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BASIC MATHEMATICS AND LOGARITHM

SECTION - A : BASIC MATHEMATICS

RATIONAL NUMBERS (Q)

* All the numbers that can be represented in the form p/q, where p and q are integers and $q \neq 0$, are called rational numbers. Integers, Fractions, Terminating decimal numbers, Non-terminating but repeating decimal numbers are all rational numbers. $Q = \left\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\right\}$
* Integers are rational numbers, but converse need not be true.
* A rational number always exists between two distinct rational numbers, hence infinite rational numbers exist between two rational numbers.

IRRATIONAL NUMBERS (Q^C)

* There are real numbers which can not be expressed in p/q form. Non-Terminating non repeating decimal numbers are irrational number e.g. $\sqrt{2}, \sqrt{5}, \sqrt{3}, \sqrt[3]{10};\ e, \pi$.
  $e \approx 2.71$ is called Napier's constant and $\pi \approx 3.14$
* Sum of a rational number and an irrational number is an irrational number e.g. $2 + \sqrt{3}$
* If $a \in Q$ and $b \notin Q$, then ab = rational number, only if a = 0.
* Sum, difference, product and quotient of two irrational numbers need not be an irrational number or we can say, result may be a rational number also.

REAL NUMBERS (R)

* The complete set of rational and irrational number is the set of real numbers, $R = Q \cup Q^C$. The real numbers can be represented as a position of a point on the real number line.

COMPLEX NUMBERS (C)

* A number of the form a + ib, where $a, b \in R$ and $i = \sqrt{-1}$ is called a complex number. Complex number is usually denoted by z and the set of all complex numbers is represented by $C = \{(x + iy) : x, y \in R, i = \sqrt{-1}\}$

$$N \subset W \subset I \subset Q \subset R \subset C$$

EVEN NUMBERS

* Numbers divisible by 2, last digit 0, 2, 4, 6, 8 & represented by 2n.

ODD NUMBERS

* Not divisible by 2, last digit 1, 3, 5, 7, 9 represented by $(2n \pm 1)$

(a) even ± even = even
(b) even ± odd = odd
(c) odd ± odd = even
(d) even × any number = even number
(e) odd × odd = odd

PRIME NUMBERS

* Let 'p' be a natural number, 'p' is said to be prime if it has exactly two distinct positive integral factors, namely 1 and itself. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 .....

COMPOSITE NUMBERS

* A number that has more than two divisors

Note :

(i) '1' is neither prime nor composite.
(ii) '2' is the only even prime number.
(iii) '4' is the smallest composite number.
(iv) Natural numbers which are not prime are composite numbers (except 1)

CO-PRIME NUMBERS/ RELATIVELY PRIME NUMBERS :

* Two natural numbers (not necessarily prime) are coprime, if their H.C.F. is one. e.g. (1, 2), (1, 3), (3, 4) (5, 6) etc.

Note :

(i) Two prime number(s) are always co-prime but converse need not be true.
(ii) Consecutive natural numbers are always co-prime numbers.

TWIN PRIME NUMBERS

If the difference between two prime numbers is two, then the numbers are twin prime numbers.
e.g. {3, 5}, {5, 7}, {11, 13} etc.

NUMBERS TO REMEMBER

Number	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400
Cube	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000
Sq. Root	1.41	1.73	2	2.24	2.45	2.65	2.83	3	3.16										

Note :

(i) Square of a real number is always non negative (i.e. $x^2 \geq 0$)

(ii) Square root of a positive number is always positive e.g. $\sqrt{4} = 2$

(iii) $\sqrt{x^2} \neq \pm x$ but $\sqrt{x^2} = |x|$

DIVISIBILITY RULES :

Divisible by	Remark.
2	Last digit 0, 2, 4, 6, 8
3	Sum of digits divisible by 3 (Remainder will be same when number is divided by 3 or sum of digits is divided by 3.)
4	Last two digits divisible by 4 (Remainder will be same whether we divide the number or its last two digits)
5	Last digit 0 or 5
6	Divisible by 2 and 3 simultaneously.
8	Last three digits is divisible by 8 (Remainder will be same whether we divide the number or its last three digits)
9	Sum of digits divisible by 9. (Remainder will be same when number is divided by 9 or sum of digit is divided by 9)
10	Last digits 0
11	(Sum of digits at even places) – (sum of digits at odd places) = 0 or divisible by 11

LCM AND HCF

(a) HCF is the highest common factor between any two or more numbers or algebraic expressions.
When dealing only with numbers, it is also called "Greatest common divisor" (GCD).

(b) LCM is the lowest common multiple of two or more numbers or algebraic expressions.

(c) The product of HCF and LCM of two numbers (or expressions) is equal to the product of the numbers.

(d) LCM of $\left(\frac{a}{b}, \frac{p}{q}, \frac{l}{m}\right) = \frac{\text{L.C.M. of (a, p, l)}}{\text{H.C.F. of (b, q, m)}}$

SOME IMPORTANT IDENTITIES

(1) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$

(2) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

(3) $a^2 - b^2 = (a + b)(a - b)$

(4) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(5) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(6) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$

(7) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$

(8) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$= a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).$$

(9) $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}\left[(a - b)^2 + (b - c)^2 + (c - a)^2\right]$$

(10) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a + b + c)\left[(a - b)^2 + (b - c)^2 + (c - a)^2\right]$$

If $(a + b + c) = 0$, then $a^3 + b^3 + c^3 = 3abc$.

(11) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a - b)(a + b)$

(12) If $a, b \geq 0$ then $(a - b) = (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

(13) $a^4 + a^2 + 1 = (a^4 + 2a^2 + 1) - a^2 = (a^2 + 1)^2 - a^2$
$= (a^2 + a + 1)(a^2 - a + 1)$

CYCLIC FACTORS

* If an expression remain same after replacing a by b, b by c & c by a, then it is called cyclic expression and its factors are called cyclic factors. e.g. $a(b - c) + b(c - a) + c(a - b)$

REMAINDER THEOREM

* Let $p(x)$ be any polynomial of degree greater than or equal to one and 'a' be any real number. If $p(x)$ is divided by $(x - a)$, then the remainder is equal to $p(a)$.

FACTOR THEOREM

* Let $p(x)$ be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$. Conversely, if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

RATIO AND PROPORTION

(a) If $\frac{a}{b} = \frac{c}{d}$, then : $\frac{a+b}{b} = \frac{c+d}{d}$ (componendo);

$\frac{a-b}{b} = \frac{c-d}{d}$ (dividendo);

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo and dividendo);

$\frac{a}{c} = \frac{b}{d}$ (alternendo); $\frac{b}{a} = \frac{d}{c}$ (invertendo)

(b) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each ratio $= \left(\frac{a^n + c^n + e^n}{b^n + d^n + f^n}\right)^{\frac{1}{n}}$

Example : $\frac{a}{b} = \frac{c}{d} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

INDICES

* The product of m factors each equal to a is represented by a^m. So, $a^m = a \cdot a \cdot a .... a$ (m times). Here a is called the base and m is the index (or power or exponent).

Law of Indices :

(1) $a^{m+n} = a^m \cdot a^n$, where m and n are rational numbers.

(2) $a^{-m} = \frac{1}{a^m}$, provided $a \neq 0$.

(3) $a^0 = 1$, provided $a \neq 0$.

(4) $a^{m-n} = \frac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$.

(5) $(a^m)^n = a^{mn}$.

(6) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

(7) $(ab)^n = a^n b^n$.

INTERVALS

* Intervals are basically subsets of R (the set of all real numbers) and are commonly used in solving inequaltities. If $a, b \in R$ such that $a < b$, then we can defined four types of intervals as follows :

Name	Representation	Description
Open interval	(a, b)	$\{x : a < x < b\}$ i.e., end points are not included.
Close interval	[a, b]	$\{x : a \le x \le b\}$ i.e., end points are also included. This is possible only when both a and b are finite.
Open-closed interval	(a, b]	$\{x : a < x \le b\}$ i.e., a is excluded and b is included.
Closed-open interval	[a, b)	$\{x : a \le x < b\}$ i.e., a is included and b is excluded.

Note :

(1) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x \mid x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(-\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = \{x : x \in R\}$

(2) If their is no value of x, then we say $x \in \phi$ (i.e., null set or void set or empty set).

SECTION - B : LOGARITHM

Definition : Every positive real number N can be expressed in exponential form as

$N = a^x$ ....(1) e.g. $49 = 7^2$

where 'a' is also a positive real different than unity and is called the base and 'x' is called the exponent.

We can write the relation (1) in logarithmic form as

$\log_a N = x$ ....(2)

Hence the two relations

$a^x = N$

and $\log_a N = x$

are identical where $N > 0, a > 0, a \neq 1$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number. Logarithm of zero does not exist and logarithm of (–) ve reals are not defined in the system of real numbers.

i.e a is raised what power to get N.

Example 1 : Find value of

(i) $\log_{81} 27$ (ii) $\log_{10} 100$ (iii) $\log_{1/3} 9\sqrt{3}$

Sol. (i) Let $\log_{81} 27 = x$

$\Rightarrow 27 = 81^x \Rightarrow 3^3 = 3^{4x}$ gives $x = 3/4$

(ii) Let $\log_{10} 100 = x$

$\Rightarrow 100 = 10^x \Rightarrow 10^2 = 10^x$ gives $x = 2$

(iii) Let $\log_{1/3} 9\sqrt{3} = x \Rightarrow 9\sqrt{3} = \left(\frac{1}{3}\right)^x$

$\Rightarrow 3^{5/2} = 3^{-x}$ gives $x = -5/2$

Note :

* Unity has been excluded from the base of the logarithm as in this case

$\log_1 N$ will not be possible and if $N = 1$

then $\log_1 1$ will have infinitely many solutions and will not be unique, which is necessary in the functional notation.

Three fundamental identities :

Using the basic definition of log we have 3 important deductions :

(i) $\log_N N = 1$

i.e. logarithm of a number to the same base is 1.

(ii) $\log_{\frac{1}{N}} N = -1$

i.e. logarithm of a number to its reciprocal is – 1.

(iii) $\log_a 1 = 0$

i.e. logarithm of unity to any base is zero.

(basic constraints on number and base must be observed.)

Note :

(a) $a^{\log_a N} = N$ is an identify for all $N > 0$ and $a > 0, a \neq 1$ e.g. $2^{\log_2 5} = 5$

(b) The number N in (2) is called the antilog of 'x' to the base 'a'. Hence, If $\log_2 512$ is 9 then $\text{antilog}_2 9$ is equal to $2^9 = 512$.

(c) Whenever the number and base are on the same side of unity then logarithm of that number to the base is (+ve), however if the number and base are located on different sides of unity then logarithm of that number to the base is (–ve) e.g. (i) $\log_{10} 100 = 2$, (ii) $\log_{1/10} 100 = -2$

(d) For a non negative number 'a' & $n \ge 2, n \in N$ $\sqrt[n]{a} = a^{1/n}$

Example 2 : Evaluate the following :

(i) $\log_{\sin 30^\circ} \cos 60^\circ$ (ii) $\log_{3/4} 1.\overline{3}$.

(iii) $\log_5 \sqrt{5\sqrt{5\sqrt{5 ......\infty}}}$

(iv) (log tan 1°) (log tan 2°) (log tan 3°) ....(log tan 89°)

Sol. (i) $\log_{\sin 30^\circ} \cos 60^\circ = \log_{\frac{1}{2}} \left(\frac{1}{2}\right) = 1$

(ii) $\log_{3/4} 1.\overline{3} = \log_{\frac{3}{4}} \left(\frac{4}{3}\right) = -1$

(iii) Let $\sqrt{5\sqrt{5\sqrt{5.....\infty}}} = x$

$\Rightarrow \sqrt{5x} = x \Rightarrow x^2 = 5x \Rightarrow x = 5 \Rightarrow \log_5 5 = 1$

(iv) Since tan 45° = 1 thus log tan 45° = 0

Example 3 :

Solve for x :

(i) $7^{\log_7 x} + 2x + 9 = 0$

(ii) $2^{\log_2(x-3)} + 2(x-3) - 12 = 0$

Sol. (i) $3x + 9 = 0 \Rightarrow (x = -3)$ as it makes initial problem undefined $x = \phi$

(ii) $x - 3 + 2x - 6 - 12 = 0$

$3x = 21 \Rightarrow x = 7$

PROPERTIES OF LOGARITHM

If m, n are arbitrary positive real numbers where $a > 0$; $a \neq 1$

(1) $\log_a m + \log_a n = \log_a mn$ $(m > 0, n > 0)$

(2) $\log_a \dfrac{m}{n} = \log_a m - \log_a n$

(3) $\log_a m^x = x \log_a m$

Example 4 :

Find the solution of $\log_2 x^2 = 4$ & $2\log_2 x = 4$ and verify solutions.

Sol. $\log_2 x^2 = 4$ | $2\log_2 x = 4$

$\Rightarrow x^2 = 16$ | $\Rightarrow \log_2 x = 2$

$\Rightarrow x = \pm 4$ | $\Rightarrow x = 4$

(two solution) | only possible soln.

BASE CHANGING THEOREM

Can be stated as "quotient of the logarithm of two numbers is independent of their common base."

Symbolically, $\dfrac{\log_c a}{\log_c b} = \log_b a$

e.g. Find value of $\log_{64} 16$

$\log_{64} 16 = \dfrac{\log_4 16}{\log_4 64} = \dfrac{2}{3}$

Case-I : $\log_b a = \dfrac{1}{\log_a b}$

Case-II : $(\log_b a).(\log_c b).(\log_d c) = \log_d a$

Case-III : $a^{\log_b c} = c^{\log_b a}$ (very Imp.)

$a^{\log_b c} = a^{(\log_b c)\,(\log_a c)\,(\log_c a)}$

$= a^{\log_a c\,(\log_b c.\,\log_c a)} = c^{\log_b a}$

$\Rightarrow \boxed{a^{\log_b c} = c^{\log_b a}}$

Case-IV : $\log_{a^x} m = \dfrac{1}{x}\log_a m$

Example 5 :

If $(\log_2 3).(\log_3 4).(\log_4 5)....\log_n(n+1) = 10$, then find the value of n.

Sol. $\log_2(n+1) = 10$

$n + 1 = 1024$; $n = 1023$

Example 6 :

Find $\log 2 + 16 \log \dfrac{16}{15} + 12 \log \dfrac{25}{24} + 7 \log \dfrac{81}{80}$.

Sol. $\log 2 + 16 \log 16 - 16 \log 15 + 12 \log 25 - 12 \log 24 + 7 \log 81 - 7 \log 80$

$= \log 2 + 64 \log 2 - 16 \log 5 - 16 \log 3 + 24 \log 5 - 12 \times 3 \log 2 - 12 \log 3 + 28 \log 3 - 7 \log 5 - 28 \log 2$

$= \log 2 + \log 5 = \log 10 = 1$

Example 7 :

Find the value of x : $3^{\log_3^2 x} + x^{\log_3 x} = 162$

Sol. $(3^{\log_3 x})^{\log_3 x} + x^{\log_3 x} = 162$

$\Rightarrow x^{\log_3 x} + x^{\log_3 x} = 162$ $\left[a^{\log_a x} = x\right]$

$\Rightarrow x^{\log_3 x} = 81$

Taking log both side with base 3

$\Rightarrow (\log_3 x)^2 = \log_3 81 = 4 \Rightarrow \log_3 x = \pm 2$

$\Rightarrow x = 9$ or $1/9$

Common and natural logarithm :

$\log_{10} N$ is referred as a common logarithm and $\log_e N$ is called as natural logarithm or logarithm of N to the base Napierian and is popularly written as ln N. Note that e is an irrational quantity lying between 2.7 to 2.8 which you will study later. **Note that** $e^{\ln x} = x$

Characteristic and Mantissa :

We observe that $\log_{10} 10 = 1$ and $\log_{10} 100 = 2$.

Hence logarithm of a number lying between 10 to 100 = 1 + a positive quantity

$\log_{10}(0.1) = -1$ and $\log_{10}(0.01) = -2$

hence log (a number between 0.01 to 0.1) = – 2 + a positive quantity

Hence the common logarithm of a number consists of two parts, integral and fractional, of which the integral part may be zero or an integer (+ve or –ve) and the fractional part, a decimal, less than one and always positive.

The integral part is called the <u>characteristic</u> and the decimal part is called the <u>mantissa</u>.

e.g. $\log_{10} 33.8 = 1.5289 \Rightarrow 33.8 = 10^{1.5289} = 10.10^{0.5289}$

$\log_{10} 0.338 = -1 + 0.5289 = \bar{1}.5289$

It should be noted that, if the characteristic of the logarithm of N is

1 ⇒ that N has two significant digits before decimal.

2 ⇒ that N has three significant digits before decimal.

(Hence number of significant digit in N = p + 1 if p is the non negative characteristic of log N.)
if characteristic
$-1 \Rightarrow$ N has no zero after decimal before a significant digit starts
$-2 \Rightarrow$ N has 1 zero after decimal before a significant digit starts and so on.
[$\log 2 = 0.3010$, $\log 3 = 0.4771$, and $\log 7 = 0.8451$]

Example 8 :
Find the number of digits $(2.5)^{200}$

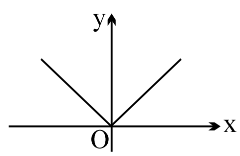
Sol. Let $N = (2.5)^{200}$
Taking log both side with base 10

$$\log_{10} N = 200 \log_{10}(2.5) = 200 \log_{10}(5/2)$$
$$= 200 [\log_{10} 5 - \log_{10} 2]$$
$$= 200 [1 - 2 \log_{10} 2]$$
$$= 200 [1 - 2 \times 0.3010] = 200 [0.3990] = 79.80$$

Characteristic = 79
Number of digits = 79 + 1 = 80

Absolute value function :

(a) $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



(b) $\sqrt{x^2} = |x|$

(c) $\log x^{2n} = 2n \log |x|$, where $n \in I$

General Note : Equations of the form
$[a(x)]^{b(x)} = [a(x)]^{c(x)}$ (Variable exponent on a variable base) with the set of permissible values defined by the condition $a(x) > 0$, can be reduced to the equivalent equation
$b(x) \log_d [a(x)] = c(x) \log_d [a(x)]$
by taking logarithms of its both sides. The last equation is equivalent to two equations.
$\log_d [a(x)] = 0$, $b(x) = c(x)$.

e.g. $|x-2|^{10x^2-1} = |x-2|^{3x}$

Taking log both side w.r.t. base 10

$$\log_{10} |x-2|^{10x^2-1} = \log |x-2|^{3x}$$

$\Rightarrow$ $(10x^2 - 1) \log |x-2| = 3x \log |x-2|$
$\Rightarrow$ $\log |x-2| (10x^2 - 3x - 1) = 0$
$\Rightarrow$ $\log |x-2| = 0$ or $10x^2 - 3x - 1 = 0$
$\Rightarrow$ $|x-2| = 1$ or $(2x-1)(5x+1) = 0$
$\Rightarrow$ $x - 2 = \pm 1$
$x = 3 ; 1$ $\quad$ $x = 1/2, -1/5$

Example 9 :

Solve for x : $|x-3|^{3x^2-10x+3} = 1$

Sol. Taking log both side with 10

$\Rightarrow$ $\log |x-3|^{3x^2-10x+3} = \log 1$

$\Rightarrow$ $(3x^2 - 10x + 3) \log_{10} |x-3| = 0$ $\quad$ $[x \neq 3]$
$\Rightarrow$ $(3x-1)(x-3) \log_{10} |x-3| = 0$
$\Rightarrow$ $x = 1/3$, $|x-3| = 1$
again when $x > 3 \Rightarrow x - 3 = 1$
$\Rightarrow$ $x = 4$ when $x < 3$
$-(x-3) = 1 \Rightarrow x = 2$
$\therefore$ $x = 1/3, 2, 4$

ADDITIONAL EXAMPLES

Example 1 :

Evaluate : $81^{1/\log_5 3} + 27^{\log_9 36} + 3^{4/\log_7 9}$

Sol. $81^{\log_3 5} + 3^{3\log_9 36} + 3^{4\log_9 7}$

$$= 3^{4\log_3 5} + 3^{\log_3 (36)^{3/2}} + 3^{\log_3 7^2} = 625 + 216 + 49 = 890.$$

Example 2 :
Find the value of x satisfying
$\log_{10}(2^x + x - 41) = x(1 - \log_{10} 5)$.

Sol. We have,
$\log_{10}(2^x + x - 41) = x(1 - \log_{10} 5)$
$\Rightarrow$ $\log_{10}(2^x + x - 41) = x \log_{10} 2 = \log_{10}(2^x)$
$\Rightarrow$ $2^x + x - 41 = 2^x \Rightarrow x = 41$

Example 3 :
For $0 < a \neq 1$, find the number of ordered pair (x, y) satisfying the equation $\log_{a^2} |x+y| = 1/2$

and $\log_a y - \log_a |x| = \log_{a^2} 4$.

Sol. We have $\log_{a^2} |x+y| = \frac{1}{2} \Rightarrow |x+y| = a$

$\Rightarrow$ $x + y = \pm a$ $\quad$ .....(1)

Also, $\log_a \left(\frac{y}{|x|}\right) = \log_{a^2} 4 \Rightarrow y = 2|x|$ $\quad$ .....(2)

If $x > 0$, then $x = \frac{a}{3}$, $y = \frac{2a}{3}$

If $x < 0$, then $y = 2a$, $x = -a$

Possible ordered pairs $= \left(\frac{a}{3}, \frac{2a}{3}\right)$ and $(-a, 2a)$

Example 4 :
Solve following log equation

$3^{\log_3 \log \sqrt{x}} - \log x + \log^2 x - 3 = 0$

Sol. $\log \sqrt{x} - \log x + \log^2 x - 3 = 0$

$\Rightarrow$ $\frac{1}{2} \log_{10} x - \log_{10} x + (\log_{10} x)^2 - 3 = 0$

$\Rightarrow$ $\log_{10} x - 2 \log_{10} x + 2(\log_{10} x)^2 - 6 = 0$
$\Rightarrow$ $2(\log_{10} x)^2 - \log_{10} x - 6 = 0$

$\Rightarrow\ 2(\log_{10}x)^2 - 4\log_{10}x + 3\log_{10}x - 6 = 0$
$\Rightarrow\ (2\log_{10}x + 3)(\log_{10}x - 2) = 0$
$\Rightarrow\ \log_{10}x = 2$ or $\log_{10}x = -3/2$
$\Rightarrow\ x = 10^2 = 100$ or $x = 10^{-3/2}$

Example 5 :

Solve the value of x : $2(\log_x\sqrt{5})^2 - 3\log_x\sqrt{5} + 1 = 0$

Sol. $\dfrac{2}{(\log_{\sqrt{5}} x)^2} - \dfrac{3}{\log_{\sqrt{5}} x} + 1 = 0$

$\Rightarrow\ \log^2_{\sqrt{5}} x - 2\log_{\sqrt{5}} x - \log_{\sqrt{5}} x + 2 = 0$

$\Rightarrow\ \log_{\sqrt{5}} x(\log_{\sqrt{5}} x - 2) - (\log_{\sqrt{5}} x - 2) = 0$

$\Rightarrow\ (\log_{\sqrt{5}} x - 1)(\log_{\sqrt{5}} x - 2) = 0$

$\Rightarrow\ \log_{\sqrt{5}} x = 1$ or $\log_{\sqrt{5}} x = 2$

$x = \sqrt{5}$ or $x = (\sqrt{5})^2 = 5$

QUESTION BANK

EXERCISE

Q.1 A circle has a radius of $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. The value of $\log_a b$ is equal to

(A) $\dfrac{1}{4\pi}$ (B) $\dfrac{1}{\pi}$
(C) π (D) 2π

Q.2 If $\log_{10}\sin x + \log_{10}\cos x = -1$ and

$\log_{10}(\sin x + \cos x) = \dfrac{(\log_{10} n) - 1}{2}$ then the value of 'n' is

(A) 24 (B) 36
(C) 20 (D) 12

Q.3 The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$
(C) $a^2 - a + 1$ (D) $a^2 + a + 1$

Q.4 The number of values of x satisfying the equation

$2^{\log_5 16\cdot\log_4 x + \log_{x_{\sqrt{2}}} 5} + 5^x + x^{\log_3 4 + 5} + x^5 = 0$ is :

(A) 0 (B) 1
(C) 2 (D) 3

Q.5 The number $N = 6\log_{10}2 + \log_{10}31$, lies between two successive integers whose sum is equal to

(A) 5 (B) 7
(C) 9 (D) 10

Q.6 If $2^a = 7^b$ then number of ordered pairs (a, b) of real numbers is

(A) zero (B) one
(C) two (D) more than 2

Q.7 The number $2^{2\log_2(3^{3\log_3 4})}$ simplifies as :

(A) 2^{12} (B) 2^{16}
(C) 2^{24} (D) 2^{72}

Q.8 If $\log_2\log_3\log_4\log_5 A = x$, then the value of A is

(A) 120^x (B) 2^{60x}

(C) $2^{3^{4^{5^x}}}$ (D) $5^{4^{3^{2^x}}}$

Q.9 Suppose $\log_a 2 = m$, $\log_a 3 = r$, $\log_a 5 = s$ and $\log_a 11 = t$. The value of $\log_a 990$, is

(A) 2mrst (B) m + 2r + s + t
(C) m + r + s + t (D) m + 2r + s + t

Q.10 $\dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc} + \dfrac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to -

(A) 1/2 (B) 1
(C) 2 (D) 4

Q.11 The value of

$6 + \log_{\frac{3}{2}}\left(\dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\ldots\ldots}}}\right)$ is

(A) 3 (B) 1
(C) 2 (D) 4

Q.12 The value of the expression, $\log_4\left(\dfrac{x^2}{4}\right) - 2\log_4(4x^4)$ when $x = -2$ is -

(A) – 6 (B) – 5
(C) – 4 (D) meaningless

Q.13 $\dfrac{1}{1 + \log_b a + \log_b c} + \dfrac{1}{1 + \log_c a + \log_c b} + \dfrac{1}{1 + \log_a b + \log_a c}$ is equal to-

(A) abc (B) $\dfrac{1}{abc}$
(C) 0 (D) 1

Q.14 Which one of the following denotes the greatest positive proper fraction ?

(A) $\left(\dfrac{1}{4}\right)^{\log_2 6}$ (B) $\left(\dfrac{1}{3}\right)^{\log_3 5}$

(C) $3^{-\log_3 2}$ (D) $8^{\left(\frac{1}{-\log_3 2}\right)}$

Q.15 If $p = \dfrac{s}{(1+k)^n}$, then n equals -

(A) $\log \dfrac{s}{p(1+k)}$ (B) $\dfrac{\log (s/p)}{\log (1+k)}$

(C) $\dfrac{\log s}{\log p\,(1+k)}$ (D) $\dfrac{\log p\,(1+k)}{\log (s/p)}$

Q.16 Value of x satisfying

$\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = \log_{10}\sqrt{1-x^2} + 2$ is -

(A) $0 < x < 1$ (B) $-1 < x < 1$
(C) $-1 < x < 0$ (D) none of these

Q.17 The number of real solution of the equation $\log_{10}(7x-9)^2 + \log_{10}(3x-4)^2 = 2$ is -
(A) 1 (B) 2
(C) 3 (D) 4

Q.18 The equation, $\log_2(2x^2) + (\log_2 x).\,x^{\log_x(\log_2 x+1)}$

$+ \dfrac{1}{2}\log_4{}^2(x^4) + 2^{-3\log_{1/2}(\log_2 x)} = 1$ has -

(A) exactly one real solution (B) two real solutions
(C) 3 real solutions (D) no solution

Q.19 Given system of simultaneous equations $2^x . 5^y = 1$ and $5^{x+1} . 2^y = 2$. Then -
(A) $x = \log_{10}5$ and $y = \log_{10}2$
(B) $x = \log_{10}2$ and $y = \log_{10}5$
(C) $x = \log_{10}(1/5)$ and $y = \log_{10}2$
(D) $x = \log_{10}5$ and $y = \log_{10}(1/2)$

Q.20 The value of $3^{\log_4 5} + 4^{\log_5 3} - 5^{\log_4 3} - 3^{\log_5 4}$ is -
(A) 0 (B) 1
(C) 2 (D) none of these

Q.21 Given that $\log_p x = \alpha$ and $\log_q x = \beta$, then value of $\log_{p/q} x$ equals -

(A) $\dfrac{\alpha\beta}{\beta-\alpha}$ (B) $\dfrac{\beta-\alpha}{\alpha\beta}$

(C) $\dfrac{\alpha-\beta}{\alpha\beta}$ (D) $\dfrac{\alpha\beta}{\alpha-\beta}$

Q.22 $\log_A B$,where $B = \dfrac{12}{3+\sqrt{5}+\sqrt{8}}$ and

$A = \sqrt{1}+\sqrt{2}+\sqrt{5}-\sqrt{10}$ is -
(A) a negative integer
(B) a prime integer
(C) a positive integer
(D) an even–natural number

Q.23 If $\log_c 2 . \log_b 625 = \log_{10}16 . \log_c 10$ where $c > 0$; $c \neq 1$; $b > 1$; $b \neq 1$ determine b -
(A) 25 (B) 5
(C) 625 (D) 16

Q.24 Number of cyphers after decimal before a significant figure comes in $(5/3)^{-100}$ is -
(A) 21 (B) 22
(C) 23 (D) none

Q.25 The sum $\sqrt{\dfrac{5}{4}+\sqrt{\dfrac{3}{2}}} + \sqrt{\dfrac{5}{4}-\sqrt{\dfrac{3}{2}}}$ is equal to

(A) $\tan(\pi/3)$ (B) $\cot(\pi/3)$
(C) $\sec(\pi/3)$ (D) $\sin(\pi/3)$

Q.26 For $N > 1$, the product

$\dfrac{1}{\log_2 N} \cdot \dfrac{1}{\log_N 8} \cdot \dfrac{1}{\log_{32} N} \cdot \dfrac{1}{\log_N 128}$ simplifies to

(A) 3/7 (B) $\dfrac{3}{7\ln 2}$

(C) $\dfrac{3}{5\ln 2}$ (D) 5/21

Q.27 If p is the smallest value of x satisfying the equation

$2^x + \dfrac{15}{2^x} = 8$ then the value of 4^p is equal to

(A) 9 (B) 16
(C) 25 (D) 1

Q.28 The sum of two numbers a and b is $\sqrt{18}$ and their difference is $\sqrt{14}$. The value of $\log_b a$ is equal to
(A) – 1 (B) 2
(C) 1 (D) 1/2

Q.29 The value of the expression
$(\log_{10}2)^3 + \log_{10}8 \cdot \log_{10}5 + (\log_{10}5)^3$ is
(A) rational which is less than 1
(B) rational which is greater than 1
(C) equal to 1
(D) an irrational number

Q.30 Let $N = 10^{3\log 2 - 2\log\left(\log 10^3\right) + \log\left(\left(\log 10^6\right)^2\right)}$

where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to
(A) 2 (B) 3
(C) 4 (D) 5

Q.31 If $x = \dfrac{\sqrt{10}+\sqrt{2}}{2}$ and $y = \dfrac{\sqrt{10}-\sqrt{2}}{2}$, then the value of $\log_2(x^2 + xy + y^2)$, is equal to
(A) 0 (B) 2
(C) 3 (D) 4

Q.32 Suppose that $x < 0$. Which of the following is equal to

$\left| 2x - \sqrt{(x-2)^2} \right|$

(A) $x-2$ (B) $3x-2$
(C) $3x+2$ (D) $-3x+2$

Q.33 If $N = \left(2^{\log_{70} 9800}\right)\left(5^{\log_{70} 140}\right)\left(7^{\log_{70} 2}\right)$, then N is equal to

(A) 20 (B) 60
(C) 18 (D) 40

Q.34 The expression $q^{\left(\sqrt{6+\sqrt{6+\sqrt{6+......\infty}}}\right)^{\log_3 p}\left[\frac{\log_q(\log_q r)}{(\log_q p)}\right]}$ simplified to

(A) p (B) q
(C) r (D) 3

Q.35 If a, b are co-prime numbers and satisfying

$$\left(2+\sqrt{3}\right)^{\frac{1}{\log_a\left(2-\sqrt{3}\right)}+\frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}} = \frac{1}{12},$$

then (a + b) can be is equal to

(A) 13 (B) 5
(C) 7 (D) 8

Q.36 If $2^{(\log_2 3)^x} = 3^{(\log_3 2)^x}$ then the value of x is equal to

(A) 1/2 (B) 1/4
(C) 1/3 (D) 1/6

Q.37 The least value of the expression $2\log_{10}x - \log_x(0.01)$, for $x > 1$ is :

(A) 10 (B) 2
(C) –0.01 (D) None of these

Q.38 The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$ has :

(A) at least one real solution
(B) exactly three real solution
(C) exactly one irrational
(D) Complex roots

Q.39 The nuber of solution of $\log_4(x-1) = \log_2(x-3)$ is :

(A) 3 (B) 1
(C) 2 (D) 0

Q.40 Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$; $3^{\ln x} = 2^{\ln y}$. Then x_0 is

(A) 1/6 (B) 1/3
(C) 1/2 (D) 6

Paragraph for Q.41-Q.43

A denotes the product xyz where x, y and z satisfy

$\log_3 x = \log 5 - \log 7$
$\log_5 y = \log 7 - \log 3$
$\log_7 z = \log 3 - \log 5$

B denotes the sum of square of solution of the equation

$\log_2(\log_2 x^6 - 3) - \log_2(\log_2 x^4 - 5) = \log_2 3$

C denotes characterstic of logarithm

$\log_2(\log_2 3) - \log_2(\log_4 3) + \log_2(\log_4 5) - \log_2(\log_6 5) + \log_2(\log_6 7) - \log_2(\log_8 7)$

Q.41 Find value of A + B + C

(A) 18 (B) 34
(C) 32 (D) 24

Q.42 Find $\log_2 A + \log_2 B + \log_2 C$

(A) 5 (B) 6
(C) 7 (D) 4

Q.43 Find $|A - B + C|$

(A) – 30 (B) 32
(C) 28 (D) 30

Q.44 Match the column

Column-I

(a) The expression

$x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$ simplifies to

(b) The number

$N = 2^{(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 .... \cdot \log_{99} 100)}$ simplifies to

(c) The expression $\frac{1}{\log_5 3} + \frac{1}{\log_6 3} - \frac{1}{\log_{10} 3}$ simplifies to

(d) The number

$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ simplifies to

Column-II

(i) an integer (ii) a prime
(iii) a natural (iv) a composite

(A) (a) i, (b) i, iii, iv, (c) i, iii, (d) i, ii, iii
(B) (a) ii, (b) i, ii, iv, (c) iii (d) ii, iii
(C) (a) iii, (b) i, iv, (c) ii, iii, (d) i, ii
(D) (a) iv, (b) iii, iv, (c) i, iv, (d) i, iii

ANSWER KEY

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	D	D	A	B	D	A	D	D	B	D	A	D	C	B	D	B	D	C	A	A	C	B	B	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44						
A	D	A	A	C	B	C	D	D	C	C	A	D	B	B	C	B	A	D	A						

EXERCISE

(1) **(C).** $C = 4\log_{10}b = 2\pi r$

$\therefore\ 4\log_{10}b = 2\pi \cdot 2\log_{10}a$ (as $r = 2\log_{10}a$)

$\dfrac{\log_{10} b}{\log_{10} a} = \pi \quad \therefore \log_a b = \pi$

(2) **(D).** Given $\log_{10}\left(\dfrac{\sin 2x}{2}\right) = -1 \Rightarrow \dfrac{\sin 2x}{2} = \dfrac{1}{10}$

$\Rightarrow\ \sin 2x = 1/5$ ....(1)

Also, $\log_{10}(\sin x + \cos x) = \dfrac{\log_{10}\left(\dfrac{n}{10}\right)}{2}$

$\Rightarrow\ \log_{10}(\sin x + \cos x)^2 = \log_{10}\left(\dfrac{n}{10}\right)$

$\Rightarrow\ 1 + \sin 2x = \dfrac{n}{10} \Rightarrow 1 + \dfrac{1}{5} = \dfrac{n}{10} \Rightarrow \dfrac{6}{5} = \dfrac{n}{10}$

$\Rightarrow\ n = 12$

(3) **(D).** $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$

$= \dfrac{2^{4\log_2 a} - 3^{3\log_{3^3}(a^2+1)}}{7^{4\log_{7^2} a} - a - 1}$

$= \dfrac{a^4 - a^2 - 2a - 1}{a^2 - a - 1}$

(4) **(A).** $2^{\log_5 16\cdot\log_4 x + \log_{x\sqrt{2}} 5} + 5^x + x^{\log_3 4 + 5} + x^5 = 0$

$2^{2\log_5 4\cdot\log_4 x + x\log_2 5} + 5^x + x^{\log_5 4}\cdot x^5 + x^5 = 0$

$2^{2\log_5 x}\, 2^{x\log_2 5} + 5^x + x^{2\log_5 2}\cdot x^5 + x^5 = 0$

$(2^{\log_5 x})^2\cdot 5^x + 5^x + (2^{\log_5 x})^2\cdot x^5 + x^5 = 0$

$5^x\left[(2^{\log_5 x})^2 + 1\right] + x^5\left[(2^{\log_5 x})^2 + 1\right] = 0$

$(5^x + x^5)\left[(2^{\log_5 x})^2 + 1\right] = 0$

$5^x + x^5 = 0$

$(2^{\log_5 x})^2 + 1 = 0$

This possible only when x will be –ve. No solution while according to question $x \geq 2$

$\therefore$ Number of values of x = zero.

(5) **(B).** $N = \log_{10}64 + \log_{10}31 = \log_{10}1984$

$\therefore\ 3 < N < 4 \Rightarrow 7$

(6) **(D).** $2^a = 7^b \Rightarrow a = b = 0$ if a and b are integers

In case a and b are not integers then

$2^{\log_2 7} = 7^b \Rightarrow a = \log_2 7$ and $b = 1$

or $2^{\log_2 49} = 7^b \Rightarrow a = \log_2 49$ and $b = 2$

or $2^a = 7^{\log_7 2}$

$\Rightarrow\ a = 1$ and $b = \log_7 2$

$\therefore$ Infinite solutions.

(7) **(A).** $2^{2\log_2\left(3^{3\log_3 4}\right)} = 2^{2\log_2\left(3^{\log_3 4^3}\right)}$

$= 2^{2\log_2\left(4^3\right)} = 2^{\log_2\left(4^3\right)^2} = 2^{\log_2\left((2)^2\right)^6} = 2^{12}$.

(8) **(D).** $\log_3\log_4\log_5 A = 2^x \Rightarrow \log_4\log_5 A = 3^{2^x}$

$\Rightarrow\ \log_5 A = 4^{3^{2^x}} \Rightarrow A = 5^{4^{3^{2^x}}}$

(9) **(D).** $a^m = 2;\ a^r = 3;\ a^s = 5$ and $a^t = 11$

$\log_a 990 = \log_a 11 + 2\log_a 3 + \log_a 2 + \log_a 5$

$= t + 2r + m + s.$

(10) **(B).**

(11) **(D).** Let

$x = \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\ \ldots\ldots}}}$

$x = \dfrac{1}{3\sqrt{2}}\sqrt{4 - x} \Rightarrow 18x^2 = 4 - x \Rightarrow 18x^2 + x - 4 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 18\times 4\times 4}}{36} = \dfrac{-1 \pm 17}{36} = \dfrac{16}{36} = \dfrac{4}{9} = \left(\dfrac{2}{3}\right)^2$

Hence, $6 + \log_{\frac{3}{2}}\left(\dfrac{2}{3}\right)^2 = 6 - 2 = 4$

(12) **(A).**

(13) **(D).** $\dfrac{\log b}{\log b + \log a + \log c} + \dfrac{\log c}{\log c + \log a + \log b}$

$+ \dfrac{\log a}{\log c + \log a + \log b} = 1$

(14) **(C).**

(15) **(B).** $(1+k)^n = \dfrac{s}{p} \Rightarrow n\log(1 + k) = \log(s/p)$

$\Rightarrow\ n = \dfrac{\log s/p}{\log(1+k)}$

(16) **(D).** $1 + x > 0,\ 1 - x > 0,\ 1 - x^2 > 0,\ x \neq 0$

(17) **(B).** $(7x - 9)^2(3x - 4)^2 = 100$

$\Rightarrow (21x^2 - 55x + 36)^2 = 100$

$\Rightarrow\ 21x^2 - 55x + 36 = \pm 10$

$21x^2 - 55x + 26 = 0$

$x = \dfrac{55 \pm \sqrt{3025 - 2184}}{42} = \dfrac{55 \pm 29}{42} = 2,\ \dfrac{13}{21}$

only two real solution

(18) **(D).** Let $\log_2 x = y$

$\Rightarrow 1 + 2y + y^2 + y + 2y^2 + y^3 = 1$

$\Rightarrow y(y^2 + 3y + 3) = 0$

$\Rightarrow y = 0$ or $y^2 + 3y + 3 = 0$

$\Rightarrow \log_2 x = 0$ or $D < 0$ no real solution

$\Rightarrow x = 1$ (which is not in domain as x is in the base in one term)

(19) **(C).** Take log on both sides of equation & solve the equation simultaneously.

(20) **(A).** Use $a^{\log_b c} = c^{\log_b a}$

$\Rightarrow 3^{\log_4 5} + 4^{\log_5 3} - 3^{\log_4 5} - 4^{\log_5 3} = 0$

(21) **(A).**

$$\frac{\log x}{\log \frac{p}{q}} = \frac{\log x}{\log p - \log q} = \frac{1}{\frac{\log p}{\log x} - \frac{\log q}{\log x}} = \frac{1}{\log_x p - \log_x q}$$

$$= \frac{1}{\frac{1}{\alpha} - \frac{1}{\beta}} = \frac{\alpha\beta}{\beta - \alpha}$$

(22) **(C).** $B = \frac{12}{3+\sqrt{5}+\sqrt{8}} = \left(\frac{12(3+\sqrt{5}-\sqrt{8})}{(3+\sqrt{5})^2 - 8}\right)$

$$= \frac{12(3+\sqrt{5}-\sqrt{8})}{6+6\sqrt{5}} = \left(\frac{2(3+\sqrt{5}-2\sqrt{2})}{1+\sqrt{5}}\right)$$

$$= \frac{6+2\sqrt{5}}{\sqrt{5}+1} - \frac{4\sqrt{2}}{\sqrt{5}+1}$$

$$= \frac{(\sqrt{5}+1)^2}{\sqrt{5}+1} - \frac{4\sqrt{2}(\sqrt{5}-1)}{4} = \sqrt{5}+1-\sqrt{10}+\sqrt{2} = A$$

$\Rightarrow \log_A B = 1$

(23) **(B).** $(\log_c 2)(\log_b 625) = (\log_{10} 16)(\log_c 10)$

$\Rightarrow \frac{\log_c 2}{\log_c 10} \times \log_b 625 = \log_{10} 16$

$\Rightarrow \log_{10} 2 \times \log_b 625 = \log_{10} 2^4$

$\Rightarrow \log_{10} 2 \times \log_b 625 = 4 \log_{10} 2$

$\Rightarrow \log_b 625 = 4$

$\Rightarrow b^4 = 625 \Rightarrow b^4 = 5^4 \Rightarrow b = 5$

(24) **(B).** $x = \left(\frac{5}{3}\right)^{-100} \Rightarrow \log_{10} x = -100(\log 5 - \log 3)$

$= -100\left(\log_{10} 10 - \log_{10} 2 - \log_{10} 3\right)$

$= -100(1 - .3010 - .4771)$

$= -22.19 = \overline{23}.81$ hence 0's $= 23 - 1 = 22$

(25) **(A).** Let $x = \sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}} + \sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}}$

$\Rightarrow x^2 = \frac{5}{2} + 2\sqrt{\frac{25}{16} - \frac{3}{2}} = \frac{5}{2} + 2\cdot\frac{1}{4} = 3$

$\Rightarrow x = \sqrt{3} = \tan\frac{\pi}{3}$.

Alternative :

Let $S = \sqrt{\frac{5}{4}+\frac{\sqrt{24}}{4}} + \sqrt{\frac{5}{4}-\frac{\sqrt{24}}{4}}$

$$= \frac{\sqrt{5+2\sqrt{6}} + \sqrt{5-2\sqrt{6}}}{2}$$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right) + \left(\sqrt{3}-\sqrt{2}\right)}{2} = \sqrt{3}$$

(26) **(D).** $\frac{1}{\log_2 N} \cdot \frac{1}{\log_N 8} \cdot \frac{1}{\log_{32} N} \cdot \frac{1}{\log_N 128}$

$$= \frac{\ln 2}{\ln N} \cdot \frac{\ln N}{3\ln 2} \cdot \frac{5\ln 2}{\ln N} \cdot \frac{\ln N}{7\ln 2} = \frac{5}{21}$$

(27) **(A).** We have,

$2^{2x} - 8 \cdot 2^x + 15 = 0 \Rightarrow (2^x - 3)(2^x - 5) = 0$

$\Rightarrow 2^x = 3$ or $2^x = 5$

Hence smallest x is obtained by equating $2^x = 3$

$\Rightarrow x = \log_2 3$. So, $p = \log_2 3$

Hence, $4^p = 2^{2\log_2 3} = 2^{\log_2 9} = 9$

(28) **(A).** We have, $a + b = \sqrt{18}$; $a - b = \sqrt{14}$

Squaring & subtract, we get $4ab = 4 \Rightarrow ab = 1$

Hence number are reciprocal of each other

$\Rightarrow \log_b a = -1$.

(29) **(C).** $\log_{10} 2 = a$ and $\log_{10} 5 = b$

$\Rightarrow a + b = 1;\ a^3 + 3ab + b^3 = ?$

Now $(a + b)^3 = 1 \Rightarrow a^3 + b^3 + 3ab = 1$

(30) **(B).** $N = 10^p$; $p = \log_{10} 8 - \log_{10} 9 + 2\log_{10} 6$

$p = \log\left(\frac{8.36}{9}\right) = \log_{10} 32$

$\therefore N = 10^{\log_{10} 32} = 32$

Hence characteristic of $\log_3 32$ is 3.

(31) **(C).** $\log_2\left((x+y)^2 - xy\right)$

but $x + y = \sqrt{10}$; $x - y = \sqrt{2}$;

$xy = \frac{10-2}{4} = 2$

$\log_2(10-2) = \log_2 8 = 3$

(32) **(D).** $y = \left|2x - |x-2|\right| = \left|2x - (2-x)\right| = |3x-2|$ as $x<0$

hence $y = 2 - 3x$

(33) **(D).** $N = \left(2^{\log_{70}\left((70)^2\times 2\right)}\right)\left(5^{\log_{70}(70\times 2)}\right)\left(7^{\log_{70}2}\right)$

$= \left(2^{2+\log_{70}2}\right)\left(5^{1+\log_{70}2}\right)\left(7^{\log_{70}2}\right)$

$= 20\,(2\times 5\times 7)^{\log_{70}2} = 20\left(70^{\log_{70}2}\right) = 20\times 2 = 40.$

(34) **(C).** Clearly, $p^{\frac{\log_q(\log_q r)}{(\log_q p)}} = p^{\log_p(\log_q r)} = \log_q r$

and let $y = \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$, $y > 0$

$\Rightarrow$ $y = \sqrt{6+y} \Rightarrow y^2 = 6 + y$

$\Rightarrow$ $y^2 - y - 6 = 0 \Rightarrow (y-3)(y+2) = 0$

But $y > 0$, so $y = 3$.

$\therefore$ Given expression

$= q^{3^{\log_3(\log_q r)}} = q^{(\log_q r)} = r.$

(35) **(C).** As, $\dfrac{1}{\log_a\left(2-\sqrt{3}\right)} + \dfrac{1}{\log_b\left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$

$= \log_{2-\sqrt{3}} a + \log_{\frac{\sqrt{3}-1}{\sqrt{3}+1}} b$

$= \log_{2-\sqrt{3}} a + \log_{2-\sqrt{3}} b = \log_{2-\sqrt{3}}(ab)$

Now, $\left(2+\sqrt{3}\right)^{\log_{2-\sqrt{3}}(ab)} = \dfrac{1}{12}$

$\Rightarrow$ $\left(2-\sqrt{3}\right)^{\log_{2-\sqrt{3}}\left(\frac{1}{ab}\right)} = \dfrac{1}{12}$

$\Rightarrow$ $\dfrac{1}{ab} = \dfrac{1}{12}$ $\Rightarrow$ $ab = 12$

As a, b are co-prime numbers, so either a = 4, b = 3 or a = 3, b = 4. Hence , (a + b) = 7.

(36) **(A).** $2^{(\log_2 3)^x} = 3^{(\log_3 2)^x}$

Taking log to the base 2 on both the sides, we get

$(\log_2 3)^x \cdot \log_2 2 = (\log_3 2)^x \log_2 3$

$(\log_2 3)^{x-1} = (\log_3 2)^x \Rightarrow \dfrac{(\log_2 3)^{x-1}}{(\log_3 2)^x} = 1$

$(\log_2 3)^{2x-1} = 1 = (\log_2 3)^0$

$\Rightarrow$ $2x - 1 = 0 \Rightarrow x = 1/2$

(37) **(D).** $2\log_{10}x - \log_x(0.01)$

$\Rightarrow$ $2\log_{10}x - \log_x(10^{-2}) \Rightarrow 2\log_{10}x + 2\log_x 10$

$\Rightarrow$ $2(\log_{10}x + \log_x 10)$

$\Rightarrow$ $2\left(\log_{10}x + \dfrac{1}{\log_{10}x}\right) \Rightarrow 2\,(\geq 2) \geq 4$

So least value is 4.

(38) **(B).** $x^{\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4}} = \sqrt{2}$

Take log both side with base 2.

$\Rightarrow \left(\dfrac{3}{4}(\log_2 x)^2 + (\log_2 x) - \dfrac{5}{4}\right)\log_2 x = \log_2\sqrt{2}$

$\Rightarrow \dfrac{3}{4}(\log_2 x)^3 + (\log_2 x)^2 - \dfrac{5}{4}(\log_2 x) = +\dfrac{1}{2}$

$\Rightarrow 3(\log_2 x)^3 + 4(\log_2 x)^2 - 5(\log_2 x) - 2 = 0$

Let $(\log_2 x) = t$

$\Rightarrow$ $3t^3 + 4t^2 - 5t - 2 = 0$

$\Rightarrow$ $(t-1)(t+2)(3t+1) = 0$

$\Rightarrow$ $t = 1$ or -2 or $-1/3$ $\therefore$ $x = 2, \dfrac{1}{4}, \dfrac{1}{\sqrt[3]{2}}$

(39) **(B).** $\dfrac{1}{2}\log_2(x-1) = \log_2(x-3) \therefore \sqrt{x-1} = (x-3)$

$\Rightarrow$ $(x-1) = (x-3)^2 \Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow$ $x^2 - 7x + 10 = 0$

$x = 5$ or 2 $\therefore$ $x = 5$ is solution. $(x > 3)$

(40) **(C).** $(2x)^{\ln 2} = (3y)^{\ln 3}$ ... (1)

$3^{\ln x} = 2^{\ln y}$ ... (2)

Take log both side w.r.t e

$\ln 2 \ln(2x) = \ln 3 \ln(3y)$ ... (3)

or $\ln x \ln 3 = \ln y \ln 2$ ... (4)

by equation (4) put the vlaue of long y in equation (3), $\ln 2[\ln 2 + \ln x] = \ln 3[\ln 3 + \ln y]$

$\Rightarrow$ $\ln 2[\ln 2 + \ln x] = \ln 3\left[\ln 3 + \dfrac{\ln x \ln 3}{\ln 2}\right]$

$\Rightarrow$ $(\ln 2)^2[\ln 2 + \ln x] = (\ln 3)^2[\ln 2 + \ln x]$

$\Rightarrow$ $(\ln 2 + \ln x)[(\ln 2)^2 - (\ln 3)^2] = 0$

$\ln x = -\ln 2$

$\ln x = \ln 2^{-1}$

or $x = 1/2$ hence by equation (4)

$y = 1/3 \therefore x_0 = 1/2$

(41) **(B), (42) (A), (43) (D).**

$x = 3^{\log 5 - \log 7}$; $y = 5^{\log 7 - \log 3}$; $z = 7^{\log 3 - \log 5}$

$\therefore$ $x \cdot y \cdot z = 1 \therefore A = 1$

$\log_2(6\log_2|x| - 3) - \log_2(4\log|x| - 5) = \log_2 3$

$\dfrac{6\log_2|x| - 3}{4\log_2|x| - 5} = 3$ Let $\log_2|x| = 1$ $\therefore$ $\dfrac{6t-3}{4t-5} = 3$

$6t - 3 = 12t - 15$, $6t = 12$ $\therefore$ $t = 2$, $\log_2|x| = 2$,

$|x| = 4$ $\therefore x = \pm 4$

$\therefore$ $B = 16 + 16 = 32$

$\log_2(\log_2 3) + \log_2(\log_3 4) + \log_2(\log_4 5) + \log_2(\log_5 6) + \log_2(\log_6 7) + \log_2(\log_7 8)$

$= \log_2(\log_2 8) = \log_2 3$

$\therefore$ $C = 1$

(44) (A).

(a) Let $t = \sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$

$\therefore\ t = \sqrt{6+t}$

$\Rightarrow t^2 - t - 6 = 0 \Rightarrow t = 3, -2 \therefore t = 3 \quad (t > 0)$

$\therefore\ x = \log_2 \log_4 3 = \log_2 \log_{3^2} 3 = \log_2 \frac{1}{2} \Rightarrow -1$ (Integer)

(b) $\log_2 3.\log_3 4.\log_4 5.\log_5 6 .... \log_{99} 100$

$\Rightarrow \frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times ... \frac{\log 100}{\log 99}$

$\Rightarrow \frac{\log 100}{\log 2} \Rightarrow \log_2 100$

$\therefore\ N = 2^{\log_2 100} = 100$ (Integer, Natural, composite)

(c) $\frac{1}{\log_3 3} + \frac{1}{\log_6 3} - \frac{1}{\log_{10} 3} = ?$

$\Rightarrow \log_3 5 + \log_3 6 - \log_3 10$

$\Rightarrow \log_3\left(\frac{5\times 6}{10}\right) = \log_3 3 = 1$ (Integer, Natural)

(d) $N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$

$14 - 6\sqrt{5} = 9 + 5 - 6\sqrt{5} = (3)^2 + (\sqrt{5})^2 - 6\sqrt{5}$

$\Rightarrow (3 - \sqrt{5})^2$

$\therefore\ \sqrt{14-6\sqrt{5}} = 3 - \sqrt{5}$

$\therefore\ \sqrt{6-3\sqrt{5}+3-\sqrt{5}} = \sqrt{9-4\sqrt{5}}$

$\sqrt{(2)^2+(\sqrt{5})^2-4\sqrt{5}} = \sqrt{(\sqrt{5}-2)^2} \quad (\sqrt{5} > 2)$

$\sqrt{2+\sqrt{5}-\sqrt{5}+2} = 2$

(Integer, Prime, Natural)