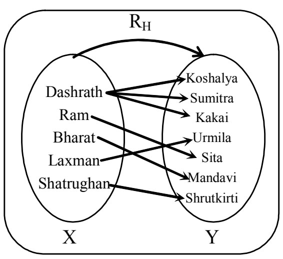
1 RELATIONS AND FUNCTIONS

RELATION

A relation R from set X to Y ($R : X \rightarrow Y$) is a correspondence between set X to set Y by which some or more elements of X are associated with some or more elements of Y. Therefore a relation (or binary relation) R, from a non-empty set X to another non-empty set Y, is a subset of $X \times Y$. i.e. $R_H : X \rightarrow Y$ is nothing but subset of $A \times B$.

e.g. Consider a set X and Y as set of all males and females members of a royal family of the kingdom Ayodhya

X = {Dashrath, Ram, Bharat, Laxman, shatrughan} and Y = {Koshaliya, Kakai, sumitra, Sita, Mandavi, Urmila, Shrutkirti} and a relation R is defined as "was husband of" from set X to set Y.



Then R_H = {(Dashrath, Koshaliya), (Ram, sita), (Bharat, Mandavi), (Laxman, Urmila), (Shatrughan, Shrutkirti), (Dashrath, Kakai), (Dashrath, Sumitra)}

Note :

(i) If a is related to b then symbolically it is written as a R b where a is pre-image and b is image

(ii) If a is not related to b then symbolically it is written as $a \not{R}\, b$.

Total number of relation from A to B :

Let number of relations from A to B be x.

Let A contain m elements and B contain n elements.

Number of elements in $A \times B \rightarrow m \times n$

Number of non-void subsets

$$= {}^{mn}C_1 + {}^{mn}C_2 + \dots\dots\dots + {}^{mn}C_{mn} = 2^{mn} - 1$$

DOMAIN, CO-DOMAIN & RANGE OF RELATION

Domain : of relation is collection of elements of the first set which are participating in the correspondence i.e. it is set of all pre-images under the relation R. e.g. Domain of R_H : {Dashrath, Ram, Bharat, Laxman, Shatrughan}

Co-Domain : All elements of set Y irrespective of whether they are related with any element of X or not constitute co-domain. e.g. Y = {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti} is co-domain of R_H.

Range : of relation is a set of those elements of set Y which are participating in correspondence i.e. set of all images. Range of R_H : {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}.

TYPES OF RELATIONS

1. **Reflexive Relation :** $R : X \rightarrow Y$ is said to be reflexive iff x R x $\forall$ $x \in X$. i.e. every element in set X, must be a related to itself therefore $\forall$ $x \in X$; $(x, x) \in R$ then relation R is called as reflexive relation.

2. **Identity Relation :** Let X be a set. Then the relation $I_x = \{(x, x) : x \in X\}$ on X is called the identity relation on X. i.e. a relation I_x on X is identity relation if every element of X related to itself only. e.g. y = x
Note : All identity relations are reflexive but all reflexive relations are not identity.

3. **Symmetric Relation :** $R : X \rightarrow Y$ is said to be symmetric iff $(x, y) \in R \Rightarrow (y, x) \in R$ for all $(x, y) \in R$ i.e. $x R y \Rightarrow y R x$ for all $(x, y) \in R$. e.g. perpendicularity of lines in a plane is symmetric relation.

4. **Transitive Relation :** $R : X \rightarrow Y$ is transitive iff $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all (x, y) and $(y, z) \in R$. i.e. x R y and y R z $\Rightarrow$ x R z. e.g.
The relation "being sister of " among the members of a family is always transitive.
Note :
(i) Every null relation is a symmetric and transitive relation.
(ii) Every singleton relation is a transitive relation.
(iii) Universal and identity relations are reflexive, symmetric as well as transitive.

5. **Anti-symmetric Relation :** Let A be any set. A relation R on set A is said to be an antisymmetric relation iff $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$ e.g. Relations "being subset of "; "is greater than or equal to" and "identity relation on any set A" are antisymmetric relations.

6. **Equivalence Relation :**
A relation R from a set X to set Y ($R : X \rightarrow Y$) is said to be an equivalence relation iff it is reflexive, symmetric as well as transitive. The equivalence relation is denoted by ~.
e.g. Relation "is equal to" Equality, Similarity and congruence of triangles, parallelism of lines are equivalence relation.

INVERSE OF A RELATION

Let A, B be two sets and let R be a relation from a set A to B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$, Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$ Also,
Dom of R = Range of R^{-1} and Range of R = Dom of R^{-1}

Example 1 :

Let R be a relation from N to N defined by
$R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Are the following true?
(i) $(a,a) \in R$, for all $a \in N$
(ii) $(a, b) \in R$ implies $(b,a) \in R$
(iii) $(a, b) \in R, (b,c) \in R$ implies $(a,c) \in R$.
Justify your answer in each case.

Sol. Here $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$
(i) No, $(3, 3) \notin R$ because $3 \neq 3^2$
(ii) No, $(9, 3) \in R$ but $(3, 9) \notin R$
(iii) No, $(81, 9) \in R, (9, 3) \in R$ but $(81, 3) \notin R$

Example 2 :

Determine whether each of the following relations are reflexive, symmetric and transitive:
(i) Relation R in the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$
(ii) Relation R in the set N of natural numbers defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$
(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y \text{ is divisible by } x\}$

Sol. **(i)** $A = \{1, 2, 3, ..., 13, 14\}, R = \{(x, y) : 3x - y = 0\}$
$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
Now $1 \in A$ but $(1, 1) \notin R$. So R is not reflexive relation.
$(1, 3) \in R$ but $(3, 1) \notin R$. So R is not symmetric relation.
$(1, 3) \in R$ and $(3, 9) \in R$ but $(1, 9) \notin R$. So R is not transitive relation.
So, R is neither reflexive nor symmetric nor transitive.

(ii) $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$
$R = \{(1, 6), (2, 7), (3, 8)\}, A = \{1, 2, 3\}$
Now $1 \in A$ but $(1, 1) \notin R$. So R is not reflexive relation.
$(1, 6) \in R$ but $(6, 1) \notin R$. So R is not symmetric relation.
In R, $(a, b) \in R$ but there is no ordered pair $(b, c) \in R$.
So R is not transitive relation.
R is neither reflexive nor symmetric nor transitive.

(iii) $A = (1, 2, 3, 4, 5, 6)$
$R = \{(x, y) : (x, y) \in A, y \text{ is divisible by } x\}$
$A = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2,4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$
Now $1, 2, 3, 4, 5, 6 \in A$
$\therefore$ $(1, 1), (2, 2) (3, 3), (4, 4), (5, 5), (6, 6) \in R$. So R is reflexive relation.
$(1, 2) \in R$ but $(2,1) \notin R$. So R is a not symmetric relation.
$(1, 1) \in R$ and $(1, 2) \in R \Rightarrow (1, 2) \in R$.
$(2, 2) \in R$ and $(2, 4) \in R \Rightarrow (2, 4) \in R$.
$(3, 3) \in R$ and $(3, 6) \in R \Rightarrow (3, 6) \in R$ and so on.
So R is transitive relation.
Thus R is reflexive and transitive but not symmetric.

TRY IT YOURSELF-1

Q.1 Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Q.2 Show that the relation R in the set A of all the books in library of a college given by R {(x, y) : x and y have same number of pages} is an equivalence relation.

Q.3 Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Q.4 Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.
(A) $(2, 4) \in R$ (B) $(3, 8) \in R$
(C) $(6, 8) \in R$ (D) $(8, 7) \in R$

Q.5 Let R be the relation in the set {1, 2, 3, 4} given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$, choose the correct answer.
(A) R is reflexive and symmetric but not transitive
(B) R is reflexive and transitive but not symmetric.
(C) R is symmetric and transitive but not reflexive.
(D) R is an equivalence relation.

Q.6 Let R be a relation on the set N of natural numbers defined by n R m if n divides m. Then R is –
(A) Reflexive and symmetric
(B) Transitive and symmetric
(C) Equivalence
(D) Reflexive, transitive but not symmetric

Q.7 Consider the set A = {1, 2, 3} and R be the smallest equivalence relation on A, then R = ...

Q.8 Check the following relations for being reflexive, symmetric, transitive and thus choose the equivalent relations if any.
(i) a R b iff $|a - b| > 1/2$; $a, b \in R$
(ii) a R b iff (a – b) is divisible by n; $a, b \in I$, n is a fixed positive integer.

Q.9 Two points A and B in a plane are related if OA = OB, where O is a fixed point. This relation is –
(A) Reflexive but not symmetric
(B) Symmetric but not transitive
(C) All equivalence relation
(D) None of these

ANSWERS

(3) R is neither reflexive nor symmetric nor transitive.
(4) (C) **(5)** (B) **(6)** (D)
(7) {(1, 1), (2, 2), (3, 3)}
(8) (i) Not reflexive, symmetric, not transitive
(ii) Reflexive, symmetric, transitive and equivalence relation. **(9)** (C)

FUNCTION

Definition : Let A and B be two given sets and if each element $a \in A$ is associated with a unique element $b \in B$ under a rule f, then this relation is called function.

Every element of A should be associated with B but vice-versa is not essential. Every element of A should be associated with a unique (one and only one) element of B but any element of B can have two or more relations in A.

KINDS OF FUNCTION

1. One -One function or Injection :

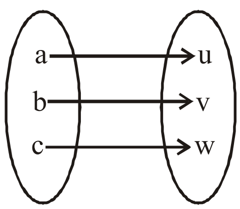
A function $f : A \to B$ is said to be one-one if different elements of A have different images in B.

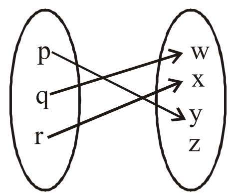
Therefore for any two elements x_1, x_2 of a set A.

$x_1 \neq x_2 \Rightarrow f(x_1) \neq (x_2)$

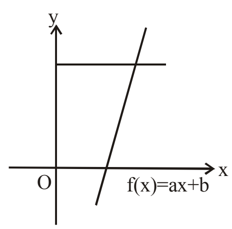
or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ then function is one - one

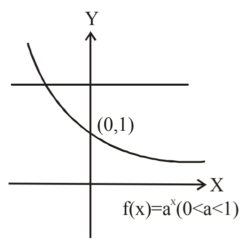
f_1 : f_2:





The above given diagrams show f_1 & f_2 one-one function. If the graph of the function y = f(x) is given, and each line parallel to x- axis cuts the given curve at maximum one point then function is one - one.





Examples of one-one functions -

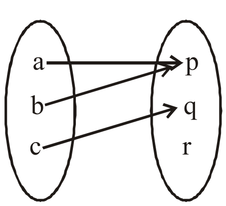
(i) $f : R \to R, f(x) = x$
(ii) $f : R \to R, f(x) = ax + b,$
(iii) $f : R \to R, f(x) = ax^n + b$, n is odd positive integer
(iv) $f : R \to R, f(x) = x|x|$
(v) $f : R \to R, f(x) = e^x,$
(vi) $f : R \to R, f(x) = a^x \ (a > 0)$
(vii) $f : R_0 \to R, f(x) = 1/x,$
(viii) $f : R_0 \to R, f(x) = \log x,$
(ix) $f : R_0 \to R, f(x) = \log_a x \ (a > 0)$

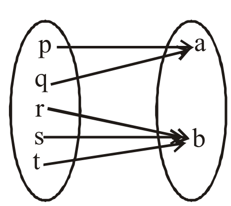
2. Many - One Function :

A function $f : A \to B$ is called many- one, if two or more different elements of A have the same f - image in B.

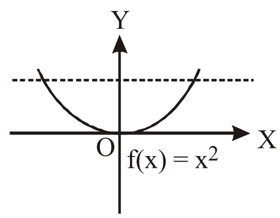
Therefore for any two elements x_1, x_2 of a set A.

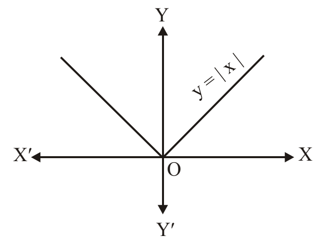
$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$ then function is many one





The above given arrow-diagrams show many-one function. If the graph of $y = f(x)$ is given and the line parallel to x- axis cuts the curve at more than one point then function is many-one.





Example of many - one function :

(i) $f : R \to R, f(x) = C$, where C is a constant
(ii) $f : R \to R, f(x) = x^2$
(iii) $f : R \to R, f(x) = ax^2 + b$
(iv) $f : R \to R, f(x) = |x|$
(v) $f : R \to R, f(x) = x + |x|$
(vi) $f : R \to R, f(x) = x - |x|$
(vii) $f : R \to R, f(x) = [x]$
(viii) $f : R \to R, f(x) = x - [x]$

Where [x] is greatest integer function.

Methods of determining whether a given function is one-one or many-one :

(a) If $x_1, x_2 \in A$ and $f(x_1), f(x_2) \in B$, equate $f(x_1)$ and $f(x_2)$ and if it implies that $x_1 = x_2$ then and only then function is one-one otherwise many-one.

(b) If there exists a straight line parallel to x-axis, which cuts the graph of the function atleast at two points, then the function is many-one, otherwise one-one

(c) If either $f'(x) \geq 0 \ \forall \ x \in$ domain of $f'(x) \leq 0 \ \forall \ x \in$ domain, where equality can hold at discrete point(s) only i.e. strictly monotonic, then function is one-one, otherwise many-one.

Note : If f and g both are one-one, then gof and fog would also be one-one (if they exist). Functions can also be classified as "Onto function (Surjective mapping)" and "Into function".

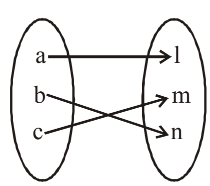
3. Onto function or Surjection :

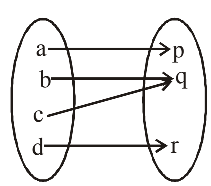
A function $f : A \to B$ is onto if the each element of B has its pre image in A.

Therefore if $f^{-1}(y) \in A, \forall \ y \in B$ then function is onto.

In other words, Range of f = co-domain of f.

The following arrow-diagrams show onto function.





Example of onto function :

(i) $f: R \to R, f(x) = x$

(ii) $f: R \to R, f(x) = ax + b, a \neq 0, b \in R$

(iii) $f: R \to R, f(x) = x^3$

(iv) $f: R \to R, f(x) = x|x|$

(v) $f: R \to R, \ f(x) = e^x$

(vi) $f: R^+ \to R, f(x) = \log x$

Example 3 :

Is the function $f: N \to N$ (N is set of the natural numbers) defined by $f(n) = 2n + 3$ for all $n \in N$ surjective ?

Sol. No, Range will consist of only odd number thus even numbers will have no pre-image.

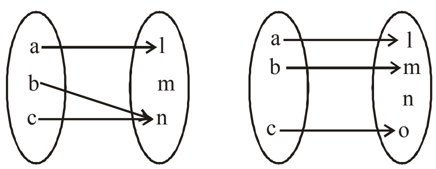
4. Into function :

A function $f: A \to B$ is into if there exit at least one element in B which is not the f - image of any element in A.

Therefore, at least one element of B such that $f^{-1}(y) = \phi$ then function is into.

In other words Range of f $\neq$ co-domain of f

The following arrow-diagrams show into function.



Example of into function :

(i) $f: R \to R, f(x) = x^2$

(ii) $f: R \to R, f(x) = |x|$

(iii) $f: R \to R, f(x) = c$ (c is constant)

(iv) $f: R \to R, f(x) = \sin x$

(v) $f: R \to R, f(x) = \cos x$

(vi) $f: R \to R, f(x) = e^x$

(vii) $f: R \to R, f(x) = a^x, a > 0$

Note : For a function to be onto or into depends mainly on their co-domain.

Ex. $f: R \to R$ then $f(x) = |x|$ is a into function

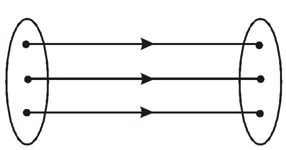
$f: R \to R^+ \cup \{0\}$ then $f(x) = |x|$ is a onto function

Ex. $f: [0, \pi] \to [-1, 1]$ then $f(x) = \sin x$ is into function

$f: [0, \pi] \to [-1, 1]$ then $f(x) = \cos x$ is onto function

5. One -One Onto function or bijection :

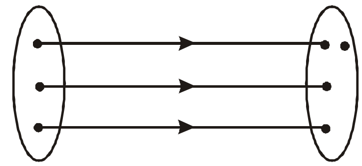
A function f is said to be one-one onto if f is one-one and onto both.



Ex. (i) If $f: R^+ \to R^+$ then $f(x) = |x|$ is a one- one onto function.

(ii) If $f: [-\pi/2, \pi/2] \to [-1, 1]$ then $f(x) = \sin x$ is a one-one onto function.

6. One-One Into function : A function is said to be one-one into if f is one-one but not onto

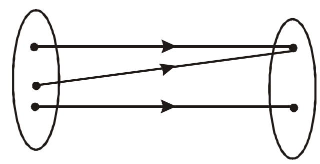


Ex.(i) If $f: [-\pi/2, \pi/2] \to R, \ f(x) = \sin x$

(ii) $f: R^+ \to R, \ f(x) = |x|$

($\therefore$ co-domain and range are not equal so function is not onto)

7. Many One -Onto function : A function $f: A \to R$ is said to be many one- onto if f is onto but not one-one.



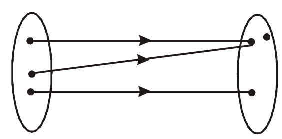
Ex. (i) $f: R \to R^+ \cup \{0\}, \ f(x) = x^2$

(ii) $f: R \to [0, \infty), \quad f(x) = |x|$

(iii) $f: R \to [-1, 1], \quad f(x) = \sin x$

8. Many One -Into function :

A function is said to be many one-into if it is neither one-one nor onto.



Ex. (i) $f: R \to R, \ f(x) = \sin x$ (ii) $f: R \to R, f(x) = |x|$

Note :

(i) If f is both injective and surjective, then it is called a bijective mapping. The bijective functions are also named as invertible, non singular or biuniform functions.

(ii) If a set A contains n distinct elements then the number of different functions defined from $A \to A$ is n^n and out of which n! are one one

(iii) If f and g both are onto, then gof or fog may or may not be onto.

(iv) The composite of two bijections is a bijection if f and g two bijections such that gof is defined, then gof is also a bijection only when co-domain of f is equal to the domain of g.

Example 4 :

Find the type of the function $f: R \to R, f(x) = x^2$

Sol. $\because 4 \neq -4$, but $f(4) = f(-4) = 16$

$\therefore$ f is many one function.

Example 5 :

Find out the type of the function

$$f: R \to R - \{1\}, f(x) = \frac{x-2}{x-3}$$

Sol. One-one/many-one : Let $x_1 x_2 \in R - \{3\}$ are the elements such that $f(x_1) = f(x_2)$: then

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$

$\Rightarrow -2x_2 - 3x_1 = -2x_1 - 3x_2$

$\Rightarrow x_2 = x_1 \;\; \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one - one function

Onto/into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ in domain is such that

$$f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x = \left(\frac{3y-2}{y-1}\right) = x \in R - \{3\}$$

$\therefore$ The pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.

$\Rightarrow$ f is onto

Example 6 :

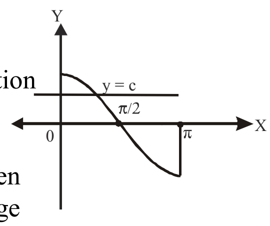
Find out the type of the function $f : [0, \pi] \to R$, $f(x) = \cos x$

Sol. One-one/many-one : -

Since line parallel to x-axis cuts the graph at one point so function is one-one

Onto/into : Since the values of cosx in interval $[0, \pi]$ lie between -1 and 1 it follows that the range of $f(x)$ is not equal to its co-domain R.

So f is not onto



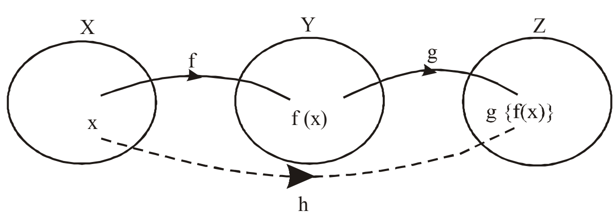
COMPOSITE FUNCTION

Consider two functions $f : X \to Y$, $g : Y \to Z$

one can define $h : X \to Z$ such that $h(x) = g\{f(x)\}$

Domain of gof(x) i. e. $g\{f(x)\} = \{x : x \in \text{Dom} f, f(x) \in \text{Dom} g\}$

Domain of fog (x) = $\{x : x \in \text{Dom} g, g(x) \in \text{Dom} f\}$



Function gof will exist only when range of f is the subset of domain of g. gof (x) is simply the g- image of f (x), where f (x) is f - image of elements $x \in A$. fog does not exist here because range of g is not a subset of domain of f.

Properties of composite functions :

(a) If both f and g are one- one, then gof is also one -one.

(b) If both f and g are onto, then gof is also onto.

(c) If gof is one-one, then f is one-one but g may not be one-one.

(d) If gof is onto, then g is onto but f may not be onto.

(e) If f and g are bijective, then gof is also bijective.

(f) If may happen that gof may exist and fog may not exist. Moreover, even if both gof and fog exist, they may not be equal.

Example 7 :

If $f : R \to R$, $f(x) = 2x + 3$ and $g : R \to R$, $g(x) = x^2 + 1$, then find the value of gof (x).

Sol. $gof(x) = g\{f(x)\}$; $f(x) = 2x + 3$

$g\{f(x)\} = g(2x+3) = (2x+3)^2 + 1 = 4x^2 + 9 + 12x + 1$

$gof(x) = 4x^2 + 12x + 10$

INVERSE FUNCTION

Let $f : X \to Y$ be a function defined by $y = f(x)$ such that f is both one-one and onto, then there exists a unique function $g : Y \to X$ such that for each $y \in Y$, $g(y) = x \Leftrightarrow y = f(x)$. The function g so defined is called the inverse of f. Further, if g is the inverse of f, then f is the inverse of g and the two functions f and g are said to be the inverse of each other. For the inverse of a function to exist, the function must be one-one and onto.

Some standard functions given below along with their inverse functions

	Function	**Inverse Function**
(i)	$f : [0, \infty) \to [0, \infty)$	$f^{-1} : [0, \infty) \to [0, \infty)$
	defined by $f(x) = x^2$	defined by $f^{-1}(x) = \sqrt{x}$
(ii)	$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$	$f^{-1} [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
	defined by $f(x) = \sin x$	defined by $f^{-1}(x) = \sin^{-1} x$
(iii)	$f : [0, \pi] \to [-1, 1]$	$f^{-1} [-1, 1] \to [0, \pi]$
	defined by $f(x) = \cos x$	defined by $f^{-1}(x) = \cos^{-1} x$

For the existence of inverse function, it should be one -one onto.

Properties of Inverse function :

Let $f : X \to Y$ and $g : Y \to Z$

(a) Inverse of a bijection is also a bijection function.

(b) Inverse of a bijection is unique.

(c) $(f^{-1})^{-1} = f$

(d) If f and g are two bijections such that (gof) exists then $(gof)^{-1} = f^{-1} og^{-1}$

(e) Inverse of an even function is not defined.

(f) In general fog (x) and gof (x) are not equal. But if f &g are inverse of each other, then gof = fog.
fog (x) and gof (x) can be equal even if f and g are not inverses of each other. e.g. $f(x) = x + 1$, $g(x) = x + 2$.
However if fog (x) = gof (x) = x, then $g(x) = f^{-1}(x)$

(g) If f (x) and g (x) are inverse function of each other, then

$$f'(x) = \frac{1}{g'(x)}$$

Example 8 :

If $f: R \to R, f(x) = 2x + 3$ then $f^{-1}(x)$

Sol. Since f is a bijection therefore its inverse mapping exists and $y = 2x + 3 \Rightarrow x = 2y + 3$

$$\Rightarrow y = \frac{x-3}{2} \quad \therefore f^{-1}(x) = \frac{x-3}{2}$$

Example 9 :

If $f: R \to R, f(x) = x^3 + 2$ then find $f^{-1}(x)$

Sol. $f(x) = x^3 + 2, x \in R$

Since this is a one-one onto function therefore inverse of this function (f^{-1}) exists.

Let $f^{-1}(x) = y$

$\therefore x = f(y) \Rightarrow x = y^3 + 2 \Rightarrow y = (x-2)^{1/3}$

$\therefore f^{-1}(x) = (x-2)^{1/3}$

Example 10 :

If $f: Q \to Q, f(x) = 2x$; $g: Q \to Q, g(x) = x + 2$, then $(fog)^{-1}$ (20) equals

Sol. $\therefore f^{-1}(x) = x/2, g^{-1}(x) = x - 2$

$\therefore (fog)^{-1}(20) = (g^{-1} o f^{-1})(20)$

$= g^{-1}[f^{-1}(20)] = g^{-1}(10) = 10 - 2 = 8$

TRY IT YOURSELF-2

Q.1 Find the type of the function $f: R \to R, f(x) = x^2$

Q.2 Find out the type of the function $f: R \to R - \{1\}, f(x) = \frac{x-2}{x-3}$

Q.3 The function $f: [2, \infty) \to y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if

(A) $y = R$ (B) $y = [1, \infty)$
(C) $y = [4, \infty]$ (D) $y = [5, \infty]$

Q.4 A function $f: A \to B$, such that set "A" contains five element and "B" contains four elements then find

(i) Total number of functions
(ii) Number of one-one functions
(iii) Number of onto functions
(iv) Number of many one functions
(v) Number of into functions

Q.5 If $f(x) + 2f(1-x) = x^2 + 2 \ \forall x \in R$, find f(x)

Q.6 Let f (x) and g (x) be functions which take integers as arguments let $f(x + y) = f(x) + g(y) + 8$ for all integer x & y. Let $f(x) = x$ for all negative integers x let $g(8) = 17$, find f(0).

Q.7 Let $f(x) = \sqrt{x}, g(x) = \sqrt{2-x}$ find the domain of

(i) (fog) (x) (ii) (gog) (x)

Q.8 Find the inverse of the following bijective function

(i) $f(x) = 3x - 5$ (ii) $f: R \to (0, 1), f(x) = \frac{2^x}{1 + 2^x}$

Q.9 If $f: R \to R$ is defined by $f(x) = x^3 + 1$, then find value of $f^{-1}(28)$.

ANSWERS

(1) many one function. **(2)** one-one, onto **(3)** (B)

(4) (i) 4^5 (ii) not possible (iii) 240, (iv) 1024 (v) 784

(5) $f(x) = \frac{x^2 + 2x - 1}{3}$ **(6)** 17

(7) (i) $x \in (-\infty, 2]$, (ii) $-2 \le x \le 4$

(8) (i) $\frac{x+5}{3}$ (ii) $\log_2\left(\frac{x}{1-x}\right)$ **(9)** 3

BINARY OPERATION

Let S be a non-void set. A function defined from S × S to S is known as a binary operation on S. In other words a binary operation on the set S is a rule or a law which gives for any ordered pair (a, b) of elements of S, a unique element of S. Generally, a binary operation is devoted by * or o or by ⊕ symbols.

Under the operation *, the element associated with the element $(a, b) \in S \times S$ is denoted as a * b.

Types of binary operations :

(i) Commutative operations : Let S be a non-void set and let * be a binary operation defined on S. If $a, b \in S$, then we know that $(a, b) \neq (b, a)$ unless a = b. Therefore it is not necessary that the image of (a, b) and (b, a) under the operation * are equal. In other words, it is not necessary that $a * b = b * a, \ \forall a, b \in S$ then the binary operation * is said to be commutative on S.

A binary operation * defined on a set S is said to be commutative if $a * b = b * a, \ \forall a, b \in S$.

(ii) Associative operation :

Let * be a binary operation, defined on a set S.

Let $a, b, c \in S$. Let us consider the expression a * b * c. Since the binary operation * can operate only on two elements at a time and we have three elements. Therefore, we should consider either a * (b * c) or (a * b) * c.

It is not necessary that a * (b * c) = (a * b) * c is always $\forall a, b, c \in S$.

If $a * (b * c) = (a * b) * c \ \forall a, b, c \in S$ then the binary operation * is said to be associative.

A binary operation *, defined on a set S is said to be associative if $a * (b * c) = (a * b) * c \ \forall a, b, c \in S$.

Identity element for an operation :

Let S be a non void set on which a binary operation * is defined. If there exists an element e_1 in S such that

$e_1 * a = a \ \forall a \in S$,

then e_1 is called a left identity in S with respect the operation *. Similarly, if there exists an element e_2 in S, such that $a * e_2 = a, \ \forall a \in S$, then e_2 is called a right identity in S with respect to the operation *.

This element is generally denoted as e. Hence, if there exist an element e in the set S such that $e * a = a * e = a, \ \forall a \in S$, then e is called the identity element in S for the operation.

Theorem : The identity element for a binary operation, if it exists, is unique.

Proof : Let * be a binary operation on a set S. If possible, let e and e' be two identities in S.

then e*e' = e' .... (1) [$\because$ e is identity in S and e' $\in$ S]

Again, e*e' = e .... (2) [$\because$ e' is identity in S and e $\in$ S]

From (1) and (2), e = e'

Inverse on an element : Let * be a binary operation on S and let e be the identity in S. let a $\in$ S. If there exists an element b in S, such that a * b = b *a= e, then b is called the inverse of a and it is denoted by a^{-1}.

If an element a of S has its inverse in S, then a is said to be an invertible element. Hence a is invertible $\Leftrightarrow a^{-1} \in S$

Note : Let e be the identity element in S for a binary operation *. Then e*e = e*e = e. Hence, the identity element in a set, if it exists, is invertible and is own inverse.

Example 11 :

Show that subtraction and division are not binary operations on N.

Sol. $N \times N \to N$, given by $(a, b) \to a - b$, is not binary operation, as the image of (3, 5) under '–' is $3 - 5 = -2 \notin N$.

Similarly, $\div : N \times N \to N$, given by $(a, b) \to a \div b$ is not a binary operation, as the image of (3, 5) under $\div$ is $3 \div 5 = 3/5 \notin N$

Example 12 :

Show that the $\vee : R \times R \to R$ given by $(a, b) \to \max\{a, b\}$ and the $\wedge : R \times R \to R$ given by $(a, b) \to \min\{a, b\}$ are binary operations.

Sol. Since $\vee$ carries each pair (a, b) in $R \times R$ to a unique element namely maximum of a and b lying in R, $\vee$ is a binary operation. Using the similar argument, one can say that $\wedge$ is also a binary operation.

Example 13 :

Show that addition and multiplication are associative binary operation on R. But subtraction is not associative on R. Division is not associative on R.

Sol. Addition and multiplication are associative, since

$(a + b) + c = a + (b + c)$

and $(a \times b) \times c = a \times (b \times c)\ \forall$ a,b,c $\in$ R.

However, subtraction and division are not associative, as $(8 - 5) - 3 \neq 8 - (5 - 3)$ and $(8 \div 5) \div 3 \neq 8 \div (5 \div 3)$.

TRY IT YOURSELF-3

For Q.1-Q.3 : Let * be a binary operation, then, find

Q.1 6 * (–5) if $a * b = a^2 + b^2$, a, b $\in$ Z

Q.2 9 * 2 if $a * b = a^2b^3$, a, b $\in$ N

Q.3 (1.5) * (3.4) if $a * b = a^2b$, a, b $\in$ R

Q.4 If a * b = 4a + 3b, find a * b and b * a, if a = 9 and b = 2. Is this binary operation commutative ?

Q.5 Let * be a binary operation on N, the set of natural numbers, defined by $a * b = a^b$ for all a, b $\in$ N.
Is * associative or commutative on N.

ANSWERS

(1) 61 **(2)** 648 **(3)** 7.65 **(4)** not commutative
(5) neither commutative nor associative

ADDITIONAL EXAMPLES

Example 1 :

Let a relation R_1 on the set R of real numbers be defined as $(a, b) \in R_1 \Leftrightarrow 1 + ab > 0$ for all a, b $\in$ R. Show that R_1 is reflexive and symmetric but not transitive.

Sol. We observe the following properties :

Reflexivity : Let a be an arbitrary element of R. Then
$a \in R \Rightarrow 1 + a\,.\,a = 1 + a^2 > 0 \Rightarrow (a, a) \in R_1$
Thus $(a, a) \in R_1$ for all a $\in$ R. So R_1 is reflexive on R.

Symmetry : Let $(a, b) \in R$. Then
$(a, b) \in R_1 \Rightarrow 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R_1$
Thus $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$ for all a, b $\in$ R
So R_1 is symmetric on R

Transitive : We observe that $(1, 1/2) \in R_1$ & $(1/2, -1) \in R_1$ but $(1,-1) \in R_1$ because $1 + 1 \times (-1) = 0 \not> 0$.
So R_1 is not transitive on R.

Example 2 :

Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function ? If this is described by the formula $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?

Sol. As $\alpha = 2, \beta = -1$; Domain A = {1, 2, 3, 4}
Range B = {1, 3, 5, 7}
Every element of domain has a unique image in B and hence g is a function. Now $g(x) = \alpha x + \beta$ but g (2) = 3, g (3) = 5
$\therefore 2 = 2\alpha + \beta$ and $5 = 3\alpha + \beta$; $\alpha = 2, \beta = -1$

Example 3 :

Let $f : R \to R$. $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the value of α for f (x) to be onto.

Sol. $y = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

$\Rightarrow (\alpha + 8y)x^2 + 6(1 - y)x - (\alpha y + 8) = 0$

According to condition, y takes all real values for all real x,

i.e., $D \geq 0\ \forall\ y \in R$

$\Rightarrow 36(1 - y)^2 + 4(\alpha y + 8)(\alpha + 8y) \geq 0\ \forall\ y \in R$

$\Rightarrow (9 + 8\alpha)y^2 + (\alpha^2 + 46)y + (9 + 8\alpha) \geq 0\ \forall\ y \in R$

i.e., $D \leq 0$ and coefficient of $y^2 > 0$

$\Rightarrow (\alpha^2 + 46)^2 \leq 4(9 + 8\alpha)^2$ and $9 + 8\alpha > 0$

$\Rightarrow \alpha^2 - 16\alpha + 28 \leq 0$ and $\alpha > -9/8$

$\Rightarrow 2 \leq \alpha \leq 14$ and $\alpha > -9/8$

Hence, $\alpha \in [2, 14]$

Example 4 :

Prove that the inverse of the function

$$f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}+2 \text{ is given by } \log_e\left(\frac{x-1}{3-x}\right)^{1/2}$$

Sol. $\frac{y-2}{1}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

Applying comp. and dividendo, $\frac{y-1}{3-y}=\frac{2e^x}{2e^{-x}}=e^{2x}$

$\therefore\ x=\frac{1}{2}\log\left(\frac{y-1}{3-y}\right)=\log\left(\frac{y-1}{3-y}\right)^{1/2}$

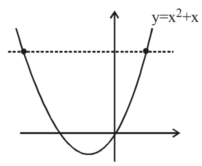
Hence, the inverse of the function

$$f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}+2 \text{ is } \log_e\left(\frac{x-1}{3-x}\right)^{1/2}$$

Example 5 :

Show $f:R\to R$ defined by $f(x)=x^2+x\ \forall\ x\in R$ many-one

Sol. By graph :



Graph of function $f(x)=x^2+x$ is cut by straight line parallel to x-axis at 2 points so $f(x)=x^2+x$ is many-one.

By calculus : $f(x)=x^2+x$; $f'(x)=2x+1$

where $f'(x)>0$ if $x>-\frac{1}{2}$; $f'(x)<0$ if $x<-\frac{1}{2}$

Not monotonic, thus function is many-one.

Example 6 :

Let $f: X\to Y$ be a function such that

$f(C)=\{f(x): x\in C\},\ C\subseteq X$ and

$f^{-1}(D)=\{f^{-1}(x): x\in D\},\ D\subseteq Y$, then

(A) $f^{-1}(f(A))=A$ only if $A\subseteq D$

(B) $f(f^{-1}(B))=B$ only if $B\subseteq D$

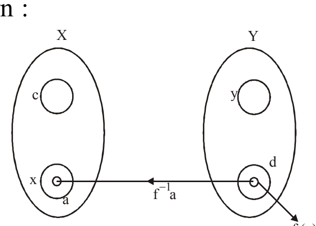
(C) $f(f^{-1}(A))=A$

(D) $f(f^{-1}(B))=B$

Sol. **(B).** Given that X and Y are two sets and $f: X\to Y$.

$\{f(c)=y; c\subset X, y\subset Y\}$ and $f^{-1}(d)=x; d\subset Y, x\subset X\}$

The pictorial representation of given information is as shown :



Since $f^{-1}(d)=x\Rightarrow f(x)=d$

Now if $a\subset x\Rightarrow f(a)\subset f(x)=d$

$\Rightarrow\ f^{-1}[f(x)]=a$

$\therefore\ f^{-1}(f(a))=a,\ a\subset x$ is the correct option.

Example 7 :

Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying $(f\circ g\circ g\circ f)(x)=(g\circ g\circ f)(x)$, where $(f\circ g)(x)=f(g(x))$, is –

(A) $\pm\sqrt{n\pi},\ n\in\{0,1,2,.....\}$

(B) $\pm\sqrt{n\pi},\ n\in\{1,2,.....\}$

(C) $\frac{\pi}{2}+2n\pi,\ n\in\{....,-2,-1,0,1,2,.....\}$

(D) $2n\pi,\ n\in\{....,-2,-1,0,1,2,.....\}$

Sol. **(A).** $f(x)=x^2$; $g(x)=\sin x$

$gof(x)=\sin x^2$

$\Rightarrow$ $gogof(x)=\sin(\sin x^2)$

$\Rightarrow$ $(fogogof)(x)=(\sin(\sin x^2))^2=\sin^2(\sin x^2)$

Now, $\sin^2(\sin x^2)=\sin(\sin x^2)$

$\Rightarrow$ $\sin(\sin x^2)=0,\ 1$

$\Rightarrow$ $\sin x^2=n\pi,\ (4n+1)\frac{\pi}{2}$; $n\in I$

$\Rightarrow$ $\sin x^2=0\Rightarrow x^2=n\pi\Rightarrow x=\pm\sqrt{n\pi}$; $n\in W$

Example 8 :

If $f: Q\to Q,\ f(x)=2x$; $g: Q\to Q,\ g(x)=x+2$, then $(fog)^{-1}(20)$ equals

Sol. $\therefore\ f^{-1}(x)=x/2,\ g^{-1}(x)=x-2$

$\therefore\ (fog)^{-1}(20)=(g^{-1}of^{-1})(20)=g^{-1}[f^{-1}(20)]$

$=g^{-1}(10)=10-2=8$

Example 9 :

Consider the binary operation $*: Q\times Q\to Q$ which is defined by $a*b=a+b-ab$, for all $a,b\in Q$

(i) Find the identity element with respect to * on Q.

(ii) Do any of the elements in Q have an inverse and what is it ?

Sol. (i) Let e be the identity element in Q with respect to * on Q. Then, for all $a\in Q$, we must have $a*e=e*a=a$

But, * is commutative therefore, $a*e=e*a$.

But, $a*e=a\Rightarrow a+e-ae=a\quad\Rightarrow e-ae=0$

$\Rightarrow$ $e(1-a)=0\Rightarrow e=0$, if $1-a\neq 0$ (or $a\neq 1$), for every $a\in Q-\{1\}$

Hence, 0 is the identity element.

(ii) Since, 0 is the identity element, therefore for a to have an inverse b, we must have $a*b=0$

Now, $a*b=0\Rightarrow a+b-ab=0$

$\Rightarrow$ $a-b(a-1)=0\Rightarrow a=b(a-1)\Rightarrow b=\frac{a}{a-1}$.

Thus, if $a\neq 1$, then a has an inverse and it is $\frac{a}{a-1}$

QUESTION BANK

CHAPTER 1 : RELATIONS AND FUNCTIONS (CLASS XII)

EXERCISE - 1 [LEVEL-1]

PART-1 - RELATIONS

Q.1 Let R be an equivalence relation defined on a set containing 6 elements. The minimum number of ordered pairs that R should contain is –
(A) 12 (B) 6
(C) 64 (D) 36

Q.2 Let R be the relation on the set N of natural numbers defined by $R : \{(x, y)\} : x + 3y = 12 \; x \in N, y \in N\}$ Find R
(A) {(8, 1), (6, 2), (3, 3)} (B) {(9, 1), (4, 2), (3, 3)}
(C) {(9, 1), (6, 2), (3, 3)} (D) {(7, 1), (6, 2), (2, 2)}

Q.3 In the above question, find Domain of R
(A) {9, 6, 3} (B) {9, 5, 3}
(C) {6, 4, 3} (D) {8, 1, 3}

Q.4 In the above question, find Range of R
(A) {9, 6, 3} (B) {9, 5, 3}
(C) {6, 4, 3} (D) {1, 2, 3}

Q.5 Let A = {1, 2, 3}, the total number of distinct relations that can be defined over A is –
(A) 2^9 (B) 6
(C) 8 (D) None

Q.6 If the number of elements in A is m and number of element in B is n then find number of relation defined from A to B
(A) 2^{mn} (B) 2^{m+n}
(C) 2^{m-n} (D) $2^{m/n}$

Q.7 Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is –
(A) 1 (B) 2
(C) 3 (D) 4

Q.8 Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is
(A) 1 (B) 2
(C) 3 (D) 4

Q.9 For n, m ∈ N, nm means that n is a factor of m, the relation
(A) Reflexive and symmetric
(B) Transitive and symmetric
(C) Reflexive, transitive and symmetric
(D) Reflexive, transitive but not symmetric

Q.10 For x, y ∈ R, $x R y \Rightarrow x - y + \sqrt{7}$ is an irrational numher, then R is –
(A) An equivalence relation
(B) Reflexive and symmetric only
(C) Symmetric and transitive only
(D) Reflexive

Q.11 Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}.
(A) R is reflexive and symmetric but not transitive
(B) R is reflexive and transitive but not symmetric.
(C) R is symmetric and transitive but not reflexive.
(D) R is an equivalence relation.

Q.12 Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}.
(A) (2, 4) ∈ R (B) (3, 8) ∈ R
(C) (6, 8) ∈ R (D) (8, 7) ∈ R

Q.13 Let R be a relation $R : A \to B$ and $R^{-1} : A \to B$ where A = { 1, 2, 3, 4}, B = { 1, 3, 5} defined by R = {(x, y) : x < y x ∈ A, y ∈ B} then ROR^{-1} is –
(A) (1, 1), (3, 3) (B) (3, 5), (5, 5), (3, 3), (5, 3)
(C) (2, 3), (2, 5), (3, 3), (3, 5) (D) (1, 1), (2, 2), (3, 3), (4, 4)

PART-2 - FUNCTIONS

Q.14 Which of the following is a function?
(A) {(2,1), (2,2), (2,3), (2,4)} (B) {(1,4), (2,5), (1,6), (3,9)}
(C) { (1,2), (3,3), (2,3), (1,4)} (D) { (1,2), (2,2), (3,2), (4,2)}

Q.15 If $f(x) = \frac{x}{x-1} = \frac{1}{y}$, then f (y) equals
(A) x (B) x – 1
(C) x + 1 (D) 1 – x

Q.16 If $f : R^+ \to R^+$, $f(x) = x^2 + 2$ and $g : R^+ \to R^+$, $g(x) = \sqrt{x+1}$ then (f + g) (x) equals
(A) $\sqrt{x^2+3}$ (B) x + 3
(C) $\sqrt{x^2+2} + (x+1)$ (D) $x^2 + 2 + \sqrt{(x+1)}$

Q.17 Find fog if $f(x) = 8x^3$ and $g(x) = x^{1/3}$
(A) x (B) $x^{1/3}$
(C) $8x^2$ (D) 8x

Q.18 Let $f : R \to R$ be defined as $f(x) = x^4$.
(A) f is one–one onto
(B) f is many–one onto
(C) f is one–one but not onto
(D) f is neither one–one nor onto.

Q.19 Let $f : R \to R$ be defined as f (x) = 3x.
(A) f is one–one onto
(B) f is many–one onto
(C) f is one–one but not onto
(D) f is neither one–one nor onto.

Q.20 If $f : R \to R$ is defined by f (x) = 2x + 3, then $f^{-1}(x)$
(A) does not exist because f is not surjective
(B) is given by $\frac{x-3}{2}$ (C) is given by $\frac{1}{2x+3}$
(D) None of these

PART-3 - BINARY OPERATIONS

Q.21 In P (X), the power set of a nonempty set X, an binary operation * is defined by
A * B A ∪ B ∀ A, B ∈ P (X). Under *, a true statement is–
(A) identity law is not satisfied
(B) inverse law is not satisfied
(C) commutative law is not satisfied
(D) associative law is not satisfied

Q.22 On the set of all nonzero reals, an operation * is defined as a * b = 3ab/2. In this group, a solution of $(2 * x) * 3^{-1} = 4^{-1}$ is –

(A) 3/2 (B) 1/6
(C) 1 (D) 6

Q.23 Number of binary operations on the set {a, b} are

(A) 10 (B) 16
(C) 20 (D) 8

Q.24 Consider a binary operation * on N defined as $a * b = a^3 + b^3$

(A) both associative and commutative
(B) commutative but not associative
(C) associative but not commutative
(D) neither commutative nor associative

Q.25 Choose the correct statement –

(i) For each binary operation * On Z^+, define $a * b = 2^{ab}$ * is commutative
(ii) For each binary operation * On Z^+, define $a * b = 2^{ab}$ * is not commutative
(iii) For each binary operation * On Z^+, define $a * b = 2^{ab}$ * is associative.
(iv) For each binary operation * On Z^+, define $a * b = 2^{ab}$ * is not associative.

(A) ii and iv (B) i and iii
(C) i, iv (D) ii and iii

Q.26 Choose the correct statement for binary operation *

(i) * On Q, define $a * b = \frac{ab}{2}$ * is commutative
(ii) * On Q, define $a * b = \frac{ab}{2}$ * is not commutative
(iii) * On Q, define $a * b = \frac{ab}{2}$ * is associative.
(iv) * On Q, define $a * b = \frac{ab}{2}$ * is not associative.

(A) ii and iv (B) i and iii
(C) i, iv (D) ii and iii

PART-4 - MISCELLANEOUS

Q.27 If $f(x) = \cos(4\pi\{x\}) + \sin(4\pi\{x\})$ where {x} denotes the fractional part of x then which of the following is true –

(A) f (x) is non-periodic (B) period of f (x) is 1/2
(C) period of f (x) is 1 (D) period of f (x) is 2

Q.28 Let f be a function with domain [–3, 5] and let g (x) = | 3x + 4 |, Then the domain of (fog) (x) is

(A) (–3, 1/3) (B) [–3, 1/3]
(C) [–3, 1/3) (D) none of these

Q.29 Which of the following relation is a function -

(A) {(a, b) (b, e) (c, e) (b, x)}
(B) {(a, d) (a, m) (b, e) (a, b)}
(C) {(a, d) (b, e) (c, d) (e, x)}
(D) {(a, d) (b, m) (b, y) (d, x)}

Q.30 Let $f : \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \to [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$. Then $f^{-1}(x)$ is given by

(A) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$

(C) $\frac{2\pi}{3} + \cos^{-1}\left(\frac{x-2}{2}\right)$ (D) none of these

Q.31 The period of the function, $f(x) = [\sin 3x] + |\cos 6x|$, (where [.] denotes the greatest integer less than or equal to x), is

(A) π (B) $2\pi/3$
(C) 2π (D) $\pi/2$

Q.32 Which of the following function is surjective but not injective

(A) $f : R \to R$ $f(x) = x^4 + 2x^3 - x^2 + 1$
(B) $f : R \to R$ $f(x) = x^3 + x + 1$
(C) $f : R \to R^+$ $f(x) = \sqrt{1+x^2}$
(D) $f : R \to R$ $f(x) = x^3 + 2x^2 - x + 1$

Q.33 If $f(x) = 2x^3 + 7x - 5$ then $f^{-1}(4)$ is

(A) equal to 1 (B) equal to 2
(C) equal to 1/3 (D) non existent

Q.34 If $F(x) = \begin{cases} \left(x - 10\left[\frac{x}{10}\right]\right).10^{[\log_{10} x]} + F\left(\left[\frac{x}{10}\right]\right), & \text{if } x \neq 0 \\ 0 & \text{, if } x = 0 \end{cases}$

where [x] stands for the greatest integer not exceeding 'x' then F(7752) =

(A) 2222 (B) 7777
(C) 7752 (D) 2577

Q.35 The range of the function

$$f(x) = \frac{e^x \ln x \; 5^{(x^2+2)}(x^2 - 7x + 10)}{2x^2 - 11x + 12} \text{ is}$$

(A) $(-\infty, \infty)$ (B) $[0, \infty)$
(C) $(3/2, \infty)$ (D) $(3/2, 4)$

Q.36 Let $f(x) = \left(\frac{2\sin x + \sin 2x}{2\cos x + \sin 2x} \cdot \frac{1 - \cos x}{1 - \sin x}\right)^{2/3}$; $x \in R$.

Consider the following statements

(I) Domain of f is R (II) Range of f is R
(III) Domain of f is $R - (4n+1)\pi/2$, $n \in I$
(IV) Domain of f is $R - (4n-1)\pi/2$, $n \in I$

Which of the following is correct?

(A) (I) and (II) (B) (II) and (III)
(C) (III) and (IV) (D) (II), (III) and (IV)

Q.37 Range of the function $f(x) = \frac{1}{\cos\{\sin^{-1}(\sin x + \cos x)\}}$ is

(A) $[-1, 1] - \{0\}$ (B) $(-\infty, -1] \cup [1, \infty)$
(C) (0, 1] (D) $[1, \infty)$

Q.38 The range of the function,

$f(x) = \cot^{-1}\log_{0.5}(x^4 - 2x^2 + 3)$ is:

(A) $(0, \pi)$ (B) $\left(0, \frac{3\pi}{4}\right]$

(C) $\left[\frac{3\pi}{4}, \pi\right)$ (D) $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

Q.39 The solution set for $[x]\{x\} = 1$ where $\{x\}$ and $[x]$ are fractional part & integral part of x, is

(A) $R^+ - (0, 1)$ (B) $R^+ - \{1\}$

(C) $\left\{m + \frac{1}{m} / m \in I - \{0\}\right\}$ (D) $\left\{m + \frac{1}{m} / m \in N - \{1\}\right\}$

Q.40 If $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$, then $f(m, n) + f(n, m) = 0$ is possible

(A) only when $m = n$ (B) only when $m \neq n$

(C) only when $m = -n$ (D) for all m and n

Q.41 The domain of $f(x) = \sqrt{e^{\cos^{-1}(\log_4 x^2)}}$ is

(A) $\left[-2, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 2\right]$ (B) $\left[-2, -\frac{1}{3}\right] \cup \left[\frac{1}{3}, 2\right]$

(C) $\left[-2, -\frac{1}{3}\right] \cup \left[\frac{1}{3}, 3\right]$ (D) none of these

Q.42 If $3f(x) - f(1/x) = \log x^4$, then $f(e^{-x})$ is

(A) $1 + x$ (B) $1/x$

(C) x (D) $-x$

Q.43 Period of $\sin 2\sqrt{x+1}$ is –

(A) 2π (B) π

(C) $\pi/2$ (D) None of these

Q.44 Which of the function defined below is one–one

(A) $f : (0, \infty) \to R$, $f(x) = x^2 - 4x + 3$

(B) $f : [0, \infty) \to R$, $f(x) = x^2 + 4x - 5$

(C) $f : R \to R$, $f(x) = e^x + \frac{1}{e^x}$

(D) $f : R \to R$, $f(x) = \ell n(x^2 + x + 1)$

Q.45 If $\log_{1/2}\left(x^2 - 5x + 7\right) > 0$, then exhaustive range of values of x is

(A) $(-\infty, 2) \cup (3, \infty)$ (B) $(2, 3)$

(C) $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$ (D) None of these

Q.46 If f(x) is defined on domain [0, 1] then f(2 sinx) is defined on

(A) $\bigcup_{n \in I}\left\{\left[2n\pi, 2n\pi + \frac{\pi}{6}\right] \bigcup \left[2n\pi + \frac{5\pi}{6}, (2n+1)\pi\right]\right\}$

(B) $\bigcup_{n \in I}\left[2n\pi, 2n\pi + \frac{\pi}{6}\right]$

(C) $\bigcup_{n \in I}\left[2n\pi + \frac{5\pi}{6}, (2n+1)\pi\right]$

(D) None of these

Q.47 Let $f(x) = \frac{x}{1+x}$ and let $g(x) = \frac{rx}{1-x}$. Let S be the set of all real numbers r such that $f(g(x)) = g(f(x))$ for infinitely many real number x. The number of elements in set S is

(A) 1 (B) 2

(C) 3 (D) 5

Q.48 If $f(x) = (a - x^n)^{1/n}$, then $f[f(x)]$ equals to

(A) $x^{1/n}$ (B) x^n

(C) $a - x$ (D) x

Q.49 The range of the function $f(x) = \sqrt{3x^2 - 4x + 5}$ is

(A) $\left[-\infty, \sqrt{\frac{11}{3}}\right]$ (B) $\left(-\infty, \sqrt{\frac{11}{3}}\right)$

(C) $\left[\sqrt{\frac{11}{3}}, \infty\right)$ (D) $\left(\sqrt{\frac{11}{3}}, \infty\right)$

Q.50 The domain of definition of the function y (x) given by the equation $\sec 2y = 2^x(2^x - 2) - 3$ is

(A) $[\log_2(1+\sqrt{5}), \infty)$

(B) $[-\log_2(1+\sqrt{5}), \log_2(1+\sqrt{5})]$

(C) $(-\infty, -\log_2 5]$

(D) $[-2, 2]$

Q.51 The range of $f(x) = \cos\frac{\pi[x]}{2}$ is

(A) $\{0, 1\}$ (B) $\{-1, 1\}$

(C) $\{-1, 0, 1\}$ (D) $[-1, 1]$

Q.52 The polynomial function f (x) satisfies the equation $f(x) - f(x-2) = (2x-1)^2$ for all x. If p and q are the coefficient of x^2 and x respectively in f (x), then p + q is equal to –

(A) 0 (B) 5/6

(C) 4/3 (D) 1

Q.53 If $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + [\log(3-x)]^{-1}$, then its domain is

(A) $[-2, 6]$ (B) $[-6, 2) \cup (2, 3)$

(C) $[-6, 2]$ (D) $[-2, 2) \cup (2, 3]$

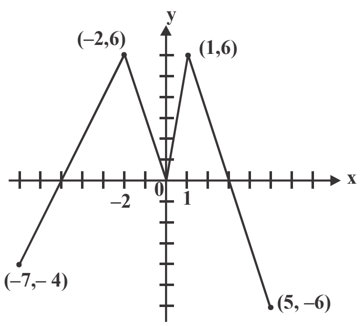
Q.54 The solution set of the equation

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is

(A) $(-1, 0)$ (B) $[-1, 0]$

(C) $\{-1, 0\}$ (D) none of these

Q.55 The graph of the function f is shown below



Number of solutions of the equation $f(f(x)) = 6$, is–
(A) 3 (B) 4
(C) 5 (D) 6

Q.56 If f is an even function and g is an odd function, then the function fog is
(A) an even function (B) an odd function
(C) neither even nor odd (D) a periodic function

Q.57 The domain of the function $f(x) = \sqrt{\frac{1-|x|}{|x|-2}}$ is
(A) $(-\infty, -1)\cup(1, \infty)$ (B) $(-\infty, -2)\cup(2, \infty)$
(C) $(-2, -1]\cup[1, 2)$ (D) none of these

Q.58 The range of the function $y = \left(\frac{\cos^{-1}(3x-1)}{\pi}+1\right)^2$ is –
(A) [1, 4] (B) $[0, \pi]$
(C) $[1, \pi]$ (D) $[0, \pi^2]$

Q.59 If $f(x) = \begin{cases} x; \text{ when x is rational} \\ 1-x; \text{ when x is irrational} \end{cases}$ then fof (x) is given as
(A) 1 (B) x
(C) 1 + x (D) None of these

Q.60 If $f: R^- \to [1, \infty)$ is defined by $f(x) = x^2 + 1$, the value of $f^{-1}(17)$ and $f^{-1}(3)$ are respectively.
(A) $\pm 4, \pm\sqrt{2}$ (B) $4, \sqrt{2}$
(C) $-4, \sqrt{2}$ (D) $-4, -\sqrt{2}$

Q.61 If the function $f : R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then A is
(A) R (B) [0, 1)
(C) (0, 1] (D) [0, 1]

Q.62 If $f(x) = \frac{1-x}{1+x}$, then domain of $f^{-1}(x)$ is
(A) R (B) $R - \{-1\}$
(C) $(-\infty, -1)$ (D) $(-1, \infty)$

Q.63 The function $f: R \to R$, defined by $f(x) = [x]$, $\forall x \in R$, is
(A) one-one
(B) onto
(C) Both one-one and onto
(D) neither one-one nor onto

EXERCISE - 2 [LEVEL-2]

Q.1 Let $A = Z \cup \{\pi\}$. R is a relation on A defined by a R b if $a + b \in Z$, then R is –
(A) reflexive (B) symmetric
(C) equivalence relation (D) None of these

Q.2 Let $R = \{(1, 3), (2, 4), (4, 2), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is –
(A) a function (B) Transitive
(C) Not symmetric (D) Reflexive

Q.3 If $X = \{x_1, x_2, x_3\}$ and $y = (x_1, x_2, x_3, x_4, x_5\}$ then find which is a reflexive relation of the following :
(1) $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)$
(2) $R_1 : \{(x_1, x_1), (x_2, x_2)$
(3) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_3), (x_2, x_4)$
(4) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)$
(A) 1, 2 and 3 are correct (B) 1 and 2 are correct
(C) 2 and 4 are correct (D) 1 and 3 are correct

Q.4 If $x = \{a, b, c\}$ and $y = \{a, b, c, d, e, f\}$ then find which of the following relation is symmetric relation:
(1) R_1 : { } i.e. void relation
(2) R_2 : {(a, b)}
(3) R_3 : {(a, b), (b, a)(a, c)(c, a)(a, a)}
(4) Every null relation is a asymmetric and transitive relation.
(A) 1, 2 and 3 are correct (B) 1 and 2 are correct
(C) 2 and 4 are correct (D) 1 and 3 are correct

Q.5 If x = {a, b, c} and y = (a, b, c, d, e} then which of the following are transitive relation.
(1) R_1 = { } (2) R_2 = {(a, a)}
(3) R_3 = {(a, a}.(c, d)} (4) R_4 = {(a, b), (b, c)(a, c)
(A) 1, 2 and 3 are correct (B) 1 and 2 are correct
(C) 2 and 4 are correct (D) 1 and 3 are correct

Q.6 $f(x) = [x]^2 + [x + 1] - 3$, where [x] = the greatest integer $\le x$.
(A) f (x) is a many-one and into function
(B) f (x) = 0 for infinite number of values of x
(C) f (x) = 0 for only two real values
(D) Both (A) and (B)

Q.7 Set of all values of p for which the function $f(x) = px + \sin x$ is bijective is
(A) $[-2, \infty)$ (B) $(-\infty, -1]\cup[1, \infty)$
(C) $(-\infty, -2]\cup[2/3, \infty)$ (D) [–2, 2/3]

Q.8 $f(x) = |x - 1|$, $f: R^+ \to R$ and $g(x) = e^x$, $g: [-1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are –
(A) $(0, \infty)$ and $[0, \infty)$ (B) $[-1, \infty)$ and $[0, \infty)$
(C) $[-1, \infty)$ and $\left[1-\frac{1}{e}, \infty\right)$ (D) $[-1, \infty)$ and $\left[\frac{1}{e}-1, \infty\right)$

Q.9 Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function, $f : S \to R^+$, $f(\Delta)$ = area of the Δ, where $\Delta \in S$ is –
(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective

Q.10 If $f : (0, \infty) \to (0, \infty)$ satisfy $f(x f(y)) = x^2 y^a$ $(a \in R)$, then find the value of a.
(A) 4 (B) 1
(C) 3 (D) 2

Q.11 If $f : R \to R$, $f(x) = \begin{cases} x|x| - 4, & x \in Q \\ x|x| - \sqrt{3} & x \notin Q \end{cases}$, then f(x) is –
(A) one to one and onto (B) many to one and onto
(C) one to one and into (D) many to one and into

Q.12 If $f : R \to R$ and $f(x) = ax + \sin x + a$, then –
(A) $f(x)$ is one–one onto function if $a \in R$
(B) $f(x)$ is one–one onto function if $a \in R - [-1, 1]$
(C) $f(x)$ is one–one onto function if $a \in R - \{0\}$
(D) $f(x)$ is one–one onto function if $a \in R - \{-1\}$

Q.13 The function $f(x) = \sec[\log(x + \sqrt{1 + x^2})]$ is
(A) even (B) odd
(C) constant (D) none of these

Q.14 The function $f : [2, \infty) \to (0, \infty)$ defined by $f(x) = x^2 - 4x + a$, then the set of values of 'a' for which f(x) becomes onto is
(A) $(4, \infty)$ (B) $[4, \infty)$
(C) $\{4\}$ (D) ϕ

Q.15 A function $f : R \to R$, $f(x) = \dfrac{2x}{1 + x^2}$ is
(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective

Q.16 Let $f : R \to R$ $f(x) = \dfrac{x}{1 + |x|}$. Then $f(x)$ is :
(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective .

Q.17 If $f : R \to R$: where $f(x) = ax + \cos x$. If $f(x)$ is bijective then
(A) $a \in R$ (B) $a \in R^+$
(C) $a \in R^-$ (D) $a \in R - (-1, 1)$

Q.18 Choose the correct statement –
(A) $f : N \to N$ given by $f(x) = x^2$: f is injective function
(B) $f : Z \to Z$ given by $f(x) = x^2$: f is injective function
(C) $f : N \to N$ given by $f(x) = x^2$: f is surjective function
(D) $f : Z \to Z$ given by $f(x) = x^2$: f is surjective function

Q.19 Choose the correct statement for Signum function
$f : R \to R$ given by $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$
(A) f is one–one function.
(B) f is onto function.
(C) f is one–one function as well as onto function.
(D) f is neither one–one nor onto function

Q.20 Choose the correct statement for $f : N \to N$ be defined by
$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$.
(A) f is one–one function.
(B) f is bijective
(C) f is not one–one, not bijective.
(D) f is one–one, not bijective.

Q.21 Consider a real valued function f(x) such that $\dfrac{1 - e^{f(x)}}{1 + e^{f(x)}} = x$.
The value of a and b for which $f(a) + f(b) = f\left(\dfrac{a + b}{1 + ab}\right)$ is satisfied are –
(A) $a \in (-\infty, 1), b \in R$ (B) $a \in (-\infty, 1), b \in (-1, \infty)$
(C) $a \in (-1, 1), b \in [-1, 1)$ (D) $a \in (-1, 1), b \in (-1, 1)$

Q.22 Suppose $f(x) = (x + 1)^2$ for $x \ge -1$. If g (x) is a function whose graph is the reflection of the graph of f (x) in the line y = x, then g (x) =
(A) $\dfrac{1}{(x+1)^2}$ $x > -1$ (B) $-\sqrt{x} - 1$
(C) $\sqrt{x} + 1$ (D) $\sqrt{x} - 1$

Directions : Assertion-Reason type questions.
This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
(A) Statement- 1 is True, Statement-2 is True, Statement2 is a correct explanation for Statement -1
(B) Statement -1 is True, Statement -2 is True; Statement2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.23 **Statement 1 :** Let : $R - \{1, 2, 3\} \to R$ be a function defined by $f(x) = \dfrac{1}{x-1} + \dfrac{2}{x-2} + \dfrac{3}{x-3}$. Then f is many-one function.
Statement 2 : If either $f'(x) > 0$ or $f'(x) < 0$, $\forall x \in$ domain of f, then y = f (x) is one-one function.

Q.24 **Statement 1 :** Relation R = {1, 1), (2, 2), (3, 3)} is a reflexive relation on set A = {1, 2, 3}
Statement 2 : Every identity relation is a reflexive relation.

Q.25 **Statement 1 :** The inverse of a strictly increasing exponential function is a logarithmic function that is strictly decreasing.
Statement 2 : ln x is inverse of e^x.

Passage 1- (Q.26-Q.28)

k (x) is a function such that k (f (x)) = a + b + c + d where

$$a = \begin{cases} 0, \text{ if } f(x) \text{ is even} \\ -1, \text{ if } f(x) \text{ is odd} \\ 2, \text{ if } f(x) \text{ is neither even nor odd} \end{cases}$$

$$b = \begin{cases} 3, \text{ if } f(x) \text{ is periodic} \\ 4, \text{ if } f(x) \text{ is aperiodic} \end{cases}$$

$$c = \begin{cases} 5, \text{ if } f(x) \text{ is one-one} \\ 6, \text{ if } f(x) \text{ is many-one} \end{cases}$$

$$d = \begin{cases} 7, \text{ if } f(x) \text{ is onto} \\ 8, \text{ if } f(x) \text{ is into} \end{cases}$$

$A = \{x^2, e^x, \sin x, |x|\}$ all the functions in set A are defined from R to R

$B = \{18, 19, 16, 17\}$

$$h : R \to R,\ h(x) = \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1}\right) \text{ and}$$

$$\phi : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R,\ \phi(x) = \tan x$$

Q.26 k (ϕ (x) is equal to –

(A) 15 (B) 16
(C) 17 (D) 18

Q.27 k (h (x)) is equal to –

(A) 15 (B) 16
(C) 17 (D) 18

Q.28 If k (x) is a function such that $k : A \to B$, $y = k(x)$, where $x \in A$, $y \in B$ then k (x) is –

(A) one-one onto (B) one-one into
(C) many-one into (D) many-one onto

Passage 2- (Q.29-Q.30)

The graph of a relation is

(i) Symmetric with respect to the x-axis provided that whenever (a, b) is a point on the graph, so is (a, – b)

(ii) Symmetric with respect to the y-axis provided that whenever (a, b) is a point on the graph, so is (– a, b)

(iii) Symmetric with respect to the origin provided that whenever (a, b) is a point on the graph, so is (– a, – b)

(iv) Symmetric with respect to the line y = x, provided that whenever (a, b) is a point on the graph, so is (b, a)

Q.29 The graph of the relation $x^4 + y^3 = 1$ is symmetric with respect to

(A) the x-axis (B) the y-axis
(C) the origin (D) the line y = x

Q.30 Suppose R is a relation whose graph is symmetric to both the x-axis and y-axis, and that the point (1, 2) is on the graph of R. Which one of the following points is not necessarily on the graph of R?

(A) (–1, 2) (B) (1, – 2)
(C) (–1, –2) (D) (2, 1)

Passage 3- (Q.31-Q.33)

If $f : [0, 2] \to [0, 2]$ is a bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non-zero real numbers then

Q.31 f (2) is equal to –

(A) 2 (B) α where $a \in (0, 2)$
(C) 0 (D) cannot be determined

Q.32 Which of the following is one of the roots f (x) = 0 –

(A) 1 / a (B) 1 / b
(C) $\frac{1}{c}$ (D) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

Q.33 Which of the following is not a value of a –

(A) –1/4 (B) 1/2
(C) – 1/2 (D) 1

Q.34 $f(x) = \frac{x}{\ln x}$ and $g(x) = \frac{\ln x}{x}$. Then identify the correct statement –

(A) $\frac{1}{g(x)}$ and f (x) are identical functions

(B) $\frac{1}{f(x)}$ and g (x) are identical functions

(C) $f(x) . g(x) = 1 \quad \forall\ x > 0$

(D) $\frac{1}{f(x).g(x)} = 1 \quad \forall\ x > 0$

Q.35 Let $f(x) = \frac{9^x}{9^x + 3}$. Evaluate the sum

$$f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \ldots.. + f\left(\frac{1995}{1996}\right).$$

(A) 997.5 (B) 1997.5
(C) 597.5 (D) 397.5

Q.36 The domain of the function $f(x) = \sqrt{x - \sqrt{1 - x^2}}$ is

(A) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$ (B) [–1, 1]

(C) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right)$ (D) $\left[\frac{1}{\sqrt{2}}, 1\right]$

Q.37 If A > 0, c, d, u, v are non-zero constants, and the graphs of f (x) = | Ax + c | + d and g (x) = – | Ax + u | + v intersect exactly at 2 points (1, 4) and (3, 1) then the value of (u + c) / A equals

(A) 4 (B) – 4
(C) 2 (D) – 2

Q.38 The set of all real numbers x for which $\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$ is defined as $\{x \mid x > c\}$. The value of c is

(A) 0 (B) $(2001)^{2002}$
(C) $(2003)^{2004}$ (D) $(2001)^{2002^{2003}}$

Q.39 The range of the function $y = 3\sin\sqrt{\frac{\pi^2}{16} - x^2}$ is

(A) $\left[0, \frac{3}{\sqrt{2}}\right]$ (B) $\left[-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right]$

(C) $\left[-\frac{3}{\sqrt{2}}, 0\right]$ (D) none of these

Q.40 The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2 - 36}$ is

(A) $(-\infty, 0) \sim \{-6\}$ (B) $(0, \infty) \sim \{1, 6\}$

(C) $(1, \infty) \sim \{6\}$ (D) $[1, \infty) \sim \{6\}$.

Q.41 Domain of the function $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$ is –

(A) [0, 2] (B) [–1, 1]

(C) [–1, 2] (D) [1, 2]

Q.42 Let $f : R \to R$ be defined by $f(x) = \frac{x-a}{x-b}$, where $a \neq b$.

Then f is

(A) Injective but not surjective

(B) Surjective but not injective

(C) Bijective

(D) none of these

Q.43 If $(3+\sqrt{8})^{[x]} + (3-\sqrt{8})^{[x]} = 34$ where [.] denotes G.I. function, then x satisfies-

(A) $[-3, -2] \cup [2, 3)$ (B) $(-3, -2] \cup [2, 3]$

(C) $[-2, -1) \cup [2, 3)$ (D) $[-2, -1] \cup [2, 3]$

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 The function $f: R \to R$ defined by $f(x) = \sin x$ is- **[AIEEE-2002]**
(A) into (B) onto
(C) one-one (D) many-one

Q.2 A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, \text{ when n is odd} \\ -\frac{n}{2}, \text{ when n is even} \end{cases}$ is **[AIEEE 2003]**
(A) neither one-one nor onto
(B) one-one but not onto
(C) onto but not one-one
(D) one-one and onto both

Q.3 If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is- **[AIEEE 2004]**
(A) [0, 3] (B) [–1, 1]
(C) [0, 1] (D) [–1, 3]

Q.4 Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is - **[AIEEE-2004]**
(A) transitive (B) not symmetric
(C) reflexive (D) a function

Q.5 Let $f: (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval - **[AIEEE-2005]**
(A) $(0, \pi/2)$ (B) $[0, \pi/2)$
(C) $[-\pi/2, \pi/2]$ (D) $(-\pi/2, \pi/2)$

Q.6 Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$, be relation on the set $A=\{3, 6, 9, 12\}$. The relation is
(A) reflexive and transitive only **[AIEEE-2005]**
(B) reflexive only
(C) an equilvalence relation
(D) reflexive and symmetric only

Q.7 Let W denote the words in the English dictionary. Define the relation R by : $R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$. Then R is –
(A) reflexive, symmetric and not transitive **[AIEEE 2006]**
(B) reflexive, symmetric and transitive
(C) reflexive, not symmetric and transitive
(D) not reflexive, symmetric and transitive

Q.8 Let R be the real line. Consider the following subsets of the plane $R \times R$: $S = \{(x, y): y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$.
Which one of the following is true ? **[AIEEE 2008]**
(A) Both S and T are equivalence relations on R.
(B) S is an equivalence relation on R but T is not.
(C) T is an equivalence relation on R but S is not.
(D) Neither S nor T is an equivalence relation on R.

Q.9 Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = |y \in N : y = 4x + 3 \text{ for some } x \in N|$. f is invertible and its inverse is **[AIEEE 2008]**
(A) $g(y) = 4 + \frac{y+3}{4}$ (B) $g(y) = \frac{y+3}{4}$
(C) $g(y) = \frac{y-3}{4}$ (D) $g(y) = \frac{3y+4}{3}$

Q.10 For real x, let $f(x) = x^3 + 5x + 1$, then - **[AIEEE 2009]**
(A) f is one – one but not onto R
(B) f is onto R but not one – one
(C) f is one – one and onto R
(D) f is neither one – one nor onto R

Q.11 Let $f(x) = (x + 1)^2 - 1, x > -1$
Statement – 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement – 2 : f is a bijection. **[AIEEE 2009]**
(A) Statement -1 is true, Statement -2 is true; Statement2 is a correct explanation for Statement -1
(B) Statement -1 is true, Statement -2 is true; Statement 2 is **not** a correct explanation for Statement -1.
(C) Statement -1 is true, Statement -2 is false.
(D) Statement -1 is false, Statement -2 is true.

Q.12 Consider the following relations:
R = {(x, y) | x, y are real numbers and x = wy for some rational number w}; **[AIEEE 2010]**
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| \begin{array}{l} \text{m, n, p and q are integers} \\ \text{such that } n, q \neq 0 \text{ and } qm = pn \end{array}\right\}$. Then
(A) neither R nor S is an equivalence relation.
(B) S is an equivalence relation but R is not an equivalence relation.
(C) R and S both are equivalence relations.
(D) R is an equivalence relation but S is not an equivalence relation.

Q.13 Let R be the set of real numbers. **[AIEEE 2011]**
Statement-1 : $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.
Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R.
(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.14 The function : $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is
(A) Surjective but not injective **[JEE MAIN 2017]**
(B) Neither injective not surjective
(C) Invertible
(D) Injective but not surjective

Q.15 Let $A = \{x \in R : x \text{ is not a positive integer}\}$.
Define a function $f: A \to R$ as $f(x) = \frac{2x}{x-1}$ then f is
(A) injective but not surjective. **[JEE MAIN 2019]**
(B) not injective.
(C) surjective but not injective.
(D) neither injective nor surjective

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	A	D	A	A	A	B	D	D	B	C	B	D	D	D	D	D	A	B	B	B	B	B	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	B	B	C	B	B	D	A	D	A	C	D	C	D	D	A	D	D	B	B	A	B	D	C	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63												
A	C	B	B	C	D	A	C	A	B	D	D	B	D												

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	D	A	D	B	B	B	A	D	B	B	D	D	A	D	A	D	C	D	D	C	A	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43							
A	A	D	C	B	D	C	A	D	A	A	D	B	B	A	C	C	A	C							

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	AD	D	D	B	D	A	A	C	C	C	B	B	C	A	A

CHAPTER - 1 : RELATIONS & FUNCTIONS
SOLUTIONS TO TRY IT YOURSELF
TRY IT YOURSELF-1

(1) Let $A = \{1, 2, 3\}$, $R = \{(1, 2), (2, 1)\}$
Now $1 \in A$ but $(1, 1) \notin R$. So R is not reflexive relation.
$(1, 2) \in R$ and $(2, 1) \in R$. So R is symmetric relation.
$(1, 2) \in R$ and $(2, 1) \notin R$ but $(1, 1) \notin R$. So R is not transitive relation.
Thus R is symmetric but neither reflexive nor transitive.

(2) $R = \{(x,y) : x$ and y have same number of pages$\}$
Now obviously, $\therefore (x, x) \in R$. So R is reflexive relation.
$(x, y) \in R \Rightarrow (y, x) \in R$ because number of pages in both the boxes is same. So R is symmetric relation.
$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ because number of pages in book x and z is same So R is transitive relation.
So R is reflexive, symmetric and transitive. Thus R is an equivalence relation.

(3) $A = \{1, 2, 3, 4, 5, 6\}$
$R = \{(a, b) : b = a + 1\} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$
Now $1, 2, 3, 4, 5, 6 \in A$ but $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$. So R is not reflexive relation.
$(1, 2) \in R$ but $(2, 1) \notin R$. So R is not symmetric relation.
$(3,4) \in R$ and $(4, 5) \in R$ but $(3, 5) \notin R$. So R is not transitive relation.
Thus R is neither reflexive nor symmetric nor transitive.

(4) **(C).** We know that the ordered pair which satisfy the equation $a = b - 2$, $b > 6$ is correct answer.
(A) is not the answer because $4 < 6$
(B) does not satisfy the equation $a = b - 2$
(C) satisfy the equation because $6 = 8 - 2$ and $8 > 6$
(D) does not satisfy the equation $a = b - 2$

(5) **(B).** $A = \{1, 2, 3, 4\}$
$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. R is reflexive because $(1, 1), (2, 2), (3, 3), (4, 4) \in R$.
R is not symmetric because $(1, 2) \in R$ but $(2, 1) \notin R$. R is transitive because
$(3, 3), (3, 2), (3, 2) \in R, (1, 2), (2, 2), (1, 2) \in R$, etc.

(6) **(D).** Since n divides n, $\forall\, n \in N$, R is reflexive. R is not symmetric since for $3, 6 \in N$, $3 R 6 \neq 6 R 3$. R is transitive since for n, m, r whenever n/m and m/r $\Rightarrow$ n/r, i.e., n divides m and m divides r, then n will divide r.

(7) $R = \{(1, 1), (2, 2), (3, 3)\}$

(8) (i) Not reflexive, symmetric, not transitive

$$|a - a| = 0 \not> \frac{1}{2} \text{ hence it is not reflexive.}$$

$$\sqrt{x^2} = |x| \text{ hence symmetric.}$$

$$\text{Let } a = 1, b = -1 \text{ and } c = 3/2, \ |a - b| = 2 > \frac{1}{2}$$

$$\text{So, } a, b \in R\ ;\ |b - c| = \frac{5}{2} > \frac{1}{2}, \text{ so } b, c \in R$$

$$\text{But } |a - c| = \left|1 - \frac{3}{2}\right| = \frac{1}{2} \not> \frac{1}{2}$$

so, $(a, c) \notin R$. Hence, R is not a transitive relation.

(ii) Reflexive, symmetric as well as transitive, hence it is an equivalence relation.

Since 0 is divisible by n $\left(\frac{0}{n} = 0\right)$ so given relation is reflexive.
If $a - b$ is divisible by n, then $(b - a)$ will also be divisible by n. Hence, symmetric.
If $a - b = nI_1$ and $b - c = nI_2$, where I_1, I_2 are integer.
Then, $a - c = (a - b) + (b - c) = n\,(I_1 + I_2)$
So, $a - c$ is also divisible by n, hence transitive.

(9) (C)

TRY IT YOURSELF-2

(1) $\because 4 \neq -4$, but $f(4) = f(-4) = 16$
$\therefore$ f is many one function.

(2) One-one/many-one : Let $x_1 x_2 \in R - \{3\}$ are the elements such that

$$f(x_1) = f(x_2) : \text{then } f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

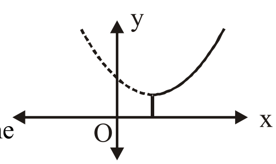
$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$
$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$
$\Rightarrow -2x_2 - 3x_1 = -2x_1 - 3x_2$
$\Rightarrow x_2 = x_1 \quad \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
$\Rightarrow$ f is one-one function
Onto/into : Let $y \in R - \{1\}$ (co-domain)
Then one element $x \in R - \{3\}$ in domain is such that

$$f(x) = y \Rightarrow \frac{x - 2}{x - 3} = y \Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x = \left(\frac{3y - 2}{y - 1}\right) = x \in R - \{3\}$$

$\therefore$ The pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.
$\Rightarrow$ f is onto

(3) **(B).** $f(x) = x^2 - 4x + 5$
Minima at $x = 2$
At $x = 2$, $y = 4 - 8 + 5 = 1$
For function to be one-one it should be monotonic.
Hence for $x \in [2, \infty)$, $f(x)$ is increasing.
At $x = 2$, $y = 1$. Hence $y \in [1, \infty)$



(4) (i) Total number of functions
Hence, number of functions $= 4 \times 4 \times 4 \times 4 \times 4 = 4^5$
(ii) Number of one-one functions
Since A contains five elements hence one-one functions is not possible.
(iii) Number of onto functions
Divide 5 elements into 4 groups of size = 1, 1, 1, 2
Number of ways mapping 4 groups with four images

$$= \left(\frac{5!}{1!1!1!2!} \times \frac{1}{3!}\right) \times 4! = 240$$

(iv) Number of many one functions
All the possible functions are many-one $= 4^5 = 1024$.

(v) Number of into functions = Total number of functions – number of onto functions = 1024 – 240 = 784

(5) $f(x)+2f(1-x)=x^2$ .... (1)

Replacing x by 1 – x

$f(1-x)+2f(x)=(1-x)^2$ .... (2)

Solving eq. (1) and (2), we get $3f(x)=2x^2-(1-x)^2$

$$f(x)=\frac{x^2+2x-1}{3}$$

(6) $f(x)=x$ for integers less than zero.

$\therefore\ f(-8)=-8\ ;\ f(x+y)=f(x)+g(y)+8$

$f(-8+8)=f(-8)+g(8)+8$

$f(0)=-8+g(8)+8=17$

(7) (i) $(fog)(x)=f(g(x))=f(\sqrt{2-x})=\sqrt{\sqrt{2-x}}$

Domain $2-x\ge 0\ ;\ x\le 2\ ;\ x\in(-\infty,2]$

(ii) $(gog)(x)=g(g(x))=g(\sqrt{2-x})=\sqrt{2-\sqrt{2-x}}$

$\therefore\ 0\le\sqrt{2-x}\le 2\Rightarrow 0\le 2-x\le 4\Rightarrow -2\le x\le 4$

(8) (i) $f(x)=3x-5$

$y=3x-5\ ;\ x=\frac{y+5}{3}\Rightarrow f^{-1}(x)=y=\frac{x+5}{3}$

(ii) $f:R\to(0,1),\ f(x)=\frac{2^x}{1+2^x}$

$y=\frac{2^x}{1+2^x}\Rightarrow y+2^x y=2^x$

$\Rightarrow\ 2^x=\frac{y}{1-y}\Rightarrow x=\log_2\left(\frac{y}{1-y}\right)$

$\Rightarrow\ f^{-1}(x)=y=\log_2\left(\frac{x}{1-x}\right)$

(9) $f^{-1}(28)=x\Rightarrow f(x)=28\Rightarrow x^3+1=28\Rightarrow x=3$

<u>TRY IT YOURSELF-3</u>

(1) We have, $a*b=a^2+b^2,\ a,b\in Z$

$\therefore\ 6*(-5)=(6)^2+(-5)^2=36+25=61\in Z$

(2) $a*b=a^2b^3,\ a,b\in N$

$\therefore\ 9*2=(9)^2(2)^3=(81)(8)=648\in N$

(3) $a*b=a^2b,\ a,b\in R$

$\therefore\ (1.5)*(3.4)=(1.5)^2(3.4)=(2.25)(3.4)=7.650\in R$

(4) We have, $a*b=4a+3b$

For a = 9 and b = 2,

$a*b=4a+3b=4(9)+3(2)=36+6=42$ .... (1)

and $b*a=4b+3a=4(2)+3(9)=8+27=35$ .... (2)

From eq. (1) and (2), we get $a*b\ne b*a$

Hence, this binary operation is not commutative.

(5) Consider that

$2*5=2^5=32$ and $5*2=5^2=25\ \therefore\ 2*5\ne 5*2$

So, * is not commutative on N.

Also, $2*(2*3)=2*2^3=2*8=2^8=256$

and $(2*2)*3=2^2*3=4*3=4^3=64$

$\therefore\ 2*(2*3)\ne(2*2)*3$

So, * is not associative on N.

Hence, * is neither commutative nor associative on N.

CHAPTER - 1 : RELATIONS & FUNCTIONS

EXERCISE-1

(1) **(B).** If $A = \{1, 2, 3, 4, 5, 6\}$
Then minimum number of ordered pairs in
$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ are 6.

(2) **(C).** We have, $x + 3y = 12 \Rightarrow x = 12 - 3y$
Putting $y = 1, 2, 3$, we get $x = 9, 6, 3$ respectively
For $y = 4$, we get $x = 0 \notin N$. Also for $y > 4$, $x \notin N$
$\therefore R = \{(9, 1), (6, 2), (3, 3)\}$

(3) **(A).** Domain of $R = \{9, 6, 3\}$

(4) **(D).** Range of $R = \{1, 2, 3\}$

(5) **(A).** $n(A \times A) = n(A) . n(A) = 3^2 = 9$.
So the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times a$ is a relation over the set A.

(6) **(A).** Since $n(A) = m$; $n(B) = n$ then $n(A \times B) = mn$
So number of subsets of $A \times B = 2^{mn}$
$\Rightarrow n(P(A \times B)) = 2^{mn}$
Number of relation defined from A to $B = 2^{mn}$
Any relation which can be defined from set A to set B will be subset of $A \times B$
$\because$ $A \times B$ is largest possible relation $A \to B$
no. of relation from $A \to B$ = no. of subsets of set $(A \times B)$

(7) **(A).** $A = \{1, 2, 3\}$. We have only one relation containing (1,2) and (1, 3) which is reflexive and symmetric but not transitive.

(8) **(B).** $A = 11, 2, 3\}$. We have two equivalence relations containing (1, 2). So answer is (B).

(9) **(D).** Since n is a factor of n, so the relation is reflexive. If however, n is a factor of m, m is not necessarily a factor of n. So the relation is not symmetric. On the other hand $n \mid m$ and $m \mid \ell$ imply $n \mid \ell$, so the relation is transitive.

(10) **(D).** $\forall x \in R$, $x R x$ because $x - x + \sqrt{7} = \sqrt{7}$ an irrational.
Therefore, R is reflexive
$\sqrt{7}$ R1 as gives $2\sqrt{7} - 1$ is irrational
but 1 R $\sqrt{7}$ as $1 - \sqrt{7} + \sqrt{7} = 1$ is not an irrational.
Also it can be verified by taking $(\sqrt{7}, 1)$ and $(1, 2\sqrt{7}) \in R$
but $\sqrt{7} R 2\sqrt{7}$ that R is not transitive.

(11) **(B).** $A = \{1, 2, 3, 4\}$
$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$.
R is reflexive because $(1, 1), (2, 2), (3, 3), (4, 4) \in R$.
R is not symmetric because $(1, 2) \in R$ but $(2, 1) \notin R$.
R is transitive because $(3, 3), (3, 2), (3, 2) \in R$, $(1, 2), (2, 2), (1, 2) \in R$, etc.

(12) **(C).** We know that the ordered pair which satisfy the equation $a = b - 2$, $b > 6$ is correct answer.
(A) is not the answer because $4 < 6$
(B) does not satisfy the equation $a = b - 2$
(C) satisfy the equation because $6 = 8 - 2$ and $8 > 6$
(D) does not satisfy the equation $a = b - 2$

(13) **(B).** $R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$
$R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$
$ROR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$

(14) **(D).** We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D).

(15) **(D).** $f(y) = \dfrac{y}{y-1} = \dfrac{(x-1)/x}{\dfrac{x-1}{x} - 1} = \dfrac{x-1}{x-1-x} = 1 - x$

(16) **(D).** $(f + g)(x) = f(x) + g(x) = x^2 + 2 + \sqrt{x+1}$

(17) **(D).** $fog(x) = f[g(x)] = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$

(18) **(D).** Let $x_1 = 2$ and $x_2 = -2 \in R$
$f(x) = x^4$ then $f(x_1) = f(2) = (2)^4 = 16$ and
$f(x_2) = f(-2) = (-2)^4 = 16$
$\therefore f(x_1) = f(x_2)$ but $x_1 \neq x_2$. So f is not one–one function.
Let $f(x) = -2 \in R$
then $x^4 = -2$, which is not possible because there is no value of x corresponding to which $x^4 = -2$.
so f is not onto function.
Thus f is neither one–one nor onto.

(19) **(A).** Let x_1 and $x_2 \in R$
$f(x) = 3x$ then $f(x_1) = 3x_1$ and $f(x_2) = 3x_2$
Now, $f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$
$\therefore f(x_1) = f(x_2)$ implies $x_1 = x_2$. So f is one–one function.
Let $f(x) = y \in R$
then $3x = y \Rightarrow x = \dfrac{y}{3} \in R$. Also, $f\left(\dfrac{y}{3}\right) = 3 \times \dfrac{y}{3} = y$.
So f is onto function.
Thus f is one–one and onto function.

(20) **(B).** $f : R \to R$; $f(s) = 2x + 3$; Let $f^{-1}(x) = y$
$x = 2y + 3 \Rightarrow 2y = x - 3$
$y = \dfrac{x-3}{2}$ $\quad \therefore f^{-1}(x) = \dfrac{x-3}{2}$

(21) **(B).** Under the binary operation,
$A * B = A \cup B, \forall A, B \in P(X)$
Inverse of A doesn't exists because $A * B \neq \phi$,
for any $B \in P(X)$
Where ϕ is the identity element in P (x)

(22) **(B).** $4^{-1} = \dfrac{(4/9)}{4} = \dfrac{1}{9}$ GE $\Rightarrow 2 * x = 4^{-1} * 3$
$\Rightarrow \dfrac{3.2x}{2} = \dfrac{1}{9} * 3 \Rightarrow 3x = \dfrac{1}{2} \Rightarrow x = \dfrac{1}{6}$

(23) **(B).** set $A = \{a, b\}$.
Number of elements in given set = 2
$\therefore n(A \times A) = 2 \times 2 = 4$
Total number of binary operations on
$\{a, b\} = [n(A)]^{n(A \times A)} = (2)^4 = 16$

(24) **(B).** Let $a, b \in N$. Now, $a * b = a^3 + b^3 = b^3 + a^3 = b * a$
$a * b = b * a$. So operation * is commutative.
Let $a, b, c \in N$.
Now, $(a * b) * c = (a^3 + b^3) * c = (a^3 + b^3)^3 + c^3$
$a * (b * c) = a * (b^3 + c^3) = a^3 + (b^3 + c^3)^3$
$\therefore (a * b) * c \neq a * (b * c)$
So operation * is not associative.

(25) **(C).** Let $a, b \in Z^+$ Now, $a * b = 2^{ab} = 2^{ba} = b * a$

$\therefore$ $a * b = b * a$. So operation * is commutative.

Let $a, b, c \in Z^+$.

Now, $(a * b) * c = (2^{ab}) * c = 2^{(2^{ab} \times c)}$

$a * (b * c) = a * 2^{bc} = 2^{(a \times 2^{bc})}$

$\therefore$ $(a * b) * c \neq a * (b * c)$

So operation * is not associative.

(26) **(B).** Let $a, b \in Q$. Now, $a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$

$\therefore$ $a * b = b * a$. So operation * is commutative.

Let $a, b, c \in Q$

Now, $(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{\frac{ab}{2} \times c}{2} = \frac{abc}{4}$

$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a \times \frac{bc}{2}}{2} = \frac{abc}{4}$

$\therefore$ $(a * b) * c = a * (b * c)$

So operation * is associative.

(27) **(B).** $f(x) = \cos(4\pi x - 4\pi[x]) + \sin(4\pi x - 4\pi[x])$

$= \cos 4\pi x + \sin 4\pi x$

(28) **(B).** $(fog)(x) = f[g(x)] = f(|3x + 4|)$.

since the domain of f is $[-3, 5]$ $\therefore -3 \le |3x + 4| \le 5$

$\Rightarrow |3x + 4| \le 5 \Rightarrow -5 \le 3x + 4 \le 5$

$\Rightarrow -9 \le 3x \le 1 \Rightarrow -3 \le x \le 1/3$.

$\therefore$ Domain of fog is $[-3, 1/3]$

(29) **(C).** Since in (C) each element is associated with unique element while in (1) element b is associated with two elements, in (2) element a is associated with three elements and in (D) element b is associated with two elements so. (C) is function.

(30) **(B).** $f : \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \Rightarrow [0, 4]$, $f(x) = \sqrt{3}\sin x - \cos x + 2$

$f(x) = 2\sin\left(x - \frac{\pi}{6}\right) + 2 \Rightarrow (f^{-1}(x)) = \sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$

(31) **(B).** Let $f_1(x) = [\sin 3x]$

$\therefore$ Period of $f_1(x) = 2\pi/3$ (least positive).

Let $f_2(x) = |\cos 6x|$ $\therefore$ period of $f_2(x) = \frac{2\pi}{12}$

Hence period of $f(x) = \text{L. C. M. of} \left(\frac{2\pi}{3}, \frac{2\pi}{12}\right) = \frac{2\pi}{3}$

(32) **(D).** (A) $f(x) = x^4 + 2x^3 - x^2 + 1 \rightarrow$ A polynomial of degree even will always be into

say $f(x) = a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + ... + a_{2n}$

$\underset{x \to \pm\infty}{\text{Limit}} f(x) = \underset{x \to \pm\infty}{\text{Limit}}$

$[x^{2n}\left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ... + \frac{a_{2n}}{x^{2n}}\right) = \left[\begin{matrix} \infty & \text{if } a_0 > 0 \\ -\infty & \text{if } a_0 < 0 \end{matrix}\right.$

Hence it will never approach $\infty / -\infty$

(B) $f(x) = x^3 + x + 1$

$\Rightarrow f'(x) = 3x^2 + 1$ – injective as well as surjective

(C) $f(x) = \sqrt{1 + x^2}$ – neither injective nor surjective

(minimum value = 1)

$f(x) = x^3 + 2x^2 - x + 1$

$\Rightarrow f'(x) = 3x^2 + 4x - 1 \Rightarrow D > 0$

Hence f(x) is surjective but not injective.

(33) **(A).** Note that f is bijective hence f^{-1} exist when y = 4

$2x^3 + 7x - 9 = 0$

$2x^2(x - 1) + 2x(x - 1) + 9(x - 1) = 0$

$(x - 1)(2x^2 + 2x + 9) = 0$

$x = 1$ only $\Rightarrow$ A

as $2x^2 + 2x + 9 = 0$ has no other roots

(34) **(D).** $F(7752) = (7752 - 10 \times 775).10^3 + F(775)$

$= 200 + F(775) = 2000 + (775 - 10 \times 77).10^2 + F(77)$

$= 2000 + 500 + F(77)$

$= 2000 + 500 + (77 - 10 \times 7).10^1 + F(7)$

$= 2000 + 500 + 70 + F(7)$

$= 2000 + 500 + 70 + (7 - 10 \times 0).10^0 + F(0)$

$= 2000 + 500 + 70 + 7 + 0 = 2577$

(35) **(A).** $f(x) = \frac{e^x \ \ln x \ 5^{(x^2+2)}.(x-2)(x-5)}{(2x-3)\ (x-4)}$

Note that at x = 3/2 & x = 4 function is not defined and in open interval (3/2,4) function is continuous.

$\underset{x \to \frac{3}{2}^+}{\text{Lim}} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \rightarrow -\infty$

$\underset{x \to 4^-}{\text{Lim}} = \frac{(+ve)(+ve)(-ve)}{(+ve)(-ve)} \rightarrow \infty$

In the open interval (3/2,4) the function is continuous & takes up all real values from $(-\infty, \infty)$

Hence range of the function is $(-\infty, \infty)$ or R

(36) **(C).** $f(x) = \left[\frac{2\sin x(1 + \cos x)}{2\cos x(1 + \sin x)} \ \frac{1 - \cos x}{1 - \sin x}\right]^{3/2}$

Hence $x \neq \frac{\pi}{2}$ and $x \neq \frac{3\pi}{2}$

$\therefore$ Domain is $R - \left\{(4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right\}$

$\therefore$ $f(x) = \left[\frac{\sin x(1 - \cos^2 x)}{\cos x(1 - \sin^2 x)}\right]^{2/3} = \tan^2 x$

$\Rightarrow$ Range is $[0, \infty)$

(37) **(D).** $\dfrac{1}{\cos\{\sin^{-1}(\sin x+\cos x)\}}$

$\Rightarrow -\sqrt{2} \le \sin x+\cos x \le \sqrt{2}$

$\Rightarrow \dfrac{\pi}{2} \le \sin^{-1} x(\sin x+\cos x) \le \dfrac{\pi}{2}$

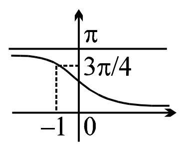
$\Rightarrow 0 \le \cos(\sin^{-1} x(\sin x+\cos x) \le 1$

$\Rightarrow$ Range $= [1, \infty)$

(38) **(C).** $y=(x^2-1)^2+2 \Rightarrow \; y_{min}=2$

$\Rightarrow \log_{0.5}(x^4-2x^2+3) \le -1$

$\Rightarrow$ range $\left[\dfrac{3\pi}{4}, \pi\right)$



(39) **(D).** $\{x\}\,[x]=1 \Rightarrow \{x\} = 1/[x]$ or $x-[x]=1/[x]$, hence $x=[x]+1/[x]$

obviously $x \ge 2$ (because $1<x<2$, $[x]\le 1$ and $2[x] < 1$)

$\therefore x=[x]+1/[x]$

(40) **(D).** $f(m, n)+f(n, m)$

$2x+\dfrac{y}{8}=m, \;\; 2x-\dfrac{y}{8}=n$;

$x=\dfrac{m+n}{4}$; $\dfrac{m+n}{2}+\dfrac{y}{8}=m \Rightarrow y=4(m-n)$

$f(m, n)+f(n, m)=0$ for all m and n.

(41) **(A).** $e^{\cos^{-1}(\log_4 x^2)} \ge 0 \;\; \Rightarrow -1 \le \log_4 x^2 \le 1$

$\Rightarrow 4^{-1} \le x^2 \le 4^1$

$\Rightarrow \dfrac{1}{4} \le x^2 \le 4 \Rightarrow \dfrac{1}{2} \le 1 \times 1 \le 2$

$\Rightarrow x \in \left[-2, -\dfrac{1}{2}\right] \cup \left[\dfrac{1}{2}, 2\right]$

(42) **(D).** $3f(x)-f\left(\dfrac{1}{x}\right)=\log x^4$; $x \equiv \dfrac{1}{x}$

$3+\left(\dfrac{1}{x}\right)-f(x)=\log\left(\dfrac{1}{x}\right)^4$

After solving we get $f(x)=\log x$

$f(e^{-x})=\log_e e^{-x}=-x$

(43) **(D).** As $\sqrt{x+1}$ is not a linear function of x i.e., of the type ax + b, the given function is not periodic.

(44) **(B).** By definition only $f(x)=x^2+4x-5$ with domain $[0, \infty)$ is one to one.

(45) **(B).** $f(x)=\log_{1/2}(x^2+5x+7)>0 \qquad x^2-5x+7>0$

$x^2+5x+7<1 \qquad x \in R$

$x^2-5x+6<0$

(2, 3)

(46) **(A).** f(x) is defined on $[0, 1] \Rightarrow 0 \le x \le 1$

Now $f(2\sin x)$ shall be defined, if $0 \le 2\sin x \le 1$

$\Rightarrow 0 \le \sin x \le 1/2$

$\Rightarrow x \in \bigcup_{n\in I}\left\{\left[2n\pi, 2n\pi+\dfrac{\pi}{6}\right] \cup \left[2n\pi+\dfrac{5\pi}{6}, (2n+1)\pi\right]\right\}$

(47) **(B).** $f(g(x))=\dfrac{rx}{1+(r-1)x}$

$g(f(x))=rx$. If $f(g(x))=g(f(x)) \Rightarrow \dfrac{rx}{1+(r-1)x}=rx$

$\Rightarrow rx=rx(1+(r-1)x) \Rightarrow r(r-1)x^2=0$

If this is to be true for infinitely many (all) x, then $r=0$ or $r-1=0$]

(48) **(D).** We have, $f(x)=(a-x^n)^{1/n}=y$

$\therefore f(y)=(a-y^n)^{1/n}=[(a-\{(a-x^n)^{1/n}\}^n]^{1/n}$

$=[a-(a-x^n)]^{1/n}=(x^n)^{1/n}=x$

(49) **(C).** f(x) is defined if $3x^2-4x+5 \ge 0$

$\Rightarrow 3\left[x^2-\dfrac{4}{3}x+\dfrac{5}{3}\right] \ge 0 \Rightarrow 3\left[\left(x-\dfrac{2}{3}\right)^2+\dfrac{11}{9}\right] \ge 0$

Which is true for all real x

$\therefore$ Domain of (f) $=(-\infty, \infty)$

Let $y=\sqrt{3x^2-4x+5} \Rightarrow y^2=3x^2-4x+5$

i.e. $3x^2-4x+(5-y^2)=0$

For x to be real, $16-12(5-y^2) \ge 0 \Rightarrow y \ge \sqrt{\dfrac{11}{3}}$

$\therefore$ Range of $y=\left[\sqrt{\dfrac{11}{3}}, \infty\right)$

(50) **(A).** Y is real provided $\sec 2y \ge 1$ i.e. $(2^x-1)^2-4 \ge 1$.

As $2^x \ge 1$, this implies $2^x \ge 1+\sqrt{5}$ or $x \ge \log_2(1+\sqrt{5})$.

(51) **(C).** $\because$ [x] is an integer, $\cos(-x)=\cos x$ and $\cos\left(\dfrac{\pi}{2}\right)=0$,

$\cos 2\left(\dfrac{\pi}{2}\right)=-1, \;\; \cos 0\left(\dfrac{\pi}{2}\right)=1, \cos 3\left(\dfrac{\pi}{2}\right)=0$

Hence range $=\{-1, 0, 1\}$

(52) **(B).** Let $\;f(x)=ax^3+px^2+qx+r$

now use $\;f(x)-f(x-2)=x^2-4x+1$

compare the coefficients to get

$a=2/3;\; p=1;\; q=-1/6;$ hence $p+q=5/6$

(53) **(B).** $f(x)=\cos^{-1}\left(\dfrac{2-|x|}{4}\right)+[\log(3-x)]^{-1}$

Domain $-1 \le \dfrac{2-|x|}{4} \le 1 \qquad x \in [-6, 6]$

$3-x>0 \qquad x \in (-\infty, 3)$

$\log(3-x) \ne 0 \qquad x \ne 2$

$x \in [-6, 2) \cup (2, 3)$

(54) **(C).** $\tan^{-1}\sqrt{x[x+1]}+\sin^{-1}\sqrt{x^2+x+1}=\dfrac{\pi}{2}$

$\sqrt{x[x+1]} \in R$ $\quad 0 \le x^2+x+1 \le 1$

$x \in R$ $\quad x \in [-1, 0]$

Domain E $\quad x \in [-1, 0]$

$\tan^{-1}\sqrt{x[x+1]} = \cos\sqrt{x^2+x+1}$

$\cos^{-1}\frac{1}{\sqrt{x^2+x+1}} = \cos^{-1}\sqrt{x^2+x+1}$

$x^2+x+1 \Rightarrow x = \{-1, 0\}$

(55) **(D).** $f(f(x)) = 6$; let $f(x) = t$

$\therefore\ f(t) = 6 \Rightarrow t = 1$ or -2

i.e. $f(x) = 1$ or -2

$f(x) = 1 \rightarrow 4$ solutions

$f(x) = -2 \rightarrow 2$ solutions

$\Rightarrow$ total solution = 6

(56) **(A).** We have,

$fog(-x) = f[g(-x)] = f[-g(x)]$ $\quad (\because$ g is odd)

$= f[g(x)]$ $\quad (\because$ f is even)

$= fog(x)\ \forall\ x \in R.$

$\therefore$ fog is an even function.

(57) **(C).** $f(x) = \sqrt{\frac{1-|x|}{|x|-2}}$ $\quad$ Domain

$|x|-2 \ne 0$ $\quad \frac{1-|x|}{|x|-2} \ge 0$

$x \ne \pm 2$ $\quad 1 \le |x| \le 2$

(58) **(A).** $-1 \le 3x-1 \le 1 \Rightarrow 0 \le x \le 2/3 \Rightarrow$ domain is $[0, 2/3]$ when $x = 0$ then $y = 1$, $x = 2/3$, $y = 4$. Hence range is $[1, 4]$

(59) **(B).** $fof(x) = \begin{cases} f(x) \text{ ;when } f(x) \text{ is rational} \\ 1-f(x); \text{ when } f(x) \text{ is irrational} \end{cases}$

$= \begin{cases} x \text{ ; when } x \text{ is rational} \\ 1-(1-x); \text{ when } x \text{ is irrational} \end{cases} = x$

(60) **(D).** $f^{-1}(17) = \{x : f(x) = 17 \text{ and } x < 0\} = \{x : x^2+1 = 17 \text{ and } x < 0\} = -4$

and $f^{-1}(3) = \{x : x^2+1 = 3 \text{ and } x < 0\} = -\sqrt{2}$

(61) **(D).** $f : R \rightarrow A$, $f(x) = \frac{x^2}{x^2+1}$ is surjection for surjection

range = codomain = A

So, we have to evaluate range

$f(x) = \frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{x^2+1}$;

$x^2+1 \ge 1 = 1 - \frac{1}{[1, \infty]} = [0, 1]$

(62) **(B).** $f(x) = \frac{1-x}{1+x}$ $\quad$ Domain of $f^{-1}(x)$

$y = \frac{1-x}{1+x} \Rightarrow y + xy = 1 - x \Rightarrow x = \frac{1-y}{1+y}$

$f^{-1}(x) = \frac{1-x}{1+x}$; $R - \{-1\}$

(63) **(D).** Let $f(x_1) = f(x_2) \Rightarrow [x_1] = [x_2]$ this not implies that $x_1 = x_2$

[For example, if $x_1 = 1.4$ and $x_2 = 1.5$, then $[1.4] = [1.5] = 1$]

$\therefore$ f is not one-one.

Also, f is not onto as its range I (set of integers) is a proper subset of its co-domain R.

EXERCISE-2

(1) **(B).** (i) $\because\ \pi R \pi : \pi + \pi = 2\pi \notin Z$

$\therefore$ It is not reflexitve

(ii) If $(a, b) \in R : a + b \in Z$

$: b + a \in R$

$\therefore$ $(b, a) \in R$ $\quad \therefore$ It is symmetric.

(2) **(C).** Clearly $(2, 3) \in R$ but $(3, 2) \notin R$

$\therefore$ R is not symmetric

(3) **(D).** (a) non-reflexive because $(x_3, x_3) \notin R_1$

(b) Reflexive $\quad$ (c) Reflexive

(d) non-reflexive because $x_4 \notin X$

(4) **(D).** (a) R_1 is symmetric relation because it has no element in it.

(b) R_2 is not symmetric because $(b, a) \in R_2$ & R_3 is symmetric.

(c) Every null relation is a symmetric and transitive relation.

(5) **(A).** (a) R_1 is transitive relation because it is null relation.

(b) R_2 is transitive relation because all singleton relations are transitive.

(c) R_3 is transitive relation

(d) R_4 is not a transitive relation

(6) **(D).** $f(x) = [x]^2 + [x+1] - 3 = \{[x]+2\}\ \{[x]-1]\}$

So, $x = 1, 1.1, 1.2, ...$

$\Rightarrow f(x) = 0$ $\quad \therefore$ f(x) is many-one

Only integral values will be attained $\quad \therefore$ f(x) is into

(7) **(B).** $f(x) = px + \sin x$

$f'(x) = p + \sin x$

either $f'(x) \le 0$ $\quad$ or $\quad f'(x) \ge 0$

$p + \sin x \le 0$ $\quad$ or $\quad p + \sin x \ge 0$

$p \le -1$ $\quad$ or $\quad p \ge 1$

$p \in (-\infty, -1] \cup [1, \infty)$

(8) **(B).** $f(x) = |x-1| = \begin{cases} 1-x & 0 < x < 1 \\ x-1 & x \ge 1 \end{cases}$

$g(x) = e^x$ $\quad x \ge -1$

$(fog)(x) = \begin{cases} 1-g(x) & 0 < g(x) < 1 \text{ i.e. } -1 \le x < 0 \\ g(x)-1 & g(x) \ge 1 \text{ i.e. } 0 \le x \end{cases}$

$= \begin{cases} 1-e^x & -1 \le x < 0 \\ e^x - 1 & x \ge 0 \end{cases}$ $\quad \therefore$ domain $= [-1, \infty]$

fog is decreasing in [–1, 0] and increasing in $(0, \infty)$

$fog(-1) = 1 - \frac{1}{e}$ and $fog(0) = 0$

$x \to \infty$ $fog(x) = \infty$ $\therefore$ range $[0, \infty)$

(9) **(B).** Two triangles may have equal areas
$\therefore$ f is not one-one
Since each positive real number can represent area of a triangle $\therefore$ f is onto.

(10) **(A).** $f : (0, \infty) \to (0, \infty)$
$f(x\,f(y)) = x^2 y^a \ (a \in R)$
Put x = 1, we get $f(f(y)) = y^a$

Put $f(y) = \frac{1}{x}$, we get $f(1) = \frac{1}{(f(y))^2} . y^a$

put y = 1 we get $(f(1))^3 = 1$ $\therefore$ $f(1) = 1$
for y = 1, we have $f(x(f(1))) = x^2$ $\therefore$ $f(x) = x^2$
$f(x\,f(x)) = x^2 x^a$; $f(x^3) = x^2 x^a$; $f(x^3) = x^6$
$\therefore x^6 = x^2 x^a$; thus a = 4

(11) **(D).**

(12) **(B).** If $a \neq 0$, Range of f(x) = R. Also, $f'(x) = a + \cos x$
If $a > 1$, $f'(x) > 0$, i.e. function is increasing
$a > -1$, $f'(x) < 0$ i.e. function is decreasing
f (x) to be one to one it must be monotonic and is possible its $a < -1$ or $a > 1$.

(13) **(B).** Let $g(x) = \log(x + \sqrt{1 + x^2})$

$\because g(-x) = \log\left(-x + \sqrt{1 + x^2}\right)$

$\therefore g(x) + g(-x)$

$= \log\left(x + \sqrt{1 + x^2}\right) + \log\left(-x + \sqrt{1 + x^2}\right)$

$= \log\left\{\left(x + \sqrt{1 + x^2}\right)\left(-x + \sqrt{1 + x^2}\right)\right\}$

$= \log(1 + x^2 - x^2) = 0 \Rightarrow g(-x) = -g(x) \Rightarrow g(x)$ is odd function

Since the function sec x is an even function and

$\log(x + \sqrt{1 + x^2})$ is an odd function therefore the

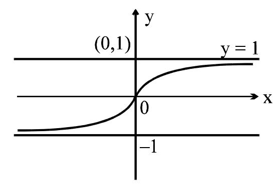
function $\sec\left[\log(x + \sqrt{1 + x^2})\right]$ is an even function.

(14) **(D).** $f(x) = x^2 - 4x + a$
always attains its minimum value. So its range must be closed. So, $a = \{\phi\}$

(15) **(D).** Put $x = \tan\theta$ or $y = \frac{2}{x + \frac{1}{x}}$ $\Rightarrow$ range is [–1, 1]

(16) **(A).** $f(x) = \begin{cases} \frac{x}{1+x} & \text{if } x \geq 0 \\ \frac{x}{1-x} & \text{if } x < 0 \end{cases}$

$f'(x) = \begin{cases} \frac{1}{(1+x)^2} & \text{if } x > 0 \\ \frac{1}{(1-x)^2} & \text{if } x < 0 \end{cases} \Rightarrow f'(x) > 0 \Rightarrow f(x)$ is $\uparrow$



As $x \to \infty$; $y \to 1$; as $x \to -\infty$; $y \to 1$
$\Rightarrow$ injective but range is [0, 1)

(17) **(D).** $f(x) = ax + \cos x$
$f'(x) = a - \sin x$

For 1–1

$f'(x) \geq 0$	$f'(x) \leq 0$
$a - \sin x \geq 0$	$a - \sin x \leq 0$
$a \geq \sin x$	$a \leq \sin x$
$a \geq 1$	$a \leq 1$

$a \in R - (-1, 1)$

(18) **(A).**
(i) $f(x) = x^2$
Let $x_1, x_2 \in N$ then $f(x_1) = {x_1}^2$ and $f(x_2) = {x_2}^2$
Now, $f(x_1) = f(x_2) \Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow {x_1}^2 - {x_2}^2$
$\Rightarrow$ $(x_1 + x_2)(x_1 - x_2) = 0$
Since $x_1, x_2 \in N$ so $x_1 + x_2 = 0$ is not possible
$\therefore$ $x_1 - x_2 = 0$ $\Rightarrow$ $x_1 = x_2$
$\Rightarrow$ $f(x_1) = f(x_2) \Rightarrow$ $x_1 = x_2$. So f is injective function.
Let $f(x) = 5 \in N$ then $x^2 = 5 \Rightarrow x = \pm\sqrt{5} \notin N$.
So f is not surjective function.
(ii) $f(x) = x^2$
Let $x_1 = 3 \in Z$ and $x_2 = -3 \in Z$
then $f(x_1) = {x_1}^2 = (3)^2 = 9$ and $f(x_2) = {x_2}^2 = (-3)^2 = 9$
$\therefore$ $f(x_1) = f(x_2)$ but $x_1 \neq x_2$. So f is not injective function.
Let $f(x) = 5 \in Z$ then $x^2 = 5 \Rightarrow x = \pm\sqrt{5} \notin N$.
So f is not surjective function.

(19) **(D).** Let $x_1 = 2 > 0$ and $x_2 = 3 > 0 \in R$
then $f(x_1) = f(2) = 1$ and $f(x_2) = f(3) = 1$
Now, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$. so f is not one–one function.
Let $f(x) = 3 \in R$
There is no value of x for which f (x) = 3 because range of f is {–1, 0, 1}. so f is not onto function.
Thus f is neither one–one nor onto function.

(20) **(C).** Let $n_1 = 3$ and $n_2 = 4 \in N$.

then $f(n_1) = f(3) = \frac{3+1}{2} = 2$ $[\because n_1$ is odd]

$f(n_2) = f(4) = \frac{4}{2} = 2$ $[\because n_2$ is even]

$\therefore$ $f(n_1) = f(n_2)$ but $n_1 \neq n_2$

So, f is not one–one function.

Since f is not one–one, it can't be bijective.

(21) **(D).** Componendo-dividendo

$$\frac{-2e^{f(x)}}{2}=\frac{x+1}{x-1}\Rightarrow e^{f(x)}=\frac{x+1}{x-1}\;;\;f(x)=\ln\frac{x+1}{x-1}$$

$$f(a)+f(b)=\ln\left[\frac{a+1}{1-a}\times\frac{b+1}{1-b}\right]\quad ....(1)$$

$$f\left(\frac{a+b}{1+ab}\right)=\ln\left[\frac{\frac{a+b}{1+ab}+1}{1-\frac{a+b}{1+ab}}\right]\quad ....(2)$$

$$\frac{(a+1)(b+1)}{(1-a)(1-b)}=\frac{a+b+1+ab}{1+ab-a-b}\;;\;\frac{ab+a+b+1}{1-a-b+ab}=\frac{a+b+1+ab}{1+ab-a-b}$$

Both are always equal

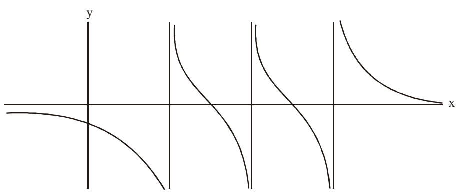
But $\frac{x+1}{1-x}>0\;;\;x\in(-1,1)$ $\quad\therefore\; a\in(-1,1),\, b\in(-1,1)$

(22) **(D).** $f(x)=(x+1)^2$ for $x\geq -1$. g (x) is the reflection of f (x) in the line y = x, then g (x) is the inverse of f (x).

Let $y=(x+1)^2\Rightarrow\sqrt{y}=x+1\Rightarrow x=\sqrt{y}-1$

i.e., $f^{-1}(y)=\sqrt{y}-1$ or $g(x)=\sqrt{x}-1$

(23) **(C).**



From the graph it is clear t hat f (x) is not one-one

$\therefore$ Statement (1) is true

Also $f'(x)<0\;\forall\; x\in D_f$ but the function is not 1-1. So the statement-2 is false.

(24) **(A).** Both statements are true and statement 2 is correct explanation of statement 1.

(25) **(D).** Statement 1 : Let g (x) is inverse of y = f (x)

$f'(g(x))=\frac{1}{g'(x)}$ $\quad\therefore$ If f (x) is strictly increasing then g (x) is also strictly increasing.

Statement 2 : e^x is mirror image of ln x wrt. line y = x

$\therefore$ ln x is inverse of e^x

(26) **(A).** ϕ (x) is odd, aperiodic, one-one and onto

So, $k(\phi(x))=-1+4+5+7=15$

(27) **(D).** h (x) is even, aperiodic, many-one and into

So $k(h(x))=0+4+6+8=18$

(28) **(C).** $k(x^2)=18$, $\quad k(e^x)=19$,

$k(\sin x)=16$, $\quad k(|x|)=18$

$\therefore$ k (x) is many-one into function.

(29) **(B).** If $x^4+y^3=1$, then we know (–x, y) is on the graph since $(-x)^4+y^3=x^4+y^3=1$. And in general, when the coordinate is raised to an even power every single time in the equation, then symmetry by the other axis occurs. Since y is raised to an odd number, then x-axis and origin symmetry are ruled out. Symmetry about y = x is another story, but since $x^4+y^3=x^3+y^4$ is not necessarily true, it is ruled out. The only symmetry is about the y-axis.

(30) **(D).** Suppose R is just a rectangle whose 4 vertices are (1, 2), (1, – 2), (–1, 2) and (–1, –2). The x-axis and y-axis symmetries in the problem are satisfied, but the point (2, 1) is not contained in R.

(31) **(C).** $f(0)=c\neq 0\;\therefore\; f(2)=0$

(32) **(A).** Since, $f(0)=2\Rightarrow c=2$ and

$f(2)=2\Rightarrow 4a+2b+c=0\Rightarrow 4a+2b+2=0$

$\Rightarrow 2a+b+1=0\Rightarrow 2+\frac{b}{a}+\frac{1}{a}=0$

$\therefore$ 1/a is a root of $ax^2+bx+c=0$

(33) **(D).** $-\frac{b}{2a}\leq 0$ or $-\frac{b}{2a}\geq 2\Rightarrow\frac{2a+1}{2a}\leq 0$

or $\frac{2a+1}{2a}\geq 2\Rightarrow -\frac{1}{2}\leq a<0$ or $0<a\leq\frac{1}{2}$

$\Rightarrow$ a = 1 is not possible.

(34) **(A).**

(A) $\frac{1}{g(x)}=\frac{1}{\ln x/x}\;;\; f(x)=\frac{x}{\ln x}$

$x>0,\, x\neq 1$ for both

(B) $\frac{1}{f(x)}=\frac{1}{x/\ln x}\;;\; g(x)=\frac{\ln x}{x}$

$\Rightarrow\frac{1}{f(x)}$ is not defined at x =1 but g (1) = 0

(C) $f(x)\,.\,g(x)=\frac{x}{\ln x}\cdot\frac{\ln x}{x}=1$

if $x>0,\, x\neq 1\;\Rightarrow$ N. I.

(D) $\frac{1}{f(x).g(x)}=\frac{1}{\frac{x}{\ln x}\cdot\frac{\ln x}{x}}=1$

only for x > 0 and $x\neq 1$

(35) **(A).** Given $f(x)=\frac{9^x}{9^x+3}$ $\quad .....(i)$

now $f(1-x)=\frac{9^{1-x}}{9^{1-x}+3}=\frac{3}{9^x+3}$ $\quad .....(ii)$

Adding (i) and (ii) $f(x)+f(1-x)=1$ $\quad .....(iii)$

Putting $x=\frac{1}{1996},\frac{2}{1996},......,\frac{998}{1996}$ in (iii)

We get $f\left(\frac{1}{1996}\right)+f\left(\frac{1995}{1996}\right)=1$

$f\left(\frac{2}{1996}\right)+f\left(\frac{1994}{1996}\right)=1$

....

....

$f\left(\frac{997}{1996}\right)+f\left(\frac{999}{1996}\right)=1$; $f\left(\frac{998}{1996}\right)+f\left(\frac{998}{1996}\right)=1$

$\Rightarrow f\left(\frac{998}{1996}\right)=\frac{1}{2}$

Adding all, we get

$f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+.....+f\left(\frac{1995}{1996}\right)$

$=(1+1+.....+997 \text{ times})+\frac{1}{2}=997.5$

(36) **(D).** For f (x) to be defined, we must have

$x-\sqrt{1-x^2}\geq 0$ or $x\geq\sqrt{1-x^2}>0$

$\therefore x^2\geq 1-x^2$ or $x^2\geq\frac{1}{2}$. Also, $1-x^2\geq 0$ or $x^2\leq 1$.

Now, $x^2\geq\frac{1}{2}\Rightarrow\left(x-\frac{1}{\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right)\geq 0$

$\Rightarrow x\leq-\frac{1}{\sqrt{2}}$ or $x\geq\frac{1}{\sqrt{2}}$

Also, $x^2\leq 1\Rightarrow(x-1)(x+1)\leq 0\Rightarrow -1\leq x\leq 1$

Thus, $x>0,\ x^2\geq\frac{1}{2}$ and $x^2\leq 1\Rightarrow 1\ x\in\left[\frac{1}{\sqrt{2}},1\right]$

(37) **(B).** $f(x)=|Ax+c|+d$

$g(x)=-|Ax+u|+v$

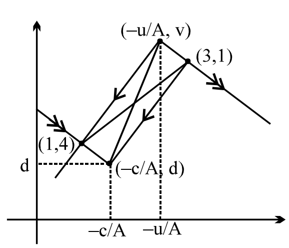


figure is parallelogram and diagonals bisect each other

$\therefore\left(-\frac{u}{A}\right)+\left(-\frac{c}{A}\right)=3+1\ \therefore\ \frac{u+c}{A}=-4$

(38) **(B).** $\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001}x)))$ is defined if

$0<\log_{2003}(\log_{2002}(\log_{2001}x))$

if $\log_{2002}(\log_{2001}x)>1$

if $\log_{2001}x>2002$

if $x>(2001)^{2002}$

(39) **(A).** For y to be defined, $\frac{\pi^2}{16}-x^2\geq 0$

$\Rightarrow\left(\frac{\pi}{4}-x\right)\left(\frac{\pi}{4}+x\right)\geq 0\Rightarrow\left(x-\frac{\pi}{4}\right)\left(x+\frac{\pi}{4}\right)\leq 0$

$\Rightarrow-\frac{\pi}{4}\leq x\leq\frac{\pi}{4}$ $\therefore$ Domain of $y=\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

Clearly, for $x\in\left[-\frac{\pi}{4},\frac{\pi}{4}\right];\sqrt{\frac{\pi^2}{16}-x^2}\in\left[0,\frac{\pi}{4}\right]$;

Since sin x is an increasing function on [0, π/4].

Therefore, $\sin 0\leq\sin\sqrt{\frac{\pi^2}{16}-x^2}\leq\sin\frac{\pi}{4}$

$\Rightarrow 0\leq 3\sin\sqrt{\frac{\pi^2}{16}-x^2}\leq\frac{3}{\sqrt{2}}\Rightarrow 0\leq y\leq\frac{3}{\sqrt{2}}$.

$\therefore$ Range of $y=\left[0,\frac{3}{\sqrt{2}}\right]$

(40) **(C).** For f(x) to be defined $x\neq-6, 6$

and $\log_{0.4}\left(\frac{x-1}{x+5}\right)\geq 0,\ \frac{x-1}{x+5}>0$.

Since $\log_a x$ for $0<a<1$ is a decreasing function, we have

$\frac{x-1}{x+5}\leq 1$ and $\frac{x-1}{x+5}>0$. For $x>-5$,

we must have $x-1\leq x+5$ and $x-1>0$.

The first inequality is always true, so we must have $x>1$ for $x<-5$.

We have $x-1\geq x+5,\ x-1<0$. These inequalities are not possible.

$\Rightarrow$ the domain of f(x) is $(1,\infty)\sim\{6\}$.

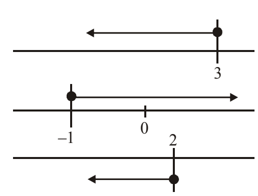
(41) **(C).**

$3-x\geq 0\Rightarrow 3\geq x\Rightarrow x\leq 3$

$2-\sqrt{3-x}\geq 0\Rightarrow 4\geq 3-x\Rightarrow x\geq-1$

and $1\geq\sqrt{2-\sqrt{3-x}}\Rightarrow\sqrt{3-x}\geq 1\Rightarrow 3-x\geq 1$

$\Rightarrow 2\geq x\Rightarrow x\leq 2$



Hence, $x\in[-1,2]$

(42) **(A).** Let $x, y\in R$ be such that

$f(x)=f(y)\Rightarrow\frac{x-a}{x-b}=\frac{y-a}{y-b}$

$\Rightarrow(x-a)(y-b)=(x-b)(y-a)$

$\Rightarrow xy-bx-ay+ab=xy-ax-by+ab$

$\Rightarrow (a-b)\,x = (a-b)\,y \Rightarrow x = y$ $\quad\therefore$ f is injective.

Let $y \in R$. Then,

$$f(x) = y \Rightarrow \frac{x-a}{x-b} = y \Rightarrow x - a = xy - by \Rightarrow x = \frac{a-by}{1-y}$$

Clearly, $x \notin R$ for $y = 1$. $\quad\therefore$ f is not surjective.

(43) **(C).** If $a^{f(x)} + b^{f(x)} = a + b$ and $a.b = 1$, then $f(x) = \pm 1$

Now $\left(3+\sqrt{8}\right)^{[x]} + \left(3-\sqrt{8}\right)^{[x]} = 34$

$$\Rightarrow \left(17+\sqrt{288}\right)^{\frac{[x]}{2}} + \left(17-\sqrt{288}\right)^{\frac{[x]}{2}} = 34$$

$\Rightarrow a^{[x]/2} + b^{[x]/2} = a + b \;\&\; a.b = 1$

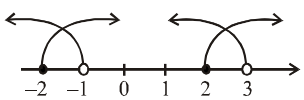
Where $a = 17 + \sqrt{288}$, $b = 17 - \sqrt{288}$

and $a.b = (17)^2 - 288 = 1$

$\therefore \frac{[x]}{2} = \pm 1 \Rightarrow [x] = \pm 2 \Rightarrow [x] = -2$ and $[x] = 2$

$\Rightarrow -2 \le x < -1$ and $2 \le x < 3$ $\therefore x \in [-2,-1) \cup [2,3)$

i.e.



EXERCISE-3

(1) **(AD).** $\because$ function of define $f : R \to R$

$\therefore$ function is many one into. If we draw lines parallel to x-axis they cuts curve of function more than one places.

$\therefore$ function is many one and in R we can draw lines those do not cuts curve of function.

$\therefore$ function is into.

(2) **(D).** $f : N \to I$

$$f(n) = \begin{cases} \frac{n-1}{2}\text{; when n is odd} \\ \frac{-n}{2}\text{; when n is even} \end{cases}$$

$\because$ n is natural no. which is always the positive if n is odd then $n-1$ will be even and $\frac{n-1}{2}$ may be even or odd but it will be positive integer.

$\therefore$ Corresponding to odd natural no. we get all the integer including 0 at $x = 1$.

and if n is even n/2 will be even or odd integer but $-n/2$ will be –ve integer.

Corresponding to even natural no. we get all –ve integer f (n) is one-one onto function.

(3) **(D).** $f : R \to S$

$f(x) = \sin x - \sqrt{3}\cos x + 1 = 2\sin(x - \pi/3) + 1$

$\because -1 \le \sin(x-\pi/3) \le 1$

$\Rightarrow -2 \le 2\sin(x-\pi/3) \le 2$

$\Rightarrow -2+1 \le 2\sin(x-\pi/3) + 1 \le 2 + 1 \Rightarrow -1 \le f(x) \le 3$

$\therefore$ Range of function $= [-1, 3]$

if function is onto then S must be range $\therefore$ Set $S = [-1, 3]$

(4) **(B).** $A = \{1, 2, 3, 4\}$

$R = \{(1,3)(4,2)(2,4),(2,3)(3,1)\}$

R is relation on set A

$\because (1,1),(2,2),(3,3),(4,4) \notin R$ $\therefore$ R is not reflexive

$\because (2,4) \in R \;\&\; (2,3) \in R$ $\quad\therefore$ R is not function

$\because (1,3) \in R$ and $(3,1) \in R$ but $(1,2) \notin R$

$\therefore$ R is not transitive

$\because (2,3) \in R \;\&\; (3,2) \notin R$ $\quad\therefore$ R is not symmetric

(5) **(D).** $f : (-1, 1) \to B$; $f(x) = \tan^{-1}\frac{2x}{1-x^2}$

for f to be both one-one and onto B must be equal to range. Let $x = \tan\theta$

$\therefore -1 < x < 1 \Rightarrow -1 < \tan\theta < 1 \Rightarrow -\pi/4 < \theta < \pi/4$

$\Rightarrow -\pi/2 < 2\theta < \pi/2$ .... (1)

$$\begin{cases} \because \tan^{-1}(\tan x) = x\,;\; -\pi/2 < x < \pi/2 \\ \therefore \tan^{-1}(\tan 2\theta) = 2\theta \\ \because -\pi/2 < 2\theta < \pi/2 \\ \text{and } -\pi/2 < 2\theta < \pi/2 \end{cases}$$

Now, $f(x) = \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{2\tan\theta}{1-\tan\theta}$

$= \tan^{-1}(\tan 2\theta) = 2\theta$

and $-\pi/2 < 2\theta < \pi/2$ from (1)

Range of $f(x) = (-\pi/2, \pi/2)$ $\therefore B \in (-\pi/2, \pi/2)$

(6) **(A).** $A = \{3, 6, 9, 12\}$

$R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$

R is relation on set A

$\because (3,3)(6,6),(9,9)(12,12) \in R$ $\therefore$ R is reflexive

$\because (6,12) \in R$ and $(12,6) \notin R$ $\quad\therefore$ R is not symmetric

$\because (3,6)$ and $(6,12) \in R \Rightarrow (3,12) \in R$ etc.

$\therefore$ R is transitive

(7) **(A).** Let $w \in W$ then $(w, w) \in R$ $\quad\therefore$ R is reflexive

also if $w_1, w_2 \in W$ and $(w_1, w_2) \in R$ then $(w_2, w_1) \in R$

$\therefore$ R is symmetric

Again Let w_1 = I N K, w_2 = L I N K, w_3 = L E T

$(w_1, w_2) \in R$ {$\because$ I, N are the common element of w_1, w_2}

and $(w_2, w_3) \in R$ {$\because$ L is common letter of w_2, w_3}

But $(w_1, w_3) \notin R$ {$\because$ there is no common letter of w_1, w_3}

$\therefore$ R is not transitive.

(8) **(C).** $S = \{(x, y)\,;\, y = x + 1$ and $0 < x < 2\}$

$T = \{(x, y)\,;\, x - y$ is an integer$\}$

For S, $(x, x) \notin S$ $\quad$ {$\because x \ne x + 1$} $\therefore$ S is not reflexive

$\therefore$ S is not equivalence relation

For T, $(x, x) \in T$ $\quad\because x - x = 0$ is an integer

$\therefore$ T is reflexive

If $(x, y) \in T$ $\;\because x - y$ is an integer

let $x - y = k$ then $(y, x) \in T \Rightarrow y - x = -k$ (is also an integer)

$\therefore$ T is symmetric relation

let $(x_1, y_1) \in T \Rightarrow x_1 - y_1 = k$ (integer)

$(y_1, z_1) \in T \Rightarrow y_1 - z_1 = \lambda$ (integer)

On adding, $x_1 - z_1 = k + \lambda$ (integer)

$\therefore (x_1, z_1) \in T$ $\therefore$ T is transitive $\therefore$ T is an equivalence

(9) **(C).** $\because\ y = 4x + 3$ $\because$ we know that $f(x) = y$

$x = f^{-1}(y)$. From (1), $y = 4x + 3$

$\Rightarrow x = \frac{y-3}{4} \Rightarrow f^{-1}(y) = \frac{y-3}{4} \Rightarrow f^{-1}(x) = \frac{x-3}{4}$

(10) **(C).** $f'(x) = 3x^2 + 5$, which is positive.

$\Rightarrow$ f (x) is strictly increasing hence it is one-one.

Also, $f(\infty) \to \infty$ and $f(-\infty) \to -\infty$

Therefore range of $f(x)$ is R.

(11) **(B).** $f(x) = y = (x+1)^2 - 1, x \geq -1, y \geq -1$

$f^{-1}(x) = 1 + \sqrt{1+x}, x \geq -1$ [$f^{-1}(x)$ exists only if f (x) is bijective]. Also, $f^{-1}(x) = f(x)$

$\Rightarrow (x+1)^2 - 1 = -1 + \sqrt{1+x} \Rightarrow x = 0, -1$

(12) **(B).** x R y need not implies y R x

$S : \frac{m}{n} s \frac{p}{q} \Leftrightarrow qm = pn \quad \frac{m}{n} s \frac{p}{q}$ reflexive

$\frac{m}{n} s \frac{p}{q} \Rightarrow \frac{p}{q} s \frac{m}{n}$ symmetry

$\frac{m}{n} s \frac{p}{q}, \frac{p}{q} s \frac{r}{s} \Rightarrow qm = pn,\ ps = rq \Rightarrow ms = rn$ transitive

S is an equivalence relation

(13) **(C). Statement - 1 :**

(i) $x - x$ is an integer $\forall\ x \in R$ so A is reflexive relation.

(ii) $y - x \in I \Rightarrow x - y \in I$ so A is symmetric relation.

(iii) $y - x \in I$ and $z - y \in I \Rightarrow y - x + z - y \in I$

$\Rightarrow z - x \in I$ so A is transitive relation.

ThereforeAis equivalence relation.

Statement - 2 :

(i) $x = \alpha x$ when $\alpha = 1 \Rightarrow$ B is reflexive relation

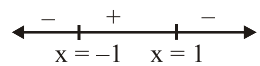
(ii) for $x = 0$ and $y = 2$, we have $0 = \alpha$ (B) for $\alpha = 0$

But $2 = \alpha (0)$ for no α

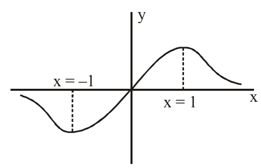
So, B is not symmetric so not equivalence.

(14) **(A).** $f(x) = \frac{x}{1+x^2}$; $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$

$f'(x) = \frac{(1+x^2)\cdot 1 - x(2x)}{(1+x^2)^2} = \frac{-(x+1)(x-1)}{(1+x^2)^2}$



sign of f ' (x)



Non-monotonic $\therefore$ Not injective

$y = \frac{x}{1+x^2}$; $x^2(y) - x + y = 0$; $D \geq 0$

$1 - 4y^2 \geq 0$; $y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

(15) **(A).** $f(x) = 2\left(1 + \frac{1}{x-1}\right)$; $f'(x) = -\frac{2}{(x-1)^2} < 0$

f (x) is strictly decreasing.

$\Rightarrow$ f is one-one but not onto.