

# MATHEMATICS

CHAPTER NUMBER :~ 6

CHAPTER NAME :~ LINES AND ANGLES

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**CHANGING YOUR TOMORROW**

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## PREVIOUS KNOWLEDGE TEST

- 1.What do you mean by co-interior angles?
- 2.What is the sum of the co-interior angles if the given lines are parallel?

## LEARNING OUTCOME:~

1. Students will be able to learn about the concept of co-interior angles.
2. Students will be able to know about the sum of co-interior angles.

## THEOREM 6.4

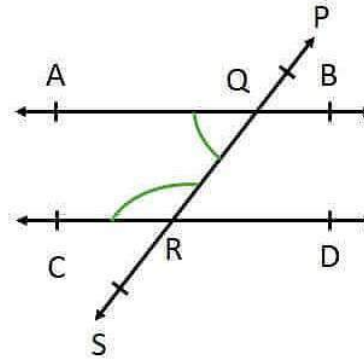
If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Given :- Two parallel lines AB and CD and a transversal PS intersecting AB at Q and CD at R.

To Prove :- Sum of interior angle on same side of transversal is supplementary.

i.e,  $\angle AQR + \angle CRQ = 180^\circ$

and  $\angle BQR + \angle DRQ = 180^\circ$



Proof:-

**For lines AB & CD,**

with transversal PS

$\angle AQP = \angle CRQ$  (Corresponding angles) ... (1)

**For lines PS**

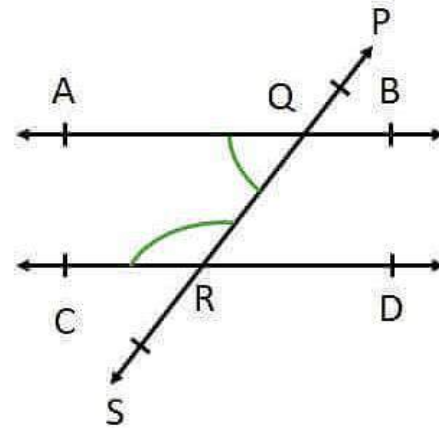
$\angle AQP + \angle AQR = 180^\circ$  (Linear pair) ... (2)

Putting (1) in (2)

$$\angle AQR + \angle CRQ = 180^\circ$$

Similarly,

$$\text{we can prove } \angle BQR + \angle DRQ = 180^\circ$$



Hence, sum of interior angles on same side of transversal is  $180^\circ$

Hence proved

## THEOREM 6.5

If a transversal intersects two lines, such that the pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

Given :- Two parallel lines AB and CD and a transversal PS

intersecting AB at Q and CD at R

such that  $\angle BQR + \angle DRQ = 180^\circ$

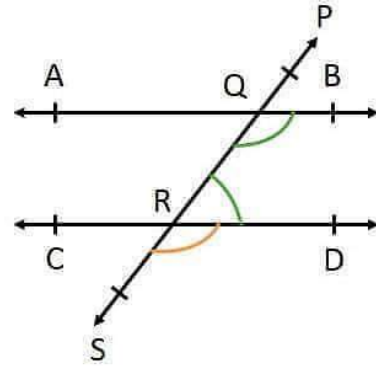
To Prove :-  $AB \parallel CD$ .

Proof :-

**For lines PS**

$$\angle DRQ + \angle DRS = 180^\circ \quad (\text{Linear pair}) \quad \dots(1)$$

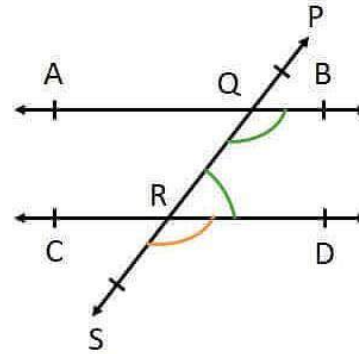
$$\text{But, } \angle BQR + \angle DRQ = 180^\circ \quad (\text{Given}) \quad \dots(2)$$



From (1) & (2)

$$\angle DRQ + \angle DRS = \angle BQR + \angle DRQ$$

$$\angle DRS = \angle BQR$$



But they are corresponding angles.

Thus, for lines AB & CD with transversal PS, corresponding angles are equal

Hence AB and CD are parallel.

Hence proved

### Theorem 6.6 :-

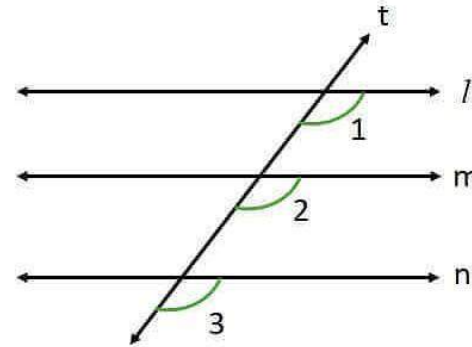
Lines which are parallel to the same lines are parallel to each other.

Given :- Three lines  $l$ ,  $m$ ,  $n$  and a transversal  $t$  such that

$l \parallel m$  and  $m \parallel n$ .

To Prove :-  $l \parallel n$ .

Proof :-



**For lines  $l$  &  $m$ ,**

with transversal  $t$

$$\angle 1 = \angle 2$$

(Corresponding  
angles) ... (1)

**For lines  $m$  &  $n$ ,**

with transversal  $t$

$$\angle 2 = \angle 3$$

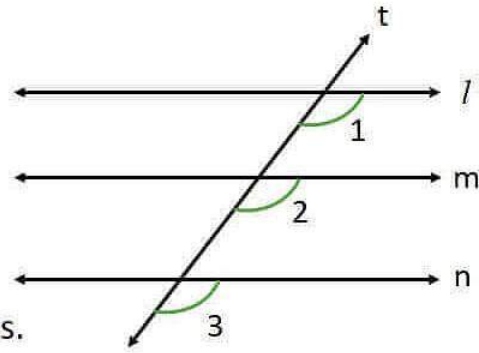
(Corresponding  
angles) ... (2)



From (1) and (2)

$$\angle 1 = \angle 3$$

But they are corresponding angles.



For lines  $l$  &  $n$  with transversal  $t$ , corresponding angles are equal

Hence  $l$  and  $n$  are parallel.

Hence  $l \parallel n$

Hence proved

### Ex 6.2, 1

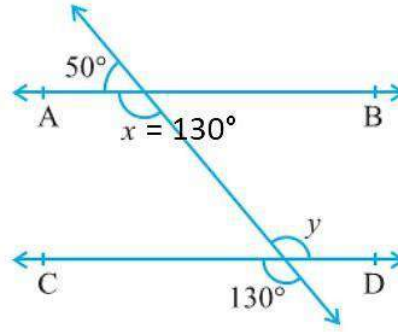
In the given figure, find the values of  $x$  and  $y$  and then show that  $AB \parallel CD$ .

Now,

$$50^\circ + x = 180^\circ \quad (\text{Linear pair})$$

$$x = 180^\circ - 50^\circ$$

$$x = 130^\circ$$



Also,

$$y = 130^\circ \quad (\text{Vertically opposite angles})$$

$$\therefore x = y$$

So, alternate angles are equal

**From theorem 6.3:** *If a transversal intersects two lines such that pair of alternate interior angles are equal, then lines are parallel.*

$$\therefore AB \parallel CD$$

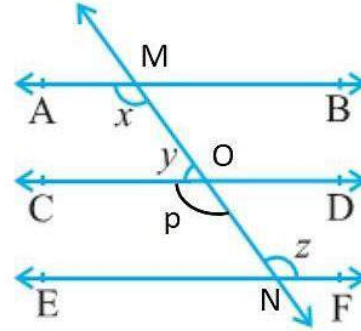
### Ex 6.2, 2

In figure, if  $AB \parallel CD$ ,  $CD \parallel EF$  and  $y : z = 3 : 7$ , find  $x$ .

It is given that

$$\frac{y}{z} = \frac{3}{7}$$

$$y = \frac{3}{7}z$$



Let  $\angle CON = p$

Now  $CD \parallel EF$

So,  $p = z$  (Alternate interior angles) ... (1)

Also,

$$y + p = 180^\circ \quad (\text{Linear pair})$$

$$y + z = 180^\circ \quad (\text{From (1)})$$

$$y + z = 180^\circ$$

Putting  $y = \frac{3}{7}z$

$$\frac{3}{7}z + z = 180^\circ$$

$$\frac{3z + 7z}{7} = 180^\circ$$

$$\frac{10z}{7} = 180^\circ$$

$$z = \frac{7}{10} \times 180^\circ$$

$$z = 126^\circ$$

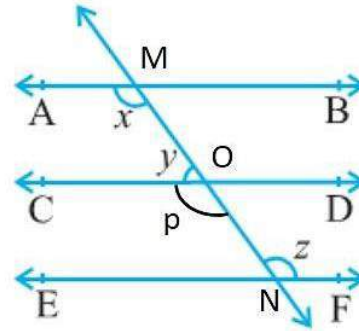
Now,

$$y = \frac{3}{7}z$$

Putting  $z = 126^\circ$

$$y = \frac{3}{7} \times 126^\circ$$

$$y = 54^\circ$$



Also,

$AB \parallel CD$  &  $MN$  is transversal

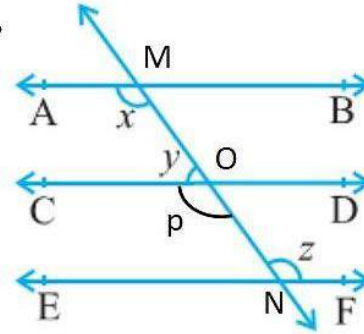
*We know that sum of interior angles on the same side of transversal is supplementary*

$$x + y = 180^\circ$$

$$x + 54^\circ = 180^\circ$$

$$x = 180^\circ - 54^\circ$$

$$x = 126^\circ$$



**Alternatively,**

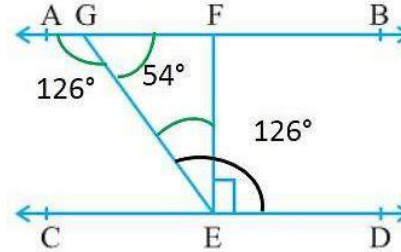
$AB \parallel EF$  &  $MN$  is transversal

$\therefore x = z$  (Alternate interior angles)

$$x = 126^\circ$$

### Ex 6.2, 3

In the given figure, If  $AB \parallel CD$ ,  $EF \perp CD$  and  $\angle GED = 126^\circ$ , find  $\angle AGE$ ,  $\angle GEF$  and  $\angle FGE$ .



Now,

$AB \parallel CD$  &  $GE$  is transversal,

Hence,

$$\angle AGE = \angle GED \quad (\text{Alternate angles})$$

$$\angle AGE = 126^\circ$$

Now,

Since  $AB$  is a line

$$\angle AGE + \angle FGE = 180^\circ \quad (\text{Linear Pair})$$

$$126^\circ + \angle FGE = 180^\circ$$

$$\angle FGE = 180^\circ - 126^\circ$$

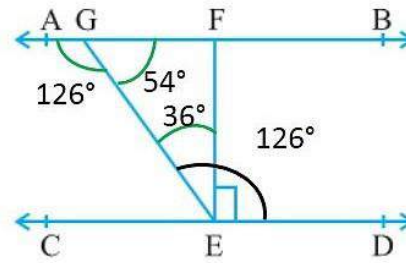
$$\angle FGE = 54^\circ$$

Now,

$$\angle GEF = \angle GED - \angle FED$$

$$\angle GEF = 126^\circ - 90^\circ$$

$$\angle FGE = 36^\circ$$

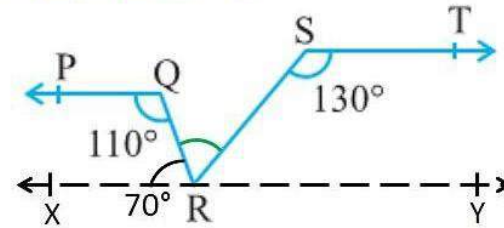


$$\therefore \angle AGE = 126^\circ, \angle GEF = 36^\circ, \angle FGE = 54^\circ$$

### Ex 6.2, 4

In the given figure, if  $PQ \parallel ST$ ,  $\angle PQR = 110^\circ$  and  $\angle RST = 130^\circ$ , find  $\angle QRS$ .

(Hint: Draw a line parallel to  $ST$  through point  $R$ .)



It is given that  $PQ \parallel ST$ ,

We draw a line  $XY \parallel ST$ ,

So,  $XY \parallel PQ$ , i.e.  $PQ \parallel ST \parallel XY$

Since  $PQ \parallel XY$  &  $QR$  is the transversal

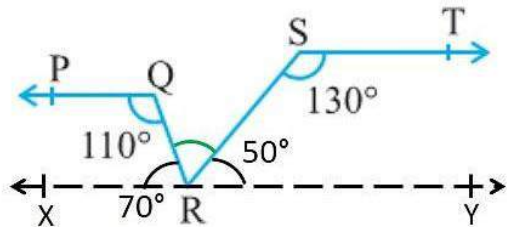
So,  $\angle PQR + \angle QRX = 180^\circ$  (Interior angles on the same side of the transversal are supplementary)

$$110^\circ + \angle QRX = 180^\circ$$

$$\angle QRX = 180^\circ - 110^\circ$$

$$\angle QRX = 70^\circ$$





Also,

$ST \parallel XY$  &  $SR$  is the transversal

$$\angle SRY + \angle RST = 180^\circ$$

*(Interior angles on the same side of the transversal are supplementary)*

$$130^\circ + \angle SRY = 180^\circ$$

$$\angle SRY = 180^\circ - 130^\circ$$

$$\angle SRY = 50^\circ$$

Since  $XY$  is a line

$$\angle QRX + \angle QRS + \angle SRY = 180^\circ \quad \text{(Linear pair)}$$

$$70^\circ + \angle QRS + 50^\circ = 180^\circ$$

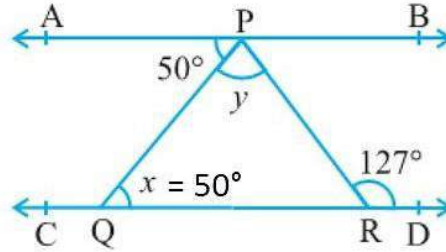
$$120^\circ + \angle QRS = 180^\circ$$

$$\angle QRS = 180^\circ - 120^\circ$$

$$\angle QRS = 60^\circ$$

### Ex6.2, 5

In the given figure, if  $AB \parallel CD$ ,  $\angle APQ = 50^\circ$  and  $\angle PRD = 127^\circ$ , find  $x$  and  $y$ .



Now,  $AB \parallel CD$

and  $PQ$  is the transversal

Hence,  $\angle APQ = x$  (Alternate angles)

$$50^\circ = x$$

$$x = 50^\circ$$

Similarly,

$AB \parallel CD$

and  $PR$  is the transversal

Hence,  $\angle APR = \angle PRD$  (Alternate angles)

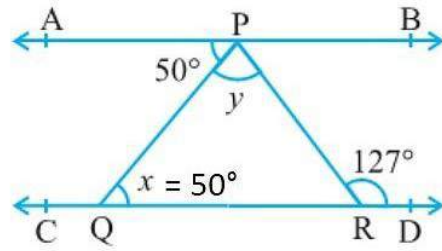
$$\angle APR = \angle PRD$$

$$50^\circ + y = 127^\circ$$

$$y = 127^\circ - 50^\circ$$

$$y = 77^\circ$$

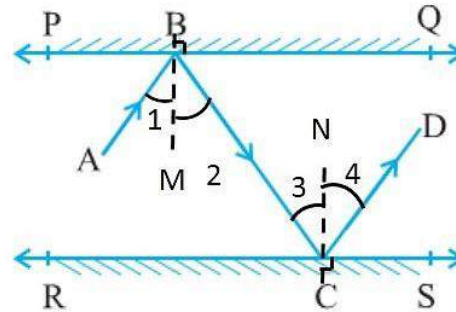
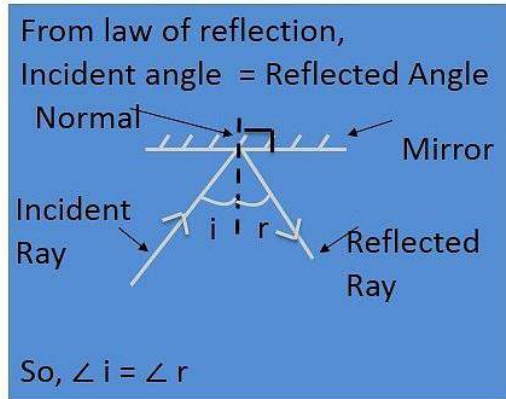
Hence,  $x = 50^\circ$  and  $y = 77^\circ$



### Ex6.2, 6

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that  $AB \parallel CD$ .

Here AB is incident ray and BC is reflected ray.



From laws of reflection,

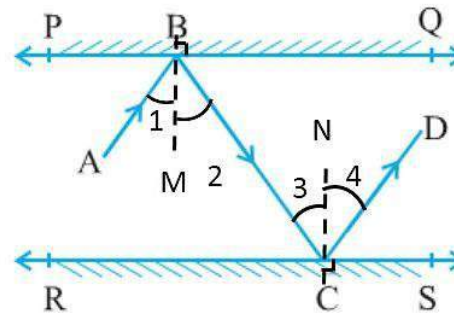
$$\angle 1 = \angle 2$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2 \quad \& \quad \angle 3 = \angle 4$$

$$\text{So, } \angle 1 = \angle 2 = \frac{1}{2} \angle ABC$$

$$\text{and } \angle 3 = \angle 4 = \frac{1}{2} \angle BCD$$



We have to prove  $AB \parallel CD$

Here,  $BM$  &  $CN$  are normal

So,  $BM \perp PQ$  and  $CN \perp RS$ .

But  $PQ \parallel RS$ ,

So,  $BM \parallel CN$

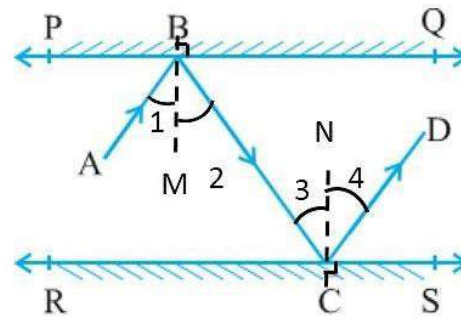
Now,  $BM \parallel CN$  &  $BC$  is the transversal

$\therefore \angle 2 = \angle 3$  (Alternate interior angles)

$$\angle 2 = \angle 3$$

$$\frac{1}{2} \angle ABC = \frac{1}{2} \angle BCD$$

$$\angle ABC = \angle DCB$$



Since,  $\angle ABC = \angle DCB$

But  $\angle ABC$  &  $\angle DCB$  are alternate interior angles for lines AB & CD with transversal BC

**From theorem 6.3:** *If a transversal intersects two lines such that pair of interior angles is equal, then lines are parallel.*

Since alternate interior angles are equal,

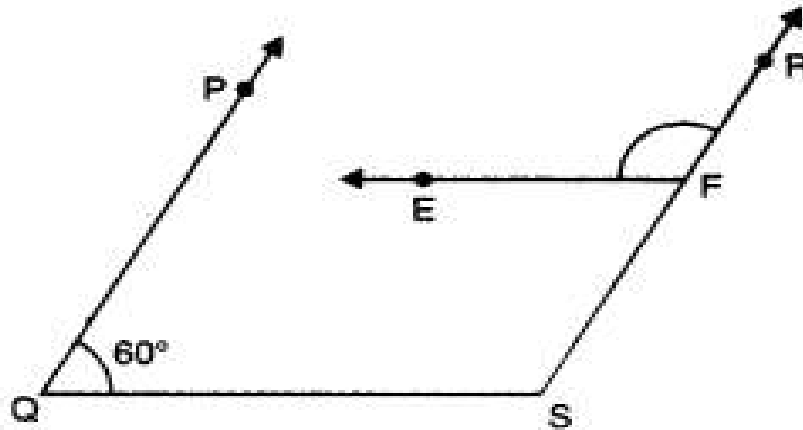
$\therefore AB \parallel CD$

# HOMEWORK ASSIGNMENT

EXERCISE 6.2 Q no-4,5,6

## AHA

In the given figure,  $PQ \parallel RS$  and  $EF \parallel QS$ . If  $\angle PQS = 60^\circ$ , then find the measure of  $\angle RFE$





**THANKING YOU**  
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