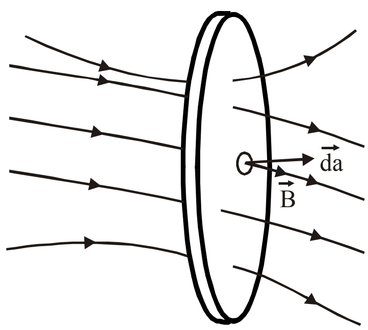
4A

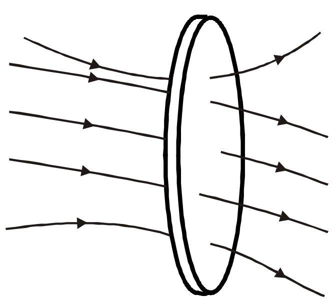
ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX

In analogy with the electric flux ϕ_E, a magnetic flux ϕ_B of the magnetic field for a surface is defined. Imagine dividing a mathematical surface into infinitesimal area elements. The direction of an area element $\vec{da}$ at a point on the surface is perpendicular to the surface at that point. A typical element for a surface is shown in fig. along with the magnetic field B at a point.



The magnetic flux for an infinitesimal area da is given by $d\phi = \vec{B}.\vec{da}$

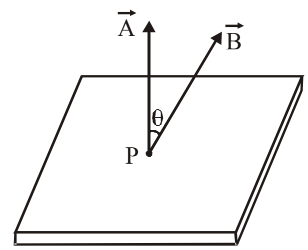


The magnetic flux for a surface is proportional to the number of field lines intersecting the surface.

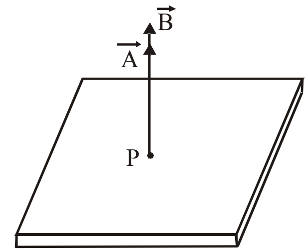
The magnetic flux $d\phi$ for the area element "da" is

$$d\phi = \vec{B}.\vec{da}\,.$$

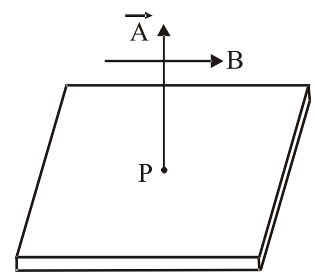
If the magnetic field $\vec{B}$ makes an angle θ with the normal to the surface as shown in fig. then the normal component of the field is B cos θ and in this case, the magnetic flux is given by $\phi = BA\cos\theta$ or $\phi = \vec{B}.\vec{A}$



ϕ = BA cosθ



ϕ_{max} = BA



$\phi_{min} = 0$

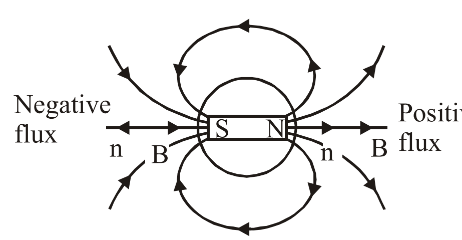
So, magnetic flux linked with a closed surface may be defined as the product of the surface area and the normal component of the magnetic field acting on that area. It may also defined as the dot or scalar product of magnetic field and surface area. The magnetic flux for a general surface is obtained by integrating (summing) the contributions $d\phi$ as the area element da ranges over the surface.

Thus, $\phi = \int \vec{B}.\vec{da}$

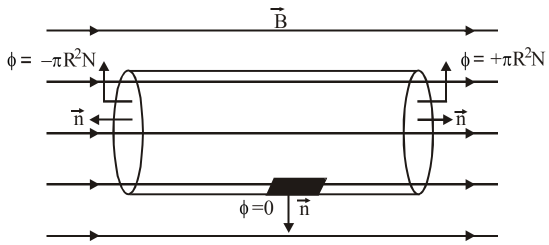
Physically it represents total lines of induction passing through a given area.

POSITIVE AND NEGATIVE FLUX

In case of a body present in a field, either uniform or non-uniform. Out ward flux is taken to be positive while inward negative.



Total flux = 0
Nonuniform field



Total flux = 0
uniform field

If the normal drawn on the surface is in the direction of the field, then the flux is taken as positive.
In this case, θ is 0° or $\theta < 90°$ then the flux is taken as positive.
If the normal on the surface is opposite to the direction of the field, then $\theta = 180°$. In this case, the magnetic flux is taken as negative.

Magnetic flux density, $B = \dfrac{\phi}{A}$

Lines of force are imaginary, but as magnetic flux associated with elemental area $\vec{da}$ in a field $\vec{B}$, so flux is a real scalar physical quantity
Dimensions : $[ML^2T^{-2}A^{-1}]$
Unit : SI : volt × sec.
It is known as weber (Wb) or $T.m^2$ (as tesla = Wb/m^2)
CGS maxwell (Mx)
1 Wb = 1 V × s = 10^8 emu of pot^n. × s = 10^8 Mx
[1 volt=10^8 emu of potential]

Special Note : $\phi_B = \int \vec{B} \cdot \vec{da}$

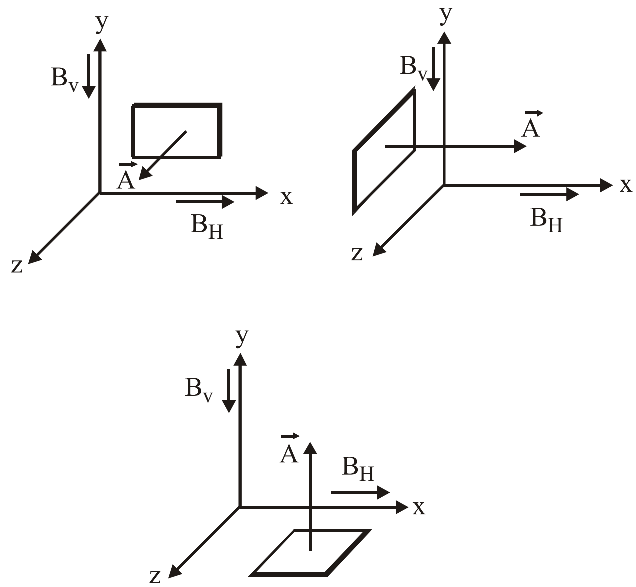
So if $\vec{B} = 0$, then $\phi_B = 0$.

But if $\phi_B = 0$, $\vec{B}$ may or may not be zero.
Because if $B \neq 0$, flux for some elements may be positive while for the other negative giving zero net flux, e.g., for a closed surface enclosing a dipole $\phi_T = 0$ but $d\phi$ and hence $\vec{B} \neq 0$.

Example 1 :
At a given place, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in fig. What is the total flux of earth's magnetic field associated with an area A, if the area A is in (a) x-y plane (b) y-z plane and (c) z-x plane?

Sol. $\because \vec{B} = \vec{i}B_H - \vec{j}B_V$ = const. and

$\phi = \int \vec{B}.d\vec{a} = \vec{B}.\vec{A}$ [as $\vec{B}$ = const.]



for area in x-y plane : $\vec{A} = A\hat{k}$

$\phi_{xy} = (\hat{i}B_H - \hat{j}B_V) . (\hat{k}A) = 0$ $[\because \hat{i} . \hat{k} = \hat{j} . \hat{k} = 0]$

for area A in y-z plane : $\vec{A} = A\hat{i}$

$\phi_{yz} = (\hat{i}B_H - \hat{j}B_V) . (\hat{i}A) = B_H A$ $[\because \hat{i} . \hat{i} = 1 \ \& \ \hat{j} . \hat{i} = 0]$

for area S in z-x plane : $\vec{A} = A\hat{j}$

$\phi_{zx} = (\hat{i}B_H - \hat{j}B_V) . (\hat{j}A) = -B_V A$

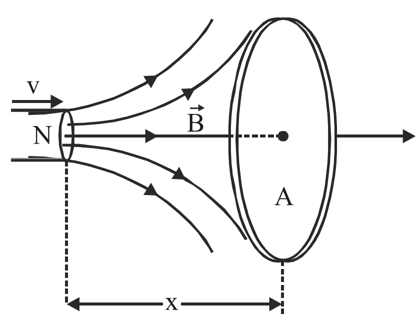
$[\because \hat{i} . \hat{j} = 0 \text{ and } \hat{j} . \hat{j} = 1]$

Negative sign implies that flux is directed vertically down.

DIFFERENT WAYS WHICH CAN VARY THE MAGNETIC FLUX

Magnetic flux in planar area $\vec{A}$ due to an uniform magnetic field $\vec{B}$, $\phi = BA\cos\theta$
So flux linked with a circuit will change only if field B, area A, orientation θ or any combination of these changes.

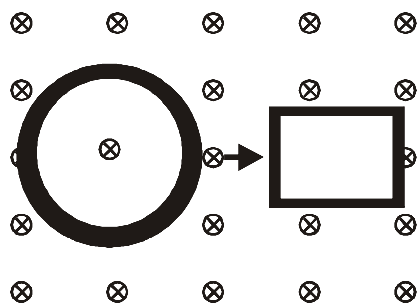
(1) By varying the magnetic field $\vec{B}$ with time :
Due to motion of magnet B will change with time.
Or if magnetic field produced due to current carrying loop then due to change in current change in B will occur with time.



Flux changes as B changes

(2) By varying the area of the conducting loop $\vec{A}$ with time:
The second method of inducing a change in flux is to vary the area of the conducting loop with time.

(a) Change in Shape of the loop : Consider a square loop of side "a" placed perpendicular to a uniform and steady magnetic field B. Suppose the square loop transforms into a circular loop of the same circumference.



Initial area of the loop = a^2

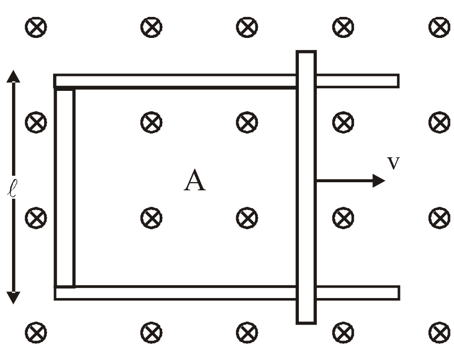
The initial flux through the loop $\phi_i = Ba^2$

The final radius of the circular loop $r = \frac{4a}{2\pi} = \frac{2a}{\pi}$

The final flux through the loop $\phi_f = B\pi r^2 = \frac{4Ba^2}{\pi}$

The net change in flux $(\phi_f - \phi_i) = Ba^2\left(\frac{4}{\pi} - 1\right)$

(b) Rod Translating in a Π Circuit : Consider a U shaped or a Π shaped conducting wire placed with its plane perpendicular to a uniform magnetic field $\vec{B}$.

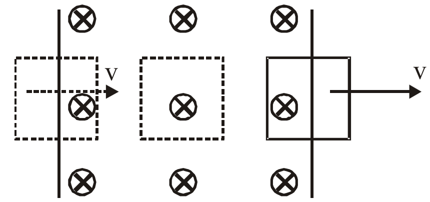


Flux changes as A changes

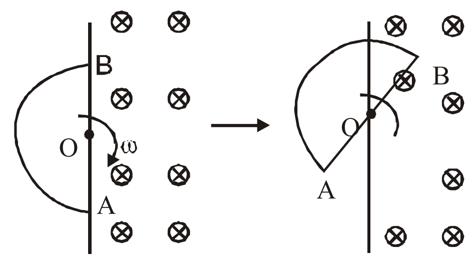
A conducting rod is placed on this wire so as to short the two arms of the U as shown in fig. Let the distance between the two arms of the U be ℓ. We now have a closed conducting loop resting in a uniform magnetic field with one of the sides of the loop free to move. Now suppose the rod translates to the right with a speed v. Then with time, the area enclosed by this loop will keep increasing and with it the flux through the loop will also change.

(3) Loop entering or leaving a finite region of magnetic field:

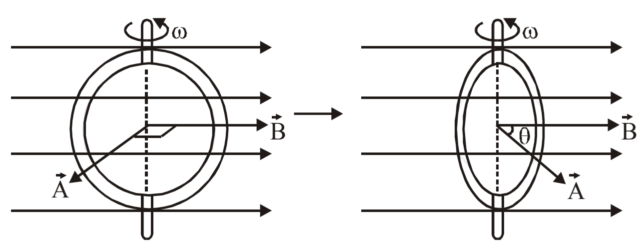
(a) Rectangular loop passing through magnetic field :
Method for creating a loop with a time varying area is to have a closed loop enter or leave a region of magnetic field. Here the actual area of the loop does not change with time. However, the effective area of the loop through which magnetic field lines pass, varies with time.



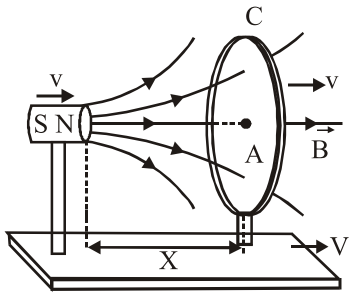
(b) Loop rotating in and out of a finite region of magnetic field : Consider a semicircular loop placed at the edge of a magnetic field. The loop rotates with an angular velocity ω about an axis perpendicular to the plane of the paper and passing through O. As the loop rotates, the area immersed in the magnetic field varies with time.



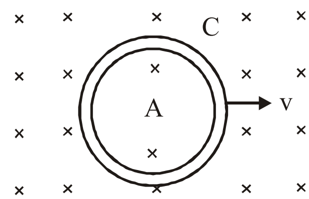
(4) Effect of time varying angle between the area vector and the magnetic field vector :
Consider a circular loop placed in an uniform magnetic field $\vec{B}$ such that the plane of the loop is initially parallel to the magnetic field. Let us now rotate the loop about its diameter with a constant angular velocity ω with time and consequently the flux through the loop also varies $(\phi = BA\cos\theta)$
Since θ varies so ϕ also varies.



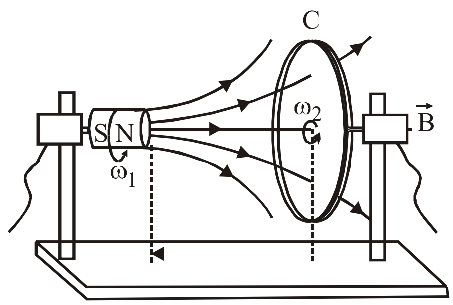
The flux linked with a loop C will not change with time :
If B, A and θ does not change with time, then
$\phi = BA\cos\theta = B(N\pi R^2)\cos 0 = N\pi R^2 B = \text{const.}$



$\phi \neq 0$ but $\frac{d\phi}{dt} = 0$



$\phi \neq 0$ but $\frac{d\phi}{dt} = 0$

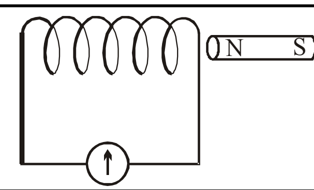
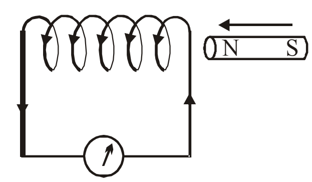
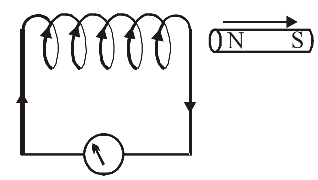
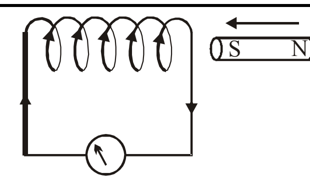
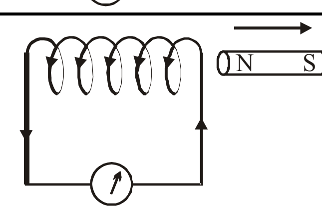


$\phi \neq 0$ but $\frac{d\phi}{dt} = 0$

FARADAY'S EXPERIMENTS

A sensitive galvanometer (G) is connected to the free ends of a coil A, a strong magnet NS is brought near the coil and away from it and the following observations are made.

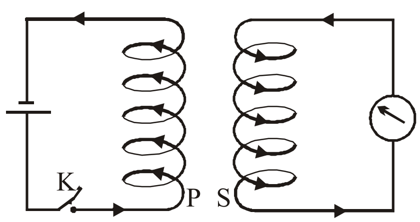
(i) When the magnet with its N-pole facing the coil is moved towards the coil, the galvanometer shows some deflection while the magnet is in motion showing that an electro current is produced in the coil even though no conventional source of e.m.f. is in the circuit. The current is called the induced current and the e.m.f. responsible for it is called the induced e.m.f.

Experiment	Observation	
1. Place a magnet near a conducting loop with a galvanometer in the circuit	No current flows through the galvanometer	
2. Move the magnet towards the loop	The galvanometer register a current	
3. Reverse the direction of motion of the magnet	The galvanometer deflection reverses	
4. Reverse the polarity of the magnet and move the magnet towards the loop	The galvanometer deflection reverses	
5. Keep magnet fixed and move the coil towards the magnet	The galvanometer register a current	

Experiment	Observation
6. Increases the speed of the magnet	The deflection increases
7. Increase the strength of the magnet	The deflection increases
8. Increase the diameter of the coil	The deflection increases
9. Fix the speed of the magnet but repeat the experiment with the magnet closer to the coil.	The deflection increases
10. Move the magnet at an angle to the plane of the coil.	Deflection decreases Deflection is maximum when the magnet moves perpendicular to the plane of the coil. Deflection is zero when the magnet moves parallel to the plane of the coil.
11. Increase the number of turns of the coil	Magnitude of current increases.

COIL-COIL EXPERIMENTS

A coil known as primary (P) is connected in series with the source battery (B) and a tap key (K). Another coil called as secondary (S) is placed closed to the primary coil but not perpendicular to one another.



The following observations are made

(i) When the key is pressed, the galvanometer shows a momentary deflection.

(ii) When the current becomes steady i.e. key is kept pressed, the deflection is zero.

(iii) When the key is released, the galvanometer again shows a momentary deflection, but now in the opposite direction.

These observations reveal that as long as there is a change in current in P., an e.m.f. is induced in S. This phenomenon in which an e.m.f. is induced in a coil due to a varying current in a neighbouring coil is called mutual induction.

It can also be seen that on keeping K pressed i.e. steady current flowing through P, but on moving S away or towards P, the galvanometer shows deflection, in either case in the opposite directions.

These observations show that an e.m.f is induced in the coil, whenever there is a relative motion between the magnet and the coil.

FARADAY'S LAWS OF ELECTRO-MAGNETIC INDUCTION

Whenever there is change in the magnetic flux associated with a circuit, an e.m.f. is induced in the circuit.

The magnitude of the induced e.m.f. (e) is directly proportional to the time rate of change of the magnetic flux through the circuit.

$$e \propto \frac{\Delta\phi}{\Delta t} \quad \text{or} \quad e = k\frac{\Delta\phi}{\Delta t}$$

In the limit $\Delta t \to 0$, $e = \frac{d\phi}{dt}$

k = const. of proportionality depending upon the system of units used.

In the S.I. system, emf 'e' is measured in volt and $\frac{d\phi}{dt}$ in Wb/sec. In MKS or SI system, these units are so chosen that k = 1, and 1 Volt = 1 Wb/sec

Induced current or e.m.f. lasts only for the time for which the magnetic flux is changing.

If the coil has N turns, then the emf will be induced in each turn and the emf's of all the turns will be added up. If the turns of coil are very close to each other, the magnetic flux passing through each turn will be same.

So, the induced emf in the whole coil $e = N\frac{\Delta\phi}{\Delta t}$

$N\phi$ = number of 'flux linkages' in the coil.

LENZ'S LAW

The direction of the induced current e.m.f. is given by Lenz's law : The direction of the induced current is such that it oppose the change in the magnetic flux that causes the induced current or e.m.f. i.e. induced current tries to maintain flux

On combining Lenz's law with Faraday's laws

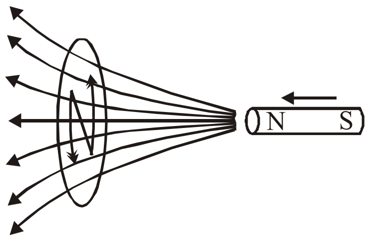
$$e = -\frac{d\phi}{dt}$$

–ve sign indicating that the induced e.m.f. opposes the change in the magnetic flux. The Lenz's Law is consistent with the law of conservation of energy.

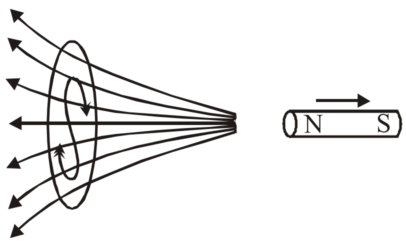
The induced e.m.f. is produced at the cost of mechanical work done by an external agent in the magnet and coil experiment. When the N-pole of the magnet is moved towards the coil, the face of the coil facing the north pole acts like a north pole. (This can be found by Flemings' Right hand Rule). As the magnet is moved towards the coil, the magnetic flux linked with the coil increases.

To oppose this increase in flux, e.m.f. induced in the coil has to be in such a direction as to reduce the increase in flux.

The external agent has to do some work against this force of repulsion between the two N-pole. This is converted into electrical energy as shown in fig.



Similarly if the magnet with its N-pole is moved away from the coil, then the face of the coil acts like a South pole and hence the flux linked with the coil tends to decrease. The induced current or e.m.f. must now be in a direction so as to increase the flux as shown in figure.



The external agent has to do work against this force of attraction between the N and S poles and this is converted into electrical energy. If suppose, on moving the N pole of the magnet towards the coil, a south polarity is induced on the face of the coil, then the magnet would be attracted to the coil, and there would be a continuous increase in magnetic flux linked with the coil leading to a continuous increase in e.m.f. without any expenditure of energy and this would violate the principle of conservation of energy.

INDUCED CURRENT AND INDUCED CHARGE

If in a coil of N turns the rate of change of magnetic flux be $\Delta\phi/\Delta t$, then the induced emf in the circuit is $e = - N (\Delta\phi/\Delta t)$. If the coil be closed and the total resistance of its circuit be R, then the induced current in the circuit will be

$$I = \frac{e}{R} = \frac{N}{R}\frac{\Delta\phi}{\Delta t}.$$

It is clear from this equation that the induced current in the circuit depends upon the resistance.

Whereas the induced emf is independent of resistance.

The charge flowed through the circuit in time-interval Δt

$$q = I \times \Delta t$$

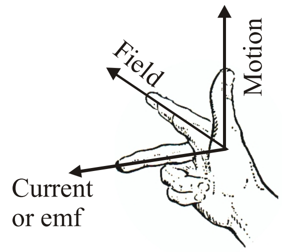
or $$q = \frac{N}{R}\frac{\Delta\phi}{\Delta t} \times \Delta t = \frac{N}{R}\Delta\phi$$

$$= \frac{\text{number of turns} \times \text{change in magnetic flux}}{\text{resistance}}$$

The induced charge does not depend upon the time-interval. Whether the change in magnetic flux be rapid or slow, the charge in the circuit will remain the same.

FLEMING'S RIGHT HAND RULE

Fleming's right hand rule gives the direction of the induced e.m.f. and current in a straight conductor moving perpendicular to the direction of magnetic field.



Statement : Stretch out the thumb, fore finger and middle finger of the right hand mutually perpendicular to each other. If the fore finger points in the direction of magnetic field, the thumb in the direction of motion of the conductor, then the middle finger will point out the direction of induced current or induced e.m.f.

DETERMINATION OF THE DIRECTION OF THE INDUCED CURRENT IN A CIRCUIT (USING LENZ'S LAW)

Lenz's law : "when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop".

The above rule can be systematically applied as follows to determine the direction of induced currents.

(A) Identify the loop in which the induced current is to be determined.

(B) Determine the direction of the magnetic field in this loop (i.e., in or out of the loop).

(C) The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (due to change in area, or change in B).

(D) Choose the appropriate current in the loop that will oppose the change in flux i.e.,

a. If the flux is into the paper and increasing, the flux due to the induced current should be out of the paper.

b. If the flux into the paper and decreasing, the flux due to the induced current should be into the paper.

c. If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.

d. If the flux is out of the paper and decreasing, the flux due to induced current should be out of the paper.

The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lens' law

$e_{ind} = -\frac{d\phi_B}{dt}$ with the appropriate sign conventions.

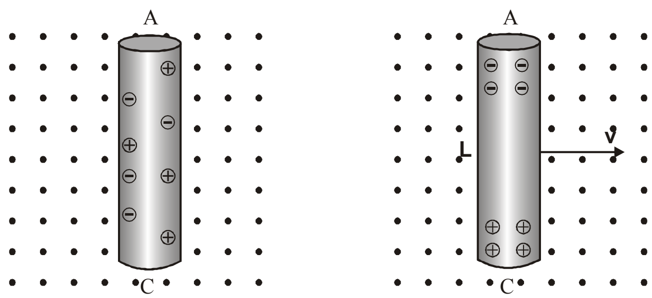
The Right Hand Sign Convention used is

- Counter clockwise current/emf is +ve
- Clockwise current/emf is –ve
- Magnetic flux out of the paper is +ve

- Magnetic flux into the plane of the paper is –ve
- The rate of change of an increasing positive flux is +ve
- The rate of change of a decreasing positive flux is –ve
- The rate of change of an increasing negative flux is –ve
- The rate of change of a decreasing negative flux is +ve

MOTION OF A WIRE IN A MAGNETIC FILED

A wire AC is moving through a magnetic field with constant velocity v at right angles to the direction of the lines of induction B. The direction of its motion is also at right angles to the length L of the wire. Because of this motion, there will be an electromotive force induced in the wire.



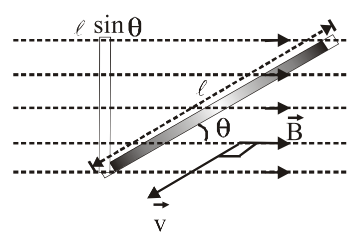
Induced potential difference between the ends of the wire

$$e = vBL$$

If v be in meter/second, B in weber/meter² and ℓ in meter, then e will be in volt.

In using this equation, it must be remembered that B, L, and v are considered to be perpendicular to one another.

If they are not, then only those components of all three that are mutually perpendicular are to be considered. In that case induced e.m.f. $e = -\vec{\ell}.(\vec{v} \times \vec{B})$



In a particular case if $\vec{v}$ and $\vec{B}$ are mutually perpendicular and $\vec{\ell}$ makes angle θ with $\vec{B}$ then magnitude of induced e.m.f. is $e = Bv(\ell\sin\theta)$

Note : If any one of these $\vec{v} \parallel \vec{B}$, $\vec{\ell} \parallel \vec{B}$, $\vec{\ell} \parallel \vec{v}$ then induced e.m.f. is zero.

Example 2 :

An artificial satellite with a metal surface has an orbit over the equator. Will the earth's magnetism induce a current in it? Explain.

Sol. The satellite will cut vertical component in the equatorial plane is zero. Consequently there will be no change in magnetic flux and hence no current will be induced.

Example 3 :

A conductor of length 10 cm is moved parallel to itself with a speed of 10 m/s at right angles to a uniform magnetic induction 10^{-4} Wb/m². What is the induced e.m.f. in it?

Sol Given : $\ell = 10$ cm $= 0.1$ m, $v = 10$ m/s
$B = 10^{-4}$ Wb/m²

e.m.f. induced in conductor

$e = B\,\ell\,V = 10^{-4} \times 0.1 \times 10 = 10^{-4}$ V

Example 4 :

A horizontal telegraph wire 10 m long oriented along the magnetic east-west direction falls freely under gravity to the ground from a height of 10 m. Find the emf induced in the wire at the instant the wire strikes ground.
($B_H = 2.5 \times 10^{-5}$ Wb/m², $g = 9.8$ m/s²)

Sol. Given : $u = 0$, $a = -g = -9.8$ m/s², $s = -10$ m,
$B_H = 2.5 \times 10^{-5}$ Wb/m², $\ell = 10$ m

Let the speed of wire at last moment = v

then $v^2 = 0 + 2(-9.8) \times (-10) = 196$ $[v^2 = u^2 + 2as]$

$\therefore$ $v = 14$ m/s

e.m.f. induced in wire

$e = B_H\,\ell\,v = 2.5 \times 10^{-5} \times 10 \times 14 = 3.5 \times 10^{-3}$ Volt

Example 5 :

A wire kept along north-south is allowed to fall freely. Will an induced emf be set up?

Sol. No. This is because there will be no change of magnetic flux in this case.

Example 6 :

A metal block and a brick of the same size are allowed to fall freely from the same height above the ground. Which of the two would reach the ground earlier and why?

Sol. The brick will reach the ground earlier. This is because the eddy currents produced in the metal block will oppose the motion of the metal block.

Example 7 :

A jet plane is travelling west at the speed of 1800 km/h. What is the voltage difference developed between the ends of the wing 25 m long if the earth's magnetic field at the location has a magnitude of 5.0×10^{-4} Tesla and the dip angle is 30°?

Sol. Given that $v = 1800 \text{ km/h} = \dfrac{1800 \times 1800}{60 \times 60}$ m/s $= 500$ m/s

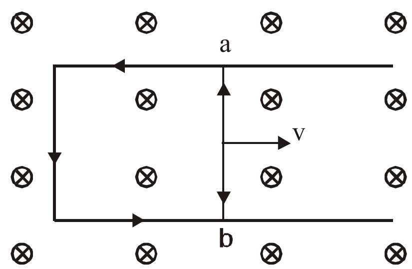
$\ell = 25$ m, $B = 5.0 \times 10^{-4}$ T and $\theta = 30°$

Vertical component of earth's magnetic field

$B_v = B\sin\theta = 5.0 \times 10^{-4} \times \sin 30° = 2.5 \times 10^{-4}$ T

MOTIONAL e.m.f.

Suppose the moving rod slides along a stationary U-shaped conductor, forming a complete circuit. Under the action of this field a counterclockwise current is established around this complete circuit.

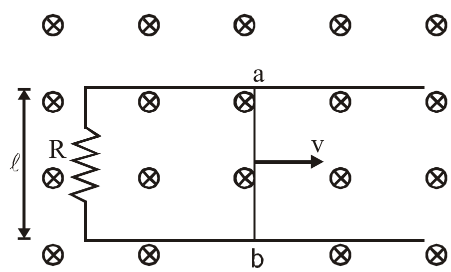


The moving rod becomes a source of electromotive force. Within it, charge moves from lower to higher potential and in the remainder of the circuit, charge moves from higher to lower potential. We call this a motional electromagnetic force denoted by e, we can write,

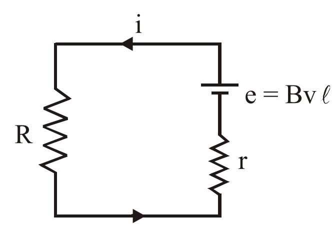
$e = Bv\ell$

If R is the resistance of the circuit, then current in the circuit

$$i = \frac{e}{R} = \frac{Bv\ell}{R}$$



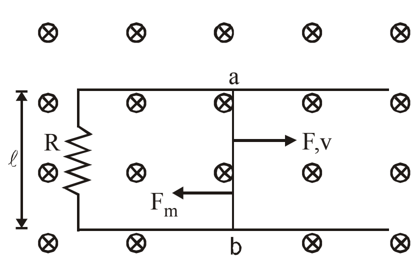
⇒



In the figure shown, we can replace the moving rod ab by a battery of emf Bvl with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery. Hence, the current in the circuit is,

$$i = \frac{e}{R+r} \quad \text{or} \quad i = \frac{B\,v\,\ell}{R+r}$$

Induction and energy transfers : In the figure shown, if you move the conductor ab with a constant velocity v, the current in the circuit is,



$$i = \frac{B\,v\,\ell}{R} \qquad (r = 0)$$

A magnetic force $F_m = i\,\ell\,B = \frac{B^2\ell^2 v}{R}$ acts on the conductor in opposite direction of velocity. So, to move the conductor with a constant velocity v an equal and opposite force F has to be applied in the conductor.

Thus, $F = F_m = \frac{B^2\ell^2 v}{R}$

The rate at which work is done by the applied force

$$P_{applied} = Fv = \frac{B^2\ell^2 v^2}{R}$$

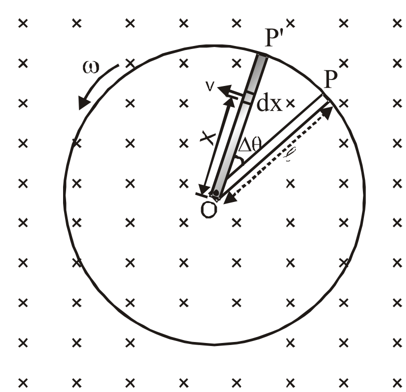
and the rate at which energy is dissipated in the circuit

$$P_{dissipated} = i^2 R = \left(\frac{B\,v\,\ell}{R}\right)^2 \; ; \; R = \frac{B^2\ell^2 v^2}{R}$$

This is just equal to the rate at which work is done by the applied force.

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD

Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.



Length of rod $= \ell$

Angular velocity of rod $= \omega$

Intensity of magnetic field $= \vec{B}$

direction of magnetic field is normal to the plane of rod.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element $de = B\,v.\,dx$ .....(i)

$\because$ This small element dx is at distance x from O (axis of rotation)

$\therefore$ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eq^n (i) $de = B\,\omega\,x\,dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field.

So, net induced e.m.f. across conducting rod

$$e = \int de = \int_0^{\ell} B\omega\, x\, dx = \omega B\left(\frac{x^2}{2}\right)_0^{\ell}$$

or $e = \frac{1}{2} B\,\omega\,\ell^2$; $e = \frac{1}{2} B \times 2\,\pi\, f \times \ell^2$

[f = frequency of rotation]

$= B\, f\, (\pi\, \ell^2)$ area traversed by the rod $A = \pi\, \ell^2$

or $e = B\,A\,f$

$e \propto B$

$\propto f$

$\propto \ell^2$ or A

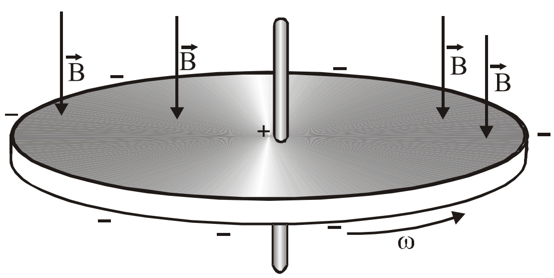
INDUCED e.m.f. DUE TO ROTATION OF A CIRCULAR METAL DISC IN A UNIFORM MAGNETIC FIELD

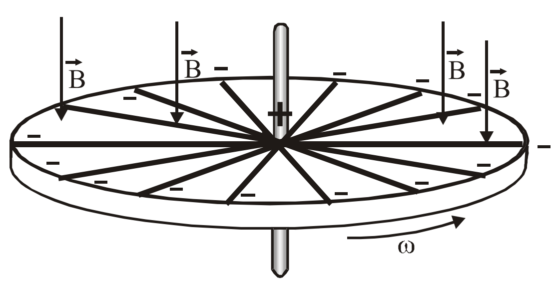
Let a metallic uniform disc of radius r is rotating about an axis passing through it centre and perpendicular to its plane.

A uniform magnetic field $\vec{B}$ is perpendicular to plane of disc normally down wards (as shown in fig.)

We can consider this disc is made up of infinite number of rods each having one end at the centre and another end at the circumference of the disc.

Length of each rod = r and rotating with angular frequency ω



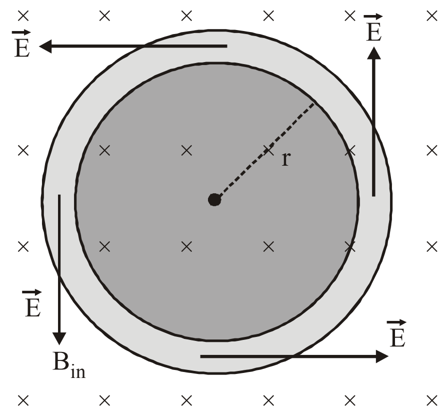


So, induced e.m.f. across each rod $e = \frac{1}{2} B\,\omega\, r^2 = B\,A\,f$

in each rod free electrons are moving with rod in magnetic field B normal to its plane. So, force on electrons is towards away from the centre and hence at centre positive potential develops whereas negative potential at circumference. All the sources of e.m.f. are connected in parallel and hence net e.m.f. will remain $e = B\,A\,f$

INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



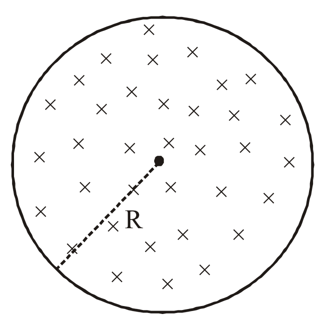
It is non-conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence, $\oint \vec{E}.d\vec{\ell} = e = -\frac{d\phi}{dt}$

This equation is valid only if the path around which we integrate is stationary.

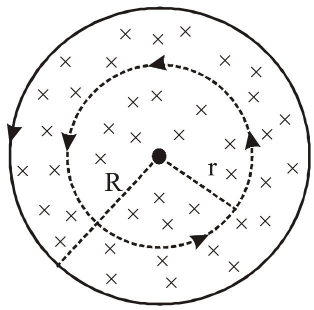
Example 8 :

The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha \frac{\text{tesla}}{\text{second}}$. Find the magnitude of electric field as a function r, the distance from the geometric centre of the region.



Sol. For $r \le R$:

$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$



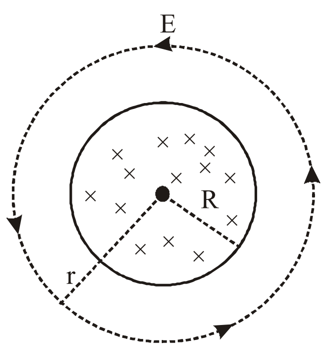
$\therefore \quad E(2\pi r) = (\pi r^2)\alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$

E-r graph is straight line passing through origin.

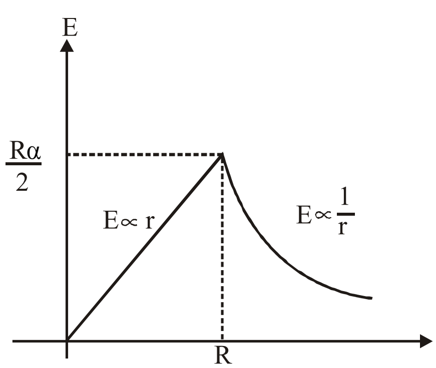
At r = R, $E = \frac{R\alpha}{2}$

For r ≥ R :

$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$



$\therefore \quad E(2\pi r) = (\pi R^2)\alpha \Rightarrow E = \frac{\alpha R^2}{2r} \Rightarrow E \propto \frac{1}{r}$



TRY IT YOURSELF-1

Q.1 A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends

(A) if $\vec{v} \parallel \vec{l}$ (B) if $\vec{v} \parallel \vec{B}$

(C) if $\vec{l} \parallel \vec{B}$ (D) none of these

Q.2 Two coils carrying current in opposite directions are placed co-axially with centres at some finite separation. If they are brought closer to each other then current flowing in them should ,

(A) decrease (B) increase

(C) remain constant

Q.3 A copper bar of length l is dropped from a small height 'h' above the Earth's surface. The axis of the bar remains horizontal and magnetic E-W throughout. The horizontal component of the Earth magnetic field is B. Immediately before striking the ground the potential difference between the ends of the bar is

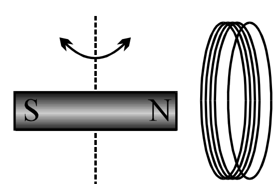
(A) $2Bl\sqrt{Bh}$ (B) $Bl\sqrt{2gh}$

(C) $Bl\,gh$ (D) $2\,Bl\,gh$

Q.4 A uniform circular ring of radius R, mass m has uniformly distributed charge q. The ring is free to rotate about its own axis (which is vertical) without friction. In the space, a uniform magnetic field B, directed vertically downwards, exists in a cylindrical region. Cylindrical region of magnetic field is coaxial with the ring and has radius, greater than R. If induction of magnetic field starts increasing at a constant rate. The angular acceleration of the ring will be :

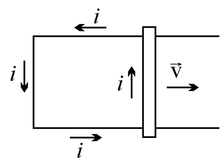
(A) directly proportional to R
(B) directly proportional to q
(C) directly proportional to m
(D) independent of R and m

Q.5 The bar magnet is oriented so that it lies along the axis of a coil of wire as shown in the diagram. If the magnet is quickly rotated about a vertical axis until the opposite end (south pole) points toward the coil, the direction of the induced current (as seen from the magnet) in the loops of wire in the coil will be –



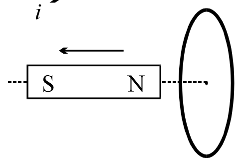
(A) without meaning because there will no induced emf or current.
(B) clockwise if the magnet is rotated clockwise, and vice versa.
(C) counter-clockwise if the magnet is rotated clockwise, and vice versa.
(D) the same, whether the magnet is rotated clockwise or counter-clockwise.

Q.6 The figure shows a conducting bar moving to the right on two conducting rails. To make an induced current *i* in the direction indicated, in what direction would the magnetic field be in the area contained within the conducting rails?



(A) out of the page
(B) into the page
(C) to the right
(D) to the left

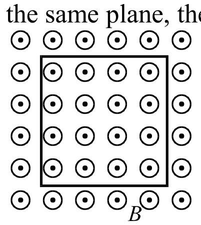
Q.7 A bar magnet with its north (N) and south (S) poles as shown is initially moving to the left, along the axis of and away from a circular conducting loop. A current I is induced in the loop and acceleration of magnet is a due to this current. As seen from the magnet looking in the direction of the loop.



(A) I runs clockwise and a points to the left.
(B) I runs counterclockwise and a points to the right.
(C) I runs clockwise and a points to the right.
(D) I runs counterclockwise and a points to the left.

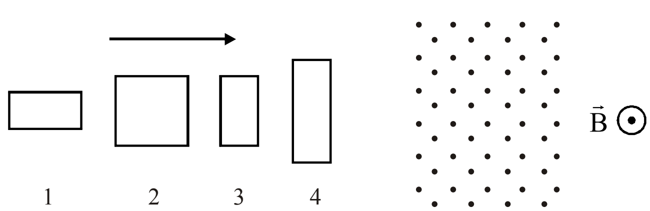
Q.8 A rectangular coil of wire rotates about an axis which is perpendicular to a uniform magnetic field at a steady rate. Consider the instant when the plane of the coil is parallel to the magnetic field lines. At that instant the induced electromotive force is,
(A) minimum (B) maximum
(C) zero (D) constant, the same at all times.

Q.9 A uniform magnetic field *B* is directed out of the page. A metallic wire has the shape of a square frame and is placed in the field as shown. While the shape of the wire is steadily transformed into a circle in the same plane, the current in the frame:



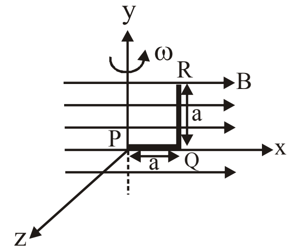
(A) is directed clockwise
(B) does not appear
(C) is directed counterclockwise
(D) is alternating

Q.10 The four wire loops shown have vertical edge lengths of either L, 2L or 3L. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced emf, greatest to least.



(A) 1 and 2 tie, then 3 and 4 tie
(B) 3 and 4 tie, then 1 and 2 tie
(C) 4, 2, 3, 1
(D) 4 then, 2 and 3 tie and then 1

Q.11 In a region there exist a magnetic field B_0 along positive x-axis. A metallic wire of length 2a and one side along x-axis and one side parallel of y-axis is rotating about y-axis with a angular velocity ω. Then at the instant shown.



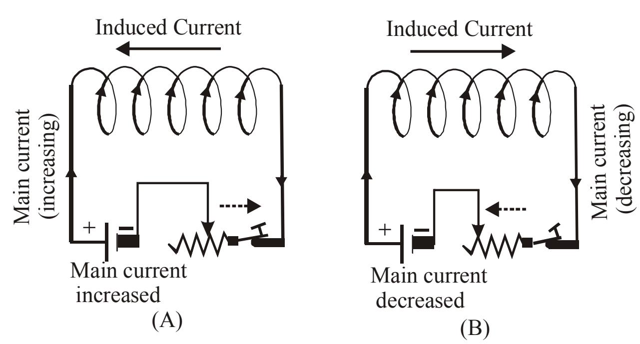
(A) Potential difference across PQ is O
(B) Potential difference across PQ is $\frac{1}{2}B_0\omega a^2$
(C) Potential difference across QR is $\frac{1}{2}B_0\omega a^2$
(D) Potential difference across QR is $B_0\omega a^2$

ANSWERS

(1) (D)	**(2)** (B)	**(3)** (B)
(4) (B)	**(5)** (D)	**(6)** (B)
(7) (C)	**(8)** (B)	**(9)** (A)
(10) (D)	**(11)** (AD)	

SELF INDUCTION

Whenever the key in the circuit is closed or opened, the current in the circuit increases or decreases. The variation of current causes a variation in magnetic flux linked with the circuit ($\because \phi \propto I$) hence, an induced emf is developed in the circuit. This emf is called induced e.m.f.. Induced emf follows Lenz's law. i.e., induced current always opposes the change in the main current. When the main current is increased (by the rheostat), the induced current flows opposite to the main current and opposes the increases in the main current (fig. A). When the main current is decreased, then the induced current flows is the same direction as the main current and opposes the decrease in the main current (fig. B).



Production of induced e.m.f. in a coil due to the changes in current in the same coil, is called self induction.
According to Lenz's law, direction of induced e.m.f. is opposite to the direction of current where it is increasing. Therefore this e.m.f. is called back e.m.f.

The magnetic flux (ϕ) linked with the coil is directly proportional to the current (I) flowing through it.

i.e. $\phi \propto I \quad \therefore \phi = LI$

The constant L is called self inductance of the coil.

L depends on the size and shape of the coil.

The induced e.m.f. $e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ then $e = -L$

The self inductance of a coil is equal to the e.m.f. induced in the coil due to the unit rate of change of current in the same coil.

S.I. unit henry (H).

$$L = \frac{e}{dI/dt} \quad \therefore \quad 1 \text{ henry (H)} = \frac{1 \text{ volt}}{1 \text{ ampere / second}}$$

1 henry : The self inductance of a coil is 1 henry, if e.m.f. of 1 volt is induced in the coil, when the current in the coil is changing at the rate of 1 ampere per second.

Relation between henry and weber :

$$L = \frac{\phi}{I} \qquad \therefore 1 \text{ H} = \frac{1 \text{ Wb}}{1 \text{ A}} = 1 \text{ Wb A}^{-1}$$

Self-inductance of a plane coil :

Let current I flowing in a plane coil of radius r.

N = number of turns in the coil.

Magnetic field at the centre of coil, $B = \frac{\mu_0 NI}{2r}$

Magnetic flux per turn of coil = BA,

cross-sectional area of the coil $A = \pi r^2$.

Total magnetic flux linked with N turns, $\phi = \frac{\mu_0 N^2 I}{2r}$

$$A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2 \qquad \text{or} \qquad \phi = \frac{\mu_0 \pi N^2 r}{2} I$$

$$\text{But } \phi = LI \qquad \therefore \qquad L = \frac{\mu_0 \pi N^2 r}{2}$$

Example 9 :

A soft iron core is introduced in an inductor. What is the effect on the self-inductance of the inductor?

Sol. Since soft iron has a large relative permeability therefore the magnetic flux and consequently the self-inductance is considerably increased.

Self-inductance of a solenoid :

Let cross-sectional area of solenoid = A

Current flowing through it = I

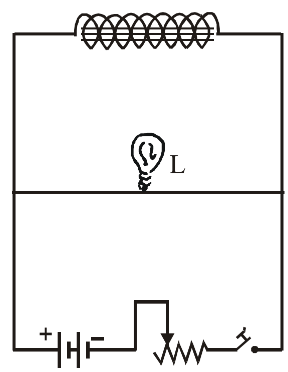
Length of the solenoid = ℓ

Then, $\phi = BAN = \frac{\mu_0 NI}{\ell} AN = \frac{\mu_0 N^2 A}{\ell} I$

But $\phi = LI \quad \therefore L = \frac{\mu_0 N^2 A}{\ell}$

If no iron or similar material is nearby, then the value of self-inductance depends only on the geometrical factors (length, cross-sectional area, number of turns).

Experimental demonstration of self-induction : Let us consider a solenoid connected in series with a battery through a rheostat and a tap key. An electric lamp L is connected in parallel as shown in fig.



Press the tap key and then adjust the current in the circuit with the help of rheostat so that the lamp just glows.

Now, when the key is released, the lamp suddenly glows brilliantly for a moment. This is because the induced emf at the time of break is greater than the original emf of the battery. When the key is pressed again, the lamp takes some time to light up properly. This is due to the opposition offered by induced emf to the growth of current.

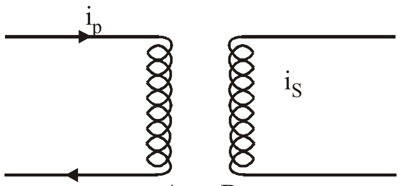
Physical significance of self-inductance :

Just as mass or inertia is responsible for slowing down the change in mechanical motion, the self-inductance in an electrical circuit slows down the change in current.

At the time of 'make' of an electrical circuit, the induced emf opposes the growth of current. Consequently, the current cannot attain its maximum value instantaneously but takes some time to do so. If inductance is large, then this time may be of several minutes. Again, at the time of 'break' of an electrical circuit, the induced emf opposes the decay of current. As an example, the field current in D.C. motor takes quite some time to attain its maximum value. This is due to the large inductance of the coil. Again, when the motor is switched off, an arc is formed at the switch due to large induced emf. So, we conclude that self-inductance plays the same role in an electrical circuit as is played by mass or inertia in mechanical motion. Thus, it is appropriate to describe self-inductance as the 'inertia of electricity'.

MUTUAL INDUCTION

Production of induced e.m.f. in coil due to the changes of current in a neighboring coil, is called mutual induction.



Consider two coil A and B placed near to each other. When current in the coil A changed, there is a change in magnetic flux. As this flux is linked with the coil B, an e.m.f. induced is B. This phenomenon is called mutual induction.
The induced e.m.f. in secondary coil B is proportional to the rate of change of current in primary coil A.

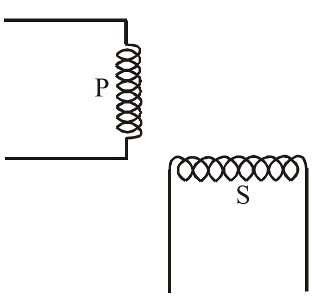
Coefficient of mutual induction or mutual inductance :
Let ϕ_s= magnetic flux linked with the secondary coil when a current I_p flows through the primary coil.
Then, $\phi_s \propto I_p$ or $\phi_s = M I_p$ ....[1]
M = constant of proportionality called mutual inductance or coefficient of mutual induction.
If $I_p = 1$, then $M = \phi_s$.
The mutual inductance of two circuits is equal to the magnetic flux linked with one circuit when a unit current flows through the other.

Differentiating eq^n. (1) w.r.t. time 't', we get $\frac{d\phi_s}{dt} = M\frac{dI_p}{dt}$

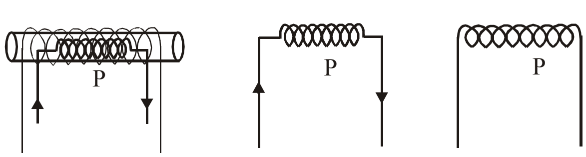
But induced emf in secondary coil $e = -\frac{d\phi_s}{dt}$

$\therefore \quad e = -M\frac{dI_p}{dt}$. If $\frac{dI}{dt} = 1$, then M = e (numerically).

The mutual inductance of two circuits or coils is equal to the induced emf set up in one circuit when the rate of change of current in the other is unity.



(A)



(B) (C)

The mutual inductance between two coil is equal to the e.m.f. induced in secondary coil due to the unit rate of change of current in the primary coil.

Factors on which mutual inductance depends
The value of mutual inductance of two coils depends upon
(i) Geometry of two coils i.e., size, shape and number of turns (N_P and N_S) of the coils.
(ii) Nature of the material on which the two coils are wound.
(iii) The distance between the two coils.
(iv) The relative placement of two coils.
If one coil is perpendicular to the other coil as (fig. A), then the mutual inductance is minimum.
Mutual inductance is maximum for the coils wound on the magnetic material (fig. B). When the entire flux due to the primary is linked with the secondary, then the mutual inductance is maximum.
This can be increased μ times when the two coils are closely wound on an iron core (laminated to avoid eddy currents) of permeability μ.
Mutual inductance has intermediate value for the configuration (fig. C).

Units of mutual inductance :
CGS : The unit of M is 'emu of mutual inductance' or 'abhenry'. The mutual inductance of two circuits is said to be 1 emu if a current changing at the rate of 1 emu per second in one circuit induces an emf of 1 emu in the other.
SI : henry (H).
The mutual inductance of two circuits is said to be 1 henry if a current changing at the rate of 1 ampere per second induces an emf of 1 volt in the other.
1 henry = 10^9 emu of mutual inductance.

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)
If M = 0, $L = L_1 + L_2$
If $M \neq 0$ $L = L_1 + L_2 + 2M$
(a) When current in both is in the same direction
Then $L = (L_1 + M) + (L_2 + M)$
(b) When current flow in two coils are mutually in opposite directions.
$L = L_1 + L_2 - 2M$
Two coils are connected in parallel :

(a) If M = 0 then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1L_2}{L_1 + L_2}$

(b) If $M \neq 0$ then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

If current in coil 1 changes, an e.m.f. will be induced in coil 2. If current in coil 2 changes, an e.m.f. will be induced in coil 1. That is why the process is called mutual induction.

COEFFICIENT OF COUPLING OF THE TWO CIRCUITS

Let us now calculate mutual inductance between two circuit in terms of the self inductance of each circuit alone.

Let us first consider a case when the total flux associated with one coil links with the other, i.e., a case of maximum flux linkage. Consider two coils placed adjacent to each other, $M_{12} = \frac{N_2\phi_{B_2}}{i_1}$ and $M_{21} = \frac{N_1\phi_{B_1}}{i_2}$

Similarly, $L_1 = \frac{N_1\phi_{B_1}}{i_1}$ and $L_2 = \frac{N_2\phi_{B_2}}{i_2}$

If all the flux of coil 2 links coil 1 and vice-versa then, $\phi_{B_2} = \phi_{B_1}$

$$M_{12}M_{21} = M^2 = \frac{N_1N_2\phi_{B_1}\phi_{B_2}}{i_1i_2} = L_1L_2$$

$$(\because M_{12} = M_{21} = M)$$

$$\therefore \quad M_{max} = \sqrt{L_1L_2}$$

this is the maximum possible value of M as the total flux associated with one coil links with the other. In general only a fraction K_2 (<1) of ϕ_{B_2} passes through the coil 1.

Similarly a fraction K_1 (<1) of ϕ_{B_1} passes through coil 2.

Hence, $\phi_{B_1} = K_2\phi_{B_2}$ and $\phi_{B_2} = K_1\phi_{B_1}$

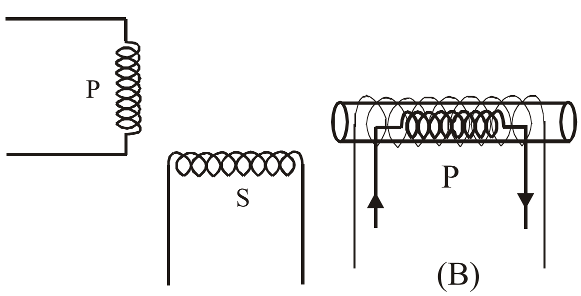
$$\therefore \quad M_{21}M_{12} = M^2 = \frac{N_1N_2K_1K_2\phi_{B_1}\phi_{B_2}}{i_1i_2} = K_1K_2L_1L_2$$

or $\quad M = K\sqrt{L_1L_2}$

Here $K = \sqrt{K_1K_2}$ is a number, depending on the geometry of the coils and their relative closeness having value between 0 and 1.

When K = 0, the coupling of two coils is loose fig. A.

When K = 1, the coupling of two coils is tight fig. B.



(A) (B)

Generally, K is always less than 1.

LC OSCILLATIONS

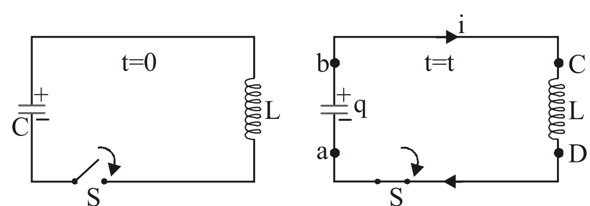
If a charged capacitor C is short-circuited though an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. Assume an idealized situation in which energy is not radiated away from the circuit. With these idealizations-zero resistance and no radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor's magnetic field back and forth. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy. Let us now derive an equation for the oscillations in an L-C circuit.

Refer figure (a) : The capacitor is charged to a potential difference V such that charge on capacitor $q_0 = CV$

Here q_0 is the maximum charge on the capacitor.

At time t = 0, it is connected to an inductor through a switch S. At time t = 0, switch S is closed.

Refer figure (b) : When the switch is closed, the capacitor



(a) (b)

starts discharging. Let at time t charge on the capacitor is q (< q_0) and since, it is further decreasing there is a current i in the circuit in the direction shown in figure.

The potential difference across capacitor = potential difference across inductor,

$$\text{or} \quad V_b - V_a = V_c - V_d \quad \therefore \quad \frac{q}{C} = L\left(\frac{di}{dt}\right) \qquad \text{...(i)}$$

Now, as the charge is decreasing,

$$i = \left(\frac{-dq}{dt}\right) \quad \text{or} \quad \frac{di}{dt} = -\frac{d^2q}{dt^2}$$

Substituting in equation (i), we get

$$\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right) \quad \text{or} \quad \frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q \qquad \text{...(ii)}$$

This is the standard equation of simple harmonic motion

$$\left(\frac{d^2x}{dt^2} = -\omega^2x\right) \quad \text{Here } \omega = \frac{1}{\sqrt{LC}} \qquad \text{...(iii)}$$

The general solution of equation (ii), is

$$q = q_0 \cos(\omega t \pm \phi)$$

In our case $\phi = 0$ as $q = q_0$ at $t = 0$.

Thus, we can say that charge in the circuit oscillates with angular frequency given by equation (iii). Thus, In L-C oscillations, q, i and $\frac{di}{dt}$ all oscillate harmonically with same angular frequency ω. But the phase difference between q and i or between i and $\frac{di}{dt}$ is $\pi/2$.

Their amplitudes are q_0, $q_0\omega$ and $\omega^2 q_0$ respectively. So

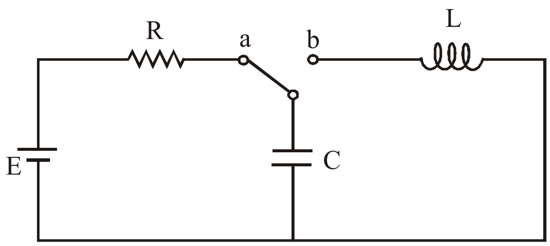
$$q = q_0 \cos\omega t, \text{ then } i = \frac{dq}{dt} = -q_0\omega \sin\omega t$$

$$\frac{di}{dt} = -q_0\omega^2 \cos\omega t$$

Similarly potential energy across capacitor (U_C) and across inductor (U_L) also oscillate with double the frequency 2ω.

Example 10 :

Consider the circuit shown in figure. Suppose the switch which has been connected to point a for a long time is suddenly thrown to b at t = 0.



Find the following quantities:
(a) the frequency of oscillation of the LC circuit.
(b) the maximum charge that appears on the capacitor.
(c) the maximum current in the inductor.

Sol. (a) The (angular) frequency of oscillation of the LC circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$.

Therefore, the frequency is $f = \frac{1}{2\pi\sqrt{LC}}$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is Q = CE

(c) The energy stored in the capacitor before the switch is thrown is $U_E = \frac{1}{2}CE^2$

On the other hand, the magnetic energy stored in the inductor is $U_L = \frac{1}{2}LI^2$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor:

$$\frac{1}{2}CE^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = E\sqrt{\frac{C}{L}}$$

GENERATOR

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

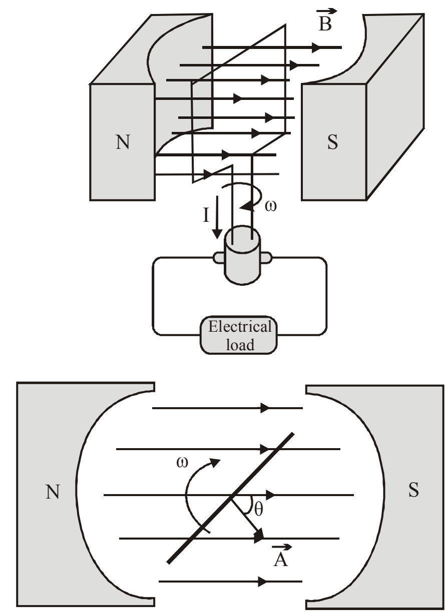


Figure (a) A simple generator.
(b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an N-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From figure (b), we see that the magnetic flux through the loop may be written as

$$\phi_B = \vec{B}.\vec{A} = BA\cos\theta = BA\cos\omega t \quad \text{........ (1)}$$

The rate of change of magnetic flux is

$$\frac{d\phi_B}{dt} = -BA\omega\sin\omega t \quad \text{........ (2)}$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\phi_B}{dt} = NBA\omega\sin\omega t \quad \text{........ (3)}$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by

$$I = \frac{|e|}{R} = \frac{NBA\omega}{R}\sin\omega t \quad \text{......... (4)}$$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = \frac{NBA\omega}{R}$.

The power delivered to this circuit is

$$P = I|e| = \frac{(NBA\omega)^2}{R}\sin^2\omega t \quad \text{......... (5)}$$

On the other hand, the torque exerted on the loop is

$\tau = \mu B \sin\theta = \mu B \sin \omega t$ .... (6)

Thus, the mechanical power supplied to rotate the loop is

$P_m = \tau\,\omega = \mu B\,\omega \sin \omega t$ .... (7)

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin\omega t \quad .... (8)$$

the above expression becomes

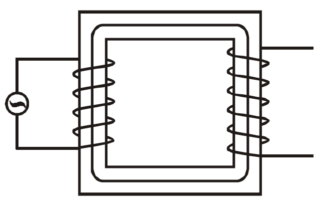
$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin\omega t\right)B\omega\sin\omega t$$

$$= \frac{(NAB\omega)^2}{R}\sin^2\omega t \quad .... (9)$$

As expected, the mechanical power put in is equal to the electrical power output.

TRANSFORMER

Principle : It work on the phenomenon of mutual induction. It is a device for transforming a low alternating voltage of high current into a high alternating voltage of lower current and vice versa, without increasing power or changing frequency.



Construction :

It has three main parts described below :

1. **A laminated core (C) of sand-mixed iron in shell type :** It has more resistance due to laminations and mixing of sand. It is done to reduce eddy current the core and minimize its heating.
2. **A primary coil P :** It is a coil of enamelled copper wire wrapped over the central arm of the core and is insulated from it by varnish.
3. **A secondary coil S :** It is also a coil of enamelled copper wire. It is wrapped over the primary and insulated from it by wax paper. Transformed alternating voltage is obtained from it. It forms output section of the transformer.

If a low voltage is to be transformed into a high voltage, then the number of turns in secondary is more than those in primary. The transformer is called a step up transformer.

If a high voltage is to be transformed into a low voltage, then the number of turns in secondary is less than those in primary. The transformer is called step-down transformer.

Transformation ratio of the transformer (K)

$$= \frac{\text{Number of turns in secondary } (N_s)}{\text{Number of turns in primary } (N_p)}$$

$$K = \frac{N_s}{N_p}$$

K > 1, for step-up transformer.

K < 1, for step-down transformer.

The whole arrangement is kept immersed in a special oil called transformer oil, taken in metallic cans. The oil provides insulation as well as cooling.

Working : As voltage applied on the primary is alternating, it produces a continuous change in magnetic flux in primary as well as in secondary. Due to mutual induction, an e.m.f. is induced in secondary. It is higher or lower than that induce in primary depending upon whether the transformer is a step or step-down transformer. The induced e.m.f. in secondary gives output voltage.

Theory : An alternating e.m.f. E_p is applied across the primary which produces current I_p in the primary circuit and a current I_s in the secondary circuit. The currents in the coils produce a magnetization in the soft-iron core and there is a corresponding magnetic field B inside the core. The field due to magnetization of the core is large as compared to the field due to the currents in the coils. We assume that the field is constant in magnitude everywhere in the core and hence its flux (BA) through each turn is same for the primary as well as for the secondary coil.

Let the flux through each turn be Φ. The emf induced in the

primary $= -N_p \dfrac{d\Phi}{dt}$

and induced in the secondary $= -N_s \times \dfrac{d\Phi}{dt} = E_s$.

If we neglect the resistance in the primary circuit, Kirchhoff's loop law applied to the primary circuit gives

$$E_p - N_p\frac{d\Phi}{dt} = 0$$

or $$E_p = N_p\frac{d\Phi}{dt} \quad ......(a)$$

Also, $$E_s = -N_s\frac{d\Phi}{dt} \quad ......(b)$$

From (a) and (b),

$$E_s = -\frac{N_s}{N_p}E_p$$

The minus sign shows that E_s is 180° out of phase with E_p. Equ^{n}s. (a) and (b) are valid for all values of currents in the primary and the secondary circuits. If there is no loss of power in output and input circuits

then **input power = output power**

$$E_p \times I_p = E_s \times I_s \quad \text{or} \quad \frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{N_s}{N_p}$$

But in practice there is always energy loss

so input power > output power

and hence $E_p \times I_p > E_s \times I_s$

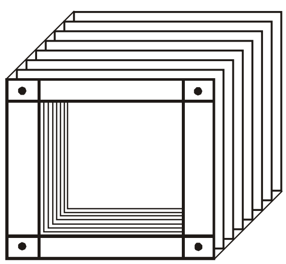
Energy Losses in a Transformer :

(i) Energy lost in winding of the transformer is known as copper loss. Primary and secondary coils of a transformer are generally made of copper wires. These copper wires have resistance (R). When current (I) flows through these wires, power loss (I^2R) takes place. This loss appears as the heat produced in the primary and secondary coils. Copper losses can be reduced by using thick wires for the windings.

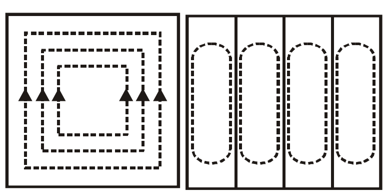
(ii) Flux Losses : In actual transformer, the coupling between primary and secondary coil is not perfect. It means that magnetic flux linked with the primary coil is not equal to the magnetic flux linked with secondary coil. so a certain amount of electric energy supplied to the primary coil is wasted.

(iii) Iron Losses :

(a) Eddy Currents Losses : When a changing magnetics flux links with the iron core of the transformer, eddy currents are set up. These eddy currents produce heat which leads to the wastage of energy. This energy loss is reduced by using laminated on cores (Fig).



Eddy currents are reduced in a laminated core because their paths are broken as compared to solid core as shown in fig.



Solid core Laminated core

(b) Hysteresis Losses : When alternating current passes through the primary coil, core of the transformer is magnetized and demagnetised over a complete cycle. Some energy is lost in magnetising and de-magnetising the iron core. The energy loss in a complete cycle is equal to area of the hysteresis loop.

This energy loss can be minimized by using suitable material having narrow hysteresis loop for the core of a transformer.

(c) Losses due to Vibration of core or humming losses : A transformer produces humming noise due to magnetostriction effect. Some electrical energy is lost in the form of mechanical energy to produce vibration in the core.

Efficiency of a transformer : In an ordinary transformer, there is some loss of energy due to coil resistance, hysteresis in the core, eddy currents in the core etc. The efficiency of a transformer is defined as

$$\eta = \frac{\text{output power}}{\text{input power}}$$

Efficiency of the order of 99% can be easily achieved.

Example 11:

A transformer having efficiency 90% is working on 100 V and at 2.0 kW power. If the current in the secondary coil is 5A, calculate (i) the current in the primary and (ii) voltage across the secondary coil.

Sol. Here, $\eta = 90\% = \frac{9}{10}$, $I_s = 5A$, $E_p = 100$ V.

$E_pI_p = 2$ kW $= 2000$ W

(i) $E_pI_p = 2000$ W

$$\therefore I_p = \frac{2000}{E_p} \quad \text{or} \quad I_p = \frac{2000}{100} = 20\text{ A}$$

$$\text{(ii) } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_sI_s}{E_pI_p}$$

$$\text{or } E_sI_s = \eta \times E_pI_p = \frac{9}{10} \times 2000 = 1800\text{ W}$$

$$\therefore \frac{1800}{I_s} = \frac{1800}{5} = 360\text{ Volt}$$

Applications : The most important application of a transformer is in long distance transmission of electric power from generating station to consumers hundreds of kilometers away through transmission lines at reduced loss of power.

Transmission lines having resistance R and carrying current I have loss of power I^2R.

This loss is reduced by reducing the current by stepping up the voltage at generating station.

This high voltage is transmitted through high-tension transmission lines supported on robust pylons (iron girder pillars).

The voltage is stepped down at consumption station.

Example 12 :

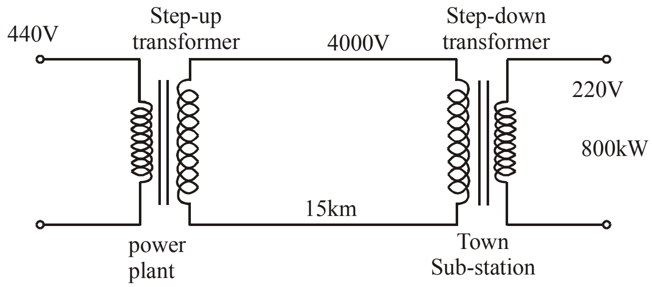
A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric power plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets from the line through a 4000 – 220 V step down transformer at a sub-station in the town.

(b) Estimate the line power loss in the form of heat.

(b) How much power must be plant supply, assuming there is a negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

Sol. The diagram shows the network :



For sub-station, $P = 800\text{ kW} = 800 \times 10^3$ watts
$V = 220\text{ V}$

$$I_S = \frac{P}{V} = \frac{800 \times 10^3}{220} = \frac{40}{11} \times 10^3 \text{ A.}$$

Primary current (I_p) in sub-station transformer will be given by $4000 \times I_P = 220 \times I_S$

$$I_P = \frac{220 \times 40 \times 10^3}{11 \times 4000} = 200\text{ A}$$

(a) Hence transmission line current = 200 A
transmission line resistance = $2 \times 15 \times 0.5 = 15\ \Omega$
transmission line power loss
$= I^2R = 200 \times 200 \times 15 = 6 \times 10^5$ watt = 600 kW.

(b) power to be supplied by plant = power required at substation + loss of power of transmission
$= 800 + 600 = 1400$ kW.

(c) Voltage in secondary at power plant has characteristics

$$= \frac{\text{Power}}{\text{Current}} = \frac{1400\text{ kW}}{200\text{ A}} = \frac{1400 \times 1000}{200} = 7000\text{ V}$$

Step-up transformer at power plant has characteristics 440 - 7000 V.

GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT

Let a battery of e.m.f. E connected to a series combination of an inductance L and resistance R as shown in fig.
The resistor R may be a separate circuit element, or it may be the resistance of the inductor windings.

Growth of Current :

If K is closed at t = 0, so at t = 0, current in the circuit I = 0
after closing the key K at time t let current in the circuit= I
for small time in the circuit, current varies with time,

so if rate of change of current with time = $\frac{dI}{dt}$

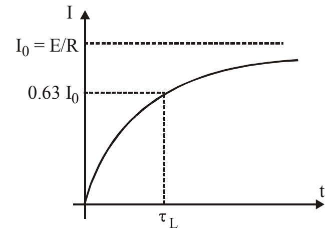
Then due to phenomenon of self induction induced emf

across inductance $= -L\frac{dI}{dt}$

Potential difference across the resistance = IR
During growth of current in L-R circuit, if we applying Kirchhoff's loop rule then

$$E + \left(-L\frac{dI}{dt}\right) = IR$$

On solving it we get the value of current at any time t during growth of current in LR-circuit.



$$I = I_0(1 - e^{-\frac{R}{L}t})$$

Time Constant :

$\frac{L}{R}$ has dimensions of time. It is called inductive time constant of LR-circuit.

$$\text{At } t = \frac{L}{R},\quad I = I_0(1 - e^{-\frac{R}{L}\cdot\frac{L}{R}}) = I_0(1 - e^{-1})$$

$$= I_0\left(\frac{e-1}{e}\right) = I_0\left(\frac{2.71-1}{2.71}\right) = 0.632\ I_0.$$

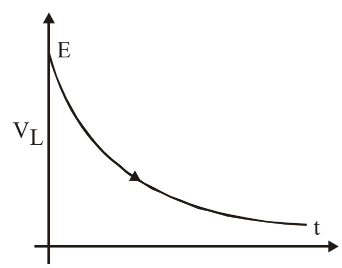
The inductive time constant of an LR-circuit is the time in which the current grows from zero to 0.632 (or 63.2%) of its maximum value.
When $t \to \infty$.

$$I = I_0(1 - e^{-\frac{R}{L}\cdot\infty}) = I_0(I - e^{-\infty}) = I_0(1-0)$$

Potential difference across resistance :

$$V_R = E(1 - e^{-\frac{R}{L}t})$$

$$V_L = L\frac{dI}{dt}$$



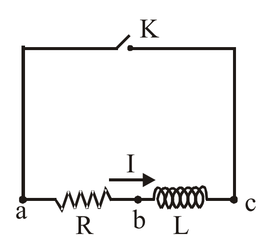
$$I = I_0 - I_0 e^{-\frac{R}{L}t}$$

$$\frac{dI}{dt} = 0 - I_0\, e^{-\frac{R}{L}t}\left(-\frac{R}{L}\right)$$

$$V_L = E\ e^{-\frac{R}{L}t}$$

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

DECAY OF CURRENT



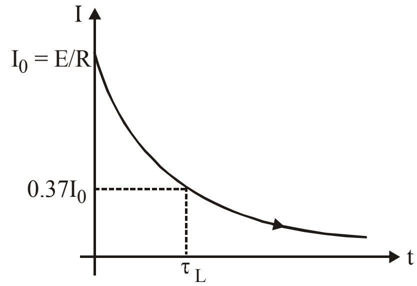
Let the current has reached its steady state value I_0 through inductor. Now switch K in the circuit shown in fig. has been closed.
Let this time is t = 0.
Let at t = 0 current in the circuit (which is maximum) = I_0
After time t current in the circuit = I
Apply Kirchhoff's loop rule to this circuit.

$$0 + \left(-L\frac{dI}{dt}\right) = IR \quad \text{(since there is no source of e.m.f.)}$$

or $L\dfrac{dI}{dt} = -IR$

or $\dfrac{dI}{I} = -\dfrac{R}{L}dt$

or $I = I_0\, e^{-\frac{R}{L}t}$



The eq^n. gives the value of current at any time t during decay of current in LR-circuit.

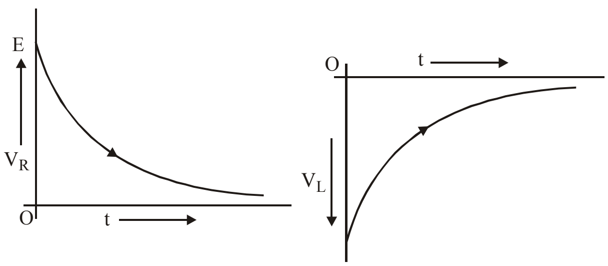
Again, dimensions of $\dfrac{L}{R}$ are same as that of time

The inductive time constant of the LR-circuit can also be defined by using eq^n.

Setting $t = \dfrac{L}{R}$ in eq^n., we get

$$I = I_0\, e^{-\frac{R}{L}\cdot\frac{L}{R}} = I_0\, e^{-1} = \frac{1}{e}I_0 \quad \text{or} \quad I \cong 0.37\, I_0.$$

as $t \to \infty$, $\quad I \to 0$



$V_R = IR \qquad$ or $V_R = e^{-\frac{R}{L}t}$; $V_L = L\dfrac{dI}{dt}$

$I = I_0\, e^{-\frac{R}{L}t}$

$\dfrac{dI}{dt} = I_0\, e^{-\frac{R}{L}t}\left(-\dfrac{R}{L}\right)$ or $V_L = -E\, e^{-\frac{R}{L}t}$

ENERGY STORED IN AN INDUCTOR

Consider that a source of e.m.f. is connected to an inductor L. As the current starts growing, an induced e.m.f. is set up in the inductor and the induced e.m.f. then, opposes the growth of current through it. The source of e.m.f. has to spend energy in sending current through the circuit against the induced e.m.f. The energy spent by the source of e.m.f. is stored in the inductor in the form of magnetic field. Initially (t = 0), the current in the inductor is zero. Let at any instant t, the current in the inductor is I

and the rate of growth of current at this time t is = $\dfrac{dI}{dt}$.

Then, induced e.m.f. produced in the inductor, $e = L\dfrac{dI}{dt}$

(in magnitude)

We can consider current through the inductor for a small time dt = I (approximately constant)
so charge dq = Idt is flown across potential difference "e" hence small amount of work done by the source

$$dW = e.dq = \left(L\frac{dI}{dt}\right) Idt = LIdI$$

The total amount of work done by the source of e.m.f, till the current increases from its initial value to its final value

I is $W = \displaystyle\int_0^I LI\, dI = L\left|\frac{I^2}{2}\right|_0^I = \frac{1}{2}LI^2$

Energy stored in the inductor $U = \dfrac{1}{2}LI^2$

This work done by the source of e.m.f. in building up current from zero to I is stored inside the inductor in the form of magnetic field energy.

Note :

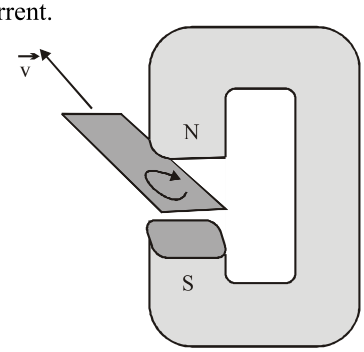
(i) The energy is stored in the inductor at the expense of the energy of the source of e.m.f.

(ii) The energy resides in the inductor in the form of magnetic field.

(iii) In case of an alternating source of e.m.f., during one half cycle energy is stored in the inductor from the source and during the next half cycle, the same amount of energy is returned to the source. It is for this reason that average electric power of an inductor is zero.
If current flowing through an inductor is I then energy stored in the form of magnetic field in the inductor is $U = \dfrac{1}{2}LI^2$, when change in I occurs in the circuit U changes it value accordingly. If I $\downarrow$ then U $\downarrow$ so loss in energy develops in the form of induced e.m.f. in the circuit and if I $\uparrow$ then U $\uparrow$ so energy required for increasing magnetic field and hence back e.m.f. induces.

EDDY CURRENTS

We know that when a conducting loop moves through a magnetic field, current is induced as the result of changing magnetic flux. If a solid conductor were used instead of a loop, as shown in figure, current can also be induced. The induced current appears to be circulating and is called an eddy current.



The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field (Figure).

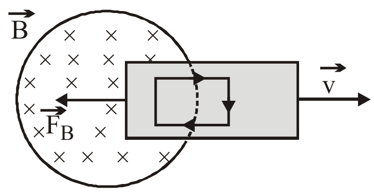
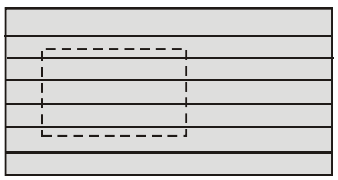


Figure : Magnetic force arising from the eddy current that opposes the motion of the conducting slab.

Since the conductor has non-vanishing resistance R, Joule heating causes a loss of power by an amount $P = e^2/R$. Therefore, by increasing the value of R, power loss can be reduced. One way to increase R is to laminate the conducting slab, or construct the slab by using gluing together thin strips that are insulated from one another (see Figure a). Another way is to make cuts in the slab, thereby disrupting the conducting path (Figure b).



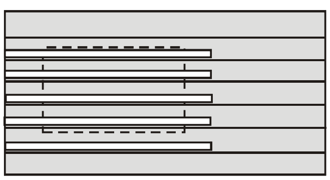


Figure : Eddy currents can be reduced by (a) laminating the slab, or (b) making cuts on the slab.

There are important applications of eddy currents. For example, the currents can be used to suppress unwanted mechanical oscillations. Another application is the magnetic braking systems in high-speed transit cars.

Example 13 :

Why a thick metal plate oscillating about a horizontal axis stops when a strong magnetic field is applied on the plate?

Sol. This is because eddy currents are produced and eddy currents oppose mechanical motion.

Example 14 :

A magnet is dropped in a very long copper tube. Even in the absence of air resistance it acquires a constant terminal velocity. Explain why?

Sol. When the magnet is dropped in a copper tube, eddy currents are produced in the tube. These eddy currents produce the magnetic field which opposes the motion of the magnet. After some time, the opposing force becomes equal to the gravitational pull on the magnet. Thus the net force acting on the magnet is zero and hence the magnet acquires a constant velocity.

Example 15 :

What happens if an iron piece is dropped between the poles of a strong magnet?

Sol. When the iron is dropped, eddy current are produced in it. These eddy currents oppose the motion of the piece of iron so it falls as it is moving through a viscous fluid.

D.C. MOTOR

A d.c. motor converts direct current energy from a battery into mechanical energy of rotation.

Principle : It is based on the fact that when a coil carrying current is held in a magnetic field, it experiences a torque, which rotates the coil.

Construction : It consists of the five parts.

Armature : The armature coil ABCD consists of a large number of turns of insulated copper wire wound over a soft iron core.

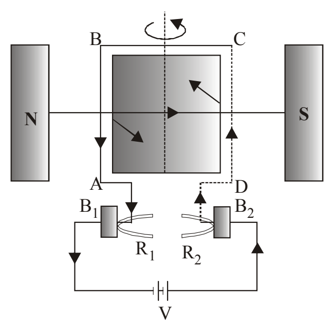
Field Magnet : The magnetic field is supplied by a permanent magnet NS.

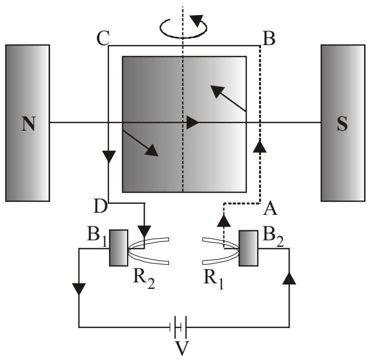
Split-rings or Commutator : These are two halves of the same ring. The ends of the armature coil are connected to these halves which also rotate with the armature.

Brushes : These are two flexible metal plates or carbon rods B_1 and B_2, which are so fixed that they constantly touch the revolving rings.

Battery : The battery consists of a few cells of voltage V connected across the brushes. The brushes convey the current to the rings, from where it is carried to the armature.

Working : The battery sends current through the armature coil in the direction shown in fig. Applying Fleming's Left Hand Rule, CD experiences a force directed inwards and perpendicular to the plane of the coil. Similarly, AB experiences a force directed outwards and perpendicular to the plane of the coil. These two forces being equal, unlike and parallel form a couple. The couple rotates the armature coil in the anticlockwise direction. After the coil has rotated through 180°, the direction of the current in AB and CD is reversed, fig. Now CD experiences an outward force and AB experiences an inward force. The armature coil thus continues rotating in the same i.e., anticlockwise direction.





Back E.M.F. : As the armature rotates in the magnetic field, the amount of magnetic flux linked it the coil changes. Therefore, an e.m.f. is induced in the coil. The direction of the induced e.m.f. is such that it opposes the battery current in the circuit. This e.m.f. is called the back e.m.f. and its magnitude goes on increasing with the speed of the armature.

Let V = e.m.f. applied across B_1 and B_2, R = resistance of the armature coil,

I=current flowing through the armature coil, at any instant t

E = back e.m.f. at that instant.

As V and E are acting in the opposite directions,

$\therefore$ effective e.m.f. across B_1 and $B_2 = V - E$

According to Ohm's law $I = \dfrac{V-E}{R}$ or $V - IR = E$

Efficiency of the d.c. motor : Since the current I is being supplied to the armature coil by the external source of e.m.f. V, therefore, Input electric power = VI

According to Joule's law of heating,

Power lost in the form of heat in the coil = $I^2 R$

If we assume that there is no other loss of power, then Power converted into external work i.e.,

Output mechanical power = $VI - I^2R = (V - IR)I = EI$

$\therefore$ Efficiency of the d.c. motor

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}}$$

or $$\eta = \frac{EI}{VI} = \frac{E}{V} = \frac{\text{back e.m.f.}}{\text{applied e.m.f.}}$$

Maximum efficiency : When E = V, then η = 1 or 100%. Therefore, for η to be maximum i.e. 100%, back e.m.f. E should be equal to the applied e.m.f. V. But in that case, the current I flowing through the armature coil becomes zero. Obviously, the motor in this case will just cease to work. This is an anomaly. Practically, the efficiency of the d.c. motor will be maximum, when output mechanical power is maximum. i.e. EI = maximum

using, $E.\dfrac{(V-E)}{R}$ = maximum

Now, $\dfrac{E(V-E)}{R}$ will be maximum, when its differential coefficient is zero.

i.e. $$\frac{d}{dE}\left[\frac{E(V-E)}{R}\right] = 0 \text{ or } \frac{d}{dE}\left[\frac{1}{R}(VE - E^2)\right] = 0$$

or $$\frac{1}{R}(V - 2E) = 0 \qquad (\because R \neq 0)$$

or $V - 2E = 0$ or $E = V/2$

From $\eta = \dfrac{E}{V}$, when $E = V/2$,

$$\eta = \frac{V/2}{V} = \frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$$

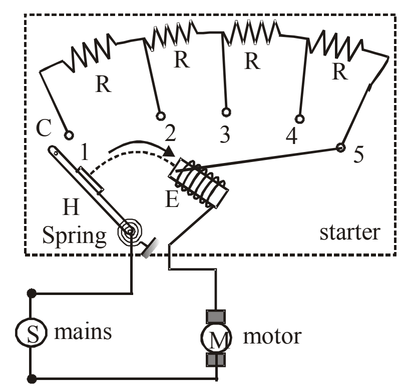
Hence a d.c. motor delivering maximum output has an efficiency of only 50%. Further, when E = V/2, then current I in the coil may become too large as R is low. Hence in practice, we do not try to get maximum output mechanical power.

Uses :

1. The d.c. motors are used in d.c. fans (exhaust, ceiling or table) for cooling and ventilation.
2. They are used for pumping water.
3. Big d.c. motors are used for running tram-cars and even trains.

MOTOR STARTER OR STARTING RESISTANCE

Motor starter regulates the current to safe limits while the motor gains the speed.



A starter is basically a variable high resistance connected in series with the motor coil in order to protect it from burning. A heavy duty motor should not be started without a starter whereas a light duty motor can be started without a starter because it gains speed in very short interval of time. When the motor just starts, there is no back e.m.f. in the coil of the motor.

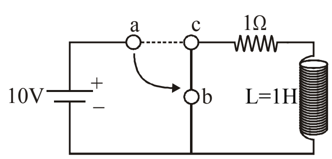
In the beginning, the motor draws a large amount of current due to full applied voltage which may burn the armature coil of the motor.

To avoid it, a starting resistance known as starter is used in series with the motor to regulate the current.

Motor starter consists of an iron handle (H) one end of which is connected to the spring system. The other end can slide over different resistances. The resistances are in series with an electromagnet (E) and armature coil (M) of the motor. Initially, whole of the resistance is in the circuit and the handle (H) makes the contact at point C of the resistances. So when the motor starts, a large resistance in the circuit and hence small starting current flows through the armature coil. As the motor picks up the speed, the resistance is switched out gradually. When the motor acquires the maximum speed all the resistance are switched out and the handle is attracted by the electromagnet.
At this stage, the current through the armature coil is normal. When the supply is switched off, the electromagnet loses its magnetisation and leaves the handle. The spring throws the handle back to its initial position C. The above process is to be repeated to start the motor again.

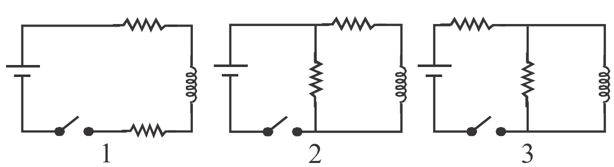
TRY IT YOURSELF-2

Q.1 In the given circuit, the switch is closed to the position bc from the earlier position of ac at t = 0. The current in the inductor after 2s of closing the switch between b and c is



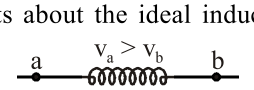
(A) $\frac{1}{10e^2}$ A
(B) $10e^{-2}$ A
(C) 10e A
(D) $10e^2$ A

Q.2 The diagrams show three circuits with identical batteries, identical inductors, and identical resistors. Rank them according to the current through the battery just after the switch is closed, from least to greatest.



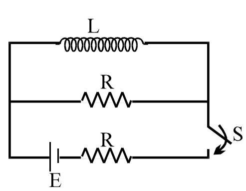
(A) 3, 2, 1 (B) 2 and 3 ties, then 1
(C) 1, 2, 3 (D) 1, 3, 2

Q.3 The potential at a is higher than the potential at b. Which of the following statements about the ideal inductor current I could be true ?



(A) I is from a to b and increasing
(B) I is from a to b and decreasing
(C) I is from b to a and increasing.
(D) I is from b to a and decreasing.

Q.4 In the circuit shown in figure switch S is closed at time t = 0, current through inductor after long time of closing of switch.



(A) E/R (B) E/2R
(C) zero (D) none

Q.5 A rectangular loop of sides 'a' and 'b' is placed in xy plane. A very long wire is also placed in xy plane such that side of length 'a' of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is 'd'. The mutual inductance of this system is proportional to
(A) a (B) b
(C) 1/d (D) current in wire

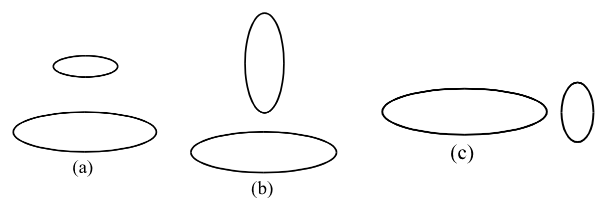
Q.6 Mutual inductance M_{12} of coil 1 with respect to coil 2
(A) increases when they are brought nearer.
(B) depends on the current passing through the coils.
(C) increases when one of them is rotated about an axis.
(D) is the same as M_{21} of coil 2 with respect to coil 1.

Q.7 Two circular coils, one of radius r and the other of radius R are placed coaxially with their centres coinciding. For R >> r, obtain expression for the mutual inductance of the arrangement

Q.8 How does the mutual inductance of a pair of coils change when
(i) distance between the coils is decreased and
(ii) number of turns in the coils is decreased ?

Q.9 A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12V battery. The current in the coil is 1.0 A at approximately the time :
(A) 500 s (B) 20 s
(C) 35 ms (D) 1 ms

Q.10 Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be :



(A) maximum in situation (a)
(B) maximum in situation (b)
(C) maximum in situation (c)
(D) the same in all situations

Q.11 Which of the following is true for an ideal transformer
(A) Total magnetic flux linked with primary coil equals flux linked with secondary coil
(B) Flux per turn in primary is equal to flux per turn in secondary
(C) Induced emf in secondary coil equals induced emf in primary
(D) Power associated with primary coil at any moment equals power associated with secondary coil.

ANSWERS

(1) (B)	**(2)** (D)	**(3)** (AD)
(4) (A)	**(5)** (A)	**(6)** (AD)
(7) $\frac{\mu_0 \pi r^2}{2R}$	**(8)** (i) increase, (ii) decrease	
(9) (D)	**(10)** (A)	**(11)** (BD)

USEFUL TIPS

* **Faraday's law** of induction imply that the emf induced in a coil of N turns is directly related to the rate of change of flux through it, $\varepsilon = -N\frac{d\phi_B}{dt}$

* **Lenz's law** states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it. The negative sign in the expression for Faraday's law indicates this fact.

* When a metal rod of length ℓ is placed normal to a uniform magnetic field B and moved with a velocity v perpendicular to the field, the induced emf (called motional emf) across its ends is $\varepsilon = B\ell v$.

* Induce emf due to rotation = $\frac{1}{2}B\omega\ell^2$

* When a current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $\varepsilon = -L\frac{dI}{dt}$

  L is the self-inductance of the coil. It is a measure of the inertia of the coil against the change of current through it.

* A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1). $\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$, The quantity M_{12} is called mutual inductance of coil 1 with respect to coil 2.

* For ideal transformer : $\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$

* LR circuit : For growth current, $i = i_0[1 - e^{-Rt/L}]$

  For decay of current, $i = i_0 e^{-Rt/L}$

* **Analogies between mechanical and electrical quantities:**

Mechanical system	Electrical system
Mass m	Inductance L
Force constant k	Reciprocal capacitance 1/C
Displacement x	Charge q
Velocity v = dx/dt	Current i = dq/dt
Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	Electromagnetic energy $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$

ADDITIONAL EXAMPLES

Example 1 :

Predict the direction of induced current in the situations described by the following fig. (1) to (5).

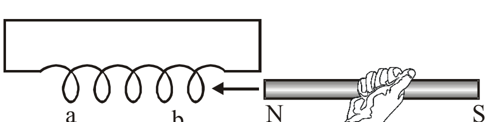


Fig. 1

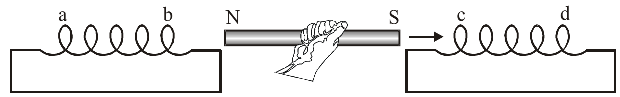


Fig. 2

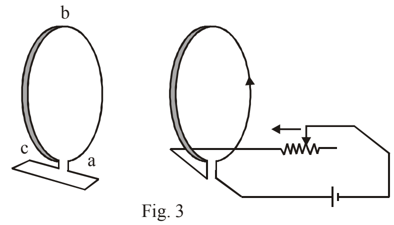


Fig. 3

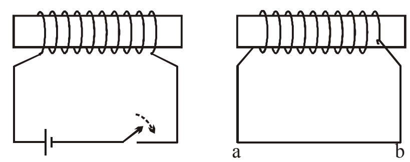


fig. 4

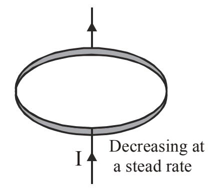


Fig.5

Sol. Applying Lenz's law

(1) along $a \to b$ (2) along $b \to a$, along $d \to c$ (3) along cba (4) along $a \to b$ (5) No induced current since field lines lie in the plane of the loop.

Example 2 :

A coil 10 turns and a resistance of 20Ω is connected in series with B.G. of resistance 30Ω. The coil is placed with its plane perpendicular to the direction of a uniform magnetic field of induction 10^{-2} T. If it is now turned through an angle of 60° about an axis in its plane. Find the charge induced in the coil. (Area of a coil = 10^{-2} m²)

Sol. Given : n = 10 turns, $R_{coil} = 20\Omega$,

$R_G = 30\Omega$, Total resistance in the circuit = 20 + 30 = 50Ω.

$A = 10^{-2}$ m², $B = 10^{-2}$ T, $\phi_1 = 0°$, $\phi_2 = 60°$

$$q = \frac{\phi_1 - \phi_2}{R} = \frac{BnA\cos\theta_1 - BnA\cos\theta_2}{R}$$

$$= \frac{BnA(\cos 0 - \cos 60)}{R} = \frac{BnA(1-0.5)}{R}$$

$$= \frac{0.5 \times 10^{-2} \times 10 \times 10^{-2}}{50} = \frac{50 \times 10^{-5}}{50}$$

$= 1 \times 10^{-5}$ C (Charge induced in a coil)

Example 3 :

A coil having 100 turns and area of 0.001 metre^2 is free to rotate about an axis. The coil is placed perpendicular to a magnetic field of 1.0 weber/metre^2. If the coil is rotate rapidly through an angle of 180°, how much charge will flow through the coil? The resistance of the coil is 10 ohm.

Sol. The flux linked with the coil when the plane of the coil is perpendicular to the magnetic field is

$\phi = nAB\cos\theta = nAB.$

The change in flux on rotating the coil by 180° is

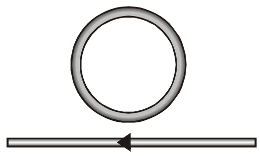
$d\phi = nAB - (-nAB) = 2nAB$

$\therefore$ Induced charge $= \frac{d\phi}{R} = \frac{2nAB}{dt} = \frac{2 \times 100 \times 0.001 \times 1}{10}$

$\therefore$ Induced charge = 0.01 coul.

Example 4 :

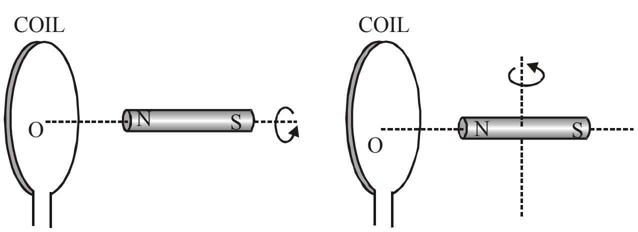
As shown in fig. the electric current in a wire in the direction is increasing. What is the direction of induced current in the metallic loop kept above the wire.



Sol. When the increasing current flows through the wire the increasing magnetic field is produced, which is directed perpendicular to the plane of the loop (or the plane of paper) and in inward direction. Due to this, induced e.m.f. is produced in the loop which opposes the magnetic field produced due to the current flowing through the wire i.e. induced current in the loop should flow in a direction so that it produces magnetic field perpendicular to the plane of the loop and in outward direction. Maxwell's cork screw rule tells that induced current in the loop will flow in anticlockwise direction.

Example 5 :

A cylindrical bar magnet is kept along the axis of a circular coil and near it as shown in fig. Will there be any induced e.m.f. at the terminals of the coil, when the magnet is rotated (a) about its own axis and (b) about an axis perpendicular to the length of the magnet?



(a) (b)

Sol. (a) When the magnet is rotated about its own axis, there is no change in the magnetic flux linked with the coil. Hence, no induced e.m.f. is produced in the coil.

(b) When the magnet is rotated about an axis perpendicular to its length, the orientation of the magnetic field due to the magnet will change continuously. Due to this, the magnetic flux linked with the coil will also change continuously and it will result in the production of induced e.m.f. in the coil.

Example 6 :

In fig. (a) and fig. (b) below show planar loops of different shapes moving out or into a region of magnetic field, which is directed normal to the plane of the loop and away from the reader. Determine the direction of induced current.

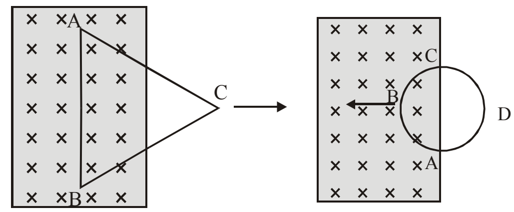


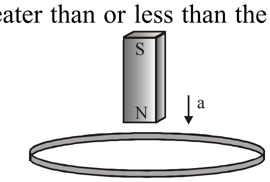
Fig. (a) Fig. (B)

Sol. (a) As the planar loop moves towards right, the magnetic flux linked with the loop decreases. The induced current will flow in a direction, so that the magnetic flux tends to increase through the loop i.e. the loop tends to move back into the magnetic field. For this, current in the arm AB (inside the magnetic field) should flow in such a direction so that the force on it acts towards left. It will happen so, if the induced current flows through the loop in the direction CBAC.

(b) As loop is coming inside the magnetic field so magnetic flux downwards through it is increasing so induced current in loop flows such that flux associate with it should reduce. Hence, the current through the loop flows in the direction CBADC. (Due to induced current flux is upwards)

Example 7 :

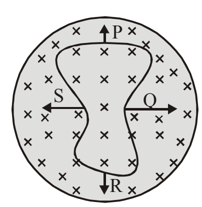
A bar magnet is freely falling along the axis of a circular loop as shown in fig. State whether its acceleration "a" is equal to, greater than or less than the acceleration due to gravity g.



Sol. According to Lenz's law, whatever may be the direction of induced current, it will oppose the cause. Here the cause is, the free fall of magnet and so the induced current will oppose it and the acceleration of magnet will be less than the acceleration due to gravity g. This can be understood in a different manner. When the magnet falls downwards with its north pole downwards. The magnetic field lines passing through the coil in the downward direction increase. Since the induced current opposes this, the upper side of the coil will become north pole, so that field lines of coil's magnetic field are upwards. Now like poles repel each other. Hence, $a < g$.

Example 8 :

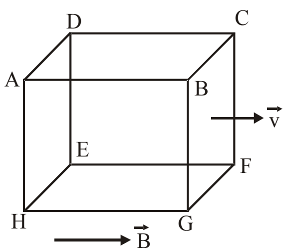
An irregular shaped wire PQRS (as shown in fig.) placed in a uniform magnetic field perpendicular to the plane of the paper changes into a circular shape. Show with reason the direction of the induced current in the loop.



Sol. When an irregular shaped wire PQRS changes to circular loop, the magnetic flux linked with the loop increases due to increase in area of the loop. The induced e.m.f. will cause current to flow in the direction, so that the wire is pulled inward from all sides. According to Fleming's left hand rule, force on wire PQRS will act inward from all sides, if the current flows in the direction PSRQ.

Example 9 :

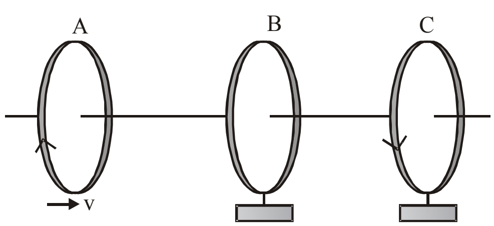
Twelve wires of equal lengths are connected in the form of a skeleton-cube which is moving with a velocity $\vec{v}$ in the direction of a magnetic field $\vec{B}$. Find the e.m.f. in each arm of the cube.



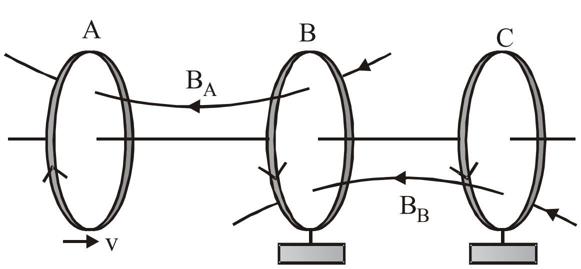
Sol. Force on a charge particle moving inside magnetic field is given by $\vec{F} = q\,(\vec{v} \times \vec{B})$. Since $\vec{v}$ and $\vec{B}$ are parallel, the force on electrons in any arm of the skeleton cube will be zero. As such, there cannot be drift of electrons in any arm from its one end to the other. Hence, no induced e.m.f. will be produced in any arm of the skeleton-cube.

Example 10 :

Three identical coils A, B and C are placed with their planes parallel to one another. Coils A and C carry currents as shown in fig. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B? If no, give reasons. If yes, mark the direction of induced current in the diagram.



Sol. Yes, the current is induced in coil B, when A moves with uniform motion towards B.



The direction of current induced is such that is opposes the approach of A towards B. For this the currents in A and B will be opposite i.e., current in B will be anticlockwise sense. As there is no relative motion between B and C, no current is induced in B due to current in C.

Example 11 :

The magnetic induction passing normally through a coil of 250 turns and area 5×10^{-3} m² is changing at the rate of 0.4 T/s. Find induced e.m.f.

Sol. Given : $n = 250, A = 5 \times 10^{-3}\ m^2, \frac{dB}{dt} = 0.4$ T/s

Induced e.m.f.

$$e = \frac{d\phi}{dt} = n\ A\frac{dB}{dt} = 250 \times 5 \times 10^{-3} \times 0.4$$
$$= 100.0 \times 5 \times 10^{-3} = 500 \times 10^{-3} = 0.5\ V$$

Example 12 :

A coil of effective area 4 m^2 is placed at right angles to the magnetic induction B. The e.m.f. of 0.32 V is induced in the coil. When the field is reduced to 20% of its initial value in 0.5 sec. Find B.

Sol. Given : $A = 4\ m^2$, $e = 0.32$ V, $dt = 0.5$sec.

B_1 is the initial magnetic induction and when it is reduced to 20% $B_2 = 0.2\ B_1$

$$e = \frac{d\phi}{dt} = \frac{A(B_1 - B_2)}{\Delta t} \quad \text{or} \quad 0.32 = \frac{4(B_1 - 0.2 B_1)}{0.5}$$

Magnetic induction $B_1 = \frac{0.16}{0.32} = 0.05\ Wb/m^2$

Example 13 :

A square wire loop with side 2 m in placed in a uniform magnetic field with its plane perpendicular to the field. The resistance of loop is 10Ω. Find at what rate the magnetic induction should be changed so that a current of 0.1 A is induced in the loop.

Sol. Given : $I = 0.1$ A, $R = 10\ \Omega$

Since, $e = IR$; $e = 0.1 \times 10 = 1$ V and $\ell = 2$ m

area, $A = \ell^2 = 2^2 = 4$

$$e = \frac{d}{dt}(BA) = A\frac{dB}{dt}\ ; \quad 1 = 4\frac{dB}{dt}$$

Rate of change of magnetic induction

$$\frac{dB}{dt} = \frac{1}{4} = 0.25\ Wb/m^2$$

Example 14 :

An earth coil is kept so that its plane is horizontal and axis of rotation in the magnetic meridian. The coil is suddenly rotated about its axis through an angle of 180°. Which magnetic field does it cut? Find the charge that has passed through coil. Given : no. of turns in the coil is 1000.
Each average area 100 π sq.m, resistance of coil $= 40\Omega$, $B_V = 2 \times 10^{-5}\ Wb/m^2$

Sol. Given : $n = 1000, A = 100\,\pi\, m^2$,

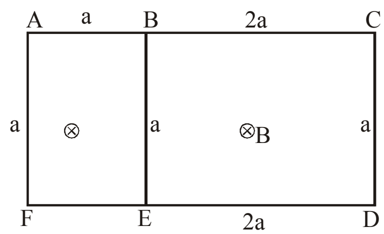
$R = 40\Omega$, $B_v = 2 \times 10^{-5}$ Wb/m^2

The coil when rotated cuts the vertical component of earth's magnetic field. The charge passed through the coil

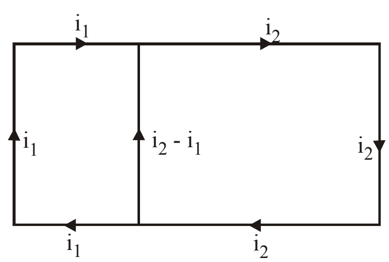
$$q = \frac{2nAB_v}{R} = \frac{2 \times 1000 \times 100\pi \times 2 \times 10^{-5}}{40} = 0.3142\text{ C}$$

Example 15 :

A closed loop with the geometry shown in fig. is placed in a uniform magnetic field directed into the plane of the paper. If the magnetic field decreases with time, determine the direction of the induced currents in this loop.



Sol. (A) There are two loops that are immersed in the magnetic field namely, ABEFA and BCDEB.



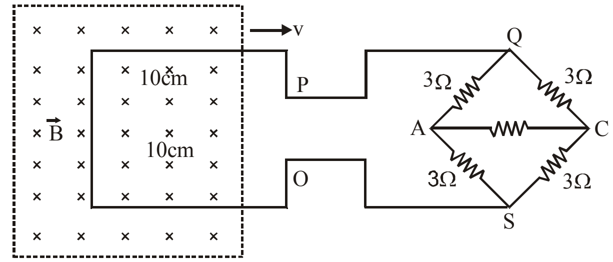
(B) Consider loop ABEFA. Magnetic flux is into the paper. For loop BCDEB too, the magnetic flux is into the paper.

(C) In both loops, the magnetic flux is decreasing with time.

(D) Hence the induced current in loop ABEFA, i_1, in the clockwise direction so as to induce a flux into the paper. Similarly in loop BCDEB the current i_2 will be in a direction so as to induce a flux into the paper. The direction of i_2 is also clockwise. The final induced currents in all the arms of the loop are shown in fig.

Example 16 :

A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v in a uniform magnetic field of induction B = 2 Wb/m² as shown in fig. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohm. The resistance of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in the loop? Give the direction of the current in the loop.



Sol. $B = 2$ Wb m^{-2} normal in the page, $\ell = 10$ cm $= 0.1$ m

Loop resistance = 1 ohm induced current I = 1 mA = 10^{-3} A

Resistance of balanced bridge, equivalent to two 6 ohm resistors in parallel = 3 ohm.

Total circuit resistance, R = 1 + 3 = 4 ohm, v = ?

Induced emf $e = B\,\ell\,v$

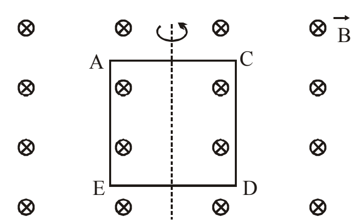
$$I = \frac{B\,\ell\,v}{R} \quad \text{and} \quad v = \frac{R\,I}{B\,\ell} = \frac{4 \times 10^{-3}}{2 \times 0.1} = 2 \times 10^{-2}\text{ m/s}$$

As flux is decreasing, the induced current in loop will set up a downward magnetic field (to support field B). Hence the current will flow in clockwise direction in the loop from O to P.

Example 17 :

A square loop ACDE of area 20 cm^2 and resistance 5Ω is rotated in a magnetic field $\vec{B} = 2T$ through 180° (a) in 0.01s and (b) in 0.02 s. Find the magnitude of e, i and Δq in both the cases.

Sol. Let us take the area vector $\vec{S}$ perpendicular to plane of loop inwards. So initially, $\vec{S}\uparrow\vec{B}\uparrow$ and when it is rotated by 180°, $\vec{S}\uparrow\vec{B}\downarrow$.



Hence, initial flux passing through the loop,

$\phi_i = BS\cos 0° = (2)(20 \times 10^{-4})(1) = 4.0 \times 10^{-3}$ Wb

Flux passing through the loop when it is rotated by 180°,

$\phi_f = BS\cos 180° = (2)(20 \times 10^{-4})(-1) = -4.0 \times 10^{-3}$ Wb

Therefore, change in flux, $\Delta\phi_B = \phi_f - \phi_i = -8.0 \times 10^{-3}$ Wb

(a) Given Δt = 0.01s, R = 5Ω

$$\therefore \quad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8.0 \times 10^{-3}}{0.01} = 0.8 \text{ volt}$$

$$i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16\text{A}$$

and $\Delta q = i\,\Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}$ C

(b) $\Delta t = 0.02$ s

$$\therefore \quad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8.0\times10^{-3}}{0.02} = 0.4 \text{ volt}$$

$$i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$

and $\Delta q = i\Delta t = (0.08)(0.02) = 1.6\times10^{-3}$ C

Example 18 :

A metal rod 1.5 m long rotates about its one end in a vertical plane at right angles to the magnetic meridian. If the frequency of rotation is 20 rev/sec., find the emf induced between the ends of the rod (H = 0.32 gauss).

Sol. When the rod rotates in a vertical plane perpendicular to the magnetic meridian, it will cut horizontal component of earth's field so that

$$e = \frac{1}{2}B_H\ell^2\omega = \pi\ell^2 fB_H \qquad (\text{as } \omega = 2\pi f)$$

Substituting the given data

$$e = \pi\times(1.5)^2\times20\times0.32\times10^{-4} = 4.5 \text{ mV}$$

Example 19 :

A metal rod of length 1 m is rotated about one of its ends in a plane right angles to a field of inductance 2.5×10^{-3} Wb/m^2. If it makes 1800 revolutions/min. Calculate induced e.m.f. between its ends.

Sol. Given : $\ell = 1$m, $B = 5\times10^{-3}$ Wb/m^2

$$f = \frac{1800}{60} = 30 \text{ rotations/sec}$$

In one rotation, the moving rod of the metal traces a circle of radius $r = \ell$

$\therefore$ Area swept in one rotation $= \pi r^2$

$$\frac{d\phi}{dt} = \frac{d}{dt}(BA) = B.\frac{dA}{dt} = \frac{B\pi r^2}{T}$$

$$= B f \pi r^2 = (5\times10^{-3})\times3.14\times30\times1 = 0.471 \text{ V}$$

$\therefore$ e.m.f. induced in a metal rod = 0.471 V

Example 20 :

A wheel with 10 metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0.40 G, what is the induced emf between the axle and the rim of the wheel?

Sol. $n = 10$, $r = 0.50$ m, $v = 120$ rev/min. = 2 rev/sec.

$B = 0.40$ G $= 0.4\times10^{-4}$ T

Area of wheel $= \pi r^2$

Area swept by each spoke per sec, $A = \pi r^2 v$.

Magnetic flux cut by each spoke per sec

$$\frac{d\phi}{dt} = B\pi r^2 v = BA = B\pi r^2 v$$

Induced e.m.f., $e = \frac{d\phi}{dt}$ (numerically)

$$= B\pi r^2 v = \frac{0.4\times10^{-4}\times22\times0.5\times0.5\times2}{7}$$

$= 6.286\times10^{-5}$ V,

Since all spokes are in parallel, total e.m.f. induced will be same as due to one spoke.

Example 21 :

Find the self inductance of a coil in which an e.m.f. of 10 V is induced when the current in the circuit changes uniformly from 1 A to 0.5 A in 0.2 sec.

Sol. Given : $e = 10$ V and $\frac{dI}{dt} = \frac{1-0.5}{0.2} = \frac{0.5}{0.2} = 2.5$ A / s

Self inductance of coil $L = \frac{e}{dI/dt} = \frac{10}{2.5} = 4$ H

$\because \; e = L\frac{dI}{dt}$ (Considering magnitude only)

Example 22 :

The mutual inductance of a pair of coils is 0.75 H. If current in the primary coil changes from 0.5 A to zero in 0.01 s find average induced e.m.f. in secondary coil.

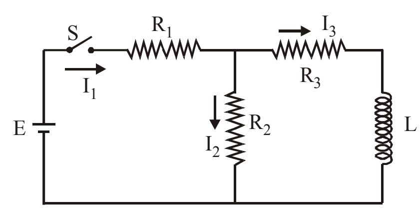
Sol. Given : $M = 0.75$ H and $\frac{dI}{dt} = \frac{0.5-0}{0.01} = 50$ A / s

$\therefore$ Average induced e.m.f. in secondary coil

$$e = M\frac{dI}{dt} = 0.75\times50 = 37.5 \text{ V}$$

Example 23 :

In fig. $E = 10$V, $R_1 = 1.0\,\Omega$, $R_2 = 2.0\,\Omega$, $R_3 = 3\Omega$ and $L = 2$H. Find the values of currents I_1, I_2 and I_3 :



(i) immediately after switch S is closed
(ii) sufficient time after the switch S is closed
(iii) immediately after opening the switch
(iv) sufficient time after opening the switch

Sol. (i) When switch S is closed, the circuit is completed, but due to electrical inertia of inductance coil, the current I_3 immediately after closing the switch is zero i.e., $I_3 = 0$.

The currents I_1 and I_2 will then be equal since R_1 and R_2 are in series to form the closed circuit.

The net resistance $R = R_1 + R_2 = 1.0 + 2.0 = 3.0\,\Omega$,

$$\therefore \quad I_1 = I_2 = \frac{E}{R} = \frac{10}{3.0} = 3.3 \text{ amp.}$$

(ii) Sufficient time after closing the switch, the inductance offers no resistance and current becomes steady in circuit containing R_3 and L. Now R_2 and R_3 are in parallel, their equivalent resistance

$$R' = \frac{R_2 R_3}{R_2 + R_3} = \frac{2 \times 3}{2+3} = \frac{6}{5} = 1.2\Omega$$

Effective resistance of circuit,

$R = R_1 + R' = 1 + 1.2 = 2.2\,\Omega.$

$\therefore$ Current in circuit is $= \frac{E}{R} = \frac{10}{2.2} = 4.5\text{A}$

Potential difference across R_2 and $R_3 = I_1 R'$

$$V = \frac{10}{2.2} \times 1.2 = \frac{60}{11} \text{ volt}$$

$$\therefore \quad I_2 = \frac{V}{R_2} = \frac{(60/11)}{2.0} = \frac{60}{22} = 2.7\text{A}$$

$$\therefore \quad I_3 = \frac{V}{R_3} = \frac{(60/11)}{3} = \frac{60}{33} = 1.8\text{A}$$

(iii) When switch S is opened, the main circuit is broken and so current delivered by cell, $I_1 = 0$.
But R_2, R_3 and L are still in closed circuit, therefore, due to electrical inertia of self inductance, immediately after opening the switch, currents I_2 and I_3 are not zero and $I_2 = I_3 = 1.8$ A.
(iv) Sufficient time after opening the switch, the inductance has no role and so all current I_1, I_2 and I_3 are zero.

Example 24 :

A simple electric motor has an armature resistance of 1 Ω and runs from a dc source of 12 volt. When running unloaded it draws a current of 2 amp. When a certain load is connected, its speed becomes one-half of its unloaded value. What is the new value of current drawn?

Sol. Let initial e.m.f. induced = e.

$$\therefore \quad \text{Initial current } i = \frac{E-e}{R} \text{ i.e., } 2 = \frac{12-e}{1}$$

This gives e = 12 – 2 = 10 volt. As $e \propto \omega$.
when speed is halved, the value of induced e.m.f. becomes

$$\frac{e}{2} = \frac{10}{2} = 5 \text{ volt}$$

$$\therefore \quad \text{Final value of current } i' = \frac{E-e}{R} = \frac{12-5}{1} = 7\,\text{A}$$

Example 25 :

An induced emf has no direction of its own. Comment.

Sol. The given statement is correct. This is because, according to Lenz's law, the direction of the induced emf is such as to oppose the cause of production of induced emf. Thus, the direction of induced emf is to be determined by the cause of emf.

Example 26 :

Two identical loops, one of copper and another of aluminium are rotated with same speed in the same magnetic field. In which case the induced (a) e.m.f. and (b) current will be more? Explain.

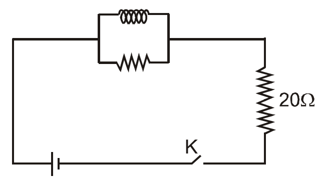
Sol. (a) The change in magnetic flux linked with both the loop will be same. So the induced e.m.f. produced in both the loops is same.

(b) $\text{Current (I)} = \frac{\text{e.m.f.(E)}}{\text{Resistance (R)}}$

Since the resistance of copper loop is less than that of the aluminium loop, so more current will flow through the copper loop than that in the aluminium loop.

Example 27 :

Two resistors of 10 Ω and 20 Ω and an ideal inductor of 10H are connected to a 2 V battery as shown. The key K is inserted at time t = 0. Find the initial (t = 0) and final ($t \to \infty$) currents through battery.



Sol. At t = 0 i.e. when the key is just pressed, no current exists inside the inductor. So 10 Ω and 20 Ω resistors are in series and a net resistance of (10 + 20) = 30 Ω exists across the circuit. Hence $I_1 = \frac{2}{30} = \frac{1}{15}\text{A}$

As $t \to \infty$, the current in the inductor grows to attain a maximum value i.e. the entire current passes through the inductor and no current passes through 10Ω resistor.

Hence $I_2 = \frac{2}{20} = \frac{1}{10}\text{A}$

Example 28 :

A 20 mH coil is connected in series with a 2000 ohm resistor and a 12-V battery. Calculate the time constant of the circuit. After what time the current attains 99% of its final value after the switch is closed?

Sol. (i) Time constant $= \frac{L}{R} = \frac{20 \times 10^{-3}}{2000} = 10 \times 10^{-6}\text{ s} = 10\ \mu\text{s}$

(ii) $I = I_0(1 - e^{-\frac{R}{L}t})$

Here, $I = 99\%,\ I_0 = \frac{99}{100} I_0$

$$\therefore \frac{99}{100} = \left(1 - e^{-\frac{2000}{20\times10^{-3}}t}\right) \Rightarrow \frac{99}{100} = 1 - e^{-10^5 t}$$

or $e^{-10^5 t} = 1 - \frac{99}{100} = \frac{1}{100}$ or $e^{-10^5 t} = 100$

Taking log of both sides $10^5 t = \log_e 100 = 2.303 \log_{10} 100$
$= 2.303 \times 2.0000 = 4.606$

or $t = 4.606 \times 10^{-5} = 46.06 \times 10^{-6}$ s $= 46.06$ μs.

Example 29 :

A coil of resistance 50 Ω is connected across a 5.0 V battery, 0.1 s after the battery is connected, the current in the coil is 60 mA. Calculate the inductance of the coil.

Sol. Here, $I_0 = \frac{E}{R} = \frac{5}{50} = 0.1$ A

$I = 60$ mA $= 60 \times 10^{-3}$ A, $t = 0.1$

Now, $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\therefore \quad 60 \times 10^{-3} = 0.1\left(1 - e^{-\frac{50}{L} \times 0.1}\right) = 0.1\left(1 - e^{-\frac{5}{L}}\right)$

or $1 - e^{-5/L} = 0.6$

$\therefore e^{-5/L} = 1 - 0.6 = 0.4 = \frac{4}{10}$ or $e^{5/L} = 10/4$

Taking log of both sides

$\frac{5}{L} = 2.303\,[\log_{10} 10 - \log_{10} 4]$

$= 2.303\,[1.0000 - 0.6021] = 2.303 \times 0.3979 = 0.9164$

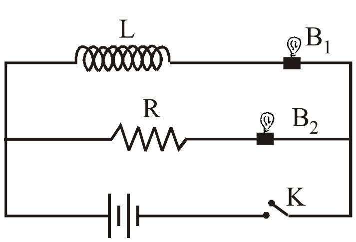
$\therefore \quad L = \frac{5}{0.9164} = 5.5\,\text{H}$

Example 30 :

Two identical bulbs are connected as shown in the fig. resistance R is equal to D.C. resistance of coil wire.

(a) Which of the bulbs light up earlier when key K is closed

(b) Will the bulbs be equally bright after some time?

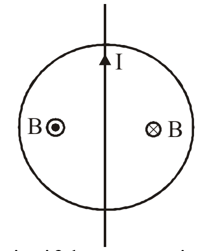


Sol. (a) When key (K) is closed. Current begins to flow through both the area. Induced e.m.f. is produced across the inductor which oppose the growth of current in the circuit. So current through B_1 is delayed. Hence bulb B_2 lights up earlier than that of bulb B_1.

(b) When current becomes constant after some time in both the arms, no induced e.m.f. is produced in L. So both the bulbs will be equally bright.

Example 31 :

A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are)



(A) The emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is infinite if the current is constant.

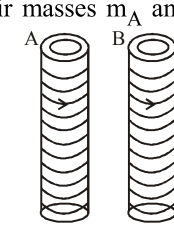
(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) The emf induced in the loop is finite if the current decreases at a steady rate.

Sol. **(AC).** Since flux of wire on the loop is zero therefore, emf will not be induced.

Example 32 :

Two metallic rings A and B, identical in shape and size but having different resistivity ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivity and their masses m_A and m_B is (are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$

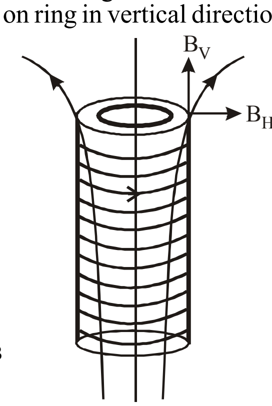
(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

(D) $\rho_A < \rho_B$ and $m_A < m_B$

Sol. **(BD).** The horizontal component of magnetic field due to solenoid will exert force on ring in vertical direction

$F = B_H i\,(2\pi r)$, $F\Delta t = MV$,



$i = \frac{\Delta\phi / \Delta t}{\left(\rho\frac{2\pi r}{A}\right)}$

$B_H i(2\pi r)\Delta t = MV$

$V = \frac{B_H \Delta\phi A}{\rho M} = \frac{K}{\rho M}$;

$h = \frac{V^2}{2g} = \frac{K^2}{\rho^2 M^2}$; $h_A > h_B$

$\Rightarrow \frac{K^2}{\rho_A^2 M_A^2} > \frac{K^2}{\rho_B^2 M_B^2} \Rightarrow \rho_B M_B > \rho_A M_A$

$\Rightarrow$ Using this we get ans (B) and (D)

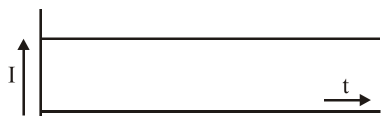
4B

ALTERNATING CURRENT

DIRECT CURRENT (D.C.)

If a circuit contents a battery and a resistance in series then e.m.f. and current in circuit are as shown below and known as direct e.m.f. and direct current.

Pure D.C. f = 0 Hz

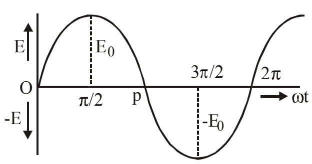


Pure D.C. f = 0 Hz

Magnitude of e.m.f. and current does not varies with time in D.C. circuits.

ALTERNATING EMF

Magnitude of emf (or voltage) changes continuously with time between zero and a maximum value and also direction reverses periodically.



A sinusoidal alternating emf is developed in a coil of wire rotating with constant angular velocity ω in a uniform magnetic field. The magnitude of the alternating emf changes with time and its direction reverses peiodically as shown in figure. The alternating emf $E = E_0 \sin \omega t$.

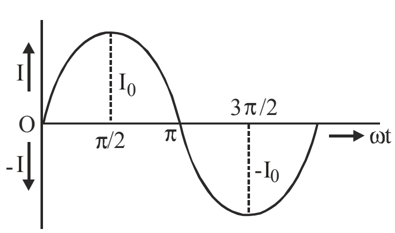
Here E = the instantaneous value of emf,

E_0 = the maximum or peak value,

ω = angular speed of the coil and ωt = phase of alternating emf.

ALTERNATING CURRENT (AC) :

When an alternating emf is applied across a coil or a bulb, it sends a similarly varying current (i.e. of the same nature as that of e.m.f.) through the coil. The currents is called alternating current.



The alternating current also varies in magnitude with time and reverses its direction periodically.

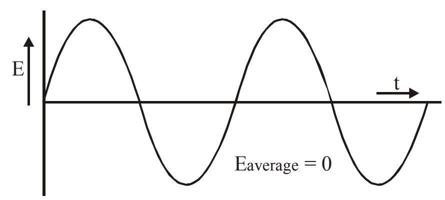
The alternating current is similarly expressed by

$I = I_0 \sin \omega t$

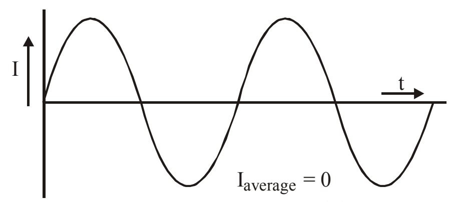
I_0 = the peak value or the amplitude of the alternating current and ωt = phase of alternating current.

The alternating current continuously varies in magnitude and periodically reverses its direction.

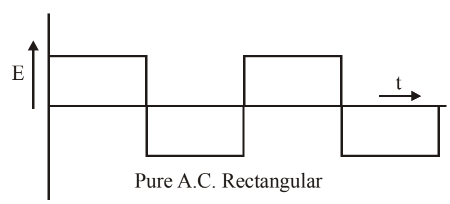
DIFFERENT TYPES OF E.M.F. AND CURRENTS



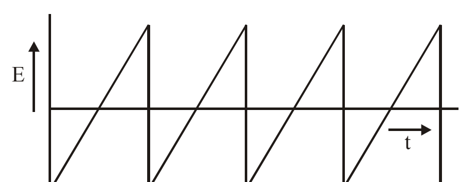
Pure Alternating e.m.f. (sinosuidal)



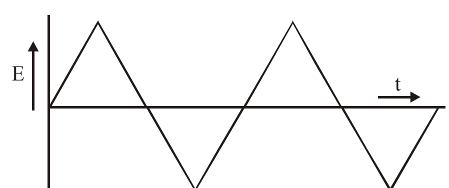
Pure A.C. (sinosuidal)



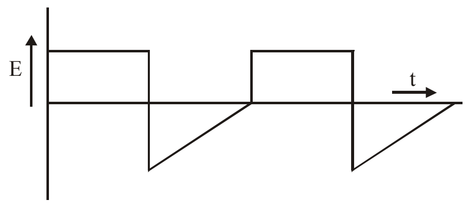
Pure A.C. Rectangular



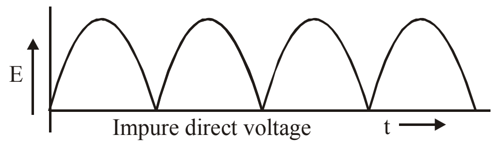
Saw tooth A.C.

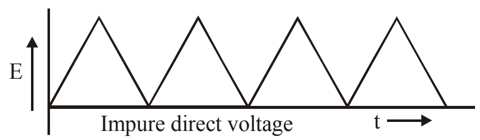


Triangular A.C.



Asymmetrical A.C.





IMPORTANT TERMS

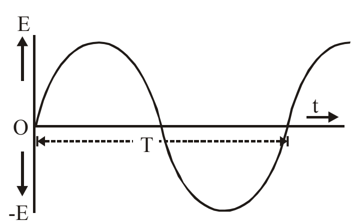
Cycle of alternating e.m.f. and current : The variation of e.m.f. from zero to maximum in one direction, through zero to maximum in the opposite direction and back to zero again is called a cycle of the alternating emf.

One cycle of a.c. to be variation of current from zero to maximum in one direction, through zero to maximum in the opposite direction and back to zero again.

Period and frequency of alternating emf (or current) : Alternating e.m.f. is defined by the equation :

$E = E_0 \sin \omega t$

This shows that after a time interval of $T = \frac{2\pi}{\omega}$, alternating e.m.f. (or current) repeats its nature.



Time Period : Time taken by the alternating emf (or current) to complete one cycle of its variations.

Time interval $\frac{2\pi}{\omega}$ is called as time period of alternating emf (or current) $T = \frac{2\pi}{\omega}$

Frequency : Number of cycles completed by alternating current (or emf) in one second.

frequency $f = \frac{1}{T}$ But $T = \frac{2\pi}{\omega}$ $\therefore$ $f = \frac{\omega}{2\pi}$

Unit of frequency : cycles/second (C/s) or hertz (Hz).

In India frequency of A.C. is 50 Hz and in America it is 60Hz.

The frequency of the domestic A.C. 50 Hz. Alternating current in electric wires or bulbs flows 50 times in one direction and 50 times in opposite direction in 1 second. Since in one cycle the current becomes zero two times, hence bulb lights up 100 times and is off 100 times in one second, but due to persistence of vision, it appears lighted continuously.

MEAN OR AVERAGE VALUE OF ALTERNATING CURRENT (OR VOLTAGE)

The mean or average value of a.c. over one complete cycle is zero. The mean value of alternating current over half a cycle is defined as the mean value. The mean or average value of alternating current over any half cycle is defined as that value of steady direct current which would send the same amount of charge through a circuit in half time period (i.e. T/2) as is sent by the a.c. through the same circuit in the same time. It is represented by I_m.

$$I_m\left(\frac{T}{2}\right) = \int_0^{T/2} I\,dt \quad \text{or} \quad I_m \frac{2}{T} = \int_0^{T/2} I\,dt \qquad \text{......(i)}$$

$$\text{i.e., } I_m = \frac{2I_0}{\pi} = 0.637\ I_0 \qquad \text{......(ii)}$$

Similarly, it can be proved that the mean or average value of alternating e.m.f. over positive half cycle (0 to T/2) is

$$E_m = \frac{2E_0}{\pi} = 0.637\ E_0$$

The average value of alternating e.m.f. over one complete cycle is also equal to zero.

Mean value for half cycle in between $t = \frac{T}{4}$ & $\frac{3T}{4}$ is zero.

ROOT MEAN SQUARE VALUE (rms) OF ALTERNATING CURRENT (VIRTUAL VALUE)

The root mean square (rms) value of alternating current is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time as is done by the a.c. when passed through the same resistance for the same time. It is represented by I_{rms}. It is also called effective value or virtual value of the alternating current. The ammeter, when connected in the circuit, always measures rms value of a.c.

$$\overline{I}^2 = \frac{1}{T}\int_0^T I^2\,dt \quad (\because \text{ instantaneous value of a.c. } I = I_0 \sin \omega t)$$

$$\therefore \quad \overline{I^2} = \frac{I_0^2}{2}\ ; \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$$

Thus the rms value or effective value or virtual value of a.c. is 0.707 times the peak value of a.c. or 70.7% of the peak value of a.c. Similarly r.m.s. value or virtual value of alternating e.m.f. $E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707\,E_0$

FORM FACTOR

$$\text{Form factor} = \frac{\text{rms value of AC}}{\text{Mean value of AC over half cycle}}$$

For sinusoidal AC :

$$\text{Form factor} = \frac{I_0/\sqrt{2}}{2I_0/\pi} \quad \text{or Form factor} = \frac{\pi}{2\sqrt{2}}$$

Note :

(i) The heating effect of a A.C. $I = I_0 \sin \omega t$ is same as that of a steady current $= 0.707\, I_0$.

(ii) Alternating current and emf are usually described in terms of their effective value or rms value.
If a bulb marked 250 V ~ and 25 Watt it means 250 V rms value.
Alternating potential difference between the supply mains of a house hold power line is 220 Volt, it means that the rms or effective potential difference is 220 Volt and hence the maximum potential difference is $220 \times \sqrt{2} = 311$ Volt. The Voltage in domestic supply varies from +311 Volt to –311 Volt in each cycle.

Example 1 :
The instantaneous current from an a.c. source is $I = 6\sin 314t$. What is the rms value of the current ?

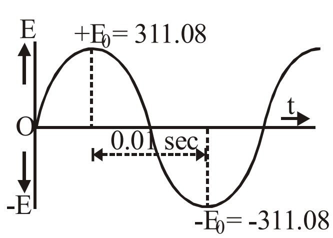
Sol. $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$ amp.

Example 2 :
What is the relation between the mean and virtual values of a.c.?

Sol. $I_m = \frac{2\sqrt{2}}{\pi} I_{rms}$

Example 3 :
Explain why A.C. is more dangerous than D.C. of same value.

Sol. There are two reasons for it :
(1) A.C. attracts while D.C. repels.



(2) A.C. gives a huge and sudden shock.
An A.C. main of 220 V has $V_{rms} = 220$ V

Hence, $V_0 = \sqrt{2}.\, V_{rms} = 12.414 \times 220 = 311.08$ V
Voltage change from $+V_0$ (positive peak) to $-V_0$ (negative peak) $= 311.08 - (-311.08) = 622.16$ V

This change takes place in half cycle i.e., in $\frac{1}{100}$ sec (for a 50 Hz A.C.) A shock of 622.16 within 0.01 sec is huge and sudden, hence fatal.

Example 4 :

If $I = I_0 \cos \omega t$, can we define $I_a = \frac{2}{T}\int_0^{T/2} I\, dt$. Why ?

Sol. No, because from 0 to T/4, I will be +ve and from T/4 to T/2, it will be –ve. So I_a will come out zero.

For $I = I_0 \cos \omega t$, average value of current $I_a = \frac{2}{T}\int_{-T/4}^{T/4} I\, dt$

Example 5 :
The effective value of current in a 50 cycle a.c. circuit is 5.0amp. What is the value of the current 1/300 sec after it is zero?

Sol. Here, $I_{rms} = 5.0$ amp, f = 50 cps
Now, $I = I_0 \sin \omega t = \sqrt{2}\, I_{rms} \sin 2\pi f t$
$= \sqrt{2} \times 5.0 \sin 2\pi \times 50 \times (1/300) = 6.124$ amp

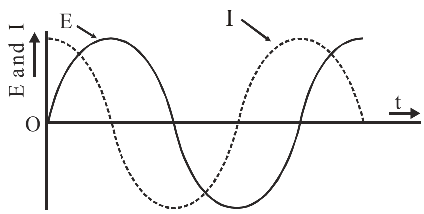
MEASUREMENT OF A.C.
Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments.

S.No.	Terms	D.C. meter	A.C. meter
1	Name	moving coil	hot wire
2.	Based on	magnetic effect of current	heating effect of current
3.	Reads	average value	r.m.s. value
4.	If used in	A.C. circuit then they reads zero ∵ average value of A.C. = zero	A.C. meter works properly in D.C. circuit also as based on heating effect of current.
5.	Deflection	def. ∝ current $\phi \propto I$ (linear)	def. ∝ heat $\phi \propto I^2_{rms}$ (non linear)
6.	Scale	Uniform Separation	Non uniform separation
	ϕ = Number of divisions	I - 1 2 3 4 5 ϕ - 1 2 3 4 5	I - 1 2 3 4 5 ϕ - 1 4 9 16 25

PHASE IN A.C.

The frequency of alternating current and alternating emf is same. Generally, when current is maximum, emf if not maximum and vice versa. This lead or lag is the phase difference between the emf (voltage) and current in A.C. circuits any may be represented by ϕ. If e.m.f. (or Voltage) in A.C. is $E = E_0 \sin \omega t$ and the current $I = I_0 \sin(\omega t + \phi)$

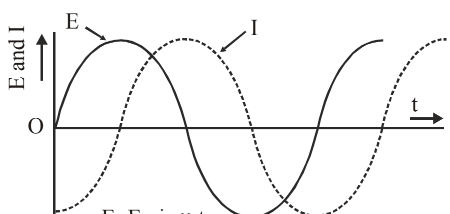
The phase difference ϕ : Positive if current leads and Negative if current lags zero if current is in phase with the emf (or voltage).



$E=E_0 \sin\omega t$

$I = I_0 \sin(\omega t + \phi)$ $\phi = \pi/2$

Current leads emf by $\pi / 2$



$E=E_0 \sin\omega t$

$I=I_0 \sin(\omega t + \phi)$ $\phi = -\pi/2$

Current lag behinds emf by $\pi / 2$

PHASORS

In order to the simplify the study of A.C. circuits, it is preferred to treat alternating current and alternating e.m.f. as vectors with the angle between the vectors equal to the phase difference between the current and the emf. The current and emf vectors are more appropriately called phasors.

PHASOR DIAGRAMS

A diagram representing alternating current and alternating voltage (of same frequency) as vectors (phasors) with the phase angle between them is called a phasor diagram.

DIFFERENT TYPES OF A.C. CIRCUITS

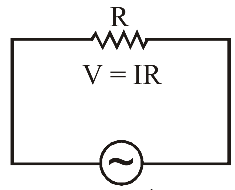
In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of three elements.

A.C. CIRCUIT CONTAINING PURE RESISTANCE

A circuit containing a pure resistance R (non-inductive) connected with a source of alternating e.m.f.



$E = E_0 \sin\omega t$

Suppose the alternating e.m.f. is represented by

$E = E_0 \sin \omega t$

Let at any instant t the current in the circuit = I.

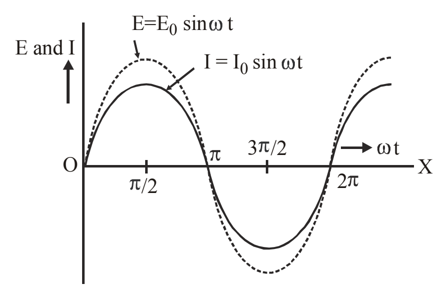
Potential difference across the resistance = I R.

with the help of kirchoff's circuital law $E - IR = 0$

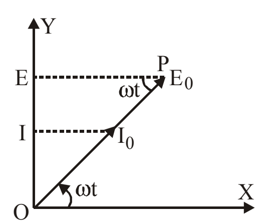
or $E_0 \sin \omega t = IR$ or $I = \dfrac{E_0}{R} \sin \omega t = I_0 \sin \omega t$

$\left(I_0 = \dfrac{E_0}{R} \right.$ = peak or maximum value of current)

Alternating current developed in a pure resistance is also of sinusoidal nature. In an a.c. circuits containing pure resistance, the voltage and current are in the same phase



The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. as shown in fig.



In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.

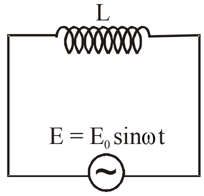
i.e. $I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$.

Since $I_0 = \dfrac{E_0}{R}$,

hence $\dfrac{I_0}{\sqrt{2}} = \dfrac{E_0}{R\sqrt{2}}$ or $I_{rms} = \dfrac{E_{rms}}{R}$

A.C. CIRCUIT CONTAINING PURE INDUCTANCE

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf.



Let the alternating e.m.f.

$E = E_0 \sin \omega t$

When a.c. flows through the circuit, emf induced across

inductance = $e = -L\frac{dI}{dt}$

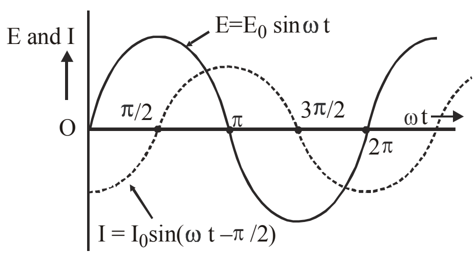
$\frac{dI}{dt}$ = the rate of change of the current.

-ive sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other then inductance so with the help of Kirchoff's

circuital law $E + \left(-L\frac{dI}{dt}\right) = 0$ or $E = L\frac{dI}{dt}$

so we get $I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

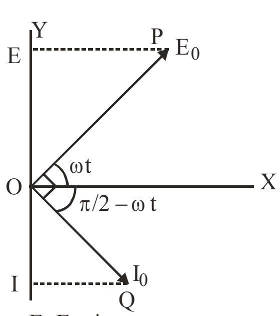


Maximum current

$I_0 = \frac{E_0}{\omega L}\times 1 = \frac{E_0}{\omega L}$ [when $\sin\left(\omega t - \frac{\pi}{2}\right)$ is max. =1]

Hence, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

In a pure inductive circuit current always lags behind the emf by $\pi/2$ or alternating emf leads the a. c. by a phase angle of $\pi/2$.



$E=E_0 \sin\omega t$
$I = I_0 \cos\omega t$

Expression $I_0 = \frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I} = R$.

This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

$X_L = \omega L = 2\pi f L$, where f = frequency of A.C.

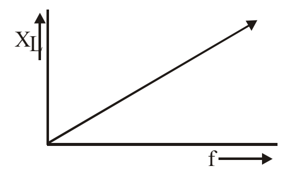
Unit of X_L : ohm

(ωL) = Unit of L × Unit of ω = henry × sec^{-1}

$$= \frac{\text{Volt}}{\text{Ampere / sec}} \times sec^{-1} = \frac{\text{Volt}}{\text{Ampere}} = \text{ohm}$$

Inductive reactance $X_L \propto f$

Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.



For d.c. circuit, f = 0

$\therefore\ X_L = \omega L = 2\pi f L = 0$

Hence, inductor offers no opposition to the flow of d.c. or where as a resistive path to a.c.

In a circuit containing resistance R only, opposition to the flow of current arises on account of obstruction to the passage of electrons through the resistor.

But in a inductor, it is the self induced emf that opposes the growth of current.

$I_0 = \frac{E_0}{\omega L} = \frac{E_0}{X_L}$ gives $\frac{I_0}{\sqrt{2}} = \frac{1}{X_L}\frac{E_0}{\sqrt{2}}$ or $I_{rms} = \frac{E_{rms}}{X_L}$

Above equation is identical to Ohm's law equation.

Example 6 :

The reactance of inductor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its reactance when connected in DC circuit? What is its consequence?

Sol. The reactance of inductor is 20 ohm. It means that the hindrance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm.

The reactance of inductance, $X_L = \omega L = 2\pi f L$. Therefore by doubling frequency, the reactance is doubled and it becomes 40 ohm.

In DC circuit f = 0; therefore reactance of inductor = 0.

Therefore inductor can not be used to control DC.

Example 7 :

A a.c. circuit consists of only an inductor of inductance 2H. If the current is represented by a sine wave of amplitude 0.25 amp. and frequency 60 Hz, calculate the effective potential difference across the inductor

Sol. The effective potential difference across the inductor is

given by $V_{eff} = I_{eff}.X_L = \frac{I_0}{\sqrt{2}}.2\pi f L$

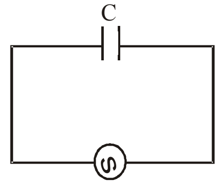
$V_{eff} = V_{rms}$

Given that $I_0 = 0.25$ amp, $f = 60$ Hz, $L = 2H$

$\therefore \quad V_{eff} = \frac{0.25}{\sqrt{2}} \times 2 \times 3.14 \times 60 \times 2 = 133.2$ Volt

A.C. CIRCUIT CONTAINING PURE CAPACITANCE

A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in figure.



$E = E_0 \sin\omega t$

The alternating e.m.f. in the circuit

$E = E_0 \sin \omega t$

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

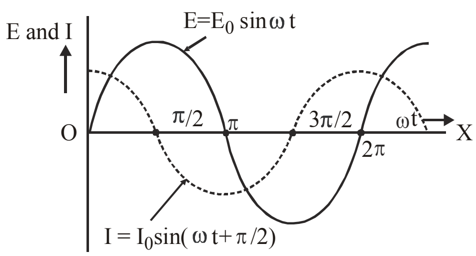
The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time.

Also the electric field between the plates of the capacitor varies sinusoidally with time.

Let at any instant t charge on the capacitor = q

Instantaneous potential difference across the capacitor

$$E = \frac{q}{C} \quad \text{or} \quad q = CE \quad \text{or} \quad q = CE_0 \sin \omega t$$



The instantaneous value of current

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t) = CE_0 \omega \cos \omega t$$

or $$I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

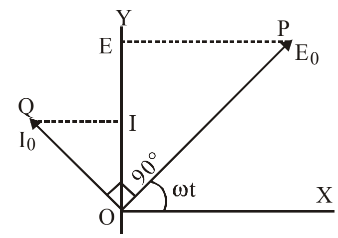
or $$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_0 = \frac{E_0}{(1/\omega C)} \times 1 = \frac{E_0}{(1/\omega C)}$$

Maximum current I_0 when $\sin\left(\omega t + \frac{\pi}{2}\right)$ is max. =1

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\frac{\pi}{2}$.

The alternating emf lags behinds the alternating current by a phase angle of $\frac{\pi}{2}$.



$\frac{E}{I}$ is the resistance R when both E and I are in phase,

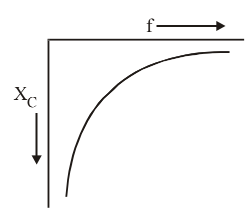
in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C.

This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequency of A.C.



X_C decreases as the frequency increases.

This is because with an increase in frequency, the capacitor charges and discharges rapidly following the flow of current.

For d.c. circuit f = 0

$$\therefore \quad X_C = \frac{1}{2\pi f C} = \infty$$

but has a very small value for a.c. This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

The physical process involved in the capacitance is entirely different to that involved in a resistance. In a resistive circuit, the resistance is due to the obstruction to the passage of electrons through the resistor. But in a capacitive circuit, resistance to the flow of current is due to the charged capacitor. $I_0 = \frac{E_0}{1/\omega C} = \frac{E_0}{X_C}$

gives that $\frac{I_0}{\sqrt{2}} = \frac{1}{X_C}\frac{E_0}{\sqrt{2}}$ or $I_{rms} = \frac{E_{rms}}{X_C}$

Above expression is similar to ohm's law equation.

In a high frequency A.C. circuit, the capacitor acts like a conductor. This is the reason that no energy is dissipated in an A.C. circuit when capacitor is present unless the dielectric has finite resistance.

Example 8 :

The reactance of capacitor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its, reactance when connected in DC circuit? What is its consequence?

Sol. The reactance of capacitor is 20 ohm. It means that the hindrance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm.

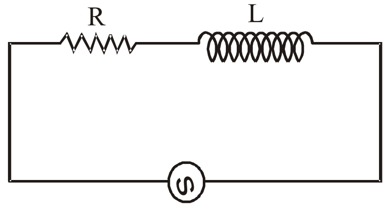
The reactance of capacitance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Therefore by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0 therefore reactance of capacitor = ∞ (infinite). Hence the capacitor can not be used to control DC.

RESISTANCE AND INDUCTANCE IN SERIES (L-R CIRCUIT)

A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in fig.



$E = E_0 \sin\omega t$

Phasor diagram For L-R circuit :

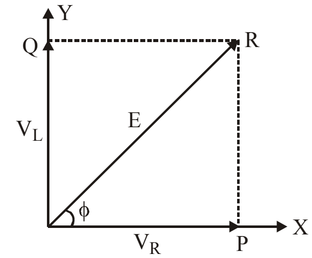
Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$.

As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$

Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$. So V_R and V_L are mutually perpendicular

(The sum of V_R and V_L cannot be put equal to applied e.m.f. E).



The phasor diagram is drawn in fig.

the vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector OR

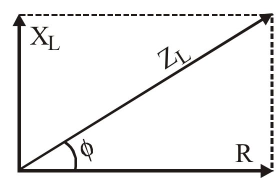
$$E = \sqrt{V_R^2 + V_L^2}$$

Thus $E^2 = V_R{}^2 + V_L{}^2 = I^2 (R^2 + X_L{}^2)$

or $I = \frac{E}{\sqrt{R^2 + X_L^2}}$

The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E. by a phase angle ϕ

$$\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I R} = \frac{X_L}{R} = \frac{\omega L}{R}$$



$$\tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Inductive Impedance Z_L : In L-R circuit the maximum value of current $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$

$\sqrt{R^2 + \omega^2 L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L.

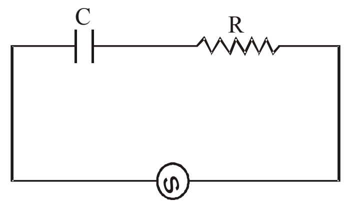
$$Z_L = \sqrt{R^2 + \omega^2 L^2} \quad \text{or} \quad Z_L = \sqrt{R^2 + (2\pi f L)^2}$$

The reciprocal of impedance is called admittance

$$Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

RESISTANCE AND CAPACITOR IN SERIES (C-R CIRCUIT)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0.

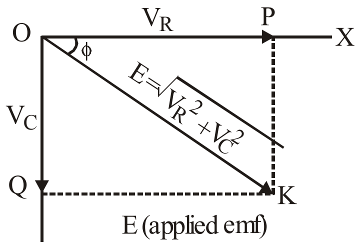


$E = E_0 \sin\omega t$

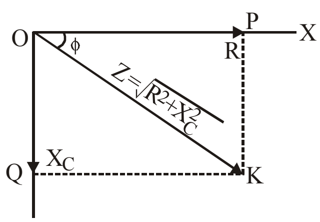
Phasor diagram For C-R circuit :
Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$V_C = I X_C$ and $V_R = I R$

where X_C = capacitive reactance and I = instantaneous current. Now, V_R is in phase with I, while V_C lags behind I by 90°. The phasor diagram is shown in fig.



The vector OP represents V_R (which is in phase with I) and the vector OQ represents V_C (which lags behind I by π/2). The vector OK represents the resultant of V_R and V_C = the applied e.m.f. E.



Hence, $V_R{}^2 + V_C{}^2 = E^2$ or $E = \sqrt{V_R^2 + V_C^2}$

or $E^2 = I^2 (R^2 + X_C{}^2)$ or $I = \dfrac{E}{\sqrt{R^2 + X_C^2}}$

The term $\sqrt{(R^2 + X_C^2)}$ represents the effective resistance of the C-R circuit and called the capacitive impedance Z_C of the circuit. Hence, in C-R circuit

$$Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

Capacitive Impedance Z_C : In C-R circuit the term $\sqrt{R^2 + X_C^2}$ effective opposition offered by C-R circuit to the flow of a.c. through it. It is known as impedance of C-R circuit and is represented by Z_C. The phasor diagram also shows that in C-R circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}$$

or $\tan\phi = \dfrac{X_C}{R} = \dfrac{1}{\omega C R}$ or $\phi = \tan^{-1}\left(\dfrac{1}{\omega CR}\right)$

Example 9 :
A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between current maximum and voltage maximum?

Sol. (a) Here, $C = 100\ \mu F = 100 \times 10^{-6}$ F, R = 40 Ω,
$V_{rms} = 110$ V, f = 60 Hz

Peak voltage, $V_0 = \sqrt{2}.V_{rms} = 100\sqrt{2} = 155.54$ V

Circuit impedance,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} = \sqrt{40^2 + \frac{1}{(2\times\pi\times 60\times 100\times 10^{-6})^2}}$$

$= \sqrt{1600 + 703.60} = \sqrt{2303.60} = 48\ \Omega$

hence, maximum current in coil,

$$I_0 = \frac{V_0}{Z} = \frac{155.54}{48} = 3.24\ A$$

(b) Phase lead angle (for current),

$$\theta = \tan^{-1}\frac{1}{\omega C R}$$

$$= \tan^{-1}\frac{1}{2\times 3.14\times 60\times 100\times 10^{-6}\times 40}$$

$= \tan^{-1} 0.66315 = 33°\ 33'$ (taken 33.5°)

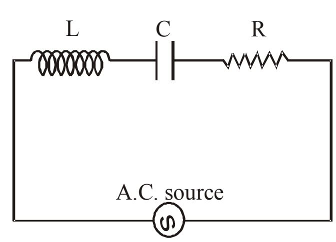
Time lead,

$$t = \frac{\theta}{\omega} = \frac{\theta}{2\pi\nu} = \frac{33.5}{360\times 60} = 0.001551 \text{ sec}$$

$= 1.551 \times 10^{-3}$ sec

Voltage will lag current by = 1.551 ms.

INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES (LCR SERIES CIRCUIT)
A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in figure.



$E = E_0 \sin\omega t$

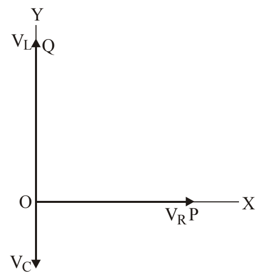
Phasor Diagram For Series LCR circuit :

Let in series LCR circuit applied alternating emf is

$E = E_0 \sin \omega t$.

As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.



Let at any instant of time t the current in the circuit is I

Let at this time t the potential differences across L, C, and R

$V_L = I X_L$, $V_C = I X_C$ and $V_R = I R$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C legs behind I by 90°.

The vector OP represents V_R (which is in phase with I)

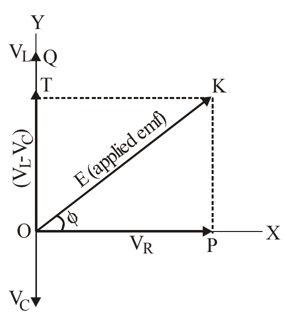
the vector OQ represent VL (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°)

V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f. Thus, $E = \sqrt{V_R^2 + (V_L - V_C)^2}$



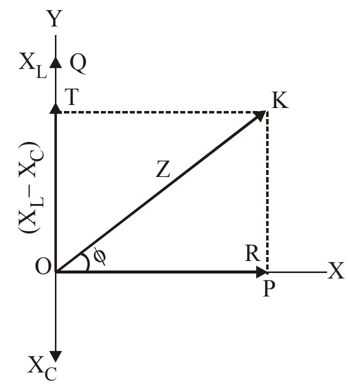
$$E^2 = V_R{}^2 + (V_L - V_C)^2 \quad \text{or} \quad E^2 = I^2 [R^2 + (X_L - X_C)^2]$$

or $$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

It is clear that the term $\sqrt{R^2 + (X_L - X_C)^2}$ represents the "effective resistance" of the circuit and is called impedance Z of the circuit.

Thus, $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ



Three cases :

(a) When $\omega L > \frac{1}{\omega C}$, then tan ϕ is +ve i.e., ϕ is positive. In this case, the alternating emf leads the current I by a phase angle ϕ. The a.c. circuit is then inductance dominated circuit.

(b) When $\omega L < \frac{1}{\omega C}$, then tan ϕ is –ve i.e., ϕ is negative. In this case, the alternating e.m.f. lags behind the current I by a phase angle ϕ. The a.c. circuit is then capacitance dominated circuit.

(c) When $\omega L = \frac{1}{\omega C}$, then tan $\phi = 0$ i.e., $\phi = 0$. In the case, the alternating e.m.f. and current I are in phase. The a.c. circuit is then non-inductive.

POWER IN AN A.C. CIRCUIT

Power in D.C. circuits P = voltage E × current I.

If current is measured in amperes and voltage in Volts, the power is measured in Watts.

The instantaneous power P of an a.c. circuit :

The instantaneous power

P = Instantaneous e.m.f. E × instantaneous current I

Let the instantaneous alternating e.m.f. $E = E_0 \sin \omega t$

and the instantaneous current in an a.c. circuit

$I = I_0 \sin (\omega t \pm \phi)$

Instantaneous power in the circuit P = E I

$= E_0 \sin \omega t . I_0 \sin (\omega t \pm \phi)$

$= E_0 I_0 \sin \omega t \sin \omega t \cos \phi \pm \cos \omega t \sin \phi$

$= E_0 I_0 \sin^2 \omega t \cos \phi \pm E_0 I_0 \sin \omega t \cos \omega t \sin \phi$

or $P = P_1 + P_2$

$P_1 = E_0 I_0 \sin^2 \omega t \cos \phi$

and $P_2 = E_0 I_0 \sin \omega t \cos \omega t \sin \phi$

First part of power is $P_1 = (E_0 \sin \omega t) (I_0 \sin \omega t) \cos \phi$

In this part of power emf ($E_0 \sin \omega t$) and current ($I_0 \cos \phi \sin \omega t$) both are in same phase.

In this part amplitude of current = ($I_0 \cos \phi$)

Average of first part $\overline{P_1} = \overline{E_0} \overline{I_0} \overline{\sin^2 \omega t} \cos \phi$

(average of $\sin^2 \omega t = \overline{\sin^2 \omega t} = \frac{1}{2}$)

$= \frac{1}{2} E_0 I_0 \cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\phi$

or $\overline{P_1} = E_{rms} \times I_{rms} \times \cos\phi$

Second part of power is

$P_2 = (E_0 \sin \omega t)(I_0 \cos \omega t) \sin\phi$

$= (E_0 \sin \omega t)[(I_0 \sin\phi)\sin(\omega t + \frac{\pi}{2})]$

In this part of power emf and current are in phase difference of $\frac{\pi}{2}$

In this part amplitude of current = $(I_0 \sin\phi)$

Average of second part of power $P_2 = \overline{P_2}$

$\overline{P_2} = \overline{E_0 I_0 \sin\omega t . \cos\omega t . \cos\phi} =$

$\overline{E_0}\ \overline{I_0}\ \overline{\sin\omega t}\ \overline{\cos\omega t}.\ \overline{\cos\phi} = \frac{1}{2} E_0 I_0\ \overline{\sin 2\omega t}$

or $\overline{P_2} = 0$ $\quad (\because \overline{\sin 2\omega t} = 0)$

Hence, average power over one complete cycle

$\overline{P} = \overline{P_1} + \overline{P_2} = E_{rms} \times I_{rms} \times \cos\phi + 0$

or $P_{av} = E_{rms} . I_{rms} . \cos\phi$

Average power is also known as True power.

The quantity $E_{rms} I_{rms}$ is known as the apparent power or virtual power.

POWER FACTOR

$\cos\phi$ is known as Power Factor.

Average power = Apparent Power × Power Factor

It is customary to express true power in KW and apparent power in KVA.

The value of power factor and hence the amount of average power consumed depends on the nature of a.c. circuit (i.e. nature of circuit element present in the circuit).

A.C. circuit contains pure resistance

Phase angle between voltage and current i.e. $\phi = 0$

$\therefore \quad P_{av} = E_{rms} I_{rms} \cos 0° = E_{rms} I_{rms} = E_{rms} \frac{E_{rms}}{R} = \frac{E_{rms}^2}{R}$

A.C. circuit contains pure inductance

Phase angle $\phi = \frac{\pi}{2}$

$\therefore \quad P_{av} = E_{rms} I_{rms} \cos\frac{\pi}{2} = E_{rms} I_{rms} \times 0 = 0$

A.C. circuit contains pure capacitance

Phase angle $\phi = \frac{\pi}{2}$

$\therefore \quad P_{av} = E_{rms} I_{rms} \cos\frac{\pi}{2} = E_{rms} \times I_{rms} \times 0 = 0$

A.C. circuit contains a series combination of L and R

$\tan\phi = \frac{\omega L}{R}$ so that $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

$\therefore \quad P_{av} = E_{rms} I_{rms} \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = E_{rms} . \frac{E_{rms}}{\sqrt{R^2 + \omega^2 L^2}} \times R$

or $P_{av} = \frac{E_{rms}^2 R}{R^2 + \omega^2 L^2}$

A.C. circuit contains a series combination of C and R

$\tan\phi = \frac{1/\omega C}{R}$, so that $\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$\therefore \quad P_{av} = E_{rms} I_{rms} \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$= E_{rms} . \frac{E_{rms}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \times \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

or $P_{av} = \frac{E_{rms}^2 R}{R^2 + \frac{1}{\omega^2 C^2}}$

It is interesting to note here, that in case of every A.C. circuit we can use the following general formula to calculate the average consumed power.

$P_{av} = E_{rms} I_{rms} . \frac{R}{Z} = E_{rms} I_{rms} \cos\phi \quad (\cos\phi = \frac{R}{Z})$

WATTLESS CURRENT

Since average of the second part of power (P_2) is zero $(\overline{P_2} = 0)$. Current related with second part of power (P_2) in which emf and current having phase difference of $\pi/2$ in known as watt less current.

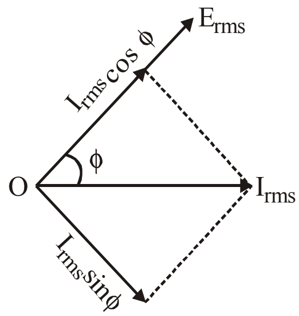
Amplitude of watt less current = $I_0 \sin\phi$

If the circuit consist of either inductance only or capacitance only (having no ohmic resistance), the phase difference ϕ between voltage and current is 90°.

The average power consumed

$P = E_{rms} I_{rms} \times \cos 90° = 0 \qquad (\cos 90° = 0)$

Thus, if the resistance of an a.c. circuit is zero, although current flows in the circuit but the average power consumed in the circuit remains zero.



The current in such a circuit is called wattless current.

Let E_{rms} leads I_{rms} by phase angle ϕ, as shown in fig. It can be assumed in general that I_{rms} is the vector sum of two perpendicular components

$I_{rms} \cos \phi$ and $I_{rms} \sin \phi$.

Since phase angle between E_{rms} and $I_{rms} \sin \phi$ is $\frac{\pi}{2}$, average power consumed in the circuit due to components $I_{rms} \sin \phi$ is zero. Only the component $I_{rms} \cos \phi$ which is to actually responsible for consumption of power in an a.c. circuit. As the circuit consumed no power due to the component $I_{rms} \sin \phi$, hence it is called the wattless component of a.c.

SERIES RESONANT CIRCUIT

For an A.C. circuit with resistance R, inductance L and capacitance C, in series. The impedance of the circuit

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

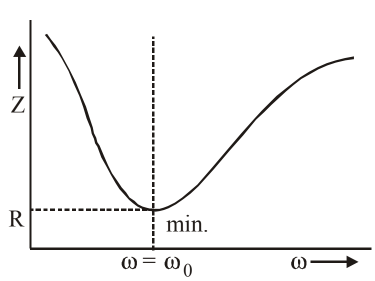
and $$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

It is clear that both the impedance and current depend on the frequency of the applied e.m.f.

At very low frequencies,

Inductive reactance $X_L = \omega L$ is negligible but capacitive reactance ($X_C = 1/\omega C$) is very high. As frequency of alternating emf applied to the circuit is increased X_L goes on increasing and X_C goes on decreasing.

For a particular value of $f = f_r$: $X_L = X_C$ 'inductive reactance' is equal to capacitive reactance.

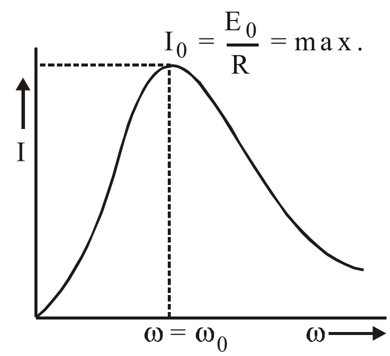


$X = X_L - X_C = 0$ i.e., 'reactance' of the circuit is zero.

$$Z = \sqrt{R^2 + X^2} = R$$

The impedance of the circuit becomes minimum $Z_{min} = R$ is equal to resistance. At frequency f_r the current becomes maximum. This particular frequency f_r at which the impedance of the circuit becomes minimum and therefore current becomes maximum, is called resonance frequency of the circuit.

$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ = max., i.e., current in the circuit is maximum.



Before Resonance ($f < f_r$) : Current in the circuit lags inphase by the applied voltage ($X_L > X_C$)

After Resonance ($f > f_r$) : Current leads is phase by the applied voltage ($X_C > X_L$)

At Resonance ($f = f_r$) :

$\phi = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{0}{R} = 0$, i.e., current is in 'phase' with applied voltage. Currents through L and C are same but voltage cross these two 180° out of phase with respect to each other so that net PD across reactance is zero

i.e. $V_X = V_L - V_C = 0$ with $V = V_R$

At this frequency f_r or angular frequency ω_r circuit is known as series resonant circuit.

The 'power factor' of the Series resonant circuit (at $f = f_r$):

Power factor $\cos\phi = \frac{R}{Z} = 1$ (max.) $\quad (\because Z = R = \text{min.})$

and hence power consumed by the circuit

$$P_{av} = V_{rms} I_{rms} \cos\phi = \frac{1}{2} V_0 I_0 \text{ (maximum.)}$$

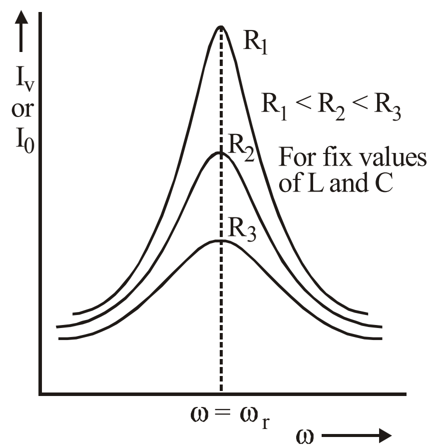
The series resonant circuit is called 'acceptor circuit' as at resonance its impedance is minimum and it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Hence, at resonance $\omega = \omega_r$,

$$\omega_r L = \frac{1}{\omega_r C} \quad \text{or} \quad \omega_r^2 = \frac{1}{LC} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

or $2\pi f_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$

The variation of I with angular frequency ω is shown in figure. It may seen that current amplitude is maximum for angular frequency ω_r and falls off to zero for both $\omega < \omega_r$ and $\omega > \omega_r$. However, the peak value of amplitude falls off with increase in the value of resistance R i.e. Higher the value of ohmic resistance present amplitude in the resonance.



In the figure, $R_3 > R_2 > R_1$.

Thus maximum value of virtual current for $\omega = \omega_r$, is given by $(I_{rms})_{max} = \frac{E_{rms}}{R}$

This value decreases with increase in value of R.

For all values of ω other than $\omega_r = 2\pi f_r$, the amplitude of current is less than peak value.

QUALITY FACTOR AND SHARPNESS OF RESONANCE

From the current versus frequency curve is quite flat for a larger value of resistance and becomes more and more sharp as the value of the resistance is decreased.

A slight change in angular frequency produces comparatively a large change in current, when the resistance in the circuit is low than in the case, when the resistance is high.

If the value of resistance in the LCR series circuit is very low; then a large current flows, when the angular frequency of a.c. source is near the resonance frequency ω_r. Such an LCR-series circuit is said to be more sharp or more selective.

The sharpness or selectively of a resonance circuit :

Sharpness of the circuit it is measured by quality-factor (Q-factor). The Q-factor of series resonant circuit is defined as the ratio of the voltage developed across the inductance or capacitance at resonance to the impressed voltage (the voltage applied across R)

$$Q = \frac{\text{voltage across L or C}}{\text{applied voltage } (= \text{voltage across R})}$$

or $Q = \frac{(\omega_0 L)\, I}{RI}$ or $Q = \frac{\omega_r L}{R}$

hence $Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$ $\left(\because\ \omega_0 = \frac{1}{\sqrt{LC}}\right)$

At resonance, $I_0 = \frac{E_0}{R}$.

So $V_L = I_0 X_L = \frac{\omega L}{R} E_0$

At resonance the voltage drops across inductance (or capacitance) is Q times the applied voltage.

The chief characteristic of series resonant circuit is 'voltage magnification'.

Note :

(a) Q is just a number.

(b) Q factor is also called voltage amplification factor of the circuit.

(c) If R is low or L is larger or C is low Q-factor of LCR series circuit will be large i.e. the circuit will have more sharpness.

(d) The electronic circuits with high Q values would respond to a very narrow range of frequencies and for low Q values circuit will respond to a wide range of frequencies.

(e) The value of Q varies from 10 to 100. However the electronic circuits dealing with very high frequencies may have Q even 200.

IMPORTANT CHARACTERISTIC OF SERIES RESONANT CIRCUIT

A unique characteristic of the series resonant circuit is that the potential differences available across the inductance L and across the capacitor C may be much more than the applied voltage.

Let us take a concrete example to make this point clear. Suppose in a LCR series circuit, R = 10 ohm, $X_L = X_C = 40$ ohm and the root mean square value of applied voltage is 100 volt.

The current in the circuit

$$I = \frac{E}{Z} = \frac{E}{R} = \frac{100}{10} = 10 \text{ ampere}$$

Potential difference across the resistance R is

$V_R = IR = 10 \times 10 = 100$ Volt

Potential difference across the inductance L is

$V_L = IX_L = 10 \times 40 = 400$ Volt

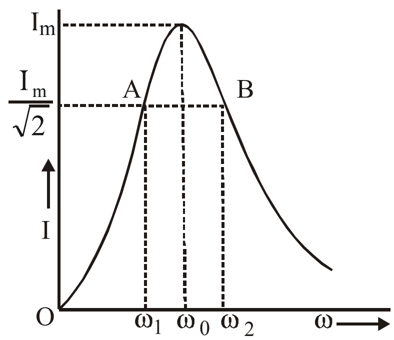
Potential difference a across the capacitor C is

$V_C = I X_C = 10 \times 40 = 400$ Volt

A series resonant circuit gives voltage amplification. It is also called voltage resonant circuit.

HALF POWER POINTS

On the resonance curve there are two points where power in the circuit is half of the power at resonance.



Since, power $P = I^2_{rms} R$,

At the half power points current becomes $\frac{I_m}{\sqrt{2}}$

so that power at half power points

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{1}{2} I^2_m R = \text{half power.}$$

On the curve these points are A and B.

The $\omega_1 = \omega_r - \frac{R}{2L}$ and $\omega_2 = \omega_r + \frac{R}{2L}$ (if $\frac{R}{2L} << \omega_r$).

Frequencies corresponding to half power points are known as half power point frequencies.
These are denoted in graph by f_1 and f_2.
At lower half power frequency f_1, the circuit is capacitive $X_C > X_L$
At upper half power frequency f_2, the circuit is inductive $X_L > X_C$
The current is 0.707 I_{0m} or 70.7% of the maximum peak current.

BAND WIDTH (Δf or $\Delta\omega$)

The band width $\Delta f = f_2 - f_1$

and $\Delta\omega = \omega_2 - \omega_1 = \left(\omega_0 + \frac{R}{2L}\right) - \left(\omega_0 - \frac{R}{2L}\right)$

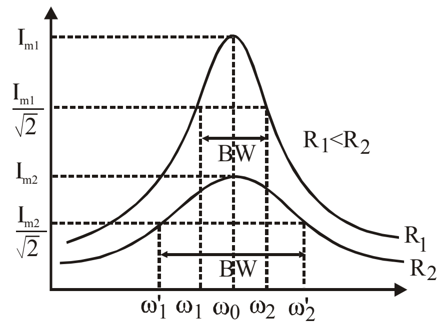
$$I_{0m} = \frac{E}{R} \; ; \; Z = \sqrt{R^2 + X^2} \; ;$$

$$I_0 = \frac{E}{Z}$$

If X = R then we get up half power current $Z = R\sqrt{2}$
It is found that the full width at half maximum (FWHM) for the resonance curve, that is the band width $\Delta\omega$, depends on R and L in the circuit.

$$\Delta\omega = \frac{R}{L} \quad \text{or} \quad \Delta f = \frac{R}{2\pi L}$$

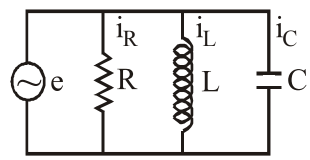
The band width is large for large R and small for small R.



The band width $\Delta\omega$ does nor depends on C in the circuit (note that resonance frequency ω_r does not depends on R while band width depends on R).

PARALLEL RESONANT CIRCUIT

The parallel LCR circuit shown in fig.



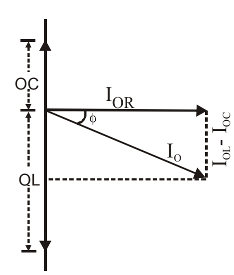
same emf. $E = E_0 \sin \omega t$ is applied across R, L and C,
$V_R = V_L = V_C = E$
but the currents in R, L and C are different.
Using phase difference knowledge currents in :
resistance R is $I_R = I_{OR} \sin \omega t$

Inductance, $I_L = I_{OL} \sin\left(\omega t - \frac{\pi}{2}\right)$

Capacitance, $I_C = I_{OC} \sin\left(\omega t + \frac{\pi}{2}\right)$

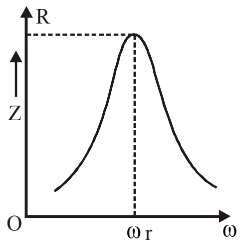
Here $I_{OR} = \frac{E_0}{R}$; $I_{OC} = \frac{E_0}{X_C}$; $I_{OL} = \frac{E_0}{X_L}$

for vector addition of currents the phasor diagram is shown in fig.



$$\vec{I}_0 = \vec{I}_{OR} + \vec{I}_{OL} + \vec{I}_{OC}$$

$$I_0 = \sqrt{I^2_{OR} + (I_{OL} - I_{OC})^2} = E_0 \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$



The admittance of the parallel LCR circuit is

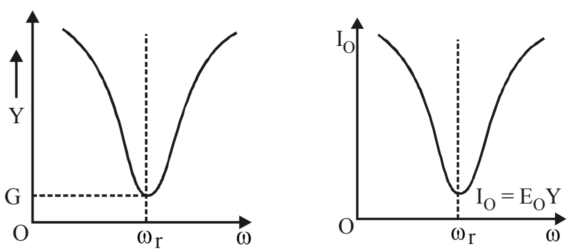
$$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\left(\frac{1}{X_L}-\frac{1}{X_C}\right)^2}\ .$$

The admittance $Y=\sqrt{G^2+(B_L-B_C)^2}$ and, $I_0=E_0Y$.

If $I_{OL}>I_{OC}$, then resultant current in the circuit lags behind the applied emf by a phase ϕ,

$$\tan\varphi=\frac{I_{OL}-I_{OC}}{I_{OR}}=\frac{B_L-B_C}{G}.$$

The variation of impedance Z, admittance Y and the peak value of current I_0, for a parallel LCR circuit is shown in fig.



RESONANCE

When the impedance of the circuit is maximum (Z = R) or admittance of the circuit becomes minimum $\left(Y=G=\frac{1}{R}\right)$, condition of resonance occurs in parallel resonance circuit.

In this condition $B_L=B_C$ or $X_L=X_C$

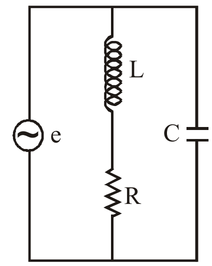
or $\omega_r=\frac{1}{\sqrt{LC}}$; $f_r=\frac{1}{2\pi\sqrt{LC}}$

In parallel circuit at resonance :

* $I_{OL}=I_{OC}$, the phase $\phi=0$, $\cos\phi=1$
* The peak current is minimum
* The quality factor $Q=\frac{R}{\omega_r L}$
* The band width $BW=\frac{f_r}{Q}$

PRACTICAL PARALLEL RESONANT CIRCUIT

The inductance coil has some resistance R and it is connected in parallel to a capacitor C.



For this circuit, the impedance is obtained from $\frac{1}{Z}=Y$,

where the admittance is $Y=\sqrt{\frac{(1-\omega^2LC)^2+(\omega CR)^2}{R^2+(\omega L)^2}}$

The resonance occurs when the admittance is minimum.

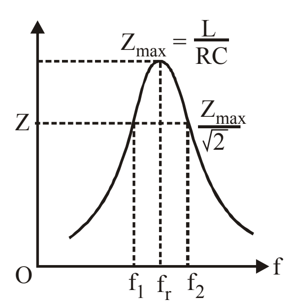
resonance frequency $\omega_r=\frac{1}{\sqrt{LC}}\left(1-\frac{R^2C}{L}\right)^{1/2}$

or $\omega_r=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$ or $f_r=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$

If R = 0 in fig. then it becomes a parallel LC circuit.

resonance occurs at $\omega_r=\frac{1}{\sqrt{LC}}$; $f_r=\frac{1}{2\pi\sqrt{LC}}$

If $\frac{1}{LC}-\frac{R^2}{L^2}$ has negative value then resonance does not occurs. A parallel LCR circuit offers maximum impedance at $f=f_r$. The impedance decreases for $f<f_r$ and $f>f_r$. Since the impedance between frequencies f_1 and f_2 is large, current between this band is small.



The parallel LCR circuit is, therefore, also called Band Rejector circuit. f_1 and f_2 are half power frequencies and impedance in between these two is $\frac{Z\geq Z_{max}}{\sqrt{2}}$. In between these frequencies (i.e. Band width) current in the circuit is frequency.

CHOKE COIL

A choke coil is simply an inductor with a large self-inductance and negligible resistance (zero in ideal case). It is used in A.C. circuits for reducing current without consuming power.

The choke coil is put in series with the electrical device, such as fluorescent tube requiring a low value of current. The inductive reactance decreases the current. Since the alternating e.m.f. leads the current by phase angle $\frac{\pi}{2}$.

The average power consumed by the choke coil

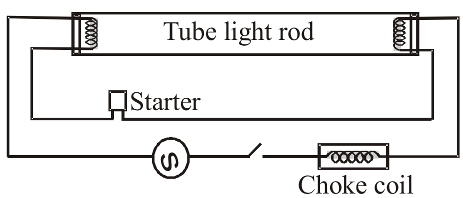
$$P_{av} = E_v I_v \cos\frac{\pi}{2} = 0.$$

However, a practical inductance possesses a small resistance i.e., a practical inductance may be treated as a series combination of inductance L and a small resistance r. Average power consumed in a practical inductance

$$P_{av} = E_v I_v \times \frac{r}{\sqrt{r^2+\omega^2L^2}}$$

for practical inductance power factor

$$\ldots \phi) = \frac{r}{\sqrt{r^2+\omega^2L^2}}$$



Uses : In a.c. circuits, a choke coil is used to control the current in place of a resistance. If a resistance is used to control the current, the electrical energy will be wasted in the form of heat. A choke coil decreases the current without wasting electrical energy in the form of heat.

ADVANTAGE OF ALTERNATING CURRENT OVER DIRECT CURRENT

(i) A.C. can be obtained over a wide range of voltages. These voltages can be easily stepped up or stepped down with the help of transformers.

(ii) The generation of a.c. is found to be economical than that of d.c.

(iii) Alternating current can be controlled by using a choke coil without any significant wastage of electrical energy.

(iv) Alternating current may be transmitted at a high voltage from the power house to any place where it can again be brought down to low voltage. The cost in such a transmission is low and energy losses are minimized. Transformers cannot be used for d.c. Hence the cost of d.c. transmission from one place to other is quite high.

(v) A.C. equipments such as electric motors etc are more durable and convenient as compared to d.c. equipments.

(vi) Alternating can be conveniently converted in to d.c. with the help of rectifiers.

DISADVANTAGE OF ALTERNATING CURRENT OVER DIRECT CURRENT

(i) Alternating current in more dangerous as compared to direct current. An a.c. wire if touched by chance, gives a more serious shock as compared to a d.c. wire.

(ii) A.C. is transmitted more by the surface of the conductor. This is called SKIN EFFECT. It is due to this reason that several strands of thin insulated wires, instead of a simple thick wire need be used.

(iii) A.C. is not available in pure frequency. It contains higher harmonics. So it is not suitable for certain purposes.

(iv) For electroplating, electrotyping only d.c. can be used and not a.c.

Comparison Among Resistance, Reactance and Impedance :

S.No.	Resistance	Reactance	Impedance
	It is opposition to the flow of any type of current.	It can be inductive or capacitive opposing the flow of alternating current	It is the total opposition offered to current due to resistance inductive reactance and capacitive reactance.
2.	It is independent on frequency of source of supply.	It depends on the frequency of the source of supply.	It depends on the frequency of the source of supply.
3.	It is denoted by R and is given by $\rho(\ell/a)$	It is denoted by X_L or X_C and is $2\pi fL$ or $1/2\pi fC$ respectively.	It is denoted by Z & is given by $Z=\sqrt{R^2+(X_L-X_C)^2}$

Expressions For Various Physical Quantities in A.C. Circuits :

Alternating Voltage	Alternating Current	Phase relationship between voltage and current	Impedance	Power Loss
(a) Ideal resistance $E = E_0 \sin \omega t$	$I = I_0 \sin \omega t$	In phase	$Z = R$	$I_{rms}^2 R$
(b) Ideal inductance $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	Current lags by $\frac{\pi}{2}$ rad or 90°	$Z = X_L = \omega L = 2\pi f L$	nil
(c) Ideal capacitor $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$	Current leads by $\frac{\pi}{2}$ rad or 90°	$Z = X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$	nil
(d) Series resistance and inductance $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t - \phi)$	Current lags by ϕ	$Z = \sqrt{R^2 + X_L^2}$	$_{rms} I_{rms} \cos\phi$
(e) Series resistance and capacitance $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t + \phi)$	Current leads by ϕ	$Z = \sqrt{R^2 + X_C^2}$	$E_{rms} I_{rms} \cos\phi$
(f) Series LCR (i) Mainly inductive $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t - \phi')$	Currrent lags by ϕ'	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$E_{rms} I_{rms} \cos\phi'$
(ii) Mainly capacitive $E = E_0 \sin \omega t$	$I = I_0 \sin (\omega t + \phi')$	Current leads by ϕ'	$Z = \sqrt{R^2 + (X_C - X_L)^2}$	$E_{rms} I_{rms} \cos\phi'$

Virtual apparent/RMS Power	Peak Power	Instantaneous Power
It is the product of RMS value of voltage and current. $P_a = V_{rms} I_{rms}$	It is the product of peak value of voltage & current. $P_0 = V_0 I_0$	It is the product of instantaneous value of voltage and current. $P_i = V_i I_i$

TRY IT YOURSELF

Q.1 110 volts (rms) is applied across a series circuit having resistance 11Ω & impedance 22 Ω. The power consumed is
(A) 366 W (B) 550 W
(C) 1100 W (D) 275 W

Q.2 A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in milli-second) of the circuit is

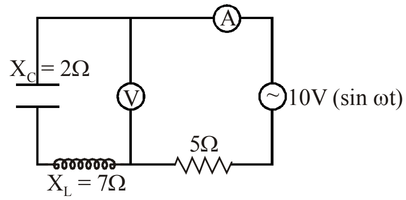
Q.3 A series LCR circuit is operated at resonance. Then
(A) voltage across R is minimum
(B) impedance is minimum
(C) power transferred is maximum
(D) current amplitude is minimum

Q.4 An inductance L, a capacitance C and a resistance R may be connected to an AC source of angular frequency ω, in three different combinations of RC, RL and RLC in series. Assume that $\omega L = 1/\omega C$. The power drawn by the three combinations are P_1, P_2, P_3 respectively. Then,
(A) $P_1 > P_2 > P_3$ (B) $P_1 = P_2 < P_3$
(C) $P_1 = P_2 > P_3$ (D) $P_1 = P_2 = P_3$

Q.5 The peak value of an alternating emf E given by $E = E_0 \cos\omega t$ is 10 V and its frequency is 50 Hz. At a time t = 1/600 s, the instantaneous value of emf is
(A) 10 V (B) $5\sqrt{3}$ V
(C) 5 V (D) 1 V

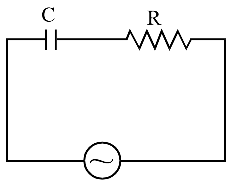
Q.6 Resonance occurs in a series L-C-R circuit when the frequency of the applied emf is 1000 Hz. Then :
(A) When f = 900 Hz, the circuit behaves as a capacitative circuit.
(B) The impedance of the circuit is maximum at f = 1000Hz.
(C) At resonance the voltage accross L and voltage accross C differ in phase by 180°.
(D) If the value of C is doubled resonance occurs at f = 2000 Hz.

Q.7 In the figure shown hot wire voltmeter and hot wire ammeter are ideal. The reading of voltmeter is



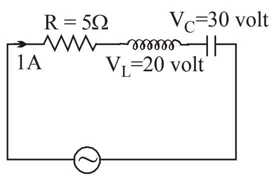
(A) $5\sqrt{2}$ V (B) 5 V
(C) 10 V (D) None of these

Q.8 In the circuit shown if the emf of source at an instant is 5V, the potential difference across capacitor at the same instant is 4 V. The potential difference across R at that instant may be



(A) 3V (B) 9V
(C) $\frac{3}{\sqrt{2}}$ V (D) none

Q.9 In the given circuit,



(A) current leads the voltage in phase
(B) voltage leads the current in phase.
(C) power factor of the circuit is $1/\sqrt{5}$
(D) Applied voltage is 15 V.

Q.10 Which of the following combinations should be selected for better tuning of an LCR circuit used for communication?
(A) $R = 20\Omega, L = 1.5$ H, $C = 35\mu F$.
(B) $R = 25\Omega, L = 2.5$ H, $C = 45\mu F$.
(C) $R = 15\Omega, L = 3.5$ H, $C = 30\mu F$.
(D) $R = 25\Omega, L = 1.5$ H, $C = 45\mu F$.

ANSWERS

(1) (D)	**(2)** 4	**(3)** (BC)
(4) (B)	**(5)** (B)	**(6)** (AC)
(7) (B)	**(8)** (B)	**(9)** (AC)
(10) (C)		

USEFUL TIPS

* For an alternating current $i = i_m \sin \omega t$ passing through a resistor R, the average power loss P (averaged over a cycle) due to joule heating is $(1/2)i_m^2R$.

Root mean square (rms) current $I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$.

The average power loss P = V I cosϕ.
The term cos ϕ is called the power factor. When a value is given for ac voltage or current, it is ordinarily the rms value.

* An ac voltage $v = v_m \sin\omega t$ applied to a pure inductor L, drives a current in the inductor
$i = i_m \sin(\omega t - \pi/2)$, where $i_m = v_m/X_L$,
$X_L = \omega L$ is called inductive reactance.
The current in the inductor lags the voltage by π/2.
The average power supplied to an inductor is zero.
An ac voltage $v = v_m \sin\omega t$ applied to a capacitor drives a current in the capacitor : $i = i_m \sin(\omega t + \pi/2)$.

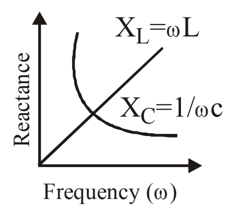
Here, $i_m = \frac{v_m}{X_C}$, $X_C = \frac{1}{\omega C}$ is called capacitive reactance.

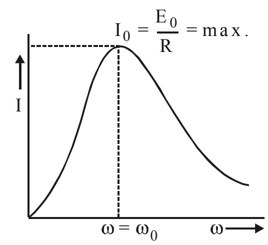
* An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} (X_L = X_C)$.

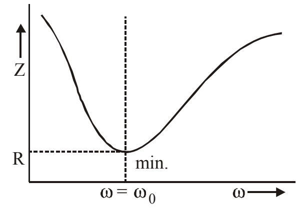
The quality factor Q defined by

$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$ is an indicator of the sharpness of the resonance, the higher value of Q indicating sharper peak in the current.

* Frequently asked graphs







ADDITIONAL EXAMPLES

Example 1 :
When a series combination of inductance and resistance are connected with a 10V, 50Hz a.c. source, a current of 1A flows in the circuit. The voltage leads the current by a phase angle of π/3 radian. Calculate the values of resistance and inductive reactance.

Sol. $Z = \sqrt{R^2 + X_L^2} = \frac{E}{I} = \frac{10}{1} = 10\Omega$

$\frac{X_L}{R} = \tan\phi = \tan\frac{\pi}{3} = \sqrt{3} \Rightarrow X_L = \sqrt{3}R$

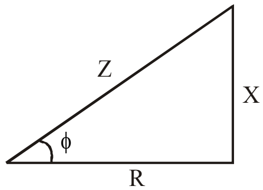
$Z = \sqrt{R^2 + 3R^2} = 10$

$\Rightarrow 2R = 10 \Rightarrow R = 5\Omega$ and $X_L = \sqrt{3}R = 5\sqrt{3}\ \Omega$

Example 2 :

For a series LCR circuit I = 100 sin (100πt – π/3) mA and V = 100 sin (100πt) volt, then
(a) Calculate resistance and reactance of circuit.
(b) Find average power loss.

Sol. (a) Impedance,



$$Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1000\ \Omega$$

$$R = Z\cos\phi = 1000\cos\left(\frac{\pi}{3}\right) = 500 \text{ ohm}$$

$$X = Z\sin\phi = 1000\sin\left(\frac{\pi}{3}\right) = 500\sqrt{3} \text{ ohm}$$

(b) $P_{av} = V_{rms} I_{rms} \cos\phi$

$$= \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2} \times 1000} \times \cos\left(\frac{\pi}{3}\right) = 5 \times \frac{1}{2} = 2.5 \text{ watts}$$

Example 3 :

A R-L circuit draws a power of 440W from a source of 220V, 50Hz. The power factor of the circuit is 0.5. To make the power factor of the circuit as 1.0, what capacitance will have to be connected with it ?

Sol. $\because$ Power $P = VI\cos\phi = \frac{V^2}{Z}\cos\phi$

$$\therefore\ Z = \frac{V^2\cos\phi}{P} = \frac{(220)^2(0.5)}{(440)} = 55\ \Omega$$

Also power factor

$$\cos\phi = \frac{R}{Z} \Rightarrow R = Z\cos\phi = (55)(0.5) = 27.5\ \Omega$$

$$\therefore\ X_L = \sqrt{Z^2 - R^2} = \sqrt{(55)^2 - \left(\frac{55}{2}\right)^2} = \frac{55\sqrt{3}}{2}\Omega$$

when power factor is 1.0 then $X_L = X_C$

$$X_L = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_L} = \frac{1}{(314)\left(\frac{55\sqrt{3}}{2}\right)} = 6.68 \times 10^{-5}\text{ F}$$

Example 4 :

A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23Ω is connected to a 230V variable frequency supply find –
(a) Source frequency for which current is maximum.
(b) Q-factor of the given circuit.

Sol. (a) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = \frac{10^5}{24}$ rad / s

$$\Rightarrow\ f = \frac{1}{2\pi} \times \frac{10^5}{24} = 6.63 \times 10^2\text{ Hz}$$

(b) Quality factor, $Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{10^5 \times 0.12}{24 \times 23} = \frac{10^3}{46} = 21.7$

Example 5 :

A variable frequency 230V alternating voltage source is connected across a series combination of L = 5.0H, C = 80μF and R = 40Ω. Calculate
(a) The angular frequency of the source at resonance.
(b) Amplitude of current at resonance frequency.

Sol. (a) Angular frequency at resonance

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5.0 \times 80 \times 10^{-6}}} = \frac{10^2}{2} = 50 \text{ rad / s}$$

(b) Amplitude of current at resonance

$$I_m = \frac{V}{R} = \frac{230\sqrt{2}}{40} = 8.13\text{ A}$$

Example 6 :

A current of 4A flows in a coil when connected to a 12V dc source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4A flows in the circuit. Determine the inductance of the coil.

Sol. A coil consists of an inductance (L) and a resistance (R). In dc only resistance is effective. Hence,

$$R = \frac{V}{i} = \frac{12}{4} = 3\Omega$$

In ac, $i_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$

$$\therefore\ L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{rms}}{i_{rms}}\right)^2 - R^2\right]$$

$$\Rightarrow\ L = \frac{1}{\omega}\sqrt{\left(\frac{V_{rms}}{i_{rms}}\right)^2 - R^2} = \frac{1}{50}\sqrt{\left(\frac{12}{24}\right)^2 - (3)^2}$$

= 0.08 henry

Example 7 :

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by π/2 radians.
(a) Name the devices X and Y.
(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X & Y.

Sol. (a) X is resistor and Y is a capacitor.
(b) Since the current in the two devices is the same (0.5A at 220 volt)
When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\Omega$$

$$\Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{200}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35\text{ A}$$

Example 8 :

A 100 Ω resistor is connected to a 220 V, 50 Hz a.c. supply.

(a) What is the rms value of current in the circuit ?

(b) What is the net power consumed over a full cycle?

Sol. (a) The rms value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2\text{ A}$$

(b) Net power consumed over full cycle,

$$P = I_{rms}^2 R = (2.2)^2 \times 100 = 484\text{ watt}$$

Example 9 :

An LCR circuit has L = 10 mH, R = 3 Ω and C = 1 μF connected in series to a source of 15 cos ωt V. Calculate the current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonance frequency.

Sol. As here resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}} = 10^4 \frac{\text{rad}}{\text{s}}$$ so,

$$\omega = \omega_0 - \frac{10}{100}\omega_0 = \frac{9}{10}\omega_0 = 9 \times 10^3 \frac{\text{rad}}{\text{s}}$$

and hence, $X_L = \omega L = 9 \times 10^3 \times 10^{-2} = 90\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{9 \times 10^3 \times 10^{-6}} = 111.11\Omega$$

so, $X = X_C - X_L = 111.11 - 90 = 21.11\Omega$

and hence, $Z = \sqrt{R^2 + X^2} = \sqrt{3^2 + (21.11)^2}$

i.e., $Z = \sqrt{9 + 445.63} = 21.32\Omega$

and as here E = 15 cos ωt, i.e., $E_0 = 15$ V,

$$I_0 = \frac{E_0}{Z} = \frac{15}{21.32} = 0.704\text{ A}$$

The average power dissipated,

$$P_{av} = V_{rms} I_{rms} \cos\phi = (I_{rms} \times Z) \times I_{rms} \times \frac{R}{Z}$$

i.e., $P_{av} = I^2_{rms} R = \frac{1}{2} I_0^2 R \qquad \left[\text{as } I_{rms} = \frac{I_0}{\sqrt{2}}\right]$

so, $P_{av} = \frac{1}{2} \times (0.704)^2 \times 3 = 0.74\text{ W}$

Now as $f = \frac{\omega}{2\pi} = \frac{9 \times 10^3}{2\pi} \frac{\text{cycle}}{\text{s}}$

So, $\frac{P_{av}}{\text{cycle}} = \frac{0.74\text{ J/s}}{(9 \times 10^3 / 2\pi)\text{ cycle/s}} = \frac{2\pi \times 0.74}{9 \times 10^3} \frac{\text{J}}{\text{cycle}}$

i.e., $\frac{P_{av}}{\text{cycle}} = 5.16 \times 10^{-4} \frac{\text{J}}{\text{cycle}}$

QUESTION BANK

CHAPTER 4 : ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART 1 : MAGNETIC FLUX

Q.1 A square of side L meters lies in the x-y plane in a region, where the magnetic field is given by $\vec{B} = B_0(2\hat{i}+3\hat{j}+4\hat{k})T$, where B_0 is constant. The magnitude of flux passing through the square is

(A) $2B_0L^2$ Wb (B) $3B_0L^2$ Wb

(C) $4B_0L^2$ Wb (D) $\sqrt{29}\,B_0L^2$ Wb

Q.2 A square coil $10^{-2}m^2$ area is placed perpendicular to a uniform magnetic field of intensity 10^3 Wb/m^2. The magnetic flux through the coil

(A) 10 weber (B) 10^{-5} weber

(C) 10^5 weber (D) 100 weber

Q.3 A circular loop of radius R carrying current I lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is

(A) Directly proportional to I

(B) Directly proportional to R

(C) Directly proportional to R^2

(D) Zero

Q.4 A loop of wire is placed in a magnetic field $\vec{B} = 0.02\hat{i}$ tesla. Then the flux through the loop is its area vector is $\vec{A} = 30\hat{i}+16\hat{j}+23\hat{k}$ cm^2, is .

(A) 60μWb (B) 32μ Wb

(C) 46μ Wb (D) 138μ Wb

Q.5 The magnetic flux linked with a coil of N turns of area of cross section A held with its plane parallel to the field B is:

(A) NAB/2 (B) NAB

(C) NAB/4 (D) Zero

PART 2 : FARADAY'S LAW OF INDUCTION

Q.6 A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if

(A) It is rotated about its axis

(B) It is rotated about a diameter

(C) It is not moved

(D) It is given translational motion in the field

Q.7 Faraday's laws are consequence of conservation of –

(A) Energy (B) Energy & magnetic field

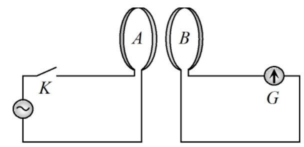
(C) Charge (D) Magnetic field

Q.8 A magnetic field of 2×10^{-2} T acts at right angles to a coil of area 100 cm^2 with 50 turns. The average emf induced in the coil is 0.1 V, when it is removed from the field in time t. The value of t is

(A) 0.1 sec (B) 0.01 sec

(C) 1 sec (D) 20 sec

Q.9 The diagram below shows two coils A and B placed parallel to each other at a very small distance. Coil A is connected to an ac supply. G is a very sensitive galvanometer. When the key is closed –



(A) Constant deflection will be observed in the galvanometer for 50 Hz supply.

(B) Visible small variations will be observed in the galvanometer for 50 Hz input.

(C) Oscillations in the galvanometer may be observed when the input ac voltage has a frequency of 1 to 2Hz.

(D) No variation will be observed in the galvanometer even when the input ac voltage is 1 or 2 Hz.

Q.10 A 50 turns circular coil has a radius of 3cm, it is kept in a magnetic field acting normal to the area of the coil. The magnetic field B increased from 0.10 tesla to 0.35 tesla in 2milliseconds. The average induced e.m.f. in the coil is

(A) 1.77 volts (B) 17.7 volts

(C) 177 volts (D) 0.177 volts

Q.11 A rectangular coil of 20 turns and area of cross- section 25sq cm has a resistance of 100 ohm. If a magnetic field which is perpendicular to the plane of the coil changes at the rate of 1000 Tesla per second, the current in the coil is

(A) 1.0 ampere (B) 50 ampere

(C) 0.5 ampere (D) 5.0 ampere

Q.12 The magnetic flux linked with a coil at any instant 't' is given by $\phi = 5t^3 - 100t + 300$, the e.m.f. induced in the coil at t = 2 second is

(A) –40 V (B) 40 V

(C) 140 V (D) 300 V

Q.13 When a small piece of wire passes between the magnetic poles of a horse-shoe magnet in 0.1sec, emf of 4×10^{-3} volt is induced in it. Magnetic flux between the poles is

(A) 4×10^{-2} weber (B) 4×10^{-3} weber

(C) 4×10^{-4} weber (D) 4×10^{-6} weber

PART 3 : LENZ'S LAW

Q.14 Two similar circular co-axial loops carry equal currents in the same direction. If the loops be brought nearer, the currents in loops

(A) Decreases (B) Increases

(C) Remains same (D) Different in each loop

Q.15 The current flowing in two coaxial coils in the same direction. On increasing the distance between the two, the electric current will –

(A) Increase

(B) Decrease

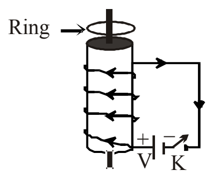
(C) Remain unchanged

(D) The information is incomplete

Q.16 Two different loops are concentric and lie in the same plane. The current in the outer loop is clockwise and increasing with time. The induced current in the inner loop then, is
(A) Clockwise
(B) Zero
(C) Counter clockwise
(D) In a direction that depends on the ratio of the loop radii.

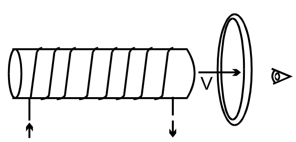
Q.17 An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z axis. The direction of induced current as seen from the z axis will be –
(A) Clockwise of the +ve z axis
(B) Anticlockwise of the +ve z axis
(C) Zero
(D) Along the magnetic field

Q.18 A conducting ring is placed around the core of an electromagnet as shown in fig. When key K is pressed, the ring



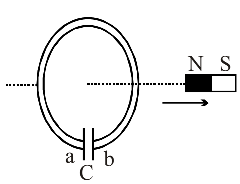
(A) Remain stationary.
(B) Is attracted towards the electromagnet.
(C) Jumps out of the core.
(D) None of the above.

Q.19 A current carrying solenoid is approaching a conducting loop as shown in the figure. The direction of induced current as observed by an observer on the other side of the loop will be -



(A) anticlockwise (B) clockwise
(C) east (D) west

Q.20 Consider the arrangement shown in figure in which the north pole of a magnet is moved away from a thick conducting loop containing capacitor. Then excess positive charge will arrive on -



(A) plate a (B) plate b
(C) On both plates a and b (D) On neither a nor b plates.

PART 4 : MOTIONAL ELECTROMOTIVE FORCE

Q.21 Direction of current induced in a wire moving in a magnetic field is found using
(A) Fleming's left hand rule (B) Fleming's right hand rule
(C) Ampere's rule (D) Right hand clasp rule

Q.22 A conductor is moving with the velocity v in the magnetic field and induced current is I. If the velocity of conductor becomes double, the induced current will be :
(A) 0.5 I (B) 1.5 I
(C) 2 I (D) 2.5 I

Q.23 A rod of length ℓ rotates with a uniform angular velocity ω about an axis passing through its middle point but normal to its length in a uniform magnetic field of induction B with its direction parallel to the axis of rotation. The induced emf between the two ends of the rod is
(A) $\frac{B\ell^2\omega}{2}$ (B) Zero
(C) $\left(\frac{B\ell^2\omega}{8}\right)$ (D) $2B\ell^2\omega$

Q.24 A conductor of 3 m in length is moving perpendicularly to magnetic field of 10^{-3} tesla with the speed of 10^2m/s, then the e.m.f. produced across the ends of conductor will be –
(A) 0.03 volt (B) 0.3 volt
(C) 3×10^{-3} volt (D) 3 volt

Q.25 An aeroplane in which the distance between the tips of wings is 50 m is flying horizontally with a speed of 360km/hr over a place where the vertical components of earth magnetic field is 2.0×10^{-4} weber/m^2. The potential difference between the tips of wings would be –
(A) 0.1 V (B) 1.0 V
(C) 0.2 V (D) 0.01 V

Q.26 A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0.4 Gauss, the induced e.m.f. between the axle and the rim of the wheel is equal to –
(A) 1.256×10^{-3} V (B) 6.28×10^{-4} V
(C) 1.256×10^{-4} V (D) 6.28×10^{-5} V

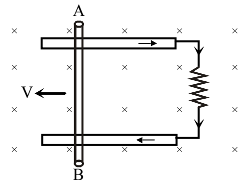
Q.27 A circular coil of mean radius of 7 cm and having 4000 turns is rotated at the rate of 1800 revolutions per minute in the earth's magnetic field (B = 0.5 gauss), the maximum e.m.f. induced in coil will be –
(A) 1.158 V (B) 0.58 V
(C) 0.29 V (D) 5.8 V

Q.28 The magnitude of the earth's magnetic field at a place is B_0 and the angle of dip is δ. A horizontal conductor of length ℓ lying along the magnetic north-south moves eastwards with a velocity v. The emf induced across the conductor is
(A) Zero (B) $B_0\ell v \sin\delta$
(C) $B_0\ell v$ (D) $B_0\ell v \cos\delta$

Q.29 A wire of length 1 m is moving at a speed of $2ms^{-1}$ perpendicular to its length and a homogeneous magnetic field of 0.5 T. The ends of the wire are joined to a circuit of resistance 6Ω. The rate at which work is being done to keep the wire moving at constant speed is
(A) W/12 (B) W/6
(C) W/3 (D) W

Q.30 A thick wire in the form of a semicircle of radius 'r' is rotated with a frequency 'f' in a magnetic field. What will be the peak value of emf induced?
(A) $B\pi r^2 f$ (B) $B\pi^2 r^2 f$
(C) $2Br^2 f$ (D) $2B\pi^2 r^2 f$

Q.31 Consider the situation shown in the figure. The wire AB is sliding on the fixed rails with a constant velocity. If the wire AB is replaced by semicircular wire, the magnitude of the induced current will –



(A) Increase
(B) Remain the same
(C) Decrease
(D) Increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

PART 5 : EDDY CURRENTS

Q.32 Induction furnace make use of
(A) Self induction (B) Mutual induction
(C) Eddy current (D) None of these

Q.33 A metal plate can be heated by
(A) Passing either a direct or alternating current through the plate.
(B) Placing in a time varying magnetic field.
(C) Placing in a space varying magnetic field, but does not vary with time.
(D) Both (A) and (B) are correct.

PART 6 : INDUCTANCE

Q.34 When the rate of change of current is unity, the induced emf is equal to
(A) Thickness of coil
(B) Number of turns in coil
(C) Coefficient of self inductance
(D) Total flux linked with coil

Q.35 A short solenoid of radius a, number of turns per unit length n_1, and length L is kept coaxially inside a very long solenoid of radius b, number of turns per unit length n_2. What is the mutual inductance of the system ?
(A) $\mu_0\pi b^2 n_1 n_2 L$ (B) $\mu_0\pi a^2 n_1 n_2 L^2$
(C) $\mu_0\pi a^2 n_1 n_2 L$ (D) $\mu_0\pi b^2 n_1 n_2 L^2$

Q.36 The unit of inductance is equivalent to
(A) $\frac{\text{volt} \times \text{ampere}}{\text{second}}$ (B) $\frac{\text{ampere}}{\text{volt} \times \text{second}}$
(C) $\frac{\text{volt}}{\text{ampere} \times \text{second}}$ (D) $\frac{\text{volt} \times \text{second}}{\text{ampere}}$

Q.37 The self inductance of a long solenoid cannot be increased by
(A) Increasing its area of cross section
(B) Increasing its length
(C) Increasing the current through it
(D) Increasing the number of turns in it

Q.38 If number of turns in primary and secondary coils is increased to two times each, the mutual inductance
(A) Becomes 4 times (B) Becomes 2 times
(C) Becomes 1/4 times (D) Remains unchanged

Q.39 If N is the number of turns in a coil, the value of self-inductance varies as :
(A) N^0 (B) N
(C) N^2 (D) N^{-2}

Q.40 Two pure inductors each of self inductance L are connected in parallel but are well separated from each other. The total inductance is –
(A) 2L (B) L
(C) L/2 (D) L/4

Q.41 A coil and a bulb are connected in series with a dc source, a soft iron core is then inserted in the coil. Then –
(A) Intensity of the bulb remains the same
(B) Intensity of the bulb decreases
(C) Intensity of the bulb increases
(D) The bulb ceases to glow

Q.42 Self induction of a solenoid is –
(A) Directly proportional to current flowing through the coil.
(B) Directly proportional to its length.
(C) Directly proportional to area of cross-section.
(D) Inversely proportional to area of cross-section.

Q.43 When the number of turns and the length of the solenoid are doubled keeping the area of cross-section same, the inductance –
(A) Remains the same (B) Is halved
(C) Is doubled (D) Becomes four times

Q.44 The mutual inductance of an induction coil is 5H. In the primary coil, the current reduces from 5A to zero in 10^{-3}s. What is the induced emf in the secondary coil –
(A) 2500V (B) 25000V
(C) 2510V (D) Zero

Q.45 The coefficient of mutual inductance of two coils is 6mH. If the current flowing in one is 2 ampere, then the induced e.m.f. in the second coil will be
(A) 3 mV (B) 2 mV
(C) 3 V (D) Zero

PART 7 : MEAN AND RMS VALUE OF AC

Q.46 An alternating current is given by the equation $i = i_1 \cos\omega t + i_2 \sin\omega t$. The r.m.s. current is given by
(A) $\frac{1}{\sqrt{2}}(i_1 + i_2)$ (B) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$
(C) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ (D) $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$

Q.47 If an ac main supply is given to be 220 V. What would be the average e.m.f. during a positive half cycle
(A) 198V (B) 386V
(C) 256V (D) None of these

Q.48 If the value of potential in an ac, circuit is 10V, then the peak value of potential is –

(A) $10/\sqrt{2}$ (B) $10\sqrt{2}$
(C) $20\sqrt{2}$ (D) $20/\sqrt{2}$

Q.49 The maximum value of a.c. voltage in a circuit is 707V. Its rms value is

(A) 70.7 V (B) 100 V
(C) 500 V (D) 707 V

Q.50 An ac source is rated at 220V, 50 Hz. The time taken for voltage to change from its peak value to zero is

(A) 50 sec (B) 0.02 sec
(C) 5 sec (D) 5×10^{-3} sec

Q.51 The peak voltage of an ac supply is 440V, then its rms voltage is

(A) 31.11 V (B) 311.1 V
(C) 41.11 V (D) 411.1 V

Q.52 When a voltage measuring device is connected to ac mains, the meter shows the steady input voltage of 220V. This means

(A) Input voltage cannot be ac voltage, but a dc voltage.
(B) Maximum input voltage is 220V.
(C) The meter reads not V but $< V^2 >$ and is calibrated to read $\sqrt{< V^2 >}$.
(D) The pointer of the meter is stuck by some mechanical defect.

Q.53 An ac source is of $\frac{200}{\sqrt{2}}$ V, 50 Hz. The value of voltage after (1 / 600) s from the start is

(A) 200 V (B) $\frac{200}{\sqrt{2}}$ V
(C) 100 V (D) 50 V

PART 8 : AC VOLTAGE TO A RESISTOR

Q.54 A 280 ohm electric bulb is connected to 200V electric line. The peak value of current in the bulb will be

(A) About one ampere (B) Zero
(C) About two ampere (D) About four ampere

Q.55 A 100Ω resistor is connected to a 220V, 50Hz ac supply. The rms value of current in the circuit is

(A) 1.56 A (B) 1.56 mA
(C) 2.2 A (D) 2.2 mA

PART 9 : AC VOLTAGE TO AN INDUCTOR

Q.56 In a circuit containing an inductance of zero resistance, the e.m.f. of the applied ac voltage leads the current by

(A) 90^o (B) 45^o
(C) 30^o (D) 0^o

Q.57 A a.c. circuit consists of only an inductor of inductance 2H. If the current is represented by a sine wave of amplitude 0.25 amp. and frequency 60 Hz, calculate the effective potential difference across the inductor

(A) 133.2 Volt (B) 122.2 Volt
(C) 143.2 Volt (D) 113.2 Volt

Q.58 The inductive reactance is directly proportional to –

(A) inductance
(B) frequency of the current
(C) amplitude of current
(D) both (A) and (B)

Q.59 An ideal inductor is in turns put across 220V, 50Hz and 220V, 100Hz supplies. The current flowing through it in the two cases will be

(A) Equal (B) Different
(C) Zero (D) Infinite

Q.60 A 44 mH inductor is connected to 220V, 50Hz ac supply. The rms value of the current in the circuit is

(A) 12.8 A (B) 13.6 A
(C) 15.9 A (D) 19.5 A

PART 10 : AC VOLTAGE TO A CAPACITOR

Q.61 An alternating e.m.f. is applied to purely capacitive circuit. The phase relation between e.m.f. and current flowing in the circuit is –

(A) e.m.f. is ahead of current by $\pi/2$
(B) Current is ahead of e.m.f. by $\pi/2$
(C) Current lags behind e.m.f. by π
(D) Current is ahead of e.m.f. by π

Q.62 A capacitor is a perfect insulator for

(A) Alternating currents (B) Direct currents
(C) Both ac and dc (D) None of these

Q.63 For high frequency, a capacitor offers

(A) More reactance (B) Less reactance
(C) Zero reactance (D) Infinite reactance

Q.64 An alternating voltage $E = 200\sqrt{2}\sin(100t)$ is connected to a 1 microfarad capacitor through an ac ammeter. The reading of the ammeter shall be

(A) 10 mA (B) 20 mA
(C) 40 mA (D) 80 mA

Q.65 The frequency for which a 5μF capacitor has a reactance of (1/1000) ohm is given by

(A) $\frac{100}{\pi}$ MHz (B) $\frac{1000}{\pi}$ Hz
(C) $\frac{1}{1000}$ Hz (D) 1000 Hz

Q.66 Reactance of a capacitor of capacitance CμF for ac frequency $\frac{400}{\pi}$ Hz is 25Ω. The value C is

(A) 50 μF (B) 25 μF
(C) 100 μF (D) 75 μF

Q.67 A 30μF capacitor is connected to a 150 V, 60Hz ac supply. The rms value of current in the circuit is

(A) 17 A (B) 1.7 A
(C) 1.7 mA (D) 2.7 A

Q.68 In a pure capacitive circuit if the frequency of ac source is doubled, then its capacitive reactance will be

(A) Remain same (B) Doubled
(C) Halved (D) Zero

Q.69 When an ac voltage of 220V is applied to the capacitor C, then
(A) The maximum voltage between plates is 220V.
(B) The current is in phase with the applied voltage.
(C) The charge on the plate is not in phase with the applied voltage.
(D) Power delivered to the capacitor per cycle is zero.

Q.70 Phase difference between voltage and current in a capacitor in an ac circuit is
(A) π (B) $\pi/2$
(C) 0 (D) $\pi/3$

PART 11 : AC VOLTAGE APPLIED TO A LR CIRCUIT

Q.71 An inductive circuit contains a resistance of 10ohm and an inductance of 2.0 henry. If an ac voltage of 120 volt and frequency of 60 Hz is applied to this circuit, the current in the circuit would be nearly –
(A) 0.32 amp (B) 0.16 amp
(C) 048 amp (D) 0.80 amp

Q.72 If an 8Ω resistance and 6Ω reactance are present in an ac series circuit then the impedance of the circuit will be
(A) 20 ohm (B) 5 ohm
(C) 10 ohm (D) $14\sqrt{2}$ ohm

Q.73 A 12 ohm resistor and a 0.21 henry inductor are connected in series to an ac source operating at 20 volts, 50 cycle/second. The phase angle between the current and the source voltage is
(A) 30° (B) 40°
(C) 80° (D) 90°

Q.74 An alternating voltage is connected in series with a resistance R and an inductance L If the potential drop across the resistance is 200 V and across the inductance is 150 V, then the applied voltage
(A) 350 V (B) 250 V
(C) 500 V (D) 300 V

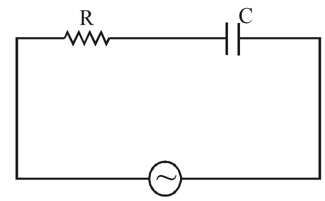
Q.75 A 220 V, 50 Hz ac source is connected to an inductance of 0.2 H and a resistance of 20 ohm in series. What is the current in the circuit
(A) 10 A (B) 5 A
(C) 33.3 A (D) 3.33 A

Q.76 A resistance of 40 ohm and an inductance of 95.5millihenry are connected in series in a 50 cycles/second ac circuit. The impedance of this combination is very nearly
(A) 30 ohm (B) 40 ohm
(C) 50 ohm (D) 60 ohm

Q.77 A circuit is made up of a resistance 1Ω and inductance 0.01 H. An alternating voltage of 200V at 50 Hz is connected, then the phase difference between the current and the voltage in the circuit is
(A) $\tan^{-1}(\pi)$ (B) $\tan^{-1}(\pi/2)$
(C) $\tan^{-1}(\pi/4)$ (D) $\tan^{-1}(\pi/3)$

PART 12 : AC VOLTAGE APPLIED TO A RC CIRCUIT

Q.78 A 50 Hz ac source of 20 volts is connected across R and C as shown in figure. The voltage across R is 12 volt. The voltage across C is –



(A) 8V
(B) 16V
(C) 10V
(D) Without values of R and C calculation is not possible.

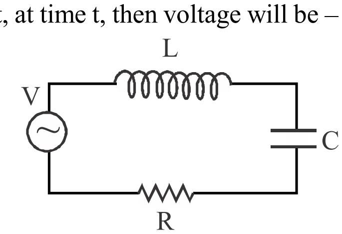
Q.79 In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit ?
(A) Only resistor (B) Resistor and inductor
(C) Resistor and capacitor (D) Only inductor

Q.80 A circuit containing a 20 Ω resistor and 0.1 μF capacitor in series is connected to 230 V ac supply of angular frequency 100 rad s^{-1}. The impedance of the circuit is
(A) 10^5 Ω (B) 10^4 Ω
(C) 10^6 Ω (D) 10^{10} Ω

Q.81 A 0.2 kΩ resistor and 15μF capacitor are connected in series to a 220V, 50Hz ac source. The impedance of the circuit is
(A) 250 Ω (B) 268 Ω
(C) 29.15 Ω (D) 291.5 Ω

PART 13 : AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

Q.82 If q is the charge on the capacitor and i the current, at time t, then voltage will be –



(A) $L\frac{di}{dt}+iR-\frac{q}{C}=V$ (B) $L\frac{di}{dt}-iR+\frac{q}{C}=V$
(C) $L\frac{di}{dt}+iR+\frac{q}{C}=V$ (D) $L\frac{di}{dt}-iR-\frac{q}{C}=V$

Q.83 In LCR circuit, the amplitude of the current is given by –
(A) $i_m=\frac{v_m}{\sqrt{R^2+(X_C-X_L)^2}}$ (B) $i_m=\frac{v_m}{\sqrt{R^2-(X_C-X_L)^2}}$
(C) $i_m=\frac{v_m}{\sqrt{2R^2+(X_C-X_L)^2}}$ (D) $i_m=\frac{v_m}{\sqrt{R^2+(X_C+X_L)^2}}$

Q.84 An alternating current source of frequency 100Hz is joined to a combination of a resistance, a capacitance and a coil in series. The potential difference across the coil, the resistance and the capacitor is 46, 8 and 40 volt respectively. The electromotive force of alternating current source in volt is
(A) 94 (B) 14
(C) 10 (D) 76

Q.85 In a series circuit R = 300 Ω, L = 0.9 H, C = 2.0 μF and ω = 1000 rad/sec. The impedance of the circuit is
(A) 1300 Ω (B) 900 Ω
(C) 500 Ω (D) 400 Ω

Q.86 In a LCR circuit the pd between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30V and that between the terminals of resistance is 40V. the supply voltage will be equal to
(A) 50 V (B) 70 V
(C) 130 V (D) 10 V

Q.87 In series LCR circuit voltage drop across resistance is 8volt, across inductor is 6 volt and across capacitor is 12 volt. Then –
(A) voltage of the source will be leading current in the circuit.
(B) voltage drop across each element will be less than the applied voltage.
(C) power factor of circuit will be 4/3.
(D) none of these.

Q.88 In a series LCR circuit R = 10 Ω and the impedance Z = 20Ω. Then the phase difference between current and voltage is :
(A) 45° (B) 90°
(C) 60° (D) 30°

Q.89 An LCR series circuit is under resonance. If I_m is current amplitude, V_m is voltage amplitude, R is the resistance, Z is the impedance, X_L is the inductive reactance and X_C is the capacitive reactance, then
(A) $I_m = \frac{Z}{V_m}$ (B) $I_m = \frac{V_m}{X_L}$
(C) $I_m = \frac{V_m}{X_C}$ (D) $I_m = \frac{V_m}{R}$

Q.90 In a series LCR circuit the voltage across an inductor, capacitor and resistor are 20V, 20V and 40V respectively. The phase difference between the applied voltage and the current in the circuit is
(A) 30º (B) 45º
(C) 60º (D) 0º

Q.91 In a series LCR circuit, the phase difference between the voltage and the current is 45º. Then the power factor will be
(A) 0.607 (B) 0.707
(C) 0.808 (D) 1

Q.92 200 V ac source is fed to series LCR circuit having X_L = 50 Ω, X_C = 50 Ω and R = 25 Ω. Potential drop across the inductor is
(A) 100 V (B) 200 V
(C) 400 V (D) 10 V

PART 14 : POWER IN AC CIRCUIT

Q.93 In an ac circuit, the current is given by $i = 5\sin\left(100\,t - \frac{\pi}{2}\right)$ and the ac potential is V = 200 sin (100) volt Then the power consumption is
(A) 20 watts (B) 40 watts
(C) 1000 watts (D) 0 watt

Q.94 In an ac circuit, the instantaneous values of e.m.f. and current are e = 200 sin 314 t volt and $i = \sin\left(314t + \frac{\pi}{3}\right)$ ampere. The average power consumed in watt is
(A) 200 (B) 100
(C) 50 (D) 25

Q.95 For an ac circuit V = 15 sin ωt and I = 20 cos ωt the average power consumed in this circuit is
(A) 300 Watt (B) 150 Watt
(C) 75 Watt (D) zero

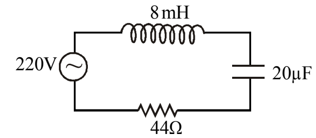
Q.96 A series LCR-circuit with R = 20Ω, L = 1.5H and C = 35μF is connected to a variable frequency 200 V ac apply. When the frequency of the supply equals the natural frequency of the circuit the average power transferred to the circuit in one complete cycle is
(A) 200 W (B) 2000 W
(C) 100 W (D) 4000 W

Q.97 An inductor of reactance 1 Ω and a resistor of 2Ω are connected in series to the terminals of a 6V (rms) ac source. The power dissipated in the circuit is
(A) 8 W (B) 12 W
(C) 14.4 W (D) 18 W

Q.98 Expression for Wattless current is –
(A) $I_{rms} \sin\phi$ (B) $I_{rms} \cos\phi$
(C) $I_{rms} \tan\phi$ (B) $I_{rms} \cot\phi$

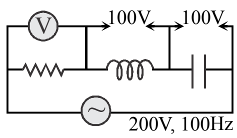
PART 15 : RESONANCE

Q.99 For the series LCR circuit shown in the figure, what is the resonance frequency and the amplitude of the current at the resonating frequency



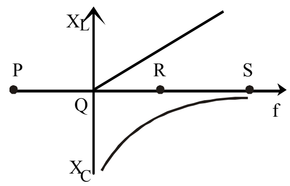
(A) 2500 rad-s^{-1} and $5\sqrt{2}$A (B) 2500 rad-s^{-1} and 5A
(C) 2500 rad-s^{-1} and $\frac{5}{\sqrt{2}}$A (D) 25 rad-s^{-1} and $5\sqrt{2}$A

Q.100 In the circuit given, what will be the reading of the voltmeter –



(A) 300 V (B) 900 V
(C) 200 V (D) 400 V

Q.101 The resonance point in $X_L - f$ and $X_C - f$ curves is



(A) P (B) Q
(C) R (D) S

Q.102 In an LCR circuit, the sharpness of resonance depends on–
(A) Inductance (L) (B) Capacitance (C)
(C) Resistance (R) (D) All of these

Q.103 In LCR circuit, if the resistance increases, the quality factor –
(A) increases finitely (B) decreases finitely
(C) remain constant (D) none of these

Q.104 At resonance frequency the impedance in series LCR circuit is
(A) Maximum (B) Minimum
(C) Zero (D) Infinity

Q.105 To reduce the resonant frequency in an LCR series circuit with a generator
(A) The generator frequency should be reduced.
(B) Another capacitor should be added in parallel to the first.
(C) The iron core of the inductor should be removed.
(D) Dielectric in the capacitor should be removed.

Q.106 Quality factor and power factor both have the dimensions of
(A) Time (B) Frequency
(C) Work (D) Angle

PART 16 : LC OSCILLATIONS

Q.107 An oscillator circuit consists of an inductance of 0.5mH and a capacitor of 20μF. The resonant frequency of the circuit is nearly
(A) 15.92 Hz (B) 159.2 Hz
(C) 1592 Hz (D) 15910 Hz

Q.108 A charged 30μF capacitor is connected to a 27mH inductor. The angular frequency of free oscillations of the circuit is
(A) 1.1×10^3 rad s^{-1} (B) 2.1×10^3 rad s^{-1}
(C) 3.1×10^3 rad s^{-1} (D) 4.1×10^3 rad s^{-1}

Q.109 An LC circuit contains a 20 mH inductor and a 25μF capacitor with an initial charge of 5mC. The total energy stored in the circuit initially is
(A) 5 J (B) 0.5 J
(C) 50 J (D) 500 J

PART 17 : AC GENERATOR AND TRANSFORMERS

Q.110 In a transformer 220 ac voltage is increased to 2200 volts. If the number of turns in the secondary are 2000, then the number of turns in the primary will be –
(A) 200 (B) 100
(C) 50 (D) 20

Q.111 The ratio of secondary to the primary turns in a transformer is 3 : 2. If the power output be P, then the input power neglecting all loses must be equal to –
(A) 5 P (B) 1.5 P
(C) P (D) (2/5) P

Q.112 The primary winding of a transformer has 100 turns and its secondary winding has 200 turns. The primary is connected to an ac supply of 120V and the current flowing in it is 10A. The voltage and the current in the secondary are –
(A) 240 V, 5 A (B) 240 V, 10 A
(C) 60 V, 20 A (D) 120 V, 20 A

Q.113 In a step up transformer, 220 V is converted into 200 V. The number of turns in primary coil is 600. What is the number of turns in the secondary coil
(A) 60 (B) 600
(C) 6000 (D) 100

Q.114 The core of a transformer is laminated to reduce
(A) Flux leakage (B) Hysteresis
(C) Copper loss (D) Eddy current

EXERCISE - 2 (LEVEL-2)

Choose one correct response for each question.

Q.1 A circular coil of radius 8 cm, 400 turns and resistance 2Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180º in 0.30 s. Horizontal component of earth magnetic field at the place is 3×10^{-5} T. The magnitude of current induced in the coil is approximately
(A) 4×10^{-2} A (B) 8×10^{-4} A
(C) 8×10^{-2} A (D) 1.92×10^{-3} A

Q.2 In a coil current falls from 5A to 0A in 0.2 s. If an average emf of 150 V is induced, then the self inductance of the coil is
(A) 4H (B) 2H
(C) 3H (D) 6H

Q.3 A metal conductor of length 1m rotates vertically about one of its ends with an angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the ends of the conductor is
(A) 5μV (B) 5mV
(C) 50μV (D) 50mV

Q.4 The line that draws power supply to your house from street has
(A) $220\sqrt{2}$ V average voltage
(B) 220V average voltage
(C) Voltage and current out of phase by π/2
(D) Voltage and current possibly differing in phase φ such that |φ| < π/2.

Q.5 A conducting wire of 100 turns is wound over 1 cm near the centre of a solenoid of 100 cm length and 2 cm radius having 1000 turns. The mutual inductance of the two coils is
(A) 1.58×10^{-4} V (B) 1.58×10^{-3} V
(C) 2.11×10^{-4} V (D) 2.11×10^{-3} V

Q.6 The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to the relation $\phi = (2t^2 + 4t + 6)$ mWb. The emf induced in the loop at t = 4s is
(A) 0.12 V (B) 2.4 V
(C) 0.02 V (D) 1.2 V

Q.7 The self inductance of an inductor coil having 100 turns is 20mH. The magnetic flux through the cross-section of the coil corresponding to a current of 4mA is
(A) 2×10^{-5} Wb (B) 4×10^{-7} Wb
(C) 8×10^{-7} Wb (D) 8×10^{-5} Wb

Q.8 If the number of turns per unit length of a coil of solenoid is doubled, the self-inductance of the solenoid will
(A) Remain unchanged (B) Be halved
(C) Be doubled (D) Become four times

Q.9 A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This is because
(A) The magnetic field is constant.
(B) The magnetic field is in the same plane as the circular coil and it may or may not vary.
(C) The magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.
(D) Both (B) and (C).

Q.10 Two coils have self-inductance L_1 = 4mH and L_2 = 1mH respectively. The currents in the two coils are increased at the same rate. At a certain instant of time both coils are given the same power. If I_1 and I_2 are the currents in the two coils at that instant of time respectively, then the value of (I_1/I_2) is
(A) 1/8 (B) 1/4
(C) 1/2 (D) 1

Q.11 The magnetic flux linked with coil, in weber is given by the equation, $\phi = 5t^2 + 3t + 16$. The induced emf in the coil in the fourth second is –
(A) 10 V (B) 30 V
(C) 45 V (D) 90 V

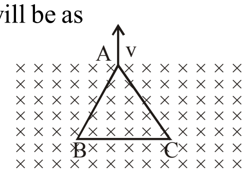
Q.12 If a coil of 40 turns and area 4.0 cm^2 is suddenly removed from a magnetic field, it is observed that a charge of 2.0×10^{-4}C flows into the coil. If the resistance of the coil is 80 Ω, the magnetic flux density in Wb/m^2 is
(A) 0.5 (B) 1.0
(C) 1.5 (D) 2.0

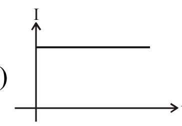
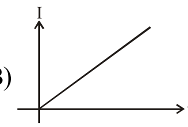
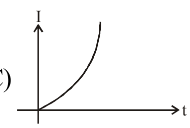
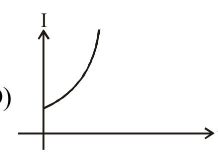
Q.13 A circular coil is radius 5 cm has 500 turns of a wire. The approximate value of the coefficient of self induction of the coil will be-
(A) 25 mH (B) 25×10^{-3} mH
(C) 50×10^{-3} mH (D) 50×10^{-3} H

Q.14 When the current in a coil changes from 8 ampere to 2 ampere in 3×10^{-2} second, the e.m.f. induced in the coil is 2volt. The self inductance of the coil (in millihenry) is
(A) 1 (B) 5
(C) 20 (D) 10

Q.15 If a current of 3.0 amperes flowing in the primary coil is reduced to zero in 0.001 second, then the induced e.m.f. in the secondary coil is 15000 volts. The mutual inductance between the two coils is
(A) 0.5 henry (B) 5 henry
(C) 1.5 henry (D) 10 henry

Q.16 An equilateral triangular loop ABC made of uniform thin wires is being pulled out of a region with a uniform speed v, where a uniform magnetic field $\vec{B}$ perpendicular to the plane of the loop exists. At time t = 0, the point A is at the edge of the magnetic field. The induced current (I) vs time (t) graph will be as

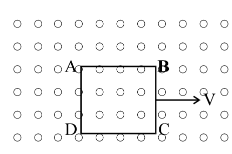


(A) 
(B) 
(C) 
(D) 

Q.17 The inductance of a coil is 60μH. A current in this coil increases from 1.0 A to 1.5 A in 0.1 second. The magnitude of the induced e.m.f. is
(A) 60×10^{-6}V (B) 300×10^{-4}V
(C) 30×10^{-4}V (D) 3×10^{-4}V

Q.18 If in a coil rate of change of area is 5 m^2/milli second and current become 1 amp from 2 amp in 2×10^{-3} sec. If magnitude of field is 1 tesla then self inductance of the coil is
(A) 2 H (B) 5 H
(C) 20 H (D) 10 H

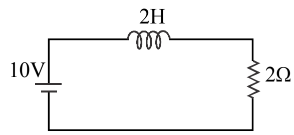
Q.19 A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Electric field is induced:



(A) in AD, but not in BC
(B) in BC, but not in AD
(C) neither in AD nor in BC
(D) in both AD and BC

Q.20 A coil resistance 20Ω and inductance 5H is connected with a 100V battery. Energy stored in the coil will be
(A) 41.5 J (B) 62.50 J
(C) 125 J (D) 250 J

Q.21 In the figure magnetic energy stored in the coil is



(A) Zero (B) Infinite
(C) 25 joules (D) None of the above

Q.22 The armature of dc motor has 20Ω resistance. It draws current of 1.5 ampere when run by 220volts dc supply. The value of back e.m.f. induced in it will be
(A) 150 V (B) 170 V
(C) 180 V (D) 190 V

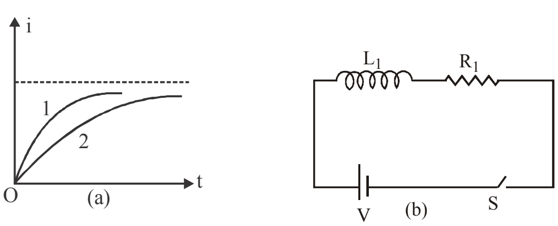
Q.23 A step-down transformer is connected to 2400 volts line and 80 amperes of current is found to flow in output load. The ratio of the turns in primary and secondary coil is 20 : 1. If transformer efficiency is 100%, then the current flowing in primary coil will be –
(A) 1600 A (B) 20 A
(C) 4 A (D) 1.5 A

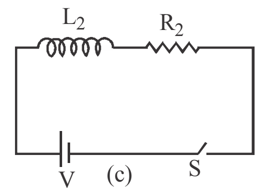
Q.24 A 100% efficient transformer has 100 turns in the primary and 25 turns in its secondary coil. If the current in the secondary coil is 4 amp, then the current in the primary coil is
(A) 1 amp (B) 4 amp
(C) 8 amp (D) 16 amp

Q.25 A bicycle wheel of radius 0.5 m has 32 spokes. It is rotating at the rate of 120 revolutions per minute, perpendicular to the horizontal component of earth's magnetic field $B_H = 4 \times 10^{-5}$ tesla. The emf induced between the rim and the centre of the wheel will be-
(A) 6.28×10^{-5} V (B) 4.8×10^{-5} V
(C) 6.0×10^{-5} V (D) 1.6×10^{-5} V

Q.26 A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be
(A) halved (B) the same
(C) doubled (D) quadrupled

Q.27 Current growth into L-R circuits (b) and (c) is as shown in figure (a). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits, then –



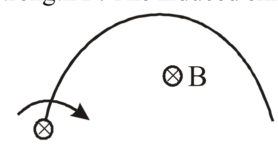


(A) $R_1 > R_2$ (B) $R_1 < R_2$
(C) $L_1 > L_2$ (D) $L_1 < L_2$

Q.28 A wire in the form of a circular loop of radius 10cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s, average induced emf in the loop is:
(A) 6.70 volt (B) 5.80 volt
(C) 6.75 volt (D) 5.75 volt

Q.29 A short circuited coil is placed in a time - varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be
(A) halved (B) the same
(C) doubled (D) quadrupled

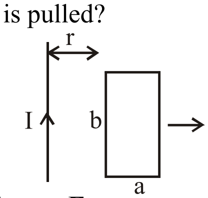
Q.30 A semicircular wire of radius R is rotated with constant angular velocity ω about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced emf between the ends is



(A) $B\omega R^2/2$
(B) $2B\omega R^2$
(C) is variable
(D) none of these

Q.31 A conducting square loop is placed in a magnetic field B with its plane perpendicular to the field. The sides of the loop start shrinking at a constant rate α. The induced emf in the loop at an instant when its side is 'a' is –
(A) $2a\alpha B$ (B) $a^2\alpha B$
(C) $2a^2\alpha B$ (D) $a\alpha B$

Q.32 A rectangular loop of wire with dimensions shown in coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and right sides of the loop as the loop is pulled?



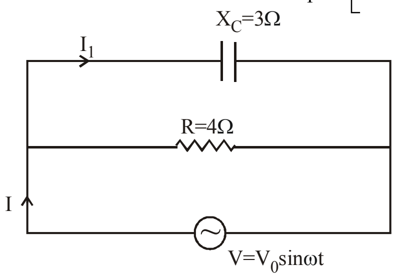
Induced current	Force on left side	Force on right side
(A) Counter clockwise	To the left	To the left
(B) Counter clockwise	To the right	To the left
(C) Clockwise	To the right	To the left
(D) Clockwise	To the left	To the right

Q.33 The emf E = 4 cos 1000t volts is applied to an L-R circuit containing inductance 3mH and resistance 4Ω. The amplitude of current is –

(A) $4\sqrt{7}$ A (B) 1.0 A

(C) (4/7) A (D) 0.8 A

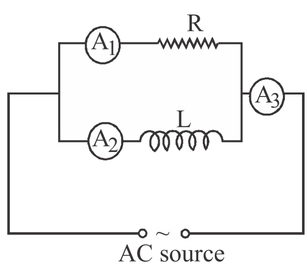
Q.34 A capacitor and a resistor are connected with an A.C. source as shown in figure. Reactance of capacitor is X_C = 3Ω and resistance of resistor is 4Ω. The phase difference between current I and I_1 is $\left[\tan\frac{4}{3} = 53°\right]$



(A) π/2 (B) zero

(C) 53° (D) 37°

Q.35 In the circuit shown, assuming all ammeters to be ideal, if readings of the hot wire ammeters A_1 and A_2 are i_1 and i_2 respectively then reading of the hot wire ammeter A_3 is –



(A) equal to $(i_1 + i_2)$ (B) greater than $(i_1 + i_2)$

(C) less than $(i_1 + i_2)$ (D) equal to 2 $(i_1 – i_2)$

Q.36 For a LCR series circuit with an A.C. source of angular frequency ω.

(A) Circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$

(B) Circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$

(C) Power factor of circuit will by unity if capacitive reactance equals inductive reactance.

(D) Current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

Q.37 An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be –

(A) clockwise of the positive z-axis

(B) anticlockwise of the positive z-axis

(C) zero

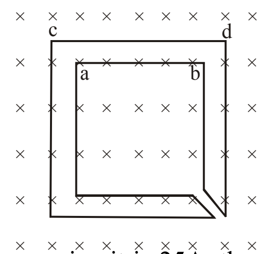
(D) along the magnetic field

Q.38 An inductor of 30 mH is connected to a 220V, 100Hz ac source. The inductive reactance is

(A) 10.58 Ω (B) 12.64 Ω

(C) 18.85 Ω (D) 22.67 Ω

Q.39 The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd.



(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction ba and I_2 is in the direction cd

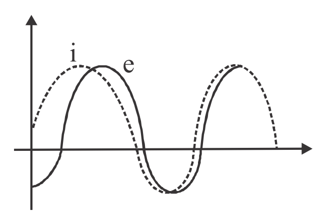
(D) I_1 is in the direction ab and I_2 is in the direction dc

Q.40 The rms value of current in an ac circuit is 25A, then peak current is

(A) 35.36 mA (B) 35.36 A

(C) 3.536 A (D) 49.38 A

Q.41 When an AC source of emf e=E_0sin (100t) is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be π/4 as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C series, find the relationship between the two elements.



(A) R = 1kΩ, C = 10μF (B) R = 1kΩ, C = 1μF

(C) R = 1kΩ, L = 10H (D) R = 1kΩ, C = 1H

Q.42 A step down transformer converts transmission line voltage from 11000 V to 220 V. The primary of the transformer has 6000 turns and efficiency of the transformer is 60%. If the output power is 9kW, then the input power will be

(A) 11 kW (B) 12 kW

(C) 14 kW (D) 15 kW

Q.43 If V = 100 sin (100 t) V and

$I = 100 \sin\left(100t + \frac{\pi}{3}\right)$ mA are the instantaneous values of voltage and current, then the rms values of voltage and current are respectively

(A) 70.7 V, 70.7 mA (B) 70.7 V, 70.7 A

(C) 141.4 V, 141.4 mA (D) 100 V, 100 mA

Q.44 The Q factor of a series LCR circuit with L = 2H, C = 32μF and R = 10 Ω is

(A) 15 (B) 20

(C) 25 (D) 30

Q.45 A 5μF capacitor is connected to a 200 V, 100 Hz ac source. The capacitive reactance is

(A) 212 Ω (B) 312 Ω
(C) 318 Ω (D) 412 Ω

Q.46 Resonance frequency of a circuit is f. If the capacitance is made 4 times the initial value, then the resonance frequency will become :

(A) f/2 (B) 2f
(C) f (D) f/4

Q.47 A pure resistive circuit element X when connected to an ac supply of peak voltage 200V gives a peak current of 5A which is in phase with the voltage. A second circuit element Y, when connected to the same ac supply also gives the same value of peak current but the current lags behind by 90º. If the series combination of X and Y is connected to the same supply, what will be the rms value of current ?

(A) $\frac{10}{\sqrt{2}}$A (B) $\frac{5}{\sqrt{2}}$A (C) $\frac{5}{2}$A (D) 5A

Q.48 In a series LCR circuit the voltage across the resistance, capacitance and inductance is 10V each. If the capacitance is short circuited, the voltage across the inductance will be

(A) 10V (B) $10\sqrt{2}$ V
(C) $(10/\sqrt{2})$V (D) 20V

Q.49 A series LCR circuit with R = 22Ω, L = 1.5H and C = 40μF is connected to a variable frequency 220V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle ?

(A) 2000 W (B) 2200 W
(C) 2400 W (D) 2500 W

Q.50 A circuit consists of a resistance of 10Ω and a capacitance of 0.1 μF. If an alternating emf of 100V, 50Hz is applied, the current in the circuit is

(A) 3.14 mA (B) 6.28 mA
(C) 1.51 mA (D) 7.36 mA

Q.51 The relation between an ac voltage source and time in SI units is V = 120 sin (100 πt) cos (100 πt) V. The value of peak voltage and frequency will be respectively

(A) 120 V and 100 Hz (B) $\frac{120}{\sqrt{2}}$V and 100Hz
(C) 60V and 200Hz (D) 60V and 100Hz

Q.52 A voltage of peak value 283 V and varying frequency is applied to series LCR combination in which R = 3Ω, L = 25 mH and C = 400 μF. Then the frequency (in Hz) of the source at which maximum power is dissipated in the above is

(A) 51.5 (B) 50.7
(C) 51.1 (D) 50.3

Q.53 A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000, 220V step down transformer at a substation in the town. The line power loss in the form of heat is

(A) 400 kW (B) 600 kW
(C) 300 kW (D) 800 W

Q.54 A coil of 0.01 H inductance and 1Ω resistance is connected to 200 V, 50 Hz ac supply. The time lag between maximum alternating voltage and current is

(A) (1/250) s (B) (1/300) s
(C) (1/200) s (D) (1/350) s

Q.55 A sinusoidal voltage of peak value 293 V and frequency 50 Hz is applied to a series LCR circuit in which R = 6Ω. L = 25 mH and C = 750 μF. The impedance of the circuit is

(A) 7.0 Ω (B) 8.9 Ω
(C) 9.9 Ω (D) 10.0 Ω

Q.56 A 60μF capacitor is connected to a 110V (rms), 60 Hz ac supply. The rms value of current in the circuit is

(A) 1.49 A (B) 14.9 A
(C) 2.49 A (D) 24.9 A

Q.57 A 100 μF capacitor in series with a 40 Ω resistor is connected to a 100 V, 60 Hz supply. The maximum current in the circuit is

(A) 2.65 A (B) 2.75 A
(C) 2.85 A (D) 2.95 A

Q.58 The equation of AC voltage is
E = 200 sin (ωt + π/6) and the A.C. current is
I = 10 sin (ωt + π/6). The average power dissipated is –

(A) 150 W (B) 550 W
(C) 250 W (D) 50 W

Q.59 If a capacitor of 8μF is connected to a 220 V, 100 Hz ac source and the current passing through it is 65 mA, then the rms voltage across it is

(A) 129.4 V (B) 12.94 V
(C) 1.294 V (D) 15 V

Q.60 An alternating voltage (in volts) given by $V = 200\sqrt{2}$ sin (100t) is connected to 1μF capacitor through an ideal ac ammeter in series. The reading of the ammeter and the average power consumed in the circuit shall be

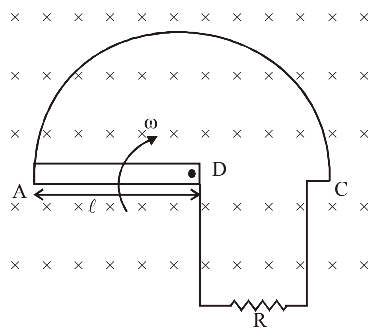
(A) 20 mA, 0 (B) 20 mA, 4W
(C) $20\sqrt{2}$ mA, 8W (D) $20\sqrt{2}$ mA, $4\sqrt{2}$ W

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

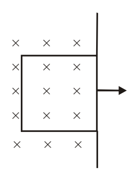
NOTE : The answer to each question is a NUMERICAL VALUE.

PART-A - Electromagnetic induction

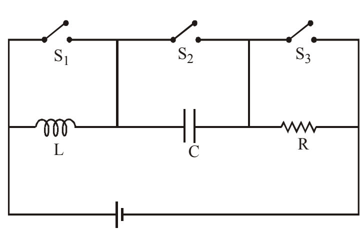
Q.1 A metallic rod of mass 1 kg and length $\sqrt{\pi}$m is hinged at point D can rotate in the plane of the paper. Other end of the rod slides on the metallic semicircular wire ABC. Point D of the rod and point C of the semicircular portion is connected by a resistance of 1Ω. If a constant inward perpendicular magnetic field of magnitude 2T exists in the region, find the minimum angular velocity (in radian/sec) given to the rod in position DA so that it reaches position DC. Neglect force due to gravity, friction, resistance of rod and semicircular wire ($\pi^2 = 10$).



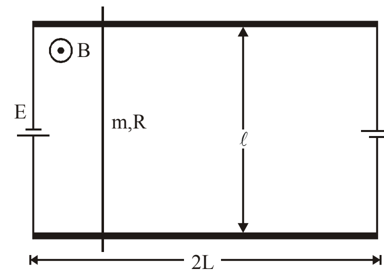
Q.2 A square loop of area 2.5×10^{-3} m^2 and having 100 turns with a total resistance of 100Ω is moved out of a uniform magnetic field of 0.40T in 1 sec. with a constant speed. Then what is the work done, in pulling the loop (in μJ)



Q.3 Consider the circuit shown in figure. With switch S_1 closed and the other two switches open, the circuit has a time constant 0.05sec. With switch S_2 closed and the other two switches open, the circuit has a time constant 2 sec. With switch S_3 closed and the other two switches open, the circuit oscillates with a period T. Find T (in sec.) (Take $\pi^2 = 10$)

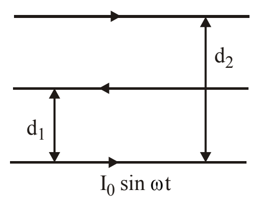


Q.4 Side rail of length 2L are fixed on a horizontal plane at a distance ℓ from each other. These ends are connected by two identical ideal batteries with emf E by resistanceless wires (see figure). On the rails is a rod of mass m, which may slide along them. The entire system is placed in a uniform vertical magnetic field B. Assuming that the resistance of the rod is R and the resistance per unit length of each of the rails equal to ρ, find the period of small oscillation (in sec.) arising from shifting the rod from the equilibrium along the rails. Neglect friction, internal resistance of batteries and induced emf in the rod. [Take : B = πT, E = π volt, ℓ = 0.5m, L = 1m, ρ = 1Ω/m, R = 0.25Ω, m = 100gm]



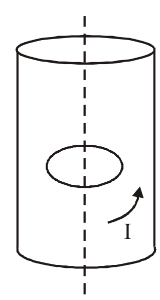
Q.5 Two long straight wires of telephone circuit inside a house are coplanar with and run parallel to a third long straight conductor that carries a current $i = i_0 \sin \omega t$. The wires are at a distance of d_1 and d_2 from the conductor. The conductor induced a noise in the telephone circuit. The rms voltage per unit length of this induced emf is 2×10^{-x} V. Find the value of x.

[Take $d_2 = 2d_1$, $i_0 = \frac{2\sqrt{2}}{\ln 2}$ A , ω = 50 rad/s]



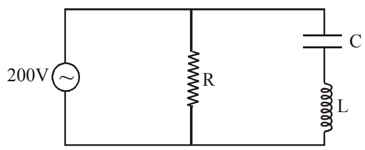
Q.6 A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5m is placed along y-axis is move along x-axis with constant speed 1 m/sec. Then induced emf in the rod 5x volt then find the value of x.

Q.7 A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos (300 t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin (300t)$, then N is



PART-B : Alternating current

Q.1 In the circuit diagram shown, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ & $R = 100\ \Omega$. The effective current through the source is $2\sqrt{x}$ A then find value of x.

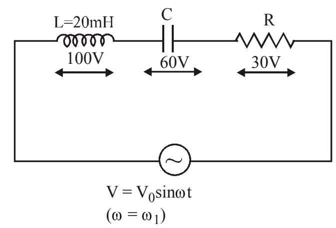


Q.2 A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Q.3 In a series L-R circuit, connected with a sinusoidal as source, the maximum potential difference across L and R are respectively 3 volts and 4 volts. At an instant the potential difference across resistor is 2 volts. The potential difference in volt, across the inductor at the same instant is (X) cos 30°. Find the value of X.

Q.4 In the above question at the same instant, the magnitude of the potential difference in volt, across the ac source is $\frac{X+3\sqrt{3}}{2}$. Find the value of X.

Q.5 Consider the RLC circuit shown below connected to an AC source of constant peak voltage V_0 and variable frequency ω_0 value of L = 20 mH. For a certain value $\omega_0 = \omega_1$, rms voltage across L, C, R are shown in the diagram. At $\omega_0 = \omega_2$, it is found that rms voltage across resistance is 50V. Then the value of V_0 is $50\sqrt{X}$ V. Find the value of X.

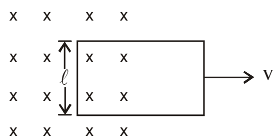


Q.6 In above question the value of ω_2 is $\sqrt{\frac{X}{5}}\omega_1$. Find the value of X.

Q.7 In above question the phase difference between voltage on L and C is $X\pi$. Find the value of X.

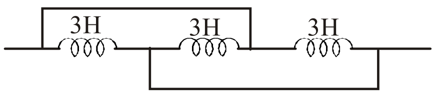
EXERCISE - 4 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 When a rectangular loop pulled out from uniform magnetic field. The induced current is proportional to **[AIEEE-2002]**



(A) B (B) ℓ^{-1}
(C) R (D) v^2

Q.2 Resultant inductance of the circuit will be **[AIEEE-2002]**



(A) 3 H (B) 9 H
(C) 1 H (D) 7.5 H

Q.3 The power factor of an A.C. circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is – **[AIEEE-2002]**

(A) $\frac{R}{\omega L}$ (B) $\frac{R}{(R^2+\omega^2 L^2)^{1/2}}$

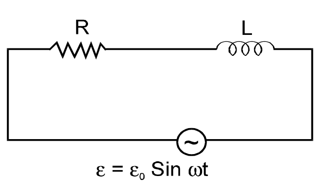
(C) $\frac{\omega L}{R}$ (D) $\frac{R}{(R^2-\omega^2 L^2)^{1/2}}$

Q.4 In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is – **[AIEEE-2002]**
(A) 4 A (B) 2 A
(C) 6 A (D) 10 A

Q.5 Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon **[AIEEE-2003]**
(A) Relative position and orientation of the two coils
(B) The materials of the wires of the coils
(C) The currents in the two coils
(D) The rates at which currents are changing in the two coils.

Q.6 When the current changes from +2A to –2A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is & **[AIEEE-2003]**
(A) 0.4 H (B) 0.8 H
(C) 0.1 H (D) 0.2 H

Q.7 Power factor of the circuit is – **[AIEEE-2003]**



(A) $\frac{R}{\omega L}$ (B) $\frac{R}{\sqrt{R^2+\omega^2 L^2}}$

(C) $\frac{R}{R^2+\omega^2 L^2}$ (D) none

Q.8 The core of any transformer is laminated so as to – **[AIEEE-2003]**
(A) Make it light weight
(B) Make it robust and strong
(C) Increase the secondary voltage
(D) Reduce the energy loss due to eddy current

Q.9 A coil having n turns and resistance RΩ is connected with a galvanometer of resistance 4RΩ. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is **[AIEEE-2004]**

(A) $-\frac{(W_2-W_1)}{5Rnt}$ (B) $-\frac{n(W_2-W_1)}{5Rt}$

(C) $-\frac{(W_2-W_1)}{Rnt}$ (D) $-\frac{n(W_2-W_1)}{Rt}$

Q.10 In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R the mean power generated per period of rotation is – **[AIEEE-2004]**

(A) $\frac{B\pi r^2\omega}{2R}$ (B) $\frac{(B\pi r^2\omega)^2}{8R}$

(C) $\frac{(B\pi r\omega)^2}{2R}$ (D) $\frac{(B\pi r\omega^2)^2}{8R}$

Q.11 A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4}T, then the e.m.f. developed between the two ends of the conductor is – **[AIEEE-2004]**
(A) 5 μV (B) 50 μV
(C) 5 mV (D) 50 mV

Q.12 Alternating current can not be measured by D.C. ammeter because – **[AIEEE-2004]**
(A) A.C. can not pass through D.C. Ammeter
(B) A.C. changes direction
(C) Average value of current for complete cycle is zero
(D) D.C. Ammeter will get damaged

Q.13 In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50 V. The voltage across the LC combination will be – **[AIEEE-2004]**

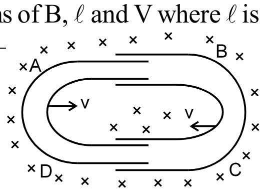
(A) 50 V (B) $50\sqrt{2}$
(C) 100 V (D) 0 V (zero)

Q.14 In a LCR circuit capacitance is changed from C to 2C. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[AIEEE-2004]**
(A) 4 L (B) 2 L
(C) L/2 (D) L/4

Q.15 A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE-2005]**
(A) 0.05 s (B) 0.1 s
(C) 0.15 s (D) 0.3 s

Q.16 The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of – **[AIEEE-2005]**
(A) 4μF (B) 8μF
(C) 1μF (D) 2μF

Q.17 One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed V, then the emf induced in the circuit in terms of B, ℓ and V where ℓ is the width of each tube, will be – **[AIEEE-2005]**



(A) BℓV (B) –BℓV
(C) zero (D) 2BℓV

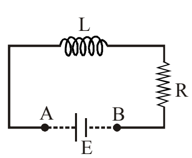
Q.18 A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be –
(A) 0.8 (B) 0.4 **[AIEEE-2005]**
(C) 1.25 (D) 0.125

Q.19 The phase difference between the alternating current and emf is π/2. Which of the following cannot be the constituent of the circuit ? **[AIEEE-2005]**
(A) C alone (B) R L
(C) L C (D) L alone

Q.20 The flux linked with a coil at any instant 't' is given by $\phi = 10t^2 - 50t + 250$. The induced emf at t = 3 s is
(A) 10 V (B) 190 V **[AIEEE 2006]**
(C) – 190 V (D) – 10 V

Q.21 In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is – **[AIEEE 2006]**
(A) N.A.B.R. (B) N.A.B.ω
(C) N.A.B.R.ω (D) N.A.B.

Q.22 An inductor (L = 100 mH), a resistor (R = 100 Ω) and a battery (E = 100 V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is – **[AIEEE 2006]**



(A) 0.1 A (B) 1 A
(C) 1/e A (D) e A

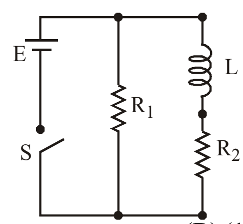
Q.23 In a series resonant LCR circuit, the voltage across R is 100 volts and R = 1 kΩ with C = 2 μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is – **[AIEEE 2006]**
(A) 250 V (B) 4×10^{-3} V
(C) 2.5×10^{-2} V (D) 40 V

Q.24 In an a.c. circuit the voltage applied is $E = E_0 \sin\omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by
(A) $P = \frac{E_0 I_0}{\sqrt{2}}$ (B) P = zero **[AIEEE 2007]**
(C) $P = \frac{E_0 I_0}{2}$ (D) $P = \sqrt{2}\, E_0 I_0$

Q.25 An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5V. 2 seconds after the connection is made, the current flowing in amperes in the circuit is - **[AIEEE 2007]**
(A) (1 – e) (B) e
(C) e^{-1} (D) $(1 - e^{-1})$

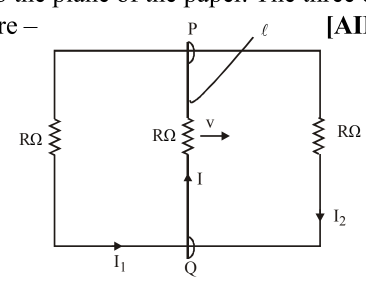
Q.26 Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is
($\mu = 4\pi \times 10^{-7}$ T m A^{-1}) **[AIEEE 2008]**
(A $4.8\,\pi \times 10^{-4}$ H (B) $4.8\,\pi \times 10^{-5}$ H
(C) $2.4\,\pi \times 10^{-4}$ H (D) $2.4\,\pi \times 10^{-5}$ H

Q.27 An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2□ and R_2 = 2□ are connected to a battery of emf 12V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is – **[AIEEE 2009]**



(A) $6\,e^{-5t}$V (B) $(12/t)\,e^{-3t}$V
(C) $6\,[1 - e^{-t/0.2})$ V (D) $12\,e^{-5t}$V

Q.28 A rectangular loop has a sliding connector PQ of length ℓ and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are – **[AIEEE 2010]**

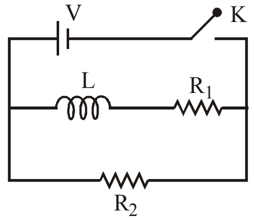


(A) $I_1 = -I_2 = \frac{B\ell v}{R}, I = \frac{2B\ell v}{R}$

(B) $I_1 = I_2 = \frac{B\ell v}{3R}, I = \frac{2B\ell v}{3R}$

(C) $I_1 = I_2 = I = \frac{B\ell v}{R}$ (D) $I_1 = I_2 = \frac{B\ell v}{6R}, I = \frac{B\ell v}{3R}$

Q.29 In the circuit shown below, the key K is closed at t = 0. The current through the battery is – **[AIEEE 2010]**



(A) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(B) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

(C) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞

(D) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

Q.30 In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is – **[AIEEE 2010]**

(A) 305 W (B) 210 W
(C) Zero (D) 242 W

Q.31 A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is – **[AIEEE 2011]**

(A) $\pi\sqrt{LC}$ (B) $\frac{\pi}{4}\sqrt{LC}$

(C) $2\pi\sqrt{LC}$ (D) $\sqrt{LC}$

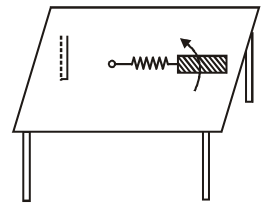
Q.32 A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is : **[AIEEE 2011]**

(A) 1 mV (B) 0.75 mV
(C) 0.50 mV (D) 0.15 mV

Q.33 A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to – **[AIEEE 2012]**

(A) development of air current when the plate is placed.
(B) induction of electrical charge on the plate
(C) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(D) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

Q.34 A metallic rod of length ℓ is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a verticalmagnetic field .B. in the region, the e.m.f. induced across the ends of the rod is – **[JEE MAIN 2013]**



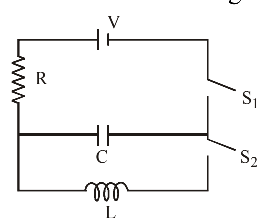
(A) $\frac{2B\omega\ell^2}{2}$ (B) $\frac{3B\omega\ell^2}{2}$

(C) $\frac{4B\omega\ell^2}{2}$ (D) $\frac{5B\omega\ell^2}{2}$

Q.35 A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0A flows through the smaller loop, then the flux linked with bigger loop is – **[JEE MAIN 2013]**

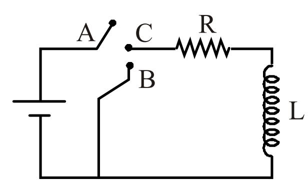
(A) 9.1×10^{-11} weber (B) 6×10^{-11} weber
(C) 3.3×10^{-11} weber (D) 6.6×10^{-9} weber

Q.36 In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct **[JEE MAIN 2013]**



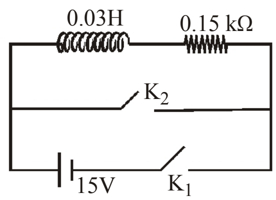
(A) Work done by the battery is half of the energy dissipated in the resistor.
(B) At t = τ, q = CV/2
(C) At t = 2τ, q = CV $(1 - e^{-2})$
(D) At t = τ/2, q = CV $(1 - e^{-1})$

Q.37 In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to **[JEE MAIN 2014]**



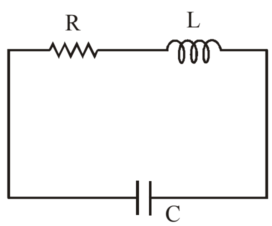
(A) –1 (B) $\frac{1-e}{e}$ (C) $\frac{e}{1-e}$ (D) 1

Q.38 An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be ($e^5 \cong 150$) **[JEE MAIN 2015]**

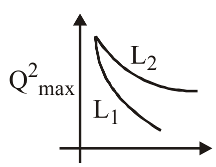


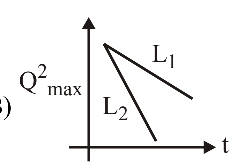
(A) 67 mA (B) 6.7 mA
(C) 0.67 mA (D) 100 mA

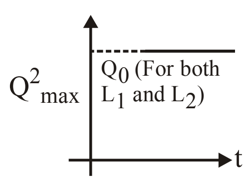
Q.39 An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown **[JEE MAIN 2015]**

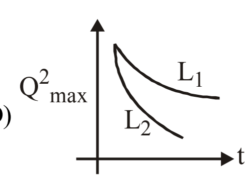


If a student plots graphs of the square of maximum charge (Q^2_{max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly?
(Plots are schematic and not drawn to scale)

(A)


(B)


(C)


(D)


Q.40 A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is **[JEE MAIN 2015]**
(A) 2.45 V/m (B) 5.48 V/m
(C) 7.75 V/m (D) 1.73 V/m

Q.41 An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to **[JEE MAIN 2016]**
(A) 0.08 H (B) 0.044 H
(C) 0.065 H (D) 80 H

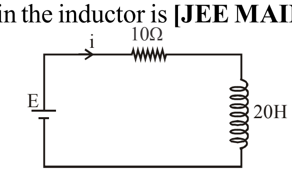
Q.42 For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = 1/\sqrt{LC}$ the current exhibits resonance. The quality factor, Q is given by : **[JEE MAIN 2018]**
(A) $\frac{R}{\omega_0 C}$ (B) $\frac{CR}{\omega_0}$
(C) $\frac{\omega_0 L}{R}$ (D) $\frac{\omega_0 R}{L}$

Q.43 In an a.c. circuit, the instantaneous e.m.f. and current are given by $e = 100\sin 30t$, $i = 20\sin\left(30t - \frac{\pi}{4}\right)$. In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively: **[JEE MAIN 2018]**
(A) $\frac{50}{\sqrt{2}}, 0$ (B) 50, 0
(C) 50, 10 (D) $\frac{1000}{\sqrt{2}}, 10$

Q.44 A conducting circular loop made of a thin wire, has area 3.5×10^{-3} m^2 and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field B(t) = (0.4T) sin (50πt). The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to: **[JEE MAIN 2019 (JAN)]**
(A) 0.14 mC (B) 21 mC
(C) 6 mC (D) 7 mC

Q.45 A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24V (50 Hz). The energy dissipated in the circuit in 60 s is : **[JEE MAIN 2019(JAN)]**
(A) 2.26×10^3 J (B) 3.39×10^3 J
(C) 5.65×10^2 J (D) 5.17×10^2 J

Q.46 A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is **[JEE MAIN 2019 (APRIL)]**



(A) $\frac{2}{\ln 2}$ (B) ln 2
(C) 2 ln 2 (D) $\frac{1}{2}\ln 2$

Q.47 A long solenoid of radius R carries a time dependent current $I = I_0\, t\,(1 - t)$. A ring of radius 2R is placed coaxially near its centre. During the time interval $0 \le t \le 1$, the induced current I_R and the induced emf V_R in the ring vary as: **[JEE MAIN 2020 (JAN)]**

(A) Current will change its direction and its emf will be zero at t = 0.25sec.
(B) Current will not change its direction & emf will be maximum at t = 0.5sec
(C) Current will not change direction and emf will be zero at 0.25sec.
(D) Current will change its direction and its emf will be zero at t = 0.5sec.

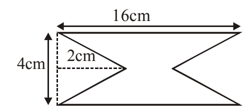
Q.48 A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at : **[JEE MAIN 2020 (JAN)]**

(A) 2.5 sec, 5 sec (B) 5 sec, 7.5 sec
(C) 2.5 sec, 7.5 sec (D) 10 sec, 5 sec

Q.49 An inductor of inductance 10 mH and a resistance of 5Ω is connected to a battery of 20 V at t = 0. Find the ratio of current in circuit at t = ∞ to current at t = 40 sec. ($e^2 = 7.389$) **[JEE MAIN 2020 (JAN)]**

(A) 1.06 (B) 1.48
(C) 1.15 (D) 0.84

Q.50 At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is : **[JEE MAIN 2020 (JAN)]**



(A) 36 μV (B) 48 μV
(C) 56 μV (D) 28 μV

Q.51 In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025ms. The self-inductance of the choke (in mH) is estimated to be _____. **[JEE MAIN 2020 (JAN)]**

Q.52 An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is : **[JEE MAIN 2019 (APRIL)]**

(A) 2.2 ms (B) 5 ms
(C) 3.3 ms (D) 7.2 ms

Q.53 A circuit connected to an ac source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of π/4 between the emf e and current i. Which of the following circuits will exhibit this ? **[JEE MAIN 2019 (APRIL)]**

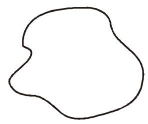
(A) RC circuit with R = 1 kΩ and C = 1μF
(B) RL circuit with R = 1kΩ and L = 1mH
(C) RL circuit with R = 1 kΩ and L = 10 mH
(D) RC circuit with R = 1kΩ and C = 10 μF

Q.54 In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10sin (314 t) is applied to the circuit, the current in the circuit is given as : **[JEE MAIN 2020 (JAN)]**

(A) 0.52 cos 314 t (B) 0.52 sin 314 t
(C) 10 cos 314 t (D) 5.2 cos 314 t

EXERCISE - 5 [PREVIOUS YEARS AIPMT / NEET QUESTIONS]

Q.1 As a result of change in the magnetic flux linked to the closed loop shown in the figure, an emf of V volt is induced in the loop. The work done (joules) in taking a charge Q coulomb once along the loop is **[AIPMT 2005]**



(A) QV (B) 2QV
(C) QV/2 (D) zero

Q.2 In a circuit L, C and R are connected in series with an alternating voltage source of frequency f. The current leads the voltage by 45°. The value of C is **[AIPMT 2005]**

(A) $\frac{1}{\pi f\,(2\pi fL - R)}$ (B) $\frac{1}{2\pi f\,(2\pi fL - R)}$

(C) $\frac{1}{\pi f\,(2\pi fL + R)}$ (D) $\frac{1}{2\pi f\,(2\pi fL + R)}$

Q.3 Two coils of self inductance 2mH and 8mH are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is – **[AIPMT 2006]**
(A) 6mH (B) 4mH
(C) 16mH (D) 10mH

Q.4 The core of a transformer is laminated because **[AIPMT 2006]**
(A) the weight of the transformer may be reduced.
(B) rusting of the core may be prevented.
(C) ratio of voltage in primary and secondary may be increased.
(D) energy losses due to eddy currents may be minimised.

Q.5 A coil of inductive reactance 31Ω has a resistance of 8Ω. It is placed in series with a condenser of capacitive reactance 25Ω. The combination is connected to an a.c. source of 110volt. The power factor of the circuit is –
(A) 0.64 (B) 0.80 **[AIPMT 2006]**
(C) 0.33 (D) 0.56

Q.6 What is the value of inductance L for which the current is maximum in a series LCR circuit with C = 10μF and $\omega = 1000 s^{-1}$. **[AIPMT 2007]**
(A) 1mH
(B) cannot be calculated unless R is known
(C) 10mH
(D) 100mH

Q.7 The primary and secondary coil of a transformer have 50 and 1500 turns respectively. If the magnetic flux ϕ linked with the primary coil is given by $\phi = \phi_0 + 4t$, where ϕ is in webers, t is time in seconds and ϕ_0 is a constant, the output voltage across the secondary coil is – **[AIPMT 2007]**
(A) 120 volts (B) 220 volts
(C) 30 volts (D) 90 volts

Q.8 A transformer is used to light a 100W and 110V lamp from a 220V mains. If the main current is 0.5 amp, the efficiency of the transformer is approximately
(A) 50% (B) 90% **[AIPMT 2007]**
(C) 10% (D) 30%

Q.9 In an a.c. circuit the e.m.f. (e) and the current (i) at any instant are given respectively by $e = E_0 \sin \omega t$; $i = I_0 \sin(\omega t - \phi)$. The average power in the circuit over one cycle of a.c. is **[AIPMT 2008]**
(A) $E_0 I_0$ (B) $E_0 I_0/2$
(C) $\frac{E_0 I_0}{2}\sin\phi$ (D) $\frac{E_0 I_0}{2}\cos\phi$

Q.10 A long solenoid has 500 turns. When a current of 2 ampere is passed through it, the resulting magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is – **[AIPMT 2008]**
(A) 4.0 henry (B) 2.5 henry
(C) 2.0 henry (D) 1.0 henry

Q.11 A circular disc of radius 0.2 meter is placed in a uniform magnetic field of induction $\frac{1}{\pi}\left(\frac{Wb}{m^2}\right)$ in such a way that its axis makes an angle of 60° with $\vec{B}$. The magnetic flux linked with the disc is **[AIPMT 2008]**
(A) 0.01 Wb (B) 0.02 Wb
(C) 0.06 Wb (D) 0.08 Wb

Q.12 A rectangular, a square, a circular and an elliptical loop, all in the (x – y) plane, are moving out of a uniform magnetic field with a constant velocity, $\vec{V} = v\hat{i}$. The magnetic field is directed along the negative z axis direction. The induced emf, during the passage of these loops, out of the field region, will not remain constant for
(A) the circular and the elliptical loops. **[AIPMT 2009]**
(B) only the elliptical loop.
(C) any of the four loops.
(D) the rectangular, circular and elliptical loops.

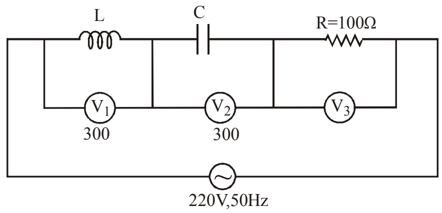
Q.13 A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s. The induced emf in the loop when the radius is 2 cm is:
(A) 4.8 π μV (B) 0.8πμV **[AIPMT 2009]**
(C) 1.6 π μV (D) 3.2 π μV

Q.14 Power dissipated in an LCR series circuit connected to an a.c source of emf ε is: **[AIPMT 2009]**

(A) $\frac{\varepsilon^2\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}{R}$ (B) $\frac{\varepsilon^2\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]}{R}$

(C) $\frac{\varepsilon^2 R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$ (D) $\frac{\varepsilon^2 R}{\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]}$

Q.15 In the given circuit the reading of voltmeter V_1 and V_2 are 300 volts each. The reading of the voltmeter V_3 and ammeter A are respectively – **[AIPMT 2010 (PRE]**



(A) 150 V, 2.2 A (B) 220 V, 2.2 A
(C) 220 V, 2.0 A (D) 100 V, 2.0 A

Q.16 A220 volt input is supplied to a transformer. The output circuit draws a current of 2.0 ampere at 440 volts. If the efficiency of the transformer is 80% the current drawn by the primary windings of the transformer is – **[AIPMT 2010 (PRE)]**

(A) 3.6 ampere (B) 2.8 ampere
(C) 2.5 ampere (D) 5.0 ampere

Q.17 A conducting circular loop is placed in a uniform magnetic field, B = 0.025 T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm s^{-1}. The induced emf when the radius is 2 cm, is **[AIPMT 2010 (PRE)]**

(A) $2\pi\mu V$ (B) $\pi\mu V$

(C) $\frac{\pi}{2}\mu V$ (D) $2\mu V$

Q.18 A condenser of capacity C is charged to a potential difference of V_1. The plates of the condenser are then connected to an ideal inductor of inductance L. The current through the inductor when the potential difference across the condenser reduces to V_2 is **[AIPMT 2010 (MAINS)]**

(A) $\left(\frac{C(V_1-V_2)^2}{L}\right)^{1/2}$ (B) $\frac{C(V_1^2-V_2^2)}{L}$

(C) $\frac{C(V_1^2+V_2^2)}{L}$ (D) $\left(\frac{C(V_1^2-V_2^2)}{L}\right)^{1/2}$

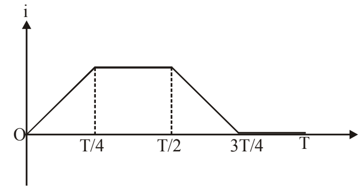
Q.19 In an ac circuit an alternating voltage $e = 200\sqrt{2}\sin 100t$ volts is connected to capacitor of capacity 1μF. The r.m.s. value of the current in the circuit is **[AIPMT 2011 (PRE)]**

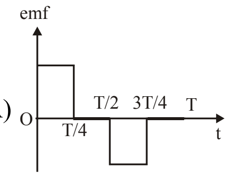
(A) 20 mA (B) 10 mA
(C) 100 mA (D) 200 mA

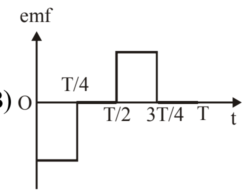
Q.20 An ac voltage is applied to a resistance R and inductor L in series. If R and the inductive reactance are both equal to 3Ω, the phase difference between the applied voltage and the current in the circuit is – **[AIPMT 2011 (PRE)]**

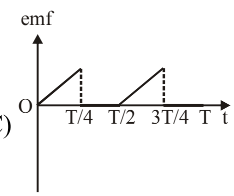
(A) Zero (B) $\pi/6$
(C) $\pi/4$ (D) $\pi/2$

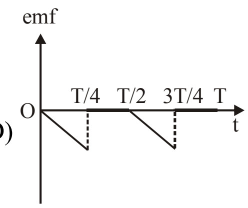
Q.21 The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would **[AIPMT 2011 (PRE)]**



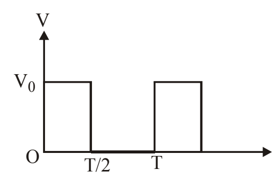
(A)


(B)


(C)


(D)


Q.22 The rms value of potential difference Vshown in the figure is– **[AIPMT 2011 (MAINS)]**



(A) V_0 (B) $V_0/\sqrt{2}$

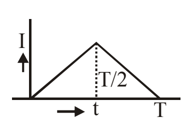
(C) $V_0/2$ (D) $V_0/\sqrt{3}$

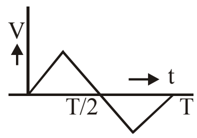
Q.23 A coil has resistance 30 ohm and inductive reactance 20ohm at 50 Hz frequency. If an ac source, of 200 volt, 100Hz, is connected across the coil, the current in the coil will be : **[AIPMT 2011 (MAINS)]**

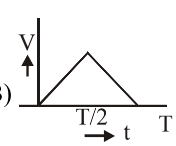
(A) 4.0 A (B) 8.0 A

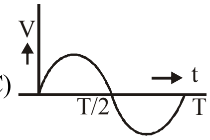
(C) $\frac{20}{\sqrt{13}}$ A (D) 2.0A

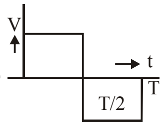
Q.24 The current (I) in the inductance is varying with time according to the plot shown in figure. Which one of the following is the correct variation of voltage with time in the coil ? **[AIPMT 2012 (PRE)]**



(A)


(B)


(C) 

(D) 

Q.25 A coil of resistance 400Ω is placed in a magnetic field. If the magnetic flux ϕ (wb) linked with the coil varies with time t (sec) as $\phi = 50t^2 + 4$. The current in the coil at t = 2 sec is : **[AIPMT 2012 (PRE)]**

(A) 0.5A (B) 0.1 A
(C) 2 A (D) 1 A

Q.26 In an electrical circuit R, L, C and an a.c. voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage the current in the circuit is $\pi/3$. If instead, C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is : **[AIPMT 2012 (PRE)]**

(A) 1/2 (B) $1/\sqrt{2}$
(C) 1 (D) $\sqrt{3}/2$

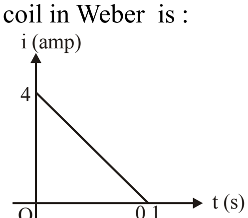
Q.27 The instantaneous values of alternating current and voltages in a circuit are given as :

$$i = \frac{1}{\sqrt{2}}\sin(100\pi t) \text{ ampere} ; \quad e = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right) \text{ volt}$$

The average power in Watts consumed in the circuit is **[AIPMT 2012 (MAINS)]**

(A) 1/4 (B) $\sqrt{3}/4$
(C) 1/2 (D) 1/8

Q.28 In a coil of resistance 10Ω, the induced current developed by changing magnetic flux through it, is shown in figure as a function of time. The magnitude of change in flux through the coil in Weber is :



[AIPMT 2012 (MAINS)]

(A) 8 (B) 2
(C) 6 (D) 4

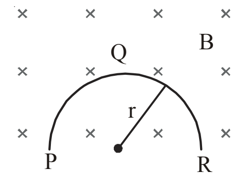
Q.29 A wire loop is rotated in magnetic field. The frequency of change of direction of the induced e.m.f. is –

(A) Six times per revolution **[NEET 2013]**
(B) Once per revolution
(C) twice per revolution
(D) four times per revolution

Q.30 A coil is self-inductance L is connected in series with a bulb B and an AC source. Brightness of the bulb decreases when **[NEET 2013]**

(A) an iron rod is inserted in the coil.
(B) frequency of the AC source is decreased.
(C) number of turns in the coil is reduced.
(D) A capacitance of reactance $X_C = X_L$ is included in the same circuit.

Q.31 A thin semicircular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in figure. The potential difference developed across the ring when its speed is v, is **[AIPMT 2014]**

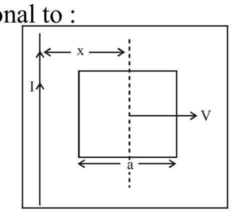


(A) Zero
(B) $Bv\pi r^2/2$ and P is at higher potential
(C) πrBv and R is at higher potential
(D) 2rBv and R is at higher potential

Q.32 A transformer having efficiency of 90% is working on 200V and 3 kW power supply. If the current in the secondary coil is 6A, the voltage across the secondary coil and the current in the primary coil respectively are – **[AIPMT 2014]**

(A) 300 V, 15 A (B) 450 V, 15 A
(C) 450 V, 13.5 A (D) 600 V, 15 A

Q.33 A conducting square frame of side 'a' and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity 'V'. The emf induced in the frame will be proportional to : **[AIPMT 2015]**



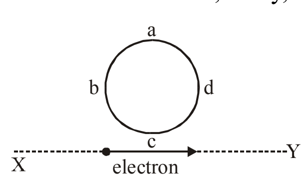
(A) $\frac{1}{(2x-a)^2}$ (B) $\frac{1}{(2x+a)^2}$

(C) $\frac{1}{(2x-a)(2x+a)}$ (D) $\frac{1}{x^2}$

Q.34 A resistance 'R' draws power 'P' when connected to an AC source. If an inductance is now placed in series with the resistance, such that the impedance of the circuit becomes 'Z', the power drawn will be: **[AIPMT 2015]**

(A) $P\sqrt{R/Z}$ (B) P (R/Z)
(C) P (D) $P(R/Z)^2$

Q.35 An electron moves on a straight line path XY as shown. The abcd is a coil adjacent to the path of electron. What will be the direction of current, if any, induced in the coil



(A) No current induced **[RE-AIPMT 2015]**
(B) abcd
(C) adcd
(D) The current will reverse its direction as the electron goes past the coil

Q.36 A series R-C circuit is connected to an alternating voltage source. Consider two situations :-
(a) When capacitor is air filled.
(b) When capacitor is mica filled.
Current through resistor is i and voltage across capacitor is V then – **[RE-AIPMT 2015]**
(A) $V_a = V_b$ (B) $V_a < V_b$
(C) $V_a > V_b$ (D) $i_a > i_b$

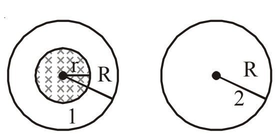
Q.37 A long solenoid has 1000 turns. When a current of 4 A flows through it, the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self inductance of the solenoid is **[NEET 2016 PHASE 1]**
(A) 4 H (B) 3 H
(C) 2 H (D) 1 H

Q.38 An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf V = 10sin 340t. The power loss in A.C. circuit is **[NEET 2016 PHASE 1]**
(A) 0.46 W (B) 0.67 W
(C) 0.76 W (D) 0.89 W

Q.39 A small signal voltage $V(t) = V_0 \sin \omega t$ is applied across an ideal capacitor C **[NEET 2016 PHASE 1]**
(A) Current I (t) lags voltage V (t) by 90°.
(B) Over a full cycle the capacitor C does not consume any energy from the voltage source.
(C) Current I (t) is in phase with voltage V (t).
(D) Current I (t) leads voltage V (t) by 180°.

Q.40 Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **[NEET 2016 PHASE 2]**
(A) R = 20 Ω, L = 1.5 H, C = 35 μF
(B) R = 25 Ω, L = 2.5 H, C = 45 μF
(C) R = 15 Ω, L = 3.5 H, C = 30 μF
(D) R = 25 Ω, L = 1.5 H, C = 45 μF

Q.41 A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{d\vec{B}}{dt}$. Loop 1 of radius R > r encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure below. Then the e.m.f. generated is **[NEET 2016 PHASE 2]**

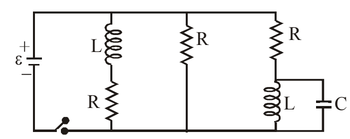


(A) Zero in loop 1 and zero in loop 2
(B) $\frac{d\vec{B}}{dt}\pi r^2$ in loop 1 and $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 2.
(C) $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 1 and zero in loop 2
(D) $-\frac{d\vec{B}}{dt}\pi r^2$ in loop 1 and zero in loop 2

Q.42 The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L-C-R circuit. The power factor of this circuit is **[NEET 2016 PHASE 2]**
(A) 0.4 (B) 0.5
(C) 0.8 (D) 1.0

Q.43 A long solenoid of diameter 0.1 m has 2×10^4 turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0A from 4 A in 0.05 s. If the resistance of the coil is $10\pi^2\Omega$. The total charge flowing through the coil during this time is – **[NEET 2017]**
(A) 16 μC (B) 32 μC
(C) 16 π μC (D) 32 π μC

Q.44 Figure shows a circuit that contains three identical resistors with resistance R = 9.0 Ω each, two identical inductors with inductance L = 2.0 mH each, and an ideal battery with emf e = 18 V. The current 'i' through the battery just after the switch closed is – **[NEET 2017]**



(A) 0.2 A (B) 4 A
(C) 0 ampere (D) 2 mA

Q.45 The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60mA. This inductor is of inductance **[NEET 2018]**
(A) 1.389 H (B) 138.88 H
(C) 0.138 H (D) 13.89 H

Q.46 An inductor 20 mH, a capacitor 100 μF and a resistor 50Ω are connected in series across a source of emf, V = 10 sin 314 t. The power loss in the circuit is **[NEET 2018]**
(A) 2.74 W (B) 0.43 W
(C) 0.79 W (D) 1.13 W

Q.47 In which of the following devices, the eddy current effect is not used? **[NEET 2019]**
(A) Induction furnace
(B) Magnetic braking in train
(C) Electromagnet
(D) Electric heater

Q.48 A 800 turn coil of effective area 0.05 m^2 is kept perpendicular to a magnetic field 5×10^{-5} T. When the plane of the coil is rotated by 90° around any of its coplanar axis in 0.1 s, the emf induced in the coil will be: **[NEET 2019]**
(A) 2 V (B) 0.2 V
(C) 2×10^{-3} V (D) 0.02 V

ANSWER KEY

EXERCISE - 1

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	C	A	D	A	D	B	A	A	C	B	C	B	C	A	A	C	C	C	A	B	B	C	B	B	B
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	D	B	B	B	B	B	C	D	C	C	D	C	A	C	C	B	C	C	B	D	C	A	B	C	D
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
A	B	C	C	A	C	A	A	D	B	C	B	B	B	B	A	A	B	C	D	B	B	C	C	B	D
Q	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
A	C	A	B	C	A	D	C	A	C	C	A	D	C	D	D	B	C	D	C	D	B	C	A	B	C
Q	**101**	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**											
A	C	D	B	B	B	D	C	A	B	A	C	A	C	D											

EXERCISE - 2

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	B	D	C	D	A	C	C	D	D	B	A	B	A	D	B	B	D	D	D	B	C	D	C	A	A
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	B	D	C	B	B	A	D	D	D	C	C	C	C	D	B	A	D	A	C	C	A	C	C	B	A
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**															
A	D	D	B	A	A	C	D	B	B	A															

EXERCISE - 3 (PART-A)

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**
A	30	100	2	1	5	5	6

EXERCISE - 3 (PART-B)

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**
A	2	4	3	4	2	3	1

EXERCISE - 4

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	A	C	B	B	A	C	B	D	B	B	B	C	D	C	B	C	D	A	B	D	B	C	A	B	D
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	D	B	B	D	B	D	D	D	A	C	D	C	B	A	C	C	D	A	D	C	D	A	C	C
Q	**51**	**52**	**53**	**54**																					
A	10	C	D	A																					

EXERCISE - 5

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	A	D	B	D	B	D	A	B	D	D	B	C	D	D	B	D	B	D	A	C	B	B	A	D	A
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**		
A	C	D	B	C	A	D	B	C	D	D	C	D	A	B	C	D	C	B	B	D	C	D	D		

ELECTROMAGNETIC INDUCTION

TRY IT YOURSELF-1

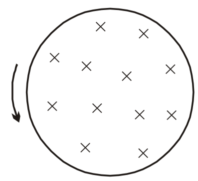
(1) (D)

(2) (B)

(3) (B)

(4) **(B).** If B is increasing at a constant rate $\phi = B.\pi R^2$

$$V_{ind} = \left|\frac{d\phi}{dt}\right|$$



$$= \pi R^2 \cdot \frac{dB}{dt} = E \cdot 2\pi R$$

$$\Rightarrow E = \frac{R}{2}\frac{dB}{dt} \text{. Acceleration of ring} = \frac{dE}{m} \; ; \; a \propto q$$

(5) (D)

(6) (B)

(7) **(C).** Magnetic field lines in right direction are decreasing, so direction of I will be clockwise. The part of the loop facing the north pole of magnet behaves as a south pole, so net force will be in right direction.

(9) **(A).** $4L = 2\pi r \; ; \; \frac{2L}{\pi} = r$

$$A = L^2 \; ; \; A = \pi\left(\frac{2L}{\pi}\right)^2 = \pi\frac{4L^2}{\sqrt{2}} \; ; \; \phi = B\frac{dA}{dt}$$

Clockwise lenz law

(10) **(D).** Perpendicular length is more so induced emf is more.

(11) **(AD).**

TRY IT YOURSELF-2

(1) **(B).** $I = I_0 e^{-t/\tau}, \quad \tau = L/R$

(2) (D)

(3) **(AD).** $Pd = -\frac{Ldi}{dt} \; ; \; \frac{di}{dt} = -ve$

(4) (A)

(5) (A)

(6) (AD)

(7) Suppose a current I_2 flows through the outer cicular coil. Magnetic field at the centre of the coil is

$$B_2 = \frac{\mu_0 I_2}{2R}$$

Field B_2 may be considered constant over the cross-sectional area of the inner smaller coil. Hence

$$\phi_1 = \pi R^2 B_2 = \frac{\mu_0 \pi r^2 I_2}{2R} = MI_2 \quad \therefore$$

$$M = \frac{\phi_1}{I_2} = \frac{\mu_0 \pi r^2}{2R}$$

(8) (i) Mutual inductance increased on decreasing distance.

(ii) Mutual inductance decreased on decreasing the number of turns.

(9) **(D).** The current-time (i-t) equation in L-R circuit is given by [Growth of current in L-R circuit]

$$i = i_0(1 - e^{-t/\tau L}) \; .... (1)$$

$$\text{where } i_0 = \frac{V}{R} = \frac{12}{6} = 2A$$

$$\text{and } \tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3} s$$

and $i = 1A$ (given), $t = ?$

Substituting theses values in eq. (1), we get

$t = 0.97 \times 10^{-3}\, s = 0.97$ ms

$t \approx 1$ ms

(10) **(A).** When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

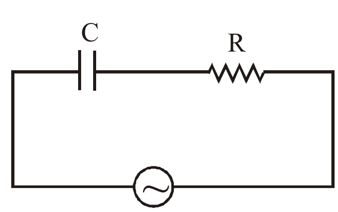
(11) (BD)

ALTERNATING CURRENT

TRY IT YOURSELF

(1) (D)

(2) **4.**



$$Z = \sqrt{\left(\frac{1}{\omega L}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega L}\right)^2 + R^2 = R^2\ (1.25)$$

$$\left(\frac{1}{\omega L}\right)^2 + R^2 = R^2 + \frac{R^2}{4} \Rightarrow \frac{1}{\omega L} = \frac{R}{2}$$

$$CR = \frac{2}{\omega} = \frac{2}{500}\sec = \frac{2}{500} \times 10^3 \sec$$

$$= \frac{2 \times 1000}{500}\text{ms} = 4\text{ ms}$$

(3) **(BC).** **(4)** (B)

(5) **(B).** $E = 10\cos(2\pi \times 50 \times \frac{1}{600}) = 5\sqrt{3}\text{ V}$

(6) **(AC).** At resonance $X_L = X_C$ and $Z = Z_{min} = R$

$$X_L = \omega L \text{ and } \frac{1}{\omega C} = X_C$$

If 'f' is decreased then 'ω' will decrease and hence X_C will increase therefore at $f < f_r$, circuit behaves as capacitive. V_L and V_C always difference in phase by 180° at any frequency.

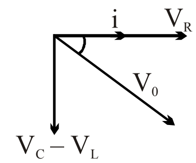
(7) **(B).** $Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{25+25} = 5\sqrt{2}$

$$I_{rms} = \frac{10/\sqrt{2}}{5\sqrt{2}} = 1$$

$$V = 1 \times (X_L - X_C) = 1 \times 5 = 5\text{ Volt}$$

(8) (B)

(9) **(AC).** Current leads the voltage



$$V_0 = \sqrt{(V_L - V_C)^2 + V_R^2}$$

$$= \sqrt{100+25} = 5\sqrt{5}\text{ V}$$

$$\text{P.F.} = \frac{V_R}{V_0} = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$$

(10) (C)

CHAPTER- 4 : ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

EXERCISE-1

(1) **(C).** Here $\vec{B} = B_0(2\hat{i}+3\hat{j}+4\hat{k})T$,

Area of the square = $L^2\hat{k}m^2$

$\therefore$ Flux passing through the square.

$\phi = \vec{B}.\vec{A} = B_0(2\hat{i}+3\hat{j}+4\hat{k}).L^2\hat{k} = 4B_0L^2Wb$

(2) **(A).** $\phi = BA = 10$ weber

(3) **(D).** Circular loop behaves as a magnetic dipole whose one surface will be N-pole and another will be S-pole.
Therefore magnetic lines a force emerges from N will meet at S. Hence total magnetic flux through x-y plane is zero.

(4) **(A).** $\phi = \vec{B}.\vec{A} = (0.02\hat{i}).(30\hat{i}+16\hat{j}+23\hat{k}) \times 10^{-4}$

$= 0.6 \times 10^{-4}\,Wb = 60\mu\,Wb$

(5) **(D).** Magnetic flux lined with a coil $\phi = NBA\cos\theta$
Since the magnetic field B is parallel to the area A, i.e., $\theta = 90^o$ $\quad \therefore \phi = 0$

(6) **(B).** An emf is induced only when magnetic flux linked with the loop changes. This is possible when the loop is rotated about a diameter.

(7) **(A).** Faraday's laws involve conversion of mechanical energy into electric energy. This is in accordance with the law of conservation of energy.

(8) **(A).** $e = -\dfrac{N(B_2-B_1)A\cos\theta}{\Delta t}$

$\Rightarrow 0.1 = \dfrac{-50\times(0-2\times10^{-2})\times100\times10^{-4}\times\cos0^o}{t}$

$t = 0.1$ s.

(9) **(C).** At low frequency of 1 to 2 Hz, oscillations may be observed as our eyes will be able to detect it.

(10) **(B).** $e = -\dfrac{N(B_2-B_1)A\cos\theta}{\Delta t}$

$= -\dfrac{50(0.35-0.10)\times\pi(3\times10^{-2})^2\times\cos0^o}{2\times10^{-3}} = 17.7V$

(11) **(C).** $i = \dfrac{|e|}{R} = \dfrac{N}{R}.\dfrac{\Delta B}{\Delta t}A\cos\theta = \dfrac{20}{100}\times1000\times(25\times10^{-4})\cos0^o$

$i = 0.5\,A$

(12) **(B).** $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(5t^3-100t+300)$

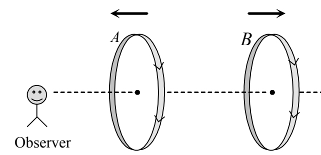
$= -(15t^2-100)$ at $t = 2$sec ; $e = 40$ V

(13) **(C).** $E = -\dfrac{d\phi}{dt}$ or $d\phi = -Edt = (0-\phi)$

or $\phi = 4\times10^{-3}\times0.1 = 4\times10^{-4}$ weber

(14) **(A).** As the loops are brought closer, the magnetic flux linked with them increases. An emf is induced in each loop which opposes the change in flux. So the current in each loop decreases.

(15) **(A).** Induced current in both the coils assist the main current so current through each coil increases.



(16) **(C).** The induced current will be in such a direction so that it opposes the change due to which it is produced.

(17) **(C).** Since the magnetic field is uniform therefore there will be no change in flux hence no current will be induced.

(18) **(C).** When key k is pressed, current through the electromagnet start increasing i.e. flux linked with ring increases which produces repulsion effect.

(19) **(A).** The direction of current in the solenoid is clockwise. On displacing it towards the loop a current in the loop will be induced in clockwise direction so as to oppose its approach. Therefore the direction of induced current as observed by the observer will be anticlockwise.

(20) **(B).** When north pole of the magnet is moved away, then south pole is induced on the face of the loop in front of the magnet i.e. as seen from the magnet side, a clockwise induced current flows in the loop. This makes free electrons to move in opposite direction, to plate a. Thus excess positive charge appear on plate b.

(21) **(B).** Direction of current induced in a wire moving in a magnetic field is found by using Fleming's right hand rule.

(22) **(C).** When the velocity of conductor becomes double, area intercepted becomes twice. Therefore induced current becomes twice.

(23) **(B).** Magnitude of induced emf between the axis and the other end is also $\left(\dfrac{Bl^2\omega}{8}\right)$. These two emf's are in opposite directions. Hence, the potential difference between the two ends of the rod is zero.

(24) **(B).** $e = Bv\ell = 3\times10^{-3}\times10^2 = 0.3$ volt

(25) **(B).** $e = B_v.v.\ell = 2\times10^{-4}\times\left(\dfrac{360\times1000}{3600}\right)\times50 \Rightarrow e = 1V$

(26) **(D).** $e = B\ell^2\pi\nu = 0.4\times10^{-4}\times(0.5)^2\times(3.14)\times\dfrac{120}{60}$

$= 6.28\times10^{-5}$ V

(27) **(B).** $e_0 = \omega NBA = (2\pi\nu)NB(\pi r^2) = 2\times\pi^2\nu\,NBr^2$

$= 2\times(3.14)^2\times\dfrac{1800}{60}\times4000\times0.5\times10^{-4}\times(7\times10^{-2})^2$

$= 0.58V$

(28) **(B).** When a conductor lying along the magnetic north-south, moves eastwards it will cut vertical component of B_0.

So induced emf

$e = vB_V\ell = v(B_0 \sin\delta\,\ell) = B_0\ell v \sin\delta$

(29) **(B).** Rate of work $= \frac{W}{t} = P = Fv$; $F = Bi\ell = B\left(\frac{Bv\ell}{R}\right)\ell$

$\Rightarrow P = \frac{B^2v^2\ell^2}{R} = \frac{(0.5)^2 \times (2)^2 \times (1)^2}{6} = \frac{1}{6}W$

(30) **(B).** $\phi = BA\cos\omega t$; $\phi = BA\cos 2\pi ft$

$\phi = \frac{B\pi r^2}{2}\cos 2\pi ft$; $e = -\frac{d\phi}{dt} = \frac{B\pi r^2}{2}.2\pi f \sin 2\pi ft$

$e = B\pi^2 r^2 f \sin 2\pi ft$

Peak value $= B\pi^2 r^2 f$

(31) **(B).** Effective length between A and B remains same.

(32) **(C).** Induction furnace make use of eddy current

(33) **(D).** When a metal plate is getting heated, it may be due to the passage of direct current, alternating current or even induced current through the plate. As time varying magnetic field produces induced current in the plate, so both (A) and (B) are correct.

(34) **(C).** As, $\varepsilon = L\frac{dI}{dt}$. When $\frac{dI}{dt} = 1$ $\therefore \varepsilon = L$

(35) **(C).** The mutual inductance of the system is $M = \mu_0\pi a^2 n_1 n_2 L$.

(36) **(D).** As $\varepsilon = L\frac{dI}{dt}$, $L = \varepsilon\frac{dt}{dI} \Rightarrow L = \frac{\text{volt} \times \text{second}}{\text{ampere}}$

(37) **(C).** The self inductance of a long solenoid is given by

$L = \frac{\mu_0 N^2 A}{\ell}$

where, N = Number of turns, ℓ = Length

A = Cross-sectional area

From the above expression it is clear that the self inductance of a long solenoid does not depend upon the current flowing through it.

(38) **(A).** $M = \frac{\mu_0 N_1 N_2 A}{\ell}$ $\therefore$ M becomes 4 times.

(39) **(C).** Self inductance $= \frac{\mu_0 N^2 A}{\ell}$

(40) **(C).** Inductors obey the laws of parallel and series combination of resistors.

(41) **(B).** There will be self induction effect when soft iron core is inserted.

(42) **(C).** $\phi = Li$; $NBA = Li$

$\mu_0\frac{N^2 i}{\ell}A = Li$; $L = \frac{\mu_0 N^2 A}{\ell}$

(43) **(C).** $L = \mu_0\frac{N^2}{\ell}A$.

When N and ℓ are doubled. L is also doubled.

(44) **(B).** $e = -M\frac{di}{dt} = -5 \times \frac{(-5)}{10^{-3}} = 25000$ V

(45) **(D).** In secondary e.m.f. induces only when current through primary changes.

(46) **(C).** $i_{rms} = \sqrt{\frac{i_1^2 + i_2^2}{2}} = \frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$

(47) **(A).** $V_{av} = \frac{2}{\pi}V_0 = \frac{2}{\pi} \times (V_{rms} \times \sqrt{2})$

$= \frac{2\sqrt{2}}{\pi}.V_{rms} = \frac{2\sqrt{2}}{\pi} \times 220 = 198$ V

(48) **(B).** $V_0 = \sqrt{2}\,V_{rms} = 10\sqrt{2}$

(49) **(C).** $E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{707}{1.41} = 500V$

(50) **(D).** Required time $t = T/4 = \frac{1}{4 \times 50} = 5 \times 10^{-3}$ sec

(51) **(B).** Here, $V_m = 400$ V

$\therefore R_{rms} = \frac{V_m}{\sqrt{2}} = \frac{440}{\sqrt{2}} = 311.1V$

(52) **(C).** The voltmeter connected to ac mains is calibrated to read root mean square value or virtual value of ac voltage.

(53) **(C).** $V_{rms} = \frac{220}{\sqrt{2}}V$; $V_0 = \sqrt{2}V_{rms} = \sqrt{2}\frac{200}{\sqrt{2}} = 200V$

$V = V_0 \sin 2\pi\nu t = 200 \sin\left(2\pi \times 50 \times \frac{1}{600}\right)$

$= 200 \sin\frac{\pi}{6} = 200 \times \frac{1}{2} = 100$ V.

(54) **(A).** $i_{rms} = \frac{200}{280} = \frac{5}{7}$A. So $i_0 = i_{rms} \times \sqrt{2} = \frac{5}{7} \times \sqrt{2} \approx 1A$.

(54) **(C).** Here, $R = 100\,\Omega$,

$V_{rms} = 220$ V, Ω, $\nu = 50$ Hz

$\therefore I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2A$

(56) **(A).** In a pure inductor (zero resistance), voltage leads the current by 90° i.e. $\pi/2$

(57) **(A).** The effective potential difference across the inductor is given by $V_{eff} = I_{eff}.X_L = \frac{I_0}{\sqrt{2}}.2\pi f L$

$V_{eff} = V_{rms}$

Given that $I_0 = 0.25$ amp, f = 60 Hz, L = 2H

$\therefore V_{eff} = \frac{0.25}{\sqrt{2}} \times 2 \times 3.14 \times 60 \times 2 = 133.2$ Volt

(58) **(D).** The inductive reactance is directly proportional to inductance and frequency of the current.

(59) **(B).** The current in the inductor coil is given by

$$I = \frac{V}{X_L} = \frac{V}{2\pi\nu L}$$

Since frequency ν in the two cases is different, hence the current in two cases will be different.

(60) **(C).** Here, $L = 44\text{ mH} = 44 \times 10^{-3}$ H
$V_{rms} = 220$ V, $\nu = 50$ Hz
The inductive reactance is $X_L = \omega L = 2\pi\nu L$
$= 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.82\ \Omega$

$$\therefore I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{13.82} = 15.9\text{A}$$

(61) **(B).** For purely capacitive circuit, $e = e_0 \sin \omega t$

$i = i_o \sin\left(\omega t + \frac{\pi}{2}\right)$ i.e. current is ahead of emf by π/2

(62) **(B).** $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

For dc $\nu = 0, \therefore X_C = \infty$

(63) **(B).** $X_C = \frac{1}{2\pi\nu C} \Rightarrow X_C \propto \frac{1}{\nu}$

(64) **(B).** Reading of ammeter $= i_{rms} = \frac{V_{rms}}{X_C} = \frac{V_0\omega C}{\sqrt{2}}$

$$= \frac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}} = 2 \times 10^{-2}\text{A} = 20\text{ mA}$$

(65) **(A).** $X_C = \frac{1}{2\pi\nu C}$

$$\frac{1}{1000} = \frac{1}{2\pi \times \nu \times 5 \times 10^{-6}} \Rightarrow \nu = \frac{100}{\pi}\text{MHz}$$

(66) **(A).** $X_C = \frac{1}{2\pi\nu C} \Rightarrow C = \frac{1}{2\pi\nu X_C} = \frac{1}{2 \times \pi \times \frac{400}{\pi} \times 25} = 50\mu\text{F}$

(67) **(B).** Here, $C = 30\mu\text{F} = 30 \times 10^{-6}$ F
$V_{rms} = 150$ V, $\nu = 60$ Hz

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = 88.46\ \Omega$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{150}{88.46} = 1.7\text{A}$$

(68) **(C).** Capacitive reactance, $X_C = \frac{1}{2\pi\nu C}$

$$\Rightarrow X_C \propto \frac{1}{\nu} \quad \therefore \frac{X_{C_1}}{X_{C_2}} = \frac{\nu_2}{\nu_1} = \frac{2\nu}{\nu} = 2$$

$$\Rightarrow X_{C_2} = \frac{X_{C_1}}{2}$$

(69) **(D).** When an ac voltage of 220V is applied to a capacitor C, the charge on the plates is in phase with the applied voltage. As the circuit is pure capacitive so, the current developed leads the applied voltage by a phase angle of 90º. Hence, power delivered to the capacitor per cycle is $P = V_{rms} I_{rms} \cos 90° = 0$.

(70) **(B).** In a capacitive ac circuits, the voltage lags behind the current in phase by π/2 radian.

(71) **(B).** $Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + (2\pi \times 60 \times 2)^2} = 753.7$

$$\therefore i = \frac{120}{753.7} = 0.159\text{ A}$$

(72) **(C).** Impedance $Z = \sqrt{R^2 + X^2} = \sqrt{(8)^2 + (6)^2} = 10\Omega$

(73) **(C).** $\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.21}{12} = 5.5 \Rightarrow \phi = 80^o$

(74) **(B).** The applied voltage is given by

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(200)^2 + (150)^2} = 250\text{ volt}$$

(75) **(D).** $i = \frac{220}{\sqrt{(20)^2 + (2 \times \pi \times 50 \times 0.2)^2}} = \frac{220}{66} = 3.33\text{ A}$

(76) **(C).** $Z = \sqrt{R^2 + (2\pi\nu L)^2}$

$$= \sqrt{(40)^2 + 4\pi^2 \times (50)^2 \times (95.5 \times 10^{-3})^2} = 50\text{ ohm}$$

(77) **(A).** $\tan\phi = \left(\frac{X_L}{R}\right)$

$X_L = \omega L = (2\pi\nu L) = (2\pi)(50)(0.01) = \pi\Omega$
Also, $R = 1\Omega$
$\therefore \phi = \tan^{-1}(\pi)$

(78) **(B).** $V_{source} = \sqrt{V_R^2 + V_C^2}$

$$\therefore V_C = \sqrt{V_{source}^2 - V_R^2} = \sqrt{(20)^2 - (12)^2} = 16\text{V}$$

(79) **(C).** Reactance of a capacitor, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

As frequency increases, X_C decreases and therefore current increases. As R does not vary with frequency, therefore, likely elements constituting the circuit may be capacitor and resistor.

(80) **(A).** Here, $R = 20\Omega$, $C = 0.1\mu\text{F} = 0.1 \times 10^{-6}\text{ F} = 10^{-7}$ F

Impedance, $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$$= \sqrt{20^2 + \frac{1}{(100)^2 \times (10^{-7})^2}} = \sqrt{400 + 10^{10}} = 10^5\Omega$$

(81) **(D).** Here, $R = 0.2\text{ k}\Omega = 200\Omega$
$C = 15\mu\text{F} = 15 \times 10^{-6}$ F
$V_{rms} = 220$V, $\nu = 50$ Hz
Capacitive reactance,

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}} = 212\ \Omega$$

The impedance of the RC circuit is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + (212)^2} = 291.5\,\Omega$$

(82) **(C).** LCR circuit connected to an AC source V. The voltage of the source to be V = Vm sin ωt. If q is the charge on the capacitor and i the current at time t, from Kirchhoff's loop rule, $L\frac{di}{dt} + iR + \frac{q}{C} = V$.

(83) **(A).** The amplitude of the current is given by

$$i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

(84) **(C).** $V_L = 46$ volts, $V_C = 40$ volts, $V_R = 8$ volts

E.M.F. of source $V = \sqrt{8^2 + (46-40)^2} = 10$ volts

(85) **(C).** For series R-L-C circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(300)^2 + \left(1000 \times 0.9 - \frac{10^6}{1000 \times 2}\right)^2} = 500\Omega$$

(86) **(A).** $V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(40)^2 + (60-30)^2}$

$= 50V$

(87) **(D).** Since $\cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also cos θ can never be greater than 1). Hence (C) is wrong. Also, $Ix_C > Ix_L \Rightarrow x_C > x_L$

∴ Current will be leading

In a LCR circuit,

$$v = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6-12)^2 + 8^2}$$

v = 10, which is less than voltage drop across capacitor.

(88) **(C).** $\cos\phi = \frac{R}{Z} = \frac{10}{20} = \frac{1}{2} \Rightarrow \phi = 60°$

(89) **(D).** Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L = X_C \quad \therefore Z = R$

$$\therefore I_m = \frac{V_m}{Z} = \frac{V_m}{R}$$

(90) **(D).** Let ϕ be the phase difference between the applied voltage and current. Then

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{I_V(X_L - X_C)}{I_V R}$$

$$= \frac{V_L - V_C}{V_R} = \frac{20V - 20V}{40V} = 0$$

∴ $\phi = \tan^{-1}(0) = 0°$

(91) **(B).** Here, $\phi = 45°$

In series LCR circuit = cos ϕ

Power factor cos ϕ

$$\therefore \cos\phi = \cos 45° = \frac{1}{\sqrt{2}} = 0.707.$$

(92) **(C).** Here, $V_{rms} = 200$ V

$X_L = 50\,\Omega$, $X_C = 50\,\Omega$, $R = 25\Omega$

Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{25^2 + (50-50)^2} = 25\Omega.$$

Current in the circuit, $I_{rms} = \frac{V_{rms}}{Z} = \frac{200V}{25\Omega} = 8A$

Voltage drop across the inductor is

$V_L = I_{rms} X_L = 8A \times 50\Omega = 400V$

(93) **(D).** $P = Vi\cos\phi$

Phase difference $\phi = \frac{\pi}{2} \Rightarrow P = \text{zero}$

(94) **(C).** $V_{rms} = \frac{200}{\sqrt{2}}$, $i_{rms} = \frac{1}{\sqrt{2}}$

$$P = V_{rms}\, i_{rms} \cos\phi = \frac{200}{\sqrt{2}} \frac{1}{\sqrt{2}} \cos\frac{\pi}{3} = 50 \text{ watt}$$

(95) **(D).** $P = V_{rms} I_{rms} \cos\phi$; since $\phi = 90^o$. So $P = 0$

(96) **(B).** If the frequency of the ac source equals the natural frequency of the circuit, the impedance $Z = R = 20\Omega$

The average power dissipated per cycle,

$$P_{av} = \frac{V_{rms}^2}{Z} = \frac{V_{rms}^2}{R} = \frac{(200)^2}{20} = 2000 \text{ W}$$

(97) **(C).** Here, $X_L = 1\Omega$, $R = 2\Omega$, $V_{rms} = 6V$

Impedance of the circuit

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{6}{\sqrt{5}} A$$

Power dissipated $P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \frac{R}{Z}$

$$= 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4W$$

(98) **(A).** Wattless current is : $I_{rms} \sin\phi$

(99) **(B).** Resonance frequency

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8 \times 10^{-3} \times 20 \times 10^{-6}}} = 2500 \text{ rad/s}$$

Resonance current $= \frac{V}{R} = \frac{220}{44} = 5A$

(100) **(C).** $V^2 = V_R^2 + (V_L - V_C)^2$

Since $V_L = V_C$ hence $V = V_R = 200V$

(101) **(C).** At resonance $X_L = X_C$

(102) (D). Since quality factor, $Q=\frac{1}{R}\sqrt{\frac{L}{C}}$

(103) (B). Q-factor $=\frac{\omega_r L}{R}$

(104) (B). At resonance frequency, the inductive and capacitive reactance are equal, i.e. $X_L = X_C$

∴ Impedance,

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+0^2}=R$$

(105) (B). Resonant frequency in a series LCR circuit is

$$\nu_r=\frac{1}{2\pi\sqrt{LC}}$$

If capacitance C increases the resonant frequency will reduce, which can be achieved by adding another capacitor in parallel to the first.

(106) (D). Both are dimensionless quantities. Angle is a dimensionless quantity.

(107) (C). $\nu_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\times3.14\sqrt{5\times10^{-4}\times20\times10^{-6}}}$

$$\nu_0=\frac{10^4}{6.28}=1592\text{ Hz}$$

(108) (A). Here, $C=30\mu F=30\times10^{-6}F$

$L=27\text{ mH}=27\times10^{-3}\text{ H}$

$$\therefore\ \omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}}$$

$$=\frac{1}{\sqrt{81\times10^{-8}}}=\frac{10^4}{9}=1.1\times10^3\text{ rad s}^{-1}$$

(109) (B). Here, $C=25\mu F=25\times10^{-6}$ F,

$L=20\text{ mH}=20\times10^{-3}$ H, $q_0=5\text{ mC}=5\times10^{-3}$ C

∴ Total energy stored in the circuit initially is

$$U=\frac{q_0^2}{2C}=\frac{(5\times10^{-3})^2}{2\times25\times10^{-6}}=\frac{25\times10^{-6}}{2\times25\times10^{-6}}=\frac{1}{2}=0.5\text{ J}$$

(110) (A). $\frac{V_p}{V_s}=\frac{N_p}{N_s}\Rightarrow N_p=\left(\frac{220}{2200}\right)2000=200$

(111) (C). Provided no power looses, being assumed.

(112) (A). $\frac{N_s}{N_p}=\frac{V_s}{V_p}\Rightarrow\frac{200}{100}=\frac{V_s}{120}\Rightarrow V_s=240\text{ V}$

Also $\frac{V_s}{V_p}=\frac{i_p}{i_s}\Rightarrow\frac{240}{120}=\frac{10}{i_s}\Rightarrow i_s=5\text{ A}$

(113) (C). $\frac{N_s}{N_p}=\frac{V_s}{V_p}\Rightarrow\frac{N_s}{600}=\frac{2200}{220}\Rightarrow N_s=6000$

(114) (D). The core of a transformer is laminated to reduce eddy current.

EXERCISE-2

(1) (B). Initial flux through the coil

$\phi_i=BA\cos\theta=3\times10^{-5}\times\pi\times(8\times10^{-2})^2\times\cos0^\circ$

$=192\,\pi\times10^{-9}$ Wb

Final flux after the rotation

$\phi_f=3\times10^{-5}\times\pi\times(8\times10^{-2})^2\times\cos180^\circ$

$=-192\pi\times10^{-9}$ Wb

∴ The magnitude of induced emf is

$$\varepsilon=N\frac{|d\phi|}{dt}=\frac{N|\phi_f-\phi_i|}{dt}=\frac{400\times(384\pi\times10^{-9})}{0.30}$$

$=1.6\times10^{-3}$ V

Current, $I=\frac{\varepsilon}{R}=\frac{1.6\times10^{-3}}{2}=8\times10^{-4}$ A

(2) (D). Here, $\frac{dI}{dt}=\frac{0-5}{0.2}=-25\text{ As}^{-1}$

$\varepsilon=150$ V

As $|\varepsilon|=L\left|\frac{dI}{dt}\right|$ $\quad\therefore L=\frac{|\varepsilon|}{|dI/dt|}=\frac{150}{25}=6H$

(3) (C). The emf developed between the ends of the conductor is $\varepsilon=\frac{1}{2}\omega B\ell^2=\frac{1}{2}\times5\times0.2\times10^{-4}\times(1)^2$

$=5\times10^{-5}\text{ V}=50\times10^{-6}\text{ V}=50\ \mu V$

(4) (D). As the line has some resistance ($R\neq0$), voltage and current differ in phase ϕ such that $|\phi|<\pi/2$.

(5) (A). Here, $N_1=1000$, $l=100$ cm $=1$ m

$A=\pi r^2=\pi\times(2\times10^{-2})^2\text{ m}^2$; $N_2=100$;

$$M=\mu_0\frac{N_1N_2A}{\ell}$$

$$=\frac{4\pi\times10^{-7}\times1000\times100\times\pi\times4\times10^{-4}}{1}$$

$=16\pi^2\times10^{-6}=16\times9.87\times10^{-6}=1.58\times10^{-4}$ V

(6) (C). Given, $\phi=(2t^2+4t+6)$ m Wb

$$\varepsilon=\frac{d\phi}{dt}=\frac{d}{dt}(2t^2+4t+6)\times10^{-3}\text{ Wb s}^{-1}$$

$=(4t+4)\times10^{-3}$ V

At t = 4s

$\varepsilon=(4\times4+4)\times10^{-3}\text{ V}=20\times10^{-3}\text{ V}=0.02$ V

(7) (C). Total magnetic flux,

$N\phi=LI=20\times10^{-3}\times4\times10^{-3}=8\times10^{-5}$ Wb

Magnetic flux through the cross-section of the coil,

$$\phi=\frac{8\times10^{-5}}{N}=\frac{8\times10^{-5}}{100}=8\times10^{-7}\text{ Wb}$$

(8) (D). Self inductance of a solenoid $=\mu_0 n^2Al$

where n is the number of turns per length

∴ Self inductance is directly proportional to n^2. Self inductance becomes 4 times when n is doubled.

(9) (D). When a circular coil expands radially in a region of magnetic field, induced emf developed is

$\varepsilon=Blv=B\times$ rate of change of area

Here, magnetic field B is in a plane perpendicular to the plane of the circular coil. As $\varepsilon = 0$, magnetic field must be in the plane of circular coil so that its component perpendicular to the plane of the coil, whose magnitude is decreasing suitably so that magnetic flux linked with the coil stays constant then

$\varepsilon = \frac{d\phi}{dt} = 0$. So both options (B) and (C) are correct.

(10) **(B).** $|\varepsilon| = L\frac{dI}{dt}$

$$L = \frac{|\varepsilon|}{dI/dt} = \frac{IR}{dI/dt} = \frac{IP/I^2}{dI/dt} = \frac{P}{I\,(dI/dt)}$$

As P and (dI/dt) are same for both the coils

$$\therefore \frac{I_1}{I_2} = \frac{L_2}{L_1} = \frac{1}{4}$$

(11) **(A).** $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^2 + 3t + 16) = (10t + 3)$

when $t = 3\,\text{sec},\ e_3 = (10 \times 3 + 3) = 33\ V$

when $t = 4\,\text{sec},\ e_4 = (10 \times 4 + 3) = 43\ V$

Hence emf induced in fourth second

$$= e_4 - e_3 = 43 - 33 = 10\ V$$

(12) **(B).** $\Delta Q = \frac{\Delta\phi}{R} = \frac{n \times BA}{R}$

$$\Rightarrow \quad B = \frac{\Delta Q.R}{nA} = \frac{2 \times 10^{-4} \times 80}{40 \times 4 \times 10^{-4}} = 1\ Wb/m^2$$

(13) **(A).** $\phi = Li \Rightarrow NBA = Li$

Since magnetic field at the centre of circular coil carrying current is given by

$$B = \frac{\mu_0}{4\pi}\frac{2\pi Ni}{r} \quad \therefore \quad N \cdot \frac{\mu_0}{4\pi}\frac{2\pi Ni}{r} \cdot \pi r^2 = Li$$

$$\Rightarrow \quad L = \frac{\mu_0 N^2 \pi r}{2}$$

Hence self inductance of a coil

$$\frac{4\pi \times 10^{-7} \times 500 \times 500 \times \pi \times 0.05}{2} = 25\ mH$$

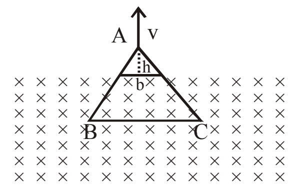
(14) **(D).** $e = -L\frac{di}{dt} \Rightarrow 2 = -L\left(\frac{8-2}{3 \times 10^{-2}}\right)$

$L = 0.01\ H = 10\ mH$

(15) **(B).** $e = M\frac{di}{dt} \Rightarrow M = \frac{15000}{3} \times 0.001 = 5\ H$

(16) **(B).** $I = \frac{\varepsilon}{R} = \frac{|d\phi/dt|}{R} = \frac{B.\frac{dA}{dt}}{R}$

$$= \frac{B\frac{d}{dt}\left(\frac{1}{2}h.b\right)}{R} \propto \frac{Bb\frac{dh}{dt}}{R} \propto Bbv$$



$\therefore\ b \propto t \Rightarrow I \propto t$

(17) **(D).** $e = L\frac{di}{dt} = 60 \times 10^{-6} \times \frac{(1.5 - 1.0)}{0.1} = 3 \times 10^{-4}$ volt

(18) **(D).** $N\phi = Li \Rightarrow \frac{Nd\phi}{dt} = \frac{Ldi}{dt} \Rightarrow NB\frac{dA}{dt} = \frac{Ldi}{dt}$

$$\Rightarrow \frac{1 \times 1 \times 5}{10^{-3}} = L \times \left(\frac{2-1}{2 \times 10^{-3}}\right) \Rightarrow L = 10H$$

(19) **(D).** Electric field will be induced in both AD and BC.

(20) **(B).** $U = \frac{1}{2}Li^2 = \frac{1}{2}L\left(\frac{E}{R}\right)^2 = \frac{1}{2} \times 5 \times \left(\frac{100}{20}\right)^2 = 62.50J$

(21) **(C).** $i = \frac{V}{R} = \frac{10}{2} = 5A$; $U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 25 = 25\,J$

(22) **(D).** $i = \frac{E - e}{R} \Rightarrow 1.5 = \frac{220 - e}{20} \Rightarrow e = 190\ V$

(23) **(C).** $\frac{N_s}{N_p} = \frac{V_s}{V_p} \Rightarrow \frac{1}{20} = \frac{V_s}{2400} \Rightarrow V_s = 120\ V$

For 100% efficiency $V_s i_s = V_p i_p$

$\Rightarrow 120 \times 80 = 2400\, i_p \Rightarrow i_p = 4A$

(24) **(A).** For 100% efficient transformer

$$V_s i_s = V_p i_p \Rightarrow \frac{V_s}{V_p} = \frac{i_p}{i_s} = \frac{N_s}{N_p}$$

$$\Rightarrow \frac{i_p}{4} = \frac{25}{100} \Rightarrow i_p = 1\ A$$

(25) **(A).** For each spoke, the induced emf between the centre O and the rim will be the same

$$e = \frac{1}{2}B\omega L^2 = B\pi L^2 f\,(\because\ \omega = 2\pi f)$$

Further for all spokes, centre O will be positive while rim will be negative. Thus all emf's are inparallel giving total emf $e = B\pi L^2 f$

independent of the number of the spokes.

Substituting the values

$e = 4 \times 10^{-5} \times 3.14 \times (.5)^2 \times 2 = 6.28 \times 10^{-5}$ volt

(26) **(B).** As number of turns are quadrupled, the induced emf will increase four times. Also, resistance of coil increases sixteen times. Hence power (e^2/R) will not change.

(27) **(D).** Steady state current for both the circuits is same.

Therefore, $\frac{V}{R_1} = \frac{V}{R_2}$ or $R_1 = R_2$

Further, $\tau_{L_1} < \tau_{L_2}$ (τ_L = time constant)

$\therefore \frac{L_1}{R_1} < \frac{L_2}{R_2}$ or $L_1 < L_2$

(28) **(C).** According to Faraday's law of electromagnetic induction, $E_{induced} = \frac{-\Delta\phi}{\Delta t} = -\frac{B(A_f - A_i)}{\Delta t}$

Let r be the radius of circle; then side of square formed

$= \frac{2\pi r}{4} = \frac{\pi r}{2}$

Change is area of loop

$= A_i - A_f = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \frac{\pi(4-\pi)r^2}{4}$

Hence average emf induced

$= \frac{\pi(4-\pi)r^2}{4} \cdot \frac{B}{t} = \frac{\pi(4-\pi)\times(0.1)^2\times 100}{4\times 0.1} = 6.75$ volt

(29) **(B).** Power $P = e^2/R$

Here, e = induced emf = – ($d\phi/dt$), where $\phi = NBA$

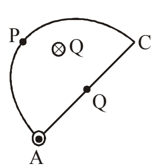
$e = -NA\left(\frac{dB}{dt}\right)$ Also, $R \propto \frac{\ell}{r^2}$

where R = resistance, r = radius, ℓ = length

$\therefore P \propto \frac{N^2 r^2}{\ell}$ $\therefore \frac{P_1}{P_2} = 1$

(30) **(B).** We connect a conducting wire from A to C and complete the semicircular loop.

The loop emf in the semicircular loop is zero because its magnetic flux does not change.



$\therefore$ emf of section APC + emf of section CQA = 0

or emf o section APC = emf of section AQC = $2BR^2\omega$

(31) **(A).** At any time t, the side of the square

$a = (a_0 - \alpha t)$, where a_0 = side at t = 0.

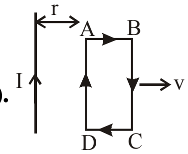
At this instant, flux through the square :

$\phi = BA\cos 0° = B(a_0 - \alpha t)^2$

$\therefore$ emf induced $E = -d\phi/dt$

$\Rightarrow E = -B.2(a_0 - \alpha t)(0 - \alpha) = +2\alpha aB$

(32) **(D).**



As flux decrease to maintain flux current in loop is clockwise. Force on DA due to long wire and BC is towards left while on wire BC toward right.

(33) **(D).** $i_0 = \frac{v_0}{Z}$,

$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{4^2 + (1000\times 3\times 10^{-3})^2} = 5\Omega$

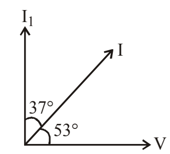
$i_0 = 4/5$; $i_0 = 0.8$ A

(34) **(D).** $\tan\phi = -\frac{4}{3}$

i.e. I is leading by 53°

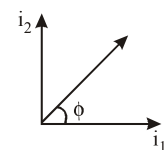
$V_c = V = V_0 \sin\omega t$

Phase difference between I_1 and I is 37°.



(35) **(C).** $i_3 = \sqrt{i_1^2 + i_2^2}$

and $i_3 < (i_1 + i_2)$



(36) **(C).** The circuit will have inductive nature if $\omega > \frac{1}{\sqrt{LC}}$

Hence A is false. Also if circuit has inductive nature the current will lag behind voltage. Hence (D) is also false.

If $\omega = \frac{1}{\sqrt{LC}}\left(\omega L = \frac{1}{\omega C}\right)$ the circuit will have resistance nature. Hence (B) is false.

Power factor

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1$$

if $\omega L = \frac{1}{\omega C}$.

(37) **(C).** Since B is constant $\therefore \frac{d\phi}{dt} = 0$ $\therefore i = 0$

(38) **(C).** Here, $L = 30$ mH $= 30\times 10^{-3}$ H

$V_{rms} = 220$ V, $\nu = 100$ Hz

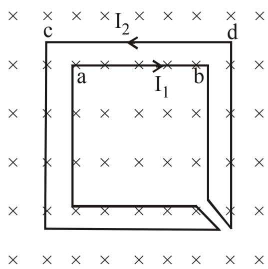
Inductive reactance,

$X_L = 2\pi\nu L = 2\times 3.14\times 100\times 30\times 10^{-3} = 18.85\Omega$

(39) **(D).** Current $I_1 = I_2$,

Since magnetic field increases with time.

So induced net flux should be outward i.e. current will flow from a to b.



(40) **(B).** Here, $I_{rms} = 25A$

$\therefore I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 25 = 35.36\,A.$

(41) **(A).** Current leads emf so circuit is R – C

$\tan\phi = X_c / R,\ \ \phi = 45°, R = 1000\Omega,$

$\omega = 100, C = ?$

since, $\tan 45° = \frac{1}{\omega CR}$ so $C = 10\ \mu F$

(42) **(D).** Here, $V_P = 11000\,V, V_s = 220V$

$N_P = 6000, \eta = 60\%$

$P_0 = 9\ kW = 9 \times 10^3\ W$

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_0}{P_i}$$

$$P_i = \frac{P_0}{\eta} = \frac{9 \times 10^3}{60/100} = 1.5 \times 10^4 = 15kW$$

(43) **(A).** The instantaneous value of voltage is

$V = 100 \sin(100\,t)\,V$

Compare it with $V = V_0 \sin(\omega t)\,V$

we get $V_0 = 100\ V, \omega = 100\ rad\ s^{-1}$

The rms value of voltage is

$$\therefore V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}} V = 70.7V$$

The instantaneous value of current is

$$I = 100 \sin\left(100t + \frac{\pi}{3}\right)\ mA$$

Compare it with $I = I_0 \sin(\omega t + \phi)$

we get $I_0 = 100\ mA, \omega = 100\ rad\ s^{-1}$

The rms value of current is

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{100}{\sqrt{2}} mA = 70.7mA$$

(44) **(C).** Here, $L = 2H, C = 32\mu F = 32 \times 10^{-6} F, R = 10\Omega$

$\therefore$ Resonance frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{10^3}{8}\ rad/s$$

$$\text{Quality factor} = \frac{\omega_r L}{R} = \frac{10^3 \times 2}{8 \times 10} = 25$$

(45) **(C).** Here, $C = 5\mu F = 5 \times 10^{-6}\ F,$

$V_{rms} = 200\ V, \nu = 100\ Hz$

The capacitive reactance is

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 100 \times 5 \times 10^{-6}}$$

$= 3.18 \times 10^2\ \Omega = 318\ \Omega$

(46) **(A).** $f = \frac{1}{2\pi\sqrt{LC}}$

i.e. $f \propto \frac{1}{\sqrt{C}} \rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}$ times

(47) **(C).** As current is in phase with the applied voltage, X must be R.

$$R = \frac{V_0}{I_0} = \frac{200V}{5A} = 40\Omega$$

As current lags behind the voltage by 90º, Y must be an inductor.

$$X_L = \frac{V_0}{I_0} = \frac{200V}{5A} = 40\Omega$$

In the series combination of X and Y,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_0}{\sqrt{2}Z} = \frac{200}{\sqrt{2}(40\sqrt{2})} = \frac{5}{2} A$$

(48) **(C).** Here, $R = X_L = X_C$ ($\because$ voltage across them is same)

Total voltage in the circuit,

$V = I\,[R^2 + (X_L - X_C)^2]^{1/2} = IR = 10V$

When capacitor is short circuited,

$$I' = \frac{10}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$\therefore$ Potential drop across inductance

$$= I'X_L = I'R = \frac{10}{\sqrt{2}} V.$$

(49) **(B).** When the frequency of the supply equals to the natural frequency of circuit, resonance occurs.

$$\therefore\ Z = R = 22\Omega \text{ and } I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{22} = 10A$$

Average power transferred per cycle,

$P = V_{rms} I_{rms} \cos 0° = 220 \times 10 \times 1 = 2200\ W$

(50) **(A).** $X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 0.1 \times 10^{-6}}$

$= 3.2 \times 10^4\ \Omega$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{100 + 10.28 \times 10^8} = 3.2 \times 10^4\ \Omega$$

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{100}{3.2 \times 10^4} = 3.14 \times 10^{-3} A = 3.14\ mA$$

(51) **(D).** $V = 120 \sin(100\pi t)\cos(100\pi t)V$
$= 60 \sin(200\pi t)$ ($\because \sin 2\theta = 2\sin\theta\cos\theta$)
Compare it with standard equation
$V = V_0 \sin \omega t$
We get, $V_0 = 60V$ and $\omega = 200\pi$
or $2\pi\nu = 200\pi$ or $\nu = 100Hz$

(52) **(D).** Here, $V_0 = 283V, R = 3\Omega, L = 25 \times 10^{-3}H$
$C = 400\mu F = 4 \times 10^{-4} F$
Maximum power is dissipated at resonance, for which

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1 \times 7}{2 \times 22\sqrt{25 \times 10^{-3} \times 4 \times 10^{-4}}}$$

$$= \frac{7 \times 10^3}{44\sqrt{10}} = 50.3\,Hz$$

(53) **(B).** Here, $P = 800\,kW = 800 \times 10^3\,W$
Total resistance of two wire line
$R = 2 \times 15 \times 0.5 = 15\,\Omega$
As supply is through 4000 – 220 V transformer
$\therefore V_{rms} = 4000\,V$

$$\therefore I_{rms} = \frac{P}{V_{rms}} = \frac{800 \times 10^3}{4000} = 200A$$

Line power loss $= I^2_{rms} R = (200)^2 \times 15$
$= 60 \times 10^4\,W = 600\,kW$

(54) **(A).** Here, $L = 0.01\,H, R = 1\Omega$
$V_{rms} = 220\,V, \nu = 50\,Hz$

$$\text{As } \tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi\nu L}{R}$$

$$\tan\phi = \frac{2 \times 3.14 \times 50 \times 0.01}{1} = 3.14$$

$$\therefore \phi = \tan^{-1}(3.14) = 72^o = 72^o \times \frac{\pi}{180}\text{rad}$$

The time lag between maximum alternating voltage

$$\text{and current is } \Delta t = \frac{\pi}{\omega} = \frac{\frac{72 \times \pi}{180}}{2\pi \times 50} = \frac{1}{250}s$$

(55) **(A).** Here, $R = 6\Omega, L = 25mH = 25 \times 10^{-3}H$,
$C = 750\mu F = 750 \times 10^{-6}F, \nu = 50\,Hz$
$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7.85\,\Omega$

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 750 \times 10^{-6}} = 4.25\,\Omega$$

$\therefore X_L - X_C = 7.85 - 4.25 = 3.6\Omega$
Impedence of the series LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(6)^2 + (3.6)^2}$$

$$= \sqrt{36 + 12.96} = 7.0\Omega$$

(56) **(C).** Here, $C = 60\mu F = 60 \times 10^{-6}\,F$
$V_{rms} = 110\,V, \nu = 60\,Hz$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{1/2\pi\nu C} = 2\pi\nu C\,V_{rms}$$

$= 2 \times 3.14 \times 60 \times 60 \times 10^{-6} \times 110 = 2.49\,A$

(57) **(D).** Here, $C = 100\mu F = 100 \times 10^{-6}\,F = 10^{-4}F$
$R = 40\Omega, V_{rms} = 100V, \nu = 60\,Hz$
$\therefore V_0 = \sqrt{2}\,V_{rms} = 100\sqrt{2}\,V$
In series RC circuit,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= \frac{1}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2C^2}}} \quad [\because \omega = 2\pi\nu]$$

Maximum current in the circuit,

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2C^2}}}$$

$$= \frac{100\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{4 \times (3.14)^2 \times (60)^2 \times (10^{-4})^2}}}$$

$$= \frac{100\sqrt{2}}{48} = 2.95A$$

(58) **(B).** We know that, $Z = E_0 / I_0$
Given, $E_0 = 220$ and $I_0 = 10$

$$\text{so } Z = \frac{220}{10} = 22\text{ ohm}$$

$$\phi = \left[\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right] = \frac{\pi}{3}$$

$$p_a = \frac{E_0}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos\phi = \frac{220}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos\frac{\pi}{3} = 550\,W$$

(59) **(B).** Here, $V_{rms} = 220V$, $I_{rms} = 65\,mA = 0.065A$
$C = 8\mu F = 8 \times 10^{-6}\,F$, $\nu = 100\,Hz$

$$\text{Capacitive reactance, } X_C = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2 \times 3.14 \times 100 \times 8 \times 10^{-6}} = 199\,\Omega$$

Then rms voltage across the capacitor is
$V_{Crms} = I_{rms} X_C = 0.065 \times 199 = 12.94V$

(60) **(A).** On comparing $V = 200\sqrt{2}\sin(100t)$ with
$V = V_0 \sin\omega t$, we get
$V_0 = 200\sqrt{2}\,V, \omega = 100\text{ rad s}^{-1}$

$\therefore V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200\sqrt{2}V}{\sqrt{2}} = 200V$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 1 \times 10^{-6}} = 10^4 \Omega$$

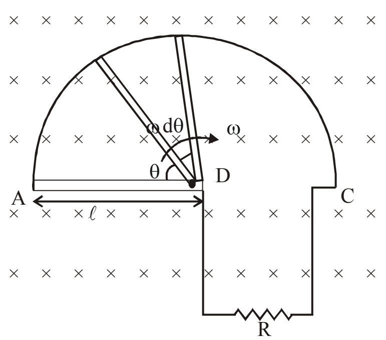
ac ammeter reads the rms value of current. Therefore, the reading of the ammeter is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{200V}{10^4 \Omega} = 20 \times 10^{-3} A = 20mA$$

The average power consumed in the circuit,
$P = I_{rms} V_{rms} \cos\phi$
In an pure capacitive circuit, the phase difference between current and voltage is $\pi/2$. $\therefore \cos\phi = 0$
$\therefore P = 0$

EXERCISE-3
PART-A

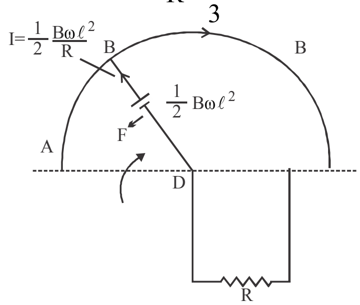
(1) **30.** Let ω_0 be the angular velocity given to rod so that it reaches position DC. Let at a time t rod is at an angular displacement θ and in further time dt is travels a distance $d\theta$. The equivalent circuit will be as shown in figure.



Current $i = \frac{1}{2}\frac{B\omega\ell^2}{R}$. Hence, $F = \frac{1}{2}\frac{B^2\omega\ell^3}{R}$

Now torque acting on the rod is $\tau = F \times \frac{\ell}{2} = \frac{1}{4}\frac{B^2\omega\ell^4}{R}$

$\therefore$ Angular retardation $\alpha = \frac{\tau}{\ell} = \frac{1}{4}\frac{B^2\omega\ell^4}{R\frac{m\ell^2}{3}} = \frac{3}{4}\frac{B^2\ell^2\omega}{mR}$



(I is moment of inertia of rod about point D)

$\Rightarrow \omega\frac{d\omega}{d\theta} = -\frac{3B^2\ell^2\omega}{4mR}$

$\int_{\omega_0}^{0} d\omega = -\int_0^{\pi} \frac{3B^2\ell^2}{4mR} d\theta$ and $\omega_0 = \frac{3B^2\ell^2\pi}{4mR} = 30$ rad / s

(2) **100.** Work done = ΔQ (heat dissipated)

$= \frac{r^2}{R}t = \frac{(N \times B\ell v)^2}{R} \times t = \frac{100^2 \times (0.4)^2 \times 2.5 \times 10^{-3}}{100} \times v^2 \times 1$

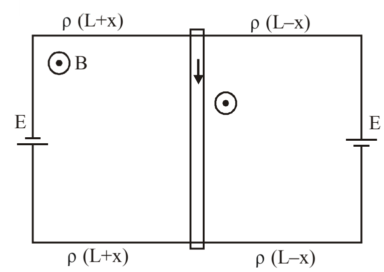
$\ell^2 = 2.5 \times 10^{-3}$; $\ell = \sqrt{25 \times 10^{-4}} = 5 \times 10^{-2}$ m

$v = \frac{5 \times 10^{-2}}{1} \Rightarrow w = 6.25 \times 10^{-6} \times 16\,J = 100\,\mu J$

(3) **2.** $\tau_1 = RC$; $\tau_2 = \frac{L}{R} \Rightarrow LC = t_1 t_2 = 0.1$ sec.

$\Rightarrow T = 2\pi\sqrt{LC} = 2\pi\sqrt{\frac{1}{10}} = 2$ sec.

(4) **1.** If rod is in middle, $i = 0 \Rightarrow F = 0$



$$\text{Eq. emf} = \frac{\frac{E}{2\rho(L-x)} - \frac{E}{3\rho(L+x)}}{\frac{1}{2\rho(L-x)} + \frac{1}{2\rho(L+x)}} = \frac{E\left[\frac{2x}{\ell^2 - x^2}\right]}{\frac{2\ell}{L^2 - x^2}} \Rightarrow \frac{Ex}{L}$$

$$\frac{1}{R_{eq.}} = \frac{1}{2\rho(L-x)} + \frac{1}{2\rho(L+x)} = \frac{1}{2\rho} \times \frac{2L}{L^2 - x^2}$$

$$\Rightarrow R_{eq.} = \frac{\rho(L^2 - x^2)}{L}$$

$$i = \frac{\frac{Ex}{L}}{\frac{\rho(L^2 - x^2)}{L} + R} = \frac{Ex}{\rho(L^2 - x^2) + RL}$$

$$ma = F = -i\ell B = \frac{-Ex\ell B}{\rho(L^2 - x^2) + RL} \approx \frac{-E\ell B}{\rho L^2 + RL}x$$

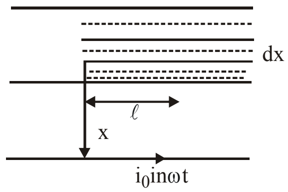
$$a = \frac{-E\ell B}{m(\rho L^2 + RL)}x \Rightarrow T = 2\pi\sqrt{\frac{m(\rho L^2 + RL)}{E\ell B}}$$

$\Rightarrow T = 1$ sec.

(5) **5.** $d\phi = \frac{\mu_0 i}{2\pi x} \times \ell\, dx$; $\phi = \frac{\mu_0 i}{2\pi} \ell \ln\left(\frac{d_2}{d_1}\right)$

$$\varepsilon = \frac{d\phi}{dt} = \frac{\mu_0}{2\pi} \ell \ln\left(\frac{d_2}{d_1}\right) \times i_0 \omega \cos\omega t$$

$$\varepsilon_{rms} = \frac{\mu_0}{2\pi} \ell \ln\left(\frac{d_2}{d_1}\right) \times \frac{i_0\omega}{\sqrt{2}}$$



$$= \frac{4\pi \times 10^{-7}}{2\pi} \times 1 \times \ln 2 \frac{2\sqrt{2}}{\ln 2} \times \frac{50}{\sqrt{2}} = 2 \times 10^{-5}\text{ V}$$

(6) **5.** $e = (\vec{v} \times \vec{B}).\ell$; $e = [\hat{i} \times (3\hat{i} + 4\hat{j} + 5\hat{k})].5\hat{j} = 25$ volt.

(7) **6.** Flux through circular ring $\phi = (\mu_0 ni)\, \pi r^2$

$$\phi = \frac{\mu_0}{L} \pi r^2 I_0 \cos 300t$$

$$i = \frac{d\phi}{Rdt} \;;\; i = \frac{\mu_0 \pi r^2 I_0}{RL} \sin 300t \times 300$$

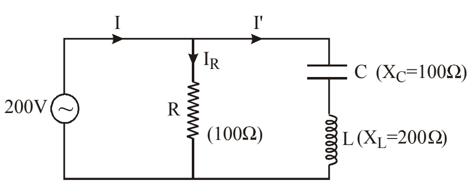
$$= \mu_0 I_0 \sin 300t \left[\frac{\pi r^2 .300}{RL}\right]$$

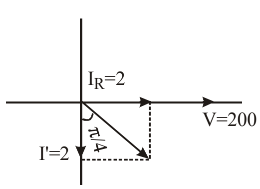
$$M = I.\pi r^2 = \mu_0 I_0 \sin 300t \left[\frac{\pi^2 r^4 .300}{RL}\right] \text{(Take } \pi^2 = 10)$$

$$= \frac{10 \times 10^{-4} \times 300}{100 \times 10}$$

PART-B

(1) **2.**

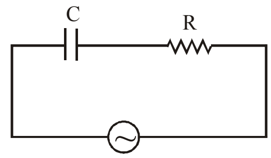




$$I_R = \frac{V}{R} = \frac{200}{100} = 2A,\; I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}\text{ amp}$$

(2) **4.**



$$Z = \sqrt{\left(\frac{1}{\omega L}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega L}\right)^2 + R^2 = R^2\,(1.25)$$

$$\left(\frac{1}{\omega L}\right)^2 + R^2 = R^2 + \frac{R^2}{4} \Rightarrow \frac{1}{\omega L} = \frac{R}{2}$$

$$CR = \frac{2}{\omega} = \frac{2}{500}\text{sec} = \frac{2}{500} \times 10^3\text{ sec}$$

$$= \frac{2 \times 1000}{500}\text{ms} = 4\text{ ms}$$

(3) **3.** **(4) 4.**

$V_{L_0} = 3, V_{R_0} = 4 \;\therefore\; e_0 = \sqrt{V_{L_0}^2 + V_{R_0}^2} = 5$ volts

If $i = i_0 \sin\omega t$, $V_R = 4\sin\omega t$, $V_L = 3\cos\omega t$

$e = 5\sin(\omega t + 37°)$

$4\sin\omega t = 2 \Rightarrow \omega t = \pi/6$

$\therefore V_L = 3\cos\omega t = 3\cos 30°$

$e = 5\sin(\omega t + 37°) \;\therefore\; 5\sin(37° + 30°) = \frac{4 + 3\sqrt{3}}{2}$

(5) **2.** **(6) 3** **(7) 1**

RMS value of supply voltage is

$$V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(30)^2 + (100 - 60)^2}$$

$$= \sqrt{(30)^2 + (40)^2} = 50V$$

$\therefore V_0 = V_{rms} \times \sqrt{2} = 50\sqrt{2}V$

If voltage across resistor is 50V then this should be the resonance condition.

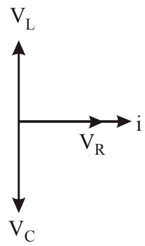
At resonance, $X_L = X_C$

$$\omega_2 L = \frac{1}{\omega_2 C} \;;\; \omega_2 = \frac{1}{\sqrt{LC}}$$

Also, At $\omega = \omega_1$

$$I = \frac{100}{X_L} = \frac{60}{X_C} \;;\; \frac{100}{\omega_1 L} = \frac{60}{1/\omega_1 C}$$

$$C = \frac{100}{\omega_1^2 L \times 60} = \frac{5}{3\omega_1^2 L}$$



$$\omega_2 = \frac{1}{\sqrt{L \times \frac{5}{3\omega_1^2 L}}} = \sqrt{\frac{3}{5}}\,\omega_1$$

Phase difference between voltage on L and C is π.

EXERCISE-4

(1) **(A).** $I_{ind} = \frac{Bv\ell}{R} \propto B$

(2) **(C).** All three coils are parallel
So $L_{eq} = 1$ H

(3) **(B).** Power factor $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

(4) **(B).** $\frac{I_S}{I_P} = \frac{N_P}{N_S}$

(5) **(A).** Relative position and orientation of the two coils.

(6) **(C).** $e = -L\frac{dI}{dt}$

(7) **(B).** Power factor

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

(8) **(D).** Reduce the energy loss due to eddy current

(9) **(B).** $I = -\frac{1}{R_{eq}}\left(n\frac{d\phi}{dt}\right) = -\frac{1}{5R}\left[\frac{n(W_2 - W_1)}{t}\right]$

(10) **(B).** Mean power per period

$$= \frac{V_{rms}^2}{R} = \frac{\left(\frac{\left(\frac{\pi}{2}r^2\right)B\omega}{\sqrt{2}}\right)^2}{R} = \frac{(B\pi\omega r^2)^2}{8R}$$

(11) **(B).** $e = \frac{1}{2}B\omega\ell^2$

(12) **(C).** D.C. Ammeter measures average value and average value for one cycle of A.C. is zero.

(13) **(D).** $V_L = V_C = V_R = 50$ volt
V_L and V_C have opposite phasors so voltage across LC combination is zero.

(14) **(C).** Frequency $= \frac{1}{2\pi\sqrt{LC}}$
$C \to 2C$; $L \to L/2$

(15) **(B).** $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$; $\frac{I_0}{2} = I_0\left(1 - e^{-\frac{2t}{300\times10^{-3}}}\right)$

t = 0.1 sec.

(16) **(C).** Frequency $= \frac{1}{2\pi\sqrt{LC}}$

(17) **(D).** $2B\ell V$

(18) **(A).** $\cos\phi = \frac{R}{Z} = \frac{12}{15} = 0.8$

(19) **(B).** RL has acute angle phase difference.

(20) **(D).** $e = \left(-\frac{d\phi}{dt}\right)_{t=3\,sec}$

(21) **(B).** In AC generator $E = NAB\omega \sin\omega t$
$\therefore\ E_{max} = NAB\omega$

(22) **(C).** Discharging of coil

$$I = \frac{E}{R}e^{-\frac{R}{L}t} = \frac{100}{100}e^{-\frac{100}{100\times10^{-3}}\times10^{-3}} = \frac{1}{e}$$

(23) **(A).** At resonance, $I = \frac{V}{Z} = \frac{V}{R} = \frac{100}{1k\Omega} = 0.1A$

$$V_L = V_C = I X_C = I\left(\frac{1}{\omega C}\right) = 250V$$

(24) **(B).** Phase difference = π/2
∴ Power factor = 0
∴ Power consumption = 0

(25) **(D).** $i = \frac{5}{5}\left(1 - e^{-\frac{5\times2}{10}}\right) = (1 - e^{-1})$

(26) **(C).** $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

$$= \frac{4\pi\times10^{-7}\times300\times400\times10\times10^{-4}}{20\times10^{-2}} = 2.4\pi\times10^{-4}\,H$$

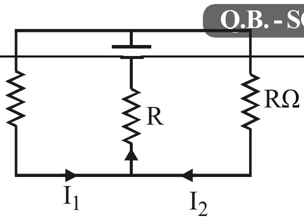
(27) **(D).** Current in LR_2 branch

$$i = \frac{E}{R_2}[1 - e^{-R_2 t/L}]\ ;\ \frac{di}{dt} = \frac{E}{R_2}\frac{R_2}{L}e^{-R_2 t/L}$$

Drop across $L = \left(\frac{E}{L}e^{-R_2 t/L}\right)L$

$$= 12e^{-2t/400\times10^{-3}} = 12e^{-5t}V$$

(28) **(B).** A moving conductor is equivalent to a battery of emf $= vB\ell$ (motion emf)
Equivalent circuit



$I = I_1 + I_2$

Applying Kirchoff's law

$I_1 R + IR - v_B \ell = 0$ ......(1)

$I_2 R + IR - v_B \ell = 0$ ......(2)

Adding (1) & (2)

$2IR + IR = 2vB\ell$

$$I = \frac{2vB\ell}{3R} \;;\; I_1 = I_2 = \frac{vB\ell}{3R}$$

(29) **(B).** At t = 0, inductor behaves like an infinite resistance

So at t = 0, $i = \frac{V}{R_2}$

At $t = \infty$, inductor behaves like a conducting wire

$$i = \frac{V}{R_{eq}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

(30) **(D).** The given circuit is under resonance as $X_L = X_C$
Hence power dissipated in the circuit is

$$P = \frac{V^2}{R} = 242\text{ W}$$

(31) **(B).** In LC oscillation energy is transfered C to L

or L to C maximum energy in L is $= \frac{1}{2} L I_{max}^2$

Maximum energy in C $= \frac{q_{max}^2}{2C}$

Equal energy will be when $\frac{1}{2} L I^2 = \frac{1}{2}\frac{1}{2} L I_{max}^2$

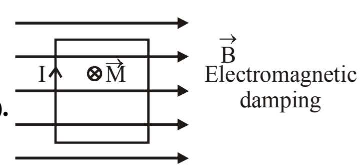
$$I = \frac{1}{\sqrt{2}} I_{max} \;;\; I = I_{max}\sin\omega t = \frac{1}{\sqrt{2}} I_{max}$$

$$\omega t = \frac{\pi}{4} \text{ or } \frac{2\pi}{T} t = \frac{\pi}{4} \text{ or } t = \frac{T}{8}$$

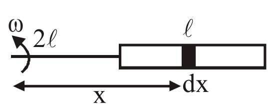
$$t = \frac{1}{8} 2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$$

(32) **(D).** $E_{ind} = B \times v \times \ell = 5.0 \times 10^{-5} \times 1.50 \times 2$
$= 10.0 \times 10^{-5} \times 1.5 = 15 \times 10^{-5}$ volt $= 0.15$ mv

(33) **(D).**



(34) **(D).**



$$e = \int_{2\ell}^{3\ell} (\omega x) B\, dx = B\omega \frac{[(3\ell)^2 - (2\ell)^2]}{2} = \frac{5B\ell^2\omega}{2}$$

(35) **(A).** $\phi = Mi = \frac{\mu_0 (2)(20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]} \times \pi (0.3 \times 10^{-2})^2$

$= 9.216 \times 10^{-11} \approx 9.2 \times 10^{-11}$ weber

(36) **(C).** $q = CV(1 - e^{-t/\tau})$
At t = 2, $q = CV(1 - e^{-2})$

(37) **(D).** Since resistance and inductor are in parallel, so ratio will be 1.

(38) **(C).** $I = I_0 e^{-t/\tau}$, $\tau = \frac{L}{R} = \frac{15}{150} e^{-\frac{1\times10^{-3}}{1/5\times10^{3}}} = 0.67\text{mA}$

(39) **(B).** For a damped pendulum, $A = A_0 e^{-bt/2m}$

$\Rightarrow A = A_0 e^{-\left(\frac{R}{2L}\right)t}$ (Since L plays the same role as m)

(40) **(A).** $I = \frac{P}{4\pi r^2} = U_{av} \times c$ .... (1) $U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$ ..... (2)

$$\Rightarrow \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 \times c \Rightarrow E_0 = \sqrt{\frac{2P}{4\pi r^2 \varepsilon_0 c}} = 2.45\text{ V/m}$$

(41) **(C).** $R = \frac{80}{10} = 8\Omega$; $i_{rms} = 10A = \frac{V_{rms}}{\sqrt{64 + X_L^2}}$

$$22^2 = 64 + X_L^2 \Rightarrow X_L = \sqrt{420}$$

$$2\pi \times 50 \times L = \sqrt{420} \;;\; L = \frac{\sqrt{420}}{100\pi} \Rightarrow \pi^2 \cong 10$$

$$\sqrt{42} \cong 6.5 \Rightarrow L \cong 0.065\text{ H}$$

(42) **(C).** Quality factor $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

(43) **(D).** $\langle P \rangle = V_{rms} I_{rms}\cos\phi$; $\phi = -\frac{\pi}{4}$

$$\langle P \rangle = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

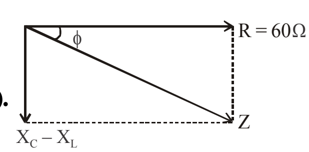
Wattless current, $I = I_{rms}\sin\phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10$

(44) **(A).** $Q = \frac{\Delta\phi}{R} = \frac{1}{10} A (B_f - B_i)$

$$= \frac{1}{10} \times 3.5 \times 10^{-3}\left(0.4\sin\frac{\pi}{2} - 0\right)$$

$$= \frac{1}{10} \times (3.5 \times 10^{-3})(0.4 - 0) = 1.4 \times 10^{-4} = 0.14\text{ mC}$$

(45) **(D).**



$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}} = 26.52\,\Omega$$

$$X_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega\ ;\ X_C - X_L = 20\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 20\sqrt{10}\ \Omega$$

$$\cos\phi = \frac{R}{Z} = \frac{3}{\sqrt{10}}\ ;\ P_{avg} = VI\cos\phi = \frac{V^2}{Z}\cos\phi = 8.64\ W$$

$$Q = P.t = 8.64 \times 60 = 5.18 \times 10^2$$

(46) **(C).** $LI\,dI = I^2R$

$$L \times \frac{E}{10}(-e^{-t/2}) \times \frac{-1}{2} = \frac{E}{10}(1 - e^{-t/2}) \times 10$$

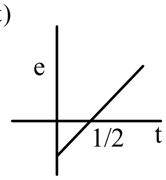
$$e^{-t/2} = 1 - e^{-t/2}$$

$$t = 2\ln 2$$

(47) **(D).** $I = I_0 t - I_0 t^2$

$\phi = BA\ ;\ \ \phi = \mu_0 nIA$

$$V_R = -\frac{d\phi}{dt} = -\mu_0 nAI_0(1 - 2t)$$



$V_R = 0$ at $t = 1/2$

$$I_R = \frac{V_R}{\text{Resistance of loop}}$$

(48) **(A).** $\omega = \frac{2\pi}{T} = \frac{\pi}{5}$. When $\omega t = \frac{\pi}{2}$

∵ ϕ will be minimum.

∴ e will be maximum

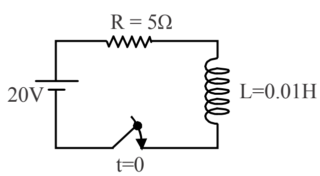
$$t = \frac{\pi/2}{\pi/5} = 2.5\ \text{sec}.$$

When $\omega t = \pi$

∵ ϕ will have maximum. ∴ e will be minimum.

$$t = \frac{\pi}{\pi/5} = 5\ \text{sec}.$$

(49) **(C).**



$$i = i_0\left(1 - e^{\frac{-t}{L/R}}\right) = \frac{20}{5}\left(1 - e^{\frac{-t}{0.01/5}}\right) = 4\,(1 - e^{-500t})$$

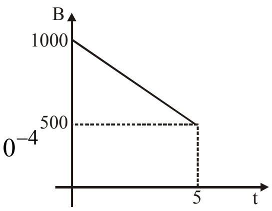
$$i_\infty = 4$$

$$i_{40} = 4\,(1 - e^{-500 \times 40})$$

$$= 4\left(1 - \frac{1}{(e^2)^{10000}}\right) = 4\left(1 - \frac{1}{7.38^{10000}}\right)$$

$$\frac{i_\infty}{i_{40}} \approx 1$$

(50) **(C).** $\frac{dB}{dt} = 100$



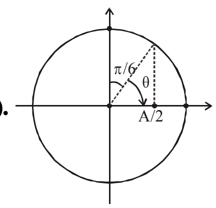
$$A = 16 \times 4 - 4 \times 2 = 56\ cm^2$$

$$\varepsilon = \frac{dB}{dt}A = 100 \times 10^{-4} \times 56 \times 10^{-4}$$

(51) **(10.00)**

$$V = \left|L\frac{di}{dt}\right|;\quad L = \frac{V}{\left|\frac{di}{dt}\right|} = \frac{100}{\frac{0.25}{0.025 \times 10^{-3}}} = 10mH$$

(52) **(C).**



$V(t) = 220\sin(100\pi t)$ volt

Time taken,

$$t = \frac{\theta}{\omega} = \frac{\pi/3}{100\pi} = \frac{1}{300}\ \text{sec} = 3.3\ \text{ms}$$

(53) **(D).** Given phase difference $= \pi/4$ and $\omega = 100$ rad/s

⇒ Reactance (X) = Resistance (R)

Now by checking options

Option (A)

$$R = 1000\ \Omega \text{ and } X_C = \frac{1}{10^{-6} \times 100} = 10^4\Omega$$

Option (B)

$R = 10^3\ \Omega$ and $X_L = 10^{-3} \times 100 = 10^{-1}\ \Omega$

Option (C)

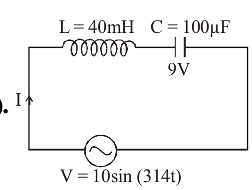
$R = 10^3\ \Omega$ and $X_L = 10 \times 10^{-3} \times 100 = 1\Omega$

Option (D)

$$R = 10^3\ \Omega\ \&\ X_C = \frac{1}{10 \times 10^{-6} \times 100} = 10^3\Omega$$

Clear option (D) matches the given condition.

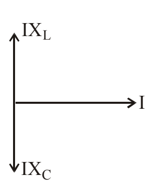
(54) **(A).**



$$X_L = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}} = \frac{10^4}{314} = 31.84\Omega$$

Phasor -

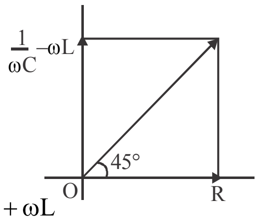


$V_m = I_m (X_C - X_L)$
$10 = I_m (31.84 - 12.56)$
$$I_m = \frac{10}{19.28} = 0.52 \text{ A}$$
$$I = 0.52 \sin\left(314t + \frac{\pi}{2}\right)$$

EXERCISE-5

(1) **(A).** $V = \frac{W}{Q} \Rightarrow W = QV$

(2) **(D).** From figure,



$$\tan 45° = \frac{\frac{1}{\omega C} - \omega L}{R}$$
$$\Rightarrow \frac{1}{\omega C} - \omega L = R \Rightarrow \frac{1}{\omega C} = R + \omega L$$
$$C = \frac{1}{\omega (R + \omega L)} = \frac{1}{2\pi f (R + 2\pi f L)}$$

(3) **(B).** Mutual Inductor of two coils
$$M = \sqrt{M_1 M_2} = \sqrt{2mh \times 8mh} = 4mH$$

(4) **(D).** Where there is change of flux in the core of a transformer due to change in current around it, eddy current is produced. The direction of this current is opposite to the current which produces it, so it will reduce the main current. We laminate the core so that flux is reduced resulting in the reduced production of eddy current.

(5) **(B).** Power factor, $\phi = \frac{R}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}}$
$$= \frac{8}{\sqrt{(31-25)^2 + 8^2}} = \frac{8}{\sqrt{6^2 + 8^2}} = 0.8$$

(6) **(D).** Condition for which the current is maximum in a series LCR circuit is
$$\omega = \frac{1}{\sqrt{LC}} \;;\; 1000 = \frac{1}{\sqrt{L(10 \times 10^{-6})}} \Rightarrow L = 100mH$$

(7) **(A).** Since $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

where, N_s = No. of turns across primary coil = 50
N_p = No. of turns across secondary coil
= 1500

and $V_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t) = 4 \Rightarrow V_s = \frac{1500}{50} \times 4 = 120V$

(8) **(B).** Efficiency of the transformer
$$\eta = \frac{P_{output}}{P_{input}} \times 100 = \frac{100}{220 \times 0.5} \times 100 = 90.9\%$$

(9) **(D).** Average power $= E_{rms} I_{rms} \cos\phi = \frac{E_0 I_0}{2} \cos\phi$

(10) **(D).** Net flux through solenoid
$\phi = 500 \times 4 \times 10^{-3} = 2$ Wb
$\phi = LI \Rightarrow 2Wb = L \times 2 \text{ Amp} \Rightarrow L = 1$ henry

(11) **(B).** $\phi = BA\cos\theta = \frac{1}{\pi} \times \pi (0.2)^2 \cos 60° = 0.02$ Wb

(12) **(C).** As the loop leaves the magnetic field, area in magnetic field decreases for all loops, so induced emf does not remain constant. (Any of four loops)

(13) **(D).** $e = -B\frac{d}{dt}(\pi r^2) = -B\pi 2r \frac{dr}{dt}$,
$e = -0.04 \times \pi \times 2 \times 2 \times 10^{-2} \times 2 \times 10^{-3}$
$= 32\pi \times 10^{-7}$
$= 3.2\pi \times 10^{-6}$ V $= 3.2\pi\mu$V

(14) **(D).** $P = I^2 R = \frac{\varepsilon^2}{|Z|^2} R = \frac{\varepsilon^2 R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$

(15) **(B).** As $V_L = V_C = 300$ V, therefore the given series LCR circuit is in resonance.
$\therefore$ $V_R = V = 220$ V ; $Z = R = 100\,\Omega$

Current, $I = \frac{V}{Z} = \frac{220V}{100\Omega} = 2.2A$

Hence, the reading of the voltmeter V_3 is 220V and the reading of ammeter A is 2.2A.

(16) **(D).** Efficiency $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_s I_s}{V_p I_p}$
$$I_p = \frac{V_s I_s}{\eta V_p} = \frac{(440V)(2A)}{\left(\frac{80}{100}\right)(220V)}$$
$$= \frac{(440V)(2A)(100)}{(80)(220V)} = 5A$$

(17) **(B).** $\frac{dr}{dt} = 1 \times 10^{-3}\ ms^{-1}$
$\phi = BA\cos\theta = B(\pi r^2)\cos 0° = B\pi r^2$
$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(B\pi r^2) = B\pi 2r \frac{dr}{dt}$
$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$
$= \pi \times 10^{-6}$V $= \pi\mu$V

(18) **(D).** In case of oscillatory discharge of a capacitor through an inductor, charge at instant t is given by
$$q = q_0 \cos\omega t, \quad \text{where } \omega = \frac{1}{\sqrt{LC}}$$

$\therefore \quad \cos\omega t = \dfrac{q}{q_0} = \dfrac{CV_2}{CV_1} = \dfrac{V_2}{V_1} \quad (\because q = CV)$ .... (1)

Current through the inductor

$I = \dfrac{dq}{dt} = \dfrac{d\,(q_0 \cos\omega t)}{dt} = -q_0 \omega \sin\omega t$

$|I| = CV_1 \dfrac{1}{\sqrt{LC}} [1 - \cos^2 \omega t]^{1/2}$

$= V_1 \sqrt{\dfrac{C}{L}} \left[1 - \left(\dfrac{V_2}{V_1}\right)^2\right]^{1/2}$

$= \left[\dfrac{C\,(V_1^2 - V_2^2)}{L}\right]^{1/2}$ (using (1))

(19) **(A).** $I_{RMS} = \dfrac{\varepsilon_0/\sqrt{2}}{1/\omega C} = 200 \times 100 \times 10^{-6}\,A$

(20) **(C).** $\tan\theta = \dfrac{X_L}{R} = 1 \;\therefore \phi = 45°$ or $\pi/4$

(21) **(B).** $\varepsilon \propto -\dfrac{di}{dt}$

(22) **(B).** $V_{rms} = \left[\dfrac{1}{T}\displaystyle\int_0^{T/2} V_0^2 dt\right]^{1/2}$

$= \left[\dfrac{V_0^2}{T}[t]_0^{T/2}\right]^{1/2} = \left[\dfrac{V_0^2}{T}\left(\dfrac{T}{2}\right)\right]^{1/2}$

$V_{rms} = \left[\dfrac{V_0^2}{2}\right]^{1/2} = \dfrac{V_0}{\sqrt{2}}$

(23) **(A).** If $\omega = 50 \times 2\pi$ then $\omega L = 20\Omega$

If $\omega' = 100 \times 2\pi$ then $\omega' L = 40\Omega$

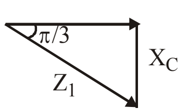
$I = \dfrac{200}{Z} = \dfrac{200}{\sqrt{R^2 + (\omega' L)^2}} = \dfrac{200}{\sqrt{30^2 + (40)^2}} = 4A$

(24) **(D).** $V = -L\dfrac{di}{dt}$. Here di/dt +ve for T/2 time and di/dt is –ve for next T/2 time.

(25) **(A).** Induced e.m.f. $e = -\dfrac{d\phi}{dt} = -(100t)$

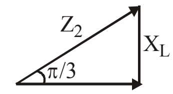
$i = \left|\dfrac{\varepsilon}{R}\right| = +\dfrac{100 \times 2}{400} = +0.5$ Amp

(26) **(C).** $\dfrac{X_C}{R} = \tan\dfrac{\pi}{3}$



$X_C = R\tan\dfrac{\pi}{3}$ .... (1)

$\dfrac{X_L}{R} = \tan\dfrac{\pi}{3}$



$X_L = R\tan\dfrac{\pi}{3}$ .... (2)

Net impedance

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

Power factor $\cos\phi = R/Z = 1$

(27) **(D).** $\langle P \rangle = V_{rms} \cdot I_{rms} \cos\phi$

$V_{rms} = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \dfrac{1}{2}$ volt ; $I_{rms} = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \left(\dfrac{1}{2}\right)$ A

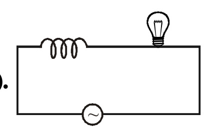
$\cos\phi = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$; $\langle P \rangle = \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{8}$ W

(28) **(B).** Area of i-t graph $= q = \dfrac{1}{2} \times 0.1 \times 4 = 0.2C$

$q = 0.2 = \dfrac{\Delta\phi}{R}$; $\Delta\phi = 2$ weber

(29) **(C).** A wire loop is rotated in magnetic field. The frequency of change of direction of the induced e.m.f. is twice per revolution. (once per half revolution)

(30) **(A).**



Brightness of the bulb

* Decreases when an iron rod is inserted in the coil as impedance of circuit increases.
* Increases when frequency of the AC source is decreased as impedance of circuit decreases.
* Increases when number of turns in the coil is reduced as impedance of circuit decreases.
* Increases when a capacitance of reactance $X_C = X_L$ is included in the circuit as impedance of circuit decreases.

(31) **(D).** $\varepsilon = BL_{eff}v$ (L_{eff} = Diameter)

$= B\,2Rv$

(32) **(B).** Power ouput $= 3kW \times \dfrac{90}{100} = 2.7$ kW ; $I_b = 6A$

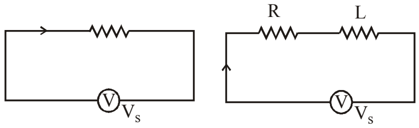
$V_S = \dfrac{2.7\,kW}{6A} = 450$ V ; $I_p = \dfrac{3\,kW}{200V} = 15A$

(33) **(C).** EMF induced $= B_1 V\ell - B_2 V\ell$

$= \dfrac{\mu_0 I}{2\pi\,(x - a/2)}\ell v - \dfrac{\mu_0 I}{2\pi\,(x + a/2)}\ell v$

$\propto \dfrac{1}{(2x - a)\,(2x + a)}$

(34) (D).



$$P = i_{rms}^2 R = \left(\frac{V_s}{R}\right)^2 R = \frac{V_s^2}{R}$$

$$P' = i_{rms}^2 R = \left(\frac{V_s}{Z}\right)^2 R = P\left(\frac{R}{Z}\right)^2$$

(35) (D). First current develops in direction of abcd but when electron moves away, then magnetic field inside loop decreases & current changes its direction.

(36) (C). When capacitor is filled with mica then capacitance C increases so X_C decreases.
In case (b) $X_C \downarrow$ so voltage across capacitor decreases. so $V_a > V_b$.

(37) (D). $N = 1000, I = 4\,A, \phi = 4 \times 10^{-3}$

$$L = \frac{\phi N}{I} = \frac{4 \times 10^{-3} \times 1000}{4} = 1\,H$$

(38) (A). $L = 20\,mH, C = 50\,\mu F, R = 40\Omega$
$X_L = \omega L = 340 \times 20 \times 10^{-3} = 6.8\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{340 \times 50 \times 10^{-6}} = 58.8\,\Omega$$

$$P_{av} = I_v^2 R = \left(\frac{E_v}{Z}\right)^2 R \;;\; E_v = \frac{10}{\sqrt{2}}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 65.6$$

$$P_{av} = \left(\frac{10}{\sqrt{2} \times 65.6}\right)^2 \times 40 = 0.46\,W$$

(39) (B). Current leads voltage by phase $\frac{\pi}{2}\,(90°)$

Power consumed = 0.

(40) (C). Better tuning means low bandwidth = R/L

(41) (D). Magnetic flux linked with area of loop 1 is πr^2

So emf in loop 1 is $-\frac{d\vec{B}}{dt}\pi r^2$

Magnetic flux linked with area of loop 2 is zero.
So emf in loop 2 = 0

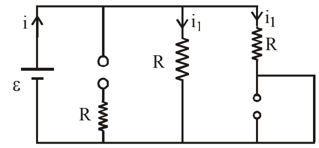
(42) (C). $\cos\theta = \frac{R}{Z} = \frac{V_R}{V} = \frac{80}{\sqrt{80^2 + (100-40)^2}} = \frac{80}{100} = 0.8$

(43) (B). $q = \left[\left(\frac{\Delta\phi}{\Delta t}\right) \cdot \frac{1}{R}\right]\Delta t \;;\; q = \left[\mu_0 n N \pi r^2 \frac{\Delta i}{\Delta t}\right]\frac{1}{R}\Delta t$

$$q = \left[4\pi \times 10^{-7} \times 2 \times 10^4 \times 100 \times \pi \times (10^{-2})^2 \times \left(\frac{4}{0.05}\right)\right]\frac{1}{10\pi^2} \times 0.05$$

$q = 32\,\mu C$

(44) (B). At t = 0, $i_1 = \frac{\varepsilon}{R} = \frac{18}{9} = 2A$



Current through the battery is, $i = 2i_1 = 2 \times 2 = 4A$

(45) (D). Energy stored in inductor, $U = \frac{1}{2}LI^2$

$$25 \times 10^{-3} = \frac{1}{2} \times L \times (60 \times 10^{-3})^2$$

$$L = \frac{25 \times 2 \times 10^6 \times 10^{-3}}{3600} = \frac{500}{36} = 13.89\,H$$

(46) (C). $P_{av} = \left(\frac{V_{rms}}{Z}\right)^2 R$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 56\,\Omega$$

$$P_{av} = \left(\frac{10}{(\sqrt{2})\,56}\right)^2 \times 50 = 0.79\,W$$

(47) (D). Electric heater does not involve Eddy currents. It uses Joule's heating effect.

(48) (D). Magnetic field $B = 5 \times 10^{-5}$ T
Number of turns in coil N = 800
Area of coil $A = 0.05\,m^2$
Time taken to rotate $\Delta t = 0.1$ s
Initial angle $\theta_1 = 0°$
Final angle $\theta_2 = 90°$
Change in magnetic flux $\Delta\phi$
$= NBA\cos 90° - BA\cos 0° = -NBA$
$= -800 \times 5 \times 10^{-5} \times 0.05 = -2 \times 10^{-3}$ weber

$$e = -\frac{\Delta\phi}{\Delta t} = \frac{-(-)\,2 \times 10^{-3}\,Wb}{0.1\,s} = 0.02\,V$$