9

DIFFERENTIAL EQUATIONS

DEFINITIONS

In certain situation we notice that the relation between the rates of change of observable quantities is simpler than the relation between the quantities themselves. In such cases differential equations are taken as model for several problems in Engineering. Physical sciences, Biological sciences. In this chapter we shall study some basic concepts.

DIFFERENTIAL EQUATION

An equation containing an independent variable, dependent variable, and differential coefficients of dependent variable with respect to independent variable is called a differential equation. For Example–

(i) $\frac{dy}{dx} = \sin x$ (ii) $\frac{dy}{dx} + xy = \cot x$

(iii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$ (iv) $\left(\frac{d^2y}{dx^2}\right)^2 + x^2\left(\frac{dy}{dx}\right)^3 = 0$

(v) $\frac{d^2y}{dx^2} + \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = 0$

(vi) $\left(\frac{d^4y}{dx^4}\right)^3 - 4\frac{dy}{dx} + 4y = 5\cos 3x$

(vii) $x^2\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 0$

Order of differential equation :
The order of a differential equation is the order of the highest derivative occurring in the differential equation.
Degree of differential equation : The degree of a differential equation is the degree of the highest derivative occurring in the differential coefficients are free from radical and fraction.

Example 1 :
Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0 .$$

Sol. Clearly order of the differential equation is 2.

Again $\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3} \Rightarrow \left(\frac{d^2y}{dx^2} + x^{1/4}\right)^3 = -\frac{dy}{dx}$

which shows that degree of the differential equation is 3.

Example 2 :

Find the order of the DE $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = e\frac{d^2y}{dx^2}$

Sol. Order = 2
[Order of the highest order derivative i.e. $\frac{d^2y}{dx^2}$ is 2.]

Example 3 :
Find the order of the differential equation of all conics whose axes coincide with the axes of co-ordinates.

Sol. Since the general equation of all conics whose axes coincide with the axes of co-ordinates is $ax^2 + by^2 = 1$ and
$\because$ it has two arbitrary constants a, b.
$\therefore$ its differential equation will be of order 2.

Example 4 :

Find the degree of the DE $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} = \ell\frac{d^2y}{dx^2}$

Sol. Clearly, we have $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = \ell^2\left(\frac{d^2y}{dx^2}\right)^2$

Its degree is 2
[degree of the highest order derivative i.e. of $\frac{d^2y}{dx^2}$ is 2]

Example 5 :
Find the degree of the differential equation.

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$

Sol. The given differential equation can be written as

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$$

$\therefore$ degree of the diff. equation = 2

Example 6 :

Find the degree of the DE $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$

Sol. Since the given differential equation is not a polynomial equation in differential coefficients,

$\therefore$ its degree is not defined

LINEAR & NON LINER DIFFERENTIAL EQUATIONS

A differential equation in which the dependent variable and its differential coefficients occur only in the first degree and are not multiplied together is called as linear differential equation.

$$P_0\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + ...P_{n-1}\frac{dy}{dx} + P_n y = Q$$

Where $P_0, P_1, P_2, ...P_{n-1}$ and Q are either constants or functions of independent variable x.

Those equations which are not linear are called non– linear differential equations. A differential equation is a non-linear differential equation if

(i) its degree is more than one.
(ii) any of the differential co-efficient has exponent more than one.
(iii) exponent of the dependent variable is more than one.
(iv) products containing dependent variable and its differential co-efficients are present.

Ex. (i) $\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^2 - 5y = x^4$ is a non-linear differential equation [$\because$ its degree is 3, more than one]

(ii) $\frac{d^2y}{dx^2} + 6\left(\frac{dy}{dx}\right)^2 + 8y = x^2$, is a non-linear differential equation.

[$\because$ differential co-efficient $\frac{dy}{dx}$ has exponent 2]

(iii) $(x^2 + y^2)\,dx - 2xy\,dy = 0$ is a non-linear differential equation.

$\because$ the exponent of the dependent variable y is 2.

(iv) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \sin x$ is non-linear

$\because x\frac{dy}{dx}$ is present

(v) $\frac{d^2y}{dx^2} + y = 0$ is a linear differential equation of order 2 and degree 1.

(vi) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x$ is a linear differential equation of order 2 and degree 1.

Note: The degree of a linear differential equation is always one. But the converse is not always true.

e.g. $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ is of degree 1 and order 1 But it is not linear. [$\because$ it can be written as $xy\frac{dy}{dx} = \frac{x^2+y^2}{2}$]

[$\because$ y and $\frac{dy}{dx}$ are multiplied together]

Again, $y\frac{dy}{dx} - 4 = x$ is of degree one but is not a linear differential equation.

FORMATION OF DIFFERENCE

(i) Write down the give equation.
(ii) Differentiate it successively with respect to x that number of times equal to the arbitrary constants.
(iii) Hence on eliminating arbitrary constants results a differential equation which involves x,y, $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ ....

Example 7 :

Form the differential equation corresponding to $y^2 = m(a^2 - x^2)$ by eliminating m and a .

Sol. We are given that $y^2 = m(a^2 - x^2)$ ....(i)

Since the given equation contains two arbitrary constants, so we shall differentiate it two times and we shall get a differential equation of second order.

Differentiating both sides of (i) w.r.t.x , we get

$$2y\frac{dy}{dx} = m(-2x) \Rightarrow y\frac{dy}{dx} = -mx \quad ....(ii)$$

Differentiating both sides of (ii) w.r.t.x , we get

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m \quad ....(iii)$$

From (ii) and (iii) , we get $x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$

Example 8 :

Find the differential equation representing the family of curves

y = A cos (x + B), where A, B are parameters.

Sol. Since y = A cos (x + B) $\therefore \frac{dy}{dx} = -A\sin(x+B)$

$$\Rightarrow \frac{d^2y}{dx^2} = -A\cos(x+B) = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0$$

Example 9 :

Find the differential equation for $y = A\cos\alpha x + B\sin\alpha x$ where A and B are arbitrary constants.

Sol. $\frac{dy}{dx} = -A\alpha \sin \alpha x + B\alpha \cos \alpha x$

$\Rightarrow \frac{d^2y}{dx^2} = -A\alpha^2 \cos \alpha x - B\alpha^2 \sin \alpha x = -\alpha^2 . y$

$\therefore \frac{d^2y}{dx^2} + \alpha^2 y = 0$

SOLUTION OF DIFFERENTIAL EQUATION

A solution of a differential equation is any function which when put into the equation changes it into an identity.

General solution : The solution which contains a number of arbitrary constants equal to the order of the equation is called general solution for complete integral or complete primitive of differential equation.

Particular Solution : Solution obtained from the general solution by giving particular values to the constants are called particular solutions.

Example 10 :

Find the general solution of $x^2 \frac{dy}{dx} = 2$.

Sol. $\frac{dy}{dx} = \frac{2}{x^2} \Rightarrow dy = \frac{2}{x^2} dx$

Now integrate it . We get $y = -\frac{2}{x} + c$

SOLVING A FIRST ORDER FIRST DEGREE DE

Differential equation of the form $\frac{dy}{dx} = f(x)$ **:**

To solve this type of differential equations we integrate both sides to obtain the general solution as discussed below

$\frac{dy}{dx} = f(x) \Rightarrow dy = f(x)\, dx$

Integrating both sides we obtain

$\int dy = \int f(x) dx + c$ or $y = \int f(x) dx + c$

Example 11 :

Find the solution of the differential equation

$\frac{dy}{dx} = \sec x (\sec x + \cot x)$

Sol. $\frac{dy}{dx} = \sec x (\sec x + \cot x) = \sec^2 x + \sec x \tan x$

Now integrating both sides, we get $y = \tan x + \sec x + c$

Example 12 :

Find the solution of the differential equation $x \cos y\, dy = (x e^x \log x + e^x)\, dx$ is

Sol. $x \cos y\, dy = (xe^x \log x + e^x)\, dx$

$\Rightarrow \cos y\, dy = \left(e^x \log x + \frac{e^x}{x}\right) dx$

On integrating, $\sin y = e^x \log x + c$

Differential equation of the form $\frac{dy}{dx} = f(x)\, g(y)$

To solve this type of differential equation we integrate both sides to obtain the general solution.

$\frac{dy}{dx} = f(x)\, g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\, dx + c$

Example 13 :

Find the solution of equation $\frac{dy}{dx} = y(e^x + 1)$

Sol. $\frac{dy}{dx} = y(e^x + 1) \Rightarrow \frac{dy}{y} = (e^x + 1)dx$

Integrating both sides, we get $\log y = e^x + x + c$

Example 14 :

Find the solution of $x \frac{dy}{dx} = y + xy + x + 1$.

Sol. $x \frac{dy}{dx} = y + xy + x + 1$

$= y(1 + x) + (x + 1) = (1 + x)(1 + y)$

$\Rightarrow \frac{dy}{1+y} = \frac{1+x}{x} dx = \left(\frac{1}{x} + 1\right) dx$

$\Rightarrow \log(1 + y) = \log x + x + A$

$\Rightarrow \log\left(\frac{1+y}{x}\right) = x + A$

$\Rightarrow \frac{1+y}{x} = e^{x+A} = e^A . e^x = Ce^x$

$\Rightarrow 1 + y = Cx\, e^x$

Example 15 :

Find the solution of differential equation

$\log\left(\frac{dy}{dx}\right) = ax + by$.

Sol. $\frac{dy}{dx} = e^{(ax+by)}$; $\frac{dy}{dx} = e^{ax} . e^{by} \Rightarrow e^{-by} \frac{dy}{dx} = e^{ax} dx$

$\Rightarrow -\frac{1}{b} e^{-by} = \frac{1}{a} e^{ax} + c$

Differential equation of the form of $\frac{dy}{dx} = f(ax+by+c)$:

To solve this type of differential equations, we put

$ax + by + c = v$ and $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right) \therefore \frac{dv}{a+bf(v)} = dx$

So solution is by integrating $\int \frac{dv}{a+bf(v)} = \int dx$

Example 16 :

Find the solution of differential equation $\frac{dy}{dx} = \cos(x+y)$.

Sol. We are given that $\frac{dy}{dx} = \cos(x+y)$

Put $x + y = v$, so that

$$1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dy}{dx} - 1$$

So, the given equation becomes

$$\frac{dv}{dx} - 1 = \cos v \Rightarrow \frac{dv}{dx} = 1 + \cos v$$

$$\Rightarrow \frac{1}{1+\cos v} dv = dx \Rightarrow \frac{1}{2}\sec^2\frac{v}{2} dv = dx$$

Integrating both sides, we get

$$\int \frac{1}{2}\sec^2\frac{v}{2} dv = \int 1.dx \Rightarrow \tan\frac{v}{2} = x + C \Rightarrow \tan\left(\frac{x+y}{2}\right) = x + C$$

which is the required solution.

Example 17 :

Find the solution of $\frac{dy}{dx} = (4x+y+1)^2$.

Sol. Put $4x + y + 1 = z$

$$\therefore \quad 4 + \frac{dy}{dx} = \frac{dz}{dx} \quad \therefore \quad \frac{dz}{dx} - 4 = z^2 \Rightarrow \int \frac{dz}{z^2+4} = \int dx + C$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\frac{z}{2} = x + C \Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x+y+1}{2}\right) = x + C$$

Differential equation of homogeneous type :

An equation in x and y is said to be homogeneous if it can be put in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where $f(x,y)$ & $g(x,y)$ are both homogeneous functions of the same degree in x & y. So to solve the homogeneous differential equation

$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, substitute $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dy}{dx}$

Thus $\quad v + x\frac{dv}{dx} = f(v) \Rightarrow \frac{dx}{x} = \frac{dv}{f(v) - v}$

Therefore solution is $\int \frac{dx}{x} = \int \frac{dv}{f(v)-v} + c$

Example 18 :

Find the solution of DE $dy/dx = y/x + \tan y/x$ is

Sol. Put $y/x = v$, or $y = xv$ or $dy/dx = x + x\, dv/dx$

$$x\frac{dv}{dx} + v = v + \tan v \Rightarrow \frac{dv}{dx} = \tan v$$

$$\Rightarrow \frac{dv}{\tan v} = \frac{dx}{x}, \qquad \cot v\, dv = \frac{dx}{x},$$

or $\ell n \sin v = \ell nC$

$$\Rightarrow \frac{\sin v}{x} = C \quad \text{or} \quad \frac{\sin(y/x)}{x} = C$$

or $x = C' \sin(y/x)$

Example 19 :

Find the solution of $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

Sol. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is homogeneous

Put $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \therefore v + x\frac{dv}{dx} = \frac{x.vx}{x^2+v^2x^2} = \frac{v}{1+v^2}$$

$$\therefore x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{v - v - v^3}{1+v^2} = -\frac{v^3}{1+v^2}$$

$$\therefore \int \frac{1+v^2}{v^3} dv + \int \frac{dx}{x} = 0 \Rightarrow \int \frac{dv}{v^3} + \int \frac{dv}{v} + \int \frac{dx}{x} = 0$$

$$\Rightarrow -\frac{1}{2v^2} + \log vx = \log C \Rightarrow -\frac{x^2}{2y^2} + \log y = \log C$$

$$\Rightarrow \log\frac{y}{C} = \frac{x^2}{2y^2} \Rightarrow y = Ce^{x^2/2y^2}$$

DIFFERENTIAL EQUATION REDUCIBLE TO HOMOGENEOUS FORM

A differential equation of the form $\frac{dy}{dx} = \frac{a_1x + b_1x + c_1}{a_2x + b_2x + c_2}$,

Where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ can be reduced to homogeneous form by adopting the following procedure

Put $x = X + h$, $y = Y + k$ so that $\frac{dY}{dX} = \frac{dy}{dx}$

The equation then transformed to

$$\frac{dY}{dX} = \frac{a_1X + bY + (a_1h + b_1k + c_1)}{a_2X + b_2Y + (a_2h + b_2k + c_2)}$$

Now choose h and k such that $a_1h + b_1k + c_1 = 0$ and $a_2h + b_2k + c_2 = 0$. Then for these values of h and k the equation becomes $\frac{dY}{dX} = \frac{a_1X + b_1Y}{a_2X + b_2Y}$

This is a homogeneous equation which can be solved by putting Y = v X and then Y and X should be replaced by y – k and x – h.

Special case :

If $dy/dx = \frac{ax + by + c}{a'x + b'y + c'}$ and $\frac{a}{a'} = \frac{b}{b'} = m$ (say) i.e

when coefficient of x and y in numerator and denominator are proportional, then the above equation can not be solved by the method discussed before because the values of h & k given vy the equation will be indeterminate . In order to solve such equations, we proceed as explained in the following example.

Solve $\frac{dy}{dx} = \frac{3x - 6y + 7}{x - 2y + 4} = \frac{3(x - 2y) + 7}{x - 2y + 4}$

$$\left(\text{obvisously } \frac{a}{a'} = \frac{b}{b'} = 3\right)$$

Put $x - 2y = v \Rightarrow 1 - 2\frac{dy}{dx} = \frac{dv}{dx}$. Now we can solve it .

Example 20 :

Find the solution of differential equation $(3x - 7x + 7)\,dx + (7y - 3x + 3)\,dy = 0.$

Sol. The given differential equation is

$$\frac{dy}{dx} = \frac{7x - 3y - 7}{-3x + 7y + 3}$$

Substituting $x = X + h$, $y = Y + k$, we obtain

$$\frac{dY}{dX} = \frac{(7X - 3Y) + (7h - 3k - 7)}{(-3X + 7Y) + (-3h + 7k + 3)}$$

Choosing h and k such that $7h - 3k - 7 = 0$ and $-3h + 7k + 3 = 0$ which gives $h = 1$, $k = 0$.

Under the above transformations equation (i) can be written as $\frac{dY}{dX} = \frac{7X - 3Y}{-3X + 7Y}$

Putting $Y = VX$ so that $\frac{dY}{dX} = V + X\frac{dV}{dX}$, we get

$$V + X\frac{dV}{dX} = \frac{-3V + 7}{7V - 3} \Rightarrow X\frac{dV}{dX} = \frac{7 - 7V^2}{7V - 3}$$

$$\Rightarrow -7\frac{dx}{X} = \frac{7}{2}\cdot\frac{2V}{V^2 - 1}dV - \frac{3}{V^2 - 1}dV$$

Integrating , we get

$$-7\log X = \frac{7}{2}\log(V^2 - 1) - \frac{3}{2}\log\frac{V - 1}{V + 1} - \log C$$

$$\Rightarrow \log C = \log x^7 + \log[(V + 1)^5 (V - 1)^2]$$

$$\Rightarrow C = (V + 1)^5 (V - 1)^2 X^7 \text{ or } C = (Y + X)^5 (Y - X)^2$$

$$C = (x + y - 1)^5 (y - x + 1)^2$$

which is the required solution.

Example 21 :

Find the solution of $(3x + 2y + 1)\,dx - (3x + 2y - 1)\,dy = 0.$

Sol. We have $\frac{dy}{dx} = \frac{3x + 2y + 1}{3x + 2y - 1}$

Put $3x + 2y = z$

$$\therefore 3 + 2\frac{dy}{dx} = \frac{dz}{dx} \quad \therefore \frac{1}{2}\left(\frac{dz}{dx} - 3\right) = \frac{z + 1}{z - 1}$$

$$\Rightarrow \frac{dz}{dx} = 3 + \frac{2z + 2}{z - 1} = \frac{5z - 1}{z - 1} \Rightarrow \frac{z - 1}{5z - 1}dz = dx$$

$$\Rightarrow \frac{1}{5}\left(\frac{5z - 1 - 4}{5z - 1}\right)dz = dx \Rightarrow$$

$$\frac{1}{5}\int dz - \frac{4}{5}\int\frac{dz}{5z - 1} = \int dx + c$$

$$\Rightarrow \frac{z}{5} - \frac{4}{25}\log(5z - 1) = x + c$$

$$\Rightarrow 5(3x + 2y) - 4\log(5(3x + 2y) - 1) = 25x + \text{constant}$$

$$\Rightarrow 4\log(15x + 10y - 1) + 10x - 10y = \text{constant}$$

$$\Rightarrow 4\log(15x + 10y + 1)\frac{5}{2}(x - y) = c$$

LINEAR DIFFERENTIAL EQUATIONS

A differential equation is linear if the dependent variable (y) and its derivative appear only in first degree. The general form of a linear differential equation of first order is

$$\frac{dy}{dx} + Py = Q \qquad \text{......(1)}$$

Where P and Q are either constants or functions of x.

This type of differential equations are solved when they are multiplied by a factor, which is called integration factor, because by multiplication of this factor the left hand side of the differential equation becomes exact differential of some function.

Multiplying both sides of (1) by $e^{\int p\,dx}$, we get

$$e^{\int p\,dx}\left(\frac{dy}{dx}+Py\right)=Q\ e^{\int p\,dx}$$

On integrating both sides with respect to x we get

$$y\ e^{\int p\,dx}=\int Q\ e^{\int p\,dx}+c$$

which is the required solution, where c is the constant and

$e^{\int p\,dx}$ is called the integration factor.

Example 22 :

Find the solution of differential equation $\frac{dy}{dx}+y=e^x$.

Sol. $\frac{dy}{dx}+y=e^x$. This is linear equation of the form

$$\frac{dy}{dx}+Py=Q,\ \text{where } P=1,\ Q=e^x$$

So integrating factor (I.F.) = $e^{\int p\,dx}=e^x$

Thus solution is given by

$$ye^{\int p\,dx}=\int Q\ e^{\int p\,dx}\,dx+c$$

$$\Rightarrow\ y\,e^x=\int e^x\,e^x\,dx+c\ \Rightarrow\ y\,e^x=\frac{e^{2x}}{2}+c$$

$$\Rightarrow\ y\,e^x=\frac{e^{2x}}{2}+ce^{-x}$$

Example 23 :

Find the solution of $(x+2y^3)\,dy=y\,dx$.

Sol. $(x+2y^3)\,dy=y\,dx$

$$\Rightarrow\ \frac{dx}{dy}=\frac{x}{y}+2y^2\ \Rightarrow\ \frac{dx}{dy}-\frac{x}{y}=2y^2$$

$$e^{\int P'dy}=e^{\int\frac{-1}{y}dy}=e^{\log y^{-1}}=\frac{1}{y}$$

Type $\frac{dx}{dy}+P'x=Q'$ $\therefore$ Sol is $\frac{x}{y}=\int 2y\,dy+C=y^2+C$

$\Rightarrow\ x=y^3+Cy$

Example 24 :

Find the solution of the differential equation

$$\frac{dy}{dx}+\frac{y}{x}=\sin x$$

Sol. $\frac{dy}{dx}+\frac{1}{x}.y=\sin x$ $\left[\text{Type } \frac{dy}{dx}+Py=Q\right]$

$$e^{\int P\,dx}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

$\therefore$ Sol. is $y\,x=\int x\sin x\,dx+C$

$$=x(-\cos x)-\int 1.(-\cos x)dx+C=-x\cos x+\sin x+C$$

$$\Rightarrow x(y+\cos x)=\sin x+C$$

EQUATION REDUCIBLE TO LINEAR FORM

(i) **Bernoulli's Equation :** A differential equation of the form $dy/dx+Py=Qy^n$, where P & Q are function of x alone id called Bernoullis equation. This form can be reduced to linear form by dividing y^n and then putting $y^{1-n}=v$

Dividing both sides by y^n, we get

$$y^{-n}\frac{dy}{dx}+P.y^{-n+1}=Q$$

Putting $y^{-n+1}=V$ so that

$$(1-n)\,y^{-n}\frac{dy}{dx}=\frac{dv}{dx}\text{, we get}$$

$$\frac{dv}{dx}+(1-n)P.Y=(1-n)Q$$

which is a linear differential equation.

(ii) If the given equation is of form $\frac{dy}{dx}+P.\,f(y)=Q.\,g(y)$, where P and Q are functions of x alone, we divide the equation by g(y), we get $\frac{1}{g(y)}\frac{dy}{dx}+P.\frac{f(y)}{g(y)}=Q$

Now substituted $\frac{f(y)}{g(y)}=v$ and solve.

Example 25 :

Find the solution of DE $\frac{dy}{dx}-y\tan x=-y^2\sec x$.

Sol. $\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$, $\frac{1}{y}=v$; $\frac{-1}{y^2}\frac{dy}{dx}=\frac{dv}{dx}$

$\therefore$ $\frac{-dv}{dx}-v\tan x=-\sec x$; $\frac{dv}{dx}+v\tan x=\sec x$,

Here P = tanx, Q = secx

I.F. = $e^{\int\tan x\,dx}=\sec x$, $v\sec x=\int\sec^2 x\,dx+c$

Hence the solution is $y^{-1}\sec x=\tan x+c$

IF DIFFERENTIAL EQUATION IS OF THE FORM OF

$\frac{d^2y}{dx^2}=f(x)$ then its solution can be obtained by integrating it with respect to x twice.

Example 26 :

Find the solution of the differential equation

$$\cos^2 x\frac{d^2y}{dx^2}=1.$$

Sol. $\cos^2 x \dfrac{d^2y}{dx^2} = 1 \Rightarrow \dfrac{d^2y}{dx^2} = \sec^2 x$

On integrating ,we get $\dfrac{dy}{dx} = \tan x \pm c_1$

Again integrating, we get $y = \log \sec x \pm c_1 x \pm c_2$

PHYSICAL APPLICATION OF DIFFERENTIAL EQUATION :

1 **Mixture Problems :**

A chemical in a liquid solution (or dispersed in a gas) runs into a container holding the liquid (or the gas) with, possibly, a specified amount of the chemical dissolved as well. The mixture is kept uniform by stirring and flows out of the container at a known rate. In this process it is often important to know the concentration of the chemical in the container at any given time. The differential equation describing the process is based on the formula.

$$\begin{array}{c}\text{Rate of change}\\ \text{of amount}\\ \text{in container}\end{array} = \begin{pmatrix}\text{rate at which}\\ \text{chemical}\\ \text{arrives}\end{pmatrix} - \begin{pmatrix}\text{rate at which}\\ \text{chemical}\\ \text{departs}\end{pmatrix} \quad ....(1)$$

If y (t) is the amount of chemical in the container at time t and V (t) is the total volume of liquid in the container at time t, then the departure rate of the chemical at time t is

$$\text{Departure rate} = \frac{y(t)}{V(t)} \cdot (\text{out flow rate})$$

$$= \begin{pmatrix}\text{concentration in}\\ \text{container at time t}\end{pmatrix} \cdot (\text{out flow rate})$$

Accordingly, Equation (1) becomes

$$\frac{dy}{dt} = (\text{chemical's given arrival rate}) - \frac{y(t)}{V(t)} \cdot (\text{out flow rate}) \quad ....(2)$$

If, say, y is measured in grams, V in liters, and t in minutes, then unit in equation (2) are

$$\frac{\text{grams}}{\text{minute}} = \frac{\text{grams}}{\text{minute}} - \frac{\text{grams}}{\text{litre}} \cdot \frac{\text{litre}}{\text{minute}}$$

Example 27 :

A tank initially contains 100 litres of brine in which 50 gms of salt dissolved. A brine containing 2 gm/litre of salt runs into the tank at the rate of 5 litre/min. The mixture is kept stirring and flows out of the tank at the rate of 4 litres/min then

(a) At what rate (gms/min) does salt enter the tank at time t.

(b) What is the volume of the brine in the tank at time t.

(c) At what rate (gms/min) does salt leave the tank at time t.

(d) form the DE of the process and solve it to find an expression for the amount of salt present at time t.

Sol. (a) Inflow rate of brine solution = (2 gm/litre) · (5 litre/min) = 10 gm/min.

(b) Volume of the brine in the tank at time t
= initial volume + (inflow – outflow) · t
= 100 + (5 – 4) t = (100 + t) litres.

(c) Let y (t) is the amount of salt at time t then outflow rate of salt = $\dfrac{4y}{(100+t)}$

(d) Rate of change of salt in container = (rate at which salt arrives) – rate of which salt leaves

$$\frac{dy}{dt} = 10 - \frac{4y}{(100+t)} \Rightarrow \frac{dy}{dx} + \frac{4y}{(100+t)} = 10$$

$$\text{Integrating factor} = e^{\int \frac{4dt}{(100+t)}} = e^{4\ln(100+t)} = (100+t)^4$$

$$y(100+t)^4 = 10\int (100+t)^4 dt = 2(100+t)^5 + C$$

at t = 0, y = 50

$\Rightarrow$ $50 \cdot (100)^4 = 2(100)^5 + C \Rightarrow C = -150(100)^4$

$\Rightarrow$ $y(100+t)^4 = 2(100+t)^5 - 150(100)^4$

$\Rightarrow$ $y = 2(100+t) - \dfrac{150}{\left(1+\dfrac{t}{100}\right)^4}$

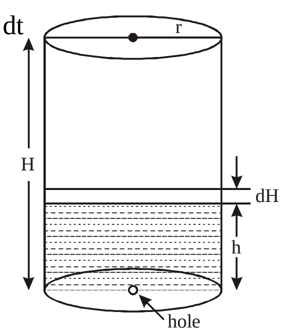
Example 28 :

Find the time required for a cylindrical tank of radius r and height H to empty through a round hole of area 'a' at the bottom. The flow through the hole is according to the law $v(t) = k\sqrt{2gh(t)}$ where v (t) and h (t) are respectively the velocity of flow through the hole and the height of the water level above the hole at time t and g is the acceleration due to gravity.

Sol. Let at time t the depth of water is h and radius of water surface is r.

If in time dt the decrease of water level is dh then

$$-\pi r^2 dh = ak\sqrt{(2gh)}\, dt$$



$\Rightarrow$ $\dfrac{-\pi r^2}{ak\sqrt{2g}\sqrt{h}} dh = dt$

$\Rightarrow$ $-\dfrac{\pi r^2}{ak\sqrt{2g}} \dfrac{dh}{\sqrt{h}} = dt$

Now when t = 0, h = H and when t = t, h = 0

then, $-\dfrac{\pi r^2}{ak\sqrt{2g}} \displaystyle\int_H^0 \frac{dh}{\sqrt{h}} = \int_0^t dt \Rightarrow -\frac{\pi r^2}{ak\sqrt{2g}} \left\{2\sqrt{h}\right\}_H^0 = t$

$\Rightarrow$ $t = \dfrac{\pi r^2 2\sqrt{H}}{ak\sqrt{2g}} = \dfrac{\pi r^2}{ak}\sqrt{\left(\dfrac{2H}{g}\right)}$

2 Statistical Applications of Differential Equation:

Example 29 :

The population of a certain country is known to increase at a rate proportional to the number of people presently living in the country. If after two years the population has doubled and after three years the population is 20,000, estimate the number of people initially living in the country.

Sol. Let N denote the number of people living in the country at any time t, and let N_0 denote the number of people initially living in the country.

Then, from equation $\frac{dN}{dt} - kN = 0$

which has the solution $N = ce^{kt}$ …(i)

At t = 0, $N = N_0$; hence equation (i) states that $N_0 = ce^{k(0)}$, or that $c = N_0$

Thus, $N = N_0e^{kt}$ …(ii)

At t = 2, $N = 2N_0$

Substituting these values into equation (ii), we have

$2N_0 = N_0e^{2k}$ from which $k = \frac{1}{2}\ln 2$

Substituting this value into equation (i) gives

$N = N_0\, e^{t/2 \ln 2}$ …(iii)

At t = 3, N = 20,000

Substituting these values into equation (iii), we obtain

$$20{,}000 = N_0e^{3/2 \ln 2} \Rightarrow N_0 = \frac{20000}{2\sqrt{2}} \approx 7071.$$

3 Geometrical Applications of Differential Equation :

We also use differential equations for finding the family of curves for which some conditiong involving the derivatives are given. For this we proceed in the following way

Equation of the tangent at a point (x, y) to the curve y = f (x) is given by $Y - y = \frac{dy}{dx}(X - x)$.

At the X axis, Y = 0, and $X = x - \frac{y}{dy/dx}$

(intercept on X-axis)

At the Y axis, X = 0, and $Y = y - \frac{dy}{dx}$ (intercept on Y-axis)

Similar information can be obtained for normals by writing equations as $(Y - y)\frac{dy}{dx} + (X - x) = 0$.

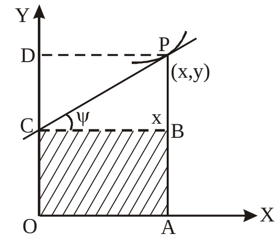
Example 30 :

Find the equation of the curve which is such that the area of the rectangle constructed on the abscissa of any point and the intercept of the tangent at this point on y-axis is equal to 4.

Sol. Equation of tangent P(x, y) is $Y - y = \frac{dy}{dx}(X - x)$

$\therefore$ Y-intercept $= y - x\frac{dy}{dx}$

$\therefore$ Area of OABC $= \left|x\left(y - x\frac{dy}{dx}\right)\right| = 4$



$\Rightarrow xy - x^2\frac{dy}{dx} = \pm 4$

$\Rightarrow \frac{dy}{dx} - \frac{1}{x}y = \pm\frac{4}{x^2}$

$\therefore$ I.F. $= e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\therefore$ the solution is $\left(\frac{y}{x}\right) = \pm 4\int \frac{1}{x^3}\,dx + c$

$\Rightarrow \frac{y}{x} = \pm\frac{2}{x^2} + c$

4. Orthogonal trajectory :

Any curve which cuts every member of a given family of curves at right angle, is called an orthogonal trajectory of the family. For example, each straight line passing through the origin, y = mx, is an orthogonal trajectory of the family of the circles $x^2 + y^2 = a^2$

Finding orthogonal trajectory of a given family of curves:

(i) Let f(x, y, c) = 0 be the equation, where c is an arbitrary parameter, of given family.

(ii) Differentiate the given equation with respect to x and then eliminate c.

(iii) Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in the equation obtained in (ii)

(iv) Solve the differential equation in (iii)

Example 31 :

Find the orthogonal trajectories of xy = c

Sol. xy = c. Differentiating w.r.t. x, we get $x\frac{dy}{dx} + y = 0$

Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ to get $x\frac{dx}{dy} - y = 0$

Integrating $x\,dx - y\,dy = 0 \Rightarrow x^2 - y^2 = c$

This is the family of required orthogonal trajectories.

Example 32 :

Find the orthogonal trajectory of $y^2 = 4ax$ (a being the parameter).

Sol. $y^2 = 4ax$; $2y\dfrac{dy}{dx} = 4a$

Eliminating a from equaiton (1) and (2)

$$y^2 = 2y\frac{dy}{dx}x.$$

Replacing $\dfrac{dy}{dx}$ by $-x\dfrac{dx}{dy}$, we get

$$y = 2\left(-\frac{dx}{dy}\right)x \ ; \quad 2x\,dx + y\,dy = 0$$

Integrating each term,

$$x^2 + \frac{y^2}{2} = c\ ; \quad 2x^2 + y^2 = 2c$$

which is required orthogonal trajectory.

TRY IT YOURSELF

Q.1 Find the order & degree of the following differential equation

(i) $\dfrac{d^2y}{dx^2} = \left[y + \left(\dfrac{dy}{dx}\right)^6\right]^{1/4}$

(ii) $e^{\frac{d^3y}{dx^3}} - x\dfrac{d^2y}{dx^2} + y = 0$

(iii) $\ln\left(\dfrac{dy}{dx}\right) = ax + by$

Q.2 From the differential equation of all circles touching the x-axis at the origin and centre on y-axis.

Q.3 From the differential equation of the family of parabolas with focus at the origin and axis of symmetry along the x-axis.

Q.4 Solve the following differential equation:

(i) $y' \sin x = y \ln y$; $y(\pi/2) = e$

(ii) $\dfrac{dy}{dx} = e^{x-y} + x^2 \cdot e^{-y}$

Q.5 Solve : $\dfrac{dy}{dx}\sqrt{1+x+y} = x + y - 1$

Q.6 Solve : $x\,dx + y\,dy = x\,(x\,dy - y\,dx)$

Q.7 Solve : $x^2\,dy + y\,(x + y)\,dx = 0$

$$\frac{dy}{dx} = -\frac{y\,(x+y)}{x^2} \quad \text{or} \quad \frac{dy}{dx} = -\frac{y}{x} - \frac{y^2}{x^2}$$

Q.8 Solve : $x\dfrac{dy}{dx} = y + 2\sqrt{y^2 - x^2}$

Q.9 Solve the following differential equation :

(i) $\cos^2 x\dfrac{dy}{dx} + y = \tan x$ (ii) $\dfrac{dy}{dx} = \dfrac{1}{x\cos y + \sin 2y}$

Q.10 Solve the following differential equation:

(i) $\dfrac{dy}{dx} = xy + x^3y^2$

(ii) $y\sin x\dfrac{dy}{dx} = \cos x\,(\sin x - y^2)$

Q.11 Suppose that a mothball volume by evaporation at a rate proportional to its instantaneous area. If the diameter of the ball decreases from 2cm to 1cm in 3 months, how long will it take until the ball has practically gone?

Q.12 What constant interest rate is required if an initial deposit placed into an account the accrues interest compounded continuously is to double its value in six years ? $(\ln|2| = 0.6930)$

Q.13 Find the equation of the curve passing through (2, 1) which has constant sub-tangent.

Q.14 Find the orthogonal tracjectory of the curve $x^2 + y^2 = a^2$.

ANSWERS

(1) (i) order = 2, degree = 4, (ii) order = 3, degree = not defined (iii) order = 1, degree = 1

(2) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$ **(3)** $y^2 = y^2\left(\dfrac{dy}{dx}\right)^2 + 2xy\dfrac{dy}{dx}$

(4) (i) $y = e^{(\text{cosec } x - \cot x)}$, (ii) $e^y = e^x + \dfrac{x^3}{3} + C$

(5) $2\left[v + \dfrac{1}{3}\log|v-1| - \dfrac{4}{3}\log|v+2|\right] = x + c$

where $v = \sqrt{1+x+y}$

(6) $(1 + y)^2 = c^2(x^2 + y^2)$

(7) $\left|\dfrac{x^2y}{2x+y}\right| = c \ \ (c > 0)$ **(8)** $y + \sqrt{y^2 - x^2} = c^2x^3$

(9) (i) $y = (\tan x - 1) + ke^{-\tan x}$,
(ii) $x = -2(\sin y + 1) + c\,e^{\sin y}$.

(10) (i) $y = \dfrac{-e^{\frac{x^2}{2}}}{(x^2 - 2)\,e^{\frac{x^2}{2}} + C}$, (ii) $y^2\sin^2 x = \dfrac{2\sin^3 x}{3} + C$

(11) 6 months **(12)** 11.55 percent

(13) $k\ln y = x - 2$. **(14)** $y = cx$

IMPORTANT POINTS

* The function f (x, y) is said to be a homogeneous function of degree n if for any real number t ($\neq 0$), we have $f(tx, ty) = t^n f(x, y)$.
For e.g. $f(x, y) = ax^{2/3} + hx^{1/3}.y^{1/3} + y^{2/3}$ is a homogeneous function of degree 2/3.

* A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is called a first order linear differential equation.

(a) Differential equation of the form $\frac{dy}{dx} = f(x)$:

$$y = \int f(x)dx + c$$

(b) $\frac{dy}{dx} = f(x)\, g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\, dx + c$

(c) $\frac{dy}{dx} = f(ax + by + c) : \int \frac{dv}{a + bf(v)} = \int dx$

(d) **Differential equation of homogeneous type :** An equation in x and y is said to be homogeneous if it can be put in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f (x ,y) and g (x ,y) are both homogeneous functions of the same degree in x & y .

$$\int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$$

(e) **Differential Equation reducible to homogeneous form:**

A differential equation of the form $\frac{dy}{dx} = \frac{a_1x + b_1x + c_1}{a_2x + b_2x + c_2}$,

where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ can be reduced to homogeneous form by using $x = X + h$, $y = Y + k$ so that $\frac{dY}{dX} = \frac{dy}{dx}$

* **Linear differential equations :**

$\frac{dy}{dx} + Py = Q$; $y\, e^{\int p\,dx} = \int Q\, e^{\int p\,dx} + c$, $e^{\int P\,dx}$ is called the integrating factor for this equation.

* A differential equation of the form $\frac{dy}{dx} = f(x, y)$ is homogeneous if f (x, y) is a homogeneous function of degree zero i.e. $f(tx, ty) = t^0 f(x, y) = f(x, y)$.

The function f does not depend on x and y separately but only on their ratio $\frac{y}{x}$ or $\frac{x}{y}$.

* Sometimes a given differential equation becomes linear if we take y as the independent variable and x as the dependent variable. e.g. the equation ; $(x + y + 1)\frac{dy}{dx} = y^2 + 3$ can be written as $(y^2 + 3)\frac{dy}{dx} = (x + y + 1)$ which is a linear differential equation.

* The differential equation of the family of circles $x^2 + y^2 + 2gx + 2fy + c = 0$; where f, g and c are arbitrary constants is $[1 + (y')^2].y''' - 3y'(y'')^2 = 0$

* A normal is drawn at a point P (x, y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then the differential equation describing such curves is,

$$y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}.$$

* **Following exact differentials must be remembered :**

(i) $xdy + y\,dx = d(xy)$

(ii) $\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$

(iii) $\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$

(iv) $\frac{x\,dy + y\,dx}{xy} = d(\ln xy)$

(v) $\frac{dx + dy}{x + y} = d(\ln(x + y))$

(vi) $\frac{x\,dy - y\,dx}{xy} = d\left(\ln\frac{y}{x}\right)$

(vii) $\frac{y\,dx - x\,dy}{xy} = d\left(\ln\frac{x}{y}\right)$

(viii) $\frac{x\,dy - y\,dx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$

(ix) $\frac{y\,dx - x\,dy}{x^2 + y^2} = d\left(\tan^{-1}\frac{x}{y}\right)$

(x) $\frac{x\,dx + y\,dy}{x^2 + y^2} = d\left[\ln\sqrt{x^2 + y^2}\right]$

(xi) $d\left(-\frac{1}{xy}\right) = \frac{x\,dy + y\,dx}{x^2 y^2}$

(xii) $d\left(\frac{e^x}{y}\right) = \frac{y e^x\,dx - e^x\,dy}{y^2}$

(xiii) $d\left(\frac{e^y}{x}\right) = \frac{x e^y\,dy - e^y\,dx}{x^2}$

ADDITIONAL EXAMPLES

Example 1 :

Find the general solution of the differential equation

$$(e^x + e^{-x})\frac{dy}{dx} = (e^x - e^{-x})$$

Sol. We have $(e^x + e^{-x})\frac{dy}{dx} = (e^x - e^{-x})$

$\Rightarrow \frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Integrating $y = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx + c$

Put $e^x + e^{-x} = t$ so that $(e^x - e^{-x})\,dx = dt$

$\therefore \quad y = \int \frac{dt}{t} + c = \log|t| + c.$

Hence $y = \log|e^x + e^{-x}| + c$

which is the reqd. general solution.

Example 2 :

Find the solution of different equation

$$(2x - 10y^3)\frac{dy}{dx} + y = 0.$$

Sol. The given equation can be written as :

$\frac{dx}{dy} + \frac{2}{y}x = 10y^2$ .....(1) [Linear Equation in x]

Here 'P' $= \frac{2}{y}$ and 'Q' $= 10y^2$

I.F. $= e^{\int P dy} = e^{\int \frac{2}{y}dy} = e^{2\log|y|} = e^{\log y^2} = y^2$

Multiplying (1) by y^2, we get :

$y^2 . \frac{dx}{dy} + 2yx = 10y^4$

$\Rightarrow \frac{d}{dy}(x.y^2) = 10y^4$

Integrating, $xy^2 = 10\int y^4 dy + c$

$\Rightarrow xy^2 = 2y^5 + c$ which is required solution

Example 3 :

Find the solution of the differential equation

$$\frac{dy}{dx} + 1 = e^{x+y}.$$

Sol. $\frac{dy}{dx} + 1 = e^x.e^y \Rightarrow e^{-y}\frac{dy}{dx} + e^{-y} = e^x$

Put $e^{-y} = z \quad \therefore -e^{-y}\frac{dy}{dx} = \frac{dz}{dx} \quad \therefore -\frac{dz}{dx} + z = e^x$

i.e. $\frac{dz}{dx} - z = -e^x$ which is linear in z. Here $P = -1$

$\therefore \int P dx = \int -dx = -x \qquad \therefore e^{\int P dx} = e^{-x}$

$\therefore$ Sol. is $z.e^{-x} = \int -e^x.e^{-x}dx + c = -x + c$

$\Rightarrow e^{-y}.e^{-x} = c - x \Rightarrow e^{-(x+y)} = -(x - c)$

$\Rightarrow (x - c)\,e^{x+y} + 1 = 0$

Example 4 :

Find the equation of family of curves for which the length of the normal is equal to the radius vector.

Sol. Length of the normal $= y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

It is given that $y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{x^2 + y^2}$

$\left[\because \text{radius vector} = r = \sqrt{x^2 + y^2}\right]$

$\Rightarrow y^2 + y^2\left(\frac{dy}{dx}\right)^2 = x^2 + y^2 \quad \Rightarrow y^2\left(\frac{dy}{dx}\right)^2 = x^2$

$\Rightarrow y\,dy \pm x dx = 0 \Rightarrow y^2 \pm x^2 = k^2$

Example 5 :

Find the solution of the equation $x dx + y dy + \frac{xdy - ydx}{x^2 + y^2} = 0$

Sol. We have, $x\,dx + y\,dy + \frac{xdy - ydx}{x^2 + y^2} = 0$

$\Rightarrow \frac{1}{2}d(x^2 + y^2) + d\ \tan^{-1}\left(\frac{y}{x}\right) = 0$

Integrating, $\frac{1}{2}(x^2 + y^2) + \tan^{-1}\frac{y}{x} = \frac{c}{2}$

$\Rightarrow x^2 + y^2 + 2\tan^{-1}(y/x) = c$

$\therefore \quad y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$ is the required solution.

Example 6 :

Find the general solution of the differential equation

$\frac{dy}{dx} + y\,g'(x) = g(x).\,g'(x)$, where g(x) is a given function of x

Sol. We have, $\frac{dy}{dx} = (g(x) - y), g'(x)$

Put $g(x) - y = V \Rightarrow g'(x) - \frac{dy}{dx} = \frac{dV}{dx}$

Hence, $g'(x) - \frac{dV}{dx} = V.g'(x)$

$\Rightarrow \frac{dV}{dx} = (1 - V).g'(x)$

$\Rightarrow \frac{dV}{1 - V} = g'(x)\,dx$

$\Rightarrow \int \frac{dV}{1 - V} = \int g'(x)\;dx$

$\Rightarrow -\log(1 - V) = g(x) - C$

$\Rightarrow g(x) + \log(1 - V) = C$

$\therefore g(x) + \log[1 + y - g(x)] = C$

Example 7 :

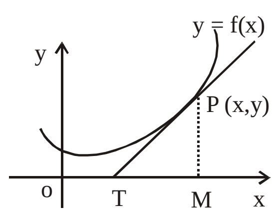
The equation of the curve which is such that the portion of the axis of x-cut off between the origin and tangent at any point is proportional to the ordinate of that point is

(A) $x = y(a - b\log x)$ (B) $\log x = by^2 + a$

(C) $x^2 = y(a - b\log y)$ (D) None of these

[b is constant of proportionality]

Sol. **(A).** Let the equation of the curve be y = f (x).



It is given that OT ∝ y

$\Rightarrow$ OT = by $\Rightarrow$ OM – TM = by

$\Rightarrow x - \frac{y}{dy/dx} = by$

[∵ TM = Length of the subtangent]

$\Rightarrow x - y\frac{dx}{dy} = b.y$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = -b$

It is linear differential equation. Its solution is given by

$\frac{x}{y} = -b\log y + a \Rightarrow x = y(a - b\log y)$

Example 8 :

A body at a temperature of 50F is placed outdoors where the temperature is 100 F. If the rate of change of the temperature of a body is proportional to the temperature difference between the body and its surrounding medium. If after 5 min. the temperature of the body is 60F, find

(a) how long it will take the body to reach a temperature of 75F and (b) the temperature of the body after 20 min.

Sol. Let T be the temperature of the body at time t and $T_m = 100$ (the temperature of the surrounding medium)

$$\frac{dT}{dt} = -k(T - T_m) \text{ or } \frac{dT}{dt} + kT = kT_m$$

where k is constant of proportionally.

$\Rightarrow \frac{dT}{dt} + kT = 100k$

This differential equation whose solution is

$T = ce^{-kt} + 100$ …(i)

Since T = 50 when t = 0, then from equation (i)

$50 = ce^{-k(0)} + 100$ or $c = -50$.

Substituting this value in equation (i), we obtain

$T = -50e^{-kt} + 100$ …(ii)

At t = 5, we are given that T = 60; hence, from equation (ii),

$60 = -50e^{-5k} + 100$.

Solving for k, we obtain $-40 = -50\,e^{-5k}$

or $k = -\frac{1}{5}\ln\frac{40}{50}$

Substituting this value in equation (ii), we obtain the temperature of the body at any time t as

$T = -50\,e^{\frac{1}{5}\ln\frac{4}{5}t} + 100$ …(iii)

(a) We require t when T = 75, Substituting T = 75 in equation (iii), we have $75 = -50\,e^{\frac{1}{5}\ln\frac{4}{5}t} + 100$, from which we get t

(b) We require T when t = 20. Substituting t = 20 in equation (iii) and then solving for T, we find

$T = -50\,e^{\frac{1}{5}\ln\frac{4}{5}(20)} + 100$

Example 9 :

Find the curve such that the intercept on the x-axis cut off between the origin and the tangent at a point is twice the abscissa and which passes through the point (1, 2).

Sol. The equation of the tangent at any point P(x, y) is

$Y - y = \frac{dy}{dx}(X - x)$

Given that intercept on X-axis

(putting Y = 0) = 2 (x-coordinate of P)

$\Rightarrow x - y\frac{dy}{dx} = 2x \Rightarrow -\frac{dy}{y} = -\frac{dx}{x}$

Integrating we get xy = c

Since the curve passes through (1, 2), c = 2

Hence, the equation of the required curve is xy = 2.

Example 10 :

Find the equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$.

Sol. Equation of normal at any point P(x, y) is

$$\frac{dy}{dx}(Y-y)+(X-x)=0$$

This meets the x-axis at $A\left(x+y\frac{dy}{dx},0\right)$

Mid point of AP is $\left(x+\frac{1}{2}y\frac{dy}{dx},\frac{y}{2}\right)$ which lies on the parabola $2y^2 = x$.

$$\therefore \quad 2\times\frac{y^2}{4}=x+\frac{1}{2}y\frac{dy}{dx} \text{ or } y^2=2x+y\frac{dy}{dx}$$

Putting $y^2 = t$, so that $2y\frac{dy}{dx}=\frac{dt}{dx}$,

we get $\frac{dt}{dx}-2t=-4x$ (linear)

I.F. $= e^{-2\int dx}=e^{-2x}$

$\therefore$ solution is $t\,e^{-2x}=-4\int xe^{-2x}\,dx+c$

$$=-4\left[\frac{1}{2}xe^{-2x}-\int-\frac{1}{2}e^{-2x}\,dx\right]+c$$

$\Rightarrow \quad y^2 e^{-2x} = 2xe^{-2x} + e^{-2x} + C$

or $\quad y^2 = 2x + 1 - e^{2x}$ is the equation of the required curve.

Example 11 :

Find the orthogonal trajectory of the curve
$x^2 + y^2 - 2ay = 0$

Sol. $x^2 + y^2 - 2ay = 0 \Rightarrow 2x + 2yy' - 2ay' = 0$

$$\Rightarrow \quad 2a=\left(\frac{2x+2yy'}{y'}\right)$$

$$\therefore \quad x^2+y^2-\left(\frac{2x+2yy'}{y'}\right)y=0$$

For orthogonal trajectory, replace y' by $\frac{-1}{y'}$

$$x^2+y^2-\frac{\left(2x-\frac{2y}{y'}\right)y}{\left(\frac{-1}{y'}\right)}=0$$

$\Rightarrow \quad x^2 + y^2 + (2xy' - 2y)\,y = 0$

$\Rightarrow \quad x^2+y^2+2xy\frac{dy}{dx}-2y^2=0$

$\Rightarrow \quad \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ (a homogeneous equation)

Put $y = vx$ $\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\Rightarrow \quad v+x\frac{dv}{dx}=\frac{v^2-1}{2v}\Rightarrow \quad x\frac{dv}{dx}=\frac{v^2-1}{2v}-v=\frac{-(v^2+1)}{2v}$$

$$\Rightarrow \quad \frac{2v}{v^2+1}dv=\frac{-dx}{x}$$

$\Rightarrow \quad \ln(v^2+1) = -\ln x + \ln c \;\Rightarrow\; x^2 + y^2 = cx$

QUESTION BANK

CHAPTER 9 : DIFFERENTIAL EQUATIONS

EXERCISE - 1 [LEVEL-1]

PART - 1 - ORDER AND DEGREE OF DE'S

Q.1 The differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^{1/2} = y^3$ has the degree

(A) 1/2 (B) 2
(C) 3 (D) 4

Q.2 The order of the differential equation whose general solution is given by

$y = c_1 \cos(2x + c_2) - (c_3 + c_4)\, a^{x+c_5} + c_6 \sin(x - c_7)$,

(A) 3 (B) 4
(C) 5 (D) 2

Q.3 Order and degree of DE $\frac{d^2y}{dx^2} = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}^{1/4}$ are

(A) 4 and 2 (B) 1 and 2
(C) 1 and 4 (D) 2 and 4

Q.4 Family $y = Ax + A^3$ of curve represented by the differential equation of degree

(A) Three (B) Two
(C) One (D) None of these

Q.5 If m and n are the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\left(\frac{d^3y}{dx^3}\right)} + \frac{d^3y}{dx^3} = x^2 - 1$, then

(A) m = 3 and n = 5 (B) m = 3 and n = 1
(C) m = 3 and n = 3 (D) m = 3 and n = 2

Q.6 Order of the differential equation of the family of all concentric circles centered at (h, k) is

(A) 1 (B) 2
(C) 3 (D) 4

Q.7 The degree of the differential equation

$\frac{d^2y}{dx^2} - \sqrt{\frac{dy}{dx} - 3} = x$

(A) 2 (B) 1
(C) 1/2 (D) 3

Q.8 The order and degree of the differential equation

$y = \frac{dp}{dx}x = \sqrt{a^2p^2 + b^2}$, where p = dy/dx

(here a and b are arbitrary constants) respectively are –

(A) 2, 2 (B) 1, 1
(C) 1, 2 (D) 2, 1

Q.9 If m and n are degree and order of $(1 + y_1^2)^{2/3} = y^2$, then the value of $\frac{m+n}{m-n}$ is –

(A) 2 (B) 5
(C) 4 (D) 3

Q.10 If 'm' and 'n' are the order and degree of the differential equation $(y'')^5 + 4 \times \frac{(y'')^3}{y'''} + y''' = \sin x$, then

(A) m = 3, n = 5 (B) m = 3, n = 1
(C) m = 3, n = 3 (D) m = 3, n = 2

Q.11 The order and degree of the differential equation

$y = x\frac{dy}{dx} + \frac{2}{dy/dx}$ is –

(A) 1, 2 (B) 1, 3
(C) 2, 1 (D) 1, 1

Q.12 The order of differential equation of all circles of given radius 'a' is –

(A) 4 (B) 2
(C) 1 (D) 3

PART - 2 - FORMATION OF DIFFERENTIAL EQUATION

Q.13 The differential equation whose solution is $y = A\sin x + B\cos x$, is

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{dy}{dx} + y = 0$ (D) None of these

Q.14 The differential equation of the family of curves

$v = \frac{A}{r} + B$, where A and B are arbitrary constants, is

(A) $\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = 0$ (B) $\frac{d^2v}{dr^2} - \frac{2}{r}\frac{dv}{dr} = 0$

(C) $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$ (D) None of these

Q.15 The differential equation obtained on eliminating A and B from the equation $y = A\cos\omega t + B\sin\omega t$ is

(A) $y'' = -\omega^2 y$ (B) $y'' + y = 0$

(C) $y'' + y' = 0$ (D) $y'' - \omega^2 y = 0$

Q.16 The differential equations of all conics whose axes coincide with the co-ordinate axis –

(A) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

(B) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}=0$

(C) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

(D) $xy\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

Q.17 The differential equations of all circles passing through origin and having their centres on the x-axis is

(A) $\frac{dy}{dx}=\frac{y^2+x^2}{2xy}$ (B) $\frac{dy}{dx}=\frac{y^2+x^2}{2x}$

(C) $\frac{d^2y}{dx^2}=\frac{y^2-x^2}{2xy}$ (D) $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$

Q.18 The differential equation of family of parabola with foci at the origin and axis along the x axes –

(A) $y\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}-y=0$ (B) $x\left(\frac{dy}{dx}\right)^2+2y\frac{dy}{dx}-y=0$

(C) $y\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}+y=0$ (D) None of these

Q.19 The differential equation satisfied by the family of curves $y=ax\cos\left(\frac{1}{x}+b\right)$, where a, b are parameters, is

(A) $x^2y_2+y=0$ (B) $x^4y_2+y=0$
(C) $xy_2-y=0$ (D) $x^4y_2-y=0$

PART - 3 - SOLUTION OF DIFFERENTIAL EQUATION

Q.20 The general solution of the differential equation $2x\frac{dy}{dx}-y=3$ is a family of –

(A) hyperbolas (B) parabolas
(C) straight lines (D) circles

Q.21 The general solution of $\frac{dy}{dx}=1-x^2-y^2+x^2y^2$ is –

(A) $2\sin^{-1}y=x\sqrt{1-y^2}+c$ (B) $\sin^{-1}y=\frac{1}{2}\sin^{-1}x+c$

(C) $\cos^{-1}y=x\cos^{-1}x+c$ (D) None of these

Q.22 Solution of $e^{dy/dx}=x$ when x = 1 and y = 0 is –

(A) $y=x(\log x-1)+1$ (B) $y=x(\log x-1)+4$
(C) $y=x(\log x-1)+3$ (D) $y=x(\log x+1)+1$

Q.23 The general solution of the differential equation $\sqrt{1-x^2y^2}\,dx=y\cdot dx+x\cdot dy$ is –

(A) $\sin(xy)=x+c$ (B) $\sin^{-1}(xy)+x=c$
(C) $\sin(x+c)=xy$ (D) $\sin(xy)+x=c$

Q.24 A gardener is digging a plot of land. As he gets tired, he works more slowly. After 't' minutes he is digging at a rate of $(2/\sqrt{t})$ square metres per minute. How long will it take him to dig an area of 40 square metres?

(A) 100 minutes (B) 10 minutes
(C) 30 minutes (D) 40 minutes

Q.25 The solution of differential equation $x\frac{dy}{dx}+2y=x^2$

(A) $y=\frac{x^2+C}{4x^2}$ (B) $y=\frac{x^2}{4}+C$

(C) $y=\frac{x^4+C}{x^2}$ (D) $y=\frac{x^4+C}{4x^2}$

Q.26 Solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$ is

(A) x (y + cosx) = cos x + C (B) x (y – cosx) = sinx + C
(C) x (y + cosx) = sinx + C (D) None of these

Q.27 The solution of $\frac{dy}{dx}=\frac{ax+h}{by+k}$ represents a parabola when

(A) a = 0, b = 0 (B) a = 1, b = 2
(C) a = 0, b ≠ 0 (D) a = 2, b = 1

Q.28 The solution of the differential equation $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$ is

(A) tan x = c tan y (B) tan x = c tan (x + y)
(C) tan x = c cot y (D) tan x sec y = c

Q.29 The degree of the differential equation of all tangent lines to the parabola $y^2=4ax$ is –

(A) 1 (B) 2
(C) 3 (D) 4

Q.30 Solution of the differential equation x dy – y dx = 0 represents

(A) parabola whose vertex is at origin
(B) circle whose centre is at origin
(C) a rectangular hyperbola
(D) straight line passing through origin

Q.31 If c is an arbitrary constant, then the general solution of the differential equation y dx – x dy = xy dx is given by –

(A) $y=cxe^{-x}$ (B) $x=cye^{-x}$
(C) $y+e^x=cx$ (D) $ye^x=cx$

Q.32 Solutions of the differential equation $\left(\frac{dy}{dx}\right)^2-\left(\frac{dy}{dx}\right)(e^x+e^{-x})+1=0$ are given by –

(A) $y^2-e^x=C$ (B) $y^3+e^x=C$
(C) $y+e^{-x}=C$ (D) $y+e^{-x}+e^x=C$

Q.33 The solution of the equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is

(A) $x\sin\left(\frac{x}{y}\right) + c = 0$ (B) $x\sin y + c = 0$

(C) $x\sin\left(\frac{y}{x}\right) = c$ (D) None of these

Q.34 An integrating factor of the differential equation

$\frac{dy}{dx} + \frac{2xy}{1-x^2} = \frac{x}{\sqrt{1-x^2}}$ is

(A) $(1+x^2)^{-1}$ (B) $(1-x^2)^{-1}$

(C) $x/(1-x^2)$ (D) $x/\sqrt{1-x^2}$

Q.35 Solution of the differential equation y' = y tan x – 2 sin x, is

(A) y = tan x + 2c cos x (B) y = tan x + c cos x

(C) y = tan x – 2c cos x (D) None of these

Q.36 A curve having the condition that the slope of tangent at some point is two times the slope of the straight line joining the same point to the origin of coordinates is a/an

(A) Circle (B) Ellipse

(C) Parabola (D) Hyperbola

Q.37 The solution of the differential equation

$(\sin x + \cos x)\,dy + (\cos x - \sin x)\,dx = 0$ is

(A) $e^x(\sin x + \cos x) + c = 0$ (B) $e^y(\sin x + \cos x) = c$

(C) $e^y(\cos x - \sin x) = c$ (D) $e^x(\sin x - \cos x) = c$

Q.38 The solution of differential equation $x\frac{dy}{dx} + y = y^2$ is

(A) y = 1 + cxy (B) y = log {cxy}

(C) y + 1 = cxy (D) y = c + xy

Q.39 The solution of the differential equation $\frac{dy}{dx} = \frac{(1+x)y}{(y-1)x}$ is

(A) log xy + x + y = c (B) $\log\left(\frac{x}{y}\right) + x - y = c$

(C) log xy + x – y = c (D) None of these

Q.40 The solution of the differential equation

$(1-x^2)(1-y)\,dx = xy\,(1+y)\,dy$ is

(A) $\log[x(1-y)^2] = \frac{x^2}{2} + \frac{y^2}{2} - 2y + c$

(B) $\log[x(1-y)^2] = \frac{x^2}{2} - \frac{y^2}{2} + 2y + c$

(C) $\log[x(1+y)^2] = \frac{x^2}{2} + \frac{y^2}{2} + 2y + c$

(D) $\log[x(1-y)^2] = \frac{x^2}{2} - \frac{y^2}{2} - 2y + c$

Q.41 The solution of $\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$ is

(A) $y^2(\log y) - e^x\sin^2 x + c = 0$

(B) $y^2(\log y) - e^x\cos^2 x + c = 0$

(C) $y^2(\log y) + e^x\cos^2 x + c = 0$

(D) None of these

Q.42 Solution of the differential equation

$\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is

(A) sec y + 2 cos x = c (B) sec y – 2 cos x = c

(C) cos y – 2 sin x = c (D) tan y – 2 sec y = c

Q.43 Solution of differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is

(A) $x^3 + y^2 = px^2$ (B) $\frac{x^2}{2} + \frac{y^3}{x} = y^2 + p$

(C) $x^2 + y^3 = px^2$ (D) $x^2 + y^2 = px^3$

Q.44 Solution of the differential equation

$\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ is

(A) $y = \tan x - 1 + ce^{-\tan x}$ (B) $y^2 = \tan x - 1 + ce^{\tan x}$

(C) $ye^{\tan x} = \tan x - 1 + c$ (D) $ye^{-\tan x} = \tan x - 1 + c$

PART - 4 - MISCELLANEOUS

Q.45 If solution of differential equation $\frac{dy}{dx} - y = 1 - e^{-x}$ and $y(0) = y_0$ has a finite value. When $x \to \infty$ then, y_0 is equal to

(A) –1/2 (B) 0

(C) 1 (D) –1

Q.46 The degree of the differential equation

$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ is

(A) 1 (B) 2

(C) 3 (D) None of these

Q.47 The equation of the curve satisfying the differential equation $y(x+y^3)\,dx = x(y^3 - x)\,dy$ and passing through the point (1, 1) is

(A) $y^3 - 2x + 3x^2y = 0$ (B) $y^3 + 2x + 3x^2y = 0$

(C) $y^3 + 2x - 3x^2y = 0$ (D) None of these

Q.48 The differential equation representing the family of hyperbolas $a^2x^2 - b^2y^2 = c^2$ is

(A) $\frac{y''}{y'}+\frac{y'}{y}=\frac{1}{x}$ (B) $\frac{y''}{y'}+\frac{y'}{y}=\frac{1}{x^2}$

(C) $\frac{y''}{y'}-\frac{y'}{y}=\frac{1}{x}$ (D) $\frac{y''}{y'}=\frac{y}{y'}-\frac{1}{x}$

Q.49 If $y = \frac{x}{\ln|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is :

(A) $\frac{x^2}{y^2}$ (B) $-\frac{x^2}{y^2}$

(C) $\frac{y^2}{x^2}$ (D) $-\frac{y^2}{x^2}$

Q.50 The curve that satisfies the differential equation $y' = \frac{x^2+y^2}{2xy}$ and passes through (2, 1) is a hyperbola with eccentricity-

(A) $\sqrt{2}$ (B) $\sqrt{3}$

(C) 2 (D) $\sqrt{5}$

Q.51 The solution of the differential equation,

$2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\frac{\pi}{2}}$ is

(A) $\sin x^2y^2 = e^{x-1}$ (B) $\sin(x^2y^2) = x$

(C) $\cos x^2y^2 + x = 0$ (D) $\sin(x^2y^2) = e.e^x$

Q.52 If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$, then

(A) $y = \cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9}+\frac{c}{x^2}$

(B) $y = -\cos x - \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9}+\frac{c}{x^2}$

(C) $y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x - \frac{x}{3}\log x - \frac{x}{9}+\frac{c}{x^2}$

(D) None of these

Q.53 Equation of a curve which satisfies the differential equation $\frac{dy}{dx}=\frac{y^2-6y+9}{6x-2xy-1}$ and passes through the point (1, 2) will be

(A) a pair of lines

(B) $x[(y-3)^2+1] = 2$

(C) an ellipse

(D) a rectangular hyperbola or a line

Q.54 The solution of the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$ is

(A) $(x + y)e^{x+y} = 0$ (B) $(x + c)e^{x+y} = 0$

(C) $(x - c)e^{x+y} = 1$ (D) $(x - c)e^{x+y} + 1 = 0$

Q.55 The general solution of the differential equation $(1 + y^2)dx + (1 + x^2)dy = 0$ is

(A) $mx - y = C(1 - xy)$ (B) $x - y = C(1 + xy)$

(C) $(x + y) = C(1 - xy)$ (D) $x + y = C(1 + xy)$

Q.56 The solution of differential equation

$\frac{dy}{dx} - y\tan x = -y^2\sec x$ is

(A) $y^{-1}\sec x = \cot x + c$ (B) $y^{-1}\cos x = \tan x + c$

(C) $y^{-1}\sec x = \tan x + c$ (D) None of these

Q.57 A tank contains 30 lit. of a chemical solution prepared by dissolving 120 gm of a soluble substance in the fresh water. Fluid containing 4 gm. of this substance per lit. runs in at the rate of 4 lit./min. and the well–stirred mixture runs out at the same rate. The amount of substance in the tank after 30 min. is

(A) 0 gm. (B) 120 gm.

(C) 60 gm. (D) 80 gm.

Q.58 The curve which satisfies the differential equation $y' = \frac{3x}{y}$ and passes through (1, 1) is a-

(A) Pair of lines through the origin

(B) Hyperbola with eccentricity 2

(C) Hyperbola with eccentricity $\frac{2}{\sqrt{3}}$

(D) None of these

Q.59 The solution of $\frac{dy}{dx} = (4x + y + 1)^2$ is

(A) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + C$

(B) $\frac{1}{2}\tan^{-1}\left(\frac{x+y-1}{2}\right) = x + C$

(C) $\frac{1}{2}\tan^{-1}\left(\frac{x+y}{2}\right) + C$

(D) None of these

Q.60 The differential equation of all ellipses centred at the origin having major and minor axes along coordinate axes is

(A) $xyy_2 - xy_1^2 + yy_1 = 0$

(B) $xyy_2 + xy_1^2 - yy_1 = 0$

(C) $xyy_2 + xy_1^2 + yy_1 = 0$

(D) none of these

Q.61 The solution of the differential equation, $(x+2y^3)\frac{dy}{dx} = y$ is :

(A) $\frac{x}{y^2} = y + c$ (B) $\frac{x}{y} = y^2 + c$

(C) $\frac{x^2}{y} = y^2 + c$ (D) $\frac{y}{x} = x^2 + c$

Q.62 The order of the differential equation associated with the primitive $y = c_1 + c_2e^x + c_3e^{-2x+c_4}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is

(A) 3 (B) 4

(C) 2 (D) none of these

Q.63 A curve passes through the point $(1, \pi/4)$ & its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation

(A) $y = x\tan^{-1}(\ln \frac{e}{x})$ (B) $y = x\tan^{-1}(\ln + 2)$

(C) $y = \frac{1}{x}\tan^{-1}(\ln \frac{e}{x})$ (D) none

Q.64 The solution of the differential equation $\frac{dy}{dx} = \sec x\,(\sec x + \cot x)$ is –

(A) $y = \sec x + \tan x + c$ (B) $y = \sec x + \cot x + c$

(C) $y = \sec x - \tan x + c$ (D) None of these

Q.65 The solution of the differential equation $x\cos y\,dy = (x\,e^x \log x + e^x)\,dx$ is

(A) $\sin y = \frac{1}{x}e^x + c$ (B) $\sin y + e^x \log x + c = 0$

(C) $\sin y = e^x \log x + c$ (D) None of these

Q.66 The general solution of the differential equation, $y' + y\,\phi'(x) - \phi(x)\,.\,\phi'(x) = 0$ where $\phi(x)$ is a known function is :

(A) $y = ce^{-\phi(x)} + \phi(x) - 1$

(B) $y = ce^{+\phi(x)} + \phi(x) - 1$

(C) $y = ce^{-\phi(x)} - \phi(x) + 1$

(D) $y = ce^{-\phi(x)} + \phi(x) + 1$

where c is an arbitrary constant .

Q.67 Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$, is

(A) $\frac{dr}{dt} + K = 0$ (B) $\frac{dr}{dt} - K = 0$

(C) $\frac{dr}{dt} = Kr$ (D) none

Q.68 The differential equation of the system of circles touching the x-axis at origin is –

(A) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

(B) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

(C) $(x^2 + y^2)\frac{dy}{dx} + 2xy = 0$

(D) a second order differential equation

Q.69 The solution of differential equation $\frac{dy}{dx} = \cos(x+y)$ is

(A) $\tan\left(\frac{x+y}{2}\right) = x + C$ (B) $\operatorname{cosec}\left(\frac{x+y}{2}\right) = x + C$

(C) $\cot\left(\frac{x+y}{2}\right) = x - C$ (D) None of these

Q.70 Solution of the differential equation

$$\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots}{x + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots} = \frac{dx - dy}{dx + dy} \text{ is}$$

(A) $2ye^{2x} = c.e^{2x} + 1$ (B) $2ye^{2x} = 2c.e^{2x} - 1$

(C) $ye^{2x} = c.e^{2x} + 2$ (D) None of these

Q.71 The differential equation $\frac{dy}{dx} + \frac{9x}{4y} = 0$ represents a family of

(A) parallel straight lines whose slope is $\tan^{-1}(3/2)$

(B) concentric circles with centre at (3, 2)

(C) ellipses with eccentricity $\sqrt{5}/3$

(D) hyperbolas with eccentricity $\sqrt{5}/2$

Q.72 If the independent variable x is change to y then the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$ is change to $x\frac{d^2y}{dx^2} + \left(\frac{dx}{dy}\right)^2 = k$ where k equals

(A) 0 (B) 1

(C) –1 (D) $\frac{dx}{dy}$

Q.73 The solution of different equation $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is

(A) $xy^2 = y^5 + c$ (B) $xy^2 + 2y^5 = c$

(C) $xy^2 = 2y^5 + c$ (D) None of these

Q.74 The solution of the differential equation $e^x (x + 1)\, dx + (ye^y – xe^x)\, dy = 0$ with initial conditions $f(0) = 0$ is –
(A) $xe^x + 2y^2e^y = 0$ (B) $2xe^x + y^2e^y = 0$
(C) $xe^x – 2y^2e^x = 0$ (D) $2xe^x – y^2e^y = 0$

Q.75 The differential equation of the curve given by $y = ax + \frac{b}{x}$ is
(A) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (B) $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$
(C) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (D) $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Q.76 The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is
(A) an ellipse (B) a parabola
(C) a rectangular hyperbola (D) none of these

Q.77 A curve which passes through (1, 2) and whose sub-normal at every point is 2, is –
(A) $2x^2 = y$ (B) $y^2 = 2x + 2$
(C) $y^2 = x + 3$ (D) $y^2 = 4x$

Q.78 If the differential equation of the family of curve given by $y = Ax + Be^{2x}$ where A and B are arbitrary constant is of the form $(1 – 2x) \frac{d}{dx}\left(\frac{dy}{dx} + \ell y\right) + k\left(\frac{dy}{dx} + \ell y\right) = 0$ then the ordered pair (k, ℓ) is –
(A) (2, –2) (B) (–2, 2)
(C) (2, 2) (D) (–2, –2)

Q.79 The tangent at a point P(x, y) on a curve meets the axes at P_1 and P_2 such that P divides $P_1 P_2$ internally in the ratio 2 : 1. The equation of the curve is-
(A) $xy = c$ (B) $x^2y = c$
(C) $xy^2 = c$ (D) None of these

Q.80 Solve the differential equation, $(x^4y^2 – y)\, dx + (x^2y^4 – x)\, dy = 0$; $(y(1) = 1)$
(A) $x^3 + y^3 + 3(xy)^{–1} = 5$ (B) $x^2 + y^2 + 3(xy)^{–1} = 5$
(C) $x^3 + y^3 + 2(xy)^{–1} = 5$ (D) $x^3 + y^3 + 3(xy)^{–1} = 15$

Q.81 The solution of the equation $\frac{dy}{dx} = \cos(x – y)$ is–
(A) $y + \cot\left(\frac{x-y}{2}\right) = C$ (B) $x + \cot\left(\frac{x-y}{2}\right) = C$
(C) $x + \tan\left(\frac{x-y}{2}\right) = C$ (D) None of these

Q.82 The solution of the differential equation $y\frac{dy}{dx} = x – 1$ satisfying $y(1) = 1$ is
(A) $y^2 = x^2 – 2x + 2$ (B) $y^2 = 2x^2 – x – 1$
(C) $y = x^2 – 2x + 2$ (D) none of these

Q.83 The solution of differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$ is
(A) $e^{-by} = \frac{1}{a}e^{ax} + c$ (B) $e^{-by} = \frac{1}{a}e^{-ax} + c$
(C) $-\frac{1}{b}e^{-by} = \frac{1}{a}e^{ax} + c$ (D) None of these

Q.84 The equation of a curve passing through (2, 7/2) and having gradient $1 - \frac{1}{x^2}$ at (x, y) is
(A) $y = x^2 + x + 1$ (B) $xy = x^2 + x + 1$
(C) $xy = x + 1$ (D) none of these

Q.85 The general solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$ is
(A) $y = \frac{1}{4}e^{-2x} + c$ (B) $y = e^{–2x} + cx + d$
(C) $y = \frac{1}{4}e^{-2x} + cx + d$ (D) $y = e^{–2x} + cx^2 + d$

Q.86 A function $y = f(x)$ satisfies the differential equation $f(x).\sin 2x – \cos x + (1 + \sin^2x)\, f'(x) = 0$ with initial condition $y(0) = 0$. The value of $f(\pi/6)$ is equal to –
(A) 1/5 (B) 3/5
(C) 4/5 (D) 2/5

Q.87 The gradient of the curve passing through the point (4, 0) is given by $\frac{dy}{dx} - \frac{y}{x} + \frac{5x}{(x+2)(x-3)} = 0$. If the point (5, a) lies on the curve, then the value of 'a', is
(A) 5 ln (7/12) (B) 67 / 12
(C) 5 sin (7/12) (D) None of these

Q.88 If gradient of a curve at any point P (x, y) is $\frac{x+y+1}{2y+2x+1}$ and it passes through origin, then curve is
(A) $2(x + 3y) = \ln\left|\left(\frac{2x+3y+2}{2}\right)\right|$
(B) $x + 3y = \ln\left|\frac{3x+3y+2}{2}\right|$
(C) $3y + x = \ln(3x + 2y + 1)$
(D) $6y – 3x = \ln\left|\left(\frac{3x+3y+2}{2}\right)\right|$

Q.89 The solution of differential equation $\frac{dy}{dx} = \sec(x + y)$ is
(A) $y - \tan\frac{x+y}{2} = c$ (B) $y + \tan\frac{x+y}{2} = c$
(C) $y + 2\tan\frac{x+y}{2} = c$ (D) None of these

EXERCISE - 2 [LEVEL-2]

Q.1 The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is
(A) 5 (B) 4
(C) 3 (D) 2

Q.2 Solution of the differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$
(A) $e^{e^y}(e^x - e^y + 1) = c$ (B) $e^{e^y}(e^y - e^x + 1) = c$
(C) $e^{e^x}(e^x - e^y + 1) = c$ (D) $e^{e^x}(e^y - e^x + 1) = c$

Q.3 The differential equation on that represents all parabolas each of which has a latusrectum 4a and whose axes are parallel to x-axis, is –
(A) $a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ (B) $2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$
(C) $2a\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$ (D) None of these

Q.4 The general solution of all the differential equation $(2\sqrt{xy} - x)\,dy + y\,dx = 0$ is –
(A) $\log x + \sqrt{\frac{y}{x}} = c$ (B) $\log y - \sqrt{\frac{x}{y}} = c$
(C) $\log y + \sqrt{\frac{x}{y}} = c$ (D) None of these

Q.5 Solution of differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is –
(A) $x^3 + y^3 = cx^2$ (B) $x^2 + y^3 = cx^2$
(C) $x^2 + y^2 = cx^3$ (D) $\frac{x^2}{2} + \frac{y^3}{x} = y^2 + c$

Q.6 Let P and A be respectively the degree and order of differential equation of the family of circles touching the lines $y^2 - x^2 = 0$ and lying in the first and second quadrant;
(A) P = 1, A = 2 (B) P = 1, A = 1
(C) P = 2, A = 1 (D) P = 2, A = 2

Q.7 The particular solution of the differential equation y' + 3xy = x which passes through (0, 4) is –
(A) $y = 1 - 11\,e^{-3x^2/2}$ (B) $3y = 1 + 11\,e^{-3x^2/2}$
(C) $3y = 1 - 11\,e^{-3x^2/2}$ (D) $3y = 1 + 11\,e^{3x^2/2}$

Q.8 If $y = C_1e^{2x} + C_2e^x + C_3e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then $\left(\frac{a^3 + b^3 + c^3}{abc}\right)$
(A) –1/4 (B) –1/2
(C) 0 (D) 1/2

Q.9 Solution of differential equation

$$x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + \ldots\ldots\ldots$$

is –
(A) $y^2 = x^2(\ln x^2 - 1) + c$ (B) $y = x^2(\ln x - 1) + c$
(C) $y^2 = x(\ln x - 1) + c$ (D) $y = x^2e^{x^2} + c$

Q.10 The orthogonal trajectory of system of curve $y = ax^2$ which does not passes through origin, is –
(A) ellipse (B) parabola
(C) circle (D) hyperbola

Q.11 A particle moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described). The time taken by a particle to traverse a distance of 99 metre is
(A) $\log_{10}e$ (B) $2\log_e 10$
(C) $2\log_{10}e$ (D) $\frac{1}{2}\log_{10}e$

Q.12 A particle starts at the origin and moves along the x-axis in such a way that its velocity at the point (x, 0) is given by the formula $\frac{dx}{dt} = \cos^2\pi x$. Then the particle never reaches the point on
(A) x = 1/4 (B) x = 3/4
(C) x = 1/2 (D) x = 1

Q.13 The solution of $y' = 1 + x + y^2 + xy^2$, y (0) = 0 is
(A) $y^2 = \exp\left(x + \frac{x^2}{2}\right) - 1$ (B) $y^2 = 1 + c\exp\left(x + \frac{x^2}{2}\right)$
(C) $y = \tan(c + x + x^2)$ (D) $y = \tan\left(x + \frac{x^2}{2}\right)$

Q.14 If y(t) is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and y (0) = –1, then y (1) is equal to –
(A) $-\frac{1}{2}$ (B) $e + \left(\frac{1}{2}\right)$ (C) $e - \frac{1}{2}$ (D) $\frac{1}{2}$

Q.15 If the gradient of the tangent at any point (x, y) of a curve which passes through the point $\left(1, \frac{\pi}{4}\right)$ is $\left\{\frac{y}{x} - \sin^2\left(\frac{y}{x}\right)\right\}$, then equation of the curve is
(A) $y = \cot^{-1}(\log_e x)$ (B) $y = \cot^{-1}\left(\log_e \frac{x}{e}\right)$
(C) $y = x\cot^{-1}(\log_e ex)$ (D) $y = \cot^{-1}\left(\log_e \frac{e}{x}\right)$

Q.16 The differential equation $y\frac{dy}{dx} + x = a$ (a is any constant)
(A) A set of circles having centre on the y-axis
(B) A set of circles centre on the x-axis
(C) A set of ellipses
(D) None of these

Q.17 The solution of $dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$ where $y = 2$ when $\pi/2$ is
(A) $y = \sin x + \text{cosec}\,x$ (B) $y = \tan\frac{x}{2} + \cot\frac{x}{2}$
(C) $y = \frac{1}{\sqrt{2}}\sec\frac{x}{2} + \sqrt{2}\cos\frac{x}{2}$ (D) None of these

Q.18 The general solution of the differential equation $(2x - y + 1)\,dx + (2y - x + 1)\,dy = 0$ is
(A) $x^2 + y^2 + xy - x + y = c$
(B) $x^2 + y^2 - xy + x + y = c$
(C) $x^2 - y^2 + 2xy - x + y = c$
(D) $x^2 - y^2 - 2xy + x - y = c$

Q.19 The solution of $ye^{-x/y}dx - (xe^{-x/y} + y^3)\,dy = 0$ is
(A) $\frac{y^2}{2} + e^{-x/y} = k$ (B) $\frac{x^2}{2} + e^{-x/y} = k$
(C) $\frac{x^2}{2} + e^{x/y} = k$ (D) $\frac{y^2}{2} + e^{x/y} = k$

Q.20 The solution of the differential equation $x\frac{dy}{dx} + y = x^2 + 3x + 2$ is
(A) $xy = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x + c$ (B) $xy = \frac{x^4}{4} + x^3 + x^2 + c$
(C) $xy = \frac{x^4}{4} + \frac{x^3}{3} + x^2 + c$ (D) $xy = \frac{x^4}{4} + x^3 + x^2 + cx$

Q.21 The solution of the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$ is
(A) $y\sin x + \cos 2x = 2c$ (B) $2y\sin x + \cos x = c$
(C) $y\sin x + \cos x = c$ (D) $2y\sin x + \cos 2x = c$

Q.22 Solution of the differential equation $\left(e^{x^2} + e^{y^2}\right)y\frac{dy}{dx} + e^{x^2}(xy^2 - x) = 0$, is
(A) $e^{x^2}(y^2 - 1) + e^{y^2} = C$ (B) $e^{y^2}(x^2 - 1) + e^{x^2} = C$
(C) $e^{y^2}(y^2 - 1) + e^{x^2} = C$ (D) $e^{x^2}(y - 1) + e^{y^2} = C$

Q.23 If $\int_a^x t\,y(t)dt = x^2 + y(x)$ then y as a function of x is
(A) $y = 2 - (2 + a^2)\,e^{\frac{x^2 - a^2}{2}}$ (B) $y = 1 - (2 + a^2)\,e^{\frac{x^2 - a^2}{2}}$
(C) $y = 2 - (1 + a^2)\,e^{\frac{x^2 - a^2}{2}}$ (D) none

Q.24 A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} - y = \cos x - \sin x$ with initial condition that y is bounded when $x \to \infty$. The area enclosed by $y = f(x)$, $y = \cos x$ and the y-axis is
(A) $\sqrt{2} - 1$ (B) $\sqrt{2}$
(C) 1 (D) $1/\sqrt{2}$

Q.25 The differential equation whose general solution is given by, $y = (c_1\cos(x + c_2)) - (c_3e^{(-x+c_4)}) + (c_5\sin x)$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is
(A) $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$
(C) $\frac{d^5y}{dx^5} + y = 0$ (D) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

Q.26 The equation of a curve passing through (1, 0) for which the product of the abscissa of a point P & the intercept made by a normal at P on the x-axis equals twice the square of the radius vector of the point P, is
(A) $x^2 + y^2 = x^4$ (B) $x^2 + y^2 = 2x^4$
(C) $x^2 + y^2 = 4x^4$ (D) none

Q.27 The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is
(A) $ye^{\frac{x}{y}} = e$ (B) $xe^{\frac{x}{y}} = e$
(C) $xe^{\frac{y}{x}} = e$ (D) $ye^{\frac{y}{x}} = e$

Q.28 Let the curve $y = f(x)$ passes through (4, –2) satisfy the differential equation, $y(x + y^3)\,dx = x(y^3 - x)\,dy$ &

$$y = g(x) = \int_{1/8}^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt,\ 0 \le x \le \frac{\pi}{2}.$$

Then find the area of the region bounded by curves, $y = f(x)$, $y = g(x)$ and $x = 0$.
(A) $\frac{1}{4}\left(\frac{3\pi}{16}\right)^4$ sq. units (B) $\frac{1}{8}\left(\frac{3\pi}{16}\right)^4$ sq. units
(C) $\frac{1}{8}\left(\frac{3\pi}{8}\right)^4$ sq. units (D) $\frac{1}{8}\left(\frac{3\pi}{16}\right)^3$ sq. units

Q.29 A tank consists of 50 litres of fresh water. Two litres of brine each litre containing 5 gms of dissolved salt are run into tank per minute; the mixture is kept uniform by stirring, and runs out at the rate of one litre per minute. If 'm' grams of salt are present in the tank after t minute, express 'm' in terms of t and find the amount of salt present after 10 minutes.

(A) $50 \cdot \frac{11}{6}$ (B) $25 \cdot \frac{11}{6}$

(C) $15 \cdot \frac{11}{6}$ (D) None of these

Q.30 Let f (x) be a differentiable function such that $f'(x) + f(x) = 4xe^{-x} \cdot \sin 2x$ and f (0) = 0. Find the value of

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(k\pi)$$

(A) $\frac{-2\pi e^{\pi}}{(e^{\pi}-1)^2}$ (B) $\frac{-\pi e^{\pi}}{(e^{\pi}-1)^2}$

(C) $\frac{-2\pi e^{\pi}}{(e^{\pi}-1)}$ (D) None of these

Directions : Assertion-Reason type questions.

(A) Statement- 1 is True, Statement-2 is True, Statement2 is a correct explanation for Statement -1
(B) Statement-1 is True, Statement -2 is True; Statement2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False.
(D) Statement -1 is False, Statement -2 is True.

Q.31 **Statement 1 :** The D.E. of all circles in a plane must be of order 3.

Statement 2 : There is only one circle passing through three non-collinear points.

Q.32 **Statement 1 :** The equation of the curve passing through (3, 9) which satisfies differential equation

$$\frac{dy}{dx} = x + \frac{1}{x^2} \text{ is } 6xy = 3x^3 + 29x - 6.$$

Statement 2 : The solution of D.E.

$$\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0 \text{ is } y = c_1e^x + c_2e^{-x}.$$

Passage (Q.33-Q.35)

A curve y = f (x) satisfies the differential equation

$$(1 + x^2)\frac{dy}{dx} + 2yx = 4x^2 \text{ and passes through the origin.}$$

Q.33 The function y = f (x)
(A) is strictly increasing $\forall\ x \in R$
(B) is such that it has a minima but no maxima
(C) is such that it has a maxima but no minima
(D) has no inflection point

Q.34 The area enclosed by $y = f^{-1}(x)$, the x-axis and the ordinate at x = 2/3 is –

(A) 2 ln 2 (B) $\frac{4}{3}\ln 2$

(C) $\frac{2}{3}\ln 2$ (D) $\frac{1}{3}\ln 2$

Q.35 For the function y = f (x) which one of the following does not hold good –
(A) f (x) is a rational function
(B) f (x) has the same domain and same range
(C) f (x) is a transcidental function
(D) y = f (x) is a bijective mapping

Passage (Q.36-Q.38)

Let y = f (x) satisfies the equation

$$f(x) = (e^{-x} + e^{x})\cos x - 2x - \int_0^x (x-t) f'(t)\,dt$$

Q.36 y satisfies the differential equation

(A) $\frac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

(B) $\frac{dy}{dx} - y = e^x(\cos x - \sin x) + e^{-x}(\cos x + \sin x)$

(C) $\frac{dy}{dx} + y = e^x(\cos x + \sin x) - e^{-x}(\cos x - \sin x)$

(D) $\frac{dy}{dx} - y = e^x(\cos x - \sin x) + e^{-x}(\cos x - \sin x)$

Q.37 The value of f '(0) + f "(0) equals –
(A) –1 (B) 2
(C) 1 (D) 0

Q.38 f (x) as a function of x equals –

(A) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x + \sin x) + \frac{2}{5}e^{-x}$

(B) $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

(C) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x - \sin x) + \frac{2}{5}e^{-x}$

(D) $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

Passage (Q.39-Q.41)

Differential equation $\frac{dy}{dx} = f(x).g(y)$ can be solved by separating variable $\frac{dy}{g(y)} = f(x)\,dx$.

Q.39 The equation of the curve to the point (1, 0) which satisfies the differential equation $(1 + y^2)\,dx - xy\,dy = 0$ –
(A) $x^2 + y^2 = 1$ (B) $x^2 - y^2 = 1$
(C) $x^2 + y^2 = 2$ (D) $x^2 - y^2 = 2$

Q.40 Solution of the differential equation $\frac{dy}{dx}+\frac{1+y^2}{\sqrt{1-x^2}}=0$ is –

(A) $\tan^{-1} y + \sin^{-1}x = c$ (B) $\tan^{-1} x + \sin^{-1}y = c$
(C) $\tan^{-1} y \,.\, \sin^{-1}x = c$ (D) $\tan^{-1} y - \sin^{-1}x = c$

Q.41 If $\frac{dy}{dx}$ = 1 + x + y + xy and y (–1) = 0 then y =

(A) $e^{\frac{(1-x)^2}{2}}$ (B) $e^{\frac{(1-x)^2}{2}}-1$
(C) $\ln(1+x)-1$ (D) $1+x$

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.42 The degree of the differential equation

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}=\ell\frac{d^2y}{dx^2} \text{ is}$$

Q.43 The order of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=e\frac{d^2y}{dx^2} \text{ is}$$

Q.44 The order of the differential equation of all conics whose axes coincide with the axes of co-ordinates is

Q.45 The differential equation $\frac{dx}{dy}=\frac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity $\sqrt{\frac{A}{2}}$ or $\sqrt{\frac{A}{3}}$. Find the value of A.

Q.46 If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3}-13\frac{dy}{dx}}{y}=K$ then the value of K is –

Q.47 Let C be the curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. If the area bounded by the curve C and x-axis in the first squadrant is $k\pi/2$ square units, then find the value of k.

Q.48 For the primitive integral equation

$ydx + y^2dy = xdy;\ x\in R,\ y>0,\ \ y = y(x),\ y(1)=1$, then y (–3) is

Q.49 Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f (–3) is equal to

Q.50 Let $y'(x)+y(x)g'(x)=g(x)g'(x), y(0)=0, x\in R$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and g (x) is a given nonconstant differentiable function on R with g (0) = g (2) = 0. Then the value of y (2) is

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 The solution of the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is- **[AIEEE 2002]**
(A) $x^2 + y^2 = cx$ (B) $x^2 - y^2 + cx = 0$
(C) $x^2 + 2xy = y^2 + cx$ (D) $x^2 + y^2 = 2xy + cx^2$

Q.2 The differential equation, which represents the family of plane curves $y = e^{cx}$, is- **[AIEEE 2002]**
(A) $y' = cy$ (B) $xy' - \log y = 0$
(C) $x \log y = yy'$ (D) $y \log y = xy'$

Q.3 The equation of the curve through the point (1, 0), whose slope is $\frac{y-1}{x^2+x}$, is- **[AIEEE-2002]**
(A) $(y-1)(x+1) + 2x = 0$ (B) $2x(y-1) + x + 1 = 0$
(C) $x(y-1)(x+1) + 2 = 0$ (D) $x(y+1) + y(x+1) = 0$

Q.4 The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively- **[AIEEE 2003]**
(A) 2 , 3 (B) 2, 1
(C) 1, 2 (D) 3, 2

Q.5 The solution of the differential equation
$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0,$$ is- **[AIEEE 2003]**
(A) $x e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$ (B) $(x-2) = k e^{\tan^{-1} y}$
(C) $2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$ (D) $x e^{\tan^{-1} y} = \tan^{-1} y + k$

Q.6 The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is- **[AIEEE 2004]**
(A) $2(x^2 - y^2)\, y' = xy$ (B) $2(x^2 + y^2)\, y' = xy$
(C) $(x^2 - y^2)\, y' = 2xy$ (D) $(x^2 + y^2)\, y' = 2xy$

Q.7 The solution of the differential equation $ydx + (x + x^2 y)\,dy = 0$ is- **[AIEEE 2004]**
(A) $-\frac{1}{xy} = C$ (B) $-\frac{1}{xy} + \log y = C$
(C) $\frac{1}{xy} + \log y = C$ (D) $\log y = Cx$

Q.8 The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows - **[AIEEE- 2005]**
(A) order 1, degree 2 (B) order 1, degree 1
(C) order 1, degree 3 (D) order 2, degree 2

Q.9 If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is - **[AIEEE-2005]**
(A) $y \log (x/y) = cx$ (B) $x \log (y/x) = cy$ (C) $\log (y/x) = cx$ (D) $\log (x/y) = cy$

Q.10 The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of– **[AIEEE 2006]**
(A) first order and second degree
(B) first order and first degree
(C) second order and first degree
(D) second order and second degree

Q.11 The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE 2007]**
(A) $x^2 = y^2 + xy\frac{dy}{dx}$ (B) $x^2 = y^2 + 3xy\frac{dy}{dx}$
(C) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (D) $y^2 = x^2 - 2xy\frac{dy}{dx}$

Q.12 The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is **[AIEEE 2008]**
(A) $(y-2)\,y'^2 = 25 - (y-2)^2$
(B) $(y-2)^2\,y'^2 = 25 - (y-2)^2$
(C) $(x-2)^2\,y'^2 = 25 - (y-2)^2$
(D) $(x-2)\,y'^2 = 25 - (y-2)^2$

Q.13 The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is - **[AIEEE 2008]**
(A) $y = x\,\ell n\,x + x^2$ (B) $y = xe^{(x-1)}$
(C) $y = x\,\ell n\,x + x$ (D) $y = \ell n\,x + x$

Q.14 The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitrary constants, is - **[AIEEE 2009]**
(A) $y' = y^2$ (B) $y'' = y'\,y$
(C) $yy'' = y'$ (D) $yy'' = (y')^2$

Q.15 Solution of the differential equation
$$\cos x\,dy = y(\sin x - y)\,dx,\ 0 < x < \frac{\pi}{2}$$ **[AIEEE 2010]**
(A) $y \sec x = \tan x + c$ (B) $y \tan x = \sec x + c$
(C) $\tan x = (\sec x + c)y$ (D) $\sec x = (\tan x + c)y$

Q.16 Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V (T) of the equipment is : **[AIEEE 2011]**
(A) $T^2 - \frac{1}{k}$ (B) $I - \frac{kT^2}{2}$
(C) $I - \frac{k(T-t)^2}{2}$ (D) e^{-kT}

Q.17 The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If p (0) = 850, then the time at which the population becomes zero is – **[AIEEE 2012]**

(A) 2 ln 18 (B) ln 9

(C) $\frac{1}{2}\ln 18$ (D) ln 18

Q.18 At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by

$\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is – **[JEE MAIN 2013]**

(A) 2500 (B) 3000

(C) 3500 (D) 4500

Q.19 Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If p (0) = 100, then p (t) equals – **[JEE MAIN 2014]**

(A) $400 - 300\,e^{t/2}$ (B) $300 - 200\,e^{-t/2}$

(C) $600 - 500\,e^{t/2}$ (D) $400 - 300\,e^{-t/2}$

Q.20 Let y (x) be the solution of the differential equation

$(x\log x)\frac{dy}{dx} + y = 2x\log x,\ (x \ge 1)$ .Then y (e) is equal to

(A) 0 (B) 2 **[JEE MAIN 2015]**

(C) 2e (D) e

Q.21 If a curve y = f (x) passes through the point (1, –1) and satisfies the differential equation, y (1 + xy) dx = xdy, then f (–1/2) is equal to – **[JEE MAIN 2016]**

(A) –4/5 (B) 2/5

(C) 4/5 (D) –2/5

Q.22 If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and y (0) = 1, then y (π/2) is equal to – **[JEE MAIN 2017]**

(A) –1/3 (B) 4/3

(C) 1/3 (D) –2/3

Q.23 Let y = y (x) be the solution of the differential equation

$\sin x\frac{dy}{dx} + y\cos x = 4x,\ x \in (0, \pi)$, If y (π/2) = 0, then y (π/6) is equal to – **[JEE MAIN 2018]**

(A) $-\frac{8}{9}\pi^2$ (B) $-\frac{4}{9}\pi^2$

(C) $\frac{4}{9\sqrt{3}}\pi^2$ (D) $\frac{-8}{9\sqrt{3}}\pi^2$

Q.24 If y = y (x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y (1) = 1, then y (1/2) is equal to : **[JEE MAIN 2019 (JAN)]**

(A) 7/64 (B) 13/16

(C) 49/16 (D) 1/4

Q.25 Let y = y (x) be the solution of the differential equation, $(x^2 + 1)^2 + 2x(x^2 + 1)y = 1$ such that y (0) = 0. If $\sqrt{ay}\,(1) = \frac{\pi}{32}$, then the value of 'a' is : **[JEE MAIN 2019 (APRIL)]**

(A) 1/2 (B) 1/16

(C) 1/4 (D) 1

Q.26 The solution of the differential equation

$x\frac{dy}{dx} + 2y = x^2\ (x \ne 0)$ with y (1) = 1, is **[JEE MAIN 2019 (APRIL)]**

(A) $y = \frac{x^3}{5} + \frac{1}{5x^2}$ (B) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$

(C) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$ (D) $y = \frac{x^2}{4} + \frac{3}{4x^2}$

Q.27 If $\cos x\frac{dy}{dx} - y\sin x = 6x,\ \left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to – **[JEE MAIN 2019 (APRIL)]**

(A) $-\frac{\pi^2}{4\sqrt{3}}$ (B) $-\frac{\pi^2}{2}$ (C) $-\frac{\pi^2}{2\sqrt{3}}$ (D) $\frac{\pi^2}{2\sqrt{3}}$

Q.28 If y = y (x) is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y (0) = 0, then y (–π/4) is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) $2 + \frac{1}{e}$ (B) $\frac{1}{2} - e$ (C) e – 2 (D) $\frac{1}{2} + e$

Q.29 Let y = y (x) be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y (0)=1. Then : **[JEE MAIN 2019 (APRIL)]**

(A) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

(B) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(C) $y\left(\frac{\pi}{4}\right) - y\left(\frac{-\pi}{4}\right) = \sqrt{2}$

(D) $y\left(\frac{\pi}{4}\right) + y\left(\frac{-\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Q.30 Consider the differential equation,

$y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when x = 1, the the value of x for which y = 2, is : **[JEE MAIN 2019 (APRIL)]**

(A) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (B) $\frac{3}{2} - \sqrt{e}$

(C) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (D) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

Q.31 The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0\ (x \neq 0)$ is : (where c is a constant of integration) **[JEE MAIN 2019 (APRIL)]**

(A) $y^2 + 2x^3 + cx^2 = 0$ (B) $y^2 + 2x^2 + cx^3 = 0$

(C) $y^2 - 2x^3 + cx^2 = 0$ (D) $y^2 - 2x^2 + cx^3 = 0$

Q.32 Let y = f (x) is a solution of differential equation

$e^y\left(\frac{dy}{dx} - 1\right) = e^x$ and f (0) = 0 then f (1) is equal to : **[JEE MAIN 2020 (JAN)]**

(A) ln 2 (B) 2 + ln 2

(C) 1 + ln 2 (D) 3 + ln 2

Q.33 Let y (x) is solution of differential equation

$(y^2 - x)\frac{dy}{dx} = 1$ and y (0) = 1, This curve intersects the x-axis at a point whose abscissa is : **[JEE MAIN 2020 (JAN)]**

(A) 2 – e (B) 2 + e

(C) 2 (D) e

Q.34 If y (x) is a solution of differential equation

$\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$, such that $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then – **[JEE MAIN 2020 (JAN)]**

(A) $y\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$ (B) $y\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$

(C) $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$ (D) $y\left(\frac{1}{2}\right) = \frac{1}{2}$

Q.35 Differential equation of $x^2 = 4b\,(y + b)$, where b is a parameter, is – **[JEE MAIN 2020 (JAN)]**

(A) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x^2$ (B) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x$

(C) $x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx} + x^2$ (D) $x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx} + 2x^2$

Q.36 If for $x \geq 0$, y = y (x) is the solution of the differential equation $(x + 1)\,dy = ((x + 1)^2 + y - 3)\,dx$, y (2) = 0, then y (3) is equal to ______. **[JEE MAIN 2020 (JAN)]**

Q.37 If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; y (1) = 1; then a value of x satisfying y (x) = e is : **[JEE MAIN 2020 (JAN)]**

(A) $\sqrt{2}e$ (B) $\frac{e}{\sqrt{2}}$ (C) $\frac{1}{2}\sqrt{3}e$ (D) $\sqrt{3}e$

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	D	C	D	A	D	A	A	A	B	D	A	B	A	C	A	C	D	A	B	B	D	A	C	A	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	C	C	B	D	D	C	C	B	D	C	B	A	C	D	A	A	D	A	A	B	C	A	D	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	A	D	D	D	C	C	B	B	A	B	B	A	A	A	C	A	A	B	A	B	C	B	C	B	C
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89											
A	C	D	A	C	A	B	A	C	B	C	D	A	D	A											

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	D	B	C	C	C	B	A	A	A	B	C	D	A	C	D	A	B	A	A	D	A	A	A	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	A	A	B	A	A	B	B	A	C	C	A	D	C	B	A	B	2	2	2	5	12	1	3	9	0

EXERCISE-3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A	A	D	A	C	C	C	B	C	C	C	C	B	C	D	D	B	A	C	A	B	C	C	A	C
Q	25	26	27	28	29	30	31	32	33	34	35	36	37											
A	B	D	C	C	B	D	A	C	A	C	B	3	D											

CHAPTER- 9: DIFFERENTIAL EQUATIONS

SOLUTIONS TO TRY IT YOURSELF

(1) (i) $\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^6\right]^{1/4} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^4 = \left[y + \left(\frac{dy}{dx}\right)^6\right]$

Hence, order is 2 and degree is 4.

(ii) $e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$

Clearly, order is 3, but degree is not defined as it cannot be written as a polynomial equation in derivatives.

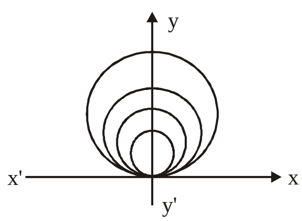
(iii) $\ln\left(\frac{dy}{dx}\right) = ax + by \Rightarrow \frac{dy}{dx} = e^{ax+by}$

Hence, order is 1 and degree is also 1.

(2) Such family of circles is given by

$x^2 + (y - a)^2 = a^2$

$\Rightarrow x^2 + y^2 - 2ay = 0$ .... (1)



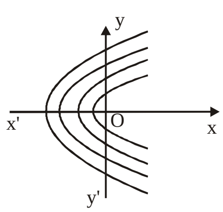
Differentiating,

$2x + 2y\frac{dy}{dx} = 2a\frac{dy}{dx}$ or $x + y\frac{dy}{dx} = a\frac{dy}{dx}$

Substituting the value of a in eq. (1),

$(x^2 - y^2)\frac{dy}{dx} = 2xy$ (order = 1 and degree = 1)

(3) Equation of such parabolas is $y^2 = 4A(A + x)$ .... (1)



Differentiating w.r.t. we get

$\Rightarrow 2y\frac{dy}{dx} = 4A \Rightarrow y\frac{dy}{dx} = 2A$ .... (2)

Eliminating A from eq. (2) and (1)

$y^2 = \left(y\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx}x$ or $y^2 = y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$

which has order 1 and degree 2.

(4) (i) $\frac{dy}{dx}\sin x = y\log y \Rightarrow \int\frac{dy}{y\ln y} = \int \operatorname{cosec} x\, dx$

$\Rightarrow \ln(\ln y) = \ln(\operatorname{cosec} x - \cot x) + \ln C$

$\Rightarrow \ln y = C(\operatorname{cosec} x - \cot x)$

At $x = \pi/2$, $y = e$; so $\ln(e) = C(1 - 0) \Rightarrow C = 1$

$\Rightarrow y = e^{(\operatorname{cosec} x - \cot x)}$

(ii) $\frac{dy}{dx} = (e^x + x^2)\cdot e^{-y} \Rightarrow e^y\, dy = (e^x + x^2)\, dx$

$\Rightarrow e^y = e^x + \frac{x^3}{3} + C$

(5) Putting $\sqrt{1 + x + y} = v$, we have

$\Rightarrow x + y - 1 = v^2 - 2 \Rightarrow 1 + \frac{dy}{dx} = 2v\frac{dy}{dx}$

Then the given equation transforms to

$\left(2v\frac{dv}{dx} - 1\right)v = v^2 - 2$

$\Rightarrow \frac{dv}{dx} = \frac{v^2 + v - 2}{2v^2} \Rightarrow \int\frac{2v^2}{v^2 + v - 2}dv = \int dx$

$\Rightarrow 2\int\left[1 + \frac{1}{3(v-1)} - \frac{4}{3(v+2)}\right]dv = \int dx$

$\Rightarrow 2\left[v + \frac{1}{3}\log|v - 1| - \frac{4}{3}\log|v + 2|\right] = x + c$

where $v = \sqrt{1 + x + y}$

(6) $x\,dx + y\,dy = x(x\,dy - y\,dx)$

Let $x = r\cos\theta$, $y = r\sin\theta$ $\therefore$ $r\,dr = r\cos\theta\,(r^2\,d\theta)$

$\int\frac{dr}{r^2} = \int\cos\theta\,d\theta \Rightarrow \frac{-1}{r} = \sin\theta + c$

$\Rightarrow 1 + r\sin\theta = -cr \Rightarrow (1 + r\sin\theta)^2 = c^2r^2$

$\Rightarrow (1 + y)^2 = c^2(x^2 + y^2)$

(7) Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Given equation transforms to $v + x\frac{dv}{dx} = -v - v^2$

$\Rightarrow \int\frac{dv}{v^2 + 2v} = -\int\frac{dx}{x} \Rightarrow \frac{1}{2}\int\left[\frac{1}{v} - \frac{1}{v+2}\right]dv = -\int\frac{dx}{x}$

$\Rightarrow \log|v| - \log|v + 2| = -2\log|x| + \log c$ $(c > 0)$

$\Rightarrow \left|\frac{vx^2}{v+2}\right| = c \Rightarrow \left|\frac{x^2y}{2x + y}\right| = c$ $(c > 0)$

(8) Putting, $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$xv + x^2\frac{dv}{dx} = vx + 2x\sqrt{v^2-1}$$

$$\Rightarrow \frac{dv}{2\sqrt{v^2-1}} = \frac{dx}{x}$$, integrating we get

$$\frac{1}{2}\ln(v+\sqrt{v^2-1}) = \ln(cx)$$

$$\Rightarrow \frac{1}{2}\ln\left(\frac{y+\sqrt{y^2-x^2}}{x}\right) = \ln(cx)$$

$$\Rightarrow y+\sqrt{y^2-x^2} = c^2x^3$$

(9) (i) $\cos^2 x\frac{dy}{dx} + y = \tan x$

$$\frac{dy}{dx} + \sec^2 x\, y = \tan x\sec^2 x$$

Integrating factor = $\int e^{\sec^2 x\,dx} = e^{\tan x}$

$$y\,e^{\tan x} = \int e^{\tan x}\tan x\sec^2 x\,dx$$

Let t = tan x, so $dt = \sec^2 x\,dx$

$$\int e^{\tan x}\tan x\sec^2 x\,dx = \int e^t t\,dt = (t-1)e^t$$

So, $y = (\tan x - 1) + ke^{-\tan x}$

(ii) $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y} \Rightarrow \frac{dx}{dy} = x\cos y + \sin 2y$

$$\frac{dy}{dx} - (\cos y)\,x = (\sin 2y)$$

Integrating factor = $e^{\int -(\cos y)\,dy} = e^{-\sin y}$

$$xe^{-\sin y} = \int e^{-\sin y}2\sin y\cos y\,dy$$

Put sin y = t, cos y = $\frac{dt}{dy}$

$$\int e^{-\sin y}2\sin y\cos y\,dy = \int e^{-t}(2t)\,dt$$

$$= 2[-t\;e^{-t} - e^{-t}] = -2e^{-t}(t+1) = -2e^{-\sin y}(\sin y+1)$$

$\therefore$ $x\,e^{-\sin y} = -2\,e^{-\sin y}(\sin y+1) + C$

$x = -2(\sin y + 1) + c\,e^{\sin y}$.

(10) (i) $\frac{dy}{dx} = xy + x^3y^2 \Rightarrow \frac{dy}{dx} - yx = x^3y^2$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{x}{y} = x^3 \quad \text{put } t = \frac{-1}{y} \quad \text{so, } dt = \frac{dy}{y^2}$$

$$\frac{dt}{dx} + tx = x^3 \Rightarrow \text{Integrating factor} = e^{\int x\,dx} = e^{\frac{x^2}{2}}$$

$$t\,e^{\frac{x^2}{2}} = \int e^{\frac{x^2}{2}}x^3dx = \int e^{\frac{x^2}{2}}x\,x^2dx$$

Put $z = \frac{x^2}{2}$; $dz = x\,dx$

$$\int e^{\frac{x^2}{2}}x^3dx = \int e^z(2z)\,dz = 2(z-1)e^z = (x^2-2)e^{\frac{x^2}{2}}$$

$$t\,e^{\frac{x^2}{2}} = (x^2-2)e^{\frac{x^2}{2}} + C \Rightarrow \frac{-e^{\frac{x^2}{2}}}{y} = (x^2-2)e^{\frac{x^2}{2}} + C$$

$$\Rightarrow y = \frac{-e^{\frac{x^2}{2}}}{(x^2-2)e^{\frac{x^2}{2}} + C}$$

(ii) $y\sin x\frac{dy}{dx} = \cos x(\sin x - y^2)$

Let $z = y^2$ $\therefore$ $dz = 2y\,dy$

$$\frac{1}{2}\frac{dz}{dx} + \cot x\,z = \cos x \Rightarrow \frac{dz}{dx} + 2\cot x\,z = 2\cos x$$

Integrating factor

$$= e^{\int 2\cot x\,dx} = e^{2\ln(\sin x)} = \sin^2 x$$

$$z\cdot\sin^2 x = \int 2\cos x\sin^2 x\,dx = \frac{2\sin^3 x}{3} + C$$

$$\therefore \quad y^2\sin^2 x = \frac{2\sin^3 x}{3} + C$$

(11) Let at any instance (t), radius of moth ball be r and v be its volume.

$$\Rightarrow v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2\frac{dr}{dt}$$

Thus as per the information,

$$4\pi r^2\frac{dr}{dt} = -k\,4\pi r^2, \text{ where } k \in R^+$$

$$\Rightarrow \frac{dr}{dt} = -k \quad \text{or } r = -kt + c \text{ at } t = 0, r = 2\text{cm}; t = 3 \text{ month}$$

r = 1 cm.

$$\Rightarrow c = 2, k = 1/3 \Rightarrow r = -\frac{1}{3}t + 2$$

Now, for $r \to 0, t \to 6$

Hence, it will take six months until the ball is practically gone.

(12) The balance N (t) in the account at any time t.

$$\frac{dN}{dt} - kN = 0 \text{, its solution is } N(t) = ce^{kt} \quad \text{...... (1)}$$

Let initial deposit be N_0.

At t = 0, N (0) = 0 = N_0, which when substituted into eq. (1) yields $N_0 = ce^{k(0)} = c$

and eq. (1) becomes $N(t) = N_0e^{kt}$ ...... (2)

We seek the value of k for which N = $2N_0$ when t = 6, substituting these values into eq. (2) and solving for k

we find $2N_0 = N_0e^{k(6)} \Rightarrow e^{6k} = 2 \Rightarrow k = \frac{1}{6}\ln|2| = 0.1155$

An interest rate of 11.55 percent is required.

(13) We are given that sub-tangent

$$= \frac{y}{dy/dx} = (\text{constant}) = k \text{ (say)} \Rightarrow k\frac{dy}{y} = dx$$

Integrating we get, k ln y = x + c

Given that curve passes through (2, 1) $\Rightarrow$ c = –2

Hence, the equation of such curve is k ln y = x – 2.

(14) $x^2 + y^2 = a^2 \Rightarrow 2x + 2yy' = 0 \Rightarrow x + yy' = 0$

For orthogonal tracjectory, replace y' by –1/y'

$$x - \frac{y}{y'} = 0 \Rightarrow xy' - y = 0$$

$$\Rightarrow x\,dy - y\,dx = 0 \Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$$

$$\Rightarrow \ln y - \ln x = \ln c \Rightarrow y = cx$$

CHAPTER- 9 : DIFFERENTIAL EQUATIONS
EXERCISE-1

(1) **(D).** From the given equation, $\left(\left(\frac{d^2y}{dx^2}\right)^2 - y^3\right)^2 = \frac{dy}{dx}$.

Hence, it is obvious from the equation that degree is 4.

(2) **(C).**

$y = c_1\cos(2x+c_2) - (c_3+c_4)\,a^{x+c_5} + c_6\sin(x-c_7)$

$y = c_1\cos(2x+c_2) - p_1a^x + c_6\sin(x-c_7)$

Since the above relation contains five arbitrary constants, so order = 5

(3) **(D).** Making fourth power both the sides, we get the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^2$

Obviously, order is 2 and degree is 4.

(4) **(A).** Differentiating the given equation, we get $\frac{dy}{dx} = A$

$\therefore y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3$ which is of degree 3.

(5) **(D).** The highest order (m) of the given equation is $\frac{d^3y}{dx^3} = 3$ and degree (n) of the given equation is $\left(\frac{d^3y}{dx^3}\right)^2 = 2$. Therefore m = 3 and n = 2.

(6) **(A).** $(x-h)^2 + (y-k)^2 = r^2$. Here r is arbitrary constant

$\therefore$ order of differential equation = 1.

(7) **(A).** $\frac{d^2y}{dx^2} - \sqrt{\frac{dy}{dx} - 3} = x \Rightarrow \frac{d^2y}{dx^2} - x = \sqrt{\frac{dy}{dx} - 3}$

Squaring both sides, we get $\left(\frac{d^2y}{dx^2} - x\right)^2 = \left(\frac{dy}{dx} - 3\right)$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 + x^2 - 2x\frac{d^2y}{dx^2} = \frac{dy}{dx} - 3$. Clearly, degree = 2.

(8) **(A).** $y = \frac{d}{dx}\left[\frac{dy}{dx}\right]x + \sqrt{a^2\left[\frac{dy}{dx}\right]^2 + b^2}$

$y = x\frac{d^2y}{dx^2} + \sqrt{a^2\left[\frac{dy}{dx}\right]^2 + b^2}$

$\therefore \left(y - x\frac{d^2y}{dx^2}\right)^2 = a^2\left[\frac{dy}{dx}\right]^2 + b^2$ $\therefore$ Order = 2, deg = 2

(9) **(B).** $(1 + y_1^2)^{2/3} = y^2$; $(1 + y_1^2)^2 = (y_2)3$

$\therefore$ m = deg = 3 ; n = order = 2

$\therefore \frac{m+n}{m-n} = \frac{3+2}{3-2} = 5$

(10) **(D).** Multiply by y''' ; Order = 3, degree = 2

(11) **(A).** $\frac{dy}{dx}y = x\left(\frac{dy}{dx}\right)^2 + 2$; Order = 1, Degree = 2

(12) **(B).** $(x-h)^2 + (y-k)^2 = a^2$, a is fixed. Hence it has two independent arbitrary constants $\therefore$ order is 2

(13) **(A).** $y = A\sin x + B\cos x \Rightarrow \frac{dy}{dx} = A\cos x - B\sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -A\sin x - B\cos x = -(A\sin x + B\cos x) = -y$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$ is the required differential equation.

(14) **(C).** $\frac{dv}{dr} = -\frac{A}{r^2} + 0 \Rightarrow \frac{d^2v}{dr^2} = \frac{2A}{r^3} \Rightarrow \frac{d^2v}{dr^2} = \frac{2}{r}\left(\frac{A}{r^2}\right)$

$\Rightarrow \frac{d^2v}{dr^2} = \frac{2}{r}\left(-\frac{dv}{dr}\right) \Rightarrow \frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$.

(15) **(A).** $y' = -A\mu\sin\mu t + B\mu\cos\mu t$

Again, $y'' = -A\mu^2\cos\mu t - B\mu^2\sin\mu t$

$= -\mu^2(A\cos\mu t + B\sin\mu t)$.

Therefore $y'' = -\mu^2 y$.

(16) **(C).** Any conic whose axes coincide with co-ordinate axis is $ax^2 + by^2 = 1$ ..(i)

Diff. both sides w.r.t. 'x', we get

$2ax + 2by\frac{dy}{dx} = 0$ i.e. $ax + by\frac{dy}{dx} = 0$ ..(ii)

Diff. again, $a + b\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0$ ..(iii)

From (ii), $\frac{a}{b} = -\frac{y\,dy/dx}{x}$

From (iii), $\frac{a}{b} = -\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)$

$\therefore \frac{y\frac{dy}{dx}}{x} = y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(17) **(D).** The equation of such circles

$x^2 + y^2 - 2hx = 0$ .....(i), where h = radius

Differentiate w.r.t. x,

$2x + 2y\frac{dy}{dx} - 2h = 0 \therefore h = x + y\frac{dy}{dx}$

Put in equation (i), $x^2 + y^2 - 2x\left(x + y\dfrac{dy}{dx}\right) = 0$

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

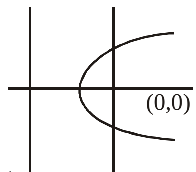
$$y^2 - x^2 - 2xy\frac{dy}{dx} = 0\ ,\ \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}.$$

(18) **(A).** Parabola

distance from focus = distance from diretrix

$x^2 + y^2 = (2a + x)^2$

$y^2 = 4a\,(a + x)$ .... (1)



$$2y\frac{dy}{dx} = 4a\,(0+1)\ ;\ a = \frac{y}{2}\frac{dy}{dx}$$

Using (1), $y^2 = 2y\dfrac{dy}{dx}\left(\dfrac{y}{2}\dfrac{dy}{dx} + x\right)$ $x = -2a$

$$y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

(19) **(B).** $y = ax\cos\left(\dfrac{1}{x} + b\right)$ ….. (i)

Differentiate (i), we get

$$y_1 = a\left[\cos\left(\frac{1}{x} + b\right) - x\sin\left(\frac{1}{x} + b\right)\left(\frac{-1}{x^2}\right)\right]$$

$$= a\left[\cos\left(\frac{1}{x} + b\right) + \frac{1}{x}\sin\left(\frac{1}{x} + b\right)\right] \quad .....(ii)$$

Again, differentiate (ii), we get $y_2 = \dfrac{-a}{x^3}\cos\left(\dfrac{1}{x} + b\right)$

$$= \frac{-ax}{x^4}\cos\left(\frac{1}{x} + b\right) = \frac{-y}{x^4} \Rightarrow x^4 y_2 + y = 0\,.$$

(20) **(B).** $2x\dfrac{dy}{dx} - y = 3\ ;\ \dfrac{dy}{y-3} = \dfrac{dx}{2x}$

$$\log(y-3) = \frac{1}{2}\log x + \log c$$

$\therefore$ $\log(y-3) = \log(\sqrt{x}) + \log c$

$\therefore$ $\dfrac{y-3}{\sqrt{x}} = c\ \therefore\ (y-3)^2 = x\,c_1 \Rightarrow$ family of parabolas.

(21) **(D).** $\dfrac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2 = (1-x^2) - y^2(1-x^2)$

$$\frac{dy}{dx} = (1-x^2)(1-y^2)$$

$$\frac{dy}{1-y^2} = (1-x^2)\,dx\ ;\ \int\frac{dy}{1-y^2} = \int(1-x^2)\,dx$$

$$\frac{1}{2}\log\left(\frac{1+y}{1-y}\right) = x - \frac{x^3}{3} + C$$

(22) **(A).** $e^{\frac{dy}{dx}} = x\ \dfrac{dy}{dx} = \log_e x \Rightarrow y = x\log x - x + c$

(on integrating)

$\therefore$ $y = x\log x - x + c$

At $x = 1,\ 0 = 0 - 1 + c$ $\therefore$ $c = 1$

$\therefore$ $y = x\log x - x + 1$ $\therefore$ $y = x(\log x - 1) + 1$

(23) **(C).** Put $xy = z$

Diff. eq. is $\sqrt{1-z^2}\,dx = dz \Rightarrow \dfrac{dz}{\sqrt{1-z^2}} = dx$ integral

$\sin^{-1} z = x + c\ ;\ z = \sin(x + c)\,;\ xy = \sin(x + c)$

(24) **(A).** $\dfrac{dA}{dt} = \dfrac{2}{\sqrt{t}} \Rightarrow \int dA = \int\dfrac{2}{\sqrt{t}}dt \Rightarrow A = 2\times 2\sqrt{t} + C$

Where $t = 0, C = 0$ $\therefore$ $4\sqrt{t} = 40 \Rightarrow t = 100$

(25) **(D).** $\dfrac{dy}{dx} + \dfrac{2}{x}y = x$; Linear in y

$$IF = e^{\int\frac{2}{x}dx} = e^{2\log x} = x^2$$

$$\therefore\ y\cdot x^2 = \int x\cdot x^2 dx + \frac{C}{4} = \frac{x^4}{4} + \frac{C}{4} \Rightarrow y = \frac{x^4 + C}{4x^2}$$

(26) **(C).** $\dfrac{dy}{dx} + \dfrac{1}{x}\,.\,y = \sin x$ $\left[\text{Type}\dfrac{dy}{dx} + Py = Q\right]$

$$e^{\int P dx} = e^{\int\frac{1}{x}dx} = e^{\log x} = x$$

$\therefore$ Sol. is $y\,x = \int x\sin x\,dx + C$

$= x(-\cos x) - \int 1.(-\cos x)dx + C = -x\cos x + \sin x + C$

$\Rightarrow x\,(y + \cos x) = \sin x + C$

(27) **(C).** $\dfrac{dy}{dx} = \dfrac{ax + h}{by + k} \Rightarrow (by + k)\,dy = (ax + h)\,dx$

$$\Rightarrow by^2 + ky = \frac{a}{2}x^2 + hx + C$$

For this to represent a parabola, one of the two terms x^2 or y^2 is zero. Therefore, either $a = 0, b \neq 0$ or $a \neq 0\,, b = 0$

(28) **(C).** $\sec^2 x\tan y\,dx + \sec^2 y\tan x\,dy = 0$

$$\Rightarrow \frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

On integrating, we get $\log\tan x + \log\tan y = \log c$

$\Rightarrow \log(\tan x\tan y) = \log c \Rightarrow \tan x = c\cot y\,.$

(29) **(B).** Equation of the tangent y = mx + a/m .... (i)
where m is arbitrary constant
$\therefore$ order = 1

$$\frac{dy}{dx} = m.1 + 0 = m$$

$$\therefore \; y = \frac{dy}{dx}x + \frac{a}{(dy/dx)} \; ; \; x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + a = 0$$

$\therefore$ degree = 2

(30) **(D).** x dy – y dx = 0 ; x dy = y dx

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

log y = log x + log c = log xc ; y = xc
Straight line passing through origin

(31) **(D).** y dx – x dy = xy dx

$$\Rightarrow \frac{ydx - xdy}{y^2} = \frac{x}{y}dx$$

$$\Rightarrow \frac{d(x/y)}{x/y} = dx \Rightarrow \ln\left(\frac{x}{y}\right) = x + c \Rightarrow \frac{x}{y} = e^x.e^c$$

$\Rightarrow y.e^x = cx$

(32) **(C).** Then given equation can be written is

$$\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^x\right) = 0$$

$$\Rightarrow \frac{dy}{dx} - e^{-x} = 0 \text{ or } \frac{dy}{dx} - e^x = 0$$

On integration we get
$\Rightarrow y + e^{-x} = C_1$ or $y - e^x = C_2$

(33) **(C).** $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ or $\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$

It is homogeneous equation, hence put y = v x

we get, $v + x\frac{dv}{dx} = v - \tan v$

$$\Rightarrow \int \cot v dv = -\int \frac{dx}{x} \Rightarrow \log(x\sin v) = \log c \Rightarrow x\sin\left(\frac{y}{x}\right) = c.$$

(34) **(B).** I.F. $= e^{\int \frac{2x}{1-x^2}dx} = e^{-\log(1-x^2)} = e^{\log(1-x^2)^{-1}} = (1-x^2)^{-1}$.

(35) **(D).** $y' = y\tan x - 2\sin x \Rightarrow \frac{dy}{dx} - y\tan x = -2\sin x$

I.F. $= e^{-\int \tan x\, dx} = e^{\log\cos x} = \cos x$

$$\therefore \; y\cos x = \int(-2\sin x)(\cos x)dx + c$$

$$\Rightarrow y\cos x = -\int \sin 2x\, dx + c$$

$$\Rightarrow 2y\cos x = \cos 2x + c.$$

(36) $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \log y = 2\log x + \log c \Rightarrow y = cx^2$.

(37) **(B).** $\frac{dy}{dx} = -\frac{\cos x - \sin x}{\sin x + \cos x} \Rightarrow dy = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)dx$

On integrating both sides, we get

$\Rightarrow y = -\log(\sin x + \cos x) + \log c$

$\Rightarrow y = \log\left(\frac{c}{\sin x + \cos x}\right) \Rightarrow e^y(\sin x + \cos x) = c$.

(38) **(A).** $x\frac{dy}{dx} + y = y^2 \Rightarrow x\frac{dy}{dx} = y^2 - y$

$$\Rightarrow \frac{dy}{y^2 - y} = \frac{dx}{x} \Rightarrow \left[\frac{1}{y-1} - \frac{1}{y}\right]dy = \frac{dx}{x}$$

On integrating, we get $\log(y-1) - \log y = \log x + \log c$

$\Rightarrow \frac{y-1}{y} = xc \Rightarrow y = 1 + cxy$.

(39) **(C).** $\frac{dy}{dx} = \frac{(1+x)y}{(y-1)x}$ can be written as

$$\frac{y-1}{y}dy = \frac{(1+x)}{x}dx \Rightarrow \left(1 - \frac{1}{y}\right)dy = \left(1 + \frac{1}{x}\right)dx$$

$\Rightarrow (y - \log y) = (x + \log x) + c \Rightarrow x - y + \log xy = c$.

(40) **(D).** $(1-x^2)(1-y)dx = xy(1+y)dy$

$\Rightarrow \int \frac{y(1+y)}{(1-y)}dy = \int \frac{(1-x^2)}{x}dx$; Now integrate it.

(41) **(A).** $\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$

$$\Rightarrow \int (2y\log y + y)dy = \int e^x(\sin^2 x + \sin 2x)dx$$

On integrating by parts, we get $y^2(\log y) = e^x \sin^2 x + c$.

(42) **(A).** $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$

$$\frac{dy}{dx}(\tan y) = 2\sin x\cos y \Rightarrow \frac{\sin y}{\cos^2 y}dy = 2\sin x dx$$

$$\Rightarrow \int \frac{\sin y}{\cos^2 y}dy = 2\int \sin x dx \Rightarrow \frac{1}{\cos y} = -2\cos x + c$$

$\therefore \sec y + 2\cos x = c$.

(43) **(D).** It is homogeneous equation $\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So, we get $x\frac{dv}{dx} = \frac{1+v^2}{2v} \Rightarrow \frac{2vdv}{1+v^2} = \frac{dx}{x}$

On integrating, we get $x^2 + y^2 = px^3$.

(44) **(A).** I.F. = $e^{\int \sec^2 x\,dx} = e^{\tan x}$

$\therefore$ Solution is $ye^{\tan x} = c + \int \tan x e^{\tan x} \sec^2 x\,dx$

$\Rightarrow y = ce^{-\tan x} + \tan x - 1$.

(45) $\frac{dy}{dx} - y = 1 - e^{-x}$, I.F. $= e^{-x}$

$\therefore$ y . $e^{-x} = \int (e^{-x} - e^{-2x})\,dx$

$y.e^{-x} = -e^{-x} + \frac{1}{2}e^{-2x} + C$

If x = 0, y = y_0

$y_0 = -1 + \frac{1}{2} + C \Rightarrow C = y_0 + \frac{1}{2}$

$\therefore\ y.e^{-x} = -e^{-x} + \frac{1}{2}e^{-2x} + y_0 + \frac{1}{2}$

If $x \to \infty$, then $y_0 = -1/2$

(46) **(B).** $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left[3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right]^3$

It is a differential equation of degree 2.

(47) **(C).** $y(x + y^3)\,dx = x(y^3 - x)\,dy$

$\Rightarrow y^3(ydx - xdy) + x(ydx + xdy) = 0$

$\Rightarrow x^2y^3\left(\frac{ydx - xdy}{x^2}\right) + xd(xy) = 0$

$\Rightarrow -\frac{y}{x}d\left(\frac{y}{x}\right) + \frac{d(xy)}{x^2y^2} = 0$

integrating, $\frac{-(y/x)^2}{2} - \frac{1}{xy} = c \Rightarrow y^3 + 2x + 2cx^2y = 0$

It passes through the point (1, 1) $\Rightarrow$ c = –3/2

the curve is $y^3 + 2x - 3x^2y = 0$

(48) **(A).** Differentiating the equation twice with respect to x, we have $2a^2x - 2b^2yy' = 0$, $a^2 - b^2(y'^2 + yy'') = 0$.

Eliminating a^2 and b^2 we have the differential equation

$\frac{y''}{y'} + \frac{y'}{y} = \frac{1}{x}$.

(49) **(D).** $\ln c + \ln|x| = \frac{x}{y}$

diff. w.r.t. x, $\frac{1}{x} = \frac{y - xy_1}{y^2}$; $\frac{y^2}{x} = y - x\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$

(50) **(A).** With y = x v, xv' = $\frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$

$\Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x}$ whose solution is $x(1 - v^2) = c$.

When x = 2, v = 1/2 so that c = 3/2.

The equation of the curve is

$x^2 - y^2 = \frac{3x}{2} \Rightarrow \left(x - \frac{3}{4}\right)^2 - y^2 = \frac{9}{16}$.

This is a rectangular hyperbola with eccentricity $\sqrt{2}$.

(51) **(A).** Put $x^2y^2 = z$

Given $x^2.2y\frac{dy}{dx} + y^2.2x = \tan(x^2y^2)$

$\frac{d}{dx}(x^2y^2) = \tan(x^2y^2)$ put $x^2y^2 = z$

now given expression transforms to $\frac{dz}{dx} = \tan z$

$\therefore \int dx = \int \cot z\,dz$; $x = \ln(\sin z) + C$

when x = 1, y = $\sqrt{\frac{\pi}{2}} \Rightarrow z = \frac{\pi}{2} \Rightarrow C = 1$

$\therefore x = \ln \sin(x^2y^2) + 1$ $\qquad \therefore \ln \sin(x^2y^2) = x - 1$

$\sin(x^2y^2) = e^{x-1}$

(52) **(D).** The given differential equation can be written as

$x\frac{dy}{dx} + 2y = x(\sin x + \log x) \Rightarrow \frac{dy}{dx} + \frac{2}{x}.\ y = \sin x + \log x$

which is linear in y i.e.

of the type $\frac{dy}{dx} + Py = Q$. Hence $P = \frac{2}{x}$

$\therefore \int Pdx = 2\log x = \log x^2$ $\quad \therefore e^{\int Pdx} = e^{\log x^2} = x^2$

$\therefore$ Sol. is y. $x^2 = \int x^2(\sin x + \log x)dx + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + \frac{x^2}{3}\log x - \frac{x^2}{9} + c$

i.e. $y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(53) **(D).** $\frac{dx}{dy} + \frac{2x}{y-3} = -\frac{1}{(y-3)^2}$

I.F = $e^{\int \frac{2}{y-3}dy} = (y-3)^2 \Rightarrow x(y-3)^2 = -y + c \Rightarrow c = 3$

$\Rightarrow (x(y-3) + 1)(y-3) = 0$

(54) **(D).** $\frac{dy}{dx}+1=e^x.e^y \Rightarrow e^{-y}\frac{dy}{dx}+e^{-y}=e^x$

Put $e^{-y}=z \therefore -e^{-y}\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore -\frac{dz}{dx}+z=e^x$ i.e. $\frac{dz}{dx}-z=-e^x$ which is linear in z.

Here $P=-1 \quad \therefore \int Pdx=\int -dx=-x \quad \therefore e^{\int Pdx}=e^{-x}$

$\therefore$ Sol. is $z.e^{-x}=\int -e^x.e^{-x}dx+c=-x+c$

$\Rightarrow e^{-y}.e^{-x}=c-x$

$\Rightarrow e^{-(x+y)}=-(x-c) \Rightarrow (x-c)\,e^{x+y}+1=0$

(55) **(C).** $(1+y^2)\,dx+(1+x^2)dy=0 \Rightarrow \frac{dx}{1+x^2}+\frac{dy}{1+y^2}=0$

On integration, we get

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}C$

$\frac{x+y}{1-xy}=C \Rightarrow x+y=C(1-xy)$

(56) **(C).** $\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$, $\frac{1}{y}=v; \frac{-1}{y^2}\frac{dy}{dx}=\frac{dv}{dx}$

$\therefore \frac{-dv}{dx}-v\tan x=-\sec x$; $\frac{dv}{dx}+v\tan x=\sec x$,

Here $P=\tan x$, $Q=\sec x$

I.F. $=e^{\int \tan x\,dx}=\sec x$, $\quad v\sec x=\int \sec^2 x\,dx+c$

Hence the solution is $y^{-1}\sec x=\tan x+c$

(57) **(B).** As concentration of input and that of tank is same (also rate of in flow = rate of out flow), then the amount of substance in the tank will remain same.

(58) **(B).** $y\,dy-3x\,dx=0 \Rightarrow y^2-3x^2=c$. This passes through (1, 1) so that $c=-2$.

The curve is the hyperbola $3x^2-y^2=2$ with eccentricity

$$\sqrt{\left(\frac{\frac{2}{3}+2}{2/3}\right)}=2.$$

(59) **(A).** Put $4x+y+1=z$

$\therefore 4+\frac{dy}{dx}=\frac{dz}{dx} \quad \therefore \frac{dz}{dx}-4=z^2$

$\Rightarrow \int \frac{dz}{z^2+4}=\int dx+C \Rightarrow \frac{1}{2}\tan^{-1}\frac{z}{2}=x+C$

$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right)=x+C$

(60) **(B).** The general equation of given family of ellipses is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

differentiating with respect to x, $\frac{2x}{a^2}+\frac{2y}{b^2}.y_1=0 \quad \ldots$(i)

Differentiating again $\frac{1}{a^2}+\frac{1}{b^2}(y_1^2+yy_2)=0 \quad \ldots$(ii)

from (i) and (ii)

$\frac{xy_1^2+xyy_2-yy_1}{b^2x}=0 \Rightarrow xyy_2+xy_1^2-yy_1=0$

(61) **(B).** $\frac{dx}{dy}=\frac{x+2y^3}{y}$; $\frac{dx}{dy}-\frac{1}{y}x=2y^2$ which is linear

I.F. $e^{\int -\frac{1}{y}dy}=e^{-\ln y}=\frac{1}{y}$

$\therefore \frac{1}{y}.x=\int \frac{1}{y}.2y^2\,dy=y^2+c \quad \therefore \frac{x}{y}=y^2+c$

(62) **(A).** $y=C_1+C_2e^x+C_3e^{-2x+c_4}=C_1+C_2e^x+C_3e^{c_4}.e^{-2x}$

So, Differential equation will be order of 3.

(63) **(A).** $\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x}$; $y=vx$

$V+x\frac{dv}{dx}=v-\cos^2 v$

$\int \frac{dv}{\cos^2 v}+\int \frac{dx}{x}=C$

$\tan v+\ln x=C$; $\tan\frac{y}{x}+\ln x=C$

if $x=1$, $y=\frac{\pi}{4} \Rightarrow C=1$; $\quad \tan\frac{y}{x}=1-\ln x=\ln\frac{e}{x}$;

$y=x\tan^{-1}\left(\ln\frac{y}{x}\right) \Rightarrow$ A]

(64) **(A).** $\frac{dy}{dx}=\sec x\,(\sec x+\cot x)$

$\Rightarrow \frac{dy}{dx}=\sec^2x+\sec x\tan x$

Now integrating both sides, we get

$y=\tan x+\sec x+c$

(65) **(C).** $x\cos y\,dy=(xe^x\log x+e^x)\,dx$

$\Rightarrow \cos y\,dy=\left(e^x\log x+\frac{e^x}{x}\right)dx$

On integrating $\sin y=e^x\log x+c$

(66) **(A).** $\frac{dy}{dx}+y\,\phi'(x)=\phi(x).\phi'(x)$

I.F. $=e^{\int \phi'(x)dx}=e^{\phi(x)}$

hence $y.e^{\phi(x)}=\int e^{\phi(x)}.\phi(x).\phi'(x)\,dx=\int e^t.t\,dt$

where $\phi(x) = t$
$= te^t - e^t + C = \phi(x).e^{\phi(x)} - e^{\phi(x)} + C$
$\therefore y = ce^{-\phi(x)} + \phi(x) - 1$

(67) **(A).** $\frac{dV}{dt} = -k4\pi r^2$ ....(1)

but $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ ....(2)

hence $\frac{dr}{dt} = -K$

(68) **(B).** $x^2 + (y-r)^2 = r^2$ .... (1)

$\therefore x + (y-r)\frac{dy}{dx} = 0 \ \therefore (r-y)\frac{dy}{dx} = x$

$\therefore r = y + \frac{x}{(dy/dx)}$

Put it in (i) we get $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

(69) **(A).** We are given that $\frac{dy}{dx} = \cos(x+y)$

Put $x + y = v$,

so that $1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dy}{dx} - 1$

So, the given equation becomes

$\frac{dv}{dx} - 1 = \cos v \Rightarrow \frac{dv}{dx} = 1 + \cos v$

$\Rightarrow \frac{1}{1+\cos v}dv = dx \Rightarrow \frac{1}{2}\sec^2\frac{v}{2}dv = dx$

Integrating both sides, we get $\int \frac{1}{2}\sec^2\frac{v}{2}dv = \int 1.dx$

$\Rightarrow \tan\frac{v}{2} = x + C \Rightarrow \tan\left(\frac{x+y}{2}\right) = x + C$

Which is the required solution.

(70) **(B).** $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{dx - dy}{dx + dy}$

By compendo and dividendo $\frac{dy}{dx} = e^{-2x}$

(71) **(C).** $y' + \frac{9x}{4y} = 0$ integrates to $4y^2 + 9x^2 = 36c$,

c being an arbitrary constant. This represents a family of ellipses whose equation in the standard form is

$\frac{x^2}{4c} + \frac{y^2}{9c} = 1$ with eccentricity $\sqrt{\frac{9c-4c}{9c}} = \frac{\sqrt{5}}{3}$.

(72) **(B).** $\frac{dy}{dx} = \frac{1}{dx/dy}$,

$\frac{d^2y}{dx^2} = \frac{d}{dy}\left(\frac{1}{dx/dy}\right)\frac{dy}{dx} = -\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2}$

Hence $x\frac{d^2x}{dy^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$

becomes $-x\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2} + \frac{1}{(dx/dy)^3} - \frac{1}{(dx/dy)} = 0$

$x\frac{d^2x}{dy^2} - 1 + \left(\frac{dx}{dy}\right)^2 = 0 \Rightarrow x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 1$

$\therefore k = 1$

(73) **(C).** The given equation can be written as :

$\frac{dx}{dy} + \frac{2}{y}x = 10y^2$ .....(1) [Linear Equation in x]

Here 'P' $= \frac{2}{y}$ and 'Q' $= 10y^2$

I.F. $= e^{\int Pdy} = e^{\int \frac{2}{y}dy} = e^{2\log|y|} = e^{\log y^2} = y^2$

Multiplying (1) by y^2, we get :

$y^2.\frac{dx}{dy} + 2yx = 10y^4 \Rightarrow \frac{d}{dy}(x.y^2) = 10y^4$

Integrating, $xy^2 = 10\int y^4 dy + c$

$\Rightarrow xy^2 = 2y^5 + c$ which is required solution.

(74) **(B).** Put $xe^x = t$

$(e^x + xe^x)\,dx = dt$

$\Rightarrow \frac{dt}{dy} + (ye^y - t) = 0 \Rightarrow \frac{dt}{dy} - t + ye^y = 0$

Integrating factor $e^{-\int dy} = e^{-y} \Rightarrow t.e^{-y} = -\int ye^y.e^{-y}dy$

$\Rightarrow xe^x.e^{-y} = \frac{-y^2}{2} + c$

$f(0) = 0 \Rightarrow c = 0,\ 2xe^x e^{-y} + y^2 = 0$

(75) **(C).** $y = ax + \frac{b}{x}$ ....(1) ; $\frac{dy}{dx} = a - \frac{b}{x^2}$ ....(2)

$\frac{d^2y}{dx^2} = \frac{2b}{x^3}$ ....(3)

From equations (1) and (2), $y + x\frac{dy}{dx} = 2ax$

From equation (3), $\frac{2b}{x} = x^2\frac{d^2y}{dx^2}$

$\therefore$ From equation (1),

$$2y = \left(y + x\frac{dy}{dx}\right) + x^2\frac{d^2y}{dx^2} \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

(76) **(C).** $\frac{dy}{dx} = \frac{x}{y} \Rightarrow \int y\,dy = \int x\,dx \Rightarrow x^2 - y^2 = C$

(77) **(D).** Length of subnormal = $y\frac{dy}{dx} = 2$; $y\,dy = 2x\,dx$

$$\frac{y^2}{2} = 2x + c$$

Since (1, 2) lies on it,

$\therefore c = 0 \quad \therefore$ the curve is $y^2 = 4x$

(78) **(A).** $y \,.\, e^{-2x} = Ax\,e^{-2x} + B$

$e^{-2x} \,.\, y_1 - 2y\,e^{-2x} = A\,(e^{-2x} - 2x\,e^{-2x})$

Cancelling e^{-2x} throughout

$y_1 - 2y = A\,(1 - 2x)$ .... (1)

differentiating againt

$$y_2 - 2y_1 = -2A \Rightarrow A = \frac{2y_1 - y_2}{2}$$

$2\,(y_1 - 2y) = (2y_1 - y_2)\,(1 - 2x)$

$2y_1 - 4y = 2y_1\,(1 - 2x) - (1 - 2x)\,y_2$

$$(1-2x)\,\frac{d}{dx}\left(\frac{dy}{dx} - 2y\right) + 2\left(\frac{dy}{dx} - 2y\right) = 0$$

hence $k = 2$ and $\ell = -2 \Rightarrow$ ordered pair $(k, \ell) \equiv (2, -2)$

(79) **(C).** Equation of the tangent at (x, y) :

$$Y - y = \frac{dy}{dx}\,(X - x)$$

$$P_1 \equiv \left(x - y\frac{dx}{dy},\, 0\right),\; P_2\left(0,\; y - x\frac{dx}{dy}\right)$$

$$\Rightarrow 3x = x - y\frac{dx}{dy} \text{ or } 3y = 2\left(y - x\frac{dy}{dx}\right)$$

Both give the differential equation

$2x\,dy + y\,dx = 0$ whose solution is $xy^2 = c$

(80) **(A).** $x^4y^2dx + x^2y^4dy = xdy + ydx$

$x^2y^2(x^2dx + y^2dy) = xdy + ydx$

$$x^2dx + y^2dy = \frac{d(xy)}{(xy)^2}$$

Integrating, $\int x^2\,dx + \int y^2\,dy = \int \frac{d(xy)}{(xy)^2}$

$$\frac{x^3}{3} + \frac{y^3}{3} = -\frac{1}{xy} + C$$

$(x^3 + y^3) + \frac{3}{xy} = C$; now if $x = 1; y = 1 \Rightarrow C = 5$,

hence, $x^3 + y^3 + 3(xy)^{-1} = 5$

(81) **(B).** Put $u = x - y$, then $\frac{du}{dx} = 1 - \frac{dy}{dx}$

$$\Rightarrow 1 - \cos u = \frac{du}{dx} \Rightarrow \int \frac{du}{1 - \cos u} = \int dx$$

$$\Rightarrow \frac{1}{2}\int \operatorname{cosec}^2\left(\frac{u}{2}\right)du = \int dx$$

$$\Rightarrow x + \cot\,(u/2) = \text{constant} \Rightarrow x + \cot\left(\frac{x-y}{2}\right) = C$$

(82) **(A).** $y\frac{dy}{dx} = x - 1 \Rightarrow ydy = (x-1)\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} - x + c$

for $x = 1, y = 1$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} - 1 + C \quad \Rightarrow \quad C = 1$$

Thus $\frac{y^2}{2} = \frac{x^2}{2} - x + 1 \Rightarrow y^2 = x^2 - 2x + 2$

(83) **(C).** $\frac{dy}{dx} = e^{(ax+by)}$

$$\frac{dy}{dx} = e^{ax}.e^{by} \Rightarrow e^{-by}\frac{dy}{dx} = e^{ax}dx \Rightarrow -\frac{1}{b}e^{-by} = \frac{1}{a}e^{ax} + c$$

(84) **(B).** $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$$\int dy = \int\left(1 - \frac{1}{x^2}\right)dx \Rightarrow y = x + \frac{1}{x} + C$$

So, $y = x + \frac{1}{x} + 1 \Rightarrow xy = x^2 + x + 1$

(85) **(C).** $\frac{d^2y}{dx^2} = e^{-2x} \Rightarrow \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$

$$\Rightarrow y = \frac{e^{-2x}}{(-2)(-2)} + cx + d = \frac{e^{-2x}}{4} + cx + d$$

(86) **(D).** $y\sin 2x - \cos x + (1 + \sin^2 x)\frac{dy}{dx} = 0$ where $y = f(x)$

$$\frac{dy}{dx} + \left(\frac{\sin 2x}{1 + \sin^2 x}\right)y = \frac{\cos x}{1 + \sin^2 x}$$

I.F. $= e^{\int \frac{\sin 2x}{1+\sin^2 x}dx} = e^{\int \frac{dt}{t}} = e^{\ln(1+\sin^2 x)} = 1 + \sin^2 x$

(by putting $1 + \sin^2 x = t$)

$y\,(1 + \sin^2 x) = \int \cos x\,dx$

$y\,(1 + \sin^2 x) = \sin x + C$; $(y\,(0) = 0) \Rightarrow C = 0$

Hence, $y = \frac{\sin x}{1 + \sin^2 x}$; $y\left(\frac{\pi}{6}\right) = \frac{2}{5}$

(87) **(A).** $\frac{dy}{dx}-\frac{y}{x}=-\frac{5x}{(x+2)(x-3)}$ which is linear in y

I.F. = $e^{\int -\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$

So the solution is $y.\frac{1}{x}=-\int\frac{5}{(x+2)\,(x+3)}dx+c$

$y.\frac{1}{x}=\ln\left(\frac{x+2}{x-3}\right)+c$

It passes through (4, 0)

$\therefore\ 0=\ln 6+c\Rightarrow c=-\ln 6$

$\therefore\ \frac{y}{x}=\ln\frac{x+2}{x-3}-\ln 6$

Then point (5, a) lies on it

$\therefore\ a=5\ln(7/12)$

(88) **(D).** $\frac{dy}{dx}=\frac{x+y+1}{2y+2x+1}$; $\frac{dy}{dx}=\frac{x+y+1}{2\,(x+y)+1}$

Let $x+y=t$; $1+\frac{dy}{dx}=\frac{dt}{dx}\Rightarrow\frac{dt}{dx}=\frac{t+1}{2t+1}+1=\frac{3t+2}{2t+1}$

$\Rightarrow\int\frac{2t+1}{3t+2}dt=\int dx$ or $\frac{2t}{3}-\frac{1}{9}\ln(3t+2)=x+c$

or $6\,(x+y)-\ln(3x+3y+2)=9x+c$

or $\ln(3x+3y+2)=6y-3x+c$

Since it passes through (0, 0) hence equation of curve is

$6y-3x=\ln\left|\left(\frac{3x+3y+2}{2}\right)\right|$

(89) **(A).** Put $x+y=v$ or $1+\frac{dy}{dx}=\frac{dv}{dx}$

$\left(\frac{dv}{dx}-1\right)=\sec v$, $\frac{dv}{dx}=\sec v+1$

$\frac{dv}{\sec v+1}=dx$ or $\frac{\cos v\,dv}{\cos v+1}=dx$

$\left(1-\frac{1}{\cos v+1}\right)dv=dx$

$\left(1-\frac{1}{2\cos^2\frac{v}{2}}\right)dv=dx$ or $\left(1-\frac{1}{2}\sec^2\frac{v}{2}\right)dv=dx$

$v-\tan\frac{v}{2}=x+c$ or $x+y-\tan\frac{x+y}{2}=x+c$ or

$y-\tan\frac{x+y}{2}=c$

EXERCISE-2

(1) **(C).** $y=(c_1+c_2)\cos(x+c_3)-c_4e^{x+c_5}$

$y_1=-(c_1+c_2)\sin(x+c_3)-c_4e^{x+c_5}$

$y_2=-(c_1+c_2)\cos(x+c_3)-c_4e^{x+C_5}=-y-2c_4e^{x+c_5}$

$\Rightarrow y_3=-y_1-2c_4e^{x+c_5}=-y_1+y_2+y$

Hence the differential equation is $y_3-y_2+y_1-y=0$, which is of order 3.

(2) **(D).** Given equation can be written as

$e^y\frac{dy}{dx}+e^x.e^y=e^{2x}$

Now, let $e^y=v$, then $e^y\frac{dy}{dx}=\frac{dv}{dx}$

So transformed equation will be

$\frac{dv}{dx}+e^xv=e^{2x}$ So, its I.F. $=e^{\int e^x dx}=e^{e^x}$

$\because$ Solution is $v.e^{e^x}=\int e^{e^x}.e^{2x}\,dx+c$

$\Rightarrow\ e^y.e^{e^x}=e^{e^x}(e^x-1)+c\ \Rightarrow\ e^{e^x}(e^y-e^x+1)=c$

(3) **(B).** Equation of the family of such parabola is

$(y-k)^2=4a\,(x-h)$ .... (1)

where h and k are two arbitrary constants

$\therefore$ order = 2

and diff. w.r.t. x $(y-k)\frac{dy}{dx}=2a$

On diff. again

$(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

Solving eq. (1), (2) and (3), $2a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$

(4) **(C).** $\frac{dy}{dx}=-\frac{y}{x-2\sqrt{xy}}$ which is homeogenous

Put $y=vx$ so that $\frac{dy}{dx}=x\frac{dv}{dx}+v\Rightarrow\ x\frac{dv}{dx}+v=\frac{v}{1-2\sqrt{v}}$

$\Rightarrow x\frac{dv}{dx}=\frac{v}{1-2\sqrt{v}}-v=\frac{2v^{3/2}}{1-2\sqrt{v}}\Rightarrow\frac{dx}{x}=\frac{1-2\sqrt{v}}{2v^{3/2}}dv$

$\Rightarrow\int\left(\frac{1}{2v^{3/2}}-\frac{1}{v}\right)dv=\int\frac{1}{x}dx$

$\Rightarrow -v^{-1/2}-\log v=\log x-c$

$\Rightarrow -\sqrt{\frac{x}{y}}-\log y+\log x=\log x-c\Rightarrow\ \log y+\sqrt{\frac{x}{y}}=c$

(5) **(C).** $\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$ .... (1)

which is homogeneous equation
Put y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ .... (2)

Now eq. (1) reduces to

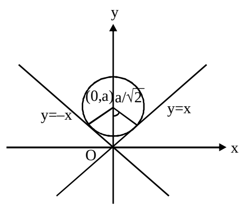
$x\frac{dv}{dx} = \frac{1+v^2}{2v} \Rightarrow \frac{2vdv}{1+v^2} = \frac{dx}{x} \Rightarrow \int \frac{2vdv}{1+v^2} = \int \frac{1}{x}dx$

$\Rightarrow \log(1+v^2) = \log x + \log c$

$\Rightarrow \left(1 + \frac{y^2}{x^2}\right) = xc \Rightarrow x^2 + y^2 = x^3 c$

(6) **(C).** $\because$ Equation of circle

$(x-0)^2 + (y-a)^2 = (a/\sqrt{2})^2$



$x^2 + y^2 - 2ay + \frac{a^2}{2} = 0$ ..... (1)

$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$

$a = \left(\frac{x + y\frac{dy}{dx}}{dy/dx}\right)$ ...... (2)

From eq. (1) and eq. (2)

$x^2 + y^2 - 2y\left(\frac{x + y\frac{dy}{dx}}{dy/dx}\right) + \frac{\left(x + y\frac{dy}{dx}\right)^2}{2.\left(\frac{dy}{dx}\right)^2}$

$\therefore$ order = 1, degree = 2 i.e. P = 2, A = 1

(7) **(B).** $\frac{dy}{dx} + 3xy = x \Rightarrow dy = x(1-3y)\,dx$

$\Rightarrow \frac{-3\,dy}{(1-3y)} = -3x\,dx$

Integrating we get, $\log(1-3y) = -\frac{-3x^2}{2} + C$

or $(1-3y) = e^{-3x^2/2} + C_1$ which passes through (0, 4)

$C_1 = -11$ $\therefore$ Required curve is $3y = 1 + 11\,e^{-3x^2/2}$

(8) **(A).** $\because$ $y = C_1e^{2x} + C_2e^{x} + C_3e^{-x}$

$\Rightarrow e^{x}y = C_1e^{3x} + C_2e^{2x} + C_3$ .... (1)

Again differentiating both the sides, we get

$e^{x}\frac{dy}{dx} + ye^{x} = 3C_1e^{3x} + 2C_2e^{2x} + 0$

$\Rightarrow e^{-x}\left(\frac{dy}{dx} + y\right) = 3C_1e^{x} + 2C_2$ .... (2)

Again, $e^{-x}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) - \left(\frac{dy}{dx} + y\right)e^{-2x} = 3C_1$

or $e^{-2x}\left(\frac{d^2y}{dx^2} - y\right) = 3C_1$ .... (3)

Finally, differentiating both the sides, we get

$\Rightarrow e^{-2x}\left(\frac{d^3y}{dx^3} - \frac{dy}{dx}\right) + \left(\frac{d^2y}{dx^2} - y\right)(-2e^{-2x}) = 0$

$\Rightarrow \frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$ .... (4)

On comparing with

$\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + Cy = 0$ .... (5)

We get a = –2, b = –1, C = 2

$\therefore \left(\frac{a^3 + b^3 + c^3}{abc}\right) = \frac{-8-1+8}{4} = -\frac{1}{4}$

(9) **(A).** $x^2 = e^{\left(\frac{x}{y}\right)^{-1}\left(\frac{dy}{dx}\right)} \Rightarrow x^2 = e^{\left(\frac{y}{x}\right)\left(\frac{dy}{dx}\right)}$

$\Rightarrow \ln x^2 = \frac{y\,dy}{x\,dx}$ or $\int x \ln x^2 dx = \int y\,dy$

$x^2 = t \Rightarrow 2x\,dx = dt$

$\frac{1}{2}\int \ln t\,dt = \frac{y^2}{2}$; $c + t\ln t - t = y^2$

or $y^2 = x^2(\ln x^2 - 1) + c$

(10) **(A).** $y = ax^2$

$\frac{dy}{dx} = 2ax$ or $a = \frac{1}{2x}\frac{dy}{dx}$

Now, equation of curve $y = \frac{1}{2x}\frac{dy}{dx}.x^2$ or $\frac{x}{2}\frac{dy}{dx} = y$

orthogonal trajectory $-\frac{dx}{dy} = \frac{2y}{x}$

$-\frac{x^2}{2} = y^2 + c$ or $y^2 + \frac{x^2}{2} = c$

This is the equation of ellipse.

(11) **(B).** $\frac{dx}{dt} = x + 1 \Rightarrow \log(x+1) = t + c$

Putting $t = 0, x = 0$, we get $\log 1 = c \Rightarrow c = 0$

$\therefore t = \log(x+1)$.

For $x = 99$, $t = \log_e 100 = 2\log_e 10$.

(12) **(C).** Given $\frac{dx}{dt} = \cos^2 ®x$. Differentiate w.r.t. t,

$$\frac{d^2x}{dt^2} = -2® \sin 2®x = -ve$$

$$\because \frac{d^2x}{dt^2} = 0 \Rightarrow -2® \sin 2®x = 0 \Rightarrow \sin 2®x = \sin ®$$

$$\Rightarrow 2®x = ® \Rightarrow x = 1/2.$$

(13) **(D).** Given $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$$\Rightarrow \frac{dy}{dx} = (1+x) + y^2(1+x) \Rightarrow \frac{dy}{dx} = (1+x)(1+y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x)dx$$

Integrating both sides, $\int \frac{dy}{1+y^2} = \int (1+x)dx$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$$

Put $y(0) = 0$, $\therefore 0 = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right).$$

(14) **(A).** Rearranging the terms, $\frac{dy}{dt} - \frac{t}{1+t}y = \frac{1}{1+t}$

$$\text{I.F.} = e^{\int -\frac{t}{1+t}dt} = e^{-t}.(1+t)$$

Solution is $ye^{-t}.(1+t) = \int (1+t).e^{-t}\frac{1}{(1+t)} + c$

$$ye^{-t}(1+t) = -e^{-t} + c$$

Also, $y(0) = -1 \Rightarrow c = 0 \Rightarrow y(1) = \frac{-1}{2}$.

(15) **(C).** Given $\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v - \sin^2 v \Rightarrow -\text{cosec}^2 v\, dv = \frac{dx}{x}$$

Integrating both sides, $-\int \text{cosec}^2 v\, dv = \int \frac{dx}{x}$

$$\Rightarrow \cot v = \log x + c,\ \cot\frac{y}{x} = \log x + c$$

This curve passes through the point $\left(1, \frac{®}{4}\right)$

$\therefore$ c = 1 $\Rightarrow \cot\frac{y}{x} = \log x + \log_e e$

$$\cot\frac{y}{x} = \log x e \Rightarrow y = x\cot^{-1}(\log x e).$$

(16) **(D).** We have $y\frac{dy}{dx} + x = a$ or $ydy + xdx = adx$

Integrating, we get $\frac{y^2}{2} + \frac{x^2}{2} = ax + c$

or $x^2 + y^2 - 2ax + k = 0$,

which represents a set of circles having centre on x-axis.

(17) **(A).** $\frac{dy}{dx} = 2\cos x - y\cot x \Rightarrow \frac{dy}{dx} + y\cot x = 2\cos x$

I.F. $= e^{\int \cot x dx} = \sin x$; $y.\sin x = \int 2\cos x.\sin x + c$

$y\sin x = \sin^2 x + c$ at $y = 2$

and $x = \frac{®}{2}$, $c = 1$; $y = \sin x + \text{cosec} x$

(18) **(B).** $(2x - y + 1)dx + (2y - x + 1)dy = 0$

$\frac{dy}{dx} = \frac{2x - y + 1}{x - 2y - 1}$, put $x = X + h$, $y = Y + k$

$$\frac{dY}{dX} = \frac{2X - Y + 2h - k + 1}{X - 2Y + h - 2k - 1}$$

$$2h - k + 1 = 0 \Rightarrow h - 2k - 1 = 0$$

On solving $h = -1$, $k = -1$; $\therefore \frac{dY}{dX} = \frac{2X - Y}{X - 2Y}$

Put $Y = vX$; $\therefore \frac{dY}{dX} = v + X\frac{dv}{dX}$

$$v + X\frac{dv}{dX} = \frac{2X - vX}{X - 2vX} = \frac{2 - v}{1 - 2v}$$

$$X\frac{dv}{dX} = \frac{2 - 2v + 2v^2}{1 - 2v} = \frac{2(v^2 - v + 1)}{1 - 2v}$$

$$\therefore \frac{dX}{X} = \frac{(1 - 2v)}{2(v^2 - v + 1)}dv$$

Put $v^2 - v + 1 = t \Rightarrow (2v - 1)dv = dt$

$$\therefore \frac{dX}{X} = -\frac{dt}{2t} \therefore \log X = \log t^{-1/2} + \log c$$

$$\therefore X = t^{-1/2}c \Rightarrow X = (v^2 - v + 1)^{-1/2}.c$$

$X^2(v^2 - v + 1) =$ constant

$(x+1)^2\left(\frac{(y+1)^2}{(x+1)^2} - \frac{(y+1)}{x+1} + 1\right) =$ constant

$$(y+1)^2 - (y+1)(x+1) + (x+1)^2 = c$$

$$y^2 + x^2 - xy + x + y = c.$$

(19) **(A).** $ye^{-x/y}dx - (xe^{-x/y} + y^3)dy = 0$

$$e^{-x/y}(ydx - xdy) = y^3 dy \Rightarrow e^{-x/y}\frac{(ydx - xdy)}{y^2} = ydy$$

$e^{-x/y} d\left(\frac{x}{y}\right) = y\,dy$. Integrating both sides, we get

$k - e^{-x/y} = \frac{y^2}{2} \Rightarrow \frac{y^2}{2} + e^{-x/y} = k$

(20) **(A).** $x\frac{dy}{dx} + y = x^2 + 3x + 2 \Rightarrow \frac{dy}{dx} + \frac{y}{x} = x + 3 + \frac{2}{x}$

Here $P = \frac{1}{x}$, $Q = x + 3 + \frac{2}{x}$, therefore I.F. $= e^{\int \frac{1}{x}dx} = x$

Now solve it.

(21) **(D).** $\frac{dy}{dx} + y\cot x = 2\cos x$

It is linear equation of the form $\frac{dy}{dx} + Py = Q$

So, I.F. $= e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

Hence the solution is $y \sin x = \int 2 \sin x \cos x dx + c$

$\Rightarrow y\sin x = -\frac{1}{2}\cos 2x + c \Rightarrow 2y \sin x + \cos 2x = c$.

(22) $y^2 = t$; $2y\frac{dy}{dx} = \frac{dt}{dx}$

Hence the differential equation becomes

$\left(e^{x^2} + e^t\right)\frac{dt}{dx} + 2e^{x^2}(x t - x) = 0$

$e^{x^2} + e^t + 2e^{x^2} \cdot x(t-1)\frac{dx}{dt} = 0$

put $e^{x^2} = z$; $e^{x^2} \cdot 2x\frac{dx}{dt} = \frac{dz}{dt}$

$z + e^t + \frac{dz}{dt}(t-1) = 0$; $\frac{dz}{dt} + \frac{z}{(t-1)} = -\frac{e^t}{(t-1)}$

I.F. $= e^{\int \frac{dt}{t-1}} = e^{ln(t-1)} = t - 1$

$z(t-1) = -\int (e^t)\,dt$; $z(t-1) = -e^t + C$

$e^{x^2}(y^2 - 1) = -e^{y^2} + C$; $e^{x^2}(y^2-1) + e^{y^2} = C$

(23) **(A).** Diff. both sides $x\,y(x) = 2x - y'(x)$

hence $\frac{dy}{dx} - xy = -2x$; I.F $= e^{\int -x\,dx} = e^{\frac{-x^2}{2}}$

$y e^{\frac{-x^2}{2}} = \int -2x e^{\frac{-x^2}{2}} dx$; $y e^{\frac{-x^2}{2}} = -2x e^{\frac{-x^2}{2}} + c$

$y = 2 + c e^{\frac{-x^2}{2}}$

if $x = a \Rightarrow a^2 + y = 0 \Rightarrow y = -a^2$

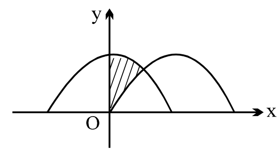
$-a^2 = 2 + c e^{\frac{-a^2}{2}}$; $c e^{\frac{-a^2}{2}} = -(2 + a^2)$

$c = -(2 + a^2) e^{\frac{-a^2}{2}}$; $y = 2 - (2 + a^2) e^{\frac{x^2 - a^2}{2}}$

(24) **(A).** I.F. $= e^{-x}$

$\therefore\ y e^{-x} = \int e^{-x}(\cos x - \sin x)\,dx$ put $-x = t$

$= -\int e^t(\cos t + \sin t)\,dt = -e^t \sin t + c$



$y e^{-x} = e^{-x}\sin x + c$

since y is bounded when $x \to \infty \Rightarrow c = 0$

$\therefore\ y = \sin x$

Area $= \int_0^{\pi/4} (\cos x - \sin x)\,dx = \sqrt{2} - 1$

(25) **(B).** $y = c_1\cos(x + c_2) - (c_3 e^{-x + c_4}) + (c_5 \sin x)$

$\therefore\ y = c_1(\cos x \cos c_2 - \sin x \sin c_2)$

$-(c_3 e^{c_4} e^{-x}) + (c_5 \sin x)$

$\therefore\ y = (c_1 \cos c_2)\cos x - (c_1 \sin c_2 - c_5)\sin x - (c_3 e^{c_4})e^{-x}$

$\therefore\ y = l\cos x + m \sin x - n e^{-x}$ ....(i)

where l, m, n are arbitrary constant

$\therefore\ \frac{dy}{dx} = -l\sin x + m\cos x + n e^{-x}$ ....(ii)

$\therefore\ \frac{d^2y}{dx^2} = -l\cos x - m \sin x - n e^{-x}$ ....(iii)

$\therefore\ \frac{d^3y}{dx^3} = l \sin x - m\cos x + n e^{-x}$ ....(iv)

(i) + (iii) gives $\frac{d^2y}{dx^2} + y = -2n e^{-x}$ ....(v)

(ii) + (iv) gives $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2n e^{-x}$ ....(vi)

From (v) and (vi) we get $\frac{d^3y}{dx^3} + \frac{dy}{dx} = -\left(\frac{d^2y}{dx^2} + y\right)$

or $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

is the required differential equation

(26) **(A).** $Y - y = -\frac{1}{m}(X - x)$ when $m = \frac{dy}{dx}$

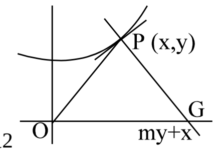
take, let $Y = 0$

$X = my + x$

hence $x(my + x) = 2(x^2 + y^2)$

$$xy\frac{dy}{dx} = x^2 + 2y^2$$

Now put $y = vx$



$$v + x\frac{dv}{dx} = \frac{x^2(1+2v^2)}{x^2 v} = \frac{1+2v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1+2v^2}{v} - v = \frac{1+v^2}{v}$$

$$\int\frac{v\,dv}{1+v^2} = \int\frac{dx}{x} \quad ; \quad \frac{1}{2}\ln(1+v^2) = \ln x + c$$

$$\ln\left(\frac{1+v^2}{x^2}\right) = c \; ; \; x^2 + y^2 = cx^4$$

(27) **(A).** $Y - y = m(X - x)$

for X-intercept $Y = 0$

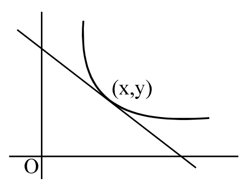
$$X = x - \frac{y}{m}, \text{ hence } x - \frac{y}{m} = y \text{ or } \frac{dy}{dx} = \frac{y}{x-y}$$

put $y = Vx$

$$V + x\frac{dV}{dx} = \frac{V}{1-V}$$

$$x\frac{dV}{dx} = \frac{V}{1-V} - V$$

$$= \frac{V - V + V^2}{1-V}$$



$$\int\frac{1-V}{V^2}dV = \int\frac{dx}{x} \quad ; \quad -\frac{1}{V} - \ln V = \ln x + c$$

$$-\frac{x}{y} - \ln\frac{y}{x} = \ln x + c \; ; \; -\frac{x}{y} = \ln y + c$$

$$x = 1, y = 1 \;\Rightarrow\; c = -1 \; ; \; 1 - \frac{x}{y} = \ln y$$

$$y = e \cdot e^{-x/y} \quad ; \quad e^{-x/y} = \frac{e}{y} \quad ; \quad y e^{x/y} = e$$

(28) **(B).** Since given differential equation is

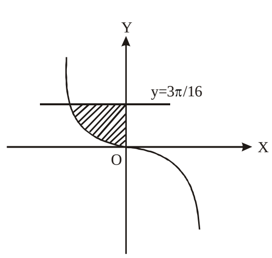
$y(x + y^3)\,dx = x(y^3 - x)\,dy$

$(xy\,dx + x^2\,dy) + y^4\,dx - y^3 x\,dy = 0$

$x(y\,dx + x\,dy) + y^3(y\,dx - x\,dy) = 0$

$x\,d(xy) = y^3(x\,dy - y\,dx)$

$$x\,d(xy) = x^2y^3 d\left(\frac{y}{x}\right) \quad ; \quad \frac{d(xy)}{(xy)^2} = \left(\frac{y}{x}\right) d\left(\frac{y}{x}\right)$$



On integrating, $-\dfrac{1}{xy} = \dfrac{1}{2}\left(\dfrac{y}{x}\right)^2 + c$

At $(4, -2) \Rightarrow \dfrac{1}{8} = \dfrac{1}{2}\left(-\dfrac{2}{4}\right)^2 + c \Rightarrow c = 0$

$\therefore\; y^3 = -2x \Rightarrow y = (-2x)^{1/3}$

Since, $y = g(x) = \displaystyle\int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$

$$\therefore\; \frac{dy}{dx} = g'(x) = x.\sin 2x + x.(-\sin 2x) = 0$$

$\therefore\; y = c_1$ (constant)

Put $\sin x = \cos x = \dfrac{1}{\sqrt{2}}$

$$\therefore\; c_1 = \int_{1/8}^{4/2}(\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t})\,dt = \frac{\pi}{2}\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\pi}{16}$$

$$\therefore\; y = g(x) = \frac{3\pi}{16}$$

Hence area between $y = \dfrac{3\pi}{16}$ and $y = (-2x)^{1/3}$

$$= \int_0^{3\pi/16} x\,dy = \left|\int_0^{3\pi/16}\left(-\frac{y^3}{2}\right)dy\right| = \frac{1}{8}\left(\frac{3\pi}{16}\right)^4 \text{ sq. units}$$

(29) **(A).** Let m gms of salt is present at time t differential equation of the process is

$$\frac{dm}{dt} = 10 - \frac{m(1)}{50+t} \; ; \; \frac{dm}{dt} + \left(\frac{1}{50+t}\right)m = 10;$$

$$\text{I.F} = e^{\int\frac{dt}{50+t}} = 50 + t;$$

$$m(50+t) = \int(50+t)dt = 10\frac{(50+t)^2}{2} + C$$

$m(50 + t) = 5(50 + t)^2 + C;\ t = 0;\ m = 0,\ C = -5.(50)^2$

$m(50 + t) = 5(50 + t)^2 - 5(50)^2$

$$m = 5(50+t)^2 - \frac{5(50)^2}{50+t} \; ; \; m(t=10) = 5 \cdot 60 - \frac{5(50)^2}{60}$$

$$m = \frac{25 \times 11}{3} = 91\frac{2}{3} = 50\left[6 - \frac{250}{60}\right] = 50 \times \frac{11}{6}$$

(30) **(A).** Let $f(x) = y$

$$\therefore\; \frac{dy}{dx} + y = 4xe^{-x} \cdot \sin 2x \quad \text{(linear differential equation)}$$

I.F. e^x $\quad ye^x = 4\int \underset{I}{x}\,\underset{II}{\sin 2x}\,dx$

$$ye^x = 4\left[x\left(-\frac{\cos 2x}{2}\right)+\frac{1}{2}\int \cos 2x\,dx\right]$$

$$ye^x = 4\left[-\frac{x\cos 2x}{2}+\frac{\sin 2x}{4}\right]+C$$

$ye^x = (\sin 2x - 2x\cos 2x) + C$

$f(0) = 0 \Rightarrow C = 0$

$\therefore\ y = e^{-x}(\sin 2x - 2x\cos 2x)$

now $f(k\pi) = e^{-k\pi}(\sin 2k\pi - 2k\pi\cdot\cos 2k\pi)$

$= e^{-k\pi}(0 - 2k\pi)$

$f(k\pi) = -2\pi(k\cdot e^{-k\pi})$

$$\sum f(k\pi) = -2\pi \underbrace{\sum_{k=1}^{\infty} ke^{-k\pi}}_{S}$$

$S = 1\cdot e^{-\pi} + 2e^{-2\pi} + 3e^{-3\pi} + \ldots\ldots + \infty$

$S e^{-\pi} = \quad + e^{-2\pi} + 2e^{-3\pi} + \ldots\ldots + \infty$

$S(1-e^{-\pi}) = e^{-\pi} + e^{-2\pi} + e^{-3\pi} + \ldots\ldots\infty$

$$S(1-e^{-\pi}) = \frac{e^{-\pi}}{1-e^{-\pi}} = \frac{1}{e^{\pi}-1}$$

$$S = \frac{1}{(e^{\pi}-1)(1-e^{-\pi})} = \frac{e^{\pi}}{(e^{\pi}-1)^2}$$

(31) **(B).** S-1 : Equation of all circles can be given by $x^2 + y^2 + 2gx + 2fy + c = 0$, will be of order 3.
S-2 is obviously true but it does not explain statement-1.

(32) **(B).** $\frac{dy}{dx} = x + \frac{1}{x^2} \Rightarrow dy = xdx + \frac{1}{x^2}dx$

$$\Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + c$$

$$\Rightarrow x = 3, y = 9 \Rightarrow 9 = \frac{9}{2} - \frac{1}{3} + c \Rightarrow \frac{9}{2} + \frac{1}{3} = c$$

$$\Rightarrow c = \frac{27+2}{6} = \frac{29}{6} \Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$$

(33) (A), (34) (C), (35) (C).

$$\left(\frac{dy}{dx}\right) + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{4x^2}{1+x^2}} = e^{\ln(1+x^2)} = (1+x^2)$$

$$\therefore\ y(1+x^2) = \int 4x^2 dx = \frac{4x^3}{3} + C$$

passing through $(0, 0) \Rightarrow C = 0 \quad \therefore\ y = \frac{4x^3}{3(1+x^2)}$

$$\frac{dy}{dx} = \frac{4}{3}\left[\frac{(1+x^2)\,3x^2 - x^3.2x}{(1+x^2)^2}\right] = \frac{4}{3}\left[\frac{3x^2+x^4}{(1+x^2)^2}\right] = \frac{4x^2(3+x^2)}{3(1+x^2)^2}$$

Hence, $\frac{dy}{dx} > 0\ \forall\ x \neq 0,$

$\frac{dy}{dx} = 0$ at $x = 0$ and it does not change sign $\Rightarrow x = 0$ is the point of inflection.

$y = f(x)$ is increasing for all $x \in R$

$x \to \infty, y \to \infty, \quad x \to -\infty, y \to -\infty$

Area enclosed by $y = f^{-1}(x)$, x-axis and ordinate at x= 2/3.

$$A = \frac{2}{3} - \frac{4}{3}\int_0^1 \frac{x^3}{1+x^2}dx$$

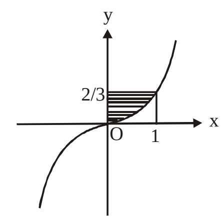
Put $1 + x^2 = t \Rightarrow 2x\,dx = dt$

$$A = \frac{2}{3} - \frac{2}{3}\int_1^2 \frac{(t-1)}{t}dt = \frac{2}{3} - \frac{2}{2}\int_1^2\left(1-\frac{1}{t}\right)dt$$

$$= \frac{2}{3} - \frac{2}{3}[t - \ln t]_1^2$$

$$= \frac{2}{3} - \frac{2}{3}[(2-\ln 2)-1]$$

$$= \frac{2}{3} - \frac{2}{3}[1-\ln 2] = \frac{2}{3}\ln 2$$



(36) **(A).** $f(0) = 2,$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[x\int_0^x f'(t)dt - \int_0^x \underset{I}{t}\,\underset{II}{\underbrace{f'(t)}}\,dt\right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - \left[xf(x) - f(0) - \left\{t.f(t)\Big|_0^x - \int_0^x f(t)dt\right\}\right]$$

$$f(x) = (e^x + e^{-x})\cos x - 2x - x\,f(x) + 2x + \left[x\,f(x) - \int_0^x f(t)dt\right]$$

$$f(x) = (e^x + e^{-x})\cos x - \int_0^x f(t)dt \qquad \ldots\ldots\ldots(1)$$

Differentiating eq. (1)

$f'(x) + f(x) = \cos x(e^x - e^{-x}) - (e^x + e^{-x})\sin x \ldots\ldots\ldots (2)$

Hence $\frac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

(37) **(D).** $f'(0) + f(0) = 0 - 2.0 = 0$

(38) **(C).** I.F. of DE (1) is e^x.

$$y.e^x = \int e^{2x}(\cos x - \sin x)\,dx - \int(\cos x + \sin x)\,dx$$

$$y.e^x = \int e^{2x}(\cos x - \sin x)\,dx - \int(\sin x - \cos x) + C$$

Let $I = \int e^{2x}(\cos x - \sin x)\,dx = e^{2x}(A\cos x + B\sin x)$

Solving A = 3/5 and B = –1/5 and C = 2/5

$\therefore\ y = e^x\left(\frac{3}{5}\cos x - \frac{1}{5}\sin x\right) - (\sin x - \cos x)\, e^{-x} + \frac{2}{5}e^{-x}$

(39) **(B).** $\frac{dx}{x} = \frac{y\,dy}{1+y^2} \Rightarrow \ln x = \frac{1}{2}.\ln(1+y^2) + c$

From the given condition c = 0

$\therefore\ x^2 - y^2 = 1$

(40) **(A).** $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0 \Rightarrow \frac{dy}{1+y^2} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \tan^{-1} y + \sin^{-1}x = c$

(41) $\frac{dy}{dx} = (1+x).(1+y)$ gives $y = e^{\frac{(1+x)^2}{2}} - 1$

(42) **2.** Clearly, we have $\left(1+\left(\frac{dy}{dx}\right)^2\right)^3 = \ell^2\left(\frac{d^2y}{dx^2}\right)^2$

Its degree is 2

[degree of the highest order derivative i.e. of $\frac{d^2y}{dx^2}$ is 2]

(43) **2.** Order = 2 [order of the highest order derivative i.e. $\frac{d^2y}{dx^2}$ is 2]

(44) **2.** Since the general equation of all conics whose axes coincide with the axes of co-ordinates is $ax^2 + by^2 = 1$ and

$\because$ it has two arbitrary constants a, b.

$\therefore$ its differential equation will be of order 2

(45) **5.** 2x dx = 3y dy, if c > 0

$x^2 = \frac{3y^2}{2} + c \Rightarrow \frac{x^2}{1} - \frac{3y^2}{2} = c \Rightarrow e = \sqrt{1+\frac{2}{3}} = \sqrt{\frac{5}{3}}$

If c < 0, $\frac{3y^2}{2} - \frac{x^2}{1} = -c \Rightarrow e = \sqrt{\frac{5}{2}}$

(46) **12.** $y = e^{4x} + 2e^{-x}$; $y_1 = 4e^{4x} - 2e^{-x}$

$y_2 = 16e^{4x} + 2e^{-x}$; $y_3 = 64e^{4x} - 2e^{-x}$

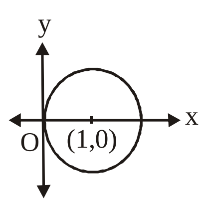
Now, $y_3 - 13y_1 = (64e^{4x} - 2e^{-x}) - 13(4e^{4x} - 2e^{-x})$

$= 12e^{4x} + 24e^{-x}$

$= 12(e^{4x} + 2e^{-x}) = 12y$

$\therefore\ \frac{y_3 - 13y_1}{y} = 12$

(47) **1.** Equation of normal is



$Y - y = -\frac{1}{m}(X - x)$

$X + mY - (x + my) = 0$..... (1)

Perpendicular distance from (0, 0) to eq. (1) is

$\left|\frac{x+my}{\sqrt{1+m^2}}\right| = |y|$

$\Rightarrow (x+my)^2 = y^2(1+m^2) \Rightarrow x^2 + 2mxy = y^2$

$\Rightarrow m = \frac{y^2 - x^2}{2x} \Rightarrow 2xy\frac{dy}{dx} = y^2 - x^2$ .... (2)

Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore$ Equation (2) becomes $x\frac{dt}{dx} = t - x^2 \Rightarrow \frac{dt}{dx} - \frac{1}{x}t = -x$

$\therefore$ I.F. $= e^{-\int\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

Now general solution is given by

$t\left(\frac{1}{x}\right) = -x + C \Rightarrow y^2\left(\frac{1}{x}\right) = -x + C$

As (1, 1) satisfy it, so C = 2

$\Rightarrow y^2 = -x^2 + 2x \Rightarrow x^2 + y^2 - 2x = 0$

Hence, required area $= \frac{k\pi}{2}$ $\therefore k = 1$

(48) **3.** The given equation is

$y\,dx + y^2\,dy = x\,dy$; $x \in R, y > 0, y(1) = 1$

$\Rightarrow \frac{y\,dx - x\,dy}{y^2} + dy = 0 \Rightarrow \frac{d}{dx}\left(\frac{x}{y}\right) + dy = 0$

On integrating we get, $\frac{x}{y} + y = C$

$y(1) = 1 \Rightarrow 1 + 1 = C \Rightarrow C = 2$

$\therefore\ \frac{x}{y} + y = 2$

To find y (–3), putting x = –3 in above eq^n we get

$-\frac{3}{y} + y = 2 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow y = 3, -1$

But given that y > 0 $\therefore$ y = 3

(49) **9.** $y - y_1 = m(x - x_1)$

Put x = 0, to get y intercept

$y_1 - mx_1 = x_1^3$

$y_1 - x_1\frac{dy}{dx} = x_1^3$; $x\frac{dy}{dx} - y = -x^3$

$\frac{dy}{dx} - \frac{y}{x} = -x^2$; $e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$y \times \frac{1}{x} = \int -x^2 \times \frac{1}{x} dx$

$\frac{y}{x} = \int -x dx \Rightarrow \frac{y}{x} = -\frac{x^2}{2} + c \Rightarrow f(x) = -\frac{x^3}{2} + \frac{3}{2}x$

$\therefore$ f (–3) = 9

(50) **0.** $y'(x) + y(x) g'(x) = g(x) g'(x), y(0) = 0 \ \forall x \in R$

$\frac{d}{dx}(y(x)) + y(x) g'(x) = g(x) g'(x), \ g(0) = g(2) = 0$

I.F. $= e^{\int g'(x) dx = e^{g(x)}}$

$y(x) e^{g(x)} = \int e^{g(x)} g(x) g'(x) dx + c$

Let $g(x) = t$

$g'(x) dx = dt$

$y(x) e^{g(x)} = \int te^t dt = te^t - e^t + c$

$y(x) = (g(x) - 1) + c e^{-g(x)}$

Let x = 0, $y(0) = (g(0) - 1) + ce^{-g(0)}$

$0 = (0 - 1) + c \Rightarrow c = 1$

$y(x) = (g(x) - 1) + e^{-g(x)}$

$y(2) = (g(2) - 1) + e^{-g(2)}$

$y(2) = (0 - 1) + e^{-(0)} = -1 + 1 = 0$

EXERCISE-3

(1) **(A).** $(x^2 - y^2) dx + 2xy dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ .... (1) $\because$ It is homogeneous D.E.

$\therefore$ Put y = v x

$\frac{dy}{dx} = v + x.\frac{dv}{dx} \quad \therefore \ v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x.vx}$ [From (1)]

$= \frac{x^2(v^2 - 1)}{x^2.2v} \Rightarrow x.\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$

$= \frac{v^2 - 1 - 2v^2}{2v} = \frac{-v^2 - 1}{2v}$

$\Rightarrow \frac{-2v}{(v^2 + 1)} dv = \frac{dx}{x} \Rightarrow \int \frac{-2v}{(v^2 + 1)} dv = \int \frac{dx}{x}$

$= -\log(v^2 + 1) = \log x + \log c$

$\Rightarrow \log \frac{1}{v^2 + 1} = \log x \Rightarrow \frac{1}{v^2 + 1} = xc \Rightarrow \frac{1}{\frac{y^2}{x^2} + 1} = xc$

$\Rightarrow \frac{x^2}{y^2 + x^2} = xc$

$\Rightarrow x = c(y^2 + x^2) \Rightarrow \frac{x}{c} = x^2 + y^2 \Rightarrow x^2 + y^2 = kx$

(2) **(D).** $y = e^{cx} \Rightarrow \log y = cx \Rightarrow c = \frac{\log y}{x}$

$\frac{dy}{dx} = e^{cx}.c \ ; \quad y' = y.\frac{\log y}{x} \Rightarrow xy' = y \log y$

(3) **(A).** $\because \frac{dy}{dx} = \frac{y - 1}{x^2 + x} \Rightarrow \frac{1}{y - 1} dy = \frac{dx}{x^2 + x}$

$\Rightarrow \int \frac{1}{y - 1} dy = \int \frac{x + 1 - x}{x(x + 1)} dx$

$\Rightarrow \log(y - 1) = \int \frac{1}{x} - \frac{1}{x + 1} dx$

$\Rightarrow \log y - 1 = \log x - \log(x + 1) + \log c$

$\Rightarrow y - 1 = \frac{xc}{x + 1}$ ($\because$ curve passes through (1, 0) it will satisfy equation of the curve)

$\Rightarrow 0 - 1 = \frac{1.c}{1 + 1} \Rightarrow c = -2$

$\therefore \ y - 1 = \frac{-2x}{x + 1} \Rightarrow (y - 1)(x + 1) = -2x$

$\Rightarrow (y - 1)(x + 1) + 2x = 0$

(4) **(C).** Equation of family of all parabolas whose axis is x axis is $y^2 = 4a(x - A)$

$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$

$\Rightarrow y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx} = 0 \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\therefore$ degree is 1 and order 2.

(5) **(C).** $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$\frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$ this is linear in x.

I.F. $= e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

$\therefore$ soln. is $x.e^{\tan^{-1} y} = \int e^{\tan^{-1} y}.\frac{e^{\tan^{-1} y}}{1 + y^2} dy + c$

Put $\tan^{-1} y = z \Rightarrow \frac{1}{1 + y^2} dy = dz = \int e^{2z} dx + c = \frac{e^{2z}}{2} + c$

$= \frac{e^{2\tan^{-1} y}}{2} + c$

$2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$ {where 2c = k}

(6) $x^2+y^2-2ay=0 \Rightarrow a=\dfrac{x^2+y^2}{2y}$ ...... (1)

$2x+2y\,y'-2ay'=0 \Rightarrow 2x+y'(2y-2a)=0$

$\Rightarrow 2x+y'\left(2y-2\dfrac{(x^2+y^2)}{2y}\right)=0$

$\Rightarrow 2xy+y'[2y^2-x^2-y^2]=0 \Rightarrow 2xy+y(y^2-x^2)=0$

$\Rightarrow y'(x^2-y^2)=2xy$

(7) **(B).** $y\,dx+(x+x^2y)\,dy=0$; $y\,dx+x\,dy+x^2y\,dy=0$

$d(xy)+x^2y\,dy=0$; $\dfrac{d(xy)}{x^2y^2}+\dfrac{dy}{y}=0$

$\Rightarrow \int\dfrac{d(xy)}{x^2y^2}=-\int\dfrac{dy}{y}=\dfrac{-1}{xy}=-\log y+c$

$\Rightarrow -\dfrac{1}{xy}+\log y=c$

(8) **(C).** $y^2=2c(x+\sqrt{c})$ ...... (1)

$2y\dfrac{dy}{dx}=2c \Rightarrow c=yy_1$ ...... (2)

From (1), $y^2=2cx+2c^{3/2}$

$\Rightarrow y^2-2cx=2c^{3/2}$

$\Rightarrow y^2-2yy_1x=2(yy_1)^{3/2}$ [From (2)]

$\Rightarrow (y^2-2yy_1x)^2=4(yy_1)^3=4y^3y_1^{\,3}$

This is of order 1 and degree 3.

(9) **(C).** $x\dfrac{dy}{dx}=y(\log y-\log x+1)$

$\dfrac{dy}{dx}=\dfrac{y}{x}\left(\log\dfrac{y}{x}+1\right)$ ..... (1)

Put $\dfrac{y}{x}=z \Rightarrow y=xz \Rightarrow \dfrac{dy}{dx}=x\dfrac{dz}{dx}+z.1$

$\therefore\ z+x\dfrac{dz}{dx}=z(\log z+1)$ [from (1)]

$x\dfrac{dz}{dx}=z\log z$; $\int\dfrac{dz}{z\log z}=\int\dfrac{dx}{x}$

$\Rightarrow \log(\log z)=\log x+\log c$

$\Rightarrow \log z=cx \Rightarrow \log(y/x)=cx$

(10) **(C).** $Ax^2+By^2=1$

$2Ax+2By\dfrac{dy}{dx}=0$

Again diff. w.r.t. x we get $-A=By.\dfrac{d^2y}{dx^2}+B\dfrac{dy}{dx}+\dfrac{dy}{dx}$

$\Rightarrow -Ax=Bxy\dfrac{d^2y}{dx^2}+Bx\left(\dfrac{dy}{dx}\right)^2$

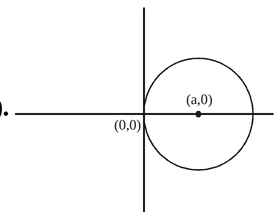
$\Rightarrow By\dfrac{dy}{dx}=Bxy\dfrac{d^2y}{dx^2}+Bx\left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow y\dfrac{dy}{dx}=xy\dfrac{d^2y}{dx^2}+x\left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow xy\dfrac{d^2y}{dx^2}+\left(x\dfrac{dy}{dx}-y\right)\dfrac{dy}{dx}=0$

this is of order 2 and degree 1.

(11) **(C).**



Equation of required circle is

$(x-a)^2+(y-0)^2=a^2$ ...... (1)

{$\because$ where (a, 0) is centre of circle and a is radius of circle}

Differentiating w.r.t. x

$2(x-a)+2yy_1=0$

$\Rightarrow x-a+yy_1=0 \Rightarrow a=x+yy_1$ .... (2)

Put this value in (1) we get

$(-yy_1)^2+y^2=(x+yy_1)^2$

$\Rightarrow y^2y_1^{\,2}+y^2=x^2+y^2y_1^{\,2}+2xyy_1$

$\Rightarrow y^2=x^2+2xyy_1=x^2+2xy\dfrac{dy}{dx}$ $\left\{\because y_1=\dfrac{dy}{dx}\right\}$

(12) **(B).** $\because$ Centre of circle lie on line y = 2

$\therefore$ Let coordinate of centre is (a, 2)

Radius of circle is 5 unit (given)

$\therefore$ equation of circle is $(x-a)^2+(y-2)^2=5^2$ .... (1)

Differentiating w.r.t. x

$2(x-a)+2(y-2)y_1=0$

$\Rightarrow (x-a)=-(y-2)y_1$ .... (2)

Put this value from (2) in (1) we get

$(-(y-2)y_1)^2+(y-2)^2=25$

$\Rightarrow (y-2)^2y_1^{\,2}+(y-2)^2=25$

$\Rightarrow (y-2)^2y_1^{\,2}=25-(y-2)^2$

(13) **(C).** $\dfrac{dy}{dx}=\dfrac{x+y}{x}$ .... (1)

Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx} \Rightarrow v+x\dfrac{dv}{dx}=\dfrac{x+vx}{x}$

$\Rightarrow v+x\dfrac{dv}{dx}=1+v \Rightarrow \int dv=\int\dfrac{dx}{x} \Rightarrow v=\log x+c$

$\Rightarrow \dfrac{y}{x}=\log x+c$ .... (2) $\because y(1)=1$

$\therefore \dfrac{1}{1}=\log 1+c \Rightarrow c=1$ $\therefore$ Solution is $\dfrac{y}{x}=\log x+1$

$\Rightarrow y=x\log x+x$

(14) **(D).** $y = c_1e^{c_2x} \Rightarrow y = c_1c_2e^{c_2x} = c_2y \Rightarrow \frac{y}{y'} = c_2$

Differentiating again we get, $y''y = (y')^2$

(15) **(D).** $\cos x\, dy = y(\sin x - y)\, dx$

$\frac{dy}{dx} = y\tan x - y^2\sec x$; $\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

Let $\frac{1}{y} = t$; $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$; $-\frac{dy}{dx} = -\tan x = -\sec x$

$\frac{dt}{dx} + (\tan x)\, t = \sec x$; I.F. $= e^{\int \tan x\, dx} = \sec x$

Solution is $t(\text{I.F.}) = \int (\text{I.F.}) \sec x\, dx$

(16) **(B).** $\frac{dV(t)}{dt} = -k(T - t)$; $\int dV(t) = \int (-kT)\, dt + \int kt\, dt$

$V(t) = -kTt + k\frac{t^2}{2} + c$

At $t = 0$, $c = I$

$V(T) = -kTt + \frac{kt^2}{2} + I$

Now, at $t = T$

$V(T) = -kT^2 + \frac{kT^2}{2} + I$; $V(T) = I - \frac{1}{2}kT^2$

(17) **(A).** $2\frac{dp(t)}{900 - p(t)} = -dt$

$-2\ln(900 - p(t)) = -t + c$

when $t = 0$, $p(0) = 850$

$-2\ln(50) = c$ $\quad\therefore\ 2\ln\left(\frac{50}{900 - p(t)}\right) = -t$

$900 - p(t) = 50\, e^{t/2}$

$p(t) = 900 - 50\, e^{t/2}$

let $p(t_1) = 0$

$0 = 900 - 50\, e^{t_1/2}$ $\quad\therefore t_1 = 2\ln 18$

(18) **(C).** $dP = (100 - 12\sqrt{x})\, dx$

By integrating, $\int dP = \int (100 - 12\sqrt{x})\, dx$

$P = 100x - 8x^{3/2} + C$

When $x = 0$ then $P = 2000 \Rightarrow C = 2000$

Now when $x = 25$ then P is

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000$

$= 2500 - 8 \times 125 + 2000 = 4500 - 1000$

$\Rightarrow P = 3500$

(19) **(A).** $\frac{dp}{dt} = \frac{p - 400}{2}$

$\frac{dp}{p - 400} = \frac{1}{2}dt$; $\ln|p - 400| = \frac{1}{2}t + c$

At $t = 0$, $p = 100$

$\ln 300 = c$

$\ln\left|\frac{p - 400}{300}\right| = \frac{t}{2} \Rightarrow |p - 400| = 300\, e^{t/2}$

$\Rightarrow 400 - p = 300\, e^{t/2}$ (as $p < 400$)

$\Rightarrow p = 400 - 300\, e^{t/2}$

(20) **(B).** It is best option. Theoretically question is wrong, because initial condition is not given.

$(x\log x)\frac{dy}{dx} + y = 2x\log x$

If $x = 1$ then $y = 0$

$\frac{dy}{dx} + \frac{y}{x\log x} = 2$; I.F. $= e^{\int \frac{1}{x\log x}dx} = e^{\log\log x} = \log x$

Solution is $y \,.\, \log x = \int 2\log x\, dx + c$

$y\log x = 2(x\log x - x) + c$

$x = 1$, $y = 0$. Then, $c = 2$, $y(e) = 2$

(21) **(C).** $y(1 + xy)\, dx = x\, dy$

$\frac{dy}{dx} = \frac{y}{x} + y^2 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = y^2$

Bernaulli's DE, $n = 2$

$\text{I.F} = \underset{e}{\int} (1-2)\left(-\frac{1}{x}\right) dx = \underset{e}{\int} \frac{1}{x}dx = x$,

Solution, $y^{1-2}x = \int (1-2)\cdot x \cdot 1\, dx$

$\frac{x}{y} = -\frac{x^2}{2} + C$. Given $f(1) = -1$

$\frac{1}{-1} = -\frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$

$\therefore$ Equation $\frac{x}{y} = -\frac{x^2}{2} - \frac{1}{2}$

When $x = -1/2$, we have

$-\frac{1}{2y} = -\frac{1}{4 \times 2} - \frac{1}{2} \Rightarrow -\frac{1}{y} = -\frac{5}{4} \Rightarrow y = \frac{4}{5}$

(22) **(C).** $\frac{dy}{dx} \equiv \frac{-(y+1)\cos x}{2 + \sin x}$; $\int \frac{dy}{y+1} = -\int \frac{\cos x}{2 + \sin x}dx$

$\ln(y + 1) = -\ln(2 + \sin x) + \ln c$

$(y + 1)(2 + \sin x) = c$; $y(0) = 1$

$(2)(2) = c \Rightarrow 4$; $y(\pi/2) = ?$

$(y + 1)(2 + 1) = 4$; $y = 1/3$

(23) **(A).** $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\, x$

$\Rightarrow d(y\sin x) = 4x\, dx$

Integrating both sides we get:

$y\sin x = 2x^2 + c$

Also, $y(\pi/2) = 0 \Rightarrow c = -\pi^2/2$

$$y\sin x = 2x^2 - \frac{\pi^2}{2} \Rightarrow y\left(\frac{\pi}{6}\right) = -\frac{8\pi^2}{9}$$

(24) **(C).** $\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x \Rightarrow \text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

$\therefore \quad yx^2 = \frac{x^4}{4} + \frac{3}{4}$ (As, y (1) = 1) $\therefore \quad y\left(x = \frac{1}{2}\right) = \frac{49}{16}$

(25) **(B).** $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore \quad \text{I.F.} = e^{\ln(x^2+1)} = x^2 + 1$

So, general solution is $y.(x^2+1) = \tan^{-1}x + c$

As $y(0) = 0 \Rightarrow c = 0$

$y(x) = \frac{\tan^{-1}x}{x^2+1}$. As $\sqrt{a}\cdot y(1) = \frac{\pi}{32}$

$\Rightarrow \quad \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

(26) **(D).** $x\frac{dy}{dx} + 2y = x^2 : y(1) = 1$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$ (LDE in y)

$\text{IF} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

$y\cdot(x^2) = \int x\cdot x^2 dx = \frac{x^4}{4} + C$

$y(1) = 1$

$1 = \frac{1}{4} + C \Rightarrow C = 1 - \frac{1}{4} = \frac{3}{4}$

$yx^2 = \frac{x^4}{4} + \frac{3}{4}$; $y = \frac{x^2}{4} + \frac{3}{4x^2}$

(27) **(C).** $\frac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\frac{\pi}{3}\right) = 0 ; y\left(\frac{\pi}{6}\right) = 7$

$e^{\int P\,dx} = e^{-\int \tan x\,dx} = e^{\ln\cos x} = \cos x$

$y . \cos x = \int 6x\sec x\cos x\,dx$

$y\cdot\cos x = \frac{6x^2}{2} + C$; $y = 3x^2\sec x + C\sec x$

$0 = 3\frac{\pi^2}{9}\cdot(2) + C(2)$; $2C = \frac{-2\pi^2}{3} \Rightarrow C = -\frac{\pi^2}{3}$

$y\left(\frac{\pi}{6}\right) = 3\frac{\pi^2}{36}\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{\sqrt{3}}\right)\cdot\left(-\frac{\pi^2}{3}\right)$; $y = -\frac{\pi^2}{2\sqrt{3}}$

(28) **(C).** $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

Now, put $\tan x = t \Rightarrow \frac{dt}{dx} = \sec^2 x$. So, $\frac{dy}{dx} + y = t$

On solving, we get $y\,e^t = e^t(t-1) + c$

$\Rightarrow \quad y = (\tan x - 1) + ce^{-\tan x} \Rightarrow y(0) = 0$; $c = 1$

$\Rightarrow \quad y = \tan x - 1 + e^{-\tan x}$. So, $y(-\pi/4) = e - 2$

(29) **(B).** $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$

$\text{I.F} = e^{\int \tan x dx} = e^{\ln.\sec x} = \sec x$

$y\cdot\sec x = \int(2x + x^2\tan x)\sec x\cdot dx$

$= \int 2x\sec x dx + \int x^2(\sec x\cdot\tan x)\,dx$

$y\sec x = x^2\sec x + \lambda \Rightarrow \quad y = x^2 + \lambda\cos x$

$y(0) = 0 + \lambda = 1 \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$; $y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$; $y'\left(\frac{-\pi}{4}\right) = \frac{-\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(30) **(D).** $y^2dx + xdy = \frac{dy}{y}$; $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

$\text{IF} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\cdot x = \int e^{-\frac{1}{y}}\cdot\frac{1}{y^3}dy + C$

$x\,e^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$; $C = -1/e$

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$ when $y = 2$

(31) **(A).** $xy\frac{dy}{dx} - y^2 + x^3 = 0$

Put $y^2 = k \Rightarrow y\frac{dy}{dx} = \frac{1}{2}\frac{dk}{dx}$

Given differential equation becomes

$\frac{dk}{dx} + k\left(-\frac{2}{x}\right) = -2x^2$. I.F. $= e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

Solution is $k \cdot \frac{1}{x^2} = \int -2x^2 \cdot \frac{1}{x^2} dx + \lambda$

$y^2 + 2x^3 = \lambda x^2$

Take $\lambda = -c$ (integration constant)

(32) **(C).** $e^y = t$

$e^y \frac{dy}{dx} = \frac{dt}{dx}$; $\frac{dt}{dx} - t = e^x$

$IF = e^{\int -1\,dx} = e^{-x}$; $t(e^{-x}) = \int e^x \cdot e^{-x} dx$

$e^{y-x} = x + c$

Put $x = 0, y = 0$ then $c = 1$

$e^{y-x} = x + 1$; $y = x + \ln(x+1)$

At $x = 1, y = 1 + \ln(2)$

(33) **(A).** $\frac{dx}{dy} + x = y^2$; $IF = e^{1.dy} = e^y$

$x \cdot e^y = \int y^2 \cdot e^y \cdot dy = y^2 \cdot e^y - \int 2y \cdot e^y \cdot dy$

$y^2e^y - 2(y.e^y - e^y) + c$

$x.e^y = y^2e^y - 2ye^y + 2e^y + C$

$x = y^2 - 2y + 2 + c.\, e^{-y}$

$x = 0, y = 1$

$0 = 1 - 2 + 2 + \frac{c}{e}$; $c = -e$

$y = 0, x = 0 - 0 + 2 + (-e)(e^{-0})$

$x = 2 - e$

(34) **(C).** $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}y = \cos^{-1}x$.

Hence, $y\left(\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

(35) **(B).** $2x = 4by' \Rightarrow b = \frac{x}{2y'}$

So, differential equation is

$$x^2 = \frac{2x}{y'} \cdot y + \left(\frac{x}{y'}\right)^2 \Rightarrow x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x$$

(36) **(3.00)**

$(x+1)\,dy - y\,dx = ((x+1)^2 - 3)dx$

$\Rightarrow \frac{(x+1)\,dy - y\,dx}{(x+1)^2} = \left(1 - \frac{3}{(x+1)^2}\right)dx$

$\Rightarrow d\left(\frac{y}{x+1}\right) = \left(1 - \frac{3}{(x+1)^2}\right)dx$

Integrating both sides

$\frac{y}{x+1} = x + \frac{3}{x+1} + C$

Given $y(2) = 0 \Rightarrow c = -3$

$\therefore\ y = (x+1)\left(x + \frac{3}{(x+1)} - 3\right)$ $\therefore y(3) = 3.00$

(37) **(D).** $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$; Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$; $v + x\frac{dv}{dx} = \frac{xvx}{x^2+v^2x^2} = \frac{v}{1+v^2}$

$x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{v - v - v^3}{1+v^2} = -\frac{v^3}{1+v^2}$

$\int \frac{1+v^2}{v^3}dv = \int -\frac{dx}{x} \Rightarrow \int v^{-3} \cdot dv + \int \frac{1}{v}dv = -\int \frac{dx}{x}$

$\Rightarrow \frac{v^2}{-2} + \ln v = -\ln x + \lambda$

$\Rightarrow -\frac{1}{2v^2} + \ln\left(\frac{y}{x}\right) = -\ln x + \lambda$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ln y - \ln x = -\ln x + \lambda$

$\Rightarrow -\frac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\frac{1}{2}$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ln y + \frac{1}{2} = 0$ at $y = e$

$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2} + 1 + \frac{1}{2} = 0 \Rightarrow \frac{x^2}{2e^2} = \frac{3}{2} \Rightarrow x^2 = 3e^2$

$\therefore\ x = \sqrt{3}e$