

= 10 m/s × 5s

= 50 m = $\vec{s}$ = displacement

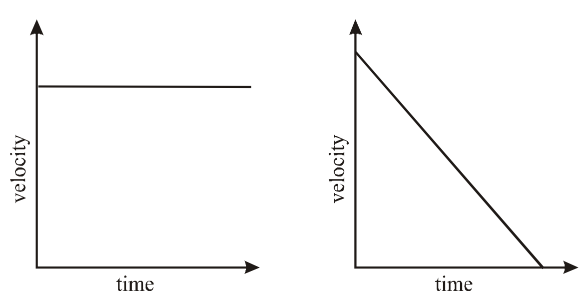
So, here we've shown that an object traveling at 10m/s for 5s has undergone a displacement of 50m.

The area between a velocity-time graph and the 'time' axis gives the displacement of the object.

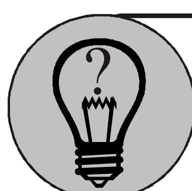
Observe following velocity-time graphs.

(a) Shows the graph for an object moving at a constant velocity over a period of time. The gradient is zero, so the object is not accelerating.

(b) Shows the graph for an object which is decelerating. You can see that the velocity is decreasing with time. The gradient, however, stays constant (remember: its the slope of a straight line), so the acceleration is constant. Here the gradient is negative, so the object is accelerating in the opposite direction to its motion, hence it is decelerating.



(a) **(b)**



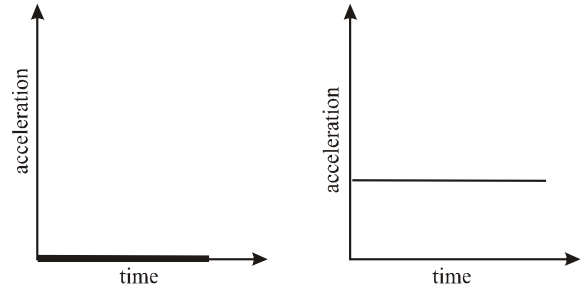
Q.1 Sketch a distance-time graph that represents the following motion : a trolley moves at a steady speed for a short time, then it moves an equal distance, speeding up.

Q.2 A car is moving along a road. Its distance-time graph gradually becomes steeper. Is the car speeding up, slowing down, or traveling at a steady speed ?

Q.3 Scientists have calculated the distance between the earth and the moon by reflecting a beam of laser light off the moon. They measured the time taken for the laser light to travel to the moon and back. What other piece of information is needed to calculate the Earth-Moon distance ? How would the distance be calculated ?

Acceleration-Time Graphs :

In this chapter on rectilinear motion we will only deal with objects moving at a constant acceleration, thus all acceleration-time graphs will look like these two:



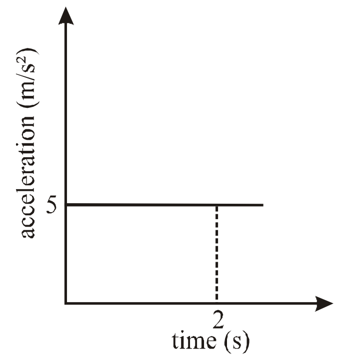
(a) (b)

Description of the graphs :

(a) shows the graph for an object which is either stationary or traveling at a constant velocity. Either way, the acceleration is zero over time.

(b) Shows the graph for an object moving at a constant acceleration. In this case the acceleration is positive - remember that it can also be negative.

We can obtain the velocity of a particle at some given time from an acceleration time graph-it is just given by the area between the graph and the time-axis. In the graph below, showing an object at a constant positive acceleration, the increase in velocity of the object after 2 seconds corresponds to the portion.



Area of rectangle $= \vec{a} \times t = 5\dfrac{m}{s^2} \times 2s = 10\dfrac{m}{s} = \vec{v}$

Its useful to remember the set of graphs below when working on problems. Figure shows how displacement, velocity and time relate to each other. Given a displacement-time graph like the one on the left, we can plot the corresponding velocity-time graph by remembering that the slope of a displacement-time graph gives the velocity. Similarly, we can plot an acceleration-time graph from the gradient of the velocity-time graph.

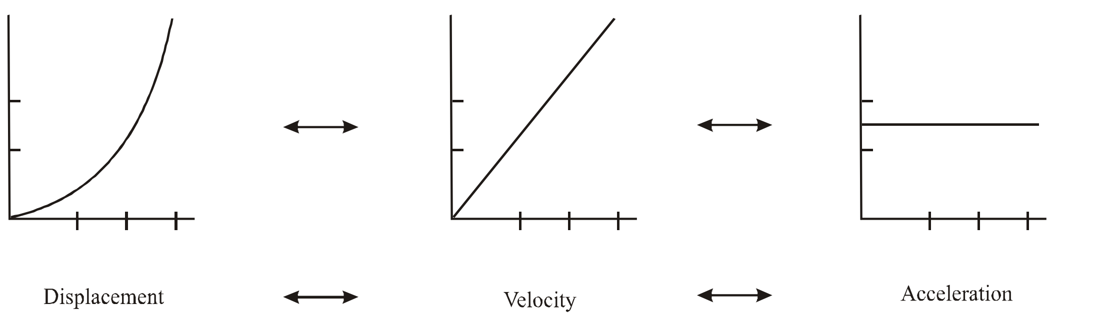


Figure : A Relationship Between Displacement, Velocity and Acceleration

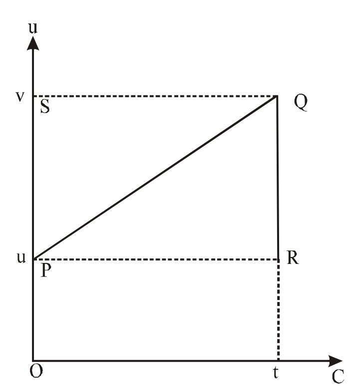
GRAPHICAL DERIVATION OF EQUATION OF MOTION

(i) First Equation :

$v = u + at$

It can be derived from u-t graph, as shown in graph.

From line PQ. The slope of the line = acceleration a

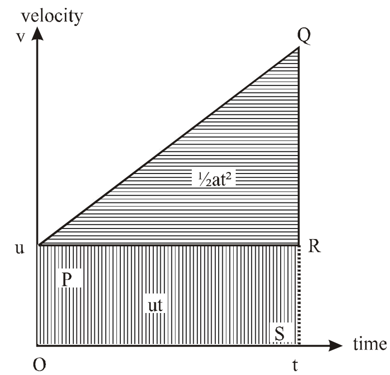


$$a = \frac{QR}{RP} = \frac{SP}{RP} \qquad \text{or} \qquad SP = a\,RP = at$$

As OS = OP + PS. Putting values, $v = u + at$

(ii) Second Equation

$$S = ut + \frac{1}{2}at^2$$



It can also be derived from u-t graph as shown in figure.
From relation,
Distance covered = Area under u-t line
S = Area of trapezium OPQS
= Area of rectangle OPRS + Area of triangle PQR

$$= OP \times PR + \frac{RQ \times PR}{2}$$

Putting values, $S = u \times t + \frac{1}{2}(v - u) \times t$ $(\because RQ = v - u \ \& \ PR = OS = t)$

$$= u \times t + \frac{1}{2}at \times t \qquad (\because v - u = at) \text{ or } S = ut + \frac{1}{2}at^2$$

(iii) Third equation : $v^2 = u^2 + 2aS$

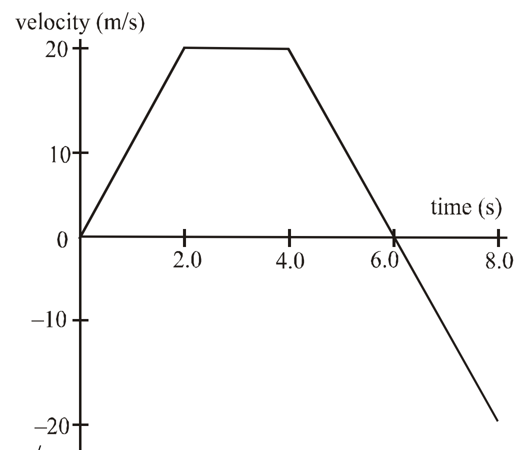
From above graph, OP = u, SQ = v, OP + SQ = u + v

$$a = \frac{QR}{PR} \quad \text{or} \quad PR = \frac{QR}{a} = \frac{v-u}{a} \ ; S = \text{Area of trapezium OPQS} = \frac{OP + SQ}{2} \times PR$$

On putting the values, $S = \frac{v+u}{2} \times \frac{v-u}{a} = \frac{v^2 - u^2}{2a}$ or $v^2 = u^2 + 2aS$

Example 10 :

The graph represents the velocity of a particle as a function of time.



(a) What is the acceleration at 1.0 s ?
(b) What is the acceleration at 3.0 s ?
(c) What is the average acceleration between 0 and 5.0 s ?
(d) What is the average acceleration for the 8.0 s interval ?
(e) What is the displacement for the 8.0 s interval ?

Sol. (a) Acceleration is the slope of the line, $a = \frac{\Delta v}{\Delta t} = \frac{20\ m/s - 0\ m/s}{2.0\ s - 0\ s} = 10\ m/s^2$

(b) The slope of the line is zero and a = 0.

(c) $\vec{a} = \frac{\Delta v}{\Delta t} = \frac{10\ m/s - 0\ m/s}{5.0\ s - 0\ s} = 2.0\ m/s^2$ **(d)** $a = \frac{\Delta v}{\Delta t} = \frac{-20\ m/s - 0\ m/s}{8.0\ s - 0\ s} = -2.5\ m/s^2$

(e) The net area equals the displacement.

The area of a rectangle is length × width and area of a triangle is $\frac{1}{2}$ × base × height

$$\Delta x_{0-2} = \frac{1}{2}(2.0\,s - 0\,s)\,(20\,m/s) = 20\,m \quad ; \quad \Delta x_{2-4} = (4.0\,s - 2.0\,s)\,(20\,m/s) = 40\,m$$

$$\Delta x_{4-6} = \frac{1}{2}(6.0\,s - 4.0\,s)\,(20\,m/s) = 20\,m \quad ; \quad \Delta x_{6-8} = \frac{1}{2}(8.0\,s - 6.0\,s)\,(-20\,m/s) = -20\,m$$

So, $\Delta x = 20m + 40m + 20m + (-20m) = 60m$

Example 11 :

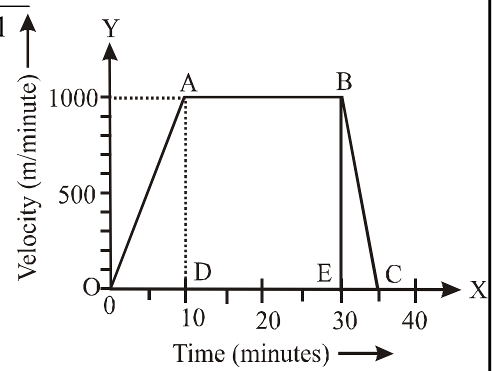
A train starts from rest and accelerates uniformly at 100 minute^{-2} for 10 minutes. It then maintains a constant velocity for 20 minutes. The brakes are then applied and the train is uniformly retarded. It comes to rest in 5 minutes. Draw a velocity-time graph use it to find :

(i) the maximum velocity reached **(ii)** the retardation in the last 5 minutes

(iii) total distance travelled, and **(iv)** the average velocity of the train

Sol. The velocity-time graph is shown in figure.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time interval}} = \frac{\text{Final velocity} - 0}{\text{Time interval}}$$



or Final velocity = acceleration × time interval

$$= \frac{100m}{\text{minute}^2} \times 10\text{ minute} = 1000m\text{ minute}^{-1}$$

(i) The maximum velocity reached = 1000m minute^{-1}.

(ii) The retardation in the last 5 minutes = slope of the line BC.

$$= \frac{BE}{EC} = \frac{(0-1000)\text{ m minute}^{-1}}{(35-30)\text{ minute}} = \frac{-1000\text{ m minute}^{-1}}{5\text{ minute}} = -200m\text{ minute}^{-2}$$

(iii) Total distance travelled = Area of trapezium OABC

$$= \frac{1}{2}(OC + AB) \times AD = \frac{1}{2}(35+20) \times 1000 = 55 \times 500 = 27500\text{ m (or 27.5 km)}$$

(iv) Average velocity $= \frac{\text{Total distance travelled}}{\text{Total time of trave}} = \frac{27500m}{35\text{ minute}} = 785.7\text{ m minute}^{-1}$

ACTIVITY-3

Amitabh and his sister Archana go to the school on their bicycles. Both of them start at the same time from their home but take different time to reach the school although they follow the same route. Table shows the distanced moved by them at different instants of time. Plot the distance-time graph for their motion on the same graph paper assuming a scale of your choice.

Table : Distance covered by Amitabh and Archana at different time on their bicycles

Time	8:00 am	8.05 am	8:10 am	8:15 am	8:20 am	8:25 am
Distance moved by Amitabh	0 km	1.0 km	1.9 km	2.8 km	3.6 km	
Distance moved by Archana	0 km	0.8 km	1.6 km	2.3 km	3.0 km	3.6 km

Compare the distance-time graph obtained by you with that shown in figure. You will find that the shape of the graph for a given set of data does not change, if the scale is changed.

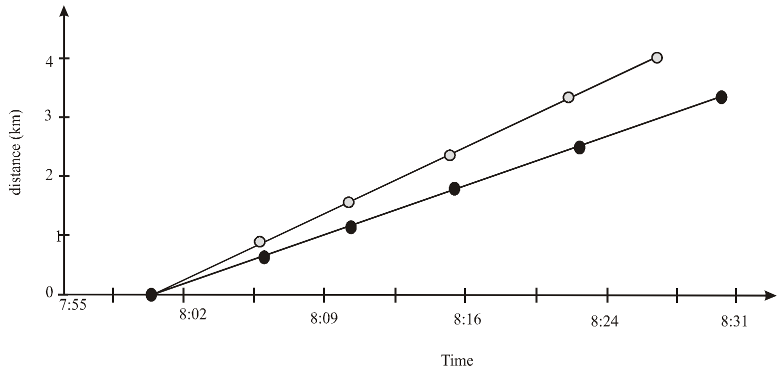


Figure : Distance-time graph for the motion of the two cyclists

SELF CHECK

Q.1 When a graph between one quantity versus another results in a straight line with positive slope, quantities are –
(1) both constant (2) equal (3) directly proportional (4) inversely proportional

Q.2 When the distance travelled by an object is directly proportional to the time, it is said to travel with –
(1) zero velocity (2) constant speed (3) constant acceleration (4) uniform velocity

Q.3 The graph plotted between two quantities is a straight line, the quantities are –
(1) equal (2) both constant (3) directly proportional (4) inversely proportional

Q.4 The distance-time graphs for two vehicles X and Y are straight lines. The slope of graph for X with time axis more than that for Y.
(1) The speed of X is more than that of Y (2) The speed of Y is more than that of X
(3) X is uniformly accelerated while Y is not
(4) The distance covered by Y is more than that by X in the same interval of time

Q.5 The velocity-time graph for a body moving with uniform velocity is a –
(1) straight line parallel to the velocity axis (2) straight line parallel to the time axis
(3) straight line passing through origin having a constant slope from time axis (4) curve

ANSWERS

(1) 3 **(2) 2** **(3) 3** **(4) 1** **(5) 2**

FREE FALL

Objects in motion solely under the influence of gravity are said to be in free fall. The magnitude of acceleration due to gravity is often expressed with a symbol g. Near the surface of the Earth, the acceleration due to gravity g = 9.80 m/s^2 (downward) and near the surface of the moon, it is g = 1.7 m/s^2.

Note that g itself is a positive quantity, 9.80 m/s^2. If we use the upward direction as our positive reference direction, then we say the acceleration due to gravity is – g = –9.80 m/s^2 (downward 9.80 m/s^2). However, if we use the downward direction as your positive reference direction , then the acceleration due to gravity + g = 9/.80 m/s^2 (still downward 9.80 m/s^2)

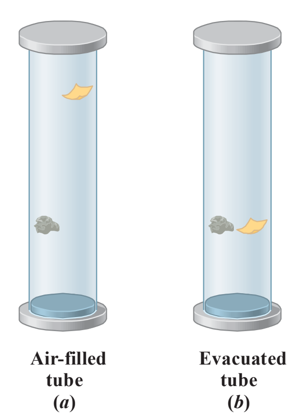


Figure : (a) In the presence of air resistance, the acceleration of the rock is greater than that of the paper. (b) In the absence of air resistance, both the rock and the paper have the same acceleration.

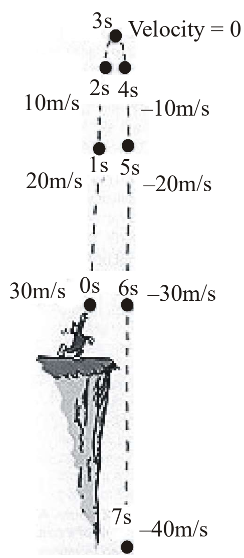
Since free fall is in the vertical direction and we often choose the upward direction as the +y axis, we replace the x's by y's and a's by –g's in the kinematic equation. The results are

$$y = \vec{v}t,\quad \vec{v} = \frac{v + v_0}{2},\quad v = v_0 - gt,\quad y = v_0 t - \frac{1}{2}gt^2,\quad v^2 = v_0^2 - 2gy \text{, where } g = 9.80\ m/s^2$$

At highest point v = 0, $t = \frac{v_0}{g},\ y = \frac{v_0^2}{2g}$.

If a ball is thrown with speed v_0 in upward direction it will reach to highest point

$\left(= \frac{v_0^2}{2g}\right)$ in time $\left(= \frac{v_0}{g}\right)$ and return back to same level in same time with same speed (= u) thus total time



taken to reach initial level = $\frac{2v_0}{g}$

Change in speed caused by Earth's pull is about 10 m/s for each second. g = 10 m/s²
Speed is subtracted while the object is rising, added when it's falling.

Example 12 :

A body is dropped from the roof of a building 10m high. Calculate the time of fall and the speed with which it hits the ground. Take g = 10 m/s^2

Sol. s = 10, u = 0, a = g = 10 m/s^2 (moving with gravity)

t = ? , v = ? use $v^2 - u^2 = 2gs$

$v^2 - (0)^2 = 2 \times 10 \times 10 \Rightarrow v^2 = 200$; $v = 10\sqrt{2} = 10 \times 1.414 = 14.14\ m/s$; t = ? use v = u + at

$\Rightarrow 10\sqrt{2} = 0 + 10 \times t \Rightarrow 10\sqrt{2} = 10t \Rightarrow \frac{10}{10}\sqrt{2} = t \Rightarrow t = \sqrt{2} = 1.41$ sec.

Example 13 :

A ball is thrown upward with an initial velocity of 10.0 m/s from the top of a 50.0m tall building.

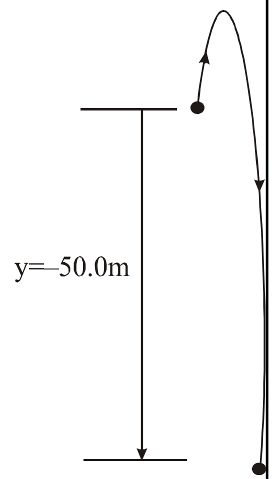
(a) With what velocity will the ball strike the ground ?

(b) How long does it take the ball to strike the ground.

Sol. Given : y = – 50.0 m (displacement), v_0 = + 10.0 m/s

Find : (a) t (b) v

The y in the kinematic equations stands for displacement from the launch point, not distance. When the ball strikes the ground, it will displace –50.0m, or 50m below the launch point.



(a) $v^2 = v_0^2 - 2gy = (+10.0\ m/s)^2 - 2\ (9.80\ m/s^2)\ (-50.0\ m) = 1.08 \times 10^3\ m^2/s^2$

So $$v = \sqrt{1.08 \times 10^3 m^2 / s^2} = \pm 32.9\ m/s$$

The positive answer is discarded since the ball is falling when it lands (moving downward).

Therefore $v = -32.9$ m/s

(b) From $v = v_0 - gt$, we have

$$t = \frac{v_0 - v}{g} = \frac{(+10.0\ m/s - (-32.9\ m/s)}{9.80\ m/s^2} = \frac{42.9\ m/s}{9.80\ m/s^2} = 4.38\ s$$

Try to solve this problem without using the overall displacement concept. You could break it into two phases. First, you would have to find out how high the ball goes, then secondly determine the velocity when it strikes the ground, and the total time it is in the air.

Example 14 :

A high-wire artist missteps and falls 9.2 meters to the ground. What is the acrobat's speed on landing ?

Sol. The motion is uniformly accelerated and starts at rest, so $v_i = 0$

$$v = \sqrt{2as} = \sqrt{2\left(9.8\frac{m}{s^2}\right)(9.2m)} = 13\ m/s$$

Example 15 :

A stone is dropped freely in the river from a bridge. It takes 5s to touch the water surface in the river. Calculate : (i) the height of the bridge from the water level, (ii) the distance covered by stone in the last second ($g = 9.8\ ms^{-2}$)

Sol. $u = 0$, $g = 9.8\ ms^{-2}$, $t = 5s$

(i) From equation of motion, $h = ut + \frac{1}{2}gt^2$; $h = 0 \times 5 + \frac{1}{2} \times 9.8 \times (5)^2 = 9.8 \times \frac{25}{2} = 122.5m$

(ii) Distance covered in last second, $S_{(t)} = u + \frac{g}{2}(2t-1) = 0 + \frac{1}{2} \times 9.8 \times (2 \times 5 - 1) = 44.1\ m$

SELF CHECK

Q.1 Drop a rock from a bridge, and it hits the water 2.3 seconds later. Find (a) the height of the bridge. (b) the velocity of the rock when it hits.

Q.2 A tennis ball is struck with a racket, firing it straight upward at 22 meters per second. After how much time will it be falling at 15 meters per second ?

Q.3 A stone is thrown in vertically upward direction with a velocity of 5 m/s. If the acceleration of the stone during its motion be 10 m/s^2, in downward direction what will be height attained by the stone and how much time will it take to reach there ?

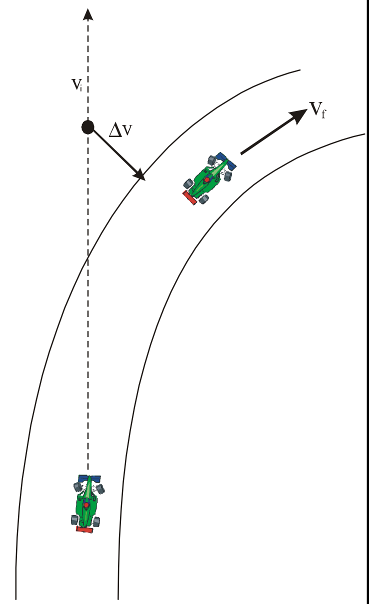
ANSWERS

(1) (a) 26 m **(b) 23 m/s** **(2) 3.8 sec.** **(3) 1.25 m, 1/2 s**

CIRCULAR MOTION

Any change of velocity-speeding up, slowing down, or turning a corner is an acceleration. We distinguish these different kinds of accelerated motion by assigning a direction to the acceleration. In other words, acceleration is a vector.

If you are traveling in a straight line and speed up, your change in velocity its ($\Delta\vec{v}$) is in the direction you are going. Then your acceleration is also in that direction. This is the first rule for the direction of an acceleration : when the acceleration is in the same direction as the velocity, the result is an increase in speed.

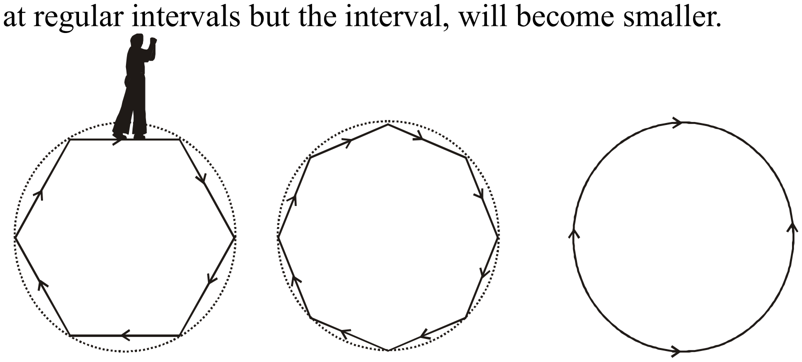


Now suppose you do not change speed but are making a turn. This is a change in only the direction of velocity, since velocity is a vector, this is surely a change in velocity-an acceleration. Figure shows how to find the direction of this acceleration. The dotted line shows what the path of the car would be if its motion were not accelerated that is, if it traveled at constant speed in a straight line.

The arrow is a vector representing the change in the velocity of the car, that is, the vector that must be added to the old velocity to get the new velocity. The acceleration is in the same direction as the change in velocity. When the car turns to the right, its acceleration is to the right, perpendicular to its velocity, this acceleration is called centripetal acceleration and its value is v^2/r (you will learn derivation of the formula in higher class)

Definition : Motion of a particle (small body) along a circle (circular path), is called a circular motion. If the body covers equal distances along the circumference of the circle, in equal intervals of time, the motion is said to be a uniform circular motion. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.

Explanation : Consider a boy running along a regular hexagonal track (path) as shown in Fig. As the boy runs along the side of the hexagon at a uniform speed, he has to take a turn at each corner changing direction but keeping the speed same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval, will become smaller.



By increasing the number of sides of the regular polygon, we find the number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of a polygon with an infinite number of sides On the circular track, the turning becomes a continuous process without any gap in between the boy running along the sides of such a track will be performing a circular motion.

Hence, circular motion is the motion of a body along the sides of a polygon of infinite number of sides with uniform speed, the direction changing continuously. Examples of Uniform Circular Motion are (i) Motion of moon around the earth.(ii) Motion of satellite around its planet.

Difference Between a Uniform linear motion and a uniform circular motion

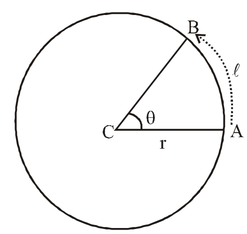
Uniform Linear Motion	Uniform Circular Motion
1. The direction of motion does not changes.	The direction of motion changes continuously.
2. The motion is non-accelerated.	The motion is accelerated.

Radian (a unit for plane angle): It is a convenient unit for measuring angle in physics.

Definition : One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to its radius.

In Fig. , the arc AB of the circle, has length ℓ and subtends an angle θ at the centre C.

If $\angle ACB = \theta$ radians. Then, $\theta = \frac{\ell}{r}$ radians. [for $\ell = r$, $\theta = 1$ radian]



Angle subtended by the circumference at the centre.

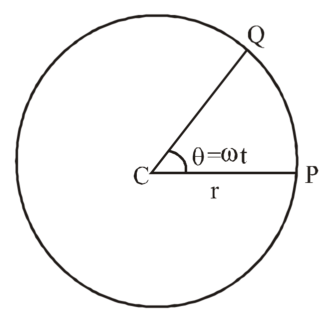
$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians } \{\text{or } 2p^C\}$$

$[^C]$ is symbol for radian, just as (°) is symbol for degree.

Relation : For complete circle at centre $2\pi^c = 360°$ or $1^c = \left|\frac{360}{2\pi}\right| = 57.3°$

Angular Displacement & Angular Velocity :

Definitions : In a circular motion, the angular displacement of a body is the angle by which the radius of the circle, joining the position of the body with the centre, rotates about the centre. It is represented by the symbol θ (theta). The angular displacement per unit time is called the angular velocity. It is represented by the symbol ω (omega).



Let a body move along a circle of radius r and perform a uniform circular motion

Let the body be at point P to start with and reach point Q after time t.

Then, angular displacement = $\angle PCQ = \theta$ and angular velocity = $\omega = \frac{\theta}{t}$ (i.e. $\theta = \omega t$) $= \frac{2\pi}{T}$

(T = time period, Time taken to complete one circular path)

Units for θ and ω : The unit for angular displacement is radian (a Supplementary quantity). The radian is defined as the angle subtended at the centre of a circle by an arc equal in length to its radius. The unit for angular velocity is radian per sec. (rad/s).

Relation between Linear and Angular Quantities:

For an arc of length ℓ, Linear displacement = ℓ. Angular displacement, $\theta = \frac{\ell}{r}$. Hence, $\ell = r\theta$

For a time interval t, Linear velocity, $v = \frac{\ell}{t}$; Angular velocity, $\omega = \frac{\theta}{t} = \frac{\ell}{r, t} = \frac{v}{r}$; Hence v = rω.

Example 16 :

Calculate the angular speed of (i) hour hand of a watch and (ii) earth about its own axis.

Sol. (i) The hour hand of a watch completes one rotation in 12 hours

angle covered in 12 × 60 × 60 sec. = 2π rad

Angular speed of hour hand $= \frac{2\pi}{12 \times 60 \times 60} = \frac{\pi}{21600}$ rad/s

(ii) Earth completes one rotation about its axis in 24 hours

angle covered in 24 × 60 × 60 sec. = 2π rad

Angular speed of earth $= \frac{2\pi}{12 \times 60 \times 60} = \frac{\pi}{43200}$ rad/s

Example 17 :

The planet Neptune travels in a nearly circular orbit of radius , r = 4.5 × 10^9 km, about the sun. It takes Neptune 165y to make a complete trip around the sun. How fast (in km/h) does not Neptune travel in its orbit ?

Sol. $v = \frac{2\pi r}{T} = \frac{2\,(3.14)(4.5 \times 10^9\,\text{km})}{(165\text{ d})\,(365\text{ d}/1\text{y})\,(24\text{ h}/1\text{d})} = 2.0 \times 10^4\,\text{km/h}$

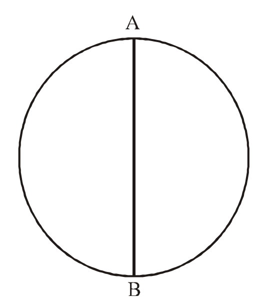
Example 18 :

A grinding wheel (radius 7.6 cm) is rotating at 1750 rpm. **(a)** What is the speed of a point on the outer edge of the wheel ? **(b)** What is the centripetal acceleration of the point ?

Sol. $v = \frac{2\pi r}{T}$. The period of motion is T = (1/1750) (60s) = 3.43 × 10^{-2} s

so $v = \frac{2\pi\,(7.6 \times 10^{-2}\,\text{m})}{3.43 \times 10^{-2}\,\text{s}} = 14\,\text{m/s}$; $a_C = \frac{v^2}{r} = \frac{14\,\text{m/s}^2}{7.6 \times 10^{-2}\,\text{m}} = 2.6 \times 10^3\,\text{m/s}^2$

<u>**SELF CHECK**</u>



Q.1 A circular cycle track has a circumference of 314m with AB as one of its diameter. A cyclist travels from A to B along the circular path with a velocity of constant magnitude 15.7 m/s. Find : (a) the distance moved by the cyclist.

(b) the displacement of the cyclist if AB represents north-south direction.

(c) the average velocity of the cyclist.

Q.2 An artificial satellite is moving in a circular orbit of radius nearly 42,240 km. Calculate its linear velocity, if it takes 24 hour to revolve round the earth.

Q.3 A body is moving with a uniform speed of 5 m/s in a circular path of radius 5m. The acceleration of tbe body is –

(1) 25 m/s^2 (2) 15 m/s^2 (3) 5 m/s^2 (4) 1 m/s^2

Q.4 A boy is running along a circular track of radius 7m. He completes one circle in 10 second. The average velocity of the boy is –

(1) 4.4 m/s (2) 0.7 m/s (3) zero (4) 70 m/s

Q.5 The wheel of a cycle of radius 50 cm. is moving with a speed 14 m/s. Calculate the angular velocity of the wheel.

Q.6 What is the period of the motion of a runner going 9.2 meters per second on a circular track whose radius is 22 meters ?

ANSWERS

(1) 157m, disp. = 100m towards south, 15.7 m/s **(2) 3.1 km/s** **(3) 3**

(4) 3 **(5) 28 rad/s** **(6) 15.0 s**

EXTRA EDGE :

A centripetal force = mv^2/r is not a new kind of force. The adjective centripetal just gives information concerning the direction of the force (i.e., inward toward the center of the circle). A centripetal force can be friction, tension in a string, a component of the lift force on an airplane, gravity, etc. The first step in analyzing a problem which deals with circular motion is to identify the centripetal force.

ADDITIONAL EXAMPLES

Example 1 :

A train travels from one station to another at a speed of 40 km/hr and returns to the first station at the speed of 60 km/hour. Calculate the average speed and average velocity of the train.

Sol. Let s be distance from one station to another. Let t_1 and t_2 be time taken by the train in going from one station to another and coming back to the first station respectively.

$$t_1 = \frac{s}{40}\text{hr} \quad \text{and} \quad t_2 = \frac{s}{60}\text{hr}$$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{t_1 + t_2} = \frac{2s}{\frac{s}{40} + \frac{s}{60}} = \frac{2s}{s\left(\frac{3+2}{120}\right)} = \frac{2}{1} \times \frac{120}{5} = \frac{240}{5} = 48 \text{ km / hr}$$

$$\text{Average velocity} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Since total displacement in going from one station to another then returns back to the same station is zero.

$\therefore$ Average velocity = zero.

Example 2 :

A car is travelling with a speed of 36 km/hr. The driver applies the brakes & retard the car uniformly. The car is stopped in 5 seconds. Find (i) the retardation of the car and (ii) distance travelled before it is stopped after applying the brakes.

Sol. Here, $u = 36\ km/hr = 36 \times \frac{5}{18}\ m/s = 10\ m/s,\ v = 0, t = 5s$

(i) Using $v = u + at$, we have $a = \frac{v - u}{t} = \frac{(0 - 10)\ ms^{-1}}{5s} = -2\ ms^{-2}$ (ii) Using $s = ut + \frac{1}{2}at^2$,

We have $s = 10\ ms^{-1} \times 5s + \frac{1}{2}(-2\ ms^{-2})(25\ s^2) = 50m - 25m = 25m$

Example 3 :

Calculate the angular speed of flywheel making 240 revolution per minute.

Sol. angular speed = $\frac{\theta}{t}$; $\theta = 240 \times 2\pi$ radian, t = 1 min = 60 sec.

$\therefore$ Angular speed = $\frac{240 \times 2\pi}{60} = 8\pi$ rad/sec.

Example 4 :

A racing car has a uniform acceleration of 4 m/s^2. What distance will it cover in 10s after the start from rest ?

Sol. Here, $u = 0$ m/s (starts from rest), s = ?, $a = 4\ ms^{-2}$, t = 10s

Using $s = ut + \frac{1}{2}at^2$, we have, $s = 0 + \frac{1}{2} \times 4 \times 10 \times 10 = 200m$

Example 5 :

A ball is thrown down vertically with an initial speed of 20 m/s from a height of 60 m. Find (a) its speed just before it strikes the ground and (b) how long it takes for the ball to reach the ground. Repeat (a) and (b) for the ball thrown directly up from the same height and with the same initial speed.
[Take g = 10 m/s^2]

Sol. **(a)** We do not know the time, but taking $y_o = 60$ m and y = 0, we can use
$v^2(y) = v_o^2 + 2a_y(y - y_o)$
$v^2(0) = (-20\ m/s)^2 - 20\ m/s^2\ (0 - 60\ m) = (400 + 1200)\ m^2/s^2$
and v = – 40 m/s, where the negative sign occurs because we have taken up as positive.
(b) $v(t) = v_o + at$ or $-40\ m/s = -20\ m/s - 10\ m/s^2\ t$ and t = 2s
(c) Now the initial and final positions are the same, but the initial velocity $v_o = +20$ m/s.
The algebra, however, will be the same as in part (a) and the answer is again v = – 40 m/s when it hits the ground. The time does change with $v(t) = v_o + at$ and $v_o = +20$ m/s:

$-40\ m/s = 20\ m/s - 10\ m/s^2\ t$ and now t = 6 s.

Example 6 :

An object is thrown vertically upward from the ground at 30 m/s. (i) What is the displacement after 4s ? (ii) What is the velocity after 4s ?

Sol. We select earth as the origin. Then $g = -9.8\ m/s^2$. For upward journey $v_0 = +30\ m/s$, $g = -9.8\ m/s^2$

(i) $h = v_0 t + \frac{1}{2}gt^2$, Here $v_0 = +30\ m/s$, $g = -9.8\ m/s^2$, $t = 4s$, $h = ?$

$h = 30 \times 4 + \frac{1}{2}(-9.8) \times (4)^2 = 42m$

(ii) $v = v_0 + gt$, Here $v_0 = +30\ m/s$, $g = -9.8\ m/s^2$, $t = 4s$

$v = 30 + (-9.8) \times 4 = -9\ m/s$

Thus four seconds after the object is thrown it is 42m above the ground and is travelling downward (negative sign shows downward motion) with an instantaneous speed 9 m/s.

Example 7 :

(a) An object moving with constant acceleration changes its speed from 20 m/s to 60 m/s in 2.0 s. What is the acceleration?

(b) How far did it move in this time ?

Sol. By the definition of constant acceleration, $a = \frac{v - v_0}{t - 0}$, where v is the velocity of the object at time t and v_o is the velocity of the object at time t = 0.

(a) $a = \{(60 - 20)\ m/s\}/(2.0\ s - 0) = 20\ m/s^2$

(b) Taking $x_o = 0$, $x(t) = v_o t + \frac{1}{2}at^2$. $x(2.0\ s) = 20\ m/s\ (2.0\ s) + \frac{1}{2}(20\ m/s^2)(2.0\ s)^2 = 80\ m$.

Example 8 :

An object moves with a constant velocity of 15 m/s.

(a) How far will it travel in 2.0 s ? (b) If the time is doubled, how far will it travel ?

Sol. **(a)** The distance moved x divided by the time t equals the constant velocity

v. $x/t = v$ or $x = vt = 15\ m/s\ (2.0\ s) = 30\ m$.

(b) Since $x/t = v$ = constant, the distance moved is directly proportional to the time elapsed. If you double the time, you double the distance.

For $t = 4.0\ s$, $x = 2 \times 30\ m = 60\ m$.

Example 9 :

An object, initially at rest, moves with a constant acceleration of $10\ m/s^2$. How far will it travel in (a) 2.0 s and (b) 4.0 s ?

If this object had an initial velocity of 4 m/s, how far will travel in (c) 2.0 s and (d) 4.0 s ?

Sol. In general for constant acceleration, the distance moved by the object

$$x = x_o + v_o t + \frac{1}{2}at^2$$, where x_o is the position of the object at t = 0 (the initial position),

v_o the velocity at t = 0 (the initial velocity), a the acceleration, and t the time at which you wish to find x.

Let's arbitrarily take $x_o = 0$. Then for an object initially at rest, $v_o = 0$, $x = 0 + 0 + \frac{1}{2}at^2$.

(a) $x\,(2.0\text{ s}) = \frac{1}{2}(10\text{ m/s}^2)(2.0\text{ s})^2 = 20\text{ m}$ (b) $x\,(4\text{ s}) = \frac{1}{2}(10\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m}$.

For constant acceleration and an initial position and initial velocity of zero,

$$x = \frac{1}{2}at^2 \text{ or } \frac{x}{t^2} = \frac{1}{2}a = \frac{a}{2} = \text{a constant.}$$

For this case x is directly proportional to the square of t.

If you double t, t^2 goes up by a factor of four and x must go up by a factor of $4 = 4 \times 20\text{ m} = 80\text{ m}$.

(c) For this part, we still take the initial position $x_o = 0$, but now the initial velocity $v_o = 4.0$ m/s.

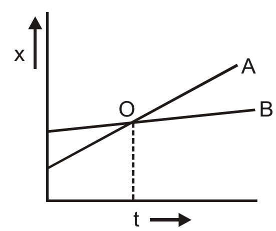
$$x\,(t) = 0 + v_o t + \frac{1}{2}at^2. \;;\; x\,(2.0\text{ s}) = 0 + (4.0\text{ m/s})(2.0\text{ s}) + \frac{1}{2}(10\text{ m/s}^2)(2.0\text{ s})^2 = 28\text{ m}$$

(d) $x\,(4.0\text{ s}) = 0 + (4.0\text{m/s})(4.0\text{ s}) + \frac{1}{2}(10\text{ m/s}^2)(4.0\text{ s})^2 = 96\text{ m}$ With $x = v_o t + \frac{1}{2}at^2$ & $\frac{x}{t^2} = \frac{v_0}{t} + \frac{a}{2}$.

Since t is a variable, the right hand side of the above equation is **not** equal to a constant and x is no longer proportional to the square of the time, that is, $96 \neq 4 \times 28$.

Example 10 :

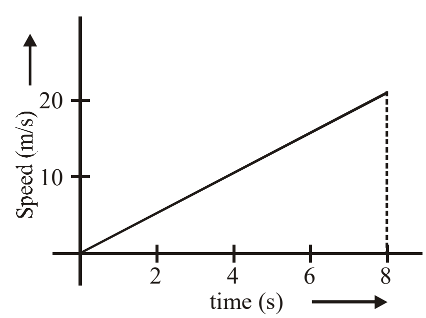
Give a position-time graph of two objects moving in the same direction with unequal velocities.



Sol. O is the time of meeting of two bodies A and B.

Example 11 :

The speed of a car as a function of time as shown in fig. Find the acceleration and distance travelled by the car in 8 seconds.



Sol. (i) Distance travelled

= Area under speed - time graph = (1/2) × 20 × 8 = 80 m

(ii) $\text{Acc} = \frac{\Delta v}{\Delta t} = \frac{20}{8} = \frac{5}{2} = 2.5\text{ m/s}^2$

Example 12 :

An airplane is flying in a horizontal circle of radius 1.0 km. What must be the speed of the plane if the pilot is to experience a centripetal acceleration three times that of gravity ?

Sol. $v^2 = a_c r$, but $a_c = 3g$ so $v^2 = 3gr = 3\,(9.80\text{ m/s}^2)(1.0 \times 10^3\text{ m})$ or v = 170 m/s = 620 km/h.