10

CIRCLE

DEFINITION

Circle is locus of a point which moves at a constant distance from a fixed point. This constant distance is called radius of the circle and fixed point is called centre of the circle.

Basic geometrical concepts related to Circle :

(i) Equal chords subtends equal angles at the centre and vice-versa.

(ii) Equal chords of a circle are equidistant from the centre and vice-versa.

(iii) Angle subtended by an arc at the centre is double the angle subtended at any point on the remaining part of the circle.

(iv) Angles in the same segment of a circle are equal.

(v) The sum of the opposite angles of a cyclic quadrilateral is 180° and vice-versa.

(vi) If a line touches a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.

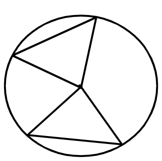
(vii) If two chords of a circle intersect either inside or outside the circle, the rectangle contained by the parts one chord is equal in area to the rectangle contained by the parts of the other. $AP \times PB = CP \times PD$

(viii) The greater of the two chords in a circle is nearer to the centre than lesser.

(ix) A chord drawn across the circular region divides it into parts each of which is called a segment of the circle.

(x) The tangents at the extremities of a chord of a circle are equal.

The angle between the tangents is bisected by the straight line, which joins their point of intersection to the centre. This straight line also bisects at right angles the chord, which joins the points where they touch the circle



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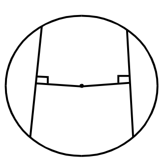

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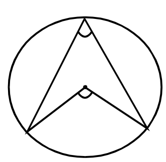

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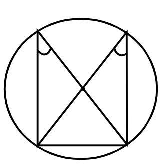

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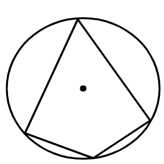

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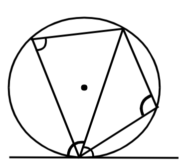

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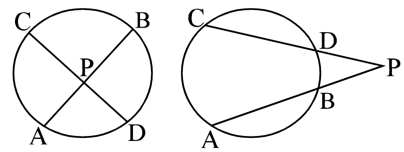

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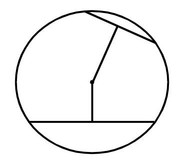

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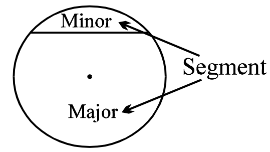

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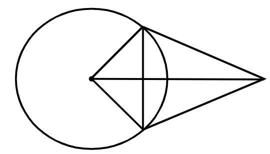


fig - (x)

STANDARD FORMS OF EQUATION OF A CIRCLE

(i) General Equation of a Circle : The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$, where g, f, c are constants. Centre of a general equation of a circle is $(-g, -f)$

i.e. $\left(-\frac{1}{2}\text{ coefficient of x}, -\frac{1}{2}\text{ coefficient of y}\right)$

Radius of a general equation of a circle is $\sqrt{g^2+f^2-c}$

The general equation of second degree $ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a circle if $a=b\neq 0$ and $h=0$.

General equation of a circle represents

(a) A real circle if $g^2+f^2-c>0$

(b) A point circle if $g^2+f^2-c=0$

(c) An imaginary circle if $g^2+f^2-c<0$

In General equation of a circle

(a) If $c=0 \Rightarrow$ The circle passes through origin

(b) If $f=0 \Rightarrow$ The centre is on x-axis

(c) If $g=0 \Rightarrow$ The centre is on y-axis

Example 1 :

If $y = 2x + k$ is a diameter to the circle $2(x^2+y^2)+3x+4y-1=0$, then find the value of k.

Sol. Centre of circle $= (-3/4, -1)$

this lies on, diameter $y = 2x + k$

$\Rightarrow -1 = -3/4 \times 2 + k \Rightarrow k = 1/2$

Example 2 :

If (4, –2) is the one extremity of diameter to the circle $x^2+y^2-4x+8y-4=0$ then find its other extremity.

Sol. Centre of circle is (2, – 4). Let the other extremity is (h, k)

$\therefore \left(\frac{4+h}{2}\right)=2, \left(\frac{-2+k}{2}\right)=-4 \Rightarrow (h, k)=(0, -6)$

(ii) Central Form of Equation of a Circle: The equation of a circle having centre (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2$$

If the centre is origin, then the equation of the circle is

$$x^2+y^2=r^2$$

If r = 0 than circle is called point circle and its equation is

$$(x-h)^2+(y-k)^2=0$$

Example 3 :

Find the equation of a circle with centre at the origin and which passes through the point (α, β).

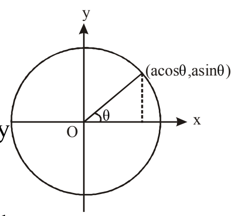
Sol. Here radius $=\sqrt{\alpha^2+\beta^2}$; so the required equation is

$$x^2+y^2=\alpha^2+\beta^2$$

(iii) Diameter form: If (x_1, y_1) and (x_2, y_2) be the extremities of a diameter, then the equation of the circle is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

(iv) Parametric Equation of a Circle :



(a) The Parametric equations of a circle $x^2+y^2=a^2$ are $x=a\cos\theta,\ y=a\sin\theta$. Hence parametric coordinates of any point lying on the circle $x^2+y^2=a^2$ are $(a\cos\theta, a\sin\theta)$

(b) The parametric equations of the circle $(x-h)^2+(y-k)^2=a^2$ are $x=h+a\cos\theta,\ y=k+a\sin\theta$

Hence parametric coordinates of any point lying on the circle are $(h+a\cos\theta, k+a\sin\theta)$

(c) Parametric equations of the circle $x^2+y^2+2gx+2fy+c=0$ is

$$x=-g+\sqrt{g^2+f^2-c}\cos\theta,$$

$$y=-f+\sqrt{g^2+f^2-c}\sin\theta$$

Example 4 :

Find the equation of circle if

(i) Centre is at origin & radius 3

(ii) Circle passes through origin & centre (1, 2)

(iii) Circle touchs x-axis & centre is (3, 2)

(iv) Circle touches the both the co-ordinates axes in first quadrant and radius = 3

(v) Circle passes through the origin centre lies on positive y-axis at (0, 3)

(vi) Circle is concentric with circle $x^2+y^2-8x+6y-5=0$ and passing through the point (–2, –7).

Sol. (i) Centre (0, 0) radius = 3

$$(x-0)^2+(y-0)^2=3^2\ ;\ x^2+y^2=9$$

(ii) Centre (1, 2) ; Radius $=\sqrt{(1-0)^2+(2-0)^2}=\sqrt{5}$

Equation of circle $(x-1)^2+(y-2)^2=(\sqrt{5})^2$

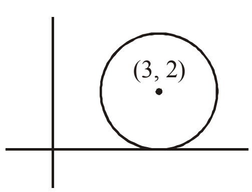
$$x^2+y^2-2x-4y=0$$

(iii) Centre (3, 2)

Circle touches x-axis

$$(x-3)^2+(y-2)^2=2^2$$

$$x^2+y^2-6x-4y+9=0$$



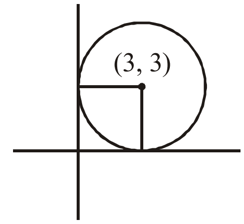
(iv) radius = 3

centre (3, 3)

Equation of circle

$$(x-3)^2+(y-3)^2=3^2$$

$$\Rightarrow x^2+y^2-6x-6y+9=0$$

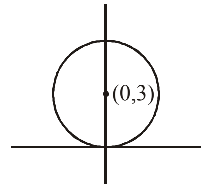


(v) Centre (0, 3)

radius = 3

$$(x-0)^2+(y-3)^2=3^2$$

$$x^2+y^2-6y=0$$



(vi) Centre (4, –3) & passes through (–2, –7)

Radius $=\sqrt{(4+2)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}$

Equation of circle $(x-4)^2+(y+3)^2=52$

$$x^2+y^2-8x+6y-27=0$$

Example 5 :

A line $\frac{x}{3}+\frac{y}{2}=1$ cuts the curve $y^2=8x$ at two distinct point A & B. Find the equation of circle taking A & B as extremities of diameter.

Sol. Equation of circle in diameteric form

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

where (x_1, y_1) & (x_2, y_2) are the extremities of diameter

Same equation can be as

$$x^2-(x_1+x_2)x+x_1x_2+y^2-(y_1+y_2)+y_1y_2=0 \quad \ldots(i)$$

Now, line $\frac{x}{3}+\frac{y}{2}=1 \Rightarrow 2x+3y=6 \quad \ldots(ii)$

intersect the curve $y^2=8x \quad \ldots(iii)$

$$\Rightarrow \left(\frac{6-2y}{3}\right)^2=8x \Rightarrow \left(\frac{3-x}{3}\right)^2=2x$$

$$\Rightarrow x^2-24x+9=0 \quad \ldots(iv)$$

$$x_1+x_2=24\ ;\ x_1x_2=9$$

Similarly $x=\frac{6-3y}{2}$ [From equation (ii)]

Now putting this value of x in equation (iii) we get quadratic in y. $y^2=8\left(\frac{6-3y}{2}\right) \Rightarrow y^2+12y-24=0 \quad \ldots(v)$

$$\Rightarrow y_1+y_2=12 \text{ or } y_1y_2=-24$$

Putting values in equation (i)

Equation of circle is

$$x^2-24x+9+y^2-12y-24=0$$

$$\Rightarrow x^2+y^2-24x-12y-15=0$$

Example 6 :

Find the Cartesian equation of the following curves whose parametric equations are :

(i) $x = 7 + 4\cos\alpha,\ y = -3 + 4\sin\alpha$

(ii) $x = \cos\theta + \sin\theta + 1,\ y = \sin\theta - \cos\theta + 2$

Sol. (i) Parametric equations of given curve are

$x = 7 + 4\cos\alpha$ ...(i)

$y = -3 + 4\sin\alpha$ ...(ii)

In order to find the Cartesian equation of the curve, we will have to eliminate parameter α.

From (i) $4\cos\alpha = x - 7$ ...(iii)

From (ii) $4\sin\alpha = y + 3$ ...(iv)

Squaring (iii) and (iv) and adding, we get

$$(x - 7)^2 + (y + 3)^2 = 4^2$$

(ii) Parametric equation of given curve are

$x = \cos\theta + \sin\theta + 1$ ...(i)

$y = \sin\theta - \cos\theta + 2$ ...(ii)

In order to find the Cartesian equation of the curve, we will have to eliminate the parameter θ,

From (i), $x - 1 = \cos\theta + \sin\theta$ ...(iii)

From (ii), $y - 2 = \sin\theta - \cos\theta$ ...(iv)

Squaring (iii) and (iv) and then adding, we get

$$(x - 1)^2 + (y - 2)^2 = 2$$

Example 7 :

Find the parametric coordinates of any point of the circle $x^2 + y^2 + 2x - 3y - 4 = 0$

Sol. Centre $= \left(-1, \frac{3}{2}\right)$, radius $= \sqrt{1 + \frac{4}{9} + 4} = \frac{7}{3}$

$\therefore$ Parametric coordinates of any point are

$$\left(-1 + \frac{7}{3}\cos\theta, \frac{3}{2} + \frac{7}{3}\sin\theta\right)$$

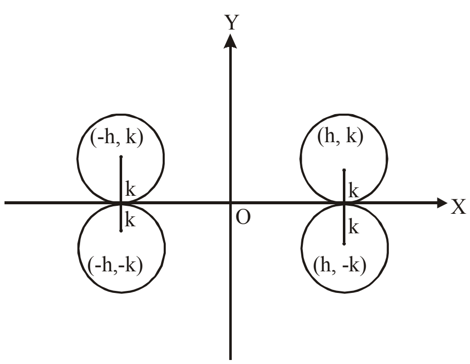
EQUATION OF A CIRCLE IN SOME SPECIAL CASES

(i) If centre of circle is (h, k) and passes through origin then its equation is

$$(x - h)^2 + (y - k)^2 = h^2 + k^2 \Rightarrow x^2 + y^2 - 2hx - 2ky = 0$$

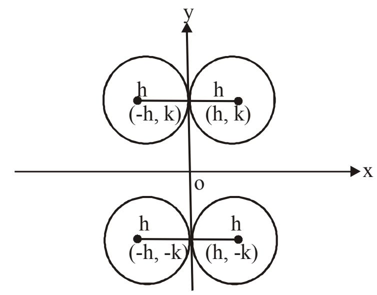
(ii) If the circle touches x– axis then its equation is (Four cases)

$$(x \pm h)^2 + (y \pm k)^2 = k^2$$

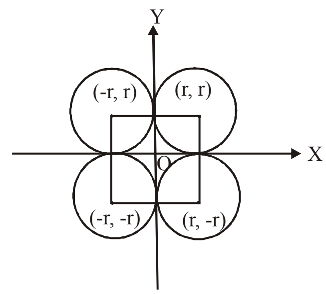


(iii) If the circle touches y axis then its equation s (Four cases)

$$(x \pm h)^2 + (y \pm k)^2 = h^2$$

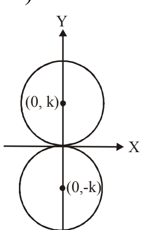


(iv) If the circle touches both the axis then its equation is (Four cases) $(x \pm r)^2 + (y \pm r)^2 = r^2$



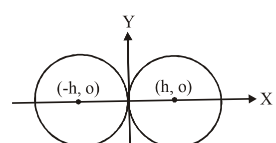
(v) If the circle touches x– axis at origin (Two cases)

$$x^2 + (y \pm k)^2 = k^2 \Rightarrow x^2 + y^2 \pm 2ky = 0$$



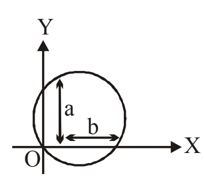
(vi) If the circle touches y axis at origin (Two cases)

$$(x \pm h)^2 + y^2 = h^2 \Rightarrow x^2 + y^2 \pm 2xh = 0$$



(vii) If the circle passes through origin and cut intercept of a and b on axes, the equation of circle is (Four cases)

$x^2 + y^2 - ax - by = 0$ and centre is (a/2, b/2)



POSITION OF A POINT WITH RESPECT TO A CIRCLE

A point (x_1, y_1) lies outside, on or inside a circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ according as

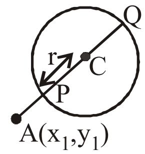
$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is positive, zero or negative

i.e. $S_1 > 0 \Rightarrow$ Point is outside the circle.

$S_1 = 0 \Rightarrow$ Point is on the circle.

$S_1 < 0 \Rightarrow$ Point is inside the circle.

The least and greatest distance of a point from a circle :
Let S = 0 be a circle and A (x_1, y_1) be a point. If the diameter of the circle which is passing through the circle at P and Q then AP = AC – r = least distance
AQ = AC + r = greatest distance
where 'r' is the radius and C is the centre of circle



POSITION OF A LINE WITH RESPECT TO A CIRCLE

Method - I : Let the equation of the circle be

$$x^2 + y^2 = a^2 \quad \ldots(i)$$

and the equation of the line be

$$y = mx + c \quad \ldots(ii)$$

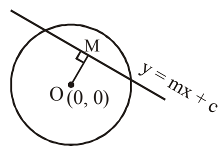
From (i) and (ii), $x^2 + (mx + c)^2 = a^2$

$$x^2(1 + m^2) + 2cmx + c^2 - a^2 = 0 \quad \ldots(iii)$$

Case-I : When points of intersection are real and distinct, then equation (iii) has two distinct roots.

$\therefore$ Discriminant > 0

or $4m^2c^2 - 4(1 + m^2)(c^2 - a^2) > 0$



or $a^2 > \dfrac{c^2}{1+m^2}$

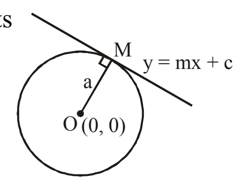
or $a > \dfrac{|c|}{\sqrt{(1+m^2)}}$ = length of perpendicular from (0, 0) to

y = mx + c $\Rightarrow$ a > length of perpendicular from (0, 0) to y = mx + c

Thus, a line intersects a given circle at two distinct points if radius of circle is greater than the length of perpendicular from centre of the circle to the line.

Case-II : When the points of intersection are coincident, the equation (iii) has two equal roots

$\therefore$ D = 0



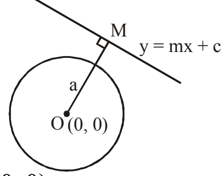
$\Rightarrow$ $a = \dfrac{|c|}{\sqrt{(1+m^2)}}$

a = length of the perpendicular from the point (0, 0) to y = mx + c

Thus, a line touches the circle if radius of circle is equal to the length of pependicular from centre of the circle to the line or called 'CONDITION OF TANGENCY'.

Case-III : When the points of intersection are imaginary. In this case (iii) has imaginary roots

$\therefore$ D < 0



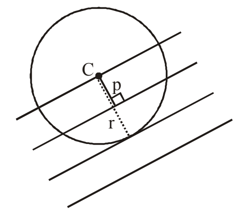
or $a < \dfrac{|c|}{\sqrt{1+m^2}}$

or a < length of perpendicular from (0, 0) to y = mx + c

Thus a line does not intersect a circle if the radius of circle is less than the length of perpendicular from centre of the circle to the line.

Method - II : Let $S = x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle and L = ax + by + c = 0 be a line.

Let r be the radius of the circle and p be the length of the perpendicular drawn from the centre (– g, – f) on the line L.

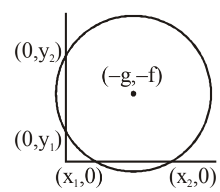


Then it can be seen easily from the figure that. If

(i) p < r $\Rightarrow$ the line intersects the circle in two distinct points.
(ii) p = r $\Rightarrow$ the line touches the circle, i.e. the line is a tangent to the circle.
(iii) p > r $\Rightarrow$ the line neither intersects nor touches the circle i.e., passes outside the circle.
(iv) p = 0 $\Rightarrow$ the line passes through the centre of the circle.

Intercepts made on coordinate axes by the circle:

Solving the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with y = 0 we get, $x^2 + 2gx + c = 0$. If discriminant $4(g^2 - c)$ is positive, i.e., if $g^2 > c$, the circle will meet the x-axis at two distinct points, say $(x_1, 0)$ and $(x_2, 0)$ where $x_1 + x_2 = -2g$ and $x_1x_2 = c$.
The intercept made on x-axis by the circle



$$\Rightarrow \quad |x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$$

Length of x intercept = $2\sqrt{g^2 - c}$

In the similar manner if $f^2 > c$,

Length of y intercept = $2\sqrt{f^2 - c}$

NOTE
(i) $g^2 - c > 0 \Rightarrow$ circle cuts the x-axis at two distinct points.
(ii) $g^2 = c \Rightarrow$ circle touches the x-axis.
(iii) $g^2 < c \Rightarrow$ circle lies completely above or below the x-axis i.e. it does not intersect x-axis.
(iv) $f^2 - c > 0 \Rightarrow$ circle cuts the y-axis at two distinct points.
(v) $f^2 = c \Rightarrow$ circle touches the y-axis.
(vi) $f^2 < c \Rightarrow$ circle lies completely on the right side or the left side of the y- axis i.e. it does not intersect y-axis.

Example 8 :
Find the length of intercept on y-axis, by a circle whose diameter is the line joining the points (–4, 3) and (12, – 1).

Sol. Here equation of the circle

$$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$$

or $x^2 + y^2 - 8x - 2y - 51 = 0$

Hence intercept on y -axis

$$= 2\sqrt{f^2 - c} = 2\sqrt{1-(-51)} = 4\sqrt{13}$$

Example 9 :

Find the equation of the circle which passes through the origin and makes intercepts of length a and b on the x and y axes respectively.

Sol. Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

Since the circle passes through the origin, we get c = 0 and given the intercepts on x and y axes a and b

then $2\sqrt{g^2 - c} = a$ or $2\sqrt{g^2 - 0} = a$

and $2\sqrt{f^2 - c} = b$ or $2\sqrt{f^2 - 0} = b$

$\therefore \quad f = \pm b/2$

Hence the equation of circle from (i) becomes

$$x^2 + y^2 \pm ax \pm by = 0$$

Example 10 :

For what value of "a" the point (a, a + 1) bounded by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ in the first quadrant.

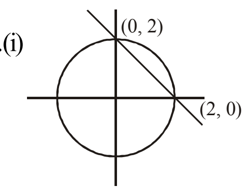
Sol. Equation of circle $x^2 + y^2 - 4 = 0$

Equation of line $x + y - 2 = 0$

Point (a, a + 1) and origin lies opposite sides with w.rt line $x + y - 2 = 0$, then $0 + 0 - 2 < 0$ therefore, $a + a + 1 - 2 > 0$

$2a - 1 > 0$

$a > 1/2 \quad \ldots(i)$



If point (a, a + 1) lies inside circle

$x^2 + y^2 - 4 = 0$

$a^2 + (a + 1)^2 - 4 = 0$

$2a^2 + 2a - 3 < 0$

$$\frac{-1-\sqrt{7}}{2} < a < \frac{-1+\sqrt{7}}{2} \quad \ldots(ii)$$

Using (i) & (ii) we get $a \in \left(\frac{1}{2}, \frac{\sqrt{7}-1}{2}\right)$

Example 11 :

Find the value of λ, such that line $2x - \lambda y + 7 = 0$ touches the circle $x^2 + y^2 + 6x + 2\lambda y + 5 + \lambda^2 = 0$. What if value of λ is equal to 3.

Sol. If line touches the circle, then perpendicular length from the centre of circle to line will be equal to radius.

centre $(-3, -\lambda)$, radius $= \sqrt{9 + \lambda^2 - 5 - \lambda^2} = 2$

So $\left|\frac{2(-3) + \lambda^2 + 7}{\sqrt{4 + \lambda^2}}\right| = 2 \Rightarrow \left|\frac{\lambda^2 + 1}{\sqrt{\lambda^2 + 4}}\right| = 2$

$\Rightarrow \quad (\lambda^2 + 1)^2 = 4(\lambda^2 + 4)$

Put $\lambda^2 + 1 = t \Rightarrow t^2 = 4(t + 3) \Rightarrow t^2 - 4t - 12 = 0$; $t = 6, -2$

$\lambda^2 + 1 = 6 \Rightarrow \lambda = \pm\sqrt{5}$

$\lambda^2 + 1 = -2 \Rightarrow$ No real value of λ

Value of λ will be $\sqrt{5}, -\sqrt{5}$

If λ = 3 then perpendicular distance will be $\frac{10}{\sqrt{13}}$

$\frac{10}{\sqrt{13}} > 2$ so line will neither touch nor cut the circle.

EQUATION OF TANGENT AND NORMAL

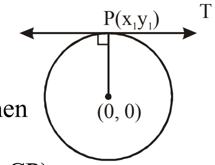
Equation of Tangent :

(A) Point form : Let $P(x_1, y_1)$ be the point on the circle

$$x^2 + y^2 = a^2 \quad \ldots(i)$$

Since C the centre of the circle has co-ordinates (0, 0),

therefore, slope of CP $= \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$



If m is the slope of the tangent at P then

$m(y_1/x_1) = -1 \quad (\because$ tangent is ⊥ CP)

or $\quad m = -x_1/y_1$

The equation of the tangent at $P(x_1, y_1)$ is

$$y - y_1 = -\frac{x_1}{y_1}(x - x_1)$$

or $\quad yy_1 - y_1^2 = -xx_1 + x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = a^2$

[$\because (x_1, y_1)$ lies on the circle $x^2 + y^2 = a^2 \quad \therefore x_1^2 + y_1^2 = a^2$]

Hence the equation of the tangent at (x_1, y_1) is

$$\mathbf{xx_1 + yy_1 = a^2 \quad or \quad T = 0}$$

(B) Slope form : Let the equation of circle is $x^2 + y^2 = a^2$ slope of tangent is m then, equation of tangent will be $y = mx + c$ when c is constant. Again if $y = mx + c$ is tangent for circle then apply the condition of tangency

$$\left|\frac{c}{\sqrt{1 + m^2}}\right| = a \quad \text{or} \quad c = \pm a\sqrt{1 + m^2}$$

Equation of tangent $\quad y = mx \pm a\sqrt{1 + m^2}$

(C) Parametric Form :

Let the equation of circle is $x^2 + y^2 = a^2$

Then equation of tangent for point (x_1, y_1) on circle is

$xx_1 + yy_1 = a^2$

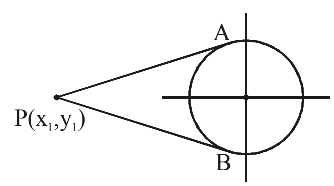
For parametric equation $x_1 = a\cos\theta$ and $y_1 = a\sin\theta$

$\therefore \quad x(a\cos\theta) + y(a\sin\theta) = a^2$

$x\cos\theta + y\sin\theta = a$

(D) Equation of Tangent From External Point :

Let the equation of circle is $x^2 + y^2 = a^2$



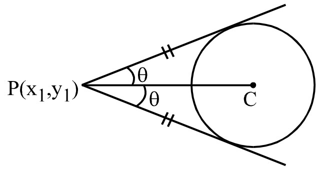
Let $P(x_1, y_1)$ is any external point for circle then equation of tangent will be $(y - y_1) = m(x - x_1)$

For m apply the condition of tangency get the two values of m

Note:

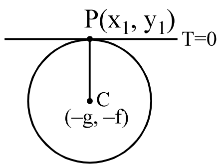
(i) For a unique value of m there will be 2 tangent which are parallel to each other.

(ii) From an external point 2 tangents can be drawn to the circle which are equal in length and are equally inclined to the line joining the point and the centre of the circle.



(iii) Equation of tangents drawn to any second degree circle at P (x_1, y_1) on it can be obtained by replacing.
$x^2 \to x x_1$; $y^2 \to y y_1$; $2x \to x + x_1$; $2y \to y + y_1$; $2xy \to xy_1 + yx_1$

(iv) **Point of Tangency :**
for P : either solve tangent and normal to get P



or compare the equation of tangent at (x_1, y_1) with the given tangent to get point of tangency.

Equation of Normal :

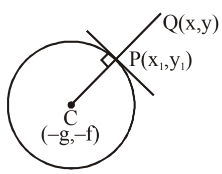
The normal to a circle at a point is defined as the straight line passing through the point and perpendicular to the tangent at that point.

Clearly every normal passes through the centre of the circle.

The equation of the normal to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

at any point (x_1, y_1) lying on the circle is



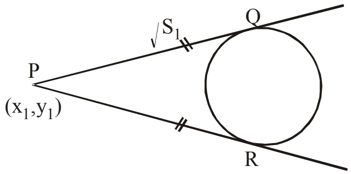
$$\boxed{\frac{y_1 + f}{x_1 + g} = \frac{y - y_1}{x - x_1}}$$

In particular, equation of the Normal to the circle

$x^2 + y^2 = a^2$ at (x_1, y_1) is $\boxed{\dfrac{y}{x} = \dfrac{y_1}{x_1}}$.

Length of Tangent :

From any point, say $P(x_1, y_1)$ two tangents can be drawn to a circle which are real, coincident or imaginary according as P lies outside, on or inside the circle.

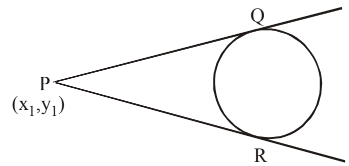


Let PQ and PR be two tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then PQ = PR is called the length of tangent drawn from point P and is given by

$$PQ = PR = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$$

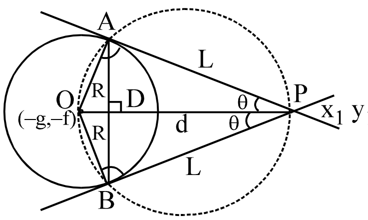
Pair of Tangents :

From a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$. Their combined equation is $SS_1 = T^2$. Where S = 0 is the equation of circle T = 0 is the equation of tangent at (x_1, y_1) and S_1 is obtained by replacing x by x_1 and y by y_1 in S.



Some important Deduction :

(i) Area of Quad PAOB $= 2\,\Delta\, POA = 2 \cdot \frac{1}{2} RL = RL$



(ii) AB i.e length of chord of contact $AB = 2L\sin\theta$

where $\tan\theta = \dfrac{R}{L} = \dfrac{2RL}{\sqrt{R^2 + L^2}}$

(iii) Area of Δ PAB (Δ formed by pair of Tangent & corresponding C.O.C.)

$$\Delta PAB = \frac{1}{2} AB \times PD = \frac{1}{2}(2L\sin\theta)(L\cos\theta) = L^2 \sin\theta\cos\theta$$

$$= \frac{RL^3}{R^2 + L^2}$$

(iv) Angle 2θ between the pair of Tangents

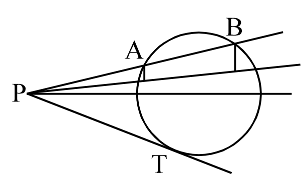
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2RL^2}{L(L^2 - R^2)}$$

$$2\theta = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$$

(v) **Power of a Point :** Square of the length of the tangent from the point P is called power of the point P w.r.t a given circle i.e. $PT^2 = S_1$

Power of a point remains constant w.r.t a circle

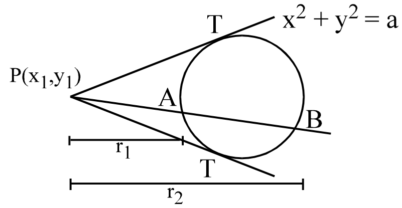
$$PA \cdot PB = (PT)^2$$



Analytical proof : $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

Substituting $x = x_1 + r\cos\theta$ and $y = y_1 + r\sin\theta$ in $x^2 + y^2 = a^2$,

we get, $r^2 + 2r(x_1\cos\theta + y_1\sin\theta) + x_1^2 + y_1^2 - a^2 = 0$



$r_1 r_2 = x_1^2 + y_1^2 - a^2 = \text{constant} = (PT)^2$

Note: Power of a point is + ve / 0 (zero) / – ve according as point 'P' lies outside / on / inside the circle.

Example 12 :

Find the equation of the tangents to the circle $x^2 + y^2 = 9$, which

(i) are parallel to the line $3x + 4y - 5 = 0$

(ii) are perpendicular to the line $2x + 3y + 7 = 0$

(iii) make an angle of 60º with the x-axis

Sol. (i) Let tangent parallel to $3x + 4y - 5 = 0$ is

$3x + 4y + \lambda = 0$ ...(1)

and circle $x^2 + y^2 = 9$

then perpendicular distance from (0, 0) to (1) = radius

$$\frac{|\lambda|}{\sqrt{(3^2+4^2)}} = 3 \text{ or } |\lambda| = 15 \quad \therefore \lambda = \pm 15$$

From (1), equations of tangents are $3x + 4y \pm 15 = 0$

(ii) Let tangent perpendicular to $2x + 3y + 7 = 0$ is

$3x - 2y + \lambda = 0$ ...(2)

and circle $x^2 + y^2 = 9$

then perpendicular distance from (0, 0) to (2) = radius

$$\frac{|\lambda|}{\sqrt{3^2+(-2)^2}} = 3 \text{ or } |\lambda| = 3\sqrt{13} \text{ or } \lambda = \pm 3\sqrt{13}$$

From (2), equations of tangents are

$3x - 2y \pm 3\sqrt{13} = 0$

(iii) Let equation of tangent which makes an angle of 60º with the x-axis is

$y = \sqrt{3}x + c$ ...(3)

or $\sqrt{3}x - y + c = 0$

and circle $x^2 + y^2 = 9$

then perpendicular distance from (0, 0) to (3) = radius

$$\frac{|c|}{\sqrt{(\sqrt{3})^2+(-1)^2}} = 3 \text{ or } |c| = 6 \text{ or } c = \pm 6$$

From (3), equations of tangents are $\sqrt{3}x - y \pm 6 = 0$

Example 13 :

If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$; then show that the line $\ell x + my + 1 = 0$ touches a fixed circle. Find the centre and radius of the circle.

Sol. Given, $4\ell^2 - 5m^2 + 6\ell + 1 = 0$ ...(i)

Given line is $\ell x + my + 1 = 0$ ...(ii)

If possible, let line (ii) touch the circle whose centre is (α, β)

and radius is a, then $\frac{|\ell\alpha + m\beta + 1|}{\sqrt{\ell^2 + m^2}} = a$

or $(\ell\alpha + m\beta + 1)^2 = a^2(\ell^2 + m^2)$

or $\ell\alpha^2 + m^2\beta^2 + 1 + 2\ell m\alpha\beta + 2\ell\alpha + 2m\beta = a^2\ell^2 + a^2m^2$

or $(\alpha^2 - a^2)\ell^2 + (\beta^2 - a^2)m^2 + 2\ell m\alpha\beta + 2\alpha\ell + 2\beta m + 1 = 0$ ...(iii)

Comparing (i) and (iii), we get

$\alpha^2 - a^2 = 4$ ...(iv), $\beta^2 - a^2 = -5$ ...(v)

$2\alpha = 6$ ...(vi), $2\beta = 0$ ...(vii)

and $2\alpha\beta = 0$ ...(viii)

From (vi), $\alpha = 3$ and from (vii), $\beta = 0$

Putting the value of α in (iv), we get

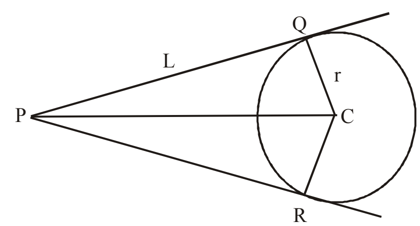
$a^2 = 3^2 - 4 = 5 \quad \therefore a = \sqrt{5}$

and the equation of circle is $(x-3)^2 + (y-0)^2 = 5$

Example 14 :

Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point P (16, 7). If the centre of the circle is C then find the area of quadrilateral PQCR.

Sol. Area PQCR = 2Δ PQC $= 2 \times \frac{1}{2} L \times r$



where L = length of tangent and r = radius of circle.

$L = \sqrt{S_1}$ and $r = \sqrt{1+4+20} = 5$

Hence the required area = 75 sq. units.

Example 15 :

A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. Then find equation of the pair of tangent.

Sol. Equation of pair of tangents is given by $SS_1 = T^2$,

or $S = x^2 + y^2 + 20(x + y) + 20$, $S_1 = 20$, $T = 10(x + y) + 20 = 0$

$\therefore SS_1 = T^2 \Rightarrow 20(x^2 + y^2 + 20(x + y) + 20) = 10^2(x + y + 2)^2$

$\Rightarrow 4x^2 + 4y^2 + 10xy = 0 \Rightarrow 2x^2 + 2y^2 + 5xy = 0$

TRY IT YOURSELF-1

Q.1 Equation of a circle which passes through (3, 6) and touches the axes is
(A) $x^2 + y^2 + 6x + 6y + 3 = 0$ (B) $x^2 + y^2 - 6x - 6y - 9 = 0$
(C) $x^2 + y^2 - 6x - 6y + 9 = 0$ (D) none of these

Q.2 The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is
(A) $x^2 + y^2 = 9a^2$ (B) $x^2 + y^2 = 16a^2$
(C) $x^2 + y^2 = 4a^2$ (D) $x^2 + y^2 = a^2$

Q.3 Find the radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Q.4 Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Q.5 Find the equation of the circle passing through the points (2,3) ; (–1,1) & whose centre is on the line $x - 3y - 11 = 0$.

Q.6 A circle is concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has area double of its area. The equation of the circle is
(A) $x^2 + y^2 - 6x + 12y - 15 = 0$
(B) $x^2 + y^2 - 6x + 12y + 15 = 0$
(C) $x^2 + y^2 - 6x + 12y + 15 = 0$
(D) None of these

Q.7 The equation of the tangent drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are
(A) $x = 0, y = 0$
(B) $(h^2 - r^2)\,x - 2rhy = 0, x = 0$
(C) $y = 0, x = 4$
(D) $(h^2 - r^2)\,x + 2\,rhy = 0, x = 0$

Q.8 A circle passes through (0, 0) and (1, 0) and touches the circle $x^2 + y^2 = 9$ then the centre of circle is –
(A) (3/2, 1/2) (B) (1/2, 3/2)
(C) (1/2, 1/2) (D) $(1/2, \pm\sqrt{2})$

Q.9 The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point –
(A) (–3/2, 0) (B) (–5/2, 2)
(C) (–3/2, 5/2) (D) (– 4, 0)

Q.10 If the tangent at the point P on the circle $x^2+y^2+ 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then length of PQ is :
(A) 4 (B) $2\sqrt{5}$
(C) 5 (D) $3\sqrt{5}$

Q.11 Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are)
(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$
(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

ANSWERS

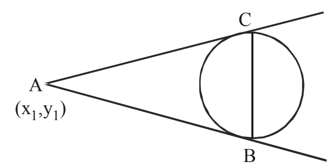
(1) (C)	**(2)** (C)	**(3)** $\sqrt{65}$
(4) Inside	**(5)** $x^2 + y^2 - 7x + 5y - 14 = 0$	
(6) (A)	**(7)** (D)	**(8)** (D)
(9) (D)	**(10)** (C)	**(11)** (AC)

DIRECTOR CIRCLE

The locus of the point of intersection of two perpendicular tangents to a circle is called the Director circle.
Let the circle be $x^2 + y^2 = a^2$, then equation of the pair of tangents to a circle from a point (x_1, y_1) is
$(x^2 + y^2 - a^2)\,(x_1^2 + y_1^2 - a^2) = (xx_1 + yy_1 - a^2)^2$.
If this represents a pair of perpendicular lines then coefficient of x^2 + coefficient of $y^2 = 0$
i.e. $(x_1^2 + y_1^2 - a^2 - x_1^2) + (x_1^2 + y_1^2 - a^2 - y_1^2) = 0$
$\Rightarrow\ x_1^2 + y_1^2 = 2a^2$
Hence the equation of director circle is $x^2 + y^2 = 2a^2$
Obviously director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the given circle.
Director circle of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
$$x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0.$$

CHORD OF CONTACT

The chord joining the two points of contact of tangents to a circle drawn from any point A is called chord of contact of A with respect to the given circle.



Let the given point is $A(x_1, y_1)$ and the circle is S = 0 then equation of the chord of contact is
$$T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

Parametric Form :

Consider the circle $x^2 + y^2 = a^2$ with its centre at the origin O and of radius 'a', then the equation of chord joining the two points whose parametric angles are α and β is
$$x\cos\frac{1}{2}(\alpha + \beta) + y\sin\frac{1}{2}(\alpha + \beta) = a\cos\frac{1}{2}(\alpha - \beta)$$

NOTE

(i) It is clear from the above that the equation to the chord of contact coincides with the equation of the tangent, if the point (x_1, y_1) lies on the circle.

(ii) The length of chord of contact $= 2\sqrt{r^2 - p^2}$

(iii) Area of Δ ABC is given by $\dfrac{a(x_1^2 + y_1^2 - a^2)^{3/2}}{x_1^2 + y_1^2}$

Example 16 :
Find the distance between the chord of contact with respect to point (0, 0) and (g, f) of circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Sol. Chord of contact with respect to (0, 0)
$gx + fy + c = 0$ ....(1)
chord of contact with respect to (g, f)
$gx + fy + g\,(x + g) + f\,(y + f) + c = 0$
$\Rightarrow 2gx + 2fy + g^2 + f^2 + c = 0$

$\Rightarrow gx + fy + \frac{1}{2}(g^2 + f^2 + c) = 0$ ....(2)

Distance between (1) and (2) is

$$= \frac{\frac{1}{2}(g^2+f^2+c) - c}{\sqrt{g^2+f^2}} = \frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$$

Example 17 :

A circle touches the line y = x at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle.

Sol. Equation of OP is y = x ...(i)

Let $P \equiv (h, h)$

Given, $OP = 4\sqrt{2}$ $\therefore h^2 + h^2 = 32$

or $h^2 = 16$ $\therefore h = \pm 4$

Thus $P \equiv (4, 4)$ or $(-4, -4)$

Let C (α, β) be the centre of the circle.

Case-I : When $P \equiv (4, 4)$:

Slope of CP $= \frac{\beta - 4}{\alpha - 4}$ and slope of OP = 1

Since CP ⊥ OP $\therefore \left(\frac{\beta-4}{\alpha-4}\right) \cdot 1 = -1$

or $\alpha + \beta = 8$ ...(ii)

Let a be the radius of the circle.

Then $CQ^2 + (3\sqrt{2})^2 = a^2$ $\therefore \left(\frac{\alpha+\beta}{\sqrt{2}}\right)^2 + 18 = a^2$

or $a^2 = \left(\frac{8}{\sqrt{2}}\right)^2 + 18 = 50$ $\therefore a = 5\sqrt{2}$ $[\because a > 0]$

Again, CP = a $\therefore \frac{|\alpha - \beta|}{\sqrt{2}} = a = 5\sqrt{2}$

or $|\alpha - \beta| = 10$ $\therefore \alpha - \beta = \pm 10$ ...(iii)

Solving (ii) and (iii), we get

$\alpha = 9, \beta = -1$ or $\alpha = -1, \beta = 9$

$\therefore C \equiv (9, -1)$ or $C \equiv (1, 9)$

If $H \equiv (-10, 2)$

When $C \equiv (9, -1)$, $CH^2 = 19^2 + (-3)^2 = 361 + 9 = 370 > a^2$

When $C \equiv (-1, 9)$, $CH^2 = 9^2 + 7^2 = 81 + 49 = 130 > a^2$

Since H lies inside the circle,

∴ Neither (9, –1) nor (–1, 9) is the centre of the circle.

Case-II : When $P \equiv (-4, -4)$:

Slope of CP $= \frac{\beta + 4}{\alpha + 4}$

Since CP ⊥ OP

$\therefore \left(\frac{\beta+4}{\alpha+4}\right) \cdot 1 = -1$ or $\alpha + \beta = -8$ ...(iv)

Again, $CQ^2 + (3\sqrt{2})^2 = a^2$

$= \left(\frac{-8}{\sqrt{2}}\right)^2 + 18 = a^2$ $\therefore a = 5\sqrt{2}$

Now, CP = a $\therefore \frac{|\alpha-\beta|}{\sqrt{2}} = 5\sqrt{2}$

or $\alpha - \beta = \pm 10$ ...(v)

Solving (iv) and (v), we get

$C \equiv (-9, 1)$ or $C \equiv (1, -9)$

When $C \equiv (-9, 1)$, $CH^2 = (1)^2 + (1-2)^2 = 2 < a^2$

When $C \equiv (1, -9)$, $CH^2 = (11)^2 + (-11)^2 = 242 > a^2$

Thus $C \equiv (-9, 1)$ and $a = 5\sqrt{2}$

Hence equation of the required circle is

$(x + 9)^2 + (y - 1)^2 = 50$

or $x^2 + y^2 + 18x - 2y + 32 = 0$

Example 18 :

Chord of contact of the tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Prove that a, b, c are in G.P.

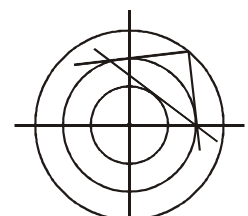
Sol. $x^2 + y^2 = a^2$

Let the point be (a cos θ, a sin θ)

Equation of chord of contact on $x^2 + y^2 = b^2$ is

$a\cos\theta \cdot x + b\sin\theta \cdot y = b^2$

Now if their line is tangent to $x^2 + y^2 = c^2$



Then $\left|\frac{b^2}{\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}}\right| = c$

$\frac{b^2}{a} = c \Rightarrow b^2 = ac$. Hence a, b, c are in G.P.

Example 19 :

Tangents are drawn to $x^2 + y^2 = 1$ from any arbitrary point P on the line 2x + y – 4 = 0. The corresponding chord of contact passes through a fixed point then find the coordinates.

Sol. Let any point on the line 2x + y – 4 = 0 be P ≡ (a, 4 – 2a).

Equation of chord of contact of the circle $x^2 + y^2 = 1$ with respect to point P is

$x \cdot a + y \cdot (4 - 2a) = 1 \Rightarrow (4y - 1) + a(x - 2y) = 0$

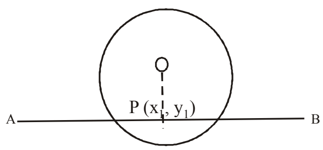
This line always passes through a point of intersection of the lines 4y – 1 = 0 and x – 2y = 0 which is fixed point whose coordinate are $y = \frac{1}{4}$ and $x = 2y = \frac{1}{2}$.

Hence coordinates are $\left(\frac{1}{2}, \frac{1}{4}\right)$

EQUATION OF A CHORD WHOSE MIDDLE POINT IS GIVEN:

The equation of the chord of the circle $x^2 + y^2 = a^2$ whose middle point $P(x_1, y_1)$ is given is

Slope of line OP $= \frac{y_1}{x_1}$; slope of AB $= -\frac{x_1}{y_1}$



So equation of chord is

$y - y_1 = -\frac{x_1}{y_1}(x - x_1)$ or $xx_1 + yy_1 = x_1^2 + y_1^2$.

Which can be represent by $T = S_1$

Example 20 :

Find the equation of chord of the circle $x^2 + y^2 = 8x$ bisected at the point (4, 3)

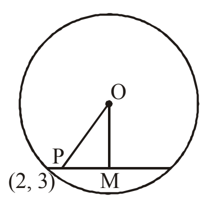
Sol. $T = S_1 \Rightarrow x(4) + y(3) - 4(x + 4) = 16 + 9 - 32$

$\Rightarrow 3y - 9 = 0 \Rightarrow y = 3$

Example 21 :

Find the equation of chord of the circle $x^2 + y^2 = a^2$ passing through the point (2, 3) farthest from the centre.

Sol. Let P (2, 3) be given point, M be the middle point of a chord of the circle $x^2 + y^2 = a^2$ through P.



Then the distance of the centre O of the circle from the chord is OM.

and $(OM)^2 = (OP)^2 - (PM)^2$ which is maximum when PM is minimum.

i.e. P coincides with M, which is the middle point of the chord. Hence, the equation of the chord is $T = S_1$,

i.e. $2x + 3y - a^2 = (2)^2 + (3)^2 - a^2 \Rightarrow 2x + 3y = 13$

DIAMETER OF A CIRCLE

The locus of middle points of a system of parallel chords of a circle is called the diameter of that circle. The diameter of the circle $x^2 + y^2 = r^2$ corresponding to the system of parallel chords $y = mx + c$ is $x + my = 0$.

CIRCLE THROUGH THE POINTS OF INTERSECTION

(i) The equation of the circle passing through the points of intersection of the circle S = 0 and line L = 0 is $S + \lambda L = 0$.

(ii) The equation of the circle passing through the points of intersection of the two circle S = 0 and S' = 0 is $S + \lambda S' = 0$ where $(\lambda \neq -1)$. In the above both cases λ can be find out according to the give problem.

Example 22 :

Find the equation of the circle passing through the origin and through the points of intersection of two circles $x^2 + y^2 - 10x + 9 = 0$ and $x^2 + y^2 = 4$

Sol. Let the circle be $(x^2 + y^2 - 10x + 9) + \lambda(x^2 + y^2 - 4) = 0$

Since it passes through (0, 0),so we have

$9 - 4\lambda = 0 \Rightarrow \lambda = 9/4$

So the required equation is

$4(x^2 + y^2 - 10x + 9) + 9(x^2 + y^2 - 4) = 0 \Rightarrow 13(x^2 + y^2) - 40x = 0$

Example 23 :

Find the equation of the circle passing through the origin and through the points of intersection of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ and the line $x + y - 1 = 0$

Sol. Let the required equation be

$(x^2 + y^2 - 2x + 4y - 20) + \lambda(x + y - 1) = 0$

Since it passes through (0, 0),so we have

$-20 - \lambda = 0 \Rightarrow \lambda - 20$

Hence the required equation is

$(x^2 + y^2 - 2x + 4y - 20) - 20(x + y - 1) = 0$

$\Rightarrow x^2 + y^2 - 22x - 16y = 0$

COMMON CHORD OF TWO CIRCLES

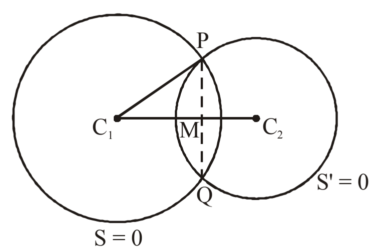
The chord joining the points of intersection of two given circles is called their common chord.

The equation of common chord of two circles

$S \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

and $S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is

$2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$ or $S - S' = 0$



Proof : $\because$ S = 0 and S' = 0 be two intersecting circles.

Then $S - S' = 0$

or $2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$ is a first degree equation in x and y.

So, it represent a straight line. Also, this equation satisfied by the intersecting points of two given circles S = 0 and S' = 0. Hence S – S' = 0 represents the common chord of circles S = 0 and S' = 0

Length of common chord :

We have $PQ = 2(PM)$ ($\because$ M is mid point of PQ)

$= 2\sqrt{\{(C_1P)^2 - (C_1M)^2\}}$

where C_1P = radius of the circle S = 0

and C_1M = length of perpendicular from C_1 on common chord PQ.

Note :

(a) The common chord PQ of two circles becomes of the maximum length when it is a diameter of the smaller one between them.

(b) Circle drawn on the common chord as a diameter then centre of the circle passing through P and Q lie on the common chord of two circles i.e., $S - S' = 0$

(c) If the length of common chord is zero, then the two circles touch each other and the common chord becomes the common tangent to the two circles at the common of contact.

Example 24 :

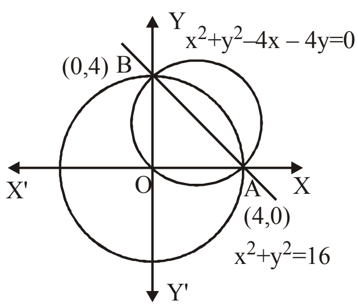
The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to

(A) $\pi/6$ (B) $\pi/4$

(C) $\pi/3$ (D) $\pi/2$

Sol. (D). The equation of the common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ is $x + y = 4$ which meets $x^2 + y^2 = 16$ at A (4, 0) and B (– 4, 0).

Obviously OA ⊥ OB.



Hence, the common chord AB makes a right angle at the centre of the circle $x^2 + y^2 = 16$

ANGLE OF INTERSECTION OF TWO CIRCLES

The angle of intersection between two circles S = 0 and S'=0 is defined as the angle between their tangents at their point of intersection.

If $S \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

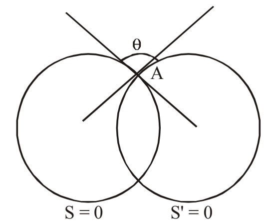
$S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

are two circles with radii r_1, r_2 and d be the distance between their centres then the angle of intersection θ between them

is given by $\cos\theta = \dfrac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$

or $\cos\theta = \dfrac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$

Condition of Orthogonality : If the angle of intersection of the two circle is a right angle (θ = 90°) then such circle are called Orthogonal circle and conditions for their orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$



When the two circles intersect orthogonally then the length of tangent on one circle from the centre of other circle is equal to the radius of the other circle.

Example 25 :

For what value of k the circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + 6y + k = 0$ cuts orthogonally

Sol. Let the two circles be $x^2 + y^2 + 2g_1 x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2 x + 2f_2y + c_2 = 0$

where $g_1 = 5/2, f_1 = 3/2, c_1 = 7,$

$g_2 = -4, f_2 = 3$ and $c_2 = k$

If the two circles intersects orthogonally, then

$2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\Rightarrow\ 2\left(-10 + \dfrac{9}{2}\right) = 7 + k$

$\Rightarrow\ 11 = 7 + k \Rightarrow k = -18$

Example 26 :

Find the equation of the circle which cuts the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and the lines $x = -g$ and $y = -f$ orthogonally

Sol. $x = -g,\ y = -f$ cuts the circle orthogonally mean these lines one normal to required circle.

Centre of required circle will be (–g, –f)

So equation of circle will be

$x^2 + y^2 + 2gx + 2fy + c' = 0$

Now it cut the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonally then $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2g^2 + 2f^2 = c + c' \Rightarrow c' = 2g^2 + 2f^2 - c$

Equation of required circle will be

$x^2 + y^2 + 2gx + 2fy + 2g^2 + 2f^2 - c = 0$

COMMON TANGENTS TO TWO CIRCLES :

(a) **Direct common tangents :** It is a tangent touching two circles at different points and not intersecting the line of centres between the centres as shown in figure.

(b) **Transverse common tangents :** It is a tangent touching two cirlces at different points and intersecting the line of centres between the centres as shown in figure.

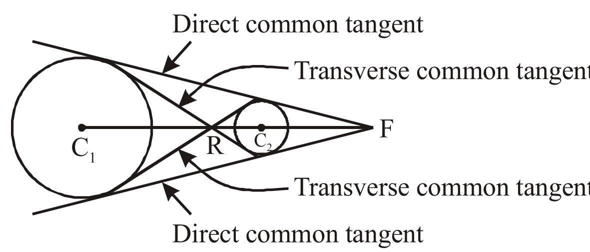
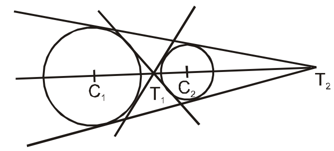
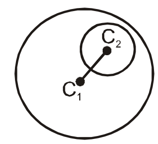
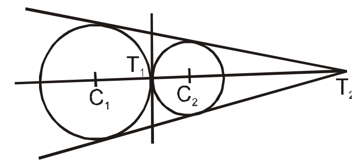
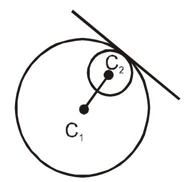
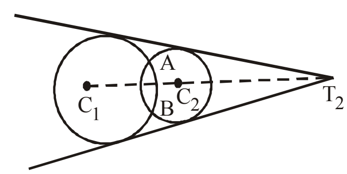


Table : Position of two circles

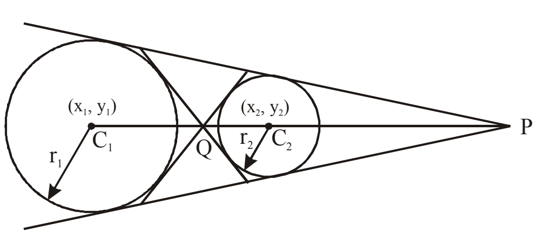
	Condition	Position	No. of common tangents	Diagram
(i)	$C_1C_2 > r_1 + r_2$	do not intersect or one out side the other	4	
(ii)	$C_1C_2 < \lvert r_1 - r_2 \rvert$	One inside the other	0	
(iii)	$C_1C_2 = r_1 + r_2$	external touch	3	
(iv)	$C_1C_2 = \lvert r_1 - r_2 \rvert$	internal touch	1	
(v)	$r_1 - r_2 < C_1C_2 < r_1 + r_2$	Intersection at two real points	2	

Points of intersection of common tangents :
The points T_1 and T_2 (points of intersection of indirect and direct common tangents) divide C_1C_2 internally and externally in the ratio $r_1 : r_2$

Equation of the common tangents at point of contact :
$S_1 - S_2 = 0$.

Point of contact : The point of contact C_1C_2 in the ratio $r_1 : r_2$ internally or externally as the case may be.

NOTE



(i) If two circles with centres $C_1\ (x_1, y_1)$ and $C_2\ (x_2, y_2)$ and radii r_1 and r_2 respectively, then direct common tangent meet at a point which divides the line joining the centre of circle externally in the ratio of their radii.

$$P \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)$$

(ii) Transverse Common tangent meets at a point which divides the line joining the centres of circles internally in the ratio of their radii.

$$Q \equiv \left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right).$$

(iii) C_1Q, C_1C_2, C_1P are in harmonic progression or Q and P are called harmonic conjugate points.

Example 27 :
Find the number of common tangents to circle
$x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$

Sol. $x^2 + y^2 + 2x + 8y - 23 = 0$

$\therefore\quad C_1\ (-1, -4), r_1 = 2\sqrt{10}$

For $x^2 + y^2 - 4x - 10y + 9 = 0$

$\therefore\quad C_2\ (2, 5), r_2 = 2\sqrt{5}$

Now, C_1C_2 = distance between centres

$\therefore \quad C_1C_2 = \sqrt{9+81} = 3\sqrt{10} = 9.486$

and $r_1 + r_2 = 2(\sqrt{10} + \sqrt{5}) = 10.6$

$r_1 - r_2 = 2\sqrt{5}(\sqrt{2}-1) = 2 \times 2.2 \times 0.4 = 4.4 \times 0.4 = 1.76$

$\Rightarrow \quad r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\Rightarrow$ Two circles intersect at two distinct points.

$\Rightarrow$ Two tangents can be drawn.

Example 28 :

Find all the common tangents to the circles $x^2 + y^2 = 1$ and $(x-1)^2 + (y-3)^2 = 4$

Sol. $C_1 : (0, 0) \quad r_1 = 1$

$C_2 : (1, 3) \quad r_2 = 2$

$C_1C_2 = \sqrt{10}$ Clearly $C_1C_2 = r_1 + r_2$

So circles neither touch nor cut each other,

there will be two direct common tangent & two transverse common tangent.

Point P divides C_1C_2 externally in the ratio of r_1 and r_2 i.e. $-1 : 2$ ('–' sign shows external division)

So coordinates of p will be

$$P : \left(\frac{-1 \times (1) + 2(0)}{-1+2}, \frac{-1 \times (3) + 2(0)}{-1+2}\right) \Rightarrow P : (-1, -3)$$

Equation of pair of tangent from the point P (–1, –3) to the circle $S : x^2 + y^2 = 1$ will be

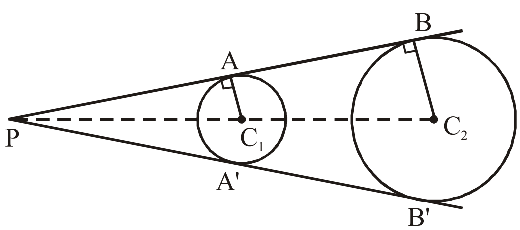
$SS_1 = T^2$

$(x^2 + y^2 - 1)((-1)^2 + (-3)^2 - 1) = (-x - 3y - 1)^2$

$9(x^2 + y^2 - 1) = x^2 + 9y^2 + 1 + 6xy + 2x + 6y$

$\Rightarrow \quad 8x^2 - 6xy - 2x - 6y - 10 = 0$

$(x + 1)(8x - 6y - 10) = 0$



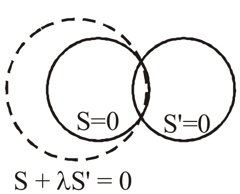
Equation of direct common tangent are

$x + 1 = 0$

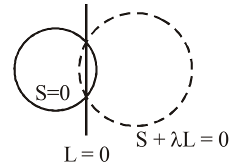
$8x - 6y - 10 = 0$

FAMILY OF CIRCLES

* **Type-1 :** The equation of the family of circles passing through the points of intersection of two given circles $S = 0$ and $S' = 0$ is given as $S + \lambda S' = 0$ (where λ is a parameter, $\lambda \neq -1$)



* **Type-2 :** The equation of the family of circles passing through the points of intersection of circle $S = 0$ and a line $L = 0$ is given as $S + \lambda L = 0$ (where λ is parameter)

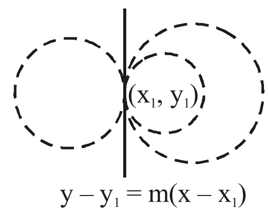


* **Type-3 :** The equation of family of circles which touch $y - y_1 = m(x - x_1)$ at (x_1, y_1) for any finite m is

$(x - x_1)^2 + (y - y_1)^2 + \lambda\{(y - y_1) - m(x - x_1)\} = 0$

and if m is infinite, the family of circles is

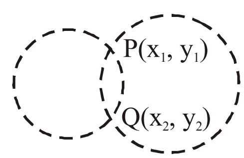
$(x - x_1)^2 + (y - y_1)^2 + \lambda(x - x_1) = 0$ (where λ is a parameter)



* **Type-4 :** The equation of a family of circles passing through two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can be written in the form

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(where λ is a parameter)



NOTE

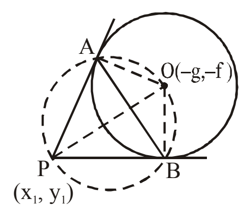
(a) Equation of the circle circumscribing the triangle PAB is

$(x_1 - x_1)(x_1 + g) + (y - y_1)(y + f) = 0$

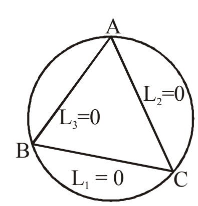
where O (–g, –f) is the centre of the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

(Here OP is diameter of the required circle)



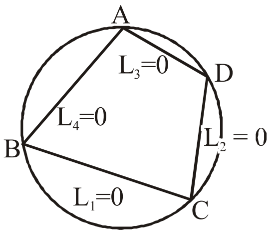
(b) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$, $L_2 = 0$ and $L_3 = 0$ is given by $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided coefficient of x^2 = coefficient to y^2.



(c) Equation of circle circumscribing a quadrilateral whose sides in order are represented by the lines $L_1 = 0, L_2 = 0, L_3 = 0$ and $L_4 = 0$ is given by

$$L_1L_3 + \lambda L_2L_4 = 0$$

provided coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$



Example 29 :

Find the equation of circle which passes through the point (–1, 2) and touches the circle $x^2 + y^2 – 8x + 6y = 0$ at the origin.

Sol. Equation of variable circle will be $S + \lambda L = 0$

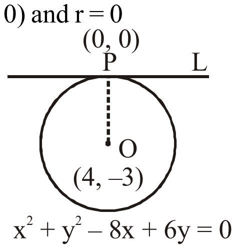
S is a point circle with centre at (0, 0) and r = 0

$S : (x – 0)^2 + (y – 0)^2 = 0$

$L : (y – 0) = \frac{4}{3}(x – 0)$

$\Rightarrow \quad 4x – 3y = 0$

[$\therefore$ L is perpendicular to OP]



Equation of family of circle is

$x^2 + y^2 + \lambda(4x – 3y) = 0$

Circle which passes through (–1, 2)

$1^2 + 2^2 + \lambda(–4 – 6) = 0 \Rightarrow 5 – 10\lambda = 0 \Rightarrow \lambda = 1/2$

Equation of required circle will be

$$x^2 + y^2 + \frac{1}{2}(4x – 3y) = 0 \Rightarrow 2x^2 + 2y^2 + 4x – 3y = 0$$

Example 30 :

If the circle $x^2 + y^2 + 2x + 3y + 1 = 0$ cuts $x^2 + y^2 + 4x + 3y + 2 = 0$ in A and B, then find the equation of the circle on AB as diameter.

Sol. The equation of the common chord AB of the two circles is $2x + 1 = 0$. [Using $S_1 – S_2 = 0$]

The equation of the required circle is

$(x^2 + y^2 + 2x + 3y + 1) + \lambda(2x + 1) = 0$ [Using $S_1 + \lambda(S_2 – S_1) = 0$]

$\Rightarrow \quad x^2 + y^2 + 2x(\lambda + 1) + 3y + \lambda + 1 = 0$

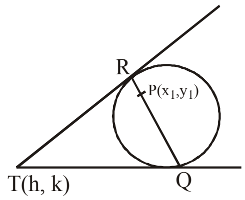
Since, AB is a diameter of this circle, therefore centre lies on it. So, $–2\lambda – 2 + 1 = 0 \Rightarrow \lambda = –1/2$

Thus, the required circle is $x^2 + y^2 + x + 3y + (1/2) = 0$

or $\quad 2x^2 + 2y^2 + 2x + 6y + 1 = 0$

POLE & POLAR

Let any straight line through the given point $P(x_1, y_1)$ intersect the circle S = 0 at two points Q and R, the locus of point of intersection of the tangents at Q and R is called the polar of the point P and the P is called the pole of the polar with respect to given circle.



Equation of Polar :

1. Equation of polar of the pole (x_1, y_1) with respect to circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$
2. Equation of polar of the pole (x_1, y_1) with respect to circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
$$x x_1 + y y_1 + g(x + x_1) + f(y + y_1) + c = 0$$

Coordinates of Pole :

1. Pole of polar $Ax + By + C = 0$ with respect to circle
$$x^2 + y^2 = a^2 \text{ is } \left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$$
2. Pole of polar $Ax + By + C = 0$ with respect to circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by the equation
$$\frac{x_1 + g}{A} = \frac{y_1 + f}{B} = \frac{gx_1 + fy_1 + c}{C}$$

Conjugate points : Two points A and B are conjugate points with respect to given circle, if each lies on the polar of the other with respect to the circle.

Conjugate lines : If two lines be such that the pole of one lies on the other, then they are called conjugate lines with respect to the given circle.

Example 31 :

Find the pole of the line $\frac{x}{a} + \frac{y}{b} = 1$ with respect to circle $x^2 + y^2 = c^2$.

Sol. Let the pole is (h, k)

Hence polar of this pole is $xh + yk – c^2 = 0$ .....(1)

but polar is $\frac{x}{a} + \frac{y}{b} = 0$ .....(2)

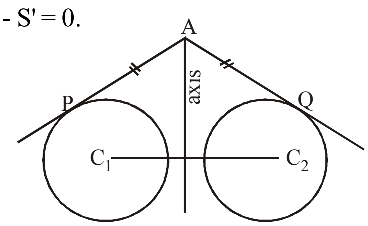
comparing the coefficient of x and y

$$\frac{h}{(1/a)} = \frac{k}{(1/b)} = \frac{-c^2}{-1} \Rightarrow h = \frac{c^2}{a}, k = \frac{c^2}{b}$$

RADICAL AXIS & RADICAL CENTRE :

Radical Axis - The radical axis of two circle is the locus of a point, which moves in such a way that the lengths of the tangents drawn from it to two given circles are equal.

The equation of radical axis of two circle S = 0 and S' = 0 is written as S - S' = 0.



NOTE

(i) Radical axis of two circle is perpendicular to the line joining their centres.

(ii) Radical axis bisects every common tangents of two circles.

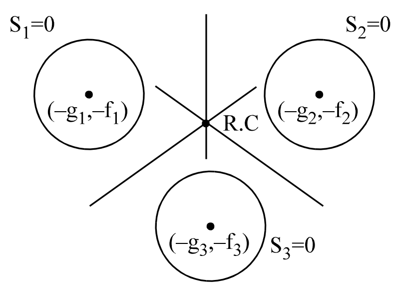
(iii) If two circles intersect a third circle orthogonally, then their radical axis passes through the centre of third circle.

(iv) Radical axis of three circle, taken two at a time meet at a point provided the centre of the circle are not collinear.

(v) If two circle touch each other, then the equation of the common tangent at the point of contact is $S - S' = 0$, which is also the equation of common chord, thus the common chord and common tangent at the point of contact are special cases of radical axis.

(vi) for two circles whose centre are not same, radical axis always exist, while common chord and common tangent may or may not exist.

Radical Centre : The point where the radical axis of three given circles taken in pairs meet,
is called the radical centre of those three circles. Thus the length of the three tangents drawn from the radical centre on the three circles are equal.
If $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ be any three given circles, then to obtain the radical centre, we solve any two of the following $S_1 - S_2 = 0$, $S_2 - S_3 = 0$, $S_3 - S_1 = 0$



NOTE

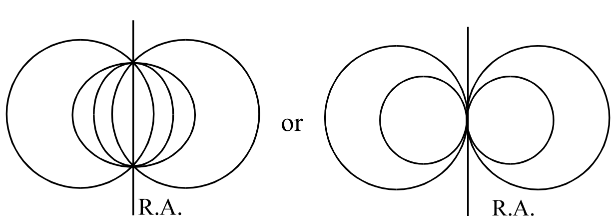
(i) If the centres of three circles are collinear then their radical centre will not exist.

(ii) The circle with centre at radical centre and radius is equal to the length of tangents from radical centre to any of the circles will cut the three circle orthogonally and is called as radical circle.

(iii) Circles are drawn on three sides of a triangle as diameter than radical centre of these circles is the orthocentre of the triangle.

COAXIAL SYSTEM OF CIRCLES

A system of circles, every 2 of which have the same radical axis, is called Coaxial system of circles.



Example 32 :

The equation of the three circles are given
$x^2 + y^2 = 1$, $x^2 + y^2 - 8x + 15 = 0$, $x^2 + y^2 + 10y + 24 = 0$.
Determine the coordinates of the point P such that the tangents drawn from it to the circles are equal in length.

Sol. We know that the point from which lengths of tangents are equal in length is radical centre of the given three circles.
Now radical axis of the first two circles is

$$(x^2 + y^2 - 1) - (x^2 + y^2 - 8x + 15) = 0,$$

i.e., $x - 2 = 0$ .... (1)

and radical axis of the second and third circles is

$$(x^2 + y^2 - 8x + 15) - (x^2 + y^2 + 10y + 24) = 0,$$

i.e., $8x + 10y + 9 = 0$ .... (2)

Solving eqs (1) and (2), the coordinates of the radical centre, i.e. of point P are P (2, –5/2).

Example 33 :

Find the locus of the centres of circles which bisect the circumference of circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x + 6y + 1 = 0$.

Sol. Let the equation of circle is

$S_1 : x^2 + y^2 + 2gx + 2fy + c = 0$

$S_2 : x^2 + y^2 = 4$

$S_3 : x^2 + y^2 - 2x + 6y + 1 = 0$

Radical axis of S_1 & S_2 is $2gx + 2fy + c + 4 = 0$

Radical axis passes through centre of $x^2 + y^2 = 4$

i.e. (0, 0)

$\Rightarrow$ $c = -4$

Radical axis of S_1 & S_3 is $(2g + 2)x + (2f - 6)y + c - 1 = 0$

it passes through centre of S_3 i.e. (1, –3)

so $2g + 2 - 6f + 18 + c - 1 = 0$, also $c = -4$

$\Rightarrow$ $2g - 6f + 15 = 0$

Now centre of circle is $(-g, -f)$, $h = -g$, $k = -f$

$\Rightarrow$ $-2h + 6k + 15 = 0$, locus is $2x - 6y - 15 = 0$

TRY IT YOURSELF-2

Q.1 Find the equation of the smallest circle passing through the intersection of the line $x + y = 1$ & the circle $x^2+y^2 = 9$.

Q.2 The locus of the centre of circle which cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is –

(A) $12x + 8y + 5 = 0$ (B) $8x + 12y + 5 = 0$
(C) $8x - 12y + 5 = 0$ (D) None of these

Q.3 If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6=0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) 3 (D) 2

Q.4 The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is-

(A) $20(x^2 + y^2) - 36x + 45y = 0$
(B) $20(x^2 + y^2) + 36x - 45y=0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$
(D) $36(x^2 + y^2) + 20x - 45y=0$

For Q.5-Q.7

Consider the equation $4\ell^2-5m^2+6\ell+1=0$, where $\ell, m \in R$, and the line $\ell x+my+1=0$ touches a fixed circle.

Q.5 Centre and radius of fixed circle respectively, are –

(A) (2, 0), 3 (B) (–3, 0), $\sqrt{3}$

(C) (3, 0), $\sqrt{5}$ (D) None of these

Q.6 Tangent PA and PB are drawn to the above fixed circle from the point P on the line $x+y-1=0$. Then chord of contact AB passes through the fixed point –

(A) (1/2, –5/2) (B) (1/3, 4/3)

(C) (–1/2, 3/2) (D) None of these

Q.7 Number of tangent which can be drawn from the point (2,–3) are –

(A) 0 (B) 1

(C) 2 (D) 1 or 2

ANSWERS

(1) $x^2+y^2-9-(x+y-1)=0$. **(2)** (C)

(3) (C) **(4)** (A) **(5)** (C)

(6) (A) **(7)** (C)

SOME IMPORTANT POINTS

1. Locus of mid point of a chord of a circle $x^2+y^2=a^2$ which subtends an angle α at the centre is $x^2+y^2=(a\cos\alpha/2)^2$
2. A variable point moves in such a way that sum of square of distances from the vertices of a triangle remains constant then its locus is a circle whose centre is the centroid of the triangle.
3. If the points where the line $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ meets the coordinate axes are concyclic then $a_1a_2=b_1b_2$.
4. If the line $lx+my+n=0$ is a tangent to the circle $x^2+y^2=a^2$, then $a^2(l^2+m^2)=n^2$.
5. If the radius of the given circle $x^2+y^2+2gx+2fy+c=0$ be r and it touches both the axes then $g=f\sqrt{c}=r$.
6. If the circle $x^2+y^2+2gx+2fy+c=0$ touches x-axis and y-axis, then $g^2=c$ and $f^2=c$ respectively.
7. The length of the common chord of the circles $(x-a)^2+y^2=a^2$ and $x^2+(y-b)^2=b^2$ is $\dfrac{2ab}{\sqrt{a^2+b^2}}$
8. If two tangents drawn from the origin to the circle $x^2+y^2+2gx+2fy+c=0$ are perpendicular to each other then $g^2+f^2=2c$
9. If the line $y=mx+c$ is a normal to the circle with radius r and centre at (a, b), then $b=ma+c$
10. If the tangent to the circle $x^2+y^2=r^2$ at the point (a, b) meets the coordinates axes at the points A and B and O is the origin. Then the area of the triangle OAB is $\dfrac{r^4}{2ab}$.
11. If O is the origin and OP, OQ are tangents to the circle $x^2+y^2+2gx+2fy+c=0$ then the circumcentre of the triangle OPQ is $(-g/2, -f/2)$.

ADDITIONAL EXAMPLES

Example 1 :

Find the equation of the circle concentric with the circle $x^2+y^2-3x+4y-c=0$ and passing through the point $(-1,-2)$.

Sol. The equation of two concentric circles differ only in constant term. So let the equation of the required circle be $x^2+y^2-3x+4y+\lambda=0$

It passes through $(-1,-2)$ so we have

$1+4+3-8+\lambda=0 \Rightarrow \lambda=0$,

Hence required equation is $x^2+y^2-3x+4y=0$

Example 2 :

If the line $x+y=1$ is a tangent to a circle with centre (2, 3), then find its equation.

Sol. Radius of the circle = perpendicular distance of (2, 3) from $x+y=1$ is $\dfrac{4}{\sqrt{2}}=2\sqrt{2}$

$\therefore$ The required equation will be

$(x-2)^2+(y-3)^2=8 \Rightarrow x^2+y^2-4x-6y+5=0$

Example 3 :

If the lines $3x-4y+4=0$ and $6x-8y-7=0$ are tangents to a circle, then find the radius of the circle.

Sol. The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x-4y+4=0$ and $3x-4y-\dfrac{7}{2}=0$ and so it is equal to -

$\dfrac{4}{\sqrt{9+16}}+\dfrac{7/2}{\sqrt{9+16}}=\dfrac{3}{2}$. Hence radius is $\dfrac{3}{4}$.

Example 4 :

The straight line $(x-2)+(y+3)=0$ cuts the circle $(x-2)^2+(y-3)^2=11$ at

(1) No points (2) One point

(3) Two points (4) None of these

Sol. **(1).** Equation of line is $x+y+1=0$. Since the perpendicular distance from centre to line is greater than radius, hence it does not cut the circle.

Example 5 :

Find the equation of the circle through the point of intersection of $x^2+y^2-1=0$, $x^2+y^2-2x-4y+1=0$ and touching the line $x+2y=0$

Sol. Family of circles is

$x^2+y^2-2x-4y+1+\lambda(x^2+y^2-1)=0$

$(1+\lambda)x^2+(1+\lambda)y^2-2x-4y+(1-\lambda)=0$

$$x^2+y^2-\frac{2}{1+\lambda}x-\frac{4}{1+\lambda}y+\frac{1-\lambda}{1+\lambda}=0$$

Centre is $\left[\dfrac{1}{1+\lambda},\dfrac{2}{1+\lambda}\right]$

and radius = $\sqrt{\left(\frac{1}{1+\lambda}\right)^2+\left(\frac{2}{1+\lambda}\right)^2-\left(\frac{1-\lambda}{1+\lambda}\right)}=\frac{\sqrt{4+\lambda^2}}{1+\lambda}$

Since it touches the line x + 2y = 0, hence
Radius = Perpendicular from centre to the line.

$$\left|\frac{\frac{1}{1+\lambda}+2\,\frac{2}{1+\lambda}}{\sqrt{1^2+2^2}}\right|=\frac{\sqrt{4+\lambda^2}}{1+\lambda}\Rightarrow\sqrt{5}=\sqrt{4+\lambda^2}\Rightarrow\lambda=\pm 1$$

$\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$
Thus, we get the equation of circle.

Example 6 :

If the straight line ax + by = 2 ; a, b $\neq$ 0 touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then find the values of a and b.

Sol. Given $x^2 + y^2 - 2x = 3$

$\therefore$ centre is (1, 0) and radius is 2 and $x^2 + y^2 - 4y = 6$

$\therefore$ centre is (0, 2) and radius is $\sqrt{10}$.

Since line ax + by = 2 touches the first circle.

$$\therefore \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}}=2 \text{ or } (a-2)=[2\sqrt{a^2-b^2}] \quad ...(i)$$

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.

$\therefore$ a (0) + b (2) = 2 or 2b = 2 $\Rightarrow$ b = 1

Putting this value in equation (i) we get

$$a-2=2\sqrt{a^2+1} \text{ or } (a-2)^2=4(a^2+1)$$

or $a^2 + 4 - 4a = 4a^2 + 4$ or $3a^2 + 4a = 0$

or a (3a + 4) = 0 or a = 0, – 4/3

$\therefore$ Values of a and b are (– 4/3, 1) respectively.

Example 7 :

Find the AM of the slopes of two tangents which can be drawn from the point (3, 1) to the circle $x^2 + y^2 = 4$.

Sol. Any tangents to the given circle, with slope m is

$$y=mx+2\sqrt{1+m^2}$$

since it passes through (3, 1) ; so

$$1=3m+2\sqrt{1+m^2}\Rightarrow 4m^2+4=(3m-1)^2$$

$\Rightarrow$ $5m^2 - 6m - 3 = 0$

If m = m_1, m_2 then AM of slopes

$$=\frac{1}{2}(m_1+m_2)=\frac{1}{2}(6/5)=3/5$$

Example 8 :

If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in A.P., then –

(1) a, b, c are in G.P. (2) a, b, c are in AP
(3) a^2, b^2, c^2 are in AP (4) a^2, b^2, c^2 are in GP

Sol. **(3).** Let P (x_1, y_1) be the given point and PT_1, PT_2, PT_3 be the lengths of the tangents from P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ respectively.

Then $PT_1=\sqrt{x_1^2+y_1^2-a^2}$, $PT_2=\sqrt{x_1^2+y_1^2-b^2}$

and $PT_3=\sqrt{x_1^2+y_1^2-c^2}$

Now, $PT_1^{\,2}$, $PT_2^{\,2}$, $PT_2^{\,3}$ are in A.P.

$\Rightarrow 2\,PT_2^{\,2} = PT_1^{\,2} + + PT_3^{\,2}$

$\Rightarrow 2\,(x_1^2+y_1^2-b^2)=(x_1^2+y_1^2-a^2)+(x_1^2+y_1^2-c^2)$

$\Rightarrow 2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2$ are in A.P.

Example 9 :

If the centre of a circle which passing through the points of intersection of the circle $x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$ is on the line y = x, then find the equation of the circle.

Sol. Family of circles through points of intersection of two circles is $S_1 + \lambda S_2 (\lambda \neq -1)$.

$x^2 + y^2 - 6x + 2y + 4 + \lambda\,(x^2 + y^2 + 2x - 4y - 6) = 0$

Centre is $(3 - \lambda, -1 + 2\lambda)$. It lies on y = x.

Therefore, $-1 + 2\lambda = 3 - \lambda \Rightarrow \lambda = 4/3$

Hence equation of circle can be found by substituting λ in the family of circles above.

Example 10 :

The line 3x – 2y = k meets the circle $x^2 + y^2 = 4r^2$ at only one point then find k^2.

Sol. Equation of line is 3x – 2y = k ...(i)

Circle is $x^2 + y^2 = 4r^2$ ...(ii)

Equation of line can be written as $y=\frac{3}{2}x-\frac{k}{2}$

Here, $c=-\frac{k}{2}$, $m=\frac{3}{2}$

Now the line will meet the circle, if

$$c=a\sqrt{1+m^2} = \frac{-k}{2}=(2r)\sqrt{1+\left(\frac{3}{2}\right)^2} \quad \text{[from (ii), a = 2r]}$$

$$=\frac{k^2}{4}=4r^2\times\frac{13}{4}\quad\therefore\ k^2=52r^2$$

Example 11 :

Find the equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on 13x + 30y = 0

Sol. The equation of required circles is $s_1 + \lambda s_2 = 0$

$$=x^2(1+\lambda)+y^2(1+\lambda)+x(2+13\lambda)-y\left(\frac{7}{2}+3\lambda\right)-\frac{25}{2}=0$$

$$\text{Centre}=\left(\frac{-(2+13\lambda)}{2},\frac{7/2+3\lambda}{2}\right)$$

Centre line on 13x + 30y = 0

$\Rightarrow\ -13\left(\frac{2+13\lambda}{2}\right)+30\left(\frac{7/2+3\lambda}{2}\right)=0 \Rightarrow \lambda=1$

$$2x^2+2y^2+15x-\frac{13}{2}y-\frac{52}{2}=0$$

Example 12 :

Locus of a point which moves such that sum of the square of its distances from the sides of a square of side unity is 9 is

(1) Straight line (2) Circle

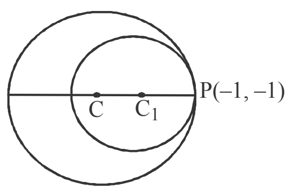
(3) Parabola (4) None of these

Sol. **(2).** $x^2+(x-1)^2+y^2+(y-1)^2=9$. Hence circle.

Example 13 :

Find the equation of the circle whose radius is 3 and which touches the circle $x^2+y^2-4x-6y-12=0$ internally at the point $(-1,-1)$.

Sol. Let C be the centre of the given circle and C_1 be the centre of the required circle.



$C=(2,3)$, $CP=\text{radius}=5$

$\because\ C_1P=3 \Rightarrow CC_1=2$

The point C_1 divided internally, the line joining C and P in the ratio 2 : 3

$\therefore$ coordinates of C_1 are (4/5, 7/5)

Example 14 :

Find the equation of the circle which is touched by y = x, has its centre on the positive direction of the x-axis and cuts off a chord of length 2 units along the line $\sqrt{3}\,y-x=0$.

Sol. Since the required circle has its centre on X-axis, So, let the coordinates of the centre be (a, 0). The circle touches y = x. Therefore, radius = length of the perpendicular from (a, 0) on $x-y=0=a/\sqrt{2}$

Circle cuts off a chord of length 2 units along $x-\sqrt{3}\,y=0$

$$\left(\frac{a}{\sqrt{2}}\right)^2=1^2+\left(\frac{a-\sqrt{3}\times 0}{\sqrt{1^2+\left(\sqrt{3}\right)^2}}\right)^2 \Rightarrow \frac{a^2}{2}=1+\frac{a^2}{4} \Rightarrow a=2$$

Thus, centre of the circle is at (2, 0) and radius $=\frac{a}{\sqrt{2}}=\sqrt{2}$

So, its equation is $x^2+y^2-4x+2=0$

Example 15 :

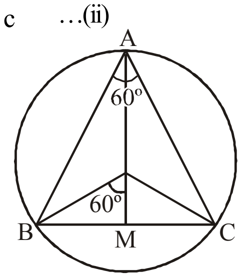
Find the area of an equilateral triangle inscribed in the circle. $x^2+y^2+2gx+2fy+c=0$

Sol. Given cirle is $x^2+y^2+2gx+2fy+c=0$ …(i)

Let O be the centre and ABC be an equilateral triangle inscribed in the circle (i).

$O\equiv(-g,-f)$

and $OA=OB=OC=\sqrt{g^2+f^2-c}$ …(ii)



In Δ OBM, $\sin 60^\circ=\frac{BM}{OB}$

$\Rightarrow\ BM=OB\sin 60^\circ=(OB)\frac{\sqrt{3}}{2}$

$\therefore\ BC=2BM=\sqrt{3}\,(OB)$ …(iii)

$\therefore$ Area of $\Delta ABC=\frac{\sqrt{3}}{4}(BC)^2=\frac{\sqrt{3}}{4}3\,(OB)^2$ from (iii)

$=\frac{3\sqrt{3}}{4}(g^2+f^2-c)$ sq. units

Example 16 :

Find the value of 'c' for which the power of a point P(2, 5) is negative w.r.t a circle $x^2+y^2-8x-12y+c=0$ and the circle neither touches nor intersects the coordinate axis.

Sol. $S\equiv x^2+y^2-8x-12y+c=0$

Power $=2^2+5^2-8\times 2-12\times 5+c<0$

$29-16-60+c<0$; $c<47$

Again $g^2-c<0$ & $f^2-c<0$

$\therefore$ $16-c<0$ & $36-c<0$

Hence $36<c<47$

Example 17 :

Find the locus of point "P" which moves such that the angle made by pair of tangents drawn to the circle $x^2+y^2=a^2$ is 60º.

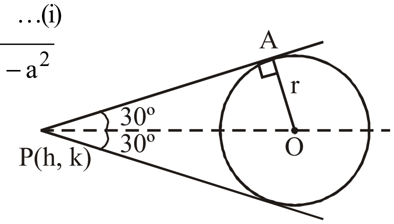
Sol. Length of tangent from P to circle

$x^2+y^2=a^2$ …(i)

$PA=\sqrt{h^2+k^2-a^2}$

radius (r) = a

$\tan 30^\circ=\frac{r}{PA}$



$\Rightarrow\ \frac{1}{\sqrt{3}}=\frac{a}{\sqrt{h^2+k^2-a^2}}$

$\Rightarrow\ h^2+k^2-a^2=3a^2 \Rightarrow h^2+k^2=4a^2$

$\Rightarrow\ x^2+y^2=4a^2$

Example 18 :

Find the locus of middle points of chords of the circle $x^2+y^2=r^2$, which subtend right angle at the point $(\lambda, 0)$.

Sol. Let N (h, k) be the middle point of any chord AB, which subtend a right angle at $P(\lambda, 0)$

Since $\angle APB=90^\circ$

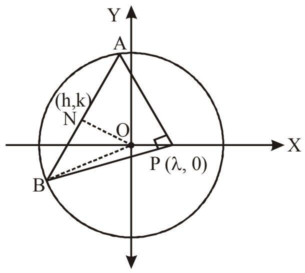
$\therefore$ NA = NB = NP (since distances of the vertices from middle point of the hypotenuse are equal)

or $(NA)^2=(NB)^2=(h-\lambda)^2+(k-0)^2$ …(i)

But also $\angle BNO = 90^\circ$

$\therefore \quad (OB)^2 = (ON)^2 + (NB)^2$

$\Rightarrow \quad -(NB)^2 = (ON)^2 - (OB)^2$



$\Rightarrow \quad -[(h-\lambda)^2 + (k-0)^2] = (h^2 + k^2) - r^2$

or $\quad 2(h^2 + k^2) - 2\lambda h + \lambda^2 - r^2 = 0$

$\therefore$ Locus of N (h, k) is

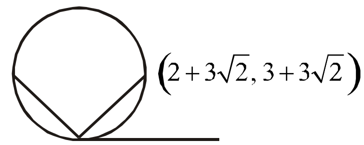
$2(x^2 + y^2) - 2\lambda x + \lambda^2 - r^2 = 0$

Example 19 :

Find the equations to the circles which pass through the point (2, 3) and cut off equal chords of length 6 units along the lines $y - x - 1 = 0$ and $y + x - 5 = 0$.

Sol. The given two lines pas through the point (2, 3) and are inclined at 45º and 135º to the x-axis. The other ends of the chords can easily be calculated as

$(2 + 3\sqrt{2}, 3 + 3\sqrt{2})$ and $(2 - 3\sqrt{2}, 3 + 3\sqrt{2})$.



There is symmetry about the line x = 2 and therefore the centres of the circles lie on x = 2. As the chords subtend right angles at the centre $2r^2 = 6^2$ gives the radius $r = 3\sqrt{2}$.

The centre is $(2, 3 + 3\sqrt{2})$.

The equations of the two circles are therefore

$(x-2)^2 + (y - 3 - 3\sqrt{2})^2 = 18$ and

$(x-2)^2 + (y - 3 + 3\sqrt{2})^2 = 18$.

QUESTION BANK

CHAPTER 10 : CIRCLE

EXERCISE - 1 [LEVEL-1]

PART 1 : EQUATION OF CIRCLE, CENTRE AND RADIUS OF CIRCLE

Q.1 Centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ is –
(A) (–1,2) (B) (1,–2)
(C) (1, 2) (D) (–1, –2)

Q.2 Radius of the circle $2(x^2 + y^2) + 4x - 3y + 1 = 0$ is –
(A) $\sqrt{17}/2$ (B) $\sqrt{17}$
(C) $\frac{2}{\sqrt{17}}$ (D) $\sqrt{17}/4$

Q.3 If (6, –3) is the one extremity of diameter to the circle $x^2 + y^2 - 3x + 8y - 4 = 0$ then its other extremity is –
(A) (3/2, –4) (B) (–3, –5)
(C) (3, –5) (D) (3, 5)

Q.4 Find the equation of a circle whose centre is (2, –1) and radius is 3.
(A) $x^2 + y^2 - 4x + 2y - 4 = 0$ (B) $x^2 + y^2 - 3x - 2y - 4 = 0$
(C) $x^2 - y^2 - 5x + y - 4 = 0$ (D) $x^2 + y^2 + 2x + y - 2 = 0$

Q.5 Find the equation of a circle with centre at the origin and which passes through (7, –2).
(A) $x^2 + y^2 = 13$ (B) $x^2 - y^2 = 53$
(C) $x^2 + y^2 = 63$ (D) $x^2 + y^2 = 53$

Q.6 If (1, 2) and (3, 4) are end points of a diameter of circle, then its equation is -
(A) $x^2 + y^2 + 4x - 6y - 11 = 0$ (B) $x^2 + y^2 - 4x + 6y - 11 = 0$
(C) $x^2 + y^2 - 4x - 6y + 11 = 0$ (D) None of these

Q.7 A circle has its equation in the form $x^2 + y^2 + 2x + 4y + 1 = 0$. Choose the correct coordinates of its centre & the right value of its radius from the following
(A) Centre (–1, –2), radius = 2 (B) Centre (2, 1), radius = 1
(C) Centre (1, 2), radius = 3 (D) Centre (–1, 2), radius=2

Q.8 Cartesian equations of a circle whose parametric equation are $x = -7 + 4\cos\theta$, $y = 3 + 4\sin\theta$ is -
(A) $(x + 7)^2 + (y - 3)^2 = 16$ (B) $(x - 7)^2 + (y - 3)^2 = 16$
(C) $(x - 7)^2 + (y + 3)^2 = 16$ (D) $(x + 7)^2 + (y + 3)^2 = 16$

Q.9 The equation of the circle touches y axis and having centre is (–2, –3) –
(A) $x^2 + y^2 - 4x - 9y - 4 = 0$ (B) $x^2 + y^2 + 4x + 9y + 4 = 0$
(C) $x^2 + y^2 + 4x + 6y + 9 = 0$ (D) $x^2 + y^2 - 4x - 6y - 9 = 0$

Q.10 A circle touches x– axis at +3 distance and cuts an intercept of 8 in +ve direction of y–axis. Its equation is –
(A) $x^2 + y^2 + 6x + 10y - 9 = 0$ (B) $x^2 + y^2 - 6x - 10y - 9 = 0$
(C) $x^2 + y^2 - 6x - 10y + 9 = 0$ (D) $x^2 + y^2 + 6x + 10y + 9 = 0$

Q.11 The lines 2x – 3y = 5 and 3x – 4y = 7 are diameters of a circle of area 154 sq. units. The equation of this circle is –
(A) $x^2 + y^2 - 2x - 2y = 47$ (B) $x^2 + y^2 - 2x - 2y = 62$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

Q.12 The equation of a circle which passes through the point (1,–2) and (4,–3) and whose centre lies on the line 3x + 4y = 7 is–
(A) $15(x^2 + y^2) - 94x + 18y - 55 = 0$
(B) $15(x^2 + y^2) - 94x + 18y + 55 = 0$
(C) $15(x^2 + y^2) + 94x - 18y + 55 = 0$
(D) None of these

Q.13 The equation of a circle passing through (–4, 3) and touching the lines x+ y = 2, x– y = 2 is –
(A) $x^2 + y^2 - 20x - 55 = 0$ (B) $x^2 + y^2 + 20x + 55 = 0$
(C) $x^2 + y^2 - 20x - 55 = 0$ (D) None of these

Q.14 The equation of the circle which touches the axis of y at the origin and passes through (3,4) is –
(A) $4(x^2 + y^2) - 25x = 0$ (B) $3(x^2 + y^2) - 25x = 0$
(C) $2(x^2 + y^2) - 3x = 0$ (D) $4(x^2 + y^2) - 25x + 10 = 0$

Q.15 The equation of a circle which touches x–axis and the line 4x – 3y + 4 = 0, its centre lying in the third quadrant and lies on the line x – y – 1 = 0, is –
(A) $9(x^2 + y^2) + 6x + 24y + 1 = 0$
(B) $9(x^2 + y^2) - 6x - 24y + 1 = 0$
(C) $9(x^2 + y^2) - 6x + 2y + 1 = 0$
(D) None of these

Q.16 The equation to a circle passing through the origin and cutting of intercepts each equal to + 5 of the axes is –
(A) $x^2 + y^2 + 5x - 5y = 0$ (B) $x^2 + y^2 - 5x + 5y = 0$
(C) $x^2 + y^2 - 5x - 5y = 0$ (D) $x^2 + y^2 + 5x + 5y = 0$

Q.17 The equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point (–1,–1) is –
(A) $\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{7}{5}\right)^2 = 3^2$ (B) $\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2$
(C) $(x - 8)^2 + (y - 1)^2 = 3^2$ (D) None of these

Q.18 The equation of a circle which passes through the three points (3, 0) (1, –6), (4, –1) is –
(A) $2x^2 + 2y^2 + 5x - 11y + 3 = 0$
(B) $x^2 + y^2 - 5x + 11y - 3 = 0$
(C) $x^2 + y^2 + 5x - 11y + 3 = 0$
(D) $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

Q.19 If (4, –2) is a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, then value of c is -
(A) – 4 (B) 0
(C) 4 (D) 1

Q.20 The abscissa of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$. The radius of the circle with AB as a diameter will be -
(A) $\sqrt{a^2 + b^2 + p^2 + q^2}$ (B) $\sqrt{b^2 + q^2}$
(C) $\sqrt{a^2 + b^2 - p^2 - q^2}$ (D) $\sqrt{a^2 + p^2}$

Q.21 Two rods of length a and b slide on the axes in such a way that their ends are always concylic. The locus of centre of the circle passing through the ends is -
(A) $4(x^2 - y^2) = a^2 - b^2$ (B) $x^2 - y^2 = a^2 - b^2$
(C) $x^2 - y^2 = 4(a^2 - b^2)$ (D) $x^2 + y^2 = a^2 + b^2$

Q.22 Circle $x^2 + y^2 + 6y = 0$ touches
(A) y-axis at the origin (B) x-axis at the origin
(C) x-axis at the point (3, 0) (D) The line $y + 3 = 0$

Q.23 The equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$, is
(A) $x^2 + y^2 + 4x - 10y + 25 = 0$
(B) $x^2 + y^2 - 4x - 10y + 25 = 0$
(C) $x^2 + y^2 - 4x - 10y + 16 = 0$
(D) $x^2 + y^2 - 14y + 8 = 0$

Q.24 The equation of the circle with centre on the x-axis, radius 4 and passing through the origin, is
(A) $x^2 + y^2 + 4x = 0$ (B) $x^2 + y^2 - 8y = 0$
(C) $x^2 + y^2 \pm 8x = 0$ (D) $x^2 + y^2 + 8y = 0$

Q.25 The centre and radius of the circle $2x^2 + 2y^2 - x = 0$ are
(A) $\left(\frac{1}{4}, 0\right)$ and $\frac{1}{4}$ (B) $\left(-\frac{1}{2}, 0\right)$ and $\frac{1}{2}$
(C) $\left(\frac{1}{2}, 0\right)$ and $\frac{1}{2}$ (D) $\left(0, -\frac{1}{4}\right)$ and $\frac{1}{4}$

Q.26 The radius of a circle which touches y-axis at (0,3) and cuts intercept of 8 units with x-axis, is –
(A) 3 (B) 2
(C) 5 (D) 8

Q.27 Radius of the circle $x^2 + y^2 + 2x\cos\theta + 2y\sin\theta - 8 = 0$ is
(A) 1 (B) 3
(C) $2\sqrt{3}$ (D) $\sqrt{10}$

Q.28 A circle has radius 3 units and its centre lies on the line $y = x - 1$. Then the equation of this circle if it passes through point (7, 3), is
(A) $x^2 + y^2 - 8x - 6y + 16 = 0$
(B) $x^2 + y^2 + 8x + 6y + 16 = 0$
(C) $x^2 + y^2 - 8x - 6y - 16 = 0$
(D) None of these

Q.29 If the coordinates of one end of the diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (–3, 2), then the coordinates of other end are
(A) (5, 3) (B) (6, 2)
(C) (1, –8) (D) (11, 2)

Q.30 The equation of the circle whose diameter lies on $2x + 3y = 3$ and $16x - y = 4$ which passes through (4, 6) is
(A) $5(x^2 + y^2) - 3x - 8y = 200$
(B) $x^2 + y^2 - 4x - 8y = 200$
(C) $5(x^2 + y^2) - 4x = 200$
(D) $x^2 + y^2 = 40$

Q.31 The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x-axis at
(A) (2, 0), (–3, 0) (B) (3, 0), (4, 0)
(C) (1, 0), (–1, 0) (D) (1, 0), (2, 0)

Q.32 If one end of the diameter is (1, 1) and other end lies on the line $x + y = 3$, then locus of centre of circle is
(A) $x + y = 1$ (B) $2(x - y) = 5$
(C) $2x + 2y = 5$ (D) None of these

Q.33 The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are
(A) (–2, –6), (4, 2) (B) (2, 6), (– 4, –2)
(C) (–2, 6) (– 4, 2) (D) None of these

Q.34 The diameter of a circle is AB and C is another point on circle, then the area of triangle ABC will be
(A) Maximum, if the triangle is isosceles
(B) Minimum, if the triangle is isosceles
(C) Maximum, if the triangle is equilateral
(D) None of these

Q.35 The locus of the centre of the circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is -
(A) $x^2 - 10x - 6y + 14 = 0$ (B) $x^2 - 6x - 10y + 14 = 0$
(C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

Q.36 The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (with $c > 0$) touch each other if -
(A) $c = |a|$ (B) $2c = a$
(C) $2a = |c|$ (D) None of these

Q.37 The equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$ is –
(A) $x^2 + y^2 + 32x - 4y + 235 = 0$
(B) $x^2 + y^2 + 32x + 4y - 235 = 0$
(C) $x^2 + y^2 + 32x - 4y - 235 = 0$
(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

Q.38 If lines $y = x + 3$ cuts the circle $x^2 + y^2 = a^2$ in two points A and B, then equation of circle with AB as diameter is -
(A) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
(B) $x^2 + y^2 + 3x - 3y + a^2 + 9 = 0$
(C) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$
(D) None of these

Q.39 If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touches each other than α –
(A) 0 (B) 1
(C) –4/3 (D) –3/4

Q.40 The centre of the circle $r^2 = 2 - 4r\cos\theta + 6r\sin\theta$ is –
(A) (2, 3) (B) (–2, 3)
(C) (–2, –3) (D) (2, –3)

Q.41 The equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5), is
(A) $x^2 + y^2 - 18x - 16y - 120 = 0$
(B) $x^2 + y^2 - 18x - 16y + 120 = 0$
(C) $x^2 + y^2 + 18x + 16y - 120 = 0$
(D) $x^2 + y^2 + 18x - 16y + 120 = 0$

Q.42 The straight line $2x + 3y - k = 0$, $k > 0$ cuts the X- and Y-axes at A and B. The area of Δ OAB, where O is the origin, is 12 sq. units. The equation of the circle having AB as diameter is –
(A) $x^2 + y^2 - 6x - 4y = 0$ (B) $x^2 + y^2 + 4x - 6y = 0$
(C) $x^2 + y^2 - 6x + 4y = 0$ (D) $x^2 + y^2 - 4x - 6y = 0$

Q.43 Equation of the circle centered at (4, 3) touching the circle $x^2 + y^2 = 1$ externally, is –
(A) $x^2 + y^2 - 8x - 6y + 9 = 0$ (B) $x^2 + y^2 + 8x + 6y + 9 = 0$
(C) $x^2 + y^2 + 8x - 6y + 9 = 0$ (D) $x^2 + y^2 - 8x + 6y + 9 = 0$

Q.44 The points (1, 0), (0, 1), (0, 0) and (2k, 3k), $k \neq 0$ are concyclic if k =
(A) 1/5 (B) –1/5
(C) –5/13 (D) 5/13

Q.45 The number of circles that touch the co-ordinate axes and the line whose slope is –1 and y-intercept is 1, is
(A) 3 (B) 1
(C) 4 (D) 2

Q.46 If $x = 2 + 3\cos\theta$ and $y = 1 - 3\sin\theta$ represent a circle then the centre and radius is
(A) (2, 1), 3 (B) (–2, –1), 3
(C) (2, 1), 9 (D) (1, 2), 1/3

PART 2 : POINT WITH RESPECT TO CIRCLE, TANGENT AND NORMAL

Q47 For what value of m the line $3x + 4y = m$ touches the circle $x^2 + y^2 - 2x - 8 = 0$
(A) –18, 12 (B) 18, 12
(C) 18, –12 (D) –18, –12

Q.48 The circle S_1 with centre C_1 (a_1, b_1) and radius r_1 touches externally the circle S_2 with centre C_2 (a_2, b_2) and radius r_2. If the tangent at their common point passes through the origin, then
(A) $(a_1^2 + a_2^2) + (b_1^2 + b_2^2) = r_1^2 + r_2^2$
(B) $(a_2^2 - a_1^2) + (b_2^2 - b_1^2) = r_2^2 - r_1^2$
(C) $(a_1^2 - b_2^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$
(D) $(a_1^2 - b_1^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$

Q.49 The point from which the tangents to the circles $x^2 + y^2 - 8x + 40 = 0$; $5x^2 + 5y^2 - 25x + 80 = 0$ $x^2 + y^2 - 8x + 16y + 160 = 0$ are equal in length is-
(A) (8, 15/2) (B) (–8, 15/2)
(C) (8, –15/2) (D) None of these

Q.50 The total number of common tangents to the two circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$, is -
(A) 1 (B) 2
(C) 3 (D) 4

Q.51 The point P (10, 7) lies outside the circle $x^2 + y^2 - 4x - 2y - 20 = 0$. The greatest distance of P from the circle is
(A) 5 (B) $\sqrt{3}$
(C) $\sqrt{5}$ (D) 15

Q.52 The equations of the tangents to the circle $x^2 + y^2 = 36$ which are inclined at an angle of 45° to the x-axis are
(A) $x + y = \pm\sqrt{6}$ (B) $x = y \pm 3\sqrt{2}$
(C) $y = x \pm 6\sqrt{2}$ (D) None of these

Q.53 If the equation of one tangent to the circle with centre at (2, –1) from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is
(A) $3x - y = 0$ (B) $x + 3y = 0$
(C) $x - 3y = 0$ (D) $x + 2y = 0$

Q.54 The equation of the tangent at the point $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$ of the circle $x^2 + y^2 = \frac{a^2b^2}{a^2+b^2}$ is
(A) $\frac{x}{a} + \frac{y}{b} = 1$ (B) $\frac{x}{a} + \frac{y}{b} + 1 = 0$
(C) $\frac{x}{a} - \frac{y}{b} = 1$ (D) $\frac{x}{a} - \frac{y}{b} + 1 = 0$

Q.55 If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same circle, then its radius will be
(A) $\frac{\sqrt{3}}{5}$ (B) $\frac{17}{6\sqrt{5}}$
(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

Q.56 The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other. The equation of their common tangent is
(A) $x = 3$ (B) $y = 6$
(C) $7x - 12y - 21 = 0$ (D) $7x + 12y + 21 = 0$

Q.57 The area of the triangle formed by the tangent at (3, 4) to the circle $x^2 + y^2 = 25$ and the co-ordinate axes is
(A) 24/25 (B) 0
(C) 625/24 (D) – (24/25)

Q.58 Tangents AB and AC are drawn from the point A (0, 1) to the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Equation of the circle through A, B and C is
(A) $x^2 + y^2 + x + y - 2 = 0$ (B) $x^2 + y^2 - x + y - 2 = 0$
(C) $x^2 + y^2 + x - y - 2 = 0$ (D) None of these

Q.59 The equation of pair of tangents drawn from the point (0,1) to the circle $x^2 + y^2 - 2x + 4y = 0$ is –
(A) $4x^2 - 4y^2 + 6xy + 6x + 8y - 4 = 0$
(B) $4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$
(C) $x^2 - y^2 + 3xy - 3x + 2y - 1 = 0$
(D) $x^2 - y^2 + 6xy - 6x + 8y - 4 = 0$

Q.60 If the length of the tangents drawn from the point (1,2) to the circles $x^2 + y^2 + x + y - 4 = 0$ and $3x^2 + 3y^2 - x - y + k = 0$ be in the ratio 4 : 3, then k =
(A) 7/2 (B) 21/2 (C) – 21/4 (D) 7/4

Q.61 Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point P (16, 7). If the centre of the circle is C, then the area of quadrilateral PQCR
(A) 75 sq. units (B) 150 sq. units
(C) 15 sq. units (D) None of these

Q.62 If the tangent to a circle $x^2 + y^2 = 5$ at point (1, –2) touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$, then its point of contact
(A) (–2, 1) (B) (3, –1) (C) (–1, –3) (D) (5, 0)

Q.63 Length of the tangent drawn from point (1, 5) to the circle $2x^2 + 2y^2 = 3$ is -
(A) 7 (B) $7\sqrt{2}$ (C) $7\sqrt{2}/2$ (D) None

Q.64 The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. The radius of the circle is –
(A) $3\sqrt{5}$ (B) $5\sqrt{3}$
(C) $2\sqrt{5}$ (D) $5\sqrt{2}$

Q.65 If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then the radius of the circle is
(A) 3/2 (B) 3/4
(C) 1/10 (D) 1/20

Q.66 If a circle S (x, y) = 0 touches at the point (2, 3) of the line $x + y = 5$ and S (1, 2)= 0, then radius of such circle.
(A) 2 units (B) 4 units
(C) 1/2 units (D) $1/\sqrt{2}$ units

Q.67 The total number of common tangents of $x^2 + y^2 - 6x - 8y + 9 = 0$ and $x^2 + y^2 = 1$ is –
(A) 1 (B) 3
(C) 2 (D) 4

Q.68 The least and the greatest distances of the point (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ are –
(A) 5, 15 (B) 10, 5
(C) 15, 20 (D) 12, 16

Q.69 If the straight line $3x + 4y = k$ touches the circle $x^2 + y^2 = 16x$, then the value of k is –
(A) 16, –64 (B) 16, 64
(C) –16, –64 (D) –16, 64

Q.70 The equations of the two tangents from (–5, – 4) to the circle $x^2 + y^2 + 4x + 6y + 8 = 0$ are –
(A) $x - 7y = 23$, $6x + 13y = 4$
(B) $x + 2y + 13 = 0$, $2x - y + 6 = 0$
(C) $2x + y + 13 = 0$, $x - 2y = 6$
(D) $3x + 2y + 23 = 0$, $2x - 3y + 4 = 0$

Q.71 A tangent is drawn to the circle $2x^2 + 2y^2 - 3x + 4y = 0$ at point 'A' and it meets the line $x + y = 3$ at B (2, 1), then AB=
(A) $\sqrt{10}$ (B) 2
(C) $2\sqrt{2}$ (D) 0

Q.72 The area of the circle having its centre at (3, 4) and touching the line $5x + 12y - 11 = 0$ is –
(A) 16 π sq. units (B) 4 π sq. units
(C) 12 π sq. units (D) 25 π sq. units

PART 3 : DIRECTOR CIRCLE, CHORD OF CONTACT, POLE AND POLAR, CHORD, ANGLE OF INTERSECTION

Q.73 The equation of the circle whose centre is (3, –1) and which cuts off a chord of length 6 on the line $2x - 5y + 18 = 0$ is
(A) $(x-3)^2 + (y+1)^2 = 38$ (B) $(x+3)^2 + (y-1)^2 = 38$
(C) $(x-3)^2 + (y+1)^2 = \sqrt{38}$ (D) None of these

Q.74 If the line $x - y + 1 = 0$ is a chord of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ then find the length of this chord
(A) 2 (B) 3
(C) 5 (D) 7

Q.75 The pole of the straight line $9x + y - 28 = 0$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$, is
(A) (2, 1) (B) (2, – 1)
(C) (3,1) (D) (3,–1)

Q.76 If the polar of a point (p, q) with respect to the circle $x^2 + y^2 = a^2$ touches the circle $(x-c)^2 + (y-d)^2 = b^2$, then
(A) $b^2 (p^2 + q^2) = (a^2 - cp - qd)^2$
(B) $b^2 (p^2 + q^2) = (a^2 - cq - dp)^2$
(C) $a^2 (p^2 + q^2) = (b^2 - cp - dq)^2$
(D) None of these

Q.77 From the origin, chords are drawn to the circle $(x - 1)^2 + y^2 = 1$, then equation of locus of middle points of these chords, is -
(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = x$
(C) $x^2 + y^2 = y$ (D) None of these

Q.78 If $y = 2x$ is a chord of the circle $x^2 + y^2 = 10x$, then the equation of the circle whose diameter is this chord is -
(A) $x^2 + y^2 + 2x + 4y = 0$ (B) $x^2 + y^2 + 2x - 4y = 0$
(C) $x^2 + y^2 - 2x - 4y = 0$ (D) None of these

Q.79 The length of the common chord of the circles $(x - a)^2 + y^2 = c^2$ and $x^2 + (y - b)^2 = c^2$ is -
(A) $\sqrt{c^2 + a^2 + b^2}$ (B) $\sqrt{4c^2 + a^2 + b^2}$
(C) $\sqrt{4c^2 - a^2 - b^2}$ (D) $\sqrt{c^2 - a^2 - b^2}$

Q.80 The angle of intersection of the two circles $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 4$, is -
(A) 30º (B) 60º
(C) 90º (D) 45º

Q.81 If a circle passes through the point (1,2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is -
(A) $x^2 + y^2 - 2x - 6y - 7 = 0$ (B) $x^2 + y^2 - 3x - 8y + 1 = 0$
(C) $2x + 4y - 9 = 0$ (D) $2x + 4y - 1 = 0$

Q.82 Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 4y + 3 = 0$
(A) touch each other externally
(B) touch each other internally
(C) intersect each other
(D) do not intersect

Q.83 The circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other -
(A) externally at (0,1) (B) internally at (0,1)
(C) externally at (1,0) (D) internally at (1,0)

Q.84 The angle between the two tangents from the origin to the circle $(x-7)^2 + (y+1)^2 = 25$ is
(A) 0 (B) π/3
(C) π/6 (D) π/2

Q.85 Middle point of the chord of the circle $x^2 + y^2 = 25$ intercepted on the line $x - 2y = 2$ is
(A) (3/5, 4/5) (B) (–2, –2)
(C) (2/5, –4/5) (D) (8/3, 1/3)

Q.86 A line through (0,0) cuts the circle $x^2 + y^2 - 2ax = 0$ at A and B, then locus of the centre of the circle drawn on AB as a diameter is
(A) $x^2 + y^2 - 2ay = 0$ (B) $x^2 + y^2 + ay = 0$
(C) $x^2 + y^2 + ax = 0$ (D) $x^2 + y^2 - ax = 0$

Q.87 Chord of contact with respect to point (2, 2) of circle $x^2 + y^2 = 1$ is -
(A) $x + y + 1$ (B) $x - y = 1/2$
(C) $x + y = 1/2$ (D) $x + y = 2$

Q.88 If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$, then $c + d =$
(A) 40 (B) 50
(C) 60 (D) 56

Q.89 Equation of polar of point (4, 4) with respect to circle $(x - 1)^2 + (y - 2)^2 = 1$ is
(A) $2x + 3y - 8 = 0$ (B) $3x + 2y + 8 = 0$
(C) $3x - 2y + 8 = 0$ (D) $3x + 2y - 8 = 0$

Q.90 The locus of the point, the chord of contact of tangents from which to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre is a circle of radius -
(A) 2a (B) a/2
(C) $\sqrt{2}\,a$ (D) a^2

Q.91 If a chord of the circle $x^2 + y^2 = 8$ makes equal intercepts of length a on the coordinate axes, then-
(A) $|a| < 8$ (B) $|a| < 4\sqrt{2}$
(C) $|a| < 4$ (D) $|a| > 4$

Q.92 The area of the triangle formed by the tangents from an external point (h, k) to the circle $x^2 + y^2 = a^2$ and the chord of contact, is -
(A) $\frac{1}{2}a\left(\frac{h^2+k^2-a^2}{\sqrt{h^2+k^2}}\right)$ (B) $\frac{a(h^2+k^2-a^2)^{3/2}}{2(h^2+k^2)}$
(C) $\frac{a(h^2+k^2-a^2)^{3/2}}{(h^2+k^2)}$ (D) None of these

Q.93 The chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at (1, 0) is perpendicular to the line –
(A) $y = x$ (B) $x + y = 0$
(C) $x = 1$ (D) $y = 1$

Q.94 Two circles centered at (2, 3) and (5, 6) intersect each other. If the radii are equal, the equation of the common chord is–
(A) $x + y + 1 = 0$ (B) $x - y + 1 = 0$
(C) $x + y - 8 = 0$ (D) $x - y - 8 = 0$

Q.95 If $2x^2 + 2y^2 + 4x + 5y + 1 = 0$ and $3x^2 + 3y^2 + 6x - 7y + 3k = 0$ are orthogonal, then value of k is –
(A) –17/12 (B) –12/17
(C) 12/17 (D) 17/12

Q.96 The center of a circle which cuts $x^2 + y^2 + 6x - 1 = 0$, $x^2 + y^2 - 3y + 2 = 0$ and $x^2 + y^2 + x + y - 3 = 0$ orthogonally is
(A) (–1/7, 9/7) (B) (1/7, –9/7)
(C) (–1/7, –9/7) (D) (1/7, 9/7)

Q.97 The number of real circles cutting orthogonally the circle $x^2 + y^2 + 2x - 2y + 7 = 0$ is –
(A) 0 (B) 1
(C) 2 (D) infinitely many

Q.98 The length of the chord of the circle $x^2 + y^2 + 3x + 2y - 8 = 0$ intercepted by the y-axis is
(A) 3 (B) 8
(C) 9 (D) 6

PART 4 : RADICAL AXIS, RADICAL CENTRE, FAMILY OF CIRCLES

Q.99 If the point (2, 0), (0, 1), (4, 5) and (0, c) are con-cyclic, then c is equal to
(A) –1, –3/14 (B) –1, –14/3
(C) 14/3, 1 (D) None of these

Q.100 If the line $y = x + 3$ meets the circle $x^2 + y^2 = a^2$ at A and B, then the equation of the circle having AB as a diameter will
(A) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
(B) $x^2 + y^2 + 3x + 3y - a^2 + 9 = 0$
(C) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$
(D) None of these

Q.101 The equation of the circle passing through the point of intersection of the circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$, and also through the point (1, 1) is -
(A) $x^2 + y^2 - 4y + 2 = 0$ (B) $x^2 + y^2 - 3x + 1 = 0$
(C) $x^2 + y^2 - 6x + 4 = 0$ (D) None of these

Q.102 The equation of the circle which passes through points of intersection of circle $x^2 + y^2 + 4x - 5y + 3 = 0$ and $x^2 + y^2 + 2x + 3y - 3 = 0$ and point (–3, 2) is -
(A) $x^2 + y^2 + 8x + 13y - 3 = 0$ (B) $x^2 + y^2 + 13x - 8y + 3 = 0$
(C) $x^2 + y^2 - 13x - 8y + 3 = 0$ (D) $x^2 + y^2 - 13x + 8y + 3 = 0$

Q.103 The radical centre of the of the three circles $x^2 + y^2 = a^2$, $(x - c)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = a^2$ is -
(A) (a/2, b/2) (B) (b/2, c/2)
(C) (c/2, b/2) (D) None of these

Q.104 The equation of the radical axis of two circles $x^2 + y^2 - x + 1 = 0$ and $3(x^2 + y^2) + y - 1 = 0$ is -
(A) $3x + y - 4 = 0$ (B) $3x - y - 4 = 0$
(C) $3x - y + 4 = 0$ (D) None of these

Q.105 Find the coordinate of the point from which the tangents to the circles $x^2 + y^2 - 8x + 40 = 0$; $5x^2 + 5y^2 - 25x + 80 = 0$ $x^2 + y^2 - 8x + 16y + 160 = 0$ are equal in length.
(A) $\left(4, -\frac{15}{8}\right)$ (B) $\left(8, -\frac{15}{4}\right)$
(C) $\left(8, -\frac{15}{8}\right)$ (D) $\left(2, -\frac{15}{8}\right)$

Q.106 The equation of circle which passes through the point of intersection of circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 6y = 0$ and has centre $\left(\frac{3}{2}, \frac{3}{2}\right)$ is –
(A) $x^2 + y^2 - 3x - 3y = 0$
(B) $x^2 + y^2 - 3x - 3y + 9 = 0$
(C) $x^2 + y^2 - 3x - 3y - 9 = 0$
(D) $x^2 + y^2 - 3x - 3y + 5 = 0$

PART 5 : MISCELLANEOUS

Q.107 Equation of a circle $S(x, y) = 0$, $(S(2, 3) = 16)$ which touches the line $3x + 4y - 7 = 0$ at $(1, 1)$ is given by

(A) $x^2 + y^2 + x + 2y - 5 = 0$ (B) $x^2 + y^2 + 2x + 2y - 6 = 0$

(C) $x^2 + y^2 + 4x - 6y = 0$ (D) none of these

Q.108 A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is –

(A) $x^2 + y^2 + ax = 0$ (B) $x^2 + y^2 + ay = 0$

(C) $x^2 + y^2 - ax = 0$ (D) $x^2 + y^2 - ay = 0$

Q.109 The pole of the line $\frac{x}{a} + \frac{y}{b} = 1$ with respect to circle $x^2 + y^2 = c^2$ is –

(A) $\left(\frac{c^2}{a}, \frac{c^2}{b}\right)$ (B) (c/a, b/c)

(C) $\left(\frac{c}{a}, \frac{c}{b}\right)$ (D) $(a^2/c, a^2/c)$

Q.110 If P(2, 8) is an interior point of a circle $x^2 + y^2 - 2x + 4y - p = 0$ which neither touches nor intersects the axes, then set for p is -

(A) $p < -1$ (B) $P < -4$

(C) $p > 96$ (D) ϕ

Q.111 The number of common tangents that can be drawn to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ is

(A) 1 (B) 2

(C) 3 (D) 4

Q.112 The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is –

(A) $9x + 10y - 7 = 0$ (B) $x - y + 2 = 0$

(C) $9x - 10y + 11 = 0$ (D) $9x + 10y + 7 = 0$

Q.113 The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through a fixed point -

(A) (2, 4) (B) (–1/2, –1/4)

(C) (1/2, 1/4) (D) (–2, – 4)

Q.114 If a chord of the circle $x^2 + y^2 = 8$ makes equal to intercepts of length a on the coordinate axes, then

(A) $|a| < 8$ (B) $|a| < 4\sqrt{2}$

(C) $|a| < 4$ (D) $|a| > 4$

Q.115 The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$ is -

(A) 0 (B) 1

(C) – 1 (D) depend on h

Q.116 Two concentric circles are such that the smaller divides the larger into two regions of equal area. If the radius of the smaller circle is 2, then the length of the tangent from any point P on the larger circle to the smaller circle is –

(A) 1 (B) $\sqrt{2}$

(C) 2 (D) None of these

Q.117 The pair of a straight lines joining the origin to the points of inersection of the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 + 2(gx + fy) = 0$ is

(A) $a^2(x^2 + y^2) - 2(gx + fy)^2 = 0$

(B) $a^2(x^2 + y^2) - 4(gx + fy)^2 = 0$

(C) $a^2(x^2 + y^2) + 4(gx + fy)^2 = 4$

(D) $x^2 + y^2 - (gx + fy)^2 = a^2$

Q.118 A circle is drawn touching the x-axis and centre at the point which is the reflection of (a, b) in the line $y - x = 0$. The equation of the circle is-

(A) $x^2 + y^2 - 2bx - 2ay + a^2 = 0$

(B) $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

(C) $x^2 + y^2 - 2ax - 2by + b^2 = 0$

(D) $x^2 + y^2 - 2ax - 2by + a^2 = 0$

Q.119 If the straight line $\frac{2x}{a} + \frac{y}{b} = 2\sqrt{2}$ touches the circle $x^2 + y^2 = 2ab$, a, b > 0, then-

(A) a = b (B) 2a = b

(C) a = 2b (D) None of these

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 The common tangents of two circles intersecting orthogonally are perpendicular. If the ratio of their radii is p then $p+\frac{1}{p}=$

(A) 3 (B) 4
(C) 5 (D) 6

Q.2 Equation of chord AB of circle $x^2+y^2=2$ passing through P(2, 2) such that PB/PA = 3, is given by-

(A) $x=3y$ (B) $x=y$
(C) $y-2=\sqrt{3}\,(x-2)$ (D) none of these

Q.3 The equation of the circle through the point of intersection of $x^2+y^2-1=0$, $x^2+y^2-2x-4y+1=0$ and touching the line $x+2y=0$, is

(A) $x^2+y^2+x+2y=0$ (B) $x^2+y^2-x+20=0$
(C) $x^2+y^2-x-2y=0$ (D) $2(x^2+y^2)-x-2y=0$

Q.4 Two circles with radii 'r_1' and 'r_2', $r_1>r_2\ge 2$, touch each other externally. If 'θ' be the angle between the direct common tangents, then

(A) $\theta=\sin^{-1}\left(\frac{r_1+r_2}{r_1-r_2}\right)$ (B) $\theta=2\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$

(C) $\theta=\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$ (D) none of these

Q.5 If the curves $ax^2+4xy+2y^2+x+y+5=0$ and $ax^2+6xy+5y^2+2x+3y+8=0$ intersect at four concyclic points then the value of a is

(A) 4 (B) –4
(C) 6 (D) –6

Q.6 Area of triangle formed by common tangents to the circle $x^2+y^2-6x=0$ and $x^2+y^2+2x=0$ is –

(A) $3\sqrt{3}$ (B) $2\sqrt{3}$
(C) $9\sqrt{3}$ (D) $6\sqrt{3}$

Q.7 If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2+y^2-2x+4y-11=0$, is 6 units, then minimum distance of T from the director circle of the given circle is –

(A) 6 (B) 12
(C) $6\sqrt{2}$ (D) $12-4\sqrt{2}$

Q.8 A chord AB drawn from the point A (0, 3) at circle $x^2+4x+(y-3)^2=0$ and it meets to M in such a way that AM = 2AB, then the locus of point M will be

(A) Straight line (B) Circle
(C) Parabola (D) None of these

Q.9 A square is inscribed in the circle $x^2+y^2-2x+4y+3=0$, whose sides are parallel to coordinate axes. One vertex of the square is –

(A) $(1+\sqrt{2},-2)$ (B) $(1-\sqrt{2},-2)$
(C) (–2, 1) (D) (2, –3)

Q.10 Set of values of m for which two points P and Q lie on the line $y=mx+8$ so that $\angle APB=\angle AQB=\pi/2$ where $A\equiv(-4,0)$, $B\equiv(4,0)$ is –

(A) $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)-\{-2,2\}$

(B) $[-\sqrt{3},-\sqrt{3}]-\{-2,2\}$

(C) $(-\infty,-1)\cup(1,\infty)-\{-2,2\}$

(D) $\{-\sqrt{3},\sqrt{3}\}$

Q.11 P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the co-ordinate axes cut at right angles, then –

(A) $a^2-6ab+b^2=0$ (B) $a^2+2ab-b^2=0$
(C) $a^2-4ab+b^2=0$ (D) $a^2-8ab+b^2=0$

Q.12 The radical centre of three circles described on the three sides of a triangle as diameter is

(A) the centroid (B) the circumcenter
(C) the incentre of the triangle (D) the orthocenter

Q.13 Minimum radius of circle which is orthogonal with both the circles $x^2+y^2-12x+35=0$ and $x^2+y^2+4x+3=0$ is–

(A) 4 (B) 3
(C) $\sqrt{15}$ (D) 1

Q.14 If the squares of the lengths of the tangents from a point P to the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$ and $x^2+y^2=c^2$ are in A.P., then

(A) a, b, c are in G.P. (B) a, b, c are in AP
(C) a^2, b^2, c^2 are in AP (D) a^2, b^2, c^2 are in GP

Q.15 If r_1 and r_2 are the radii of smallest and largest circles which passes through (5, 6) and touches the circle $(x-2)^2+y^2=4$, then r_1r_2 is –

(A) 4 / 41 (B) 41 / 4
(C) 5 / 41 (D) 41 / 6

Q.16 From a point R (5, 8) two tangents RP and RQ are drawn to a given circle S = 0 whose radius is 5. If circumference of the triangle PQR is (2, 3), then the equation of circle S = 0 is–

(A) $x^2+y^2+2x+4y-20=0$
(B) $x^2+y^2+x+2y-10=0$
(C) $x^2+y^2-x-2y-20=0$
(D) $x^2+y^2-4x-6y-12=0$

Q.17 If $C_1: x^2+y^2=(3+2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents on C_1, where P is any point on the director circle of C_1, then the radius of smallest circle which touch C_1 externally and also the two tangents PA and PB is –

(A) $2\sqrt{2}-3$ (B) $2\sqrt{2}-1$
(C) $2\sqrt{2}+1$ (D) 1

Q.18 A (1, 0) and B (0, 1) and two fixed points on the circle $x^2 + y^2 = 1$. C is a variable point on this circle. As C moves, the locus of the orthocentre of the triangle ABC is –

(A) $x^2 + y^2 - 2x - 2y + 1 = 0$ (B) $x^2 + y^2 - x - y = 0$
(C) $x^2 + y^2 = 4$ (D) $x^2 + y^2 + 2x - 2y + 1 = 0$

Q.19 The locus of the middle points of the chords of the circle $x^2 + y^2 = 4a^2$, which subtends a right angle at the centre of the circle is

(A) $x + y = 2a$ (B) $x^2 + y^2 = 2a^2$
(C) $x^2 + y^2 = a^2$ (D) $x^2 + y^2 = a^2\sqrt{2}$

Q.20 Circum circle of the quadilateral ABCD ,where
$AB \equiv x + y - 10$, $BC \equiv x - 7y + 50 = 0$,
$CD \equiv 22x - 4y + 125 = 0$, $DA \equiv 2x - 4y - 5 = 0$, is

(A) $x^2 + y^2 = 125$ (B) $2x^2 + 2y^2 = 125$
(C) $x^2 + y^2 = 225$ (D) none of these

Q.21 Suppose $f(x) = x^2 - 3x + 1$. If c_1 and c_2 are the two values of 'c' for which the tangent line to the graph of f (x) at the point [c, f (x)] intersects at the point (–3, 0) then $(c_1 + c_2)$ equals –

(A) 6 (B) –6
(C) $2\sqrt{19}$ (D) $-2\sqrt{10}$

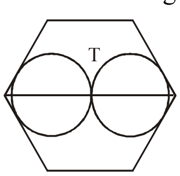
Q.22 The equation of a tangent from the origin to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ is

(A) $y = 0$ (B) $y = \left(\frac{b^2 - a^2}{2ab}\right)x$
(C) $y = \left(\frac{a^2 - b^2}{2ab}\right)x$ (D) $y = \left(\frac{b^2 - a^2}{ab}\right)x$

Q.23 If the line $3x - 4y - k = 0$ touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ at (a, b), then k + a + b is equal to

(A) 20 or – 28 (B) 22 or – 26
(C) –30 or 24 (D) 28 or –20

Q.24 Two circles of equal radii are inscribed within a regular hexagon, as shown in figure. The sides of the hexagon are of length ℓ, and the circles are tangent at T. The common radius of these circles can be expressed as $a\sqrt{3} + b$, where a and b are integers. The value of (a + b) is equal to –



(A) –2 (B) –1
(C) 1 (D) 2

Q.25 If (α, β) is a point on the circle whose centre is on the y-axis and which touches x + y = 0 at (–2, 2), then the greatest value of β is

(A) $4 - \sqrt{2}$ (B) 6
(C) $4 + 2\sqrt{2}$ (D) $4 + \sqrt{2}$

Q.26 Radius (R < 4) of a circle which touches the circle $x^2 + y^2 = 16$ externally and angle between the direct common tangents is $\tan^{-1}(24/7)$ is –

(A) 3 (B) 2
(C) 1/2 (D) 1

Q.27 Consider a family of circles passing through the intersection point of the lines $\sqrt{3}(y - 1) = x - 1$ & $y - 1 = \sqrt{3}(x - 1)$ and having its centre on the acute angle bisector of the given lines. The common chords of each member of the family and the circle $x^2 + y^2 + 4x - 6y + 5 = 0$ are concurrent. Find the point of concurrency.

(A) (1/2, 3/2) (B) (1, 2)
(C) (2, 3) (D) (1, 1)

Q.28 The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is:

(A) (–16/5, 27/10) (B) (–16/7, 53/10)
(C) (–16/5, 53/10) (D) none of these

Q.29 Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point at a distance of 4 from the point (8, 6) on the circle, is –

(A) $4x + 3y - 50 = 0$ (B) $4x + 3y - 100 = 0$
(C) $4x + 3y - 46 = 0$ (D) None of these

Q.30 In a circle with centre 'O' PA and PB are two chords. PC is the chord that bisects the angle APB. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R respectively. If QC = 3, QA = 2 and RC = 4, then length of RB equals –

(A) 2 (B) 8/3
(C) 10/3 (D) 11/3

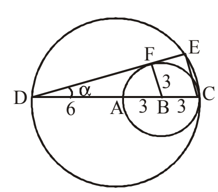
Q.31 Let C_1 and C_2 are circles defined by $x^2 + y^2 - 20x + 64 = 0$ and $x^2 + y^2 + 30x + 144 = 0$. The length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q is –

(A) 15 (B) 18
(C) 20 (D) 24

Q.32 The minimum value of $(x_1 - x_2)^2 + (\sqrt{1 - x_1^2} - (3 - x_2))^2$ for all possible real values of x_1 and x_2 is –

(A) $\frac{3}{\sqrt{2}} - 1$ (B) $\frac{11}{2} - 3\sqrt{2}$
(C) $\frac{3}{\sqrt{2}}$ (D) $\frac{11}{2} + 3\sqrt{2}$

Q.33 In the diagram, DC is a diameter of the large circle centred at A, and AC is a diameter of the smaller circle centred at B. If DE is tangent to the smaller circle at F and DC = 12 then the length DE is –



(A) $8\sqrt{2}$ (B) 16
(C) $9\sqrt{2}$ (D) $10\sqrt{2}$

Q.34 If θ is the angle between the two radius (one to each circle) drawn from one of the point of intersection of two circles $x^2+y^2=a^2$ and $(x-c)^2+y^2=b^2$, then the length of common chord of two circles is

(A) $\frac{ab\sin\theta}{\sqrt{a^2+b^2-2ab\cos\theta}}$ (B) $\frac{2ab\sin\theta}{\sqrt{a^2+b^2-2ab\cos\theta}}$

(C) $\frac{2ab}{\sqrt{a^2+b^2-2ab\cos\theta}}$ (D) none of these

Q.35 A circle is tangent to the y-axis at (0, 2) and cuts the positive x-axis at two distinct points A and B (OB > OA), the coordinate of the point B being (8, 0). The radius of the circle is –

(A) 9/2 (B) 15/4

(C) 17/4 (D) $\sqrt{17}/2$

Q.36 Triangle ABC is right angled at A. The circle with centre A and radius AB cuts BC and AC internally at D and E respectively. If BD = 20 and DC = 16 then the length AC equals –

(A) $6\sqrt{21}$ (B) $6\sqrt{26}$

(C) 30 (D) 32

Q.37 The point A(2 , 1) is outside the circle $x^2+y^2+2gx+2fy+c=0$ and AP,AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is

(A) $(x+g)(x-2)+(y+f)(y-1)=0$

(B) $(x+g)(x-2)-(y+f)(y-1)=0$

(C) $(x-g)(x+2)+(y-f)(y+1)=0$

(D) none of these

Q.38 A rhombus is inscribed in the region common to the two circles $x^2+y^2-4x-12=0$ and $x^2+y^2+4x-12=0$ with two of its vertices on the line joining the centres of the circles . The area of the rhombus is

(A) $8\sqrt{3}$ sq. units (B) $4\sqrt{3}$ sq. units

(C) $16\sqrt{3}$ sq. units (D) none of these

ASSERTION AND REASON QUESTIONS

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement -1 is True, Statement-2 is False.

(D) Statement -1 is False, Statement-2 is True.

(E) Statement -1 is False, Statement-2 is False.

Q.39 Tangents are drawn from the origin to the circle $x^2+y^2-2hx-2hy+h^2=0$.

Statement 1: Angle between the tangents is π/2.

Statement 2: The given circle is touching the co-ordinate axes.

Q.40 Number of common tangents of $x^2+y^2-2x-4y-95=0$ and $x^2+y^2-6x-8y+16=0$ is zero.

Statement 2 : If $C_1C_2<|r_1-r_2|$, then there will be no common tangent. (where C_1, C_2 are the centre and r_1, r_2 are radii of circles).

Q.41 **Statement-1 :** Number of circles passing through (1, 2), (4, 7) and (3, 0) is one.

Statement-2 : One and only circle can be made to pass through three non-collinear points.

MATCH THE COLUMN TYPE QUESTIONS

Q.42 Consider two circles C_1 of radius a and C_2 of radius b (b>a) both lying in the first quadrant and touching the coordinate axes. In each of the conditions listed in column I, the ratio of b/a is given in column II.

Column I	Column II
(a) C_1 and C_2 touch each other	(p) $2+\sqrt{2}$
(b) C_1 and C_2 are orthogonal	(q) 3
(c) C_1 and C_2 intersect so that the common chord is longest	(r) $2+\sqrt{3}$
(d) C_2 passes through the centre	(s) $3+2\sqrt{2}$
of C_1	(t) $3-2\sqrt{2}$

Code :

(A) a-s, b-r, c-q, d-p (B) a-p, b-q, c-r, d-s

(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q

Q.43 Match the column –

Column I	Column II
(a) The greatest distance between $x^2+y^2-2x-2y+1=0$, $x^2+y^2-10x+4y+20=0$ is 3λ, then λ is	(p) 3
(b) Numerical value of radius of the circle circumscribed about a square of area 200 sq. units is	(q) 7
(c) Minimum value of $\cos^4x-6\cos^2x+5$ is	(r) 0
(d) If a chord of the circle $x^2+y^2-4x-2y+k=0$ is trisected at the points (1/3, 1/3) and (8/3, 8/3) and ℓ is the length of the chord then $\ell/\sqrt{2}$ =	(s) 10

Code :

(A) a-r, b-p, c-q, d-r (B) a-p, b-q, c-r, d-s

(C) a-r, b-q, c-s, d-p (D) a-p, b-s, c-r, d-q

Q.44 Match the column –

Column I	Column II
(a) Length of direct common tangent between circles $x^2+y^2+14x-4y+28=0$ and $x^2+y^2-14x+4y-28=0$ is	(p) 4
(b) Number of values of m for which line $(y-2)=m(x-1)$ cuts the circle $x^2+y^2=5$ at two real points are	(q) 14
(c) The maximum number of points with rational coordinates on a circle whose centre is $(0,\sqrt{2})$ are	(r) 2
(d) Number of circles touching both the axes and the line $x+y=4$ are	(s) infinite

Code :

(A) a-r, b-p, c-q, d-r (B) a-p, b-q, c-r, d-s

(C) a-q, b-s, c-r, d-p (D) a-r, b-s, c-p, d-q

PASSAGE BASED QUESTIONS

Passage 1- (Q.45-Q.47)

Let $x^2 + y^2 = 1$ be the equation of the circumcircle of a ΔABC. If $P(\alpha, \beta)$ be a point on the circle but not a vertex of ΔABC, perpendiculars PD, PE and PF are drawn to the three sides BC, CA and AB of triangle ABC. X, Y and Z are feet of perpendiculars from A, B and C to the sides BC, CA and AB respectively and H is the orthocentre of ΔABC and I, I_1, I_2 and I_3 are incentre and ex-centres of ΔABC. Let R, R_1, R_2, R_3 are radii of circumcircle of $\Delta I_1I_2I_3$, ΔII_2I_3, ΔII_1I_3, ΔII_1I_2.

Q.45 Points D, E and F –
(A) form a right angled Δ (B) form an equilateral Δ
(C) form an isosceles Δ (D) are collinear

Q.46 I_1 is the orthocentre of –
(A) $\Delta I_1I_2I_3$ (B) ΔII_1I_2
(C) ΔII_1I_3 (D) ΔII_2I_3

Q.47 Ex-centred of ΔXYZ –
(A) lie inside the ΔABC.
(B) are the corresponding vertices of the ΔABC.
(C) may lie inside or outside depending on ΔABC is acute or obtuse angled.
(D) None of these

Passage 2 : (Q.48-Q.50)

Three circles are given by $S_1 \equiv x^2 + y^2 = 4$,
$S_2 \equiv (x-4)^2 + (y-4)^2 = 4$,
$S_3 \equiv x^2 + y^2 - 6x + 8y + 24 = 0$

Q.48 Centre of that circle which cuts the circles S_1, S_2, S_3 orthogonally is
(A) $\left(\frac{2}{7}, \frac{30}{7}\right)$ (B) $\left(-\frac{30}{7}, \frac{2}{7}\right)$
(C) $\left(\frac{30}{7}, \frac{2}{7}\right)$ (D) $\left(\frac{30}{7}, -\frac{2}{7}\right)$

Q.49 Radius of the circle obtained above is
(A) $4\frac{\sqrt{177}}{7}$ (B) $2\frac{\sqrt{177}}{7}$
(C) $\frac{\sqrt{177}}{7}$ (D) $8\frac{\sqrt{177}}{7}$

Q.50 Point of intersection of direct tangents between S_1 and S_3 always lies on the line
(A) $3y - 8x = 0$ (B) $4y + 3x = 0$
(C) $3y + 4x = 0$ (D) $3y + 4x + 2 = 0$

Passage 3 : (Q.51-Q.53)

P is a variable point on the line L = 0. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQRS is completed.

Q.51 If $L \equiv 2x + y - 6 = 0$, then the locus of circumcentre of ΔPQR is –
(A) $2x - y = 4$ (B) $2x + y = 3$
(C) $x - 2y = 4$ (D) $x + 2y = 3$

Q.52 If $P \equiv (6, 8)$, then the area of ΔQRS is –
(A) $\frac{(6)^{3/2}}{25}$ sq. units (B) $\frac{(24)^{3/2}}{25}$ sq. units
(C) $\frac{48\sqrt{6}}{25}$ sq. units (D) $\frac{196\sqrt{6}}{25}$ sq. units

Q.53 If $P \equiv (3, 4)$, then coordinate of S is –
(A) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (B) $\left(-\frac{51}{25}, -\frac{68}{25}\right)$
(C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

Passage 4 : (Q.54-Q.56)

Consider the circles : $S_1 : x^2 + y^2 - 6y + 5 = 0$, $S_2 : x^2 + y^2 - 12x + 35 = 0$ and a variable circle $S : x^2 + y^2 + 2gx + 2fy + c = 0$.

Q.54 Number of common tangents to S_1 and S_2 is –
(A) 1 (B) 2
(C) 3 (D) 4

Q.55 Length of a transverse common tangent to S_1 and S_2 is –
(A) 6 (B) $2\sqrt{11}$
(C) $\sqrt{35}$ (D) $11\sqrt{2}$

Q.56 If the variable circle S = 0 with centre C moves in such a way that it is always touching externally the circles $S_1 = 0$ and $S_2 = 0$ then the locus of the centre C of the variable circle is –
(A) a circle (B) a parabola
(C) an ellipse (D) a hyperbola

Passage 5 : (Q.57-Q.59)

Let f (x, y) = 0 be the equation of a circle such that f (0, y) = 0 has equal real roots has f (x, 0) = 0 has two distinct real roots. Let g (x, y) = 0 be the locus of point P from where tangents to circle f (x, y) = 0 make angle $\pi/3$ between them and $g(x, y) = x^2 + y^2 - 5x - 4y + c,\ c \in R$.

Q.57 Let Q be a point from where tangents drawn to circle g (x, y) = 0 are mutually perpendicular. If A, B are the points of contact of tangents drawn from Q to circle g (x, y) = 0, then area of triangle QAB is –
(A) 25/12 (B) 25/8
(C) 25/4 (D) 25/2

Q.58 The area of region bounded by circle f (x, y) = 0 with axis in the first quadrant is –
(A) $3 + \frac{25}{8}\left(\pi - \tan^{-1}\frac{1}{2}\right)$ (B) $3 + \frac{25}{8}\tan^{-1}\left(\frac{24}{11}\right)$
(C) $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\frac{3}{4}\right)$ (D) $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\frac{24}{7}\right)$

Q.59 The number of points with positive integral coordinates satisfying $f(x, y) > 0$, $g(x, y) < 0$; $y > 3$ and $x < 6$ is –
(A) 7 (B) 8
(C) 10 (D) 11

Passage 6 -(Q.60-Q.62)

A ball is moving around the circle $14x^2 + 14y^2 + 216x - 69y + 432 = 0$ in clockwise direction leaves it tangentially at the point P(–3, 6). After getting reflected from a straight line L = 0 it passes through the center of the circle. The perpendicular distance of this straight line L = 0 from the point P is $\frac{11}{13}\sqrt{130}$. You can assume that the angle of incidence is equal to the angle of reflection.

Q.60 The equation of tangent to the circle at P is
(A) $2x - y + 12 = 0$ (B) $4x + 3y - 6 = 0$
(C) $3x - 2y + 21 = 0$ (D) $2x + 5y - 24 = 0$

Q.61 Radius of the circle is
(A) 165/14 (B) 165/46
(C) 165/28 (D) none of these

Q.62 If angle between the tangent at P and the line through 'P' perpendicular to the line L = 0 is θ, then tan θ is
(A) 2/11 (B) 3/11
(C) 4/11 (D) None of these

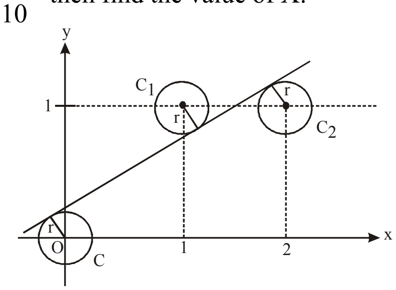
EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Two circles each of radius 5 units, touch each other at (1, 2). If the equation of their indirect common tangent is 4x + 3y = 10 and the equations of two circles are $x^2 + y^2 + \alpha x + \beta y - 15 = 0$, $x^2 + y^2 + \gamma x + \delta y + 25 = 0$, then find the value of $(\alpha + \beta) - (\gamma + \delta)$.

Q.2 If the tangents are drawn from any point on the line x + y = 3 to the circle $x^2 + y^2 = 9$, then the chord of contact passes through the point (3, a) then find the value of a.

Q.3 As shown in the figure, three circles which have the same radius r, have centres at (0, 0) ; (1, 1) and (2, 1). If they have a common tangent line, as shown then, their radius r is $\frac{\sqrt{X}}{10}$ then find the value of X.



Q.4 The circle passing through the distinct points (1, t), (t, 1) & (t, t) for all values of 't', passes through the point (a, b). Find the value of (a + b).

Q.5 If p_1 and p_2 are the two values of p for which two perpendicular tangents can be drawn from the origin to the circle $x^2 - 6x + y^2 - 2py + 17 = 0$, then find the value of $(p_1^2 + p_2^2)$.

Q.6 Let A (– 4, 0) and B (4, 0). Number of points C = (x, y) on the circle $x^2 + y^2 = 16$ such that the area of the triangle whose vertices are A, B and C is a positive integer, is.

Q.7 A point moving around a circle $x^2 + y^2 + 8x + 4y - 5 = 0$ with centre C broke away from it either at the point A or at the point B on the circle and moved along a tangent to the circle passing through the point D (3, –3). Find the area of the quadrilateral ABCD.

Q.8 Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and centres (–3, 0), (–1, 0), (1, 0) and (3, 0) respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x.

Q.9 Circles A and B are externally tangent to each other and to line *t*. The sum of the radii of the two circles is 12 and the radius of circle A is 3 times that of circle B. The area in between the two circles and its external tangent is $a\sqrt{3} - \frac{b\pi}{2}$ then find the value of a + b.

Q.10 A circle lying in 1^{st} quadrant touches x and y axis at point P and Q respectively. BC and AD are parallel tangents to the circle with slope –1. If the points A and B are on the y axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ square units then the radius of circle is –

Q.11 Let W_1 and W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the smallest positive value of 'a' for which the line y = ax contains the centre of a circle that is externally tangent to W_2 and internally tangent to W_1. Given that $m^2 = p/q$ where p and q are relatively prime integers, find (p + q).

Q.12 If the tangent at the point P on the circle $x^2+y^2+ 6x + 6y = 2$ meets the straight line 5x – 2y + 6 = 0 at a point Q on the y-axis, then length of PQ is :

Q.13 If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

Q.14 Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

Q.15 Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of π/k and 2π/k, where k > 0, then the value of [k] is : [Note : [k] denotes the largest integer less than or equal to k].

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 The square of the length of tangent from (3, – 4) on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$- **[AIEEE-2002]**
(A) 20 (B) 30
(C) 40 (D) 50

Q.2 Radical axis of the circle $x^2 + y^2 + 6x - 2y - 9 = 0$ and $x^2 + y^2 - 2x + 9y - 11 = 0$ is – **[AIEEE-2002]**
(A) $8x - 11y + 2 = 0$ (B) $8x + 11y + 2 = 0$
(C) $8x - 11y - 2 = 0$ (D) $8x + 11y - 2 = 0$

Q.3 If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then **[AIEEE-2003]**
(A) $r > 2$ (B) $2 < r < 8$
(C) $r < 2$ (D) $r = 2$

Q.4 The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is **[AIEEE-2003]**
(A) $x^2 + y^2 - 2x + 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 62$
(C) $x^2 + y^2 + 2x - 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 47$

Q.5 If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is- **[AIEEE-2004]**
(A) $2ax + 2by + (a^2 + b^2 + 4) = 0$
(B) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(C) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(D) $2ax - 2by - (a^2 + b^2 + 4) = 0$

Q.6 A variable circle passes through the fixed point A(p, q) and touches x- axis. The locus of the other end of the diameter through A is- **[AIEEE-2004]**
(A) $(x-p)^2 = 4qy$ (B) $(x-q)^2 = 4py$
(C) $(y-p)^2 = 4qx$ (D) $(y-q)^2 = 4px$

Q.7 If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- **[AIEEE-2004]**
(A) $x^2 + y^2 - 2x + 2y - 23 = 0$
(B) $x^2 + y^2 - 2x - 2y - 23 = 0$
(C) $x^2 + y^2 + 2x + 2y - 23 = 0$
(D) $x^2 + y^2 + 2x - 2y - 23 = 0$

Q.8 The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is: **[AIEEE-2004]**
(A) $x^2 + y^2 - x - y = 0$ (B) $x^2 + y^2 - x + y = 0$
(C) $x^2 + y^2 + x + y = 0$ (D) $x^2 + y^2 + x - y = 0$

Q.9 If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q then the line $5x + by - a = 0$ passes through P and Q for - **[AIEEE-2005]**
(A) exactly one value of a (B) no value of a
(C) infinitely many values of a (D) exactly two values of a

Q.10 A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is- **[AIEEE-2005]**
(A) an ellipse (B) a circle
(C) a hyperbola (D) a parabola

Q.11 If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is - **[AIEEE-2005]**
(A) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(B) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(C) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$
(D) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

Q.12 If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then – **[AIEEE-2005]**
(A) $3a^2 - 10ab + 3b^2 = 0$ (B) $3a^2 - 2ab + 3b^2 = 0$
(C) $3a^2 + 10ab + 3b^2 = 0$ (D) $3a^2 + 2ab + 3b^2 = 0$

Q.13 If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is– **[AIEEE-2006]**
(A) $x^2 + y^2 + 2x - 2y - 62 = 0$
(B) $x^2 + y^2 - 2x + 2y - 62 = 0$
(C) $x^2 + y^2 - 2x + 2y - 47 = 0$
(D) $x^2 + y^2 + 2x - 2y - 47 = 0$

Q.14 Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $2\pi/3$ at its centre is – **[AIEEE-2006]**
(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = \frac{27}{4}$
(C) $x^2 + y^2 = \frac{9}{4}$ (D) $x^2 + y^2 = \frac{3}{2}$

Q.15 Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval- **[AIEEE-2007]**
(A) $0 < k < 1/2$ (B) $k \geq 1/2$
(C) $-1/2 \leq k \leq 1/2$ (D) $k \leq 1/2$

Q.16 The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is - **[AIEEE-2008]**
(A) (–3, 4) (B) (–3, –4)
(C) (3, 4) (D) (3, – 4)

Q.17 If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for – **[AIEEE-2009]**
(A) exactly one value of p (B) all values of p
(C) all except one value of p (D) all except two values of p

Q.18 The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if – **[AIEEE 2010]**
(A) $-35 < m < 15$ (B) $15 < m < 65$
(C) $35 < m < 85$ (D) $-85 < m < -35$

Q.19 The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if : **[AIEEE 2011]**
(A) $2|a| = c$ (B) $|a| = c$
(C) $a = 2c$ (D) $|a| = 2c$

Q.20 The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is – **[AIEEE 2012]**
(A) 10/3 (B) 3/5
(C) 6/5 (D) 5/3

Q.21 The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point – **[JEE MAIN 2013]**
(A) (–5, 2) (B) (2, –5)
(C) (5, –2) (D) (–2, 5)

Q.22 Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to – **[JEE MAIN 2014]**
(A) $\sqrt{3}/\sqrt{2}$ (B) $\sqrt{3}/2$
(C) 1/2 (D) 1/4

Q.23 Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in R$, is a
(A) Straight line parallel to y-axis **[JEE MAIN 2015]**
(B) Circle of radius $\sqrt{2}$
(C) Circle of radius $\sqrt{3}$
(D) Straight line parallel to x-axis

Q.24 The number of common tangents to the circles
$x^2 + y^2 - 4x - 6y - 12 = 0$ and
$x^2 + y^2 + 6x + 18y + 26 = 0$, is **[JEE MAIN 2015]**
(A) 2 (B) 3
(C) 4 (D) 1

Q.25 If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$ is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is :
(A) $5\sqrt{3}$ (B) 5 **[JEE MAIN 2016]**
(C) 10 (D) $5\sqrt{2}$

Q.26 The radius of a circle, having minimum area , which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is **[JEE MAIN 2017]**
(A) $4(\sqrt{2} - 1)$ (B) $4(\sqrt{2} + 1)$
(C) $2(\sqrt{2} + 1)$ (D) $2(\sqrt{2} - 1)$

Q.27 Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then **[JEE MAIN 2019 (JAN)]**
(A) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (B) a, b, c are in A. P.
(C) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P. (D) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Q.28 The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n, n \in N$, where N is the set of all natural numbers, is : **[JEE MAIN 2019 (APRIL)]**
(A) 320 (B) 160
(C) 105 (D) 210

Q.29 The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is : **[JEE MAIN 2019 (APRIL)]**
(A) 1/3 (B) $4/\sqrt{3}$
(C) $1/\sqrt{3}$ (D) $2/\sqrt{3}$

Q.30 If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is **[JEE MAIN 2019 (APRIL)]**
(A) $x^2 + y^2 - 2xy = 0$ (B) $x^2 + y^2 - 16x^2y^2 = 0$
(C) $x^2 + y^2 - 4x^2y^2 = 0$ (D) $x^2 + y^2 - 2x^2y^2 = 0$

Q.31 The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point : **[JEE MAIN 2019 (APRIL)]**
(A) (–4, 6) (B) (6, –2)
(C) (–6, 4) (D) (4, –2)

Q.32 If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0, (K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for : **[JEE MAIN 2019 (APRIL)]**
(A) exactly two values of K
(B) exactly one value of K
(C) no value of K.
(D) infinitely many values of K

Q.33 The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is : **[JEE MAIN 2019 (APRIL)]**
(A) $3\sqrt{2}$ (B) 3
(C) $2\sqrt{2}$ (D) 2

Q.34 The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is : **[JEE MAIN 2019 (APRIL)]**
(A) $y = \sqrt{1 + 4x}, x \geq 0$ (B) $x = \sqrt{1 + 4y}, y \geq 0$
(C) $x = \sqrt{1 + 2y}, y \geq 0$ (D) $y = \sqrt{1 + 2x}, x \geq 0$

Q.35 If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is **[JEE MAIN 2019 (APRIL)]**
(A) 60/13 (B) 120/13
(C) 13/2 (D) 13/5

Q.36 A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point : **[JEE MAIN 2019 (APRIL)]**
(A) (3, 10) (B) (2, 3)
(C) (1, 5) (D) (3, 5)

Q.37 Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to : **[JEE MAIN 2020 (JAN)]**
(A) 64 / 5 (B) 24 / 5
(C) 8 / 5 (D) 8 / 13

Q.38 If $y = mx + c$ is a tangent to the circle $(x-3)^2 + y^2 = 1$ and also the perpendicular to the tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then **[JEE MAIN 2020 (JAN)]**

(A) $c^2 + 6c + 7 = 0$ (B) $c^2 - 6c + 7 = 0$
(C) $c^2 + 6c - 7 = 0$ (D) $c^2 - 6c - 7 = 0$

Q.39 A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle ? **[JEE MAIN 2020 (JAN)]**

(A) $3x - 4y - 24 = 0$ (B) $3x + 4y - 6 = 0$
(C) $4x + 3y - 8 = 0$ (D) $4x - 3y + 17 = 0$

Q.40 If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is _______. **[JEE MAIN 2020 (JAN)]**

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	D	B	A	D	C	A	A	C	C	C	B	D	B	A	C	B	D	A	A	A	B	B	C	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	B	A	D	A	D	C	A	A	D	A	D	A	C	B	B	A	A	D	D	A	C	B	C	D
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	C	C	A	B	A	C	B	B	C	A	B	C	A	B	D	B	A	D	B	B	A	A	A	D
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	A	B	C	C	D	C	C	A	D	C	D	C	B	D	C	C	C	D	C	A	A	A	D	C	A
Q	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119						
A	B	B	C	A	C	A	A	C	A	D	C	C	C	C	C	C	B	B	C						

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	B	C	B	B	A	D	B	D	A	C	D	C	C	B	A	D	A	B	B	B	B	A	B	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	A	C	C	B	C	A	A	B	C	B	A	A	A	A	D	A	D	C	D	D	B	D	B	C
Q	51	52	53	54	55	56	57	58	59	60	61	62													
A	B	D	B	D	A	D	D	D	D	B	C	B													

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	28	3	5	2	50	62	25	63	69	15	169	5	3	2	2

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	A	B	D	B	A	A	A	B	D	D	D	C	C	B	B	B	A	B	A
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	C	D	B	B	A	A	A	D	D	C	B	C	C	D	B	A	A	A	C	36

CHAPTER- 10 :
CIRCLE
SOLUTIONS TO TRY IT YOURSELF
TRY IT YOURSELF-1

(1) (C)

(2) **(C).** Centroid of the triangle coincides with the centre of the circle and the radius of the circle is 2/3 of the length of the median]

(3) The given equation is $x^2 + y^2 - 4x - 8y - 45 = 0$

$\Rightarrow \quad (x^2 - 4x) + (y^2 - 8y) = 45$

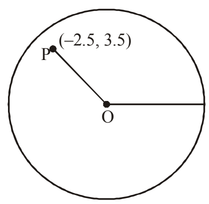
Adding 4 and 16 to make perfect squares, we get

$\Rightarrow \quad (x^2 - 4x + 4) + (y^2 - 8y + 16) = 45 + 4 + 16$

$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 65$

Radius = $\sqrt{65}$.

(4) **Inside.**



Here the given circle is $x^2 + y^2 = 25$.

Its centre O is (0, 0) and radius r is 5.

Let P be a point (–2.5, 3.5).

$OP^2 = (-2.5 - 0)^2 + (3.5 - 0)^2$

$OP^2 = 6.25 + 12.25 = 18.5$

Here, r = 5 and OP = $\sqrt{18.5} = 4.3$

OP < r

Hence, the point (–2.5, 3.5) lies inside the circle, since the distance of the point to the centre of the circle is less than the radius of the circle.

(5) Let the equation of the circle be

$(x - h)^2 + (y - k)^2 = r^2$ .... (1)

Since the circle (1) passes through (2, 3) and (–1, 1)

We have, $(2 - h)^2 + (3 - k)^2 = r^2$

$\Rightarrow \quad h^2 - 4h + 4 + k^2 - 6k + 9 = r^2$ .... (2)

$(-1 - h)^2 + (1 - k)^2 = r^2$

$\Rightarrow \quad h^2 + 2h + 1 + k^2 - 2k + 1 = r^2$ .... (3)

Centre of the circle as on

$x - 3y - 11 = 0$, so $h - 3k - 11 = 0$ .... (4)

On subtracting (3) from (2), we get

$-6h - 4k + 3 + 8 = 0$

$\Rightarrow \quad 6h + 4k = 11$ .... (5)

On solving, (4) and (5), we have

$$h = \frac{7}{2},\ k = \frac{-5}{2}$$

On putting the value of h and k in (2), we get

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2 \Rightarrow r^2 = \frac{65}{2}$$

Therefore, the equation of the circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}$$

$$x^2 - 7x + \frac{49}{4} + y^2 + 5y + \frac{25}{4} = \frac{65}{2}$$

$x^2 + y^2 - 7x + 5y - 14 = 0$

(6) **(A).** If area of circle is double then $R' = \sqrt{2}R$

(R' = radius of new circle)

then $R'^2 = 2R^2$

or $\quad (\sqrt{9+36-k})^2 = 2(\sqrt{9+36-15})^2$

or $\quad 45 - k = 2\,(30)$

or $k = -15$

(7) **(D).** Let the equation of tangent is y = mx then

$$\left|\frac{mr - h}{\sqrt{1+m^2}}\right| = \sqrt{r^2 + h^2 - h^2}$$

$$\Rightarrow \left|\frac{mr - h}{\sqrt{1+m^2}}\right| = r \Rightarrow m^2r^2 + h^2 - 2mrh = r^2\,(1 + m^2)$$

$\Rightarrow \quad h^2 - r^2 - 2mrh = 0$

$$\Rightarrow \quad m = \frac{h^2 - r^2}{2rh} \text{ or one root is } \infty$$

$\therefore \quad (h^2 - r^2)\,x - 2rhy = 0,\ x = 0$

(8) **(D).** Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

If it is passes through (0, 0) and (1, 0).

$1 + 2g = 0$ .... (1)

or $\quad g = -1/2$

If circle touches $x^2 + y^2 = 9$ then distance between centre = sum of radii or difference of radii.

$\therefore \quad \sqrt{g^2 + f^2} = \sqrt{g^2 + f^2} \pm 3$ and $f = \pm\sqrt{2}$

$\therefore \quad$ Centre is $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

(9) **(D).** Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ as it passes through (–1,0) & (0,2)

$\therefore \quad 1 - 2g + c = 0$ and $4 + 4f + c = 0$

Also $f^2 = c$

$\Rightarrow \quad f = -2,\ c = 4\ ;\ g = 5/2$

$\therefore \quad$ Equation of circle is $x^2 + y^2 + 5x - 4y + 4 = 0$ which passes through (– 4, 0)

(10) **(C).** Line $5x - 2y + 6 = 0$ is intersected by tangent at P to circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis at Q (0, 3).

In other words tangent passes through (0, 3).

$\therefore \quad$ PQ = length of tangent to circle from (0, 3).

$$= \sqrt{0 + 9 + 0 + 18 - 2} = \sqrt{25} = 5$$

(11) **(AC).** Equation of circle can be written as

$(x - 3)^2 + y^2 + \lambda\,(y) = 0$

$\Rightarrow \quad x^2 + y^2 - 6x + \lambda y + 9 = 0.$

Now, $(\text{radius})^2 = 7 + 9 = 16$

$$\Rightarrow \quad 9 + \frac{\lambda^2}{4} - 9 = 16 \Rightarrow \lambda^2 = 64 \Rightarrow \lambda = \pm 8.$$

$\therefore \quad$ Equation is $x^2 + y^2 - 6x \pm 8y + 9 = 0.$

TRY IT YOURSELF-2

(1) Any circle passing through the point of intersection of the given line and circle has the equation $x^2 + y^2 - 9 + \lambda (x + y - 1) = 0$. Its centre $= (-\lambda/2, -\lambda/2)$
The circle is the smallest if $(-\lambda/2, -\lambda/2)$ is on the chord $x + y = 1$.

$$\Rightarrow \; -\frac{\lambda}{2} - \frac{\lambda}{2} \Rightarrow \lambda = -1$$

Putting this value for λ, the equation of the smallest circle is $x^2 + y^2 - 9 - (x + y - 1) = 0$.

(2) **(C).** Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ then for circles

$x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$

$2g(2) + 2f(-3) = c + 9$ .... (1)

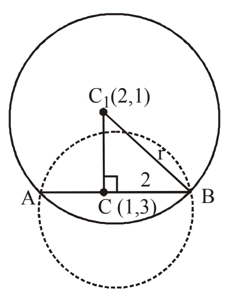
$2g(-2) + 2f(3) = c + 4$ .... (2)

or eliminating c, we get, $8x - 12y - 5 = 0$

(3) **(C).** The given circle is $x^2 + y^2 - 2x - 6y + 6 = 0$

with centre C (1, 3) and radius $= \sqrt{1 + 9 - 6} = 2$.

Let AB be one of its diameter which is the chord of other circle with centre at C_1 (2, 1).



Then in ΔC_1CB, $C_1B^2 = CC_1^{\,2} + CB^2$

$\Rightarrow \; r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$

$\Rightarrow \; r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3$

(4) **(A).** Let mid point be (h, k), then chord of contact :

$hx + ky = h^2 + k^2$ ....(i)

Let any point on the line $4x - 5y = 20$ be

$$\left(x_1, \frac{4x_1 - 20}{5}\right)$$

$\therefore$ Chord of contact : $5x_1x + (4x_1 - 20)y = 45$ ....(ii)

(i) and (ii) are same

$$\therefore \; \frac{5x_1}{h} = \frac{4x_1 - 20}{k} = \frac{45}{h^2 + k^2}$$

$$\Rightarrow \; x_1 = \frac{9h}{h^2 + k^2} \text{ and } x_1 = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$$\Rightarrow \; \frac{9h}{h^2 + k^2} = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$\Rightarrow \; 20(h^2 + k^2) - 36h + 45k = 0$

$\Rightarrow$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

(5) **(C).** $4\ell^2 - 5m^2 + 6\ell + 1 = 0$

$\ell x + my + 1 = 0$

Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$, $r = \sqrt{g^2 + f^2 - c}$

$$\left| \frac{-g\ell - mf + 1}{\sqrt{\ell^2 + m^2}} \right| = \sqrt{g^2 + f^2 - c}$$

$g^2\ell^2 + m^2f^2 + 1 + 2fg\ell m - 2g\ell - 2mf = (g^2 + f^2 - c)(\ell^2 + m^2)$

$g^2\ell^2 + m^2f^2 + 1 + 2fg\ell m - 2g\ell - 2mf$

$= g^2\ell^2 + g^2m^2 + f^2\ell^2 + f^2m^2 + c\ell^2 - cm^2$

$\ell^2(f^2 - c) + m^2(g^2 - c) + 2g\ell + 2mf - 2g\ell m - 1 = 0$

$4\ell^2 - 5m^2 + 6\ell + 1 = 0$

On comparing we get $f = 0$

$$\text{Also, } \frac{f^2 - c}{4} = \frac{g^2 - c}{-5} = \frac{2g}{6} = \frac{-1}{1}$$

$f^2 - c = -4$, $g^2 - c = 5$, $g = -3$

$c = 4$, $f = 0$

Centre (3, 0), radius $= \sqrt{9 - 4} = \sqrt{5}$

(6) **(A).** Equation of circle becomes

$x^2 + y^2 - 6x + 4 = 0$

Let a point on $x + y - 1 = 0$ be $(h, 1 - h)$

Chord of contact from $(h, 1 - h)$

$hx + (1 - h)y - 3(x + h) + 4 = 0$

$h(x - y - 3) + y - 3x + 4 = 0$

On solving, $x - y - 3 = 0$ and $y - 3x + 4 = 0$

We get, $x = 1/2$, $y = 5/2$

(7) **(C).** $S = x^2 + y^2 - 6x + 4$

$S_1 = (2)^2 + (-3)^2 - 6(2) + 4 = 4 + 9 - 12 + 4 > 0$

Hence, two tangents can be drawn from (2, –3).

CHAPTER- 10 : CIRCLE
EXERCISE-1

(1) **(B).** $2g=-2 \Rightarrow g=-1$
$2f=4 \Rightarrow f=2 \Rightarrow$ Centre is $(1,-2)$

(2) **(D).** First making the coefficient of x^2 and y^2, 1 by dividing the equation with 2

$$\Rightarrow x^2+y^2+2x-\frac{3}{2}y+\frac{1}{2}=0$$

$2g=2 \Rightarrow g=1$

$$2f=-\frac{3}{2} \Rightarrow f=-\frac{3}{4},\ c=\frac{1}{2}$$

$$\Rightarrow r=\sqrt{(1)^2+\left(-\frac{3}{4}\right)^2-\frac{1}{2}}=\sqrt{\frac{17}{16}}=\frac{\sqrt{17}}{4}$$

(3) **(B).** Centre of circle is $\left(\frac{3}{2},-4\right)$

Let the other extremity is (h, k)

$$\therefore \left(\frac{6+h}{2}\right)=\frac{3}{2};\left(\frac{-3+k}{2}\right)=-4 \Rightarrow (-3,-5)$$

(4) **(A).** $(x-2)^2+(y+1)^2=3^2$
$\Rightarrow x^2-4x+4+y^2+2y+1=9$
$\Rightarrow x^2+y^2-4x+2y-4=0$

(5) **(D).** Radius $r=\sqrt{7^2+(-2)^2}=\sqrt{53}$
Equation of circle is $x^2+y^2=53$

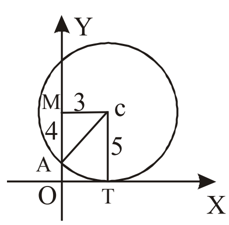
(6) **(C).** $(x-1)(x-3)+(y-2)(y-4)=0$
$\Rightarrow x^2+y^2-4x-6y+11=0$

(7) **(A).** Centre $(-1,-2)$, radius $(\sqrt{1^2+2^2-1})=2$.

(8) **(A).** $\because x=-7+4\cos\theta,\ y=3+4\sin\theta$
or $x+7=4\cos\theta,\ y-3=4\sin\theta$
Squaring and adding
$(x+7)^2+(y-3)^2=16(\cos^2\theta+\sin^2\theta)$
$\Rightarrow (x+7)^2+(y-3)^2=16$

(9) **(C).** Here radius of circle $|-2|=2$
$\therefore$ Equation is $(x+2)^2+(y+3)^2=2^2$
or $x^2+y^2+4x+6y+9=0$

(10) **(C).** From figure.



Radius of Circle $\sqrt{3^2+4^2}=5$ and centre is (3,5)
Hence equation is
$(x-3)^2+(y-5)^2=5^2$
$\Rightarrow x^2+y^2-6x-10y+9=0$

(11) **(C).** The point of intersection of the given lines is (1,–1) which is the centre of the required circle. Also if its radius be r, then as given $\pi r^2=154$

$$\Rightarrow r^2=\frac{154\times 7}{22}=49 \Rightarrow r=7$$

$\therefore$ reqd. equation is $(x-1)^2+(y+1)^2=49$
$\Rightarrow x^2+y^2-2x+2y=47$

(12) **(B).** Let the circle be
$x^2+y^2+2gx+2fy+c=0$ ...(1)
Substituting the points, (1,–2) and (4,–3) in equation (1)

$$\left.\begin{array}{l}5+2g-4f+c=0 \quad ...(2)\\ 25+89-6f+c=0 \quad ...(3)\end{array}\right\}$$

centre (–g,–f) lies on line $3x+4y=7$
solving for g,f,c
Hence $-3g-4f=7$ ...(4)

$$\text{Here } g=\frac{-47}{15},\ f=\frac{9}{15},\ c=\frac{55}{15}$$

Hence the equation is
$15(x^2+y^2)-94x+18y+55=0$

(13) **(D).** Let the circle be $x^2+y^2+2gx+2fy+c=0$.
Passes through (–4, 3)
$25-8g+6f+c=0$ ...(1)
Touches both lines

$$\frac{-g-f-2}{\sqrt{2}}=\sqrt{g^2+f^2-c}=\frac{-g+f-2}{\sqrt{2}}$$

$\therefore f=0 \therefore g^2-4g-4-2c=0$
Also $c=8g-25$

$\therefore g=10\pm 3\sqrt{6},\ f=0,\ c=55\pm 24\sqrt{6}$

It is easy to see that the answers given are not near to the values of g,f,c. Hence none of these is the correct option.

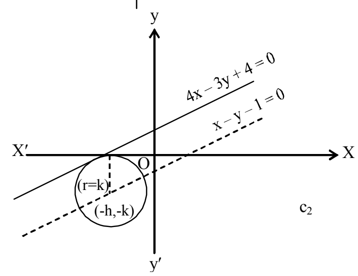
(14) **(B).** The centre of the circle lies on x– axis. Let a be the radius of the circle. Then, coordinates of the centre are (a, 0). The circle passes through (3,4). Therefore,

$$\sqrt{(a-3)^2+(0-4)^2}=a \Rightarrow -6a+25=0 \Rightarrow a=\frac{25}{6}$$

So, equation of the circle is $(x-a)^2+(y-0)^2=a^2$
or, $x^2+y^2-2ax=0$ or $3(x^2+y^2)-25x=0$

(15) **(A).** Let centre be (–h,–k) equation
$(x+h)^2+(y+k)^2=k^2$ ...(1)
Also $-h+k=1$ ...(2)
$\therefore$ $h=k-1$ radius = k (touches x– axis)
Touches the line $4x-3y+4=0$

$$\left|\frac{-4h-3(-k)+4}{5}\right|=k \quad ...(3)$$



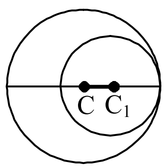
Solving (2) and (3), $h = \frac{1}{3}, k = \frac{4}{3}$

Hence the circle is $\left(x + \frac{1}{3}\right)^2 + \left(y + \frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^2$

$\Rightarrow 9(x^2 + y^2) + 6x + 24y + 1 = 0$

(16) **(C).** Let the circle cuts the x – axis and y– axis at A and B respectively. If O is the origin, then $\angle AOB = 90^\circ$, and A (5,0); B (0,5) is the diameter of the circle.
Then using diameter from the equation to the circle, we get $(x - 5)(x - 0) + (y - 0)(y - 5) = 0$
$\Rightarrow x^2 + y^2 - 5x - 5y = 0$

(17) **(B).** Let C be the centre of the given circle and C_1 be the centre of the required circle. Now C = (2,3), C = radius = 5
$\because C_1P = 3 \Rightarrow CC_1 = 2$
$\therefore$ The point C_1 divides internally, the line joining C and P in the ratio 2: 3



$\therefore$ coordinates of C_1 are $\left(\frac{2\times(-1)+3\times 2}{2+3}, \frac{2\times(-1)+3\times 3}{2+3}\right)$

Hence (B) is the required circle.

(18) **(D).** Let the circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)
$9 + 0 + 6g + 0 + c = 0$ ...(2)
$1 + 36 + 2g - 12f + c = 0$ ...(3)
$16 + 1 + 8g - 2f + c = 0$ ...(4)
from (2) – (3), $-28 + 4g + 12f = 0$
$\Rightarrow g + 3f - 7 = 0$ ...(5)
from (3) – (4), $20 - 6g - 10f = 0$
$\Rightarrow 3g + 5f - 10 = 0$ ...(6)
Solving

$$\frac{g}{-30+35} = \frac{f}{-21+10} = \frac{1}{5-9} \therefore g = -\frac{5}{4}, f = \frac{11}{4}, c = -\frac{3}{2}$$

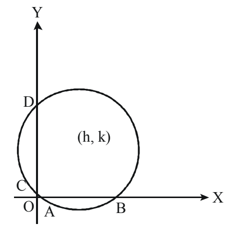
Hence the circle is
$2x^2 + 2y^2 - 5x + 11y - 3 = 0$

(19) **(A).** Since the first circle is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, therefore its equation can be written as $x^2 + y^2 - 2x + 4y + c = 0$
If it passes through (4,–2), then $16 + 4 - 8 - 8 + c = 0$
$\Rightarrow c = -4$

(20) **(A).** Let $A \equiv (\alpha, \beta)$; $B \equiv (\gamma, \delta)$.
Then $\alpha + \gamma = -2a$, $\alpha\gamma = -b^2$ and $\beta + \delta = -2p$, $\beta\delta = -q^2$
Now equation of the required circle is
$(x - \alpha)(x - \gamma) + (y - \beta)(y - \delta) = 0$
$\Rightarrow x^2 + y^2 - (\alpha + \gamma)x - (\beta + \delta) + \alpha\gamma + \beta\delta = 0$
$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Its radius $= \sqrt{a^2 + b^2 + p^2 + q^2}$

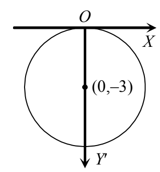
(21) **(A).** Let a rod AB of length 'a' slides on x-axis and rod CD of length 'b' slide on y - axis so that ends A, B, C and D are always concyclic.



Let equation of circle passing through these ends is $x^2 + y^2 + 2gx + 2fy + c = 0$

Obviously $2\sqrt{g^2 - c} = a$ and $2\sqrt{f^2 - c} = b$
$\therefore 4(g^2 - f^2) = a^2 - b^2 \Rightarrow 4[(-g)^2 - (-f)^2] = a^2 - b^2$
therefore locus of centre (–g, –f) is $4(x^2 - y^2) = a^2 - b^2$.

(22) **(B).** Centre is (0, –3) and $R = \sqrt{0^2 + 9 + 0} = 3$.



(23) **(B).** First find the centre. Let centre be (h, k), then

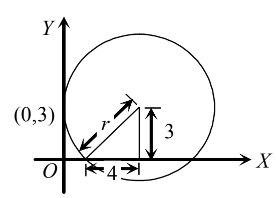
$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2}$ ....(i)
and $k - 4h + 3 = 0$ ....(ii)
From (i), we get $-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$
or $4h + 4k - 28 = 0$ or $h + k - 7 = 0$ ....(iii)
From (iii) and (ii), we get (h, k) as (2, 5). Hence centre is (2, 5) and radius is 2. Now find the equation of circle.
Obviously, circle $x^2 + y^2 - 4x - 10y + 25 = 0$ passes through (2, 3) and (4, 5).

(24) **(C).** As the centre may be $(\pm 4, 0)$ and radius = 4.

(25) **(A).** The circle is $x^2 + y^2 - \frac{1}{2}x = 0$.

Centre $(-g, -f) = \left(\frac{1}{4}, 0\right)$ and $R = \sqrt{\frac{1}{16} + 0 - 0} = \frac{1}{4}$.

(26) **(C).** Obviously from figure,



Radius is $r = \sqrt{4^2 + 3^2} = 5$

(27) **(B).** $R = \sqrt{\cos^2\theta + \sin^2\theta + 8} = 3$

(28) **(A).** Let its centre be (h, k), then $h - k = 1$ ...(i)
Also radius $a = 3$
Equation is $(x-h)^2 + (y-k)^2 = 9$
Also it passes through (7, 3)
i.e., $(7-h)^2 + (3-k)^2 = 9$ ....(ii)
We get h and k from (i) and (ii) solving simultaneously as (4, 3). Equation is $x^2 + y^2 - 8x - 6y + 16 = 0$.
Since the circle $x^2 + y^2 - 8x - 6y + 16 = 0$ satisfies the given conditions.

(29) **(D).** Obviously the centre of the circle is (4, 2) which should be the middle point of the ends of diameter.
Hence the other end is (11, 2).

(30) **(A).** Let point (x_1, y_1) on the diameter.
$\Rightarrow 2x_1 + 3y_1 = 3$ ....(i)
$16x_1 - y_1 = 4$ ....(ii)
On solving (i) and (ii), we get centre,
$\Rightarrow x_1 = \frac{3}{10}, y_1 = \frac{4}{5}$
$\therefore$ Equation of circle,
$(x-x_1)^2 + (y-y_1)^2 = r^2 \Rightarrow \left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = r^2$
$\because$ Circle passes through (4, 6).
So, $r^2 = \left(\frac{37}{10}\right)^2 + \left(\frac{26}{5}\right)^2 \Rightarrow r^2 = \frac{4073}{100}$
$\therefore$ Required equation of circle is
$\left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{4073}{100}$
$\Rightarrow 5(x^2 + y^2) - 3x - 8y = 200$.

(31) **(D).** Given, equation of circle is $x^2 + y^2 - 3x - 4y + 2 = 0$ and it cuts the x-axis. $\therefore$ $x^2 + 0 - 3x + 2 = 0$ or $x^2 - 3x + 2 = 0$ or $(x-1)(x-2) = 0$ or $x = 1, 2$.
Therefore the points are (1,0) and (2, 0).

(32) **(C).** The other end is $(t, 3-t)$
So the equation of the variable circle is
$(x-1)(x-t) + (y-1)(y-3+t) = 0$
or $x^2 + y^2 - (1+t)x - (4-t)y + 3 = 0$
$\therefore$ The centre (α, β) is given by
$\alpha = \frac{1+t}{2}, \beta = \frac{4-t}{2} \Rightarrow 2\alpha + 2\beta = 5$
Hence, the locus is $2x + 2y = 5$.

(33) **(A).** Substituting $x = \frac{3y+10}{4}$ in equation of circle, we get a quadratic in y. Solving, we get two values of y as 2 and – 6 from which we get value of x.

(34) **(A).** As base is constant and height varies and is maximum for isosceles Δ.

(35) **(D).** Let the centre of the required circle $C_1 \equiv (h, k)$.
Since it touches y–axis, so its radius $r_1 = h$.
For the given circle centre $C_2 \equiv (3, 3)$, radius $r_2 = \sqrt{9+9-14} = 2$. Since the circle touch externally, so
$C_1C_2 = r_1 + r_2$
$\Rightarrow (h-3)^2 + (k-3)^2 = (h+2)^2$
$\Rightarrow k^2 - 10h - 6k + 14 = 0$.
Hence the locus of the centre (h , k) will be
$y^2 - 10x - 6y + 14 = 0$

(36) **(A).** The centres of the two circles are $C_1(-a/2, 0)$ and $C_2(0, 0)$, and their redii are $\frac{|a|}{2}$ and c.
So, the two circles will touch each other if
C_1C_2 = sum or difference of radii
$\Rightarrow \sqrt{(-a/2-0)^2 + (0-0)^2} = \left|c \pm \frac{|a|}{2}\right|$
$\Rightarrow \frac{|a|}{2} = \left|c \pm \frac{|a|}{2}\right| \Rightarrow c \pm \frac{|a|}{2} = \frac{|a|}{2}$
$\Rightarrow c - \frac{|a|}{2} = \frac{|a|}{2}$ & $c + \frac{|a|}{2} = \frac{|a|}{2}$
$\Rightarrow c = |a|$ or $c = 0$
$\Rightarrow c = |a| \quad [\because c > 0]$

(37) **(D).** The centre of the required circle is the image of the centre (–8, 12) with respect to the line mirror $4x + 7y + 13 = 0$ and radius is equal to the radius of the given circle.
Let (h, k) be the image of the point (–8, 12) with respect to the line mirror. Then the mid-point of the line joining $C(-8, 12)$ and $P(h, k)$ lies on the line mirror.
$\therefore 4\left(\frac{h-8}{2}\right) + 7\left(\frac{k+12}{2}\right) + 13 = 0$
or $4h + 7k + 78 = 0$ ....(i)
Also CP is perpendicular to $4x + 7y + 13 = 0$
$\therefore \frac{k-12}{h+8} \times -\frac{4}{7} = -1$ or $7h - 4k + 104 = 0$ ....(ii)
Solving (i) and (ii), $h = -16, k = -2$.
Thus the centre of the image circle is $(-16, -2)$. The radius of the image circle is same as the radius of $x^2 + y^2 + 16x - 24y + 183 = 0$ i.e., 5.
Hence the equation of the required circle is
$(x+16)^2 + (y+2)^2 = 5^2$ i.e. $x^2 + y^2 + 32x + 4y + 235 = 0$.

(38) **(A).** The equation of circle passing through the point of intersection of circle and line can be written as
$x^2 + y^2 - a^2 + \lambda(x - y + 3) = 0$
The centre of this circle is $\left(-\frac{\lambda}{2}, \frac{\lambda}{2}\right)$, which lies on the line y = x + 3 because this line is a diameter of the circle.

$\therefore -\frac{\lambda}{2} - \frac{\lambda}{2} + 3 = 0 \Rightarrow \lambda = 3$

Thus equation of required circle is

$(x^2 + y^2 - a^2) + 3(x - y + 3) = 0$

$\Rightarrow x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(39) **(C).** c_1 (0, 0), $r_1 = 3$ and c_2 $(-\alpha, -1)$, $r_2 = |\alpha|$

Circles touches each other if $c_1c_2 = r_1 \pm r_2$

$\sqrt{\alpha^2 + 1} = 3 \pm |\alpha|$; $\alpha^2 + 1 = 9 + \alpha^2 \pm 6|\alpha|$

$6|\alpha| = \pm 8$; $\alpha = \pm 4/3$

(40) **(B).** Put $x = r\cos\theta$ & $y = r\sin\theta \Rightarrow x^2 + y^2 = 2 - 4x + 6y$

$\Rightarrow x^2 + y^2 + 4x - 6y - 2 = 0$

$\Rightarrow$ Centre $= (-2, 3)$

(41) **(B).** Let the centre of the required circle be (x_1, y_1) and the centre of given circle is (1, 2). Since radii of both circles are same, therefore, point of contact (5, 5) is the mid point of the line joining the centres of both circles. Hence $x_1 = 9$ and $y_1 = 8$. Hence the required equation is $(x-9)^2 + (y-8)^2 = 25$

$\Rightarrow x^2 + y^2 - 18x - 16y + 120 = 0$.

The point (5, 5) must satisfy the required circle. Hence the required equation is given by (B).

(42) **(A).** x- and y- intercepts of $2x + 3y \cdot k = 0$ are k/2 and k/3.

$\therefore$ Area of the triangle $= \frac{1}{2}\left(\frac{k}{2}\right)\left(\frac{k}{3}\right) = 12 \Rightarrow k = 12$

and $2x + 3y - 12 = 0$ is diameter to the circle

$x^2 + y^2 - 6x - 4y = 0$

Because it passes through the center (3, 2)

(43) **(A).** By inspection

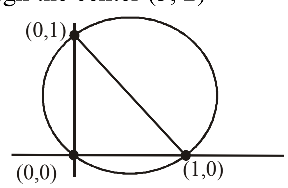
(44) **(D).** $x(x-1) + y(y-1) = 0$

$x^2 + y^2 - x - y = 0$

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

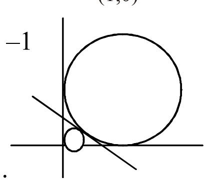
$13k = 5 \Rightarrow k = 5/13$



(45) **(D).** Equation of line whose slope is –1 and y-intercept 1 is

$y = -x + \phi \Rightarrow x + y - 1 = 0$

From the diagram, it is clear two circles can be drawn.



(46) **(A).** $(x-2)^2 = 9\cos^2\theta$ and $(y-1)^2 = 9\sin^2\theta$

$\Rightarrow (x-2)^2 + (y-1)^2 = 9$

Centre (2, 1) and r = 3

(47) **(C).** Here centre is (1, 0) and radius is $\sqrt{1^2 + 8} = 3$

given line will touch the circle if p = r

$\Rightarrow \frac{3-m}{\sqrt{9+16}} = 3 \Rightarrow 3 - m = \pm 15$

$\Rightarrow m = 18, -12$

(48) **(B).** The two circles are

$S_1 = (x - a_1)^2 + (y - b_1^2) = r_1^2$ ...(i)

$S_2 = (x - a_2)^2 + (y - b_2^2) = r_2^2$ ...(ii)

The equation of the common tangent of these two circles is given by $S_1 - S_2 = 0$

i.e., $2x(a_1 - a_2) + 2y(b_1 - b_2) + (a_2^2 + b_2^2) - (a_1^2 + b_1^2) + r_1^2 - r_2^2 = 0$

If this passes through the origin, then

$(a_2^2 + b_2^2) - (a_1^2 + b_1^2) + r_1^2 - r_2^2 = 0$

$(a_2^2 - a_1^2) + (b_2^2 - b_1^2) = r_2^2 - r_1^2$

(49) **(C).** The required point is the radical centre of the three given circles. The radical axes of these three circles taken in pairs are : $3x - 24 = 0$; $16y + 120 = 0$ and $-3x + 16y + 80 = 0$

Solving any two of these three equations, we get

$x = 8, y = -\frac{15}{8}$. Hence, the required point is $\left(8, -\frac{15}{2}\right)$.

(50) **(D).** Here c_1 (1,3), $r_1 = \sqrt{1+9-9} = 1$

$c_2(-3,1)$, $r_2 = \sqrt{9+1-1} = 3$

Now $c_1c_2 = \sqrt{(1+3)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17}$

$c_1c_2 > r_1 + r_2$

Hence the circles are non- intersecting externally.

Hence 4 tangents, two direct and two transverse tangents may be drawn.

(51) **(D).** $r = \sqrt{4+1+20} = 5$; $C \equiv (2, 1)$

$\therefore CP = \sqrt{(10-2)^2 + (7-1)^2} = 10$

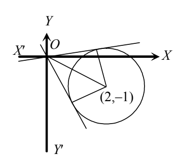
$\therefore$ Maximum distance $= 10 + 5 = 15$.

(52) **(C).** $y = mx + c$ is a tangent, if $c = \pm a\sqrt{1+m^2}$, where $m = \tan 45^o = 1$

$\therefore$ The equation is $y = x \pm 6\sqrt{2}$.

(53) **(C).** Centre is $(2, -1)$.

Therefore $r = \left|\frac{3(2)-1}{\sqrt{10}}\right| = \frac{5}{\sqrt{10}}$



Now draw a perpendicular on $x - 3y = 0$, we get

$r = \left|\frac{2 - 3(-1)}{\sqrt{10}}\right| = \frac{5}{\sqrt{10}}$.

(54) **(A).** From formula of tangent at a point,

$x\left(\frac{ab^2}{a^2+b^2}\right) + y\left(\frac{a^2b}{a^2+b^2}\right) = \frac{a^2b^2}{a^2+b^2} \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$.

(55) **(B).** Since the tangents are parallel, therefore the distance between these two tangents will be its diameter i.e.,

diameter $= \frac{34}{\sqrt{180}} = \frac{17}{3\sqrt{5}}$. Hence, radius $= \frac{17}{6\sqrt{5}}$.

(56) **(A).** Let $S_1 \equiv x^2 + y^2 - 2x + 6y + 6 = 0$
and $S_2 \equiv x^2 + y^2 - 5x + 6y + 15 = 0$,
then common tangent is $S_1 - S_2 = 0$
$\Rightarrow 3x = 9 \Rightarrow x = 3$.

(57) **(C).** The equation of the tangent at $P(3,4)$ to the circle $x^2 + y^2 = 25$ is $3x + 4y = 25$, which meets the co-ordinate axes at $A\left(\frac{25}{3}, 0\right)$ and $B\left(0, \frac{25}{4}\right)$. If O be the origin, then the ΔOAB is a right angled triangle with $OA = 25/3$ and $OB = 25/4$.

Area of the $\Delta OAB = \frac{1}{2} \times OA \times OB$

$$= \frac{1}{2} \times \frac{25}{3} \times \frac{25}{4} = \frac{625}{24}.$$

(58) **(B).** Equation of BC (chord of contact) is
$0.x + 1.y - (x + 0) + 2(y + 1) + 1 = 0$ or $-x + 3y + 3 = 0$
Equation of circle through B and C i.e., intersection of the given circle and chord of contact is
$(x^2 + y^2 - 2x + 4y + 1) + \lambda(-x + 3y + 3) = 0$.
It passes through $A(0, 1)$, so the equation of the required circle is $x^2 + y^2 - x + y - 2 = 0$.

Aliter : Centre of the required circle is mid-point of $A(0, 1)$ and centre of the given circle i.e., $(1, -2)$.

Therefore, centre $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and radius $\sqrt{\frac{5}{2}}$.

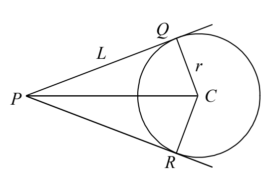
Hence the circle is $x^2 + y^2 - x + y - 2 = 0$.

(59) **(B).** Let $S = x^2 + y^2 - 2x + 4y$ then
$S_1 = 0^2 + 1^2 - 2.0 + 4.1 = 5$
$T = x.0 + y.1 - (x + 0) + 2(y + 1) = (-x + 3y + 2)$
$\therefore$ The equation of the pair of tangent $SS_1 = T^2$
$(x^2 + y^2 - 2x + 4x + 4y)5 = (-x + 3y + 2)^2$
$\Rightarrow 4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$

(60) **(C).** Given $\frac{T_1}{T_2} = \frac{4}{3}$, where T_1 and T_2 are the length of tangents drawn to the given circle.

$$\Rightarrow \frac{\sqrt{1 + 4 + 1 + 2 - 4}}{\sqrt{(1)^2 + (2)^2 - \frac{1}{3} - \frac{2}{3} + \frac{k}{3}}} = \frac{4}{3} \Rightarrow k = -\frac{21}{4}.$$

(61) **(A).** Area $PQCR = 2.\Delta PQC = 2 \times \frac{1}{2} L \times r$



Where L = length of tangent and r = radius of circle.

$L = \sqrt{S_1}$ and $r = \sqrt{1 + 4 + 20} = 5$

Hence the required area = 75 sq. units

(62) **(B).** Tangent at (1,–2) to $x^2 + y^2 = 5$ is $x - 2y = 5$
To find the point of contact with second circle, we solve this equation with the equation of the second circle, so we have $(2y + 5)^2 + y^2 - 8(2y + 5) + 6y + 20 = 0$
$\Rightarrow 5y^2 + 10y + 5 = 0 \Rightarrow (y + 1)^2 = 0 \Rightarrow y = -1$
Also then x = 3. So the required point is (3, –1)

(63) **(C).** Dividing the equation of the circle by 2, we get

$$x^2 + y^2 = \frac{3}{2} \Rightarrow \left(x^2 + y^2 - \frac{3}{2}\right) = 0$$

$\therefore$ length of the tangent $= \sqrt{(1)^2 + (5)^2 - \frac{3}{2}}$

$$= \sqrt{26 - \frac{3}{2}} = \sqrt{\frac{49}{2}} = \sqrt{\frac{7}{2}} = \frac{7\sqrt{2}}{2}$$

(64) **(A).** Let centre is (4 + 2B, B).

$$r = \left|\frac{8 + 4B - B + 1}{\sqrt{5}}\right|^2 = (2B + 2)^2 + (5 - B)^2 ; B = 1$$

Centre (8, 2), Radius $= 3\sqrt{5}$

(65) **(B).** The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and $3x - 4y - \frac{7}{2} = 0$ and so it is equal to

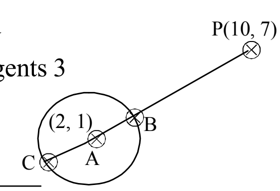
$\frac{4}{\sqrt{9 + 16}} + \frac{7/2}{\sqrt{9 + 16}} = \frac{3}{2}$. Hence radius is $\frac{3}{4}$.

(66) **(D).** Desired equation of the circle is
$(x - 2)^2 + (y - 3)^2 + \lambda(x + y - 5) = 0$
$1 + 1 + \lambda(1 + 2 - 5) = 0 \Rightarrow \lambda = 1$
$x^2 - 4x + 4 + y^2 - 6y + 9 + x + y - 5 = 0$
$\Rightarrow x^2 + y^2 - 3x - 5y + 8 = 0$

$$\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = -8 + \frac{25}{4} + \frac{9}{4} = \frac{2}{4} = \frac{1}{2} ; r = \frac{1}{\sqrt{2}}$$

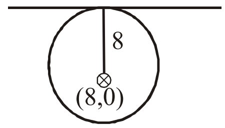
(67) **(B).** $c_1 = (3, 4)$; $c_2 = (0, 0)$
$r_1 = 4$; $r_2 = 1$
$c_1c_2 = 5$; $r_1 + r_2 = 5$
Circles touch externally
$\therefore$ No. of common tangents 3

(68) **(A).** Radius,



$r = \sqrt{4 + 1 + 20} = 5$
$AP = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = 10$
Greatest distance = 10 + r = 10 + 5 = 15
Least distance = 10 – r = 10 – 5 = 5

(69) **(D).** $3x + 4y - k = 0$ touches $x^2 + y^2 - 16x = 0$
condition is



$\perp$ r distance from centre to $3x + 4y - k = 0$ } = radius of the circle

$$\left|\frac{3(8) + 4(0) - k}{\sqrt{9+16}}\right| = 8 \text{ ; Centre} = (8, 0)$$

$\therefore 24 - k = 40$ or $24 - k = -40 \Rightarrow k = -16 ; k = 64$

(70) **(B).** Let the equation of tangent be
$y + 4 = m(x + 5) \Rightarrow mx - y + (5m - 4) = 0$
Clearly $C = (-2, -3)$, $r = \sqrt{4+9-8} = \sqrt{5}$
Since (1) is a tangent,

$$\left|\frac{m(-2) + 3 + 5m - 4}{\sqrt{m^2+1}}\right| = \sqrt{5}$$

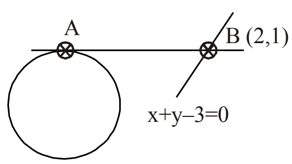
($\because$ perpendicular distance from center = radius)

$$\Rightarrow \left|\frac{3m-1}{\sqrt{m^2+1}}\right| = \sqrt{5} \Rightarrow 9m^2 + 1 - 6m = 5(m^2 + 1)$$

$\Rightarrow 4m^2 - 6m - 4 = 0 \Rightarrow 2m^2 - 3m - 2 = 0$
$\Rightarrow 2m^2 - 4m + m - 2 = 0 \Rightarrow 2m(m - 2) + (m - 2) = 0$
$\Rightarrow m = 2, -1/2$
$\therefore$ Equations of tangents are
$y + 4 = 2(x + 5) \Rightarrow 2x - y + 6 = 0$

$$y + 4 = -\frac{1}{2}(x + 5) \Rightarrow x + 2y + 13 = 0$$

(71) **(B).** AB = length of Tangent to the circle from B.



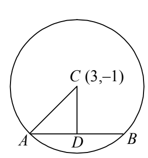
$$AB = \sqrt{x^2 + y^2 - \frac{3}{2}x + 2y} = \sqrt{4+1-3+2} = 2 \text{ units}$$

(72) **(A).** $C \equiv (3, 4)$

$$r = \left|\frac{5(3) + 12(4) - 11}{\sqrt{25+144}}\right| = \left|\frac{15+48-11}{\sqrt{169}}\right| = \left|\frac{52}{13}\right| = 4$$

$A = \pi r^2 = 16\pi \text{ units}^2$

(73) **(A).** Let AB (= 6) be the chord intercepted by the line $2x - 5y + 18 = 0$ from the circle and let CD be the perpendicular drawn from centre (3, –1) to the chord AB.



i.e. $AD = 3$, $CD = \dfrac{2.3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$.

(74) **(A).** Centre of the circle = (1, –2)
Radius = $\sqrt{1+4+4} = 3$

Here $p = \dfrac{1+2+1}{\sqrt{2}} = 2\sqrt{2}$

Length of chord = = $2\sqrt{a^2 - p^2} = 2\sqrt{9-8} = 2$.

(75) **(D).** Let (x_1, y_1) be the pole

$\therefore$ Polar $\quad 2xx_1 + 2yy_1 - \dfrac{3}{2}(x + x_1) + \dfrac{5}{2}(y + y_1) - 7 = 0$

or $(4x_1 - 3)x + (4y_1 + 5)y + \dfrac{-3x_1 + 5y_1 - 14}{1} = 0$

Comparing with given line

$$\frac{4x_1 - 3}{9} = \frac{4y_1 + 5}{1} = \frac{-3x_1 + 5y_1 - 14}{-28} = k \text{ say}$$

$$\therefore x_1 = \frac{9k+3}{4}, y_1 = \frac{k-5}{4}$$

$$\text{Hence} -3\left(\frac{9k+3}{4}\right) + 5\left(\frac{k-5}{4}\right) - 14 = -28k$$

$\Rightarrow -27k - 9 + 5k - 25 - 56 = -112k$
$\Rightarrow (-27 + 5 + 112)k = 90 \Rightarrow k = 1$

Pole is $x = \dfrac{9+3}{4} = 3, y = \dfrac{1-5}{4} = -1 \quad \therefore (3, -1)$

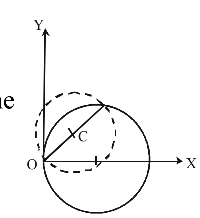
(76) **(A).** The polar of point (p, q) with respect to the circle $x^2 + y^2 = a^2$ is $\quad px + qy = a^2$
This line touches $(x - c)^2 + (y - d)^2 = b^2$

$$\therefore \left|\frac{cp + dq - a^2}{\sqrt{p^2 + q^2}}\right| = b$$

$\Rightarrow (a^2 - cp - dq)^2 = b^2(p^2 + q^2)$.

(77) **(B).** Here equation of the given circle is $x^2 + y^2 - 2x = 0$
This clearly passes through origin
Hence if (x_1, y_1) be midpoint of the chord then its equation is given by $T = S_1$
$xx_1 + yy_1 - (x + x_1) = x_1^2 + y_1^2 - 2x_1$
or $xx_1 + yy_1 - x = x_1^2 + y_1^2 - x_1$
This passes through the origin (0, 0)
$\therefore x_1^2 + y_1^2 - x_1 = 0$
$\therefore$ Locus reqd. is $x^2 + y^2 = x$

(78) **(C).** Here equation of the circle
$(x^2 + y^2 - 10x) + \lambda(y - 2x) = 0$
Now centre C $(5 + \lambda, -\lambda/2)$ lies on the Chord again



$\therefore \frac{-\lambda}{2} = 2(5+\lambda) \Rightarrow \frac{-5\lambda}{2} = 10$

$\therefore \lambda = -4$

Hence $x^2 + y^2 = 10x - 4y + 8x = 0$

or $x^2 + y^2 - 2x - 4y = 0$

(79) **(C).** The equation of the common chord is

$[(x-a)^2 + y^2 - c^2] - [x^2 + (y-b)^2 - c^2] = 0$

$\Rightarrow 2ax - 2by - a^2 + b^2 = 0$ ...(1)

Now p = length of perpendicular from (a, 0) on (1)

$= \frac{2a^2 - a^2 + b^2}{\sqrt{4a^2 + 4b^2}} = \frac{1}{2}\sqrt{a^2 + b^2}$

$\therefore$ length of common chord

$= 2\sqrt{c^2 - p^2} = 2\sqrt{c^2 - \frac{a^2 + b^2}{4}} = \sqrt{4c^2 - a^2 - b^2}$

(80) **(D).** Here circles are

$x^2 + y^2 - 2x - 2y = 0$ ...(1)

$x^2 + y^2 = 4$ ...(2)

Now $c_1\ (1, 1), r_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$

$c_2\ (0, 0), r_2 = 2$

If θ is the angle of intersection then

$\cos\theta = \frac{r_1^2 + r_2^2 - (c_1c_2)^2}{2r_1r_2} = \frac{2 + 4 - (\sqrt{2})^2}{2.\sqrt{2}.2.} = \frac{1}{\sqrt{2}}$

$= \theta = 45^{\circ}$

(81) **(C).** Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0,$

Since it passes through (1, 2), so

$1 + 4 + 2g + 4f + c = 0$

$\Rightarrow 2g + 4f + c + 5 = 0$ ...(1)

Also this circle cuts $x^2 + y^2 = 4$

orthogonally, so $2g(0) + 2f(0) = c - 4$

$\Rightarrow c = 4$ ...(2)

From (1) and (2) eliminating c, we have

$2g + 4f + 9 = 0$

Hence locus of the centre (–g, –f) is

$2x + 4y - 9 = 0$

(82) **(C).** Here C_1 (0, 0) and C_2 (1, 2)

$\therefore C_1C_2 = \sqrt{1+4} = \sqrt{5} = 2.23.$

Also $r_1 = 2, r_2 = \sqrt{1+4-3} = \sqrt{2} = 1.41$

$\therefore r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\Rightarrow$ circles intersect each other.

(83) **(A).** The centres of the two circles are C_1 (–1, 1) and C_2 (1, 1) and both have radii equal to 1.

We have: $C_1C_2 = 2$ and sum of the radii = 2

So, the two circles touch each other externally.

The equation of the common tangent is obtained by subtracting the two equations.

The equation of the common tangent is $4x = 0 \Rightarrow x = 0.$

Putting x = 0 in the equation of the either circle, we get

$y^2 - 2y + 1 = 0 \Rightarrow (y-1)^2 = 0 \Rightarrow y = 1.$

Hence, the points where the two circles touch is (0,1).

(84) **(D).** Any line through (0, 0) be $y - mx = 0$ and it is a tangent to circle $(x-7)^2 + (y+1)^2 = 25$, if

$\frac{-1 - 7m}{\sqrt{1+m^2}} = 5 \Rightarrow m = \frac{3}{4}, -\frac{4}{3}.$

Therefore, the product of both the slopes is –1.

i.e., $\frac{3}{4} \times -\frac{4}{3} = -1.$

Hence the angle between the two tangents is π/2.

(85) **(C).** Here the intersection point of chord and circle can be found by solving the equation of circle with the equation of given line, therefore, the points of intersection are

$(-4, -3)$ and $\left(\frac{24}{5}, \frac{7}{5}\right).$

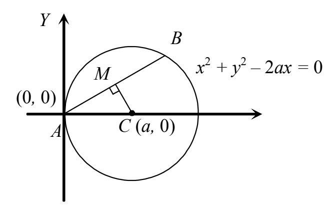
Hence the midpoint is $\left(\frac{-4 + \frac{24}{5}}{2}, \frac{-3 + \frac{7}{5}}{2}\right) = \left(\frac{2}{5}, -\frac{4}{5}\right).$

(86) **(D).** Let chord AB is $y = mx$ .....(i)

Equation of CM, $x + my = \lambda$

It is passing through (a, 0)

$\therefore x + my = a$ .....(ii)



From (i) and (ii), $x + y.\frac{y}{x} = a \Rightarrow x^2 + y^2 = ax$

$\Rightarrow x^2 + y^2 - ax = 0$ is the locus of the centre of the circle.

(87) **(C).** $T = 0 \Rightarrow 2x + 2y = 1$

$\Rightarrow x + y = 1/2$

(88) **(B).** The common chord of the given circles is

$6x^2 + 14y + c + d = 0$

Since $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0.$

So, (i) passes through the centre of the second circle i.e. (1, –4). $\therefore 6 - 56 + c + d = 0 \Rightarrow c + d = 50$

(89) **(D).** $(x-1)^2 + (y-2)^2 = 1$; $x^2 + y^2 - 2x - 4y + 4 = 0$

equation of polar of point (4, 4) is

$4x + 4y - (x+4) - 2(y+4) + 4 = 0$

$\Rightarrow 3x + 2y - 8 = 0$

(90) **(C).** Let P (h, k) be the point. Then, the chord of contact of tangents drawn from P to the circle

$x^2 + y^2 = a^2$ is $hx + ky = a^2.$

The combined equation of the lines joining the (centre) origin to the points of intersection of the circle $x^2 + y^2 = a^2$ and the chord of contact of tangents drawn from P (h, k) is a homogeneous equation of second degree given by

$$x^2+y^2=a^2\left(\frac{hx+ky}{a^2}\right)^2 \quad \text{or } a^2(x^2+y^2)=(hx+ky)^2$$

The lines given by the above equation will be perpendicular if coeff. of x^2 + coeff. of $y^2 = 0$

$\Rightarrow h^2-a^2+k^2-a^2=0 \; h^2+k^2=2a^2$

So, locus of (h,k) is $x^2+y^2=2a^2$.

Clearly, it is a circle of radius $\sqrt{2}$ a.

(91) **(C).** Since the chord makes equal intercepts of length a on the coordinate axes.

So, its equation can be written as $x \pm y = \pm a$.

This line meets the given circle at two distinct points.

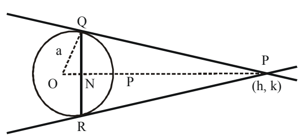
So, length of the perpendicular from the centre (0, 0) of the given circle must be less than the radius.

i.e. $\left|\frac{\pm a}{\sqrt{2}}\right| < \sqrt{8} \Rightarrow a^2 < 16 \Rightarrow |a| < 4$.

(92) **(C).** Here area of Δ PQR is required

Now chord of contact w.r. to circle $x^2+y^2=a^2$,

and point (h, k) $hx+ky-a^2=0$



Perp. from (h, k), $PN=\dfrac{h^2+k^2-a^2}{\sqrt{h^2+k^2}}$

Also length QR

$$=2\sqrt{a^2-\frac{(a^2)^2}{h^2-k^2}}=\frac{2a\sqrt{h^2+k^2-a^2}}{\sqrt{h^2+k^2}}$$

$$\therefore \Delta PQR=\frac{1}{2}(QR)(PN)$$

$$=\frac{1}{2}\,2a\sqrt{\frac{h^2+k^2-a^2}{h^2+k^2}}\;\frac{(h^2+k^2-a^2)}{\sqrt{h^2+k^2}}$$

$$=\frac{a\,(h^2+k^2-a^2)^{3/2}}{(h^2+k^2)}$$

(93) **(D).** $x^2+y^2 \; 4x=0$ $\therefore$ centre = (2, 0)

$\therefore$ Slope of the line joining (1, 0) and (2, 0) = 0

= slope of the radius

$\therefore$ y = 1 is perpendicular to the chord, because it is parallel to radius.

(94) **(C).** Given circles are $(x-2)^2+(y-3)^2=r^2$ .... (1)

$(x-5)^2+(y-6)^2=r^2$ .... (2)

Radical axis is, Eq. (1) – Eq. (2)

$-4x+10x-6y+12y+4+9-25-36=0$

$6x+6y-48=0\;;\; x+y-8=0$

(95) **(A).** $2g_1g_2+2f_1f_2=c_1+c_2$

$$\Rightarrow 2(1)+\frac{5}{2}\left(-\frac{7}{6}\right)=\frac{1}{2}+k \Rightarrow k=-\frac{17}{12}$$

(96) **(A).** Req. point = radical center

$S_1-S_2=0 \Rightarrow 6x+3y-3=0$

$S_2-S_3=0 \Rightarrow -x-4y+5=0$ | $\therefore x=5-4(9/7)=-1/7$

$\Rightarrow -6x-24y+30=0$ $\Rightarrow (x, y)=(-1/7, 9/7)$

$\Rightarrow -21y+27=0$

$y=27/21=9/7$

(97) **(A).** $x^2+y^2+2x-2y+7=0$

$r=\sqrt{1+1-7}=\sqrt{-5}$, Imaginary

$\therefore$ Number of real circles cutting orthogonally given imaginary circle is zero.

(98) **(D).** $x^2+y^2+3x+2y-8=0$

Intercept made by y-axis = $2\sqrt{f^2-C}=2\sqrt{(1)+8}=6$

(99) **(C).** Circle with (2, 0), (0, 1) as end points of diameter is $(x-2)x+(y-1)y=0$ and line through these two points is $y-0=\left(\frac{-1}{2}\right)(x-2)$ or $2y+x-2=0$

Family of circles through these two points are

$x(x-2)+y(y-1)+\lambda(2y+x-2)=0$.

It passes through (4, 5).

i.e., $4(2)+5(4)+\lambda(10+4-2)=0 \Rightarrow \lambda=\frac{-7}{3}$.

Hence equation of circle is

$$x(x-1)+y(y-1)-\frac{7}{3}(2y+x-2)=0$$

It passes through (0, c), therefore

$$c(c-1)-\frac{7}{3}(2c-2)=0$$

$\Rightarrow 3c^2-17c+14=0$ or $c=\frac{14}{3}$ and 1.

(100) **(A).** Let the equation of the required circle be

$(x^2+y^2-a^2)+\lambda(y-x-3)=0$

since its centre ($\lambda/2$, $-\lambda/2$) lies on the given line, so we have $-\lambda/2=\lambda/2+3=-3$

Putting this value of in (A) we get the reqd. eqn. as

$x^2+y^2+3x-3y-a^2+9=0$

(101) **(B).** Let the equation of the required circle be

$(x^2+y^2-6x+8)+(x^2+y^2-6)=0$

Since it passes through (1, 1), so we have

$1+1-6+8+\lambda(1+1-6)=0=1$

$\therefore$ the required equation is $x^2+y^2-3x+1=0$

(102) **(B).** The equation of circle through the points of intersection of given circle is -

$x^2+y^2+4x-5y+3+\lambda(x^2+y^2+2x+3y-3)=0$

Since it passes through point (–3, 2) therefore

$-6+10\lambda=0 \Rightarrow \lambda=3/5$

Hence equation of required circle is

$5x^2+5y^2+20x-25y+15+3x^2+3y^2+6x+9y-9=0$

$\Rightarrow 2x^2+2y^2+26x-16y+6=0$

$\Rightarrow x^2+y^2+13x-8y+3=0$

(103) **(C).** Radical axis of first and second circle is given by

$(x^2+y^2)-(x^2+y^2-2cx+c^2)=0$

or $x=c/2$

Also the radical axis of first and third circle is given by
$(x^2 + y^2) - (x^2 + y^2 - 2by + b^2) = 0$ or $y = b/2$
$\therefore$ their radical centre = (c/2, b/2)

(104) **(A).** The given equations may be written as
$3x^2 + 3y^2 - 3x + 3 = 0$
$3x^2 + 3y^2 + y - 1 = 0$
Now required equation is given by S – S' = 0
$\Rightarrow -3x + 3 - y + 1 = 0 \Rightarrow 3x + y - 4 = 0$

(105) **(C).** The required point is the radical centre of the three given circles.
The radical axes of these three circles taken in pairs are
$3x - 24 = 0$
$16y + 120 = 0$
and $-3x + 16y + 80 = 0$
Solving any two of these three equations, we get
$x = 8, y = -15/8$
Hence, the required point is (8, –15/8)

(106) **(A).** $x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 6y) = 0$
$(1 + \lambda)x^2 + (1 + \lambda)y^2 - 6x - 6\lambda y = 0$

$$x^2 + y^2 - \frac{6}{1+\lambda}x - \frac{6\lambda}{1+\lambda}y = 0$$

$$\text{Centre}\left(\frac{3}{1+\lambda}, \frac{3\lambda}{1+\lambda}\right) \Rightarrow \frac{3}{1+\lambda} = \frac{3}{2}, \frac{3\lambda}{1+\lambda} = \frac{3}{2} \Rightarrow \lambda = 1$$

(107) **(A).** Any circle which touches $3x + 4y - 7 = 0$ at (1, 1) will be of the form
$S(x, y) \equiv (x - 1)^2 + (y - 1)^2 + \lambda(3x + 4y - 7) = 0$
Since $S(2, 3) = 16 \Rightarrow \lambda = 1$, so required circle will be
$x^2 + y^2 + x + 2y - 5 = 0$.

(108) **(C).** Let (h, k) be the centre of the required circle. Then (h,k) being the mid-point of the chord of the given circle, its equation is $hx + ky - a(x + h) = h^2 + k^2 - 2ah$
Since it passes through the origin, we have
$-ah = h^2 + k^2 - 2ah \Rightarrow h^2 + k^2 - ah = 0$
Hence locus of (h, k) is $x^2 + y^2 - ax = 0$

(109) **(A).** Let the pole is (h, k)
Hence polar of this pole is $xh + yk - c^2 = 0$ .....(1)

but polar is $\frac{x}{a} + \frac{y}{b} = 0$ .....(2)

comparing the coefficient of x and y

$$\frac{h}{(1/a)} = \frac{k}{(1/b)} = \frac{-c^2}{-1} \Rightarrow h = \frac{c^2}{a}, k = \frac{c^2}{b}$$

(110) **(D).** For internal point p (2, 8) $4 + 64 - 4 + 32 - p < 0$
$\Rightarrow p > 96$ and x intercept $= 2\sqrt{1+p}$ therefore $1 + p < 0$
$\Rightarrow p < -1$ and y intercept $= 2\sqrt{4+p} \Rightarrow p < -4$

(111) **(C).** The two circles are $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$
Centre : $C_1 \equiv (2, 3), C_2 \equiv (-1, -1)$ radii : $r_1 = 4, r_2 = 1$
We have $C_1C_2 = 5 = r_1 + r_2$, therefore there are 3 common tangents to the given circles.

(112) **(C).** $x^2 + y^2 + 4x - 6y + 9 = 0$
$x^2 + y^2 - 5x + 4y - 2 = 0$
$9x - 10y + 11 = 0$

(113) **(C).** The chord of contact of tangents from (α, β) is
$\alpha x + \beta y = 1$ ....(1)

Hence, (1) passes through $\left(\frac{1}{2}, \frac{1}{4}\right)$.

(114) **(C).** Since the chord makes equal intercepts of length a on the coordinates axes. So, its equation can be written as $x \pm y = \pm a$. This line meets the given circle at two distinct points.
So, length of the perpendicular from the centre (0, 0) of the given circle must be less than the radius. i.e.

$$\left|\frac{\pm a}{\sqrt{2}}\right| < \sqrt{8} \Rightarrow a^2 < 16 \Rightarrow |a| < 4.$$

(115) **(C).** The equation of the tangent at (h, h) to $x^2 + y^2 = a^2$ is $hx + hy = a^2$. Therefore slope of the tangent = –h/h = – 1

(116) **(C).** $\pi r_1^2 = \pi r_2^2 - \pi r_1^2 \Rightarrow 2r_1^2 = r_2^2 \Rightarrow r_2 = \sqrt{2}r_1$
Note P lies on the director circle of radius r_1
$\Rightarrow L = r_1 = 2$ cm.

(117) **(B).** $x^2 + y^2 + 2gx + 2fy = 0$
$x^2 + y^2 - a^2 = 0$
Equation of common chord is $2gx + 2fy + a^2 = 0$

$$\text{Homogenization } x^2 + y^2 - a^2\left(\frac{2gx + 2fy}{a^2}\right)^2 = 0$$

$\Rightarrow a^2(x^2 + y^2) - 4(gx + fy)^2 = 0$

(118) **(B).** The reflection of (a, b) in $y - x = 0$ is (b, a) so that the equation of the circle is $(x - b)^2 + (y - a)^2 = a^2$ as it touches the x-axis.

(119) **(C).** Condition for tangency is

$$c^2 = a^2(1 + m^2) \Rightarrow 8b^2 = 2ab\left(1 + \frac{4b^2}{a^2}\right)$$

$\Rightarrow 4b^2 - 4ab + a^2 = 0 \Rightarrow a = 2b$

EXERCISE-2

(1) **(B).** Angle between direct common tangents

$$= 2\sin^{-1}\left(\frac{r_1 \sim r_2}{d}\right) = 90°$$

$$\Rightarrow \frac{r_1 \sim r_2}{d} = \frac{1}{\sqrt{2}} \Rightarrow 2(r_1 - r_2)^2 = d^2 \quad \text{......(1)}$$

circles are orthogonal $\Rightarrow d^2 = r_1^2 + r_1^2$ ...... (2)

From (1) and (2), we get $2(r_1 - r_2)^2 = r_1^2 + r_2^2$

$$\Rightarrow r_1^2 + r_2^2 = 4r_1r_2 \Rightarrow \frac{r_1}{r_2} + \frac{r_2}{r_1} = 4 \Rightarrow p + \frac{1}{p} = 4$$

(2) **(B).** Any line passing through (2, 2) will be of the form

$$\frac{y-2}{\sin\theta} = \frac{x-2}{\cos\theta} = r$$

When this line cuts the circle

$x^2 + y^2 = 2,\ (r\cos\theta + 2)^2 + (2 + r\sin\theta)^2 = 2$

$\Rightarrow r^2 + 4(\sin\theta + \cos\theta)\, r + 6 = 0$

$\dfrac{PB}{PA} = \dfrac{r_2}{r_1}$, now if $r_1 = \alpha,\ r_2 = 3\alpha$,

then $4\alpha = -4(\sin\theta + \cos\theta),\ 3\alpha^2 = 6 \Rightarrow \sin 2\theta = 1$

$\Rightarrow \theta = \pi/4.$

So required chord will be $y - 2 = 1(x - 2) \Rightarrow y = x$.

(3) **(C).** Family of circles is

$x^2 + y^2 - 2x - 4y + 1 + \lambda(x^2 + y^2 - 1) = 0$

$(1+\lambda)x^2 + (1+\lambda)y^2 - 2x - 4y + (1-\lambda) = 0$

$$x^2 + y^2 - \frac{2}{1+\lambda}x - \frac{4}{1+\lambda}y + \frac{1-\lambda}{1+\lambda} = 0$$

Centre is $\left[\dfrac{1}{1+\lambda}, \dfrac{2}{1+\lambda}\right]$ and radius

$$= \sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \left(\frac{1-\lambda}{1+\lambda}\right)} = \frac{\sqrt{4+\lambda^2}}{1+\lambda}$$

Since it touches the line $x + 2y = 0$, hence

Radius = Perpendicular from centre to the line.

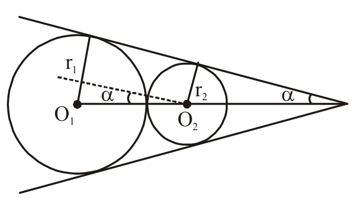
$$\text{i.e., } \left|\frac{\frac{1}{1+\lambda} + 2\,\frac{2}{1+\lambda}}{\sqrt{1^2 + 2^2}}\right| = \frac{\sqrt{4+\lambda^2}}{1+\lambda}$$

$\Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$

$\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$

Thus, we get the equation of circle.

(4) **(B).** $\sin\alpha = \dfrac{r_1 - r_2}{r_1 + r_2} \Rightarrow \theta = 2\sin^{-1}\left(\dfrac{r_1 - r_2}{r_1 + r_2}\right)$



(5) **(B).** Any second degree curve passing through the intersection of the given curves is

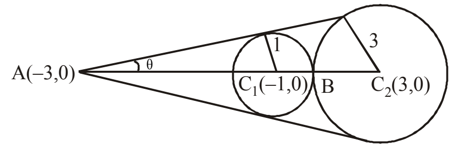
$ax^2 + 4xy + 2y^2 + x + y + 5$
$\qquad + \lambda(ax^2 + 6xy + 5y^2 + 2x + 3y + 8) = 0$

If it is a circle, then coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$

$a(1+\lambda) = 2 + 5\lambda$ and $4 + 6\lambda = 0$

$\Rightarrow a = \dfrac{2+5\lambda}{1+\lambda}$ and $\lambda = -\dfrac{2}{3} \Rightarrow a = \dfrac{2-(10/3)}{1-(2/3)} = -4.$

(6) **(A).** A divides C_1C_2 externaly in the ratio 1 : 3.



$\therefore$ coordinate of A are $(-3, 0)$

We have $\sin\theta = 1/2 \quad \therefore \theta = 30°$

Area $= 3 \times 3 \tan 30° = 3\sqrt{3}$

(7) **(D).** $(x-1)^2 + (y+2)^2 = 16$

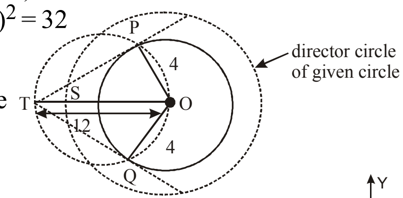
$(x-1)^2 + (y-2)^2 = 32$

$\Rightarrow OS = 4\sqrt{2}$

Required distance

$TS = OT - SO$

$TS = 12 - 4\sqrt{2}$

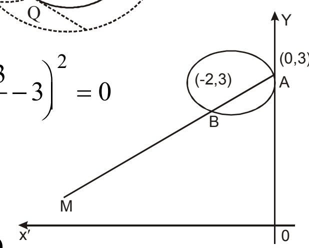


(8) **(B).** $\left(\dfrac{h}{2}\right)^2 + 4\left(\dfrac{h}{2}\right) + \left(\dfrac{k+3}{2} - 3\right)^2 = 0$

$\Rightarrow \dfrac{h^2}{4} + \dfrac{8h}{4} + \dfrac{(k-3)^2}{4} = 0$

or $x^2 + y^2 + 8x - 6y + 9 = 0$

This is a circle.



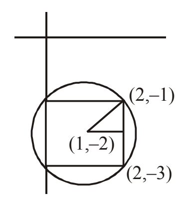
(9) **(D).** Centre $(1, -2)$, radius $\sqrt{2}$

$\therefore$ vertices are

$\left(1 \pm \sqrt{2}\cos 45°, -2 \pm \sqrt{2}\sin\dfrac{\pi}{4}\right)$

$\equiv (1 \pm 1, -2 \pm 1) \equiv (0, -1)$ and $(0, -3)$

$(2, -1)$ and $(2, -3)$

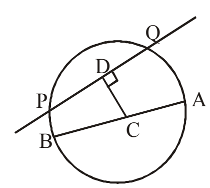


(10) **(A).** Since $\angle APB = \angle AQB = \dfrac{\pi}{2}$ so $y = mx + 8$ intersect the circle whose diameter is AB.

Equation of circle is $x^2 + y^2 = 16$

$CD < 4$

$\Rightarrow \dfrac{8}{\sqrt{1+m^2}} < 4 \Rightarrow 1 + m^2 > 4$



$\Rightarrow m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

If the line passing throw the point A $(-4, 0)$, B $(4, 0)$, then $\angle APB = \angle AQB = \pi/2$ does not formed.

$\therefore m \neq \pm 2$

(11) **(C).** Equation of the two circles be $(x - r)^2 + (y - r)^2 = r^2$

i.e., $x^2 + y^2 - 2rx - 2ry + r^2 = 0$, where $r = r_1$ and r_2.

Condition of orthogonality gives

$2r_1r_2 + 2r_1r_2 = r_1^2 + r_2^2 \Rightarrow 4r_1r_2 = r_1^2 + r_2^2$

Circle passes through (a, b)

$\Rightarrow a^2 + b^2 - 2ra - 2rb + r^2 = 0$

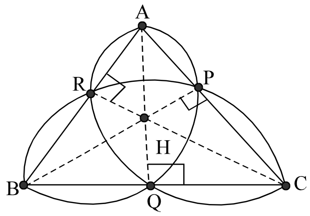
i.e., $r^2 - 2r(a+b) + a^2 + b^2 = 0$

$r_1 + r_2 = 2(a+b)$ and $r_1r_2 = a^2 + b^2$

$\therefore\ 4(a^2+b^2) = 4(a+b)^2 - 2(a^2+b^2)$

i.e., $a^2 - 4ab + b^2 = 0$

(12) **(D)**



(13) **(C).** $x^2+y^2-12x+35=0$ .... (1)

and $x^2+y^2+4x+3=0$ .... (2)

Equation of radical axis of (1) and (2) is

$-16x+32=0$ i.e., $x=2$

It intersect the line joining the centers i.e., $y=0$ at the point (2, 0)

$\therefore$ required radius $= \sqrt{4-24+35} = \sqrt{15}$

(14) Let P (x_1, y_1) be the given point and PT_1, PT_2, PT_3 be the lengths of the tangents from P to the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$ and $x^2+y^2=c^2$ respectively.

Then, $PT_1 = \sqrt{x_1^2+y_1^2-a^2}$, $PT_2 = \sqrt{x_1^2+y_1^2-b^2}$ and

$PT_3 = \sqrt{x_1^2+y_1^2-c^2}$

Now, $PT_1^{\,2}, PT_2^{\,2}, PT_2^{\,3}$ are in A.P.

$\Rightarrow 2\,PT_2^{\,2} = PT_1^{\,2} + + PT_3^{\,2}$

$\Rightarrow 2\,(x_1^2+y_1^2-b^2) = (x_1^2+y_1^2-a^2) + (x_1^2+y_1^2-c^2)$

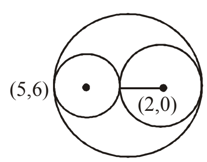
$\Rightarrow 2b^2 = a^2+c^2 \Rightarrow a^2, b^2, c^2$ are in A.P.

(15) **(B).** $(x-2)^2+b^2=4$

centre is (2, 0) and radius 2.

Distance between (2, 0) and (5, 6) is

$\sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

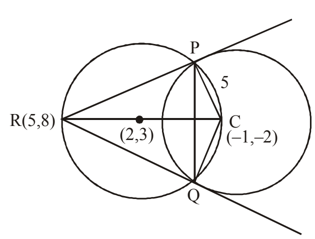


$\therefore\ r_1 r_2 = \frac{3\sqrt{5}-2}{2} \cdot \frac{3\sqrt{5}+2}{2} = \frac{45-4}{4} = \frac{41}{4}$

(16) **(A).** Let C be the centre of the given circle.

Then circumcircle of the Δ RPQ passes through C.

$\therefore$ (2, 3) is the mid point of RC



$\therefore$ Coordinates of C are (–1 , –2)

$\therefore$ Equation of the circle is $x^2+y^2+2x+4y-20=0$

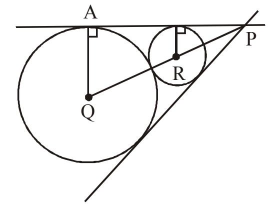
(17) **(D).** $AQ = 3+2\sqrt{2}$

$PQ = 3\sqrt{2}+4$

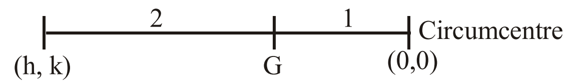
Let r be required radius

$3\sqrt{2}+4 = 3+2\sqrt{2}+r+r\sqrt{2}$

$\sqrt{2}+1 = r(1+\sqrt{2}) \Rightarrow r = 1$

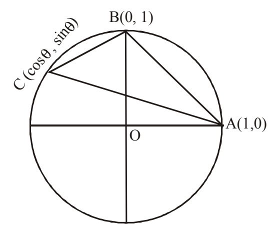


(18) **(A).** Let C (cos θ, sin θ), H (h, k) is the orthocentre of the ΔABC



$\frac{\cos\theta+1}{3}, \frac{\sin\theta+1}{3}$

$h = 1+\cos\theta,\ k = 1+\sin\theta$



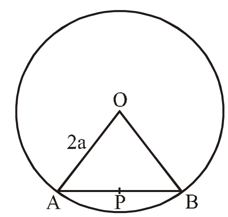
$(x-1)^2+(y-1)^2=1$

$x^2+y^2-2x-2y+1=0$

(19) **(B).** Since $\angle AOB = 90^\circ$

$PA = PB = PO = AO\cos 45^\circ$

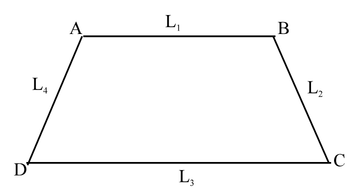
$= \frac{2a}{\sqrt{2}} = a\sqrt{2}$



Since $OP = a\sqrt{2}$, locus of P is the circle with O as origin and radius $a\sqrt{2}$ and its equation is $x^2+y^2=2a^2$.

(20) **(B).** Equation of circum circle be $L_1 . L_3 + \lambda L_2 L_4 = 0$

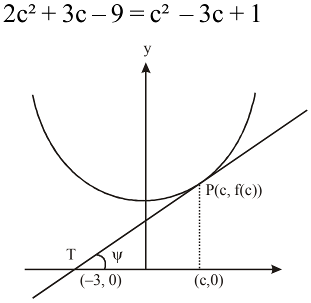
For circle a = b, h = 0. Put λ and find circle $2x^2+2y^2=125$



(21) **(B).** $\left.\frac{dy}{dx}\right]_P = \frac{f(c)}{c+3}$

$(2c-3)(c+3) = c^2 - 3c + 1$

$2c^2 + 3c - 9 = c^2 - 3c + 1$



$c^2+6c-10=0 \Rightarrow c_1+c_2=-6$

(22) **(B).** y = mx is a tangent to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$

if "p = r", (i.e.) $\left|\dfrac{b - ma}{\sqrt{1+m^2}}\right| = \sqrt{a^2 + b^2 - b^2}$

$\therefore\ b^2 - 2abm = a^2$ or $m = \dfrac{b^2 - a^2}{2ab}$

Equation of the tangent is $y = \left(\dfrac{b^2 - a^2}{2ab}\right)x$

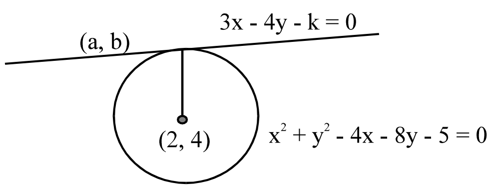
Also x = 0 is a tangent, since $y^2 - 2by + b^2$ is a perfect square.

(23) **(A).** Perpendicular distance from centre upon line equal to radius

$\Rightarrow (x-2)^2 + (y-4)^2 = 25$

$\Rightarrow 4y - 16 = 3x - 6 \pm 25 \ \Rightarrow K = -35,\ K = +15$

Slope of tangent $= \dfrac{3}{4} \Rightarrow \dfrac{b-4}{a-2}.\dfrac{3}{4} = -1$



$\Rightarrow\ x = 2 \pm 5\left(-\dfrac{3}{5}\right),\ y = 4 \pm 5\left(\dfrac{4}{5}\right)$

$\Rightarrow a + b + K \Rightarrow -1 + 8 - 35 = -28$ and $5 + 15 = 20$

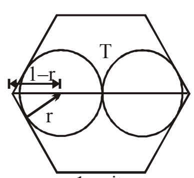
(24) **(B).** $\sin 60° = \dfrac{r}{1-r} = \dfrac{\sqrt{3}}{2}$

$2r = \sqrt{3} - \sqrt{3}r$;

$r = \dfrac{\sqrt{3}}{2+\sqrt{3}} = \sqrt{3}\,(2 - \sqrt{3})$

$= 2\sqrt{3} - 3$

$\Rightarrow a = 2, b = -3 \ \Rightarrow\ (a + b) = -1$



1 unit

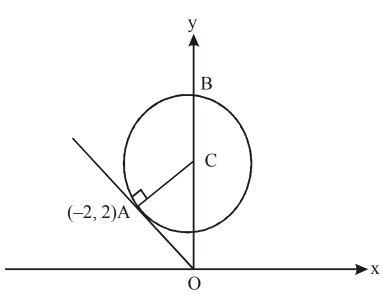
(25) **(C).** A = (–2, 2)

Equation of AC is $y - 2 = 1\,(x + 2)$

i.e. $x - y + 4 = 0$. Hence C = (0, 4)

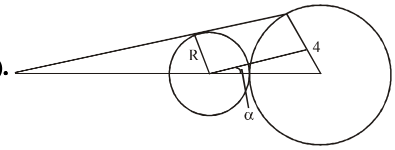
Radius = CA = $2\sqrt{2}$

For any point (α, β) on this circle β is maximum when (α, β) corresponds to point B and then



$\beta = OB = OC + CB = 4 + 2\sqrt{2}$

(26) **(D).**



$\tan 2\alpha = \dfrac{24}{7}$; $\therefore\ \tan\alpha = \dfrac{3}{4}$ $\therefore\ \sin\alpha = \dfrac{3}{5}$

$\therefore\ \dfrac{4-R}{4+R} = \dfrac{3}{5}$ $\therefore\ \dfrac{R}{4} = \dfrac{5-3}{5+3} = \dfrac{2}{8} = \dfrac{1}{4}$ $\therefore\ R = 1$

(27) **(A).** The given lines $\sqrt{3}\,(y-1) = x - 1$ ....(1)

$y - 1 = \sqrt{3}\,(x-1)$ ......(2)

intersect at the point (1, 1).

Rewriting the equation of the given lines such that their constant terms are both positive, we have

$x - \sqrt{3}\,y + \sqrt{3} - 1 = 0$ ......(3)

and $-\sqrt{3}\,x + y + \sqrt{3} - 1 = 0$ ......(4)

Here, we have

(product of coeff.'s of x) + (product of coeff.'s of y)

$= -\sqrt{3} - \sqrt{3} = -$ ve quantity

which implies that the acute angle between the given lines contains the origin.

Therefore, equation of the acute angle bisector of the given lines is

$\dfrac{x - \sqrt{3}y + \sqrt{3} - 1}{2} = + \dfrac{-\sqrt{3}x + y + \sqrt{3} - 1}{2}$ i.e. y = x

Any point on the above bisector can be chosen as (α, α) and equation of any circle passing through (1, 1) and having centre at (α, α) is

$(x - \alpha)^2 + (y - \alpha)^2 = (1 - \alpha)^2 + (1 - \alpha)^2$

i.e. $x^2 + y^2 - 2\alpha x - 2\alpha y + 4\alpha - 2 = 0$ ....(6)

The common chord of the given circle

$x^2 + y^2 + 4x - 6y + 5 = 0$ ....(7)

and the circle represented by equation (6) is

$(4 + 2\alpha)\,x + (2\alpha - 6)y + (7 - 4\alpha) = 0$

i.e. $(4x - 6y + 7) + 2\alpha(x + y - 2) = 0$ ....(8)

which represents a family of straight lines passing through the intersection point of the lines

$4x - 6y + 7 = 0$ ....(9)

and $x + y - 2 = 0$ ....(10)

Solving equation (9), (10) gives the coordinates of the fixed point as (1/2, 3/2).

(28) **(C).** Let centre of circle be P (h, k). So, that $PA^2 = PB^2$

where A = (2, 4) and B = (0, 1)

and (slope of OA) (slope of tangent at A) = –1

$\Rightarrow\ h^2 + (k-1)^2 = (h-2)^2 + (k-4)^2$

or $4h + 6k - 19 = 0$ .... (1)

also slope of OA $= \frac{k-4}{h-2}$ and slope of tangent at (2, 4) to $y = x^2$ is 4

$\therefore \frac{k-4}{h-2}.4 = -1$

or $4k - 16 = -h + 2 \Rightarrow h + 4k = 18$ .....(2)

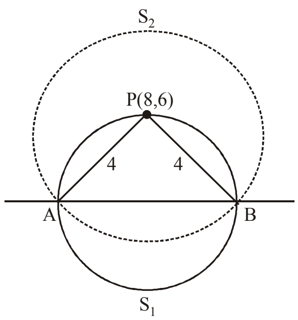
solving (1) and (2), we get

$k = 53/10$ and $h = -16/5$

$\therefore$ Centre co-ordinates are (–16/5, 53/10)

(29) **(C).** $S_1 : x^2 + y^2 = 100$

equation of S_2 centred at (8, 6) is $(x-8)^2 + (y-6)^2 = 16$



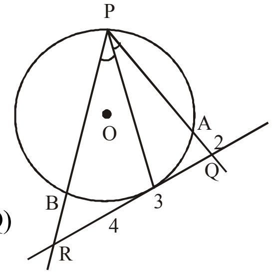
$x^2 + y^2 - 16x - 12y + 84 = 0$

$\therefore$ required line AB, (i.e. common chord) $S_1 - S_2 = 0$

$x^2 + y^2 - 16x - 12y + 84 - x^2 + y^2 - 100 = 0$

$-16x - 12y + 184 = 0$ or $4x + 3y - 46 = 0$

(30) **(B).** $(PQ)(AQ) = (QC)^2$



$PQ = \frac{9}{2}$ .... (1)

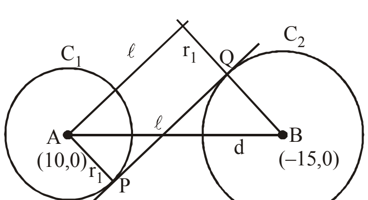
$\frac{RC}{QC} = \frac{RP}{PQ}$

($\because$ PC is angle bisector of RPQ)

$RP = \frac{4}{3} \times \frac{9}{2} = 6$ .... (2)

$(RB)(RP) = (RC)^2 \Rightarrow RB = \frac{16}{6} = \frac{8}{3}$

(31) **(C).**



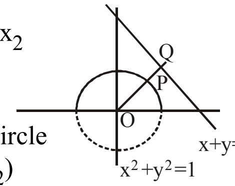
Centres are (10, 0) and (–15, 0)

$r_1 = 6; r_2 = 9, d = 25; r_1 + r_2 < d$

$\Rightarrow$ circles are separated

$PQ = \ell = \sqrt{d^2 - (r_1 + r_2)^2} = \sqrt{625 - 225} = 20$

(32) **(A).** $y_1 = \sqrt{1 - x_1^2}$ and $y_2 = 3 - x_2$



$x_1^2 + y_1^2 = 1$ and $y_2 + x_2 = 3$

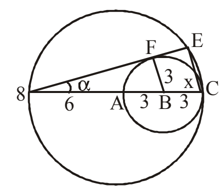
So P (x_1, y_1) is a point on semicircle $x^2 + y^2 = 1$ $(y \geq 0)$ and Q (x_2, y_2) is a point on line $x + y = 3$.

So the minimum value of $(PQ)^2$ mininmum of

$(x_1 - x_2)^2 + (y_1 - y_2)^2 = OQ - OP$ [as shown in figure]

$= \frac{3}{\sqrt{2}} - 1$

(33) **(A).** $\sin\alpha = \frac{3}{9} = \frac{1}{3}$



also $\sin\alpha = \frac{x}{12}$ (where EC = x)

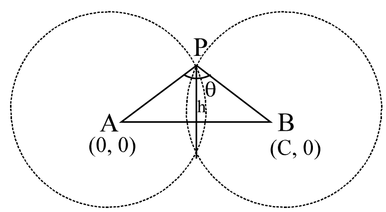
$\frac{1}{3} = \frac{x}{12} \Rightarrow x = 4$

$(DE)^2 = 144 - 16 = 128 \Rightarrow DE = 8\sqrt{2}$

(34) **(B).** $\cos\theta = \frac{-c^2 + a^2 + b^2}{2ab}$

$\Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

In Δ PAB , $\frac{1}{2}ab\sin\theta = \frac{1}{2}c.h$



Length of common chord $= 2h = \frac{2ab\sin\theta}{c}$

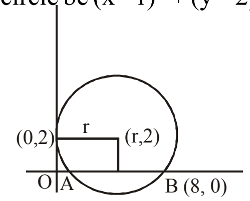
(35) **(C).** Let the equation of the circle be $(x-r)^2 + (y-2)^2 = r^2$

$x^2 + y^2 - 2xr - 4y + 4 = 0$

at A or B $y = 0$

$x^2 - 2xr + 4 = 0$

$x_1x_2 = 4 ; 8x_1 = 4$



$\Rightarrow x_1 = \frac{1}{2}$ and $x_2 = 8$

$\therefore$ sum $= 2r = 17/2 \Rightarrow r = 17/4$

(36) **(B).** $b^2 + r^2 = (36)^2$ .... (1)

Also, CD . CB = CE . CX

(using power of the point C)

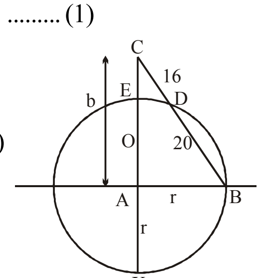
$16 . 36 = (b - r)(b + r)$

$\therefore b^2 - r^2 = 16 . 36$ .... (2)

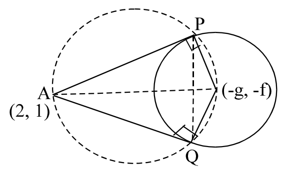
from (1) and (2)

$2b^2 = 36(36 + 16) = 36 . 52$

$b^2 = 36 . 26 \Rightarrow b = 6\sqrt{26}$

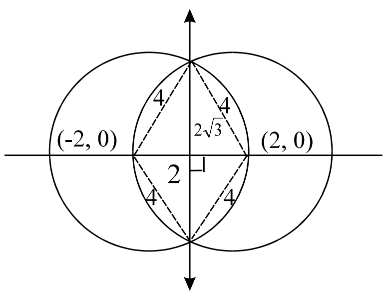


(37) **(A).** Equation of required circle is



$(x-2)(x+g)+(y-1)(y+f)=0$

(38) **(A)** Area $= 4.\left(\frac{1}{2}\times 2\times 2\sqrt{3}\right) = 8\sqrt{3}$ square units



(39) **(A).** The centre of circle is (h, h) and radius = h

$\Rightarrow$ The circle is touching the co-ordinate axes.

(40) **(A).** $C_1(1, 2), r_1 = 10$

$C_2(3, 4), r_2 = 3$

$\therefore\ C_1C_2 = 2\sqrt{2} < |r_1 - r_2| = 7$

$\therefore$ the statement is true

(41) **(D).** Slope of line joining its (1, 2) & (-4, 7) $= \frac{7-2}{-4-1} = -1$

Slope of line joining points (1, 2) & (3, 0) $= \frac{0-2}{3-1} = -1$

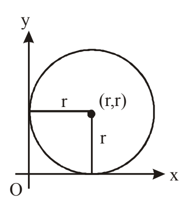
$\therefore$ points are collinear

$\therefore$ no circle can be drawn

(42) **(A).**

Equation of circle touching the coordinates axes and centre (r, r) in the first quadrant is

$x^2 + y^2 - 2xr - 2yr + r^2 = 0$



For r = a or b

Hence $C_1 : x^2 + y^2 - 2ax - 2ay + a^2$ ....(1)

Centre (a, a), radius = a, a > 0

$C_2 : x^2 + y^2 - 2bx - 2by + b^2$ ....(2)

Centre (b, b), radius b, b > 0

(a) C_1 and C_2 touch each other radical axis between (1) and (2) is (1) – (2) = 0

$2(b-a)x + 2(b-a)y - (b^2 - a^2) = 0$

$2x + 2y - (b+a) = 0$ ....(3)

If it touches both C_1 and C_2 then perpendicular from (a, a) = radius 'a'

$\left|\frac{2a+2a-(b+a)}{\sqrt{8}}\right| = a$ ....(4)

$|3a - b| = 2\sqrt{2}\,a$ ....(5)

now origin and (a, a) must lie on the same side of (3) but (0, 0) gives – ve sign with (3).

hence (a, a) should also give the same sign

i.e. $4a - b - a < 0 \Rightarrow 3a - b < 0$

Hence (5) becomes

$b - 3a = 2\sqrt{2}a \Rightarrow \frac{b}{a} = 3 + 2\sqrt{2}$

Alternativly: (A) As C_1 and C_2 touch each other externally so, distance between their centre = sum of their radius

$\Rightarrow \sqrt{(a-b)^2 + (a-b)^2} = (a+b)$

$\Rightarrow 2(a-b)^2 = (a+b)^2 \Rightarrow a^2 + b^2 - 6ab = 0$

$\therefore \frac{b}{a} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$

but $\frac{b}{a} = 3 - 2\sqrt{2}$ (rejected as $\frac{b}{a} > 1$.

Hence $\frac{b}{a} = 3 + 2\sqrt{2}$

(b) If (1) and (2) are orthogonal then

$2g_1g_2 + 2f_1f_2 = C_1 + C_2$

i.e. $2(-a)(-b) + 2(-a)(-b) = a^2 + b^2$

$4ab = a^2 + b^2$

$\left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 1 = 0$

If $\frac{b}{a} = t,\ t^2 - 4t + 1 = 0$

$\Rightarrow (t-2)^2 = 3 \Rightarrow t - 2 = +\sqrt{3}$ or $-\sqrt{3}$

$t = 2 + \sqrt{3}$

as $t > 1 \Rightarrow 2 - \sqrt{3}$ is not possible

$\therefore \frac{b}{a} = 2 + \sqrt{3} \Rightarrow r$

(c) If common chord is longest then (3) must pass through the centre (a, a) of C_1.

i.e. $4a - b - a = 0$

$3a = b \Rightarrow \frac{b}{a} = 3 \Rightarrow q$

(d) If C_2 passes through the centre of C_1 then (a, a) must satisfy (2)

i.e. $a^2 + a^2 - 2b(2a) + b^2 = 0$

$\Rightarrow 2a^2 - 4ab + b^2 = 0$

$\left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 2 = 0$

Put $\frac{b}{a}=t$; $t^2-4t+2=0$

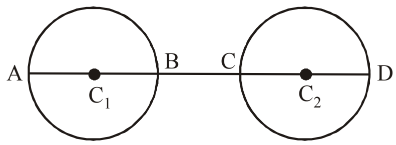
$\Rightarrow (t-2)^2=4-2=2 \Rightarrow t-2=\sqrt{2}$ or $-\sqrt{2}$

$t=2+\sqrt{2}, t\neq 2-\sqrt{2}$ (as $t>1$) $\Rightarrow$ p

(43) (D).

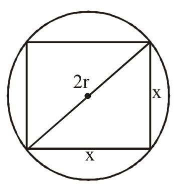
(a) Greatest distance is

$AD=C_1C_2+AC_1+DC_2=5+1+3=9$



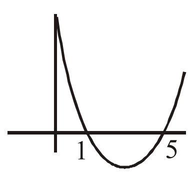
$9=3\lambda \Rightarrow \lambda=3$

(b) $x^2=200$; $2x^2=4r^2$; $2r^2=200 \Rightarrow r=10$



(c) $y=\cos^4 x-6\cos^2 x+5\cos^2 x=t$

$y=t^2-6t+5 \qquad 0\le t\le 1$



For $t\in[0,1]$ min. occurs at $t=1$

$y_{min}=0$

(d) Distance between $\left(\frac{1}{3},\frac{1}{3}\right)$ and $\left(\frac{8}{3},\frac{8}{3}\right)$ is $\frac{7}{3}\sqrt{2}$

$\therefore \frac{\ell}{\sqrt{2}}=7$

(44) (C).

(a) Centre and radius of the circle

$x^2+y^2+14x-4y+28=0$ are $(-7,2)$, 5 respectively

Centre and radius of the circle.

$x^2+y^2-14x+4y-28=0$ are $(7,-2)$, 9

$\therefore$ length of direct common tangent

$=\sqrt{(7+7)^2+(-2-2)^2-(9-5)^2}=14$

(b) the line is $mx-y+2-m=0$

$\left|\frac{2-m}{\sqrt{m^2+1}}\right|<5$ which is true for all real values of m

(c) $x^2+(y-\sqrt{2})^2=r^2$ i.e., $x^2+y^2-2\sqrt{2}y+2=r^2$

$\therefore$ for $y=0$, we have $x^2+2=r^2$

$\therefore$ if r is rational and $r^2>2$, then there are 2 points on the circle which have rational co-ordinates.

further if there are three point, then circumcentre of the triangle fromed by these three point has rational coordinates, which is not so.

$\therefore$ maximum number of points is 2.

(d) Let (h, k) be the centre, then

$|h|=|k|$ and $|h+k-4|=\sqrt{2}|h|$

Case - 1 : If $h=k$, then $|2h-4|=\sqrt{2}|h|$ i.e. $2h-4=\pm\sqrt{2}h$

It gives two different values of (h, k)

Case 2 : If $h=-k$, then $|-4|=\sqrt{2}|h|$ i.e. $h=\pm 2\sqrt{2}$

it a gain gives two different points (h, k) thus there are 4 different circles.

(45) (D). Feet of perpendicular are collinear.

(46) (D). I_1 is the orthocentre of $\Delta\, I I_2 I_3$ by property of triangle.

(47) (B). As Δ XYZ is pedal triangle of Δ ABC, ex-centers of Δ XYZ lie on vertices of Δ ABC.

(48) (D). $S_1-S_2=0 \Rightarrow x+y=4$ (Radical axis)

$S_1-S_3=0 \Rightarrow 3x-4y=14$ (Radical axis)

Radical centre = intersection point of radical axis

$\Rightarrow \left(\frac{30}{7},\frac{-2}{7}\right)$

(49) (B). Radius of circle is nothing but length of tangent from radical centre to any of the given circle.

$\Rightarrow r=\sqrt{\left(\frac{30}{7}\right)^2+\left(\frac{-2}{7}\right)^2-4}=2\frac{\sqrt{177}}{7}$

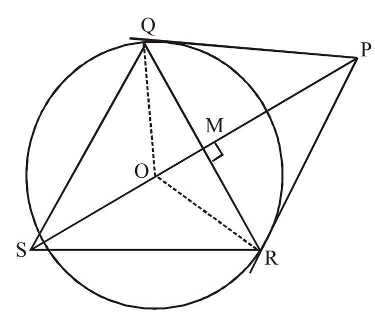
(50) (C). Point of intersection of direct tangent always lie on the line joining there centre

$\Rightarrow$ (0, 0) and (3, - 4)

$\Rightarrow$ line is $3y+4x=0$

(51) (B), (52) (D), (53) (B).

$\because$ PQ = PR i.e. parallelogram PQRS is a rhombus



$\therefore$ Mid point of QR = Midpoint of PS and QR $\perp$ PS

$\therefore$ S is the mirror image of P w.r.t. QR

$\because L\equiv 2x+y=6$, Let $P\equiv(k, 6-2k)$

$\because \angle PQO=\angle PRO=\frac{\pi}{2}$

$\therefore$ OP is diameter of circumcircle PQR,

then centre is $\left(\frac{k}{2}, 3-k\right)$

$\therefore\ x = \frac{k}{2} \Rightarrow k = 2x$

$y = 3 - k \therefore 2x + y = 3.$

P (6, 8)

$\therefore$ Equation of QR is $6x + 8y = 4 \Rightarrow 3x + 4y - 2 = 0$

$\therefore\ PM = \frac{48}{5}$ and $PQ = \sqrt{96}$

$QM = \sqrt{96 - \frac{(48)^2}{25}} = \sqrt{\frac{96}{25}} \therefore QR = 2\sqrt{\frac{96}{25}}$

$\therefore$ Area of $\Delta PQR = \frac{1}{2}.PM.QR = \frac{196\sqrt{6}}{25}$

$\because$ PQRS is a rhombus

$\therefore$ Area of ΔQRS = Area of $\Delta PQR = \frac{196\sqrt{6}}{25}$ sq. units.

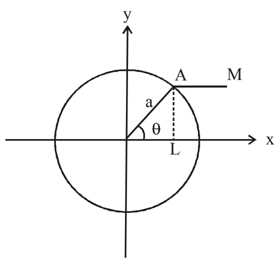
$P \equiv (3, 4)$

$\therefore$ equation of QR is $3x + 4y = 4$ .... (i)

Let $S \equiv (x_1, y_1)$

$\because$ S is mirror image of P w.r.t. eq. (i)

then $\frac{x_1 - 3}{3} = \frac{y_1 - 4}{4} = \frac{-2\,(3 \times 3 + 4 \times 4 - 4)}{3^2 + 4^2} = -\frac{42}{25}$



$\therefore\ x_1 = -\frac{51}{25}, y_1 = -\frac{68}{25}$; $S \equiv \left(-\frac{51}{25}, -\frac{68}{25}\right)$

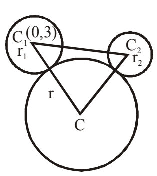
(54) (D), (55) (A), (56) (D).

$r_1 = 2, r_2 = 1, C_1 = (0, 3), C_2 = (6, 0),\ C_1C_2 = 3\sqrt{5}$

Clearly the circle with centre C_1 and C_2 are separated

$CC_1 = r + r_1$; $CC_2 = r + r_2$

$CC_1 - CC_2 = r_1 - r_2$ = constant



(57) (D), (58) (D), (59) (D).

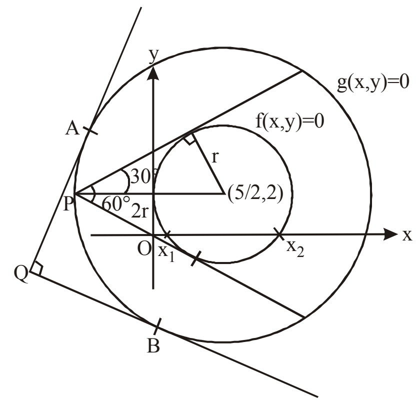
Given $f(x, y) = 0$ is circle. As $f(0, y)$ has equal roots hence $f(x, y) = 0$ touches the y-axis and as $f(x, 0) = 0$ has two distinct real roots hence $f(x, y) = 0$ cuts the x-axis in two distinct points. Hence $f(x, y) = 0$ will be as shown now, given $g(x, y) = x^2 + y^2 - 5x - 4y + c$

centre $= \left(\frac{5}{2}, 2\right)$, radius $= \sqrt{\frac{25}{4} + 4 - c}$

Note that radius of $g(x, y)$ = twice the radius of $f(x, y) = 0$ but as it is clear from the adjacent figure $r = 5/2$

$\therefore$ radius of $g(x, y) = 5$

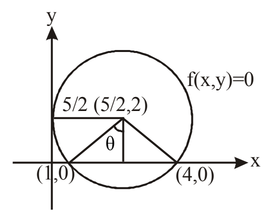
hence, $\frac{25}{4} + 4 - c = 25 \Rightarrow c = -\frac{59}{4}$



Equation of $g(x, y)$ is $x^2 + y^2 - 5x - 4y - \frac{59}{4} = 0$

equation of $f(x, y) = 0$

$\left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \frac{25}{4}$



$y = 0, \left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 4 = \frac{9}{4}$

$x - \frac{5}{2} = \frac{3}{2}$ or $-\frac{3}{2} \Rightarrow x = 4$ or $x = 1$

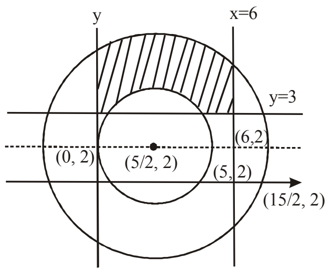
Area of $\Delta QAB = \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

$\theta = \tan^{-1}\frac{3}{4}$; $2\theta = \tan^{-1}\left(\frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{27}{4}\right)$

Area of region inside $f(x, y) = 0$ above the x-axis is

x-axis $= \frac{1}{2}\left(\frac{5}{2}\right)^2\left(2\pi - \tan^{-1}\left(\frac{27}{4}\right)\right) + \frac{1}{2} \times 3 \times 2$

$= 3 + \frac{25}{8}\left(2\pi - \tan^{-1}\left(\frac{27}{4}\right)\right)$



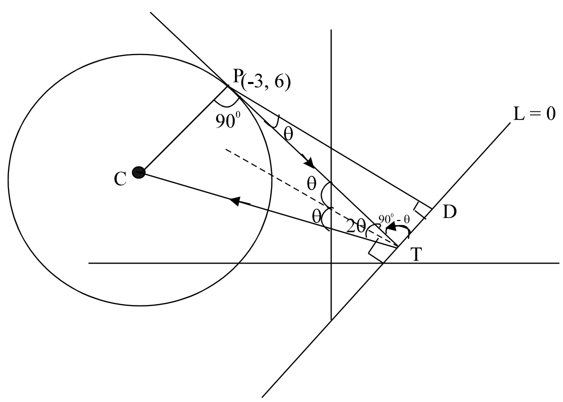
(11) Points satisfying the conditions are
(1, 5) (1, 6), (2, 5), (2, 6) (3, 5), (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6).

(60) **(B).** $14\,.\,x\,.\,(-3)+14\,.\,y\,.\,6+108(x-3)-\dfrac{69}{2}(y+6)+432=0$

$\Rightarrow\ x(108-42)+y\left(84-\dfrac{69}{2}\right)+(432-531)=0$

$\Rightarrow\ 4x+3y-6=0$

(61) **(C).** $g=\dfrac{216}{28},\ f=-\dfrac{69}{28},\ c=\dfrac{432}{14}$



radius $=\sqrt{g^2+f^2-c}=\dfrac{165}{28}$

(49) **(B).** $\angle DPT=\theta$, Slope of PT = – 4/3

Let $PT=\ell$, $\tan 2\theta=\dfrac{165}{28\ell}$ ....(i)

$\cos\theta=\dfrac{11\sqrt{130}}{13\ell}$ ....(ii)

Dividing (i) by (ii) $\dfrac{\tan 2\theta}{\cos\theta}=\dfrac{15.13}{28.\sqrt{13}.\sqrt{10}}$

$\sin\theta=\dfrac{-56\sqrt{10}\pm 74\sqrt{10}}{60\sqrt{13}}$ (only positive value is possible)

$\Rightarrow\ \tan\theta=\dfrac{3}{11}$

EXERCISE-3

(1) **28.** Equation of common tangent is $4x + 3y = 10$

$\therefore$ equation of a circle is

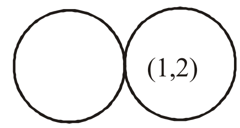
$(x-1)^2+(y-2)^2+\lambda\,(4x+4y-10)=0$

i.e. $x^2+y^2+(4\lambda-2)\,x+(3\lambda-4)\,y+5-10\lambda=0$

Comparing it with

$x^2+y^2+\alpha x+\beta y-15=0$, we get

$\alpha=4\lambda-2,\ \beta=3\lambda-4$ and $15=10\lambda-5\ \therefore\ \alpha=6,\ \beta=2$



Comparing with $x^2+y^2+\gamma x+\delta y+25=0$, we get

$\gamma=4\lambda-2,\ \delta=3\lambda-4$ and $25=5-10\lambda$

$\therefore\ \gamma=-10,\ \delta=-10$

Thus $\alpha+\beta-(\gamma+\delta)=28$

(2) **3.** Let $(\alpha, 3-\alpha)$ be any point on $x + y = 3$

$\therefore$ equation of chord of contact is $\alpha x+(3-\alpha)\,y=9$

i.e., $\alpha\,(x-y)+3y-9=0$

$\therefore$ the chord passes through the point (3, 3) for all values of α.

(3) **5.** Equation of line joining origin and centre of circle

$C_2\equiv(2, 1)$ is, $y=\dfrac{x}{2}\Rightarrow x-2y=0$

Let equation of common tangent is

$x-2y+c=0$ .... (1)

$\therefore$ Perpendicular distance from (0, 0) on this line
= perpendicular distance from (1, 1)

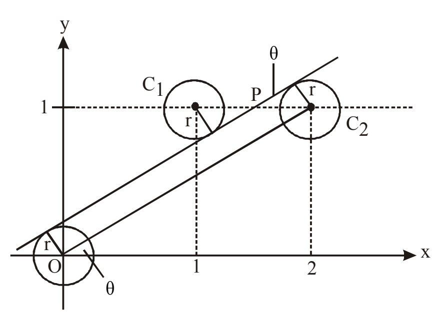
$\left|\dfrac{c}{\sqrt{5}}\right|=\left|\dfrac{c-1}{\sqrt{5}}\right|\Rightarrow c=1-c\Rightarrow c=\dfrac{1}{2}$

Equation of common tangent is

$x-2y+\dfrac{1}{2}=0\ \Rightarrow\ 2x-4y+1=0$ .... (2)

perpendicular from (2, 1) on the line (2)

$r=\left|\dfrac{4-4+1}{\sqrt{20}}\right|=\dfrac{1}{2\sqrt{5}}=\dfrac{\sqrt{5}}{10}$



Alternative sol 1 : P is the mid point of C_1C_2

$\therefore$ P (3/2, 1)

hence eq. of the common tangent is $y-1=\frac{1}{2}\left(x-\frac{3}{2}\right)$

$2x-4y+1=0$ now proceed

Alternative sol 2 : $\sin\theta = 2r$ as $(PC_2 = 1/2)$

$\sin\theta = \frac{1}{\sqrt{5}}$ as $(CC_2 = \sqrt{5})$.

Hence, $2r = \frac{1}{\sqrt{5}}$ $\quad\therefore\ r = \frac{1}{2\sqrt{5}}$

(4) **2.** Equation of circle is $x^2+y^2+2gx+2fy+c=0$

$\Rightarrow 1+t^2+2g+2ft+c=0$

$(t, t) \Rightarrow t^2+t^2+2gt+2ft+c=0$

$(t, 1) \Rightarrow 1+t^2+2gt+2f+c=0$

subtract $\quad 1+2g-t^2-2gt=0$

$\Rightarrow 1-t^2+2g(1-t)=0$

$\Rightarrow (1-t)(1+t+2g)=0 \Rightarrow t=1$

$\therefore$ one point (t, t) $\quad\therefore$ passes through (1, 1)

(5) **50.** The equation of given circle is

$S(x, y) = x^2+y^2-6x-2py+17=0$

$\Rightarrow (x-3)^2+(y-p)^2=(p^2-8)$

$S(0, 0) = 17 > 0 \quad\therefore$ (0, 0) lies outside the circle.

Equation of director circle of S = 0 will be

$(x-3)^2+(y-p)^2=2(p^2-8)$.

$\therefore$ Tangents drawn from (0, 0) to S = 0 are perpendicular to each other

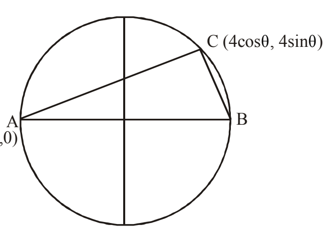
$\therefore$ (0, 0) must lie on director circle.

$\Rightarrow (0-3)^2+(0-p)^2=2(p^2-8)$

$\Rightarrow p^2=25 \Rightarrow p=\pm 5$

Hence $p_1^{\,2}+p_2^{\,2}=(5)^2+(-5)^2=25+25=50$

(6) **62.**



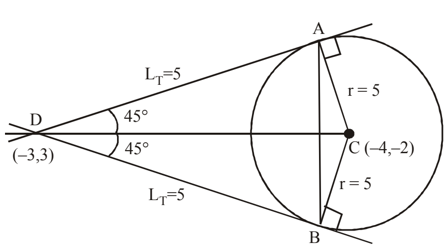
$A = \frac{1}{2}\times 8\times 4\sin\theta = |16\sin\theta|$

Now $\sin\theta$ can be $\frac{1}{16}, \frac{2}{16}, ...\frac{15}{16}$

i.e. 15 points in each quadrant

$\Rightarrow$ 60 + 2 more with $\sin\theta = 1 \Rightarrow$ total = 62

(7) **25 .**



$L = \sqrt{S_1} = 5$

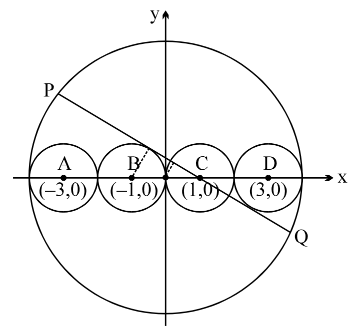
Area of quadrilateral ABCD

$= 2$ Area of $\Delta ACD = 2\left(\frac{1}{2}\times 5\times 5\right) = 25$ sq. units

(8) **63.** Triangles BCM and OCN are similar

now let ON = p. N will be mid point of chord PQ

$\therefore\ \frac{p}{1}=\frac{1}{2} \Rightarrow p=\frac{1}{2}$



Now $R = 2\sqrt{r^2-p^2}$

for large circle = $2\sqrt{16-(1/4)} = \sqrt{63}$

Alternatively: Equation of large circle as $x^2+y^2=16$

now C = (1, 0) with slope PQ = $-\frac{1}{\sqrt{3}}$ (think !)

equation of PQ : $\sqrt{3}\,y+x=1$

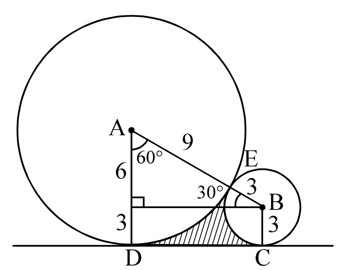
P (from origin) = $\frac{1}{2} \Rightarrow$ result

(9) **69.** Let r be the radius of circle A

and R be the radius of circle B

$\therefore r+R=12$ and $r=3R$

$\therefore 4R=12;\ \therefore R=3$ and $r=9$



Area of trapezium ABCD = $\frac{1}{2}(3+9)\sqrt{(12)^2-6^2}$

$= 6\sqrt{108} = 36\sqrt{3}$

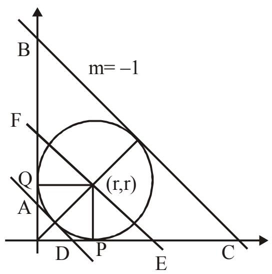
Area of arc ADC = $\frac{1}{2}\times 81\times\frac{\pi}{3} = \frac{27\pi}{2}$

Area of arc BCE = $\frac{1}{2}\times 9\times\frac{2\pi}{3} = 3\pi$

$\therefore$ required area $= 36\sqrt{3}-\left(\frac{27\pi}{2}+3\pi\right) = 36\sqrt{3}-\frac{33\pi}{2}$

$\therefore a=36,\ \ b=33 \quad \therefore a+b=69$

(10) 15.



Δ = Area of ABCD

$= \frac{1}{2}(a+b)h$ {as ABCD is a trapezium}

where $\frac{a+b}{2} = EF$ (median) and $h = 2r$

Hence $\Delta = 2r\,(EF)$

Equation of EF is $y = -x + c$ passes through $(r, r) \Rightarrow c = 2r$

Hence EF is $y = -x + 2r$

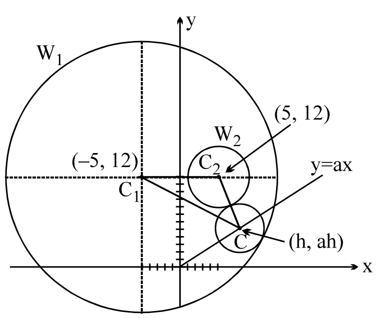
$\Rightarrow E = (2r, 0)$ and $F \equiv (0, 2r) \Rightarrow EF = 2\sqrt{2}r$

$\Delta = (2r)(2\sqrt{2}r) = 4\sqrt{2}r^2 = 900\sqrt{2} \Rightarrow r = 15$

(11) 169. W_1: $C_1 = (-5, 12)$, $r_1 = 16$ $\quad$ W_2: $C_2 = (5, 12)$, $r_2 = 4$

Now, $CC_2 = r + 4$

$CC_1 = 16 - r$



let $C(h, k) = c(h, ah)$

$CC_1^2 = (16 - r)^2$

$\Rightarrow (h + 5)^2 + (12 - ah)^2 = (16 - r)^2$

$CC_2^2 = (4 + r)^2$

$\Rightarrow (h - 5)^2 + (12 - ah)^2 = (4 + r)^2$

By subtraction, $20h = 240 - 40r$

$\Rightarrow h = 12 - 2r \Rightarrow 12r = 72 - 6h$ ...(1)

By addition

$2[h^2 + 25 + a^2h^2 - 24ah + 144] = 272 - 24r + 2r^2$

$h^2(1 + a^2) - 24ah + 169 = 136 - 12r + r^2 = 136 + (6h - 72)$

$+ \left(\frac{12-h}{2}\right)^2$ [using (1)]

$\Rightarrow 4[h^2(1 + a^2) - 24ah + 169] = 4[64 + 6h] + (12 - h)^2$

$= 256 + 144 + h^2$

$\Rightarrow h^2(3 + 4a^2) - 96ah + 105 \cdot 4 - 36 \cdot 4 = 0$

$\Rightarrow h^2(3 + 4a^2) - 96ah + 69 \cdot 4 = 0$; for 'h' to be real $D \ge 0$

$\Rightarrow (96a)^2 - 4 \cdot 4 \cdot 69(3 + 4a^2) \ge 0$

$\Rightarrow 576a^2 - 69.3 - 276a^2 \ge 0$

$300a^2 \ge 207 \Rightarrow a^2 \ge \frac{69}{100}$; hence m (smallest) $= \frac{13}{10}$

So, $m^2 = \frac{69}{100}$ $\quad\therefore p + q = 169$

(12) 5. Line $5x - 2y + 6 = 0$ is intersected by tangent at P to circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis at Q (0, 3).

In other words tangent passes through (0, 3).

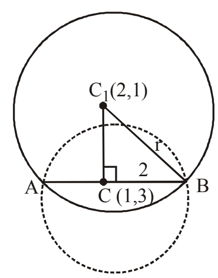
$\therefore$ PQ = length of tangent to circle from (0, 3).

$= \sqrt{0 + 9 + 0 + 18 - 2} = \sqrt{25} = 5$

(13) 3. The given circle is $x^2 + y^2 - 2x - 6y + 6 = 0$

with centre C (1, 3) and radius $= \sqrt{1 + 9 - 6} = 2$.

Let AB be one of its diameter which is the chord of other circle with centre at C_1 (2, 1).

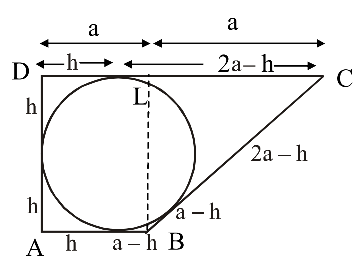


Then in $\Delta\,C_1CB$, $C_1B^2 = CC_1^2 + CB^2$

$\Rightarrow r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$

$\Rightarrow r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3$

(14) 2.



Let AB = a and AD = 2h

In triangle BCL, $a^2 + 4h^2 = (3a - 2h)^2$; $a = 3h/2$

$\frac{1}{2} \times 3a \times 2h = 18 \Rightarrow h = 2$; Radius = 2 unit.

(15) 2. $2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1$

$\cos\frac{\pi}{2k} + \cos\frac{\pi}{k} = \frac{\sqrt{3}+1}{2}$

Let $\frac{\pi}{k} = 0$, $\cos\theta + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$

$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$

$\cos\frac{\theta}{2} = t$; $2t^2 + t - \frac{\sqrt{3}+3}{2} = 0$

$$t=\frac{-1\pm\sqrt{1+4(3+\sqrt{3})}}{4}$$

$$=\frac{-1\pm(2\sqrt{3}+1)}{4}=\frac{-2-2\sqrt{3}}{4},\frac{\sqrt{3}}{2}$$

$$\because\ t\in[-1,1],\ \cos\frac{\theta}{2}=\frac{\sqrt{3}}{2}\ ;\ \frac{\theta}{2}=\frac{\pi}{6}\Rightarrow k=3$$

EXERCISE-4

(1) **(C).** Length of tangent from any point (x_1, y_1) to the circle is $\sqrt{S_1}$.

$\Rightarrow$ Length of tangent from (3, – 4) on the circle $x^2+y^2-4x-6y+3=0$

is $\sqrt{3^2+(-4)^2-4(3)-6(-4)+3}$

$=\sqrt{9+16-12+24+3}=\sqrt{40}$ and is square is 40

(2) **(A).** Given equation of circle are
$x^2+y^2+6x-2y-9=0$
and $x^2+y^2-2x+9y-11=0$
$\therefore$ equation of radical axis is $S_1-S_2=0$
$\Rightarrow 8x-11y+2=0$

(3) **(B).** Given equation of two circle are
$(x-1)^2+(y-3)^2=r^2$ ..... (1)
Coordinate of centre is (1, 3)
and radius is r
and $x^2+y^2-8x+2y+8=0$ ..... (2)
$\therefore$ Coordinate of centre is (4, –1)
$\therefore$ radius $=\sqrt{16+1-8}=3$

Now, $C_1C_2=\sqrt{(1-4)^2+[3-(-1)]^2}=5$

If circle intersect in two distinct points then
$C_1C_2<r_1+r_2$ and $C_1C_2>r_1-r_2$
$5<r+3$ and $5>r-3$
$2<r$ and $8>r$
$\Rightarrow 2<r<8$

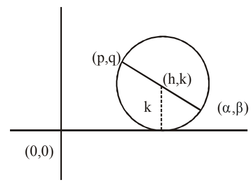
(4) **(D).** If lines $2x-3y=5$ and $3x-4y=7$ are diameter of a circle then their intersection point will be centre of circle.
$\therefore$ Intersection point of these two lines is (1, –1)
$\therefore$ Coordinate of centre of circle is (1, –1)
Now let radius of circle is r
Area is $\pi r^2=154$ (given Area = 154 sq^2 unit)

$$\Rightarrow r^2=\frac{154}{\pi}=\frac{154}{22}\times 7\Rightarrow r^2=7\times 7\Rightarrow r=7\text{ unit}$$

$\therefore$ equation of circle will be
$(x-1)^2+[y-(-1)]^2=7^2$
$\Rightarrow x^2-2x+1+y^2+1+2y=49$
$\Rightarrow x^2+y^2-2x+2y=47$

(5) **(B).** Let the equation of circle whose centre is (– g, – f) is
$x^2+y^2+2gx+2fy+c=0$ .... (1)
$\Rightarrow$ this circle passes through (a, b)
$\Rightarrow a^2+b^2+2ag+2bf+c=0$ .... (2)
Now circle (1) cut circle $x^2+y^2-4=0$
Orthogonally $\therefore 2g(0)+2f(0)=c-4$
{if two circle cuts orthogonally then condition is $2g_1g_2+2f_1f_2=c_1+c_2$} $\Rightarrow c-4=0\Rightarrow c=4$
Put this value in (2) we get $a^2+b^2+2ga+2bf+4=0$
$\Rightarrow$ for locus of centre replace (–g, –f) with (x, y)
$\Rightarrow g=-x$ and $f=-y$
$\therefore$ Locus is $a^2+b^2-2ax-2by+4=0$
$\Rightarrow 2ax+2by-(a^2+b^2+4)=0$

(6) **(A).** Equation of circle which touches x axis is
$(x-h)^2+(y-k)^2=k^2$ where let h, k a re coordinate of centre $\Rightarrow (x-h)^2+y^2-2ky=0$ .... (1)



$\because$ (p, q) lies on circle
$\therefore (p-h)^2+q^2-2kq=0$ .... (2)
Let coordinate of other end of diameter α, β

$$\therefore\ h=\frac{\alpha+p}{2}\ \text{and}\ k=\frac{\beta+q}{2}$$

Put this in (2) we get

$$\left[p-\frac{(\alpha+p)}{2}\right]^2+q^2-2\left[\frac{\beta+q}{2}\right]q=0$$

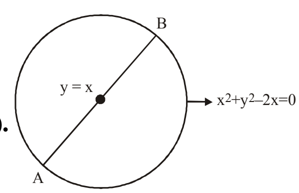
$$\Rightarrow\left[\frac{p-\alpha}{2}\right]^2+q^2-\beta q-q^2=0$$

$$\Rightarrow\frac{(p-\alpha)^2}{4}-\beta q=0\Rightarrow(p-\alpha)^2=4\beta q$$

$\therefore$ Locus of (α, β) is $(p-x)^2=4yq\Rightarrow(x-p)^2=4qy$

(7) **(A).** If lines $2x+3y-1=0$ and $3x-y-4=0$ are diameter of circle then their intersection point will be centre of circle.
$\therefore$ their interpoint is (1, –1)
$\therefore$ coordinate of circle is (1, –1)
Now circumcircle of circle is $2\pi r=10\pi$ (given)
$\Rightarrow r=5$ unit
$\therefore$ equation of circle is $(x-1)^2+[y-(-1)^2]=25$
$x^2+1-2x+y^2+1+2y=25$
$\Rightarrow x^2+y^2-2x+2y=23$
$x^2+y^2-2x+2y-23=0$

(8) **(A).**



Given circle is $x^2+y^2-2x=0$ .... (1)
Given line is $y=x$ .... (2)
Putting $y=x$ in (1) we get
$2x^2-2x=0\Rightarrow x=0, 1$

From (1), $y = 0, 1$

$\therefore$ Intersection points are $(0, 0)$ and $(1, 1)$

Let A $(0, 0)$ and B $(1, 1)$

equation of required circle is

$(x-0)(x-1)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

(9) **(B).** Given equation of circles are

$x^2+y^2+2ax+cy+a=0$

and $x^2+y^2-3ax+dy-1=0$

and their intersection point are P and Q

$\therefore$ equation of line passing through P and Q is

$S_1-S_2=0 \Rightarrow 5ax+(c-d)y+a+1=0$

$\Rightarrow 5x+\left(\frac{c-d}{a}\right)y+\frac{a+1}{a}=0$ .... (1)

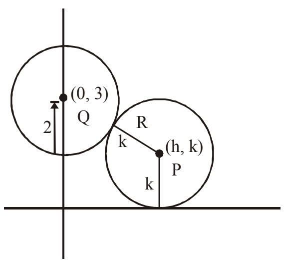
Given line is $5x+by-a=0$ .... (2)

Comine eq. (1) and (2) we get

$\frac{a+1}{a}=-a \Rightarrow a+1=-a^2 \Rightarrow a^2+a+1=0$ $\{\therefore$ d is $c>0\}$

which is not possible.

(10) **(D).**



$PR = k$ and $QR = 2$

$\Rightarrow PR + QR = k + 2$

Let centre of circle which touches x axis (h, k)

$\therefore$ equation of this circle is

$(x-h)^2+(y-k)^2=k^2$

Now, $PQ = PR + RQ$ $\therefore$ $PQ^2=(PR+RQ)^2$

$\Rightarrow (h-0)^2+(k-3)^2=(k+2)^2$

$\Rightarrow h^2+k^2-6k+9=k^2+4+4k \Rightarrow h^2-10k+5=0$

$\therefore$ Locus of (h, k) is $x^2-10y+5=0$

$x^2=10y-5$ equation of parabola

(11) **(D).** Let the equation of circle whose centre is $(-g - f)$ is

$x^2+y^2+2gx+2fy+c=0$ .... (1)

$\because$ this circle passes through (a, b)

$\therefore$ $a^2+b^2+2ag+2bf+c=0$ .... (2)

Now circle (1) cut circle

$x^2+y^2-p^2=0$

Orthogonally, $2g.(0)+2f.(0)=c-p^2 \Rightarrow c-p^2=0$

$\Rightarrow$ $c=p^2$ {if two circle cuts orthogonally then condition is $2g_1g_2+2f_1f_2=c_1+c_2$} $\Rightarrow$ $c-4=0 \Rightarrow$ $c=4$

Put this value in (2) we get

$a^2+b^2+2ga+2bf+p^2=0$

$\Rightarrow$ for locus of centre replace $(-g, -f)$ with (x, y)

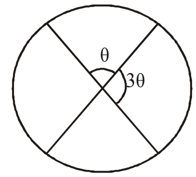
$\Rightarrow$ $g=-x$ and $f=-y$

$\therefore$ Locus is $a^2+b^2-2ax-2by+p^2=0$

$\Rightarrow$ $2ax+2by-(a^2+b^2+p^2)=0$

(12) **(D).** Equation of pair of lines is

$ax^2+2(a+b)xy+by^2=0$



$\because$ Area of one sector is thrice of the area of other section

$\therefore 4\theta=\pi \Rightarrow \theta=\pi/4$

Angle between lines is given by

$\tan\theta=\frac{2\sqrt{h^2-ab}}{|a+b|}$ $\left\{\because \tan\theta=\tan\frac{\pi}{4}=1 \text{ and } h=a+b\right\}$

$1=\frac{2\sqrt{(a+b)^2-ab}}{|a+b|} \Rightarrow (a+b)^2=4[(a+b)^2-ab]$

$\Rightarrow 3a^2+3b^2+2ab=0$

(13) **(C).** If lines $3x-4y-7=0$ and $2x-3y-5=0$ are diameter of a circle then their intersection point will be centre of circle. $\therefore$ Intersection point of these two lines is (1, 1)

$\therefore$ Coordinate of centre of circles $(1, -1)$

Now let radius of circle r

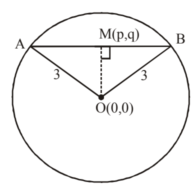
Area is $\pi r^2=154 \Rightarrow$ given area $=154$ sq^2 unit

$\Rightarrow r^2=\frac{154}{\pi}=\frac{154}{22}\times 7 \Rightarrow r^2=7\times 7 \Rightarrow r=7$ unit

$\therefore$ equation of circle will be $(x-1)^2+[(y-(-1)]^2=7^2$

$\Rightarrow x^2-2x+1+y^2+1+2y=49 \Rightarrow x^2+y^2-2x+2y=47$

(14) **(C).** Let M (p, q) be the mid point of chord AB of circle subtending an angle of $2\pi/3$ at centre as Δ AOB is an isoscleles triangle $OM \perp AB$



$\therefore$ $AM^2=OA^2-OM^2=9-(p^2+q^2)$

$\Rightarrow AM=\sqrt{9-(p^2+q^2)}$

$\Rightarrow AB=2AM=2\sqrt{9-(p^2+q^2)}$

By law of cosine

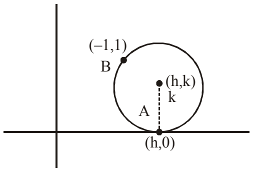
$\cos\frac{2\pi}{3}=\frac{OA^2+OB^2-AB^2}{2(OA)(OB)}$

$\Rightarrow -\frac{1}{2}=\frac{9+9-4(9-(p^2+q^2))}{2\times 3\times 3}$

$\Rightarrow -9=18-36+4p^2+4q^2 \Rightarrow p^2+q^2=\frac{9}{4}$

thus required locus is $x^2+y^2=\frac{9}{4}$

(15) (B).



$\because$ Circle touches the x-axis and coordinate of centre is (h,k) $\therefore$ radius will be k

Now equation of circle will be

$(x-h)^2+(y-k)^2=k^2$ .... (1)

$\because$ Circle passes through (–1, 1), $\therefore$ it will satisfy (1)

$\therefore\ (-1-h)^2+(1-k)^2=k^2$

$\Rightarrow\ 1+h^2+2h+1+k^2-2k=k^2$

$\Rightarrow\ 1+h^2+2h=2k-1$ ...... (2)

From the figure we see that k > 0 and

$AB\le 2k \Rightarrow \sqrt{(h+1)^2+(0-1)^2}\le 2k$

$\Rightarrow\ h^2+1+2h+1\le 4k^2 \Rightarrow\ h^2+1+2h\le 4k^2-1$

From (2), $2k-1\le 4k^2-1$

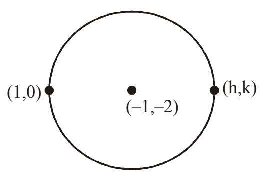
$\Rightarrow\ 4k^2-2k\ge 0 \Rightarrow\ 2k(k-1)\ge 0$

$\Rightarrow$ either $k\le 0$ or $k\ge 1/2$ but $k>0$

$\therefore\ k\ge 1/2$

(16) (B). Given equation of circle is $x^2+y^2+2x+4y-3=0$

$\therefore$ Coordinate of centre of circle is (–1, –2)



Let coordinate of point diameterically opposite to point P (1, 0) is (h, k)

$\therefore\ \frac{h+1}{2}=-1 \Rightarrow\ h+1=-2 \Rightarrow\ h=-3$

and $\frac{k+0}{2}=-2 \Rightarrow\ k=-4$

$\therefore\ (h,k)=(-3,-4)$

(17) (B). Let the circle be $S_1+\lambda S_2=0$

$x^2+y^2+3x+7y+2p-5+\lambda(x^2+y^2+2x+2y-p^2)=0$

passes through (1, 1)

$7+2p+\lambda(6-p^2)=0$, when $p=\pm\sqrt{6}$ required circle become $S_2=0$

(18) (A). Circle $x^2+y^2-4x-8y-5=0$

Centre = (2, 4), Radius = $\sqrt{4+16+5}=5$

If circle is intersecting line 3x – 4y = m at two distinct points.

$\Rightarrow$ length of perpendicular from centre < radius

$\Rightarrow \frac{|6-16-m|}{5}<5 \Rightarrow |10+m|<25$

$\Rightarrow -25<m+10<25 \Rightarrow -35<m<15.$

(19) (B). $x^2+y^2=ax$ ....(1)

$\Rightarrow$ centre $c_1\left(-\frac{a}{2},0\right)$ and radius $r_1=\left|\frac{a}{2}\right|$

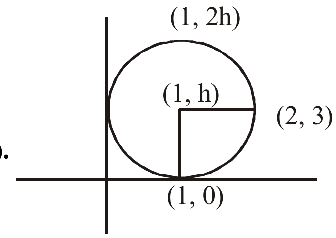
$x^2+y^2=c^2$ .... (2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2=c$

both touch each other if

$\frac{a^2}{4}=\left(\pm\frac{a}{2}\pm c\right)^2 \Rightarrow \frac{a^2}{4}=\frac{a^2}{4}\pm|a|c+c^2 \Rightarrow |a|=c$

(20) (A).



$h^2=(1-2)^2+(h-3)^2$

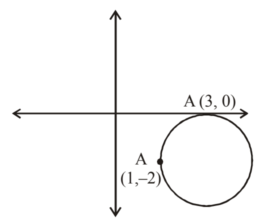
$0=1-6h+9$

$6h=10$; $h=5/3$

Now, diameter is $2h=10/3$

(21) (C). Let the equation of circle be

$(x-3)^2+(y-0)^2+\lambda y=0$



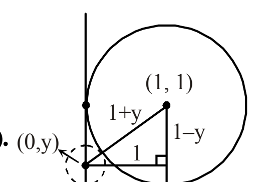
As it passes through (1, –2)

$\therefore\ (1-3)^2+(-2)^2+\lambda(-2)=0 \Rightarrow \lambda=4$

$\therefore$ Equation of circle is $(x-3)^2+y^2-8=0$

So, (5, –2) satisfies equation of circle.

(22) (D).



According to the figure

$(1+y)^2=(1-y)^2+1$ $(y>0)$

$\Rightarrow\ y=1/4$

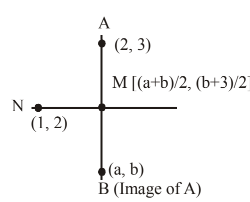
(23) (B). After solving equation (i) & (ii)

$2x-3y+4=0$ ...(i) $2x-4y+6=0$ ...(ii)

x = 1 and y = 2

Slope of AB × Slope of MN = –1

$\frac{b-3}{a-2}\times\frac{\frac{b+3}{2}-2}{\frac{a+2}{2}-1}=-1$



$(y-3)(y-1)=-(x-2)x$

$y^2-4y+3=-x^2+2x$

$x^2+y^2-2x-4y+3=0$

Circle of radius $=\sqrt{2}$.

(24) **(B).** $x^2+y^2-4x-6y-12=0$

C_1 (center) $=(2,3)$, $r=\sqrt{2^2+3^2+12}=5$

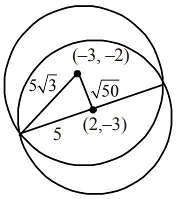
$x^2+y^2+6x+18y+26=0$

C_2 (center) $(-3,-9)$, $r=\sqrt{9+81-26}=\sqrt{64}=8$

$C_1C_2=13$, $C_1C_2=r_1+r_2$

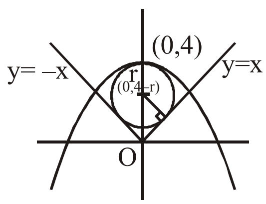
Number of common tangent is 3.

(25) **(A).**



(26) **(A).** $x^2+x-4=0$

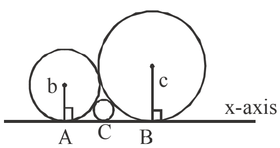
$x=\dfrac{-1\pm\sqrt{1+16}}{2}=\dfrac{-1+\sqrt{17}}{2}$



$\dfrac{(4-r)-0}{\sqrt{2}}=r$; $4-r=r\sqrt{2}$

$r=\dfrac{4}{\sqrt{2}+1}=\dfrac{4(\sqrt{2}-1)}{1}$

(27) **(A).**



$AB=AC+CB$

$\sqrt{(b+c)^2-(b-c)^2}$

$=\sqrt{(b+a)^2-(b-a)^2}+\sqrt{(a+c)^2-(a-c)^2}$

$\sqrt{bc}=\sqrt{ab}+\sqrt{ac}$

$\dfrac{1}{\sqrt{a}}=\dfrac{1}{\sqrt{c}}+\dfrac{1}{\sqrt{b}}$

(28) **(D).** $p=\dfrac{n}{\sqrt{2}}$, but $\dfrac{n}{\sqrt{2}}<4 \Rightarrow n=1,2,3,4,5.$

Length of chord AB $=2\sqrt{16-\dfrac{n^2}{2}}$

$=\sqrt{64-2n^2}=\ell$ (say)

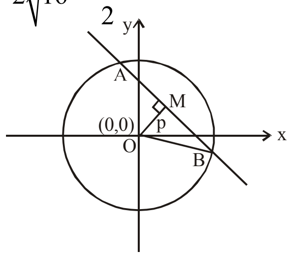
For $n=1$, $\ell^2=62$

$n=2$, $\ell^2=56$

$n=3$, $\ell^2=46$

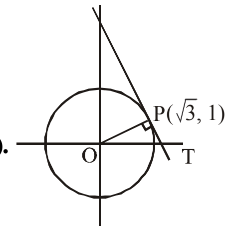
$n=4$, $\ell^2=32$

$n=5$, $\ell^2=14$



Required sum $=62+56+46+32+14=210$

(29) **(D).**



Given $x^2+y^2=4$

Equation of tangent $\sqrt{3}x+y=4$ ...(1)

Equation of normal $x-\sqrt{3}y=0$ ...(2)

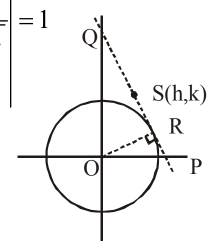
Coordinate of $T\left(\dfrac{4}{\sqrt{3}},0\right)$ $\therefore$ Area of triangle $=\dfrac{2}{\sqrt{3}}$

(30) **(C).** Let the mid point be S (h, k), P(2h, 0) and Q (0, 2k)

Equation of PQ : $\dfrac{x}{2h}+\dfrac{y}{2k}=1$

PQ is tangent to circle at R(say)

$\therefore$ $OR=1 \Rightarrow \left|\dfrac{-1}{\sqrt{\left(\dfrac{1}{2h}\right)^2+\left(\dfrac{1}{2k}\right)^2}}\right|=1$



$\dfrac{1}{4h^2}+\dfrac{1}{4k^2}=1$

$\Rightarrow x^2+y^2-4x^2y^2=0$

Aliter : Tangent to circle $x\cos\theta+y\sin\theta=1$

P : $(\sec\theta, 0)$; Q : $(0, \operatorname{cosec}\theta)$

$2h=\sec\theta \Rightarrow \cos\theta=\dfrac{1}{2h}$ & $\sin\theta=\dfrac{1}{2k}$

$\dfrac{1}{(2x)^2}+\dfrac{1}{(2y)^2}=1$

(31) **(B).** Circle touches internally

C_1 $(0,0)$; $r_1=2$

C_2 : $(-3,-4)$; $r_2=7$

$C_1C_2=|r_1-r_2|$

$S_1-S_2=0 \Rightarrow$ eqn. of common tangent

$6x+8y-20=0$

$3x+4y=10$

$(6,-2)$ satisfy it

(32) **(C).** Equation of common chord

$4kx+\dfrac{1}{2}y+k+\dfrac{1}{2}=0$ ....(1)

and given line is $4x+5y-k=0$ .....(2)

On comparing (1) & (2), we get

$k=\dfrac{1}{10}=\dfrac{k+\dfrac{1}{2}}{-k} \Rightarrow$ No real value of k exist

(33) **(C).** Equation of circle can be written as

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

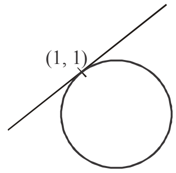
It passes through (1, –3)

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

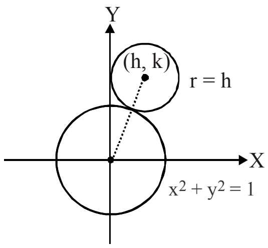
$\ldots^2 + (y-1)^2 - 4(x-y) = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$



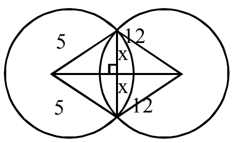
(34) **(D).**



$\sqrt{h^2 + k^2} = |h| + 1$; $x^2 + y^2 = x^2 + 1 + 2x$

$y^2 = 1 + 2x$; $y = \sqrt{1+2x}$; $x \geq 0$

(35) **(B).**



Let length of common chord = 2x

$\sqrt{25 - x^2} + \sqrt{144 - x^2} = 13$

After solving, $x = \frac{12 \times 5}{13}$; $2x = \frac{120}{13}$

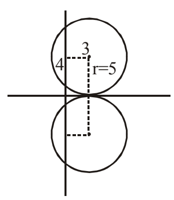
(36) **(A).** Equaiton of circles are

$(x-3)^2 + (y-5)^2 = 25$

$(x-3)^2 + (y+5)^2 = 25$

$x^2 + y^2 - 6x - 10y + 9 = 0$

$x^2 + y^2 - 6x + 10y + 9 = 0$



(37) **(A).** $L = \sqrt{S_1} = \sqrt{16} = 4$; $R = \sqrt{16 + 4 - 16} = 2$

Length of Chord of contact

$= \frac{2LR}{\sqrt{L^2 + R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}}$

Square of length of chord of contact = 64 / 5.

(38) **(A).** Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x^2 + y^2 = 1$

$2x + 2yy' = 0$

$y' = -\frac{x}{y} = -1$

$y = mx + c$ is tangent of $x^2 + y^2 = 1$

so, m = 1

$y = x + c$

Now distance of (3, 0) from $y = x + c$ is

$\left|\frac{c+3}{\sqrt{2}}\right| = 1$

$c^2 + 6c + 9 = 2$

$c^2 + 6c + 7 = 0$

(39) **(C).** Equation of family of circle touching y-axis at (0, 4) is given by $(x-0)^2 + (y-4)^2 + \lambda x = 0$.

$\because$ It passes through (2, 0)

$\Rightarrow \lambda = -10$.

$\Rightarrow$ Required circle is $(x-0)^2 + (y-4)^2 - 10x = 0$

$\Rightarrow x^2 + y^2 - 10x - 8y + 16 = 0$

Center of circle ≡ (5, 4) and radius = 5

Distance of $4x + 3y - 8 = 0$ from (5, 4) $= \left|\frac{24}{5}\right| \neq$ radius

(40) **(36)** Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for I[st] circle

$\Rightarrow \frac{|-18 - 8 + k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \Rightarrow k_{max} = 36$