LINES AND ANGLES

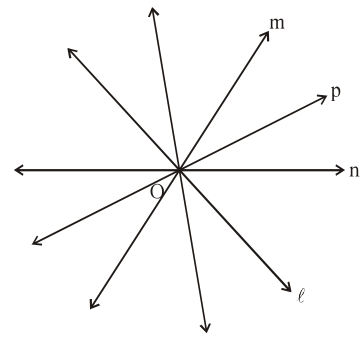
BASIC CONCEPTS IN GEOMETRY

A 'point', a 'line' and a 'plane' are the basic concepts to be used in geometry.

Axioms :

The basic facts which are granted without proof are called axioms.

The word "axiom"comes from a Greek word meaning "worthy".

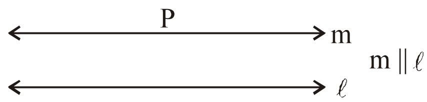


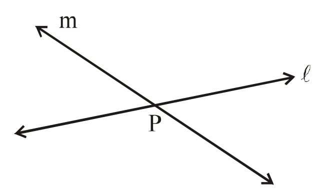
1. A line segment can be produced to any desired length.

2. A line is the collection of infinite number of points.

3. Through a given point, an infinite lines can be drawn.

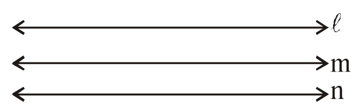
4. Given two distinct points, there is one and only one line that contains both the points.





5. If P is a point outside a line ℓ, then one and only one line can be drawn through P which is parallel to ℓ.

6. Two distinct lines cannot have more than one point in common.

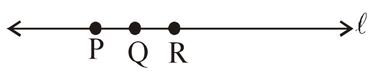


7. Two lines which are both parallel to the same line, are parallel to each other.

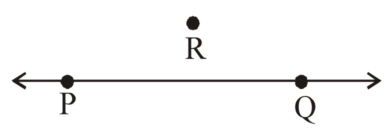
i.e., $\ell \parallel n,\ m \parallel n \Rightarrow \ell \parallel m$

DEFINITIONS

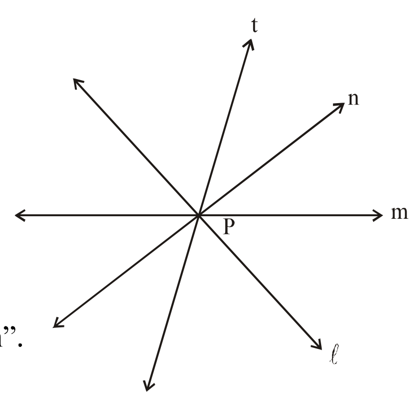
Collinear points : Three or more points are said to be collinear if there is a line which contains all to them. P, Q and R in fig (a) are collinear while in fig. (b) are non-collinear.



(a)

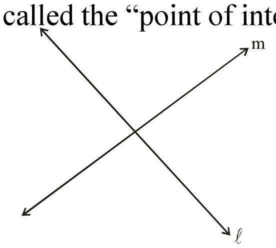


(b)

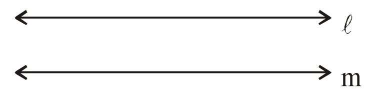


Concurrent lines : Three or more lines are said to be concurrent if there is a point which lies on all of them.

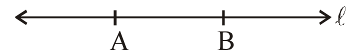
Intersecting lines : Two lines are intersecting if they have a common point. The common point is called the "point of intersection".



Parallel lines : The straight lines, which lie in the same plane and do not meet at any point on producing on either side, are called parallel lines.
The distance between the two parallel lines always remains the same. If two straight lines are parallel to any other line, then they are parallel to each other also.



Line segment : Given two points A and B on a line ℓ, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

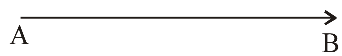


Interior point of a line segment : A point R is called an interior point of a line segment PQ if R lies between P and Q but R is neither P nor Q.

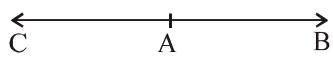
Congruence of line segment : Two line segments AB and CD are congruent if trace copy of one can be superposed on the other so as to cover it completely and exactly in this case we write AB $\cong$ CD. In other words we can say two lines are congruent if their lengths is same.
Distance between the two points: The distance between two points P and Q is the length of line segment PQ.

RAY

A line segment AB when extended indefinitely in one direction is called the ray $\overrightarrow{AB}$. It has one end point A. A ray has no definite length. A ray cannot be drawn on a piece of paper, it can simply be represented.

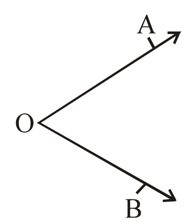


Opposite rays : Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of the two rays.

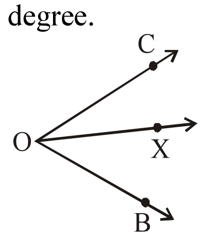


ANGLE

An angle is the union of two non-collinear rays with a common initial point. The common initial point is called the vertex of the angle and the two rays are called the 'arms' of the angles.

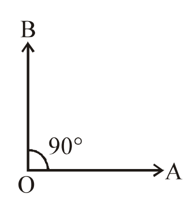


Every angle has a measure and unit of measurement is degree.
One right angle = 90°
1° = 60' (minutes) ; 1' = 60" (Seconds)
Angle addition axiom :
If x is a point in the interior of $\angle$ BAC,
then m $\angle$ BAC = m $\angle$ BAX + m $\angle$ XAC

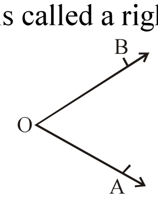


TYPES OF ANGLES

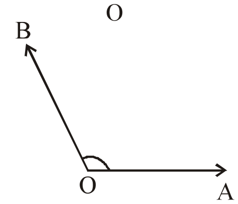
Right angle: An angle whose measure is 90° is called a right angle.



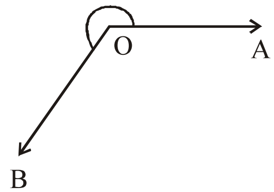
Acute angle : An angle whose measuring is less than 90° is called an acute angled.
$0° < \angle BOA < 90°$



Obtuse angle : An angle whose measure is more than 90° but less than 180° is called an obtuse angle.
$90° < \angle AOB < 180°$



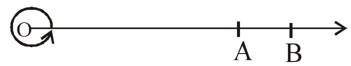
Straight line : An angle whose measure is 180° is called a straight angle.

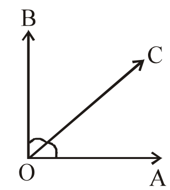


Reflex angle : An angle whose measure is more that 180° is called a reflex angle.

$180° < \angle AOB < 360°$

Complete angle : An angle whose measure is 360° is called a complete angle.



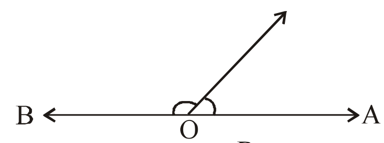


Complementary angles : Two angles, whose sum measure is 90° are called complementary angles.

$\angle AOC$ & $\angle BOC$ are complementary as $\angle AOC + \angle BOC = 90°$

Example (1) : Complement of 36° = 90° – 36° = 54°

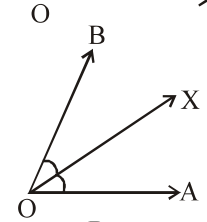
Example (2) : Complement of 23°45'19' = 90° – 23°45'19" = 66°14'41"



Supplementary angles: Two angles, the sum of whose measures is 180°, are called the supplementary angles.

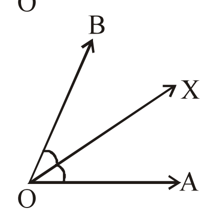
$\angle AOC$ & $\angle BOC$ are supplementary as their sum is 180°

Supplement of an angle of 115° = An angle of (180° – 115°) = 65°



Angle bisectors :

A ray OX is said to be the bisector $\angle AOB$, if X is a point in the interior of $\angle AOB$ and $\angle AOX = \angle BOX$



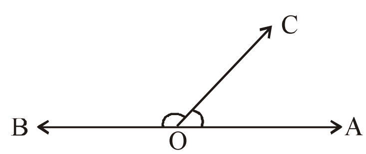
Adjacent angles :

Two angles are called adjacent angles, if (i) they have the same vertex, (ii) they have a common arm and (iii) non common arms are on either side of the common arm.

$\angle AOX$ and $\angle BOX$ are adjacent angles, OX is common arm, OA and OB are common arms and lies on either side of OX.

Linear pair of angles :

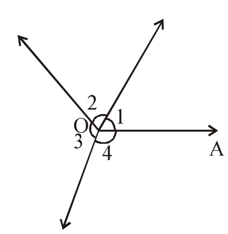
Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays. If the sum of two adjacent angles is 180°, they are said to form a linear pair.



Angle about a point :

The sum of all the angles formed about a point is 360°.

In the given figure, we have : $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

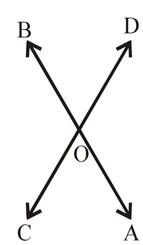


Vertically opposite angles :

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.

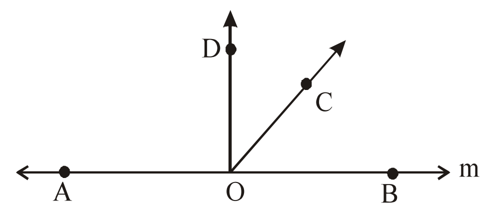
$\angle AOC$ & $\angle BOD$ form a pair of vertically opposite angles.

Also $\angle AOD$ and $\angle BOC$ form a pair of vertically opposite angles.



Theorem 1 :

If a ray stands on a line, then the sum of the adjacent angles so formed is 180°.



Given : A ray OC stands on a st. line AB forming adjacent angles AOC and BOC.

To prove : $\angle AOC + \angle BOC = 180°$

Construction : Draw $OD \perp AB$

Proof : $\angle AOC = \angle AOD + \angle DOC$ .... (1) [Adding axiom of adj. $\angle$ s]

$\angle BOC = \angle BOD - \angle DOC$ .... (2) [Adding axiom of adj. $\angle$ s]

Adding (1) and (2), we get

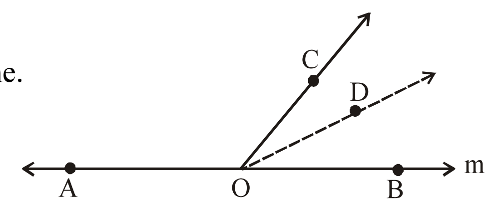
$\angle AOC + \angle BOC = \angle AOD + \angle BOD$ [$\angle AOD = \angle BOD = 90°$, since $OD \perp AB$]

$= 90° + 90° = 180°$

Hence, $\angle AOC + \angle BOC = 180°$

Theorem 2: If the sum of two adjacent angles is 180°, the non-common arms the angles are in a straight line.

Given : Given two adjacent angles $\angle AOC$ and $\angle BOC$ such that $\angle AOC + \angle BOC = 180°$



To prove : OA and OB are in the same st. line, AOB is st. line.

Construction : If possible, let AOB be not a st. line.

Then produce AO to D, such that AOD is a st. line.

Proof : $\angle AOC + \angle BOC = 180°$ .... (1) [Given]

$\angle AOC = \angle DOC = 180°$ .... (2) [Ray OC stands on line AOD]

$\angle AOC + \angle BOC = \angle AOC + \angle DOC$

$\Rightarrow \angle BOC = \angle DOC$ [$\angle AOC$ is common to both sides]

But $\angle BOC \neq \angle DOC$ [A part cannot be equal to the whole]

$\therefore$ Our assumption is wrong. Hence, AOB is a st. line.

Example 1 :

The supplement of an angle is one-fifth of itself. Determine the angle and its supplement.

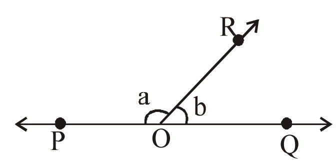
Sol. Let the measure of the angle be x°. Then the measure of its supplementary angle is 180° – x°.

It is given that $180 - x = \frac{1}{5}x \Rightarrow 5(180° - x) = x \Rightarrow 900 - 5x = x \Rightarrow 900 = 5x + x$

$\Rightarrow 900 = 6x \Rightarrow 6x = 900 \Rightarrow x = 900/6 = 150$

Example 2 :

In figure, $\angle POR$ and $\angle QOP$ form a linear pair. If a – b = 80°, find the values of a and b.



Sol. $\because \angle POR$ and $\angle QOR$ for a linear pair

$\therefore \angle POR + \angle QOR = 180°$ (Linear pair axiom)

or a + b = 180° .... (1)

But a – b = 80° .... (2) [Given]

Adding eq (1) and (2), we get 2a = 260° $\therefore$ a = 260/2 = 130°

Substituting the value of a in (1), we get, 130° + b = 180° ; b = 180° – 130° = 50°

SELF CHECK

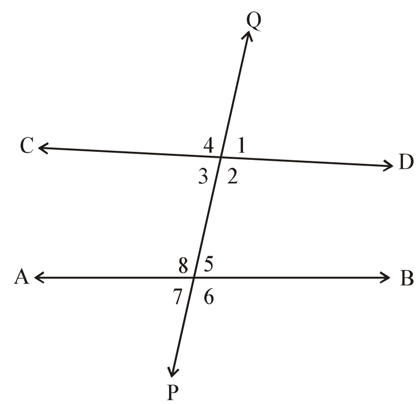
Q.1 Which angle is twice its supplement ? **Q.2** Which angle is half of its complement ?

ANSWERS

(1) 120° **(2) 30°**

TRANSVERSAL

A line which intersects two or more given lines at distinct points is called a transversal of the given lines.
Straight lines AB and CD are cut by transversal PQ.



Corresponding angles : Two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines, in figure ∠ 1 & ∠ 5, ∠ 4 & ∠ 8, ∠ 2 & ∠ 6, ∠ 3 & ∠ 7 are the pairs of corresponding angles.

Alternate interior angles : ∠ 3 & ∠ 5, ∠ 2 & ∠ 8, are the pairs of alternate interior angles.

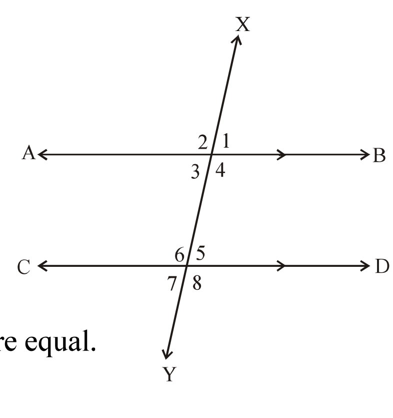
Consecutive interior angles :
The pair of interior angles on the same side of the transversal are called pairs of consecutive interior angles. In figure ∠ 2 & ∠ 5, ∠ 3 & ∠ 8, are the pairs of consecutive interior angles.

Corresponding angles axiom :
In a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely, if a transversal intersects two lines, making a pair of equal corresponding angles, then the lines are parallel.
If AB // CD and transversal XY cuts them (figure), then :

(i) Alternate angles are equal.
 i.e. ∠ 3 = ∠ 5 and ∠ 4 = ∠ 6.

(ii) Corresponding angles are equal.
 i.e. ∠ 1 = ∠ 5, ∠ 2 = ∠ 6, ∠ 3 = ∠ 7 and ∠ 4 = ∠ 8

(iii) Co-interior angles are supplementary
 i.e. ∠ 3 + ∠ 6 = 2 rt. angles and ∠ 4 + ∠ 5 = 2 rt. angles.



Theorem 3 :
If two lines intersect each other, then the vertically opposite angles are equal.
Given : Two lines AB and CD intersecting at a point O.
To prove : (i) ∠ AOC = ∠ BOD
(ii) ∠ BOC = ∠ AOB
Proof : Since ray OD stands on AB
∴ ∠ AOD + ∠ DOB = 180° .... (1) (linear pair)

Again, ray OA stands on CD

$\therefore$ $\angle AOC + \angle AOD = 180°$ .... (2) (linear pair)

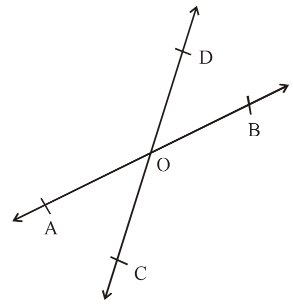
by (1) and (2) we get,

$\angle AOD + \angle DOB = \angle AOC + \angle AOD$

$\Rightarrow \angle DOB = \angle AOC$

$\Rightarrow \angle AOC = \angle DOB$

Similarly, $\angle BOC = \angle DOA$



Theorem 4 : If a transversal intersect two parallel lines, then each of alternate interior angles is equal.

Given : AB and CD are two parallel lines, Transversal ℓ intersects AB and CD at P and Q respectively making two pairs of alternate interior angles, $\angle 1, \angle 2$ & $\angle 3, \angle 4$

To prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof : clearly $\angle 2 = \angle 5$ (Vertically opposite angles)

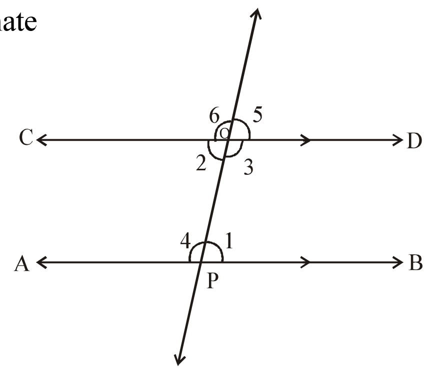
and $\angle 1 = \angle 5$

$\therefore$ $\angle 1 = \angle 2$

Also $\angle 3 = \angle 6$

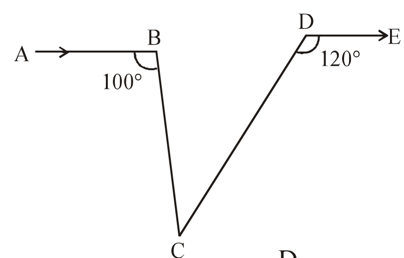
and $\angle 4 = \angle 6$

$\therefore$ $\angle 3 = \angle 4$



Example 3 :

In the figure, AB // DE : find the value of $\angle BCD$.



Sol. Draw PCQ parallel to AB and DE.

1. $\angle a + 100° = 180°$ [Co-interior angles are supplementary)

$\Rightarrow$ $\angle a = 80°$

2. $\angle b + 120° = 180°$ [Co-interior angles are supplementary)

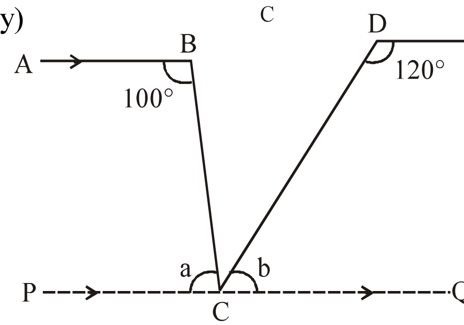
$\Rightarrow \angle b = 60°$

Since, PCQ is a straight line

$\Rightarrow \angle a + \angle BCD + \angle b = 180°$

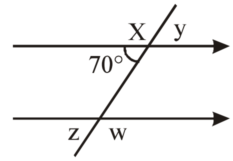
$\Rightarrow 80° + \angle BCD + 60° = 180°$

$\Rightarrow \angle BCD = 40°$



Example 4 :

From the adjoining diagrams, calculate $\angle x, \angle y, \angle z$ and $\angle w$.



Sol. $\angle y = 70°$

$\angle x + 70 = 180°$ ..... (vertical opp. angle)

$\therefore \angle x = 180 - 70 = 110°$ .... (adjacent angles on a st. line or linear pair)

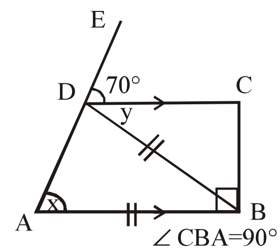
$\angle z = 70°$ .....(corresponding angles)

$\angle z + \angle w = 180°$ ..... (adjacent angles on a st. line or linear pair)

$\therefore 70 + \angle w = 180°$ $\therefore$ $\angle w = 180° - 170° = 110°$

Example 5 :

From the adjoing diagram (i) $\angle x$ (ii) $\angle y$ (iii) $\angle z$



Sol. $\angle x = \angle EDC = 70°$ .....(corresponding angles)

Now, $\angle ADB = x = 70°$ ... [AD = DB]

In Δ ABD, $\angle ABD = 180 - \angle x - \angle x = 180 - 70 - 70 = 40°$

$\Rightarrow \angle BDC = \angle ABD = 40°$ (alternate angles) $\Rightarrow \angle y = 40°$

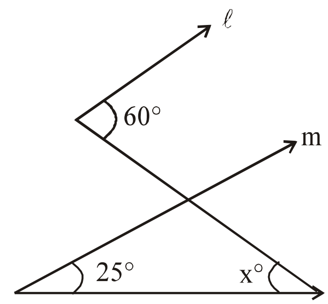
Since, AB / / DC $\Rightarrow \angle z + \angle 90 = 180°$ (Cojoined angles)

$\Rightarrow \angle z = 180 - 90 = 90°$ $\angle z = 90°$

ADDITIONAL EXAMPLES

Example 1 :

In figure, if $\ell \parallel m$, then find the value of x.



Sol. As $\ell \parallel m$ and DC is transversal

$\therefore$ $\angle D + \angle 1 = 180°$

$60° + \angle 1 = 180°$

$\angle 1 = 120°$

Here, $\angle 2 = \angle 1 = 120°$ (vertically opposite angles)

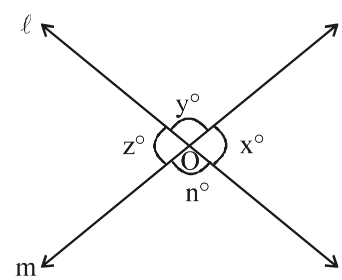
In the Δ ABC, $\angle A + \angle B + \angle C = 180°$

$25° + x° + 120° = 180°$ or $x = 35°$

Example 2 :

In the figure, lines ℓ and m intersect at O.

If x = 63° then find the values of y, z and n.



Sol. x = 63°

Now, $\angle x = \angle z$ (V.O. angles)

$\therefore$ $\angle z = 63°$

As $\angle x + \angle y = 180°$

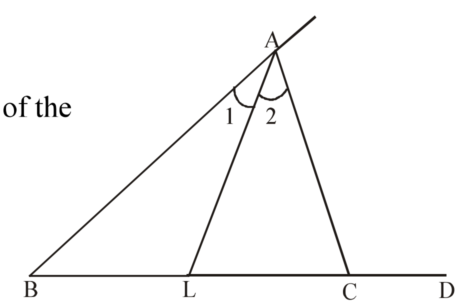
$63° + \angle y = 180°$

$\angle y = 117°$

Now, $\angle y = \angle n$ (V.O. angles) $\therefore$ $\angle n = 117°$

Example 3 :

The side BC of a triangle ABC is produced to D. The bisector of the $\angle A$ meets BC in L. Prove that $\angle ABC + \angle ACD = 2 \angle ALC$.



Sol. In the Δ ACD $= \angle A + \angle B$ .... (1)

Now in the Δ ABL, $\angle ALC = \angle 1 + \angle B$ .... (2)

$\Rightarrow$ $2 \angle ALC = 2 \angle 1 + 2 \angle B$

$\Rightarrow$ $2 \angle ALC = \angle A + 2 \angle B$

$\Rightarrow$ $2 \angle ALC = (\angle ACD - \angle B) + 2 \angle B$

$\Rightarrow$ $2 \angle ALC = \angle ACD + \angle B = \angle ACD + \angle ABC$

Example 4 :

In the figure, find the value of x°.

Sol. In the Δ ABC, $\angle A + \angle B + \angle ACB = 180°$

$\Rightarrow$ $25° + 35° + \angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 120°$

Now $\angle ACB + \angle ACD = 180°$ (linear pair)

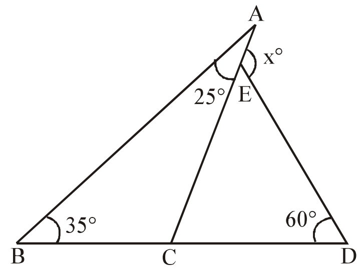
or $120° + \angle ACD = 180°$

or $\angle ACD = 60° = \angle ECD$

Again in the Δ CDE, CE is produced to A.

Hence, $\angle AED = \angle ECD + \angle EDC$

$\Rightarrow$ $x = 60° + 60° = 120°$



Example 5 :

If the ratio of three angles of a triangle is 1 : 2 : 3, find the angles.

Sol. Ratio of the three angles of a Δ = 1 : 2 : 3. Let the angles be x, 2x and 3x.

$\therefore$ $x + 2x + 3x = 180°$ $\therefore$ $6x = 180°$ $\therefore$ $x = 30°$

Hence the first angle = x = 30°, The second angle = 2x = 60°, The third angle = 3x = 90°

Example 6 :

Which of the following pairs of angles are supplementary : (a) 75°, 105° (b) 110°, 105°

Sol. (a) $75° + 105° = 180°$ $\therefore$ These are supplementary angles.

(b) $110° + 105° = 215° \neq 180°$ $\therefore$ These are not supplementary angles.

Example 7 :

In the figure, show that AB || EF.

Sol. $\angle ABC = 70°$

$\angle BCD = \angle BCE + \angle ECD$

$= 30° + 40° = 70°$

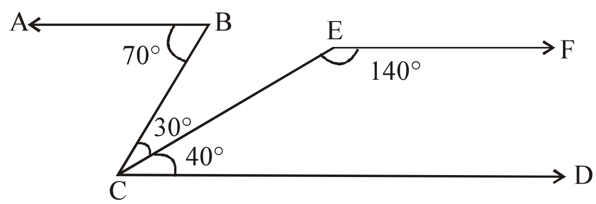
$\therefore \angle ABC = \angle BCD$

But these are alternate agles

$\therefore$ AB || CD

Now, $\angle DCE + \angle FEC = 40° + 140° = 180°$

But these are consecutive interior angles $\therefore$ CD || EF But AB || CD $\therefore$ AB || EF



Example 8 :

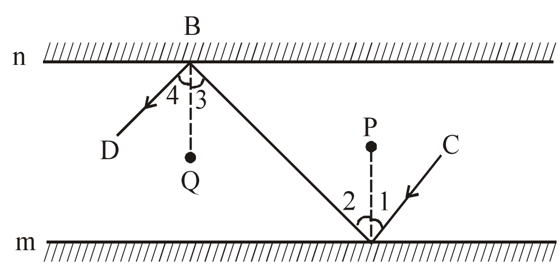
An angle is 16° more than its complement. What is its measure ?

Sol. Let one angle = a°. Then second angle = (a + 16)°. Now, $a + (a + 16) = 90 \Rightarrow 2a + 16 = 90 \Rightarrow a = 37°$

Second angle = 37 + 16 = 53°

Example 9 :

In figure, m and n are two plane mirrors parallel to each other. Show that the incident ray CA is parallel to the reflected ray BD.



Sol. Given m and n are two plane mirrors parallel to each other. CA is the incident ray and BD is the reflected ray as shown in the figure.

To prove : CA || BD

Construction : Through A and B draw respectively, AP ⊥ M and BQ ⊥ n, so that AP || BQ

Proof : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ But $\angle 2 = \angle 3$

$\therefore$ $\angle 1 = \angle 4$ or $2\angle 1 = 2\angle 4$, i.e., $\angle CAB = \angle ABD$

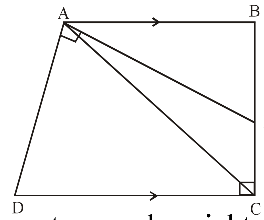
But they form a pair of alt. ∠ s $\therefore$ CA || BD

QUESTION BANK

EXERCISE - 1

Q.1 Name the following angles in the diagram.

(a) An acute angle at A (b) An acute angle at C
(c) A right angle at A (d) A right angle at C
(e) An obtuse angle at A (f) An obtuse angle at E
(g) Four pairs of conjoined angle in ABCD
(h) A straight line



Q.2 Define : Adjacent angle, linear pair of angles, supplementary angles, complementary angles, right angle, acute angle, obtuse angle, straight angle, reflex angle, line segment, interior point, distance between two points, betweenness, mid-point, bisector of a line, half-plane, angle, vertex of an angle, arms of an angle, congruent lines.

Q.3 Prove that if two lines intersect, then the vertically opposite angles are equal.

Q.4 Explain the terms : transversal, pairs of corresponding angles and pairs of alternate angles and parallel lines.

Q.5 Two parallel lines are cut by a transversal such that one of the interior angle is 57°. Find each of the other interior angles.

Q.6 A transversal cuts two straight lines, such that the bisectors of one pair of corresponding angles are parallel to each other. Prove that the lines are also parallel to each other.

Q.7 Two straight lines are perpendicular to the same line, prove that they are parallel to each other.

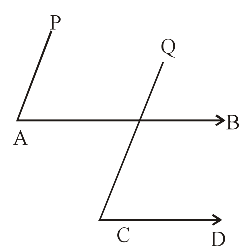
Q.8 Prove that the bisectors of vertically opposite angles are in one and the same straight line.

Q.9 Two straight lines AB, CD intersect at O. PO is the bisector of $\angle$ AOC. Prove that PO produced bisects $\angle$ BOD.

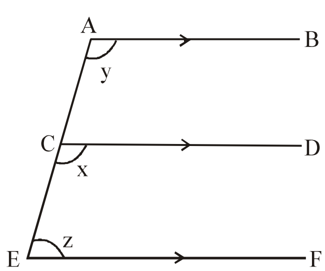
Q.10 Two straight lines AB and CB intersect at O, and AB is parallel to CD, prove that triangles ABO and CDO are equiangular.

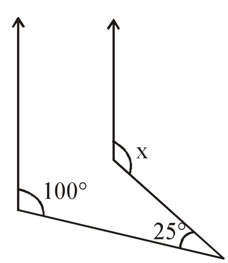
Q.11 Find the adjoining diagram AB and CD are parallel

$\angle$ BAP = $\angle$ DCQ, show that AP and CQ are parallel.



Q.12 In the given figure if $\angle$ z = 70, find $\angle$ x and $\angle$ y.





Q.13 Find $\angle$ x : **(Figure Q.13)**

Q.14 The supplement of an angle is one third of itself. Determine the angle and its supplement.