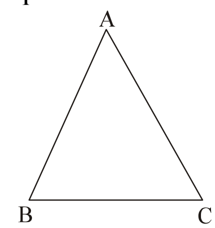
7

TRIANGLES

INTRODUCTION

A triangle is formed by three line segments obtained by joining three pairs of points taken from a set of three non-collinear points in the plane.



In figure, three non-collinear points, A, B, C have been joined and the figure ABC, enclosed by three line segments, AB, BC and CA is called triangle.

The symbol Δ (delta) is used to denote a triangle. Thus triangle ABC is written as Δ ABC.

The three given points are called the vertices of the triangle. The three line segments are called the sides of the triangle. The angles made by the line segments at the vertices are called the angles of the triangle.

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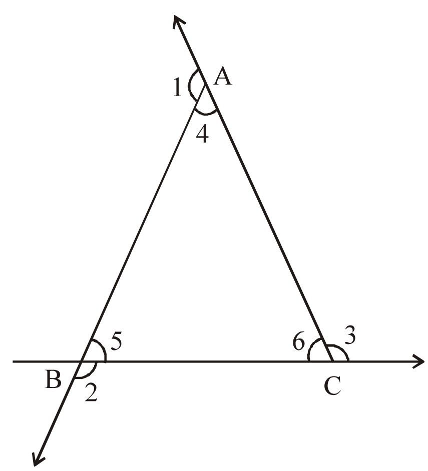
It is clear from figure that Δ ABC

(i) A, B and C are its vertices.

(ii) The line segments AB, BC and CA are its sides.

(iii) ∠ CAB, (or ∠ A), ∠ ABC (or ∠ B), ∠ BCA (or ∠ C) are the angle of the triangle.



If the sides of the Δ ABC are extended in order, then the angle between the extended and the adjoining side is called the exterior angle of the triangle.

In figure, ∠ 1, ∠ 2 and ∠ 3 are the exterior angles of the triangle while ∠ 4, ∠ 5 and ∠ 6 are the interior angles of the triangle.

For any exterior angle of a triangle, the two interior angles, other than the adjacent supplementary angle, are called interior opposite angles. Thus for ∠ 1, the exterior opposite angles are ∠ 5 and ∠ 6, for ∠ 2, the interior angles are ∠ 4 and ∠ 6 and for ∠ 3, the interior opposite angles are ∠ 4 and ∠ 5. The exterior angle of a triangle is equal to the sum of two interior opposite angles.

CLASSIFICATION OF TRIANGLES ON THE BASIS OF SIDES

(i) Scalene triangle : If all the three sides of a triangle are unequal, it is called a scalene triangle.

AB ≠ BC ≠ CA, so Δ ABC is a scalene triangle

(ii) Isosceles triangle : If any two sides of a triangle are equal, it is called an isosceles triangle

AB = AC, so Δ ABC is a isosceles triangle.

Usually, equal sides are indicated by putting marks on each of them.

(iii) Equilateral triangle : If all the three sides of a triangle are equal, it is called a equilateral triangle.
AB = BC = CA, so Δ ABC is a equilateral triangle.

CLASSIFICATION OF TRIANGLES ON THE BASIS OF ANGLES

(i) Acute angled triangle : If all the three angles of a triangle are acute (less than 90°), it is called an acute angled triangle.
Each angle is less than 90°, so Δ ABC is an acute angled triangle.

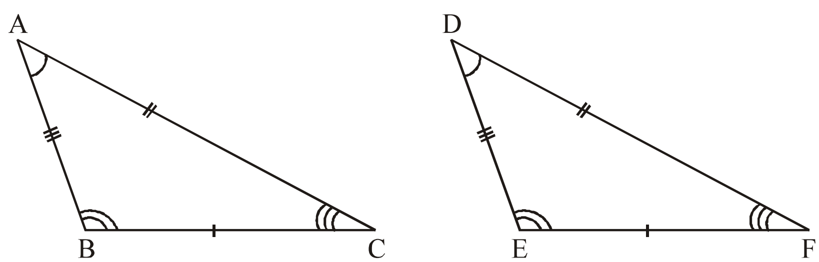
(ii) Right angled triangle : If one angle of a triangle is right angle (= 90°), it is called a right angled triangle.
In a right angled triangle, the side opposite to right angle is called hypotenuse.
$\angle$ B = 90°, so Δ ABC is the hypotenuse.

(iii) Obtuse angled triangle : If one angle of a triangle is obtuse (greater than 90°), it is called an obtuse angled triangle.
$\angle$ B is obtuse (greater than 90°), so Δ ABC is an obtuse angled triangle.

CONGRUENCE OF TRIANGLE

In our daily life, we come across many objects of different shapes and sizes. Some of these are of the same shape and different sizes and some are of the same shape and the same size. If two figures in a plane are such that if we superpose one figure over the another, and they cover each other exactly then both are said to be congruent. The relation of two figures being congruent is called "Congruence". Two line-segments are congruent if they are of equal length and two angles are congruent, if they have the same measure.

Two triangles are congruent if and only if three sides and three angles of the one are equal to the corresponding sides and the angles of the other. The congruence of two triangles follows immediately from the congruence of three line segments and three angles.



In Fig., two triangles ABC and DEF are shown, where AB = DE. BC = EF, AC = DF, $\angle$A= $\angle$D, $\angle$B = $\angle$E and $\angle$C = $\angle$F then Δ ABC and Δ DEF are congruent and we write Δ ABC $\cong$ Δ DEF. Δ ABC $\cong$ Δ DEF is read as "triangle ABC is congruent to triangle DEF". Here the vertices A, B and C of triangle ABC correspond to the vertices D, E and F of triangle DEF. Thus, out of the six different correspondences of vertices only Δ ABC and Δ DEF on superposing will cover each other exactly. The above correspondence could also be written as Δ BCA $\cong$ Δ EFD or Δ CAB $\cong$ Δ FDE. But it will not be correct to write Δ ABC $\cong$ Δ EFD or Δ BCA $\cong$ Δ DEF because it is very important to write the congruence relation between two triangles symbolically in a correct correspondence of sides and angle and also to identify their corresponding parts correctly.

Different properties of congruence of triangles :

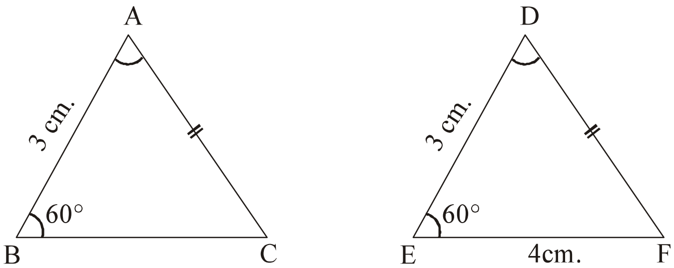
Two triangles are congruent if all the six parts (three sides and three angles) of one triangle are equal to the corresponding six parts of the other triangle.

In order to prove the congruence of two triangles, it is not necessary that all the given six parts of one triangle are equal to the corresponding six parts of the other triangle, because it is possible to prove the congruence of two triangle on selected three pairs of their corresponding parts. The remaining three are automatically become congruent

Axiom Side-Angle-Side Rule (SAS Rule) :

Two triangles are congruent, if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.

As this rule involves two sides and the angle included between them, therefore it is generally known as SAS (Side-Angle-Side) Rule for congruence of triangles.



In Figure, AB = DE, BC = EF and $\angle B = \angle E$ then by SAS Rule $\Delta ABC \cong \Delta DEF$. It means that the corresponding parts of the two triangles are equal and hence

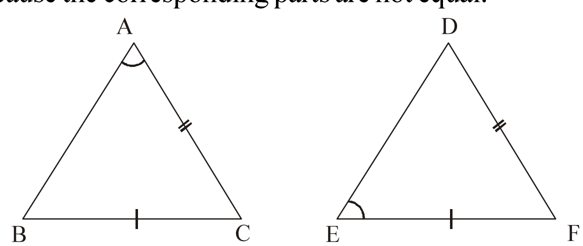
$\Rightarrow$ AC = DF = 3.5 cm, $\angle A = \angle D = 70°$ and $\angle C = \angle F = 50°$

Note : An important point in the Side-Angle-Side Rule is that the equality of included angle is essential. Suppose two sides and one angle (not the included angle) between two sides are equal to two sides and one angle of the other triangle then the triangles are not congruent because the corresponding parts are not equal.

In Fig. let BC = EF; AC = DF and $\angle A = \angle E$

then ΔABC and ΔDEF are not congruent.

Hence Side-Side-one Angle is not a rule (criterion).



Angle-Side-Angle Rule (ASA Rule) :

Two triangles are congruent if any two angles and the included side of one triangle are equal of the two angles and the included side of the other triangle.

Given : In ΔABC and ΔPQR, $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and BC = EF

To prove : $\Delta ABC \cong \Delta DEF$

Proof : On comparing the sides AB and DE of ΔABC and ΔDEF there are three possibilities :

(i) AB = DE (ii) AB < DE and (iii) AB > DE

Case (i) : When AB = DE then in ΔABC and ΔDEF

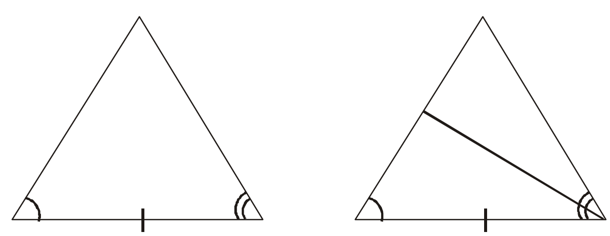
AB = DE (Assumed)

$\angle ABC = \angle DEF$ (Given)

BC = EF (Given)

Hence ΔABC and ΔDEF are congruent by SAS rule.

Hence, $\Delta ABC \cong \Delta DEF$



Case (ii) : When AB < DE, then we take point G on DE such that AB = GE and join GF as shown in figure.

Now in ΔABC and ΔGEF

AB = GE (Assumed)

BC=EF (Given)

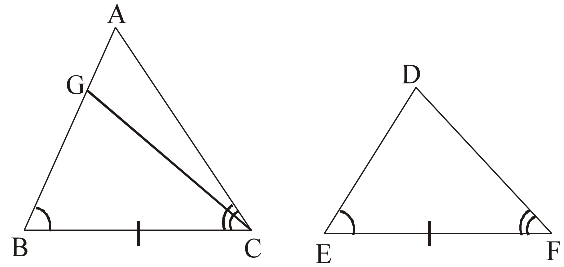
$\angle ABC = \angle GEF$ (Given) [$\because \angle GEF = \angle DEF$]

$\therefore$ $\Delta ABC \cong \Delta GEF$ (By SAS Rule)

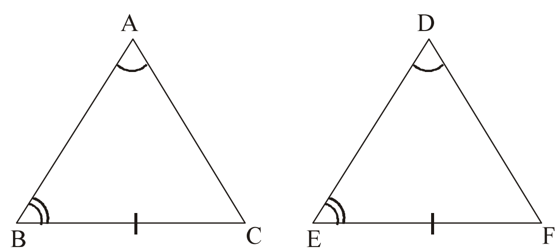
Hence $\angle ACB = \angle GFE$ .... (1)

and $\angle ACB = \angle DFE$ (Given) .... (2)

Therefore from (1) and (2), we have $\angle$ DFE = $\angle$ GFE
which is impossible unless GF coincide with DF then G coincides with D.
$\therefore$ AB = DE . Hence Δ ABC $\cong$ Δ DEF
Case (iii) : When AB > DE , then we take a point G on AB such that BG = DE (Fig,) and show as in case (ii) that G must coincide with A,
i.e., AB = DE and Δ ABC $\cong$ Δ DEF (SAS rule)
Hence in all three cases Δ ABC $\cong$ Δ DEF
Since the sum of the three interior angles of a triangle is 180°, therefore if two angles of one triangle are equal to the two angles of another triangle, then the third angle of the first triangle will automatically be equal to the third angle of the second triangle. On the basis of this property, we can state the following corollary to the above theorem.



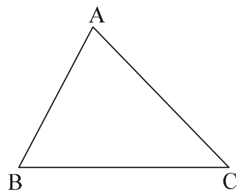
Corollary : Angle-Angle-Side Rule (AAS Rule)
If any two angles and a side of one triangle are equal to two angles and the corresponding side of the other triangle, the two triangles are congruent.
Given : In Δ ABC and Δ DEF, $\angle$ B = $\angle$ E, $\angle$ A = $\angle$ D and BC = EF
To prove : Δ ABC $\cong$ Δ DEF
Proof : The sum of three interior angle of a triangle is 180°, then

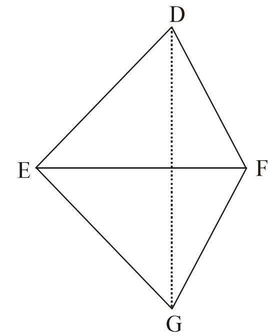


$\angle$ A + $\angle$ B + $\angle$ C = 180° .... (1)
$\angle$ D + $\angle$ E + $\angle$ F = 180° .... (2)
From (1) and (2),
$\angle$ A + $\angle$ B + $\angle$ C = $\angle$ D + $\angle$ E + $\angle$ F......... (3)
Given that $\angle$ B = $\angle$ E ; $\angle$ A = $\angle$ D
Hence $\angle$ C = $\angle$ F [By (3)] .... (4)
Now in Δ ABC and Δ DEF
$\angle$ B = $\angle$ E (Given)
BC = EF (Given)
$\angle$ C = $\angle$ F [By (4)]
Hence, Δ ABC $\cong$ Δ DEF (By ASA rule)

Side-Side-Side Rule (SSS Rule) : Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.
Given : In Δ ABC and Δ DEF, AB = DE, BC = EF and AC = DF
To prove Δ ABC $\cong$ Δ DEF
Construction : Draw a line segment EG at the other side of Δ DEF such that EG = AB and $\angle$ ABC = $\angle$ FEG then join EG, GF and DG.

Proof : In Δ ABC and Δ GEF





AB = GE (By construction)

$\angle$ ABC = $\angle$ GEF (By construction)

BC = EF (Given)

Hence by SAS rule

$\Delta ABC \cong \Delta GEF$

Hence the corresponding sides and angles are equal

$\Rightarrow$ $\angle A = \angle G$, AB = GF .... (1)

Now AB = EG (By construction)

AC = DE (Given)

$\Rightarrow$ AB = DE .... (2)

Similarly AC = GF and AC = DF (Given)

$\Rightarrow$ GF = DF .... (3)

Now in Δ EDG. the opposite angle of equal sides EG and DE are equal

$\Rightarrow$ $\angle EDG = \angle EGD$ .... (4)

Similarly in Δ FDG the opposite angle of equal, sides GF and DF are equal

$\angle GDF = \angle DGF$ .... (5)

$\angle EDG + \angle GDF = \angle EGD + \angle DGF$

$\Rightarrow$ $\angle D = \angle G$ .... (6)

From eq. (1) $\angle A = \angle G$ .... (7)

Hence, from (6) and (7), we have

$\angle A = \angle D$ .... (8)

In Δ ABC and Δ DEF, AB = DE (Given) ; $\angle A = \angle D$ [By (8)]

AC = DF (Given)

By SAS rule $\Delta ABC \cong \Delta DEF$

Right-Hypotenuse-Side Rule (RHS Rule)

Two right triangles are congruent if the hypotenuse and a side of one triangle are equal to hypotenuse and a side of the other triangle are equal to hypotenuse and a side of the other triangle respectively.

Given : In two right triangles Δ ABC and Δ DEF

$\angle B = \angle E = 90°$

Hypotenuse AC = Hypotenuse DF and Side AB = Side DE

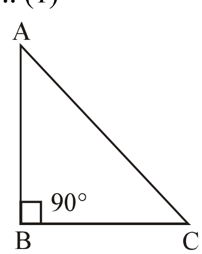
To prove : $\Delta ABC \cong \Delta DEF$

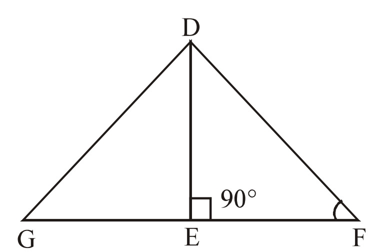
Construction : Produce FE to G such that GE = BC and join G and D.

Proof : In Δ DEF

$\angle DEF = 90°$ then $\angle DEG = 90°$ .... (1)

Now in Δ ABC and Δ DEG





AB = DE (Given)

BC = GE (By construction)

$\angle ABC = \angle DEG = 90°$ [By (1)]

Therefore, by SAS rule, $\Delta ABC \cong \Delta DEG$

Hence the corresponding sides and angles are equal

AC = DG and $\angle C = \angle G$ .... (2)

But given that AC = DF .... (3)

From (2) and (3)   DG = DF   .... (4)

∵ Angles opposite to the equal sides (DG = DF) of Δ DGF are equal

Hence   ∠ G = ∠ F   .... (5)

Therefore, from (2) and (5)

∠ C = ∠ F   .... (6)

Now in Δ ABC and Δ DEF

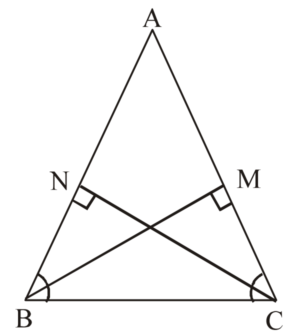
AB = DE   (Given)

∠ C = ∠ F   [From (6)]   and ∠ ABC ≅ ∠ DEF = 90° (Given)

Hence by ASA Rule   Δ ABC ≅ Δ DEF

Example 1 :

In the adjoining figure, AB = AC. Prove that BM = CN



Sol. In   Δ ABC, AB = AC (given)

∴   ∠ ABC = ∠ ACB   (angles opposite equal sides are equal)

In   Δ s   BCM and BCN,

∠ N = ∠ M   (each = 90°)

∠ ABC = ∠ ACB   (from above)

BC = BC   (common)

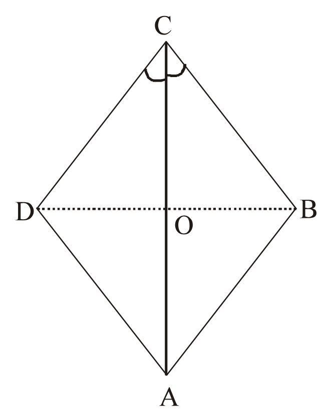
∴   Δ BCM ≅ Δ BCN   (A.A.S. axiom of congruency)

⇒   BM = CN   (c.p.c.t.)

Example 2 :

In a quadrilateral ABCD, AC bisects ∠ C and BC = CD.

Prove that (i) AB = AD   (ii) AC is the perpendicular bisector of BD.



Sol. Given : AC is the bisector of ∠ C i.e. ∠ DCA = ∠ BCA, CD = BC

To prove :   (i) AB = AD (ii) AC = BD

(i) In triangles ADC and ABC,

CD = BC   (Given)

CD = CA (Common)

incl. ∠ DCA = incl. ∠ BCA (Given)

∴   Δ DCA ≅ Δ BCA

∴   AB = AD

(ii)   **Construction :** Join DB. In triangles DCO and BCO

CD = BC   (Proved in (i) above)

CO = CO   (common)

incl. ∠ DCO = incl. ∠ BCO (Given)

∴   Δ DCO ≅ Δ BCO

∴   ∠ COD = ∠ COB

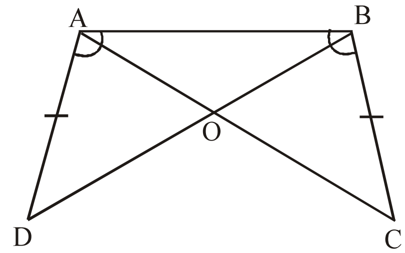
Since ∠ COD and ∠ COB are adjacent angles ∠ DOC = ∠ BOC = 90°

∴   CO is perpendicular to DB

∴   CA is perpendicular to DB

Example 3 :

In the adjoining figure, ABC and BAD are two triangles on the same base AB such that BC = AD and $\angle$ ABC = $\angle$ BAD. Prove that : (i) AC = BD (ii) $\angle$ ACB = $\angle$ BDA (iii) CO = DO.



Sol. In Δ s ABC and BAD, we have :

AB = BA (Common)

BC = AD (Given)

$\angle$ ABC = $\angle$ BAD (Given)

$\therefore$ Δ ABC $\cong$ Δ BAD (SAS rule]

$\therefore$ AC = BD and $\angle$ ABC = $\angle$ BDA and therefore, $\angle$ OCB = $\angle$ ODA

Again, in Δ s BOC and AOD, we have :

BC = AD (Given)

$\angle$ OCB = $\angle$ ODA (Proved)

and $\angle$ BOC = $\angle$ AOD (Vertically opposite angles]

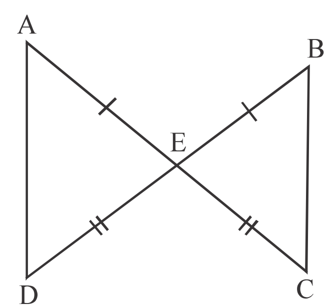
$\therefore$ Δ BOC $\cong$ Δ AOD (AAS rule)

Hence, CO = DO.

Example 4 :

In figure, AE = EC and DE = BE. Prove that

(i) Δ AED $\cong$ Δ CEB (ii) $\angle$ A = $\angle$ C



Sol. Given that as in figure

AE = EC

DE = BE .... (1)

In Δ AED and Δ BEC

AE = EC (Given)

$\angle$ AED = $\angle$ CEB (Opposite angle)

DE = EB (Given)

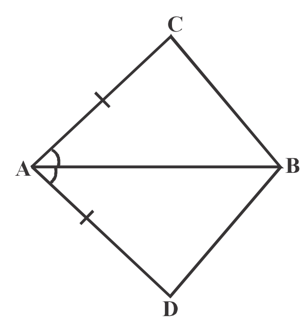
Then by SAS Rule, Δ AED $\cong$ Δ CEB

Therefore corresponding angles are equal

Hence $\angle$ A = $\angle$ C and $\angle$ D = $\angle$ B

SELF CHECK

Q.1 In quadrilateral ABCD, AC = AD and AB bisects $\angle$ A (figure). Show that Δ ABC $\cong$ Δ ABD. What can you say about BC and BD ?



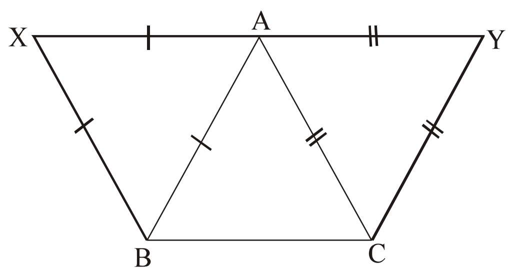
Q.2 In the following pairs of triangles, find out whether the triangles in each pair are congruent or not.

(i) Δ ABC : AC = 2 cm, BC = 3 cm and $\angle$ C = 72°,
Δ DEF : DE = 2 cm, DF = 3 cm and $\angle$ D = 72°

(ii) Δ ABC : AB = 4 cm, AC = 8 cm and $\angle$ A = 90°,
Δ PQR : PQ = 4 cm, QR = 8 cm and $\angle$ Q = 90°

(iii) Δ ABC and Δ DEF in which BC = EF, $\angle$ A = 90°, $\angle$ B = $\angle$ E = 50° and $\angle$ F = 40°

Q.3 From the given figure, prove that



(i) ∠ CAX = ∠ YAB (ii) CX = YB

Q.4 Prove that the following triangles are congruent to each other.

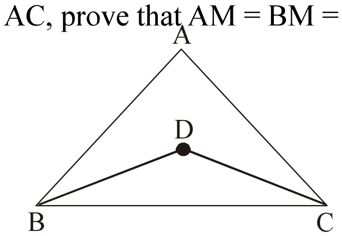
(i) Triangle ABC in which AB = 5 cm, AC = 6 cm and ∠ A = 50°

(ii) Triangle DEF in which DF = 6 cm, EF = 5 cm and ∠ F = 50° and ∠ D = 63°

(iii) Triangle PQR in which PR = 5cm, ∠ R = 50° and ∠ Q = 63°.

Q.5 In a right-angled triangle, if M is the mid-point of the hypotenuse AC, prove that AM = BM = CM.

Q.6 In figure, Δ ABC and Δ DBC are on the same base, in which AB = AC and DB = DC.
Show that ∠ ABD = ∠ ACD.



Q.7 AB is a line segment line ℓ is its perpendicular bisector.
If a point P lies on ℓ, show that P is equidistant from A and B.

ANSWERS

(1) They are equal **(2) (i) Yes (SAS)** **(ii) Yes (SAS)** **(iii) Yes (AAS)**

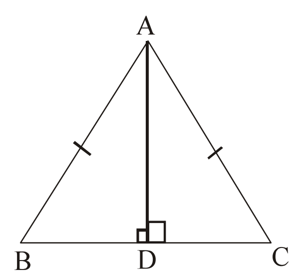
SOME PROPERTIES OF TRIANGLES

(1) An isosceles triangle whose two sides are equal. Here, we prove some properties related to isosceles triangles. The angles opposite to equal sides of a triangle are equal.

Given : Δ ABC is an isosceles triangle in which AB = AC.

To prove : ∠ B = ∠ C

Construction : Draw AD, bisector of angle ∠ BAC which meets BC at D.



Proof : In Δ ABD and Δ ACD

AB = AD (Given)

∠ BAD = ∠ CAD (By construction)

AD = AD (Common side)

Therefore Δ ABD ≅ Δ ACD (By SAS Rule)

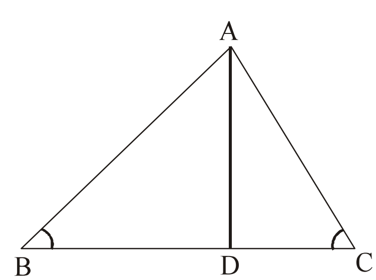
Hence corresponding angles ∠ B = ∠ C

(2) The sides opposite to equal angles of a triangles are equal.

Given : Δ ABC is an isosceles triangle in which ∠ B = ∠ C

To prove : AB = AC

Construction : Draw AD, the bisector of angle ∠ BAC which meets BC at D.



Proof : In Δ ABD and Δ ACD

$\angle$ B = $\angle$ C (Given)

AD = AD (Common side)

$\angle$ BAD = $\angle$ CAD (By construction)

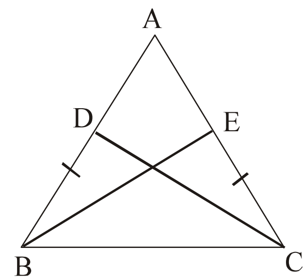
Therefore Δ ABD $\cong$ Δ ACD (By ASA)

Hence corresponding sides, AB = AC

Example 5 :

Prove that the medians bisecting the equal sides of an isosceles triangle are equal.

Sol. Given : In Δ ABC, D and E are mid point of AB and AC respectively.



To prove : BE = CD

Proof : Δ ABC is an isosceles triangle, then

AB = AC .... (1)

and $\angle$ ABC = $\angle$ ACB .... (2)

and D and E are mid points of AB and AC

then DB = DA = EC = AE

Now, in Δ BCD and Δ BCE

BC = BC (By construction)

$\angle$ DBC = $\angle$ ECB [by (2)]

BD = CE

Therefore by SAS rule Δ BCD $\cong$ Δ BCE

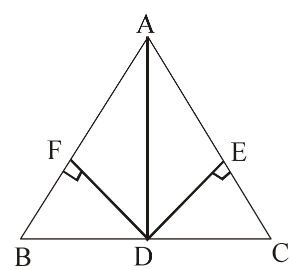
Consequently corresponding sides are equal i.e. BE = CD

Example 6 :

If the perpendiculars drawn from the mid point of one side of a triangle to its other two sides are equal, then show that the triangle is isosceles.

Sol. Given Δ ABC is the triangle in which D is the mid point of side BC. DE and DF are perpendiculars on AC and AB respectively.

To prove : Δ ABC is isosceles, i.e. AB = AC



Construction : Join AD

Proof : In Δ BDF and Δ CDE

Hypotenuse BD = Hypotenuse CD (Given)

$\angle$ DFB = $\angle$ DEC = 90° and DF = DE (Given)

Therefore by RHS Rule

Δ BDF $\cong$ Δ CDE

Consequently $\angle$ B = $\angle$ C

Hence the opposite sides of angles $\angle$ B and $\angle$ C are equal i.e., AB = AC.

SELF CHECK

Q.1 In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle$ B and $\angle$ C intersect each other at O. Join A to O. Show that :

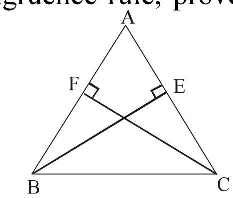
(i) OB = OC (ii) AO bisects $\angle$ A

Q.2 AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects $\angle$ A

Q.3 BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Q.4 In the given figure, AB = AC, BE and CF are the altitudes. Prove that (i) BE = CF (ii) AF = AE



Q.5 Prove that the bisector of the vertical angle of an isosceles triangle bisects the base at right angles.

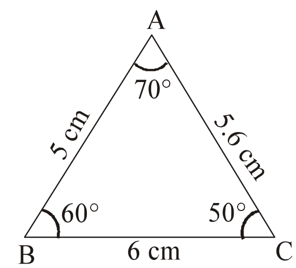
Q.6 In an isosceles triangle ABC, PQ is drawn parallel to BC cutting AB, AC at P and Q. Prove that triangle APQ is an isosceles.

INEQUALITIES

When two quantities are unequal then on comparing these quantities, we obtain a relation between their measures, called "Inequality" relation.

Activity :

Let us construct a triangle ABC with unequal sides BC = 6 cm, AC = 5.6 cm and AB = 5 cm. (figure). Now we measure the three angles with the help of a protractor, and find that $\angle A = 70°$, $\angle B = 60°$ and $\angle C = 50°$.



Similarly we can repeat the above process for triangles of other measurements and unequal sides. Every time we shall find that the angle opposite the larger side is greater.

Theorem 1 :

If two sides of a triangle are unequal, the larger side has the greater angle opposite to it.

Given : In triangle ABC, AB > AC

To prove : $\angle C > \angle B$

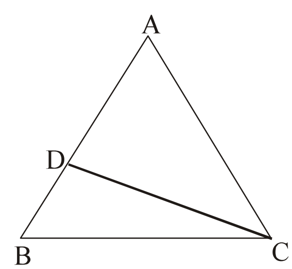
Construction : Draw a line segment CD from vertex C such that AC = AD.

Proof : In Δ ACD, AC = AD

Therefore, $\angle ACD = \angle ADC$ .... (1)

But $\angle ADC$ is an exterior angle of Δ BDC

$\therefore$ $\angle ADC > \angle B$ .... (2)



From (1) and (2), we have

$\angle ACD > \angle B$ .... (3)

By figure, $\angle ACB > \angle ACD$ .... (4)

From (3) and (4), we have

$\angle ACB > \angle ACD > \angle B$

$\Rightarrow$ $\angle ACB > \angle B$

$\Rightarrow$ $\angle C > \angle B$

Theorem 2 :

In a triangle, the greater angle has a larger side opposite to it.

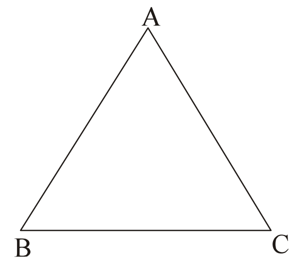
Given : A triangle ABC in which $\angle B > \angle C$

To prove : AC > AB

Proof : We have the following three possibilities for sides AB and AC of Δ ABC.

(i) AC = AB (ii) AC < AB and (iii) AC > AB

Case (i) : If AC = AB :



If AC = AB, then opposite angles of equal sides are equal then $\angle B > \angle C$. Hence $AC \neq AB$

Case (ii) : If AC < AB : We know that the larger side has greater angle opposite to it

$\therefore \quad AC < AB \quad \Rightarrow \quad \angle C > \angle B$

which is also contrary to given ($\angle B > \angle C$). Hence $AC \nless AB$

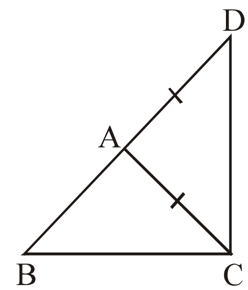
Case (iii) : If AC > AB :

We are left only this possibility which must be true. Hence, AC > AB.

<u>Theorem 3</u> :

The sum of any two sides of a triangle is greater than its third side.

Given : A triangle ABC.



To prove :

(i) $AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

Construction : Produce BA to D, such that AD = AC and join DC.

Proof : In ΔADC, by construction AD = AC, then opposite angles are equal,

$\therefore \quad \angle ACD = \angle ADC$ .... (1)

and $\quad \angle BCD > \angle ACD$ .... (2)

From (1) and (2), we have

$\angle BCD > \angle ADC = \angle BDC$

Therefore, $BD > AC$ [greater angle has a larger opposite side]

$\Rightarrow \quad BA + AD > BC$ [$\because BD = BA + AD$]

$\Rightarrow \quad BA + AC > BC$ [By construction AD = AC]

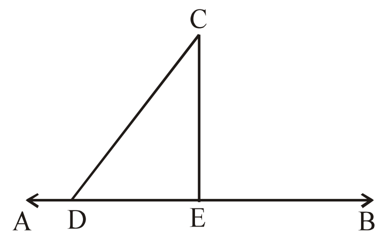
Similarly, we may show that $AB + BC > AC$; $BC + AC > AB$

<u>Theorem 4</u> :

Of all the line segments that can be drawn to a given line from an external point, the perpendicular line segment is the shortest.

Note : Distance between a line and an external point (point not on the line) is the length of the perpendicular drawn from the point on the line.

Given : A line AB and an external point C



Join CD and draw $CE \perp AB$

To prove : $CE < CD$

Proof : In ΔCED, $\angle CED = 90°$

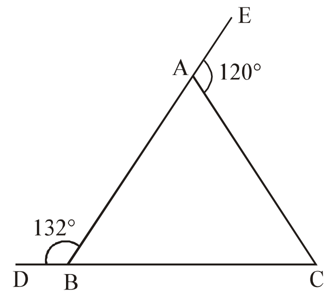
Then $\angle CDE < \angle CED$

$\Rightarrow \quad CD > CE$ (greater angle has larger opposite side)

Hence the perpendicular line segment is the shortest.

Example 7 :

In figure, $\angle DBA = 132°$ and $\angle EAC = 120°$. Show that AB > AC.



Sol. As DBC is a straight line,

$132° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 180° - 132° = 48°$

For ΔABC, $\angle EAC$ is an exterior angle

$120° = \angle ABC + \angle BCA$ (ext. $\angle$ = sum of two opp. it. $\angle$ s)

$\Rightarrow \quad 120° = 48° + \angle BCA$

$\Rightarrow \quad \angle BCA = 120° - 48° = 72°$

Thus, we find that $\angle BCA > \angle ABC$

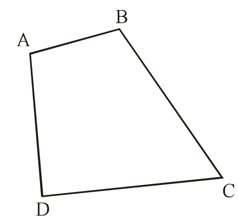
$\Rightarrow \quad AB > AC$ (side opposite to greater angle is greater)

Example 8 :

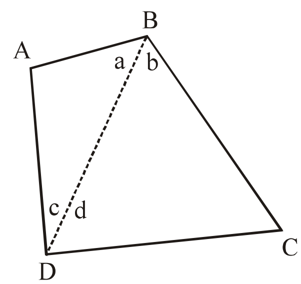
In quadrilateral ABCD, AB is the shortest side and DC is the longest side.

Prove that :

(i) $\angle B > \angle D$ (ii) $\angle A > \angle C$



Sol. Join B and D



In ΔABD, AD > AB (given, AB is the shortest side)

$\therefore \quad \angle a > \angle c$ .... (1) (angle opposite to greater side is greater)

In ΔBCD, CD > BC (given, CD is the longest side)

$\therefore \quad \angle b > \angle d$ .... (2) (angle opposite to greater side is greater)

$\therefore \quad \angle a + \angle b > \angle c + \angle d$ (Adding (1) and (2))

$\Rightarrow \quad \angle B > \angle D$

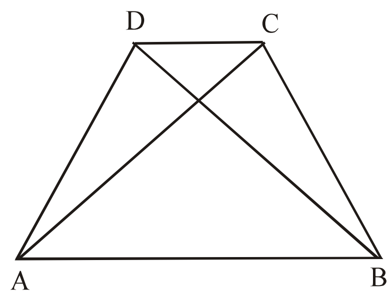
Similarly, by joining AC, it can be proved that $\angle A > \angle C$.

Example 9 :

In figure, ABCD is a quadrilateral. Show that

(i) AB + BC + CD + DA > 2 AC

(ii) AB + BC + CD + DA > AC + BD



Sol. Given : A quadrilateral ABCD. (figure)

To prove : (i) AB + BC + CD + DA > 2 AC

(ii) AB + BC + CD + DA > AC + BD

Construction : Join AC and BD.

Proof : We know that the sum of the two sides of a triangle is greater than its third side.

In ΔABC, AB + BC > AC .... (1)

In ΔADC, AD + DC > AC .... (2)

In ΔABD, AB + AD > BD .... (3)

In ΔBCD, BC + CD > BD .... (4)

Adding (1) and (2), we get

AB + BC + AD + CD > 2 AC .... (i)

Again adding (1), (2), (3) and (4), we get

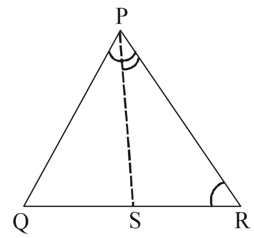
2 (AB + BC + AD + DC > 2 (AC + BD)

$\Rightarrow$ AB + BC + AD + DC > AC + BD .... (ii)

SELF CHECK

Q.1 In an equilateral triangle ABC, if D is any point on AB and E is any point on AB produced, prove that
(i) DC < SB (ii) CE > AB > CD.

Q.2 If D be any point on the base BC of an isosceles triangle ABC, prove that AB > AD and AC > AD.



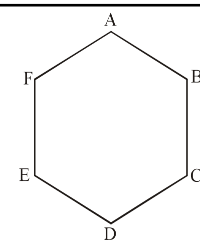
Q.3 In figure, PR > PQ and PS bisects ∠ QPR. Prove that ∠ PSR > ∠ PSQ.

Q.4 In Δ ABC, AB = 7.6 cm, BC = 6.9 cm and CA = 6.1 cm. Which is,
(i) the greatest angle ? **(ii)** the smallest angle ? Give reason.

ANSWERS

(4) (i) angle C (ii) angle B

POLYGON



A Polygon is a closed plane figure bounded by straight lines.
Thus, ABCDEF is a polygon of 6 sides.
An angle formed by two consecutive sides of a polygon is called an interior angle or simply an angle of the polygon.
Line segment joining any two nonconsecutive vertices of a polygon is called its diagonal.

Convex Polygon : A polygon in which none of its interior angles is more than 180°, is called a convex polygon.

Concave Polygon : A polygon in which atleast one angle is more than 180°, is called a concave (or re-entrant) polygon.

Regular Polygon : A regular polygon has all its sides and angles equal. Polygons are classified as under :

No. of sides	3	4	5	6	7	8
Name	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
No. of sides	9	10				
Name	Nonagon	Decagon				

In a convex polygon of 'n' sides, the sum of the interior angles is equal to (2n – 4) right angles.
The sum of the exterior angles of a polygon is equal to 4 right angles.

(i) Exterior angle of a regular polygon = $\frac{360}{n}$, where n = number **(ii)** Interior angle = 180° – exterior angle

ADDITIONAL EXAMPLES

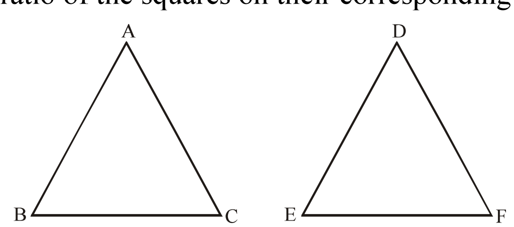
Example 1 :

In the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let Δ ABC ~ Δ DEF and area (Δ ABC) = area (Δ DEF)

Since the ratio of the areas of two similar Δ s is equal to the ratio of the squares on their corresponding sides, we have

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$



$$\Rightarrow \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = 1 \qquad [\because \text{Area } (\Delta ABC) = \text{Area } (\Delta DEF)]$$

$\Rightarrow AB^2 = DE^2,\ AC^2 = DF^2$ and $BC^2 = EF^2$

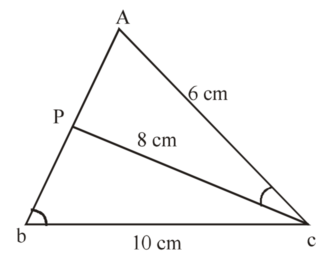
$\Rightarrow AB = DE,\ AC = DF$ and $BC = EF$

$\therefore \Delta ABC \cong \Delta DEF$ (By SSS-congruence)

Example 2 :

From the adjoining diagram, calculate

(i) AB (ii) AP (iii) $\Delta APC : \Delta ABC$



Sol. In ΔAPC and ΔABC :

$\angle ACP : \angle ABC$; $\angle A = \angle A$

$\Rightarrow \Delta ACP \sim \Delta ABC$

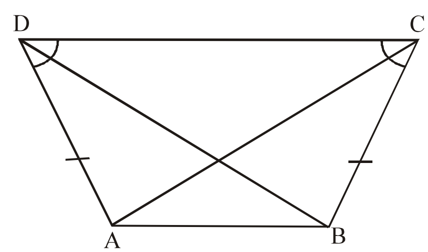
$$\Rightarrow \frac{AP}{AC} = \frac{PC}{BC} = \frac{AC}{AB} \qquad \therefore \frac{AP}{6} = \frac{8}{10} = \frac{6}{AB} \quad \Rightarrow AP = 6 \times \frac{8}{10} = 4.8 \quad \text{and} \quad AB = \frac{60}{8} = 7.5$$

$$\Rightarrow AP = 4.8 \text{ cm and } AB = 7.5 \text{ cm.} \quad ; \quad \frac{\Delta ACP}{\Delta ABC} = \frac{CP^2}{BC^2} = \frac{8^2}{10^2} = 0.64$$

Example 3 :

In figure, ABCD is a quadrilateral in which BC = AD and $\angle ADC = \angle BCD$, then show that

(i) AC = BD (ii) $\angle ACD = \angle CDB$.



Sol. In figure, it is given that

BC = AD and $\angle ADC = \angle BCD$

Hence in ΔADC and ΔBCD

AD = BC (Given)

CD = CD (common)

$\angle ACD = \angle BCD$ (Given)

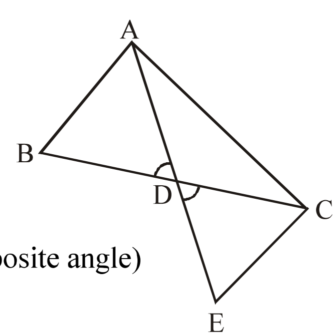
Therefore, by SAS rule $\Delta ADC \cong \Delta BCD$

Hence corresponding sides and angles are equal.

i.e., AC = BD and $\angle ACD = \angle CDB$

Example 4 :

In figure, AD is a median of ΔABC. Prove that AB + AC > 2AD.



Sol. **Given :** AD is median of ΔABC

To prove : AB + AC > 2AD

Construction : Produce AD to E such that AD = DE and join CE.

Proof : In ΔADB and ΔEDC

AD = DE (By construction)

BD = DC (Given) and $\angle ADB = \angle EDC$ (Vertically opposite angle)

Therefore, $\Delta ADB \cong \Delta EDC$ (By SAS rule)

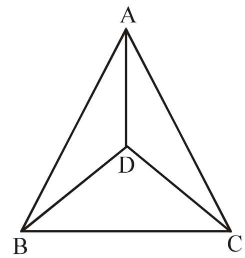
Consequently AB = CE

Now in ΔACE, AC + CE > AE $\Rightarrow$ AC + AB > AE [$\because$ CE = AB]

$\Rightarrow$ AC + AB > 2AD [$\because$ AE = 2AD]

Example 5 :

In figure, AB = AC. D is a point in the interior of Δ ABC such that ∠ DBC = ∠ DCB. Prove that AD bisects ∠ BAC.



Sol. Given : In Δ ABC, AB = AC and ∠ DBC = ∠ DCB

To prove : AD is bisector of ∠ BAC i.e. ∠ BAD = ∠ CAD

Proof : In Δ BDC, ∠ DBC = ∠ DCB then the opposite sides are equal.

i.e. CD = BD .... (1)

Now in Δ ABD and Δ ACD

BD = CD [by (1)]

AD = AD (common side)

AB = AC (Given)

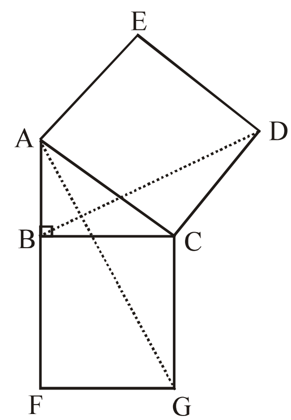
Therefore by SSS Rule, Δ ABD ≅ Δ ACD.

Consequently, ∠ BAD = ∠ CAD ⇒ AD bisects ∠ BAC

Example 6 :

In the figure, Δ ABC is right angled at B. ACDE and BFGC are squares. Prove that

(i) Δ BCD ≅ Δ ACG **(ii)** BD = AG



Sol. ∠ BCA = ∠ BCA (same angle)

⇒ 90° + ∠ BCA = 90° + ∠ BCA (adding 90° to both sides)

⇒ ∠ ACD + ∠ BCA = ∠ GCB + ∠ BCA (each angle in a square = 90°)

⇒ ∠ BCD = ∠ ACG

In Δ s BCD and ACG

CD = AC (sides of square ACDE)

BC = CG (sides of square BFGC)

∠ BCD = ∠ ACG (from above)

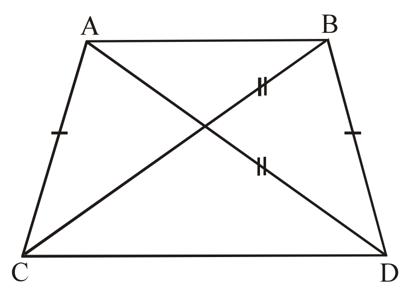
(i) Δ BCD ≅ Δ ACG (SAS axiom congruency)

(ii) BD = AG

Example 7 :

In figure, AD = BC and BD = CA.

Prove that ∠ ADB = ∠ BCA and ∠ DAB = ∠ CBA



Sol. Given that AD = BC and BD = CA

In Δ ABD and ∠ ABC

$$\left.\begin{array}{l} AD = BC \\ BD = CA \end{array}\right\} \text{(Given)}$$

AB = AB (Common)

Therefore by SSS rule Δ ABD ≅ Δ ABC

Hence corresponding angles are equal,

i.e. ∠ ADB = ∠ BCA and ∠ DAB = ∠ CBA