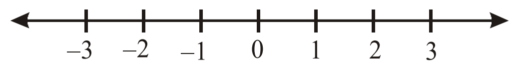
1

RATIONAL NUMBERS

INTRODUCTION

In earlier classes, you have learnt addition, subtraction, multiplication and division with the help of number line.
Number line is geometrical straight line with arbitrarily define zero (origin).
Let us first review that number line.



To obtain (+10) + (+5) start from (+10) and count 5 points to the right you come to +15.

$\therefore$ (+10) + (+5) = +15

To obtain (+7) – (+4) start from (+7) and count 4 points to the left you come to +3.

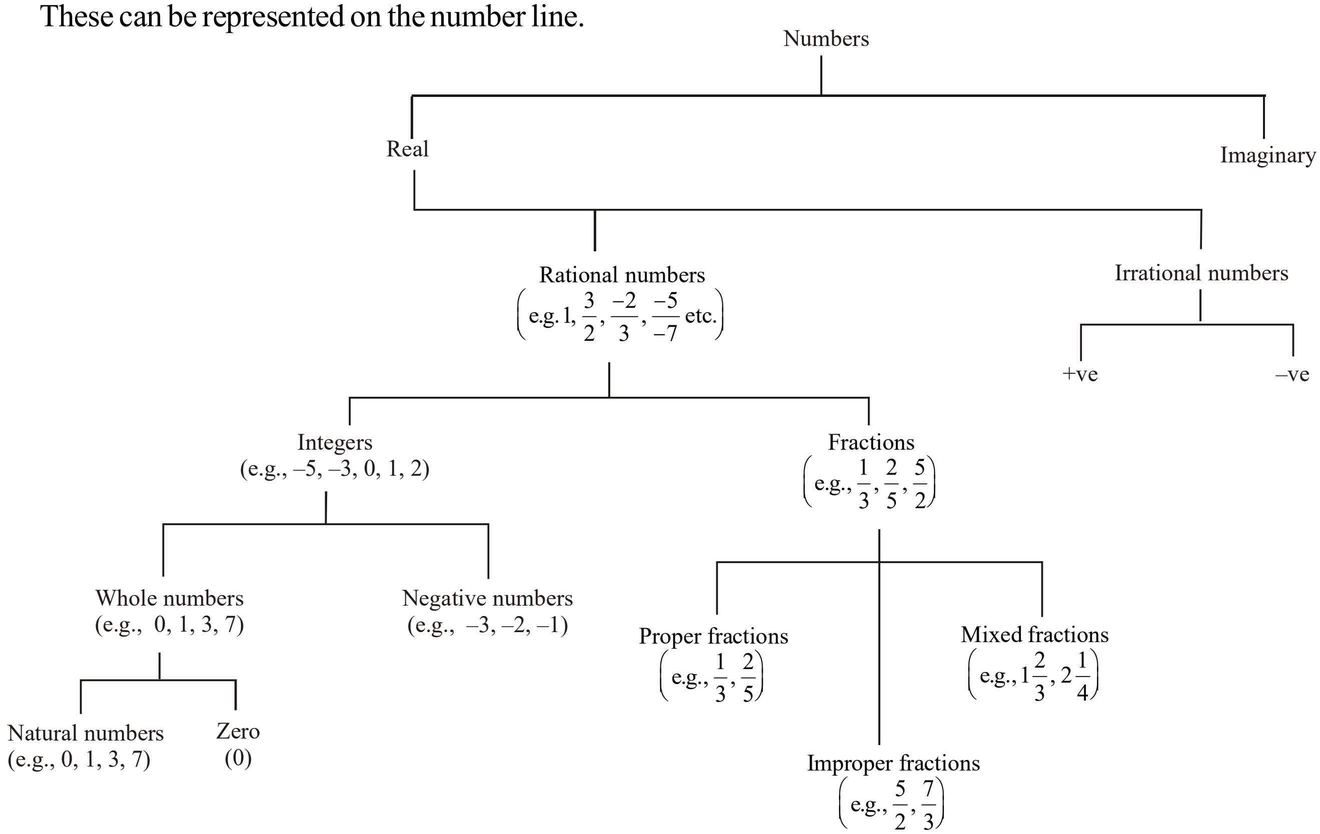
$\therefore$ (+7) – (+4) = 3

and similarly you can multiply and divide simple numbers but now let say we wish to calculate – 8 ÷ 3 it will be not be possible to operate on a line that contains only integers (– ∞,......–3, –2, –1, 0, 1, 2, 3.......∞) therefore we need a new system in which we should be able carry out all types of multiplication, division and hence rational number, irrational number were created.

Number tree : Numbers are basically of 2 types : Real no. and Imaginary no.

You will study about an imaginary number in higher classes. Presently we will study only Real numbers.

Real Numbers : These are the numbers which can represent actual physical quantities in a meaningful way. These can be represented on the number line.



Natural numbers : Set of all non-fractional numbers from 1 to N = {1, 2, 3, 4...........}.
If we add 1 to any natural number, we get its successor. Thus, every natural number has its successor. Consequently, the set of all natural numbers is an infinite set, given by N = {1, 2, 3, 4, 5, ...}
Clearly, the sum of two natural numbers is always a natural number. We express this property by saying that the set N of all natural numbers is closed for addition.
However, if we subtract a natural number from another natural number, the result is not always a natural number.
e.g. $3 \in N, 7 \in N$ but $(3-7) \notin N$
Thus, the set N is not closed for subtraction.

Even natural numbers : Natural numbers which are exactly divisible by 2 are called even natural numbers.
E = {2, 4, 6, 8, 10, 12, ...} is the set of all even natural numbers.

Odd natural numbers : Natural numbers which are not even, are called odd natural numbers.
O = {1, 3, 5, 7, 9, 11,...........} is the set of all odd natural numbers.

Whole numbers :
If 0 (zero) is adjoined to the set N, we obtain a new set W = {0, 1, 2, 3, 4, 5, ...} called the set of whole numbers. Clearly, every natural number is a whole number.

Integers : All natural numbers, negatives of natural numbers and 0, together form the set Z or I of all integers.
$Z = \{........., -3, -2, -1, 0, 1, 2, 3,..........\} = N \cup \{0\} \cup N^-$
$Z^+ = \{1, 2, 3, 4, 5, 6, ...\} = N$ is the set of all positive integers.
$Z^- = \{-1, -2, -3, -4, ...\}$ is the set of all negative integers.

Prime numbers : All natural numbers that have one & itself as their factors are prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. Eg. 2, 3, 5, 7, 11, 13, 19, 23 identification of prime number.
Step 1 : Find approximate square root of given no.
Step 2 : Divide the given no. by prime numbers less than approximately square root of number. If given number is not divisible by any of these prime numbers then the number is prime otherwise not eg. : 571.
Sol. : Approximately square root = 24
Prime nos. < 24 are 2, 3, 5, 7, 11, 13, 17, 19 & 23. 571 is not divisible by any of these prime nos. so 571 is a prime number.

Composite numbers : Natural numbers having more than two factors are called composite numbers.
e.g. 4, 6, 8, 9, 10, 12, 14, 15, 16, etc.
Note that 1 is neither prime nor composite.
Twin Primes : Prime numbers differing by 2 are called twin primes.
e.g., 5 and 7, 11 and 13, 17 and 19 etc.
Prime Triplet : The set {3, 5, 7} of three consecutive primes is called the prime triplet.
Co-primes : Every pair of two natural numbers having no common factor, other than 1, is called a pair of co-primers. e.g., (5, 6), (6, 11), (14, 17), (16, 21), (20, 27), etc.

RATIONAL NUMBERS

Rational numbers sound like they should be very sensible numbers. In fact, they are. Rational numbers are simply numbers that can be written as fractions or **ratio**s (this tells you where the term **ratio**nal comes from). This includes integers, terminating decimals, and repeating decimals as well as fractions.

* An **integer** can be written as a fraction simply by giving it a denominator of one, so any integer is a rational number. $6 = \frac{6}{1}$, $4 = \frac{4}{1}$, $0 = \frac{0}{1}$

* A **terminating decimal** can be written as a fraction simply by writing it the way you say it: 3.75 = three and seventy-five hundredths = $3\frac{75}{100}$, then adding if needed to produce a fraction : $\frac{300}{100} + \frac{75}{100} = \frac{375}{100}$.

So, any terminating decimal is a rational number.

* A **repeating decimal** can be written as a fraction using algebraic methods, so any repeating decimal is a rational number. **Examples :**

(i) 0 can be written as $\frac{0}{1}$, which is rational. $\therefore$ 0 is a rational number.

(ii) Every integer a can be written as $\frac{a}{1}$, which is rational. $\therefore$ Every integer is a rational number.

(iii) The square root of every perfect square number is rational.

e.g., $\sqrt{4} = 2$, which is rational similarly, $\sqrt{9}, \sqrt{16}, \sqrt{25}$ etc. are all rational.

(iv) Every terminating decimal is a rational number.

e.g., $0.7 = \frac{7}{10}$, which is rational ; $0.375 = \frac{375}{1000}$, which is rational

(v) Every recurring decimal is a rational number. Let us consider the recurring decimal 0.333 ....

Let x = 0.3333 .... (1) Then, 10x = 3.3333 .... (2)

On subtracting (1) from (2) , we get, $9x = 3 \Leftrightarrow x = \frac{3}{9} = \frac{1}{3}$ $\therefore$ 0.333.......... = $\frac{1}{3}$, which is rational.

Positive rational numbers : A rational number is said to be positive if its numerator and denominator are either both positive integers or both negative integers. Example : $\frac{3}{8}, \frac{5}{12}, \frac{-9}{-14}, \frac{-5}{-17}$ are positive rational number.

Negative rational numbers : A rational number is said to be negative if its numerator and denominator are such that one of them is positive integer and the other one is a negative integer.

Example : $\frac{-7}{9}, \frac{13}{-16}, \frac{-15}{29}$ are negative rational number.

IRRATIONAL NUMBER

Every non-terminating and non-repeating decimal number is known as an irrational number e.g. 0.101001000100001.... **Example :** $\sqrt{2}, \sqrt{5}, \pi$ etc.

Pythagoras' Student

The ancient greek mathematician *Pythagoras* believed that all numbers were rational (could be written as a fraction), but one of his students *Hippasus* proved (using geometry, it is thought) that you could **not** represent the square root of 2 as a fraction, and so it was *irrational*.

However *Pythagoras* could not accept the existence of irrational numbers, because he believed that all numbers had perfect values. But he could not disprove *Hippasus'* "irrational numbers" and so Hippasus was thrown overboard and drowned!

Example : Classify according to number type; some numbers may be of more than one type.

(a) 0.45 : This is a terminating decimal, so it can be written as a fraction: $^{45}/_{100} = ^{9}/_{20}$. Since this fraction does not reduce to a whole number, then it's not an integer or a natural. And everything is a real, so the answer is: **rational, real**

(b) 3.14159265358979323846264338327950288419716939937510...

You probably recognize this as being pi, though this may be more decimal places than you customarily use. The point, however, is that the decimal does not repeat, so pi is an irrational. And everything (that you know about so far) is a real, so the answer is: **irrational, real.**

(c) 3.14159 **:** Don't let this fool you! Yes, you often use something like this as an *approximation* of pi, but it isn't pi! This is a rounded decimal approximation, and, since this approximation *terminates*, this is actually a rational, unlike pi which is irrational! The answer is: **rational, real**

(d) 10 :Obviously, this is a counting number. That means it is also a whole number and an integer. Depending on the text and teacher (there is some inconsistency), this may also be counted as a rational, which technically-speaking it is. And of course it's also a real. The answer is: **natural, whole, integer, rational** (possibly)**, real**

(e) $\frac{5}{3}$ **:** This is a fraction, so it's a rational. It's also a real, so the answer is: **rational, real**

(f) $1\frac{2}{3}$ **:** This can also be written as $\frac{5}{3}$, which is the same as the previous problem. The answer is: **rational, real**

(g) $-\sqrt{81}$ **:** Your first impulse may be to say that this is irrational, because it's a square root, but notice that this square root simplifies: $-\sqrt{81} = -9$, which is just an integer. The answer is: **integer, rational, real**

(h) $-\frac{9}{3}$: This is a fraction, but notice that it reduces to –3, so this may also count as an integer. The answer is: **integer** (possibly)**, rational, real**

BASIC NUMBER PROPERTIES

Distributive Property :

The Distributive Property is easy to remember, if you recall that "multiplication distributes over addition". Formally, write this property as "a(b + c) = ab + ac". In numbers, this means, that 2(3 + 4) = 2×3 + 2×4.

Why is the following true? 2(x + y) = 2x + 2y

Since they distributed through the parentheses, this is true **by the Distributive Property.**

Use the Distributive Property to rearrange: 4x – 8

The Distributive Property either takes something through a parentheses or else factors something out. Since there aren't any parentheses to go into, you must need to factor out of.

Then the answer is "**By the Distributive Property, 4x – 8 = 4(x – 2)"**

Associative Property : The word "associative" comes from "associate" or "group";the Associative Property is the rule that refers to grouping. For addition, the rule is "a + (b + c) = (a + b) + c"; in numbers, this means 2 + (3 + 4) = (2 + 3) + 4.

For multiplication, the rule is "a(bc) = (ab)c"; in numbers, this means 2 (3 × 4) = (2 × 3) 4.

Rearrange, using the Associative Property: 2(3x)

Regroup things, not simplify things. In other words, we do not want you to say "6x". We want you to see the following regrouping: **(2×3)x**

Commutative Property : The word "commutative" comes from "commute" or "move around", so the Commutative Property is the one that refers to moving stuff around. For addition, the rule is "a + b = b + a"; in numbers, this means 2 + 3 = 3 + 2. For multiplication, the rule is "ab = ba"; in numbers, this means 2×3 = 3×2.

Use the Commutative Property to restate "3×4×x" in at least two ways.

We want you to move stuff around, not simplify. In other words, the answer is not "12x"; the answer is any two of the following: 4 × 3 × x, 4 × x × 3, 3 × x × 4, x × 3 × 4, and x × 4 × 3

Example 1 :

Simplify 3a – 5b + 7a. Justify your steps.

Sol.

3a – 5b + 7a	Original (given) statement	3a + 7a – 5b	Commutative Property
(3a + 7a) – 5b	Associative Property	a(3 + 7) – 5b	Distributive Property
a(10) – 5b	Simplification (3 + 7 = 10)	10a – 5b	Commutative Property

Identity and Inverse : The identity" is whatever doesn't change your number at all, and "the inverse" is whatever turns your number into the identity.

For addition, "the identity" is zero, because adding zero to anything doesn't change anything. The "inverse" is the additive inverse: it's the same number, but with the opposite sign. For instance, suppose your number is –6, and you're adding. The identity is zero, and the inverse is 6, because –6 + 6 = 0.

For multiplication, "the identity" is one, because multiplying by one doesn't change anything. The "inverse" is the multiplicative inverse: the same number, but on the opposite side of the fraction line. For instance, suppose your number is –6, and you're multiplying. The identity is one, and the inverse is $\frac{-1}{6}$, because $(-6)\left(\frac{-1}{6}\right) = 1$.

Determine which property was used.

(i) 1×7 = 7 : Multiplied, and didn't change anything: **the multiplicative identity**

(ii) x + 0 = x : Added, and didn't change anything: **the additive identity.**

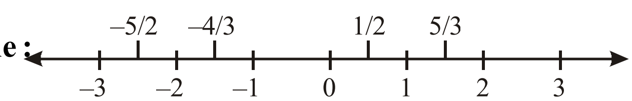
(iii) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 1$: Multiplied, and ended up with one: **the multiplicative inverse.**

PLAYING WITH RATIONAL NUMBERS

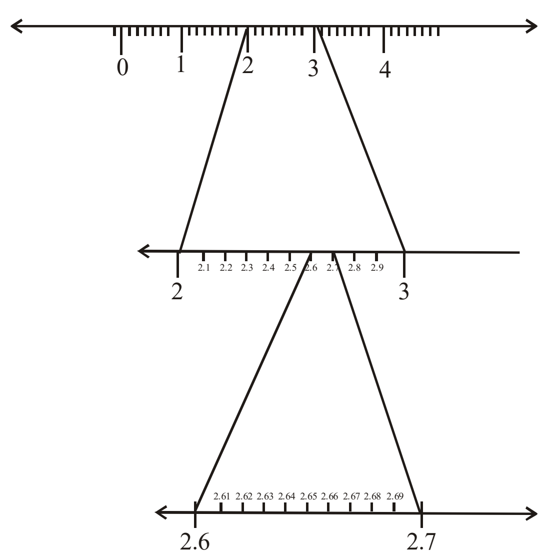
1. Representation of rational number on a number line :

Rational numbers can be represented by points on the number line.

Even we can represent terminating decimal fraction as 2.65 (process of magnification)



Number Line

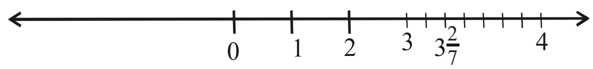


Example 2 :

Represent $3\frac{2}{7}$ on the number line.

Sol. In order to represent $3\frac{2}{7}$ on the number line, take 3 unit lengths between 0 and 3 and divide the unit length between 3 and 4 into seven equal parts and take the end of 2nd part on it.

This point represents the rational number $3\frac{2}{7}$.

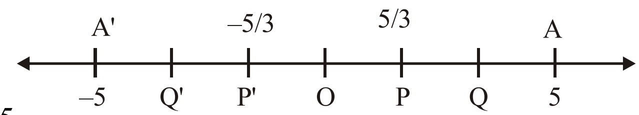


Example 3 :

Represent $\frac{5}{3}$ and $-\frac{5}{3}$ on the number line.

Sol. We draw a number line and choose a point O on it to present 0. Now, we mark a point A on the number line at a distance of 5 units to represent the positive integer 5.

Since, the denominator of $\frac{5}{3}$ is 3, we divide segment OA into three equal parts such that OP = PQ = QA. Now, P represent the rational number $\frac{5}{3}$.



Again choose a point A' on the left of O at a distance equal to OA to represent –5. Again divide line segment OA' into three equal parts such that OP' = P'Q' = Q'A'. Now point P' represents $-\frac{5}{3}$

2. Standard form of a rational number : A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and the integers p and q have no common divisor other than 1.

In order to express a given rational number is standard form, we take the following steps :

Step 1 : See whether the denominator of the given rational number is positive or not. If it is negative, multiply or divide numerator and denominator both by –1 so that the denominator becomes positive.

Step 2 : Find the greatest common divisor (GCD) or HCF of the absolute values of the numerator and the denominator.

Step 3 : Divide the numerator and denominator of the given rational number by the HCF obtained in step 2. The rational number so obtained is the standard form of the given rational number.

Example 4 :

Express the following in standard form

(i) $\frac{19}{38}$ (ii) $\frac{6}{9}$ (iii) $-\frac{4}{6}$ (iv) $\frac{45}{-70}$

Sol. A rational number $\frac{p}{q}$ (where p and q are integers and $q \neq 0$) is said to be in standard form when p and q are coprime (i.e. do not have any common factor other than 1) and q is positive.

(i) $\frac{19}{38}$: Now, the above can be written as, $\frac{19}{38}=\frac{19\times 1}{19\times 2}=\frac{1}{2}$, which is in standard form.

(ii) $\frac{6}{9}$: Now, the above can be written as $\frac{6}{9}=\frac{3\times 2}{3\times 3}=\frac{2}{3}$, which is in standard form.

(iii) $-\frac{4}{6}$: Now, the above can written as, $\frac{-4}{6}=\frac{-2\times 2}{3\times 2}=\frac{-2}{3}$, which is in standard form.

(iv) Make denominator positive : $\frac{45}{-70}=\frac{-45}{70}$

Divide both numerator and denominator by the HCF. HCF of 45 and 70 is 5. $\therefore \frac{-45}{70}=\frac{(-45)\div 5}{70\div 5}=\frac{-9}{14}$.

3. Comparison of rational numbers :

Method 1 : To compare any two rational numbers, we can use the following steps

Step 1 : Write the given rational numbers so that their denominators are positive.

Step 2 : Find the LCM of the positive denominators of the rational numbers obtained in step 1.

Step 3 : Express each rational number (obtained in step 1) with the LCM (obtained in step 2) as common denominator.

Step 4 : Compare the numerators of rational numbers obtained in step 3. The number having greater numerator is the greater rational number.

Method 2 : If a and b are integers and c and d are positive integers, then

$\frac{a}{c}>\frac{b}{d}$, if and only if ad > bc. ; $\frac{a}{c}<\frac{b}{d}$, if and only if ad < bc. ; $\frac{a}{c}=\frac{b}{d}$, if and only if ad = bc.

Example 5 :

Which of the numbers $\frac{3}{-4}$ and $\frac{-5}{6}$ is greater ?

Sol. First we write each of the given numbers with positive denominator.

One number = $\frac{3}{-4}=\frac{3\times(-1)}{(-4)\times(-1)}=\frac{-3}{4}$. The other number = $\frac{-5}{6}$

LCM of 4 and 6 = 12

$\therefore \frac{-3}{4}=\frac{(-3)\times 3}{4\times 3}=\frac{-9}{12}$ and $\frac{-5}{6}=\frac{(-5)\times 2}{6\times 2}=\frac{-10}{12}$. Clearly, $\frac{-9}{12}>\frac{-10}{12}$. Hence, $\frac{-3}{4}>\frac{-5}{6}$

Example 6 :

Arrange the following in ascending order : $\frac{-2}{3},\frac{-5}{6},2,\frac{7}{6},\frac{13}{7}$

Sol. We are given the following rational numbers $\frac{-2}{3},\frac{-5}{6},2,\frac{7}{6},\frac{13}{7}$

Now, LCM (3, 6, 1, 7) = 6 × 7 = 42

Let us express the given rational numbers with common denominator

(i) $\frac{-2}{3} = \frac{-2}{3} \times \frac{14}{14} = \frac{-28}{42}$ (ii) $\frac{-5}{6} = \frac{-5}{6} \times \frac{7}{7} = \frac{-35}{42}$ (iii) $2 = \frac{2}{1} \times \frac{42}{42} = \frac{84}{42}$

(iv) $\frac{7}{6} = \frac{7}{6} \times \frac{7}{7} = \frac{49}{42}$ (v) $\frac{13}{7} = \frac{13}{7} \times \frac{6}{6} = \frac{78}{42}$

Now, we can arrange the above numbers in ascending order by comparing their numerators

$\frac{-35}{42}, \frac{-28}{42}, \frac{49}{42}, \frac{78}{42}, \frac{84}{42}$ i.e. $\frac{-5}{6}, \frac{-2}{3}, \frac{7}{6}, \frac{13}{7}, 2$; which is in ascending order.

Example 7 :

Compare $\frac{-2}{5}$ and $\frac{5}{-6}$.

Sol. $\frac{-2}{5} \times \frac{-5}{6}$ $\left[\because \frac{5}{-6} = \frac{5 \times (-1)}{-6 \times (-1)} = \frac{-5}{6}\right]$

-2×6 5×-5

$-12 > -25$ [$\because$ Number with less magnitude is greater between negative numbers]

$\Rightarrow \frac{-2}{5} > \frac{5}{-6}$

4. Absolute value of a rational number : For a rational number x,

$$|x| = \begin{Bmatrix} x & \text{if} & x > 0 \\ 0 & \text{if} & x = 0 \\ -x & \text{if} & x < 0 \end{Bmatrix}.$$

Thus, $\left|\frac{5}{7}\right| = \frac{5}{7}$, $\left|\frac{-12}{19}\right| = \frac{12}{19}$ and $|0| = 0$

5. Additional of Rational numbers with Distinct denominators :

In order to find the sum of two rational numbers which do not have the same denominator, take the following steps :

Step 1 : Make the denominator of the rational numbers positive.

Step 2 : Find the LCM of the denominators obtained in step 1.

Step 3 : Express each one of the rational numbers in step 2 so that the LCM obtained in step 2 becomes their common denominator.

Step 4 : Write a rational number whose numerator is equal to the sum of the numerators of rational numbers obtained in step 3 and denominator as the LCM obtained in step 2.

Step 5 : The rational number obtained in step 4 is the required sum.

Note : If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that q and s do not have a common fraction other than 1, i.e. HCF of q and s is 1, then $\frac{p}{q} + \frac{r}{s} = \frac{p \times s + r \times q}{q \times s}$

Example 8 :

Evaluate : (i) $\frac{3}{5} + \frac{7}{3} + \frac{-11}{5} + \frac{-2}{3}$ (ii) $\frac{-7}{11} + \frac{1}{6}$ (iii) $\frac{3}{4} + \frac{2}{-5}$

Sol. (i) $\frac{3}{5} + \frac{7}{3} + \frac{-11}{5} + \frac{-2}{3} = \left(\frac{3}{5} + \frac{-11}{5}\right) + \left(\frac{7}{3} + \frac{-2}{3}\right) = \frac{\{3 + (-11)\}}{5} + \frac{\{7 + (-2)\}}{3} = \frac{-8}{5} + \frac{5}{3} = \frac{(-24 + 25)}{15} = \frac{1}{15}$

(ii) $\frac{-7}{11}+\frac{1}{6}=\frac{(-7)\times 6+1\times 11}{11\times 6}=\frac{-42+11}{66}=\frac{-31}{66}$

(iii) $\frac{3}{4}+\frac{2}{-5}=\frac{3}{4}+\left[\frac{2}{-5}\times\frac{(-1)}{(-1)}\right]=\frac{3}{4}+\left(-\frac{2}{5}\right)=\frac{3}{4}-\frac{2}{5}=\frac{3\times 5-2\times 4}{4\times 5}=\frac{15-8}{20}=\frac{7}{20}$

6. To find rational numbers between two given rational numbers :

Step 1 : Find the sum of the two numbers

Step 2 : Divide the sum by two. The result so obtained is the rational number between the two given numbers.

Step 3 : Take one of the given rational numbers and add it to the result obtained in the previous step. Now divide the sum by 2. This gives the another rational number.

Step 4 : Repeat step 3 for finding some more rational numbers.

Example 9 :

Find a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Sol. Required number $=\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{2+3}{6}\right)=\left(\frac{1}{2}\times\frac{5}{6}\right)=\frac{5}{12}$.

Hence, $\frac{5}{12}$ is a rational no. lying b/w $\frac{1}{3}$ and $\frac{1}{2}$.

Example 10 :

Find ten rational numbers lying between $\frac{1}{15}$ and $\frac{1}{17}$.

Sol. To determine ten rational numbers between $\frac{1}{15}$ and $\frac{1}{17}$.

Let us take LCM (15, 17) i.e. LCM (15, 17) = 15 × 17 = 255

Now, given rational numbers can be expressed with common denominator,

i.e., $\frac{1}{15}=\frac{1}{15}\times\frac{17}{17}=\frac{17}{255}$ and $\frac{1}{17}=\frac{1}{17}\times\frac{15}{15}=\frac{15}{255}$. So, the numbers are $\frac{15}{255}$ and $\frac{17}{255}$

First rational number between $\frac{15}{255}$ and $\frac{17}{255}$ is $\frac{16}{255}$

To determine other rational numbers.

Let us multiply numerator and denominator of both the numbers by 10.

i.e., $\frac{15}{255}=\frac{15}{255}\times\frac{10}{10}=\frac{150}{2550}$ and $\frac{17}{255}=\frac{17}{255}\times\frac{10}{10}=\frac{170}{2550}$

So, other 9 required rational numbers between $\frac{150}{2550}$ and $\frac{170}{2550}$ are

$\frac{151}{2550},\frac{152}{2550},\frac{153}{2550},\frac{154}{2550},\frac{155}{2550},\frac{156}{2550},\frac{157}{2550},\frac{158}{2550},\frac{159}{2550}$

which gives us the required solution.

SELF CHECK

Q.1 Represent on the number line : (i) $\frac{1}{3}$ (ii) $4\frac{2}{3}$ (iii) $-1\frac{2}{3}$ (iv) $-2\frac{7}{8}$

Q.2 Express $\frac{-42}{98}$ as a rational number with denominator.

Q.3 Choose the greater rational numbers: (i) $\frac{-4}{3}$ or $\frac{-8}{7}$ (ii) $\frac{9}{-13}$ or $\frac{7}{-12}$ (iii) $\frac{-12}{5}$ or -3

Q.4 Arrange the rational numbers in descending order : (i) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$ (ii) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$

Q.5 Find the sum of following rational numbers : (i) $\frac{-2}{5}$ and $\frac{4}{5}$ (ii) $\frac{5}{6}$ and $\frac{-1}{6}$ (iii) $\frac{5}{8}$ and $\frac{-7}{12}$

ANSWERS

(2) $\frac{-3}{7}$ **(3)** (i) $\frac{-8}{7}$ (ii) $\frac{7}{-12}$ (iii) $\frac{-12}{5}$

(4) (i) $\frac{1}{3} > -2 > \frac{-13}{6} > \frac{8}{-3}$ (ii) $\frac{-23}{33} > \frac{-19}{22} > \frac{-39}{44} > \frac{-10}{11}$ **(5)** (i) $\frac{2}{5}$ (ii) $\frac{2}{3}$ (iii) $\frac{1}{24}$

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

1. Closure property : Sum of any two rational numbers is always a rational number.

$\frac{a}{b}+\frac{c}{d}$ is a unique rational number. e.g. $\left(-\frac{2}{3}\right)+\frac{1}{4}=\frac{(-8)+3}{12}=\left(-\frac{5}{12}\right)$. Here, $\left(-\frac{5}{12}\right)$ is a rational number.

2. Commulative property : $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$

If we add two rational numbers in any order, the result is the same. e.g., $\frac{4}{5}+\left(-\frac{3}{4}\right)=\left(-\frac{3}{4}\right)+\frac{4}{5}$

3. Associative property : Sum of three rational numbers can be carried out in two ways :
(i) preserving the same order and adding the sum of the first and second number to the third number,
(ii) preserving the same order and adding the first number to the sum of the second and third number. The result is the same in both the cases.

For any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$, we have $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

e.g., $\left[\left(-\frac{1}{4}\right)+\frac{1}{2}\right]+\frac{1}{3}=\left(-\frac{1}{4}\right)+\left[\frac{1}{2}+\frac{1}{3}\right]$

4. Existence of neutral number (Property of zero) : $\frac{a}{b}+0=0+\frac{a}{b}=\frac{a}{b}$

The sum of any rational number and zero is the same rational no. e.g., $\left(-\frac{1}{3}\right)+0=\left(-\frac{1}{3}\right), 0+\frac{4}{5}=\frac{4}{5}$

Note : 0 is called the additive identity or the identity element for the addition of rational numbers.

5. Existence of opposite number (Negative of rational number) : $\frac{a}{b}+\left(-\frac{a}{b}\right)=\left(-\frac{a}{b}\right)+\frac{a}{b}=0$

$-\frac{a}{b}$ is called negative of $\frac{a}{b}$. It is also called the additive inverse of $\frac{a}{b}$.

For every rational number, there is an opposite rational number.

e.g., $\left(-\frac{2}{7}\right)+\frac{2}{7}=0$. So, $\left(-\frac{2}{7}\right)$ and $\frac{2}{7}$ are opposite of each other.

Note :

* We have 0 + 0 = 0 = 0 + 0, so 0 is the additive inverse of itself, that is, – 0 = 0
  0 is the only rational number which is its own additive inverse.
* Whole number are closed under addition. For example, 0 + 5 = 5, a whole number.
* Integers are closed under addition. For example, – 6 + 5 = –1, an integer.

Subtraction : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}-\frac{c}{d}=\frac{a}{b}+$ (additive inverse of $\frac{c}{d}$) $=\frac{a}{b}+\left(-\frac{c}{d}\right)$

For example, $\frac{-7}{12}-\frac{1}{8}=\frac{-7}{12}+$ (additive inverse of $\frac{1}{8}$) $=\frac{-7}{12}+\left(-\frac{1}{8}\right)=\frac{-7\times 2+(-1)\times 3}{24}=\frac{-14-3}{7}=\frac{-17}{24}$

[LCM of 12 and 8 is 24]

Properties of Subtraction :

(i) Closure property : If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}-\frac{c}{d}$ is also a rational number.

For example, $\frac{4}{5}-\frac{3}{10}=\frac{4\times 2-3\times 1}{10}=\frac{8-3}{10}=\frac{5}{10}=\frac{1}{2}$, is a rational number.

(ii) Existence of right identity : In case of addition, we have $\frac{a}{b}+0=0+\frac{a}{b}=\frac{a}{b}$, but in case of subtraction, for any rational number $\frac{a}{b}$, $\frac{a}{b}-0=\frac{a}{b}$ but $0-\frac{a}{b}=-\frac{a}{b}$ (not equal to $\frac{a}{b}$). Therefore, 'only the right identity exists for subtraction.'

Note :

* Whole numbers are **not** closed under subtraction. For example, 5 – 7 = – 2, which is not a whole number.
* Integers are closed under subtraction. For example, – 6 – (–8) = 2, an integer.
* Addition is commutative for whole numbers. For example, 0 + 7 = 7 + 0 = 7
* Subtraction is not commutative for whole numbers. For example, $0-7\neq 7-0$
* Addition is commutative for an integer.
  Subtraction is not commutative for an integer. For example, $5-(-3)\neq -3-5$
* Addition is associative for whole numbers.
* Subtraction is not associative for whole numbers.
* Addition is associative for an integer.
* Subtraction is not associative for an integer. For example, $5-(7-3)\neq(5-7)-3$

Example 11 :

Find the additive inverse of :

(i) $\frac{5}{9}$ (ii) $\frac{-15}{8}$ (iii) $\frac{9}{-11}$ (iv) $\frac{-6}{-7}$ (v) $\frac{7}{6}$ (vi) $\frac{-3}{11}$

Sol. (i) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$ (ii) Additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$

(iii) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$ Hence, its additive inverse is $\frac{9}{11}$

(iv) We may write, $\frac{-6}{-7}=\frac{(-6)\times(-1)}{(-7)\times(-1)}=\frac{6}{7}$ Hence, its additive inverse is $\frac{-6}{7}$

(v) $\frac{7}{6}$: Let, the additive inverse be x. Now, $\frac{7}{6}+x=0\Rightarrow x=-\frac{7}{6}$, which is the required additive inverse.

(vi) $\frac{-3}{11}$: Let, the additive inverse be y. Now, $\frac{-3}{11}+y=0\Rightarrow y=\frac{3}{11}$, which is the required additive inverse.

Example 12 :

What number should be added to $\frac{5}{-8}$ to get $\frac{-3}{2}$?

Sol. According to given question, Required number $=-\frac{3}{2}-\left[\frac{5}{-8}\right]=-\frac{3}{2}-\left[\frac{5}{-8}\times\left(\frac{-1}{-1}\right)\right]$

$=-\frac{3}{2}-\left[-\frac{5}{8}\right]=-\frac{3}{2}+\frac{5}{8}=\frac{(-3\times4)+5}{8}=\frac{-12+5}{8}=-\frac{7}{8}$, which is the required solution.

Example 13 :

Check if $\frac{7}{18}-\frac{10}{27}=\frac{10}{27}-\frac{7}{18}$

Sol. $\frac{7}{18}-\frac{10}{27}=\frac{21-20}{54}=\frac{1}{54}$; $\frac{10}{27}-\frac{7}{18}=\frac{20-21}{54}=\frac{-1}{54}$; $\frac{1}{54}$ is not the same as $-\frac{1}{54}$

So, two rational numbers are not commutative under subtraction.

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBER

1. **Closure property :** The product of any two rational numbers is a rational number.

   If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational numbers.

   e.g. $\left(-\frac{3}{4}\right)\times\frac{5}{7}=\left(-\frac{15}{28}\right)$. Here, $\left(-\frac{15}{28}\right)$ is a rational number.

2. **Commutative property :** If we multiply two rational numbers in any order, the result is the same.

   If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$. e.g., $\left(-\frac{2}{3}\right)\times\frac{11}{17}=\frac{11}{17}\times\left(-\frac{2}{3}\right)$

3. **Associative property :** The product of three rational numbers can be found out in two ways : (i) preserving the same order and multiplying the product of the first and second number to the third number, (ii) preserving the same order and multiplying the first number to the product of the second and third number. The result is the same in both the cases. If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then

$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ e.g., $\left[\left(-\frac{1}{6}\right)\times\frac{1}{7}\right]\times\frac{3}{5}=\left(-\frac{1}{6}\right)\times\left[\frac{1}{7}\times\frac{3}{5}\right]$

4. **Existence of neutral number :** The product of any rational number with 1 is that same rational number.

If $\frac{a}{b}$ is a rational number, then $\frac{a}{b}\times 1 = 1\times\frac{a}{b}=\frac{a}{b}$ .e.g., $\left(-\frac{4}{5}\right)\times 1=\left(-\frac{4}{5}\right)$

Thus 1 is the neutral number for multiplication.

5. **Existence of inverse number :** For every non-zero rational number $\frac{a}{b}$, there exists a rational number such that the product of the two is 1. Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1$

$\frac{a}{b}$ is called the reciprocal of $\frac{a}{b}$. It is denoted by $\left(\frac{a}{b}\right)^{-1}$. Thus, $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$.

e.g. $\left(-\frac{7}{12}\right)\times\left(-\frac{12}{7}\right)=1$. Thus, $\left(-\frac{7}{12}\right)$ and $\left(-\frac{12}{7}\right)$ are inverse of each other. The inverse of 0 does not exist.

6. **Distributive law :** In rational numbers, the multiplication is distributed over the addition.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are rational numbers then $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$.

e.g., $\left(-\frac{3}{4}\right)\times\left[\left(-\frac{1}{2}\right)+\frac{7}{3}\right]=\left[\left(-\frac{3}{4}\right)\times\left(-\frac{1}{2}\right)\right]+\left[\left(-\frac{3}{4}\right)\times\frac{7}{3}\right]$

Note :

* Whole numbers are closed under multiplication. For example, $0 \times 3 = 0$, a whole number.
* Integers are closed under multiplication. For example, $5 \times 8 = 40$, an integer.
* Multiplication is commutative for whole numbers.
* Multiplication is commutative for an integer.
* Multiplication is associative for whole numbers.
* Multiplication is associative for an integer. For example, $5 \times [(-7) \times (-8)] = [5 \times (-7)] \times (-8)$

Example 14 :

Let $\frac{3}{7}, \frac{4}{9}$ and $\frac{7}{12}$ be three rational numbers, then check out if the associative property of multiplication holds true for rational numbers.

Sol. LHS $=\left(\frac{3}{7}\times\frac{4}{9}\right)\times\frac{7}{12}=\left(\frac{3\times4}{7\times9}\right)\times\frac{7}{12}=\frac{12}{63}\times\frac{7}{12}=\frac{84}{756}=\frac{1}{9}$

RHS $=\frac{3}{7}\times\left(\frac{4\times7}{9\times12}\right)=\frac{3}{7}\times\frac{28}{108}=\frac{84}{756}=\frac{1}{9}$. So, $\left(\frac{3}{7}\times\frac{4}{9}\right)\times\frac{7}{12}=\frac{3}{7}\times\left(\frac{4}{9}\times\frac{7}{12}\right)$

Example 15 :

Show that : $\frac{1}{2}\times\left(\frac{3}{4}\times\frac{5}{6}\right)=\left(\frac{1}{2}\times\frac{3}{4}\right)\times\frac{5}{6}$

Sol. L.H.S. $=\frac{1}{2}\times\left(\frac{3}{4}\times\frac{5}{6}\right)=\frac{1}{2}\times\left(\frac{3\times5}{4\times6}\right)=\frac{1}{2}\times\left(\frac{15}{24}\right)=\frac{1}{2}\times\frac{15}{24}=\frac{1}{2}\times\frac{5}{8}=\frac{5}{16}$

R.H.S. $=\left(\frac{1}{2}\times\frac{3}{4}\right)\times\frac{5}{6}=\left(\frac{1\times3}{2\times4}\right)\times\frac{5}{6}=\frac{3}{8}\times\frac{5}{6}=\frac{3\times5}{8\times6}=\frac{1\times5}{8\times2}=\frac{5}{16}=$ L.H.S.

Example 16 :

Find the reciprocal of the following :

(i) $-\frac{14}{17}$ (ii) 6 (iii) 12 (iv) -8 (v) $\frac{5}{16}$

Sol. (i) $-\frac{14}{17}$;Let, reciprocal be x. Now, $-\frac{14}{17}\times x=1\Rightarrow x=-\frac{17}{14}$, which is the required reciprocal.

(ii) 6 : Let, the reciprocal be y. So, $6\times y=1\Rightarrow y=1/6$, which is the required reciprocal.

(iii) Reciprocal of 12 is $\frac{1}{12}$ (iv) Reciprocal of -8 is $\frac{1}{-8}$ i.e., $\frac{-1}{8}$ (v) Reciprocal of $\frac{5}{16}$ is $\frac{16}{5}$

DIVISION OF RATIONAL NUMBER

Division of rational number is the inverse of multiplication.

If x and y are two rational numbers such that $y\neq0$, then the result of dividing x by y is the rational number obtained on multiplying x by the reciprocal of y. When x is divided by y, we write $x\div y$. Thus, $x\div y=x\times\frac{1}{y}$.

In general, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d}\neq0$, then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{a\times d}{b\times c}$

Here, $\frac{a}{b}$ is called dividend, $\frac{c}{d}$ is called divisor and $\frac{a\times d}{b\times c}$ is called the quotient.

Properties of Division of rational number :

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d}\neq0$, then $\frac{a}{b}\div\frac{c}{d}$ is always rational number. That is, the set of all non-zero rational numbers is closed under division.

e.g. $\frac{27}{16}\div\frac{9}{8}=\frac{27}{16}\times\frac{8}{9}=\frac{27\times8}{16\times9}=\frac{3\times1}{2\times1}=\frac{3}{2}$ is a rational number.

2. For any rational numbers $\frac{a}{b}$, we have $\frac{a}{b}\div1=\frac{a}{b}$ and $\frac{a}{b}\div(-1)=-\frac{a}{b}$

e.g., $\frac{8}{21}\div1=\frac{8}{21}$, $\frac{8}{21}\div(-1)=\frac{8}{21}\div\frac{-1}{1}=\frac{8}{21}\times\frac{1}{-1}=\frac{8\times1}{21\times-1}=\frac{8}{-21}=\frac{-8}{21}$

3. For every non-zero rational number $\frac{a}{b}$, we have (i) $\frac{a}{b} \div \frac{a}{b} = 1$ (ii) $\frac{a}{b} \div \left(-\frac{a}{b}\right) = -1$ (iii) $-\frac{a}{b} \div \frac{a}{b} = -1$

Note :

* Whole numbers are not closed under division. For example, $5 \div 8 = \frac{5}{8}$, which is not a whole number.
* Integers are not closed under division. For example, $5 \div 8 = \frac{5}{8}$, which is not an integer.
* Division is not commutative for whole numbers.
* Division is not commutative for an integer.
* Division is not associative for whole numbers.
* Division is not associative for an integer. For example, $[(-10) \div 2]\,(-5) \neq (-10) \div [2 \div (-5)]$

Example 17 :

Find $\frac{7}{2} \div \frac{5}{9}$

Sol. $\frac{7}{2} \div \frac{5}{9} = \frac{7}{2} \times \frac{9}{5} = \frac{63}{10} = 6\frac{3}{10}$ (Reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$)

Example 18 :

Show that : $\frac{-7}{24} \div \frac{3}{-8} \neq \frac{3}{-8} \div \frac{-7}{24}$

Sol. $\frac{-7}{24} \div \frac{3}{-8} \neq \frac{3}{-8} \div \frac{-7}{24}$. Now, L.H.S. $= \frac{-7}{24} \div \frac{3}{-8} = \frac{-7}{24} \times \frac{-8}{3} = \frac{-7}{3} \times \frac{(-1)}{3} = \frac{7}{9}$

Also, R.H.S. $= \frac{3}{-8} \div \frac{-7}{24} = \frac{3}{-8} \times \frac{24}{-7} = \frac{3}{-1} \times \frac{3}{-7} = \frac{9}{7}$. $\therefore$ L.H.S. $\neq$ R.H.S.

Example 19 :

Prove that division of rational numbers is not commutative, using $\frac{1}{4} \div \frac{1}{2} \neq \frac{1}{2} \div \frac{1}{4}$

Sol. $\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$; $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$; $\frac{1}{2} \neq 2$. So, $\frac{1}{4} \div \frac{1}{2} \neq \frac{1}{2} \div \frac{1}{4}$

So division of rational numbers is not commutative.

SELF CHECK

Q.1 State which property of rational number is used in each of the following statements :

(i) $\frac{3}{8} + \frac{4}{11} = \frac{4}{11} + \frac{3}{8}$ (ii) $\left(\frac{-3}{11}\right) \times 0 = 0$ (iii) $\left(\frac{-2}{5}\right) \times \frac{8}{13} = \frac{8}{13} \times \left(\frac{-2}{5}\right)$ (iv) $\frac{4}{9} \times 1 = \frac{4}{9}$

(v) $\frac{2}{5} + \left(-\frac{2}{5}\right) = 0$ (vi) $\frac{5}{17} + 0 = \frac{5}{17}$ (vii) $\frac{2}{7} \times \frac{7}{2} = 1$ (viii) $\left(-\frac{1}{2}\right) + 0 = \left(-\frac{1}{2}\right)$

(ix) $\left(\frac{2}{9}+\frac{3}{8}\right)+\frac{4}{11}=\frac{2}{9}+\left(\frac{3}{8}+\frac{4}{11}\right)$ (x) $\frac{1}{4}\times\left(\frac{3}{5}\times\frac{2}{7}\right)=\left(\frac{1}{4}\times\frac{3}{5}\right)\times\frac{2}{7}$

(xi) $\left(\frac{-3}{4}\right)\times\left(\frac{5}{7}+\frac{7}{11}\right)=\left[\left(\frac{-3}{4}\right)\times\frac{5}{7}\right]+\left[\left(\frac{-3}{4}\right)\times\frac{7}{11}\right]$

Q.2 Fill in the blanks by selecting the proper alternative from those given in the brackets :

(i) 1 < .... < 2 $\left[\left(-\frac{3}{2}\right),(-8),\frac{7}{4}\right]$

(ii) (– 4) < .... < (–3) $\left[\frac{7}{2},(-3.8),(-4.8)\right]$

(iii) Inverse of (–0.6) is ... $\left[0.6,\left(-\frac{6}{10}\right),\left(-\frac{10}{6}\right)\right]$

(iv) The opposite of $\left(-\frac{8}{15}\right)$ is ..... $\left[\frac{8}{15},\frac{15}{8},\left(-\frac{15}{8}\right)\right]$

(v) Inverse of (–1) is .... [1, (–1), 0]

(vi) $\left(-1\frac{4}{5}\right)\times\left(-\frac{5}{9}\right)=$ .... [(–1), 1, 0]

(vii) $2\frac{3}{5}+\left(-\frac{13}{5}\right)=$ .... $\left[\frac{11}{2},5\frac{1}{2},0\right]$

(viii) 0.6 and .... are equal rational nos. $\left[(-0.6),\frac{10}{6},\frac{3}{5}\right]$

(ix) 2.15 is expressed as .... in the form $\frac{p}{q}$. $\left[\frac{215}{10},2\frac{15}{10},\frac{215}{100}\right]$

(x) The inverse of 0.05 is .... $\left[\frac{5}{100},\frac{10}{5},\frac{100}{5}\right]$

ANSWERS

(1) (i) commutative property for addition
(ii) product with zero
(iii) commutative property for multiplication
(iv) neutral number for multiplication
(v) existence of opposite
(vi) neutral number for addition
(vii) existence in inverse
(viii) neutral number for addition
(ix) associative property for addition
(x) associative property for multiplication
(xi) distributive law

(2) (i) $\frac{7}{4}$ (ii) (–3.8) (iii) $\left(-\frac{10}{6}\right)$ (iv) $\frac{8}{15}$ (v) (–1) (vi) 1

(vii) 0 (viii) $\frac{3}{5}$ (ix) $\frac{215}{100}$ (x) $\frac{100}{5}$

WORD PROBLEMS BASED ON RATIONAL NUMBERS

Example 20 :

An express train travels $\frac{455}{2}$ km in $\frac{7}{2}$ hours. Find the speed of the train in km/hr.

Sol. Speed in km/hr. $=\frac{455}{2}\div\frac{7}{2}=\frac{455}{2}\times\frac{2}{7}=65$ km/h.

Example 21 :

Radhika has a certain amount of money in her piggy bank. She spent Rs. $10\frac{1}{4}$ in the school canteen, gave Rs. $15\frac{1}{2}$ to her friend and bought a gift worth Rs. $25\frac{3}{4}$ for her brother. Radhika then had a balance of Rs. $200\frac{1}{8}$. How much did she have to begin with.

Sol. To begin with Radhika had

$$= \text{Rs.}\,10\frac{1}{4} + \text{Rs.}\,15\frac{1}{2} + \text{Rs.}\,25\frac{3}{4} + 200\frac{1}{8} = \text{Rs.}\left[(10+15+25+200)+\left(\frac{1}{4}+\frac{1}{2}+\frac{3}{4}+\frac{1}{8}\right)\right]$$

$$= \text{Rs.}\left[250+\frac{2+4+6+1}{8}\right] = \text{Rs.}\left[250+\frac{13}{8}\right] = \left[250+1\frac{5}{8}\right] = \text{Rs.}\,251\frac{5}{8}$$

SELF CHECK

Q.1 Find the cost of $3\frac{2}{5}$ metres of cloth at Rs. $36\frac{3}{4}$ per metre.

Q.2 After reading $\frac{7}{9}$ of a book, 40 pages are left. How many pages are there in the book ?

Q.3 If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, find the number.

Q.4 The product of two rational number is $\frac{-8}{9}$. If one of the numbers is $\frac{10}{3}$, find the other.

Q.5 From a rope 40 metre long, pieces of equal size are cut. If the length of one piece is $\frac{10}{3}$ metre, find the number of such pieces.

ANSWERS

(1) Rs. $124\frac{19}{20}$ **(2)** 180 **(3)** 140 **(4)** $\frac{-4}{15}$ **(5)** 12

USEFUL TIPS

1. While comparing rational numbers we have to keep in mind that the rational number is in standard form and make sure that denominator is positive

2. Always remember that, LCM is defined only for positive numbers which excludes all negative numbers as well as zero. e.g., LCM for (0, –3) and (3, –4) is not defined.
This means always make the denominator positive before taking LCM of rational numbers.

3. (a) Additive inverse exists in natural numbers, whole numbers and fractions also.
(b) Additive inverse of zero is zero.

4. Natural numbers, whole numbers and integers.
(a) Are closed under addition (b) Show commutativity in addition (c) Show associativity in addition.

5. Subtraction of zero from a rational number gives the rational number itself.

6. Always remember that when we multiply two rational numbers, we multiply the numerator with numerator and denominator with denominator.

7. Always remember that division of a rational number by zero is not defined.

8. Like rational numbers, natural numbers, whole numbers and integers do not show the following properties are division. (i) Closure property (ii) Commutative property (iii) Associative property

9. The table shows which of the given properties are shown under subtraction, by natural numbers, whole numbers and integers.

Property	Natural number	Whole number	Integers
Closure	×	×	√
Commutative property $a - b = b - a$	×	×	×
Associative property $a - (b - c) = (a - b) - c$	×	×	×

10. The table shows which of the given properties are shown under multiplication, by natural nos, whole nos. & integers.

Property	Natural number	Whole number	Integers
Closure	✓	✓	✓
Commutative property $a \times b = b \times a$	✓	✓	✓
Associative property $(a \times b) \times c = a \times (b \times c)$	✓	✓	✓

SOLVED NCERT TEXT BOOKS QUESTIONS (TRY THESE/EXERCISES)

Q.1 Fill in the blanks in the following table

Numbers	Closed under Addition	Subtraction	Multiplication	Division
Rational numbers	Yes	Yes	....	No
Integers	....	Yes	....	No
Whole numbers	....	....	Yes	....
Natural numbers	....	No	....	....

Sol.

Numbers	Closed under Addition	Subtraction	Multiplication	Division
Rational numbers	Yes	Yes	Yes	No
Integers	Yes	Yes	Yes	No
Whole numbers	Yes	No	Yes	No
Natural numbers	Yes	No	Yes	No

Q.2 Complete the following table :

Numbers	Commutative for Addition	Subtraction	Multiplication	Division
Rational numbers	Yes	....	....	....
Integers	....	No	....	....
Whole numbers	....	....	Yes	....
Natural numbers	....	....	....	No

Sol.

Numbers	Commutative for Addition	Subtraction	Multiplication	Division
Rational numbers	Yes	No	Yes	No
Integers	Yes	No	Yes	No
Whole numbers	Yes	No	Yes	No
Natural numbers	Yes	No	Yes	No

Q.3 Complete the following table :

Numbers	Associative for Addition	Subtraction	Multiplication	Division
Rational numbers	....	....	....	No
Integers	....	....	Yes	....