

# MATHEMATICS

CHAPTER NUMBER :~ 6

CHAPTER NAME :~ LINES AND ANGLES

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**CHANGING YOUR TOMORROW**

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## PREVIOUS KNOWLEDGE TEST

1. What do mean by transversal of lines?
2. Can there be a transversal for non parallel lines?

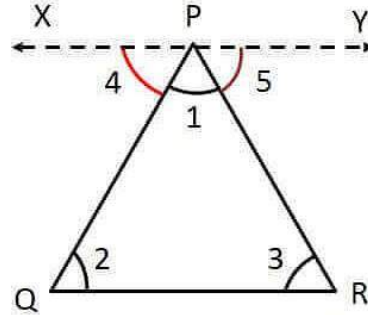
## LEARNING OUTCOME:~

1. Students will be able to learn and prove angle sum property of triangles .
2. Students will develop a relation between interior angles and exterior angles of a triangle.

**Theorem 6.7 :-**

The sum of all angles are triangle is  $180^\circ$ .

Given :-  $\Delta PQR$  with angles  $\angle 1$ ,  $\angle 2$  and  $\angle 3$



Prove :-  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Construction:- Draw a line  $XY$  passing through  $P$  parallel to  $QR$

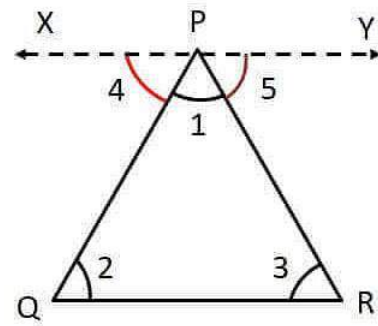
Proof:

For lines  **$XY$  &  $QR$** ,  
with **transversal  $PQ$**

$$\angle 2 = \angle 4 \quad (\text{Alternate angles}) \quad \dots(1)$$

For lines  **$XY$  &  $QR$** ,  
with **transversal  $PR$**

$$\angle 3 = \angle 5 \quad (\text{Alternate angles}) \quad \dots(2)$$



Also, for line XY

$$\angle 1 + \angle 4 + \angle 5 = 180^\circ \quad (\text{Linear pair})$$

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ \quad (\text{From (1) and (2)})$$

Hence, sum of all angle of a triangles are equal.

Hence proved

### Theorem 6.8 :-

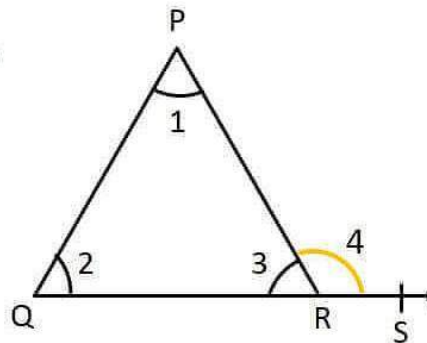
If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given :- A  $\triangle PQR$ ,  $QR$  is produced to point  $S$ .

where  $\angle PRS$  is exterior angle of  $\triangle PQR$ .

To Prove :-  $\angle 4 = \angle 1 + \angle 2$

Proof:-



**For line QS**

$$\angle 3 + \angle 4 = 180^\circ \quad (\text{Linear pair})..(1)$$

**In  $\triangle PQR$**

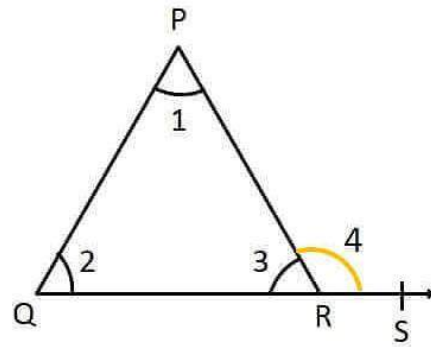
$$\angle 1 + \angle 2 + \angle 3 = 180^\circ \quad (\text{Angle sum property of triangle})..(2)$$

From (1) and (2)

$$\angle 3 + \angle 4 = \angle 1 + \angle 2 + \angle 3$$

$$\angle 4 = \angle 1 + \angle 2$$

$$\angle PRS = \angle RPQ + \angle PQR$$

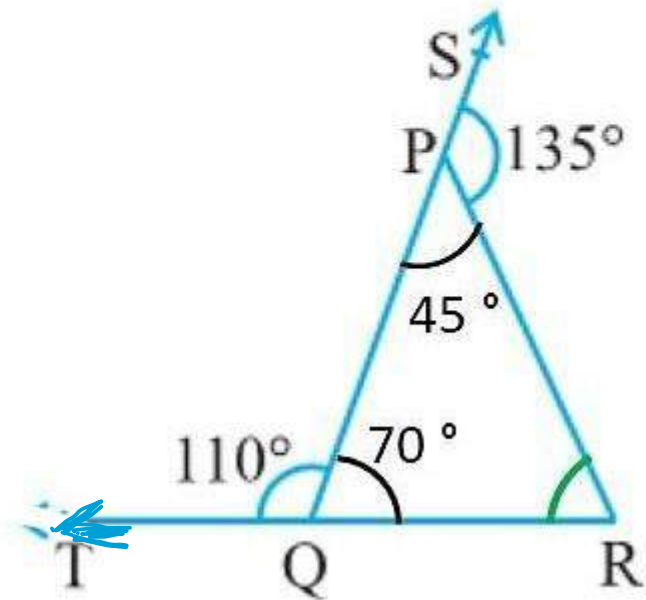


Hence, exterior angle is sum of two opposite interior angles.

Hence proved

### Ex 6.3, 1

In the given figure, sides QP and RQ of  $\Delta PQR$  are produced to points S and T respectively. If  $\angle SPR = 135^\circ$  and  $\angle PQT = 110^\circ$ , find  $\angle PRQ$ .





Since TR is a line

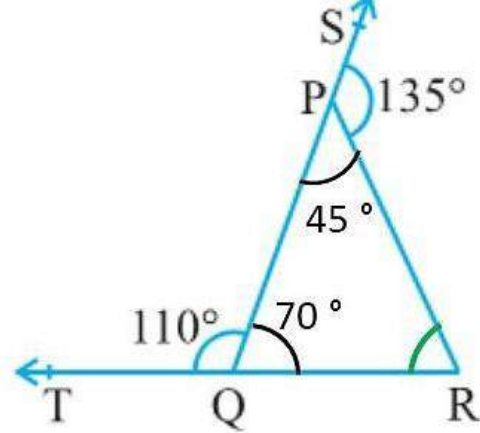
$$\angle PQT + \angle PQR = 180^\circ$$

$$110^\circ + \angle PQR = 180^\circ$$

$$\angle PQR = 180^\circ - 110^\circ$$

$$\angle PQR = 70^\circ$$

(Linear Pair)



Since QS is a line

$$\angle SPR + \angle QPR = 180^\circ$$

$$135^\circ + \angle QPR = 180^\circ$$

$$\angle QPR = 180^\circ - 135^\circ$$

$$\angle QPR = 45^\circ$$

(Linear Pair)

In  $\Delta PQR$ ,

$$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$$

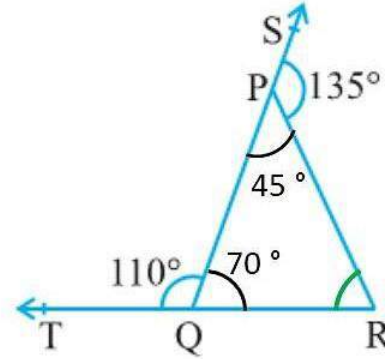
$$45^\circ + 70^\circ + \angle PRQ = 180^\circ$$

$$115^\circ + \angle PRQ = 180^\circ$$

$$\angle PRQ = 180^\circ - 115^\circ$$

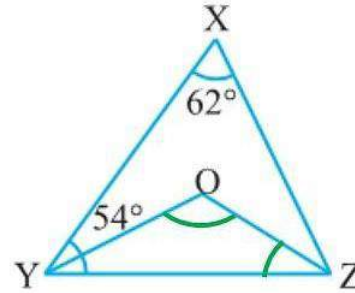
$$\angle PRQ = 65^\circ$$

*(Angle sum property of triangle)*



### Ex 6.3, 2

In the given figure,  $\angle X = 62^\circ$ ,  $\angle XYZ = 54^\circ$ . If YO and ZO are the bisectors of  $\angle XYZ$  and  $\angle XZY$  respectively of  $\triangle XYZ$ , find  $\angle OZY$  and  $\angle YOZ$ .



Given,

OY is the angle bisector of  $\angle XYZ$

$$\text{So, } \angle XYO = \angle OYZ = \frac{1}{2}(\angle XYZ)$$

$$\angle XYO = \angle OYZ = \frac{1}{2}(54^\circ)$$

$$\angle XYO = \angle OYZ = 27^\circ \quad \dots(1)$$

Also,

OZ is the angle bisector of  $\angle XZY$

$$\text{So, } \angle XZO = \angle OZY = \frac{1}{2}(\angle XZY) \quad \dots(2)$$

In  $\triangle XYZ$ ,

$$\angle YXZ + \angle XYZ + \angle XZY = 180^\circ$$

(Angle sum property of triangle)

$$62^\circ + 54^\circ + \angle XZY = 180^\circ$$

$$\angle XZY = 180^\circ - 116^\circ$$

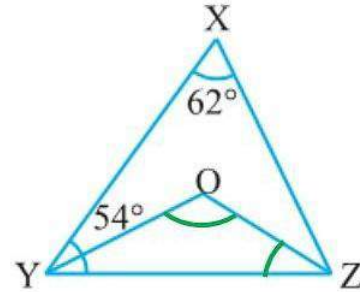
$$\angle XZY = 64^\circ$$

From (2)

$$\angle XZO = \angle OZY = \frac{1}{2}(\angle XZY)$$

$$\angle XZO = \angle OZY = \frac{1}{2} \times 64^\circ$$

$$\angle XZO = \angle OZY = 32^\circ$$



In  $\triangle OYZ$

$$\angle OYZ + \angle OZY + \angle YOZ = 180^\circ$$

(Angle sum property of triangle)

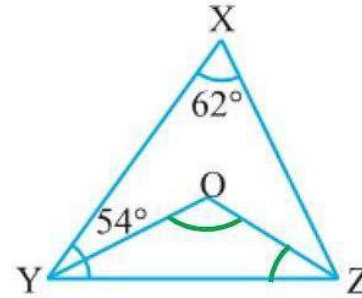
$$27^\circ + 32^\circ + \angle YOZ = 180^\circ$$

$$59^\circ + \angle YOZ = 180^\circ$$

$$59^\circ + \angle YOZ = 180^\circ$$

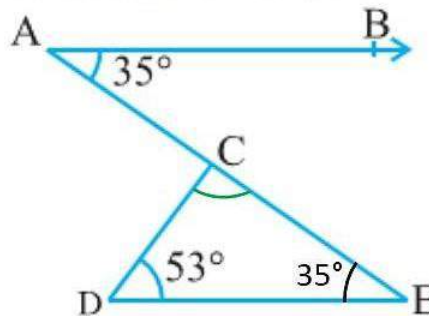
$$\angle YOZ = 180^\circ - 59^\circ$$

$$\angle YOZ = 121^\circ$$



**Ex6.3, 3**

In the given figure, if  $AB \parallel DE$ ,  $\angle BAC = 35^\circ$  and  $\angle CDE = 53^\circ$ , find  $\angle DCE$ .



Since  $AB \parallel DE$  and  $AE$  is a transversal.

$$\angle BAC = \angle CED \quad (\text{Alternate interior angles})$$

$$35^\circ = \angle CED$$

$$\therefore \angle CED = 35^\circ$$

In  $\triangle CDE$ ,

$$\angle CDE + \angle CED + \angle DCE = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$53^\circ + 35^\circ + \angle DCE = 180^\circ$$

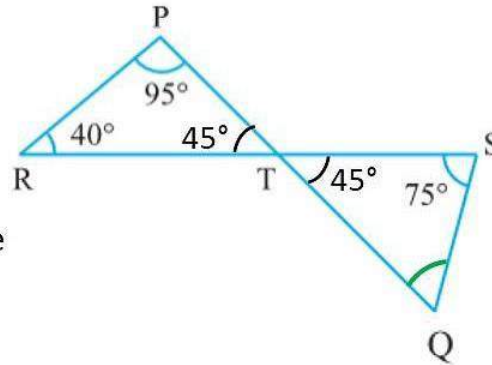
$$88^\circ + \angle DCE = 180^\circ$$

$$\angle DCE = 180^\circ - 88^\circ$$

$$\angle DCE = 92^\circ$$

### Ex 6.3, 4

In the given figure, if lines PQ and RS intersect at point T, such that  $\angle PRT = 40^\circ$ ,  $\angle RPT = 95^\circ$  and  $\angle TSQ = 75^\circ$ , find  $\angle SQT$ .



In  $\Delta PRT$

By angle sum property of triangle

$$\angle PRT + \angle RPT + \angle PTR = 180^\circ$$

$$90^\circ + 45^\circ + \angle PTR = 180^\circ$$

$$135^\circ + \angle PTR = 180^\circ$$

$$\angle PTR = 180^\circ - 135^\circ$$

$$\angle PTR = 45^\circ$$

Also,

$$\angle STQ = \angle PTR$$

$$\angle STQ = 45^\circ \quad (\text{Vertically opposite angles})$$

In  $\Delta SQT$

By angle sum property of triangle

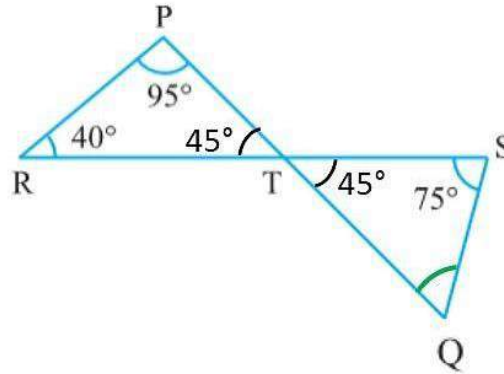
$$\angle SQT + \angle STQ + \angle TSQ = 180^\circ$$

$$\angle SQT + 75^\circ + 45^\circ = 180^\circ$$

$$\angle SQT + 120^\circ = 180^\circ$$

$$\angle SQT = 180^\circ - 120^\circ$$

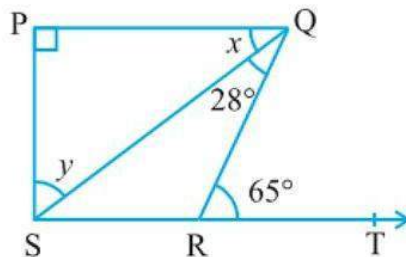
$$\angle SQT = 60^\circ$$





### EX 6.3, 5

In the given figure, if  $PQ \perp PS$ ,  $PQ \parallel SR$ ,  $\angle SQR = 28^\circ$  and  $\angle QRT = 65^\circ$ , then find the values of  $x$  and  $y$ .



Since

$PQ \parallel SR$  and  $QR$  is a transversal.

$$\angle PQR = \angle QRT \quad (\text{Alternate interior angles})$$

$$x + 28^\circ = 65^\circ$$

$$x = 65^\circ - 28^\circ$$

$$x = 37^\circ$$

In  $\triangle SPQ$ ,

$$\angle SPQ + x + y = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$90^\circ + 37^\circ + y = 180^\circ \quad (\text{Given } \angle SPQ = 90^\circ \text{ as } PQ \perp PS)$$

$$y = 180^\circ - 127^\circ$$

$$y = 53^\circ$$

### Ex 6.3 ,6

In the given figure, the side QR of  $\Delta PQR$  is produced to a point S.  
If the bisectors of  $\angle PQR$  and  $\angle PRS$  meet at point T, then prove  
that  $\angle QTR = \frac{1}{2} \angle QPR$

Given

TQ is the bisector of  $\angle PQR$ .

$$\text{So, } \angle PQT = \angle TQR = \frac{1}{2} \angle PQR$$

Also,

TR is the bisector of  $\angle PRS$

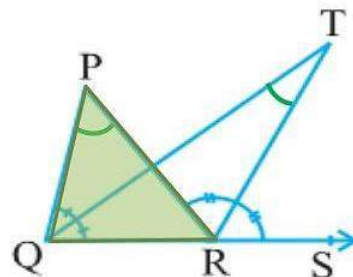
$$\text{So, } \angle PRT = \angle TRS = \frac{1}{2} \angle PRS$$

In  $\Delta PQR$ ,

$\angle PRS$  is the external angle

$$\angle PRS = \angle QPR + \angle PQR$$

$$\left( \begin{array}{l} \text{External angle is sum of two} \\ \text{interior opposite angles} \end{array} \right) \quad \dots(1)$$



In  $\Delta TQR$ ,

$\angle TRS$  is the external angle

$$\angle TRS = \angle TQR + \angle QTR \quad \left( \begin{array}{l} \text{External angle is sum of two} \\ \text{interior opposite angles} \end{array} \right) \quad \dots(2)$$

Putting  $\angle TRS = \frac{1}{2} \angle PRS$  &  $\angle TQR = \frac{1}{2} \angle PQR$

$$\frac{1}{2} \angle PRS = \frac{1}{2} \angle PQR + \angle QTR$$

$$\frac{1}{2} \angle PRS = \frac{1}{2} \angle PQR + \angle QTR$$

Putting  $\angle PRS = \angle QPR + \angle PQR$  from (1)

$$\frac{1}{2} (\angle QPR + \angle PQR) = \frac{1}{2} \angle PQR + \angle QTR$$

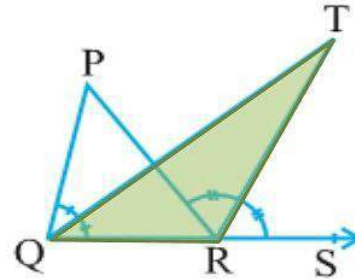
$$\frac{1}{2} \angle QPR + \frac{1}{2} \angle PQR = \frac{1}{2} \angle PQR + \angle QTR$$

$$\frac{1}{2} \angle QPR + \frac{1}{2} \angle PQR - \frac{1}{2} \angle PQR = \angle QTR$$

$$\frac{1}{2} \angle QPR = \angle QTR$$

$$\angle QTR = \frac{1}{2} \angle QPR$$

Hence proved

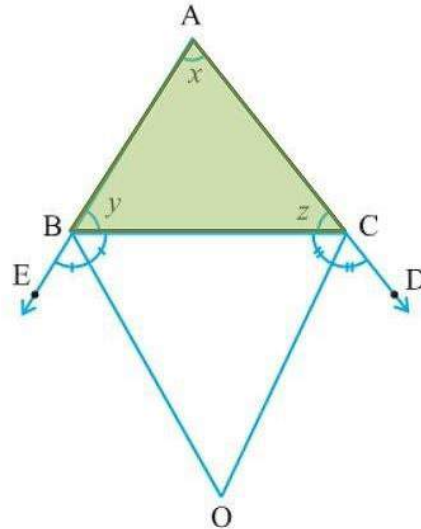


# HOMEWORK ASSIGNMENT

## EXERCISE 6.3

## AHA

In figure, the sides AB and AC of  $\triangle ABC$  are produced to points E and D respectively. If bisectors BO and CO of  $\angle CBE$  and  $\angle BCD$  respectively meet at point O, then prove that  $\angle BOC = 90^\circ - \frac{1}{2}\angle BAC$ .



**THANKING YOU**  
**ODM EDUCATIONAL GROUP**