8

AREA BOUNDED BY CURVE

INTRODUCTION

The process of finding area of some plane region is called Quadrature. In this chapter we shall find the area bounded by some simple plane curves with the help of definite integral. For solving the problems on quadrature easily, if possible first draw the rough sketch of the required area.

CURVE TRACING

In chapter function, we have seen graphs of some simple elementary curves. Here we introduce some essential steps for curve tracing which will enable us to determine the required area.

(i) Symmetry :

The curve $f(x, y) = 0$ is symmetrical

* about x–axis if all terms of y contain even powers.
* about y-axis if all terms of x contain even powers.
* about the origin if $(-x, -y) = f(x, y)$.

For example, $y^2 = 4ax$ is symmetrical about x-axis, $x^2=4ay$ is symmetrical about y–axis and the curve $y=x^3$ is symmetrical about the origin.

(ii) Origin: If the equation of the curve contains no constant term then it passes through the origin.

For example $x^2 + y^2 + 2ax = 0$ passes through origin.

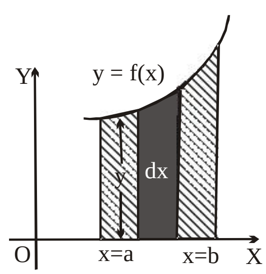
(iii) Points of intersection with the axes : If we get real value of x on putting y = 0 in the equation of the curve, then real values of x and y = 0 give those points where the curve cuts the x-axis. Similarly by putting x = 0, we can get the points of intersection of the curve and y-axis. For example, the curve $x^2/a^2 + y^2/b^2 = 1$ intersects the axes at point $(\pm a, 0)$ & $(0, \pm b)$.

(iv) Region Write the given equation as $y = f(x)$, and find minimum and maximum value of x which determine the region of the curve. $y = a\sqrt{\dfrac{a-x}{x}}$. Now, y is real, if $0 < x \le a$, so its region lies between the lines x = 0 and x = a.

AREA BOUNDED BY A CURVE

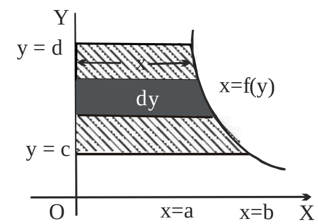
(i) The area bounded by a Cartesian curve y = f(x), x-axis and ordinates x = a and x = b is given by,

$$\text{Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx$$



(ii) The area bounded by a Cartesian curve x = f(y), y-axis and abscissa y = c and y = d is

$$\text{Area} = \int_c^d x\,dy = \int_c^d f(y)\,dy$$



Example 1 :

Find the area bounded by the curve $y = x^3$, x-axis and ordinates x = 1 and x =2

Sol. Required Area $= \int_{x=1}^{2} y dx = \int_1^2 x^3 dx = \left[\dfrac{x^4}{4}\right]_1^2 = \dfrac{15}{4}$

Example 2 :

Find the area bounded by the curve y = sinx, x-axis and the ordinates x = 0 and x = π/2.

Sol. Area $= \int_0^{\pi/2} y\,dx = \int_0^{\pi/2} \sin x\,dx = [-\cos x]_0^{\pi/2} = 1$

Example 3 :

Find the area bounded by the curve y = mx, x-axis and ordinates x = 1 and x =2

Sol. Required Area $= \int_1^2 ydx = \int_1^2 mxdx = \left[\dfrac{mx^2}{2}\right]_1^2$

$$= \frac{m}{2}(4-1) = \left(\frac{3}{2}\right)m$$

Example 4 :

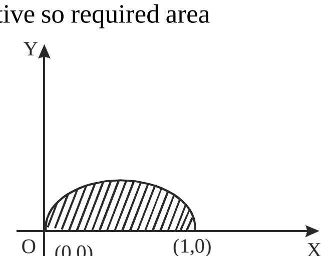
Find the area bounded by the curve $y = x(1-x)^2$ and x -axis,

Sol. Clearly the given curve meets the x-axis at (0, 0) and (1, 0) and for x = 0 to 1, y is positive so required area

$$= \int_0^1 x(1-x)^2 dx$$

$$= \int_0^1 (x - 2x^2 + x^3)dx$$

$$= \left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{2}{3} + \frac{1}{4} = \frac{1}{12}$$



Example 5 :

Find the area bounded by the curve $y^2 = 4x$, y-axis & y = 3.

Sol. $\text{Area} = \int_0^3 x\,dy = \int_0^3 \frac{y^2}{4} dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}(27-0) = 9/4$ units

Example 6 :

Find the area bounded by the curve y = log x ; y-axis and the line y = 2.

Sol. Required Area $= \int_0^2 x\,dy = \int_0^2 e^y\,dy = (e^y)^2_0 = e^2 - 1$

(iii) If the equation of a curve is in parametric form, say x = f(t), y = g(t), then the area $= \int_a^b y\,dx = \int_{t_1}^{t_2} g(t) f'(t)\,dt$

where t_1 and t_2 are the values of t respectively corresponding to the values of a & b of x.

SYMMETRICAL AREA

If the curve is symmetrical about a coordinate axis (or a line or origin), then we find the are of one symmetrical portion and multiply it by the number of symmetrical portion to get the required area.

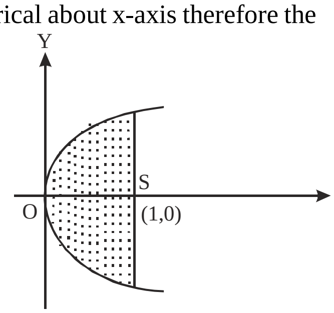
Example 7 :

Find the area bounded by the parabola $y^2 = 4x$ and its latus rectum.

Sol. Since the curve is symmetrical about x-axis therefore the required Area

$= 2\int_0^1 y\,dx = 2\int_0^1 \sqrt{4x}\,dx$

$= 4.\frac{2}{3}\left[x^{3/2}\right]_0^1 = \frac{8}{3}$



POSITIVE AND NEGATIVE AREA

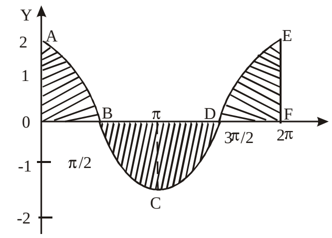
Area is always taken as positive. If some part of the area lies in the positive side i.e., above x-axis and some part lies in the negative side i.e. below x-axis then the area of two parts should be calculated separately and then add their numerical values to get the desired area.

Example 8 :

Find the area bounded by the curve y = 2 cosx and the x-axis from x = 0 to x = 2π.

Sol. The graph of y = 2 cos x from x = 0 to x = 2π is shown in the figure. We have to find out the shaded area. If we integrate directly from x = 0 to x = 2π, the net result will be zero as half of the area is above the x-axis and therefore positive and remaining half is below the x-axis and therefore negative.

Thus to avoid incorrect result, we will find the area from x=0 to x = π/2 (Area OAB) and multiply it by 4.



Area OAB

$= \int_0^{\pi/2} y\,dx = \int_0^{\pi/2} 2\cos x\,dx$

$= \left[2\sin x\right]_0^{\pi/2} = 2\sin\frac{\pi}{2} - 2\sin 0 = 2$

∴ Total area from x = 0 to x = 2π is 4 × 2 = 8q. units.

Example 9 :

Find the area between the curve y = x (x –1) (x – 2) & x-axis.

Sol. Given curve meets x-axis at x = 0,1, 2

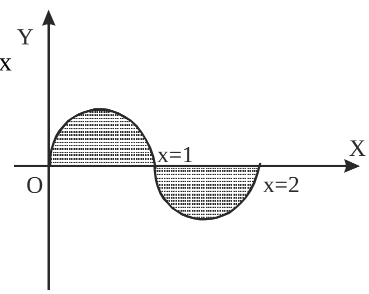
The required area is symmetrical about the point x = 1 as shown in the diagram.

So, reqd. area $= 2\int_0^1 y\,dx$

$= 2\int_0^1 (x^3 - 3x^2 + 2x)\,dx$

$= 2\left[\frac{x^4}{4} - x^3 + x^2\right]_0^1$

$= 2\left(\frac{1}{4} - 1 + 1\right) = \frac{1}{2}$

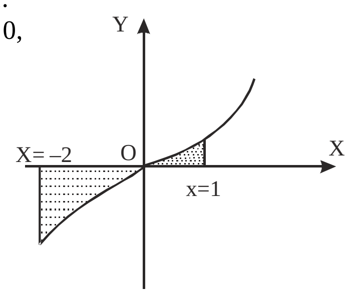


Example 10 :

Find the area bounded by the curve $y = x^3$, x-axis and ordinates x = –2 and x = 1.

Sol. Obviously when $-2 \le x < 0$, then y < 0 and when $0 < x \le 1$, then y > 0

Hence area between x = –2 and x = 0 lies below x-axis and area between x = 0 and x = 1 lies above x-axis. So required area

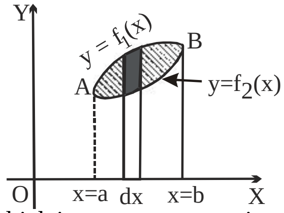


$= \left|\int_{-2}^0 x^3 dx\right| + \int_0^1 x^3 dx$

$= \left|\left[\frac{x^4}{4}\right]_{-2}^0\right| + \left[\frac{x^4}{4}\right]_0^1 = 4 + \frac{1}{4} = \frac{17}{4}$

AREA BETWEEN TWO CURVES

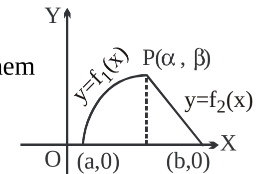
1. When two curves intersect at two points and their common area lies between these points.



If $y = f_1(x)$ and $y = f_2(x)$ are two curves where $f_1(x) > f_2(x)$ which intersect at two points A (x = a) and B(x = b) and their common area lies between A & B, then their common area

$$= \int_a^b (y_1 - y_2)\,dx \quad = \int_a^b [f_1(x) - f_2(x)]\,dx$$

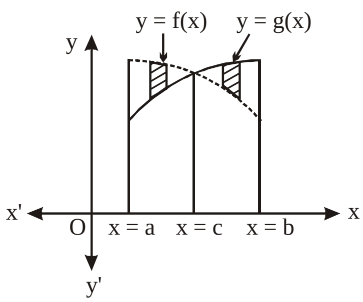
2. When two curves intersect at a point and the area between them is bounded by x-axis.



If $y = f_1(x)$ and $y = f_2(x)$ are two curves which intersect at $P(\alpha,\beta)$ and meet x-axis at A(a, 0) B(b,0) respectively, then area between them and x-axis is given by

$$\text{Area} = \int_a^{\alpha} f_1(x)dx + \int_{\alpha}^{b} f_2(x)$$

3. The area bounded by $y = f(x)$ and $y = g(x)$ (where $a \le x \le b$), when they intersect at $x = c \in (a, b)$ is given by



$$A = \int_a^b |f(x) - g(x)|\,dx$$

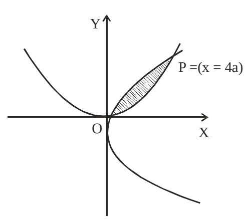
or $\int_a^c (f(x) - g(x))\,dx + \int_c^b (g(x) - f(x))\,dx$

Example 11 :

Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

Sol. Solving the equation of the given curves for x, we get x = 0 and x = 4a.

$$\text{So, reqd area} = \int_0^{4a} \left(\sqrt{4ax} - \frac{x^2}{4a}\right) dx$$



$$= \left[2\frac{2}{3}\sqrt{a}\,x^{3/2} - \frac{x^3}{12a}\right]_0^{4a}$$

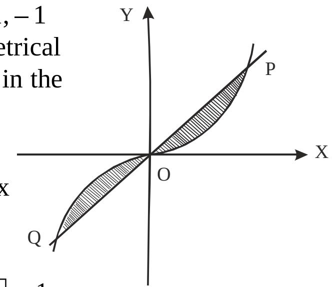
$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2$$

Example 12 :

Find the area between the curves $y = x$ and $y = x^3$.

Sol. Solving the equation of the given curves for x, we get x = 0, 1, – 1

The required area is symmetrical about the origin as shown in the diagram. So



$$\text{Reqd. area} = 2\int_0^1 (x - x^3)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$$

Example 13 :

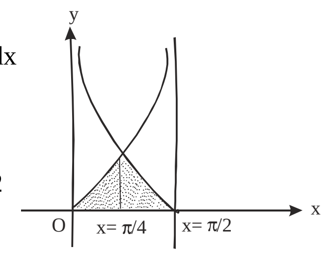
Find the area between the curves y = tanx, y = cotx and x-axis in the interval $[0, \pi/2]$.

Sol. In first quadrant tanx and cotx meet at $x = \pi/4$. Also as shown in the diagram, desired area is bisected at $x = \pi/4$.

$$\text{So, reqd. area} = 2\int_0^{\pi/4} \tan x\,dx$$

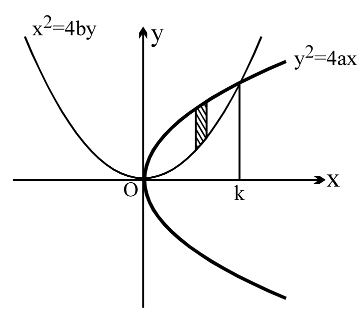
$$= 2[\log \sec x]_0^{\pi/4}$$

$$= 2\left[\log\sqrt{2} - 0\right] = \log 2$$



STANDARD AREAS TO BE REMEMBERED :

(1) Area bounded by the curve $y^2 = 4ax$; $x^2 = 4by$ is equal to $\frac{16\,ab}{3}$:



At point of intersection $\left(\frac{x^2}{4b}\right)^2 = 4ax$

$\Rightarrow \quad x^4 = 64\,ab^2\,x \Rightarrow x = 0,\ (64\,ab^2)^{1/3}$

Let $k = 4\,(ab^2)^{1/3}$

$$A = \int_0^k \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4b}\right) dx = \left[2\sqrt{a}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12b}\right]_0^k$$

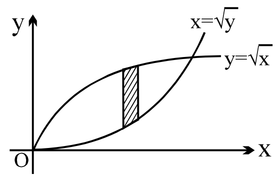
$$= \frac{4\sqrt{a}}{3}k^{\frac{3}{2}} - \frac{k^3}{12b} = \frac{4}{3}\sqrt{a}\ 8\ \left(ab^2\right)^{\frac{1}{2}} - \frac{64(ab^2)}{12b}$$

$$= \frac{32}{3}ab - \frac{16}{3}ab = \frac{16ab}{3}$$

Example 14 :

Find the area bounded by the curve $y = \sqrt{x}$; $x = \sqrt{y}$

Sol. a = 1/4 ; b = 1/4



Required area $= \frac{16ab}{3} = \frac{16 \cdot \frac{1}{4} \cdot \frac{1}{4}}{3}$

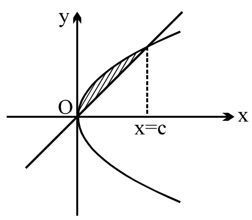
Area = 1/3

(2) Area bounded by the parabola $y^2 = 4ax$ and $y = mx$ is equal

$\frac{8a^2}{3m^3}$ **:** $y^2 = 4ax$ and $y = mx$

At point of intersection

$m^2x^2 = 4ax \Rightarrow x = 0,\ \frac{4a}{m^2}$



Area $= \int_0^c (2\sqrt{a}\sqrt{x} - mx)\,dx$, where $c = \frac{4a}{m^2}$

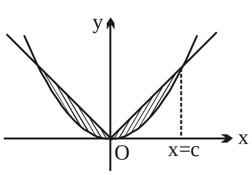
$$= \left(2\sqrt{a}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{mx^2}{2}\right)_0^c = \frac{4\sqrt{a}}{3}c^{\frac{3}{2}} - \frac{mc^2}{2}$$

$$= \frac{4\sqrt{a}}{3} \cdot \frac{8a\sqrt{a}}{m^3} - \frac{m}{2} \cdot \frac{16a^2}{m^4} = \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3}$$

Example 15 :

Find the area bounded by the curves $x^2 = y$; $y = |x|$.

Sol. Area $= 2\left(\frac{8a^2}{3m^3}\right) = 16\frac{(1/4)^2}{3\,(1)^3} = \frac{1}{3}$

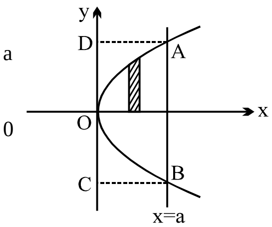


(3) Area enclosed by $y^2 = 4ax$ and its double ordinate at $x = a$:

(chord perpendicular to the axis of symmetry)

Required area = OABO

$$= 2 \cdot \int_0^a (2\sqrt{ax})\,dx = 4\sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^a$$



$$= \frac{8}{3}\sqrt{a} \cdot (a\sqrt{a}) = \frac{8a^2}{3}$$

Area of rectangle ABCD = $4a^2$

$\Rightarrow$ Area of AOB $= \frac{2}{3}$ (area □ABCD)

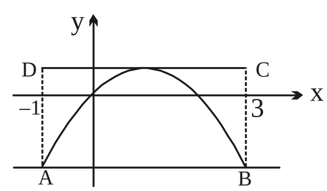
Example 16 :

Find the area bounded by the curve. $y = 2x - x^2$, $y + 3 = 0$

Sol. For point of intersection of $y = 2x - x^2$ and $y + 3 = 0$

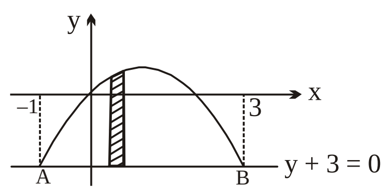
Area (□ABCD) = 4 × 4 = 16

Required area $= \frac{2}{3} \times 16 = \frac{32}{3}$



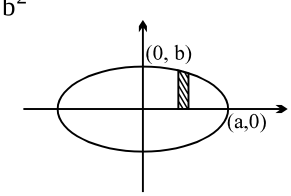
Alternative method :

By integration $A = \int_{-1}^{3} [\,(2x - x^2) - (-3)\,]\,dx = \frac{32}{3}$



(4) Whole area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to ® ab :

$$A = 4\int_0^a \left(b\sqrt{1 - \frac{x^2}{a^2}}\right)dx$$



Put $x = a\sin\theta$

$$A = 4\int_0^{\pi/2} ab\cos^2\theta\,d\theta = 4ab\int_0^{\pi/2}\cos^2\theta\,d\theta$$

$$= 4ab\int_0^{\pi/2}\left(\frac{1 + \cos 2\theta}{2}\right)d\theta = 4ab\left(\frac{\pi}{4}\right) = \pi\ ab$$

Example 17 :

Find the area of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Sol. Area of ellipse = $\pi ab = \pi(4)(3) = 12\pi$

SHIFTING OF ORIGIN

Since area remains invariant even if the coordinates axes are shifted, hence shifting of origin in many cases proves to be very convenient in computing the areas.

Example 18 :

Area enclosed between the parabolas $y^2 - 2y + 4x + 5 = 0$ and $x^2 + 2x - y + 2 = 0$.

Sol. $y^2 - 2y + 1 \Rightarrow (y-1)^2 = -4(x+1)$ ... (1)

$x^2 + 2x + 1 = y - 1 \Rightarrow (x+1)^2 = (y-1)$ ... (2)

Let $y - 1 = Y$ and $x + 1 = X$

So equation $Y^2 = -4X$ and $X^2 = Y$

$a = 1,\ b = 1/4$

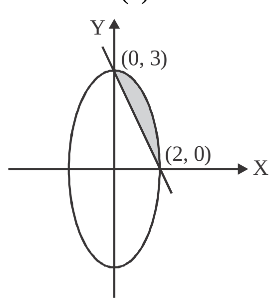
so required area = $\frac{16ab}{3} = \frac{16}{3}\times 1\times\frac{1}{4} = \frac{4}{3}$

Example 19 :

Area enclosed between the ellipse $9x^2 + 4y^2 - 36x + 8y + 4 = 0$ and the line $3x + 2y - 10 = 0$ in the first quadrant.

Sol. $9x^2 + 4y^2 - 36x + 8y + 4 = 0 \Rightarrow 9(x-2)^2 + 4(y+1)^2 = 36$

$\Rightarrow \frac{(x-2)^2}{2^2}+\frac{(y+1)^2}{3^2}=1$ ... (1)



Let $X = x - 2$ and $Y = y + 1$

So equation of ellipse will be

$\frac{X^2}{2^2}+\frac{Y^2}{3^2}=1$

and equation of line

$3x + 2y - 10 = 0$ ... (2)

$3(X+2) + 2(Y-1) - 10 = 0$

$3X + 2Y - 6 = 0$

So required area (shaded region)

$$= \frac{\pi ab}{4} - \frac{1}{2}(ab) = \frac{\pi}{4}(2)(3) - \frac{1}{2}(2)(3) = \frac{3\pi}{2} - 3 = \frac{3(\pi-2)}{2}$$

VARIABLE AREA GREATEST AND LEAST VALUE :

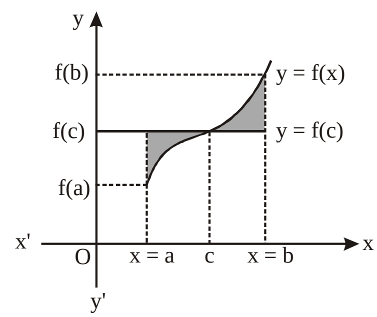
An important concept :

If y = f (x) is a monotonic function in (a, b) then the area bounded by the ordinates at x = a, x = b, y = f (x) and y = f (c), [where $c \in (a, b)$] is minimum when $c = \frac{a+b}{2}$.

Proof : $A = \int_a^c (f(c) - f(x))\,dx + \int_c^b (f(x) - f(c))\,dx$

$$= f(c)(c-a) - \int_a^c (f(x))\,dx + \int_c^b (f(x))\,dx - f(c)(b-c)$$

$$A = [2c - (a+b)]\,f(c) + \int_c^b (f(x))\,dx - \int_a^c (f(x))\,dx$$



Differetiating w.r.t. c,

$$\frac{dA}{dc} = [2c - (a+b)]\ f'(c) + 2f(c) + 0 - f(c) - (f(c))$$

for maxima and minima $\frac{dA}{dc} = 0$

$\Rightarrow\ f'(x)\,[2c - (a+b)] = 0$ (as $f'(c) \neq 0$)

hence $c = \frac{a+b}{2}$

Also $c < \frac{a+b}{2},\ \frac{dA}{dc} < 0$ and $c > \frac{a+b}{2},\ \frac{dA}{dc} > 0$.

Hence A is minimum when $c = \frac{a+b}{2}$.

Note : Let f (x) be the bijective functon and g (x) be the inverse of it then area bounded by y = g(x), and the ordinate at x = a and x = b is same as area bounded by y = f (x) and the abscissa at y = a and y = b as f (x) and g (x) are mirror image with respect to line y = x.

Example 20 :

If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ and the straight lines $x = 0$; $x = 2$ and the x-axis is minimum then find the value of 'a'.

Sol. $f(x) = \frac{x^3}{3} - x^2 + a$

$f'(x) = x^2 - 2x = x(x-2) < 0$ (note that f(x) is monotonic in (0, 2)). Hence for the minimum and f(x) must cross the x-axis at $\frac{0+2}{2} = 1$. Hence $f(1) = \frac{1}{3} - 1 + a = 0 \Rightarrow a = \frac{2}{3}$.

Example 21 :

The value of the parameter a for which the area of the figure bounded by the abscissa axis, the graph of the function $y = x^3 + 3x^2 + x + a$ and the straight lines, which are parallel to the axis of ordinates and cut the abscissa axis at the point of extremum of the function, is the least, is

(A) 2 (B) 0

(C) – 1 (D) 1

Sol. $f(x) = x^3 + 3x^2 + x + a$

$$f'(x) = 3x^2 + 6x + 1 = 0 \Rightarrow x = -1 \pm \frac{\sqrt{6}}{3}$$

Hence, f (x) cuts the x-axis at

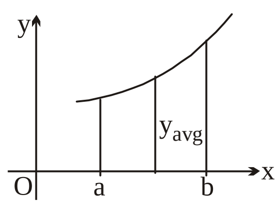
$$\frac{1}{2}\left[\left(-1+\frac{\sqrt{6}}{3}\right)+\left(-1-\frac{\sqrt{6}}{3}\right)\right] = -1$$

$f(-1) = -1 + 3 - 1 + a = 0$; $a = -1$

AVERAGE VALUE OF A FUNCTION

Average value of the function in y = f (x) w.r.t. x over an interval $a \le x \le b$ is defined as

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$



Note :

(i) Average value can be + ve, – ve or zero.

(ii) If the function is defined in $(0, \infty)$ then

$$y_{avg} = \lim_{b\to\infty} \frac{1}{b}\int_0^b f(x)\,dx$$ provided the limit exists.

Root mean square value (RMS) is defined as

$$\rho = \left[\frac{1}{b-a}\int_a^b f^2(x)\,dx\right]^{1/2}$$

Example 22 :

Compute the average value of $f(x) = \dfrac{\cos^2 x}{\sin^2 x + 4\cos^2 x}$ in $[0, \pi/2]$.

Sol. Average value of $f(x) = \dfrac{\cos^2 x}{\sin^2 x + 4\cos^2 x}$

$$y_{average} = \frac{1}{\left(\frac{\pi}{2}-0\right)}\int_0^{\pi/2}\frac{\cos^2 x}{(\sin^2 x + 4\cos^2 x)}dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\frac{1}{(\tan^2 x + 4)}dx = \frac{2}{\pi}\int_0^{\pi/2}\frac{\sec^2 x}{\sec^2 x(\tan^2 x + 4)}dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\frac{\sec^2 x\,dx}{(1+\tan^2 x)(4+\tan^2 x)}$$

Put $t = \tan x$ so $dt = \sec^2 x\,dx$

$$y_{average} = \frac{2}{\pi}\int_0^{\infty}\frac{dt}{(t^2+1)(4+t^2)} = \frac{2}{3\pi}\int_0^{\infty}\left[\frac{1}{t^2+1}-\frac{1}{t^2+4}\right]dt$$

$$= \frac{2}{3\pi}\left[\tan^{-1} t - \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty} = \frac{2}{3\pi}\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{2}{3\pi}\frac{\pi}{4} = \frac{1}{6}$$

DETERMINATION OF FUNCTION

The area function A(x) satisfies the differential equation $\dfrac{dA(x)}{dx} = f(x)$ with initial condition A(a) = 0 i.e. derivative of the area function is the function itself.

Note : If F (x) is any integral of f (x) then ,

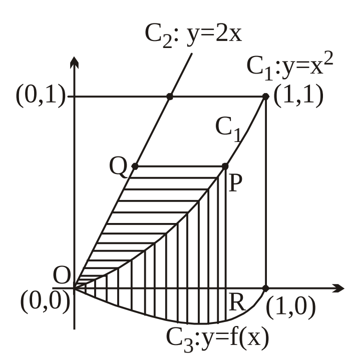
$A(x) = \int f(x)\,dx = [F(x) + c]$

$A(a) = 0 = F(a) + c \Rightarrow c = -F(a)$

hence $A(x) = F(x) - F(a)$. Finally by taking x = b we get , $A(b) = F(b) - F(a)$.

Example 23 :

Let C_1 & C_2 be the graphs of the functions $y = x^2$ & $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 & C_3 at Q & R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ & ORP are equal, determine the function f(x).



Sol. Let $P(h, h^2)$ be a point on the curve C_1.

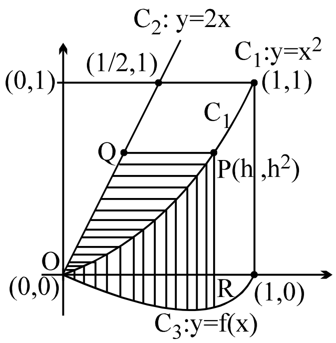
$\Rightarrow$ $R(h, f(h))$

Area OPQO = Area OPRO

$$\int_0^{h^2}\left(\sqrt{y}-\frac{y}{2}\right)dy = \int_0^{h}\left(x^2 - f(x)\right)dx$$

Differenting w.r.t. h

$$\left(\sqrt{h^2}-\frac{h^2}{2}\right).2h = h^2 - f(h)$$



$\Rightarrow$ $2h^2 - h^3 = h^2 - f(h) \Rightarrow f(h) = h^3 - h^2 \Rightarrow f(x) = x^3 - x^2$

AREA ENCLOSED IN CASE ONE CURVE ARE EXPRESSED IN POLAR FORM :

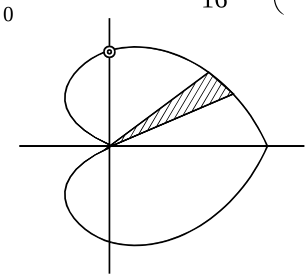
Area of any curve = $\frac{1}{2}\int r^2 d\theta$

Example 24 :

Find the area of the cardioid $r = a(1 + \cos\theta)$

Sol. $A = \frac{1}{2}\int_0^{2\pi} r^2\, d\theta = \frac{a^2}{2}\int_0^{2\pi} 4\cos^4\frac{\theta}{2}d\theta$ put $\frac{\theta}{2} = t$

$$A = a^2\int_0^{\pi}4\cos^4 t\,dt = 8\times\frac{3\pi a^2}{16} = \left(\frac{3\pi a^2}{2}\right)$$



AREA IN RESPECT OF CURVE REPRESENTED PARAMETRICALLY

Example 25 :

Find the area enclosed by the curves $x = a\sin^3 t$ and $y = a\cos^3 t$

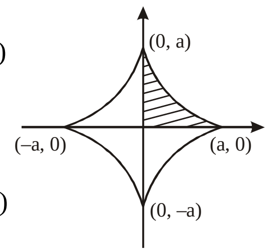
Sol. $x^{2/3} + y^{2/3} = a^{2/3}$

Required area = $4\int_0^a (a^{2/3} - x^{2/3})^{3/2}\,dx$

Put $x = a\sin^3 t$; $dx = 3a\sin^2 t\cos t\,dt$

Area = $4\int_0^{\pi/2}(a^{2/3} - a^{2/3}\sin^2 t)^{3/2}\ 3a\sin^2 t\cos t\,dt$

$A = 12a^2\int_0^{\pi/2}\sin^2 t\cos^4 t\,dt$ ... (1)



$A = 12a^2\int_0^{\pi/2}\sin^4 t\cos^2 t\,dt$ ... (2)

Adding (i) and (ii)

$$A = \frac{12a^2}{2}\int_0^{\pi/2}\cos^2 t\sin^2 t\ (\cos^2 t + \sin^2 t)\,dt$$

$$= 6a^2\int_0^{\pi/2}\sin^2 t\cos^2 t\,dt = 6a^2\int_0^{\pi/2}\frac{\sin^2 2t}{4}$$

$$= \frac{3a^2}{2}\int_0^{\pi/2}\left(\frac{1-\cos 4t}{2}\right)dt$$

$$= \frac{3a^2}{4}\left(t - \frac{\sin 4t}{4}\right)_0^{\pi/2} = \frac{3a^2}{4}\left(\frac{\pi}{2}\right) = \frac{3\pi a^2}{8}$$

TRY IT YOURSELF

Q.1 Compute the area enclosed between $y = \tan^{-1}x$; $y = \cot^{-1}x$ and y-axis.

Q.2 Compute the larger area bounded by $y = 4 + 3x - x^2$ and the coordinates axes.

Q.3 The area of the region enclosed by the curves $y = x\log x$ and $y = 2x - 2x^2$ is –

(A) (7/12) sq. units (B) (1/2) sq. units

(C) (5/12) sq. units (D) None of these

Q.4 The area bounded by the curve $a^2y = x^2(x + a)$ and x-axis is

(A) $(a^2/3)$ sq. units (B) $(a^2/4)$ sq. units

(C) $(3a^2/4)$ sq. units (D) $(a^2/12)$ sq. units

Q.5 Find the area enclosed by the curve $(y - \sin^{-1}x)^2 = x - x^2$.

Q.6 If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ and the straight lines x = 0 ; x = 2 and the x-axis is minimum then find the value of a.

Q.7 The area from 0 to x under a certain graph is given to be $A = \sqrt{1+3x} - 1,\ x \geq 0$:

(a) Find the average of change of A w.r.t. x as x increases from 1 to 8.

(b) Find the instantaneous rate of change of A w.r.t. x at x = 5.

(c) Find the ordinate (height) y of the graph as a function of x.

(d) Find the average value of the ordinate (height) y, w.r.t. x as x increases from 1 to 8.

ANSWERS

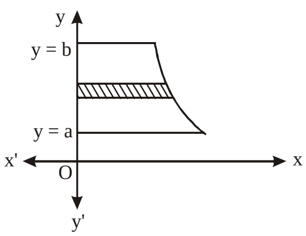
(1) ln 2	**(2)** 56/3	**(3)** (A)
(4) (D)	**(5)** $\pi/4$	**(6)** 2/3

(7) (a) 3/7, (b) 3/8, (c) $\dfrac{3}{2\sqrt{1+3x}}$ (d) 3/7

IMPORTANT POINTS

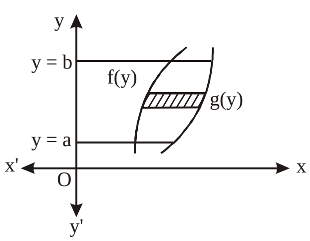
DIFFERENT CASES OF BOUNDED AREA

1. The area bounded by the continuous curve $x = f(y)$, the axis of y and the abscissa y = a and y = b (where b > a) is given by $A = \int_a^b f(y)dy = \int_a^b x\,dy$



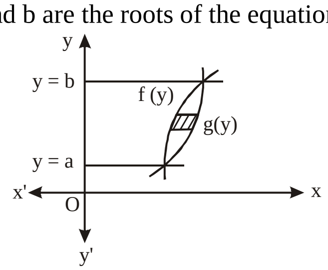
2. The area bounded by the straight line $y = a$, $y = b$ $(a < b)$ and the curves $x = f(y)$ and $x = g(y)$, provided $f(y) < g(y)$ (where $a \le y \le b$), is given by

$$A = \int_a^b [g(y) - f(y)]dy$$



3. When two curves $x = f(y)$ and $x = g(y)$ intersect, the bounded area is $A = \int_a^b [g(y) - f(y)]\,dy$; where a < b.

where a and b are the roots of the equation $f(y) = g(y)$

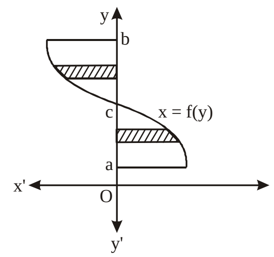


4. If some part of a curve lies left to y-axis, then its area becomes negative but area cannot be negative. Therefore, we take its modulus.

If the curves crosses the y-axis at c, then the area bounded by the curve $x = f(y)$ and abscissae $y = a$ and $y = b$ (where $b > a$) is given by

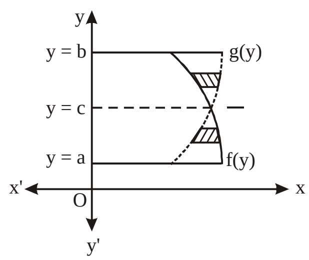
$$A = \left|\int_a^c f(y)\,dy\right| + \left|\int_c^b f(y)\,dy\right|$$

$$= A = \int_a^c f(y)\,dy - \int_c^b f(y)\,dy$$



5. The area bounded by $x = f(y)$ and $x = g(y)$ (where $a \le y \le b$), when they intersect at $y = c \in (a, b)$ is given by $A = \int_a^b |f(y) - g(y)|\,dy$

or $\int_a^c (f(y) - g(y))\,dy + \int_c^b (g(y) - f(y))\,dy$

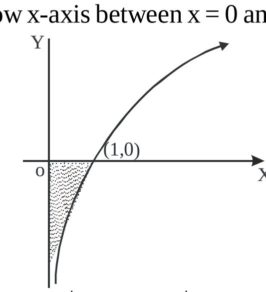


ADDITIONAL EXAMPLES

Example 1 :

Find the area bounded by the curve y = log x and the coordinate axes.

Sol. Observing to the graph of log x, we find that the required area lies below x-axis between x = 0 and x = 1.



So required area = $\left|\int_0^1 \log x\,dx\right| = |(x\log x - x)_0^1| = |-1| = 1$

$$\left[\begin{array}{l} \because \lim\limits_{x\to 0}(x\log x) = \lim\limits_{x\to 0}\dfrac{\log x}{1/x}\left(\dfrac{\infty}{\infty}\right) \\ = \lim\limits_{x\to 0}\dfrac{1/x}{-1/x^2} = \lim\limits_{x\to 0}(-x) = 0 \end{array}\right]$$

Example 2 :

Find the area bounded by the curve $x = at^2$, $y = 2at$ and the x-axis in $1 \le t \le 3$.

Sol. Eliminating t, we get $y^2 = 4ax$

For t = 1, x = a and for t = 3, x = 9 a

$$\therefore \quad \text{Reqd. area} = \int_a^{9a} |y|\,dx = \int_a^{9a} 2\sqrt{a}\sqrt{x}\,dx$$

$$= 2\sqrt{a}\;2\left|\frac{x^{3/2}}{3}\right|_a^{9a} = \frac{4}{3}\sqrt{a}[(9a)^{3/2} - a^{3/2}]$$

Example 3 :

Find the area between the curves $y = \sqrt{x}$ and y = x.

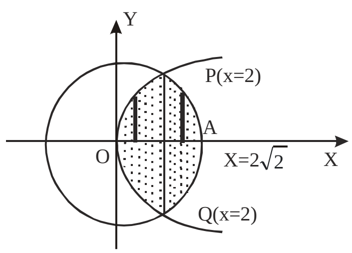
Sol. The points of intersection of curves are x = 0 and x = 1

$$\text{Reqd area } = \int_0^1 (\sqrt{x} - x)\,dx = \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

Example 4 :

Find the area of the smaller portion between curves $x^2 + y^2 = 8$ and $y^2 = 2x$.

Sol. Two curves meet at P and Q where x = 2.



Obviously the required area lies between x = 0 and $x = 2\sqrt{2}$. It is symmetrical about x-axis and bounded by two given curves. So required area

$$= 2\left[\int_0^2 \sqrt{2}\sqrt{x}\,dx + \int_2^{2\sqrt{2}} \sqrt{8 - x^2}\,dx\right]$$

$$= 2\left[\left(\frac{2\sqrt{2}}{3}x^{3/2}\right)_0^2 + \left(\frac{x}{2}\sqrt{8 - x^2} + 4\sin^{-1}\frac{x}{2\sqrt{2}}\right)_2^{2\sqrt{2}}\right]$$

$$= 2\left[\left(\frac{8}{3} - 0\right) + (2\pi - 2 - \pi)\right] = 2\pi + \frac{4}{3}$$

Example 5 :

Find the area bounded by the curve y = (x – 1) (x – 2) (x – 3) lying between the ordinates x = 0 and x = 3.

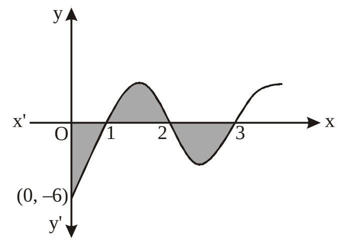
Sol. y = (x – 1) (x – 2) (x – 3)

The curves will intersect the x-axis, when y = 0.

$\Rightarrow$ (x – 1) (x – 2) (x – 3) = 0 $\Rightarrow$ x = 1, 2, 3

And the curve intersects the y-axis, when x = 0 $\Rightarrow$ y = –6

Thus, the graph of the given function for $0 \le x \le 3$ is as shown in figure.



Hence, the required area A = shaded area

$$= \left|\int_0^1 y\,dx\right| + \left|\int_1^2 y\,dx\right| + \left|\int_2^3 y\,dx\right|$$

Since $\int y\,dx = \int (x - 1)(x - 2)(x - 3)\,dx$

$$= \int (x^3 - 6x^2 + 11x - 6)\,dx = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x$$

From equation (1)

$$A = \left|\left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right]_0^1\right| + \left|\left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right]_1^2\right|$$

$$+ \left|\left[\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right]_2^3\right|$$

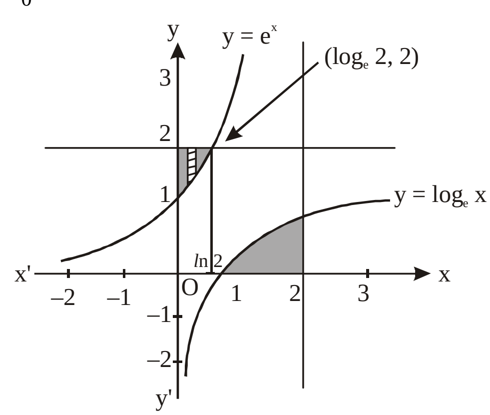
= | –9/4 | + (1/4) + | –1/4 | = 11/4 sq. units

Example 6 :

Consider the region formed by the linex x = 0, y = 0, x = 2, y = 2. Area enclosed by the curves $y = e^x$ and y = ln x, within this region, is being removed. Then, find the area of the remaining region.

Sol. Required area = shaded region

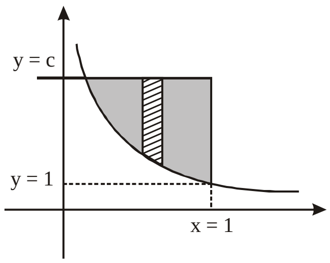
$$= 2\int_0^{\ln 2} (2 - e^x)\,dx = 2\,[2x - e^x]_0^{\ln 2} = 2\,(2\ln 2 - 1) \text{ sq. units}$$



Example 7 :

Find the value of c for which the area of the figure bounded by the curves $y = \frac{4}{x^2}$; $x = 1$ and $y = c$ is equal to $\frac{9}{4}$.

Sol.



Required area = $\int_{2/\sqrt{c}}^{1} \left(c - \frac{4}{x^2}\right) dx = \left[cx + \frac{4}{x}\right]_{2/\sqrt{c}}^{1}$

Area = $c\left(1 - \frac{2}{\sqrt{c}}\right) + 4 - 2\sqrt{c} = c - 4\sqrt{c} + 4 = \frac{9}{4}$

$\Rightarrow (\sqrt{c} - 2)^2 = \frac{9}{4} \Rightarrow \sqrt{c} = 2 \pm \frac{3}{2}$

$\sqrt{c} = \frac{1}{2}, \frac{7}{2}$; $c = \frac{1}{4}, \frac{49}{4}$

Example 8 :

If the area bounded by $y = x^2 + 2x - 3$ and the line $y = kx + 1$ is the least, find k and also the least area.

Sol. x_1 and x_2 are the roots of the equation

$x^2 + 2x - 3 = kx + 1$, or $x^2 + (2-k)x - 4 = 0$

$$\Rightarrow \left.\begin{array}{l} x_1 + x_2 = k - 2 \\ x_1 x_2 = -4 \end{array}\right\}$$

$$A = \int_{x_1}^{x_2} [(kx+1) - (x^2 + 2x - 3)]\, dx$$

$$= \left[(k-2)\frac{x^2}{2} - \frac{x^3}{2} + 4x\right]_{x_1}^{x_2}$$

$$= \left[(k-2)\frac{x_2^2 - x_1^2}{2} - \frac{1}{3}\left(x_2^3 - x_1^3\right) + 4(x_2 - x_1)\right]$$

$$= (x_2 - x_1)\left[\frac{(k-2)^2}{2} - \frac{1}{3}\left((x_2 + x_1)^2 - x_1 x_2\right) + 4\right]$$

$$= \sqrt{(x_2 + x_1)^2 - 4x_1 x_2}\left[\frac{(k-2)^2}{2} - \frac{1}{3}\left((k-2)^2 + 4\right) + 4\right]$$

$$= \frac{\sqrt{(k-2)^2 + 16}}{6}\left[\frac{1}{6}(k-2)^2 + \frac{8}{3}\right]$$

$$= \frac{\left[(k-2)^2 + 16\right]^{3/2}}{6}$$

which is least when k = 2 and A_{least} = 32/3 sq. units.

QUESTION BANK

CHAPTER 8 : AREA BOUNDED BY CURVE

EXERCISE - 1 [LEVEL-1]

Q.1 The area bounded by the curve $y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$ and the line y = 4 is –
(A) 32/3 (B) 8/3
(C) 40/3 (D) 16/3

Q.2 If the area between $y = mx^2$ and $x = my^2$ (m > 0) is 1/4 sq units, then value of m is –
(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) $\pm 2/\sqrt{3}$ (D) $\pm 3\sqrt{2}$

Q.3 The area bounded by the curve y = sin (x/3), x-axis and lines x = 0 and x = 3π is –
(A) 9 (B) 0
(C) 6 (D) 3

Q.4 The area of the region bounded by the lines y = mx, x = 1, x = 2, and x axis is 6 sq. units, then 'm' is
(A) 3 (B) 1
(C) 2 (D) 4

Q.5 Area of the region bounded by two parabolas $y = x^2$ and $x = y^2$ is –
(A) 1/4 (B) 1/3
(C) 4 (D) 3

Q.6 Area bounded by $y = x^3$, y = 8 and x = 0 is –
(A) 2 sq. units (B) 14 sq. units
(C) 12 sq. units (D) 6 sq. units

Q.7 Area bounded by the curves $y = |x| - 2$ and $y = 1 - |x - 1|$ is equal to
(A) 4 sq. units (B) 6 sq. units
(C) 2 sq. units (D) 8 sq. units

Q.8 For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line y = mx equals 9/2?
(A) – 4 (B) 1/2
(C) 2 (D) 4

Q.9 The area bounded by the curve f (x) = (x + sinx) and its inverse between the ordinates x = 0 to x = 2π is –
(A) 4 square units (B) 8 square units
(C) 4π square unit (D) 8π square units

Q.10 The area of the region bounded between the curves $|y| - |\sin x| \geq 0$ and $x^2 + y^2 - \pi^2 \leq 0$ is $\pi^3 - A$. Find the value of A.
(A) 8 (B) 7
(C) 6 (D) 5

Q.11 Area common to the curve $y = \sqrt{9 - x^2}$ & $x^2 + y^2 = 6x$ is–
(A) $\frac{\pi + \sqrt{3}}{4}$ (B) $\frac{\pi - \sqrt{3}}{4}$
(C) $3\left(\pi + \frac{\sqrt{3}}{4}\right)$ (D) $3\left(\pi - \frac{3\sqrt{3}}{4}\right)$

Q.12 The area between the curves y = x and $y = x^3$ is
(A) 1/4 (B) 1/2
(C) 1/3 (D) 1

Q.13 The total area of the curve $a^2y^2 = x^2 (a^2 - x^2)$ is
(A) $\frac{2}{3}a^2$ (B) a^2
(C) $\frac{\pi a^2}{2}$ (D) $\frac{4a^2}{3}$

Q.14 The area of the figure bounded by $y = e^x$, $y = e^x$ and x=1
(A) 2 (e – 1) (B) $e + \frac{1}{e} - 2$
(C) $e - \frac{1}{e} + 2$ (D) $e + \frac{1}{e}$

Q.15 The area of the region bounded by y = | x – 1 | and y = 1 is
(A) 2 (B) 1
(C) 1/2 (D) 1/4

EXERCISE - 2 [LEVEL-2]

Q.1 Find the area between the parabola $x^2 = 4y$ and line x = 4y – 2.
(A) 7/4 (B) 9/8
(C) 5/4 (D) 8/4

Q.2 The area enclosed by $y = x^3$, its normal at (1, 1) and x-axis is equal to –
(A) 7/4 (B) 9/4
(C) 5/4 (D) 8/4

Q.3 The area bounded by the curve $y = (x^2 + 2x + 1)$ and tangent at (1, 4) and y-axis is –
(A) 2/3 square unit (B) 1/3 square unit
(C) 2 square unit (D) 4/3 square unit

Q.4 The area bounded between normals drawn to the circle $x^2 + y^2 = 4$ at its point of intersection with curve $y = \sqrt{\sqrt{2}\,|x|}$ and curve $y^4 = 2x^2$ is –
(A) 1/3 (B) 2/3
(C) 1 (D) 4/3

Q.5 The area bounded by the curve $y = x^2 - 1$ & the straight line x + y = 3 is :
(A) 9/2 (B) 4
(C) $\frac{7\sqrt{17}}{2}$ (D) $\frac{17\sqrt{17}}{6}$

Q.6 If A_m represents the area bounded by the curve $y = \ln x^m$, the x-axis and the lines x = 1 and x = e, then $A_m + m A_{m-1}$ is-
(A) m (B) m^2
(C) $m^2/2$ (D) $m^2 - 1$

Q.7 The area bounded by the curve
$y^2 = 4\sqrt{3}\left(\sqrt{3} - |x - \sqrt{3}|\right)$ is (in sq.units)
(A) 8 (B) 16
(C) 24 (D) 32

Q.8 The area enclosed by the curve $y = \sqrt{x}$ & $x = -\sqrt{y}$, the circle $x^2 + y^2 = 2$ above the x-axis, is –
(A) $\pi/4$ (B) $3\pi/2$
(C) π (D) $\pi/2$

Q.9 The area bounded by the circle $x^2 + y^2 = 1$ and the curve $|x| + |y| = 1$ is
(A) $\pi - 2$ (B) $\pi - 2\sqrt{2}$
(C) $2(\pi - 2\sqrt{2})$ (D) None of these

Q.10 The area bounded by the curves $y = x(1 - \ln x)$; $x = e^{-1}$ and positive X-axis between $x = e^{-1}$ and x = e is :
(A) $\left(\frac{e^2 - 4e^{-2}}{5}\right)$ (B) $\left(\frac{e^2 - 5e^{-2}}{4}\right)$
(C) $\left(\frac{4e^2 - e^{-2}}{5}\right)$ (D) $\left(\frac{5e^2 - e^{-2}}{4}\right)$

Q.11 Find the area of the region bounded by $y = \log_e x$ and $y = \sin^4 \pi x$.
(A) 1/8 (B) 11/8
(C) 3/8 (D) 2/7

Q.12 Find the area (in sq. units) of the figure enclosed by the curve $5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0$
(A) $\pi/2$ (B) $\pi/4$
(C) $\pi/3$ (D) $\pi/6$

Directions : Assertion-Reason type questions.
(A) Statement-1 is True, Statement-2 is True, Statement2 is a correct explanation for Statement -1
(B) Statement-1 is True, Statement -2 is True; Statement2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.13 **Statement -1 :** Area bounded by parabola $y = x^2 - 4x + 3$ and y = 0 is 4/3 sq. units.
Statement -2 : Area bounded by curve $y = f(x) \geq 0$ and y=0 between ordinates x = a and x = b (b > a) is $\int_a^b f(x)\,dx$.

Q.14 **Statement–1 :** The area enclosed by the curves $y = \cos x$, $y = 1 + \sin 2x$ and $x = \frac{3\pi}{2}$ equals $2 + \frac{3\pi}{2}$
Statement–2 : $A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)\,dx = 2$

Passage (Q.15-Q.17)
Consider one side AB of a square ABCD, (read in order) on the line y = 2x – 17, and the other two vertices C, D on the parabola $y = x^2$.

Q.15 Minimum intercept of the line CD on y-axis is –
(A) 3 (B) 4
(C) 2 (D) 6

Q.16 Maximum possible area of the square ABCD can be –
(A) 980 (B) 1160
(C) 1280 (D) 1520

Q.17 The area enclosed by the line CD with minimum y-intercept and the parabola $y = x^2$ is –
(A) 15/3 (B) 14/3
(C) 22/3 (D) 32/3

The answer to each question is a NUMERICAL VALUE.

Q.18 If A be the area bounded by the curves y = | x – 1 | and $y + \frac{3}{|x+1|} = 2$, then find the value of (2A + 3 ln 3).

Q.19 Let $0 \leq a \leq 4$. If the maximum area bounded by the curves $y = 1 - |x - 1|$ and $y = |2x - a|$ is A then A = 1/P. Find the value of P.

Q.20 If $y = 2\sin x + \sin 2x$ for $0 \leq x \leq 2\pi$, then the area enclosed by the curve and the x-axis is

Q.21 The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$ where $x, y \leq 0$ is 1/A. Find the value of A.

Q.22 The value of 'a' (a>0) for which the area bounded by the curves $y = \frac{x}{6} + \frac{1}{x^2}$, y = 0, x = a and x = 2a has the least value is –

Q.23 The area bounded by the curves y = | x | – 1 and y = – | x | + 1 is

Q.24 The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x–axis in the 1st quadrant is

Q.25 The area enclosed between the curves $y = ax^2$ and $a = ay^2$ (a > 0) is 1 sq. unit, then the value of a is $1/\sqrt{X}$. Find the value of X.

Q.26 The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line y = 1/4 is 1/X sq. units. Find the value of X.

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 If the area bounded by the x- axis, curve y = f(x) and the lines x = 1, x = b is equal to $\sqrt{b^2+1}-\sqrt{2}$ for all b > 1, then f (x) is **[AIEEE 2002]**

(A) $\sqrt{(x-1)}$ (B) $\sqrt{(x+1)}$

(C) $\sqrt{(x^2+1)}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Q.2 The area of the region bounded by the curves $y=|x-1|$ and $y = 3 - |x|$ is- **[AIEEE 2003]**

(A) 6 sq. units (B) 2 sq. units
(C) 3 sq. units (D) 4 sq. units

Q.3 The area of the region bounded by the curves $y = |x - 2|$, x = 1, x = 3 and the x- axis is- **[AIEEE 2004]**

(A) 1 (B) 2
(C) 3 (D) 4

Q.4 Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is - **[AIEEE-2005]**

(A) 2ab (B) ab
(C) $\sqrt{ab}$ (D) a/b

Q.5 The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate axes is - **[AIEEE-2005]**

(A) 1 (B) 2
(C) 3 (D) 4

Q.6 The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is - **[AIEEE-2005]**

(A) 1 : 2 : 1 (B) 1 : 2 : 3
(C) 2 : 1 : 2 (D) 1 : 1 : 1

Q.7 Let f (x) be a non-negative continuous function such that the area bounded by the curve y = f (x), x-axis and the ordinates x = π/4 and x = β > π/4 is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is -**[AIEEE-2005]**

(A) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (B) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(C) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (D) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

Q.8 The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[AIEEE 2007]**

(A) 2/3 (B) 1
(C) 1/6 (D) 1/3

Q.9 The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to - **[AIEEE 2008]**

(A) 1/3 (B) 2/3
(C) 4/3 (D) 5/3

Q.10 The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x – axis is- **[AIEEE 2009]**

(A) 3 (B) 6
(C) 9 (D) 12

Q.11 The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and x = 3π/2 is **[AIEEE 2010]**

(A) $4\sqrt{2}+2$ (B) $4\sqrt{2}-1$

(C) $4\sqrt{2}+1$ (D) $4\sqrt{2}-2$

Q.12 The area of the region enclosed by the curves y = x, x = e, y = 1/x and the positive x-axis is – **[AIEEE 2011]**

(A) 1/2 square units (B) 1 square units
(C) 3/2 square units (D) 5/2 square units

Q.13 The area bounded between the parabolas $x^2 = y/4$ and $x^2 = 9y$ and the straight line y = 2 is – **[AIEEE 2012]**

(A) $20\sqrt{2}$ (B) $\frac{10\sqrt{2}}{3}$ (C) $\frac{20\sqrt{2}}{3}$ (D) $10\sqrt{2}$

Q.14 The area (in square units) bounded by the curves $y=\sqrt{x}$, 2y – x + 3 = 0, x-axis, and lying in the first quadrant is – **[JEE MAIN 2013]**

(A) 9 (B) 36
(C) 18 (D) 27/4

Q.15 The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is **[JEE MAIN 2014]**

(A) $\frac{\pi}{2}+\frac{4}{3}$ (B) $\frac{\pi}{2}-\frac{4}{3}$ (C) $\frac{\pi}{2}-\frac{2}{3}$ (D) $\frac{\pi}{2}+\frac{2}{3}$

Q.16 The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is **[JEE MAIN 2015]**

(A) 5/64 (B) 15/64
(C) 9/32 (D) 7/32

Q.17 The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is – **[JEE MAIN 2016]**

(A) $\pi-\frac{8}{3}$ (B) $\pi-\frac{4\sqrt{2}}{3}$ (C) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$ (D) $\pi-\frac{4}{3}$

Q.18 The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is :

(A) 7/3 (B) 5/2 **[JEE MAIN 2017]**
(C) 59/12 (D) 3/2

Q.19 Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve y = (gof) x & the lines x = α, x = β and y = 0, is **[JEE MAIN 2018]**

(A) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ (B) $\frac{1}{2}(\sqrt{2}-1)$

(C) $\frac{1}{2}(\sqrt{3}-1)$ (D) $\frac{1}{2}(\sqrt{3}+1)$

Q.20 The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is : **[JEE MAIN 2019 (JAN)]**
(A) 14/3 (B) 56/3
(C) 8/3 (4D 32/3

Q.21 The area (in sq. units) of the region $A = \{(x, y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is : **[JEE MAIN 2019 (APRIL)]**
(A) 53 / 6 (B) 59 / 6
(C) 8 (D) 26 / 3

Q.22 Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0<\lambda<4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals **[JEE MAIN 2019 (APRIL)]**
(A) $2\,(4/25)^{1/3}$ (B) $4\,(4/25)^{1/3}$
(C) $2\,(2/5)^{1/3}$ (D) $2\,(4/5)^{1/3}$

Q.23 The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is **[JEE MAIN 2019 (APRIL)]**
(A) 10/3 (B) 9/2
(C) 31/6 (D) 13/6

Q.24 The area (in sq. units) of the region
$A = \{(x,y) : \frac{y^2}{2} \le x \le y + 4\}$ is **[JEE MAIN 2019 (APRIL)]**
(A) 53/3 (B) 18
(C) 30 (D) 16

Q.25 The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is : **[JEE MAIN 2019 (APRIL)]**
(A) $\frac{3}{2} - \frac{1}{\log_e 2}$ (B) $\frac{1}{2}$ (C) $\log_e 2 + \frac{3}{2}$ (D) $\frac{3}{2}$

Q.26 If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to : **[JEE MAIN 2019 (APRIL)]**
(A) 8/3 (B) 10/3
(C) 6 (D) –2/3

Q.27 If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is 1/9, then λ is equal to **[JEE MAIN 2019 (APRIL)]**
(A) 24 (B) 48
(C) $4\sqrt{3}$ (D) $2\sqrt{6}$

Q.28 The area that is enclosed in the circle $x^2 + y^2 = 2$ which is not common area enclosed by $y = x$ and $y^2 = x$ is **[JEE MAIN 2020 (JAN)]**
(A) $\frac{1}{12}(24\pi - 1)$ (B) $\frac{1}{6}(12\pi - 1)$
(C) $\frac{1}{12}(6\pi - 1)$ (D) $\frac{1}{12}(12\pi - 1)$

Q.29 The area bounded by $4x^2 \le y \le 8x + 12$ is - **[JEE MAIN 2020 (JAN)]**
(A) 127 / 3 (B) 128 / 3 (C) 124 / 3 (D) 125 / 3

Q.30 If $y^2 = ax$ and $x^2 = ay$ intersect at A & B. Area bounded by both curves is bisected by line x = b (given a > b > 0). Area of triangle formed by line AB, x = b and x-axis is 1/2. Then **[JEE MAIN 2020 (JAN)]**
(A) $a^6 - 12a^3 - 4 = 0$ (B) $a^6 + 12a^3 - 4 = 0$
(C) $a^6 - 12a^3 + 4 = 0$ (D) $a^6 + 12a^3 + 4 = 0$

Q.31 Let P be the set of points (x, y) such that $x^2 \le y \le -2x + 3$. Then area of region bounded by points in set P is **[JEE MAIN 2020 (JAN)]**
(A) 16/3 (B) 32/3
(C) 29/3 (D) 20/3

Q.32 Given : $f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$.
Then the area (in sq. units) of the region bounded by the curves, y = f (x) and y = g (x) between the lines, 2x = 1 and $2x = \sqrt{3}$, is : **[JEE MAIN 2020 (JAN)]**
(A) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (C) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (D) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	C	C	D	B	C	A	D	B	A	D	B	D	B	B

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	B	A	B	D	D	B	B	D	A	B	B	A	B	C	A	C	D
Q	18	19	20	21	22	23	24	25	26								
A	4	3	8	3	1	2	9	3	3								

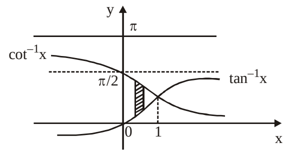
EXERCISE-3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	D	D	A	A	A	D	D	C	C	C	D	C	C	A	A	C	A	B	C	C
Q	21	22	23	24	25	26	27	28	29	30	31	32								
A	B	B	B	B	A	C	A	B	B	C	B	B								

CHAPTER- 8:
AREA BOUNDED BY CURVE

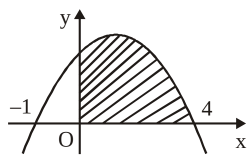
SOLUTIONS TO TRY IT YOURSELF

(1) $A = \int_0^1 (\cot^{-1} x - \tan^{-1} x)\, dx$



$A = \int_0^{\pi/4} (\tan y)\, dy + \int_{\pi/4}^{\pi/2} (\cot y)\, dy = \ln 2$

(2) $A = \int_0^4 y\, dx = \int_0^4 (4 + 3x - x^2)\, dx$



$= \left[4x + \frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^4 = \frac{56}{3}$

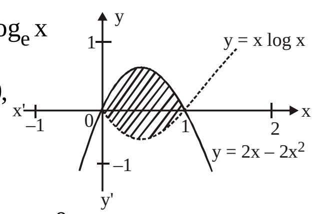
(3) **(A).** Curve tracing : $y = x \log_e x$

Clearly, $x > 0$,

For $0 < x < 1$, $x \log_e x < 0$,

and for $x > 1$, $x \log_e x > 0$

Also, $x \log_e x = 0 \Rightarrow x = 1$



Further, $\frac{dy}{dx} = 0 \Rightarrow 1 + \log_e x = 0$

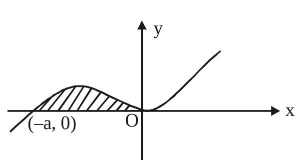
$\Rightarrow$ $x = 1/e$, which is a point of minima.

Required area $= \int_0^1 (2x - 2x^2)\, dx - \int_0^1 x \log x\, dx$

$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1$

$= \left(1 - \frac{2}{3}\right) - \left[0 - \frac{1}{4} - \frac{1}{2}\lim_{x \to 0} x^2 \log x\right] = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

(4) **(D).** The curve is $y = \frac{x^2(x+1)}{a^2}$, which is a cubic polynomial.



Since, $\frac{x^2(x+a)}{a^2} = 0$ has repeated root $x = 0$,

it touches x-axis at (0, 0) and intersects at (–a, 0).

Required area

$= \int_{-a}^{0} y\, dx = \int_{-a}^{0} \left[\frac{x^2(x+a)}{a^2}\right] dx = \frac{a^2}{12}$ sq. units

(5) $(y - \sin^{-1}x)^2 = x - x^2$

$y = \sin^{-1} x \pm \sqrt{x - x^2} \Rightarrow$ domain $x \in [0, 1]$

Area enclosed by the curve

$= \int_0^1 (\sin^{-1} x + \sqrt{x - x^2}) - (\sin^{-1} x - \sqrt{x - x^2})\, dx$

$= 2\int_0^1 \sqrt{x - x^2}\, dx = 2\int_0^1 \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}\, dx$

$= 2\left[\frac{1}{2}\left(x - \frac{1}{2}\right)\sqrt{x - x^2} + \frac{1}{2}\left(\frac{1}{4}\right)\sin^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right)\right]_0^1$

$= 2\left[\left(0 + \frac{1}{8}\frac{\pi}{2}\right) - \left(0 + \frac{1}{8}\left(-\frac{\pi}{2}\right)\right)\right] = 2\left(\frac{\pi}{16} + \frac{\pi}{16}\right) = \frac{\pi}{4}$

(6) $f(x) = \frac{x^3}{3} - x^2 + a$

$f'(x) = x^2 - 2x = x(x-2) < 0$ (note that f (x) is monotonic in (0, 2)). Hence, for the minimum and f (x) must cross the x-axis at $\frac{0+2}{2} = 1$. Hence, $f(1) = \frac{1}{3} - 1 + a = 0 \Rightarrow a = \frac{2}{3}$

(7) $A = \sqrt{1 + 3x} - 1 = \int_0^x f(x)\, dx$

(a) $\left.\frac{dA}{dx}\right|_{avg} = \frac{1}{(8-1)}\int_1^8 \left(\frac{dA}{dx}\right) dx$

$= \frac{1}{7}(\sqrt{1+3x} - 1)_1^8 = \frac{1}{7}(4 - 1) = \frac{3}{7}$

(b) $\left.\frac{dA}{dx}\right|_{x=5} = \frac{3}{2\sqrt{1+3x}} = \frac{3}{2\sqrt{1 + 3(5)}} = \frac{3}{8}$

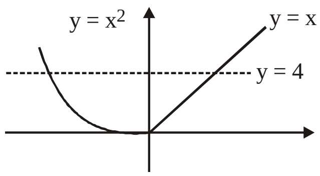
(c) $A(x) = \int_0^x f(x)\, dx = \sqrt{1+3x} - 1$

Differentiating w.r.t. x, $f(x) = \frac{3}{2\sqrt{1+3x}}$

(d) $y_{avg} = \frac{1}{(8-1)}\int_1^8 f(x)\, dx = \frac{1}{7}(\sqrt{1+3x} - 1)_1^8 = \frac{3}{7}$

CHAPTER- 8 : AREA BOUNDED BY CURVE
EXERCISE-1

(1) **(C).** $A_2 = \frac{1}{2} \times 4 \times 4 = 8$



$$A_1 = \int_0^4 \sqrt{y}dy = \frac{2}{3}y^{3/2}\Big|_0^4 = \frac{2}{3}(2^2)^{3/4} = \frac{16}{3}$$

Area $= A_1 + A_2 = 8 + \frac{16}{3} = \frac{40}{3}$ sq. units

(2) **(C).** $y = mx^2$ $\quad x = my^2$

$x^2 = \frac{1}{m}y \quad y^2 = \frac{1}{m}x$

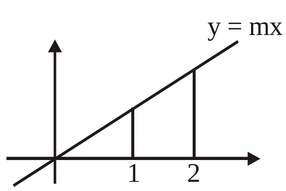
$4a = m, \quad a = \frac{1}{4}m$

Area $= \frac{16a^2}{3} = \frac{1}{4} \Rightarrow 16.\frac{\frac{1}{16m^2}}{3} = \frac{1}{4}$

$\Rightarrow \frac{1}{3m^2} = \frac{1}{4} \Rightarrow m^2 = \frac{4}{3} \Rightarrow m = \pm\frac{2}{\sqrt{3}}$

(3) **(C).** Put $\frac{x}{3} = t$, Given integral $= 3\int_0^{\pi} \sin t\, dt = 3 \times 2 = 6$

(4) **(D).** Area $= \int_1^2 mx\, dx = 6$



$m\frac{x^2}{2}\Big|_1^2 = 6$

$\Rightarrow m(2^2 - 1^2) = 12$

$\Rightarrow 3m = 12 \Rightarrow m = 4$

(5) **(B).** $y^2 = 4ax$, $x^2 = 4by$ is $\frac{4a \times 4b}{3}$

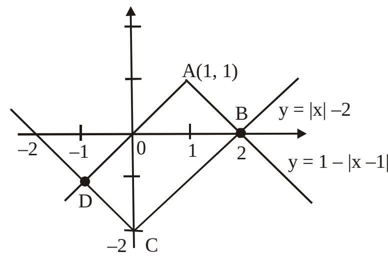
Required $= \frac{1 \times 1}{3} = \frac{1}{3}$

(6) **(C).** $A = \int_0^8 x\, dy = \int_0^8 y^{1/3}\, dy = \frac{3}{4}y^{4/3}\Big|_0^8 = \frac{3}{4}(8^{4/3}) = \frac{3}{4} \times 16$

= 12 sq. units

(7) **(A).** Bounded figure ABCD is a rectangle.

$AB = \sqrt{1+1} = \sqrt{2}$; $BC = \sqrt{4+4} = 2\sqrt{2}$



Thus, bounded area = $(\sqrt{2})\,(2\sqrt{2})$ = 4 sq. units.

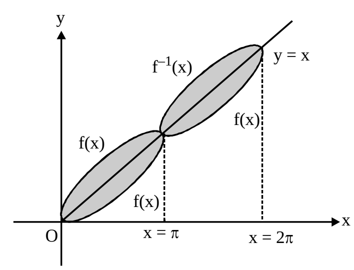
(8) **(D).** $\because \left|\int_0^{(1-m)} \left(x - x^2 - mx\right)dx\right| = \frac{9}{2}$

$\Rightarrow \left[\frac{(1-m)x^2}{2} - \frac{x^3}{3}\right]_0^{(1-m)} = \frac{9}{2} \Rightarrow \left|\frac{(1-m)^3}{6}\right| = \frac{9}{2}$

$\Rightarrow |1 - m| = 3$

$1 - m = 3$ or $-3 \Rightarrow m = -2$ or $m = 4$

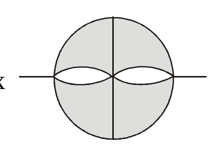
(9) **(B).** Graph of $f^{-1}(x)$ is the mirror image of $f(x)$ about the line $y = x$.



Required area $= 4\left(\int_0^{\pi} f(x)\,dx - \frac{1}{2}\pi.\pi\right)$

$= 4\left\{\left[\frac{x^2}{2} - \cos x\right]_0^{\pi} - \frac{\pi^2}{2}\right\} = 4\left[\frac{\pi^2}{2} + 1 + 1 - \frac{\pi^2}{2}\right] = 8$

(10) **(A).** Reqd area $= \pi(\pi^2) - 4\int_0^{\pi} \sin x dx$



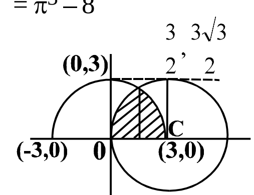
$= \pi^3 - 8$

(11) **(D).** $x^2 + y^2 = 9$ ....(1)

$x^2 + y^2 - 6x = 0$ ....(2)

On solving $x = 3/2$

$y^2 = 9 - 9/4 = 27/4$

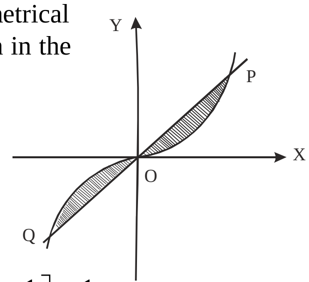


$\Rightarrow y = \frac{3\sqrt{3}}{2}$ $\quad \therefore A = 2\int_{3/2}^{3} \sqrt{9 - x^2}\, dx$

(12) **(B).** Solving the equation of the given curves for x, we get $x = 0, 1, -1$

The required area is symmetrical about the origin as shown in the diagram.

So reqd. area



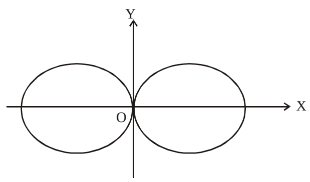
$$= 2\int_0^1 (x - x^3)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$$

(13) **(D).** The curve has two loops is symmetrical about the

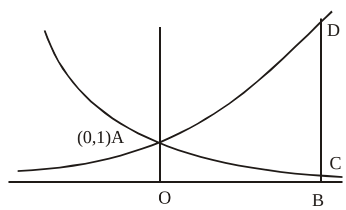
$$\text{x- axis } A = 4\int_0^a \frac{x}{a}\sqrt{a^2 - x^2}\,dx = \frac{2}{a}$$

$$\left[\frac{(a^2 - x^2)^{3/2}}{\frac{3}{2}(-1)}\right]_0^a = \frac{4a^2}{3}$$



(14) **(B).** Required area $= \int_0^1 \left(e^x - e^{-x}\right)dx = \left[e^x + e^{-x}\right]_0^1$

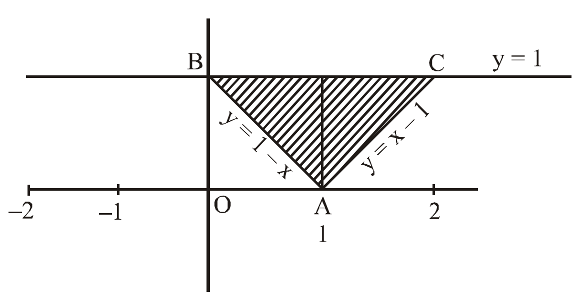
$$= (e^1 + e^{-1}) - (1 + 1) = e + \frac{1}{e} - 2$$



(15) **(B).** The given region is represented by the equations

$y = 1 - x, x \le 1 = x - 1, x \ge 1$

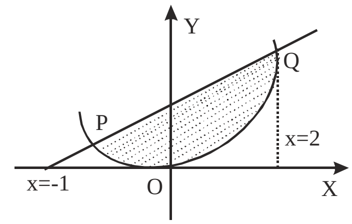
and $y = 1$; $C = (2, 1)$ and $B = (0, 1)$



$\therefore$ the shaded area in the figure $= \frac{1}{2}$ BC . AC $= \frac{1}{2}\,2 . 1 = 1.$

EXERCISE-2

(1) **(B).** Solving the equation of the given curves for x, we get $x^2 = x + 2 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$



$$\text{Reqd. area} = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\,[(2 + 4 - 8/3) - (1/2 - 2 + 1/3)] = 9/8$$

(2) **(A).** $y = x^3 \quad \frac{dy}{dx} = 3x^2; \quad \left(\frac{dy}{dx}\right) = 3$

Normal at P (1, 1); $y - 1 = -\frac{1}{3}(x - 1)$

$3y + x = 4$ .... (1)

So intersecting point of normal at x-axis is (4, 0)

$$\text{Area} = \int_0^4 x^3 dx + \int_1^4 \frac{(4 - x)}{3} dx$$

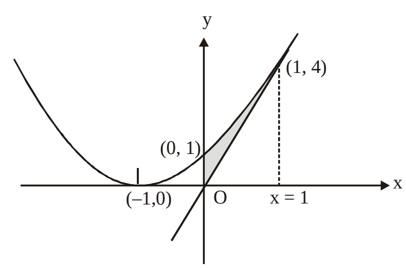
$$= \left[\frac{x^4}{4}\right]_0^1 + \frac{1}{3}\left[4x - \frac{x^2}{2}\right]_1^4 = \frac{7}{4}$$

(3) **(B).** Since, $y = x^2 + 2x + 1$

$$\therefore \frac{dy}{dx} = (2x + 2) \quad \therefore \left(\frac{dy}{dx}\right)_{x=1} = 4$$

Equation of the tangent at (1, 4) is

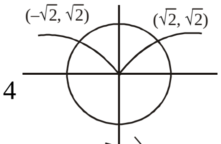
$(y - 4) = 4(x - 1) \Rightarrow 4x - y = 0$



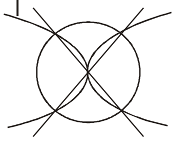
$$\text{Required area} = \int_0^1 y_p dx - \frac{1}{2} \times 1 \times 4$$

$$= \int_0^1 (x^2 + x + 1)\,dx - 2 = \frac{1}{3} \text{ sq. unit}$$

(4) **(D).** $y = \sqrt{\sqrt{2}\,|x|}$, $x^2 + y^2 = 4$



Normals are $x = y,\ x + y = 0$



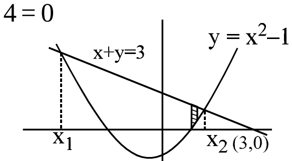
Required area = $4\int_0^{\sqrt{2}} (2^{1/4}x^{1/2} - x)\,dx$

$$= 4\left[\frac{2^{1/4}x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^{\sqrt{2}} = 4\left(\frac{2.2^{1/4}2^{3/4}}{3} - \frac{2}{2}\right) = 4\left(\frac{4}{3} - 1\right) = \frac{4}{3}$$

(5) **(D).** $3 - x = x^2 - 1 \Rightarrow x^2 + x - 4 = 0$

$x_1 + x_2 = -1$

$x_1 x_2 = -4$ ....(1)



$$A = \int_{x_1}^{x_2} [(3 - x) - (x^2 - 1)]\,dx = \int_{x_1}^{x_2} (4 - x - x^2)\,dx \quad \text{use (1)}$$

(6) **(B).** $A_1 = \int_1^e \ln x\,dx = (x\ln x - x)_1^e = 1$ sq. unit then

$A_m = mA_1 = m$

$A_{m-1} = (m - 1)A_1 = (m - 1) \Rightarrow A_m + mA_{m-1} = m^2$

(7) **(B).** For $x < \sqrt{3}$, $y^2 = 4\sqrt{3}x$

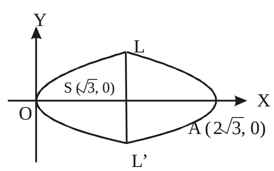
For $x \ge \sqrt{3}$, $y^2 = -4\sqrt{3}(x - 2\sqrt{3})$

The two parabolas have common latusrectum, common axis and opposite in concavities.

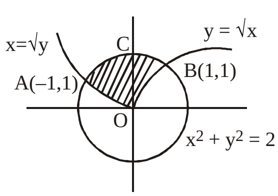
Area bounded by parabola $y^2 = 4ax$ and its latusrectum is $\frac{8a^2}{3}$ sq. units.

$\therefore$ Answer is $2 \times 8a^2/3$

where $a = \sqrt{3}$ = 16 sq. units.



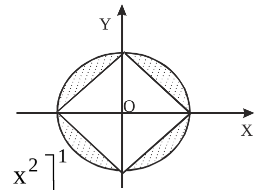
(8) **(D).**



$$A = \int_{-1}^{0}\left[\sqrt{2 - x^2} - x^2\right]dx + \int_0^1\left[\sqrt{2 - x^2} - \sqrt{x}\right]dx = \frac{\pi}{2}$$

(9) **(A).** By changing x as –x and y as–y, both the given equation remains unchanged so required area will be symmetric w.r.t. both the axis, which is shown in the fig., So required area

$$= 4\int_0^1\left[\sqrt{1 - x^2} - (1 - x)\right]dx$$



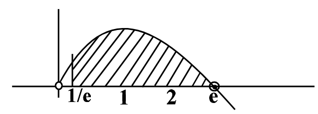
$$= 4\left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$$

$$= 4\left[0 + \frac{1}{2}.\frac{\pi}{2} - 1 + \frac{1}{2}\right] = \pi - 2$$

(10) **(B).** $y = x(1 - \ln x) = 0 \Rightarrow x = e$ (as $x > 0$)

$\frac{dy}{dx} = -\ln x \Rightarrow \uparrow$ in $(0,1)$ and $\downarrow$ in $(1, \infty)$

also $\lim_{x\to 0} x(1 - \ln x) = 0$

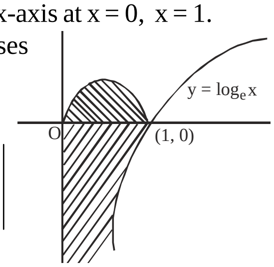


$$A = \int_{1/e}^{e} x(1 - \ln x)\,dx$$

(11) **(B).** $y = \sin^4 \pi x$ intersects the x-axis at $x = 0$, $x = 1$.

The curve, y = log x also passes through (1, 0)

Required area



$$= \left|\int_0^1 \sin^4 \pi x\,dx\right| + \left|\int_0^1 \log_e x\,dx\right|$$

$$= \int_0^{\pi} \sin^4\theta \frac{d\theta}{\pi} + \left|[x\log_e x - x]_0^1\right|$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\sin^4\theta\,d\theta + 1 = \frac{2}{\pi} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} + 1 = \frac{11}{8}.$$

(12) **(A).** Equation of curve can be re-written as

$2y^2 + 6(1 + x)y + 5x^2 + 7x + 6 = 0$

$$y_1 = \frac{-3(1 + x) - \sqrt{(3 - x)(x - 1)}}{2},$$

$$y_2 = \frac{-3(1 + x) + \sqrt{(3 - x)(x - 1)}}{2}$$

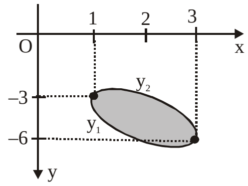
Therefore the curves (y_1 and y_2) are defined for values of x for which $(3 - x)(x - 1) \ge 0$

i.e., $1 \le x \le 3$ (Actually the given equation denotes an ellipse, because $\Delta \ne 0$ and $h^2 < ab$).

Required area will be given by

$$A = \int_1^3 (y_1 - y_2)\,dx \Rightarrow A = \int_1^3 \sqrt{(3 - x)(x - 1)}\,dx$$

Put $x = 3\cos^2\theta + \sin^2\theta$ i.e., $dx = -2\sin 2\theta\,d\theta$



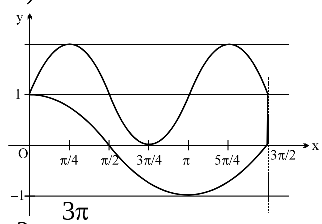
$$A = 2\int_0^{\pi/2} \sin^2 2\theta \, d\theta = \frac{\pi}{2} \text{ sq. units}$$

(13) **(B).** Area $= \int_1^3 -(x^2-4x+3)\,dx = -\left(\frac{x^3}{3}-\frac{4x^2}{2}+3x\right)\Big|_1^3 = \frac{4}{3}$

$\therefore$ S-1 is true. S-2 is true but does not explain S-1.

(14) **(C).** $A = \int_0^{3\pi/2} (1+\sin 2x - \cos x)\,dx$

$= \left(x - \frac{1}{2}\cos 2x - \sin x\right)_0^{\frac{3\pi}{2}}$



$\left(\frac{3\pi}{2}+\frac{1}{2}+1\right) - \left(0-\frac{1}{2}\right) = 2 + \frac{3\pi}{2}$

(15) (A), (16) (C) (17) (D).

Let the equation of the line CD be y = 2x + b ....(1)
and side of the square ABCD be 'a'

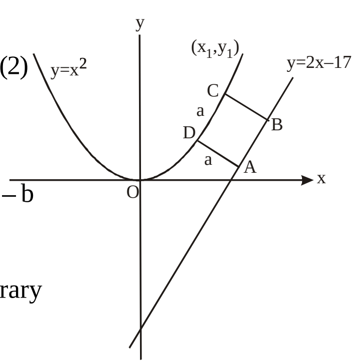
but $\frac{y_1-x_1}{x_1-x_2} = 2$ (CD || AB)

$a^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (x_2-x_1)^2 \left[1+\left(\frac{y_2-y_1}{x_2-x_1}\right)^2\right] = 5(x_1-x_2)^2$

$\therefore\ a^2 = 5(x_1-x_2)^2$

$= 5[(x_1+x_2)^2 - 4x_1x_2]$ ....(2)



Solving (1) with $y = x^2$

$x^2 = 2x + b$

or $x^2 - 2x - b = 0$

$x_1 + x_2 = 2$; $x_1x_2 = -b$

$\therefore\ a^2 = 5[4+4b]$

$= 20(b+1)$ ....(3)

Now assume any arbitrary point on the line $y = 2x - 17$ say (x_1, y_1)

Now perpendicular distance from (x_1, y_1) on $2x - y + b = 0$

$\therefore\ a = \left|\frac{2x_1 - y_1 + b}{\sqrt{5}}\right|$ where $y_1 = 2x_1 - 17 \therefore\ a = \left|\frac{17+b}{\sqrt{5}}\right|$

$5a^2 = (17+b)^2$; $5 \cdot 20(b+1) = (17+b)^2$

$100b + 100 = 289 + b^2 + 34b$

$b^2 - 66b + 189 = 0$; $b^2 - 3b - 63b + 189 = 0$

$b(b-3) - 63(b-3) = 0$

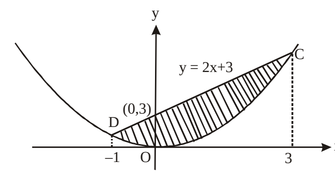
$b = 3$ or $b = 63 \Rightarrow a^2 = 80$ or $a^2 = 1280$

$A_{max} = 1280$ Ans.

(iii) Solving $y = 2x + 3$ and $y = x^2$

$x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0 \Rightarrow x = 3$ or $x = -1$

$\therefore$ Area $= \int_{-1}^{3} [(2x+3) - x^2]\,dx = \left[x^2 + 3x - \frac{x^2}{3}\right]_{-1}^{3}$



$= (9+9-9) - \left(1-3+\frac{1}{3}\right) = 9 - \left(\frac{5}{3}\right) = 9 + \frac{5}{3} = \frac{32}{3}$

(18) **4.** Solving the two functions we get x = 2, $\sqrt{3}-1$

So, required area $= \int_{\sqrt{3}-1}^{2} \left[2 - \frac{3}{(x+1)} - |x-1|\right] dx$

$= \int_{\sqrt{3}-1}^{1} \left[2 - \frac{3}{(x+1)} + (x-1)\right] dx + \int_{1}^{2} \left[2 - \frac{3}{(x+1)} + (1-x)\right] dx$

$A = \left(2 - \frac{3}{2}\ln 3\right)$ sq. units

$2A = 4 - 3\ln 3$

$2A + 3\ln 3 = 4$

(19) **3. Case I :** If $a \in [0, 1]$ the curves intersect at $\left(\frac{a}{3}, \frac{a}{3}\right)$ and (a, a).

The bounded region is contained in the triangle with vertices (a, a), $\left(\frac{a}{3}, \frac{a}{3}\right)$ and $\left(\frac{a}{2}, 0\right)$ with area $= \frac{a^2}{3}$.

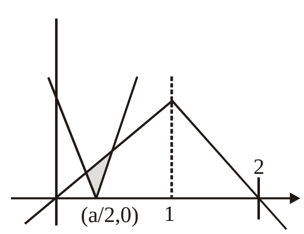
Hence, the area cannot exceed 1/3.

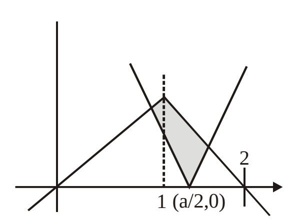
Case II : If $a \in [1, 3]$ in this case, the bounded region in a quadrilateral with four vertices

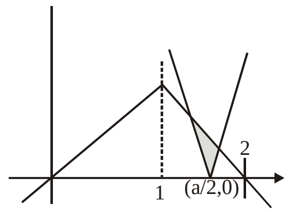
$\left(\frac{a}{3}, \frac{a}{3}\right)$ and $\left(\frac{a}{2}, 0\right)$, $\left(\frac{a+2}{3}, \frac{4-a}{3}\right)$ and (1, 1).

In this case area bounded $= \frac{1}{6}[2-(a-2)^2]$

Case III : If $a \in [3, 4]$. This case is symmetric with case I.







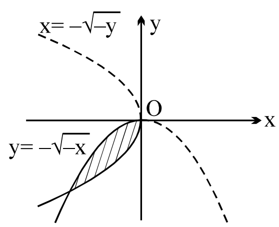
(20) **8.** $A = 2\int_0^{\pi}(2\sin x + \sin 2x)dx$

$= 4\int_0^{\pi}\sin x\,dx + 2\int_0^{\pi}\sin 2x\,dx = 8 + 0 = 8$

(21) **3.** $y = -\sqrt{-x} \Rightarrow y^2 = -x$ where x & y both (–) ve

$x = -\sqrt{-y} \Rightarrow x^2 = -y$ where x & y both (–) ve

Hence, $A = \dfrac{16ab}{3}$

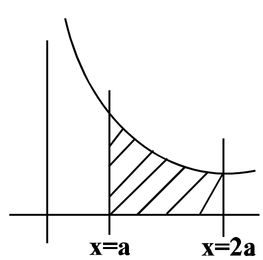


where a = b = 1/4 ∴ A = 1/3

(22) **1.** $A = \int_a^{2a}\left(\dfrac{x}{6}+\dfrac{1}{x^2}\right)dx = \left[\dfrac{x^2}{12} - \dfrac{1}{x}\right]_a^{2a}$

$= \left(\dfrac{a^2}{3} - \dfrac{1}{2a}\right) - \left(\dfrac{a^2}{12} - \dfrac{1}{a}\right)$; $f(a) = \dfrac{a^2}{4} + \dfrac{1}{2a}$

Now, $f'(a) = \dfrac{a}{2} - \dfrac{1}{2a^2} = 0 \Rightarrow a^3 = 1 \Rightarrow a = 1$



(23) **2.** The given lines are

$y = x - 1$; $y = -x - 1$

$y = x + 1$; $y = -x + 1$

which are two pairs of parallel lines and distance between the lines of each pair is $\sqrt{2}$.

Thus lines represents a square of side $\sqrt{2}$.

Hence, area = $(\sqrt{2})^2 = 2$ sq. units.

(24) **9.** The curves given are

$y = \sqrt{x}$ .... (1)

$2y + 3 = x$ .... (2)

and x-axis $y = 0$ .... (3)

Eq^n (1) [$y^2 = x$] represents right handed parabola but with +ve values of y i.e., part of curve lying above x-axis.

Solving eq. (1) and (2), we get

$2y + 3 = y^2$

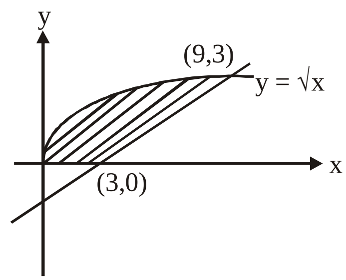
$\Rightarrow y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = 3$ (as $\neq$ –ve)

$\Rightarrow x = 9$

Also, (2) meets x-axis at (3, 0).



Shaded area is the required area given by

$$A = \int_0^9 \sqrt{x}\,dx - \int_3^9 \frac{x-3}{2}dx = \left[\frac{2x^{3/2}}{3}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9$$

$$= \frac{2\times 27}{3} - \frac{1}{2}\left[\frac{81}{2} - 27 - \frac{9}{2} + 9\right] = \frac{54}{3} - \frac{1}{2}[18]$$

= 18 – 9 = 9 sq. units.

(25) **3.** $y = ax^2$ and $x = ay^2$

Points of intersection are O (0, 0) and A (1/a, 1/a)

$$\text{Area} = \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = \frac{2}{3a^2} - \frac{1}{3a^2} = \frac{1}{3a^2} = 1$$

$\Rightarrow a = \pm\dfrac{1}{\sqrt{3}}$

(26) **3.** The given curves are $y = (x + 1)^2$ .... (1)

upward parabola with vertex at (–1, 0) meeting y-axis at (0, 1)

$y = (x - 1)^2$ .... (2)

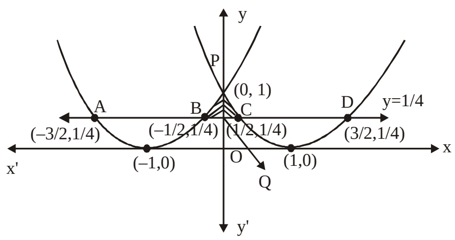
upward parabola with vertex at (1, 0) meeting y-axis at (0, 1).

$y = 1/4$ .... (3)

a line parallel to x-axis meeting (1) at (–1/2, 1/4), (–3/2, 1/4)

and meeting (2) at (3/2, 1/4), (1/2, 1/4).

The graph is as shown



The required area is the shaded portion, given by ar (BPCQB) = 2ar (PQCP) (by symmetry)

$$= 2\left[\int_0^{1/2}\left((x-1)^2 - \frac{1}{4}\right)dx\right] = 2\left[\left(\frac{(x-1)^3}{3} - \frac{x}{4}\right)_0^{1/2}\right]$$

$$= 2\left[\left(-\frac{1}{24} - \frac{1}{8}\right) - \left(-\frac{1}{3}\right)\right]$$

$$= 2\left[\frac{-1-3+8}{24}\right] = \frac{1}{3} \text{ sq . units}$$

EXERCISE-3

(1) **(D).** $\int_1^b f(x) = \sqrt{b^2+1} - \sqrt{2}$

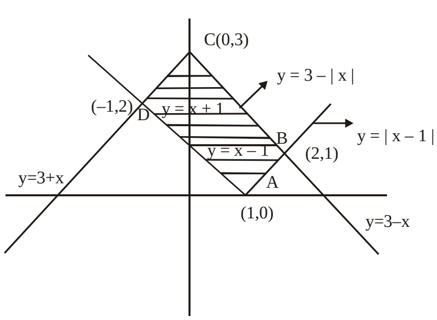
$$\frac{d}{db}\int_1^b f(x) = \frac{d}{db}(\sqrt{b^2+1} - \sqrt{2})$$

$$f(b) . 1 - f(1) . 0 = \frac{1}{2\sqrt{b^2+1}} \times 2b = f(b) = \frac{b}{\sqrt{b^2+1}}$$

$$\Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$$

(2) **(D).** $y = |x-1| = \begin{cases} x-1 & ; x \ge 1 \\ -(x-1) & ; x < 1 \end{cases}$

and $y = 3 - |x| = \begin{cases} 3-x & ; x \ge 0 \\ 3+x) & ; x < 0 \end{cases}$

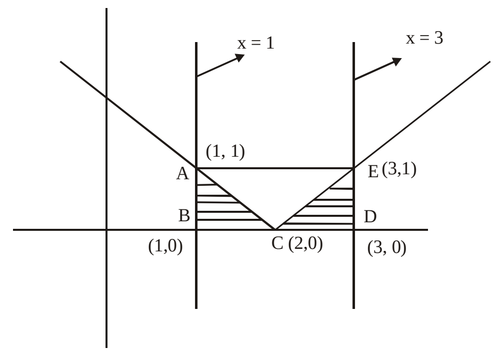


Solving, $y = x - 1$ and $y = 3 - x \Rightarrow x - 1 = 3 - x$
$\Rightarrow 2x = 4 \Rightarrow x = 2$ and $y = 2 - 1 = 1$
$AB^2 = (2-1)^2 + (1-0)^2 = 1 + 1 = 2$; $AB = \sqrt{2}$

$BC^2 = (0-2)^2 + (3-1)^2 = 4 + 4 = 8$; $BC = 2\sqrt{2}$
Area of rectangle ABCD

$= AB \times BC = \sqrt{2} \times 2\sqrt{2} = 4$ sq. unit

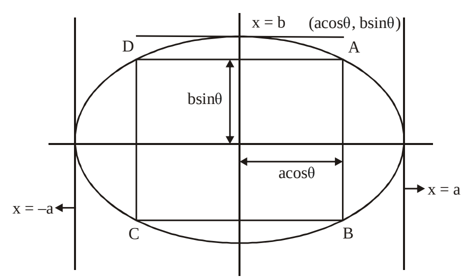
(3) **(A).** $y = |x-2| = \begin{cases} x-2 & ; x \ge 2 \\ -x+2 & ; x < 2 \end{cases}$



Required area = Area of Δ ABC + Area of Δ CDE

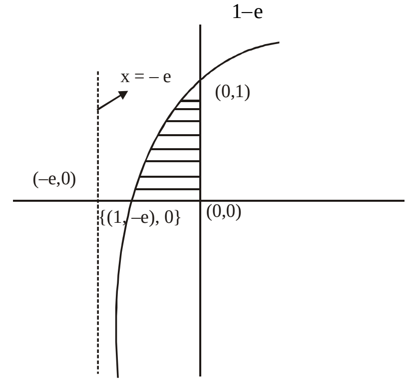
$$= \frac{1}{2}\times1\times1 + \frac{1}{2}\times1\times1 = \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. unit}$$

(4) **(A).** Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Let polar coordinate of point A on ellipse is
$x = a\cos\theta$ and $y = b\sin\theta$
$\therefore$ Area of rectangle ABCD = $2a\cos\theta \times 2b\sin\theta = 2ba\sin 2\theta$
it will be max. if $2\theta = \pi/2 \Rightarrow \theta = \pi/4$
$\therefore$ Area = $2ab\sin\pi/2 = 2ab$

(5) **(A).** Required area = $\int_{1-e}^{0} \log_e(x+e)\,dx$



Put $x + e = t \Rightarrow dx = dt$

$$\int_1^e \log_e t\, dt = [t \log_e t - t]_1^e$$

$= [e \log e - e - 1 \log 1 + 1] = (e - e - 0 + 1) = 1$

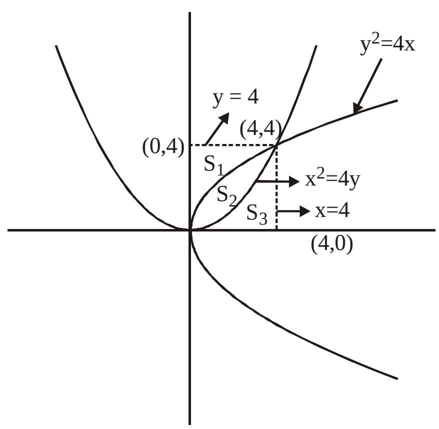
(6) **(D).** $S_1 = S_3 = \int_0^4 \frac{x^2}{4} dx$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{12} \times 64 = \frac{16}{3} \text{sq. unit}$$

$\because S_1 + S_2 + S_3 = 4 \times 4 = 16$

$\therefore S_2 = 16 - (S_1 + S_3)$ $\quad \{\because S_1 = S_3 = \frac{16}{3}\text{sq. unit}\}$

$$= 16 - \left(2 \times \frac{16}{3}\right) = \frac{16}{3}\text{sq. unit}$$



$\therefore S_1 : S_2 : S_3 = 1 : 1 : 1 \quad \{\because S_1 = S_2 = S_3\}$

(7) **(D).** $\int_{\pi/4}^{\beta} f(x)\, dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$

Differentiate with respect to β on both side

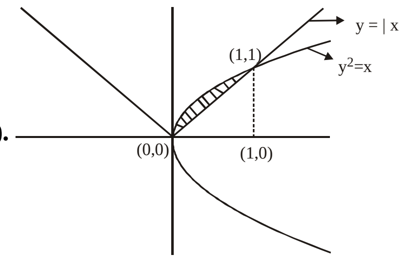
$f(\beta) . 1 - f(\pi/4) \times 0 = \sin\beta . 1 + \beta \cos\beta - \pi/4 \sin\beta + \sqrt{2}$

$\Rightarrow f(\beta) = \sin\beta + \beta\cos\beta - \pi/4 \sin\beta + \sqrt{2}$

$$\therefore f(\pi/2) = \sin\frac{\pi}{2} + \beta\cos\frac{\pi}{2} - \frac{\pi}{4}\sin\frac{\pi}{2} + \sqrt{2}$$

$$1 + 0 - \frac{\pi}{4} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$$

(8) **(C).**

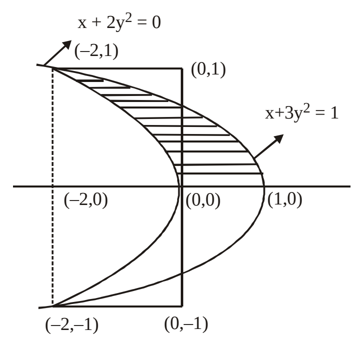


Required area $= \int_0^1 (\sqrt{x} - x)\, dx$ $\quad \{\text{if } x \geq 0 ; |x|\}$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \left[\left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0)\right] = \frac{4-3}{6} = \frac{1}{6}$$

(9) **(C).** $x + 2y^2 = 0 \Rightarrow y^2 = x/2$

$$x + 3y^2 = 1 \Rightarrow y^2 = \frac{1-x}{3}$$



Required area $= 2\int_0^1 [(1 - 3y^2) - (-2y^2)]\, dy$

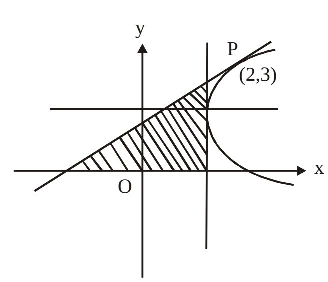
$$= 2\int_0^1 (1 - y^2)\, dy = 2\left[y - \frac{y^3}{3}\right]_0^1 = 2\left[1 - \frac{1}{3}\right] = 2 \times \frac{2}{3} = \frac{4}{3}$$

(10) **(C).** $(y - 2)^2 = x - 1$

$$2(y - 2)\,y' = 1 \Rightarrow y' = \frac{1}{2(y-2)} \Rightarrow y' = \frac{1}{2} \text{ at } y = 3$$

Equation of tangent $y - 3 = \frac{1}{2}(x - 2)$

$\Rightarrow 2y - 6 = x - 2 \Rightarrow x = 2y - 4$

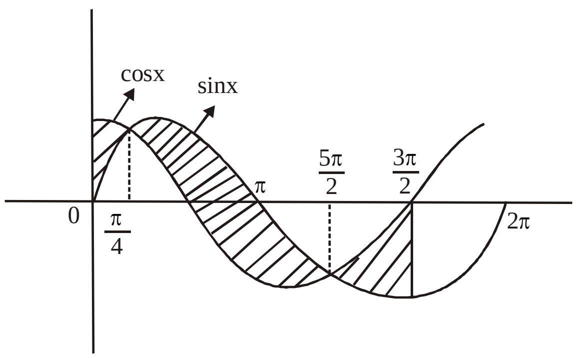


$$A = \int_0^3 ((y-2)^2 + 1 - (2y-4))\, dy$$

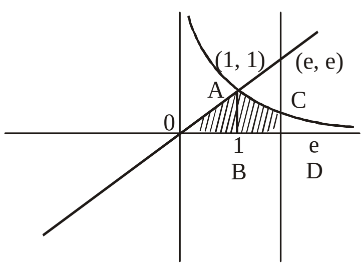
$$= \int_0^3 (y^2 - 6y + 9)\, dy = \frac{27}{3} = 9$$

(11) **(D).** $\displaystyle\int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$

$$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) = 4\sqrt{2} - 2$$



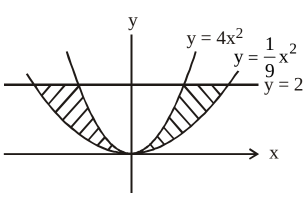
(12) **(C).**



Required area = OAB + ACDB

$$= \frac{1}{2} \times 1 \times 1 + \int_1^e \frac{1}{x} dx = \frac{1}{2} + (\ln x)_1^e = \frac{3}{2} \text{ square units}$$

(13) **(C).**



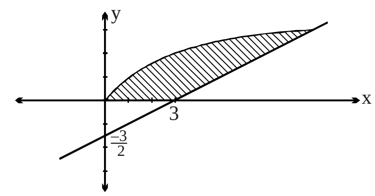
$$y = 4x^2;\ y = \frac{1}{9}x^2$$

$$\text{Area} = 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy = 2\left[\frac{5y\sqrt{y}}{23/2}\right]_0^2 = 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

(14) **(A).** $y = \sqrt{x}$ ....(1)

and $2y - x + 3 = 0$ ....(2)

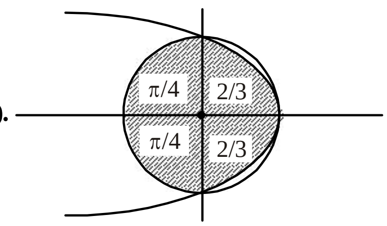
On solving both y = – 1, 3



$$\text{Required area} = \int_0^3 \{(2y+3) - y^2\}\, dy$$

$$= y^2 + 3y - \frac{y^3}{3}\Big|_0^3 = 9 + 9 - 9 = 9$$

(15) **(A).**



$$A = \frac{1}{2} \times \pi + 2\int_0^1 \sqrt{1-x}\, dx = \frac{\pi}{2} + \frac{4}{3}$$

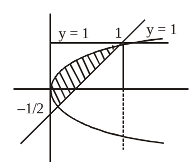
(16) **(C).** After solving $y = 4x - 1$ and $y^2 = 2x$

$$y = 4 \cdot \frac{y^2}{2} - 1\ ;\ 2y^2 - y - 1 = 0$$

$$y = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4}\ ;\ y = 1, -1/2$$

$$A = \int_{-1/2}^{1} \left(\frac{y+1}{4}\right) dy - \int_{-1/2}^{1} \frac{y^2}{2}\, dy$$

$$= \frac{1}{4}\left[\frac{y^2}{y} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$



$$= \frac{1}{4}\left[\frac{4+8-1+4}{8}\right] - \frac{1}{2}\left[\frac{8+1}{24}\right]$$

$$= \frac{1}{4}\left[\frac{15}{8}\right] - \frac{9}{48} = \frac{15}{32} - \frac{6}{32} = \frac{9}{32}$$

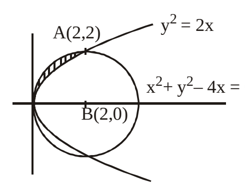
(17) **(A).** $x^2 + y^2 - 4x \le 0$

$y^2 \ge 2x$

$x^2 + 2x - 4x = 0$

$\Rightarrow$ $x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$\Rightarrow$ $x = 0, x = 2$



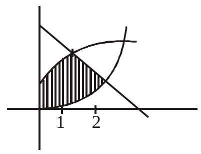
$$\text{Area} = \int_0^2 \left[\sqrt{4x - x^2} - \sqrt{2}\sqrt{x}\right] dx$$

$$= \int_0^2 \left[\sqrt{2^2 - (x-2)^2} - \sqrt{2}\sqrt{x}\right] dx$$

$$= \left[\left|\frac{x-2}{2}\sqrt{4x-x^2} + \frac{4}{2}\sin^{-1}\frac{x-2}{2} - \sqrt{2}\times\frac{2}{3}x^{3/2}\right|_0^2\right]$$

$$= \left[-\frac{2\sqrt{2}}{3}\times 2\sqrt{2} - \left\{-2\times\frac{\pi}{2}\right\}\right] = \left[\pi - \frac{8}{3}\right]$$

(18) **(B).** Solving $x^2 = 4y$ and $x + y = 3$



We get, $\frac{x^2}{4} + x = 3$

$\Rightarrow$ $x^2 + 4x - 12 = 0$

$\Rightarrow$ $(x+6)(x-2) = 0 \Rightarrow x = 2, y = 1$

Solving $y = 1 + \sqrt{x}$ and $y = 3 - x$

We get $1 + \sqrt{x} = 3 - x \Rightarrow x = 1, y = 2$

$\therefore$ Area = $\int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)\,dx - \int_0^2 \frac{x^2}{4}dx$

$$= x + \frac{2}{3}x^{3/2}\Big|_0^1 + 3x - \frac{x^2}{2}\Big|_1^2 - \frac{x^3}{12}\Big|_0^2 = \frac{5}{2}$$

(19) **(C).** $g(x) = \cos x^2$; $f(x) = \sqrt{x}$; $g(f(x)) = \cos x$

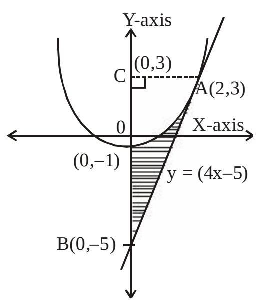
Given, $18x^2 - 9\pi x + \pi^2 = 0$

$(6x - \pi)(3x - \pi) = 0 \quad \therefore \quad x = \frac{\pi}{6}, \frac{\pi}{3}$

Area = $\int_{\pi/6}^{\pi/3} \cos x\,dx = \frac{\sqrt{3}-1}{2}$

(20) **(C).** Equation of tangent at (2,3) on $y = x^2 - 1$, is $y = (4x - 5)$

Required shaded area

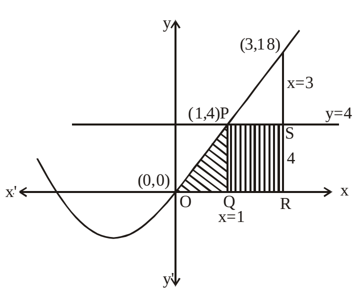


$= \text{ar}(\Delta ABC) - \int_{-1}^{3}\sqrt{y+1}\,dy$

$= \frac{1}{2}\times 8\times 2 - \frac{2}{3}[(y+1)^{3/2}]_{-1}^{3}$

$= 8 - \frac{16}{3} = \frac{8}{3}$ Square units

(21) **(B).**



Required area $= \int_0^1 (x^2 + 3x)\,dx$ + Area of rectangle

$PQRS = \frac{11}{6} + 8 = \frac{59}{6}$

(22) **(B).** $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$A(\alpha) = 2\int_0^{\alpha}\sqrt{x}\,dx = 2\alpha^{3/2}$

$A(4) = 2\times 4^{3/2} = 16$; $A(\lambda) = 2\times\lambda^{3/2}$

$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4\cdot\left(\frac{4}{25}\right)^{1/3}$

(23) **(B).** $x^2 \le y \le x + 2$

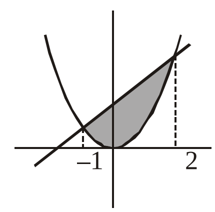
$x^2 = y$; $y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x-1) = 0$

$x = 2, -1$



Area = $\int_{-1}^{2} (x+2) - x^2 dx = \frac{9}{2}$

(24) **(B).** $y^2 = 2x$

$x - y - 4 = 0$

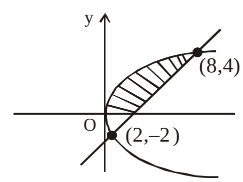
$(x-4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

$x = 8, 2$

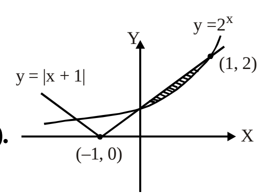
$y = 4, -2$



$$A = \int_{-2}^{4}\left(y + 4 - \frac{y^2}{2}\right)dy = \frac{y^2}{2}\Big|_{-2}^{4} + 4y\Big|_{-2}^{4} - \frac{y^3}{6}\Big|_{-2}^{4}$$

$$= (8-2) + 4(6) - \frac{1}{6}(64+8) = 6 + 24 - 12 = 18$$

(25) **(A).**



$$\int_0^1 ((x+1) - 2^x)\,dx = \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$$

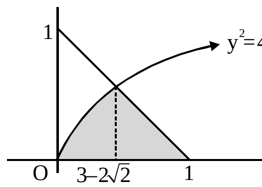
$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right) = \frac{3}{2} - \frac{1}{\ln 2}$$

(26) **(C).** $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$

$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2}(1-(3-2\sqrt{2}))\,(1-(3-2\sqrt{2}))$$

$$= \frac{2\,[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}(2\sqrt{2}-2)\,(2\sqrt{2}-2)$$

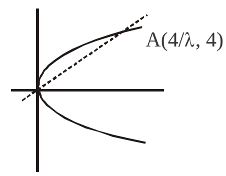
$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$



$$a = \frac{8}{3},\quad b = -\frac{10}{3}$$

$a - b = 6$

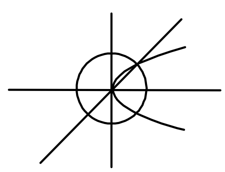
(27) **(A).** Area $= \frac{1}{9} = \int_0^{4/\lambda} (\sqrt{4\lambda x} - \lambda x)\,dx$



$\Rightarrow \lambda = 24$

(28) **(B).** Total area – enclosed area

$$2\pi - \int_0^1 \sqrt{x} - x\,dx$$



$$2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$2\pi - \left(\frac{2}{3} - \frac{1}{2}\right) = 2\pi - \frac{1}{6} = \frac{12\pi - 1}{6}$$

(29) **(B).** $4x^2 = y$

$y = 8x + 12$

$4x^2 = 8x + 12$

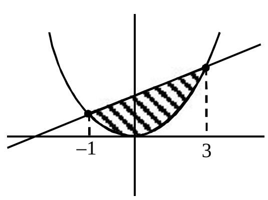
$x^2 - x - 3 = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$



$$A = \int_{-1}^{3} (8x + 12 - 4x^2)\,dx\ ;\ A = \left.\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right|_{-1}^{3}$$

$$= (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3}$$

(30) **(C).** $\int_0^b \left(\sqrt{a}\,x^{1/2} - \frac{x^2}{a}\right) dx = \frac{a^2}{6}$

$\Rightarrow \frac{2}{3}\sqrt{a}\,b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6}$ ….(i)

Also area of Δ OQR = 1/2

$\frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$

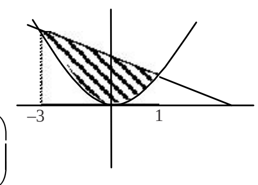
Put in (i), $4a\sqrt{a} - 2 = a^3$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$

(31) **(B).** Point of intersection of $y = x^2$ & $y = -2x + 3$

Obtained by $x^2 + 2x - 3 = 0 \Rightarrow x = -3, 1$

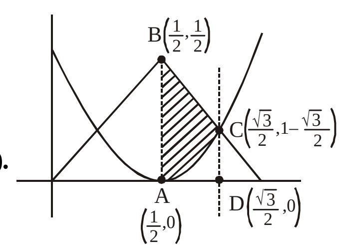
Area $= \int_{-3}^{1} (3 - 2x - x^2)\,dx$



$$= 3(4) - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right)$$

$$= 12 + 8 - \frac{28}{3} = \frac{32}{3}$$

(32) **(B).**



Required area = Area of trepezium ABCD

– Area of parabola between $x = \frac{1}{2}$ & $x = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4} - \frac{1}{3}$$