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MATRICESAND DETERMINANTS

INTRODUCTION

Elementary matrix already has now becomes as integral part of the mathematical background necessary in field of electrical / computer engineering / chemistry.

A matrix is any rectangular array of numbers written within brackets. A matrix is usually represented by a capital letter on a closeified by its dimension of the (D) and classified by its dimensions. The dimension of the matrices are the number of rows and columns.

A $m \times n$ matrix is usually written as

$$
A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}
$$
 (E) Vertical Matrix : A matrix of order $m \times n$ is a vertical matrix

(where a_{ii} represents any number which lies ith row (from top) $\&$ jth column form left)

- (i) The matrix is not a number. It has got no numerical value.
- (ii) The determinant of matrix

$$
A_{m \times m} = |A_{m \times m}| = \begin{vmatrix} a_{11} \dots \dots \dots a_{1m} \\ \dots \dots \dots \dots \dots \\ a_{m1} \dots \dots \dots a_{mn} \end{vmatrix}
$$
 Note: Ever converse.

Abbreviated as :

 $A = [a_{i,j}]$ $1 \le i \le m$; $1 \le j \le n$, i denotes the row and j denotes the column is called a matrix of order $m \times n$. The (F) elements of a matrix may be real or complex numbers. If all the elements of a matrix are real, the matrix is called real matrix. the column is called a matrix of order m × n. 1 me

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is called positive matrix. A r or Null Matrix : A

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SPECIAL TYPE OF MATRICES :

(A) Row Matrix :

 $A = [a_{11}, a_{12}, \dots, a_{1n}]$ having one row . $(1 \times n)$ matrix. (i) (or row vectors)

(B) Column Matrix : A =
$$
\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}
$$

having one column. $(m \times 1)$ matrix (or column vectors)

(C) Zero or Null Matrix : $(A = O_{m \times n})$

An $m \times n$ matrix all whose entries are zero.

$$
A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$
 is a 3 × 2 null matrix &

MINANTS
\nB =
$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$
 is 3 × 3 null matrix
\nrizontal Matrix : A matrix of order m × n is a horizontal
\ntrix if n > m.
\n
$$
\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}
$$
\n
$$
1 > n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}
$$
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1 > n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}
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1 > n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}
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1 = n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}
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1 = n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}
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1 = n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}
$$
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$$
1 = n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \end{bmatrix}
$$
\n
$$
1 = n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \end{b
$$

Horizontal Matrix : A matrix of order m × n is a horizontal matrix if $n > m$.

written as
$$
\begin{bmatrix} 1 & 2 & 3 & 4 \ 2 & 5 & 1 & 1 \end{bmatrix}
$$

 a_{m1} a_{m2} a_{mn} **(E)** Vertical Matrix : A matrix of order m × n is a vertical matrix

$$
if m > n. \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}
$$

 $\frac{1}{2}$ Note: Every row matrix is also a Horizontal but not the converse.

 $|||$ ly every column matrix is also a vertical matrix but not the converse.

(F) Square Matrix : (Order n)

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or. It has got no numerical value.
 $\lim_{n=1,...,n}$ Note: Every row matrix is also a Horizonta
 $\lim_{n=1,...,n}$ Note: Every row matrix is also If number of rows = number of columns \Rightarrow a square matrix. A real square matrix all whose elements are positive is called a positive matrices. Such matrices have application in mechanics and economics.

NOTE :

In a square matrix the pair of elements $a_{ii} \& a_{i}$ are called **Conjugate Elements**.

e.g. in the matrix
$$
\begin{pmatrix} a_{11} & a_{12} \ a_{21} & a_{22} \end{pmatrix}
$$
, a_{21} and a_{12} are conjugate elements.

x

 $\left[\begin{array}{c} 1 \\ 2 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 3 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 2 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 3 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c} 1 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c}$ bents of a matrix and to contract the matrix is called real

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is called a positive matrices

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vectors)
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 $\mathbf{A} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$
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Fundments of a matrix are real, the matrix is called real
 $a_{11}, a_{12}, \dots, a_{1n}$ having one row (ii) The elements a_{11} , a_{22} , a_{33} , a_{nn} are called **Diagonal Elements** . The line along which the diagonal elements lie is called **" Principal or Leading "** diagonal. The quantity Σa_{ii} = trace of the matrix written as, $(t_r) A = t_r(A)$

NOTE

(i) Minimum number of zeros in an upper or lower triangular matrix of order n

$$
= 1 + 2 + 3 + \dots + (n - 1) = \frac{n(n-1)}{2}
$$

(ii) Minimum number of cyphers in a diagonal/scalar/unit matrix of order $n = n (n - 1)$

and maximum number of cyphers $= n^2 - 1$.

"It is to be noted that with every square matrix there is a corresponding determinant formed by the elements of A in the same order." If $|A| = 0$ then A is called a **singular matrix**

and if $|A| \neq 0$ then A is called a **non singular matrix.** (i)

Note: If
$$
A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$
 then det. $A = 0$ but not conversely.

ALGEBRA OF MATRICES : ADDITION :

 $A + B = [a_{i} + b_{i}]$ where $A \& B$ are of the same type. (same order)

If A and B are square matrices of the same type then, $t_r(A + B) = t_r(A) + t_r(B)$

- **(a) Addition of matrices is commutative :** i.e. $A + B = B + A$ where A and B must have the same order
- **(b) Addition of matrices is associative :** $(A + B) + C = A + (B + C)$ Provided A, B & C have the same order.

(c) Additive inverse :

If $A + B = 0 = B + A$ $[A = m \times n]$

and both A and B have the same order then A and B are said to be the to be the additive inverse of each other where **O** is the null matrix of the same order as that of A and B. '**O**' is the additive identity element.

 \mathcal{L} cancellation laws hold good.

$\frac{(1-1)}{2}$ **MULTIPLICATION OF A MATRIX BY A SCALAR :**

If
$$
A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}
$$
; $kA = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$

i.e. $k(A + B) = kA + kB$

Note:

(i) If A is a square matrix then
$$
t_r(kA) = k[t_r(A)]
$$

$$
\begin{vmatrix}\na & 0 & 0 \\
0 & a & 0 \\
0 & 0 & a\n\end{vmatrix} = I_3
$$
\nIf A + B = A + C \Rightarrow B = C
\nand If B + A = C + A \Rightarrow B = C
\ncancellation laws hold good.
\nMULTIPLICATION OF A MATRIX BY A SCALAR :
\nIf A =
$$
\begin{bmatrix}\na & b & c \\
b & c & a \\
c & a & b\n\end{bmatrix}
$$
; kA =
$$
\begin{bmatrix}\nka & kb & kc \\
kb & kc & ka \\
kc & ka & kb\n\end{bmatrix}
$$
\ni.e. k(A + B) = kA + kB
\nNote:
\n(i) If A is a square matrix then t_r(kA) = k[t_r(A)]
\n(ii)
$$
A = \begin{bmatrix}\n1 & -2 \\
2 & 3\n\end{bmatrix} + \begin{bmatrix}\n1 & -2 \\
2 & 3\n\end{bmatrix} + \begin{bmatrix}\n1 & -2 \\
2 & 3\n\end{bmatrix} = \begin{bmatrix}\n3 & -6 \\
6 & 9\n\end{bmatrix} = 3A
$$
\nExample 1:
\nA matrix A = [a_{ij}] of order 2 × 3 whose elements are such that a_{ij} = i + j is –
\n(i)
$$
\begin{bmatrix}\n2 & 3 & 4 \\
3 & 4 & 5\n\end{bmatrix}
$$
\n(i)
$$
\begin{bmatrix}\n2 & 3 & 4 \\
5 & 5 & 4\n\end{bmatrix}
$$
\n(j) None of these
\nSol. (1), a_{ij} is the element of ith row and jth column of matrix A
\n
$$
\therefore a_{11} = 1 + 1 = 2, a_{12} = 1 + 2 = 3, a_{13} = 1 + 3 = 4
$$

Example 1 :

A matrix $A = [a_{ij}]$ of order 2 \times 3 whose elements are such that $a_{ii} = i + j$ is –

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\nLTIPLLCATION OF A MATRIX BY A SCALAR :
\nIf A =
$$
\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}
$$
; kA = $\begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$
\ni.e. k(A+B)=kA+kB
\nNote:
\nIf A is a square matrix then t_r(kA)=k[t_r(A)]
\nA= $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ then A+A+A
\n $= \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & 9 \end{bmatrix} = 3A$
\n**mple 1**:
\nA matrix A = [a_{ij}] of order 2 × 3 whose elements are such
\nthat a_{ij} = i + j is -
\n(I) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \end{bmatrix}$
\n(3) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 5 & 4 \end{bmatrix}$ (4) None of these
\n(1), a_{ij} is the element of ith row and jth column of matrix A
\n∴ a₁₁ = 1 + 1 = 2, a₁₂ = 1 + 2 = 3, a₁₃ = 1 + 3 = 4
\n a₂₁ = 2 + 1 = 3, a₂₂ = 2 + 2 = 4, a₂₃ = 2 + 3 = 5
\nA = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

Sol. (1). a_{ij} is the element of ith row and jth column of matrix A \therefore a₁₁ = 1 + 1 = 2, a₁₂ = 1 + 2 = 3, a₁₃ = 1 + 3 = 4

$$
a_{21} = 2 + 1 = 3. \ a_{22} = 2 + 2 = 4, \ a_{23} = 2 + 3 = 5
$$

$$
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}
$$

Example 2 :

If A = 1 3 2 2 k 5 4 2 1 is a singular matrix, then find the value 1 3 2 2 k 5 4 2 1

of k.

Sol. A is singular $\Rightarrow |A| = 0$

$$
\Rightarrow \begin{vmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{vmatrix} = 0
$$

\Rightarrow 1(k-10)+3(2-20)+2(4-4k)=0

 \Rightarrow 7k + 56 = 0 \Rightarrow k = -8

MULTIPLICATION OF MATRICES

If A and B be any two matrices, then their product AB will be defined only when number of column in A is equal to the number of rows in B. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ then their product $AB = C = [c_{ij}]$, will be matrix of order m x

p, where,
$$
(AB)_{ij} = c_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}
$$

\nEx. If $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$
\nthen $AB = \begin{bmatrix} 1.1 + 4.2 + 2.1 & 1.2 + 4.2 + 2.3 \\ 2.1 + 3.2 + 1.1 & 2.2 + 3.2 + 1.3 \end{bmatrix}$
\n $AB = \begin{bmatrix} 11 & 16 \\ 9 & 13 \end{bmatrix}$

Properties of Matrix Multiplication : If A, B and C are three matrices such that their product is defined, then

- (i) $AB \neq BA$ (Generally not commutative)
- (ii) $(AB) C = A(BC)$ (Associative Law)
- (iii) $IA = A = AI$
	- (I is identity matrix for matrix multiplication
- (iv) $A (B+C) = AB + AC$ (Distributive Law)
- (v) If $AB = AC$ this not implies that $B = C$ (Cancellation Law is not applicable)
- (vi) If $AB = 0$ It does not mean that $A = 0$ or $B = 0$, again product of two non-zero matrix may be zero matrix. (vii) tr $(AB) = tr (BA)$

NOTE

- (i) The multiplication of two diagonal matrices is again a diagonal matrix.
- (ii) The multiplication of two triangular matrices is again a triangular matrix.
- (iii) The multiplication of two scalar matrices is also a scalar matrix.

- (iv) If A and B are two matrices of the same order, then
	- (a) $(A + B)^2 = A^2 + B^2 + AB + BA$ (b) $(A - B)^2 = A^2 + B^2 - AB - BA$ (c) $(A - B)(A + B) = A^2 - B^2 + AB - BA$ (d) $(A + B)(A - B) = A^2 - B^2 - AB + BA$
	- (e) $A(-B) = (-A)B = -(AB)$

Positive Integral Powers of a Matrix : The positive integral powers of a matrix A are defined only when A is a square matrix. Also then

 $A^2 = A$. A $A^3 = A$. A $A = A^2 A$ Also for any positive integers m, n (i) $A^m A^n = A^{m+n}$ (ii) $(A^m)^n = A^{mn} = (A^n)^m$ $(iii) Iⁿ = I, I^m = I$ (iv) $A^{\circ} = I_n$ where A is a square matrices of order n.

Example 3 :

ir rj r 1 a b 1 4 2 2 3 1 and B = 1 2 2 2 1 3 1.1 4.2 2.1 1.2 4.2 2.3 2.1 3.2 1.1 2.2 3.2 1.3 11 16 9 13 If A = 2 1 1 2 and A2–4A – nI = 0, then find the value of n. **Sol.** A² ⁼ 5 4 4 5 , 4A = 8 4 4 8 , nI = n 0 0 n A² – 4A – nI ⁼ 5 8 n 4 4 0 4 4 0 5 8 n ⁼ 3 n 0 0 3 n A² – 4A – nI = 0 3 n 0 0 3 n ⁼ 0 0 0 0 – 3 – n = 0 n = – 3 1 2 3 4

Example 4 :

If
$$
A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}
$$
 then find element a_{21} of A^2 .

Sol. The element a_{21} is product of second row of A to the first column of A

$$
\therefore \quad a_{21} = [3 \quad -4] \begin{bmatrix} -1 \\ 3 \end{bmatrix} = -3 - 12 = -15
$$

TRANSPOSE OFA MATRIX

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of Matrix A and is denoted by A^T or A'. From the definition it is obvious that e element a_{21} is product of second row of A to the first

ultim of A
 $a_{21} = [3 \t -4] \begin{bmatrix} -1 \\ 3 \end{bmatrix} = -3 - 12 = -15$
 POSE OF AMATRIX

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 SPOSE OF AMATRI \Rightarrow -3 - n = 0 \Rightarrow n = - 3

ple4:

ff A = $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then find element a_{21} of A².

The element a_{21} is product of second row of A to the first

column of A
 \therefore a_{21} = [3 -4] $\begin{bmatrix} -1 \\ 3 \end{$ \Rightarrow $-3 - n = 0 \Rightarrow n = -3$

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 $-3 - 12 = -15$

a given matrix A by changing its

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b by A^T or A'.

bvious that

order of A^T is n x m.
 $\begin{bmatrix} 1 & b_1 \\ 2$ t of second row of A to the first
 $-3-12=-15$

a given matrix A by changing its

mns into rows is called transpose

d by A^T or A'.

by outs that

order of A^T is n x m.
 $\begin{bmatrix} 1 & b_1 \\ 2 & b_2 \\ 3 & b_3 \end{bmatrix}_{3\times 2}$ $-3-12 = -15$
a given matrix A by changing its
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d by A^T or A' .
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order of A^T is n x m.
 $\begin{bmatrix} 1 & b_1 \\ 2 & b_2 \\ 3 & b_3 \end{bmatrix}_{3\times 2}$ and element a_{21} of A^2 .

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a order of A^T is n x m.
 a $\begin{bmatrix} 0 & 0 \end{bmatrix}$

3

3
 $\begin{bmatrix} 0 & 0 \end{bmatrix}$
 $= -3 - 12 = -15$
 $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 &$ 3
 3
 $\frac{1}{2}$

and element a_{21} of A^2 .
 a_{21}
 a_{32}
 a_{33}
 b_{33}
 b_{32}
 b_{33}
 b_{34}
 $\frac{1}{2}$ find element a_{21} of A^2 .

uct of second row of A to the first
 $= -3 - 12 = -15$
 \therefore

m a given matrix A by changing its

lumns into rows is called transpose

ted by A^T or A' .

obvious that

en order of A^T is

If order of A is m x n, then order of A^T is n x m.

Ex. Transpose of Matrix

$$
\begin{bmatrix} a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \end{bmatrix}_{2 \times 3}
$$
 is
$$
\begin{bmatrix} a_1 & b_1 \ a_2 & b_2 \ a_3 & b_3 \end{bmatrix}_{3 \times 2}
$$

Properties of Transpose

(i)
$$
(A^T)^T = A
$$

\n(ii) $(A \pm B)^T = A^T \pm B^T$
\n(iii) $(AB)^T = B^T A^T$
\n(iv) $(kA)^T = k(A)^T$
\n(v) $I^T = I$
\n(vi) tr $(A) = tr (A)^T$
\n(vii) $(A_1A_2A_3.....A_{n-1}A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$

Example 5 :

SOLUTION ANTER
\n**6** (i)
$$
(A^T)^T = A
$$

\n(ii) $(AB)^T = B^T A^T$
\n(iii) $(AB)^T = B^T A^T$
\n(iv) $(kA)^T = k(A)^T$
\n(v) $I^T = I$
\n(vi) $(A_1A_2A_3.....A_{n-1}A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$
\n**6** (i) A^{2n} is a symmetric matrix for n
\n(ii) $(A_1A^T)B^T = B^T A^T$
\n(iv) $(kA)^T = k(A)^T$
\n(v) $I^T = I$
\n(vii) $(A_1A_2A_3.....A_{n-1}A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$
\n**6** (i) $B^T AB$ is also skew-symmetric matrix
\nsquare matrix of order that of A.
\n**6** (iii) If A, B are two symmetric matrices,
\n(iv) If A, B are two symmetric matrices,
\n(iii) $A^T = B^T A^T$
\n(v) If A is a skew-symmetric matrices,
\n(iv) If A, B are two symmetric matrix when A
\n(c) AB is a symmetric matrix when A
\n(c) AB is a symmetric matrix when A
\n(d) AB - BA is a skew-symmetric matrix
\n(e) AB - BA is a skew-symmetric matrix
\n(f) AB - BA is a skew-symmetric matrix
\n(g) AB + BA is a skew-symmetric matrix
\n(h) AB - BA is a symmetric matrix and
\n(iii) B-A is a skew-symmetric matrix
\n(iiii) If A is a skew-symmetric matrix
\n(iiv) If A, B are two skew-symmetric matrix
\n(iv) If A is a skew-symmetric matrix
\n(vi) If A is a skew-symmetric matrix
\n(vii) Every square matrix A can uniquely
\n(g) AB + BA is a symmetric matrix
\n(h) AB + BA is a symmetric matrix
\n(iii) B-BA is a symmetric matrix
\n(iiii) B-BA is a symmetric matrix
\n(iv) If A is a skew-symmetric matrix
\n(v) If A is a skew-symmetric matrix
\n(v) If A is a skew-symmetric matrix
\n(vi) If A is a skew-symmetric matrix
\n(vii) Every square matrix A can uniquely
\nof a symmetric and skew-symmetric

Example 6 :

Solution
\n**EXAMPLE 1** (i)
$$
(A^T)^T = A
$$

\n(ii) $(A \pm B)^T = B^T A^T$
\n(iii) $(A \pm B)^T = A^T \pm B^T$
\n(ii) $(A \lambda)^{2n}$ is a symmetric matrix
\n(iii) $(A)^T = B^T A^T$
\n(iv) $(A \lambda)^{2n-1}$ is a skew-symmetric matrix
\n(v) $I^T = I$
\n(vi) $I^T = I$
\n(vii) $(A_1 A_2 A_3.....A_{n-1} A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$
\n**Example 5:**
\nIf $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then find $(AB)^T$.
\n**Example 6:**
\nIf $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then find $(AB)^T$.
\n**Example 6:**
\n**Example 1**
\n**Example 1**
\n**Example 1**
\n**Example 2**
\n**Example 3**
\n**Example 4**
\n**Example 5:**
\nIf $A = \begin{bmatrix} 1 & 2 \\ 9 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 16 \\ 9 & 12 \end{bmatrix}$
\n \therefore $(AB)^T = \begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$
\n \therefore $(AB)^T = \begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$
\n \therefore $(A^T)^T = A_1^T A_2^T B_3^T B_4^T B_5^T B_6^T B_7^T B_7^T B_8^T B_8^T B_9^T B_9^T B_9^T B_9^T B_9^T B_9^T B_9$

SYMMETRICAND SKEW - SYMMETRIC MATRIX

Symmetric Matrix: A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = a_{ji}$ for all i, j or $A^T = A$

$$
\begin{bmatrix} a & h & g \ h & b & f \ g & f & c \end{bmatrix}
$$

NOTE

- (i) Every unit matrix and square zero matrix are symmetric matrices.
- (ii) Maximum number of different element in a symmetric matrix $+1)$

2 a set of \sim 3 a set of \sim

Skew-Symmetric Matrix : A square matrix $A = [a_{ij}]$ is called
skew-symmetric matrix, if $a_{ij} = -a_{ij}$ for all i, i or $A^T = -A$ skew-symmetric matrix. if $a_{ii} = -a_{ii}$ for all i, j or $A^T = -A$

NOTE

(i) All Principal diagonal elements of a skew-symmetric matrix are always zero because for any diagonal element

$$
a_{ii} = -a_{ii} \implies a_{ii} = 0
$$

(ii) Trace of a skew symmetric matrix is always 0

Properties of Symmetric and skew-symmetric matrices

- (i) If A is a square matrix, then $A + A^T$, AA^T , A^T A are symmetric $\overrightarrow{(v)}$ matrices while $A - A^T$ is Skew-Symmetric Matrices.
- (ii) If A is a Symmetric Matrix, then $-A$, KA, A^T , $Aⁿ$, $A⁻¹$, B^T (vii) AB are also symmetric matrices where $n \in N$, $K \in R$ and B is a square matrix of order that of A
- T **STUDY MATERIAL:** M.

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) If A is a skew symmetric matrix, then $(vi) (kA) = k(A)^T = k(A)^T$

(iv) $k(A) = m(A)^T$

(iv) $k(A) = m(A)^T$

(c) $k(A) = m(A)^T$

(c) $k(A) = m(A)^T$

(d) $B^T AB$ is a skew-symmetric matrix where
 STUDY MATERIAL: M

(ii) $(A \pm B)^T = A^T \pm B^T$

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a shew symmetric matrix, then = N

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a skew-symmetric matric stor

(c) kA is also skew - symmetric matrix **STUDY MATERIAL: MATE STUDY MATERIAL: (v) (kA)^T = k₀J^T = EXERIBING**

Tries of Transpose

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(A \pm B)^T = A^T \pm B^T$

(b) $A^2 A^2$ is a symmetric matrix of the n=
 $= 1$
 $\frac{1}{1} \Delta_$ **STUDY MATERIAL: MATE STUDY MATERIAL: MATH**

(ii) $(A \pm B)^T = A^T \pm B^T$

(iv) $(kA)^T = k(A)^T$

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a symmetric matrix, then $\in N$

(vi) tr $(A) = \text{tr}(A)^T$

(c) $A^T A_{n-1}^T ... A_3^T A_2^T A_1^T$

(d) $B^T AB$ is also skew-symmet **STUDY MATERIAL: MATHEMATICS**

(ii) $(A \pm B)^T = A^T \pm B^T$ (a) A^2a^{2n+1} is a symmetric matrix, then –

(iv) $(kA)^T = k(A)^T$ (b) A^2a^{2n+1} is a skew-symmetric matrix for $n \in \mathbb{N}$

(vi) $\ln(A) = \ln(A)^T$ (c) kA is also skew-s **STUDY MATERIAL: MATHEMATICS**
 $\pm B^T$ (ii) If A is a skew symmetric matrix, then $-$
 $(2) A^{2n}$ is a symmetric matrix for $n \in \mathbb{N}$

(b) A^{2n+1} is a skew-symmetric matrices for $n \in \mathbb{N}$

(c) A^{2n+1} is a skew-**STUDY MATERIAL: MATHEMATICS**

(ii) If A is a skew symmetric matrix, then -
 $T_{\pm}B^{T}$ (a) A^{2n} is a symmetric matrix for n $\in N$

(b) A^{2n+1} is a skew-symmetric matrix where $k \in R$

(c) kA is also skew - symmetri **STUDY MATERIAL: MATHEMATICS**

T+B^T (a) If A is a skew symmetric matrix, then –

(a) A^{2n} is a symmetric matrix for n \in N

(b) A^{2n+1} is a skew-symmetric matrices for n \in N
 ${}^{T}A_2{}^{T}A_1{}^{T}$ (c) kA is al **STUDY MATERIAL: MATI

(ii)** $(A \pm B)^T = A^T \pm B^T$

(iii) A^2^n is a symmetric matrix, then –

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a skew-symmetric matrix for n e N

(vi) It $(A) = \text{tr}(A)^T$

(c) k as lass bsew-symmetric matrix **STUDY MATERIAL: MATE**

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) If A is a skew symmetric matrix, then -

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a skew-symmetric matrix for n e N

(c) kA is a skew-symmetric matrix where k
 $= A_n^T A_{n-1$ **STUDY MATERIAL: MATHI**

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) If A is a skew symmetric matrix, then –

(iv) $(kA)^T = k(A)^T$

(b) A^2 n is a skew-symmetric matrix for n \in

(vi) If $(A) = tr(A)^T$

(c) kA is a skew-symmetric matrix wh **STUDY MATERIAL: MATHI**

(ii) $(A \pm B)^T = A^T \pm B^T$

(iv) $B^T = A^T \pm B^T$

(iv) $B^T = A^T \pm B^T$

(iv) $B^T = A^T \pm (A)^T$

(iv) $B^T = A^T \pm (A)^T$

(iv) $B^T = A^T A_{n-1}^T \pm A_1^T$

(b) A^{2n+1} is a skew-symmetric matrix where k =
 $T =$ spose

(ii) $(A \pm B)^T = A^T \pm B^T$

(ii) If A is a skew symmetric matrix for n = N

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a symmetric matrix for n = N

(b) A^{2n+1} is a symmetric matrix for n = N

(b) A^{2n+1} is a symmetric m **spose**

(ii) $(A \pm B)^T = A^T \pm B^T$

(iv) $k(A)^T = k(A)^T$

(b) A^{2n+1} is a skew-symmetric matrix of $n \in \mathbb{N}$
 A_n)^T = $A_n^{-T} A_{n-1}^{-T} \dots A_3^{-T} A_2^{-T} A_1^{-T}$

(b) A^{2n+ **STUDY MATERIAL:**
 STUDY MATERIAL:
 STUDY MATERIAL:
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 STUDY MATERIAL:
 A_{n-1} (i) $(kA)^T = k(A)^T$ (b) A^{2n+1} is a skew-symmetric matrix for n $\in \mathbb{N}$
 A_{n-1} A_n)^T = A_n **SIUDY MATERIAL: N**
 SIUDY MATERIAL: N

(ii) $(A \pm B)^T = A^T \pm B^T$

(iv) $(kA)^T = k(A)^T$

(iv) $(kA)^T = k(A)^T$

(b) A^{2n+1} is a skew-symmetric matrics, then -

(vi) Ir $(A) = \text{tr}(A)^T$

(c) kA is also skew-symmetric matrics whe (i) $(A \pm B)^T = A^T + B^T$

(ii) $F(A \pm B)^T = A^T + B^T$

(ii) $B(A \pm B)^T = A^T + B^T$

(ii) $B(A \pm B)^T = A^T + B^T$

(iv) $B(A \pm B)^T = A^T + B^T$

(c) kA is a skew-symmetric matrix where $k \in R$
 $= A_n^{-T} A_{n-1}^{-T} \dots A_3^{-T} A_2^{-T} A_1^{-T}$
 $= A_n^{-T} A_{n-1}^{$ ii) $(A \pm B)^T = A^T \pm B^T$

(a) A^{2n} is a symmetric matrix, then $=$ Niv) $(kA)^T = k(A)^T$

(b) A^{2n} is a skew-symmetric matrices for $n \in \mathbb{N}$

vi) If $(A) = tr(A)^T$

(c) $kA^{n-1}A_{n-1}T...A_3^{-1}A_2^{-1}A_1^{-T}$

(d) B^T AB is also **STUDY MATERIAL: MATHEMATICS**

(ii) $(A \pm B)^T = A^T + B^T$

(iii) If A is a skew symmetric matrix, then $-(a) \lambda^{2n+1}$ is a symmetric matrix for n e N

(v) $(\kappa A)^T = k(A)^T$

(v) $k^2 - k(A)^T$

(v) $k^2 - k(A)^T$

(c) kA is also skew-sym **STUDY MATERIAL:** MATHEMATICS

(ii) $(A \pm B)^T = A^T + B^T$

(iv) $(kA)^T$

(iv) $(kA)^T$

(iv) $k^T = k(A)^T$

(b) A^{2n} is a symmetric matrix of ne N

(v) $(kA)^T - k(A)^T$

(b) A^{2n+1} is a skew-symmetric matrix where $k \in \mathbb{R}$
 $A_n^$ (iii) If A is a skew symmetric matrix, then – (a) A^{2n} is a symmetric matrix for $n \in N$ (b) A^{2n+1} is a skew-symmetric matrices for $n \in N$ (c) kA is also skew -symmetric matrix where $k \in R$ (d) $B^T AB$ is also skew - symmetric matrix where B is a square matrix of order that of A. **STUDY MATERIAL: MATHEMATICS**

a skew symmetric matrix, then –

ⁿ is a symmetric matrix for n = N

ⁿ⁺¹ is a skew-symmetric matrix shore n = N

is also skew-symmetric matrix where k = R

is also skew-symmetric matrix w **STUDY MATERIAL: MATHEMATICS**

s a skew symmetric matrix, then -

²ⁿ is a symmetric matrix for n \in N

²ⁿ⁺¹ is a skew-symmetric matrices for n \in N

A is also skew -symmetric matrix where k \in R

A is also skew **DY MATERIAL: MATHEMATICS**

c matrix, then –

c matrix, then –

matrix for n = N

mmetric matrices for n = N

mmetric matrix where k = R

v - symmetric matrix where B is a

that of A.

ric matrices, then –

also symmetric **IDY MATERIAL: MATHEMATICS**

ric matrix, then—

matrix for n \in N

mmetric matrices for n \in N

mmetric matrix where k \in R

w - symmetric matrix where B is a

that of A.

tric matrices, then—

e also symmetric matr as a skew-symmetric matrices for $n \in N$
is a skew-symmetric matrices for $n \in N$
also skew - symmetric matrix where $k \in R$
B is also skew - symmetric matrix where B is a
datrix of order that of A.
intitive for the actions, is a skew symmetric matrix, then –

is a symmetric matrix for n ∈ N

is a skew-symmetric matrices for n ∈ N

is also skew -symmetric matrices for n ∈ N

A B is also skew - symmetric matrix where k ∈ R

AB is also skew is a symmetric matrix for $n \in \mathbb{N}$
 $+1$ is a skew-symmetric matrices for $n \in \mathbb{N}$

is also skew-symmetric matrix where $k \in \mathbb{R}$

AB is also skew-symmetric matrix where B is a

matrix of order that of A.

are two
	- (iv) If A, B are two symmetric matrices, then (a) A \pm B , AB + BA are also symmetric matrices. (b) AB – BA is a skew-symmetric matrix (c) AB is a symmetric matrix when $AB = BA$. also skew -symmetric matrix where $k \in R$

	B is also skew - symmetric matrix where B is a

	natrix of order that of A.

	re two symmetric matrices, then—

	i, AB + BA are also symmetric matrices.

	BA is a skew-symmetric matri
	- (v) If A, B are two skew-symmetric matrices, then (a) A \pm B, AB – BA are skew-symmetric matrices (b) AB + BA is a symmetric matrix
	- (vi) If A is a skew-symmetric matrix and C is a column matrix, then $C^T AC$ is a zero matrix.
	- (vii) Every square matrix A can uniquely be expressed as sum of a symmetric and skew symmetric matrix i.e.

$$
A = \left[\frac{1}{2}(A + A^{T})\right] + \left[\frac{1}{2}(A - A^{T})\right]
$$

Example 7 :

If
$$
A = \begin{bmatrix} -1 & 7 \\ 2 & 3 \end{bmatrix}
$$
, then find skew-symmetric part of A.

Sol. Let $A = B + C$, where $B = \frac{1}{2}(A + A^{T})$ and $C = \frac{1}{2}(A - A^{T})$ are $\frac{1}{2}$ (A + A^T) and C = $\frac{1}{2}$ (A - A^T) are $\frac{1}{2}$ (A-A^T) are respectively symmetric and skew-symmetric parts of A. save symmetric matrix

service matrix

then AB = BA.

kew-symmetric matrix

symmetric matrix

metric matrix

metric matrix

metric matrix

metric matrix

and C is a column matrix,

zero matrix.

atrix A can uniquely be ex skew-symmetric matrices, then –

BA are skew-symmetric matrices

a symmetric matrix

a symmetric matrix

zero matrix.

zero matrix.

and skew symmetric matrix i.e.

and skew symmetric matrix i.e.
 $\begin{bmatrix} \Gamma \end{bmatrix} + \begin{bmatrix} 1$ B + BA are also symmetric matrices.

is a skew-symmetric matrix

constants when AB = BA.

constants when AB = BA.

constants when AB = BA.

3 – BA are skew-symmetric matrices

is a symmetric matrix

is a zero matrix.

is A is a symmetric matrix

w-symmetric matrix

cive a column matrix,

ci is a zero matrix.

are matrix A can uniquely be expressed as sum

tric and skew symmetric matrix i.e.
 $+A^T$) + $\left[\frac{1}{2}(A - A^T) \right]$
 $\left[\frac{1}{2}(A - A^$ w-symmetric matrix and C is a column matrix,

is a zero matrix.

re matrix A can uniquely be expressed as sum

ric and skew symmetric matrix i.e.
 (A^T) + $\left[\frac{1}{2}(A - A^T)\right]$
 $\left[\frac{1}{2}(A - A^T)\right]$
 $\left[\frac{1}{2}(A - A^T)\right]$
 $\$ e two second-structure matrices, then

AB – BA are skew-symmetric matrices

BA is a symmetric matrix

Rew-symmetric matrix

dev-symmetric matrix

C is a zero matrix.

C is a zero matrix.

C is a zero matrix.

C is a zero AB – BA are skew-symmetric matrices

BA is a symmetric matrix

bew-symmetric matrix

devantors are matrix.

C is a zero matrix.

uare matrix A can uniquely be expressed as sum

netric and skew symmetric matrix i.e.

A + A xammetric matrix

metric matrix

metric matrix,

ro matrix,

ro matrix,

rix A can uniquely be expressed as sum

skew symmetric matrix i.e.
 $\left| + \left[\frac{1}{2} (A - A^{T}) \right] \right|$

nen find skew-symmetric part of A.

re B = $\frac{1}{$ c matrix and C is a column matrix,
trix.
can uniquely be expressed as sum
v symmetric matrix i.e.
 $\frac{1}{2}(A - A^{T})$
and skew-symmetric part of A.
 $= \frac{1}{2}(A + A^{T})$ and $C = \frac{1}{2}(A - A^{T})$ are
and skew-symmetric parts of A.
 \begin symmetric matrices, then-

e skew-symmetric matrix

netric matrix

tric matrix and C is a column matrix,

matrix.

A can uniquely be expressed as sum

tew symmetric matrix i.e.
 $\left[\frac{1}{2}(A - A^{T})\right]$

find skew-symmetric p re skew-symmetric matrices

metric matrix

and C is a column matrix,

A can uniquely be expressed as sum

dew symmetric matrix i.e.
 $\left[\frac{1}{2}(A - A^{T})\right]$

find skew-symmetric part of A.
 $B = \frac{1}{2}(A + A^{T})$ and $C = \frac{1}{2}(A -$ ¹ 1 $\binom{7}{2}$, then find skew-symmetric part of A.

¹ + C, where B = $\frac{1}{2}$ (A + A^T) and C = $\frac{1}{2}$ (A - A^T) are

ely symmetric and skew-symmetric parts of A.
 $\frac{1}{2} \left\{ \begin{bmatrix} -1 & 7 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} -1 &$ 2 3 with the subset of A.
 $+C$, where $B = \frac{1}{2}(A + A^{T})$ and $C = \frac{1}{2}(A - A^{T})$ are

ely symmetric and skew-symmetric parts of A.
 $\frac{1}{2} \begin{bmatrix} -1 & 7 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 7 & 3 \end{bmatrix}$
 $\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} 0$ $A + C$, where $B = \frac{1}{2}(A + A^{T})$ and $C = \frac{1}{2}(A - A^{T})$ are

ely symmetric and skew-symmetric parts of A.
 $\frac{1}{2}\begin{bmatrix} -1 & 7 \ 2 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 2 \ 7 & 3 \end{bmatrix}$
 $\begin{bmatrix} 0 & 5 \ -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5/2 \ -5/2 & 0 \end{bmatrix}$

NT OF

$$
\begin{bmatrix} 2 & -1 \ -7 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 1 \ 7 & 2 \end{bmatrix} \text{ then find } B^{T}A^{T}
$$
\n
$$
\begin{bmatrix} 1 & 0 \ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -7 \ -1 & 4 \end{bmatrix} = \begin{bmatrix} 8-7 & -28+28 \ 2-2 & -7+8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}
$$
\n
$$
\begin{bmatrix} 1 & 0 \ 1 & 2 \end{bmatrix} \begin{bmatrix} 2(1) & -7 \ -1 & 4 \end{bmatrix} = \begin{bmatrix} 8-7 & -28+28 \ 2-2 & -7+8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}
$$
\nExample 7:
\n**RICANDSEVE-V-SVMMETRCMATRX**\n
$$
\begin{bmatrix} 1 & 6 \ -1 & 7 \ 1 & 2 \end{bmatrix}, \text{ then find skew-symmetric part of } A.
$$
\n**EXECANDSEVE-V-SVMMETRCMATRX**\n
$$
\begin{bmatrix} a & b & b \ b & c \end{bmatrix}
$$
\n
$$
\begin{bmatrix} a & b & b \ b & c \end{bmatrix}
$$
\n
$$
\begin{bmatrix} a & b & b \ b & c \end{bmatrix}
$$
\n
$$
\begin{bmatrix} a & b & b \ b & c \end{bmatrix}
$$
\n
$$
\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 \ 2 & 3 \end{bmatrix}, \text{ then find skew-symmetric part of } A.
$$
\n
$$
\begin{bmatrix} 1 & 6 & 1 \ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 7 \ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \ 2 &
$$

DETERMINANT OFA MATRIX

Now C=
$$
\frac{1}{2}
$$
 $\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$ = $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$
\n**EXAMPLEMINANT OFAMATRIX**
\nIf A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix, then its determinant, denoted by |A| or det (A) is defined as
\n $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$
\n**erties of the Determinant of a matrix :**
\n|A| exists \Leftrightarrow A is a square matrix
\n|AB| = |A||B|

determinant, denoted by $|A|$ or det (A) is defined as

$$
|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$

Properties of the Determinant of a matrix :

- (i) $|A|$ exists \Leftrightarrow A is a square matrix
- (ii) $|AB| = |A||B|$
- (iii) $|A^T| = |A|$
- (iv) $|kA| = k^n |A|$, if A is a square matrix of order n.
- If A and B are square matrices of same order then $|AB| = |BA|$
- (vi) If A is a skew symmetric matrix of odd order then $|A|=0$
- (vii) If $A = diag(a_1, a_2, \dots, a_n)$ then $|A| = a_1 a_2 \dots a_n$
- (viii) $|A|^n = |A^n|, n \in N$.

MATRICES AND DETERMINANTS

If every element of a square matrix A be replaced by its cofactor in $|A|$, then the transpose of the matrix so obtained S_6 is called the adjoint of matrix A and it is denoted by adj A. Thus if $A = [a_{ij}]$ be a square matrix and F_{ii} be the cofactor of a_{ij} in | A | , then Adj. $A = [F^{ij}]^T$

ATRICES AND DETERMINANTS)
\n**IONTOFAMATRX**
\nIf every element of a square matrix A be replaced by its
\ncofactor in |A|, then the transpose of the matrix so obtained
\nis called the adjoint of matrix A and it is denoted by adj A.
\nThis if A = [a_{ij}] be a square matrix and F_{ij} be the cofactor
\nof a_{ij} in |A|, then Adj. A = [F^{ij}]^T
\nIf A =
$$
\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}
$$
, then adj. A = $\begin{bmatrix} F^{11} & F^{12} & \ldots & F^{1n} \\ F^{21} & F^{22} & \ldots & F^{2n} \\ F^{n1} & F^{n2} & \ldots & F^{nn} \end{bmatrix}$
\n**INVERSE OFAMATRX**
\nIf A and B are two matrices such that
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthen B is called the inverse of A and
\nthe inverse matrix of a given matrix
\nfor mula A⁻¹ = B \Leftrightarrow AB = 1 = BA
\nfor find inverse matrix of a given matrix
\nfor mula A⁻¹ = $\frac{adj(A)}{A}$. Thus A⁻¹ exist
\nfor example, A⁻¹ = $\frac{adj(A)}{A}$. Thus A⁻¹ exists
\nand (A⁻¹) = |A|ⁿ⁻¹
\nand (A⁻¹) = |A|ⁿ⁻¹
\nand (A⁻¹) = |A|ⁿ⁻¹
\nand (A⁻¹) = |A|ⁿ⁻¹
\nand (A⁻¹) = (adj A)ⁿ
\nand (A⁻¹) = (adj A)ⁿ
\nand (A⁻¹) = 0
\nand (A⁻¹) = 0
\nand (A⁻¹) = 0
\nand (A⁻¹

Properties of adjoint matrix :

If A, B are square matrices of order n and I_n is corresponding unit matrix, then

- (i) $A(adj.A) = |A| I_n = (adj A) A$
- (ii) $| \text{adj } A | = |A|^{n-1}$
- (Thus A (adj A) is always a scalar matrix) (iii) adj (adj A) = $|A|^{n-2} A$

(iv) | adj (adj A) $= |A|^{(n-1)^2}$ 2

- (v) adj $(A^T) = (adj A)^T$
- (vi) adj $(AB) = (adj B) (adj A)$
- (vii) adj $(A^m) = (ad \nvert A)^m$, $m \in N$
- (viii) adj (kA) = k^{n-1} (adj. A), $k \in R$
- (ix) adj $(I_n) = I_n$
- (x) adj $0 = 0$
- (xi) A is symmetric \Rightarrow adj A is also symmetric
- (xii) A is diagonal \Rightarrow adj A is also diagonal
- (xiii) A is triangular \Rightarrow adj A is also triangular
- (xiv) A is singular \Rightarrow | adj A | = 0

Example 8 :

Propence Var in
$$
A = 0
$$
 in $A = 0$ **in** $A = 1$ **in** $$

$$
=8\begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix} = 16\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}
$$

Example 9 :

If
$$
A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}
$$
, then find | adj (adj A)|.

T = 2⁴ = 16 11 12 1n 21 22 2n n1 n2 nn F F ...F F F ...F F F ...F ^T 3 2 4 9 ⁼ 3 4 2 9 **Sol.** |A| = 1 0 3 2 1 1 0 0 2 = 2 |adj (adj A) | = 2 2 (n 1) 2 | A | | A | [Here n = 3] | A | . Thus A–1 exists | A | ⁰

INVERSE OFA MATRIX

If A and B are two matrices such that $AB = I = BA$ then B is called the inverse of A and it is denoted by A^{-1} . , thus $A^{-1} = B \iff AB = I = BA$ To find inverse matrix of a given matrix A we use following

thus A⁻¹ = B
$$
\Leftrightarrow
$$
 AB = I = BA
\nTo find inverse matrix of a given matrix A we use following
\nformula A⁻¹ = $\frac{adj.A}{|A|}$. Thus A⁻¹ exists $\Leftrightarrow |A| \neq 0$
\n**Note :** (i) Matrix A is called invertible if A⁻¹ exists.
\n(ii) Inverse of a matrix is unique.
\n**erties of Inverse Matrix :**
\nLet A & B are two invertible matrices of the same order,
\nthen
\n(A^T)⁻¹ = (A⁻¹)^T
\n(AB)⁻¹ = B⁻¹A⁻¹
\n(A^k)⁻¹ = (A⁻¹)^k, k ∈ N
\nadj (A⁻¹) = (adj A)⁻¹
\n(A⁻¹)⁻¹ = A
\n|A⁻¹| = $\frac{1}{|A|} = |A|^{-1}$
\nIf A = diag (a₁, a₂,..., a_n), then A⁻¹
\n= diag (a₁⁻¹, a₂⁻¹, a_n⁻¹)
\nAs symmetric matrix \Rightarrow A⁻¹ is symmetric matrix.
\nA is triangular matrix and | A | ≠ 0 \Rightarrow A⁻¹ is a triangular

Note : (i) Matrix A is called invertible if A^{-1} exists. (ii) Inverse of a matrix is unique.

Properties of Inverse Matrix :

Let $A \& B$ are two invertible matrices of the same order, then

- (i) $(A^T)^{-1} = (A^{-1})^T$
- (ii) $(AB)^{-1} = B^{-1}A^{-1}$
- (iii) $(A^k)^{-1} = (A^{-1})^k$, $k \in N$
- (iv) adj $(A^{-1}) = (adj A)^{-1}$
- (v) $(A^{-1})^{-1} = A$

(vi)
$$
|A^{-1}| = \frac{1}{|A|} = |A|^{-1}
$$

(vii) If A = diag (a¹ ,a²,aⁿ), then A–1 = diag (a¹ –1, a² –1,aⁿ –1)

- (viii) A is symmetric matrix \Rightarrow A⁻¹ is symmetric matrix.
- (ix) A is triangular matrix and $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular matrix. n A⁻¹

".......a_n⁻¹)

¹ is symmetric matrix.
 $\lambda | \neq 0 \Rightarrow A^{-1}$ is a triangular

calar matrix

is diagonal matrix

0.

2 -3

-4 2

2 m |A|= −8

3

2 A⁻¹

......a_n⁻¹)

is symmetric matrix.
 $|\neq 0 \Rightarrow A^{-1}$ is a triangular

calar matrix

is diagonal matrix

0.

2

4 2

4 2

n |A|=-8

3

2

3

3 en A⁻¹

.........a_n⁻¹)

⁻¹ is symmetric matrix.

|A | ≠ 0 ⇒ A⁻¹ is a triangular

scalar matrix
 \neq 0.

-4 2

-4 2

hen |A | = -8

3 en A⁻¹

........a_n⁻¹)

-¹ is symmetric matrix.

|A | ≠ 0 ⇒ A⁻¹ is a triangular

scalar matrix

+ 0.

-4 0.

[2 -3]

-4 2]

hen |A | = -8

3

2 then A⁻¹

,a_n⁻¹)

A⁻¹ is symmetric matrix.
 $| |A | \neq 0 \Rightarrow A^{-1}$ is a triangular

is scalar matrix
 x^{-1} is diagonal matrix
 $| \neq 0$.
 $\left[\begin{matrix} 2 & -3 \\ -4 & 2 \end{matrix}\right]$

then $|A| = -8$
 $\begin{matrix} 2 & 3 \\ 4 & 2 \end{matrix}$
 then A⁻¹

⁻¹,a_n⁻¹)

A⁻¹ is symmetric matrix.

Id $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

¹ is scalar matrix

A⁻¹ is diagonal matrix

A| \neq 0.

A| \neq 0.

Frack C₋₄ 2

A₁, then $|A| = -8$
 $\begin{bmatrix}$ then A⁻¹

⁻¹,a_n⁻¹)

A⁻¹ is symmetric matrix.

Id $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

¹ is scalar matrix

A⁻¹ is diagonal matrix

A₁ \neq 0.

A₁ \neq 0.

A₁ \neq 0.

A₁ $\left[\begin{array}{cc} 2 & -3 \\ -4 &$ A⁻¹ is symmetric matrix.

d | A | \neq 0 \Rightarrow A⁻¹ is a triangular

is scalar matrix

A⁻¹ is diagonal matrix

A | \neq 0.

f $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

, then | A | = -8
 $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$
 $\frac{1}{8} \begin{bmatrix} 2$ hen A⁻¹

,a_n⁻¹)
 A^{-1} is symmetric matrix.
 $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

s scalar matrix
 $\begin{bmatrix} -1 \\ +0 \end{bmatrix}$ is diagonal matrix
 $|\neq 0$.
 $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

then $|A| = -8$
 $\begin{bmatrix} 2 & 3 \\ 4 &$ A^{-1} is symmetric matrix.
 $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

scalar matrix
 $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is diagonal matrix
 $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

then $|A| = -8$
 $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$
- (x) A is scalar matrix \Rightarrow A⁻¹ is scalar matrix
- (xi) A is diagonal matrix \Rightarrow A⁻¹ is diagonal matrix
- (xii) $AB = AC \implies B = C$, iff $|A| \neq 0$.

Example 10 :

Find the inverse matrix of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

Sol. Let the given matrix is A, then $|A| = -8$

ER

(v) $(A^{-1})^{-1} = A$

also symmetric

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

also tiangular

(vii) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(vii) $|A = \text{diag}(a_1, a_2, ..., a_n)$, then A^{-1}
 $= \text{diag}(a_1^{-1}, a_2^{-1},, a_n^{-1})$

(viii) A is symmetric matrix $\Rightarrow A$ also symmetric

also symmetric

also diagonal

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(dii) If A = diag (a₁, a₂,...., a_n, then A⁻¹
 $=$ diag (a₁⁻¹, a₂⁻¹,, a_n⁻¹)

(viii) A is symmetric matrix \Rightarrow also symmetric

also symmetric

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

also triangular

(vii) If A = diag (a₁, a₂, ..., a_n, then A⁻¹

diag (a₁⁻¹, a₂⁻¹, ..., ..., a_n⁻¹)

(viii) A is symmetric matrix \Rightarrow A⁻¹ s also symmetric

also diagonal

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

also diagonal

(0) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(1) If A = diag $(a_1, a_2, ..., a_n)$, then A^{-1}
 $= diag (a_1^{-1}, a_2^{-1}, ..., a_n^{-1})$

(iii) A is symmetric matrix $\Rightarrow A^{-1}$ is sy and adj A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}^T$ = $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ = $|A|^{-1}$

1, a₂...., a_n), then A⁻¹

c matrix \Rightarrow A⁻¹ is symmetric matrix.

r matrix and $|A| \neq 0 \Rightarrow$ A⁻¹ is a triangular

trix \Rightarrow A⁻¹ is scalar matrix

matrix \Rightarrow A⁻¹ is diagonal matrix

matrix \Rightarrow A 1, a₂.....,a_n), then A⁻¹

c matrix \Rightarrow A⁻¹ is symmetric matrix.

r matrix and |A | ≠ 0 \Rightarrow A⁻¹ is a triangular

trix \Rightarrow A⁻¹ is scalar matrix

matrix \Rightarrow A⁻¹ is diagonal matrix

matrix \Rightarrow A⁻¹ is dia $\begin{aligned}\n\begin{aligned}\n\mathbf{a} &= |\mathbf{A}|^{-1} \\
\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\end{aligned}\n\end{aligned}$ A₁, $\mathbf{a}_2, \dots, \mathbf{a}_n, \mathbf{b}_n$ then \mathbf{A}^{-1}
 $\mathbf{a} \mathbf{g} \left(\mathbf{a}_1^{-1}, \mathbf{a}_2^{-1}, \dots, \mathbf{a}_n^{-1} \right)$
 $\mathbf{b} \mathbf{c} \mathbf{a}_1 \mathbf{b}_1 \mathbf{c}_2 \mathbf{c}_3$ $\begin{aligned}\n\begin{bmatrix}\n= |A|^{-1} \\
1, a_2, ..., a_n\n\end{bmatrix},\n\end{aligned}$ then A^{-1}

ag $(a_1^{-1}, a_2^{-1}, \dots, a_n^{-1})$

ic matrix $\Rightarrow A^{-1}$ is symmetric matrix.

ar matrix and $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

atrix $\Rightarrow A^{-1}$ is scalar matrix
 $B = C$, iff $\therefore A^{-1} = \frac{1}{|A|}$ adj $A = -\frac{1}{8} \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ netric matrix \Rightarrow A⁻¹ is symmetric matrix.
gular matrix and $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular
r matrix \Rightarrow A⁻¹ is scalar matrix
mal matrix \Rightarrow A⁻¹ is diagonal matrix
 \Rightarrow B = C, iff $|A| \neq 0$.
nverse matrix of $\$ g $(a_1^{-1}, a_2^{-1}, \dots, a_n^{-1})$

anatrix $\Rightarrow A^{-1}$ is symmetric matrix.

anatrix and $|A| \neq 0 \Rightarrow A^{-1}$ is a triangular

rix $\Rightarrow A^{-1}$ is scalar matrix

matrix $\Rightarrow A^{-1}$ is diagonal matrix
 $B = C$, iff $|A| \neq 0$.

se matrix of $\begin{bmatrix$

Example 11 :

EXAMPLEARINIS
\n**Example 11 :**
\n**Step-V :** Introduce unity at the intersect
\nthird column.
\n**Step-V :** Finally introduce zeros at all
\n
$$
\text{Step-V1 : Finally introduce zeros at all\nthird column except at the intersection\nthird column.\nExample 12 :\n1. $M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$
\n**Example 13 :**
\n**Example 12 :**
\nUsing elementary transformation, find
\nmatrix $A = \begin{bmatrix} a & b \\ c & \frac{1 + bc}{a} \end{bmatrix}$.
$$

EXAMPLE 13
\n**Example 11:**
\n**Example 13**
\nIf
$$
A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}
$$
, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M=AB, then find M⁻¹.
\n**Example 12:**
\n**Sol.** $M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$
\n $|M| = 6$, adj $M = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$
\n**Method of finding the inverse of a matrix by Elementary transformation:**
\n**Method of finding the inverse of a matrix by Elementary transformation:**
\nLet A be a non-singular matrix of order n. Then A can be
\nreduendone identity, the inverse of a matrix is a finite sequence of
\nelementary transformation only A's we have discussed.

Method of finding the inverse of a matrix by Elementary transformation :

Let A be a non singular matrix of order n. Then A can be reduced to the identity matrix I_n by a finite sequence of elementary transformation only. As we have discussed every elementary row transformation of a matrix is equivalent to pre-multiplication by the corresponding elementary matrix. Therefore there exist elementary matrices E_1, E_2, \dots, E_4 such that $(E_k E_{k-1} \dots E_2 E_1) A = I_n$
 $\Rightarrow (E_k E_{k-1} \dots E_2 E_1) A A^{-1} = I_n A^{-1}$ (post multiplying by A^{-1})

⇒
$$
(E_k E_{k-1} \dots E_2 E_1) I_n = A^{-1}
$$

\n(∴ $I_n A^{-1} = A^{-1}$ and $AA^{-1} = I_n$)
\n⇒ $A^{-1} = (E_k E_{k-1} \dots E_2 E_1) I_n$

Algorithm for finding the inverse of a non singular matrix by elementary row transformations :

Let A be non-singular matrix of order n

Step-I : Write $A = I_n A$

Step-II : Perform a sequence of elementary row operations successively on A on the LHS and the prefactor I_n on the RHS till we obtain the result $I_n = BA$ **Step-III :** Write $A^{-1} = B$

The following steps will be helpful to find the inverse of a square matrix of order 3 by using elementary row transformations.

Step-I : Introduce unity at the intersection of first row and first column either by interchanging two rows or by adding a constant multiple of elements of some other row to first (i) row.

Step-II : After introducing unity at $(1, 1)$ place introduce zeros at all other places in first column.

Step-III : Introduce unity at the intersection 2nd row and 2nd column with the help of 2nd and 3rd row.

Step-IV : Introduce zeros at all other places in the second column except at the intersection of $2nd$ and $2nd$ column

Step-V: Introduce unity at the intersection of 3rd row and third column.

STUDY MATERIAL: M
 SEP-V: Introduce unity at the intersection

third column.
 SEP-VI: Finally introduce zeros at all of
 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M=AB, then find M⁻¹.
 Example 12:
 Example 12:

Using elem **STUDY MATERIAL:** M
 SEP-V: Introduce unity at the intersection

third column.
 SEP-VI: Finally introduce zeros at all of
 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M=AB, then find M⁻¹.
 Example 12:

Using elementary transfor **STUDY MATERIAL:M**
 SEP-V: Introduce unity at the intersection

third column.
 SEP-VI: Finally introduce zeros at all of
 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M=AB, then find M⁻¹.
 Example 12:

Using elementary transform **STUDY MATERIAL: MANU STUDY MATERIAL: MANU STUDY MATERIAL: MANU SEEP-V : Introduce unity at the intersection

for 1

1 0

1 1 3

and M=AB, then find M⁻¹.

Example 12 :**
 Example 12 : STUDY MATERIAL: MATERIAL: MAT **STUDY MATERIAL: MANUSION STUDY MATERIAL: MANUSION SEP-V:** Introduce unity at the intersection
 $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M=AB, then find M⁻¹.
 Example 12:
 $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$
 E STUDY MATERIAL: MATHER

Step-V: Introduce unity at the intersection of 3rd

third column.
 Step-V: Introduce zeros at all other place
 $\begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$ and M=AB, then find M⁻¹.
 Example 12:

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 STUDY MATERIAL: MATHEM
 SEP-V: Introduce unity at the intersection of 3rd represented that the intersection of 3rd represented that dolumn.
 SEP-VI: Finally introduce zeros at all other places that dolumn.
 Exa STUDY MATERIAL: MATHEM
 SEP-V: Introdue unity at the intersection of 3rd re

third column.
 SEP-VI: Finally introduce zeros at all other places

third column.
 Example 12:

Using elementary transformation, find **Step-VI :** Finally introduce zeros at all other places in the third column except at the intersection of third row and third column. **STUDY MATERIAL: MATHEMATICS**

coduce unity at the intersection of 3^{rd} row and

anally introduce zeros at all other places in the

n except at the intersection of third row and

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that is the intersection of the inver **STUDY MATERIAL: MATHEMATICS**
vee unity at the intersection of 3rd row and
introduce zeros at all other places in the
ept at the intersection of third row and
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 $\frac{b}{a}$
 $\frac{1 + bc}{$ **STUDY MATERIAL: MATHEMATICS**
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troduce unity at the intersection of 3^{rd} row and
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entary transformation, find the inve **STUDY MATERIAL: MATHEMATICS**
troduce unity at the intersection of 3^{rd} row and
n.
inally introduce zeros at all other places in the
in except at the intersection of third row and
n.
entary transformation, find the inve **STUDY MATERIAL: MATHEMATICS**
 V : Introduce unity at the intersection of 3^{rd} row and

column.
 VI : Finally introduce zeros at all other places in the

column except at the intersection of third row and

column.
 STUDY MATERIAL: MATHEMATICS
troduce unity at the intersection of 3^{rd} row and
nn.
Finally introduce zeros at all other places in the
nn except at the intersection of third row and
nn.
nentary transformation, find the **STUDY MATERIAL: MATHEMATICS**

-V: Introduce unity at the intersection of 3^{rd} row and

column.

-VI: Finally introduce zeros at all other places in the

column except at the intersection of third row and

column.

2:
 STUDY MATERIAL: MATHEMATICS
 -V : Introduce unity at the intersection of 3rd row and

column.
 -VI : Finally introduce zeros at all other places in the

column except at the intersection of third row and

column.
 STUDY MATERIAL: MATHEMATICS

-V: Introduce unity at the intersection of 3rd row and

column.

-VI: Finally introduce zeros at all other places in the

column except at the intersection of third row and

column.

2:

g **STUDY MATERIAL: MATHEMATICS**
duce unity at the intersection of 3rd row and
ally introduce zeros at all other places in the
except at the intersection of third row and
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intersection of 3rd row and
os at all other places in the
rsection of third row and
on, find the inverse of the
 $\left[\frac{1}{R_1} \rightarrow \frac{R_1}{R_2}\right]$ unity at the intersection of 3rd row and
throduce zeros at all other places in the
pt at the intersection of third row and
transformation, find the inverse of the
 $\frac{b}{a-b}$
 $\frac{b}{a}$
 $\left[\frac{1}{a} \quad 0\right]$
 A
 $\left[R_1 \rightarrow \frac{R$

Example 12 :

.

Using elementary transformation, find the inverse of the

matrix
$$
A = \begin{bmatrix} a & b \\ c & \left(\frac{1+bc}{-a}\right) \end{bmatrix}
$$
.

Sol.
$$
A = \begin{bmatrix} a & b \\ c & \left(\frac{1+bc}{a}\right) \end{bmatrix}
$$

We write,
$$
\begin{bmatrix} a & b \\ c & \left(\frac{1+bc}{a}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A
$$

Step-V: Introduce unity at the intersection of 3rd row and third column.
\n**Step-V1**: Finally introduce zeros at all other places in the third column except at the intersection of third row and third column.
\n**lnple 12**:
\nUsing elementary transformation, find the inverse of the matrix
$$
A = \begin{bmatrix} a & b \ c & \left(\frac{1+bc}{-a}\right) \end{bmatrix}
$$
.
\n
$$
A = \begin{bmatrix} a & b \ c & \left(\frac{1+bc}{a}\right) \end{bmatrix}
$$
\nWe write,
$$
\begin{bmatrix} a & b \ c & \left(\frac{1+bc}{a}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix} A
$$
\n
$$
\Rightarrow \begin{bmatrix} 1 & \frac{b}{a} \\ c & \left(\frac{1+bc}{a}\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \rightarrow \frac{R_1}{a})
$$
\nor
$$
\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & \frac{1}{a} \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ \frac{-c}{a} & 1 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 - cR_1)
$$
\nor
$$
\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ -c & a \end{bmatrix} A \qquad (R_2 \rightarrow aR_2)
$$
\nor
$$
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} A \qquad (R_1 \rightarrow R_1 - \frac{b}{a}R_2)
$$

$$
or \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & \frac{1}{a} \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ -c & 1 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 - cR_1)
$$

or
$$
\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ -c & a \end{bmatrix} A
$$
 $(R_2 \rightarrow aR_2)$

$$
[A \times B] = \begin{bmatrix} 1 & b \\ c & d + bc \\ c & d + bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A
$$

\n
$$
\Rightarrow \begin{bmatrix} 1 & \frac{b}{a} \\ c & d + bc \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \rightarrow \frac{R_1}{a})
$$

\nor
$$
\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & \frac{1}{a} \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ -\frac{c}{a} & 1 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 - cR_1)
$$

\nor
$$
\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & 0 \\ -c & a \end{bmatrix} A \qquad (R_2 \rightarrow aR_2)
$$

\nor
$$
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1 + bc}{a} & -b \\ -c & a \end{bmatrix} A \qquad (R_1 \rightarrow R_1 - \frac{b}{a}R_2)
$$

\n
$$
\Rightarrow A^{-1} = \begin{bmatrix} \frac{1 + bc}{a} & -b \\ -c & a \end{bmatrix}
$$

\n**IE SPECIAL CASES OF MATRX**
\n**Orthogonal Matrix :** Aguare Matrix A is called orthogonal
\nif AA^T = I = A^TA i.e. if A⁻¹ = A^T
\nEx A =
$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
 is a orthogonal matrix because here
\nA⁻¹ = A^T

SOME SPECIAL CASES OF MATRIX

(i) Orthogonal Matrix : A square Matrix A is called orthogonal if $AA^T = I = A^T A$ i.e. if $A^{-1} = A^T$

Ex
$$
A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
 is a orthogonal matrix because here
 $A^{-1} = A^{T}$

MATRICES AND DETERMINANTS

(ii) Idempotent Matrix : A square matrix A is called an Idempotent Matrix if $A^2 = A$

Ex. $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is a Idempotent Matrix because here **ES AND DETERMINANTS**
 potent Matrix : A square matrix A is called an

botent Matrix if $A^2 = A$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 EXECUTE A
 EXECUT ES AND DETERMINANTS
 potent Matrix : A square matrix A is called an

botent Matrix if $A^2 = A$
 $1/2$ $1/2$ $1/2$ is a Idempotent Matrix because here
 component Matrix and
 component Matrix Ais called an involutory **ES AND DETERMINANTS**
 npotent Matrix : A square matrix A is called an

potent Matrix if A² = A
 $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is a Idempotent Matrix because here

A
 (xi) Rank of a Matrix : A number
 (xi) Ran **ES AND DETERMINANTS**
 outing the Matrix if $A^2 = A$

[1/2 1/2] is a Idempotent Matrix because here
 a
 is a Idempotent Matrix because here
 is a Idempotent Matrix because here
 is in Parally Allutory Matrix : A $A^2=$ **ND DETERMINANTS**

Int Matrix : A square matrix A is called an

It Matrix if $A^2 = A$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 2 & 5 & 3 & 1 \end{bmatrix}$

If Matrix is a Idempotent Matrix because here
 Solution $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 &$ **ND DETERMINANTS**

Int Matrix : A square matrix A is called an

11 Matrix if $A^2 = A$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 2 & 5 & 3 & 1 \end{bmatrix}$
 $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$ is a Idempotent Matrix because here
 Matrix: A square matrix A is c **ND DETERMINANTS**

Int Matrix : A square matrix A is called an

1/2 is a Idempotent Matrix because here
 $\begin{bmatrix}\n2 & 1 & 0 & -1 \\
1/2 & 5 & 3 & 1\n\end{bmatrix}$
 Matrix : A square matrix A is called an involutory
 Matrix : A square m **AND DETERMINANTS**

ent Matrix : A square matrix A is called an
 $2 \tln 1/2$
 $2 \tln 2$
 $1/2$
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 $1/2$
 $1/2$
 $1/2$

is a Idempotent Matrix because here

(xi) Rank of a Matrix : A number r is

(xi) Rank of a M **AND DETERMINANTS**
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ent Matrix : A square matrix A is called an

ent Matrix if $A^2 = A$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 2 & 1 & 2 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 Matrix : A square matrix A is ca **AND DETERMINANTS**
 AND DETERMINANTS
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 AND
 AND

(iii) Involutory Matrix : A square matrix A is called an involutory Matrix if $A^2 = I$ or $A^{-1} = A$

Ex
$$
A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
 is a Involutory Matrix.

(iv) Nilpotent Matrix : A square matrix A is called a nilpotent Matrix if there exist $p \in N$ such that $A^p = 0$

Example 2 Ex
$$
A = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}
$$
 is a Nilpotent Matrix

- **ND DETERMINANTS**
 Int Matrix : A square matrix A is called an

1/2] is a Idempotent Matrix because here
 $\begin{bmatrix}\n1/2 \\
1/2\n\end{bmatrix}$ is a Idempotent Matrix because here
 SUP ATRIX
 **EXECUTE 10 A⁻¹ = A

(SUP STARK of a NATE ANTIXE SET ANTIFORMUSE SET ANTIFORM ANTISHT AT A SQUARE METALLY ASSAULT ANTIFORM ANTISH AND MATTIX A SQUARE METALLY ASSAULT AND MATTIX A SQUARE METALLY ASSAULT AND METALLY AND METALLY AND METALLY AND METALLY AND MET AND DETERMINANTS**
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and **Matrix**: A square matrix A is called an
 $\frac{1}{2}$ 2 1/2] is a Idempotent Matrix because here
 EXECUTE AND MATRIX: A square matrix A is called an inv **(v) The conjugate of a Matrix :** The conjugate of a matrix A is a matrix \overline{A} whose each element is a conjugate complex number of corresponding element of Matrix A. **Note :** Conjugate transpose Matrix of matrix A is a Transpose Matrix of conjugate of matrix A and it is denoted by A* or A^{θ} . i.e. $A^* = (\overline{A})^T$ i.e. r (A) c \overline{A} whose each element is a conjugate complex

of Corresponding element of Matrix A.

of Corresponding element of Matrix A.

of Corresponding terms of matrix A.

of $\lambda^* = (\overline{A})^T$

(ii) The rank of every non null of corresponding element of Matrix A.

of conjugate transpose Matrix of matrix A sa Transpose

of conjugate of matrix A and it is denoted by A^{*} or
 $A^* = (\overline{A})^T$

iiii Elementary transformation
 $A^* = (\overline{A})^T$

iiiii E mjugate of a Matrix : The conjugate of a matrix A is
 $\sum_{i=1}^{n} P_{i} = 1$
 $\sum_{i=1}^{n} P_{i} = 1$
 2. Hence r(A) = 2.

ix \overline{A} whose each element is a conjugate complex

conjugate complex

corresponding element of Matrix A.

corresponding element of Matrix A.

corresponding element of Matrix A.

corresponding ele
- **(vi) Hermition Matrix :** A square Matrix is Hermition Matrix if $\theta = A$. i.e. $a_{ij} = \overline{a}_{ji} \forall i, j$
- **(vii) Skew Hermition Matrix :** A Square Matrix A is Skew-Hermition is $A = -A^{\theta}$ e.q. $a_{ij} = -\overline{a}_{ij} \forall i, j$.
- **(viii) Period of a Matrix :** If for any Matrix A, $A^{k+1} = A$ then k is called period of Matrix (where k is a least positive integer) Ex. If $A^3 = A$, $A^5 = A$, $A^7 = A$, then it is a periodic matrix Q.2 and $A^{2+1} = A$ so its period is $= 2$ (A)¹ (i.e. r(A)=r(A¹).
 Iatrix : A square Matrix is Hermition Matrix if
 $\vec{u} = \vec{a}_{j\parallel} \forall i, j$
 Iatrix : A Square Matrix A is Skew
 $A = -A^0$ e.q. $a_{ij} = -\vec{a}_{ij} \forall i, j$.
 IXVITYOURSEL
 IATRIX if for any Matrix **EXECUTE:**
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 $\vec{u} = \vec{a}_{ji} \forall i, j$
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 onjugate of matrix A and it is denoted by A^* or
 $\overline{A} = (\overline{A})^T$
 $\overline{A} = (\overline{A})^T$ Superior of Alternative Assessment and Substitution Matrix : A square Matrix is Hermition Matrix if $\sin \theta$ (ii) The rank of matrix.
 $\vec{A} = \vec{a} \vec{b} \vec{b} \vec{c}$ (ii) Elementary transformation do no
 $\vec{A} = -A^0$ e.g. $a_{$ = (A)⁻¹ (a) $\arctan x$ (a) = (a) $\arctan x$ (a) = ($A = -A^0$ e.q. a_{ij} $= -\overline{a}$ ij \forall i, j.
 Matrix: If for any Matrix $A, A^{k+1} = A$ then k is
 Matrix: If for any Matrix $A, A^{k+1} = A$ then k is
 $A^5 = A, A^7 = A, ...$then it is a periodic matrix
 $A^5 = A, A^7 = A, ...$the **Matrix :** If for any Matrix A, $A^{k+1} = A$ then k is

d of Matrix (where k is a least positive integer)
 $A^{5} = A$, $A^{7} = A$,.....then it is a periodic matrix
 $A^{5} = A$, $A^{7} = A$,.....then it is a periodic matrix
 $A^{5} = A$ **Matrix :** If for any Matrix A, $A^{k+1} = A$ then k is
 $A^5 = A$, $A^7 = A$, then it is a periodic matrix
 $A^5 = A$, $A^7 = A$, then it is a periodic matrix
 $A^5 = A$, $A^7 = A$, then it is a periodic matrix
 A 5 mition Matrix : A Square Matrix A is Skew-

is $A = -A^0$ e.q. $a_{ij} = -\overline{a}_{ij} \,\forall i, j$.
 (1) The matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ be a

Matrix : If for any Matrix A , $A^{k+1} = A$ then k is

do of Matrix (w is $A = -A^0$ e.q. $a_{ij} = -\overline{a}_{ij} \forall i,j$.
 Q.1 The matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ be a
 Matrix: If for any Matrix $A_i A^{k+1} = A$ then k is
 $A_i A^3 = A_i A^7 = A_i$,..... then it is a periodic matrix
 $A_i A^5 = A_i A$ Matrix: If for any Matrix A, $A^{k+1} = A$ then k is

do of Matrix (where k is a least positive integer)

by polynomial $f(x) = x^2 - 4x - 5$. Find A, $A^5 = A$, $A^7 = A$,..... then it is a periodic matrix

(A, $A^5 = A$, $A^7 = A$,....

(ix) Differentiation of a Matrix :

If A=
$$
\begin{bmatrix} f(x) & g(x) \\ h(x) & \ell(x) \end{bmatrix}
$$
 then $\frac{dA}{dx} = \begin{bmatrix} f'(x) & g'(x) \\ h'(x) & \ell'(x) \end{bmatrix}$ is a

differentiation of Matrix A

Ex. if
$$
A = \begin{bmatrix} x^2 & \sin x \\ 2x & 2 \end{bmatrix}
$$
 then $\frac{dA}{dx} = \begin{bmatrix} 2x & \cos x \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$. Find n

(x) Submatrix : Let A be m x n matrix, then a matrix obtained by leaving some rows or columns or both of A is called a sub matrix of A

Ex. if
$$
A' = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 2 \\ 2 & 5 & 3 \end{bmatrix}
$$
 and $\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}$ are sub matrices of

$$
Matrix A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}
$$

Rank of a Matrix : A number r is said to be the rank of a

 $\begin{array}{c|c}\n\text{OMADVANCEDIEARNING}\n\hline\n\text{DOMADVANCEDIEARNING}\n\end{array}$ 2 1 0 -1

2 5 3 1

Latrix : A number r is said to be the rank of a

Aif **SPON ADVANCED LEARNING**
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2 2 2 4

2 5 3 1
 Extrix: A number r is said to be the rank of a

Aif

quare sub matrix of order $(r + 1)$ or more is **SPONADVANCED LEARNING**

2 1 0 -1

2 3 2 2 4

2 5 3 1]
 Eatrix : A number r is said to be the rank of a

Aif **SPON ADVANCED LEARNING**
 $\begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 Matrix : A number r is said to be the rank of a

KA if

square sub matrix of order $(r + 1)$ or more is **(xi) Rank of a Matrix :** A number r is said to be the rank of a $m \times n$ matrix A if

(a) Every square sub matrix of order $(r + 1)$ or more is singular and

(b) There exists at least one square submatrix of order r which is non-singular. Thus, the rank of matrix is the order of the highest order non-singular sub matrix. **EDIMADVANCED LEARNING**
 EDIMADVANCED LEARNING
 x of order $(r + 1)$ or more is
 e square submatrix of order r

the rank of matrix is the order

gular sub matrix.
 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$ is
 (A) is **EDENADEABLEARNING**
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 EXECUTE ARRIVALUATED IN THE ARRY OF THE ARRY SUBSEMERTARY SUBSEMERTARY 3 4 5 5 6 16 16 16 1 SUPERBANDING

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SUPERBANDING

2 square submatrix of order r

the rank of matrix is the order

gular sub matrix.
 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$ is

(A) is less then 3, we observe

ar square sub matri **EDENTADYANCED LEARNING**
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 ODM ADVANCED LEARNING

there is said to be the rank of a

rix of order (r + 1) or more is

ne square submatrix of order r

rgular sub matrix.
 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$ is

r(A) i **Example 18**
 $xA = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$
 of a Matrix : A number r is said to be the rank of a

matrix A if

very square sub matrix of order $(r + 1)$ or more is

are and

are rank and the stree $\mathbf{x} = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$

of a Matrix : A number r is said to be the rank of a

matrix Aif

very square sub matrix of order (r + 1) or more is

are and

are mation

enere exists at least one **EXERCISE AND SUBLIMATE SET AND SUBLIMATE SET AND SUBLIMATION AND SUBDIMATED LARGED LEARNING

is** $A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$ **

and** A **of a Matrix**: A number r is said to be the rank of a

indirix **EXECUTE 18**
 $\begin{bmatrix}\n2 & 1 & 0 & -1 \\
3 & 2 & 2 & 4 \\
2 & 5 & 3 & 1\n\end{bmatrix}$
 k of a Matrix : A number r is said to be the rank of a

matrix A if

Every square sub matrix of order (r + 1) or more is

lar and

his non-singular. Thus,

Ex. The rank of matrix
$$
A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}
$$
 is

We have $|A| = 0$, therefore r(A) is less then 3, we observe

that $\begin{vmatrix} 5 & 6 \\ 4 & 5 \end{vmatrix}$ is a non-singular square sub matrix of order

2. Hence $r(A) = 2$.

Note: (i) The rank of the null matrix is not defined and the rank of every non null matrix is greater than or equal to one.

(ii) The rank of matrix is same as the rank of its transpose i.e. $r(A) = r(A^T)$.

(iii) Elementary transformation do not alter the rank of matrix.

TRY IT YOURSELF-1

volutiony Matrix.
\n
$$
x_0 = 0
$$
\n
$$
x_1 = 0
$$
\n
$$
x_2 = 0
$$
\n
$$
x_3 = 0
$$
\n
$$
x_4 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \end{bmatrix}
$$
\n
$$
x_5 = 2
$$
\n
$$
x_6 = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \end{bmatrix}
$$
\n
$$
x_7 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}
$$
\n
$$
x_8 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}
$$
\n
$$
x_9 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 5 & 5 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 5 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 5 & 5 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 5 & 5 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 6 & 6 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 6 \\ 10 & 1 & 6 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 6 \\ 1 & 2 & 2 & 1 \end{bmatrix}
$$
\n
$$
x_1 = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 6 \\
$$

polynomial $f(x) = x^2 - 4x - 5$. Find the trace of matrix A^3 . . **Q.2** If A, B are symmetric matrixes of same order than AB– BA

is a (A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

Q.3 The product of n matrices

$$
\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots \dots \begin{bmatrix} 1 & n \\ 0 & n \end{bmatrix}
$$
 is equal to

$$
\begin{bmatrix} 1 & 576 \\ 0 & 1 \end{bmatrix}
$$
. Find n

$$
a_{ij} = -\overline{a}_{ij} \forall i, j.
$$
\nQ.1 The matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ be a zero divisor
\nany Matrix A, A^{k+1} = A then k is
\nere k is a least positive integer)
\nso: m, m, then it is a periodic matrix
\n $\overline{a} = 2$
\n $\overline{b} = 2$
\n $\overline{c} = 2$
\n $\overline{d} = \begin{bmatrix} f'(x) & g'(x) \\ h'(x) & \ell'(x) \end{bmatrix}$ is a
\n(A) Skew symmetric matrix
\n(C) Zero matrix
\n(D) Identity matrix
\n $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & n \\ 0 & n \end{bmatrix}$ is equal to m
\n $\overline{a} = \begin{bmatrix} 2x & cos x \\ 2 & 0 \end{bmatrix}$
\n $\begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$. Find n
\nmatrix, then a matrix obtained
\n $\overline{a} = \begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}$ are sub matrices of
\n $\overline{b} = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$ and the transpose of matrix $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$
\n $\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}$ are sub matrices of

Q.5 If α and β are roots of the equation

S(TUDY MATERIAL: MATHEMATICS
\n**Q.5** If
$$
\alpha
$$
 and β are roots of the equation
\n
$$
\begin{bmatrix}\n1 & 25 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\n1 & -1 \\
2 & 0\n\end{bmatrix}^{10}\n\begin{bmatrix}\n0 & \frac{1}{2} \\
2 & 0\n\end{bmatrix}^{5}\n\begin{bmatrix}\nx^2 - 5x + 20 \\
x + 2\n\end{bmatrix} = [40]\n\begin{bmatrix}\n40 & 1 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\nx^2 - 5x + 20 \\
x + 2\n\end{bmatrix} = [40]\n\begin{bmatrix}\n40 & 1 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\nx^2 - 5x + 20 \\
x + 2\n\end{bmatrix} = [40]\n\begin{bmatrix}\n40 & 1 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\n5 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\nx^2 - 5x + 20 \\
x + 2\n\end{bmatrix} = [40]\n\begin{bmatrix}\n40 & 1 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\n5 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\n6 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{bmatrix}\n6 \\
-1 & \frac{1}{2}\n\end{bmatrix}\n\begin{
$$

then find the value of $(1 - \alpha) (1 - \beta)$.

Q.6 Using elementary transformation, find the inverse of the

SOLUTION MATERIAL: MATHEMATICS
\nQ.5 If
$$
\alpha
$$
 and β are roots of the equation
\n
$$
[1 \t25] \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}^{-10} \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x^2 - 5x + 20 \\ x + 2 \end{bmatrix} = [40] \qquad \qquad \mathbf{D} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$
\nthen find the value of $(1 - \alpha)(1 - \beta)$.
\nQ.6 Using elementary transformation, find the inverse of the
\nmatrix $A = \begin{bmatrix} a & b \\ c & \frac{1 + bc}{a} \end{bmatrix}$.
\nQ.7 If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then find the value of $|A|$ |adj A].
\nQ.8 Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.
\nD.8 Find Without finding B⁻¹, the value of K for which
\nX⁻¹ is the number itself.
\nEVALUATE: A
\nEVALUATE: A
\n $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \end{bmatrix}$, then find the value of |A| |adj A|.
\n $\begin{bmatrix} a & b \\ b_2 & c_2 \\ c_2 & 0 \end{bmatrix} = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & b_3 \end{bmatrix} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$
\n $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \end{bmatrix}$, then find the value of |A| |adj A|.
\n

Q.8 Matrices A and B satisfy $AB = B^{-1}$, where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

 $KA = 2B^{-1} + I = 0$

$$
\begin{bmatrix} -1 & \frac{1}{2} \end{bmatrix}^{L} \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}^{L} \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}^{L} \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}^{L} \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}
$$
\nmatrix A = \begin{bmatrix} a & b \\ c & d & b \end{bmatrix}

\nmatrix A = \begin{bmatrix} 1 & b \\ 0 & 0 & a \end{bmatrix}

\nMatrix A = \begin{bmatrix} 1 & b \\ c & d & d \end{bmatrix}

\nmatrix A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}

\nMatrix B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix B = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix C = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}

\nMatrix D = \begin{bmatrix} 1 +

DETERMINANTS

HISTORICAL DEVELOPMENT

Development of determinants took place while mathematicians were trying to solve a system of simultaneous linear equations.

$$
(4) \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}
$$
 (5) 51 (6) $\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$ So. $\begin{vmatrix} 1+cos\theta & sin\theta \\ sin\theta & 1-cos\theta \end{vmatrix} = (1+c)(7)a^9$ (8) 2
\n**EXECALDEVELOPMENTS**
\n**ORICALDEVELOPMENTS**
\nDetermine of determinants took place while mathematicians were trying to solve a system of simultaneous linear equations.
\n
$$
a_1x + b_1y = c_1
$$
\n
$$
e.g. a_2x + b_2y = c_2
$$
\n
$$
a_2x + b_2y = c_2
$$
\n
$$
f(x) = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$
\n
$$
f(x) = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$
\n
$$
f(x) = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$
\n
$$
f(x) = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$
\n
$$
f(x) = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$
\n
$$
f(x) = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}
$$
\n
$$
f(x) = \begin{vmatrix} a_1 & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
f(x) = \begin{vmatrix} a_1 & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
f(x) = \begin{vmatrix} a_1 & a_{12} & a_{13} \\ a_2 & a_{23} & a_{33} \end{vmatrix}
$$
\n
$$
f(x) = \begin{vmatrix} a
$$

Mathematicians defined the symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

determinant of order 2 and the four numbers arranged in row and column were called its elements. Its value was taken as $a_1b_2 - a_2b_1$ which is the same as denominator. This kind of definition helped then to state the solution of the simultaneous equation as

EXAMPLEMATICS
\nβ are roots of the equation
\n
$$
\frac{1}{2} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x^2 - 5x + 20 \\ x + 2 \end{bmatrix} = [40]
$$
\n
$$
D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$
\nthe value of (1-α)(1-β).
\nHence, transformation, find the inverse of the
\n
$$
= \begin{bmatrix} a & b \\ c & (\frac{1+bc}{2}) \end{bmatrix}.
$$
\n
$$
D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$
\n
$$
D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$
\n
$$
D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & c_1 \\ a_2 & c_2 \\ a_3 & b_3 \end{vmatrix}
$$
\nwhere c_1 is the number itself.
\n
$$
D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = \begin{vmatrix} c_1 & c_1 \\ c_2 & c_2 \\ c_3 & c_3 \end{vmatrix}
$$
\n
$$
D = \begin{vmatrix} c_1 & c_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = \begin{vmatrix} c_1 & c_1 \\ c_2 & c_2 \\ c_3 & c_3 \end{vmatrix}
$$
\n
$$
D = \begin{
$$

NOTE : A determinant of order 1 is the number itself.

The symbol
$$
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
$$
 is called the determinant of

order 3. Its value can be found as

STUDY MATERIAL: MATHEMATICS
\n
$$
x = \frac{D_1}{D} \text{ and } y = \frac{D_2}{D} \text{ where}
$$
\n
$$
\left| = [40] \qquad D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$
\n**NOTE**: A determinant of order 1 is the number itself.
\n**PROOF** The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order 3. Its value can be found as
\n
$$
D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}
$$
\n**if** A|
\n
$$
= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}
$$
\nIn the way we can expand a determinant in 6 ways using elements of R₁, R₂, R₃, C₁, C₂, C₃.
\n**Example 13**:
\nFind the value of $\begin{vmatrix} 1 + \cos \theta & \sin \theta \\ \sin \theta & 1 - \cos \theta \end{vmatrix}$

In the way we can expand a determinant in 6 ways using elements of $R_1, R_2, R_3, C_1, C_2, C_3$.

Example 13 :

$$
+20\begin{vmatrix} 1 & 0 \ 0 & 1 \end{vmatrix} = [40] \qquad D = \begin{vmatrix} a_1 & b_1 \ a_2 & b_2 \end{vmatrix}; D_1 = \begin{vmatrix} c_1 & b_1 \ c_2 & b_2 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & c_1 \ a_2 & c_2 \end{vmatrix}
$$

\n**NOTE**: A determinant of order 1 is the number itself.
\n**Inverse** of the
\nThe symbol $\begin{vmatrix} a_1 & b_1 & c_1 \ a_2 & b_2 & c_2 \ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of
\norder 3. Its value can be found as
\n
$$
D = a_1 \begin{vmatrix} b_2 & c_2 \ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \ a_3 & b_3 \end{vmatrix}
$$

\n
$$
|\mathbf{A}|| \mathbf{A}| = a_1 \begin{vmatrix} b_2 & c_2 \ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \ b_2 & c_2 \end{vmatrix}
$$

\n
$$
= \begin{vmatrix} 2 & -1 \ 2 & 0 \end{vmatrix}.
$$
 In the way we can expand a determinant in 6 ways using
\nelements of R₁, R₂, R₃, C₁, C₂, C₃.
\n**Example 13**:
\n
$$
27
$$
 Find the value of $\begin{vmatrix} 1 + \cos \theta & \sin \theta \\ \sin \theta & 1 - \cos \theta \end{vmatrix} = (1 + \cos \theta)(1 - \cos \theta) - (\sin \theta)(\sin \theta)$
\n
$$
= 1 - \cos^2 \theta - \sin^2 \theta = 0
$$

\n**Example 14**:
\n
$$
= \begin{vmatrix} \frac{1}{2} & 2 & 3 \ -3 & 6 \end{vmatrix}.
$$
 Find the value of $\begin{vmatrix} 1 &$

Example 14 :

Find the value of
$$
\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}
$$
.

| Let $x = 0$ and $y = 0$ and < |
|---|
|---|

MINOR & COFACTOR

Minor : The Determinant that is left by cancelling the row and column intersecting at a particular element is called the minor of that element.

$$
\begin{vmatrix} 1 & 3 & 0 \\ 2 & -7 & 9 \end{vmatrix} = 1 \begin{vmatrix} -7 & 9 \end{vmatrix} = 2 \begin{vmatrix} 2 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & -7 \end{vmatrix}
$$

= 1 (3 × 9 – 6(-7)) – 2 (-4 × 9 – 2 × 6) +3 ((-4)(-7) – 3 × 2)
= (27 + 42) – 2 (-36 – 12) + 3 (28 – 6) = 231
OR & COFACTOR
Minor : The Determinant that is left by cancelling the row
and column intersecting at a particular element is called
the minor of that element.

$$
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
 then Minor of a_{11} is

$$
M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}
$$
, Similarly $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Using this concept the value of Determinant can be

 $\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ or $\Delta = -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}$ or $\Delta = a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}$
Cofactor : The cofactor of an element a_{ij} is denoted by F_{ij} and is equal to $(-1)^{i+j} M_{ii}$ where M is a minor of element a ij **TRICES AND DETERMINANTS**

Using this concept the value of Determinant can be
 $\Delta = a_{11}M_{11} - a_{12} M_{12} + a_{13} M_{13}$

or $\Delta = -a_{21}M_{21} + a_{22} M_{22} - a_{23} M_{23}$

or $\Delta = a_{31}M_{31} - a_{32} M_{32} + a_{33} M_{33}$
 Cofactor: The co **ND DETERMINANTS**

concept the value of Determinant can be
 ${}_{1}M_{11} - a_{12} M_{12} + a_{13} M_{13}$
 ${}_{21}M_{21} + a_{22} M_{22} - a_{23} M_{23}$
 ${}_{31}M_{31} - a_{32} M_{32} + a_{33} M_{33}$

The cofactor of an element a_{ij} is denoted by F_{ij} **ND DETERMINANTS**

concept the value of Determinant can be
 ${}_{1}M_{11} - a_{12} M_{12} + a_{13} M_{13}$
 ${}_{21}M_{21} + a_{22} M_{22} - a_{23} M_{23}$

The cofactor of an element a_{ij} is denoted by F_{ij}
 ${}_{11}$ to $(-1)^{i+j} M_{ij}$ where M i **NO DETERMINANTS**

soncept the value of Determinant can be
 $\begin{vmatrix} 11 & 11 & 1 & 21 \ 0 & 1 & 1 & 1 & 1 \end{vmatrix}$
 $\begin{vmatrix} 11 & 1 & 1 & 21 \ 0 & 1 & 1 & 1 \end{vmatrix}$
 $\begin{vmatrix} 21 & 1 & 2 & 1 \ 0 & 2 & 1 & 1 \end{vmatrix}$
 $\begin{vmatrix} 21 & 1 & 2 & 2 \ 0 & 2 & 1 & 1 \end{vm$ **AND DETERMINANTS**

soncept the value of Determinant can be
 $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$
 $\begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$
 $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$
 $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\$ **NO DETERMINANTS**

soncept the value of Determinant can be
 $\begin{vmatrix} 11 \ 101 \ 11 \end{vmatrix} = a_{12} M_{12} + a_{13} M_{13}$
 $a_{21} M_{21} + a_{22} M_{22} - a_{23} M_{23}$
 $a_{31} M_{31} - a_{32} M_{32} + a_{33} M_{33}$

and $D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a$ Ex $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & c_2 & c_3 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{$

INRICES AND DETERMINANTS)
\nUsing this concept the value of Determinant can be
\n
$$
\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}
$$
\nor
$$
\Delta = a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}
$$
\nor
$$
\Delta = a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}
$$
\n**Cofactor:** The cofactor of an element a_{ij} is denoted by F_{ij} then D' = -D
\nand is equal to $(-1)^{i+j}M_{ij}$ where M is a minor of element
\n a_{ij}
\nif
$$
\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{12} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{11} & b_{1} & c_{1} \\ a_{12} & b_{2} & c_{2} \\ a_{21} & b_{2} & c_{2} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{11} & b_{1} & c_{1} \\ a_{12} & b_{2} & c_{2} \\ a_{21} & b_{2} & c_{2} \end{vmatrix}
$$
\n
$$
= \begin{vmatrix} a_{11} & b_{1} & c_{1} \\ a_{12} & b_{2} & c_{2} \\
$$

NOTE :

- (i) The sum of products of the element of any row with their corresponding cofactor is equal to the value of determinant i.e. $\Delta = a_{11}F_{11} + a_{12}F_{12} + a_{13}F_{13}$
- (ii) The sum of the product of element of any row with corresponding cofactor of another row is equal to zero i.e. $a_{11}F_{21} + a_{12}F_{22} + a_{13}F_{23} = 0$
- (iii) If order of a determinant (Δ) is 'n' then the value of the determinant formed by replacing every element by its cofactor is Δ^{n-1} . .

Example 15 :

Find the cofactor element 0 in Determinant
$$
\begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & -3 \\ 4 & 0 & -4 \end{vmatrix}
$$
 to the elements
\nof the corners
\ncolumn).
\n $F_{32} = (-1)^{3+2} \begin{vmatrix} -1 & 1 \\ -2 & -3 \end{vmatrix} = -[(-1)(-3) - (-2)(1)]$
\n $= [3+2] = -5$
\n**PERTIES OF DETERMINANT**
\n $\begin{vmatrix} P-1 \end{vmatrix}$ The value of Determinant remains unchanged, if the
\nrows and the column are interchanged.
\nThis is always denoted by ' and is also called transpose
\n $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
\nThen $D' = D$, D and D' are transpose of each other
\nNote: Since the Determinant remains unchanged when rows
\n $\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix}$ and $\begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & -3 \\ a_2 & -3 & -3 \end{vmatrix}$
\n $\begin{vmatrix} P-6 : The value \\ a_1 & 2 \\ a_2 & a_3 \end{vmatrix}$
\n $\begin{vmatrix} P-1 & 1 \\ -2 & -3 \end{vmatrix} = -[(-1)(-3) - (-2)(1)]$
\n $\begin{vmatrix} P-1 & 2 \\ a_1 & a_2 \\ a_2 & a_3 \end{vmatrix}$
\n $\begin{vmatrix} P-1 & 1 \\ P-1 & 2 \\ P-1 & 3 \end{vmatrix}$
\n $\begin{vmatrix} P-1 & 1 \\ P-1 & 2 \\ P-1 & 1 \end{vmatrix} = D$
\n $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$
\n $\begin{vmatrix} P-1 & 1 \\ P-1 & 1 \\ P-1 & 2 \end{vmatrix} = D$
\n $\begin{vmatrix} P-1 & 1 \\$

Sol.
$$
F_{32} = (-1)^{3+2} \begin{vmatrix} -1 & 1 \ -2 & -3 \end{vmatrix} = -[(-1)(-3) - (-2)(1)]
$$

= $[3+2] = -5$

PROPERTIES OF DETERMINANT

P-1 The value of Determinant remains unchanged, if the rows and the column are interchanged.

This is always denoted by ' and is also called transpose

$$
\begin{array}{ccc} \n\text{Ex} & \mathbf{D} = & \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } \mathbf{D}' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \n\end{array}
$$

actor element 0 in Determinant $\begin{vmatrix} -1 & 2 & 1 \ -2 & 3 & -3 \end{vmatrix}$ to the elements

of the correspondent of the correspondent of the correspondent of the correspondent of the column).
 $\begin{vmatrix} 2 & -1 & 1 \ -2 & -3 & -2 \end{vmatrix} = -[(-1)(-3)$ actor element 0 in Determinant $\begin{vmatrix} -2 & 3 & -3 \ 4 & 0 & -4 \end{vmatrix}$ of the correspondent of the correspondent $\begin{vmatrix} 2 & -1 & 1 \ -2 & -3 & -\end{vmatrix} = -[(-1)(-3) - (-2)(1)]$
 $\begin{vmatrix} 2 & -5 \ -5 & 2 \end{vmatrix}$
 EX $D = \begin{vmatrix} a_1 + ma & 2 \ a_2 + a_3 \end{vmatrix}$
 DU Then $D' = D$, D and D' are transpose of each other **Note:** Since the Determinant remains unchanged when rows and columns are interchanged, it is obvious that any theorem which is true for 'rows' must also be true for 'Columns'

P- 2 If any two rows (or columns) of a determinant be inter changed, the determinant is unaltered in numerical value, but is changed in sign only.

determinant can be
\n
$$
M_{13}
$$
\n
$$
M_{23}
$$
\n
$$
M_{33}
$$
\

P- 3 If a Determinant has two rows (or columns) identical, then its value is zero.

Ex Let D =
$$
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}
$$
 then, D = 0

Forminant can be
 $\begin{vmatrix}\n13 \\
14 \\
13 \\
13\n\end{vmatrix}$
 $\begin{vmatrix}\n11 \\
12 \\
13 \\
13\n\end{vmatrix}$
 $\begin{vmatrix}\n13 \\
13 \\
14 \\
15\n\end{vmatrix}$
 $\begin{vmatrix}\n14 \\
15 \\
16 \\
18\n\end{vmatrix}$
 $\begin{vmatrix}\n15 \\
16 \\
17\n\end{vmatrix}$
 $\begin{vmatrix}\n16 \\
17 \\
18\n\end{vmatrix}$
 $\begin{vmatrix}\n18 \\
18 \\
19\n\end{vmatrix}$
 $\begin{vmatrix}\n1$ Find the same of $\frac{13}{123}$
 $\frac{1}{233}$
 $\frac{1}{23$ **EXECUTE ARNING**
 EXECUTE ARNING

¹ $\begin{bmatrix} 1 & C_1 \\ 2 & C_2 \\ 2 & C_2 \\ 3 & C_3 \end{bmatrix}$ and $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

inant has two rows (or columns) identical,

zero.

¹ $\begin{bmatrix} b_1 & c_1 \\ b_1 & c_1 \\$ **SOBRADYANCED LEARNING**
 $\begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \\ b_3 & c_3 \end{bmatrix}$ and $D' = \begin{bmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{bmatrix}$

minant has two rows (or columns) identical,

s zero.
 $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{b$ **SPON ADVANCED LEARNING**
 $\begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \\ b_3 & c_3 \end{bmatrix}$ and $D' = \begin{bmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{bmatrix}$

minant has two rows (or columns) identical,

s zero.
 $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{$ **b**₁ c₁

b₂ c₂ and D' = $\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

minant has two rows (or columns) identical,

s zero.

a₁ b₁ c₁ then, D = 0

ements of any row (or column) be multiplied

umber, then **P- 4** If all the elements of any row (or column) be multiplied by the same number, then the value of Determinant is multiplied by that number. a_1 b_1 c_1
 a_3 b_3 c_3
 ≥ 0

So (or columns) identical,
 $D = 0$

or column) be multiplied

alue of Determinant is
 $\begin{array}{c} 1 \text{ kb}_1 \text{ kc}_1 \\ 1 \text{ b}_2 \text{ c}_2 \\ 0 \text{ s}_3 \text{ c}_3 \end{array}$ then $D' = k D$. | a₃ b₃ c₃ |

ws (or columns) identical,

D = 0

(or column) be multiplied

value of Determinant is

a₁ kb₁ kc₁

₂ b₂ c₂ | then D'= k D.

row (or column) can be

then the determinant can ws (or columns) identical,
 $D=0$

(or column) be multiplied

value of Determinant is
 $a_1 k b_1 k c_1$
 $b_2 b_2 c_2$
 $b_3 c_3$ then $D' = k D$.

row (or column) can be

then the determinant can

Determinants **EXECUTE ARISING**
 EXECUTE ARISING
 $= \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

solves (or columns) identical,
 $a_1, D = 0$
 w (or column) be multiplied
 x value of Determinant is
 $ka_1 kb_1 kc_1$
 $a_2 b_2 c_2$ th **EXECUTE ARISING**
 EXECUTE ARI EXECUTE ARNING
 a $\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

b contained by the property of the p s) identical,

e multiplied

erminant is
 $\text{len D'} = \text{k D}.$

mn) can be

rminant can
 $\begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

d by adding

ee multiples

er row (or

**EXAMPLEAIRING
\nEx** D =
$$
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
$$
 and D' = $\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$
\nthen D' = -D
\n**P-3** If a Determinant has two rows (or columns) identical, then its value is zero.
\n**Ex** Let D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ then, D = 0
\n**P-4** If all the elements of any row (or column) be multiplied by the same number, then the value of Determinant is multiplied by that number.
\n**Ex** D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and D' = $\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then D' = k D.
\n**P-5**: If each elements of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the Determinants $|a_1 + x, b_1 + y, c_1 + z| = |a_1, b_1, c_1| \quad |x, y, z|$

P- 5 : If each elements of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the Determinants

1 a₃ b₃ c₃ | a₃ b₃ c₃ |
\nthen D'=-D
\nP-3 If a Determinant has two rows (or columns) identical,
\nthen its value is zero.
\nEx. Let D=
$$
\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}
$$
 then, D=0
\nP-4 If all the elements of any row (or column) be multiplied
\nby the same number, then the value of Determinant is
\nmultiplied by that number.
\nEx. D= $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and D'= $\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then D' = k D.
\nP-5: If each elements of any row (or column) can be
\nexpressed as a sum of two terms, then the determinant can
\nbe expressed as the sum of the Determinants
\nEx. $\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$
\nP-6: The value of a Determinant is not altered by adding
\nto the elements of any row (or column) the same multiples
\nof the corresponding elements of any other row (or

Ex Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ then, $D = 0$
 P_4 If all the elements of any row (or column) be multiplied

by the same number, then the value of Determinant is

multiplied by that number.

with their

Ex ²
 P_4 If all the elements of any row (or column) be multiplied

by the same number, then the value of Determinant is

multiplied by that number.

with their

Ex. D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ **P_4** If all the elements of any row (or column) be multiplied
by the same number, then the value of Determinant is
multiplied by that number.
We with their
 $E_x D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin$ Fx. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \ a_2 & b_2 & c_2 \ a_3 & b_3 & c_3 \end{vmatrix}$ then, $D = 0$
 P_4 If all the elements of any row (or column) be multiplied

by the same number, then the value of Determinant is

multiplied by that number.

N -4 column). Fall the elements of any row (or column) be multiplied

a same number, then the value of Determinant is

blied by that number.
 $= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 &$ If a Determinant has two rows (or columns) identical,

its value is zero.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \ a_1 & b_1 & c_1 \ a_2 & b_2 & c_2 \end{vmatrix}$ then, $D = 0$

If all the elements of any row (or column) be multiplied

the same number, t If a Determinant has two rows (or columns) identical,

its value is zero.

Let D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ then, D = 0

If all the elements of any row (or column) be multiplied

the same number, v (or column) be multiplied
value of Determinant is
 $\begin{vmatrix} a_1 & k b_1 & k c_1 \ a_2 & b_2 & c_2 \ 3 & b_3 & c_3 \end{vmatrix}$ then D'=k D.
row (or column) can be
then the determinant can
Determinants
 $\begin{vmatrix} 1 & b_1 & c_1 \ 2 & b_2 & c_2 \ 3 & b_3 & c_3 \end{vmatrix$ bows (or columns) identical,
 $n, D = 0$

w (or column) be multiplied

e value of Determinant is
 $ka_1 kb_1 kc_1$
 $a_2 b_2 c_2$ then $D' = k D$.

y row (or column) can be

s, then the determinant can

Determinants
 $a_1 b_1 c_1 | x y z |$ b continuity of columns) identical,
 $\mathbf{a}_1 \mathbf{b}_1 = 0$

w (or column) be multiplied

e value of Determinant is
 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then $D' = k D$.

y row (or column) can be

is, then the det multiplied

en D'=k D.

an) can be

minant can
 $\begin{vmatrix} y & z \\ 2 & b_2 & c_2 \\ 3 & b_3 & c_3 \end{vmatrix}$

by adding

e multiples

r row (or s) identical,

e multiplied

erminant is

nen D'=k D.

mn) can be

rminant can

x
 $\begin{bmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

d by adding

ne multiples

er row (or s) identical,

e multiplied

erminant is

nen D'=k D.

mn) can be

rminant can

x y z

a₂ b₂ c₂

a₃ b₃ c₃

d by adding

ne multiples

er row (or **P- 6 :** The value of a Determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or $\begin{vmatrix} 6 & 3 & 3 \\ 0 & 3 & 1 \end{vmatrix}$ $\begin{vmatrix} 4 & 3 & 0 \\ 4 & 3 & 0 \end{vmatrix}$ of column) can be

a sum of two terms, then the determinant can

b₁ + y c₁ + z

b₂ c₂ = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ + $\begin{vmatrix} x &$ ch elements of any row (or column) can be

a sum of two terms, then the determinant can

las the sum of the Determinants
 $b_1 + y c_1 + z$
 $\begin{vmatrix} b_1 + y c_1 + z \\ b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vm$

⁺2 M₁₂ = - M₁₂ = -
$$
\begin{vmatrix} a_{21} & a_{23} \ a_{33} \end{vmatrix}
$$

\nby the same number, then the value of Determinant multiplication.
\nproduces of the element of any row with their
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} \ b_{11} & a_{12} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} \ b_{11} & a_{12} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} \ b_{11} & a_{12} \end{vmatrix}$
\n $\begin{vmatrix} a_{12} & a_{12} & a_{13} \ b_{12} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{12} & a_{12} & a_{13} & a_{13} \ b_{12} & a_{13} & b_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{13} \ b_{13} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{12} & a_{13} & a_{13} & a_{13} \ b_{13} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{13} \ b_{13} & a_{13} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{13} \ b_{11} & a_{12} & a_{13} & a_{13} \ b_{12} & a_{13} & a_{13} \ b_{13} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{13} \ b_{13} & a_{13} & a_{13} \ b_{13} & a_{13} & a_{13} \ b_{13} & a_{13} \end{vmatrix}$
\n $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{13} \ b_{13} & a_{13} & a_{13} \ b_{13} & a_{13} \end{vm$

then $D' = D$

Note: It should be noted that while applying P-6 at least one row (or column) must remain unchanged

P-7: If
$$
\Delta = f(x)
$$
 and $f(a) = 0$ then $(x-a)$ is a factor of Δ .

Ex.
$$
D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}
$$

 $D' = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 - na_1 & b_3 - nb_1 & c_3 - nc_1 \end{vmatrix}$
then $D' = D$
Note: It should be noted that while applying P-6 at least
one row (or column) must remain unchanged
P-7: If $\Delta = f(x)$ and $f(a) = 0$ If we replace a by b then $D = \begin{vmatrix} 1 & 1 & 1 \\ b & b & c \\ b^2 & b^2 & c^2 \end{vmatrix} = 0$ \Rightarrow (a – b) is a factor of D

$$
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$$

 \mathbf{r}

P-8 : In a determinant the sum of the products of the elements of any row (column) with their corresponding cofactors is equal to the value of determinant. SI

determinant the sum of the products of the

f any row (column) with their corresponding

sequal to the value of determinant.

1 b₁ c₁

2 b₂ c₂

3 b₃ c₃

 C_i be the cofactors of the elements

1, 2, 3) SI

determinant the sum of the products of the

f any row (column) with their corresponding

equal to the value of determinant.

(3) $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)$
 $\begin{vmatrix} 1 & b_1 & c_1 \\ 2 & b_2 & c_2 \\ 3 & b_3 & c_3 \end{$ SIDE determinant the sum of the products of the

f any row (column) with their corresponding

equal to the value of determinant.
 $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)$
 $\begin{vmatrix} 1 & b_1 & c_1 \\ 2 & b_2 & c_2 \\ 3 & b_3 & c_3 \end{vmatrix}$

Let
$$
D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}
$$

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TO a determinant the sum of the products of the

tents of any row (column) with their corresponding
 $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$
 $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_$ **EXERUINTER**

In a determinant the sum of the products of the

and the value of determinant.

In a determinant the sum of the products of the

sis equal to the value of determinant.
 $\begin{vmatrix} 3 & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$ Let A_i , B_i , C_i be the cofactors of the elements a_i, b_i, c_i (i = 1, 2, 3) Then, $a_1A_1 + b_1B_1 + c_1C_1 = D$ $a_2A_2 + b_2B_2 + c_2C_2 = D$

Similarly, in a determinant the sum of the products of the elements of any row (column) with the cofactors of corresponding elements of any other row (column) is zero. i.e., $a_1A_2 + b_1B_2 + c_1C_2 = 0$ or $a_2A_1 + b_2B_1 + c_2C_1 = 0$

SOME IMPORTANT DETERMINANTSTO REMEMBER :

(1)
$$
\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)
$$

\n**Proof:** Let $D = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$
\n $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$
\n $\Rightarrow D = \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$
\n $\Rightarrow D = \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$
\n $D = (x-y)(y-z) \begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$
\n $D = (x-y)(y-z)(z-x)$. Hence proved.
\nExample 16:

(2)
$$
\begin{vmatrix} 1 & x & x^{3} \\ 1 & y & y^{3} \\ 1 & z & z^{3} \end{vmatrix} = (x - y)(y - z)(z - x)(x + y + z)
$$

Proof: Let
$$
D = \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}
$$

Sol. If ABe
 $a = AR$

Apply $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$. Given

$$
D = \begin{vmatrix} 0 & x - y & x^3 - y^3 \\ 0 & y - z & y^3 - z^3 \\ 1 & z & z^3 \end{vmatrix} = (x - y)(y - z) \begin{vmatrix} 0 & 1 & x^2 + xy + y^2 \\ 0 & 1 & y^2 + yz + z^2 \\ 1 & z & z^3 \end{vmatrix} \quad \therefore \Delta = \begin{vmatrix} \log A & p \\ \log A & q \\ \log A & r \end{vmatrix}
$$

\n
$$
D = (x - y)(y - z) [y^2 + yz + z^2 - x^2 - xy - y^2]
$$

\n
$$
D = (x - y)(y - z) [y(z - x) + z^2 - x^2]
$$

\n
$$
= (x - y)(y - z) (z - x) (x + y + z).
$$

\n
$$
= 0 + \log R \begin{vmatrix} p \\ q \\ q \end{vmatrix}
$$

(3)
$$
\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)
$$

| SUBING | STUDY MATERIAL: MATHEMATICS | |
|--|--|--|
| a determinant the sum of the products of the | 1 | |
| of any row (column) with their corresponding | 2 | |
| is equal to the value of determinant. | 3 | \n $\begin{vmatrix}\nx & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy\n\end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$ \n |
| a1 $\begin{vmatrix}\n b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3\n\end{vmatrix}$ \n | \n Proof: Let $D = \begin{vmatrix}\nx & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy\n\end{vmatrix} = \begin{vmatrix}\n1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & y^4\n\end{vmatrix}$ \n | |
| C _i be the cofactors of the elements | Apply $R_1 \rightarrow xR_1$; $R_2 \rightarrow yR_2$, $R_3 \rightarrow zR_3$ divide by xyz | |

Apply $R_1 \rightarrow xR_1$; $R_2 \rightarrow yR_2$, $R_3 \rightarrow zR_3$ divide by xyz balancing.

$$
D = \frac{1}{xyz} \begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix} = \frac{xyz}{xyz} \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}
$$

$$
= \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}
$$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$.

$$
= \begin{vmatrix} 0 & x^2 - y^2 & x^3 - y^3 \\ 0 & y^2 - z^2 & y^3 - z^3 \\ 1 & z^2 & z^3 \end{vmatrix}
$$

$$
= (x - y) (z - x) (y - z) (xy + yz + zy)
$$

(4)
$$
\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc) < 0
$$
 if a, b, c are

different and positive

Proof : c a b b c a a b c = a [bc – a²] – [b² –ac] + c(ab – c²) = 3abc – (a³ + b³ + c³). log a p 1 log b q 1 log c r 1

Example 16 :

If a, b, c are pth, qth and rth, terms of a G.P., then find

-
- $\begin{array}{c|c}\n1 & y & y^3 \\
1 & z & z^3\n\end{array}$ **Sol.** If A be the first term and R be the c.r. of G.P., then
 $a = ARP^{-1} b = AR^{q-1} c = AR^{r-1}$ $a = AR^{p-1}$, $b = AR^{q-1}$, $c = AR^{r-1}$ $\log a = \log A + (p-1)\log R$

(4)
$$
\begin{vmatrix} b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc) < 0
$$
 if a, b, c are
\ndifferent and positive
\ny)(y-z)(z-x)
\nProof: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a [bc - a^2] - [b^2 - ac] + c(ab - c^2)$
\n= 3abc - (a³ + b³ + c³).
\nExample 16:
\nIf a, b, c are pth, qth and rth, terms of a G.P., then find
\n $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$
\nSo. If A be the first term and R be the c.r. of G.P., then
\na = ARP⁻¹, b=AR⁻¹, c=AR^{r-1}
\nlog a = logA + (p - 1)log R
\n $\begin{vmatrix} 0 & 1 & x^2 + xy + y^2 \\ 0 & 1 & y^2 + yz + z^2 \\ 1 & z & z^3 \end{vmatrix}$
\n $\therefore \Delta = \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1) \log R & p & 1 \\ (q-1) \log R & q & 1 \\ (r-1) \log R & r & 1 \end{vmatrix}$
\n= 0 + log R $\begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix} = 0$ [by C₂-C₁]
\n72

Example 17 :

Find determinant
$$
\begin{vmatrix} a+b+nc & (n-1)a & (n-1)b \ (n-1)c & b+c+na & (n-1)b \ (n-1)c & (n-1)a & c+a+nb \end{vmatrix}
$$

Sol. Applying $C_1 + (C_2 + C_3)$ and taking n(a+b+c) common from C_1 , we get

Applying C₁ + (C₂ + C₃) and taking n(a+b+c) common
\nfrom C₁, we get
\n
$$
\Delta = n (a+b+c) \begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 1 & b+c+na & (n-1)b \\ 1 & (n-1)a & c+a+nb \end{vmatrix} = \begin{vmatrix} = (a+b+c) \begin{vmatrix} c-a & b-a \\ c-b & c-a \end{vmatrix} + c \\ = (a+b+c) \begin{vmatrix} c-a^2 - (c-d-b) \\ c-b & c-a \end{vmatrix} + c \\ = (a+b+c) \begin{vmatrix} c-a^2 - (c-d-b) \\ c-b^2 + a^2 - 2ac \end{vmatrix} + c^2 - a \\ = a^3 + b^3 + c^3 - 3abc = R.H.
$$
\n
$$
= n (a+b+c) \begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} = Rx + b
$$
\n
$$
= n (a+b+c) \begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = 4abc
$$
\n
$$
= (a+b+c) \begin{vmatrix} a-b-c & 2a & 2b & b-c-a \\ 2b & b-c-a & 2c & c \\ 2c & 2c & c & c \end{vmatrix}
$$
\n
$$
= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2c & 2c \\ 2c & 2c & c & c \end{vmatrix}
$$
\n
$$
= -2a \begin{vmatrix} 0 & c & b \\ c+a & a \\ 1 & a & a+b \end{vmatrix} = -2a \begin{vmatrix} 0 & c & b \\ 1 & a & a+b \end{vmatrix} = -2a \begin{vmatrix} 0 & c & b \\ 1 & a & a+b \end{vmatrix} = R_2 - R_3
$$
\n
$$
= -2a \begin{vmatrix} c & b \\ c & -b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & 0 \\ 2b & -b-c-a \\ 2c & 0 \end{vmatrix}
$$
\n
$$
= -2a \begin{vmatrix} c & b \\ c & -b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & 0 \\ 2b & b-c-a \\ 2c
$$

Example 18 :

Prove that
$$
\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = 4abc
$$
 Sol. L.H.S.

Sol. L.H.S. =
$$
\begin{vmatrix} 0 & c & b \\ -2a & c+a & a \\ -2a & a & a+b \end{vmatrix}
$$
 $[C_1 \rightarrow C_1 - (C_2 + C_3)]$
\n $\begin{vmatrix} 0 & c & b \end{vmatrix}$ $\begin{vmatrix} 0 & c & b \end{vmatrix}$

$$
= -2a \begin{vmatrix} 1 & c+a & a \\ 1 & a & a+b \end{vmatrix} = -2a \begin{vmatrix} 0 & c & -b \\ 1 & a & a+b \end{vmatrix} [R_2 \to R_2 - R_3]
$$

for b

$$
= -2a \begin{vmatrix} c & 0 \\ c & -b \end{vmatrix}
$$
 [expanding along C₁]
= -(-2a) (-2bc) = 4abc = R.H.S.

Example 19 :

Show that
$$
\begin{vmatrix} b+c & a+b & a \ c+a & b+c & b \ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.
$$

Sol. We have

L.H.S. =
$$
\begin{vmatrix} -2a & c+a & a \\ -2a & a & a+b \end{vmatrix}
$$
 $[C_1 \rightarrow C_1 - (C_2 + C_3)]$
\n $\begin{vmatrix} 0 & c & b \\ 1 & a & a+b \end{vmatrix} = -2a \begin{vmatrix} 0 & c & b \\ 0 & c & -b \\ 1 & a & a+b \end{vmatrix}$ $[R_2 \rightarrow R_2 - R_3]$
\n $\begin{vmatrix} -2a & b \\ b & c \end{vmatrix}$ $[2a - b - c - a]$ $\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b - c - a & 0 \\ 2c & 0 & -c - a - b \end{vmatrix}$
\n $\begin{vmatrix} -2a & b \\ c & b \end{vmatrix}$ $[expanding along C_1]$
\n $\begin{vmatrix} -2a & c + a & b \\ c + a & b + c & b \\ a + b & c + a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.
\n $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.
\n $\begin{vmatrix} 2c & 2c & c - a - b \\ 2c & 0 & -c - a \\ 0 & -c - a & 0 \end{vmatrix}$ $[expan B_2]$
\n $\begin{vmatrix} 2c & 2c & c - a - b \\ 2c & 0 & -c - a \\ 0 & -c - a & 0 \end{vmatrix}$ $[expan B_2]$
\n $\begin{vmatrix} 2c & 2c & c - a - b \\ 2c & 0 & -c - a \\ 0 & -c - a & 0 \end{vmatrix}$ $[expan B_2]$
\n $\begin{vmatrix} 2c & 2c & c - a - b \\ 2c & 0 & -c - a \\ 0 & -c - a & 0 \end{vmatrix}$ $[expan B_2]$
\n $\begin{vmatrix} 2c & 2c & c - a - b \\ 2c & 0 & -c - a \\ 0 & -c - a & 0 \end{vmatrix}$ $[expan B_2]$
\n $\begin{vmatrix} 2c & 2c & c - a - b$

a b nc (n 1)a (n 1)b (n 1)c b c na (n 1)b (n 1)c (n 1)a c a nb 1 (n 1)a (n 1)b 1 b c na (n 1)b 1 (n 1)a c a nb 1 (n 1)a (n 1)b 0 a b c 0 0 0 a b c b c c b 1 a b a (a b c) 0 c a b a 0 c b c a 2 2 1 3 3 1 R R R R R R c a b a (a b c) c b c a [expending along C¹] = (a + b + c) [(c – a)² – (c – b) (b – a)] = (a + b + c) [(c² + a² – 2ac)² – (cb – ca – b² + ab)] = (a + b + c) [a² + b² + c² – ab – bc – ca] = a³ + b³ + c³ – 3abc = R.H.S. a b c 2a 2a 2b b c a 2b 2c 2c c a b a b c 2a 2a 2b b c a 2b 2c 2c c a b

Example 20 :

Show that
$$
\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3
$$
.

Find determinant
$$
\begin{vmatrix}\n\text{Find determinant } \begin{vmatrix}\n\text{Find determinant } \begin{vmatrix}\n\text{Find determinant } \begin{vmatrix}\n\text{Find determinant } \begin{vmatrix}\n\text{Find the equation } \end{vmatrix} & \text{if } \begin{vmatrix}\n\text{If } \begin{vmatrix}\n\text{If } \begin{vmatrix}\n\text{If } \begin{bmatrix}\n\text{If } \begin{bmatrix}\n\
$$

$$
|2c - 2c - c - a - b|
$$

\n
$$
= (a + b + c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b - c - a & 0 \\ 2c & 0 & -c - a - b \end{vmatrix}
$$

\n
$$
= (a + b + c) \begin{vmatrix} -b - c - a & 0 \\ 0 & -c - a - b \end{vmatrix}
$$

\n
$$
= (a + b + c) (a + b + c)^2 = (a + b + c)^3 = R.H.S.
$$

\nExample 21:
\n
$$
\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.
$$

\nSoI. L.H.S. = $\frac{1}{abc} \begin{vmatrix} a(a^2 + 1) & ab^2 & ac^2 \\ a^2b & b(b^2 + 1) & bc^2 \\ a^2c & b^2c & c(c^2 + 1) \end{vmatrix}$
\n
$$
\begin{bmatrix} C_1 \rightarrow aC_1 \\ C_2 \rightarrow bC_2 \\ C_3 \rightarrow cC_3 \end{bmatrix}
$$

Example 21 :

Show that
$$
\begin{vmatrix} a^2 + 1 & ab & ac \ ab & b^2 + 1 & bc \ ac & bc & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.
$$

Sol. L.H.S. =
$$
\frac{1}{abc} \begin{vmatrix} a(a^2 + 1) & ab^2 & ac^2 \\ a^2b & b(b^2 + 1) & bc^2 \\ a^2c & b^2c & c(c^2 + 1) \end{vmatrix}
$$
 $\begin{bmatrix} C_1 \rightarrow aC_1 \\ C_2 \rightarrow bC_2 \\ C_3 \rightarrow cC_3 \end{bmatrix}$

2 2 2 2 2 2 2 2 2 a 1 b c abc a b 1 c abc a b c 1 1 2 3 taking a, b, c common from **Example 23 :** R , R , R respectively 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 a b c b c 1 a b c b 1 c 1 a b c b c 1 [C¹ C¹ + C² + C³] **Sol.** 2 2 2 2 2 2 2 2 2 1 b c (1 a b c) 1 b 1 c 1 b c 1 2 2 2 2 2 1 b c (1 a b c) 0 1 0 0 0 1 2 2 1 3 3 1 R R R R R R = (1 + a² 1 0 0 1 [expanding along C¹] 2 2 2 1 1 1 1 sin A 1 sin B 1 sin C sin A sin A sin B sin B sin C sin C 1 1 1 1 sin A 1 sin B 1 sin C

=
$$
(1 + a^2 + b^2 + c^2)
$$
 $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ [expanding along C₁]
= $1 + a^2 + b^2 + c^2$ = R.H.S.

Example 22 :

If A, B, C are the angle of a triangle and

$$
\begin{vmatrix}\n1 & 1 & 1 \\
1 + \sin A & 1 + \sin B & 1 + \sin C \\
\sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C\n\end{vmatrix} = 0
$$

prove that ABC must be isosceles

Sol. Let
$$
\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix}
$$

$$
= (1 + a^{2} + b^{2} + c^{2}) \begin{vmatrix} 1 & 0 \ 0 & 1 \end{vmatrix}
$$
 [expanding along C₁]
\n
$$
= 1 + a^{2} + b^{2} + c^{2} = R.H.S.
$$

\n**Example 22:**
\nIf A, B, C are the angle of a triangle and
\n
$$
\begin{vmatrix} 1 & 1 & 1 \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin A & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C & 1+ \sin B & 1+ \sin C \ 1+ \sin B & 1+ \sin C & 1
$$

 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ In all the three cases, the triangle will be isosceles. Now, since Δ is given to be zero, therefore we have $(\sin B - \sin A) (\sin C - \sin A) (\sin C - \sin B) = 0$ i.e.sin $B - \sin A = 0$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$ i.e. $\sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$ i.e. $B = A$ or $C = A$ or $C = B$ **STUDY MATERIAL: MATHEMATICS**

nce Δ is given to be zero, therefore we have
 $-\sin A$ (sin $C - \sin A$) (sin $C - \sin B$) = 0
 $B = \sin A$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$
 $B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$
 $\in A$ or $C = A$ or $C = B$

i **STUDY MATERIAL: MATHEMATICS**
ince Δ is given to be zero, therefore we have
 $-\sin A$ (sin $C - \sin A$) (sin $C - \sin B = 0$
 $B = \sin A = 0$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$
 $B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$
 $= A$ or $C = A$ or $C = B$
ne thr **STUDY MATERIAL: MATHEMATICS**

ince Δ is given to be zero, therefore we have
 $= \sin A$) (sin C – sin A) (sin C – sin B) = 0

3 – sin A = 0 or sin C – sin A = 0 or sin C – sin B = 0

B = sin A or sin C = sin A or sin C = **STUDY MATERIAL: MATHEMATICS**

Vow, since Δ is given to be zero, therefore we have
 $\sin B - \sin A$) ($\sin C - \sin A$) ($\sin C - \sin B$) = 0

e. $\sin B = \sin A = 0$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$

e. $\sin B = \sin A$ or $\sin C = A$ or $\sin C = \sin B$

e. $B = A$ **STUDY MATERIAL: MATHEMATICS**

Sow, since Δ is given to be zero, therefore we have
 $\sin B - \sin A$) ($\sin C - \sin A$) ($\sin C - \sin B = 0$
 $\cos A = 0$ and $\cos A = 0$ or $\sin C = \sin A$
 $\cos A = 0$ and $\cos A = 0$ and $\cos A = 0$
 $\sin C = \sin B$
 $\sin A = \cos A$ or **STUDY MATERIAL: MATHEMATICS**

Sow, since A is given to be zero, therefore we have
 $\sin B - \sin A$ (sin C – $\sin A$) (sin C – $\sin B = 0$
 $\sin B = \sin A$ or $\sin C - \sin A - 0$ or $\sin C - \sin B = 0$
 $\sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$
 $\sin B = A$ or **STUDY MATERIAL: MATHEMATICS**

w, since Δ is given to be zero, therefore we have
 $1B - \sin A$) (sin $C - \sin A$) (sin $C - \sin B = 0$
 $\sin B - \sin A = 0$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$
 $\sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$
 $B = A$ or $C =$ **STUDY MATERIAL: MATHEMATICS**

w, since Δ is given to be zero, therefore we have

n B – sin A) (sin C – sin A) (sin C – sin B) = 0

sin B – sin A = 0 or sin C – sin A = 0 or sin C – sin B = 0

sin B = sin A or sin C = **STUDY MATERIAL: MATHEMATICS**

w, since Δ is given to be zero, therefore we have
 $B - \sin A$) (sin $C - \sin A$) (sin $C - \sin B$) = 0

sin $B = \sin A = 0$ or sin $C = \sin A = 0$ or sin $C = \sin B$
 $B = A$ or $C = A$ or $C = B$

all the three cases, **STUDY MATERIAL: MATHEMATICS**

Now, since Δ is given to be zero, therefore we have
 $(\sin B - \sin A)(\sin C - \sin A)(\sin C - \sin B) = 0$
 $(\sin B - \sin A - 0 \text{ or } \sin C - \sin A - 0 \text{ or } \sin C - \sin B = 0$
 $(\sin B - \sin A - \sin A) = 0$
 $(\sin B - \sin A - \sin A) = 0$
 $(\sin B - \sin A - \sin A) = 0$
 $(\$ **STUDY MATERIAL: MATHEMATICS**

Now, since Δ is given to be zero, therefore we have
 $(\sin B - \sin A) (\sin C - \sin A) (\sin C - \sin B) = 0$
 \therefore $\sin B = \sin A = 0$ or $\sin C - \sin A = 0$ or $\sin C - \sin B = 0$
 \therefore $\sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$
 \therefore $\$ Sin C – sin A) (sin C – sin B) = 0

= 0 or sin C – sin A = 0 or sin C – sin B = 0

A or sin C = sin A or sin C = sin B

or C = A or C = B

or C = A or C = B

cases, the triangle will be isosceles.

eient of x in the det

Find the coefficient of x in the determinant

$$
\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \ \end{vmatrix} = 0.
$$

ol. If f(x) be a polynomial in x,
then coefficient of xⁿ in f(x) =
$$
\frac{f^{n}(0)}{n!}
$$

Let the given determinant be denoted by $f(x)$, then

1. e.sin B – sin A = 0 or sin C – sin A = 0 or sin C – sin B = 0
\ni.e. sin B = sin A or sin C = sin A or sin C = sin B
\ni.e. B = A or C = A or C = B
\nIn all the three cases, the triangle will be isosceles.
\n**uple 23 :**
\nFind the coefficient of x in the determinant
\n
$$
(1+x)^{a_1b_1} (1+x)^{a_1b_2} (1+x)^{a_1b_3}
$$
\n
$$
(1+x)^{a_2b_1} (1+x)^{a_2b_2} (1+x)^{a_2b_3}
$$
\nIf f(x) be a polynomial in x,
\nthen coefficient of xⁿ in f(x) = $\frac{f^n(0)}{n!}$
\nLet the given determinant be denoted by f(x), then
\n
$$
a_1b_1(1+x)^{a_1b_1-1} (1+x)^{a_1b_2} (1+x)^{a_1b_3}
$$
\nf'(x) =
$$
\begin{vmatrix}\na_1b_1(1+x)^{a_1b_1-1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\
a_2b_1(1+x)^{a_2b_1-1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\
a_3b_1(1+x)^{a_3b_1-1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3}\n\end{vmatrix}
$$
\n
$$
+ \begin{vmatrix}\n(1+x)^{a_1b_1} & a_1b_2(1+x)^{a_1b_2-1} & (1+x)^{a_1b_3} \\
(1+x)^{a_2b_1} & a_2b_2(1+x)^{a_2b_2-1} & (1+x)^{a_2b_3} \\
(1+x)^{a_3b_1} & a_3b_1(1+x)^{a_3b_2-1} & (1+x)^{a_3b_3}\n\end{vmatrix}
$$

$$
\begin{bmatrix}\n\text{t\,} & \text{a} & \text{b}\n\end{bmatrix}\n+ a^2 + b^2 + c^2\n\begin{bmatrix}\n\frac{1}{2} & \text{b} & \text{c}\n\end{bmatrix}\n+ a^2 + b^2 + c^2\n\begin{bmatrix}\n\frac{1}{2} & \text{c}\n\end{bmatrix}\n\end{bmatrix} = 0
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{c}\n\end{bmatrix}\n+ a^2 + b^2 + c^2\n\begin{bmatrix}\n\frac{1}{2} & \text{d}\n\end{bmatrix}\n+ a^2 + b^2 + c^2\n\begin{bmatrix}\n\frac{1}{2} & \text{e}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f}^2 \\
\frac{1}{2} & \text{f}^2 & \text{f}^2\n\end{bmatrix} = 0.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2} & \text{f}^2 & \text{f
$$

$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 1 \end{vmatrix}
$$
 [expanding along C₁]
\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 1 \end{vmatrix}
$$
 [expanding along C₁]
\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 1 \end{vmatrix}
$$

$$
c^{2}\begin{vmatrix} 1+x^{2b_{1}} & 0 & 0 \ 0 & 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

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$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

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c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
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\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0 \end{vmatrix}
$$

\n
$$
c^{2}\begin{vmatrix} 1 & 0 \ 0 & 0
$$

Thus, we have

$$
f'(0) = \begin{vmatrix} a_1b_1 & 1 & 1 \ a_2b_1 & 1 & 1 \ a_3b_1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & a_1b_2 & 1 \ 1 & a_2b_2 & 1 \ a_3b_2 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & a_1b_3 \ 1 & 1 & a_2b_3 \ 1 & 1 & a_3b_3 \end{vmatrix} = 0
$$

Hence, we have

Coeff. of x in
$$
f(x) = \frac{f'(0)}{1!} = 0
$$

SYMMETRIC & SKEW SYMMETRIC DETERMINANT

Symmetric determinant : A determinant is called symmetric Determinant if for its every element.

$$
a_{ij} = a_{ji} \quad \forall \ i, j \qquad \qquad \text{Ex} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}
$$

Skew Symmetric determinant : A determinant is called skew Symmetric determinant if for its every element

$$
a_{ij} = -a_{ji} \forall i, j
$$

Ex.
$$
\begin{vmatrix} 0 & 3 & -1 \\ -3 & 0 & 5 \\ 1 & -5 & 0 \end{vmatrix}
$$

NOTE

- (i) Every diagonal element of a skew symmetric determinant is always zero.
- (ii) The value of a skew symmetric determinant of even order is always a perfect square and that of odd order is always zero. EVALUATION OF TWO DETERMINANTS
 $\begin{bmatrix}\n1 & 0 & 1 \\
- & 1 & 0 \\
- & 1 & 0\n\end{bmatrix}$
 $\begin{bmatrix}\n0 & 3 & -1 \\
1 & -5 & 0 \\
1 & -5 & 0\n\end{bmatrix}$

TIE

TIE

TIE

TIE

TE erry diagonal element of a skew symmetric determinant

always zero.

In the value of = - a_{ji} \forall 1, 1
 $\begin{vmatrix} 0 & 3 & -1 \\ -3 & 0 & 5 \\ 1 & -5 & 0 \end{vmatrix}$
 PIE

PIE

PIE

PIE

PIE

PIE

ery diagonal element of a skew symmetric determinant

evalue of a skew symmetric determinant of even order

always a perfect s diagonal element of a skew symmetric determinant

ys zero.

lue of a skew symmetric determinant of even order

ys a perfect square and that of odd order is always

CATION OF TWO DETERMINANTS

lication of two second orde

MULTIPLICATION OFTWO DETERMINANTS

Multiplication of two second order determinants is

$$
\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 \\ \ell_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1 \ell_1 + b_1 \ell_2 & a_1 m_1 + b_1 m_2 \\ a_2 \ell_1 + b_2 \ell_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}
$$

Multiplication of two third order determinants is

$$
= \begin{vmatrix} a_1 \ell_1 + b_1 \ell_2 + c_1 \ell_3 & a_1 m_1 + b_1 m_2 + c_1 m_3 & a_1 n_1 + b_1 n_2 + c_1 n_3 \\ a_2 \ell_1 + b_2 \ell_2 + c_2 \ell_3 & a_2 m_1 + b_2 m_2 + c_2 m_3 & a_2 n_1 + b_2 n_2 + c_2 n_3 \\ a_3 \ell_1 + b_3 \ell_2 + c_3 \ell_3 & a_3 m_1 + b_3 m_2 + c_3 m_3 & a_3 n_1 + b_3 n_2 + c_3 n_3 \end{vmatrix}
$$

Note: In above case the order of Determinant is same, if the order is different then for their multiplication first of all they should be expressed in the same order.

To express a determinants as a product of two determinants :

To express a determinant as product of two determinants one requires a lot of practice and this can be done only by inspection and trial. It can be understood by the following examples.

Example 24 :

$$
\int_{\Gamma}^{2} f(t) \, ds = \int_{\Gamma}^{2} \int_{\Gamma}^{2} f(t) \, dt
$$
\n
$$
= \int_{\Gamma}^{
$$

expressed as

(A)
$$
\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2
$$
 (B) $\begin{vmatrix} c & b & a \\ a & b & c \\ c & a & b \end{vmatrix}^2$

| MATRICES AND DETERMINANTS | SSUMIETRC OETERMINANT |
|--|-----------------------|
| Symmetric determinant : A determinant is called symmetric determinant if of <i>i</i> is even and symmetric determinant if \overrightarrow{b} is a the <i>i</i> th term in <i>i</i> th | |

SUMMATION OF DETERMINANTS

Let
$$
\Delta = \begin{vmatrix} f(r) & a & \ell \\ g(r) & b & m \\ h(r) & c & n \end{vmatrix}
$$
 where a, b, c, l, m and n are
constants, independent of r. Then

$$
\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} f(r) & a & \ell \\ \sum_{r=1}^{n} g(r) & b & m \\ \sum_{r=1}^{n} h(r) & c & n \end{vmatrix}.
$$
Here function of r can be the elements of only one row or
one column.

constants, independent of r. Then

10FTWODETERMINANTS
\nof two second order determinants is
\n
$$
\frac{1}{2} \text{ m}_1 \begin{vmatrix} 1 & t & 1 & t \ 2 & 1 & 2 \ 2 & 1 & 2 \end{vmatrix} = \begin{vmatrix} a_1 t & t & b_1 t & 2 & a_1 m_1 + b_1 m_2 \\ a_2 t & t & b_2 t & 2 & a_2 m_1 + b_2 m_2 \end{vmatrix}
$$

\nof two third order determinants is
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & t \ 2 & 1 & 2 & 1 \ 3 & 3 & 3 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & t \ 2 & 1 & 2 & 1 \ 4 & 5 & 3 & 1 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & t \ 2 & 1 & 2 & 1 \ 5 & 1 & 3 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 2 & 1 \ 5 & 1 & 3 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 2 & 1 \ 5 & 1 & 3 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 2 & 1 \ 5 & 1 & 3 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 3 & 4 \ 2 & 1 & 1 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 3 & 4 \ 2 & 1 & 1 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 3 & 4 \ 2 & 1 & 1 & 4 \end{vmatrix}$
\n $\frac{1}{3} \begin{vmatrix} 1 & t & 1 & 1 \ 2 & 1 & 1 & 4 \ 2 & 1 & 1 & 4 \end{vmatrix}$
\n

Here function of r can be the elements of only one row or one column.

LIMIT OFA DETERMINANT

2 2 2 2bc a c b c 2ca b a b a 2ab c ² a b c b c a c a b ² c b a a b c c a b Let f (x) g(x) h(x) (x) (x) m(x) n(x) , u(x) v(x) w(x) then x a x a x a x a x a x a x a x a x a x a lim f (x) lim g(x) lim h(x) lim (x) lim (x) lim m(x) lim n(x) , lim u(x) lim v(x) lim w(x)

provided each of nine limiting values exist finitely.

DIFFERENTIATION OF DETERMINANTS :

Let
$$
\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ \ell(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}
$$
, then

EXAMPLEMATION OF DETERMINANTS:
\n**ERENTIATION OF DETERMINANTS:**
\nLet
$$
\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}
$$
, then
\n
$$
\Delta'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ g(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & m'(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & m'(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}
$$
\n**Example 26:**
\n
$$
+ \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ g(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & w(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & h(x) \\ g(x) & h(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x
$$

Example 25 :

Let
$$
f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}
$$
, where p is a constant.

Then find
$$
\frac{d^3}{dx^3}[f(x)]
$$
 at x = 0.

Sol.
$$
\frac{d}{dx}f(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + 0 + 0
$$

Let
$$
f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}
$$
, where p is a constant.
\nThen find $\frac{d^3}{dx^3} [f(x)] dx = 0$.
\nThen find $\frac{d^3}{dx^3} [f(x)] dx = 0$.
\n $\frac{d}{dx} f(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + 0 + 0$
\n $= -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\sin^2 x \frac{\sin^2 x + \cos^3 x}{\sin^2 x} = -\left(\frac{1}{2} \cdot \frac{\pi}{2} + \frac{4.2}{5.3.1}\right) = -\left(\frac{\pi}{4} + \frac{8}{15}\right)$
\n $\frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & -1 & 0 \\ p & p^2 & p^3 \\ p & p^2 & p^3 \end{vmatrix}$
\n**APILLCATIONS OFDETERMINANT**
\n $\frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & -1 & 0 \\ p & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0$
\n $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - z_2 + z_3)] = -\frac{1}{2} \frac{x_1}{2} \frac{$

i.e. independent of p.

DETERMINANTS INVOLVING INTEGRATIONS

 $3^{1(A)}$ at α $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $\Delta(x)$ ℓ m n | where a, b, c, ℓ , m and (i) ℓ

are constants.

.

THEOREMING
\n**THEMATION OF DETERMINANTS:**
\n
$$
\Delta(x) = \begin{vmatrix}\nf(x) & g(x) & h(x) \\
g(x) & m(x) & n(x) \\
u(x) & v(x) & w(x)\n\end{vmatrix}, \text{ then } \qquad \Rightarrow \int_{a}^{b} \Delta(x) dx = \begin{vmatrix}\n\int_{a}^{b} f(x) dx & \int_{a}^{b} g(x) dx & \int_{a}^{b} h(x) dx \\
\int_{a}^{b} f(x) dx & \int_{a}^{b} g(x) dx & \int_{a}^{b} h(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} h(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} h(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} h(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x) dx & \int_{c}^{d} g(x) dx \\
\int_{c}^{d} f(x) dx & \int_{c}^{d} g(x)
$$

Example 26 :

Let
$$
f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos \sec x \\ \cos^2 x & \cos^2 x & \csc^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}
$$
.

Prove that
$$
\int_{0}^{\pi/2} f(x) dx = -\left(\frac{\pi}{4} + \frac{8}{15}\right)
$$
.

Sol. Operate $R_1 \rightarrow R_1$ – sec x R_3

S(TIDV MATERAI: MATEMAI: S
\n[*f(x)* ∞F) (5)
\n
$$
\Delta(x) = \begin{vmatrix}\nf(x) & g(x) & h(x) \\
g(x) & m(x) & n(x) \\
g(x) & m(x) & n(x)\n\end{vmatrix}, then\n= $\int_{a}^{b} A(x) dx = \int_{a}^{b} \frac{f(rx) dx}{m} \int_{a}^{b} \frac{f(rx) dx}{m} \int_{a}^{b} h(x) dx$
\n
$$
= \int_{a}^{b} A(x) dx = \int_{a}^{b} \frac{f(rx) dx}{m} \int_{a}^{c} \frac{f(rx) dx}{m} \int_{a}
$$
$$

$$
\begin{vmatrix}\nf(x) & g(x) & h(x) \\
\ell(x) & m(x) & n(x) \\
u'(x) & v'(x) & w'(x)\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\nx^3 & \sin x & \cos x \\
a & 1 & 0 \\
p & p^2 & p^3\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\nx^3 & \sin x & \cos x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\nx^2 & \sin x & \cos x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\cos^2 x & \cos^2 x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\sin x & \cos x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\sin x & \cos x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\sin x & \cos x \\
\frac{\pi}{2} & 1\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n3x^2 & \cos x & -\sin x \\
6 & -1 & 0 \\
p & p^2 & p^3\n\end{vmatrix} + 0 + 0
$$
\n
$$
\begin{vmatrix}\n\sin x & -\cos x \\
\frac{\pi}{2} & \sin^2 x & \cos^2 x\n\end{vmatrix}
$$
\n
$$
= -\sin^2 x \frac{\sin^2 x + \cos^3 x - \cos^3 x \sin^2 x}{\sin^2 x}
$$
\n
$$
= -\left(\sin^2 x + \cos^2 x\right) \left(\cos^2 x - 1\right)
$$
\n
$$
= -\sin^2 x \frac{\sin^2 x + \cos^3 x - \cos^3 x \sin^2 x}{\sin^2 x}
$$
\n
$$
\begin{vmatrix}\n6 & -\cos x & \sin x \\
6 & -1 & 0 \\
0 & -1 & 0 \\
0 & p^2 & p^3\n\end{vmatrix}
$$
\n
$$
= -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x}\right) dx
$$
\n
$$
= -\left(\frac{1}{2}, \frac{\pi}{2}, \frac{4, 2}{3, 3}\right) = -\left(\frac{\pi}{4}, \frac{8}{15}\right)
$$

APPLICATIONS OF DETERMINANT

1. Area of triangle : The area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , is given by the expression

$$
\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]
$$

Now this expression can be written in the form of a

$$
\int_{0}^{\pi/2} f(x) dx = -\int_{0}^{\pi/2} (\sin^{2} x + \cos^{5} x) dx
$$

= $-\left(\frac{1}{2} \cdot \frac{\pi}{2} + \frac{4.2}{5.3.1}\right) = -\left(\frac{\pi}{4} + \frac{8}{15}\right)$
ICATIONS OF DETERMINANT
Area of triangle : The area of a triangle whose vertices are
 $(x_{1}, y_{1}), (x_{2}, y_{2})$ and (x_{3}, y_{3}) , is given by the expression
 $\frac{1}{2} [x_{1}(y_{2} - y_{3}) + x_{2}(y_{3} - y_{1}) + x_{3}(y_{1} - y_{2})]$
Now this expression can be written in the form of a
determinant as $\Delta = \frac{1}{2} \begin{vmatrix} x_{1} & y_{1} & 1 \\ x_{2} & y_{2} & 1 \\ x_{3} & y_{3} & 1 \end{vmatrix}$ (1)
Since area is a positive quantity, we always take the absolute
value of the determinant in (1).
If area is given, use both positive and negative values of

- (i) Since area is a positive quantity, we always take the absolute value of the determinant in (1).
- (ii) If area is given, use both positive and negative values of the determinant for calculation.
- (iii) The area of the triangle formed by three collinear points is zero.

MATRICES AND DETERMINANTS

2. System of linear equations :

Definition-1 :

A system of linear equations in n unknowns $x_1, x_2, x_3, \ldots, x_n$ is of the form :

$$
\begin{cases}\na_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
\dots \\
\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n\n\end{cases}
$$
\n
$$
\begin{cases}\n\text{(a)} \quad \text{If } a_1 - \frac{a_2}{b_2} \quad b_1 \neq 0 \text{ i.e., if } a_1b_2 - a_2b_1 \neq 0. \\
\text{then we find from the first equation of system (iv) that}\n\end{cases}
$$

If b_1 , b_2 ,, b_n are all zero, the system is called **homogeneous** and non-homogeneous if at least one b_i is non-zero.

Definition-2 :

The solution set of the system (A) is an n type (α_1, α_2) , $..., \alpha_n$) of real numbers (or complex numbers if the coefficients are complex) which satisfy each of the equations of the system.

Definition-3 :

A system of equations is called **consistent** if it has at least one solution; **inconsistent** if it does not have any solution; **determinate** if it has a unique solution; **indeterminate** if it has more than one solution. For the system (A) is an n type $(\alpha_1, \alpha_2, \alpha_n)$

1 ution set of the system (A) is an n type $(\alpha_1, \alpha_2, \alpha_n)$
 α_n) of real numbers (or complex numbers if the

tients are complex) which satisfy each of the

for convenie 2 2 2 $a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$
 b_2, \dots, b_n are all zero, the system is called
 b_2, \dots, b_n are all zero, the system is called
 $\frac{c_1b_2-a_2b_1}{a_1b_2-a_2b_1}$
 $\frac{c_2b_1}{a_1b_2-a_2b_1}$
 $\frac{c_3b_2-a_3b_1}{a_1b_2-a_2b_1}$ x₁ + a_{n2}x₂ ++ a_{nn}x_n = b_n (A) (a) If a₁ - b₂ b₁ ≠ 0 i.e., if a₁b₂

x₁ + a_{n2}x₂ ++ a_{nn}x_n = b_n (A) then we find from the first equ

cous and non-homogeneous if at least one b₁ $x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$
 $\dots b_n$ are all zero, the system is called
 Example 18 and the most of the system (A) is an n type $(\alpha_1, \alpha_2, \beta_1)$
 $\alpha = \frac{a_1b_2 - a_2b_1}{a_1b_2 - a_2b_1}$

Substituting this value of x in

sy

(A) Non-homogeneous Equations in two unknowns : Consider the system of equations

$$
\begin{cases}\na_1x + b_1y = c_1 \\
a_2x + b_2y = c_2\n\end{cases}
$$
...(i)

We consider the following cases.

a x b y c **(1) ai, bi, ci (i = 1, 2) are all zero :** Then any pair of numbers (x, y) is a solution of the system (i) since in this case equation reduces to an identity. So, in this case equations are always **consistent and indeterminate.** 1 1 1 2 2 2 2 a x b y c a c x y b b a c a b x c ·b b b …(iii)

(2) a_{**i**}, **b**_{**i**} (**i** = **1**, **2**) are all zero, but at least one **c**₁ and **c**₂ is nonzero. Then the system has solution i.e. the equation are **inconsistent. EVALUATE:**

2) are all zero, but at least one **c₁ and c₂** is non-

the system has solution i.e. the equation are
 $\neq 0$. Then system (i), is equivalent to the
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow = 1, 2) are all zero, but at least one **c₁** and **c**₂ is non-

then the system has solution i.e. the equation are

tone of a_tb₁ (i = 1, 2) is non-zero

to the order a_tb₁ (i = 1, 2) is non-zero

to the order a For example the system has solution i.e. the equation are
 $\mathbf{r} \times \mathbf{r} = \mathbf{r}$
 $\mathbf{r} \times \mathbf$

(3) Al least one of $a_i b_i$ **(i = 1, 2) is non-zero**

Suppose $b_2 \neq 0$. Then system (i), is equivalent to the system.

$$
\begin{cases}\n a_1 x + b_1 y = c_1 \\
 \frac{a_2}{b_2} x + y = \frac{c_2}{b_2}\n\end{cases}
$$
...(ii)

i.e., if the pair (x_0, y_0) is a solution of system (i) then it is also a solution of system (ii), and vice-versa.

Multiplying the second equation of system (ii) by b_1 and (i) 1 subtracting from first, we get

$$
\left(a_1 - \frac{a_2}{b_2}b_1\right) x = c_1 - \frac{c_2}{b_2} \cdot b_1 \qquad \dots (ii)
$$

Now replacing the first equation of system (ii) by equation (i) (iii), we obtain the system

**CDMADVANGED LEARNNING
\n
$$
\left\{\left(a_1 - \frac{a_2}{b_2}b_1\right)x = c_1 - \frac{c_2}{b_2} \cdot b_1
$$
\n
$$
\left\{\begin{array}{c}\frac{a_2}{b_2}x + y = \frac{c_2}{b_2} \\ \frac{a_2}{b_2}y + y = \frac{c_2}{b_2}\end{array}\right\} \dots (iv)
$$
\nthen we find from the first equation of system (iv) that**

(a) If
$$
a_1 - \frac{a_2}{b_2} b_1 \neq 0
$$
 i.e., if $a_1b_2 - a_2b_1 \neq 0$.

$$
x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \qquad \qquad ...(v)
$$

Substituting this value of x into the second equation of system (iv), we obtain

$$
y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}
$$

For convenience, we write

$$
\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \ \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \ \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \dots (vi)
$$

[Note that Δ_x and Δ_y are obtained by replacing the first and second columns in Δ by the column of c_1 and c_2 respectively]. Then (v) and (vi) can be written as

$$
x = \frac{\Delta_x}{\Delta},
$$
 $y = \frac{\Delta_y}{\Delta}$...(vii)

 $\begin{pmatrix} a_2x + b_2y = c_2 \end{pmatrix}$...(1) by (vii). Hence in this case, the equations are **consistent** This is known as **Cramer's rule**. If $a_1b_2 - a_2b_1 \neq 0$ then the system (iv) or system (i) has the unique solution given **and determinate.** $\begin{vmatrix} b_1 \\ b_2 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$...(vi)

x and Δ_y are obtained by replacing the first

columns in Δ by the column of c_1 and c_2

...Then (v) and (vi) $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$
 $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{vm$ If Δ_x and Δ_y are obtained by replacing the list

and columns in Δ by the column of c_1 and c_2

ely]. Then (v) and (vi) can be written as
 Δ
 $\frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$...(vii)
 Δ
 $\frac{\Delta_x}{\Delta}$, $y = \frac{\Delta$ = $\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$

mvenience, we write

= $\begin{vmatrix} a_1 & b_1 \ a_2 & b_2 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} c_1 & b_1 \ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \ a_2 & c_2 \end{vmatrix}$...(vi)

that Δ_x and Δ_y are obtained by replacing the f $a_1b_2 - a_2b_1$

venience, we write
 $\begin{vmatrix} a_1 & b_1 \ a_2 & b_2 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} c_1 & b_1 \ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \ a_2 & c_2 \end{vmatrix}$...(vi)

ant Δ_x and Δ_y are obtained by replacing the first

ond columns in Δ y_2 a₂ v_1
ience, we write
 b_1 , $\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$...(vi)
 Δ_x and Δ_y are obtained by replacing the first

i columns in Δ by the column of c_1 and c_2 venience, we write
 $\begin{vmatrix} a_1 & b_1 \ a_2 & b_2 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} c_1 & b_1 \ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \ a_2 & c_2 \end{vmatrix}$...(vi)

aat Δ_x and Δ_y are obtained by replacing the first

ond columns in Δ by the column o m (iv), we obtain
 $y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$

convenience, we write
 $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$...(vi)

e that Δ_x and Δ_y are obta

(b) Now let $\Delta = a_1 b_2 - a_2 b_1 = 0$. Then the system (iv) has the form

$$
\begin{cases}\n0.x = c_1b_2 - c_2b_1 \\
\frac{a_2}{b_2}x + y = \frac{c_2}{b_2}\n\end{cases}
$$
...(viii)

Obviously this system has no solution if

$$
c_1b_2 - c_2b_1 = \Delta_x \neq 0
$$

thus in this case, the equations are inconsistent.

But if $\Delta_{\mathbf{x}} = 0$, then any pair of numbers (\mathbf{x}, \mathbf{y}) ,

where
$$
y = \frac{c_2}{b_2} - \frac{a_2}{b_2}x
$$
, $x \in R$, is a solution of system (viii).

In this case, the equations are consistent and indeterminate. We **summarize** the whole discussion given in (A) as follows:

(i) If $\Delta \neq 0$, then the system is consistent and determinant and its solution is given by

$$
x = \frac{\Delta_x}{\Delta}
$$
, $y = \frac{\Delta_y}{\Delta}$ (i.e., unique solution)

(ii) If $\Delta = 0$, but at least one of the numbers Δ_x , Δ_y is nonzero, then the system is inconsistent i.e., it has no solution.

- (iii) If $\Delta = 0$, and $\Delta_x = \Delta_y = 0$ but at least one of the numbers a_1 , b_1 , a_2 , b_2 is non-zero, then the system has infinite number of solutions and hence it is consistent and indeterminante.
- (iv) If $a_i = b_i = c_i = 0$ (i = 1, 2), then system has infinite number of solutions and so it is consistent and indeterminante.

(B) Homogenous linear equations in two unknowns :

Consider the system of equations

$$
\begin{cases} a_1x + b_1y = 0\\ a_2x + b_2y = 0 \end{cases}
$$
(ix)

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 $\frac{\text{EASTNING}}{2}$
 $\frac{\text{EASTNING}}{2}$
 $\frac{\text{$ The system always has the solution $x = 0$, $y = 0$. If follows from the discussion in part (A) that if $\Delta \neq 0$, then the system (ix) has the unique solution $x = 0$, $y = 0$.

And if $\Delta = 0$, and at least one of a_1 , a_2 , b_1 , b_2 is non-zero then system (i) reduced to the single equation so that any pair of numbers (x, y) is a solution. Thus system (ix) is always consistent. 1 1 1 1 $[x + b_1y = 0]$
 $\Delta = 0$, and at least one of a_1 , a_2 by $-c_2$
 $\begin{cases}\nx + b_1y = 0 \\
b_1x + b_2y = 0\n\end{cases}$
 $\begin{cases}\n\Delta = 0 \\
\Delta = 0\n\end{cases}$ and $\Delta = 0$, and at least one of a_1 , a_2 , b_1 , b_2 is non-zero
 $\Delta = 0$, and at lea $|x + b_1y = 0$
 $\ge x + b_2y = 0$
 $\ge x + b_2y = 0$
 $\ge x + b_1y = 0$
 $\ge x + b_2y = 0$
 $\ge x = 0$, and at least one of a_1, a_2, b_1, b_2 is non-zero

(ix) has the unique solution $x = 0, y = 0$. If follows
 $\triangle = 0$, and at least one of a_1 and the discussion in the unit of the system in the system is a series of equations and so it is consistent and indeterminante.

b₁ = c₁ = 0 (i = 1, 2), then system has infinite number

digital equation and any two va $x_1 = x_1 - 0$ and at least one of x_1 , x_2 , then is possible equation and y wo variables can
be the system of equations of the consistent and indeterminante. The values So equations are consistent and
example with $x =$ ations and so it is consistent and modernminante.

So equations are consistent and

der the system of equations
 $(a_1x + b_1y = 0)$ (ix)
 $a_1x + b_1y = 0$
 $a_2x + b_2y = 0$

(ix) as the solution $x = 0$, $y = 0$. If follows by is non-zero, then the system has infinite number
 $= b_1 = c_1 = 0$ (i=1, 2), then system and indeterminante.

Intions and hence it is consistent and indeterminante.

It are not all zero, then in this case the system

is =b₁ = c₁ = 0 (i = 1, 2), then system has infinite number

single equation and any two variables can be

ultions and so it is consistent and indeterminante.

identically the system of equations in two unknowns:

(a₁

(C) Non-homogeneous linear equations in three unknowns : Consider the system of equations

$$
\begin{cases}\na_1x + b_1y + c_1z = d_1 \\
a_2x + b_2y + c_2z = d_2 \\
a_3x + b_3y + c_3z = d_3\n\end{cases}
$$
(1)

Let us introduce the following notations

$$
\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}
$$

Without going into details, we give the following rule for testing the consistency of the system (1).

- **(1)** Let $a_i = b_i = c_i = d_i = 0, i = 1, 2, 3$ In this case any triplet (x, y, z) is a solution of the system. Hence equations are consistent and indeterminate.
- **(2)** If $a_i = b_i = c_i = 0$, $i = 1, 2, 3$ and at least one d_i ($i = 1, 2, 3$) is non-zero, then the system has no solution, i.e., the equations in this case are inconsistent. **(3)** Let $\Delta \neq 0$. In this case the system (1) has the unique (i)

solution $x = \frac{\Delta_x}{\Delta_y} = \frac{\Delta_y}{z} = \frac{\Delta_z}{z}$ (2) (ii)
-

solution $x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$ (ii) Inconsistent : S Δ_y Δ_z (2) $z = \frac{z}{\Delta}$ (2) (ii) Inconsistent : Solution c $\frac{\Delta_z}{\Delta}$ (2) (ii)

This is known as **Crammer's rule**. So equations in this case are consistent and determinate.

- **(4)** If $\Delta = 0$, $\Delta_{\mathbf{v}} \neq 0$ (or $\Delta_{\mathbf{v}} \neq 0$ or $\Delta_{\mathbf{z}} \neq 0$), then the system has no solution so the equations are inconsistent.
- **(5)** If $\Delta = \Delta_{\rm x} = \Delta_{\rm y} = \Delta_{\rm z} = 0$ and at least one of the cofactors of Δ is non-zero, then the system will have an infinite number of solutions. In this case, any one of the variables can be given arbitrary value and other variables can be expressed in terms of that variable.

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 $\int_{0}^{\frac{\pi}{2}} \cos \theta_{\infty} \cos \theta_{\infty} \cos \theta_{\infty} \cos \theta_{\infty} \cos \theta_{\infty} \sin \theta_{\infty}$
 $\int_{0}^{\frac{\pi}{2}} \sin 2\theta_{\infty} \cos \theta_{\infty} \cos \theta_{\infty} \cos \theta_{\infty} \sin \theta_{\infty} \sin \theta_{\infty} \sin \theta_{\infty} \sin \theta_{\infty} \sin \theta_{\infty} \sin \theta_{\infty} \sin$ **STUDY MATE**
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 C, and $\Delta_x = \Delta_y = 0$ but at least one of the numbers a_1 ,
 b_2 is non-zero, then the system has infinite number
 $b_1 = c_1 = 0$ ($i = 1, 2$), then system has infinite number
 $b_1 = c_1 = 0$ (i **STUDY MAT**

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In such cases, the three equation

is non-zero, then the system has infinite number

and $\Delta_x = \Delta_y = 0$ but at least one of the numbers a_1 ,

is and hence it is consistent and indeterminante.

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In such cases, the three equation

is non-zero, then the system has infinite numbers a_1 ,

and $\Delta_x = \Delta_y = 0$ but at least one of the numbers a_1 .

In such cases, the three equation

is non-zero, In such cases, the three equations reduce to two equations If all the cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ are not all zero, then in this case the system will reduced to single equation and any two variables can be given arbitrary values. So equations are consistent and indeterminate. cases, the three equations reduce to two equations

e cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ

all zero, then in this case the system will reduced to

call zero, then in this case the system will **STUDY MATERIAL: MATHEMATICS**

h cases, the three equations reduce to two equations

he cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ

tall zero, then in this case the system will reduced to

equation **STUDY MATERIAL: MATHEMATICS**

h cases, the three equations reduce to two equations

he cofactors Δ , Δ_y , Δ_y are zero but elements of Δ

tall zero, then in this case the system will reduced to

equation and any **STUDY MATERIAL: MATHEMATICS**
h cases, the three equations reduce to two equations
he cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ
t all zero, then in this case the system will reduced to
equation and **STUDY MATERIAL: MATHEMATICS**

ch cases, the three equations reduce to two equations

the cofactors Δ , Δ , Δ , Δ , Δ are zero but elements of Δ

ot all zero, then in this case the system will reduced to

e **STUDY MATERIAL: MATHEMATICS**

ch cases, the three equations reduce to two equations

the cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ

ot all zero, then in this case the system will reduced to

e equ

(D) Homogeneous linear equations :

 $\begin{cases} \alpha_1 x + b_1 y = 0 \\ a_2 x + b_2 y = 0 \end{cases}$ (ix) For such a system if $\Delta \neq 0$, then it has the unique solution If in (1), we take $d_i = 0$ (i = 1, 2, 3) then the system is called the homogenous system of equations. $x = 0, y = 0, z = 0.$ (Trivial) So such system of equations is always consistent. e cofactors Δ , Δ_x , Δ_y , Δ_z are zero but elements of Δ
all zero, then in this case the system will reduced to
quation and any two variables can be given arbitrary
So equations are consistent and indeterminate all zero, then in this case the system will reduced to
quation and any two variables can be given arbitrary
So equations are consistent and indeterminate.

seneous linear equations:

seneous die q = 0 (i = 1, 2, 3) then t

(1) Three equations in two unknowns :

Consider the equations

$$
\begin{cases}\na_1x + b_1y = c_1 \\
a_2x + b_2y = c_2 \\
a_3x + b_3y = c_2\n\end{cases}
$$
(3)

The system (3) will be consistent if the solutions set of any satisfies the third equations, i.e., if

$$
\begin{vmatrix} a_1 & b_1 & c_1 \ a_2 & b_2 & c_2 \ a_3 & b_3 & c_3 \end{vmatrix} = 0.
$$

Note : The factors of the following two determinants be remembered.

$$
f \circ \text{F} \text{or such a system if } \Delta \neq 0, \text{ then it has the unique solution}
$$
\n
$$
x = 0, y = 0. \text{ If follows that if } \Delta \neq 0, \text{ then the}
$$
\n
$$
x = 0, y = 0. \text{ or } 0. \text{ or } 0. \text{ So such system of equations is always consistent.}
$$
\n
$$
a_1, a_2, b_1, b_2 \text{ is non-zero}
$$
\n
$$
a_1, a_2, b_1, b_2 \text{ is non-zero}
$$
\n
$$
a_2, a_2, b_1, b_2 \text{ is non-zero}
$$
\n
$$
a_3x + b_3y = c_2
$$
\n
$$
a_3x + b_3y = c_3
$$
\n
$$
a_3
$$

(2) Gist of discussion in simple language :

- Consistent : Solution exists whether unique infinite number of solutions.
- Inconsistent : Solution does not exist.
- (iii) Homogeneous Equations : constant terms zero.
- Trivial solution : All variables zero i.e., $x = 0$, $y = 0$, $z = 0$.
- (v) Non-trivial solution : Infinite number of solutions. For example

$$
a_1x + b_1y = c_1
$$

\n
$$
a_2x + b_2y = c_2
$$

\n
$$
\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \Delta_1 \text{ or } \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix},
$$

$$
\Delta_2 \text{ or } \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}
$$

- **(3) Case-I :** Intersecting lines $2x + 3y = 10$ and $x + y = 4$ \therefore $x = 2, y = 2$ $\Delta = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1, \Delta \neq 0.$
- **(4) Case II :** $2x + 3y = 10$ $4x + 6y = 20$

Here $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$, or

but $\Delta_1 = \begin{vmatrix} 20 & 4 \end{vmatrix} = 0, \Delta_2 = 0$ $|10 \t2|$ $= 0, \Delta_2 = 0$

As a matter of fact on division by 2 the second equation reduces to first. Thus we have got only one line

 $2x + 3y = 10$ on which lie infinite number of points. Thus there are infinite number of solutions and the system is

consistent. $\left(k, \frac{10-3k}{2}\right)$ are infinite number of solutions

by giving different values to k.

Case-III $2x + 3y = 10$
 $4x + 16y = 15$ or $2x + 3y = 15/2$ or $\frac{3}{2}$ $+16y=15$ or $2x+3y=15/2$ or $\overline{-31}$ $x + 3y = 10$
 $x + 16y = 15$ or $2x + 3y = 15/2$ or $\frac{x}{-31} = \frac{-y}{62} = \frac{1}{-31}$,

i.e. parallel lines which we know do not intersect and hence no solution.

i.e. inconsistent. Here $\Delta = 0$ but $\Delta_1 \neq 0$, $\Delta_2 \neq 0$

NOTE

- (i) $\Delta \neq 0$ Unique (Intersecting lines) Consistent
- (ii) $\Delta = 0$, $\Delta_1 = 0$, $\Delta_2 = 0$ (Identical lines) Consistent, Infinite solution.
- (iii) $\Delta = 0$, $\Delta_1 \neq 0$ (Parallel lines) Inconsistent. No solution. Homogeneous : $a_1x + b_1y = 0$ $a_2x + b_2y = 0$ $\Delta \neq 0$, Unique x = 0, y = 0, Trivial. $\Delta = 0$, Identical line through origin, Non-trivial solution.

(5) Concurrent lines : Two variable, three equations :

 $a_1x + b_1y = c$, $a_2x + b_2y = c_2$, $a_3x + b_3y = c_3$ The point of intersection of any two lines should satisfy the third.

$$
\therefore \quad \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0
$$

is the required condition.

Example 27 :

For what value of λ the equations $2x + 3y = 8$, $7x - 5y + 3 = 0$ and $4x - 6y + \lambda = 0$ are consistent ? Also find the solution of the system of equations for the values of λ . **EDIMADVANCED LEARNING**

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2 3 -8

7 -5 3

4 -6 λ

-8(-42+20)=0

76=0

76=0

76

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 $\ln 4x - 6y + \lambda = 0$
he solution of the system of
 λ .
ar. We have 3 equations in 2
 $\begin{vmatrix} 2 & 3 & -8 \\ 7 & -5 & 3 \\ 4 & -6 & \lambda \end{vmatrix} = 0$
 $\begin{vmatrix} -8(-42+20) = 0 \\ 76 = 0 \end{vmatrix}$
aas a solution which c **EDENTADYANCED LEARNING**

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 $\left|\frac{4x-6y+\lambda=0}{4-6} \right|$

be solution of the system of
 $\left|\frac{2}{x-1} \cdot \frac{3}{4-6} \cdot \frac{3}{4}\right| = 0$
 $\left|\frac{-8(-42+20)=0}{-6-0}\right|$

76=0

3

and a solution which can be

of the t SO COMADVANCED LEARNING

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Ne have 3 equations in 2
 $\begin{vmatrix} 3 & -8 \\ -5 & 3 \\ -6 & \lambda \end{vmatrix} = 0$
 $\begin{vmatrix} -42 + 20 \end{vmatrix} = 0$
 $= 0$ **SPON ADVANCED LEARNING**
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Sol. Here the equations are linear. We have 3 equations in 2 unknowns.

$$
\therefore \text{ they are consistent if } \begin{vmatrix} 2 & 3 & -8 \\ 7 & -5 & 3 \\ 4 & -6 & \lambda \end{vmatrix} = 0
$$

or
$$
2(-5\lambda + 18) - 3(7y - 12) - 8(-42 + 20) = 0
$$

or
$$
-10\lambda + 36 - 21\lambda + 36 + 176 = 0
$$

or $-31\lambda + 248 = 0$; $\therefore \lambda = 8$

 \therefore for $\lambda = 8$ the system has a solution which can be obtained by solving any two of the three equations. Solving $2x + 3y - 8 = 0$

 $7x - 5y + 3 = 0$ by Cramer's rule,

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\na₁ c₁
\na₂ c₂
\nb)
$$
2x + 3y = 8, 7x - 5y + 3 = 0
$$
 and $4x - 6y + \lambda = 0$
\n $y = 10$ and $x + y = 4$
\n $y = 2$
\n $y = 2$
\n**EXECUTE:** For what value of λ the equations
\n $y = 10$ and $x + y = 4$
\n $y = 2$
\n $y = 2$
\n $2x + 3y = 10$
\n $4x + 6y = 20$
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $2x + 3y = 10$
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $2x + 3y = 10$
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $2x + 3y = 10$
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $2x + 3y = 0$
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$,
\n $\Delta = \begin{vmatrix} 2$

Example 28 :

- For what values of p and q the system of equations $2x + py + 6z = 8$ $x + 2y + qz = 5$ $x + y + 3z = 4$
- has (i) unique solution (ii) no solution
-
- (iii) infinite number of solutions? **Sol.** Here the system of linear equations in x, y, z are

2x + py + 6z – 8 = 0

$$
x + 2y + qz - 5 = 0
$$

x + y + 3z - 4 = 0

$$
\begin{vmatrix} -5 & 3 \ 0 & 7 & 3 \ \end{vmatrix} = \begin{vmatrix} 7 & -5 \ 7 & -5 \end{vmatrix}
$$

\nor $\frac{x}{9-40} = \frac{-y}{6+56} = \frac{1}{-10-21}$
\nor $\frac{x}{-31} = \frac{-y}{62} = \frac{1}{-31}$, $\therefore x = 1, y = 2$
\n**ple 28 :**
\nFor what values of p and q the system of equations
\n $2x+py+6z = 8$
\n $x+2y+qz = 5$
\n $x+y+3z = 4$
\nhas (i) unique solution (ii) no solution
\n(iii) infinite number of solutions?
\nHere the system of linear equations in x, y, z are
\n $2x+py+6z-8=0$
\n $x+2y+qz-5=0$
\n $x+2y+qz-5=0$
\n $x+y+3z-4=0$
\n $\therefore \Delta = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & p-2 & 0 \\ 1 & 1 & q-3 \\ 1 & 0 & 0 \end{vmatrix}$,
\n $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - 3 \times C_1$
\n $= \begin{vmatrix} p-2 & 0 \\ 1 & q-3 \end{vmatrix} = (p-2)(q-3)$
\n \therefore If $p \neq 2, q \neq 3$ then $D \neq 0$
\nand so the system will have unique solution, i.e., the system
\nwill be independent/solvable/consistent.
\nIf $p = 2$ or $q = 3$ then $\Delta = 0$.
\nand so the system cannot have unique solution.

 \therefore If $p \neq 2$, $q \neq 3$ then $D \neq 0$

and so the system will have unique solution, i.e., the system will be independent/solvable/consistent.

If $p = 2$ or $q = 3$ then $\Delta = 0$.

and so the system cannot have unique solution.

When $p = 2$,

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| When p = 2, | TRY IT YOLIR | |
| $\Delta_x = \begin{vmatrix} p & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 2 & 1 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ \n | Q.1 Show that $\begin{vmatrix} a-b-c & 2a & b-c \\ 2b & b-c-a & 2c \\ 2c & 2c & 2c \end{vmatrix}$ | |
| $\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & q & -5 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & q & -5 \end{vmatrix} = 0$ \n | $\therefore R_1 = R_3$ \n | Q.2 If a, b, c are in A.P., then the c |
| $\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & q & -5 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & q & -5 \end{vmatrix} = 0$ \n | $\therefore R_1 = R_2$ \n | $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ 1 & 1 & q+2b \end{vmatrix} = 1$ \n |

| EXAMPLE 3 | STUDY MA | |
|---|--|---|
| When p = 2, | TRY IT YOLIR | |
| $\Delta_x = \begin{vmatrix} p & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 2 & 1 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | Q.1 Show that | $\begin{vmatrix} a-b-c & 2a \\ 2b & b-c-d \\ 2c & 2c \end{vmatrix}$ |
| $\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | $(: R_1 = R_3)$ | Q.2 If a, b, c are in A.P., then the c |
| $\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | $(: R_1 = R_3)$ | $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is |
| $\Delta_z = \begin{vmatrix} 2 & p & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 2 & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & -4 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 0$ | Q.3 If α, β, γ are the roots of $x^3 - 2\pi$ | |

| When p = 2, | STUDY MA | | |
|---|---|--|---------------|
| When p = 2, | TRY IT YOLIE | | |
| $\Delta_x = \begin{vmatrix} p & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 2 & 1 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | Q.1 Show that $\begin{vmatrix} a-b-c & 2a \\ 2b & b-c-2c \\ 2c & 2c \end{vmatrix}$ | | |
| $\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | $(\because R_1 \equiv R_3)$ | Q.2 If a, b, c are in A.P., then the c | |
| $\Delta_z = \begin{vmatrix} 2 & 6 & -8 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0$ | $(\because R_1 \equiv R_3)$ | $(\because R_1 \equiv R_3)$ | $(A) 0$ |
| $\Delta_z = \begin{vmatrix} 2 & p & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 2 & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & -4 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 0$ | Q.3 If α, β, γ are the roots of x^3 – $(\because R_1 \equiv R_3)$ | $(\therefore R_1 \equiv R_3)$ | $(\therefore$ |

$$
\therefore \text{ when } p = 2, \Delta = 0, \Delta_x = \Delta_y = \Delta_z.
$$

\n
$$
\therefore \text{ the system of equations will have infinite number of solutions (the system of equations will be dependent) for } p = 2 \text{ and any real value of q.}
$$

When $q = 3$,

$$
\Delta_x = \begin{vmatrix} 2 & 9 & -5 \\ 1 & 3 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 9 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0
$$
 Q.1 Show that
$$
\begin{vmatrix} 2b & b-c-c \\ 2c & 2c \end{vmatrix}
$$

\n
$$
\Delta_y = \begin{vmatrix} 2 & 6 & -8 \\ 1 & 9 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & -4 \\ 1 & 9 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 0
$$
 $(\because R_1 = R_3)$
\n
$$
\Delta_z = \begin{vmatrix} 2 & p & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = \begin{vmatrix} 2 & 2 & -8 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & -4 \\ 1 & 2 & -5 \\ 1 & 1 & -4 \end{vmatrix} = 0
$$
 Q.3 If α, β, γ are the roots of $x^3 - 2$
\n \therefore the system of equations will have infinite number of solutions (the system of equations will have infinite number of solutions (the system of equations will be dependent) for $p = 2$ and any real value of q.
\nWhen $q = 3$,
\n
$$
\Delta_x = \begin{vmatrix} p & 6 & -8 \\ 2 & q & -5 \\ 1 & 3 & -4 \end{vmatrix} = \begin{vmatrix} p & 6 & -8 \\ 2 & 3 & -5 \\ 1 & 3 & -4 \end{vmatrix} = 2 \begin{vmatrix} p-2 & 0 & 0 \\ 2 & 3 & -5 \\ 1 & 3 & -4 \end{vmatrix}
$$
 Q.4 Without expanding the deter-ations with have no solutions, i.e., the system is solvable/incons with have no solutions, i.e., the system is solvable/incons with have $Q_2 = 12x + 3$ and $2x^2 + 3x - 1 = 3x$ and $3x - 3 = -3$ and $2x^2 + 3x - 1 = 3x$ and $3x - 3 = -3$ and

 \therefore p \neq 2, Δ x \neq 0 and so the system of equations will have no solutions, i.e., the system is solvable/inconsistent when $q = 3$ but $p \neq 2$.

- Thus we find that the system of equations will have
- (i) unique solution if $p \neq 2$ and $q \neq 3$
- (ii) no solution if $p \neq 2$ and $q = 3$
- (iii) infinite number of solutions if $p = 2$.

Example 29 :

Find values of k so that the following system of equations has non-trivial solution

$$
x + ky + 3z = 0; kx + 2y + 2z = 0; 2x + 3y + 4z = 0
$$

| (iii) infinite number of solutions if p = 2. | determinant | cos (α – β) |
|---|---|------------------------|
| Example 29 : | Find values of k so that the following system of equations has non-trivial solution x + ky+3z = 0; kx+2y+2z = 0; 2x+3y+4z = 0 | (A) a + b (C) a – b |
| Sol. Here Δ = 0 ⇒ $\begin{vmatrix} 1 & k & 3 \\ k & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = 0$ | Q.6 For what value of λ the equa- 2x + 3y = 8, 7x – 5y + 3 = 0 2x + 3y = 8, 7x – 5y + 3 = 0 2x + 3y = 8, 7x – 5y + 3 = 0 2x + 3y = 8, 7x – 5y + 3 = 0 2x + 3y = 8, 7x – 5y + 3 = 0 2x – 3x = 0 2x + 2y = 8, 7x – 5y + 3 = 0 2x = 0 2x = 0 2x = 0 3x = 2y + kz = 4 has unique solution if 2x = 2, 5/4 | |
| Example 30 : | Q.7 If f(x) = $\begin{vmatrix} 1 & 2 & x & x \ 2x - 1 & 2x & x + 2y + kz = 4 has unique solution if3x = 2x – 13x = 2x – 14x = 2x – 15x = 2, 5/4$ | |
| Sol. (2). Given system will have unique solution, if 2x = 2, 2x + y – z = 3, 3x + 2y + kz = 4 has unique solution, if 2x = 2, 2x + y – z = 3, 4x = 0 4x = 0 4x = 0 5x = 0 6x = 0 6x = 0 6x = 0 6x = | | |

Example 30 :

The system of equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has unique solution if $(1) k = 0$ (2) $k \neq 0$ $(3)-1 < k < 1$ (4) $-2 < k < 2$

Sol. (2). Given system will have unique solution, if

$$
\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0
$$

TRY IT YOURSELF-2

STUDY MATERIAL: MATHEMATICS
\n**TRY IT YOLURSELE-2**
\n
$$
= \begin{vmatrix}\n2 & 6 & -8 \\
2 & 9 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 2 \begin{vmatrix}\n1 & 3 & -4 \\
2 & 1 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 0
$$
\nQ.1 Show that
$$
\begin{vmatrix}\na - b - c & 2a & 2a \\
2b & b - c - a & 2b \\
2c & 2c & c - a - b\n\end{vmatrix} = (a + b + c)^3.
$$
\n
$$
= 2 \begin{vmatrix}\n1 & 3 & -4 \\
1 & 9 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 0 \quad (\because R_1 = R_3)
$$
\n
$$
= 2 \begin{vmatrix}\nx + 2 & x + 3 & x + 2a \\
x + 3 & x + 4 & x + 2b \\
x + 4 & x + 5 & x + 2c\n\end{vmatrix}
$$
\nis -
\n
$$
= 2 \begin{vmatrix}\n1 & 3 & -4 \\
1 & 9 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 0 \quad (\because R_1 = R_3)
$$
\n
$$
= 2 \begin{vmatrix}\n1 & 1 & -4 \\
1 & 4 & 0 \\
2 & 2 & -8\n\end{vmatrix} = 0 \quad (3) \quad (4) \quad (5) \quad (6) \quad (7) \quad (8) \quad (9) \quad (1) \quad (1) \quad (1) \quad (2) \quad (3) \quad (4) \quad (5) \quad (6) \quad (6) \quad (7) \quad (8) \quad (9) \quad (1) \quad (1) \quad (2) \quad (3) \quad (4) \quad (5) \quad (6) \quad (6) \quad (7) \quad (8) \quad (9) \quad (1) \quad (1) \quad (1) \quad (2) \quad (3) \quad (4) \quad (5) \quad (5) \quad (6) \quad (6) \quad (7) \quad (8) \quad (9) \quad (1) \quad (1) \quad (1) \quad (2) \quad (3) \quad (3) \quad (4) \quad (5) \quad (5) \quad (6) \quad (6) \quad (7) \quad (8) \quad (9) \quad (1) \quad (1) \quad (1) \quad (1) \quad (2) \quad (3
$$

 $(X: R_1 = R_3)$ **Q.2** If a, b, c are in A.P., then the determinants

STUDY MATERIAL: MATHEMATICS
\n
$$
= \begin{vmatrix}\n2 & 6 & -8 \\
2 & q & -5 \\
1 & 3 & -4\n\end{vmatrix} = 2 \begin{vmatrix}\n1 & 3 & -4 \\
2 & 1 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 0
$$
\n**Q.1** Show that
$$
\begin{vmatrix}\na-b-c & 2a & 2a \\
2b & b-c-a & 2b \\
2c & 2c & c-a-b\n\end{vmatrix} = (a+b+c)^3.
$$
\n
$$
= 2 \begin{vmatrix}\n1 & 3 & -4 \\
1 & q & -5 \\
1 & 3 & -4\n\end{vmatrix} = 0
$$
\n
$$
(\because R_1 = R_3)
$$
\n
$$
= \begin{vmatrix}\nx+2 & x+3 & x+2a \\
x+3 & x+4 & x+2b \\
x+4 & x+5 & x+2c\n\end{vmatrix} = 0
$$
\n
$$
= \begin{vmatrix}\n2 & 2 & -8 \\
1 & 2 & -5 \\
1 & 1 & -4\n\end{vmatrix} = 2 \begin{vmatrix}\n1 & 1 & -4 \\
1 & 2 & -5 \\
1 & 1 & -4\n\end{vmatrix} = 0
$$
\n**Q.3** If α, β, γ are the roots of $x^3 - 3x + 2 = 0$, then the value of $(\because R_1 = R_3)$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0
$$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0
$$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0
$$
\n
$$
= 0, \Delta_x = \Delta_x = \Delta_x
$$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0, \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0, \Delta_x = \Delta_x = \Delta_y = \Delta_z.
$$
\n
$$
= 0, \Delta_x = \Delta_x = \Delta_x = \Delta_x = \Delta_x
$$
\n
$$
= 0, \Delta_x = \
$$

 -4 | 1 -4 **Q.3** If α , β , γ are the roots of $x^3 - 3x + 2 = 0$, then the value of

 -5 that **Q.4** Without expanding the determinant at any stage show that

$$
\begin{vmatrix} x^2 + x & x + 1 & x - 2 \ 2x^2 + 3x - 1 & 3x & 3x - 3 \ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = Ax + B
$$

Q.5 If α , β are the roots of $ax^2 + bx + c = 0$, then the value of the

$$
\begin{vmatrix}\n2 & 2 & -8 \\
1 & 2 & -5 \\
1 & 1 & -4\n\end{vmatrix} = 0
$$
\n(C) x\n(D)2x
\n
$$
\begin{vmatrix}\n2 & 2 & -5 \\
1 & 2 & -5 \\
1 & 1 & -4\n\end{vmatrix} = 0
$$
\n(C) x\n(D)2x
\n(1) 2x
\n(2) 3x
\nEquations will be dependent) for
\n
$$
\begin{vmatrix}\n2 & 3 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 2
$$
\n
$$
\begin{vmatrix}\n5 & -8 \\
2 & 3 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 2
$$
\n
$$
\begin{vmatrix}\n9 & -2 & 0 & 0 \\
2 & 3 & -5 \\
1 & 3 & -4\n\end{vmatrix}
$$
\n(d) 4x
\n
$$
\begin{vmatrix}\n6 & -8 \\
1 & 3 & -4\n\end{vmatrix} = 2
$$
\n(e) 0
\n
$$
\begin{vmatrix}\n9 & 6 & -8 \\
2 & 3 & -5 \\
1 & 3 & -4\n\end{vmatrix} = 2
$$
\n
$$
\begin{vmatrix}\n9 & -2 & 0 & 0 \\
2 & 3 & -5 \\
1 & 3 & -4\n\end{vmatrix}
$$
\n(d) -3
\n(d) -3
\n(d) -3
\n(e) 2x
\n
$$
\begin{vmatrix}\n6 & 9 & x \\
9 & x & 8\n\end{vmatrix} = 8\nand solve the system of equations will have not\nthe system of equations will have\nthe system of equations will have\nthe system of equations will have\n
$$
\begin{vmatrix}\nx^2 + x & x + 1 & x - 2 \\
2x^2 + 3x - 1 & 3x & 3x - 3 \\
x^2 + 2x + 3 & 2x - 1 & 2x - 1\n\end{vmatrix} = Ax + B
$$
\n
$$
\begin{vmatrix}\n1 & \cos(\alpha - \beta) & \cos \alpha \\
\cos \alpha & \cos \beta & 1\n\end{vmatrix} = B
$$
\nso that the following system of equations
\nso that the following system of equations
\n
$$
\begin{vmatrix}\n1 & 2 & 3 \\
1 & 2 & 4 \\
1 & 3 & -4\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 &
$$
$$

Q.6 For what value of λ the equations $2x + 3y = 8$, $7x - 5y + 3 = 0$ and $4x - 6y + \lambda = 0$ are consistent? Also find the solution of the system of equations for the values of λ .

Q.7 If
$$
f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x-1)x(x-1) \end{vmatrix}
$$

then $f(100)$ is equal to –

(A) 0
\n(B) 1
\n(C) 100
\n(D) -100
\n(D) 100
\n(D) 100
\n3) (D)
\n(5) (B)
\n(6)
$$
\lambda = 8, x = 1, y = 2
$$

\n(7) (A)

USEFUL TIPS

Some important determinants to remember :

1.
$$
\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)
$$

2.
$$
\begin{vmatrix} 1 & x & x^{3} \\ 1 & y & y^{3} \\ 1 & z & z^{3} \end{vmatrix} = (x - y) (y - z) (z - x) (x + y + z)
$$

3.
$$
\begin{vmatrix} x & x^{2} & yz \\ y & y^{2} & zx \\ z & z^{2} & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)
$$

4.
$$
\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + a^3 - 3abc) < 0
$$

if a, b, c are different and positive.

ADDITIONAL EXAMPLES

Example 1 :

If A =
$$
\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}
$$
 then find adj (adj A)

Sol. We know adj (adj. A) = $|A|^{n-2}$ A Now if $n = 3$ then adj (adj A) = |A| A

$$
= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} A = \{1(6-1)-2(4-3)+3(2-9)\} A
$$

= (5-2-21) A=-18 A

Example 2 :

If
$$
M(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
; $M(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$
then find $[M(\alpha) M(\beta)]^{-1}$.

Sol. $[M(\alpha) M(\beta)]^{-1} = M(\beta)^{-1} M(\alpha)^{-1}$

| RICES AND DETERMINANTS | ESERUL TIPS | cos $(-\alpha)$ - sin $(-\alpha)$ 0] | | | |
|------------------------|--------------|--------------------------------------|---|--------------|--------------|
| 1 $x x^2$ | 1 $y y^2$ | = $(x-y)(y-z)(z-x)$ | $log(A) = 0$ | $log(A) = 0$ | $log(A) = 0$ |
| 1 $x x^2$ | 1 $y y^3$ | = $(x-y)(y-z)(z-x)$ | $log(B)^{-1} = \begin{bmatrix} cos(-\beta) & 0 & sin(-\beta) \\ 0 & 1 & 0 \\ sin\beta & 0 & cos\beta \end{bmatrix}$ | | |
| 1 $x x^3$ | 1 $y y^3$ | = $(x-y)(y-z)(z-x)(x+y+z)$ | $log(A) = 0$ | | |
| 2 z^2 | 3 y^2 | $log(A) = 0$ | | | |
| 3 y^2 | z^2 | $log(A) = 0$ | | | |
| 4 z^2 | $log(A) = 0$ | $log(A) = 0$ | | | |
| 5 z^2 | $log(A) = 0$ | | | | |
| 6 z^2 | z^2 | $log(A) = 0$ | | | |
| 7 z^2 | $log(A) = 0$ | | | | |
| 8 z^2 | $log(A) = 0$ | | | | |
| 9 z^2 | $log(A) = 0$ | | | | |
| 10 z^2 | $log(A) = 0$ | | | | |
| 2 | | | | | |

Example 3 :

If
$$
I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$
, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$,
then find B in terms of I and J.

$$
\begin{aligned}\n&= \begin{bmatrix}\n\cos(-\beta) & 0 & \sin(-\beta) \\
0 & 1 & 0 \\
-\sin(-\beta) & 0 & \cos(-\beta)\n\end{bmatrix} = M(-\beta) \\
[M(\alpha) M(\beta)]^{-1} &= M(-\beta) M(-\alpha)\n\end{aligned}
$$
\nExample 3:
\nIf $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$,
\nthen find B in terms of I and J.
\nSol. Here $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ 0 & \cos \theta \end{bmatrix} + \begin{bmatrix} 0 & \sin \theta \\ -\sin \theta & 0 \end{bmatrix}$
\n $= \cos \theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin \theta \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = I \cos \theta + J \sin \theta$
\nExample 4:
\nIf $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then find A^{-n} .
\nSol. $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$; $A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

Example 4 :

$$
\begin{vmatrix}\n1 & x & x^3 \\
1 & y & y^3 \\
1 & z & z^3\n\end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z) \n= \begin{bmatrix}\n\cos(-\beta) & 0 & \sin(-\beta) \\
0 & 1 & 0 \\
-\sin(-\beta) & 0 & \cos(-\beta)\n\end{bmatrix} = M(-\beta)
$$
\n
$$
\begin{vmatrix}\nx & x^2 & yx \\
y & y^2 & zx \\
z & z^2 & xy\n\end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)
$$
\nExample 3:
\n
$$
\begin{vmatrix}\na & b & c \\
b & c & a \\
c & a & b\n\end{vmatrix} = -(a^3+b^3+a^3-3abc) < 0
$$
\n
$$
\begin{vmatrix}\na & b & c \\
b & c & a \\
c & a & b\n\end{vmatrix} = -(a^3+b^3+a^3-3abc) < 0
$$
\nSolution
\n**ADDITIONAL EXAMPLES**
\n**Example 4:**
\nIf A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$,
\n
$$
\begin{vmatrix}\na & b & c \\
b & c & a \\
c & a & b\n\end{vmatrix} = -(a^3+b^3+a^3-3abc) < 0
$$
\n**SoL Here** $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$
\n
$$
\begin{vmatrix}\n\cos\theta & \sin\theta \\ 0 & 1\n\end{vmatrix} = I\cos\theta + J\sin\theta
$$
\n**Example 4:**
\nIf A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$
\nNow if n =

Example 5 :

$$
\begin{vmatrix}\n\mathbf{c} & \mathbf{a} & \mathbf{b} \\
\mathbf{c} & \mathbf{a} & \mathbf{b}\n\end{vmatrix} = -(\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{a}^2 - 3\mathbf{a}\mathbf{b}) \mathbf{c} \n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 2 & 3 \\
2 & 3 & 1 \\
3 & 1 & 2\n\end{bmatrix} \text{ then find } \mathbf{a} \mathbf{d} \mathbf{j} \text{ (a d) A}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 2 & 3 \\
2 & 3 & 1 \\
3 & 1 & 2\n\end{bmatrix} \text{ then find } \mathbf{a} \mathbf{d} \mathbf{j} \text{ (a d) A}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
2 & 3 & 1 \\
3 & 1 & 2\n\end{bmatrix} \text{ then find } \mathbf{d} \mathbf{d} \mathbf{j} \text{ (a d) A}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
1 & 1\n\end{bmatrix} \text{ then find } \mathbf{A}^{-n}.
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix} = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix} = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix} = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix} = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix}
$$
\n
$$
\mathbf{F}(\mathbf{A}) = \begin{bmatrix}\n1 & 0 \\
-1 & 1\n\end{bmatrix} = \begin{bmatrix}\n1 & 0 \\
-2 & 1\n\end{bmatrix}; \quad \mathbf{A}^{-n} = \begin{bmatrix}\n1 & 0
$$
$$

Hence transposing [A_{ij}] we get adj A =
$$
\begin{vmatrix} -24 & 4 & 8 \\ -27 & 1 & 11 \\ 30 & -2 & -10 \end{vmatrix}
$$

Example 6 :

If
$$
\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}
$$
, then find $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$

Sol. We know that if any row of a determinant is multiplied by k, then the value of the determinant is also multiplied by k, Here all the three rows are multiplied by k, therefore the value of new determinant will be $k^3 \Delta$.

Example 7 :

Find
$$
\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \ b^2 & c^2 + a^2 & b^2 \ c^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$

Sol. Applying $R_1 - (R_2 + R_3)$, we get

If
$$
\Delta = \begin{vmatrix} x & y & z \\ p & q & r \end{vmatrix}
$$
, then find $\begin{vmatrix} kx & ky & kz \\ kp & ka & rr \end{vmatrix}$
\nWe know that if any row of a determinant is multiplied by k,
\nk, then the value of the determinant will be k³ A.
\nHere all the three rows are multiplied by k, therefore the
\nvalue of new determinant will be k³ A.
\nHence
\n
$$
\begin{vmatrix} 6^2 + x^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ b^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ b^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ b^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6^2 - x^2 & a^2 & b^2 \\ c^2 & 6 & a^2 \end{vmatrix}
$$
\n
$$
\begin{vmatrix} 6x - 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1
$$
\n
$$
\begin{vmatrix} 6x - 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1
$$
\n
$$
\begin{vmatrix} 6x - 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1
$$
\n
$$
\begin{vmatrix} 6x - 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1
$$
\n
$$
\begin{vmatrix} 6x - 1 & 1 & 1 \\ 1 & 1 &
$$

Example 8 :

$$
\begin{vmatrix}\n e^{2} & e^{2} & a^{2} + b^{2} & e^{2} & a^{2} + b^{2} & e^{2} & a^{2} + b^{2}\n\end{vmatrix}
$$
\n
$$
= 2 \begin{vmatrix}\n 0 & -c^{2} & -b^{2} \\
 b^{2} & a^{2} & 0 \\
 c^{2} & 0 & a^{2}\n\end{vmatrix}
$$
\n
$$
(by R_{2} + R_{1}, R_{3} + R_{1})
$$
\n
$$
= 2 (a^{2}b^{2}c^{2} + a^{2}b^{2}c^{2}) = 4a^{2}b^{2}c^{2}
$$
\n
$$
= 2 (a^{2}b^{2}c^{2} + a^{2}b^{2}c^{2}) = 4a^{2}b^{2}c^{2}
$$
\n
$$
= 4a^{2}c^{2}c^{2}
$$
\n
$$
= 4a^{2}
$$

Sol. (A). C_1 : C_1 + C_2 + C_3 gives,

$$
D = \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}
$$

Taking $2(a + b + c)$ as common factor and then $R_2 : R_2 - R_1$ and R_3 : $R_3 - R_1$. gives

D=2 (a+b+c)
$$
\begin{vmatrix} 1 & b+c & c+a \\ 0 & a-b & b-c \\ 0 & a-c & b-a \end{vmatrix}
$$

=2 (a+b+c) [-(a-b)²-(b-c) (a-c)]
=-2(a+b+c) {a²+b+ +c²-ab-bc-ca}
=-2(a³+b³+c³-3 abc)

Example 9 :

**EXAMPLEMATICS
\n
$$
F(A) = \begin{vmatrix}\n1 & b & c \\
x & y & z \\
p & q & 1\n\end{vmatrix}
$$
, then find $\begin{vmatrix}\nka & kb & kc \\
kx & ky & kz \\
kp & kx & kx\n\end{vmatrix}$
\nWe know that if any row of a determinant is multiplied by k,
\nk, then the value of the determinant is also multiplied by k,
\nwhere all the three rows are multiplied by k, therefore the
\nvalue of new determinant will be $k^3\Delta$.
\n
$$
A^{-1} = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}
$$

\n
$$
P(A^{-1}) = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}
$$

\n
$$
P(B^{-1})
$$

\n
$$
P(C)
$$

\n
$$
P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
$$

\n
$$
P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
$$

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$$
P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
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P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
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P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
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P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
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P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
$$

\n
$$
P(D \times 1)^2 = \begin{bmatrix} 1 & -\tan x \\ \frac{1}{2} & \tan x & 1 \end{bmatrix}
$$

\n
$$
P(D \times 1)^2 = \begin{bmatrix} 1 & -\
$$**

Example 10 :

Find the number of positive integral solutions of the

$$
\begin{vmatrix}\nc^2 & -b^2 \\
a^2 + a^2 & b^2 \\
c^2 & a^2 + b^2\n\end{vmatrix}
$$
 equation
$$
\begin{vmatrix}\nx^3 + 1 & x^2y & x^2z \\
xy^2 & y^3 + 1 & y^2z \\
xz^2 & yz^2 & z^3 + 1\n\end{vmatrix} = 11.
$$

$$
A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix} \Rightarrow |A'A^{-1}| = 1
$$

\n**Example 10 :**
\nFind the number of positive integral solutions of the
\nequation
$$
\begin{vmatrix} x^3 + 1 & x^2y & x^2z \\ xy^2 & y^3 + 1 & y^2z \\ xz^2 & yz^2 & z^3 + 1 \end{vmatrix} = 11.
$$

\n**SoI.** LHS =
$$
\begin{vmatrix} x^3 & x^2y & x^2z \\ xy^2 & y^3 + 1 & y^2z \\ xz^2 & yz^2 & z^3 + 1 \end{vmatrix} + \begin{vmatrix} 1 & x^2y & x^2z \\ 0 & y^3 + 1 & y^2z \\ 0 & yz^2 & z^3 + 1 \end{vmatrix}
$$

\n
$$
\begin{vmatrix} x^2 & 0 & 0 \\ y^2 & 1 & 0 \end{vmatrix}
$$

1 + tan² x [tan x 1]²
\nA'A⁻¹ =
$$
\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix} \Rightarrow |A'A^{-1}| = 1
$$

\n
\n**Example 10:**
\n
\n**Example 10:**
\nFind the number of positive integral solutions of the
\n
$$
= 2 \begin{vmatrix} 0 & -c^2 & -b^2 \\ b^2 & c^2 & a^2 + b^2 \end{vmatrix}
$$
\nequation $\begin{vmatrix} x^3 + 1 & x^2y & x^2z \\ xy^2 & y^3 + 1 & y^2z \\ xz^2 & yz^2 & z^3 + 1 \end{vmatrix} = 11.$
\n(b) $R_2 + R_1, R_3 + R_1$)
\n**Example 10:**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(by R_2 + R_1, R_3 + R_1)$
\n**Substituting the equation**
\n $(x^2 - 0, 0)$
\n**Substituting the equation**
\n $(x^2 - 0, 0)$
\n**Substituting the equation**
\n $(x^2 - 0, 0)$
\n**Substituting the equation**
\n $(x$

+ c³ - 3abc) As $10 = 2^3 + 1^3 + 1^3$, the solutions are $(2,1,1), (1,2,1), (1,1,2).$

Example 11 :

Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if –

(A) The first column of M is the transpose of the second row of M.

(B) The product of entries in the main diagonal of M is not the square of an integer.

(C) M is a diagonal matrix with nonzero entries in the main diagonal.

(D) Both
$$
(B)
$$
 and (C)

2c²) = 4a²b²c²
\n
$$
c + a
$$

\na + b + c
\n
$$
c + a
$$

\nb + c $c + a$
\nc + a a + b
\n
$$
a + b + c
$$
\n
$$
c^3
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MATRICES AND DETERMINANTS

Example 12 :

- Let M and N be two 3×3 matrices such that MN = NM. \therefore \therefore \therefore \Rightarrow N and M² = N⁴, then
- (A) Determinant of $(M^2 + MN^2)$ is 0
- (B) There is a 3×3 non-zero matrix U such that (M^2+MN^2) U is the zero matrix.
- (C) Determinant of $(M^2 + MN^2) \ge 1$
- (D) Both (A) and (B)
- **Sol. (D).** $M^2 N^4 = 0 \Rightarrow (M N^2)(M + N^2) = 0$ $M - N^2 = 0$ not Possible $M + N^2 = 0$; $|M + N^2| = 0$ $M - N^2 \neq 0$; $|M - N^2| = 0$ In any case $|M + N^2| = 0$ (A) $(|M^2 + MN^2| = |M| |M + N^2| = 0$
	- (B) If $|A| = 0$ then $AU = 0$ will have solution. Thus $(M^2 + MN^2)$ U = 0 will have many 'U'.

Example 13 :

If α is a characteristic root of a non-singular matrix,

then prove that $\left| \frac{\ }{a} \right|$ is a characteristic root of adj A.

Sol. Since a is a characteristic root of a non-singular matrix, therefore $a \neq 0$. Also a is a characteristic root of A implies that there exists a non-zero vector X such that
 $A Y = \alpha Y$

In any case |M + N²| = 0
\n(A) (|M² + MN²| = |M| |M + N²| = 0
\n(B) If |A| = 0 then AU = 0 will have solution.
\nThus (M² + MN²)U = 0 will have many 'U'.
\n
$$
\Rightarrow X = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} =
$$
\n
$$
\Rightarrow \text{where } 13:
$$
\nIf α is a characteristic root of a non-singular matrix,
\nthen prove that $\begin{bmatrix} A \\ \alpha \end{bmatrix}$ is a characteristic root of adj A.
\nSince a is a characteristic root of a non-singular matrix,
\ntherefore a ≠ 0. Also a is a characteristic root of A implies
\n
$$
AX = \alpha X
$$
\n
$$
\Rightarrow (\text{adj } A)(AX) = (\text{adj } A)(\alpha X)
$$
\n
$$
\Rightarrow [(\text{adj } A)A] X = \alpha(\text{adj } A)X
$$
\n
$$
\Rightarrow |A| IX = \alpha(\text{adj } A)X
$$
\n[\because (adj) A] $A = |A| I$]
\n
$$
\Rightarrow |A| X = \alpha(\text{adj } A)X \Rightarrow \frac{|A|}{\alpha} X = (\text{adj } A)X
$$
\nIf $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then find the v
\n
$$
\Rightarrow |\text{adj } A| = |A| I |
$$
\n
$$
\Rightarrow |A| X = \alpha(\text{adj } A)X \Rightarrow \frac{|A|}{\alpha} X = (\text{adj } A)X
$$
\n
$$
\Rightarrow (\text{adj } A)X = \frac{|A|}{\alpha} X
$$
\nSince X is a non-zero vector, therefore $\begin{bmatrix} A \\ \alpha \end{bmatrix}$ is a
\ncharacteristic root of the matrix adj A.
\n
$$
\Rightarrow \text{Since } X \text{ is a non-zero vector, therefore } \begin{bmatrix} A \\ \alpha \end{bmatrix}
$$
 is a
\ncharacteristic root of the matrix adj A.
\nSolve the following system of equations using matrix
\n
$$
\begin{bmatrix
$$

Since X is a non-zero vector, therefore $\left| \frac{\ }{8} \right|$ is a characteristic root of the matrix adj A.

Example 14 :

Solve the following system of equations, using matrix method : $x + 2y + z = 7$, $x + 3z = 11$, $2x - 3y = 1$.

Sol. The given system of equation is

⇒ [(adj A) A] X = α(adjd A) X
\n⇒ |A| IX = α(adjd A)X [∴ (adj) A] A = |A| I]
\n⇒ |A| X = α(adjd A)X [∴ (adj) A] A = |A| I]
\n⇒ |A| X = α(adjd A)X
\n⇒ (adj A) X =
$$
\frac{|A|}{\alpha}
$$
 X
\nSince X is a non-zero vector, therefore $\left[\frac{A}{\alpha}\right]$ is a characteristic root of the matrix adj A.
\n $\frac{d}{\alpha} = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$
\n= $\begin{vmatrix} |A| & 0 & 0 \\ 0 & 0 & |A| \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = \begin{vmatrix} 9 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = \begin{vmatrix} 9 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} \begin{vmatrix} x_1$

So, the given system of equation has a unique solution given by $X = A^{-1} B$

So, the given system of equation has a unique solution
\ngiven by
$$
X = A^{-1}B
$$

\n \therefore adj $A = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$
\n $\Rightarrow A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{8} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$
\nNow, $X = A^{-1}B$
\n $\Rightarrow X = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$
\n $\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \Rightarrow x = 2, y = 1, z = 3$

Now, $X = A^{-1}B$

THERMINANTS
\ntwo 3 × 3 matrices such that MN = NM.
\nand M² = N⁴; then
\nand M² = N⁴; then
\n3 non-zero matrix U such that
\n3 non-zero matrix U such that
\n
$$
\therefore \text{ adj } A = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}
$$
\n
$$
\Rightarrow A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{8} \begin{bmatrix} 9 & -3 & 6 \\ -3 & -2 & 7 \\ -3 & 7 & -2 \end{bmatrix}
$$
\n
$$
\Rightarrow (M+N^2) = 0
$$
\n
$$
\Rightarrow (M+N^2) = 0
$$
\n
$$
M+N^2 = 0
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\n
$$
M+N^2 = 0
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\n
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M+N^2 = 0
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\n
$$
M=N^2U = 0
$$
\nNow, X = A⁻¹B
\n
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M=N^2U = 0
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\nNow, X = A⁻¹B
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M=N^2U = 0
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\n
$$
M=N^2U = 0
$$
\nNow, X = A⁻¹B
\n
$$
M=N^2U = 0
$$
\n
$$

$$

Example 15 :

If
$$
A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}
$$
, then find the value of |A| |adj A|.

Sol. $|A| |adj A| = |A adj A| = |A| |I|$

$$
= \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3 = (a^3)^3 = a^9
$$

Example 16 :

By the method of matrix inversion, solve the system.

3.1 a characteristic root of a non-singular matrix,
\nprove that
$$
\left[\frac{\alpha}{\alpha}\right]
$$
 is a characteristic root of a adj A.
\n $\Rightarrow \left[\frac{x}{z}\right] = \frac{1}{18}\left[\frac{36}{18}\right] = \frac{1}{2} = \frac{1}{18}\left[\frac{36}{18}\right] = \frac{1}{2} = \frac{1}{2} = \frac{1}{28}$
\n \Rightarrow a is a characteristic root of a nonsingular matrix,
\nfor a $\neq 0$. Also a is a characteristic root of A implies
\nA(x - \alpha X)
\nA(x) A (X) = (\alpha d) A (X)
\nA(x) = (\alpha d) A (X)
\nB(x) = (\alpha d) A (X

$$
\therefore A^{-1} = \frac{\text{adj. A}}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}
$$

$$
A^{-1}B = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}
$$

$$
= \frac{-1}{4} \begin{bmatrix} -4 & 4 \\ -12 & -8 \\ -20 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}
$$

From equation (i),

$$
X = A^{-1}B \implies \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}
$$

\n
$$
\implies x_1 = 1, x_2 = 3, x_3 = 5 \text{ or } y_1 = -1, y_2 = 2, y_3 = 1
$$

Example 17 :

If A, B and C are $n \times n$ matrix and $det(A) = 2$, $det(B) = 3$ and det $(C) = 5$, then find the value of the det $(A^{2}BC^{-1})$. **Sol.** Given that $|A| = 2$, $|B| = 3$, $|C| = 5$.

$$
\det (A^{2}BC^{-1}) = |A^{2}BC^{-1}| = \frac{|A|^{2}||B|}{|C|} = \frac{4 \times 3}{5} = \frac{12}{5}
$$

Example 18 :

STUDY MATERIAL: MATHEMATIC
 $\therefore A^{-1} = \frac{adj. A}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $A^{-1}B = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 16 & -3 & -5 \end{bmatrix}$
 $A^{-1}B = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 16 & -3 & -5 \end{bmatrix} \begin{bmatrix} 9 &$ **STUDY MATERIAL: MATHEMATIC**

adj. A $= -1$ $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 16 & -3 & -5 \end{bmatrix}$
 Example 18 :

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$
 STUDY MATERIAL:
 $\frac{d\vec{j} \cdot A}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18:
 Example 18:
 Example 18:
 Example 18:

Matrices A and B satisfy $AB = B^{-1}$, where $\vec{k}A = 2B^{-1} + I = 0$, (ii) without **STUDY MATERIAL: MATHEMATIC**

8 1 3 3
 Example 18 :

8 50 4 1 1 0 4 5 8 4 5 8 4 7 A 5 = B⁻¹, **STUDY MATERIAL: MATHEMATICS**
 Example 18 :
 $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18 :

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

Find (i) without finding B⁻¹, the val **STUDY MATERIAL: MATHEMATICS**
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18:

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

Find (i) without finding B⁻¹, the value of K for **STUDY MATERIAL: MATHEMATICS**
 Example 18 :
 $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18 :

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

Find (i) without finding B⁻¹, the val **STUDY MATERIAL: MATHEMATI**
 $\frac{A}{1} = -1\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $= -3\begin{bmatrix} 9 & 2 \\ 2 & 15 \\ 0 & -1 \end{bmatrix}$
 $= -4\begin{bmatrix} -4 & 4 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18:
 Example 18:
 Example 18:
 Example 18:
 Example STUDY MATERIAL: MATHEMAT
 $\frac{1}{4}A = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 52 & 15 \\ 16 & -1 \end{bmatrix}$
 $\begin{bmatrix} 52 & 15 \\ 2 & -1 \end{bmatrix}$
 $\begin{bmatrix} -4 & 4 \\ -8 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 5 & 1 \end{bmatrix}$
 $\begin{bmatrix} -4 & 4 \\ -20 & -4 \end{bmatrix$ **STUDY MATERIAL: MATHEMATION**
 $\frac{A}{1} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18:
 $\frac{A}{1} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 Example 18:
 Example 18:
 Example 18:
 Example 18: STUDY MATERIAL: MATHEMATIC
 $\frac{di}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{aligned}\n&\text{Example 18:} \\
&\text{Matrices A and B satisfy AB} = B^{-1}, \text{ where } B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} \\
&\text{Find (i) without finding } B^{-1}, \text{ the value of K for which} \\
&\text{KA} = 2B^{-1} + I = 0, \text{ (ii) without finding } A^{-1}, \text{ the matrix }$ STUDY MATERIAL: MATHEMATIC
 $= \frac{adj. A}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $= \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$
 $= \frac{-1}{2} \begin{bmatrix} -4 & 4 \\ -12 & -8 \end{bmatrix} \begin{$ STUDY MATERIAL: MATHEMATIC
 $\frac{d\vec{j} \cdot A}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 52 & 15 \\ 2 & 15 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ 52 & 15 \\ -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$
 STUDY MATERIAL: MATHEMA
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $=\frac{1}{4}\begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$
 $=\frac{1}{4}\begin{bmatrix} 52 & 15 \\ 0 & -1 \end{bmatrix}$
 $=\frac{1}{4}\begin{bmatrix} 1 & -1$ **STUDY MATERIAL: MATHEMA**
 $\begin{bmatrix}\n1 & -1 \\
2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$ **Example 18:**
 $\begin{bmatrix}\n2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$ **Example 18:**
 $\begin{bmatrix}\n2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$ **Example 18:**
 $\begin{bmatrix}\n2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$ **Example 18:** STUDY MATERIAL: MATHEMA
 $\begin{bmatrix}\n-1 & -1 & 2 & 2 \\
-8 & -1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n-12 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n2 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n2 & 2 & 2 \\
2 & 15 \\
-3 & -5\n\end{bmatrix}$
 $\begin{bmatrix}\n2 & 2 & 2 \\
5$ STUDY MATERIAL: MATHEMAT
 $\frac{A}{A} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $\frac{12}{3} \begin{bmatrix} 9 & 2 \\ 16 & -3 \end{bmatrix}$
 $\frac{12}{3} \begin{bmatrix} 9 & 2 \\ 2 & 15 \\ 0 & -1 \end{bmatrix}$
 $\frac{16}{3} \begin{bmatrix} -4 & 4 \\ -20 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$ STUDY MATERIAL: MATHEMAT
 $=\frac{adj. A}{|A|} = \frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $=\frac{-1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $=\frac{-1}{4} \begin{bmatrix} -1 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $=\frac{-1}{4} \begin{bmatrix} -4 & 4 \\ -20 & -4 \end{bmatrix$ STUDY MATERIAL: MATHEMAT
 $\frac{A}{N_1} = \frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$
 $= \begin{bmatrix} 9 & 2 \\ 5 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$
 $= \begin{bmatrix} 9 & 2 \\ 2 & 15 \end{bmatrix}$
 $= \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$
 $= \begin{bmatrix} 4 & 4 \\ 5 & 1 \end$ $\begin{bmatrix}\n-\frac{1}{4} \begin{bmatrix}\n-12 & 2 & 2 \\
16 & -3 & -5 \\
-8 & 1 & 3\n\end{bmatrix}\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
52 & 15 \\
1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
52 & 15 \\
0 & -1\n\end{bmatrix}$
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-8 & 1\n\end{bmatrix}$
 $\begin{bmatrix}\n1 & -1 \\
5 & 1\n\end{bmatrix}$
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-8 & 1\n\end{bmatrix}$ $\begin{bmatrix}\n\frac{-1}{4} \end{bmatrix}\n\begin{bmatrix}\n1 & -3 & -5 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
-3 & -5 \\
1 & 3\n\end{bmatrix}\n\begin{bmatrix}\n9 & 2 \\
52 & 15 \\
0 & -1\n\end{bmatrix}$
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-8 & -3 & -2 \\
-8 & -4\n\end{bmatrix}\n\begin{bmatrix}\n1 & -1 \\
5 & 1\n\end{bmatrix}$
 $\begin{bmatrix}\n1 & -1 \\
-8 & 1\n\end{bmatrix}$
 \begin 4 $\begin{bmatrix} -8 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 9 & 2 \\ -3 & -5 \end{bmatrix}$
 $\begin{bmatrix} 9 & 2 \\ 52 & 15 \end{bmatrix}$
 $\begin{bmatrix} 52 & 15 \\ 0 & -1 \end{bmatrix}$
 $\begin{bmatrix} 1 & -1 \\ 5 & 1 \end{bmatrix}$
 $\begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$
 $\begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$
 $\begin{bmatrix} 1 & -1 \\ 3 &$ STUDY MATERIAL: MATHEMAT
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -2 \\ 2 & 0 \end{bmatrix}$

Find (i) without finding B⁻¹, the value of K for which
 $\begin{bmatrix} 2 & 2 \\$ STUDY MATERIAL: MATHEMAI
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$ Example 18:
 $\begin{aligned}\n&\text{Matrices A and B satisfy AB} = B^{-1}, \text{ where } B = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \\
&\text{Find (i) without finding } B^{-1}, \text{ the value of K for which KAP and K.}\n\end{aligned}$
 $\begin{aligned}\n&\text{SOL} &\text{in } B \text{ is the sum of } B \text{ with } B \text{ with } B \text{$ STUDY MATERIAL: MATHEMAT
 $=\frac{-1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$ Example 18:

Matrices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & 2 \\ 2 & \text{Find (i) without finding B}^{-1}$, the value of K for which
 $\begin{bmatrix} 2 & 2 \\ -3 & -5 \\ 1 & 3 \end{$ STUDY MATERIAL: MATHEMATION
 $\begin{bmatrix}\n\frac{1}{2} & -1 \\
-1 & 16 & -3 & -5 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & -1 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & -1 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & -1 \\
-8 & 1 & 3\n$ STUDY MATERIAL: MATHEMATIC
 $\begin{bmatrix}\n-1 & -1 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n-12 & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & 2 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & 0 & 2 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{2} & 0 & 2 \\
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 $\begin{b$ STUDY MATERIAL: MATHEMATIP
 $\begin{bmatrix}\n-1 & -1 \\
-4 & 16 & -3 & -5 \\
-8 & 1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
2 & 2 \\
-8 & -1\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
2 & 15 \\
-1 & 3\n\end{bmatrix}$
 $\begin{bmatrix}\n9 & 2 \\
2 & 15 \\
-1 & 13\n\end{bmatrix}$
 $\begin{bmatrix}\n1 & -1 \\
5 & 1\n\end{bmatrix}$
 $\begin{bmatrix}\n1 & -1 \\
 = 1$ Sol. (i) $\overline{AB} = \overline{B}^{-1} \Rightarrow AB^2 = I$

A $BA - 2B^{-1} + I = 0 \Rightarrow KAB - 2B^{-1}B + IB = O$
 $\Rightarrow KAB - 2I + B = \Rightarrow KAB - 2B + B^2 = O$
 $\Rightarrow KI - 2B + B^2 = O$
 $\Rightarrow K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $KAP = 2B^{-1} + 1 = O \Rightarrow KAB = 2B^{-1}B + IB = O$
 -1
 $\Rightarrow KAB = 2I + B = O \Rightarrow KAB^2 - 2B + B^2 = O$
 $\Rightarrow K[-2B + B^2] = O \Rightarrow K[B^2 - 2B + B^2] = O$
 $\Rightarrow K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} K$ satisfying A⁻¹XA = B.

Sol. (i) AB = B⁻¹ ⇒ AB² = I
 $KA-2B^{-1} + 1 = O \Rightarrow KAB-2B^{-1}B + IB = O$

⇒ KAB - 21 + B = O ⇒ KAB - 2B + B² = O

⇒ KI-2B + B² = O

⇒ K[0 1] - 2[2 - 0] + [2 - 1] [2 - 1] = [0 0]

⇒ [(0 1) - 2[2 0] + [2 Matrices A and B satisfy $AB = B^{-1}$, where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$. MATICS
 $2 \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

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matrix X MATICS
 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

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 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.
which
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ple 18:

Matrices A and B satisfy $AB = B^{-1}$, where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

Find (i) without finding B^{-1} , the value of K for which
 $KA = 2B^{-1} + I = 0$, (ii) without finding A^{-1} , the m $KA = 2B^{-1} + I = 0$, (ii) without finding A^{-1} , the matrix X satisfying $A^{-1}XA = B$. **Sol.** (i) $AB = B^{-1} \Rightarrow AB^2 = I$ $KA-2B^{-1}+I=O \Rightarrow KAB-2B^{-1}B+IB=O$ \Rightarrow KAB-2I+B=O \Rightarrow KAB²-2B+B²=O \Rightarrow KI-2B+B²=O $\Rightarrow K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **STUDY MATERIAL: MATHEMATICS**

s A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

without finding B⁻¹, the value of K for which

B⁻¹ + I = 0, (ii) without finding A⁻¹, the matrix X

ng A⁻¹XA = B.
 STUDY MATERIAL: MATHEMATICS

s A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

without finding B⁻¹, the value of K for which

B⁻¹ + I = 0, (ii) without finding A⁻¹, the matrix X

ng A⁻¹XA = B.

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 $\begin{bmatrix}\n\text{without finding } \text{B}^{-1} \text{, the value of K for which} \\
\text{BB}^{-1} + \text{I} = 0, \text{ (ii) without finding } \text{A}^{-1}, \text{ the matrix X} \\
\text{ing } \text{A}^{-1} \text{XA} = \text{B}. \\
\text{A} = 2\text{B}^{-1}$ TUDY MATERIAL: MATHEMATICS

atisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

ding B⁻¹, the value of K for which

(ii) without finding A⁻¹, the matrix X

= B.

AB² = I

= O \Rightarrow KAB -2B⁻¹B + IB = O

O

2 -1 TUDY MATERIAL: MATHEMATICS

atisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

ding B⁻¹, the value of K for which

(ii) without finding A⁻¹, the matrix X

= B.

AB² = I

= O \Rightarrow KAB -2B⁻¹B + IB = O

O

2 -1 STUDY MATERIAL: MATHEMATICS

satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

nding B⁻¹, the value of K for which

0, (ii) without finding A⁻¹, the matrix X
 $x = B$.
 $B^2 = I$
 $I = O \Rightarrow KAB - 2B^{-1}B + IB = O$
 $I = O \Rightarrow KAB^2$ STUDY MATERIAL: MATHEMATICS

satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

nding B⁻¹, the value of K for which

0, (ii) without finding A⁻¹, the matrix X
 $x = B$.
 $AB^2 = I$
 $I = O \Rightarrow KAB - 2B^{-1}B + IB = O$
 $= O \Rightarrow KAB^2 -$ TERIAL: MATHEMATICS
 B^{-1} , where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

the value of K for which

at finding A⁻¹, the matrix X
 $B - 2B^{-1}B + IB = O$
 $B^2 - 2B + B^2 = O$
 $2 \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} -2 \\ -2$ TERIAL: MATHEMATICS
 B^{-1} , where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

the value of K for which

at finding A⁻¹, the matrix X
 $B - 2B^{-1}B + IB = O$
 $B^2 - 2B + B^2 = O$
 $2 \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} -2 \\ -2$ ATERIAL: MATHEMATICS

= B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

the value of K for which

out finding A⁻¹, the matrix X

AB - 2B⁻¹B + IB = O

AB² - 2B + B² = O
 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ **ATERIAL: MATHEMATICS**
 $= B^{-1}$, where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

the value of K for which

out finding A⁻¹, the matrix X

AB $-2B^{-1}B + IB = O$

AB² $-2B + B^2 = O$
 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\begin{b$ **MATHEMATICS**

ere $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

of K for which
 A^{-1} , the matrix X
 $B + IB = O$
 $+ B^2 = O$
 $2 \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $0 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **MATHEMATICS**

ere B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

of K for which

A⁻¹, the matrix X

B + IB = O

+ B² = O

2 -1] = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2 0] = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$: **MATHEMATICS**

nere $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

e of K for which

g A⁻¹, the matrix X

¹B + IB = O

+ B² = O
 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

[0 0]

0 0] : **MATHEMATICS**

nere $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

e of K for which

g A⁻¹, the matrix X

¹B + IB = O

+ B² = O
 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **ATICS**
 $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

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 $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

which

matrix X
 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix}\n\cos A \text{ and } B \text{ satisfy } AB = B^{-1}, \text{ where } B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}.\n\end{bmatrix}$

(i) without finding B⁻¹, the value of K for which
 $2B^{-1} + I = 0$, (ii) without finding A⁻¹, the matrix X
 $\begin{bmatrix}\n\sin A^{-1}$ STUDY MATERIAL: MATHEMATICS
 \therefore
 $\cos A$ and B satisfy $AB = B^{-1}$, where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

(i) without finding B^{-1} , the value of K for which
 $2B^{-1} + I = 0$, (ii) without finding A^{-1} , the matrix X
 $\sin A^{-1}XA$ **STUDY MATERIAL: MATHEMATICS**
 8:

ices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

(i) without finding B⁻¹, the value of K for which
 $-2B^{-1} + I = 0$, (ii) without finding A⁻¹, the matrix X

fyi 8:

STUDY MATERIAL: MATHEMATICS

8:

ices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

(i) without finding B⁻¹, the value of K for which
 $2B^{-1} + I = 0$, (ii) without finding A⁻¹, the matrix X

fyin \Rightarrow 0 $K - 2$ $=$ 0 \rightarrow K = 2 STUDY MATERIAL: MATHEMATICS
 \therefore
 $\cos A$ and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

i) without finding B⁻¹, the value of K for which
 $2B^{-1} + I = 0$, (ii) without finding A⁻¹, the matrix X
 $\begin{aligned}\n$ S A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

without finding B⁻¹, the value of K for which
 $2^{n-1} + 1 = 0$, (ii) without finding A⁻¹, the matrix X
 $B = B^{-1} \Rightarrow AB^2 = I$
 $-2B^{-1} + I = O \Rightarrow KAB - 2B^{-1}B + IB = O$ **STUDY MATERIAL: MATHEMATICS**

8:

 8:

 6:
 9:
 9 STUDY MATERIAL: MATHEMATICS

8:

ices A and B satisfy AB = B⁻¹, where B = $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$.

(i) without finding B⁻¹, the value of K for which
 $2B^{-1} + I = 0$, (ii) without finding A⁻¹, the matrix X

fying A (ii) $A^{-1} X A = B$ \Rightarrow AA⁻¹XA = AB \Rightarrow IXA = AB \Rightarrow XAB = AB² \Rightarrow XAB=I \Rightarrow XAB²=B \Rightarrow XI=B \Rightarrow X=B

MATRICES AND DETERMINANTS QUESTION BANK

85 PART - 1 - MATRICES Q.1 If I is a unit matrix, then 3I will be (A) A unit matrix (B) A triangular matrix (C) A scalar matrix (D) None of these **Q.2** If A is a symmetric matrix, then matrix M'AMis (A) Symmetric (B) Skew-symmetric (C) Hermitian (D) Skew-Hermitian **Q.3** If A is a square matrix, then A + A^T is (A) Non singular matrix (B) Symmetric matrix (C) Skew-symmetric matrix (D) Unit matrix **Q.4** If A is a square matrix satisfying the equation A² – 4A – 5I = 0 then A–1 is equal to – (A) A – 4I (B) ¹ 3 (A – 4I) (C) ¹ 4 (A – 4I) (D) ¹ 5 (A – 4I) **Q.5** If A = 1 a 0 1 , then Aⁿ (where n N) equals (A) 1 na 0 1 (B) ² 1 n a 0 1 (C) 1 na 0 0 (D) n na 0 n **Q.6** If A and B are two square matrices of the same order such that AB = B and BA = A then A² + B² is always equal to (A) I (B) A + B (C) 2 AB (D) 2 BA **Q.7** Inverse of a diagonal non-singular matrix is – (A) diagonal matrix (B) scalar matrix (C) skew symmetric matrix (D) zero matrix **Q.8** If the multiplicative group of 2 × 2 matrices of the form a a a a , for a 0 and a R , then the inverse of 2 2 2 2 (A) 1 1 8 8 1 1 8 8 (B) 1 1 4 4 1 1 4 4 (C) 1 1 2 2 1 1 2 2 (D) Does not exist **Q.9** If 1 2 3 1 2 P 2 3 4 2 0 3 4 5 0 4 4 5 6 0 0 1 then P22⁼ (A) 40 (B) – 40 (C) – 20 (D) 20 **Q.10** If A = p q q p , B = r s s r then (A) AB = BA (B) AB BA (C) AB = – BA (D) None of these **Q.11** If A = 0 1 0 0 and a and b are arbitrary constants then – (aI + bA)² = (A) a² I + abA (B) a² I + 2abA (C) a² I + b2A (D) None of these **Q.12** If is square root of identity matrix of order 2 then – (A) 1 + 2 + = 0 (B) 1 + 2 – = 0 (C) 1 – 2 + = 0 (D) 2 + = 1 **Q.13** If 2 2 A a b and A² = O, then (a, b) = (A) (–2, –2) (B) (2, –2) (C) (–2, 2) (D) (2, 2) **Q.14** If 1 A 2 , 3 then AA' = (A) 14 (B) 1 4 3 (C) 1 2 3 2 4 6 3 6 9 (D) None **Q.15** If 3 5 A 2 0 and 1 17 B 0 10 then | AB | is equal to (A) 80 (B) 100 (C) –110 (D) 92 **Q.16** If A is non singular matrix, then – (A) | A–1 | = | A | (B) | A–1 | = A–1 (C) | A–1 | = 0 (D) | A–1 | = 1 / | A | **Q.17** If A is a 3 × 3 nonsingular matrix and if | A | = 3, then | (2A)–1 | = (A) 24 (B) 3 (C) 1/3 (D) 1/24 **EXERCISE - 1 [LEVEL-1] QUESTION BANK CHAPTER 3 : MATRICES AND DETERMINANTS**

PART - 2 - DETERMINANTS

MATRICES AND DETERMINANTS QUESTION BANK

Q.32 Let 1 1 1 1 2 2 2 3 3 3 a b c a b c a b c and , then 1 2 can be expressed as the sum of how many determinants (A) 9 (B) 3 (C) 27 (D) 2 **Q.33** If C 2cos , then the value of the determinant C 1 0 1 C 1 6 1 C is (A) sin 4 sin (B) sin (C) ² 4cos (2cos 1) (D) None of these **Q.34** If ¹ x b b a x b a a x and ² determinants, then (A) ² 1 2 3() (B) 1 2 d dx (C) ² 1 2 d () 2() dx (D) 3/2 **Q.35** Find the value of (A) 2 (B) 3 (C) 0 (D) 4 **Q.36** Find the value of the determinant . (A) 2 (B) 3 (C) 1 (D) 0 **Q.37** If x, y, z are unequal and = 0 then find the value of xyz. (A) 2 (B) –1 (C) 4 (D) – 4 **Q.38** If in the multiplication of

EXAMPLEMANSUS
\n**REMINANTS**
\n**1 a**
$$
\begin{vmatrix} a_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_2 & \gamma_3 & \gamma_4 \end{vmatrix}
$$
 the elements of the first row then the elements of the
\n*c* 0D₂.
\n**b c** 0D₂.
\n**2 a** $\begin{vmatrix} a_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \end{vmatrix}$,
\n**b c d e e f f g g h i** $\begin{vmatrix} a_1 & \beta_1 & \beta_1 & \beta_1 \\ \beta_1 & \beta_2 & \beta_2 & \beta_2 \\ \beta_2 & \beta_2 & \beta_2 & \beta_1 \end{vmatrix}$ **ii** $\begin{vmatrix} a_1 & \beta_1 & \beta_1 & \beta_1 \\ \beta_1 & \beta_2 & \beta_2 & \beta_2 \\ \beta_2 & \beta_2 & \beta_2 & \beta_3 \end{vmatrix}$ **iii** (3a) 2a + 3b) (3b) 22
\n**7 8 1**

(A)
$$
1 + abc = 0
$$

\n(B) $a + b + c + 1 = 0$
\n(C) $(a - b) (b - c) (c - a) = 0$ (D) None of these

QUESTION BANK STUDY MATERIAL : MATHEMATICS

Q.25 If a, b, c are real then the value of determinant

| MATRICES AND DETERMINANTS | QUBSTIONBANK | ES | |
|---|--|---------------------|---------|
| Q.25 If a, b, c are real then the value of determinant | | | |
| $a^2 + 1$ ab $b^2 + 1$ be $a^2 + 1$ | Q.31 Suppose $a_1, a_2, \ldots, \text{real numbers, with } a_1 \neq 0$. | | |
| $a^2 + 1$ ab $b^2 + 1$ be $c^2 + 1$ | Q.33 Suppose $a_1, a_2, \ldots, \text{real numbers, with } a_1 \neq 0$. | | |
| (A) $a + b + c = 0$ | (B) $a + b + c = 1$ | (C) $a + b + c = 1$ | |
| Q.26 Let $a = \lim_{x \to 1} \frac{x}{\ln x} - \frac{1}{x \ln x}$; $b = \lim_{x \to 0} \frac{x^3 - 16x}{4x + x^2}$; | (B) The system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_3y + a_3z = 0$, $a_4x + a_5y + a_5z = 0$ has infinite | | |
| $c = \lim_{x \to 0} \frac{\ln(1 + \sin x)}{x} \& d = \lim_{x \to 1} \frac{x}{3(\sin(x + 1) - (x + 1))}$ | (C) $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_6 & a_7 & a_7 \end{bmatrix}$ is non singular; where $i = \sqrt{-1}$ | | |
| then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is | Q.32 If a determinant of order 3 × 3 is formed by using the number of solutions | | |
| (A) Idempotent | (B) Invblutary | (A) – 2 | (B) – 4 |

(A) - 2 (B) - 8
\n(C) - 16 (D) 8
\nQ.28 The characteristic equation of a matrix A is
\n
$$
\lambda^3 - 5\lambda^2 - 3\lambda + 2 = 0
$$
 then | adj (A)|
\n(A) 4 (B) 9
\n(C) 25 (D) 21
\n $|x + 1|$

Q.29 If A =
$$
\begin{vmatrix} 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}
$$
 and B = $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$, then $\frac{dA}{dx}$ =
\n(A) 3B + 1
\n(B) 3B
\n(C) -3B
\n(D) 1 - 3B

Q.30 If
$$
\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)
$$
 then the

solution of the equation

$$
\begin{vmatrix} 1 & 1 & 1 \ (x-a)^2 & (x-b)^2 & (x-c)^2 \ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \ \end{vmatrix} = 0
$$
, is
(A) $\frac{a+b+c}{3}$ (B)1

(C)
$$
\frac{a+b+c}{2}
$$
 (D) $\sqrt[3]{abc}$

SPARADVANCED LEARNING
 $\frac{a+b+c}{2}$ (D) $\sqrt[3]{abc}$

pose a_1, a_2, \dots real numbers, with $a_1 \neq 0$.
 a_2, a_3, \dots are in A.P. then
 $\Delta = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_1 & a_7 & a_8 \\ a_1 & a_2 & a_9 \\ a_1 & a_2 & a_1 \\ a_2 & a_2 & a_3 \\ a_4 & a_$ $+ b + c$
 2

ODMADVANCED LEARNING
 $+ b + c$

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OD $\sqrt[3]{abc}$

OD $\sqrt[3]{abc}$

OSE a_1, a_2, \dots real numbers, with $a_1 \neq 0$.
 a_2, a_3, \dots are in A.P. then **Q.31** Suppose a_1, a_2, \ldots real numbers, with $a_1 \neq 0$. If a_1, a_2, a_3, \dots and in A.P. then

$$
(C) \frac{a+b+c}{2}
$$
\n(D) $\sqrt[3]{abc}$
\nSuppose a_1, a_2, \dots real numbers, with $a_1 \neq 0$.
\nIf a_1, a_2, a_3, \dots are in A.P. then
\n(A) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular
\n(B) The system of equations $a_1x + a_2y + a_3z = 0$,
\n $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_0z = 0$ has infinite

 $\frac{1}{2}$; (B) The system of equations $a_1x + a_2y + a_3z = 0$, + x² $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite **EXECUTE ARRIVANCED LEARNING**
 C

(D) $\sqrt[3]{abc}$

(a₂, real numbers, with $a_1 \neq 0$.

............are in A.P. then
 $\begin{bmatrix} 1 & a_2 & a_3 \\ a & a_6 & a_7 \end{bmatrix}$ is singular

stem of equations $a_1x + a_2y + a_3z = 0$,
 $5y + a$ 5 6 7 **EXECUTE AGAINING**

FORMADY ANGELE AGAINING

1, a₂, real numbers, with a₁ \neq 0.

3, are in A.P. then

a₁ a₂ a₃

a₄ a₅ a₆ a₇

a₃ a₆ a₇ is singular

a₃ a₆ a₇ is singular
 EXECUTE ABINING

FORMADVANCED LEABNING

1, a₂, real numbers, with a₁ \neq 0.

3,are in A.P. then

a₁ a₂ a₃

a₄ a₅ a₆ a₇

ystem of equations a₁x + a₂y + a₃z = 0,

a₅y + a₆z **EXECUTE ARIVING**

For a symple of the set of solutions
 a_1 , a_2 , a_3
 a_4 , a_5 , a_6
 a_7
 $a_8y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$, **EDENTADYANCED LEARNING**
 $+ c$

(D) $\sqrt[3]{abc}$
 $a_1, a_2,$ real numbers, with $a_1 \neq 0$.
 a_3 ,are in A.P. then
 $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

system of equations $a_1x + a_2y + a_$ **EXECUTE ARNING**

FODM ADVANCED LEARNING
 a_1, a_2, \dots, a_{10} real numbers, with $a_1 \neq 0$.
 a_3, \dots, a_{10} real numbers, with $a_1 \neq 0$.
 $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

system of equation number of solutions 1 2 **EDIMADIVANCED LEARNING**
 $+ c$ (D) $\sqrt[3]{abc}$
 a_1, a_2, \dots real numbers, with $a_1 \neq 0$.
 a_3, \dots, a_r in A.P. then
 $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

system of equations $a_1x + a_2y + a_3z = 0$,

(C)
$$
B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}
$$
 is non singular ; where $i = \sqrt{-1}$

EDENTADVANCED LEARNING

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 EXECUTE ARISTS
 **EXECUTE ASSAURE AND ASSAURE AND SET ON EXECUTE AND SET OF SOLUTIONS

EXECUTE ASSAURE ASSAURE SOLUTAD VANCED LEARNING**
 $\begin{array}{ll}\n & \text{6DUL A D U A D U A D U A D U B D D E}\n\hline\n\end{array}$
 $\begin{array}{ll}\n & \text{1} & \text{2} & \text{3} \\
\text{3} & \text{4} & \text{4} & \text{5} \\
\text{4} & \text{4} & \text{5} & \text{6} \\
\text{4} & \text{4} & \text{6} & \text{7} \\
\end{array}$ **Solutions**
 $\begin{array}{ll}\n & \text{4} & \text{4} \\
\text{4} &$ **EDENADVANCED LEARMING**
 $\begin{bmatrix}\n+ c & (D) \sqrt[3]{abc} \\
a_1, a_2, \dots a_1c_1c_1d_1c_2 \end{bmatrix}$
 $\begin{bmatrix}\na_1 & a_2 & a_3 \\
a_4 & a_5 & a_6 \\
a_5 & a_6 & a_7\n\end{bmatrix}$ is singular

system of equations $a_1x + a_2y + a_3z = 0$,
 $\begin{bmatrix}\na_1 & ia_2 \\
ia_2 & a_1\n\end{bmatrix}$ is (D) All of these **Q.32** If a determinant of order 3×3 is formed by using the numbers of 1 or – 1 then minimum value of determinant is (A) –2 (B) – 4
(C) 0 (D) – 8 $(D) - 8$ **Q.33** If A is a square matrix of order 3 such that $|A| = 2$ then $\begin{bmatrix} a_1 & a_2 & a_3 \ a_4 & a_5 & a_6 \end{bmatrix}$ is singular
 a_5 a a_6 a a_7
 a_8 system of equations $a_1x + a_2y + a_3z = 0$,
 $a_1a_2 + a_3y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite

there of solutions
 $\begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1$ $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ is singular
 $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ is singular
 $\begin{vmatrix} 4 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{vmatrix}$ is non singular; where $i = \sqrt{-1}$

of these
 $\begin{vmatrix} 4 & 1 & 0 \\ 0 & 2 & 0 \\ 0 &$ $\begin{bmatrix} a_5 & a_6 & a_7 \end{bmatrix}$

e system of equations $a_1x + a_2y + a_3z = 0$,
 $+ a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite

there of solutions
 $\begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is non singular; where $i = \sqrt{-1}$

of these

ermin 2, a₃, are in A.P. then
 $=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

ne system of equations $a_1x + a_2y + a_3z = 0$,
 $x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite

tember of solution $\begin{bmatrix}\na_1 & a_2 & a_3 \\
a_4 & a_5 & a_6 \\
a_5 & a_6 & a_7\n\end{bmatrix}$ is singular

e system of equations $a_1x + a_2y + a_3z = 0$,
 $x + a_3y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite
 $=\begin{bmatrix}\na_1 & ia_2 \\
ia_2 & a_1\n\end{bmatrix}$ is non singular; where $i = \sqrt{-1$ = $\begin{bmatrix} a_4 & a_5 & a_6 \ a_5 & a_6 \end{bmatrix}$ is singular

ne system of equations $a_1x + a_2y + a_3z = 0$,
 $x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite

timber of solutions

= $\begin{bmatrix} a_1 & ia_2 \ ia_1 \end{bmatrix}$ is non singular; where $i =$

$$
|(adj A^{-1})^{-1}| is -
$$

(A) 1 \t(B) 2
(C) 3 \t(D) 4

$$
\begin{bmatrix} 6 & 8 & 5 \end{bmatrix}
$$

Q.34 If $A = \begin{pmatrix} 4 & 2 & 3 \end{pmatrix}$ is the sum of a symmetric matrix B and

skew symmetric matrix C, then B is –

et a =
$$
\lim_{x\to 1} \frac{x}{\ln x} - \frac{1}{x \ln x}
$$
; b = $\lim_{x\to 0} \frac{x^3 - 16x}{4x + x^2}$;
\na₁x + a₂y + a₃z = 0,
\na₁x + a₃y + a₃z = 0 has infinite number of solutions
\n $x \to 0$ $\frac{(x + 1)^3}{x}$ (a) $\frac{(x + 1)^3}{x^2}$ (b) The system of equations a₁x + a₂y + a₃z = 0 has infinite number of solutions
\n $\frac{1}{x \to 0}$ $\frac{\ln(1 + \sin x)}{x}$ & (b) $\frac{1}{x \to 1}$ $\frac{1}{3}(\sin(x + 1) - (x + 1))$
\n $\frac{1}{x \to 0}$ (c) $\frac{1}{x \to 0}$ $\frac{1}{x \to 2}$ (d) $\frac{1}{x \to 0}$ (e) $\frac{1}{x \to 2}$ (f) $\frac{1}{x \to 0}$ (g) $\frac{1}{x \to 2}$ (h) $\frac{1}{x \to 0}$ (i) $\frac{1}{x \to 2}$ (ii) $\frac{1}{x \to 0}$ (b) $\frac{1}{x \to 2}$ (c) $\frac{1}{x \to 2}$ (d) $\frac{1}{x \to 2}$ (e) $\frac{1}{x \to 2}$ (f) $\frac{1}{x \to 2}$ (g) $\frac{1}{x \to 2}$ (h) $\frac{1}{x \to 2}$ (i) $\frac{1}{x \to 2}$ (ii) $\frac{1}{x \to 2}$ (iii) $\frac{1}{x \to 2}$ (iv) $\frac{1}{x \to 2}$ (v) $\frac{1}{x \to 2}$ (vi) $\frac{1}{x \to 2}$ (v) $\frac{1}{x \to 2}$ (vi) $\frac{1}{x \to 2}$ (v) $\$

Q.43 If $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ then $PQ =$

1 1 ad 1 bd 1 1 ae 1 be | C | ⁼ (A) 2 2 1 1 1 1 (B) 2 2 1 1 1 1 (C) 2 2 1 1 (D) 2 2 0 0 1 0 0 0 1 2 2 2 2 2 2 sin 13 sin 77 tan135 sin 77 tan135 sin 13 tan135 sin 13 sin 77

Q.44 The value of determinant

 $is (A) -1$ (B) 0 $(C) 1$ (D) 2 sin² 77° tan 135°

tan 135° sin² 13° is -

sin² 13° sin² 77° is -

(B) 0

(D) 2
 ion-Reason type questions.

th-1 is True, Statement-2 is true, statement-2

ect explanation for Statement -1.

action-1 is True, S sin⁻⁷/² tan135°

tan135° sin² 13° is -

sin² 13° sin² 77° is -

(B) 0

(D) 2
 ion-Reason type questions.

th-1 is True, Statement-2 is true, statement-2

correct explanation for Statement-1.

correct explanati

Directions : Assertion-Reason type questions.

- (A) Statement- 1 is True, Statement-2 is true, statement-2 is a correct explanation for Statement -1.
- (B) Statement -1 is True, Statement-2 is true; statement-2 is NOT a correct explanation for Statement - 1. (C) Statement - 1 is True, Statement- 2 is False.
- (D) Statement -1 is False, Statement -2 is True.
- **2.45 Statement 1 :** For a singular square matrix A, if $AB = AC \implies B = C$.

Statement 2: If $|A| = 0$ then A^{-1} does not exist.

Q.46 Statement 1: If $f_1(x)$, $f_2(x)$, $f_9(x)$ are polynomials whose degree ≥ 1 , where $f_1(\alpha) = f_2(\alpha)$ = $f_9(\alpha) = 0$

$$
\begin{vmatrix}\nsin^2 77^\circ & \tan 135^\circ & \sin^2 13^\circ \\
\tan 135^\circ & \sin^2 13^\circ & \sin^2 77^\circ\n\end{vmatrix}
$$
\n(A) -1 (B) 0
\n(C) 1 (D) 2
\n**ons : Aserrion-Reason type questions.**
\n(A) Statement -1 is True, Statement -2 is true, statement -2
\nis a correct explanation for Statement -1.
\n(B) Statement -1 is True, Statement -2 is true; statement -2
\nis NOT a correct explanation for Statement -1.
\n(C) Statement -1 is True, Statement -2 is False.
\n**Statement -1** is True, Statement -2 is False.
\nStatement -1 is True, Statement -2 is True.
\nStatement 1 : For a singular square matrix A, if
\nAB = AC \Rightarrow B = C.
\nStatement 2 : If |A| = 0 then A⁻¹ does not exist.
\nStatement 1 : If $f_1(x), f_2(x)$, $f_0(x)$ are polynomials
\nwhose degree ≥ 1 , where $f_1(\alpha) = f_2(\alpha)$ = $f_0(\alpha) = 0$
\nand $A(x) = \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{bmatrix}$ and $\frac{A(x)}{x - \alpha}$ is also
\na matrix of 3 × 3 whose entries are also polynomials.
\nStatement 2 : x - α is a factor of polynomial f(x) iff $(\alpha) = 0$.
\nLet x, y, z are three integers lying between 1 and 9 such
\nthat x 51, y 41, z 31 are three digit numbers.
\nStatement 1 : The value of determinant is zero. If the
\nentries any two rows (or columns) of the determinants
\nare correspondingly proportional.
\nLet A = $\begin{bmatrix} \sin \alpha & -\cos \alpha & 1 \\ \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$
\nStatement -1 : A⁻¹ = adj (A)
\nStatement -2 : |A| = 1

a matrix of 3×3 whose entries are also polynomials. **Statement 2 :** $x - \alpha$ is a factor of polynomial $f(x)$ if $f(\alpha) = 0$.

Q.47 Let x, y, z are three integers lying between 1 and 9 such that $x 51, y 41, z 31$ are three digit numbers.

Statement 1 : The value of determinant

$$
\begin{vmatrix} 5 & 4 & 3 \ x 51 & y 41 & z 31 \ x & y & z \end{vmatrix}
$$
 is zero.

Statement 2 : The value of determinant is zero. If the entries any two rows (or columns) of the determinants are correspondingly proportional.

Q.48 Let
$$
A = \begin{bmatrix} \sin \alpha & -\cos \alpha & 1 \\ \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Statement–1: A^{-1} = adj (A) **Statement–2** : $|A| = 1$

MATRICES AND DETERMINANTS QUESTION BANK

Consider the determinant
$$
\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}
$$

$$
M_{ij}
$$
 = Minor of the element if ith row and jth column
C_{ii} = Cofactor of the element if ith row and jth column

Q.49 Value of b₁.C₃₁ + b₂.C₃₂ + b₃.C₃₃ is
\n(A) 0
\n(B)
$$
\triangle
$$

\n(C) 2 \triangle
\n(D) \triangle ²

Q.50 If all the elements of the determinants are multiplied by 2, then the value of new determinant is – $(A) 0$ (B) 8 Δ

$$
(C) 2\Delta \qquad (D) 2^9 \Delta
$$

Q.51 a₃ $M_{13} - b_3 M_{23} + d_3 M_{33}$ is equal to – $(A) 0$ (B) 4Δ $(C) 2\Delta$ (D) Δ

Passage (Q.52-Q.53) :

Let A and B are two matrices of same order 3×3 where

Q.52 If matrix $(A + 2B)$ is singular, then the value of K is –

(A)
$$
\frac{7}{12}
$$
 (B) $\frac{22}{13}$ (C) $\frac{35}{13}$ (D) $\frac{-35}{13}$
\n**Q.53** If C = A – B and Tr(C) = 2, then K is equal to –
\n(A) 11 (B) 9
\nX²
\n(A) 11

$$
(C) 10 \t\t (D) 5
$$

NOTE : The answer to each question is a NUMERICAL VALUE.

- **Q.54** If the system of the equations : $x + y + 2z = 6$ (1), $x+3y+3z=10$ (2) $x+2y+\lambda z=\mu$ (3) has infinite number of solutions, then find the value of $4(\lambda + \mu)$. $\begin{bmatrix} 1 & -2 & 2 \ 3 & 1 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 & 1 \ 4 & 4 & 2 \end{bmatrix}$
 $\begin{bmatrix} 1 & -2 & 2 \ 3 & 1 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 & 1 \ 4 & 4 & 2 \end{bmatrix}$
 $\begin{bmatrix} 1 & -2 & 2 \ 1 & 3 & 5 & 2 \end{bmatrix}$
 $\begin{bmatrix} 1 & -2 & 2 \ 1 & 3 & 5 & 2 \end{bmatrix}$
 x (A+2B) is singular, then the value of K is –

ind B are two matrices of same order 3 × 3 where
 $\begin{vmatrix}\n-2 & 2 \\
k & 6\n\end{vmatrix}$ and B = $\begin{bmatrix}\n2 & 3 & 1 \\
4 & 4 & 2 \\
3 & 5 & 2\n\end{bmatrix}$

ix (A+2B) is singular, then the value of K is Let A and B are two matrices of same order 3 × 3 where
 $A = \begin{bmatrix} 1 & -2 & 2 \ 3 & 1 & -2 \ 3 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 1 \ 4 & 4 & 2 \ 3 & 5 & 6 \end{bmatrix}$
 $A = \begin{bmatrix} 1 & -2 & 2 \ 3 & 6 & 6 \ 3 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 1 \ 4 & 2 & 4 \$
- **Q.55** If α , β , γ are different from 1 and are the roots of $ax^3 + bx^2 + cx + d = 0$ and $(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = 25/2$ then

A =
$$
\begin{bmatrix} 5 & k & 6 \\ 3 & 1 & -2 \end{bmatrix}
$$
 and B = $\begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 2 \end{bmatrix}$
\nIf matrix (A+2B) is singular, then the value of K is-
\n(A) $\frac{7}{12}$ (B) $\frac{22}{13}$ (C) $\frac{35}{13}$ (D) $\frac{-35}{13}$
\nIf C = A – B and Tr (C) = 2, then K is equal to-
\n(A) 11 (B) 9
\n(C) 10 (B) 9
\n(C) 10 (D) 5
\n**11** The answer to each question is a NUMBERICAL VALUE.
\n $\begin{aligned}\n\text{If the system of the equations : } x + y + 2z &= 6 \quad \text{........(1)}, \\
\text{If the system of the equations : } x + y + 2z &= 6 \quad \text{........(2)}, \\
\text{If the system of the equations : } x + y + 2z &= 6 \quad \text{........(3)}, \\
\text{If A + μ .\n\end{aligned}$
\n $\begin{aligned}\n\text{If A + μ .\n\end{aligned}$

Q.56 Let the matrix
$$
A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}
$$
 be a zero divisor of the

polynomial $f(x) = x^2 - 4x - 5$. Find the sum of all the elements in the matrix A^3 . .

Q.57 A is an involutary matrix given by

1 2 3 1 2 3 1 2 3 a a a b b b d d d A = 0 1 1 4 3 4 3 3 4 then the inverse of A 2 is xA. Find the 2 2 2 2 2 2 2 2 2 1 a x (1 b)x (1 c)x (1 a)x 1 b x (1 c)x (1 a)x (1 b)x 1 c x 2 2 c c

value of x.
Q.58 If
$$
a^2 + b^2 + c^2 = -2
$$
 and

$$
f(x) = \begin{vmatrix} 1 + a^2 x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}
$$

then $f(x)$ is a polynomial of degree

Q.59 For a non - zero, real a, b and c,

1 2 2 5 k 6 3 1 2 2 3 1 4 4 2 3 5 2 2 2 2 2 a b ^c b c a a a c a b b b = abc, 3 2 2 2 3 2 2 2 3 x 1 x y x z xy y 1 y z xz yz z 1 2 1 4 1 ; B = 3 4 2 3 + t^r ² A(BC)

then the values of α is

Q.60 The number of positive integral solutions of the equation

$$
\begin{vmatrix} x^3 + 1 & x^2y & x^2z \\ xy^2 & y^3 + 1 & y^2z \\ xz^2 & yz^2 & z^3 + 1 \end{vmatrix} = 11
$$
 is

$$
\begin{vmatrix} (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}
$$

then f(x) is a polynomial of degree
Q.59 For a non - zero, real a, b and c,

$$
\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \ \frac{b^2+c^2}{a} & \frac{b^2+c^2}{b} \end{vmatrix} = \alpha \text{ abc},
$$

then the values of α is
Q.60 The number of positive integral solutions of the equation

$$
\begin{vmatrix} x^3+1 & x^2y & x^2z \ xy^2 & y^3+1 & y^2z \ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11 \text{ is}
$$

$$
\begin{vmatrix} x^3 & -4 \ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11 \text{ is}
$$

$$
C = \begin{bmatrix} 3 & -4 \ -2 & 3 \end{bmatrix} \text{ then}
$$

$$
C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}
$$
 then

Q.59 For a non - zero, real a, b and c,
\n
$$
\begin{vmatrix}\na^2 + b^2 & c & c \\
a & \frac{b^2 + c^2}{a} & a \\
b & b & \frac{c^2 + a^2}{b}\n\end{vmatrix} = \alpha \text{ abc},
$$
\nthen the values of α is
\nQ.60 The number of positive integral solutions of the equation
\n
$$
\begin{vmatrix}\nx^3 + 1 & x^2y & x^2z \\
x^3y^2 & y^3 + 1 & y^2z \\
xz^2 & yz^2 & z^3 + 1\n\end{vmatrix} = 11 \text{ is}
$$
\nQ.61 Let three matrices A = $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; B = $\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and
\n
$$
C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}
$$
 then
\n $t_r(A) + t_r \left(\frac{ABC}{2}\right) + t_r \left(\frac{A(BC)^2}{4}\right) + t_r \left(\frac{A(BC)^3}{8}\right) + ... + \infty =$
\nQ.62 A is a 2 × 2 matrix such that
\n
$$
A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}
$$
 and $A^2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$
\nThe sum of the elements of A, is
\nQ.63 A matrix has 12 elements. Find the possible number of orders it can have.
\nQ.64 If matrices A and B satisfy AB = A, BA = B, A² = kA,
\nB² = (B and (A + B)³ = m (A + B), then find the value of

$$
A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \text{ and } A^2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
$$

The sum of the elements of A, is

- **Q.63** A matrix has 12 elements. Find the possible number of orders it can have.
- (B) 9

unction is a NUMERICALVALUE.

unction is a NUMERICALVALUE.

unctions: $x + y + 2z = 6$ (3)

...... (2) $x + 2y + \lambda z = \mu$ (3)

f solutions, then find the value of

from 1 and are the roots of
 $C = \begin{bmatrix$ 2 1 2 2 2 1 c) 2, then is a NUMERICALVALUE.

(B) 9

(B) 5

(B) 5

(B) 2 x 2² y2² y² y² y²

(D) 5

x 2² y y₂² y² y²

x² y₂² y² y²

(a) 1

c) s + c₂² (3) and the sum of the sum of all the

t from 1 and (D) 3

question is a NUMERICAL VALUE.

equation is a NUMERICAL VALUE.
 $\lim_{x \to 2x \to 6} f(x) = 1$ (b) $\lim_{x \to 2x \to 6} f(x) = 1$ (d) $\lim_{x \to 2x \to 6} f(x) = \frac{1}{2}$ (d) $\lim_{x \to 2x \to 6} f(x) = \frac{1}{2}$ (d) of solutions, then find the valu **Q.64** If matrices A and B satisfy $AB = A$, $BA = B$, $A^2 = kA$, $B^2 = \ell B$ and $(A + B)^3 = m(A + B)$, then find the value of $k + \ell + m$.

Q.65 Let ω be the complex number $\cos \frac{2\pi}{\omega} + i \sin \frac{2\pi}{\omega}$. Then

the number of distinct complex numbers z satisfying

| QUESTION BANK | STUDY MATER |
|---|--|
| Let ω be the complex number $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$. Then | Q.68 The total number of distinct $x \in F$ |
| the number of distinct complex numbers z satisfying | \n $\begin{vmatrix}\nx & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 2x & 4x^2 & 1+27x^3\n\end{vmatrix} = 10$ is $-\begin{vmatrix}\nx & x^2 & 1 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3\n\end{vmatrix} = 10$ is $-3x - 9x^2$ |
| Let k be a positive real number and let | Q.69 Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the i |

Q.66 Let k be a positive real number and let

| OMMGE 0.145.018.18 | STUDY MATERIAL: MA |
|---|---|
| Let ω be the complex number $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$. Then | Q.68 The total number of distinct $x \in R$ for which $x \in R$ |

If det (adj A) + det (adj B) = 10^6 , then [k] is equal to [Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Q.67 Let M be a 3×3 matrix satisfying

Then the sum of the diagonal entries of M is

Q.68 The total number of distinct $x \in R$ for which

QUESTION BANK
\n
$$
\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}
$$
\n**Q.68** The total number of distinct x \in R for which numbers z satisfying
\n
$$
\begin{vmatrix}\nx & x^2 & 1+x^3 \\
2x & 4x^2 & 1+8x^3 \\
3x & 9x^2 & 1+27x^3\n\end{vmatrix} = 10
$$
 is –
\nequal to
\n
$$
\begin{bmatrix}\n1 & 0 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0\n\end{bmatrix}
$$

EXAMPLEMATIES
\nIt to be the complex number
$$
\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}
$$
. Then
\nnumber of distinct complex numbers z satisfying
\n $\begin{vmatrix}\n2x + 1 & 0 & 0 \\
2x + 4 & 2 & 1 + 8x^3 \\
3x & 9x^2 & 1 + 2x^3\n\end{vmatrix} = 10 \text{ is } -1$
\n $\begin{vmatrix}\n2x - 1 & 2\sqrt{k} & 2\sqrt{k} \\
-2\sqrt{k} & 1 & -2k \\
-2\sqrt{k} & 2k & -1\n\end{vmatrix} = 0$ is equal to
\n $\begin{vmatrix}\n2x - 1 & 2\sqrt{k} & 2\sqrt{k} \\
-2\sqrt{k} & 1 & -2k \\
-2\sqrt{k} & 2k & -1\n\end{vmatrix} = 0$ is equal to
\n $\begin{vmatrix}\n2x - 1 & 2\sqrt{k} & 2\sqrt{k} \\
-2\sqrt{k} & 1 & -2k \\
-2\sqrt{k} & 2k & -1\n\end{vmatrix} = 0$ is equal to
\n $\begin{vmatrix}\n2x - 1 & 2\sqrt{k} & 2\sqrt{k} \\
-2\sqrt{k} & 2k & -1 \\
-2\sqrt{k} & 2k & -1\n\end{vmatrix} = 0$ is equal to
\n $\begin{vmatrix}\n2x - 1 & 2\sqrt{k} & 2\sqrt{k} \\
-2\sqrt{k} & 2\sqrt{k} & -2\sqrt{k} \\
-2\sqrt{k} & 2\sqrt{k} & 0\n\end{vmatrix} = 0$ is $\begin{vmatrix}\n3x - 16 & 2x - 1 & \sqrt{k} \\
16 & 4 & 1 & 0 \\
21 & 22 & 0 & 0\n\end{vmatrix}$ and I be the identity matrix of order
\n $\begin{vmatrix}\n2x - 1 & -2x - 2x - 1 & -2x - 2x - 1 & -2x - 2x - 1 \\
-2x - 2x - 1 & 0 & 0\n\end{vmatrix} = 0$.
\nLet (adj A) + det (adj B) = 10⁶, then [k] is equal to
\n $\begin{vmatrix}\n1 & 0 & 0 & 2x - 1 & \sqrt{k} \\
3x + 1 & 3x + 3 & 0 & 0\n\end{vmatrix} = 0$

3. If
$$
Q = [q_{ij}]
$$
 is a matrix such that $P^{50} - Q = I$, then
 $q_{31} + q_{32}$

$$
\frac{q_{21}}{q_{21}}
$$
 equals:

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

- **Q.71** How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of M^TM is 5 ? et P = $\begin{bmatrix} 4 & 1 & 0 \ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order

If Q = $\begin{bmatrix} q_{ij} \end{bmatrix}$ is a matrix such that P⁵⁰ – Q = I, then
 $\frac{31+932}{921}$ equals:

or a real number α , if the system $\begin{bmatrix} 1 &$
- **Q.72** If a, b, c are in A.P., then the determinants

$$
\begin{array}{|cccc|}\nx+2 & x+3 & x+2a \\
x+3 & x+4 & x+2b \\
x+4 & x+5 & x+2c\n\end{array}
$$
 is -

MATRICES AND DETERMINANTS QUESTION BANK

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 If $A = \begin{pmatrix} 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \end{pmatrix}$ **AND DETERMINANTS**

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION
 $\begin{bmatrix} \mathbf{i} & 0 \\ 0 & -\mathbf{i} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 & \mathbf{i} \\ \mathbf{i} & 0 \end{bmatrix}$, then
 $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$ and **AND DETERMINANTS**

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION

i 0
 $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $I = C^2 =$

(B) I

(D) 21

(D) 42 = I

(B) 2

(D) 42 = I

(**EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION**
 $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\begin{bmatrix} i(0) & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\begin{bmatrix} i(0) & 0 \\$ **EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION**
 EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION
 $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\begin{bmatrix} i(0) & -1 \\ 1 & 0 \end{bmatrix}$ an MINANTS

ERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $\frac{0}{1}$
 $\frac{-1}{0}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\frac{(A) A is a zero matrix}{(B) A = (-1) I}$, where I is a unit matrix
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 MINANTS

ERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $0 \t -1 \t 0 \t 0 \t 1$
 $0 \t 1 \t 0 \t 0 \t 21$

(B) 1

(B) 1

(D) 2 I
 $\begin{bmatrix} -5 & 7 & 1 \ 1 & -5 & 7 \end{bmatrix}$, 0.9

(B) 2

(B **ERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]**
 $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $\begin{bmatrix} (A) A is a zero matrix \\ (B) A = (-1) I, where I is a unit matrix \\ (C) A^{-1} does not exist \end{bmatrix}$
 $\begin{bmatrix} (B) I \\ (D) 2 I \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 4 \end{bmatrix}$ **ERCISE -3 [PREVIOUS YEARS JEE MAIN QUESTIONS]**
 $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $\begin{bmatrix} (A) A is a zero matrix \\ (B) A = (-1) I, where I is a unit matrix \\ (C) A^{-1} does not exist \end{bmatrix}$
 $\begin{bmatrix} (B) I \\ (D) 2 I \\ (D) 4 \end{bmatrix}$
 $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\$ $A^2 = B^2 = C^2 =$ = **[AIEEE 2002]** (A) I^2 (B) I $(C) - I$ (D) 2 I **Q.2** If $A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \end{pmatrix}$ then $AB =$ **AND DETERMINANTS**
 EXERCISE - 3 [PREVIOUS VEARS JEE MAIN QUESTIONS
 \vec{i} 0
 $\vec{0}$ -i], $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & \mathbf{i} \\ \mathbf{i} & 0 \end{bmatrix}$, then
 $\begin{bmatrix} (A) A \text{ is a zero matrix} \\ (B) A = (-1) I, \text{ where } I \text{ is a unit} \\ (C) A^{-1} \text{$ **AND DETERMINANTS**
 EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS
 $\begin{bmatrix}\ni & 0 \\
0 & -i\n\end{bmatrix}, B = \begin{bmatrix}0 & -1 \\
1 & 0\n\end{bmatrix}$ and $C = \begin{bmatrix}0 & i \\
i & 0\n\end{bmatrix}$, then
 $\begin{bmatrix}\n\text{(A) A is a zero matrix} \\
\text{(B) A = (1) I}, \text{ where I is a unit} \\
\text{(C) A⁻¹ does not exist}\n\end{b$ **AND DETERMINANTS**

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS
 $\begin{bmatrix}\ni & 0 \\
0 & -i\n\end{bmatrix}$, $B = \begin{bmatrix}\n0 & -1 \\
1 & 0\n\end{bmatrix}$ and $C = \begin{bmatrix}\n0 & i \\
i & 0\n\end{bmatrix}$, then
 $\begin{bmatrix}\n\text{(A)} \text{A} \text{ is a zero matrix} \\
\text{(B)} \text{A} = (-1)I$, where I is a un **SAND DETERMINANTS**

EXERCISE - 3 [PREVIOUS VEARS JEE MAIN QUESTIONS]
 $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $(2^2 = C^2 =$
 $(3)^2 = C^2 =$
 $(4)^2 = C^2 =$
 $(5)^2 = C^2 =$
 $(6)^2 =$ **SAND DETERMINANTS**
 EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $\begin{bmatrix} 1 & 0 \ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \ i & 0 \end{bmatrix}$, then
 $\begin{bmatrix} 18 & 0 \ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \ 1 & 0 \end{bmatrix}$ **EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS**
 EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS
 $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $\begin{bmatrix} (A) A is a zero matrix \\ (B) A = (-1) I$, w **EXECUTE: SUPREVIOUS VEARS JEE MAIN QUESTIONS**

CISE - 3 [PREVIOUS VEARS JEE MAIN QUESTIONS]
 $^{-1}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $\begin{array}{c} (A) A \text{ is a zero matrix} \\ (B) A = (-1) I$, where I is a unit matrix
 $(A) A \text{ is a zero matrix} \\ (C) A^{-1} \text{ does$ **NANTS**
 CISE -3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $\begin{bmatrix}\n-1 \\
0\n\end{bmatrix}$ and $C = \begin{bmatrix}\n0 & i \\
i & 0\n\end{bmatrix}$, then
 $\begin{bmatrix}\n(1) \text{ A} \text{ B} \text{ A} = \begin{bmatrix}\n0 & 1 \\
0 & 0\n\end{bmatrix}$, then
 $(1) \text{ A} = \begin{bmatrix}\n0 & 1 \\
0 & 0\n\end{bmatrix}$, then
 $(2) \$ **EXAMPLE 3** PREVIOUS VEARS JEE MAIN QUESTIONS

CISE - 3 [PREVIOUS VEARS JEE MAIN QUESTIONS]
 $\begin{bmatrix}\n-1 \\
0\n\end{bmatrix}$ and $C = \begin{bmatrix}\n0 & i \\
i & 0\n\end{bmatrix}$, then (B) A = (-1) I, where I is a unit matrix

(B) I

(D) 21
 $\begin{bmatrix}\n1 & -5 &$ **INANTS CUESTIONBANK**
 REISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $\begin{bmatrix}\n-1 \\
0\n\end{bmatrix}$ and $C = \begin{bmatrix}\n0 & i \\
i & 0\n\end{bmatrix}$, then
 $\begin{bmatrix}\n(1) \text{ A is a zero matrix} \\
(2) \text{ A is a zero matrix}\n\end{bmatrix}$
 $\begin{bmatrix}\n-1 \\
0\n\end{bmatrix}$ and $C = \begin{bmatrix}\n0 & i \\
i & 0\n\$ INANTS CUESTIONBANK

RCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $^{-1}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $(B)A = (-1)1$, where I is a unit matrix
 $(D)A^2 = I$
 $(D)1$
 $(-5 \t 7 \t 1)$
 $1 \t -5 \t 7$ then AB =
 $1 \t -5 \t 1$
 $(D)2$ INANTS CUESTIONBANK

RCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]
 $^{-1}$
 $(A) I_2$ $(B) 2 I_3$ **[AIEEE 2002]** (C) 4 I₃ (D) 18 I₃ **Q.3** If p^{th} , q^{th} , r^{th} term of a GP are ℓ , m, n then the value of CES AND DETERMINANTS

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION
 $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\begin{aligned}\n&= B^2 = C^2 = \begin{cases}\n\text{(B)} & 1 \\
\text{(C)} & \text{(D)}^2 + \text{(D)}^2 \\
\text{(E)} & \text$ **ICES AND DETERMINANTS**
 EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION
 $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $\begin{aligned} (A) A is a zero matrix \\ (B) A = (-1) I, \text{ where I is a unit} \\ (C) A^{-1} does not exist \end{aligned}$
 \begin EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTION
 $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then
 $A = \begin{bmatrix} 0 & -1 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \$ $\begin{vmatrix} \log \ell & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ is equal to is equal to- **[AIEEE 2002]** n 2n 3 1 2 3 1 3 1 - 5 7 determinant

2 3 1 3 1 - 5 7 den AB = the inverse of

(A)-2

(B) 2 I₃ [AIEEE 2002] (C) 2

(D) 18 I₃

h, rth term of a GP are ℓ , m, n then the value of

p 1

q 1

is equal to-

(B) 1

(B) (D) N 2 3 1 1 $\begin{bmatrix} 1 & -3 \end{bmatrix}$ (A) -2

(B) 21₃ (AIEEE 2002] (C) 181₃

th, rth term of a GP are ℓ , m, n then the value of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is equal to **AIEEE 2002**] (B) 1
 AIEEE 2002 (A) 0 1

and C= [1 o] when

(C) A⁻¹ does not exist

(C) A⁻¹ does not exist

(D) A² = I

(B) 1

(D) 2 1

3

(B) 1

D) 2 1

3

(B) 1

D) 2 1

(B) 1

(A² = B² = C² = (AIEEE 2002)

(B)1

(B)1

(B)21

(fA = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$ then AB = the inverse of matrix A, then

(A) -2

(A) -2

(A) -2

(A) -2

(A) -2

(1

(b)1

(b)21
 $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$ then AB =

(b)21

(c)21

(e)21

(e)2

(e) 1₃ (B) 21₃ [AIEEE 2002] (C) 2

a $\frac{1}{a^2 + a^3}$ [B) 181₃ (B) 181₃ (B) 181₄ (B) 4 h, d^h, t^h term of a GP are ξ , m, n then the value of

b (d) to 1₃

g m q 1

(b) None of these

(A) 0

 $(A) 0$ (B) 1 (C) $\ell + m + n$ (D) None of these

Q.4 If 1,
$$
\omega
$$
, ω^2 are the cube roots of unity, then

1 1 1 is equal to – **[AIEEE 2003]** 2 3 2 3 2 3 c c 1 c

$$
(A) \omega^2
$$
 (B) 0
(C) 1 (D) ω

Q.5 If
$$
\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0
$$
 and vectors (1, a, a²), (1, b, b²) &

 $(1, c, c²)$ are non-coplanar, then the product abc equals-(A) 0 (B) 2 **[AIEEE 2003]** $(C) - 1$ (D) 1

(A) 0
\n(A) 0
\n(A) 0
\n(A) 0² are the cube roots of unity, then
\n
$$
\Delta = \begin{vmatrix}\n1 & \omega^n & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix}
$$
\nis equal to - [AIEEE 2003] (A) -2
\n(A) 0² (B) -2
\n(C) 1
\n(D) 0
\n(E) 1
\n3. (A) -2
\n(A) 0² (B) -2
\n(C) 1
\n(D) 0
\n(E) 1
\n3. (B) 0
\n5. If $\begin{vmatrix}\na & a^2 & 1+a^3 \\
b & b^2 & 1+b^3 \\
c & c^2 & 1+c^2\n\end{vmatrix} = 0$ and vectors (1, a, a²), (1, b, b²) & (A) 1
\n(A) 0
\n(B) 0
\n(C) -1
\n(D) 1
\n(D) 0
\n3. (a, c²) are non-coplanar, then the product abc equals-
\n(a, b)
\n(b) 0
\n(c) -1
\n(d) 0
\n(e) 0
\n(f) 1
\n(g) 2
\n(g) 2
\n4. (a, b)
\n(b) 1
\n(c) -1
\n(d) 0
\n(e) 2
\n(f) 3
\n(g) 4
\n(h) 4
\n(i, c, c²) are non-coplanar, then the product ab equals-
\n(i, a, a²), (1, b, b²) & (A) 1
\n(B) 0
\n(C) -1
\n(D) 1
\n(E) 2
\n(E) 3
\n(E) 4
\n(E) 2003
\n(E) 4
\n(E) 2003
\n(E) 4
\n(E) 4
\n(E) 2004
\n(E)

Q.7 Let
$$
A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}
$$
. The only correct statement about

the matrix A is- **[AIEEE 2004]**

OULSTIONBANK
 EXEVIOUS YEARS JEE MAIN QUESTIONS
 $\begin{bmatrix} 0 & \mathbf{i} \\ \mathbf{i} & 0 \end{bmatrix}$, then $\begin{bmatrix} (A) A \text{ is a zero matrix} \\ (B) A = (-1) I, \text{ where } I \text{ is a unit matrix} \\ (C) A^{-1} \text{ does not exist} \end{bmatrix}$

[AIEEE 2002] $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ $\begin{bmatrix} 4 & 2 & 2 \\$ (A) A is a zero matrix $(B) A = (-1) I$, where I is a unit matrix (C) A⁻¹ does not exist $(D) A² = I$

| QUESTION BANK | CDIMADVANICED LEARNING DDMADVANICED LEARNING [1 0], then |
|---|---|
| \n $\begin{bmatrix}\n 0 & i \\ i & 0\n \end{bmatrix}$, then | \n $\begin{array}{c}\n (A) \text{A is a zero matrix} \\ (B) \text{A} = (-1) \text{I}, \text{ where I is a unit matrix} \\ (C) \text{A}^{-1} \text{ does not exist}\n \end{array}$ \n |
| \n $\begin{bmatrix}\n 0 & i \\ 1 & 0\n \end{bmatrix}$, then | \n $\begin{array}{c}\n (B) \text{A} = (-1) \text{I}, \text{ where I is a unit matrix} \\ (C) \text{A}^{-1} \text{ does not exist}\n \end{array}$ \n |
| \n $\begin{bmatrix}\n 0.8 & \text{Let } A = \begin{bmatrix}\n 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1\n \end{bmatrix}$ and\n \end{array}\n \end{array} and\n \begin{array}{c}\n (10) \text{B} = \begin{bmatrix}\n 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3\n \end{bmatrix}. \n If B is the inverse of matrix A, then α is-\n $\begin{bmatrix}\n (A) - 2 & (B) - 1 \\ (C) 2 & (D) 5 \\ (D) 5 & (D) 5\n \end{bmatrix}$ \n | |
| \n Let B = 2002]\n | \n $\begin{bmatrix}\n \log a_1 & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+1} & \log a_{n+2} & \log a_{n+3} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\n \end{bmatrix}$, is-\n $\begin{bmatrix}\n \text{AIEEE 2002]}\n \end{bmatrix}$ \n |

the inverse of matrix A, then
$$
\alpha
$$
 is
\n(A) - 2 (B) - 1
\n(C) 2 (D) 5
\n(D) 5

Q.9 If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in G.P., then the value of the

determinant n n 1 n 2 n 3 n 4 n 5 n 6 n 7 n 8 log a log a log a log a log a log a , is-

$$
(A) 0 \t\t (B) 1 \t\t [AIEEE 2005]
$$

(B) 1

(B) 1

(C) 2

(D) None of these

The proots of unity, then
 $\begin{vmatrix}\n\mathbf{B} & \mathbf{B} & \mathbf{C} \\
\mathbf{C} & \mathbf{D} & \mathbf{C} \\
\mathbf{D} & \mathbf{C} & \mathbf{D}$ (B) 1

(D) None of these

(D) None of these
 $x + ay + z = \alpha - 1, x + y + \alpha z = \alpha - 1$ has no solutions
 $x + ay + z = \alpha - 1, x + y + \alpha z = \alpha - 1$ has no solutions
 $x + ay + z = \alpha - 1, x + y + \alpha z = \alpha - 1$ has no solutions
 $A = 1$.

(A) $B = 0$

(B) 0

(B) 0

(B) **EXECUTIONS**

EXECUTIONS

In matrix

i, where I is a unit matrix

not exist

not exist

not exist
 $\begin{bmatrix}\n1 & -3 \\
1 & -3 \\
1 & 1\n\end{bmatrix}$ and $(10)B = \begin{bmatrix}\n4 & 2 & 2 \\
-5 & 0 & \alpha \\
1 & -2 & 3\n\end{bmatrix}$. If B is
 B
 $\begin{bmatrix}\n1 & -3 \\
1 & -2\n\end{bmatrix}$ $(D) - 2$ **Q.10** The system equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solution, if α is - **[AIEEE 2005]** (A) –2 (B) either –2 or 1 (C) not -2 (D) 1 **Q.11** If $a^2 + b^2 + c^2 = -2$ and (D) 5

......, a_n,..... are in G.P., then the value of the
 $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B) 1 [AIEEE 2005]

equations $\alpha x + y + z = \alpha - 1$,
 $\alpha - 1, x + y + \alpha z$, a_n ,..... are in G.P., then the value of the
 $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B)1 [AIEEE 2005]

(D)-2

equations $\alpha x + y + z = \alpha - 1$,
 $\alpha - 1, x + y + \alpha z = \$ $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B) 1 [AIEEE 2005]

(D)-2

equations $\alpha x + y + z = \alpha - 1$,
 $\alpha - 1, x + y + \alpha z = \alpha - 1$ has no solution, if α

[AIEEE 2005]

(B) eit 1 and (10)B = $\begin{pmatrix} 1 & 1 & 1 \ 1 & -2 & 3 \end{pmatrix}$. ITB is

1 stress of matrix A, then α is-

(B) - 1

(D) 5

(B) - 1

(D) 5

1 and (B) - 1

(D) 5

1 and (B) - 2

(B) 1

(B) 1

2 erse of matrix A, then α is-

(B) - 1

(D) 5

2, a₃,....., a_n,..... are in GP, then the value of the
 $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B) 1 [AIEEE 20 (B) - 1

(D) 5

(B) - 1

(D) 5

(log a_n a_n,..... are in GP, then the value of the
 $\begin{vmatrix} \log a_{n} & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B) 1 [AIEEE 2005]

(D) -2
 2 1 -3

1 1 -3

1 -5

1 -5

1 -5

1 -5

1 -2

1 -3

1 -5

1 -5

1 -5

1 -5

1 -5

1 -1

1 -2

1 -1

1 -2

1 -1

2 -1

1 1 1 1 \int and (10)B = $\begin{pmatrix} 1 & -2 & 3 \end{pmatrix}$. ITB IS

se of matrix A, then α is-

(B) - 1

(D) 5

a₃,....., a_n,.... are in G.P., then the value of the
 $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{$ se of matrix A, then α is-

(B) - 1

(D) 5

a₃,....., a_n,..... are in G.P., then the value of the
 $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is-

(B) 1 [AIEEE 2005] 1 0 em equations $\alpha x + y + z = \alpha - 1$,
 $z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solution, if α

[AIEEE 2005]

(B) either -2 or 1
 $\alpha + 2 + z = -2$ and
 $\alpha + 2 + 3 + 2$ and $\alpha + 2 + 3 + 2$ and $\alpha + 3 + 2$ and $\alpha + 2 + 3 + 2$ and $\alpha + 3 + 2$ and cos^{10} (B) 1 [AIEEE 2005]

(B) 1 [AIEEE 2005]

(B) -2

tem equations αx + y + αz = α - 1,

+z = α - 1, x + y + αz = α - 1 has no solution, if α

[AIEEE 2005]

(B) either -2 or 1

-2

-2

-2

-2

-2

(D) 1

-3

-3

(AIE (B) 1 **(AIEEE 2005**

(D)-2

tem equations $\alpha x + y + z = \alpha - 1$,
 $z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solution, if α
 [AIEEE 2005]

(B) either -2 or 1
 $z^2 + c^2 = -2$ and
 $1 + a^2 x (1 + b^2)x (1 + c^2)x$
 $(1 + a^2)x (1 + b^2)x (1 + c^2)x$
 $(1 + a$ (b) 1

(b) -2
 $+x+z=\alpha-1$,
 $\alpha z = \alpha - 1$ has no solution, if α

[AIEEE 2005]

(B) either -2 or 1

(D) 1

x (1+c²)x</sup>

(1+c²)x

x 1+c²x

of degree [AIEEE 2005]

(B) 0

(D) 2

inverse of A is -[AIEEE-2005]

(B) A

(D) + y + z = α - 1,
 $\alpha z = \alpha - 1$,

(a) either -2 or 1

(b) 1

(b) 1

x (1+c²)x</sup>

(1+c²)x

x 1+c²x

of degree [AIEEE 2005]

(b) 0

(b) 2

inverse of A is -[AIEEE-2005]

(b) A

(b) 1-A

1 0

0 1], then which one of th $cos α_{n+/-} cos α_{n+8}$

(B) 1 [AIEEE 2005]

(D)-2
 $x + y + z = α - 1$,
 $+αz = α - 1$ has no solution, if α

[AIEEE 2005]

(B) either-2 or 1

(D) 1
 $)x (1+c²)x$
 $x (1+c²)x$
 $x (1+c²)x$

of degree - [AIEEE 2005]

(B) 0 (B) 1 [AIEEE 2005]
 $(b)-2$
 $(c+y+z=\alpha-1$,
 $+\alpha z=\alpha-1$ has no solution, if α

[AIEEE 2005]

(B) either -2 or 1

(D) 1
 α (1+c²)x
 x (1+c²)x
 x (1+c²)x

(f) 1

(D) 2

inverse of A is -[AIEEE-2005]

(B) 0

(D) 1-A

$$
f(x) = \begin{vmatrix} 1 + a^2 x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}
$$

 (C) 3 then $f(x)$ is a polynomial of degree - $[AIEEE 2005]$ $(A) 1$ (B) 0 (D) 2 **Q.12** If $A^2 - A + I = 0$, then the inverse of A is $\text{-}[AIEEE\text{-}2005]$

 $(A) A + I$ (B) A (C) A – I (D) I – A **Q.13** If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the

following holds for all $n \ge 1$, by the principle of mathematical induction-
[AIEEE-2005] mathematical induction - $(A) Aⁿ = nA - (n-1) I$ (B) $Aⁿ = 2ⁿ⁻¹$ $(B) Aⁿ = 2ⁿ⁻¹ A - (n - 1) I$

- $(C) Aⁿ = nA + (n I) I$ (D) $Aⁿ = 2ⁿ⁻¹$ $(D) Aⁿ = 2ⁿ⁻¹A + (n - 1) I$ **Q.14** If A and B are square matrices of size $n \times n$ such that
	- $A^2 B^2 = (A B) (A + B)$, then which of the following will be always true– **[AIEEE 2006]** (A) $AB = BA$ (B) Either of A or B is a zero matrix
		- (C) Either of A or B is an identity matrix
		- (D) $A = B$

(C) Statement-1 is false, Statement -2 is true.

(D) Statement-1 is true, Statement-2 is true Statement-2 is a correct explanation for Statement-1. **Q.22** Let a, b, c be such that $b(a+c) \neq 0$. If

QUISION BANK
\n(a) 0)
\n(0) 1, a, b ∈ N. Then
\n(10) 22 Let a, b, c be such that
$$
bt + c) + 0
$$
. If
\n(10) 1, a, b ∈ N. Then
\n(10) 22 Let a, b, c be such that $bt + c) + 0$. If
\n(10) 22 Let a, b, c be such that $bt + c) + 0$. If
\n(10) 22 Let a, b, c be such that $bt + c) + 0$. If
\n(10) 22 Let a, b, c be such that $bt + b + 1$ and 0 . If
\n(10) 22 1. If $2 + b + 1$ and 0 . If $2 - b + 1$ and 0 . If $2 - b + 1$ are
\n(10) 10. If $2 - 25$, then $|a|$ equals.
\n(2) 23 The number of 3 × 3 non-singular matrices, with four
\nentrices 1 and all other entries, 10, 15
\n(3) 3
\n(4) 4
\n(5) 4 × 4 × 1, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 2 × 2 identity matrix. Define 1 ft(A) = sum
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n $At^2 = 1$, where 1 is 1 ft(A) = 0
\n At^2

| MATRICES AND DETERMINANTS | QUISION BANS | Self |
|--|--------------|------|
| Q.28 Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices equations $2x_1 - 2x_2 + x_3 = \lambda x_1$, $2x_1 - 3x_2 + \lambda x_4 = \lambda x_1$, $2x_1 - 3x_2 - \lambda x_2 = \lambda x_1$, $2x_1 - 3x_2 - \lambda x_3 = \lambda x_1$, $2x_1 - x_2 = 0$; $x_1 + x_2 = 0$; | | |
| Q.29 Let P and Q be 3 × 1 matrices, P ≠ Q. IFPE = Q ³ If $x = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is the set of distinct values of λ for which the | | |

TERMINANTS) (QUSTION BANX
\n**QUSTION BANY**
\nQ.35 The set of all values of
$$
\lambda
$$
 for which the system of linear equations $2x_1 - 2x_2 + x_3 = \lambda x_1$; $2x_1 - 3x_2 + 2x_3 = \lambda x_2$;
\n λ is a singular non-trivial solution for:
\n(A) Is a singular
\n(B) and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is Q.36 The system of linear equations,
\n(C) Continuous two elements
\n(D) and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is Q.36 The system of linear equations,
\n $x + \lambda y = 2e$; $x + y = 2e$; $x + y = 2e$ (11)
\n(A) exactly one value of λ .
\n**EXERCISE 2012**
\n(A) exactly two values of λ .
\n(A) exactly one values of λ .
\n(B) If λ is a real solution for:
\n(A) exactly two values of λ .
\n(B) If λ is the set of distinct values of λ
\n(b) infinitely many values of λ
\n(c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ (f) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ (g) 13
\n(g) 14
\n(h) 166
\n(g) 160
\n(h) -1
\n(g) 160
\n(h) -1
\n(g) 170
\n(h) 181
\n31 420
\n43 45 40
\n54 50 46, for which the system of
\nthe system of
\nthe system of linear equations for 'for which the
\n $x + 8y = 4k$; $kx + (k+3)y = 3k-1$
\n(B) 1
\n(B) 1
\n(C) an empty set
\n(a) a similar vector is

ANSWER KEY

21 42 42 42 21 21 42 42 21 n (n 1) 378 n 27 1 5 6 1 3 2 3 5 6 5 5 3 2 3 1 6 6 1 1 0 1 b / a 1/ a 0 1 bc ^A c 0 1 a 1 1 R R a or 1 b / a 1/ a 0 A 0 1/ a c / a 1 [R² R² – cR¹] or 1 b / a 1/ a 0 A 0 1 c a [R² aR²] or 1 bc 1 0 ^b a A 0 1 c a 1 1 2 ^b R R R a 1 1 bc ^b ^A ^a c a **(7)** | A | | adj A | = | A adj A | = | | A | I | ⁼ | A | 0 0 0 | A | 0 0 0 | A | = | A |³ = (a³)³ = a⁹ **(8)** AB = B–1 AB² = I Now, KA – 2B–1 + I = O KAB – 2B–1B + IB = O KAB – 2I + B = O KAB² – 2B + B² = O KI – 2B + B² = O 1 0 2 1 2 1 2 1 0 0 K 2 0 1 2 0 2 0 2 0 0 0 K 0 4 2 2 2 0 0 0 K 4 0 4 2 0 0 K 2 0 0 0 0 K 2 0 0 K = 2 **TRY IT YOURSELF-2** a b c 2a 2a 2b b c a 2b 2c 2c c a b [R¹ R¹ + R² + R³] 1 1 1 (a b c) 2b b c a 2b 2c 2c c a b 1 0 0 (a b c) 2b b c a 0 2c 0 c a b 2 2 1 3 3 1 C C C C C C

MATRICES AND DETERMINANTS TRY SOLUTIONS

| MATRICES AND DETERMINANTS | IPANSOLUTIONS | ESINSOLUTIONS | | | | | | | | | | | | | | | | | | | | |
|---|---|--|--|--|--|--|--|-----------------------|--|-----------------------|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| \n $=(a+b+c)$ \n $0 - c - a - b$ \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n | \n $1 - c - a - b$ \n |
| \n $1 - 2 - 2 - 2$ \n | \n $1 - 3 - 2 - 2$ \n </td | | | | | | | | | | | | | | | | | | | | | |

$$
\therefore 2b = a + c = \frac{1}{2} \begin{vmatrix} x + 2 & x + 3 & x + 2a \\ 0 & 0 & 0 \\ x + 4 & x + 5 & x + 2c \end{vmatrix} = 0
$$

(3) **(D).** We have,
$$
\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} \alpha + \beta + \gamma & \beta & \gamma \\ \alpha + \beta + \gamma & \gamma & \alpha \\ \alpha + \beta + \gamma & \alpha & \beta \end{vmatrix}
$$

$$
[C_1 \to C_1 + C_2 + C_3]
$$

= 0 [:: $\alpha + \beta + \gamma = 0$ from the equation $x^3 - 3x + 2 = 0$]

Operating
$$
R_2 \rightarrow 2R_2 - R_1 - R_3
$$

\n
$$
= \frac{1}{2} \begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 2b-a-c \\ x+4 & x+5 & x+2c \end{vmatrix}
$$
\n= sin²β + cos (α – β) cos (α – β

TS
\n**TRY SOLUTIONS**
\n
$$
0
$$
\n
$$
-c-a-b
$$
\nExpanding along C₁]
\n
$$
a+b+c
$$
³=R.H.S.
\n
$$
c+a
$$

[expanding along
$$
R_1
$$
]

cos cos 1 = (1 – cos2) + cos (–) [cos cos – cos (–)] + cos [cos (–) cos – cos] = sin2 + cos (–) [2 cos cos – cos (–)] 2 3 8 7 5 3 0 4 6

$$
= \sin^2 \beta + \cos (\alpha - \beta) \cos (\alpha + \beta) - \cos^2 \alpha
$$

$$
= \sin^2 \beta + (\cos^2 \alpha - \sin^2 \beta) - \cos^2 \alpha = 0
$$

(6) Here the equations are linear. We have 3 equations in 2 unknowns.

$$
\therefore \quad \text{They are consistent if } \begin{vmatrix} 2 & 3 & -8 \\ 7 & -5 & 3 \\ 4 & -6 & \lambda \end{vmatrix} = 0
$$

or
$$
2(-5\lambda + 18) - 3(7\lambda - 12) - 8(-42 + 20) = 0
$$

- or $-10\lambda + 36 21\lambda + 36 + 176 = 0$
- or $-31\lambda + 248 = 0$ $\therefore \lambda = 8$
- \therefore for $\lambda = 8$ the system has a solution which can be obtained by solving any two of the three equations. Solving, $2x + 3y - 8 = 0$

 $7x - 5y + 3 = 0$ By Cramer's rule,

$$
= \sin^2 \beta + \cos (\alpha - \beta) [2 \cos \alpha \cos \beta - \cos (\alpha - \beta)]
$$

\n
$$
= \sin^2 \beta + \cos (\alpha - \beta) \cos (\alpha + \beta) - \cos^2 \alpha
$$

\n
$$
= \sin^2 \beta + (\cos^2 \alpha - \sin^2 \beta) - \cos^2 \alpha = 0
$$

\n(6) Here the equations are linear. We have 3 equations in 2 unknowns.
\n
$$
\begin{array}{rcl}\nx + 2a \\
0 \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\n
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\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\n
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\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\n
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\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
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\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx + 2a \\
x + 2c\n\end{array} = 0
$$
\n
$$
\begin{array}{rcl}\nx
$$

$$
\begin{vmatrix} x^2 \\ 2 \\ 2 \end{vmatrix}
$$
 (7) (A). We have, $f(x) = x(x+1)(x-1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix} = 0$

$$
[C_1 \rightarrow C_1 - C_3 \text{ and } C_2 \rightarrow C_2 - C_3]
$$

Hence, $f(100) = 0$

CHAPTER-3: MATRICES AND DETERMINANTS EXERCISE-1

- **(1) (C).** It is based on fundamental concept.
- {A is symmetric. Hence M'AM is a symmetric matrix). **(3) (B).** $A + A^T$ is a square matrix.

$$
(A + A^{T})^{T} = A^{T} + (A^{T})^{T} = A^{T} + A
$$

Hence A is a symmetric matrix.

(4) **(D).**
$$
A^2 - 4A - 5I = 0
$$
; $A(A - 4I) = 5I$; $A^{-1} = \frac{1}{5}(A - 4I)$

4 (b) (b)
$$
A^2 - 4A - 5I = 0
$$
; $A(A - 4I) = 5I$; $A^{-1} = \frac{1}{5}(A - 4I)$
\n**5** (a) (b) $A^2 - 4A - 5I = 0$; $A(A - 4I) = 5I$; $A^{-1} = \frac{1}{5}(A - 4I)$
\n**6** (b) (c) (d) (e) $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$
\n**6** (d) $A^3 = A^2A = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$
\n**6** (e) $A^2 + B^2 = A$. $A + B$. $B = A(BA) + B(AB)$
\n**6** (f) $A^2 + B^2 = A$. $A + B$. $B = A(BA) + B(AB)$
\n**6** (g) $A^2 + B^2 = A$. $A + B$. $B = A(BA) + B(AB)$
\n**6** (h) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow A^{-1} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$
\n**6** (i) (j) $A^2 + B^2 = A$. $A + B$. $B = A(BA) + B(AB)$
\n**6** (k) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow A^{-1} \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix}$
\n**6** (l) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1/d_2 \end{pmatrix} \Rightarrow A^{-1} \begin{pmatrix} 1/d_1 &$

$$
A3 = A2A = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}
$$

- In general by induction, $A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$, $\forall n \in N$]
- **(6) (B).** $A^2 + B^2 = A \cdot A + B \cdot B = A (BA) + B (AB)$ $=(AB) A + (BA) B = BA + AB = A + B$

(7) **(A).**
$$
A = \begin{pmatrix} d_1 & 0 \ 0 & d_2 \end{pmatrix} \Rightarrow A^{-1} \begin{pmatrix} 1/d_1 & 0 \ 0 & 1/d_2 \end{pmatrix}
$$

(8) (D). Given, A multiplicative group of 2×2 matrices of the

 A^T is a square matrix.
 A^T is a square matrix.

A² - A form $\begin{bmatrix} 0 & 0 \end{bmatrix}$. Let $A = \begin{bmatrix} 0 & 0 \end{bmatrix}$ since $|A| = 0$, therefore (17) (D), $|(2A)^{-1}$ Alsa symmetric matrix.
 $-4A-5I=0; A(A-4I)=5I; A^{-1}=\frac{1}{5}(A-4I)$
 $\Rightarrow a^2+β\gamma=1$
 $\Rightarrow a^2+β\$ 2-4A-5I= 0; A(A-4I) = 5I; A⁻¹ = $\frac{1}{5}$ (A-4I)

a have A² = $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

be have A² = $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0$ A¹ $y' = A^2 + (A^2)^2 = A^2 + A$
 $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

We have $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

We have $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end$ Example the matrix (and $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

We have $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$

We have $A^2 = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin$ inverse of A does not exist.

(9) (A).

(A). We have A² =
$$
\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}
$$

\n $\Rightarrow 4 + 2a = 0, 4 + 2b = 0, 2a + 2b = 0$
\n $2a + b^2 = 0$ must be consistent.
\nA³ = A²AA = $\begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$
\n $\Rightarrow 4 + 2a = 0, 4 + 2b = 0, 2a + 2b = 0$
\n $2a + b^2 = 0$ must be consistent.
\n $\Rightarrow a = -2, b = -2$
\n $2a + b^2 = 0$ must be consistent.
\n $\Rightarrow a = -2, b = -2$
\n $2a + b^2 = 0$ must be consistent.
\n(A) A = $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, \forall n ∈ N
\n \Rightarrow (A) B = BA + AB = A+B
\n(A) B = BA + AB = A+B
\n(B) A² + B² - A² + B² + A = A+B
\n(B) A² + B² - A² + B² + B = AB
\n(A) B = $\begin{pmatrix} 1 & a \\ 0 & a \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ since |A| = 0, therefore A⁻¹ = $\begin{pmatrix} 1 & a \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 6 \end{pmatrix}$.
\n \therefore |AB| = |A||B| | \neq (therefore A⁻¹ + 2² - 3

$$
P = \begin{bmatrix} 12 & 15 & 4 \\ 32 & 40 & 28 \\ 44 & 55 & 40 \end{bmatrix}_{3 \times 3} \Rightarrow P_{22} = 40
$$

(10) (A). Here AB =
$$
\begin{bmatrix} pr - qs & ps + qr \ -qr - ps & -qs + pr \end{bmatrix}
$$

Also BA =
$$
\begin{bmatrix} rp - qs & qr + sp \ -sp - qr & -qs + pr \end{bmatrix}
$$
 Clearly AB = BA

UTIONS

\n**STUDY MATERIAL: MATHEMATICS**

\n(11) **(B).** Here
$$
al + bA = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}
$$

\n
$$
\therefore (al + bA)^2 = \begin{pmatrix} a^2 + 0 & ab + ba \\ 0 + 0 & 0 + a^2 \end{pmatrix} = \begin{pmatrix} a^2 & 2ab \\ 0 & a^2 \end{pmatrix}
$$

Q.B.-SOLUTIONS STUDY MATERIAL: MATHEMATICS
\n**MATRICES AND DETERMINANTS** (11) (B). Here al +bA =
$$
\begin{pmatrix} a & 0 \ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \ 0 & a \end{pmatrix}
$$

\n(1) (C). It is based on fundamental concept.
\n(2) (A). (M'AM) = M'AM = M'AM
\n(A) is symmetric. Hence M'AM is a symmetric matrix.
\n(A + A^T)^T = A^T + (A^T)^T = A^T + A
\nHence A is a symmetric matrix.
\n(A + A^T)^T = A^T + (A^T)^T = A^T + A
\nHence A is a symmetric matrix.
\n(3) (B). A + A^T is a square matrix.
\n(A + A^T)^T = A^T + (A^T)^T = A^T + A
\nHence A is a symmetric matrix.
\n(4) (D). A² - 4A – 5I = 0; A (A – 4I) = 5I; A⁻¹ = $\frac{1}{5}$ (A – 4I)
\n(5) (A). We have A² = $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$
\n(6) \Rightarrow 4 + 2a = 0, 4 + 2b = 0, 2a + 2b = 0
\n2a + b² = 0 must be consistent.
\nA³ = A²A = $\begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$
\n(14) (C). A' = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 1 & 3 \end{bmatrix}$, therefore
\n

RICES AND DETERMINANTS
\n**RICES AND DETERMINANTS**
\n**EXERCISES AND DETERMINANTS**
\n**EXAMPLER**
\n**1** (11) (B). Here
$$
aI + bA = \begin{pmatrix} a & 0 \ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \ 0 & a \end{pmatrix}
$$

\n $\begin{pmatrix} 1 \ \ \end{pmatrix} \times 10^{-1} = M^2 \text{ A M}$
\n $\begin{pmatrix} 1 \ \ \end{pmatrix} = M^2 \text{ A M} = M^2 \text{ A M}$
\n $\begin{pmatrix} 1 \ \ \end{pmatrix} = A^T + (A^T)^T = A^T + A$
\n $\begin{pmatrix} 1 \ \ \end{pmatrix} = (A - 4I)^T = A^T + A$
\n $\begin{pmatrix} 1 \ \ \end{pmatrix} = (A - 4I)^T = (A - 5I) = 0; A(A - 4I) = 5I; A^{-1} = \frac{1}{5}(A - 4I)$
\n $\begin{pmatrix} 1 \ \ \end{pmatrix} = \begin{pmatrix} 1 & a \ b \end{pmatrix} = \begin{pmatrix} 1 & a \ b \end{pmatrix} = \begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = \begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pmatrix} 1 & a \ b \end{pmatrix} = 1$
\n $\begin{pm$

$$
\begin{array}{ccc} \n\frac{1}{5} & (A-41) & \\
\end{array}
$$
\n
$$
\n\begin{array}{ccc} (A) & A^2 = \begin{bmatrix} 2 & 2 \\ a & b \end{bmatrix} \begin{bmatrix} 2 & 2 \\ a & b \end{bmatrix} = \begin{bmatrix} 4+2a & 4+2b \\ 2a+ab & 2a+b^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

 $2a + b² = 0$ must be consistent. \Rightarrow a = -2, b = -2

$$
AA' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [1 \ 2 \ 3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}.
$$

- I=0; A(A-4I) = 51; A⁻¹ = $\frac{1}{5}$ (A-4I)
 $= (\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = (\begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix})$
 $= (\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = (\begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix})$
 $= 4 + 2a = 0, 4 + 2b = 0, 2a +$ ana.

A-41) = 51; A⁻¹ = $\frac{1}{5}$ (A-41)

(13) (A). $A^2 = \begin{bmatrix} 2 & 2 \ a & b \end{bmatrix}$ $B = \begin{bmatrix} 4+2a & 4+2b \ 2a+ab & 2a+b^2 \end{bmatrix} = 0$

(13) (A). $A^2 = \begin{bmatrix} 2 & 2 \ a & b \end{bmatrix}$ $B = \begin{bmatrix} 4+2a & 4+2b \ 2a+ab & 2a+b^2 \end{bmatrix} = 0$
 $2a+b^2 = 0$ A(A-4I) = 5I; $A^{-1} = \frac{1}{5}$ (A-4I)

(13) (A). $A^{2} = \begin{bmatrix} 2 & 2 \ a & b \end{bmatrix} \begin{bmatrix} 2 & 2 \ a & b \end{bmatrix} = \begin{bmatrix} 4+2a & 4+2b \ 2a+8b & 2a+b^2 \end{bmatrix} = 0$

(a) $\begin{bmatrix} 1 & a \ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2a \ 0 & 1 \end{bmatrix}$
 $\Rightarrow 4+2a = 0, 4+2b = 0, 2a+2b$ **(15) (B).** Since A and B are square matrix \therefore $|AB| = 100$.
	- **(16) (D).** Since $|A| \neq 0$ therefore A^{-1} exist such that $AA^{-1} = I = A^{-1}A$

17) (D).
$$
|(2A)^{-1}| = \frac{1}{|2A|} = \frac{1}{2^3 \cdot |A|} = \frac{1}{2^3 \cdot 3} = \frac{1}{24}
$$

- 2a + b² = 0 must be consistent.

(1 3a)

⇒ a = -2, b = -2

(0 1)

(14) (C). A' = [1 2 3], therefore

a

(14) (C). A' = [1 2 3], therefore

AA' = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$.

+ HB (AB)

(15) (B) 2a + b² = 0 must be consistent.

⇒ a = -2, b = -2
 (14) (C). A' = [1 2 3], therefore

AA' = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ [1 2 3] = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.
 (15) (B). Since A and B are square matrix
 $\begin{bmatrix} 2 \\ b \end{bmatrix} = \begin{bmatrix} 4+2a & 4+2b \\ 2a+ab & 2a+b^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2b = 0, 2a + 2b = 0

be consistent.

therefore
 $= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.

are square matrix

; $|A| = -10$; $|B| = -10$

th $\begin{bmatrix} 2 & 2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4+2a & 4+2b \\ 2a+ab & 2a+b^2 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $+2b = 0, 2a + 2b = 0$
 $+2b = 0, 2a + 2b = 0$

t be consistent.

2
 $\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.

B are **(18) (D).** Expanding: $x-2-1-2(1-x)-1(1-x^2+2x)=0$ $x-3-2+2x-1+x^2-2x=0$ $x^{2} + x - 6 = 0$. $x = 2$ satisfies the above equation e A⁻¹ exist such that
 $\frac{1}{|A|} = \frac{1}{2^3 \cdot 3} = \frac{1}{24}$

2(1-x)-1(1-x²+2x)=0

2x=0

eatisfies the above equation

B-B²

BA
 $\left[\begin{array}{c} 3 \\ -1 \end{array}\right] - \left[\begin{array}{c} 2 & 5 \\ 3 & -1 \end{array}\right]$
 $\left[\begin{array}{c} -1 \\ 1 \end{array}\right]$
 $\left[\begin{array}{c}$
	- **(19) (A).** AB BA is skew symmetric

(20) (C).
$$
A^2 - B^2 = A^2 - BA + AB - B^2
$$

\n $\Rightarrow 0 = -BA + AB \Rightarrow AB = BA$

1)
$$
\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}
$$

\n2) $\begin{pmatrix} -1 & a \\ 0 & 1 \end{pmatrix}$, $\forall n \in \mathbb{N}$
\n3) $B = A(BA) + B(AB)$
\n4) $A^2 = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix}$, $\forall n \in \mathbb{N}$
\n5) $B = A(BA) + B(AB)$
\n6) $\begin{pmatrix} -1 & a \\ 3 & 1 \end{pmatrix} = BA + AB = A + B$
\n7) $\begin{pmatrix} -1 & a \\ 0 & 1/a \end{pmatrix}$
\n8) $\begin{pmatrix} 1 & a \\ 1 & 0 \\ 0 & 1/a \end{pmatrix}$
\n9) $\begin{pmatrix} -1 & a \\ 1 & 0 \\ 0 & 1/a \end{pmatrix}$
\n10) $\begin{pmatrix} 1 & a \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$
\n11) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$
\n12) $\begin{pmatrix} -1 & 1 & a \\ 1 & 1 & 4 \end{pmatrix} = BA + BA$
\n13) $\begin{pmatrix} -1 & 1 & a \\ 1 & 1 & 4 \end{pmatrix} = BA + AB$
\n14) $\begin{pmatrix} -1 & 1 & a \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$
\n15) $\begin{pmatrix} -1 & 4 & -5 & -6 \\ 0 & 0 & 1 & 2 \end{pmatrix}$
\n16) $\begin{pmatrix} 10 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 3$

(22) (B).

(23) **(D).** Symmetric part of
$$
A = \frac{1}{2}(A + A')
$$

(16) (D). Since |A| ≠ 0 therefore A⁻¹ exist such that
\n
$$
AA^{-1} = I = A^{-1}A
$$
\n(17) (D). |(2A)⁻¹| = $\frac{1}{|2A|} = \frac{1}{2^3 \cdot |A|} = \frac{1}{2^3 \cdot 3} = \frac{1}{24}$
\n(18) (D). Expanding: x-2-1-2(1-x)-1(1-x²+2x)=0
\nx-3-2+2x-1+x²-2x=0
\nx²+x-6=0. x=2 satisfies the above equation
\n(19) (A). AB-BA is skew symmetric
\n(20) (C). A²-B² = A²-BA+AB-B²
\n⇒ 0=-BA+AB ⇒ AB = BA
\n(21) (D). B = $\frac{1}{2}$ (A-A') = $\frac{1}{2}$ $\begin{bmatrix} 2 & 3 \ 5 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 5 \ 3 & -1 \end{bmatrix}$
\n= $\frac{1}{2}$ $\begin{bmatrix} 0 & -2 \ 2 & 0 \end{bmatrix}$ = $\begin{bmatrix} 0 & -1 \ 1 & 0 \end{bmatrix}$
\n(22) (B).
\n(23) (D). Symmetric part of A = $\frac{1}{2}$ (A+A')
\n= $\frac{1}{2}$ $\begin{bmatrix} 1 & 2 & 4 \ 6 & 8 & 2 \ 2 & -2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 2 \ 2 & 8 & -2 \ 4 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \ 4 & 8 & 0 \ 3 & 0 & 7 \end{bmatrix}$
\n(24) (C). We know A. Adj A = |A|I
\nClearly |A| = 10
\n|Adj A| = |A|³⁻¹ = |A|² = 10² = 100

$$
|\text{Adj A}| = |\text{A}|^{3-1} = |\text{A}|^{2} = 10^{2} = 100
$$

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

(MATRICES AND DETERMINANTS)

(25) **(D).** If $A = \begin{bmatrix} a & 0 & 0 \ 0 & b & 0 \ 0 & 0 & c \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \ 0 & 1/b & 0 \ 0 & 0 & c \end{bmatrix}$

When $a \ne 0$, $b \ne 0$, $c \ne 0$

(26) **(D).** $|A| = \begin{vmatrix} 4 & 1 \ 2 & 3 \end{vmatrix} = (4 \times 3 - 1 \times 2$ AND DETERMINANTS

A = $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$, A⁻¹ = $\begin{pmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{pmatrix}$

A = $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1/c \end{pmatrix}$, A⁻¹ = $\begin{pmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{pmatrix}$

A = $\begin{pm$ **DETERMINANTS** (Q.B.-SOLUTIONS CORRECTED FRIME AGNISED CORRECTED FRIME 2013
 $\begin{bmatrix}\n0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0\n\end{bmatrix}, A^{-1} = \begin{bmatrix}\n1/a & 0 & 0 \\
0 & 1/b & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 $\begin{bmatrix}\n\end{bmatrix}$ (1/4 0 0 1/ c)
 $\begin{bmatrix}\n\end{bmatrix}$ (1/4 0 0 1 (25) **(D).** If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \end{pmatrix}$, $A^{-1} = \begin{pmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/a \end{pmatrix}$ **D DETERMINANTS**

(a 0 0)

(a 0 0)

(a 0 0)

(b 0 0)

(a 0 0)

(a 0 0)

(b 0 1/b 0

(a 0 0)

(b 0 1/b 0

(a 0 0)

(b 0 1/b 0

(c 0 0 1/c)

(e.g., $\cos P \cos Q \cos R$

(cos B $\cos B$ $\cos B$

(cos C $\cos C$)

(cos C $\cos C$)

(cos C $\cos C$ **ND DETERMINANTS**

(**Q.B.-SOLUTIONS**

This determinant can be written as 8 determinants and
 $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$
 $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{$ **D DETERMINANTS**

(**Q.B.-SOLUTIONS**

(a 0 0)

(a 0 0)

(a b 0)

(a + 0)

(a + 0)

(a + 0)

(c e s control b 0 0 1/c

(a + 0)

(c e s control b 0 0 1/c

(c e s control b 0 0 1/c

(c e s control b 0 0 1/c

(c e s control b When $a \neq 0$, $b \neq 0$, $c \neq 0$ **(26) (D).** $|A| = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$ $= (4 \times 3 - 1 \times 2) = 12 - 2 = 10$ **(D) DETERMINANTS**

(**Q,B,-SOLUTIONS**

(a) 0

(a) 0

(a) 1/b

(a) 0

(a) 1/b

(a) 1/c

(a) 0

(a) 2

(a) 2

(a) 2

(a) 1/c

(a) 1/c **EXERMINANTS**

(**O.B.-SOLUTIONS**)

(**a** 0 0)

(**b** 0 0)

(**b** 0 0)

(**a** + 0, b + 0, c + 0

(**a** + 3 – 1 × 2) = 12 – 2 = 10

(**31)**

(**A)**. Given determinant
 $\begin{bmatrix}\n a_{11} & a_{12} \\
 a_{21} & a_{22}\n\end{bmatrix}$, then

(**a** + 1

(**a** DETERMINANTS

0 0)

b 0)

b 0)

0 0 1/e

0 e.g., cos P cos Q c

e.g., cos P cos Q c

e.g., cos P cos Q c

(31) (A). Given determini

(31) (A). Given determini DETERMINANTS

0 0)

b 0)

b 0)

0 0 1/e

0 1/b 0

0 1/e

0 1/e

0 1/e

21 a₂₂

(31) (A). Given determinant

c g., cos P cos Q c

(31) (A). Given determinant

(31) (A). Given determinant
 $\begin{cases}\n\cos \theta + \cos \theta = (4 \times 3 - 1 \times 2) =$ **D DETERMINANTS**

(a 0 0)

(b 0 b 0)

(a + 0)

(b - 0 c + 0

(a + 0)

(a + 0)

(a + 0)

(a + 0)

(a + 2 minuminant = $\begin{bmatrix} 1 & a_1 \\ a_2 & a_2 \end{bmatrix}$, then

(31)

(a + 0 c + 2 minuminant = $\begin{bmatrix} 1 & a_1 \\ a_2 & a_2 \end{bmatrix}$, then
 INANTS

(**Q.B.-SOLUTIONS**)

This determinant can be written as

the value of each of these 8 determ
 (31) (A). Given determinant
 (31) (A). Given determinant
 (31) (A). Given determinant
 (31) (A). Given determi **D DETERMINANTS**

(a 0 0)

(a 0 0)

(b 0)

(a + 0)

(b + 0)

(a + 2)

(a + 0)

(a + 0) **DETERMINANTS**
 O.B. SOLUTIONS
 $\begin{bmatrix}\n0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0\n\end{bmatrix}, A^{-1} = \begin{bmatrix}\n1/a & 0 & 0 \\
0 & 1/b & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 $= (4 \times 3 - 1 \times 2) = 12 - 2 = 10$
 B. SOLUTIONS

the value of each of these 8 determinant
 $\begin{bmatrix}\n\cos A & \cos B & \$ S AND DETERMINANTS **1988**
 $\begin{array}{ccc}\n\text{S AND DETERMINANTS} & \text{Q.B.-SOLUTIONS} & \text{SUSY} \\
\text{This determinant can be written as 8 d\n the value of each of these 8 determinant \\
\text{When } a \neq 0, b \neq 0, c \neq 0\n\end{array}$

When $a \neq 0, b \neq 0, c \neq 0$
 $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 2 & 1 \\ 4 & 21 \\ 4 & 21 \\ 4 & 21 \end{bmatrix} = (4 \times 3 - 1 \times$ **DETERMINANTS**
 O.B.-SOLUTIONS
 o 0)
 o. (1/a 0 0)
 o. (1/b 0)
 e.g.. $\cos P \cos Q \cos R$
 $\cos C$
 $\cos C$
 o. (31)
 (A). Given determinant can be w **ND DETERMINANTS**

(a 0 0)

(a 0 0)

(a 0 0)

(a + 0 0)

(a + 0 0 0)

(a + 0 0 0 1/c)

(a + 0 0 0 1/c)

(a 0 0 0 1/c)

(a 0 0 0 1/c)

(a 0 0 1/c)

(a 0 0 1/c)

(b 0 0 1/c)

(e.g., cos Pcos Qcos R cos B

(cos A cos B

(cos **ESAND DETERMINANTS**
 $\begin{array}{ccc}\n\textbf{1} & \textbf{1} & \textbf{1} & \textbf{1} & \textbf{1} \\
\textbf{1} & \textbf{1} & \textbf{1} & \textbf{1} & \textbf{1} \\
\textbf{2} & \textbf{3} & \textbf{2} & \textbf{1} & \textbf{1} \\
\textbf{3} & \textbf{4} & \textbf{5} & \textbf{6} & \textbf{6} \\
\textbf{5} & \textbf{6} & \textbf{7} & \textbf{6} & \textbf{7} \\
\textbf{6} & \textbf{7} & \$ **ND DETERMINANTS**
 (O.B.-SOLUTIONS)

This determinant can be writted
 $\begin{bmatrix}\n a & 0 & 0 \\
 0 & b & 0 \\
 0 & 0 & c\n\end{bmatrix}, A^{-1} = \begin{bmatrix}\n 1/a & 0 & 0 \\
 0 & 1/b & 0 \\
 0 & 0 & 1/c\n\end{bmatrix}$
 $= (4 \times 3 - 1 \times 2) = 12 - 2 = 10$
 $\begin{bmatrix}\n a_{11} & a_{12} \\
 a_{21} & a_{22$ **CESAND DETERMINANTS**
 O.B. SOLUTIONS

(O.B. SOLUTIONS)

This determinant can be written as 8 determinants are
 $\left\{\begin{array}{l}\text{so } 0 & 0 \\ \text{0} & 0 \\ \text{0} & 0 \end{array}\right\} \times 1^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/b \end{bmatrix}$

When $a \ne$ ∴ if A = $\begin{bmatrix} a_{11} & a_{12} \ a_{21} & a_{22} \end{bmatrix}$, then
 $|A| = \begin{vmatrix} a_{11} & a_{12} \ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22} - a_{12}a_{21})$ **(27) (A).** Determinant = $\begin{bmatrix} 0x + 2 & 9x + 3 & 12x \\ 0 & 0 & 12x \end{bmatrix}$, **MINANTS**
 (Q.B.-SOLUTIONS)
 $x^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$ This determinant can be written as 8 determinants as $x = 0$;
 $x^2 + 0 = 12 - 2 = 10$

(4 x 3 - 1 x 2) = 12-2 = 10

(31) (A). Given determinants **ENIEVANTS**
 EXECUTIONS
 $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$
 $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$
 $= (4 \times 3 - 1 \times 2) = 12 - 2 = 10$
 EXECUTE SURE A COS A COS A COS A COS A
 $\cos B$ cos A cos A cos **IINANTS**
 IDENTIFY
 IDENTIFY
 IDENTIFY
 $\begin{pmatrix} 1/\pi & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/b \end{pmatrix}$
 IDENTIFY
 IDENTIFY
 $\begin{pmatrix} 1/\pi & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/b \end{pmatrix}$
 IDENTIFY
 IDENTIFY
 IDENTIFY
 IDENTIFY
 IDENTIFY IINANTS
 IEVANTS
 $\begin{bmatrix}\n\frac{1}{4} & \frac{1}{4} & 0 & 0 \\
0 & 1/b & 0 & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{4} & 0 & 0 \\
0 & 1/b & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{4} & 0 & 0 \\
0 & 1/b & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 $\begin{bmatrix}\n\frac{1}{4} & 0 & 0 \\
0 & 0 & 1/c\n\end{bmatrix}$
 MINANTS
 (O.B.-SOLUTIONS

This determinant can be written as 8 determinants is
 $e^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

This determinants can be written as 8 determinants is
 $e.g., \cos P \cos Q \cos R$ dos A cos A co
 (D). If $A = \begin{bmatrix} a & 0 & 0 \ 0 & b & 0 \ 0 & 0 & 0 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \ 0 & 1/b & 0 \ 0 & 0 & 0 \end{bmatrix}$

When $a \ne 0$, $b \ne 0$, $c \ne 0$, $c \ne 0$
 (D). $|A| = \begin{vmatrix} 4 & 1 \ 2 & 3 \ 2 & 1 \end{vmatrix} = (4 \times 3 - 1 \times 2) = 12 - 2 = 10$
 (D). $|$ Lif A = $\begin{bmatrix} 0 & b & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $K^{-1} = \begin{bmatrix} 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

When $a \ne 0$, $b \ne 0$, $c \ne 0$

When $a \ne 0$, $b \ne 0$, $c \ne 0$

When $a \ne 0$, $b \ne 0$, $c \ne 0$

When $a \ne 0$, $b \ne 0$, $c \ne 0$

When $a \ne 0$, (a 0 d) (a 0 l(e)
 $\left.\frac{1}{2}$
 $\left.\frac{1}{3}\right|$ = (4 x 3 - 1 x 2) - 12 - 2 = 10

(31) (A). Given determinants can be show
 $f A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then
 $\left.\begin{bmatrix} 4 & 1 \\ a_{21} & a_{22} \\ a_{22} & a_{22} \end{bmatrix}\right|$, the **AND DETERMINANTS**
 AND DETERMINANTS
 $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/b \end{bmatrix}$
 $= (4 \times 3 - 1 \times 2) = 12 - 2 = 10$
 $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then
 $A = \begin{bmatrix} a_{11}$ A = $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, A⁻¹ = $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$
 $\begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 1/c \\ 0 & 0 & 1/c \end{bmatrix}$
 $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$
 $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1$ $= \begin{vmatrix} 12x+1 & 14x+3 & 50x+10 \\ 18x+2 & 21x+3 & 75x+9 \\ 0 & -1 & 0 \end{vmatrix} \begin{array}{l} C_1 \leftarrow C_3 + C_1 \\ C_2 \leftarrow C_3 + C_2 \\ C_3 \leftarrow 4C_3 + 3C_2 \end{array}$ $=-97x-11.$ So that $x=\frac{-11}{97}$. 11 **(28) (D).** Given $A = \begin{vmatrix} \sin (\theta + \alpha) & \cos (\theta + \alpha) & 1 \\ \sin (\theta + \beta) & \cos (\theta + \beta) & 1 \\ \sin (\theta + \gamma) & \cos (\theta + \gamma) & 1 \end{vmatrix}$ 1 x if A = $\begin{bmatrix} \sin(6) = 1 & x & x+1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

(A) Determinant = $\begin{bmatrix} 4x & 6x+2 & 8x+1 \\ 8x & 2 & 2 \end{bmatrix}$

(A) Operation = $\begin{bmatrix} 4x & 6x+2 & 8x+1 \\ 8x+2 & 9x+3 & 12x \\ -3 & -4 & 3 \end{bmatrix}$,

(A) Operation = $\begin{b$ $\begin{vmatrix} 1 & 0 & -1 \ 1 & 0 & 0 \ 2 & 2 & 0 \end{vmatrix}$
 $\begin{vmatrix} 1 & 0 & 0 \ 2 & 1 & 0 \ 3 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 4x + 2 & 8x + 1 \ 3 & 2 & 4 \ 3 & 3 \end{vmatrix}$

(A). Determinant = $\begin{vmatrix} 4x + 2 & 8x + 1 \ 6x + 2 & 8x + 1 \end{vmatrix}$
 $\begin{vmatrix} 4x + 2 & 1 \ 3x + 3 & 7 \ 5x + 4 &$ (A). Determinant = $\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 8x+2 & 9x+3 & 12x \\ -3 & -4 & 3 \\ 0 & -1 & 0 \end{vmatrix}$
 $\begin{vmatrix} 1 & x & 1 \\ 2x+1 & 14x+3 & 50x+10 \\ 13x+2 & 21x+3 & 75x+9 \\ 0 & -1 & 0 \end{vmatrix}$
 $C_1 + C_2 + C_3 + C_1$
 $C_2 + C_3 + C_2$
 $C_3 + C_4 + C_5 + C_1$

(D). Give $\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 8x+2 & 9x+3 & 12x \\ -3 & -4 & 3 \end{vmatrix}$, $\begin{vmatrix} 1 & x & 1 \\ 2x & x-1 & x \\ -3x & -1 & (x-1)(x-2) & x(x-1) \end{vmatrix}$
 $R_3 \leftarrow R_3 + R_1 - 2R_2$
 $\begin{vmatrix} 3 & 50x+10 \\ 2x+6x+6x+2 \\ -3x+6x+3x-2 \end{vmatrix}$ = $x(x+1)(x-1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x$ (A). Determinant = $\begin{vmatrix} 6x+2 & 9x+3 & 12x \\ -3 & -4 & 3 \\ 18x+2 & 21x+3 & 50x+10 \end{vmatrix}$ = $x(x+1)$ = $x(x-1)$ (x-1) (x-1) (x-2) $x(x-1)$ = $x(x-1)(x-2)$ $x(x-1)$ = $x(x-1)(x-2)$ $x(x-1)$ = $x(x-1)(x-2)$ $x(x-1)$ = $x(x-1)(x-2)$ $x(x-1)$ = $x(x$ which is independent of θ . $R_3 \leftarrow R_3 + R_1 - 2R_2$
 $= \begin{vmatrix} 12x + 1 & 14x + 3 & 50x + 10 \ 18x + 2 & 21x + 3 & 75x + 9 \end{vmatrix}$ C₂ + C₃ + C₃
 $= 0$
 $= -97x - 11$. So that $x = \frac{-11}{97}$.

(28) (D). Given A = $\begin{vmatrix} \sin(\theta + \alpha) & \cos(\theta + \alpha) & 1 \ \sin(\theta + \alpha) & \cos(\theta + \alpha) & 1$ 2 2 1 a 2a cosdx a a So that $x = \frac{-11}{97}$.

So that $x = \frac{-11}{97}$.
 $\begin{vmatrix} \sin(0+ \alpha) & \cos(\theta + \alpha) & 1 \\ \sin(\theta + \beta) & \cos(\theta + \beta) & 1 \end{vmatrix}$
 $\begin{vmatrix} \sin(\theta + \alpha) & \cos(\theta + \beta) & 1 \\ \sin(\theta + \gamma) & \cos(\theta + \beta) & 1 \end{vmatrix}$
 $\begin{vmatrix} \sin(\theta + \alpha) & \cos(\theta + \beta) & 1 \\ \cos(\theta + \alpha) & \sin(\theta + \gamma) & \cos(\theta + \gamma) \end{vmatrix$ So that $x = \frac{-11}{97}$.
 $\begin{vmatrix} \sin (\theta + \alpha) & \cos (\theta + \alpha) & 1 \\ \sin (\theta + \beta) & \cos (\theta + \beta) & 1 \\ \sin (\theta + \gamma) & \cos (\theta + \gamma) & 1 \end{vmatrix}$
 $= x(x+1)(x-1)\begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x & -2 & x \end{vmatrix}$
 $\Rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$
 $\therefore f(x) = 0 \Rightarrow f(100) = 0$
 $\Rightarrow R_2 - R_1, R_3 \rightarrow R_3 -$ +1 14x + 3 30x +10

+2 21x +3 75x + 0

-2 11. So that x = $\frac{-11}{97}$.

x = $\frac{1}{27}$
 $\begin{vmatrix} \sin(0+ \beta) & -\sin(0+ \alpha) & -\sin(0+ \beta) & -\sin(0+ \alpha) & -\sin$ $\Delta = \begin{vmatrix} 1 + a^2 - 2a \cos dx & a & a^2 \\ 0 & \cos px & \cos (p + d)x \\ 0 & \sin px & \sin (p + d)x \end{vmatrix}$ $= (1 + a² – 2a cos dx) sin dx$, (which is independent of p). 97

(1 + a² - 2 a cos (θ + α) is in (θ + α) cos (θ + β) 1

(33) (D). Δ_i $\begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & -2 \\ 0 & 0 & 1 \end{vmatrix}$

(1 + a² - 2 a cos (θ + α)) + (cos(θ + β)

(32) (C). Each term in A₁ × A₂ is the sum of (θ + β) **(30) (A).** The determinant can be expanded as Dependent $P_2 \rightarrow R_2 - R_1R_3 \rightarrow R_3 - R_1$
 $\sin(\theta + \beta) - \sin(\theta + \alpha)$) $-\cos(\theta + \alpha)$
 $\sin(\theta + \beta) - \sin(\theta - \alpha)$
 $-\cos(\theta + \alpha)$ (32) (C). Each term in A₁ × A₂ is the sum of three terms. So
 $\sin(\theta + \beta) - \sin(\beta - \alpha) - \sin(\alpha - \gamma)$
 $-\cos(\theta + \alpha)$ {sin (θ + berate R₂ \rightarrow R₂ - R₁, R₃ - R₃ - R₁
 $\sin(\theta + \beta) - \sin(\theta - \alpha) - \cos(\theta + \alpha))$
 $\sin(\theta + \beta) - \sin(\theta - \alpha) - \sin(\alpha - \gamma)$
 $\sin(\theta + \beta) - \sin(\theta - \alpha) - \sin(\alpha - \gamma)$
 $\sin(\theta - \gamma) - \sin(\beta - \alpha) - \sin(\alpha - \gamma)$
 $\sin(\theta - \gamma) - \sin(\beta - \alpha) - \sin(\alpha - \gamma)$

Anche is independent : A = {cos(θ + y) - cos(θ + α)}

sin(θ + β) - sin(θ + α)} - {cos(θ + β)

sin(θ + β) - sin(θ + α)} {sin(θ + γ) - sin(θ + α)}

= - cos(θ + α)} {sin(θ + γ) - sin(θ + α)}

sin(θ - γ) - sin(θ - γ) - sin(θ + γ) - sin(θ + γ) -Nally $\sin(\theta + y) \cos(\theta + y)$
 $\Rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$
 $\Rightarrow (6 + y) \cos(\theta + \alpha)$
 $\sin(\theta + \alpha) - \cos(\theta + \alpha)$
 $\sin(\theta + \alpha) - \cos(\theta + \alpha)$
 $\cos(\theta + \alpha) + \sin(\theta + \alpha)$
 $\Rightarrow \cos(\theta + \alpha) + \sin(\theta + \alpha)$
 $\Rightarrow \cos(\theta + \alpha) + \sin(\theta + \alpha)$
 $\Rightarrow \cos(\theta + \alpha) + \sin(\theta + \alpha)$
 $\Rightarrow \cos(\theta + \alpha) + \sin(\theta + \alpha)$ (sin(0+r) cos (0+r) + |

= x(x+1)(x-1)[-2x+2x] = 0

→ R₂ - R₁, R₃ → R₃ - R₁

= x(x+1)(x-1)[-2x+2x] = 0

= x(x+1)(x-1)[-2x+2x] = 0

= cos(0+α)} (30) (C). Each term in A₁ × A₂ is the sum of three terms.

= c → R₂ - R₁, R₃ → R₃ - R₁

(θ + γ) - cos(θ + α)}

sin(θ + α) - {cos(θ + β)

- cos(θ + α)}

- cos(θ + α)}

(32) (C). Each term in $\Delta_1 \times \Delta_2$ is the sum of three terms.

- sin(θ - α) - sin(θ + γ) - sin(θ + γ) - s color-p)
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\sin(\theta + \gamma) - \sin(\theta + \alpha)\right\}$
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\sin(\theta + \gamma) - \sin(\theta + \alpha)\right\}$
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\sin(\theta + \gamma) - \sin(\theta + \alpha)\right\}$
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\cos(\theta + \alpha)\right\}$
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\cos(\theta + \alpha)\right\}$
 $\left(\frac{\partial}{\partial x}\right)^n \left\{\cos(\theta + \$ cos Beos R + sin Asin R

cos Bcos R + sin Bsin R

cos Ccos R + sin Bsin R

cos Cos R + sin Csin R

co $(\alpha - \gamma)$
 $(\alpha + \gamma)$
 $\cos A \cos R + \sin A \sin R$ $\cos B \cos R + \sin B \sin R$ $\cos C \cos R + \sin C \sin R$

This determinant can be written as 8 determinants and the value of each of these 8 determinants is zero;

S
\n**o_{DMADVANCED LEARNING}**
\nThis determinant can be written as 8 determinants and
\nthe value of each of these 8 determinants is zero;
\n**e.g.,**
$$
cos P cos Q cos R
$$
 $\begin{vmatrix} cos A & cos A & cos A \\ cos B & cos B & cos B \\ cos C & cos C & cos C \end{vmatrix} = 0$
\nSimilarly other determinants can be shown zero.
\n**(A).** Given determinant
\n
$$
f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 2x & x(x-1) & (x+1)x \end{vmatrix}
$$

Similarly other determinants can be shown zero. **(31) (A).** Given determinant

DDETERMINANTS
\na 0 0 b 0, A⁻¹ =
$$
\begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/x & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$
, A⁻¹ = $\begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/x & 0 \\ 0 & 0 & 1/x \end{bmatrix}$
\n(b) b 0, c ≠ 0
\n⇒ 0, b ≠ 0, c ≠ 0
\n⇒ 0, d → 0
\n⇒ 0, d → 0
\n⇒ 0, e. g. cos P cos Q cos R
\n $\begin{vmatrix} 6x + 1 & 8x - 1 & 8x - 1 & 1x \\ 3 & 1 & 2 & 2 \\ 1 & 1 & 8 & 2 \\ 1 & 1 & 8 & 2 \end{vmatrix}$
\n $\begin{vmatrix} 4x & 6x + 2 & 8x + 1 \\ 1 & 4x & 2 \\ -3 & -4 & 3 \end{vmatrix}$
\n $\begin{vmatrix} 4x & 6x + 2 & 8x + 1 \\ -3 & -4 & 3 \\ -1 & -4 & 3 \end{vmatrix}$
\n $\begin{vmatrix} 1 & x & x + 1 \\ 1 & x & x + 1 \\ 2x & x & -1 & x \\ 3x(x - 1) & (x - 1)(x - 2) & (x + 1)x(x - 1) \end{vmatrix}$
\n $\begin{vmatrix} 1 & x & 1 \\ 1 & x & 1 \\ 3x(x - 1) & (x - 1)(x - 2) & x(x - 1) \end{vmatrix}$
\n $\begin{vmatrix} 1 & x & 1 \\ 1 & x & 1 \\ 3x(x - 1) & (x - 1)(x - 2) & x(x - 1) \end{vmatrix}$
\n $\begin{vmatrix} 1 & x & 1 \\ 2x & x - 1 & x \\ 3x(x - 1) & (x - 1)(x - 2) & x(x - 1) \end{vmatrix}$
\n $\begin{vmatrix} 1 & 1 & 1 \\ 2x & x - 1 & x \\ 3x(x - 1) & (x - 1)(x - 2) & x(x - 1) \end{vmatrix}$

each entry in C₁ or C₂ or C₃ in Δ_1 $3 \times 3 \times 3 = 27$ determinants.

$$
= x(x+1)(x-1) \begin{vmatrix} x & -1 & x \\ 2x & -2 & x \end{vmatrix}
$$

\n
$$
= x(x+1)(x-1)[-2x+2x] = 0
$$

\n
$$
\therefore f(x) = 0 \Rightarrow f(100) = 0
$$

\n(32) (C). Each term in $\Delta_1 \times \Delta_2$ is the sum of three terms. So each entry in C₁ or C₂ or C₃ in $\Delta_1 \times \Delta_2$ is the sum of three terms. Hence, $\Delta_1 \times \Delta_2$ can be written as the sum of 3 × 3 × 3 = 27 determinants.
\n(33) (D). $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C[C^2 -1]-1[C-6] \therefore C = 2\cos\theta$
\n
$$
\Rightarrow \Delta = 2\cos\theta(4\cos^2\theta - 1) - (2\cos\theta - 6)
$$

\n
$$
\Rightarrow \Delta = 8\cos^3\theta - 4\cos\theta + 6
$$

\n(34) (B). $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx \Rightarrow \frac{d}{dx}\Delta_1 = 3(x^2 - ab)$
\n
$$
\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab \Rightarrow \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2
$$

$$
(34) \quad (B). \ \Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx \implies \frac{d}{dx} \Delta_1 = 3(x^2 - ab)
$$

$$
\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab \Rightarrow \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2
$$

(35) (C). Determinant

$$
\begin{array}{ccc}\n1 & 1 & 1 \\
b+c & c+a & a+b \\
-(a+b+c) & -(a+b+c) & -(a+b+c)\n\end{array}
$$

Applying $[R_3 - 2R_2]$, We get

$$
= -(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ 1 & 1 & 1 \end{vmatrix} = 0
$$

(36) (D). Applying $C_1 - (C_2 + C_3)$ we get $\begin{vmatrix} 6 & 6 & 7 \\ 3 & 3 & 15 \end{vmatrix}$

$$
Det = \begin{vmatrix} 3 & 3 & 13 \\ 11 & 11 & 6 \end{vmatrix} = 0 \quad (:.C_1 = C_2)
$$

(B). Writing the given determinant as the

(37) (B). Writing the given determinant as the sum of two determinants, we have

Det =
$$
\begin{vmatrix}\n37 & 4B \\
11 & 11 & 6 \\
12 & 12 & 12\n\end{vmatrix} = 0 \quad (\therefore C_1 - C_2)
$$

\n33. 10.2
\n34.31.
$$
\begin{vmatrix}\n3a & 3b & c \\
-c & b & c \\
-1 & 2b & 3c \\
-1 & 2b & 2c\n\end{vmatrix} = 0
$$

\n34.4
\n35.5
\n46.6
\n47.6 (A) (C). Applying
$$
R_2 - xR_1, R_3 - xR_2
$$
 then

\n47.6 (A) (B) A =
$$
\begin{vmatrix}\n1 & 2 & 1 \\
-1 & 2 & -1 \\
-1 & 2 & 2\n\end{vmatrix} = 0
$$

\n5. 6 (A)
$$
\begin{vmatrix}\n3a & x & p \\
a & x & p \\
a & x & p\n\end{vmatrix} = 0
$$

\n6. 10.42
\n7. 22.
$$
\begin{vmatrix}\n1 & 2 & 2 \\
1 & 2 & 2\n\end{vmatrix} = 0
$$

\n6. 22.
$$
\begin{vmatrix}\n1 & 2 & 2 \\
-1 & 2 & 3\n\end{vmatrix} = 0
$$

\n7. 23.
$$
\begin{vmatrix}\n3x & x^2 & 1 \\
x & x^2 & 1 \\
x & x^2 & 2\n\end{vmatrix} = 0
$$

\n8.
$$
\begin{vmatrix}\n3x & x^2 & 1 \\
x & x^2 & 2\n\end{vmatrix} = 0
$$

\n9.
$$
\begin{vmatrix}\n3x & x^2 & 1 \\
x & x^2 & 2\n\end{vmatrix} = 0
$$

\n10.
$$
\begin{vmatrix}\n3a & b & c \\
a & b & c \\
a & c & 2\n\end
$$

(39) (D). $\begin{vmatrix} x & 2y & 2 \\ p & 5 & 5 \end{vmatrix} = \begin{vmatrix} 30 & 2y & 5 \\ 2 & 2 & 5 \end{vmatrix}$

[changing rows into columns] (46)

$$
=\frac{1}{3}\begin{vmatrix} 3a & x & p \\ 3b & 2y & 5 \\ 3c & 3z & 15 \end{vmatrix} = \frac{3}{3} \times \frac{1}{5} \begin{vmatrix} a & 5x & p \\ b & 10y & 5 \\ c & 15z & 15 \end{vmatrix} = \frac{1}{5} (125) = 25
$$

(40) (C). Applying $R_2 - xR_1$, $R_3 - xR_2$ then

$$
f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & 0 & a-x \end{vmatrix} = a(a+x)^2
$$

35.6. Determine the given determinant is the sum of two
\n
$$
\begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
-1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_2 = C_3
$$
\n
$$
C_3 = C_4 + C_5
$$
\n
$$
C_4 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
-1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_2 = C_3
$$
\n
$$
C_3 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
-1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_1 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
-1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_1 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
-1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_1 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_3 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_4 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_4 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_3 = \begin{vmatrix}\n1 & 1 & 1 \\
-1 & 1 & 1 \\
1 & 1 & 1\n\end{vmatrix} = 0 \qquad \therefore C_1 = C_2
$$
\n
$$
C_3 = \begin{
$$

 $= 2 + x - 4 - x = -2.$ **(45) (A).** Splitting the determinant into two determinants, we

$$
\begin{vmatrix} 1 & 2 & 2 \end{vmatrix} \qquad \begin{vmatrix} 1 & 2 & 2 \end{vmatrix}
$$

get
$$
\Delta = \begin{vmatrix} 1 & a & a \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0
$$

Because a, b, c are different, the second factor cannot be zero. Hence, option (A) , $1 + abc = 0$, is correct.

.

(B). Since it is an identity in λ so satisfied by every value of λ . Now put $\lambda = 0$ in the given equation, we have

$$
= \frac{1}{5} (125) = 25
$$

$$
t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -4 \\ -3 & 4 & 0 \end{vmatrix} = -12 + 30 = 18
$$

(47) (B). Put $x = 1$, then we have

$$
\begin{bmatrix}\n- \begin{bmatrix}\n-3 & -1 & x+2\n\end{bmatrix}, & 0 & \begin{bmatrix}\n0 & 0 & x+2\n\end{bmatrix}, & 0 & 0 & 0 \\
-3 & -2 & -2 & -2 & -2 \\
-2 & -3 & -4 & -x & -2\n\end{bmatrix}
$$
\n**(A).** Splitting the determinant into two determinants, we get\n
$$
\Delta = \begin{vmatrix}\n1 & a & a^2 \\
1 & b & b^2 \\
1 & c & c^2\n\end{vmatrix} + abc \begin{vmatrix}\n1 & a & a^2 \\
1 & b & b^2 \\
1 & c & c^2\n\end{vmatrix} = 0
$$
\nBecause a, b, c are different, the second factor cannot be zero. Hence, option (A), 1 + abc = 0, is correct.\n**(B).** Since it is an identity in λ so satisfied by every value of λ . Now put $\lambda = 0$ in the given equation, we have\n
$$
t = \begin{vmatrix}\n0 & -1 & 3 \\
1 & 2 & -4 \\
-3 & 4 & 0\n\end{vmatrix} = -12 + 30 = 18
$$
\n**(B).** Put $x = 1$, then we have\n
$$
\begin{vmatrix}\n2 & 2 & -1 \\
4 & 3 & 0 \\
6 & 1 & 1\n\end{vmatrix} = A - 12 \Rightarrow \begin{vmatrix}\n0 & 2 & -1 \\
1 & 3 & 0 \\
5 & 1 & 1\n\end{vmatrix} = A - 12
$$

{Apply C C C 1 1 2 } 2 (1)(14) A 12 A 24 . **(48) (A).** Apply R² – R³ and note that 2 2 (x y) (x y) 4xy 2 2 2 2 2 2 a b c 4 a b c ⁼ 2 2 2 a b c 4 a b c 1 1 1 **(49) (A).** We have ⁼ 2 2 2 ⁼ 2 2 2 2 2 2 2 abc Applying C (a),C (b),C (c) 1 2 3 2 2 2 2 abc . **(50) (C).** 2 2 2 2 2 2 2 2 2 4a b c Ka b c , (given) K = 4. **(51) (C).** a 2b 2c 3 b c 0, 4 a b [R³ R¹ – 2R²]

(a 1) (b 1) (c 1) {Applying R (R 2R)} 3 1 2 . 2 2 2 1 1 1 2 a b c a bc b ac c ab 1 1 1 1 1 1 2 a b c 2 a b c bc ac ab a b c a b c 1 1 1 2 a b c a b c abc abc abc a b c 2 2 2 a b c 1 1 1 2 a b c (abc) a b c 0 1 1 1 a b c a ab ac a b c ab b bc abc a b c a b c ac bc c 2 2 2 1 1 1 (abc)(abc) 1 1 1 a b c (1)(4) 1 1 1 a 6 0 0 3 b c 0 4 a b 2 = 9 = ± 3 2 2 (a 6)(b ac) 0 b ac 0 (^a 6) 2 3 ac b abc b . **(52) (B).** Let a,b,c are in G.P. and assume a = 1, b = 2, c = 4 1 2 3 A 2 4 6 0 3 6 0 . **(53) (A).** 2 2 ² 1 1 1 1 3() ¹ 1 3i 1 3i 3 3 3 i 2 2 . **(54) (C).** 1 1 1 bc ca ab b c c a a b ⁼ 0 0 1 c(b a) a(c b) ab b a c a a b 1 1 2 2 2 3 {C C C ,C C C } ⁼ 0 0 1 (b a) (c b) c a ab 1 1 a b ⁼ (b a)(c a) (c a) = (a – b) (b – c) (c – a) **(55) (C).** 2 2 2 a b c b c a a(bc a) b(b ca) c(ab c) c a b ⁼ 3 3 3 a b c 3abc⁼ 3 3 3 1 [a b c 3abc] ⁼ 2 2 2 [(a b c)(a b c ab bc ca)] k = –1. **(56) (B).** = cos 15 cos 75 – sin 15 sin 75 = cos (15 + 75)= cos 90 **(57) (A).** Using C³ C³ – (C¹ + C²), D¹ ⁼ ^a ^b ^a ^b c d c d a b a b and D² ⁼ ^a ^c ^a ^b ^c b d b d a c a c 2 1 D D ⁼ b(ad bc) 2b(ad bc) = – 2 **(58) (D).** C¹ C¹ + C³ D = 0 0 1 1 1 1 x 1 x x 1 x y = –1[x – 1 – x] = 1 **(59) (C).** 1 [² – – 1] – ² [1 – 1 – ²] + (1 –) [² – 1] = ² – – 1 + + ² – 1 – 1 + = ² – 3 + + ² = ² – 4 **(60) (C).** | A³ | = |A|³ = 125 = 5³ | A | = 5 a² – 4 = 5

**EXAMPLEMATEMATE RIAL: MATIERAL: MATIERAL: MATIERMATERAL: MATIERMATERAL: MATIERMATERAL: MATIERMATERAL: MATHEMATICS
\n**(61)** (b) | adj A| = |A|ⁿ⁻¹
\n25 = |A|² \Rightarrow |A| = ±5 : |A^{-1}| =
$$
\frac{1}{|A|} = \pm \frac{1}{5} = \pm 0.2
$$

\n26 (c). Since A is orthogonal :: $AA^T = I$
\n36 (d) (e). $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$
\n $\Rightarrow k^2 \begin{vmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 2$
\n $\Rightarrow k^2 \begin{vmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 2$
\n $\Rightarrow k^2 \begin{vmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 2$
\n $\Rightarrow k^2 \begin{vmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 2$
\n $\Rightarrow k^2 \begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 2$
\n $\Rightarrow k^2 \begin{vmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ 2+4-2 & 4-2 & 4-2 \end{vmatrix}$
\n**(64)** (A). $\Delta = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$
\n $\Rightarrow (\alpha-1)^2(\alpha+2) = 0; \quad \alpha = 1, -2$
\n(b) The given system of homogeneous equations has a
\n**(65)** (D). Given set of equations will have a non trivial solution
\n**(66)** (D). Given set of equations will have a non trivial solution
\n**(67)** (N) $\alpha = 3$
\n $\begin{vmatrix} 1 & 1 & -1 \\$**

i.e.,
$$
\begin{vmatrix} 1 & 1 & 1 \\ 3 & -\alpha & -3 \\ 1 & -3 & 1 \end{vmatrix} = -2\alpha - 6 = 0
$$
, i.e. if $\alpha = -3$.

(66) (D). Given set of equations will have a non trivial solution if the determinant of coefficient of x, y, z is zero

i.e.,
$$
\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow 2k - 33 = 0
$$
 or $k = \frac{33}{2}$.

EXERCISE

(1) (B). $f(\alpha) f(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$ i.e., $\begin{vmatrix} 3 & -\alpha & -3 \\ 1 & -3 & 1 \end{vmatrix} = -2\alpha - 6 = 0$, i.e. if $\alpha = -3$.

(D). Given set of equations will have a non trivial solution

(D). Given set of equations will have a non trivial solution

if the determinant of coeffi 1 -3 1 |

Siven set of equations will have a non trivial solution

determinant of coefficient of x, y, z is zero

1 k 3

3 k -2 = 0 = 2k -33 = 0 or k = $\frac{33}{2}$.

Since $|A| \ne 0$. Hence unique solution.

(4) (C). Her ... $\begin{vmatrix} 1 & 1 & -1 \\ 1 & -3 & 1 \end{vmatrix} = -2\alpha - 6 = 0$, i.e. if $\alpha = -3$.

(AB)₁₁ = 8-7 = 1, (BA)₁₁ = 8-7 = 1, (BA)₁₁ = 8-7 = 1, (BA)₁ = 8

(AB ≠ BA may be not tru Similarly $f(\alpha) f(\beta) f(\gamma)$ if the determinant of coefficient of x, y, x is zero

i.e., $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow 2k - 33 = 0$ or $k = \frac{33}{2}$.

(A). Here $|A| \neq 0$. Hence unique solution.

(B). I(α) I(β) = $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \$ 1 k 3
 $\frac{1}{3}$ k -2
 $\frac{1}{2}$ = 0 = 2k - 33 = 0 or k = $\frac{33}{2}$.
 $\frac{1}{2}$ = 0 = 2k - 33 = 0 or k = $\frac{33}{2}$.
 $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{2}{3}$.
 $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{1}{2}$
 $\frac{1}{2}$ = $\frac{2}{2$ 0. Given set of equations will have a non trivial solution

Now AB = (-7 a) (7 2)

he determinant of coefficient of x, y, z is zero
 $\begin{vmatrix} 1 & k & 3 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow 2k - 33 = 0$ or $k = \frac{33}{2}$.

D. Here $|A| \ne 0$. Hen y. Uncertainty fix of the determinant of coefficient of x, y, z is zero
 $\left[\begin{array}{ccc|c} 1 & k & 3 \ 2 & 3 & -4 \end{array}\right] = 0 \Rightarrow 2k - 33 = 0$ or $k = \frac{33}{2}$.
 $\left[\begin{array}{ccc|c} 1 & k & 3 \ 2 & 3 & -4 \end{array}\right] = 0 \Rightarrow 2k - 33 = 0$ or $k = \frac{33}{2}$.
 $\left[\begin{array}{$ (A). Here $|\mathbf{A}| \neq 0$. Hence unique solution.
 $|\mathbf{A}| \neq 0$. Hence unique solution.
 $(\mathbf{A}) \cdot \text{Here } |\mathbf{A}| \neq 0$. Hence unique solution.
 $\mathbf{EXERCISE-2}$
 $(\mathbf{B}) \cdot \mathbf{f}(\alpha) \cdot \mathbf{f}(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix$ 2 3 -4|

Here $|A| \ne 0$. Hence unique solution.
 $\begin{bmatrix}\n\cos \alpha & \sin \alpha \\
2 & 1 & 2\n\end{bmatrix}$
 $\begin{bmatrix}\n\cos \alpha & \sin \alpha \\
2 & 2 & 1\n\end{bmatrix}$
 $\begin{bmatrix}\n\cos \alpha & \sin \alpha \\
2 & 1 & 2\n\end{bmatrix}$

(a) (f) $\begin{bmatrix}\n\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha\n\end{bmatrix}\begin{bmatrix}\n\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta$ -1 k 3

-1 k 3

-2 a -4

-2 a -2 + 28 -7+8)⁼(0 1)

-1 -28+28 -7+8)⁼(0 1)

-1 -28+28 -7+8)⁼(0 1)

-1 -38+28 -7+8)⁼(0 1)

-1 -5inφ cost sinφ -

-3inφ cost sinφ -

-3inφ cost sinφ -

-sinφ cost sinφ -

-sinφ cost $\frac{1}{2}$ 3 k -2 = 0 ⇒ 2k - 33 = 0 or k = $\frac{33}{2}$.

A Here | A | ≠ 0 . Hence unique solution.

(4) (C). Here $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$
 \therefore EXERCISE-2

D. f(α) f(β) = $\begin{bmatrix} \$ $=\begin{bmatrix} \cos \pi & \sin \pi \\ -\sin \pi & \cos \pi \end{bmatrix}$ as $\alpha + \beta + \gamma = \pi$ (A). Here $|A| \neq 0$. Here $\ln |\pm 0|$. Here $\ln |\cos \theta| = \left[\cos \alpha + \sin \beta\right]$

(B). $f(\alpha) f(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$
 $= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \end{bmatrix}$
 $= \begin{bmatrix} \cos \alpha$ EXERCISE-2

f(α) f(β) = $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$

= $\begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$

cos α cos β – sin α cos β – sin α sin β – sin α b. Here | A | ≠ 0 . Hence unique solution.
 $\mathbf{f}(\alpha) \hat{\mathbf{f}}(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \alpha & \cos \alpha \end{bmatrix}$
 $\mathbf{F}(\alpha) \hat{\mathbf{f}}(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$ b. Here | A | ≠ 0 . Here unique solution.

EXERCISE-2

b. f(α) f(β) = $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \alpha & \cos \alpha \end{bmatrix}$
 $= \sin \alpha \cos \beta - \sin \alpha \sin \beta - \cos \alpha \sin \beta + \cos \alpha \cos \beta$
 $= \sin \alpha \cos \beta - \sin \alpha \sin \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta$
 $= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -I_2$

(2) **(A).** Let $A = k \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$ $\therefore A^{T} = k \begin{bmatrix} 2 & -1 \end{bmatrix}$ TUDY MATERIAL: MATHEMATICS

1 2 2

2 -1 2

2 -1 2

cond : AA^T = I

2 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

cond : AA^T = I

2 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ \therefore $A^{T} = k \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

gonal \therefore $AA^{T} = I$
 $2 \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$:: $A^T = k \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

gonal :: $AA^T = I$
 $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$:: $A^T = k \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

ogonal :: $AA^T = I$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ \therefore $A^T = k \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

ogonal \therefore $AA^T = I$
 $2 \text{ } T-1$ 2 2 $\overline{1}$ STUDY MATERIAL: MATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ \therefore $A^{T} = k \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

ogonal \therefore $AA^{T} = I$
 $2 \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$ THEMATICS
 $\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ THEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ ATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ ATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ ATHEMATICS
 $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

Since A is orthogonal \therefore AA^T = I

$$
\begin{array}{llll}\n\text{SVD} & \text{SHD} \text{ MATIEMA-1.} \text{ INHEMA-1.} \text{ INHEMA
$$

$$
= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \ 2+2+4 & 4+1+4 & 4+2+2 \ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \ 8 & 9 & 8 \ 8 & 8 & 9 \end{bmatrix}
$$

$$
A^{2}-4A = \begin{bmatrix} 9-4 & 8-8 & 8-8 \\ 8-8 & 9-4 & 8-8 \\ 8-8 & 8-8 & 9-4 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 5I
$$

$$
= \begin{pmatrix} 8-7 & 2-2 \ -28+28 & -7+8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \ 0 & 1 \end{pmatrix} ; (AB)^{T} = \begin{pmatrix} 1 & 0 \ 0 & 1 \end{pmatrix} = I
$$

(4) **(C)** Here $A^2 = \begin{bmatrix} 1 & 2 & 2 \ 2 & 1 & 2 \ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \ 2 & 1 & 2 \ 2 & 2 & 1 \end{bmatrix}$

$$
= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \ 2+2+4 & 4+1+4 & 4+2+2 \ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \ 8 & 9 & 8 \ 8 & 8 & 9 \end{bmatrix}
$$

$$
A^2 - 4A = \begin{bmatrix} 9-4 & 8-8 & 8-8 \ 8-8 & 9-4 & 8-8 \ 8-8 & 8-9-4 & 8-8 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & 0 & 1 \end{bmatrix} = 5I
$$

(5) **(C)** We have $(AB)_{11} = 1.3 + 2.1 = 5$
 $(BA)_{11} = 3.1 \ 4.3 = 15$
 $\therefore AB \neq BA$ Again $(A^2)_{11} = 1.1 + 2.3 = 6 \neq 3 = (B)_{11}$
Also $(AB)^{T} = B^{T}A^{T} = \begin{bmatrix} 3 & 1 \ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 3 \ 2 & 0 \end{bmatrix}$

$$
= \begin{bmatrix} 3+2 & 9+0 \ 4+12 & 12+0 \end{bmatrix} = \begin{bmatrix} 5 & 9 \ 16 & 12 \end{bmatrix}
$$
 is correct.

 (4)

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

| MATRICES AND DETERMINANTS | Q.B.S.OIMITOSS | Self comcomcreteness | |
|---|--|--|---|
| (6) (C). Det= $\begin{vmatrix}\nx & \frac{x(x-1)}{2} & \frac{x(x-1)(x-2)}{6} \\ x & \frac{d(x-1)}{2} & 6\n\end{vmatrix} = \frac{xy}{12}$ \n | 100 | | |
| (a) (C). Det= $\begin{vmatrix}\nx & x-1 & (x-1)(x-2) \\ y & y-1 & (y-1)(y-2) \\ z & z-1 & (z-1)(z-2)\n\end{vmatrix} = \begin{vmatrix}\ny+\frac{x}{7} & 9 & 1 \\ 9 & 11 & 1\n\end{vmatrix} = 0$ \n | [Applying R ₃ - R ₂ and R ₂ - R ₁ in second det] | | |
| $\begin{vmatrix}\nx & x-1 & (x-1)(x-2) \\ y & y-1 & (y-1)(y-2) \\ 1 & z & z^2\n\end{vmatrix} = \begin{vmatrix}\nx & x^2 \\ y & y^2 \\ z & z^2\n\end{vmatrix} = 0$ \n | [Applying R ₃ - R ₂ in a 10] (Applying R ₂ - C ₁]\n | | |
| $= \frac{xyz}{12}$ \n | [x - y)(y - z)(z - x) | [x - y - z)(z - x - z - 0] (z - x - 0 - 0) = -1 \n(z - z - 0) = 0 ⇒ 0 = 1 \n(z - 1) = 1 + 2 ++(n - 1) = $\frac{n(n - 1)}{2}$ \n | After solving R ₁ + R ₂ and R ₂ + R ₂ - R ₃ \n |
| (7) (A). ∴ $\frac{x}{t-1}$ (t ⁻¹) – 1 + 2 ++(n-1) ² = $\frac{n(n-1)(2n-1)}{6}$ \n </td | | | |

(8) (C).Breaking the given determinant into two, determinants, we get

$$
\begin{vmatrix} 3^2 + k & 4^2 & 3^2 + k \ 4^2 + k & 5^2 & 4^2 + k \ 5^2 + k & 6^2 & 5^2 + k \ \end{vmatrix} + \begin{vmatrix} 3^2 + k & 4^2 & 3 \ 4^2 + k & 5^2 & 4 \ 5^2 + k & 6^2 & 5 \ \end{vmatrix} = 0
$$
 (12) (A). A

| AND DETERMINANTS | Q.B.-SOLUTIONS | EXAMPLE RMINANS |
|---|---|-----------------|
| \n $\begin{vmatrix}\n x & x(x-1) & x(x-1)(x-2) \\ x & \frac{y(y-1)}{2} & \frac{y(y-1)(y-2)}{6} \\ y & \frac{z(z-1)}{2} & \frac{z(z-1)(z-2)}{6}\n \end{vmatrix} = \frac{xyz}{12}$ \n | \n $\begin{vmatrix}\n 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 9 & 11 & 1\n \end{vmatrix} = 0$ \n | |
| \n $\begin{vmatrix}\n 1 & x & 1 \\ 1 & y & 2 \\ 1 & z & 2\n \end{vmatrix}\n \begin{vmatrix}\n x & y & 1 \\ x & x & 2 \\ 1 & y & y^2 \\ 1 & z & z^2\n \end{vmatrix}\n \begin{vmatrix}\n 6x^2 - 1(x-2) & 1 \\ 1 & y & y^2 \\ 1 & z & z^2\n \end{vmatrix}\n \begin{vmatrix}\n 9+k & 16 & 3 \\ 9 & 11 & 1 \\ 2 & 2 & 0\n \end{vmatrix}\n = 0$ \n | \n (Applying R ₃ - R ₂ and R ₂ - R ₁ in second det.)\n | |
| \n $\begin{vmatrix}\n 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2\n \end{vmatrix}\n \begin{vmatrix}\n 6x^2 - 1(x-2) & 1 \\ 1 & y & y^2 \\ 1 & z & z^2\n \end{vmatrix}\n \begin{vmatrix}\n 6x^2 - 1(x-2) & 1 \\ 1 & z & z^2\n \end{vmatrix}$ \n | \n $\begin{vmatrix}\n 9+k & 16 & 3 \\$ | |

(10) (D). Write 1 as $\sin^2 \alpha + \cos^2 \alpha$ etc. to get

6
\n
$$
\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \cos \beta + \sin \alpha \sin \beta}
$$
\n
$$
\cos \beta \cos \alpha + \sin \beta \sin \alpha
$$
\n
$$
\cos \gamma + \sin \alpha \sin \gamma
$$
\n
$$
\cos \beta + \sin \alpha \sin \beta
$$
\n
$$
\cos^2 \beta + \sin^2 \beta
$$
\n
$$
\cos \gamma \cos \beta + \sin \alpha \sin \gamma
$$
\n
$$
\cos \beta \cos \gamma + \sin \beta \sin \gamma
$$
\n
$$
\sin^2 \gamma + \cos^2 \gamma
$$

can be factorized into 2 determinant

$$
\begin{vmatrix}\n\cos \alpha & \sin \alpha & x \\
\cos \beta & \sin \beta & x \\
\cos \gamma & \sin \gamma & x\n\end{vmatrix}\n\begin{vmatrix}\n\cos \alpha & \cos \beta & \cos \gamma \\
\sin \alpha & \sin \beta & \sin \gamma \\
x & x & x\n\end{vmatrix} = 0
$$

(11) (A). Observe that the sum of all the elements in a column is $x^2 - 4$. Therefore the determinant

1(x)
$$
\begin{vmatrix} \sin^2 x & \cos^2 x & 5 + 4 \sin 2x \end{vmatrix}
$$

\nAfter solving, f(x)=150 + 100 sin 2x
\nClearly, domain → (−∞, ∞)
\nRange → [50, 250]; Period → π
\n10). Write 1 as sin² α + cos² α etc. to get
\n $\begin{vmatrix} \sin^2 α + cos^2 α & cos β cos α + sin β sin α & cos γ cos α + sin γ sin α \\ cos α cos β + sin α sin β & cos β + sin α sin γ & cos β cos γ + sin β sin γ & sin² γ + cos² γ \end{vmatrix}$
\ncan be factorized into 2 determinant
\n $\begin{vmatrix} \cos α & \sin α & x \\ \cos q & \sin β & x \\ \cos γ & \sin β & x \end{vmatrix} = 0$
\n10, 0. Observe that the sum of all the elements in a column
\nis x² - 4. Therefore the determinant
\n $= (x^2-4) \begin{vmatrix} 1 & 1 & 1 \\ 10 & x^2+2 & 1 \\ -2 & 0 & x^2 \end{vmatrix} = (x^2-4) \begin{vmatrix} 1 & 0 & 0 \\ 10 & x^2-8 & -9 \\ -2 & 14 & x^2+2 \end{vmatrix}$
\n= (x²-4)(x⁴-6x²+110) = x⁴-6x²+110 = (x²-3)²+101 > 0
\nso that real roots are ± 2.
\n21. (A). Aⁿ = $\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$; $\frac{1}{n}A^n = \begin{bmatrix} \frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ -\frac{\sin n\theta}{n} & \frac{\cos n\theta}{n} \end{bmatrix}$
\nBut -1 ≤ cos nθ ≤ 1 and -1 ≤ sin nθ ≤ 1

$$
(n-1)^{3} = \frac{n^{2}(n-1)^{2}}{4}
$$

\n
$$
\begin{vmatrix}\n\cos \alpha \cos \beta + \sin \alpha \sin \beta & \cos^{2} \beta + \sin^{2} \beta & \cos \gamma \cos \beta + \sin \gamma \sin \beta \\
\cos \alpha \cos \gamma + \sin \alpha \sin \gamma & \cos \beta \cos \gamma + \sin \beta \sin \gamma & \sin^{2} \gamma + \cos^{2} \gamma\n\end{vmatrix}
$$
\n
$$
\begin{vmatrix}\n\cos \alpha & \sin \alpha & x \\
\cos \beta & \sin \beta & x \\
\cos \gamma & \sin \gamma & x\n\end{vmatrix} \begin{vmatrix}\n\cos \alpha & \cos \beta & \cos \gamma \\
\sin \alpha & \sin \beta & \sin \gamma \\
x & x & x\n\end{vmatrix} = 0
$$
\n6\n10\n2(2n-1)\n
$$
\begin{vmatrix}\n1 & 1 & 1 \\
3n(n-1)\n\end{vmatrix} = 0
$$
\n
$$
= (x^{2}-4) \begin{vmatrix}\n1 & 1 & 1 \\
1 & 0 & x^{2} + 2 \\
-2 & 0 & x^{2}\n\end{vmatrix} = (x^{2}-4) \begin{vmatrix}\n1 & 0 & 0 \\
1 & 0 & x^{2} + 8 \\
-2 & 14 & x^{2} + 2\n\end{vmatrix}
$$
\n
$$
= (x^{2}-4) (x^{4}-6x^{2}+110)
$$
\n
$$
= x^{4}-6x^{2}+110=(x^{2}-3)^{2}+101>0
$$
\nso that real roots are ± 2.\n
$$
3^{2}+k + 4^{2} - 3
$$
\n
$$
3^{2}+k + 4^{2} - 3
$$
\n
$$
3^{2}+k + 6^{2} - 3
$$
\n
$$
4^{2}+k + 5^{2} - 4
$$
\n
$$
5^{2}+k + 6^{2} - 5
$$
\n
$$
6^{2}+k + 6^{2} - 5
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6^{2}+k + 6^{2} - 5
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107
$$
\n
$$
3^{2}+k + 6^{2} - 5
$$
\n
$$
5^{2}+k + 6^{2} - 5
$$
\n $$

Q.B. - SOLUTIONS STUDY MATERIAL : MATHEMATICS

3.20.30.20.30.21.22
\nAnswer,
$$
x^2 - x^2 + xy = 0
$$

\n $x^2 - 1$
\n $x^3 - 1 = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$
\n $x^4 - 1 = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$
\n $x^4 - 1 = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$
\n $x^4 - 1 = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$
\n $x^4 - 1 = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix} = |A^2 - 1 - 1$
\n $x^4 - 1 = \begin{bmatrix} \cos 2x & -\sin 2x \\ \cos 2x & -\sin 2x \\ \cos 2x & -\sin 2x \end{bmatrix} = |A^2 - 1 - 1 - 1|$
\n $x^4 - 1 = \begin{bmatrix} \log a + (a) - \log r & \log a + \log r \\ \log a + (a) - \log r & \log a + (a) + 2 \log r \\ \log a + (a) - \log r & \log a + (a) + 2 \log r \end{bmatrix}$
\n $x^4 - 1 = \begin{bmatrix} \log a + (a) - \log r & \log a + \log r \\ \log r & \log a + (a) + 2 \log r & \log a + (a) + 2 \log r \\ \log r & \log r & \log r \end{bmatrix}$
\n $x^4 - 1 = \begin{bmatrix} \log a + (a) - \log r & \log a + \log r \\ \log r & \log a + (a) + 2 \log r & \log a + (a) + 2 \log r \\ \log r & \log r & \log r \end{bmatrix}$
\n $x^4 - 1 = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$, $a d(A) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$, $a d(A)$

EXAMPLE 1.1 A1.1
\n
$$
\int_{\sin x/2}^{1} \frac{1}{n} A^{n} = \begin{pmatrix} 0 & 0 \ 0 & 0 \ 0 & 0 \end{pmatrix}
$$
\n
$$
A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
$$
\n
$$
A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
$$
\n
$$
A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
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A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
$$
\n
$$
A^{n} = \begin{pmatrix} 1 & -\tan x \ -\sin x & 1 \end{pmatrix}
$$
\n
$$
A^{n} = \begin{pmatrix} 1 & -\tan x \ -\tan x
$$

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

| MATRICES AND DETERMINANTS | Q.B. SOLUTIONS | | |
|---|--|--|---|
| (22) (C). A = $\begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$ | (26) (D). a ≠ 2; b = -1 Let A = $\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$ | | |
| = 2(1 + sin ² θ) | $\begin{vmatrix} \sin \theta & \sin \theta \\ \sin \theta & \sin \theta \end{vmatrix}$ | $\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix}$ | |
| = 2(1 + sin ² θ) | $\begin{vmatrix} \sin \theta & \sin \theta \\ \sin \theta & \sin \theta \end{vmatrix} \le 1 \Rightarrow -1 \le \sin \theta \le 1 \Rightarrow 0 \le \sin^2 \theta \le 1$ where A is nilpoten where A is nilpoten of a in A = 2 (a - 2) ⇒ a ≠ 2 cofactor of 0 in A is 2 – 3a. According to value of A ⁻¹ , A = $\frac{2-3a}{ A } = \frac{1}{2} \Rightarrow \frac{2-3a}{2(a-2)} = \frac{1}{2}$ $\Rightarrow 2-3a = a - 2 \Rightarrow a = 1$ | $\begin{vmatrix} a & b & c \\ c & d & d \end{vmatrix} = -1$ \Rightarrow $\begin{vmatrix} a & b_1 \\ c & d_1 \end{vmatrix} + \begin{vmatrix} a & b_2 \\ c & d_2 \end{vmatrix} = \begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly | $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly |
| $\begin{vmatrix} a & b_1 \\ c & d_1 \end{vmatrix} + \begin{vmatrix} a & b_2 \\ c & d_2 \end{vmatrix} = \begin{vmatrix} a & b_1 +$ | | | |

Again
$$
c = \frac{\text{cofactor of a in } |A|}{|A|} = \frac{\begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}}{2(a-2)} = \frac{2}{2(1-2)}
$$

= -1

Alternative : $AA^{-1} = I$ **(24) (A).** Applying the result

$$
\begin{vmatrix} a & b_1 \ c & d_1 \end{vmatrix} + \begin{vmatrix} a & b_2 \ c & d_2 \end{vmatrix} = \begin{vmatrix} a & b_1 + b_2 \ c & d_1 + d_2 \end{vmatrix}
$$
 repeatedly (29) (B). A =

$$
\sum_{1}^{n} f(r) = \begin{pmatrix} n(2n+1) & 2n+1 & 6n(n+1)\sum_{1}^{n} r^{2} \\ n+1 & 2n+2 & 2n(n+1)\sum_{1}^{n} r \\ n & 2n+1 & 4\sum_{1}^{n} r^{3} \end{pmatrix}
$$
 (30)

Again
$$
c = \frac{\text{cofactor of a in }|A|}{|A|} = \frac{1}{2(a-2)} = \frac{2}{2(1-2)}
$$

\n $= -1$
\n(A) Applying the result
\n(A) Applying the result
\n
$$
\begin{vmatrix} a & b_1 \\ c & d_1 \end{vmatrix} + \begin{vmatrix} a & b_2 \\ c & d_2 \end{vmatrix} = \begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}
$$
 repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ c & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ d & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ d & d_1 + d_2 \end{vmatrix}$ repeatedly
\n $= -1$
\n $\begin{vmatrix} a & b_1 + b_2 \\ d & d_1 + d_2 \end{vmatrix}$ typically column 2*i* by $(x - i)$

(25) (D). Multiply R_1 by a, R_2 by b & R_3 by c & divide the determinant by abc. Now take a, b $\&$ c common from c_1 , $c_2 \& c_3$. Now use $C_1 \rightarrow C_1 + C_2 + C_3$ to get

$$
(a2 + b2 + c2 + 1)\begin{vmatrix} 1 & 1 & 1 \ b2 & b2 + 1 & b2 \ c2 & c2 + 1 \end{vmatrix} = 1.
$$

Now use $c_1 \rightarrow c_1 - c_2 \& c_2 \rightarrow c_2 - c_3$ we get $1 + a^2 + b^2 + c^2 = 1 \implies a = b = c = 0$

EXECUTEATION SET UP: (a) 1
\n
$$
E(CES AND DETERMINANTS) \n(C) (D), a = +2; b = -4; c = 1; d = -2\n(C) + 1 = -\frac{1}{4} = 0\n(C) + 1 = -\frac{1}{4} = 0\n(C) + 1 = -\frac{1}{4} = 0\n(C) + 1 = -\frac{1}{4} = 0\n(D) - 1 =
$$

Order of A = 3 × 3
$$
\therefore
$$
 n = 3
\n
$$
|adj A| = |A|^{n-1} = (-2)^{3-1} = 4
$$
\n(29) (B). A = x (x² - 1) - 1 (x - 1) + 1 (1 - x) = x³ - x - x + 1 + 1 - x

$$
A = x3 - 3x + 2 ; \frac{dA}{dx} = 3x2 - 3 (B = x2 - 1) = 3B
$$

 (30) n $\sqrt{2}$ $\begin{array}{ccc} 1 & \vert & \vert & \vert & \vert & \vert \end{array}$ **(A).** Multiply column 1^{st} by $(x - a)$ Multiply column 2^{nd} by $(x - b)$ Multiply column 3^{rd} by $(x-c)$

$$
\therefore \frac{1}{\prod(x-a)} \begin{vmatrix} (x-a) & (x-b) & (x-c) \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ \prod(x-a) & \prod(x-a) & \prod(x-a) \end{vmatrix} = 0
$$

$$
= -1
$$
\n
$$
= -1
$$

Q.B. - SOLUTIONS STUDY MATERIAL : MATHEMATICS

 $(A (adj A)A^{-1})A = (A (adj A)) (A^{-1}A) = (|A|I) I = |A|I$

(31) (D). Let We have |A| = 1 2 3 1 2 3 4 5 6 5 6 7 a a a a a a a a a 3d 3d 3d a a a d d d = 0 [Using R³ R³ – R² , and R² R² – R¹] A is singular The given system of homogeneous equations has infinite number of solutions. Also |B| = a¹ 2 + a² ² 0. Thus B is non- singular. **(32) (B).** Let 11 12 13 21 22 23 31 32 33 a a a a a a a a a ; 12 11 a 13 3 3 1 ¹¹ ^a C C C ^a 11 12 13 21 22 21 23 21 11 11 12 13 31 32 31 32 31 11 11 a 0 0 a a a a a a a a a a a a a a a a a a so minimum value = – 4 **(33) (D).** | (adj A–1 | = | A–1 |² = ² 1 | (adj A–1)–1 | = 2 2 1 1 **(34) (A).** We know that every square matrix A can be written as sum of a symmetric & skew-symmetric matrix T T A A A A ^A 2 2 T B 2 2 **(35) (D).** (1 + x) (1 + x)⁴ (1 + x)⁷ 2 2 2 = a⁰ + a1x + a2x 2 + Since all the rows are identical so the value of determinant is zero. a¹ = 0 **(36) (A).** We have = 6

2 2 1 ^a C C C | A | | A | 2 4 | adj A | 6 8 5 6 4 9 4 2 3 8 2 7 A A 9 7 1 5 3 1 ⁼ 6 6 7 6 2 5 7 5 1 1 (1 x) (1 x) 1 (1 x) (1 x) 1 (1 x) (1 x) 0 1 1 | A | 2 1 3 3 2 1 ⁼ 6 0 0 3 0 0 0 6 0 2 0 3 0 0 0 6 0 0 3 **(37) (B).** We have, ⁼ 1 1 ac 1 bc 1 1 ad 1 bd 1 1 ae 1 be Applying C² C² – C¹ , C³ C³ – C¹ 1 ac bc 1 ad bd 1 ae be = ab 1 c c 1 d d 1 e e **(38) (B).** 2 2 2 x x x 1 x 2 2x 3x 1 3x 3x 3 x 2x 3 2x 1 2x 1 = Ax – 12 C² C² – C³ 2 2 2 x x 3 x 2 2x 3x 1 3 3x 3 x 2x 3 0 2x 1 = Ax – 12 R² R² – R¹ 2 2 2 x x 3 x 2 x 2x 1 0 2x 1 x 2x 3 0 2x 1 = Ax – 12 R³ R³ – R² 2 2 x x 3 x 2 x 2x 1 0 2x 1 4 0 0 = Ax – 12 4 (6x – 3 – 0) = Ax – 12 24x – 12 = Ax – 12 A = 24 **(39) (D).** Given A = 1 1 1 0 2 3 2 1 0 , B = (adj A) and C = 5A | A | = 1 (0 + 3) + 1 (0 + 6) + 1 (– 4) = 5 2 2 (n 1) (3 1) 3 3 3 | adj B | | adj.adj A | | A | (5) | C | 5 | A | 5 | A | 5 .5 ⁼ ⁴ 4 5 1 ⁵

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

(40) (B). D = n! (n+1)! (n + 2)! 1 (n 3) (n 4) (n 3) Applying R¹ R¹ – R² , R² R² – R³ D = n! (n+1)! (n + 2)! 0 1 2 (n 2) 0 1 2 (n 3) 0 (n 3) (n 4) (n 3) Now expanding along C¹ D = 2n!(n + 1)! (n + 2)! So, 3 3 D 2n!(n 1)! (n 2)! 4 4 (n!) (n!) = 2.(n + 1) (n + 2) (n + 1) – 4 = n (2n² + 8n + 10). So divisible by n **(41) (B).** i 1 2 i 1 det (A) det (A) det (A) ⁼ 2 2 3 3 2 2 3 3 a b a b a b b a b a b a = (a² – b²) + (a⁴ – b⁴) + (a⁶ – b⁶) + (a² + a⁴ + a⁶ +) – (b² + b⁴ + b⁶ +) 2 2 2 2 a b 1 a 1 b 2 2 2 2 a b (1 a) (1 b) 2 2 2 2 2 2 2 2 i 0 i i 0 i PQ 0 0 i 0 0 i i 0 0 i 0 0 2 2 2 2 sin 13 cos 13 1 cos 13 1 sin 13 1 sin 13 cos 13

(42) (B). It is obvious form the properties of symmetric & skew symmetric matrices.

(43) **(B).** PQ =
$$
\begin{pmatrix} -i^2 + 0 - i^2 & i^2 + 0 + i^2 \ 0 + 0 + i^2 & 0 + 0 - i^2 \ i^2 + 0 + 0 & -i^2 + 0 + 0 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & -2 \ -1 & 1 \ -1 & 1 \end{pmatrix}
$$

$$
\begin{pmatrix} 5(K-4) & 5(K-4) \ 5K-23 & -13K+1 \end{pmatrix}
$$

(44) (B). Applying $C_1 \to C_1 + C_2 + C_3$

⇒
$$
(a^2 + a^4 + a^6 +) - (b^2 + b^4 + b^6 +)
$$

\n⇒ $\frac{a^2}{1-a^2} - \frac{b^2}{1-b^2} \Rightarrow \frac{a^2-b^2}{(1-a^2)(1-b^2)}$
\n**B)**. It is obvious form the properties of symmetric & skew
\nsymmetric matrices.
\n**B)**. PQ = $\begin{pmatrix} -i^2 + 0 - i^2 & i^2 + 0 + i^2 \\ 0 + 0 + i^2 & 0 + 0 - i^2 \\ i^2 + 0 + 0 & -i^2 + 0 + 0 \end{pmatrix}$
\n $z = \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$
\n**B)**. PQ = $\begin{pmatrix} -i^2 + 0 - i^2 & i^2 + 0 + i^2 \\ 0 + 0 + i^2 & 0 + 0 - i^2 \\ i^2 + 0 + 0 & -i^2 + 0 + 0 \end{pmatrix}$
\n $z = \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$
\n $z = \begin{pmatrix} 5(K+8-55) - 4(5(K+8-55)) - 4(-5K+142) - 5(K-47) - 4(-32) - 5(K-27) + 128 + 142 \\ 5(K-47) - 4(-32) - 5(K-47) -$

(45) (D). A^{-1} exist only for non-singular matrix. $AB = AC \implies B = C$ if A^{-1} exist. If A^{-1} exist

(46)
$$
(A). A(\alpha) = \begin{bmatrix} f_1(\alpha) & f_2(\alpha) & f_3(\alpha) \\ f_4(\alpha) & f_5(\alpha) & f_6(\alpha) \\ f_7(\alpha) & f_8(\alpha) & f_9(\alpha) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

 $x - \alpha$ is a factor of $f_1(x)$, $f_2(x)$, $f_9(x)$ $f(x) = (x - \alpha) \phi(x)$ $f(\alpha) = 0 \Rightarrow x - \alpha$ is a factor of $f(x)$

1 (n 1) (n 2) (n 1) 1 (n 2) (n 3) (n 2) **(47) (D).** 5 4 3 100x 50 1 100y 40 1 100z 30 1 x y z ⁼ 5 4 3 1 1 1 x y z (i) 1 31 2 32 3 33 b .C b .C b .C **⁼** 2 3 1 3 1 2 1 2 3 2 3 1 3 1 2 a a a a a a b b b 0 b b b b b b

$$
= \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} (R_2 \rightarrow R_2 - 100 R_3 - 10 R_1) \neq 0
$$

(48) (A). Since $|A|=1$ We know $A^{-1} = 1 / |A|$ adj $A \Rightarrow A^{-1} = adj(A)$ **(49) (A), (50) (C), (51) (D).**

i)
$$
b_1.C_{31} + b_2.C_{32} + b_3.C_{33}
$$

$$
\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - b_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + b_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0
$$

(ii) Value of new determinants = $2^3\Delta = 8\Delta$ (iii) $a_3M_{13} - b_3M_{23} + d_3M_{33} = a_3C_{13} + b_3C_{23} + d_3C_{33} = \Delta$ By definition

(52) (C). Given matrix A + 2B is singular
$$
\Rightarrow
$$
 |A + 2B| = 0

3 2 ⁼ 2 2 1 1 1 1 0 cos 13 1 0 1 sin 13 0 0 sin 13 cos 13 A + 2B = 1 2 2 4 6 2 5 K 6 8 8 4 3 1 2 6 10 4 ⁼ 5 4 4 13 K 8 10 9 11 2 5 4 2 13 K 8 5 9 11 1 1 5 1 1 K 4 4 0 4 4

$$
|A+2B|=0 \Rightarrow 2\begin{bmatrix} 5 & 4 & 2 \\ 13 & K+8 & 5 \\ 9 & 11 & 1 \end{bmatrix}=0
$$

$$
2 [5 (K+8-55) -4 (13-45) + 2 (143-9K-72)] = 0
$$

5 (K-47) - 4 (-32) + 2 (71-9K) = 0
5K-235 + 128 + 142 - 18 K = 0

$$
-13K + 35 = 0 \Rightarrow K = \frac{35}{13}
$$

(53) (A). Given
$$
C = A - B
$$
 and $Tr(C) = 2$

+.....)
\n
$$
[3 \t1^{-1} - 2] [6 \t10^{-4}] [3
$$
\n
$$
[3 \tK + 8^{-5}] = 0
$$
\n
$$
[3 \tK + 8^{-5}] = 0
$$
\n
$$
2[5(K + 8 - 55) - 4(13 - 45) + 2(143 - 9K - 7)]
$$
\n
$$
= \begin{pmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{pmatrix}
$$
\n
$$
3 \times 2
$$
\n
$$
5(K - 47) - 4(-32) + 2(71 - 9K) = 0
$$
\n
$$
5K - 235 + 128 + 142 - 18K = 0
$$
\n
$$
-13K + 35 = 0 \Rightarrow K = \frac{35}{13}
$$
\n
$$
(53) (A). Given C = A - B and Tr(C) = 2
$$
\n
$$
213^{\circ} -1
$$
\n
$$
-1 \sin^{2} 13^{\circ} = 0
$$
\n
$$
C = \begin{bmatrix} -1 & -5 & 1 \\ 1 & K - 4 & 4 \\ 0 & -4 & -4 \end{bmatrix}, Tr(C) = 2
$$
\n
$$
\Rightarrow -1 + K - 4 - 4 = 2 \Rightarrow K = 11
$$
\n
$$
3 \times 2
$$
\n
$$
3 \times 2
$$
\n
$$
13^{\circ} \cos^{2} 13^{\circ}
$$
\n
$$
= 0
$$
\n
$$
C = \begin{bmatrix} -1 & -5 & 1 \\ 1 & K - 4 & 4 \\ 0 & -4 & -4 \end{bmatrix}, Tr(C) = 2
$$
\n
$$
\Rightarrow -1 + K - 4 - 4 = 2 \Rightarrow K = 11
$$
\n
$$
(54) 42. \begin{bmatrix} 1 & 1 & 2 & 6 \\ 1 & 3 & 3 & 10 \\ 0 & 2 & 1 & 4 \end{bmatrix}
$$

$$
\Rightarrow -1 + K - 4 - 4 = 2 \Rightarrow K = 11
$$

$$
\begin{vmatrix}\n0 & a & \mu & 0 \\
a^{6} + \dots & 0^{4}\n\end{vmatrix} + (a^{6} - b^{6}) + \dots \\
a^{6} + \dots & - (b^{2} + b^{4} + b^{6} + \dots) \\
a^{6} + \dots & - (b^{2} + b^{4} + b^{6} + \dots) \\
b^{2} - a^{2} - b^{2} \\
\hline\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
b^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} & 1 & 2\n\end{vmatrix} = 0
$$
\n
$$
\begin{vmatrix}\n1 & a^{2} & \mu & 0 \\
a^{2} &
$$

$$
2[5(K+8-55)-4(13-45)+2(143-9K-72)] = 0
$$

\n
$$
5(K-47)-4(-32)+2(71-9K) = 0
$$

\n
$$
5K-235+128+142-18K = 0
$$

\n
$$
-13K+35 = 0 \Rightarrow K = \frac{35}{13}
$$

\n(A) Given C = A – B and Tr(C) = 2
\n
$$
C = \begin{bmatrix} -1 & -5 & 1 \\ 1 & K-4 & 4 \\ 0 & -4 & -4 \end{bmatrix}, Tr(C) = 2
$$

\n
$$
\Rightarrow -1+K-4-4 = 2 \Rightarrow K = 11
$$

\n42. $\begin{bmatrix} 1 & 1 & 2 & 6 \\ 1 & 3 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 2 & 1 & 4 \\ 0 & 1 & \lambda-2 & \mu-6 \end{bmatrix}$
\n
$$
\sim \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 1 & 1/2 & 2 \\ 0 & 1 & \lambda-2 & \mu-6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 1 & 1/2 & 2 \\ 0 & 0 & \lambda-\frac{5}{2} & \mu-8 \end{bmatrix}
$$

\n
$$
\therefore \lambda = \frac{5}{2}, \mu = 8 \therefore 4(\lambda + \mu) = 42
$$

EXAMPLE 3.1 (35) 25. A = α βγ
$$
\begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} & \frac{1}{1-\gamma} \\ \alpha & \beta & \gamma \end{vmatrix}
$$
 (39) 4<sup>- $\frac{1}{\alpha} \begin{vmatrix} \frac{1}{\alpha^2} + b^2 & c^2 & c^2 \\ \frac{1}{\beta^2} & b^2 + c^2 & a^2 \\ b^2 & b^2 - c^2 + a^2 \end{vmatrix}$ use R₁ → R
\n
$$
= \frac{\alpha \beta \gamma}{\begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} & \frac{1}{1-\alpha} & \frac{1}{1-\gamma} \\ \frac{1}{\alpha} & \beta & \gamma & 2 \end{vmatrix}}{\begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} & \frac{1}{1-\alpha} \\ \frac{1}{\alpha} & \frac{1}{\beta} & -\alpha & \gamma - \alpha \\ \frac{\alpha \beta \gamma}{\alpha} & -\alpha & 2 \end{vmatrix}} = \frac{\frac{1}{\alpha b c} \begin{vmatrix} 0 & -2b^2 & -2a^2 \\ b^2 & b^2 & c^2 + a^2 \\ b^2 & 0 & c^2 \end{vmatrix}}{\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ b^2 & 0 & c^2 \\ 0 & 0 & c^2 \end{vmatrix}}
$$

\n
$$
= \frac{\frac{\alpha \beta \gamma}{\alpha}(-b)(1-\beta)(1-\gamma)}{(1-\alpha)(1-\beta)(1-\gamma)}
$$

\nSince, α, β, γ are the roots of ax³ + bx² + cx + d = 0
\n \therefore ax³ + bx² + cx + d = a(x - α)(x - β)(x - γ) and 0
\n \therefore a³ + bx² + cx + d = 0
\n $A^3 = A \cdot A^2 = A(AA + 5I) = 4A^2 + 5A$
\n $= 4(AA + 5I) = 4A^2 + 5A$
\n $= 4(A + 5I) + 5A = 2A + 2A + 5A$
\n $= 4(A + 5I) + 5A =$</sup>

35. A = a, b = 1
\n
$$
35A = a, b = 1
$$
\n
$$
a + b = 2
$$
\n
$$
a + b =
$$

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

A² 1 1 ⁼ 1 1 1 A A A 1 2 0 . This gives – a + 2b = 1(5) and – c + 2d = 0(6) (3) + (5) b = 0 and a = – 1 (4) + (6) d = 2 and c = 4 so the sum a + b + c + d = 5. **(63) 6.** Possible orders (1 × 12) ; (12 × 1) ; (2 × 6) ; (6 × 2) ; (3 × 4) ; (4 × 3) **(64) 6.** A² = A.A = (AB).A = A (BA) = BA = A k = 1 B 2 = B.B = (BA) B = B (AB) = BA = B = 1 (A + B)² = (A + B) (A + B) = A² + AB + BA + B²) = (A + A + B + B) = 2 (A + B) (A + B)³ = (A + B)² . (A + B) = 2 (A + B) (A + B) = 2² (A + B) k + + m = 6 **(65) 1.** i2 /3 ^e 2 2 2 z 1 z 1 0 1 z ; 2 2 1 z z 1 0 1 1 z z [(z + ²) (z +) – 1 – (z + – 1) + ² (1 – z – ²)]=0 z 3 = 0 z = 0 is only solution. **(66) 5.** | A | = (2k + 1)³ , | B | = 0 (Since B is a skew.symmetric matrix of order 3) det (adj A) = |A|n–1 = ((2k + 1)³)² = 10⁶ 2k + 1 = 10 2k = 9 [k] = 4. **(67) 9.** Let M = 11 12 13 21 22 23 31 32 33 a a a a a a a a a then a12 = –1, a22 = 2, a32 = 3 a11 – a12 = 1 a11 = 0, a21 – a22 = 1 a21 = 3, a31 – a32 = 1 a31 = 2, a11 + a12 + a¹³ = 0 a13 = 1 a21 + a22 + a²³ = 0 a23 = –5 a31 + a32 + a³³ = 12 a33 = 7 Hence sum of diagonal of M is = a11 + a22 + a³³ = 0 0 + 2 + 7 = 9 **(68) 2.** 2 3 2 3 2 3 x x 1 x 2x 4x 1 8x 10 3x 9x 1 27x 2 2 3 2 2 3 2 2 3 x x 1 x x x 2x 4x 1 2x 4x 8x 10 3x 9x 1 3x 9x 27x 2 2 2 2 2 2 1 x x 1 x x 1 2x 4x x 2x 3x 1 2x 4x 10 1 3x 9x 1 3x 9x –x × – x × 2x (1 + 6x³) 10 2x³ (1 + 6x³) 10 x 3 (1 + 6x³) 5 ; x³ = t t + 6t² 5 ; 6t² + t – 5 = 0 6t² + 6t – 5t – 5 = 0 6t (t + 1) – 5 (t + 1) = 0 ; t –1, t = 5/6 x 3 = –1, x³ = 5/6 x = –1 and x (5/6)1/3 **(69) 103.** ² 1 0 0 1 0 0 P 4 1 0 , P 8 1 0 16 4 1 48 8 1 3 4 1 0 0 1 0 0 P 12 1 0 , P 16 1 0 96 12 1 160 6 1 Pattern of element P31 is 16 [1, 3, 6, 10,......] 50th term is 16 × 1275 [By observing that Tⁿ of S = 1 + 3 + 6 + 10 + ….. is ² n n 2] 50 1 0 0 P 200 1 0 16 275 200 1 50 0 0 0 Q P I 200 0 0 16 275 200 0 31 32 21 q q 16 1275 200 q 200 = 102 + 1 = 103 **(70) 1.** 2 2 2 1 1 1 (1) 0 1 1 2 = 1 ; = ± 1 = 1 (two planes are parallel) (Rejected)

 $\alpha = -1$ (two planes are coincident)

Q.B. - SOLUTIONS STUDY MATERIAL : MATHEMATICS

I and the set of the set

(71) 198. 1 2 3 1 2 3 1 2 3 T T 5 = 1² + 1² + 1² + 1² + 0² + 0² + 0² + 0² s s 9 ⁵ 5, 1 , 4, 0 C 1 2 + 2² + 0² s 9 2 9 8 126 2 198 2 **(72) 0.** Let A = . Operating R² 2R² – R¹ – R³ ⁼ x 2 x 3 x 2a 1 2 x 4 x 5 x 2c But a, b, c are in A.P., 2b = a + c ⁼ x 2 x 3 x 2a 1 2 x 4 x 5 x 2c **EXERCISE-3 (1) (C).** ² A A ⁼ ² 2 B B I ² 0 i 0 i 0 i C C i 0 i 0 i 0

33.34
\n198. M =
$$
\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}
$$

\n199. M = $\begin{bmatrix} a_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$
\n190. M = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$
\n191. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{bmatrix}$
\n192. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n193. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n194. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n195. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n196. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n197. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n198. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n199. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n190. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n191. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n193. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$
\n194. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end$

MATRICES AND DETERMINANTS Q.B. - SOLUTIONS

(4) (B). 1, $\omega \& \omega^2$ are the cube roots of unity, \therefore 1 + ω + ω ² = 0 and ω ³ = 1 $\Rightarrow \omega$ ³ⁿ = 1

RICES AND DETERMINANTS
\n(B). 1,
$$
\omega \& \omega^2
$$
 are the cube roots of unity,
\n $\therefore 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1 \Rightarrow \omega^{3n} = 1$
\n $\therefore \Delta = \begin{vmatrix}\n1 & \omega^n & \omega^{2n} \\
\omega^n & \omega^{2n} & 1 \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix}$
\nWe write 1 as ω^{3n} in R₁ and ω^n common from R₁
\n $\begin{vmatrix}\n0 & 0 & -1 \\
-1 & 0 & 0 \\
-1 & 0 & 0\n\end{vmatrix}$
\n $|A| = \begin{vmatrix}\n0 & 0 & -1 \\
0 & -1 & 0 \\
0 & -1 & 0 \\
-1 & 0 & 0\n\end{vmatrix} = -1 (0 - 1)$

We write 1 as ω^{3n} in R₁ and ω^{n} common from R₁

RICES AND DETERMINANTS) (Q.B.-SOLUTIONS
\n**(B).** 1,
$$
\omega
$$
 & ω^2 are the cube roots of unity,
\n $\therefore 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1 \Rightarrow \omega^{3n} = 1$
\n $\therefore \Delta = \begin{vmatrix}\n1 & \omega^n & \omega^{2n} \\
\omega^n & \omega^{2n} & 1 \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix}$
\nWe write 1 as ω^{3n} in R₁ and ω^n common from R₁
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = \omega^n \begin{vmatrix}\n\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = \omega^n \begin{vmatrix}\n\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{2n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{3n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n} & 1 & \omega^n \\
\omega^{3n} & 1 & \omega^n\n\end{vmatrix} = 0$
\n $\begin{vmatrix}\n\omega^{3n} & \omega^{2n} \\
\omega^{2n$

 $\{ \because R_1, R_3 \text{ are identical} \}$

| MATRICES AND DIFFENINANTS | Q1.5-SOLUTION | Q2.5-SOLUTION | EXAMPLE A | | | | |
|--|--|---------------|-----------------------|-------------|-------------|-------------|-------------|
| (4) (B) 1, ω & ω ° are the cube roots of unity. | | | | | | | |
| (5) (B) ω & ω ° are the cube roots of unity. | | | | | | | |
| \therefore $\Delta = \begin{vmatrix} 1 & \omega^0 & \omega^{2n} \\ \omega^{2n} & 1 & \omega^0 \end{vmatrix}$ | $2\omega^2$ | | | | | | |
| $\begin{vmatrix} 1 & \omega^0 & \omega^{2n} \\ \omega^{2n} & 1 & \omega^0 \end{vmatrix}$ | $2\omega^2$ | $2\omega^2$ | | | | | |
| $\begin{vmatrix} \omega^{2n} & \omega^{2n} \\ \omega^{2n} & 1 & \omega^0 \end{vmatrix}$ | $2\omega^2$ | $2\omega^2$ | | | | | |
| $\begin{vmatrix} \omega^{2n} & \omega^{2n} \\ \omega^{2n} & 1 & \omega^0 \end{vmatrix}$ | $2\omega^2$ | $2\omega^2$ | | | | | |
| (5) (C) $\begin{vmatrix} \omega & \omega^{2n} \\ \omega & \omega^{2n} \end{vmatrix}$ | $\begin{vmatrix} \omega^0 & \omega^{2n} \\ \omega^0 & \omega^{2n} \end{vmatrix}$ | $2\omega^0$ | $2\omega^0$ | $2\omega^0$ | $2\omega^0$ | $2\omega^0$ | $2\omega^0$ |
| (7) $\begin{pmatrix} 1 & \omega & \omega^{2n} \\ \omega & 2 & 1 + \omega^3 \\ \omega & 2 & 1 + \omega^3 \end{pmatrix}$ | $2\omega^0$ | $2\omega^0$ | <math< td=""></math<> | | | | |

ND DETERMINANTS) (Q.B.-SOLUTIONS)
\n
$$
\& \omega^2
$$
 are the cube roots of unity,
\n $+\omega^2 = 0$ and $\omega^3 = 1 \Rightarrow \omega^{3n} = 1$
\n ω^{n} (7) (D). $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ (1)
\n ω^{2n} (1)
\n ω^{2n} 1
\n ω^{2n} 2
\n ω^{2n} 3
\n ω^{2n} 1
\n ω^{2n} 2
\n ω^{2n} 3
\n ω^{2n} 1
\n ω^{2n} 2
\n ω^{2n} 3
\n ω^{2n} 1
\n ω^{2n} 4
\n ω^{2n} 1
\n ω^{2n} 2
\n ω^{2n} 3
\n ω^{2n} 4
\n ω^{2n} 1
\n

Clearly from (1) A \neq 0 (first option cancelled)

$$
|A| = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = -1 (0 - 1) = \neq 0
$$

1 ω^n \therefore A⁻¹ exist (third option cancelled)

$$
A \neq (-1) I \begin{cases} \cdot & -I = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} \end{cases}
$$

(IInd option cancelled)

EXECUTERMIIAXITS) (O.13.5010110) ×
\nare the cube roots of unity,
\n
$$
0^{-1} \text{ and } \omega^3 = 1 \Rightarrow \omega^3 = -1
$$

\n $\omega^3 = \frac{\omega^2 a}{1 - 1} = \frac{1}{0}$
\n $\omega^3 = \frac{1}{1 - 0}$
\n $\omega = \omega^3 = \frac{1}{1 - 0}$
\n $\omega = \omega^3 = \frac{1}{1 -$

 $\frac{1}{2}(0) = 0$ { : C₂ is the sum of two elements, first identical with C_1 and second with C_3

Solution
\n**EXAMPLE 3.1** A
\n**EXERCISE 2A**
\n**EXAMPLE 3.2**
\n**EXERCISE 3A**
\n**EXERCISE 4A**
\nSimplify:
\n
$$
\begin{vmatrix}\na+1 & -1 & -1 \\
a+1 & 0 \\
1 & 1 & a \\
1 & a & 0\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n1 & 1 & 1 \\
a & 1 & 0 \\
1 & 1 & a \\
1 & 1 & a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n1 & 1 & 1 \\
a & 1 & 1 \\
1 & 1 & a \\
1 & 1 & a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n1 & 1 & 1 \\
0 & 1 & 1-a \\
0 & 1 & 1-a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)\begin{vmatrix}\n0 & 1 & -1-a \\
0 & 1 & 1-a \\
1 & 1 & a\n\end{vmatrix} = 0
$$
\n
$$
\Rightarrow (a+2)(1-a)(1-a)(1-a) = 0
$$
\n
$$
\Rightarrow (a+2)(1-a)(1-a) = 0
$$
\n
$$
\Rightarrow (a+2)(1-a) = 0 \Rightarrow a = -2 \text{ R}
$$
\n**10.1**
\n**21**
\n**23**
\n**23**
\n**24**
\n**25**
\n**26**
\n**27**
\n**28**
\n**29**
\n**20**
\n**21**
\n**23**
\n**23**
\n**24**
\n**25**
\n**2**

$$
\begin{array}{llll}\n\hline\n\text{matrix} & \text{SPLDVM ATIERMA: MATHEMATICS} \\
\text{of equation has no solution} & \text{Similarity: } A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \text{........(1)} \\
\hline\n1 & 1 & 0 \\
1 & 2 & 1 \end{bmatrix} \\
\begin{vmatrix}\n1 & 1 & 1 \\ 1 & 0 \end{vmatrix} = 0 & \text{Now, } nA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \text{........(2)} \\
\hline\n2 & 1 & 1 \end{vmatrix} \\
\begin{vmatrix}\n1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 & \text{Now, } nA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} & \text{........(3)} \\
\hline\n3 & 2 & 1 \end{vmatrix} \\
\begin{vmatrix}\n1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 & \text{Now, } nA = \begin{bmatrix} 0.2 \text{ and } 0.1 \text{ and } 0
$$

if det $A = I$

$$
\therefore A^{-1} = \frac{\text{adj } A}{\text{det } A} \quad \{ |A| = \pm 1 \} = \pm (\text{adj } A)
$$

 \therefore A⁻¹ exist and all its entries are integers.

| MATRICES AND DETERMINANTS | Q.B.SOD E I 100N | EXAMPLE 25.1 | | | | | | | | | | | | | | | |
|--|--|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|-------|
| \n $\therefore A^{-1} = \frac{adj \Delta}{det A} \{ A = \pm 1 \} = \pm (adj A)$ \n | \n $\therefore A^{-1} = \frac{adj \Delta}{det A} \{ A = \pm 1 \} = \pm (adj A)$ \n | \n $A^{-1} = \frac{adj \Delta}{det A} \{ A = \pm 1 \} = \pm (adj A)$ \n | \n $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $A^2 = \begin{bmatrix} a & b \\ d & d \end{bmatrix}$ \n | \n $$ |

$$
y = az + c (cy + bz) \Rightarrow y = \frac{(a + bc) z}{1 - c^2}
$$
........(4)
 $\Rightarrow 8 - k^2 + c (cy + bz) \Rightarrow 9 - k^2 - 2$
 $\Rightarrow -k^2 - 2 \Rightarrow (k + 6)$

and
$$
z = b (cy + bz) + ay \Rightarrow y = \frac{1 - b^2}{a + bc}
$$
(5)
Number of
From eq. (4) $k(5)$

$$
\text{round}(4) \propto (3)
$$
\n
$$
(2 + \ln 2) \times (1 + \ln^2)
$$

$$
y = az + c(cy + bz) \Rightarrow y = \frac{(a + bc)z}{1 - c^2} \dots \dots \dots (4) \Rightarrow x = K(K-2)-2(2K-8) = 0
$$

\n
$$
\Rightarrow k^2 - 2k + 24 = 0 \Rightarrow k^2 + 2k = 0
$$

\nand $z = b(cy + bz) + ay \Rightarrow y = \frac{1 - b^2}{a + bc} \dots \dots \dots (5)$
\nFrom eq. (4) & (5)
\n
$$
\frac{(a + bc)z}{1 - c^2} = \frac{1 - b^2}{a + bc}z
$$

\n
$$
\Rightarrow (a + bc)^2 = (1 - b^2)(1 - c^2)
$$

\n
$$
\Rightarrow a^2 + a^2b^2 + 2abc = 1 - c^2 - b^2 + b^2c^2
$$

\n
$$
\Rightarrow a^2 + b^2 + c^2 + 2abc = 1
$$

\n(21) (A). Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ add $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
\n
$$
\Rightarrow \text{adj } (adj A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}
$$
 add $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
\n
$$
\Rightarrow \text{discrete } (P^2 + Q^2)(P - Q) = 0
$$

\n
$$
\Rightarrow P - Q = 0 \text{ contradiction}
$$

\n(22) (B). The given equation can be written as
\n
$$
\begin{vmatrix} a & a + 1 & a - 1 \\ -b & b + 1 & b - 1 \\ c & c - 1 & c + 1 \end{vmatrix} + (-1)^n \begin{vmatrix} a & a + 1 & a - 1 \\ -b & b + 1 & b - 1 \\ c & c - 1 & c + 1 \end{vmatrix} = 0
$$

\n
$$
\Rightarrow P = \begin{vmatrix} 30 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.
$$
 For no solution, $\frac{k + 1}{k} = \frac{1}{k}$
\n
$$
\Rightarrow n \text{ has to be any odd integer.}
$$

$$
\Rightarrow \text{adj }(\text{adj }A) = \begin{bmatrix} c & d \end{bmatrix}
$$

Also $|$ adj A $| = |A|$ but this does not explain the S-1. **(22) (B).** The given eqaution can be written as

$$
\begin{vmatrix} a & a+1 & a-1 \ -b & b+1 & b-1 \ c & c-1 & c+1 \ \end{vmatrix} + (-1)^n \begin{vmatrix} a & a+1 & a-1 \ -b & b+1 & b-1 \ c & c-1 & c+1 \ \end{vmatrix} = 0
$$

\n \Rightarrow n has to be any odd integer.

ES AND DETERMINANTS

et A = I (23) (C). First row with exactly on
 $A^{-1} = \frac{adj A}{det A}$ {|A|=±1} = ± (adj A) (23) (C). First row with exactly on

Total number of cases = 6

First row 2 zeros we get more

Total we get more t **(23) (C).** First row with exactly one zero; Total number of cases $= 6$ First row 2 zeros we get more cases Total we get more than 7.

det A a b **(24) (B).** Let a b A , abcd 0 c d ² a b a b A . c d c d 2 2 2 a bc ab bd A ac cd bc d a 2 + bc = 1, bc + d² = 1 1 2 1 D 2 3 1 0 3 5 2 ; ¹ 3 2 1 D 3 3 1 0 1 5 2 4 k 2 k 4 1 0

 $ab + bd = ac + cd = 0$; $c \ne 0$ and $b \ne 0 \Rightarrow a + d = 0$ Trace $A = a + d = 0$ $|A| = ad - bc = -a^2 - bc = -1.$

(25) (C).
$$
D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0
$$
; $D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

 \Rightarrow Given system, does not have any solution. \Rightarrow No solution.

(26) (B). $A' = A$, $B' = A$; $P = A(BA)$; $P' = (A(BA))'$ $= (BA)' A' = (A'B') A' = (AB) A = A(BA)$ \therefore A(BA) is symmetric. similarly (AB) A is symmetric. S-2 is correct but not correct explanation of S-1. d) (c d) – A - $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ (e d) – A - $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$

= 1, bc + d² = 1

a + d = 0

a + d = 0

bc = -a² - bc = -1.

1

2

2

3

3

5

2

3

5

2

system, does not have any solution.

1, S = A (26) **(B).** A¹ = A, B¹ = A, B¹ = A(BA); P¹ = (A(BA))

= (BA)'A' = (A'B')A' = (AB)A = A(BA)

∴ A(BA) is symmetric. similarly (AB) A is symmetric

S-2 is correct but not correct explanation of S-1.
 (27) (B). $D = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$;

Siven system, does not have any solution.
 $A' = A, B' = A; P = A(BA); P' = (A(BA))'$
 $A')'A' = (A'B')A' = (AB)A = A(BA)$
 $A)$ is symmetric, similarly (AB) A is symmetric

scorrect but not correct explanati $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ = $\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix}$ ≠ 0

does not have any solution.

P = A(BA); P' = (A(BA))'

V' = (AB) A = A(BA)

Niric. similarly (AB) A is symmetric

of correct explanation of S-1.
 1 2 1

2 3 1 = 0; D₁ = $\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

3 5 2

3 1 = 0; D₁ = $\begin{vmatrix} 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

5 = A; P = A(BA); P' = (A(BA))'

5 = A; P = A(BA), P' = (A(BA))'

5 symmetric. simila $\begin{vmatrix} = 0; & 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix}$
 $\begin{aligned} P &= A(BA); P &= (A(BA))' \\ Y &= (AB)A = A(BA) \\ Y &= (AB)A \end{aligned}$

Therefore the sum of S-1.
 $\begin{vmatrix} = 0 \\ 0 \\ 1 \\ 1 \end{vmatrix} = 0$
 $2k - 8 = 0$
 $2k - 8 = 0$
 $2k - 8 = 0$
 $2k^2 +$ 7, $\begin{vmatrix} 1 & 5 & 2 \end{vmatrix}$

not have any solution.

A(BA); P'=(A(BA))'

(AB)A = A(BA)

similarly (AB)A is symmetric

rrect explanation of S-1.

9

9 = 0
 $\begin{vmatrix} 8 \end{vmatrix} = 0$
 $\begin{vmatrix} 2+2k-24=0 \\ k=-6,4 \\ 8 \end{vmatrix}$
 $\begin{vmatrix} 2+2k$ $\begin{bmatrix} 5 & 2 \end{bmatrix}$

solution.

(A(BA))'

A)

A is symmetric

ion of S-1.
 $\begin{bmatrix} 0 \\ -1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$ adj A
 $\begin{bmatrix} 0 & 0 \\ -1 \\ -1 \\ -1 \end{bmatrix}$ a, does not have any solution.

A; P = A(BA); P' = (A(BA)')

')A' = (AB) A = A(BA)

metric. similarly (AB) A is symmetric

not correct explanation of S-1.

2

1

2

2(2k-8) = 0

k + 16 = 0

= 0 ⇒ k = -6, 4

s of k is 2
 No solution.

A' = A, B' = A; P = A(BA); P' = (A(BA))'

A' A' e (A'B') A' = (AB) A = A(BA)

(BA) is symmetric. similarly (AB) A is symmetric

is correct but not correct explanation of S-1.

 $\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 &$ P' = (A(BA)'

A(BA)

(AB) A is symmetric

lanation of S-1.

 $24 = 0$

1; A⁻¹ = $\frac{1}{|A|}$ adj A

 $\frac{1}{|A|}$ adj A

 $\frac{1}{|A|}$ $\frac{0}{-2}$ $\frac{0}{1}$ = $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$
 $-Q^2P$ ve any solution.
 $\begin{aligned}\n\therefore P' &= (A(BA))' \\
&= A(BA) \\
\text{ly (AB) A is symmetric} \\
\text{vplanation of S-1.} \\
\therefore -24 &= 0 \\
4\n\end{aligned}$
 $\begin{aligned}\n\frac{1}{|A|} &= \frac{1}{|A|} \text{adj } A \\
\frac{1}{|A|} &= \frac{1}{|A|} \text{adj } A \\
\frac{1}{|A|} &= \frac{1}{|A|} \text{adj } A \\
\frac{1}{|A|} &= \frac{1}{|A|} \end{aligned}$ A); P' = (A(BA))'

A = A(BA)

arly (AB) A is symmetric

explanation of S-1.

0

0

2k – 24 = 0

6, 4
 $\begin{vmatrix} = 1 \\ 1 \\ -2 \\ 1 \\ -2 \\ 1 \end{vmatrix}$ adj A
 $\begin{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 \\ -1 \\ -1 \\ -1 \end{vmatrix}$
 $Q^3 - Q$

(27) **(B).**
$$
\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0
$$

$$
+ bc) z
$$

\n
$$
- c2
$$

\n
$$
- c2
$$

\n
$$
- b2
$$

\n
$$
1 - b2
$$

\n
$$
= 8 - k(k-2) - 2(2k-8) = 0
$$

\n
$$
3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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4 - k2 + 2k - 24 = 0
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4 - k2 + 2k - 4k + 16 = 0
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3 - k2 + 2k - 4k + 16 = 0
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4 - k2 + 2k - 4k + 16 = 0
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4 - k2 + 2k - 4k + 16 = 0
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4 - k2 + 2k - 4k + 16 = 0
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4 - k2 + 2k - 4k + 16 = 0
$$

\n
$$
4 - k
$$

(28) **(D).** A
$$
(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}
$$
; $|A| = 1$; $A^{-1} = \frac{1}{|A|}$ adj A

(b), A = A, B = A, F = A(BA), F = (ABA)
\n= (BA)'A' = (AB')A' = (AB)A = A(BA)
\n∴ A(BA) is symmetric, similarly (AB) A is symmetric
\nS-2 is correct but not correct explanation of S-1.
\n**(B).**
$$
\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0
$$

\n⇒ 8-k(k-2)-2(2k-8)=0
\n⇒ k-2k-2k+24=0 ⇒ k² + 2k-24=0
\n⇒ (k+6) (k-4) = 0 ⇒ k = -6, 4
\nNumber of values of k is 2
\n**(D).** A (u₁ + u₂) = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$; |A|=1; A⁻¹ = $\frac{1}{|A|}$ adj A
\n(u₁ + u₂) = A⁻¹ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; A⁻¹ = $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$
\n(C). Subtracting P³ – P²Q = Q³ – Q²P
\nP²(P-Q)+Q²(P-Q)=0
\nIf | P²+Q² | ≠ 0 then P² + Q² is invertible
\n⇒ P-Q= 0 contradiction
\nHence |P²+Q²|=0
\n(B). For no solution, $\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$
\nk²+4k+3=8k; k²-4k+3=0; k=1,3

- a b d b -bc = -1

S-2 is correct but not correct explanation of S

..............(1)
 $we get$
 $\frac{(a + bc) z}{1 - c^2}$(4)
 $y = \frac{1 - b^2}{a + bc}$(5)

(28) (D). A (u₁ + u₂) = A⁻¹ + 2k - 2k - 2k - 2k - 2k - 2k - 2k cet.

S-2 is concet out not concet expansion of S-1.

(a)
 $\& (3)$ we get
 $\Rightarrow 2 \times 6 \times 10^{-12}$ (b). $\Delta = \begin{vmatrix} 1 & 1 \ 1 & 4 \end{vmatrix} = 0$
 $\Rightarrow 3 - k^2 + 2k - 4k + 16 = 0$
 $\Rightarrow -k^2 - 2k + 2k - 4k + 16 = 0$
 $\Rightarrow -k^2 - 2k + 24 = 0 \Rightarrow k^2 + 2k - 24 =$ (from (2))
 $\therefore A(\text{BA})$ is symmetric. similarly (AB) A is symmetric

rect.
 $\therefore A(\text{BA})$ is symmetric. similarly (AB) A is symmetric
 $\therefore A(\text{BA})$ is symmetric. similarly (AB) A is symmetric
 $\therefore A(\text{BA})$ is symmetric. simil a b c d orrect.

(1)

(2) & (3) we get

(3)

(27) (B). $A = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

(27) (B). $A = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

(27) (B). $A = \begin{vmatrix} k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

(27) (B). $A = \begin{vmatrix} k &$ + ay ⇒ y = $\frac{1-6^2}{a + bc}$ = $\frac{3-6^2-2k+24=0}{2(k+6)(k-4)=0 \Rightarrow k=6,4}$

+ ay ⇒ y = $\frac{1-b^2}{a + bc}$ = $\frac{-b^2}{a + bc}$ = $\frac{1-b^2}{b^2}$ = $\frac{1-b^2}{a^2}$ = $\frac{1-b^2}{a^2}$ = $\frac{1-b^2}{a^2}$ = $\frac{1-b^2}{a^2}$ = $\frac{1-b^2}{a^2}$ = $\frac{1-b$ ay \Rightarrow $y = \frac{1-b^2}{a + bc}$ (5)
 $\Rightarrow (k+6)(k-4) = 0 \Rightarrow k = -6, 4$
 $\Rightarrow (k+6)(k-4) = 0 \Rightarrow 6k = 2$
 $\Rightarrow (k-6)(k-4) = 0 \Rightarrow 6k = 2$
 $\Rightarrow (k-1)c^2 - b^2 + b^2c^2$
 $\Rightarrow (k-1)(k-4) = 0$

(a) b b c c -1 b $\Rightarrow (k-2)(k-4) = 0 \Rightarrow 6k = 2$

(b) adj A = $\begin{bmatrix} 1 \\ -c & a \$ c = $\frac{2}{a + bc}$ (28) (D). A (u₁ + u₂) = $\begin{pmatrix} 1 \ 1 \ 0 \end{pmatrix}$; |A| = 1; A⁻¹ = $\frac{1}{|A|}$ adj A

c 28) (D). A (u₁ + u₂) = Λ^{-1} $\begin{pmatrix} 1 \ 1 \ 0 \end{pmatrix}$; |A| = 1; A⁻¹ = $\frac{1}{|A|}$ adj A

c $\begin{pmatrix} 2 \ -2-b^2 + b^$ <u>Be) z</u>(4)

⇒ 8 - k² - 2k- -4k + 16 = 0

⇒ - k² - 2k- + 2t - 4k + 16 = 0

⇒ k + 6) (k-4) = 0 ⇒ k + 2k - 24 = 0

⇒ (k+ 6) (k-4) = 0 ⇒ k = -6, 4

Number of values of k is 2

(28) (D). A (u₁ + u₂) = \begin 1-c²
 $\Rightarrow -k^2 - 2k + 24 = 0 \Rightarrow k^2 + 2k - 24 = 0$
 $\Rightarrow (k+6)(k-4) = 0 \Rightarrow k-6, 4$
 $\Rightarrow (k+6)(k-4) = 0 \Rightarrow k-6, 4$

Number of values of k is 2
 $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$ adj A = $\begin{bmatrix} 1 \\ -c^2 - b^2 + b^2c^2 \\ -c & a \end{bmatrix}$

(28) (D). A (u₁+u₂) = $\frac{1}{2}$ ⇒ $\frac{1}{2}$
 $\frac{1}{2}$
 (29) (C). Subtracting $P^3 - P^2Q = Q^3 - Q^2P$ $P^2 (P - Q) + Q^2 (P - Q) = 0$ $(P^2+Q^2)(P-Q)=0$ If $|P^2 + Q^2| \neq 0$ then $P^2 + Q^2$ is invertible \Rightarrow P – Q = 0 contradiction Hence $|P^2 + Q^2| = 0$ k 1 8 4k $k^2 + 2k - 24 = 0$
 $k = -6, 4$
 $k = 2, 4$
 $k = 1; A^{-1} = \frac{1}{|A|}$ adj A
 $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$
 $k^2Q = Q^3 - Q^2P$
 $k^2 + Q^2$ is invertible
 $\frac{1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$
 $k^2 - 4$
	- **(30) (B).** For no solution, $\frac{1}{1}$ $+1$ 8 4k $k^2 + 4k + 3 = 8k$; $k^2 - 4k + 3 = 0$; $k = 1, 3$

If k = 1 then $\frac{8}{1+3} = \frac{4.1}{3-1}$ False And If $k = 3$ then $\frac{6}{5} \neq \frac{4!}{9!}$ True -1 and -1 True Therefore, $k = 3$ Hence only one value of k. **(31) (B).** $|P| = 1 (12 - 12) - \alpha (4 - 6) + 3 (4 - 6) = 2\alpha - 6$ $|P| = |A|^2 = 16$ $2\alpha - 6 = 16 \Rightarrow \alpha = 11$. **(32) (B).** $B = A^{-1}A' \Rightarrow AB = A'$ $ABB' = A'B' = (BA)' = (A^{-1}A'A)' = (A^{-1}AA')' = A$ \Rightarrow BB'=I (33) **(C).** $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$ $=\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha - 1 & \beta - 1 \\ 1 & \alpha^2 - 1 & \beta^2 - 1 \end{vmatrix}$ $= ((\alpha - 1) (\beta^2 - 1) - (\beta - 1) (\alpha^2 - 1))^2$ $= (\alpha - 1)^2 (\beta - 1)^2 (\alpha - \beta)^2$ $\Rightarrow k=1$ **(34) (C).** $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $a+4+2b=0$; $2a+2-2b=0$; $a+1-b=0$ $2a - 2b = -2$; $a + 2b = -4$ Solving, $b = -1$, $a = -2$; $(-2, -1)$ **(35) (B).** $x_1(2-\lambda)-2x_2+x_3=0$ $2x_1 + x_2(-\lambda - 3) + 2x_3 = 0$; $-x_1 + 2x_2 - \lambda x_3 = 0$ 1+α⁻ +β⁻ 1+α⁻ +β⁻ 1+α⁻ +β⁻ 1+α⁻ +β⁻ 1

(39) (**D**). A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

(39) (**D**). A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

(a b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ \alpha^2 & \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \alpha -1 & \beta -1 \\$ (a) $\begin{bmatrix}\n1 & 1 & 1 \\
\alpha & \beta & 1 \\
\alpha^2 & \beta^2 & 1\n\end{bmatrix}\n\begin{bmatrix}\n1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \beta & \beta^2\n\end{bmatrix} = \n\begin{bmatrix}\n1 & 0 & 0 \\
1 & \alpha - 1 & \beta - 1 \\
1 & \alpha^2 - 1 & \beta^2 - 1\n\end{bmatrix}^2\n\begin{bmatrix}\n2 & -3 \\
-4 & 1\n\end{bmatrix} = \n\begin{bmatrix}\n4 + 12 & -6 \\
-8 - 4 & 12 + 12 \\
-8 - 4 & 12 + 12\n\$ 1 1 1 1 1 1 1 0 0 $A^2 = \begin{bmatrix} 2 & -3 \ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \ 1 & \alpha & \alpha^2 \ 1 & \beta & \beta^2 \end{bmatrix}^2$
 $\alpha^2 - 1 \begin{bmatrix} 6 & -1 \ 1 & 6 \end{bmatrix} (\alpha^2 - 1) (\alpha^2 - 1)^2$
 $\alpha - 1 \begin{bmatrix} 6 & -1 \ 2 & 0 \end{bmatrix} (\alpha - 1) (\alpha^2 - 1)^2$
 $\alpha - 1 \begin{bmatrix} 6 & -1 \end{bmatrix} (\alpha - 1)$ + α + p 1+ α + p 1+ α + p 1+ α + p

+ α - p 2 + α 3+ p 3 + α 4+ p⁴

(39) (D). A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

(a β $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha -1 & \beta -1 \\ 1 & \alpha^2 -1 & \beta^2 -1 \end{vmatrix}$

(a -1) (39) (b), $A = \begin{bmatrix} 1 & 1 \ -4 & 1 \end{bmatrix}$
 $\alpha = \begin{bmatrix} 1 & 1 \ 2 & 1 \ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \ 1 & \alpha & \alpha^2 \ 1 & \beta & \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \ 1 & \alpha-1 & \beta-1 \ 1 & \alpha^2-1 & \beta^2-1 \end{bmatrix}^2$
 $\alpha = \begin{bmatrix} 2 & -3 \ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \ -4 & 1 \end{bmatrix}$ $(2 - \lambda)(\lambda^2 + 3\lambda - 4) + 2(-2\lambda + 2) + (4 - \lambda - 3) = 0$ $(\lambda - 1) (\lambda + 3) (\lambda - 1) = 0 \Rightarrow \lambda = 1, 1, -3$ Two elements **(36) (C).** $\begin{vmatrix} \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$ $α^2$ $β^2$ $β^$ -1) (β²-1)-(β-1)(α²-1)²
 $\frac{1}{2}$ (β-1)² (α-β)²
 $\frac{1}{2}$ (β-1)² (α-β)²
 $\frac{1}{2}$ (β-1)² (α-β)²
 $\frac{1}{2}$ (β-1)² (α-β)²
 $\frac{1}{2}$ (1 2 a] $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$
 $\begin{bmatrix} 3\lambda^2 + 12$ (a-1)²(β-1)²(a-β)²
 $=1$
 $\begin{bmatrix} 2 & 2 \\ 2 & 1 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix}$
 $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 6 & 9 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

Adj (3A² + 12A = $\begin{bmatrix} 22 \\ -84 \\ -84 \end{bmatrix}$

Adj (3A² B $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \beta^2 & 1 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & \alpha -1 & \beta -1 \\ 1 & \alpha^2 -1 & \beta^2 -1 \end{vmatrix}$
 $-1)(\beta^2 - 1) - (\beta - 1)(\alpha^2 - 1)^2$
 $-1)^2(\beta - 1)^2(\alpha - \beta)^2$
 $-1)^2(\beta - 1)^2(\alpha - \beta)^2$
 $\begin{vmatrix} 2 & 1 & 2 \\ 2 & 1 & 2 \\ 0 & 9 & 0 \end{vmatrix} = \begin{vmatrix} 9 & 0 & 0 \\$ $α² β² || 1 β β² || 1 α² - 1 β² - 1 |
\n
$$
= 1
$$
\n(α-1) (β² - 1) – (β-1) (α² - 1)²
\n= 1
\n1 2 2 || 2 1 2 || 2 1 2 || 2 0 0 0]
\n= 1
\n1 2 2 || 2 1 2 || 2 1 2 || 0 0 0]
\n= 1
\n1 2 2 || 2 1 2 || 2 0 0 0$ $(\lambda + 1) - \lambda (\lambda^2 + 1) - (\lambda + 1) = 0$ $(\lambda + 1)(1 + \lambda(\lambda - 1) - 1) = 0$; $\lambda = -1$ or 0 or 1 (4) **(37) (A).** A (adj A) = $|A|I_n = AA^T$ (Given); $|A| = 10a + 3b$ (a 2 b)[2 -2 b] (b 0 9]

a + 4 2 b = 0; 2 a + 2 - 2 b = 0; a + 1 - b = 0

2a - 2b = -2; a + 2b = -4

Solving, b = -1, a = -2; (-2,-1)

5) (B). $x_1(2-\lambda)-2x_2+x_3=0$

2x + $x_2(-\lambda)-2x_2+x_3=0$

2x + $x_2(-\lambda)-2x_2+x_3=0$

2x + x $A^1 = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$; $AA^T \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ a + 4 + 2b = 0 ; 2a + 2 - 2b = 0 ; a + 1 - b = 0

Solving, b = -1, a = -2 ; (-2, -1)

3). $x_1 (2-\lambda) - 2x_2 + x_3 = 0$
 $2x_1 + x_2 (-\lambda - 3) + 2x_3 = 0$; $-x_1 + 2x_2 - \lambda x_3 = 0$
 $\begin{vmatrix}\n2 & \lambda & 1 \\
2 & -\lambda & -2 \\
-1 & 2 & -\lambda\n\end{vmatrix} = 0$
 $(2-\lambda)(\lambda^$ (C) $\begin{vmatrix} 2 & 1 & -2 \ 2 & 1 & 2 \end{vmatrix} = 2 - 2$ b $\begin{vmatrix} 0 & 9 & 0 \ 0 & 0 & 9 \end{vmatrix}$
 $a + 4 + 2b = 0$; $2a + 2b = -2$; $a + 2b = -4$

Solving, $b = -1$, $a = -2$; $(-2, -1)$
 $2x + x_2(-\lambda - 3) + 2x_3 = 0$; $-x_1 + 2x_2 - \lambda x_3 = 0$
 $2x_1 + x_2(-\lambda - 3) + 2x_$ $\Rightarrow \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$

8 4 .1 1 3 3 1 8 4 . 3 6 9 1 2 2 2 2 3 3 2 2 3 3 4 4 3 1 1 1 1 1 1 1 1 2 2 2 2 2 2 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 2 2 1 2 a 9 0 0 2 1 2 2 1 2 0 9 0 a 2 b 2 2 b 0 0 9 ^T 5a b 5a 3 10a 3b 0 3 2 b 2 0 10a 3b 2 2 25a b 15a 2b 10a 3b 0 15a 2b 13 0 10a 3b 15a – 2b = 0 a = 2b/15 and 10a + 3b = 13 13 3b a 10 2b 13 3b 15 10 4b = 39 – 9b 13b = 39 b = 3 2 6 2 a 3 5a 2 15 15 5 5a + b = 2 + 3 = 5 **(38) (B).** 2 1 1 1 1 a 1 (a b) (1 a) (b a) a b 1 = a – b – 1 + a + b – a² = –(a² – 2a + t) = – (a– 1)² = 0 a = 1 x + y + z = 1 x + by + z = 0 Two plane should be parallel, b = 1 **(39) (D).** 2 3 A 4 1 ² 2 3 2 3 4 12 6 3 16 9 A 4 1 4 1 8 4 12 1 12 13 ² 48 27 24 36 3A ; 12A 36 39 48 12 ² 72 63 3A 12A 84 51 ² 51 63 Adj (3A 12A) 84 72 **(40) (D).** 1 k 3 7 3 k 2 0 k 2 2 4 3 x + ky + 3z = 0 …..(i) 3x + ky – 2z = 0 .….(ii) 2x + 4y – 3z = 0 …..(iii) On solving (i) and (ii), 2x – 5z = 0 …..(iv) On solving (iii) and (iv), 4y = –2z 2 2 5 z z xz 2 10 y z / 4 **(41) (A).** x 4 2x 2x 2x x 4 2x 2x 2x x 4 = (A + Bx) (x – A)² , Put x = 0, 3 4 0 0 0 4 0 A 0 0 4 ; A = – 4

| MATRICES AND DETERMINANS | Q.B.S. SOLUTIONS | Self parameters | | |
|--|--|--|--|--|
| \n $Pux = 1, \quad\n \begin{vmatrix}\n -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & -3 & -2\n \end{vmatrix} = (A + B) (1 - A)^2$ \n | \n $\begin{vmatrix}\n -3(9-4) - 2(-6-4) - 2(4-6) \\ -15+20-2(0-4-1+1)25\n \end{vmatrix}$ \n | \n $\begin{vmatrix}\n -3(9-4) - 2(-6-4) - 2(4-6) \\ -15+20-2(0-4-1+1)25\n \end{vmatrix}$ \n | \n $\begin{vmatrix}\n -3(9-4) - 2(-6-4) - 2(4-6) \\ -15+20-2(0-4-1+1)25\n \end{vmatrix}$ \n | \n $\begin{vmatrix}\n -3(9-4) - 2(9-4) - 2(9-4-1) & -3(9-2)(9-1-1) \\ -3(9-2) - 2(9-4-1) & -3(9-2)(9-2-1) & -3(9-2-2)(9-2-1) \\ 2(9-2) - 2(9-2-1) & 2(9-2-1) & -3(9-2-1) & -3(9-2-1) & -3(9-2-1) & 0\n \end{vmatrix}$ \n |
| \n $P_1 = \begin{vmatrix}\n 2 & 1 & 1 \\ 5 & 3 & 2 \\ 2 & 2 & 1\n \end{vmatrix} = a^2 - a + 1$ \n | \n $\begin{vmatrix}\n 45 & 10 & 0 & 0 & 0 & 0 & 0 \\ 44 & 1 & 3 & 0 & 0 & 0 & 0 \\ 45 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 46 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 47 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 49 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\$ | | | |

 D 2 5 2 a 3 2 a 1 a 1 1 1 2 D 2 3 5 a 4 2 3 a 1 = ± 3 4 0 ^A–1 = AT⁼ cos sin sin cos Also, ⁿ cos (n) sin (n) ^A sin (n) cos (n) ⁵⁰ cos (50) sin (50) ^A sin (50) cos (50) 3 / 2 1/ 2 1/ 2 3 / 2 cos sin sin cos ² cos sin cos sin sin cos sin cos cos 2 sin 2 sin 2 cos 2 ³ cos 2 sin 2 cos sin sin 2 cos 2 sin cos cos3 sin 3 sin 3 cos3 ³² cos32 sin 32 0 1 A sin 32 cos32 1 0 cos 32 = 0 & sin 32 = 1 ³² = (4n + 1) , n I ² (4n 1) , n I ⁶⁴ ; for n 0 64 **(45) (D).** Put 2 c ^b 2 in determinant of A 3 2 c 6c 12c 8 | A | [2,16] ⁴ (c – 2)³ [8, 64] c [4, 6] **(46) (A).** Roots of the equation x² + x + 1 = 0 are = and = ² where , ² are complex cube roots of unity 2 2 2 y 1 y 1 1 y R R R R 1 1 2 3 2 2 1 1 1 y y 1 1 y Expanding along R¹ , we get = y.y² D = y³ **(47) (A).** 1 1 1 2 1 3 1 n 1 0 1 0 1 0 1 0 1 ⁼ 0 78 0 1 1 1 2 3 n 1 1 78 0 1 0 1 n (n 2) ⁷⁸ 2 n = 13, –12 (reject) We have to find inverse of 1 13 0 1 1 13 0 1 **(48) (C).** 2 2 2 2 2 1 1 1 1 1 1 ¹ A 1 1 3 1 1 1 0 0 0 0 1 0 1 0 A⁴ = I A30 = A28 × A³ = A³

 $=\begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$

(49) (C). For non-trivial solution

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\n(C). For non-trivial solution
\n(C). For non-trivial solution
\n
$$
\left|\begin{array}{ccc|c}\n2 & 2a & a \\
2 & 3b & b \\
2 & 4c & c\n\end{array}\right| = 0 \quad , \quad \left|\begin{array}{ccc|c}\n1 & 2a & a \\
1 & 3b & b \\
1 & 4c & c\n\end{array}\right| = 0
$$
\n
$$
\left|\begin{array}{ccc|c}\n1 & 2a & a \\
2 & 3b & b \\
2 & 4c & c\n\end{array}\right| = 0 \quad , \quad \left|\begin{array}{ccc|c}\n1 & 2a & a \\
1 & 3b & b \\
1 & 4c & c\n\end{array}\right| = 0
$$
\n
$$
\left|\begin{array}{ccc|c}\n54 \text{ of } 1 & 2 & 3 \\
-1 & -1 & 3\n\end{array}\right| \Rightarrow |A| = 6
$$
\n
$$
\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ in A.P.}
$$
\n
$$
\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ in A.P.}
$$
\n**6**
\n**6**

(50) **(A).**
$$
|B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \ b_{21} & b_{22} & b_{23} \ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{12} & 3^2 a_{13} \ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}
$$

\n $\Rightarrow 81 = 3^3 \cdot 3 \cdot 3^2 |A| \Rightarrow 3^4 = 3^6 |A|$
\n $\Rightarrow |A| = 1/9$

$$
(51) \quad 672.00
$$

Trace $(AA^T) = \Sigma a_{ij}^2 = 3$ Hence, number of such matrices = ${}^9C_3 \times 2^3$ = 672.00

(52) (A).
$$
D = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix}
$$

\n $-2(1-x)+(y'+x') = \pm 10$
\n $-2+2x'+y'+x' = \pm 10$
\n $3x'+y'=12 \text{ or } 3x'+y' = -8$
\n $\lambda = 3, -2$
\n(53) (C). $D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$ (R₃ \rightarrow R₃ - 2R₁ + 3R₂)
\n $= \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$

(Q.B.-SOLUTIONS STUDY MATERIAL: MATHEMATICS

n

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite
 $1 \quad 2a$ a
 $1 \quad 3b$ b
 $= 0$
 $1 \quad 4c$ c
 $= 1 \quad 4c$ c
 $= 2 \times \beta = -2$

So for infinite 1 3b b 0 **(Q.B.- SOLUTIONS** STUDY MATERIAL: MATHEMATICS

1 2a a

1 2a a

1 3b b = 0

1 4c c

1 4c c (54) (C), A = $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ = $|A| = 6$

1 4c c (54) (C), A = $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ **(Q.B.- SOLUTIONS** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$

Hence $3\alpha + \beta = 4 \& 4\alpha + 2\beta = 4 \Rightarrow \alpha = 2 \& \beta = -2$

So for in **(O.B.- SOLUTIONS)** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$
 $\alpha b = 0$

Hence $3\alpha + \beta = 4 \& 4\alpha + 2\beta = 4 \Rightarrow \alpha = 2 \& \beta = -2$ **(O.B.-SOLUTIONS** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions in will have infinite

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So for inf **(O.B.- SOLUTIONS**)

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$

Hence $3\alpha + \beta = 4 \approx \alpha - 2 \approx \beta = -2$

So for infinite solution $2\mu - 2 = \delta \Rightarrow 2\mu \neq \delta + 2$ **(Q.B.-SOLUTIONS** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$
 \therefore Hence $3\alpha + \beta = 4 \& 4\alpha + 2\beta = 4 \Rightarrow \alpha = 2 \& \beta = -2$

So f **(O.B.-SOLUTIONS)** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$

Hence $3\alpha + \beta = 4 \approx \alpha - 2 \approx \beta - -2$

So for infinite solutio **(O.B.-SOLUTIONS** STUDY MATERIAL: MATHEMATICS

Now let $P_3 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 \equiv \alpha P_1 + \beta P_2$
 $+ \epsilon_2 = 0$
 $+ \epsilon_3 = 0$
 $+ \epsilon_4 = 0$
 $+ (2ab-3ab) = 0$
 $+ (2ab$ **(2).B. SOLUTIONS**

Now let $P_2 = 4x + 4y + 4z - \delta = 0$.

If the system has solutions it will have infinite

solution, so $P_3 = \alpha P_1 + \beta P_2$
 $\Rightarrow \alpha = 2 \& \beta = -2$

So for infinite solution $2\mu - 2 - \delta = 2 \& \beta = -2$
 $+(2ab - 3ab) = 0$

Sys Now let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution, so $P_3 \equiv \alpha P_1 + \beta P_2$ Hence $3\alpha + \beta = 4 \& 4\alpha + 2\beta = 4 \Rightarrow \alpha = 2 \& \beta = -2$ So for infinite solution $2\mu - 2 = \delta \Rightarrow 2\mu \neq \delta + 2$ System inconsistent STUDY MATERIAL: MATHEMATICS

let P₃ = 4x + 4y + 4z - δ = 0.

system has solutions it will have infinite

ion, so P₃ = α P₁ + β P₂
 ϵ 3 $\alpha + \beta = 4$ & $4\alpha + 2\beta = 4 \Rightarrow \alpha = 2$ & $\beta = -2$

r infinite solution 2STUDY MATERIAL: MATHEMATICS

let P₃ = 4x + 4y + 4z - δ = 0.

system has solutions it will have infinite

ion, so P₃ = α P₁ + β P₂

e 3 α + β = 4 & 4 α + 2 β = 4 $\Rightarrow \alpha$ = 2 & β = -2

r infinite STUDY MATERIAL: MATHEMATICS

let P₃ = 4x + 4y + 4z - δ = 0.

system has solutions it will have infinite

ion, so P₃ = α P₁ + β P₂

e 3 $\alpha + \beta = 4$ & $4\alpha + 2\beta = 4 \Rightarrow \alpha = 2$ & $\beta = -2$

r infinite solution $2\mu -$ STUDY MATERIAL: MATHEMATICS

let P₃ = 4x + 4y + 4z - δ = 0.

e system has solutions it will have infinite

tion, so P₃ = α P₁ + β P₂

ce $3\alpha + \beta = 4$ & $4\alpha + 2\beta = 4 \Rightarrow \alpha = 2$ & $\beta = -2$

or infinite solution STUDY MATERIAL: MATHEMATICS

r let P₃ = 4x + 4y + 4z - δ = 0.

e system has solutions it will have infinite

tion, so P₃ = α P₁ + β P₂

ce 3α + β = 4 & 4α + 2β = 4 ⇒ α = 2 & β = -2

or infinite solution 2μ - 2 = δ STUDY MATERIAL: MATHEMATICS

let P₃ = 4x + 4y + 4z - δ = 0.

e system has solutions it will have infinite

tion, so P₃ = α P₁ + β P₂

ce 3α + β = 4 & 4α + 2β = 4 ⇒ α = 2 & β = -2

or infinite solution 2μ - 2 = δ ⇒ AL: MATHEMATICS

= 0.

will have infinite
 $\therefore 4 \Rightarrow \alpha = 2 \& \beta = -2$
 $= \delta \Rightarrow 2\mu \neq \delta + 2$
 $\frac{4}{\text{A}} = \frac{|\text{A}|^3}{3^3} = \frac{(6)^3}{(3)^3} = 8$

ne

tion of the given ERIAL: MATHEMATICS
 $\delta = 0$.

it will have infinite
 P_2
 $\beta = 4 \Rightarrow \alpha = 2 \& \beta = -2$
 $-2 = \delta \Rightarrow 2\mu \neq \delta + 2$
 $\frac{A|^4}{^3 |A|} = \frac{|A|^3}{3^3} = \frac{(6)^3}{(3)^3} = 8$

a line

bution of the given STUDY MATERIAL: MATHEMATICS

Now let P₃ = 4x + 4y + 4z - δ = 0.

If the system has solutions it will have infinite

solution, so P₃ = α P₁ + β P₂

Hence 3 α + β = 4 & α + 2 β = 4 $\Rightarrow \alpha$ = 2 & $\$ STUDY MATERIAL: MATHEMATICS

by let $P_3 = 4x + 4y + 4z - \delta = 0$.

the system has solutions it will have infinite

lution, so $P_3 = \alpha P_1 + \beta P_2$

for infinite solution $2\mu - 2 = \delta \Rightarrow 2\mu \neq \delta + 2$

stem inconsistent

for infinite HEMATICS

nfinite

2 & $\beta = -2$
 $\epsilon \delta + 2$
 $= \frac{(6)^3}{(3)^3} = 8$

given

(54) (C).
$$
A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow |A| = 6
$$

$$
\frac{|\text{adj }B|}{|\text{ c}|} = \frac{|\text{adj }(\text{adj }A)|}{|\text{ 9A }|} = \frac{|A|^4}{3^3 |A|} = \frac{|A|^3}{3^3} = \frac{(6)^3}{(3)^3} = 8
$$

(55) (A). For planes to intersect on a line There should be infinite solution of the given system of equations For infinite solutions

3 a 3 a 3 a ij a 3 0 2 1 ¹ D 1 1 1 x y 1 **(53) (C).** 3 3 1 2 3 4 5 D 1 2 3 (R R 2R 3R) 4 4 4 3 4 5 1 2 3 0 0 0 0 1 4 2 1 7 5 0 1 5 3 + 9 = 0 = –3 ^z 1 4 1 1 7 0 1 5 5 13 – = 0 = 13 + = –3 + 13 = 10 **(56) (A).** 7x + 6y – 2z = 0 (1) 3x + 4y + 2z = 0 (2) x – 2y – 6z = 0 (3) 7 6 2 3 4 2 0 1 2 6 Infinite solutions Now (1) + (2) y = –x put in (1), (2) & (3) all will lead to x = 2z **(57) (C).** R¹ R¹ + R³ – 2R² a c 2b 0 0 x b x 3 x 2 x c x 4 x 3

$$
f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \ x+b & x+3 & x+2 \ x+c & x+4 & x+3 \end{vmatrix}
$$

= (a+c-2b)((x+3)²-(x+2)(x+4))
= x² + 6x + 9 - x² - 6x - 8 = 1
 \Rightarrow f(x) = 1 \Rightarrow f(50) = 1