3

COORDINATE GEOMETRY

INTRODUCTION

Analytic geometry, is done by representing points in the plane by ordered pairs of real numbers, called Cartesian co-ordinates and representing lines and curved by algebraic equations. In analytic geometry because of use of co-ordinate, it is called coordinate geometry.

CO-ORDINATES SYSTEMS

Cartesian Co-ordinates :

Let XOX' and YOY' be two perpendicular straight lines drawn through point O in the plane of the paper. Then –

X–Axis: The line XOX' is called X–axis

Y–Axis: The line YOY' is called Y–axis

Co-ordinate axes : x–axis and y–axis together are called axis of co-ordinates or axis of reference

Origin : The point 'O' is called the origin of co-ordinates or the Origin.

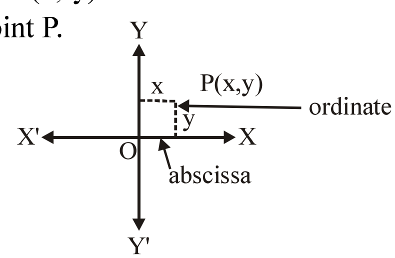
Cartesian Co-ordinates :

The ordered pair of perpendicular distance from both axis of a point P lying in the plane is called Catesian co-ordinates of P. If the cartesian co-ordinates of a point P are (x, y) then x is called abscissa or x co-ordinate of P and y is called the ordinate or y co-ordinate of point P.

(i) Co-ordinates of the origin is (0, 0)

(ii) y co-ordinate on x-axis is zero.

(iii) x co-ordinate on y–axis is zero.



QUADRANTS

The two axes XOX' and YOY' divide the plane into four parts, these are called quadrants. XOY, YOX', X'OY' and Y'OX are, called the first, second, third and fourth quadrants respectively. We usually take the direction of OX and OY as positive, while the direction of OX' and OY' as negative.

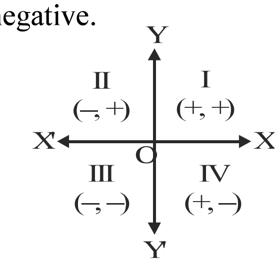
If the coordinates of a point P in the plane are (x, y) , then

In first quadrant; $x > 0, y > 0$; coordinates (+, +)

Second quadrant; $x < 0, y > 0$; coordinates (-, +)

Third quadrant; $x < 0, y < 0$; coordinates (-, -)

Fourth quadrant; $x > 0, y < 0$; coordinates (+, -)



(i) If the coordinates of a point P are (x, y), we generally write it as f '(x, y).

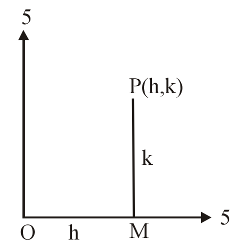
(ii) The abscissa of a point is its perpendicular distance from y-axis.

(iii) The ordinate of a points is its perpendicular distance from x-axis.

(iv) The abscissa of a point situated on the right side of y-axis is positive and the abscissa of a point situated on the left side of y-axis is negative.

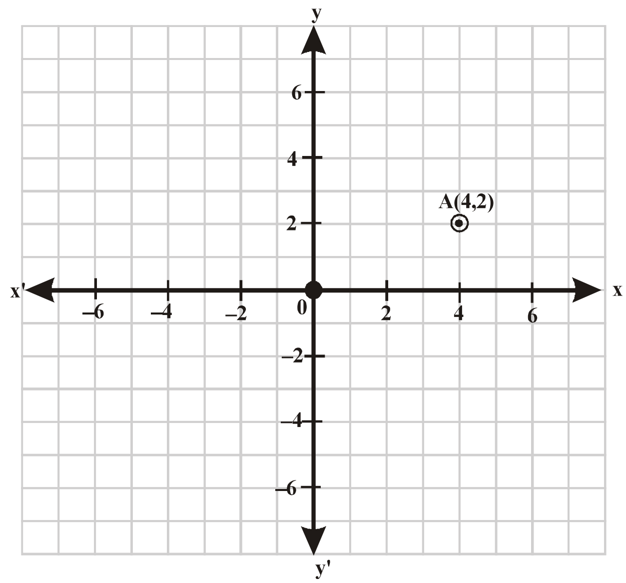
(v) The ordinate of a point situated above x-axis is positive and that below the x-axis is negative.
(vi) If y = 0, the point situated on x-axis.
(vii) If x = 0, the point situated on y-axis.
(viii) If x = 0, y = 0, the point is a Origin.

Plotting of a point whose coordinates are known :
The point can be plotted by measuring its proper distance from the axes. Thus, any point (h, k) can be plotted as follows :
(i) Measure OM equal to h along the xi-axis.
(ii) Now measure MP perpendicular to OM and equal K.



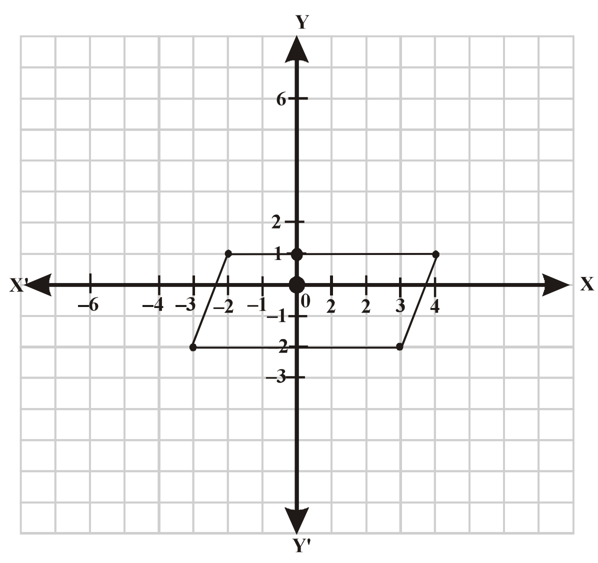
Example 1 :
Plot the points A (4, 2).

Sol. On a graph paper, draw the co-ordinate axes xox' and yoy' intersecting at origin O. With proper scale, mark the numbers on the two co-ordinate axes. For plotting any point, two steps are to be adopted. e.g. to plot point A (4, 2).
Step 1 : Starting from the origin O, move 4 units along the positive direction of x-axis i.e. to the right of the origin O.
Step 2 : Now, from there, move 2 units up (i.e. parallel to positive direction of y-axis) and place a dot at the point reached. Label this point as A (4, 2)



Example 2 :
Plot the points A (4, 1), B (–2, 1), C (–3, –2) and D (3, –2).
Name the figure ABCD. Find its area.

Sol. Points A, B, C, D are marked in the adjoining figure. It is easy to see that it is a parallelogram.
Area = base × height = 6 units × 3 units
= 18 square units



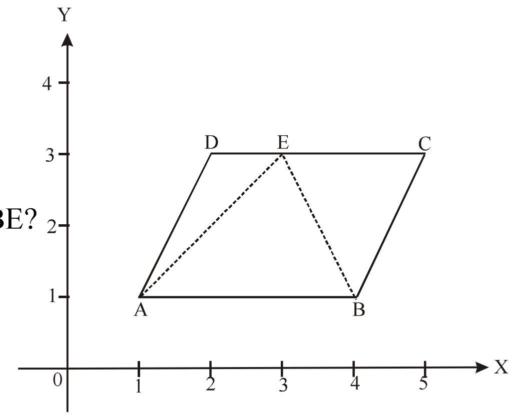
Example 3 :

Using the adjoining diagram, write down the co-ordinates of the points A, B, C, D and E.

(i) Name the figure ABCD and find its area.

(ii) Name the figure ABE, find its area.

(iii) Is area of the figure ABCD twice the area of the figure ABE? What conclusion can you derive from the above result ?



Sol. Clearly from the diagram, we have

A (1, 1), B (4, 1), C (5, 3), D (2, 3), E (3, 3).

(i) ABCD is a parallelogram, since AB = DC = 3 units, and AB is parallel to DC.

Area = base × height = 3 × 2 = 6 sq. units

(ii) ABE is a triangle, since, it is a three-sided figure. Area = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 3 × 2 = 3 sq. units

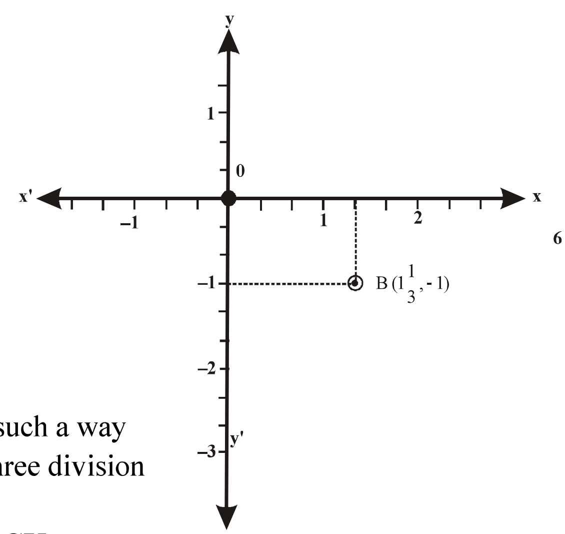
(iii) Yes, Area of // gm ABCD = 2 (Area of triangle ABE)

Conclusion : The area of parallelogram is twice the area of a triangle on the same base and between the same parallels.

Example 4 :

Plotting of the points whose abscissa or ordinate or both are fractional numbers.

Plot the points : $B\left(1\frac{1}{3}, -1\right)$



Sol. Plotting the points Plot the points : $B\left(1\frac{1}{3}, -1\right)$

By taking suitable scale, mark the co-ordinate axes in such a way that the fraction 1/3 can easily be read. For this, take three division equal to one unit.

SELF CHECK

Q.1 Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5), (5, –3) and (6, 1) in the Cartesian plane.

Q.2 Plot each of the following points on graph **(i)** $\left(-2\frac{1}{3}, \frac{3}{4}\right)$ **(ii)** $\left(0, -2\frac{1}{4}\right)$ **(iii)** $\left(2\frac{2}{3}, -1\frac{2}{5}\right)$

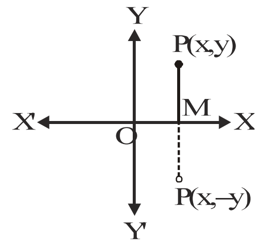
Q.3 A (–2, 2), B (8, 2) and C (4, – 4) are the vertices of a parallelogram ABCD. By plotting the given points on a graph paper. Find the co-ordinates of the fourth vertex D. Also, from the same graph, state the co-ordinates of the mid-points of the sides AB and CD.

Image of an object in a mirror : When an object is placed in front of a plane mirror, then its image is formed at the same distance behind the mirror as the distance of the object from the mirror.

Reflection : The transformation R_1 which maps a point P to its image P' in a given line (or point) ℓ, is called a reflection in ℓ. Thus, $R_\ell(P) = P'$. We shall represent :

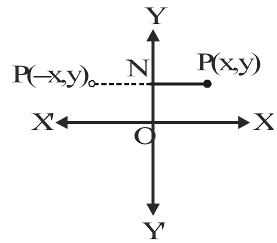
(i) Reflection in x-axis by R_x, **(ii)** Reflection in y-axis by R_y, **(iii)** Reflection in the origin by R_o,

Reflection in x-axis :



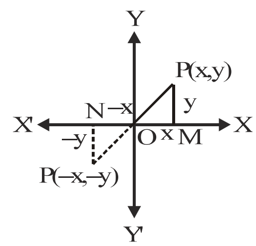
Let P (x, y) be a point in a plane. Draw PM ⊥ OX, meeting it at M.
Produce PM to P' such that MP = MP'.
Then, P' is the image of P when reflected in x-axis.
Clearly, the co-ordinates of P' are P' (x, –y)
∴ P (x, y) when reflected in x-axis, have the image P' (x, –y)
∴ R_x (x, y) = (x, –y)

Reflection in y-axis :



Let P (x, y) be a point in a plane. Draw PN ⊥ OY, meeting it at N.
Produce PN to P' such that NP = NP'.
Then, P' is the image of P when reflected in y-axis.
Clearly, the co-ordinates of P' are P' (x, –y)
∴ P (x, y) when reflected in x-axis, have the image P' (– x, y) ∴ R_y (x, y) = (– x, y)

Reflection in the origin :



Let P (x, y) be a point in a plane.
Join PO and produce it to P' such that OP' = OP.
Then, P' is the image of P when reflected in the origin.
Clearly, the co-ordinate of P' are P' (–x, –y)
∴ P (x, y) when reflect in the origin, has the image P' (–x, –y)
∴ R_0 (x, y) = (– x, –y)

Example 5 :

(i) Point P (a, b) is reflected in the x-axis to P'(5, –2). Write down the values of a and b.
(ii) P'' is the image of P when reflected in y-axis. Write down the coordinates of P''.
(iii) Name a single transformation that maps P' to P''.

Sol. (i) We know that, R_x (a, b) = (a, –b)
∴ R_x (a, b) = (5, –2) ⇒ (a, –b) = (5, –2) ⇒ a = 5 and – b = – 2 ⇒ a = 5 and b = 2
Hence, the coordinates of P are P (5, 2)

(ii) We know that R_y (x, y) = (–x, y) ∴ R_y (5, 2) = (–5, 2). Hence the coordinate of P'' are P'' (5, 2)

(iii) P' → P'' means (5, –2) → (–5, 2). Also, R_0 (5, –2) = (–5, 2)
∴ Reflection in the origin is the single transformation that maps P' to P''.

EXTRA EDGE

Distance Formula : The distance between two points P(x_1, y_1) and Q(x_2, y_2) is given by

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(i) Distance of point P(x,y) from the origin $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

(ii) Distance between two polar co-ordinates A (r_1, θ_1) and B (r_2, θ_2) is given by

$$AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

Note : Always the distance is taken in positive sign.

Applications of distance formula :

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. After finding AB, BC, and CA we shall find that the point are

(a) Collinear, if the sum of any two distances is equal to the third.

(b) Vertices of an equilateral triangle if AB = BC = CA

(c) Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB

(d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A, B, C, D

(a) AB = BC = CD = DA ; AC = BD $\Rightarrow$ ABCD is a square

(b) AB = BC = CD = DA ; AC $\neq$ BD $\Rightarrow$ ABCD is a rhombus

(c) AB = CD , BC = DA ; AC = BD $\Rightarrow$ ABCD is a rectangle

(d) AB = CD , BC = DA ; AC $\neq$ BD $\Rightarrow$ ABCD is a parallelogram

S.No.	**Quadrilateral**	**Diagonals**	**Angle between diagonals**
(i)	Parallelogram	Not equal	$\theta \neq \pi/2$
(ii)	Rectangle	Equal	$\theta \neq \pi/2$
(iii)	Rhombus	Not equal	$\theta = \pi/2$
(iv)	Square	Equal	$\theta = \pi/2$

(a) Diagonal of square, rhombus, rectangle and parallelogram always bisect each other

(b) Diagonal of rhombus and square bisect each other at right angle.

(c) Four given points are collinear, if area of quadrilateral is zero.

Example 6 :

If distance between the point (x, 2) and (3, 4) is 2, then the value of x.

Sol. $2 = \sqrt{(x-3)^2 + (2-4)^2} \Rightarrow 2 = \sqrt{(x-3)^2 + 4}$

Squaring both sides $4 = (x-3)^2 + 4 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

Example 7 :

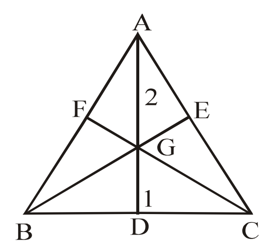
Prove that points A (1, 1), B (–2, 7) and C (3, –3) are collinear.

Sol. $AB = \sqrt{(1+2)^2 + (1-7)^2} = \sqrt{9+36} = 3\sqrt{5}$. $BC = \sqrt{(-2-3)^2 + (7+3)^2} = \sqrt{25+100} = 5\sqrt{5}$

$CA = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = 2\sqrt{5}$. Clearly BC = AB + AC. Hence A, B, C are collinear.

CO-ORDINATE OF SOME PARTICULAR POINTS

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of any triangle ABC, then

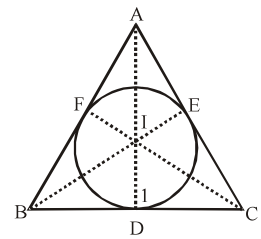


1. Centroid : The centroid is the point of intersection of the medians (Line joining the mid point of sides and opposite vertices)

Centroid divides the median in the ratio of 2 : 1

Co-ordinates of centroid G $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

2. **Incentre :** The incentre is the point of intersection of internal bisector of the angle. Also it is a centre of circle touching all the sides of a triangle.



Co-ordinates of incentre I $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

where a, b, c are the sides of triangle ABC

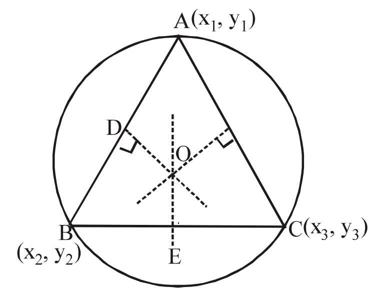
(i) Angle bisector divides the opposite sides in the ratio of remaining sides

Ex. $\frac{BD}{DC}=\frac{AB}{AC}=\frac{c}{b}$

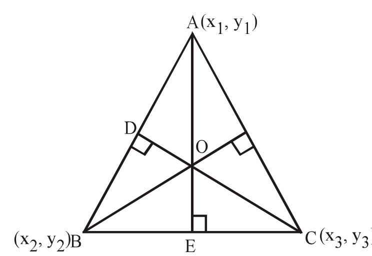
(ii) Incentre divides the angle bisectors in the ratio (b + c) : a, (c + a) : b and (a + b) : c

3. **Excentre :** Point of intersection of one internal angle bisector and other two external angle bisector is called as excentre. There are three excentre in a triangle. Co-ordinate of each can be obtained by changing the sign of a, b, c respectively in the formula of Incentre.

4. **Circumcentre :** It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of a circle passing vertices of the triangle. If O is the circumcentre of any triangle ABC, then
$OA^2 = OB^2 = OC^2$
If a triangle is right angle, then its circumcentre is the mid point of hypotenuse.



5. **Orthocentre :**
It is the point of intersection of perpendicular drawn from vertices on opposite sides (called altitudes) of a triangle and can be obtained by solving the equation of any two altitudes. If a triangle is right angled triangle, then ortho centre is the point where right angle is formed.



(i) If the triangle is equilateral, the centroid, incentre, orthocentre, circumcentre, coincides
(ii) Ortho centre, centroid and circumcentre are always colinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.
(iii) In an isosceles triangle centroid, orhtocentre, incentre, circumecentre lies on the same line

Example 8 :
If (0, 1), (1, 1) and (1, 0) are middle points of the sides of a triangle, find its incentre is -

Sol. Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of a triangle, then
$x_1 + x_2 = 0$, $x_2 + x_3 = 2$, $x_3 + x_1 = 2$
$y_1 + y_2 = 2$, $y_2 + y_3 = 2$, $y_3 + y_1 = 0$
Solving these equations, we get, A (0, 0), B (0, 2) and C (2, 0)
Now, a = BC = $2\sqrt{2}$, b = CA = 2, c = AB = 2
Thus incentre of a Δ ABC is $(2-\sqrt{2},\ 2-\sqrt{2})$

ADDITIONAL EXAMPLES

Example 1 :

If the distance between points (x, 3) and (5, 7) is 5, find the value of x.

Sol. Let P (x, 3) and Q (5, 7) be the given points then PQ = 5

$$\sqrt{(x-5)^2 + (3-7)^2} = 5$$

Squaring both sides $(x-5)^2 + (-4)^2 = 25$ or $x^2 - 10x + 25 + 16 = 25$

or $x^2 - 10x + 16 = 0$ or $(x-2)(x-8) = 0$ $\therefore$ x = 2, 8

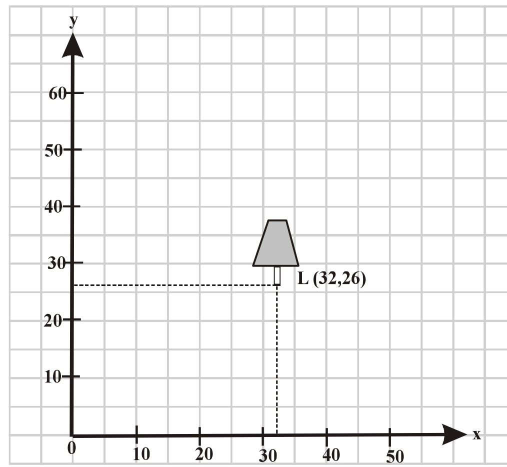
Example 2 :

How will you describe the position of a table lamp on your study table to another person ?

Sol. In order to locate the position of the lamp, we take a fixed point (i.e. corner of the table) for reference.

Let lamp be a point and table as plane.

Draw perpendicular from the lamp point of two edges (perpendicular from the lamp point to each other) of the table and measure the distance of these perpendiculars from the fixed point (i.e. corner of the table). Let these distances are 26 cm. and 32 cm.



Then position of lamp can be written as L (26, 32) or L (32, 26) with respect to origin i.e. fixed point i.e. corner of table. (see figure).

Example 3 :

Write the coordinate of a point.

(i) Above x-axis lying on y-axis and at a distance of 3 units.

(ii) Below x-axis and on y-axis at a distance of 8 units

(iii) Right of origin and on x-axis at a distance of 2 units.

(iv) Left of y-axis and on x-axis at a distance of units.

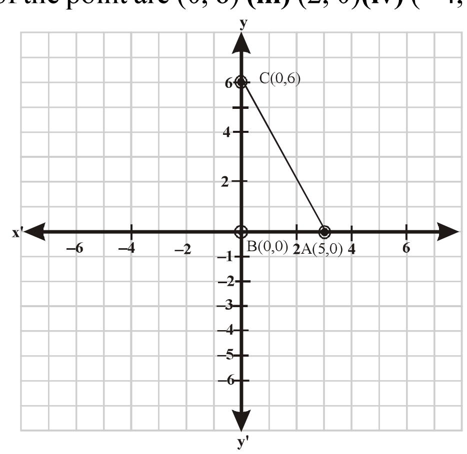
Sol. **(i)** The coordinates of the point are (0, 3) **(ii)** The coordinates of the point are (0, 8) **(iii)** (2, 0)**(iv)** (– 4, 0)

Example 4 :

Find the area of the figure formed by joining the points (5, 0), (0, 0), (0, 6).

Sol. Given points A (5, 0), B (0, 0), C (0, 6). After plotting the points A, B, C on rectangular boxes, we get (figure)

Now join AB, BC and CA. It is a right-triangle in which $\angle$ ABC = 90°, AB = 5 units and BC = 6 units.



$\therefore$ Area of rt Δ ABC = $\frac{1}{2}$ base × height = $\frac{1}{2} \times 5 \times 6$

$\therefore$ ar Δ ABC = 15 sq. units.

Example 5 :

If the point (x, y) is equidistant from the points (a + b, b – a) and (a – b, a + b), prove that bx = ay.

Sol. Let P (x, y), Q (a + b, b – a) and R (a – b, a + b) be the given points, therefore according to the questions.

$PQ = PR$ or $PQ^2 = PR^2$

or $[x - (a + b)]^2 + [y - (b - a)]^2 = [x - (a - b)]^2 + [y - (a + b)]^2$

or $x^2 - 2(a + b)x + (a + b)^2 + y^2 - 2(b - a)y + (b - a)^2$

$= x^2 - 2(a - b)x + (a - b)^2 + y^2 - 2(a + b)y + (a + b)^2$

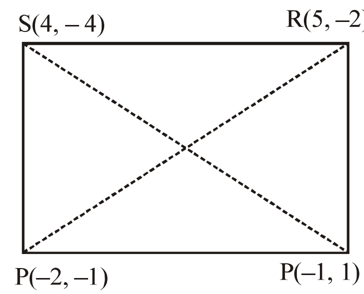
or $-2(a + b)x - 2(b - a)y = -2(a - b)x - 2(a + b)y$

or $ax + bx + by - ay = ax - bx - ay - by$ or $2bx = 2ay \Rightarrow bx = ay$

Example 6 :

Prove that the point (–2, –1), (–1, 1), (5, –2) and (4, – 4) are the vertices of a rectangle.

Sol. Let P (–2, –1), Q (–1, 1), R (5, –2) and S (4, – 4) be the given points.



Now, $PQ = \sqrt{[-2-(-1)]^2 + [-1-1]^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$QR = \sqrt{[5-(-1)]^2 + [-2-1]^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$RS = \sqrt{[4-5]^2 + [-4-(-2)]^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$SP = \sqrt{[4-(-2)]^2 + [-4-(-1)]^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$\therefore$ PQ = RS and QR = SP. Hence, the opposite sides are equal

Again, diagonal $PR = \sqrt{[5-(-2)]^2 + [-2-(-1)]^2} = \sqrt{(7)^2 + (-1)^2} = \sqrt{50}$

$QS = \sqrt{[4-(-1)]^2 + [-4-1]^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{50}$

Hence the diagonals are equal. Hence the given point P, Q, R, S are the vertices of rectangle.

Example 7 :

Plot the points (2, 0), (2, 3), (0, 6), (–2, 3) and (–2, 0) and join them in order. Find the type of figure thus formed.

Sol. Given points A (2, 0), B (2, 3), C (0, 6), D (–2, 3) and E (–2, 0). After plotting and joining, we get the figure.

After plotting A, B, C, D and E and joining we get a 'Pentagon'.