10

CIRCLE

DEFINITION

Circle is locus of a point which moves at a constant distance from a fixed point. This constant distance is called radius of the circle and fixed point is called centre of the circle.

Basic geometrical concepts related to Circle :

- (i) Equal chords subtends equal angles at the centre and viceversa.
- (ii) Equal chords of a circle are equidistant from the centre and vice-versa.
- (iii) Angle subtended by an arc at the centre is double the angle subtended at any point on the remaining part of the circle.
- (iv) Angles in the same segment of a circle are equal.
- (v) The sum of the opposite angles of a cyclic quadrilateral is 180° and vice-versa.
- (vi) If a line touches a circle and from the point of contact a (i) chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.
- (vii) If two chords of a circle intersect either inside or outside the circle, the rectangle contained by the parts one chord is equal in area to the rectangle contained by the parts of the
Radius of a general equation of a circle is $\sqrt{g^2 + f^2 - c}$ other. $AP \times PB = CP \times PD$
- (viii) The greater of the two chords in a circle is nearer to the centre than lesser.
- (ix) A chord drawn across the circular region divides it into parts each of which is called a segment of the circle.
- (x) The tangents at the extremities of a chord of a circle are equal.

The angle between the tangents is bisected by the straight line, which joins their point of intersection to the centre. This straight line also bisects at right angles the chord, which joins the points where they touch the circle

STANDARD FORMS OF EQUATION OFA CIRCLE

General Equation of a Circle : The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$, where g, f, c are constants. Centre of a general equation of a circle is $(-g, -f)$

 $fig - (ix)$ $fig - (x)$

i.e.
$$
\left(-\frac{1}{2}\right)
$$
 coefficient of x, $-\frac{1}{2}$ coefficient of y)

The general equation of second degree

 $ax^2 + by^2 + 2hxy + 2gx + 2fy+c = 0$ represents a circle if $a = b \neq 0$ and $h = 0$.

General equation of a circle represents

- (a) A real circle if $g^2 + f^2 c > 0$
- (b) A point circle if $g^2 + f^2 c = 0$
- (c) An imaginary circle if $g^2 + f^2 c < 0$
- In General equation of a circle
- (a) If $c = 0 \implies$ The circle passes through origin
- (b) If $f = 0 \implies$ The centre is on x-axis
- (c) If $g = 0 \implies$ The centre is on y-axis

Example 1 :

If $y = 2x + k$ is a diameter to the circle

 $2(x^2+y^2) + 3x + 4y - 1 = 0$, then find the value of k.

Sol. Centre of circle $=(-3/4, -1)$ this lies on, diameter $y = 2x + k$ $\Rightarrow -1 = -3/4 \times 2 + k \Rightarrow k = 1/2$

Example 2 :

If $(4, -2)$ is the one extremity of diameter to the circle $x^2 + y^2 - 4x + 8y - 4 = 0$ then find its other extremity. **Sol.** Centre of circle is $(2, -4)$. Let the other extremity is (h, k) A real circle if $g^2 + f^2 - c > 0$

A point circle if $g^2 + f^2 - c = 0$

An imaginary circle if $g^2 + f^2 - c = 0$

An imaginary circle if $g^2 + f^2 - c < 0$

An imaginary circle if $g^2 + f^2 - c < 0$

If $f = 0 \Rightarrow$ The circle passes through b \neq 0 and h = 0.

example adquation of a circle represents

A real circle if $g^2 + f^2 - c > 0$

A point circle if $g^2 + f^2 - c > 0$

A point circle if $g^2 + f^2 - c > 0$

An imaginary circle if $g^2 + f^2 - c < 0$

If $c = 0 \Rightarrow$ The c elaid equation of a cite elephesents
 α real circle if $g^2 + f^2 - c > 0$

A point circle if $g^2 + f^2 - c > 0$

A point circle if $g^2 + f^2 - c > 0$

Hence if $g^2 + f^2 - c < 0$

Hence the set of $g^2 + f^2 - c < 0$

Exerced equation of a

$$
\therefore \ \left(\frac{4+h}{2}\right) = 2, \ \left(\frac{-2+k}{2}\right) = -4 \Rightarrow (h, k) = (0, -6)
$$

CIRCLE

(ii) Central Form of Equation of a Circle: The equation of a circle having centre (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$

If the centre is origin, then the equation of the circle is $x^2 + y^2 = r^2$

If $r = 0$ than circle is called point circle and its equation is $(x-h)^2 + (y-k)^2 = 0$

Example 3 :

Find the equation of a circle with centre at the origin and which passes through the point (α, β) .

- $x^2 + y^2 = \alpha^2 + \beta^2$
- (iii) **Diameter form:** If (x_1, y_1) and (x_2, y_2) be the extremities of a diameter, then the equation of the circle is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

y

- **(iv) Parametric Equation of a Circle :**
- (a) The Parametric equations of a circle $x^2 + y^2 = a^2 \arex = acos\theta, y = asin\theta.$ O Department of the contract o $\left(\text{acos}\theta,\text{asin}\theta\right)$ $\mathbf x$ Hence parametric coordinates of any point lying on the circle $x^2 + y^2 = a^2$ are (acos θ , asin θ) Find the equation of a circle with centre at the origin and

Which passes through the point (α, β) .

Here radius $\sqrt{(\alpha^2 + \beta^2)^2} = 2^2 + \beta^2$

Diameter form: If (x_1, y_1) and (x_2, y_2) be the extremities of

a diameter
- (b) The parametric equations of the circle $(x-h)^{2} + (y-k)^{2} = a^{2}$ are $x = h + a \cos\theta$, $y = k + a \sin\theta$ Hence parametric coordinates of any point lying on the circle are $(h + a\cos\theta, k + a\sin\theta)$
- (c) Parametric equations of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

is
$$
x=-g+\sqrt{g^2+f^2-c}\cos\theta
$$
,
 $y=-f+\sqrt{g^2+f^2-c}\sin\theta$

Example 4 :

Find the equation of circle if

- (i) Centre is at origin & radius 3
- (ii) Circle passes through origin $&$ centre $(1, 2)$
- (iii) Circle touchs x-axis & centre is $(3, 2)$
- (iv) Circle touches the both the co-ordinates axes in first quadrant and radius $= 3$
- (v) Circle passes through the origin centre lies on positive yaxis at $(0, 3)$
- (vi) Circle is concentric with circle $x^2 + y^2 8x + 6y 5 = 0$ and passing through the point $(-2, -7)$.

circle are (h + acos9, k + asin9)
\n(c) Parametric equations of the circle
$$
x^2 + y^2 + 2gx + 2fy + c = 0
$$

\nis $x = -g + \sqrt{g^2 + f^2 - c \cos \theta}$,
\n $y = -f + \sqrt{g^2 + f^2 - c \sin \theta}$
\n $y = -f + \sqrt{g^2 + f^2 - c \sin \theta}$
\nExample 4:
\nFind the equation of circle if
\nand $g = 3$
\n $y = -f + \sqrt{g^2 + f^2 - c \sin \theta}$
\n $y = -f + \sqrt{g^2 + f^2 - c \sin \theta}$
\nExample 5.
\nExample 6.
\nExample 6.
\nExample 1:
\nNow, line $\frac{x}{3} + \frac{y}{2} = 1 \Rightarrow 2x + 3y = 6$...(ii)
\n $y = 8x$...(iii)
\n $y = 8x$...(iv)
\n $y = 8x$...(v)
\n $y =$

(vi) Centre $(4, -3)$ & passes through $(-2, -7)$

Radius =
$$
\sqrt{(4+2)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52}
$$

Equation of circle $(x-4)^2 + (y+3)^2 = 52$
 $x^2 + y^2 - 8x + 6y - 27 = 0$

Example 5 :

A line
$$
\frac{x}{3} + \frac{y}{2} = 1
$$
 cuts the curve $y^2 = 8x$ at two distinct point

 $A & B$. Find the equation of circle taking $A & B$ as extremities of diameter.

Sol. Equation of circle in diameteric form

(vi) Centre (4, -3) & passes through (-2, -7)
\nRadius =
$$
\sqrt{(4+2)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52}
$$

\nEquation of circle (x-4)² + (y+3)² = 52
\nx²+y²-8x+6y-27=0
\n**nple 5:**
\nA line $\frac{x}{3} + \frac{y}{2} = 1$ cuts the curve y² = 8x at two distinct point
\nA & B. Find the equation of circle taking A & B as extremities
\nof diameter.
\nEquation of circle in diameteric form
\n(x-x₁) (x-x₂) + (y-y₁) (y-y₂) = 0
\nwhere (x₁, y₁) & (x₂, y₂) are the extremities of diameter
\nSame equation can be as
\nx² – (x₁ + x₂)x + x₁x₂ + y² – (y₁ + y₂) + y₁y₂ = 0 ...(i)
\nNow, line $\frac{x}{3} + \frac{y}{2} = 1 \Rightarrow 2x + 3y = 6$...(ii)
\nintersect the curve y² = 8x ...(iii)
\n $\Rightarrow (\frac{6-2y}{3})^2 = 8x \Rightarrow (\frac{3-x}{3})^2 = 2x$...(iv)

$$
x^{2} - (x_{1} + x_{2})x + x_{1}x_{2} + y^{2} - (y_{1} + y_{2}) + y_{1}y_{2} = 0 \dots (i)
$$

Now, line
$$
\frac{x}{3} + \frac{y}{2} = 1 \Rightarrow 2x + 3y = 6
$$
 ...(ii)
intersect the curve $y^2 = 8x$...(iii)

$$
y^2 = 8x \qquad \qquad \dots (iii)
$$

be the extremities of
\n
$$
= 0
$$
\n $$

Similarly
$$
x = \frac{6-3y}{2}
$$
 [From equation (ii)]

Now putting this value of x in equation (iii) we get quadratic

in y.
$$
y^2 = 8\left(\frac{6-3y}{2}\right) \Rightarrow y^2 + 12y - 24 = 0
$$
 ...(v)
\n⇒ $y_1 + y_2 = 12$ or $y_1y_2 = -24$

 $=(\sqrt{5})^2$ Future values in equation Putting values in equation (i)

 $x^2 - 24x + 9 + y^2 - 12y - 24 = 0$ \implies $x^2 + y^2 - 24x - 12y - 15 = 0$

Example 6 :

Find the Cartesian equation of the following curves whose parametric equations are : (i) $x = 7 + 4 \cos \alpha$, $y = -3 + 4 \sin \alpha$ (ii) $x = cos \theta + sin \theta + 1$, $y = sin \theta - cos \theta + 2$ **Sol.** (i) Parametric equations of given curve are $x = 7 + 4 \cos \alpha$...(i) $y = -3 + 4 \sin \alpha$ …(ii) In order to find the Cartesian equation of the curve, we will have to eliminate parameter α . From (i) $4 \cos \alpha = x - 7$ …(iii) From (ii) $4 \sin \alpha = y + 3$...(iv) Squaring (iii) and (iv) and adding, we get $(x-7)^2 + (y+3)^2 = 4^2$ (ii) Parametric equation of given curve are $x = \cos \theta + \sin \theta + 1$...(i) $y = \sin \theta - \cos \theta + 2$...(ii) In order to find the Cartesian equation of the curve, we will have to eliminate the parameter θ , From (i), $x - 1 = \cos \theta + \sin \theta$...(iii) From (ii), $y - 2 = \sin \theta - \cos \theta$...(iv) Squaring (iii) and (iv) and then adding, we get $(x-1)^2 + (y-2)^2 = 2$ + a cos α - ...(ii)

a 3 + 4 sin α ...(ii)

imate parameter α...

d on $\alpha = y + 3$...(ii)

4 sin α = y + 3 ...(ii)

4 sin α = y + 3 ...(iii)

(iv) If the circle touches both the ax

cos α = x - 7 ...(iii)

4 sin α = y + 3-y + sinc at

in find the Cartesian equation of the curve, we will

iminate parameter α .

4 cos $\alpha = x - 7$...(iii)

4 sin $\alpha = y + 3$...(iii)

4 sin $\alpha = y + 3$...(iii)

(iii) and (iv) and adding, we get

certic equatio ation of the curve, we will

(iii)

..(iii)

..(ii (a)

(b) If the circle touches both the axis then its equation is (Four

cases) $(x \pm r)^2 + (y \pm r)^2 = r^2$

we are

we are

(b) If the circle touches x-axis at origin (Two cases)

(c) If the circle touches x-axis at origi (ii)

(ii)

(iii)

(iii)

(iv)

(iv)

we get

(iv)

we get

(iv)

we get

(iv)

(iii)

x

x

x
 cing (iii) and (iv) and adding, we get
 $x = 0.97^2 + (y + \pm t)^2 = t^2$
 $x = 7y^2 + (y + \pm t)^2 = t^2$
 $x = 10^2 + 0.29^2 = 4^2$
 $x = \tan \theta - \cos \theta + \sin \theta + 1$...(i)
 $\sin \theta = \tan \theta + \cos \theta + \sin \theta$
 $\cos \theta = \sin \theta - \cos \theta$...(iv)
 $\cos \theta = \sin \theta - \cos \theta$...(iv)
 $\$ (ii) $4 \sin \alpha = y + 3$...(iv)

(iv) If the circle touches both the axis then its eq
 $(\alpha - \beta)^2 + (y + 3)^2 = 4^2$
 $(\alpha - \gamma)^2 + (y + 3)^2 = 4^2$
 $(\alpha - \gamma)^2 + (y + 3)^2 = 4^2$
 $(\alpha - \gamma)^2 + (y + 3)^2 = 4^2$
 $(\alpha - \gamma)^2 + (y + 3)^2 = 4^2$
 $= \sin \theta - \cos \theta + 2$...(

Example 7 :

Find the parametric coordinates of any point of the circle $x^2 + y^2 + 2x - 3y - 4 = 0$

Sol. Centre =
$$
\left(-1, \frac{3}{2}\right)
$$
, radius = $\sqrt{1 + \frac{4}{9} + 4} = \frac{7}{3}$

Parametric coordinates of any point are

$$
(-1 + \frac{7}{3}\cos\theta, \frac{3}{2} + \frac{7}{3}\sin\theta
$$

EQUATION OF A CIRCLE IN SOME SPECIAL CASES

- **(i)** If centre of circle is (h, k) and passes through origin then its equation is
- $(x-h)^2 + (y-k)^2 = h^2 + k^2 \implies x^2 + y^2 2hx 2ky = 0$ **(ii)** If the circle touches x– axis then its equation is (Four cases)
- $(x \pm h)^2 + (y \pm k)^2 = k^2$ Y

(iii) If the circle touches y axis then its equation s (Four cases) $(x \pm h)^2 + (y \pm k)^2 = h^2$

(iv) If the circle touches both the axis then its equation is (Four cases) $(x \pm r)^2 + (y \pm r)^2 = r^2$

(v) If the circle touches x– axis at origin (Two cases) $x^2 + (y \pm k)^2 = k^2 \Rightarrow x^2 + y^2 \pm 2ky = 0$

(vi) If the circle touches y axis at origin (Two cases) $(x \pm h)^2 + y^2 = h^2 \Rightarrow x^2 + y^2 \pm 2xh = 0$

(vii) If the circle passes through origin and cut intercept of a and b on axes, the equation of circle is (Four cases) $x^2 + y^2 - ax - by = 0$ and centre is (a/2, b/2)

POSITION OF A POINT WITH RESPECT TO A CIRCLE

A point (x_1, y_1) lies outside, on or inside a circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ according as $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is positive, zero or negative i.e. $S_1 > 0 \Rightarrow$ Point is outside the circle.

- $S_1 = 0 \Rightarrow$ Point is on the circle.
- $S_1 < 0 \Rightarrow$ Point is inside the circle.

The least and greatest distance of a point from a circle : Let S = 0 be a circle and $A(x_1, y_1)$ be a point. If the diameter

of the circle which is passing through the circle at P and Q Q

then $AP = AC - r =$ least distance $AQ = AC + r =$ greatest distance where 'r' is the radius and C is the centre of circle

POSITION OF A LINE WITH RESPECT TO A CIRCLE

Method - I : Let the equation of the circle be

 $x^2 + y^2 = a^2$...(i) and the equation of the line be $y = mx + c$ …(ii)

From (i) and (ii), $x^2 + (mx + c)^2 = a^2$

 $x^2(1+m^2)+2$ cmx + c² – a² = 0 \dots (iii)

Case-I : When points of intersection are real and distinct, then equation (iii) has two distinct roots.

 \therefore Discriminant > 0

Let S = 0 to each area of
$$
(x_1, y_1)
$$
 be a point. If the diameter
of the circle which is passing through the circle at P and Q
then AP = AC - r = least distance
AC = AC + r = greatest distance
and the equation of the line be
 $x^2 + y^2 = a^2$...(i)
and the equation of the line be
 $x^2 + y^2 = a^2$...(ii)
and the equation of the line be
 $y = mx + c$
 $x^2(1 + m^2) + 2$ cmx + c² - a² - 4 (1 + m²) (c² - a²) - 2 (ii)
and the equation of the line be
 $x^2 + y^2 = a^2$...(ii)
 $x^2(1 + m^2) + 2$ cmx + c² - a² - 4 (1)
 $x^2(1 + m^2) + 2$ cmx + c² - a² - 4 (1)
Then (iii) has two distinct roots.
Therefore, the equation (iii) has two distinct roots.
or $a^2 > \frac{c^2}{1 + m^2}$
or $a^2 > \frac{c^2}{1 + m^2}$
or $a > \frac{|c|}{\sqrt{(1 + m^2)}} =$ length of perpendicular from (0, 0) to
 $y = mx + c$
or $ax = 1$: When points of intersection are coincident.
Thus, a line intersects a given circle at two distinct points
if radius of circle is greater than the length of perpendicular
from centre of the circle to the line.
Case-II : When the points of intersection are coincident.
 $2a =$ length of the perpendicular from the point (0, 0) to
 $y = mx + c$
 $2a = \frac{|c|}{\sqrt{(1 + m^2)}}$
 $2a =$ length of the perpendicular from the point (0, 0) to
 $2a = 1$ and the equation (iii) has two equal roots
 $2a = 1$ and the equation (iv) has two equal roots
 $2a = 1$ when the points of intersection are coincident.
 $2a = -1$ when the points of intersection are coincident.
 $2a = \frac{|c|}{\sqrt{(1 + m^2)}}$
 $2a =$ length of the perpendicular from the point (0, 0) to
 $a = 1$

or $a > \frac{1}{\sqrt{(1 + m^2)}}$ = length of perpendicular from (0, 0) to

 $y = mx + c \implies a >$ length of perpendicular from (0, 0) to $y = mx + c$

Thus, a line intersects a given circle at two distinct points if radius of circle is greater than the length of perpendicular from centre of the circle to the line.

Case-II : When the points of intersection are coincident, the equation (iii) has two equal roots

Thus, a line intersects a given circle at two distinct points
\nif radius of circle is greater than the length of perpendicular
\nfrom centre of the circle to the line.
\nCase-II : When the points of intersection are coincident,
\n
$$
a = \frac{|c|}{\sqrt{(1 + m^2)}}
$$
\n
$$
a
$$

a = length of the perpendicular from the point $(0, 0)$ to (i) $y = mx + c$

Thus, a line touches the circle if radius of circle is equal to the length of pependicular from centre of the circle to the line or called 'CONDITION OF TANGENCY'.

Case-III : When the points of intersection are imaginary. In this case (iii) has imaginary roots $M \sim$

$$
\therefore D < 0
$$

or $a < \frac{|c|}{\sqrt{1 + m^2}}$

or a < length of perpendicular from $(0, 0)$ to $y = mx + c$ Thus a line does not intersect a circle if the radius of circle is less than the length of perpendicular from centre of the circle to the line.

Method - II : Let $S = x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle and $L = ax + by + c = 0$ be a line.

Let r be the radius of the circle and p be the length of the perpendicular drawn from the centre $(-g, -f)$ on the line L.

Then it can be seen easily from the figure that. If

- (i) $p < r \Rightarrow$ the line intersects the circle in two distinct points.
- (ii) $p = r \implies$ the line touches the circle, i.e. the line is a tangent to the circle.
- (iii) $p > r \Rightarrow$ the line neither intersects nor touches the circle i.e., passes outside the circle.
- (iv) $p = 0 \implies$ the line passes through the centre of the circle.

Intercepts made on coordinate axes by the circle:

Solving the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with y = 0 we get, $x^2 + 2gx + c = 0$. If discriminant $4(g^2 - c)$ is positive, i.e., if $g^2 > c$, the circle will meet the x-axis at two distinct points, say $(x_1, 0)$ and $(x_2, 0)$ where $x_1 + x_2 = -2g$ and $x_1x_2 = c$. can be seen easily from the figure that. If
 ϕ the line intersects the circle in two distinct points.
 ϕ the line intersects the circle in two distinct points.
 \Rightarrow the line neither intersects nor touches the circ p > r ⇒ the line neither intersects nor touches the circle i.e.,
passes outside the circle.
p = 0 ⇒ the line passes through the centre of the circle.
cepts made on coordinate axes by the circle:
Solving the circle $x^2 + y$ **Length of y intercept =** ² 2 f c

The intercept made on x-axis by the circle (0,y)² (0,y)¹ (x ,0) ¹ (x ,0) ² (–g,–f)

$$
\Rightarrow |x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}
$$

In the similar manner if $f^2 > c$,

Length of y intercept =
$$
2\sqrt{f^2 - q}
$$

NOTE

- (i) $g^2 c > 0$ \Rightarrow circle cuts the x-axis at two distinct points.
- (ii) $g^2 = c \implies$ circle touches the x-axis.
- (iii) $g^2 < c \Rightarrow$ circle lies completely above or below the x-axis i.e. it does not intersect x-axis.
- (iv) $f^2 c > 0 \Rightarrow$ circle cuts the y-axis at two distinct points.
- (v) $f^2 = c \implies$ circle touches the y-axis.
- (vi) $f^2 < c \Rightarrow$ circle lies completely on the right side or the left side of the y- axis i.e. it does not intersect y-axis.

Example 8 :

Find the length of intercept on y-axis, by a circle whose diameter is the line joining the points $(-4, 3)$ and $(12, -1)$. e similar manner if $f^2 > c$,

e similar manner if $f^2 > c$,

th of y intercept = $2\sqrt{f^2 - c}$
 $c > 0 \Rightarrow$ circle cuts the x-axis at two distinct points.

c \Rightarrow circle touches the x-axis.
 \Rightarrow snot intersect x-axis.
 \Rightarrow

Sol. Here equation of the circle

 $(x+4)(x-12)+(y-3)(y+1)=0$ or $x^2 + y^2 - 8x - 2y - 51 = 0$ Hence intercept on y -axis

$$
=2\sqrt{f^2-c}=2\sqrt{1-(-51)}=4\sqrt{13}
$$

Example 9 :

Find the equation of the circle which passes through the origin and makes intercepts of length a and b on the x and y axes respectively.

Sol. Let the equation of the circle be

$$
x^2 + y^2 + 2gx + 2fy + c = 0
$$

Since the circle passes through the origin, we get $c = 0$ and given the intercepts on x and y axes a and b

 \ldots (i)

**S(TUDY MATERIAL: MATHEMAX
\ninple 9:
\nFind the equation of the circle which passes through the Equation of Tangent:
\norigin and makes intercepts of length a and b on the x and
\n0 x + y² + y² + 2gx + 2fy + c = 0 ...(i)
\nSince the circle passes through the origin, we get c = 0 and
\ngiven the intercepts on x and y axes a and b
\nthen
$$
2\sqrt{g^2 - c} = a
$$
 or $2\sqrt{g^2 - 0} = a$
\nand $2\sqrt{f^2 - c} = b$ or $2\sqrt{f^2 - 0} = b$
\n $x^2 + y^2 + 2gx + 2fy + c = 0$...(ii)
\nHence the direction of the circle has co-ordinates (there has co-ordinates of
\ntherefore, slope of CP = $\frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$
\nIf m is the slope of the tangent at P then
\n $\frac{m(y_1/x_1) = -1}{x_1 + x_1^2}$ (\because tangent is \pm CP)
\n \therefore f= ±b/2
\nThe equation of the tangent at P(x₁, y₁) is
\n $x^2 + y^2 \pm ax \pm by = 0$
\n**where** the equation of circle from (i) becomes
\n $y - y_1 = -\frac{x_1}{y_1}(x - x_1)$
\n $y_1 - y_1^2 = -xx_1 + x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = x_1^2 + y_1^2$
\nFor what value of "a" the point (a, a + 1) bounded by the**

Hence the equation of circle from (i) becomes $x^2 + y^2 \pm ax \pm by = 0$

Example 10 :

For what value of "a" the point $(a, a + 1)$ bounded by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ in the first quadrant. **Sol.** Equation of circle $x^2 + y^2 - 4 = 0$

Equation of line $x+y-2=0$

Point (a, $a + 1$) and origin lies opposite sides with w.rt line (B) $x + y - 2 = 0$, then $0 + 0 - 2 < 0$ therefore, $a + a + 1 - 2 > 0$

then
$$
2\sqrt{g^2-c} = a
$$
 or $2\sqrt{g^2-0} = a$
\nand $2\sqrt{f^2-c} = b$ or $2\sqrt{f^2-0} = b$
\n \therefore $f = \pm b/2$
\nHence the equation of circle in (1) becomes
\n $x^2 + y^2 = ax + b$ y = 0
\n**Table 10:**
\nFor what value of "a" the point (a, a + 1) bounded by the
\ncircle $x^2 + y^2 = 4$ and the line x + y = 2 in the first quadrant.
\nEquation of circle $x^2 + y^2 = 4$ and the line x + y = 2 in the first quadrant.
\nEquation of circle $x^2 + y^2 = 4$ and the line x + y = 2 in the first quadrant.
\nEquation of circle $x^2 + y^2 = 4$
\nEquation of circle x + y - 2 = 0
\n $x + y - 2 = 0$
\n $x^2 + y^2 - 4 = 0$
\n<

Using (i) & (ii) we get
$$
a \in \left(\frac{1}{2}, \frac{\sqrt{7}-1}{2}\right)
$$

Example 11 :

Find the value of λ , such that line $2x - \lambda y + 7 = 0$ touches the circle $x^2 + y^2 + 6x + 2\lambda y + 5 + \lambda^2 = 0$. What if value of λ is equal to 3.

Sol. If line touches the circle, then perpendicular length from the centre of circle to line will be equal to radius.

If point (a, a + 1) lies inside circle
\n
$$
x^2 + y^2 - 4 = 0
$$

\n $x^2 + y^2 - 4 = 0$
\n $2a^2 + 2a - 3 < 0$
\nUsing (i) & (ii) we get a $\in \left(\frac{1}{2}, \frac{\sqrt{7} - 1}{2}\right)$
\nUsing (i) & (ii) we get a $\in \left(\frac{1}{2}, \frac{\sqrt{7} - 1}{2}\right)$
\nHence the circle, the perpendicular length from
\nline circles the circle, then perpendicular length from
\nthe equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x = 0$ with $x = 1$ and $x = 1$
\n $x = 0$ with $x = 1$ with the equation of circle is $x^2 + y^2 = a^2$
\n $x^2 + 1 = -1$ with $x^2 + 1$
\n $\Rightarrow (x^2 + 1)^2 = 4(x + 4)$
\n $\Rightarrow (x^2 + 1)^2 = 4(x + 4)$
\n $\Rightarrow (x^2 + 1)^2 =$

EQUATION OF TANGENT AND NORMAL

Equation of Tangent :

(A) **Point form**: Let $P(x_1, y_1)$ be the point on the circle $x^2 + y^2 = a^2$...(i)

Since C the centre of the circle has co-ordinates (0, 0),

STUDY MATERIAL: MATHEMATICS
\n**JATION OF TANGENT AND NORMAL**
\n**Point form:** Let
$$
P(x_1, y_1)
$$
 be the point on the circle
\n $x^2 + y^2 = a^2$...(i)
\nSince C the centre of the circle has co-ordinates (0, 0),
\ntherefore, slope of $CP = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$
\nIf m is the slope of the tangent at P then
\n $m(y_1/x_1) = -1$ (: tangent is $\perp CP$)
\nor $m = -x_1/y_1$
\nThe equation of the tangent at $P(x_1, y_1)$ is

If m is the slope of the tangent at P then

$$
m(y_1/x_1) = -1 \quad (\because \text{ tangent is } \perp CP)
$$

or
$$
m = -x_1/y_1
$$

The equation of the tangent at $P(x_1, y_1)$ is

$$
y - y_1 = -\frac{x_1}{y_1} (x - x_1)
$$

or $yy_1 - y_1^2 = -xx_1 + x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = a^2$ [: (x_1, y_1) lies on the circle $x^2 + y^2 = a^2$ $\therefore x_1^2 + y_1^2 = a^2$] Hence the equation of the tangent at (x_1, y_1) is

$$
xx_1 + yy_1 = a^2 \quad or \quad T = 0
$$

(B) Slope form : Let the equation of circle is $x^2 + y^2 = a^2$ slope of tangent is m then, equation of tangent will be $y = mx + c$ when c is constant. Again if $y = mx + c$ is tangent for circle then apply the condition of tangency the slope of the tangent at P then
 $(y_1/x_1) = -1$ (: tangent is $\perp CP$)
 $= -x_1/y_1$

antion of the tangent at $P(x_1, y_1)$ is
 $y_1 = -\frac{x_1}{y_1} (x - x_1)$
 $y_1 - y_1^2 = -xx_1 + x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = a^2$
 y_1) lies on th = $\frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$

tangent at P then

(\therefore tangent is $\perp CP$)

mgent at P(x_1, y_1) is
 $-x_1$)
 $+x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = a^2$

ircle $x^2 + y^2 = a^2$ $\therefore x_1^2 + y_1^2 = a^2$]

the tangent at (x_1, y_1) m $(y_1/x_1) = -1$ (\because tangent is \bot CP)

or $m = -x_1/y_1$

The equation of the tangent at $P(x_1, y_1)$ is
 $y - y_1 = -\frac{x_1}{y_1} (x - x_1)$

or $yy_1 - y_1^2 = -xx_1 + x_1^2$ or $xx_1 + yy_1 = x_1^2 + y_1^2 = a^2$
 $[\because (x_1, y_1)]$ ieso on the circle

$$
\left|\frac{c}{\sqrt{1+m^2}}\right| = a \quad \text{or} \quad c = \pm a\sqrt{1+m^2}
$$

(C) Parametric Form :

Let the equation of circle is $x^2 + y^2 = a^2$ Then equation of tangent for point (x_1, y_1) on circle is $xx_1 + yy_1 = a^2$

For parametric equation $x_1 = a \cos \theta$ and $y_1 = a \sin \theta$ \therefore x (a cos θ) + y (a sin θ) = a² $x \cos \theta + y \sin \theta = a$

(D) Equation of Tangent From External Point :

Let the equation of circle is $x^2 + y^2 = a^2$

Let $P(x_1, y_1)$ is any external point for circle then equation of tangent will be $(y - y_1) = m(x - x_1)$

For m apply the condition of tangency get the two values of m

Note:

- (i) For a unique value of m there will be 2 tangent which are
- 13 (ii) From an external point 2 tangents can be drawn to the circle which are equal in length and are equally inclined to the line joining the point and the centre of the circle.

(iii) Equation of tangents drawn to any second degree circle at $P(x_1, y_1)$ on it can be obtained by replacing.

$$
x2 \rightarrow x x1 ; y2 \rightarrow y y1 ; 2x \rightarrow x + x1 ; 2y \rightarrow y + y1 ;2xy \rightarrow xy1 + yx1
$$

(iv) **Point of Tangency :**

for P : either solve tangent and normal to get P

$$
\frac{P(x_1, y_1)}{C}
$$

or compare the equation of tangent at (x_1, y_1) with the given tangent to get point of tangency.

Equation of Normal :

The normal to a circle at a point is defined as the straight line passing through the point and perpendicular to the tangent at that point. compare the equation of tangent at (x_1, y_1) with the

en tangent to get point of tangency.
 nof Normal:
 nof Normal:
 nof Normal:
 nof Normal:
 nof Normal:
 e normal to a circle at a point is defined as the compare the equation of tangent at (x_1, y_1) with the

en tangent to get point of tangency.
 nof Normal :
 notion is def For Fangenty:
 $\frac{P(x_1, y_1)}{P} = 0$

Some important Deduction :
 $\frac{P(x_1, y_1)}{P} = 0$

Some important Deduction :

(a) Area of Quad PAOB = 2 Δ

(b) Area of Quad PAOB = 2 Δ

or Normal :

or Normal :

or Normal :

or F(x₁, y₁) = 0

Some important Deduction :

(i) Area of Quad PAOB = 2 Δ

($\left(\frac{1}{\epsilon_8}-\epsilon_9\right)$)

ompare the equation of tangent at (x_1, y_1) with the

nangent to get point of tangency.

of Normal :

or Some import

Clearly every normal passes through the centre of the circle.

The equation of the normal to the circle

 $x^2 + y^2 + 2gx + 2fy + c = 0$ at any point (x_1, y_1) lying on the circle is

$$
\boxed{\frac{y_1 + f}{x_1 + g} = \frac{y - y_1}{x - x_1}}
$$

In particular, equation of the Normal to the circle

$$
x^{2} + y^{2} = a^{2}
$$
 at (x_{1}, y_{1}) is $\left[\frac{y}{x} = \frac{y_{1}}{x_{1}}\right]$. (iv) Angle

From any point, say $P(x_1, y_1)$ two tangents can be drawn to a circle which are real, coincident or imaginary according as P lies outside, on or inside the circle.

Let PQ and PR be two tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then PQ = PR is called the length of tangent drawn from point P and is given by

$$
PQ = PR = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}
$$

Pair of Tangents :

From a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$. Their combined equation is $SS_1 = T^2$. Where $S = 0$ is the equation of circle T = 0 is the equation of tangent at (x_1, y_1) and S_1 is obtained by replacing x by x_1 and y by y_1 in S.

Some important Deduction :

(i) Area of Quad PAOB =
$$
2 \triangle
$$
POA = $2 \cdot \frac{1}{2}$ RL = RL

(ii) AB i.e length of chord of contact $AB = 2 L \sin\theta$

here
$$
\tan \theta = \frac{R}{L} = \frac{2R L}{\sqrt{R^2 + L^2}}
$$

(iii) Area of \triangle PAB (\triangle formed by pair of Tangent & corresponding C.O.C.)

$$
(\frac{-g - 1}{B})\sqrt{R} = \frac{d}{L} = \frac{Q}{L}
$$

AB i.e length of chord of contact AB = 2 L sin θ
where tan $\theta = \frac{R}{L} = \frac{2R L}{\sqrt{R^2 + L^2}}$
Area of \triangle PAB (\triangle formed by pair of Tangent & corresponding C.O.C.)

$$
\triangle PAB = \frac{1}{2}AB \times PD = \frac{1}{2}(2 L \sin\theta)(L \cos\theta) = L^2 \sin\theta \cos\theta
$$

$$
= \frac{R L^3}{R^2 + L^2}
$$
Angle 2 θ between the pair of Tangents
tan $2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2R L^2}{L (L^2 - R^2)}$
$$
2\theta = \tan^{-1} \left(\frac{2R L}{L^2 - R^2}\right)
$$

$$
2\theta = \tan^{-1} \left(\frac{2R L}{L^2 - R^2}\right)
$$
Power of a Point : Square of the length of the tangent from the point P is called power of the point P w.r.t a given circle
i.e. $PT^2 = S_1$
Power of a point remains constant w.r.t a circle
PA · PB = (PT)²

$$
=\frac{R L^3}{R^2 + L^2}
$$

 (iv) Angle 2 θ between the pair of Tangents

$$
\begin{array}{ll}\n\text{Lip} & \text{for } \text{R} \text{Normal to the circle} \\
\text{Syl is } \frac{y}{x} = \frac{y_1}{x_1} & \text{(iv) Angle 20 between the pair of Tangents} \\
\text{tan } 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2R L^2}{L (L^2 - R^2)} \\
\text{Syly P(x1, y1) two tangents can be drawn to\n\end{array}
$$
\n
$$
\text{Lily P(x1, y1) two tangents can be drawn to\n\begin{align*}\n\text{Cyly P(x1, y1) two tangents can be drawn to\n\end{align*}
$$
\n
$$
\text{Lily P(x1, y1) two tangents can be drawn to\n\begin{align*}\n\text{Cyly P(x1, y1) two tangents according to the right of the tangent from the point P is called power of the length of the tangent from the point P is called power of the point P. What is given by\n\end{align*}
$$
\n
$$
\text{Cyly P(x1, y1) to the right of the tangent from the point P is called the right of the tangent from the point P. The right of the tangent from the point P is called power of a point. The right of the tangent from the point P is called power of a point. The right of the tangent is P and P is given by\n\end{align*}
$$
\n
$$
\text{Lily P(x1, y1) to the right of the tangent from the point P. The right of the tangent from the point P. The right of the tangent is P and P is given by\n\end{align*}
$$
\n
$$
\text{Lily P(x1, y1) to the right of the tangent is P and P is given by\n\end{align*}
$$
\n
$$
\text{Lily P(x1, y1) to the right of the tangent is P and P and is given by\n\end{align*}
$$
\n
$$
\text{Lily P(x1, y1) to the right of the tangent is P and P and P and P are given by\n\end{align*}
$$
\n $$

(v) Power of a Point : Square of the length of the tangent from the point P is called power of the point P w.r.t a given circle i.e. $PT^2 = S_1$

Power of a point remains constant w.r.t a circle $PA \cdot PB = (PT)^2$

Analytical proof:
$$
\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r
$$

Substituting $x = x_1 + r \cos \theta$ and $y = y_1 + r \sin \theta$ in $x^2 + y^2 = a^2$,

we get,
$$
r^2 + 2r(x_1 \cos\theta + y_1 \sin\theta) + x_1^2 + y_1^2 - a^2 = 0
$$

$$
r_1r_2 = x_1^2 + y_1^2 - a^2 = \text{constant} = (PT)^2
$$

Note: Power of a point is + ve $/0$ (zero) $/$ – ve according as point 'P' lies outside / on / inside the circle.

Example 12 :

Find the equation of the tangents to the circle $x^2 + y^2 = 9$, which

(i) are parallel to the line $3x + 4y - 5 = 0$

- (ii) are perpendicular to the line $2x + 3y + 7 = 0$ (iii) make an angle of 60º with the x-axis
- **Sol.** (i) Let tangent parallel to $3x + 4y 5 = 0$ is

 $3x + 4y + \lambda = 0$ …(1) and circle $x^2 + y^2 = 9$

then perpendicular distance from $(0, 0)$ to (1) = radius

$$
\frac{|\lambda|}{\sqrt{(3^2+4^2)}} = 3 \quad \text{or} \quad |\lambda| = 15 \qquad \therefore \lambda = \pm 15
$$

From (1), equations of tangents are $3x + 4y \pm 15 = 0$ (ii) Let tangent perpendicular to $2x + 3y + 7 = 0$ is

e: Power of a point is + ve/0 (zero)/ – ve according as
\n
$$
2a = 6
$$
 ...
\nand 2αβ = 0
\nfrom (vi), α = 2
\n-4
\n-12:
\n112:
\n12: Putting the value of the tangents to the circle x² + y² = 9,
\n-13x + 4y - 5 = 0
\n-14
\n-15
\n16: A graph of the tangents to the circle
\n-17: The graph of the equation of the tangents to the circle
\n-18: The graph of the equation
\nthe equation of the tangents to the circle x² + y² = 9,
\n-19: A graph of the x-axis
\n-10: A graph of the x-axis
\n-11: A graph of the x-axis
\n-12: A graph of the x-axis
\n-13: A graph of the x-axis
\n-14: A graph of the x-axis
\n-15: A graph of the y-axis
\n-20: B graph of the y-axis
\n-3x + 4y + λ = 0
\n-15: A graph of the x-axis
\n-3x + 4y + λ = 0
\n-16: B graph of the x-axis
\n-3x + 4y + λ = 0
\n-17: B graph of the y-axis
\n-3x + 4y + λ = 0
\n-18: A graph of the y-axis
\n-19: B graph of the y-axis
\n-10: B graph of the y-axis
\n-11: B graph of the y-axis
\n-11: B graph of the y-axis
\n-12: B graph of the y-axis
\n-13: A graph of the y-axis
\n-14: B graph of the y-axis
\n-15: A graph of the y-axis
\n-16: B graph of the y-axis
\n-17: B graph of the y-axis
\n-18: B graph of the y-axis
\n-19: B graph of the y-axis
\n-10: B graph of the y-axis
\n-11: B graph of the y-axis
\n-12: B graph of the y-axis
\n-13: B graph of the y-axis
\n-14: B graph of the y-axis
\n-15: B graph of the y-axis
\n-16: B graph of the y-axis
\n-17: B graph of the y-axis
\n-18: B graph of the y-axis
\n-19: B graph of the y-axis
\n-10: B graph of the y-axis
\n-11: B graph of the y-axis
\n-10: B graph of the y-axis
\n-

then perpendicular distance from (0, 0) to (2) = radius
\n
$$
\frac{|\lambda|}{\sqrt{3^2 + (-2)^2}} = 3 \text{ or } |\lambda| = 3\sqrt{13} \text{ or } \lambda = \pm 3\sqrt{13}
$$
\nwhere L = length of tangent of the line
$$
L = \sqrt{51} \text{ and } r = \sqrt{1 + 4 + 2}
$$
\nFrom (2), equations of tangents are
\n
$$
3x - 2y \pm 3\sqrt{13} = 0
$$
\nLet equation of tangent which makes an angle of 60°
\nwith the x-axis is
\n
$$
y = \sqrt{3}x + c
$$
\n
$$
\sqrt{3}x - y + c = 0
$$
\n
$$
\sqrt{3}x - y + c = 0
$$
\n
$$
\sqrt{3}x - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n
$$
\sqrt{3}y - y + c = 0
$$
\n<

From (2), equations of tangents are

$$
3x-2y\pm 3\sqrt{13}=0
$$

(iii) Let equation of tangent which makes an angle of 60° with the x-axis is

$$
y = \sqrt{3} x + c \qquad \qquad ...(3)
$$

or $\sqrt{3} x - y + c = 0$

and circle $x^2 + y^2 = 9$

then perpendicular distance from $(0, 0)$ to (3) = radius

$$
\frac{|c|}{\sqrt{(\sqrt{3})^2 + (-1)^2}} = 3 \quad \text{or} \quad |c| = 6 \quad \text{or} \quad c = \pm 6
$$

From (3), equations of tangents are $\sqrt{3} x - y \pm 6 = 0$

Example 13 :

STUDY MATERIAL: MATHEN
 $= \frac{y - y_1}{\sin \theta} = r$
 θ and $y = y_1 + r \sin \theta$ in
 $y_1 \sin \theta + x_1^2 + y_1^2 - a^2 = 0$
 Example 13:
 Example 13:

If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$; then show that the line
 $(x + my + 1 = 0$ touches a fixed circl STUDY MATERIAL: MATHEMATIC

From Pape 13:

If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$; then show that the line
 $x_1 + r \sin \theta$ in
 $x_1^2 + y_1^2 - a^2 = 0$

Sol. Given, $4\ell^2 - 5m^2 + 6\ell + 1 = 0$...(i)

Sol. Given line is $\ell x + my + 1 = 0$...(i)
 If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$; then show that the line ℓ x + my + 1 = 0 touches a fixed circle. Find the centre and radius of the circle. **Y MATERIAL: MATHEMATICS**

(; then show that the line

a fixed circle. Find the centre and
 $1=0$...(i)
 $=0$...(ii)

(c) the circle whose centre is (α, β)
 $\frac{\alpha + m\beta + 1}{\sqrt{\ell^2 + m^2}} = a$
 $(\ell^2 + m^2)$
 $\frac{(n\beta + 2\ell\alpha + 2m$

OMMIGI	STUDY MATERIAL: MATHEMATICS		
Analytical proof: $\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$	Example 13:		
Substituting $x = x_1 + r \cos \theta$ and $y = y_1 + r \sin \theta$ in $x^2 + y^2 = a^2$,			
we get, $r^2 + 2r(x_1 \cos \theta + y_1 \sin \theta) + x_1^2 + y_1^2 - a^2 = 0$	SoL. Given, $4\ell^2 - 5m^2 + 6\ell + 1 = 0$...(i) If possible, let line (ii) touch the circle whose centre is (α, β)		
$r(x_1, y_1)$	$\frac{x^2 + y^2}{4}$	and radius is a, then $\frac{1}{\ell}(\alpha + m\beta + 1)$	= a

and radius is a, then
$$
\frac{|\ell\alpha + m\beta + 1|}{\sqrt{\ell^2 + m^2}} = a
$$

MATERIAL: MATHEMATICS

nen show that the line

xed circle. Find the centre and
 $= 0$...(i)

(ii)

the circle whose centre is (α, β)
 $+\frac{m\beta + 1}{2} = a$
 $\alpha + m^2$
 $\alpha\beta + 2\ell\alpha + 2m\beta = a^2\ell^2 + a^2m^2$
 $\ell^2 + 2\ell m\alpha\beta + 2$ JDY MATERIAL: MATHEMATICS

= 0; then show that the line

es a fixed circle. Find the centre and
 $\ell + 1 = 0$...(i)
 $+ 1 = 0$...(ii)

touch the circle whose centre is (α, β)
 $\frac{|\ell\alpha + m\beta + 1|}{\sqrt{\ell^2 + m^2}} = a$
 $a^2(\ell^2 + m^$ or $(\ell \alpha + m\beta + 1)^2 = a^2 (\ell^2 + m^2)$ or $\ell \alpha^2 + m^2 \beta^2 + 1 + 2\ell m \alpha \beta + 2\ell \alpha + 2m \beta = a^2 \ell^2 + a^2 m^2$ or $(\alpha^2 - a^2) \ell^2 + (\beta^2 - a^2) m^2 + 2\ell m\alpha\beta + 2\alpha\ell + 2\beta m + 1 = 0$ \dots (iii)

or $(\alpha^2 + m^2)^2 + 1 + y^2 = 4^{\circ}$
 $\alpha^2 - a^2 = 4$ or $(\alpha^2 - a^2) (\alpha^2 + (\beta^2 - a^2))$
 $\alpha^2 - a^2 = 4$ or $(\alpha^2 - a^2)^2 + (\beta^2 - a^2)$

wer of a point is + ve /0 (zero)/- ve according as
 $\alpha^2 - a^2 = 4$...(iv), $\beta^2 - a^2 = 4$...(iv), $\beta^2 - a$ Comparing (i) and (iii), we get $\alpha^2 - a^2 = 4$...(iv), $\beta^2 - a^2 = -5$ \ldots (v) $2\alpha = 6$...(vi), $2\beta = 0$...(vii) and $2\alpha\beta = 0$...(viii) From (vi), $\alpha = 3$ and from (vii), $\beta = 0$ Putting the value of α in (iv), we get

 $a^2 = 3^2 - 4 = 5$ $\therefore a = \sqrt{5}$ and the equation of circle is $(x-3)^2 + (y-0)^2 = 5$

Example 14 :

Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point P (16, 7). If the centre of the circle is C then find the area of quadrilateral PQCR.

Sol. Area PQCR = 2Δ PQC= $2 \times \frac{1}{2}$ L \times r $\frac{1}{2}$ L × r

where $L =$ length of tangent and $r =$ radius of circle.

 $L = \sqrt{S_1}$ and $r = \sqrt{1 + 4 + 20} = 5$

Hence the required area $= 75$ sq. units.

Example 15 :

A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. Then find equation of the pair of tangent.

Sol. Equation of pair of tangents is given by $SS_1 = T^2$, , or $S = x^2 + y^2 + 20(x + y) + 20$, $S_1 = 20$, $T = 10(x + y) + 20 = 0$ \therefore SS₁ = T² \Rightarrow 20(x² + y² + 20(x⁺ y) + 20) = 10²(x+y+2)² \Rightarrow 4x² + 4y² + 10xy = 0 \Rightarrow 2x² + 2y² + 5xy = 0

CIRCLE

TRY IT YOURSELF-1

Q.1 Equation of a circle which passes through (3, 6) and touches the axes is

 $(A) x² + y² + 6x + 6y + 3 = 0$ (B) $x² + y² - 6x - 6y - 9 = 0$ L $(C) x^2 + y^2 - 6x - 6y + 9 = 0$ (D) none of these

Q.2 The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

(A) $x^2 + y^2 = 9a^2$ (B) $x^2 + y^2 = 16a^2$ (C) $x^2 + y^2 = 4a^2$ (D) $x^2 + y^2 = a^2$

- **Q.3** Find the radius of the circle $x^2 + y^2 4x 8y 45 = 0$
- **Q.4** Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?
- **Q.5** Find the equation of the circle passing through the points $(2,3)$; $(-1,1)$ & whose centre is on the line $x - 3y - 11 = 0$.
- **Q.6** A circle is concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has area double of its area. The equation of the circle is $(A) x² + y² - 6x + 12y - 15 = 0$ $(B) x^2 + y^2 - 6x + 12y + 15 = 0$ $(C) x^2 + y^2 - 6x + 12y + 15 = 0$ (D) None of these
- **Q.7** The equation of the tangent drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are $(A) x = 0, y = 0$ (B) (h²-r²) x - 2rhy = 0, x = 0 $(C) y = 0, x = 4$ (D) (h² – r²) x + 2 rhy = 0, x = 0 (C) $x^2 + y^2 - 2x + 12y + 15 = 0$

(D) None of these

The equation of the tangent drawn from the origin to the

(A) $x^2 - y^2 - 2x - 2hy + h^2 = 0$ are

(B) (b)²-r³ y = 0 are

(C) $y = 3y - 2$ are (0.0) and (1, 0) and touches The equation of the tangent drawn from the origin to the

(A) $x = 2 - 2$ concerned $x^2 + y^2 - 2$ concerned by the concerned by (x, y_1) and the circle is $S = 0$

(C) $y = 0$, $x = 4$

(C) $y = 0$, $x = 2$ channel control of the
- **Q.8** A circle passes through (0, 0) and (1, 0) and touches the circle $x^2 + y^2 = 9$ then the centre of circle is – (A) (3/2, 1/2) (B) (1/2, 3/2)

(C)
$$
(1/2, 1/2)
$$
 (D) $(1/2, \pm \sqrt{2})$

- **Q.9** The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point – $(A)(-3/2, 0)$ (B) $(-5/2, 2)$ (C) (–3/2, 5/2) (D) (–4, 0) (A) $(-3,2)$

(B) (th²-r²) $x = 0$

(C) $y = 0$, $x = 4$

(C) $y = 0$, $x = 0$

(C) $y = 0$, $y = 0$ and (1, 0) and buches the

circle passes through (0, 0) and (1, 0) and buches the

circle $x^2 + y^2 = 9$ then the center of ci
- **Q.10** If the tangent at the point P on the circle $x^2+y^2+6x+6y=2$ meets the straight line $5x-2y+6=0$ NOTE at a point Q on the y-axis, then length of PQ is :

(A) 4
\n(B)
$$
2\sqrt{5}
$$

\n(C) 5
\n(D) $3\sqrt{5}$

Q.11 Circle(s) touching x-axis at a distance 3 from the origin (

 $(A) x^2 + y^2 - 6x + 8y + 9 = 0$ $(B) x^2 + y^2 - 6x + 7y + 9 = 0$ (iii) A $(C) x^2 + y^2 - 6x - 8y + 9 = 0$ $(D) x^2 + y^2 - 6x - 7y + 9 = 0$

ANSWERS

DIRECTOR CIRCLE

The locus of the point of intersection of two perpendicular tangents to a circle is called the Director circle.

Let the circle be $x^2 + y^2 = a^2$, then equation of the pair of tangents to a circle from a point (x_1, y_1) is

$$
(x2 + y2 – a2) (x12 + y12 – a2) = (xx1 + yy1 – a2)2.
$$

If this represents a pair of perpendicular lines then coefficient of x^2 + coefficient of y^2 = 0

i.e.
$$
(x_1^2 + y_1^2 - a^2 - x_1^2) + (x_1^2 + y_1^2 - a^2 - y_1^2) = 0
$$

\n $\Rightarrow x_1^2 + y_1^2 = 2a^2$

Hence the equation of director circle is $x^2 + y^2 = 2a^2$ Obviously director circle is a concentric circle whose radius

is $\sqrt{2}$ times the radius of the given circle.

$$
2x + y^2 + 2gx + 2fy + c = 0
$$
is

$$
x^2 + y^2 + 2gx + 2fy + c = 0
$$
is

CHORD OF CONTACT

The chord joining the two points of contact of tangents to a circle drawn from any point A is called chord of contact of A with respect to the given circle.

Let the given point is $A(x_1, y_1)$ and the circle is $S = 0$ then equation of the chord of contact is

 $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

Parametric Form :

(C) (1/2, 1/2) (D) (1/2, $\pm \sqrt{2}$) Consider the circle $x^2 + y^2 = a^2$ with its centre at the origin O and of radius 'a', then the equation of chord joining the two points whose parametric angles are α and β is

$$
x \cos \frac{1}{2}(\alpha + \beta) + y \sin \frac{1}{2}(\alpha + \beta) = a \cos \frac{1}{2}(\alpha - \beta)
$$

NOTE

 $x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0.$
 RD OF CONTACT

The chord joining the two points of contact of tangents to

circle drawn from any point A is called chord of contact of

a with respect to the given circle.
 $\begin{array}{c}\n\lambda$ (i) It is clear from the above that the equation to the chord of contact coincides with the equation of the tangent, if the point (x_1, y_1) lies on the circle. Example 16:

(iii) Area of Δ and the circle is $S = 0$ then

equation of the chord of contact is
 $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
 Parametric Form :

Consider the circle $x^2 + y^2 = a^2$ with its centre at the origin

C (y+y₁)+c-o
with its centre at the origin
ation of chord joining the
gles are α and β is
= $a cos \frac{1}{2} (\alpha - \beta)$
e equation to the chord of
tion of the tangent, if the
= $2\sqrt{r^2 - p^2}$
 $\frac{2 + y_1^2 - a^2}{x_1^2 + y_1^2}$ with its centre at the origin

lation of chord joining the

gles are α and β is
 $= a \cos \frac{1}{2} (\alpha - \beta)$

he equation to the chord of

ation of the tangent, if the
 $= 2\sqrt{r^2 - p^2}$
 $\frac{1^2 + y_1^2 - a^2}{x_1^2 + y_1^2}$

and its centre at the origin

n of chord joining the

are α and β is
 $\cos \frac{1}{2} (\alpha - \beta)$

quation to the chord of

of the tangent, if the
 $\sqrt{r^2 - p^2}$
 $y_1^2 - a^2 y_1^2$
 $y_1^2 + y_1^2$

of contact with respect
 $y^2 + 2gx$ h its centre at the origin
on of chord joining the
s are α and β is
 $\cos \frac{1}{2}(\alpha - \beta)$
quation to the chord of
n of the tangent, if the
 $2\sqrt{r^2 - p^2}$
 $+\frac{y_1^2 - a^2}{a^2 + y_1^2}$
 $\frac{y_1^2 + y_1^2}{a^2 + y_1^2}$ y_1) and the circle is S = 0 then
tated is
()+f(y+y₁)+c=0
a² with its centre at the origin
equation of chord joining the
c angles are α and β is
 β) = a cos $\frac{1}{2}(\alpha - \beta)$
at the equation to the chord of
eq d the circle is S = 0 then

s

y + y₁) + c = 0

ith its centre at the origin

ion of chord joining the

es are α and β is

a cos $\frac{1}{2}(\alpha - \beta)$

equation to the chord of

on of the tangent, if the
 $2\sqrt{r^2 - p^2}$ d the circle is S = 0 then
s
 $y + y_1$ + c = 0

th its centre at the origin

ion of chord joining the

es are α and β is

a cos $\frac{1}{2}(\alpha - \beta)$

equation to the chord of

on of the tangent, if the
 $2\sqrt{r^2 - p^2}$
 $+ y_1^2$

ii) The length of chord of contact
$$
= 2\sqrt{r^2 - p^2}
$$

(iii) Area of
$$
\triangle
$$
 ABC is given by
$$
\frac{a(x_1^2 + y_1^2 - a^2)^{3/2}}{x_1^2 + y_1^2}
$$

Example 16 :

(1) (C) (2) (C) (3) $\sqrt{65}$ find the distance between the chord of contact with respect
to point (0, 0) and (g, f) of circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Find the distance between the chord of contact with respect **Sol.** Chord of contact with respect to (0, 0)

$$
gx + fy + c = 0
$$
\n
$$
gx + fy + c = 0
$$
\n
$$
gx + fy + g(x + g) + f(y + f) + c = 0
$$
\n
$$
\Rightarrow 2gx + 2fy + g^{2} + f^{2} + c = 0
$$

$$
\Rightarrow gx + fy + \frac{1}{2} (g^2 + f^2 + c) = 0
$$
(2)

Distance between (1) and (2) is

$$
=\frac{\frac{1}{2}(g^2+f^2+c)-c}{\sqrt{g^2+f^2}}=\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}
$$

Example 17 :

A circle touches the line $y = x$ at a point P such that

point (–10, 2) in its interior and the length of its chord on circle.

Sol. Equation of OP is $y = x$ …(i) Let $P = (h,h)$

> $2 + h^2 = 32$ or $h^2 = 16$ \therefore h = \pm 4 Thus $P = (4, 4)$ or $(-4, -4)$ Let C (α, β) be the centre of the circle. **Case-I :** When $P = (4, 4)$:

Slope of CP =
$$
\frac{\beta - 4}{\alpha - 4}
$$
 and slope of OP = 1

Since
$$
\text{CP} \perp \text{OP}
$$
 $\therefore \left(\frac{\beta - 4}{\alpha - 4} \right) \cdot 1 = -1$

or $\alpha + \beta = 8$ …(ii) Let a be the radius of the circle.

the line x + y = 0 is 6y2. Determine the equation of the
\ncircle.
\n
$$
C = (1, -9), CH^2 = (11)
$$
\nEquation of OP is y = x ...(i)
\nEquation of OP is y = x ...(i)
\nEquation of OP is y = x ...(i)
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -9), CH^2 = (11)
$$
\n
$$
C = (1, -2)
$$
\n
$$
C = (1, -2)
$$
\n
$$
C = (1, -2)
$$
\n
$$
C = (1, -9), CH^2 = (10)
$$
\n
$$
C = (1, -9), CH^2 = (1
$$

$$
\therefore C \equiv (9, -1) \quad \text{or} \quad C \equiv (1, 9)
$$

If $H = (-10, 2)$ When $C \equiv (9, -1)$, $CH^2 = 19^2 + (-3)^2 = 361 + 9 = 370 > a^2$

When C (–1, 9), CH² = 9² + 7² = 81 + 49 = 130 > a² Since H lies inside the circle,

 \therefore Neither (9, -1) nor (-1, 9) is the centre of the circle.

Case-II : When $P \equiv (-4, -4)$:

Slope of CP =
$$
\frac{\beta + 4}{\alpha + 4}
$$

Since CP \perp OP

EXAMPLEMATENATION MATERIAL: MATIERIAL: MATHERMATICS
\n
$$
\Rightarrow gx + fy + \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = -c
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) = 0
$$
\n
$$
\Rightarrow \frac{1}{2}(g^2 + f^2 + c) =
$$

Example 18 :

Chord of contact of the tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Prove that a, b, c are in G.P.

2, where O is the origin. The circle contains the
\n3, 2) in its interior and the length of its chord on
\n3, 2) in its interior and the length of its chord on
\n
$$
y = 0
$$
 is 6 $\sqrt{2}$. Determine the equation of the
\nwhen $C = (-9, 1)$, $0 \text{ H}^2 = (11)^2 + (-1)^2 = 242 \times a^2$
\nWhen $C = (-9, 1)$, $0 \text{ H}^2 = (11)^2 + (-1)^2 = 242 \times a^2$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + (y - 1)^2 = 50$
\n $(x + 9)^2 + y^2 = 6^2$
\n $(x + 9)^2 + y^2 = 6^2$
\n $(x + 9)^2 + y^2 = 2^2$
\n $(x + 9$

Example 19 :

 2^2 a $-5\sqrt{2}$ Tangents are drawn to $x^2 + y^2 = 1$ from any arbitrary point P on the line $2x + y - 4 = 0$. The corresponding chord of contact passes through a fixed point then find the coordinates. Hence a, b, c are in G.P.
 $+ y^2 = 1$ from any arbitrary point
 $= 0$. The corresponding chord of

h a fixed point then find the
 $2x + y - 4 = 0$ be $P = (a, 4 - 2a)$.

tact of the circle $x^2 + y^2 = 1$ with
 $\Rightarrow (4y - 1) + a(x - 2y) = 0$

Sol. Let any point on the line $2x + y - 4 = 0$ be $P = (a, 4 - 2a)$. Equation of chord of contact of the circle $x^2 + y^2 = 1$ with respect to point P is

 $x \cdot a + y \cdot (4 - 2a) = 1 \implies (4y - 1) + a(x - 2y) = 0$ This line always passes through a point of intersection of the lines $4y - 1 = 0$ and $x - 2y = 0$ which is fixed point whose

.

coordinate are
$$
y = \frac{1}{4}
$$
 and $x = 2y = \frac{1}{2}$.

Hence coordinates are $\left(\frac{1}{2}, \frac{1}{4}\right)$

EQUATION OF A CHORD WHOSE MIDDLE POINT IS GIVEN:

The equation of the chord of the circle $x^2 + y^2 = a^2$ whose middle point $P(x_1, y_1)$ is given is

So equation of chord is

$$
y-y_1 = -\frac{x_1}{y_1}(x-x_1)
$$
 or $xx_1 + yy_1 = x_1^2 + y_1^2$.

Which can be represent by $T = S_1$

Example 20 :

Find the equation of chord of the circle $x^2 + y^2 = 8x$ bisected at the point $(4, 3)$

Sol. T = $S_1 \Rightarrow x(4) + y(3) - 4(x+4) = 16 + 9 - 32$ \Rightarrow 3y – 9 = 0 \Rightarrow y = 3

Example 21 :

Find the equation of chord of the circle $x^2 + y^2 = a^2$ passing c through the point (2, 3) farthest from the centre.

Sol. Let P (2, 3) be given point, M be the middle point of a chord of the circle $x^2 + y^2 = a^2$ through P.

Then the distance of the centre O of the circle from the chord is OM.

and $(OM)^2 = (OP)^2 - (PM)^2$ which is maximum when PM is minimum.

i.e. P coincides with M, which is the middle point of the chord. Hence, the equation of the chord is $T = S_1$, i.e. $2x + 3y - a^2 = (2)^2 + (3)^2 - a^2 \implies 2x + 3y = 13$

DIAMETER OF A CIRCLE

The locus of middle points of a system of parallel chords of a circle is called the diameter of that circle. The diameter of the circle $x^2 + y^2 = r^2$ corresponding to the system of parallel chords $y = mx + c$ is $x + my = 0$.

CIRCLE THROUGH THE POINTS OF INTERSECTION

- **(i)** The equation of the circle passing through the points of intersection of the circle $S = 0$ and line $L = 0$ is $S + \lambda L = 0$.
- **(ii)** The equation of the circle passing through the points of intersection of the two circle $S = 0$ and $S' = 0$ is $S + \lambda S' = 0$ where $(\lambda \neq -1)$. In the above both cases λ can be find out according to the give problem.

Example 22 :

Find the equation of the circle passing through the origin and through the points of intersection of two circles $x^2 + y^2 - 10x + 9 = 0$ and $x^2 + y^2 = 4$

y₁ Since it passes through (0, 0),so we have Sol. Let the circle be $(x^2 + y^2 - 10x + 9) + \lambda (x^2 + y^2 - 4) = 0$ $9 - 4\lambda = 0 \Rightarrow \lambda = 9/4$ So the required equation is $4(x^2+y^2-10x+9)+9(x^2+y^2-4)=0 \Rightarrow 13(x^2+y^2)-40x=0$

Example 23 :

Find the equation of the circle passing through the origin and through the points of intersection of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ and the line $x + y - 1 = 0$

Sol. Let the required equation be $(x^2 + y^2 - 2x + 4y - 20) + \lambda (x + y - 1) = 0$ Since it passes through $(0, 0)$, so we have

$$
-20 - \lambda = 0 \Longrightarrow \lambda - 20
$$

Hence the required equation is

$$
(x2+y2-2x+4y-20)-20(x+y-1)=0
$$

\n⇒ $x2+y2-22x-16y=0$

COMMON CHORD OF TWO CIRCLES

The chord joining the points of intersection of two given circles is called their common chord.

The equation of common chord of two circles

 $S = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S' = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is

Proof: \because S = 0 and S' = 0 be two intersecting circles. $S - S' = 0$

or $2x (g_1 - g_2) + 2y (f_1 - f_2) + c_1 - c_2 = 0$ is a first degree equation in x and y.

So, it represent a straight line. Also, this equation satisfied by the intersecting points of two given circles $S = 0$ and $S' = 0$. Hence $S - S' = 0$ represents the common chord of circles $S = 0$ and $S' = 0$ $S' = 0$
 -1
 $S' = 0$
 -1
 -1
 -1
 -1
 -1
 -1
 -2
 -1
 -1
 -2
 -1
 -1
 -2
 -1
 -3
 -1

Length of common chord :

We have $PQ = 2(PM)$ (\therefore M is mid point of PQ)

$$
=2\sqrt{\{(C_1P)^2-(C_1M)^2\}}
$$

where C_1P = radius of the circle S = 0

and C_1M = length of perpendicular from C_1 on common chord PQ.

Note :

- (a) The common chord PQ of two circles becomes of the maximum length when it is a diameter of the smaller one between them.
- (b) Circle drawn on the common chord as a diameter then centre of the circle passing through P and Q lie on the common chord of two circles i.e., $S - S' = 0$
- (c) If the length of common chord is zero, then the two circles touch each other and the common chord becomes the common tangent to the two circles at the common of contact.

Example 24 :

The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to

- (A) $\pi/6$ (B) $\pi/4$
- (C) $\pi/3$ (D) $\pi/2$
- **Sol. (D).** The equation of the common chord of the circles $x^{2} + y^{2} - 4x - 4y = 0$ and $x^{2} + y^{2} = 16$ is $x + y = 4$ which meets $x^2 + y^2 = 16$ at A $(4, 0)$ and B $(-4, 0)$. Obviously $OA \perp OB$.

Hence, the common chord AB makes a right angle at the centre of the circle $x^2 + y^2 = 16$

ANGLE OF INTERSECTION OF TWO CIRCLES

The angle of intersection between two circles $S = 0$ and S'=0 is defined as the angle between their tangents at their point of intersection.

If $S = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ $S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

are two circles with radii r_1, r_2 and d be the distance between their centres then the angle of intersection θ between them

is given by
$$
\cos \theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}
$$

or $\cos \theta = \frac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{g_1g_2 + f_1g_2 - c_1} \sqrt{g_2g_2 + f_2g_1^2 - c_2}}$

Condition of Orthogonality : If the angle of intersection of the two circle is a right angle $(\theta = 90^{\circ})$ then such circle are called Orthogonal circle and conditions for their orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

When the two circles intersect orthogonally then the length of tangent on one circle from the centre of other circle is equal to the radius of the other circle. 1. The two circles intersects of the conditionally then the length

1. the two circles intersect of other circle is

to the radius of the other circle.

5:

5:

2. $8x + 6y + k = 0$ cuts orthogonally

2. $-8x + 6y + k = 0$ cuts o

= 16 **Example 25 :**

For what value of k the circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + 6y + k = 0$ cuts orthogonally

Sol. Let the two circles be $x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$ and $x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$

where
$$
g_1 = 5/2
$$
, $f_1 = 3/2$, $c_1 = 7$,

 $g_2 = -4$, $f_2 = 3$ and $c_2 = k$ If the two circles intersects orthogonally, then

the two circles intersects orthogonal
2
$$
(g_1g_2 + f_1f_2) = c_1 + c_2
$$

$$
\Rightarrow 2\left(-10 + \frac{9}{2}\right) = 7 + k
$$

\n
$$
\Rightarrow 11 = 7 + k \Rightarrow k = -18
$$

Example 26 :

Find the equation of the circle which cuts the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and the lines $x = -g$ and $y = -f$ orthogonally

Find the equation of the circle

Find the equation of the circle v

x²+y²=16

Tend the equation of the circle v

x²+y²+2gx + 2fy + c = 0 and

the centre of the circle x² + y² = 16

So a x = -g, y = -f cuts the Find the equation of the control of the sequence of the set of the x-+y-=16

and AB makes a right angle at

sol. x=-g, y=-f cuts the circl

one normal to required circl

one normal to required circl

DFTWOCIRCLES

DFTWOCIRCLES

DFTWOCIRCLES
 $x^2+y^2+2gx+2fy+c$

between two circles S = 0 and Example 26:

(4,0)
 $x^2+y^2=16$

Frigo Find the equation of the circle which cuts the
 $x^2+y^2=16$

Frigo Find the equation of the circle which cuts the
 $x^2+y^2=16$

Follow $x^2+y^2+2gx+2fy+c=0$ and the lines $x=-x$

orthog $\frac{1}{2} + \frac{1}{2} + \frac{1$ Hence, the common chord AB makes a right angle at

the centre of the circle $x^2 + y^2 = 16$

the centre of the circle $x^2 + y^2 = 16$

The angle of intersection between two circles S = 0 and

The angle of intersection between mond contains are a signifiable to the normal of equired circle will be $x^2 + y^2 + 2gx + 2fy + c = 0$

ecircle $x^2 + y^2 = 16$

Ennotice will be $x^2 + y^2 + 2gx + 2fy + c = 0$

ecircle between two circles S = 0 and

a magne between their 1 1 1 2 2 2 A

(A,g)

Find the equation of the circle which cuts the circle
 $x^2 + y^2 = 16$

Find the equation of the circle which cuts the circle

x²+y²=16

section between the single at

one normal to required circle.

DIDNOFTWO x²⁺y²=16

x²⁺y² + 2² = 16

common cha B makes a right angle at

solution corresponding to equation of circle will be $(-g, -f)$

SECTION OF TWO CIRCLE Example 26:
 $\sqrt{40}$
 $x^2+y^2=16$
 $x^2+y^2=16$
 $x^2+y^2=16$
 $x^2+y^2+2gx+2fy+c=0$ and the lines $x = -g$ and y

orthogonally mean these

shord AB makes a right angle at

sol. $x = -g$ and f or the times $x = -g$ and y

sol. \cos Example 26:

(4.0)

Find the equation of the circle which cuts the circle

x²+y²-16

x²+y²-12g x + 2f y + c = 0 and the lines x = -g and y = -1

solto xore = g, y = -1 cuts the circle orthogonally mean these lines **Sol.** $x = -g$, $y = -f$ cuts the circle orthogonally mean these lines one normal to required circle. Centre of required circle will be $(-g, -f)$ So equation of circle will be $x^2 + y^2 + 2gx + 2fy + c' = 0$ Now it cut the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonally then $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ $2g^2 + 2f^2 = c + c' \Rightarrow c' = 2g^2 + 2f^2 - c$ Equation of required circle will be $x^2 + y^2 + 2gx + 2fy + 2g^2 + 2f^2 - c = 0$

COMMON TANGENTS TO TWO CIRCLES :

- **(a) Direct common tangents :** It is a tangent touching two circles at different points and not intersecting the line of centres between the centres as shown in figure.
- **(b) Transverse common tangents :** It is a tangent touching two cirlces at different points and intersecting the line of centres between the centres as shown in figure.

Table : Position of two circles

Points of intersection of common tangents :

The points T_1 and T_2 (points of intersection of indirect and direct common tangents) divide C_1C_2 internally and externally in the ratio $r_1 : r_2$

Equation of the common tangents at point of contact : $S_1 - S_2 = 0$.

Point of contact : The point of contact C_1C_2 in the ratio r_1 : r_2 internally or externally as the case may be.

NOTE

(i) If two circles with centres C_1 (x_1 , y_1) and C_2 (x_2 , y_2) and F radii r_1 and r_2 respectively, then direct common tangent meet at a point which divides the line joining the centre of circle externally in the ratio of their radii.

$$
P \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)
$$

(ii) Transverse Common tangent meets at a point which divides the line joining the centres of circles internally in the ratio of their radii.

$$
Q \equiv \left(\frac{r_1 x_2 + r_2 x_1}{r_1 + r_2}, \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2} \right).
$$

(iii) C_1Q, C_1C_2, C_1P are in harmonic progression or Q and P are called harmonic conjugate points.

Example 27 :

Find the number of common tangents to circle $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$ **Sol.** $x^2 + y^2 + 2x + 8y - 23 = 0$ $C_1(-1, -4), r_1 = 2\sqrt{10}$ x₁, $\frac{r_1y_2 - r_2y_1}{r_1 - r_2}$

in tangent meets at a point which divides

centres of circles internally in the ratio
 $\frac{x_1}{r_1 + r_2}$, $\frac{r_1y_2 + r_2y_1}{r_1 + r_2}$.

in harmonic progression or Q and P are

giugate poi For $x^2 + y^2 - 4x - 10y + 9 = 0$ $C_2(2, 5)$, $r_2 = 2\sqrt{5}$ $\frac{r_2 x_1}{r_2}$, $\frac{r_1 y_2 - r_2 y_1}{r_1 - r_2}$

mon tangent meets at a point which divides

he centres of circles internally in the ratio
 $\frac{r_1 x_2}{r_1}$, $\frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}$.

are in harmonic progression or Q an

Now, C_1C_2 = distance between centres

$$
\therefore C_1C_2 = \sqrt{9+81} = 3\sqrt{10} = 9.486
$$

and
$$
r_1 + r_2 = 2(\sqrt{10} + \sqrt{5}) = 10.6
$$

$$
r_1 - r_2 = 2\sqrt{5} (\sqrt{2} - 1) = 2 \times 2.2 \times 0.4 = 4.4 \times 0.4 = 1.76
$$

$$
\Rightarrow r_1 - r_2 < C_1 C_2 < r_1 + r_2
$$

- \Rightarrow Two circles intersect at two distinct points.
- \Rightarrow Two tangents can be drawn.

Example 28 :

Find all the common tangents to the circles $x^2 + y^2 = 1$ and y $(x-1)^2 + (y-3)^2 = 4$

Sol. C_1 : (0, 0) $r_1 = 1$ C_2 : (1,3) $r_2 = 2$

 $C_1C_2 = \sqrt{10}$ Clearly $C_1C_2 = r_1 + r_2$

So circles neither touch nor cut each other,

there will be two direct common tangent & two transverse common tangent.

Point P divides C_1C_2 externally in the ratio of r_1 and r_2 i.e. $-1:2$ ($-$ ' sign shows external division) So coordinates of p will be

$$
P: \left(\frac{-1 \times (1) + 2(0)}{-1 + 2}, \frac{-1 \times (3) + 2(0)}{-1 + 2} \right) \Rightarrow P: (-1, -3)
$$

Equation of pair of tangent from the point $P(-1, -3)$ to the circle $S: x^2 + y^2 = 1$ will be

$$
SS1 = T2(x2+y2-1)((-1)2 + (-3)2-1) = (-x-3y-1)29(x2+y2-1) = x2+9y2+1+6xy+2x+6y\n\Rightarrow 8x2-6xy-2x-6y-10=0
$$

$$
(x+1)(8x-6y-10)=0
$$

Equation of direct common tangent are

 $x + 1 = 0$

 $8x - 6y - 10 = 0$

FAMILY OF CIRCLES

Type-1 : The equation of the family of circles passing through the points of intersection of two given circles $S = 0$ and $S' = 0$ is given as $S + \lambda S' = 0$ (where λ is a parameter, $\lambda \neq -1$)

STUDY MATERIAL: MATER **STUDY MATERIAL: MA**
 \therefore C₁C₂ = $\sqrt{9+81} = 3\sqrt{10} = 9.486$
 \therefore **Type-2** : The equation of the family of circle S

and $r_1 + r_2 = 2(\sqrt{10} + \sqrt{5}) = 10.6$
 $r_1 - r_2 \le C_1 C_2 \le r_1 + r_2$
 \Rightarrow Two circles intersect at two **Type-2**: The equation of the family of circles passing through the points of intersection of circle $S = 0$ and a line L = 0 is given as $S + \lambda L = 0$ (where λ is parameter)

In a fir + F₂ = 2(x|10 + x|5) = 10.6
 $\frac{1}{2}$ = 2(x|10 + x|5) = 10.6
 \Rightarrow T₁ - F₂ - 2(x| (2) - 1) - 2 × 2 × 0.4 = 4.4 × 0.4 = 1.76
 \Rightarrow T₁ - F₂ < C₁ < C₁ - f₂ + x₂ = 0
 \Rightarrow Two circles intersect at U=1 $\sqrt{3}$ through the points of intersection of circle S = 0 and
 $r_1 + r_2 \ge 2\sqrt{3}(\sqrt{2} - 1) = 2 \times 2 \times 0.4 = 4.4 \times 0.4 = 1.76$
 $r_1 - r_2 \le C_1 C_2 \le r_1 + r_2$

Two circles intersect at two distinct points.

The $r_1 - r_2 \le C_1 C_$ **Type-3 :** The equation of family of circles which touch $y-y_1 = m(x-x_1)$ at (x_1, y_1) for any finite m is $(x-x_1)^2 + (y-y_1)^2 + \lambda \{(y-y_1) - m(x-x_1)\} = 0$ and if m is infinite, the family of circles is $(x-x_1)^2 + (y-y_1)^2 + \lambda(x-x_1) = 0$ (where λ is a parameter) les which touch
te m is
 $-x_1$)} = 0
s
 $x = \lambda$ is a parameter)
les passing through
) can be written in
 $x - y$
 $x_1 - y_1$
 $x_1 - y_1$
 $x_2 - y_2 - 1$
 $y_2 - 1$
 $y_1 = 0$ les which touch
ite m is
 $-x_1$ } = 0
is
re λ is a parameter)
les passing through
2. les passing through
2. les passing through
2. x_1 y_1 1
 x_1 y_1 1
 x_2 y_2 1 cles which touch
ite m is
 $(-x_1)$ } = 0
is
re λ is a parameter)
cles passing through
 y) can be written in
 x_1 y_1 x_2 y_2 x_1
 x_2 y_2 x_1

Type-4 : The equation of a family of circles passing through two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can be written in the form

$$
(x-x_1)^2 + (y-y_1)^2 + \lambda \{(y-y_1) - m(x-x_1)\} = 0
$$

and if m is infinite, the family of circles is

$$
(x-x_1)^2 + (y-y_1)^2 + \lambda(x-x_1) = 0 \text{ (where } \lambda \text{ is a parameter)}
$$

$$
\begin{cases}\n(x_1, y_1) \\
\vdots \\
y-y_1 = m(x-x_1)\n\end{cases}
$$

Type-4: The equation of a family of circles passing through
two given points P(x₁, y₁) and Q(x₂, y₂) can be written in
the form

$$
(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0
$$

(where λ is a parameter)

$$
\begin{cases}\n(-1)^x & y & 1 \\ y & y & 1 \\ z & y & 1\end{cases} = 0
$$

(where λ is a parameter)

NOTE

(a) Equation of the circle circumscribing the triangle PAB is

 $(x_1-x_1)(x_1+g)+(y-y_1)(y+f)=0$ where $O(-g, -f)$ is the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

(Here OP is diameter of the required circle)

(b) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0, L_2 = 0$ and $L_3 = 0$ is given by $\left(\begin{array}{c} L_1 = 0 \\ 0 \end{array} \right)$ $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided B coefficient of x^2 = coefficient to y^2 .

CIRCLE

(c) Equation of circle circumscribing a quadrilateral whose sides in order are represented by the lines

 $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ and $L_4 = 0$ is given by $L_1L_3 + \lambda L_2L_4 = 0$ provided coefficient of \bar{x}^2 = coefficient of y^2

Example 29 :

Find the equation of circle which passes through the point 1 . $(-1, 2)$ and touches the circle $x^2 + y^2 - 8x + 6y = 0$ at the origin.

Sol. Equation of variable circle will be $S + \lambda L = 0$ S is a point circle with centre at $(0, 0)$ and $r = 0$

S:
$$
(x-0)^2 + (y-0)^2 = 0
$$

\n
$$
L:(y-0) = \frac{4}{3}(x-0)
$$

 \Rightarrow 4x – 3y = 0

 $[\cdot]$. L is perpendicular to OP]

Equation of family of circle is

 $x^2 + y^2 + \lambda (4x - 3y) = 0$ Circle which passes through $(-1, 2)$

 $1^2 + 2^2 + \lambda (-4 - 6) = 0 \implies 5 - 10 \lambda = 0 \implies \lambda = 1/2$ Equation of required circle will be

$$
x^{2} + y^{2} + \frac{1}{2}(4x - 3y) = 0 \Rightarrow 2x^{2} + 2y^{2} + 4x - 3y = 0
$$

Example 30 :

If the circle $x^2 + y^2 + 2x + 3y + 1 = 0$ cuts $x^2 + y^2 + 4x + 3y + 2 = 0$ in A and B, then find the equation of the circle on AB as diameter.

Sol. The equation of the common chord AB of the two circles is $2x + 1 = 0$. [Using $S_1 - S_2 = 0$]

The equation of the required circle is

$$
(x^2+y^2+2x+3y+1)+\lambda(2x+1)=0
$$

[Using S₁ +
$$
\lambda
$$
(S₂-S₁) = 0]
\n \Rightarrow x²+y²+2x(\lambda + 1)+3y+\lambda+1=0

(0, 0)

 Ω $(4, -3)$ /

 $-8x + 6y = 0$

 $\mathbf L$

Since, AB is a diameter of this circle, therefore centre lies on it. So, $-2\lambda - 2 + 1 = 0 \Rightarrow \lambda = -1/2$ Thus, the required circle is $x^2 + y^2 + x + 3y + (1/2) = 0$ or $2x^2 + 2y^2 + 2x + 6y + 1 = 0$

POLE & POLAR

Let any straight line through the given point $P(x_1,y_1)$ intersect the circle $S = 0$ at two points Q and R, the locus of point of intersection of the tangents at Q and R is called the polar of the point P and the P is called the pole of the polar with respect to given circle.

Equation of Polar :

- **1.** Equation of polar of the pole (x_1, y_1) with respect to circle $x^{2} + y^{2} = a^{2}$ is $xx_{1} + yy_{1} = a^{2}$
- **2.** Equation of polar of the pole (x_1, y_1) with respect to circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x x_1 + y y_1 + g (x + x_1) + f (y + y_1) + c = 0$

Coordinates of Pole :

Pole of polar $Ax + By + C = 0$ with respect to circle

$$
x^2 + y^2 = a^2
$$
 is $\left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$

2. Pole of polar $Ax + By + C = 0$ with respect to circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by the equation

$$
\frac{x_1 + g}{A} = \frac{y_1 + f}{B} = \frac{gx_1 + fy_1 + c}{C}
$$

1 1 1 1 x g y f gx fy c T(h, k)
 $\frac{1}{T(h, k)}$

on of Polar :

a $\frac{2}{3}$ is $xx_1 + yy_1 = a^2$

on $\frac{2}{3}$ is $xx_1 + yy_1 = a^2$

on $\frac{2}{3}$ is $\$ T(h, k)

T(h, k)

Q
 nof Polar of the pole (x_1, y_1) with respect to circle

a 2 is $xx_1 + yy_1 = a^2$

of polar of the pole (x_1, y_1) with respect to circle
 $-2gx + 2fy + c = 0$ is
 $+ y y_1 + g(x + x_1) + f(y + y_1) + c = 0$
 ates of Pole From Reflective to the pole (x₁, y₁) with respect to circle

Bolar:

s xx₁ + yy₁ = a²

polar of the pole (x₁, y₁) with respect to circle
 $x + 2fy + c = 0$ is
 $y_1 + g(x + x_1) + f(y + y_1) + c = 0$

of Pole :
 $-ax + By + C = 0$ w **Conjugate points :** Two points A and B are conjugate points with respect to given circle, if each lies on the polar of the other with respect to the circle. $x x_1 + y y_1 + g (x + x_1) + f (y + y_1) + c = 0$
 Coordinates of Pole :

Pole of polar Ax + By + C = 0 with respect to circle
 $x^2 + y^2 = a^2$ is $\left(\frac{Aa^2}{C}, \frac{Ba^2}{C} \right)$

Pole of polar $Ax + By + c = 0$ with respect to circle
 $x^2 + y^2 + 2$ $x^2 = 0$ with respect to circle
 $\left(\frac{a^2}{c}, \frac{Ba^2}{c}\right)$
 $x^2 = 0$ with respect to circle

0 is given by the equation
 $\frac{x_1 + f y_1 + c}{c}$

cooints A and B are conjugate points

le, if each lies on the polar of the

sircle s
 $+ f(y + y_1) + c = 0$
 $\frac{1}{z} - \frac{Ba^2}{C}$
 $\frac{1}{z} - \frac{Ba^2}{C}$
 $\frac{1}{z} - \frac{C}{C}$
 $\frac{1$ $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$

Pole of polar Ax + By + C = 0 with respect to circle
 $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by the equation
 $\frac{x_1 + g}{A} = \frac{y_1 + f}{B} = \frac{gx_1 + fy_1 + c}{B}$

Conjugate points Two points A a b $x^2 2gx + 2ly + c - 0$ is given by the equation
 $\frac{+g}{+g} = \frac{gx_1 + fy_1 + c}{c}$
 $\frac{+g}{+g} = \frac{gx_1 + fy_1 + c}{c}$

are **the points** : Two points A and B are conjugate points

there to given circle, if each lies on the polar of the

it Fix 1 by $V = 0$ winterspect to circle
 $x + 2fy + c = 0$ is given by the equation
 $= \frac{y_1 + f}{B} = \frac{gx_1 + fy_1 + c}{B}$

points : Two points A and B are conjugate points

to given circle, if each lies on the polar of the

sepect to

Conjugate lines : If two lines be such that the pole of one lies on the other, then they are called conjugate lines with respect to the given circle.

Example 31 :

 $x^2 + y^2 = c^2$.

Sol. Let the pole is (h, k) Hence polar of this pole is $xh + yk - c^2 = 0$ (1)

out polar is
$$
\frac{x}{a} + \frac{y}{b} = 0
$$
(2)

comparing the coefficient of x and y

$$
\frac{h}{(1/a)} = \frac{k}{(1/b)} = \frac{-c^2}{-1} \Rightarrow h = \frac{c^2}{a}, k = \frac{c^2}{b}
$$

RADICAL AXIS & RADICAL CENTRE :

 $\frac{x_1 + g}{A} = \frac{y_1 + f}{B} = \frac{gx_1 + fy_1 + c}{C}$

expect to given circle, if each lies on the polar of the

sespect to given circle, if each lies on the polar of the

with respect to the circle.

urgate lines : If two lines be su $\frac{v_1 + c}{1 + c}$

A and B are conjugate points

aach lies on the polar of the

be such that the pole of one

called conjugate lines with
 $= 1$ with respect to circle
 $yk - c^2 = 0$ (1)

.....(2)

and y
 $h = \frac{c^2}{a}$, are precised to the equation
 $\frac{+c}{ }$

and B are conjugate points

sh lies on the polar of the

such that the pole of one

lled conjugate lines with

1 with respect to circle
 $k - c^2 = 0$ (1)

.....(2)

d y
 $\frac{c^2$ **Radical Axis -** The radical axis of two circle is the locus of a point, which moves in such a way that the lengths of the tangents drawn from it to two given circles are equal.

The equation of radical axis of two circle $S = 0$ and $S' = 0$ is written as $S - S' = 0$.

NOTE

- **(i)** Radical axis of two circle is perpendicular to the line joining their centres.
- **(ii)** Radical axis bisects every common tangents of two circles.
- **(iii)** If two circles intersect a third circle orthogonally, then their radical axis passes through the centre of third circle.
- **(iv)** Radical axis of three circle, taken two at a time meet at a point provided the centre of the circle are not collinear.
- **(v)** If two circle touch each other, then the equation of the common tangent at the point of contact is $S - S' = 0$, which is also the equation of common chord, thus the common chord and common tangent at the point of contact are special cases of radical axis.
- **(vi)** for two circles whose centre are not same, radical axis always exist, while common chord and common tangent may or may not exist.

Radical Centre : The point where the radical axis of three given circles taken in pairs meet,

is called the radical centre of those three circles. Thus the length of the three tangents drawn from the radical centre on the three circles are equal.

If $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ be any three given circles, then to obtain the radical centre, we solve any two of the following $S_1 - S_2 = 0$, $S_2 - S_3 = 0$, $S_3 - S_1 = 0$

NOTE

- **(i)** If the centres of three circles are collinear then their radical centre will not exist.
- **(ii)** The circle with centre at radical centre and radius is equal to the length of tangents from radical centre to any of the circles will cut the three circle orthogonally and is called as radical circle.
- **(iii)** Circles are drawn on three sides of a triangle as diameter **Q.3** than radical centre of these circles is the orthocentre of the triangle.

COAXIAL SYSTEM OF CIRCLES

A system of circles, every 2 of which have the same radical
axis is called Coaxial system of circles **Q.4** axis, is called Coaxial system of circles.

Example 32 :

The equation of the three circles are given $x^2 + y^2 = 1$, $x^2 + y^2 - 8x + 15 = 0$, $x^2 + y^2 + 10y + 24 = 0$. Determine the coordinates of the point P such that the tangents drawn from it to the circles are equal in length.

Sol. We know that the point from which lengths of tangents are equal in length is radical centre of the given three circles. Now radical axis of the first two circles is

$$
(x2+y2-1)-(x2+y2-8x+15)=0,i.e., x-2=0(1)and radical axis of the second and third circles is
$$
(x2+y2-8x+15)-(x2+y2+10y+24)=0,
$$
i.e., 8x+10y+9=0(2)
Solving eqs (1) and (2), the coordinates of the radical cent
$$

cal centre, i.e. of point P are $P(2, -5/2)$.

Example 33 :

Find the locus of the centres of circles which bisect the circumference of circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x + 6y + 1 = 0$.

Sol. Let the equation of circle is S_1 : $x^2 + y^2 + 2gx + 2fy + c = 0$ S_2 : $x^2 + y^2 = 4$ S_3 : $x^2 + y^2 - 2x + 6y + 1 = 0$ Radical axis of $S_1 \& S_2$ is $2gx + 2fy + c + 4 = 0$ Radical axis passes through centre of $x^2 + y^2 = 4$ i.e. $(0, 0)$ \Rightarrow c=-4 Radical axis of $S_1 \& S_3$ is $(2g+2)x + (2f-6)y+c-1=0$ it passes through centre of S_3 i.e. $(1, -3)$ so $2g + 2 - 6f + 18 + c - 1 = 0$, also $c = -4$ \implies 2g – 6f + 15 = 0 Now centre of circle is $(-g, -f)$, $h = -g$, $k = -f$ \implies -2h + 6k + 15 = 0, locus is 2x – 6y – 15 = 0

TRY IT YOURSELF-2

Q.1 Find the equation of the smallest circle passing through the intersection of the line $x + y = 1$ & the circle $x^2+y^2=9$.

Q.2 The locus of the centre of circle which cuts the circle $x^{2} + y^{2} + 4x - 6y + 9 = 0$ and $x^{2} + y^{2} - 4x + 6y + 4 = 0$ orthogonally is – (A) $12x + 8y + 5 = 0$ (B) $8x + 12y + 5 = 0$

- (C) $8x 12y + 5 = 0$ (D) None of these
- **Q.3** If one of the diameters of the circle $x^2 + y^2 2x 6y + 6 = 0$ is a chord to the circle with centre $(2, 1)$, then the radius of the circle is

(A)
$$
\sqrt{3}
$$
 (B) $\sqrt{2}$
(C) 3 (D) 2

The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line

$$
4x - 5y = 20
$$
to the circle $x^2 + y^2 = 9$ is
(A) 20 $(x^2 + y^2) - 36x + 45y = 0$

(B)
$$
20(x^2+y^2)+36x-45y=0
$$

- (C) 36 ($x^2 + y^2$) 20x + 45y = 0
- (D) 36 (x^2+y^2) + 20x 45y=0

194

.

CIRCLE

For Q.5-Q.7

Consider the equation $4\ell^2$ –5m²+6 ℓ + 1 = 0, where ℓ , m \in R, and the line ℓ x + my + 1 = 0 touches a fixed circle.

Q.5 Centre and radius of fixed circle respectively, are –

- **Q.6** Tangent PA and PB are drawn to the above fixed circle from the point P on the line $x + y - 1 = 0$. Then chord of contact AB passes through the fixed point – (A) (1/2, -5/2) (B) (1/3, 4/3) (C) (-1/2, 3/2) (D) None of these
- **Q.7** Number of tangent which can be drawn from the point $(2,-3)$ are $-$

(A) 0 (B) 1 (C) 2 (D) 1 or 2

ANSWERS
(1)
$$
x^2+y^2-9-(x+y-1)=0.
$$
 (2)(C)

$$
(3) (C) (4) (A) (5) (C)
$$

(6) (A) **(7)**(C)

SOME IMPORTANT POINTS

- **1.** Locus of mid point of a chord of a circle $x^2 + y^2 = a^2$ which subtends an angle α at the centre is $x^2 + y^2 = (\text{acos}\alpha/2)^2$
- **2.** A variable point moves in such a way that sum of square of distances from the vertices of a triangle remains constant then its locus is a circle whose centre is the centroid of the triangle.
- **3.** If the points where the line $a_1x + b_1y + c_1 = 0$ and $a_2x+b_2y+c_2=0$ meets the coordinate axes are concyclic then $a_1 a_2 = b_1 b_2$.
- 4. If the line $1x + my + n = 0$ is a tangent to the circle $x^2 + y^2 = a^2$, then $a^2(1^2 + m^2) = n^2$.
- **5.** If the radius of the given circle $x^2 + y^2 + 2gx + 2fy + c = 0$ be r and it touches both the axes then $g = f\sqrt{c} = r$.
- 6. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches x-axis and yaxis, then $g^2 = c$ and $f^2 = c$ respectively.
- **7.** The length of the common chord of the circles

$$
(x-a)^2 + y^2 = a^2
$$
 and $x^2 + (y-b)^2 = b^2$ is $\frac{2ab}{\sqrt{a^2 + b^2}}$

- **8.** If two tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are perpendicular to each other then $g^2 + f^2 = 2c$
- **9.** If the line $y = mx + c$ is a normal to the circle with radius r and centre at (a, b) , then $b = ma + c$
- **10.** If the tangent to the circle $x^2 + y^2 = r^2$ at the point (a, b) meets the coordinates axes at the points A and B and O is

the origin. Then the area of the triangle OAB is $\frac{r^4}{2!}$.

11. If O is the origin and OP, OQ are tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ then the circumcentre of the triangle OPQ is $(-g/2, -f/2)$.

ADDITIONAL EXAMPLES

Example 1 :

- Find the equation of the circle concentric with the circle $x^2 + y^2 - 3x + 4y - c = 0$ and passing through the point $(-1, -2)$.
- **Sol.** The equation of two concentric circles differ only in constant term. So let the equation of the required circle be $x^2 + y^2 - 3x + 4y + \lambda = 0$ It passes through $(-1, -2)$ so we have $1+4+3-8+\lambda=0 \Rightarrow \lambda=0,$ **EXAMPLES**
 EXAMPLES

of the circle concentric with the circle
 $-c = 0$ and passing through the point

two concentric circles differ only in

two concentric circles differ only in
 $\rightarrow \lambda = 0$
 $(-1, -2)$ so we have
 $+\lambda = 0 \$

Hence required equation is $x^2 + y^2 - 3x + 4y = 0$

Example 2 :

If the line $x + y = 1$ is a tangent to a circle with centre (2, 3), then find its equation.

Sol. Radius of the circle = perpendicular distance of $(2, 3)$ from

$$
x + y = 1
$$
 is $\frac{4}{\sqrt{2}} = 2\sqrt{2}$

:. The required equation will be

$$
(x-2)^2 + (y-3)^2 = 8 \Rightarrow x^2 + y^2 - 4x - 6y + 5 = 0
$$

Example 3 :

If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

Sol. The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and

14.4
$$
1
$$
 3-6 1 $−$ 4 $−$ 50 $−$

Example 4 :

The straight line $(x-2)+(y+3)=0$ cuts the circle $(x-2)^2 + (y-3)^2 = 11$ at (1) No points (2) One point (3) Two points (4) None of these

2ab
does not cut the circle. **Sol.** (1). Equation of line is $x + y + 1 = 0$. Since the perpendicular distance from centre to line is greater than radius, hence it

Example 5 :

Find the equation of the circle through the point of intersection of $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and touching the line $x + 2y = 0$

Sol. Family of circles is

d
\nd
\nconcyclic
\n
$$
\frac{4}{\sqrt{9+16}} + \frac{7/2}{\sqrt{9+16}} = \frac{3}{2}
$$
\nHence radius is $\frac{3}{4}$.
\n+ $c = 0$ be
\nExample 4:
\nThe straight line $(x-2) + (y+3) = 0$ cuts the circle
\n $(x-2)^2 + (y-3)^2 = 11$ at
\n(1) No points (2) One point
\n(3) Two points (4) None of these
\nSol. (1). Equation of line is $x + y + 1 = 0$. Since the perpendicular
\ndistance from centre to line is greater than radius, hence it
\ndoes not cut the circle.
\nExample 5:
\nExample 5:
\nExample 5:
\nExample 6:
\nExample 6
\nExample 7:
\nFind the equation of the circle through the point of
\nintersection of $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and
\ntouching the line $x + 2y = 0$
\nSol. Family of circles is
\noint (a, b)
\n $x^2 + y^2 - 2x - 4y + 1 + \lambda (x^2 + y^2 - 1) = 0$
\nand O is
\n $(1 + \lambda) x^2 + (1 + \lambda) y^2 - 2x - 4y + (1 - \lambda) = 0$
\n $x^2 + y^2 - \frac{2}{1 + \lambda} x - \frac{4}{1 + \lambda} y + \frac{1 - \lambda}{1 + \lambda} = 0$
\ncircle
\ncircle
\nCheck the
\nCentre is $\left[\frac{1}{1 + \lambda}, \frac{2}{1 + \lambda}\right]$

and radius =
$$
\sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \left(\frac{1-\lambda}{1+\lambda}\right)} = \frac{\sqrt{4+\lambda^2}}{1+\lambda}
$$

Since it touches the line $x + 2y = 0$, hence

Radius = Perpendicular from centre to the line.

EXAMPLEAIRINING
\nand radius =
$$
\sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \left(\frac{1-\lambda}{1+\lambda}\right)} = \frac{\sqrt{4+\lambda^2}}{1+\lambda}
$$
 the lengths of the tangents
\nSince it touches the line x + 2y = 0, hence
\nRadius = Perpendicular from centre to the line.
\n $\left|\frac{1+\lambda}{\sqrt{1^2+2^2}}\right| = \frac{\sqrt{4+\lambda^2}}{1+\lambda} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\lambda = -1$ cannot be possible in case of circle. So λ = 1
\n $\lambda = -1$ cannot be possible in case of circle. So λ = 1
\n $\lambda = -1$ cannot be possible in case of circle. So λ = 1
\n $\lambda = -1$ cannot be possible in case of circle. So λ = 1
\n $\lambda^2 = -2$ and is normal to the circle.
\nIf the straight line ax + by = 2; a, b ≠ 0 touches the circle
\n $x^2 + y^2 - 2x = 3$ and is normal to the circle x² + y² - 4y = 6
\nGiven $\frac{x^2 + y^2 - 2x = 3}{(1, 0)}$ and radius is 2 and x² + y² - 4y = 6
\n∴ centre is (0, 2) and radius is $\sqrt{10}$.
\nSince line ax + by = 2 touches the first circle.
\n $\therefore \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}} = 2$ or (a-2) = $\left[2\sqrt{a^2-b^2}\right]$...(i)
\nHence equation of circle.
\n $\therefore \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}} = 2$ or (a-2) = $\left[2\sqrt{a^2-b^2}\right]$...(i)
\n \therefore The line 3x – 2y = k meets the
\nthe equation (i) we get
\n \therefore a(0) + b(2) = 2 or 2b = 2 \Rightarrow b = 1
\nPutting this value in equation (i) we get
\n \therefore a = 2

 $\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$ Thus, we get the equation of circle.

Example 6 :

If the straight line $ax + by = 2$; $a, b \ne 0$ touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then find the values of a and b. $\sqrt{1^2 + 2^2}$ $\sqrt{1 + \lambda}$ $\rightarrow \sqrt{5} = \sqrt{4 + \lambda^2}$ $\rightarrow \lambda - \pm 1$
 $\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$
 $\Rightarrow 20^2 = a^2$

Thus, we get the equation of circle.

If the centre
 $x^2 + y^2 - 2x = 3$ and is normal to t 1 cannot be possible in case of circle. So $\lambda = 1$
 \Rightarrow $2b^2 = a^2 + c^2 \Rightarrow a^2$, b $\neq 0$ straight line ax + by = 2; a, b ≠ 0 touches the circle

if the certic fa circle which passing the equation of circle.
 $y^2 - 2x = 3$ an

Sol. Given $x^2 + y^2 - 2x = 3$

$$
\therefore
$$
centre is (1, 0) and radius is 2 and $x^2 + y^2 - 4y = 6$

 \therefore centre is (0, 2) and radius is $\sqrt{10}$.

Since line $ax + by = 2$ touches the first circle.

$$
\therefore \frac{a(1) + b(0) - 2}{\sqrt{a^2 + b^2}} = 2 \text{ or } (a - 2) = [2\sqrt{a^2 - b^2}] \quad ...(i)
$$

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.

 \therefore a (0) + b (2) = 2 or 2b = 2 \Rightarrow b = 1

Putting this value in equation (i) we get

$$
a-2 = 2\sqrt{a^2 + 1}
$$
 or $(a-2)^2 = 4(a^2 + 1)$

or
$$
a^2 + 4 - 4a = 4a^2 + 4
$$
 or $3a^2 + 4a = 0$

or $a(3a+4) = 0$ or $a = 0, -4/3$

 \therefore Values of a and b are $(-4/3, 1)$ respectively.

Example 7 :

Find the AM of the slopes of two tangents which can be drawn from the point (3, 1) to the circle $x^2 + y^2 = 4$.

Sol. Any tangents to the given circle, with slope m is

$$
y = mx + 2\sqrt{1 + m^2}
$$

since it passes through (3, 1) : so

$$
∴ centre is (0, 2) and radius is $\sqrt{10}$.
\nSince line ax + by = 2 touches the first circle.
\n
$$
∴ \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}} = 2 \text{ or } (a-2) = [2\sqrt{a^2-b^2}]
$$
\n
$$
∴ \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}} = 2 \text{ or } (a-2) = [2\sqrt{a^2-b^2}]
$$
\n
$$
∴ \frac{a(0)+b(2)=2 \text{ or } 2b=2 \Rightarrow b=1}{a(0)+b(2)=2 \text{ or } 2b=2 \Rightarrow b=1}
$$
\n
$$
∴ a(0)+b(2)=2 \text{ or } 2b=2 \Rightarrow b=1
$$
\nPutting this value in equation (i) we get
\n
$$
a-2=2\sqrt{a^2+1} \text{ or } (a-2)^2=4(a^2+1)
$$
\nor $a^2+4-4a = 4a^2+4 \text{ or } 3a^2+4a=0$
\nor $a(3a+4)=0$ or $a=0, -4/3$
\n
$$
∴ Values of a and b are (-4/3, 1) respectively.\nwhere, c = $-\frac{k}{2}$, m = $\frac{3}{2}$
\n
$$
⇒ max + 2\sqrt{1+m^2}
$$
\n
$$
y=mx + 2\sqrt{1+m^2}
$$
\n
$$
y= 3m + 2\sqrt{1+m^2} \Rightarrow 4m^2+4=(3m-1)^2
$$
\n
$$
⇒ 5m^2-6m-3=0
$$
\nIf m = n₁, m₂ then AM of slopes
\n
$$
f(x) = 2x^2+2y^2+4x-7y-25=0
$$
\n
$$
f(x) = 2x^2+2y^2+4x-7y-25=0
$$
\n
$$
f(x) = 2x^2+2y^2+4x-7y-25=0
$$
\n
$$
f(x) = \frac{1}{2}(6/5)=3/5
$$
\n
$$
f(x) = \frac{1}{2}(6/5)=3/5
$$
\n
$$
f(x) = \frac{1}{2}(x^2
$$
$$
$$

$$
= \frac{1}{2} (m_1 + m_2) = \frac{1}{2} (6/5) = 3/5
$$

Example 8 :

If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in A.P., then (1) a, b, c are in G.P. (2) a, b, c are in AP $(3)a²$, $b²$, $c²$ are in A are in AP (4) a^2 , b^2 , c^2 are in GP

STUDY MATERIAL : MATHEMATICS

the lengths of the tangents from P to the circles $x^{2} + y^{2} = a^{2}$, $x^{2} + y^{2} = b^{2}$ and $x^{2} + y^{2} = c^{2}$ respectively.

EXAMPLEARBIMIGG
\nand radius =
$$
\sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \left(\frac{1-\lambda}{1+\lambda}\right)} = \frac{\sqrt{4+\lambda^2}}{1+\lambda}
$$

\nSince it touches the line x + 2y = 0, hence the line.
\n $\frac{1}{\sqrt{1^2 + 2^2}} = \frac{\sqrt{4+\lambda^2}}{1+\lambda} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\frac{1}{\sqrt{1^2 + 2^2}} = \frac{\sqrt{4+\lambda^2}}{1+\lambda} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\frac{1}{\sqrt{1^2 + 2^2}} = \frac{\sqrt{4+\lambda^2}}{1+\lambda} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\frac{1}{\sqrt{1^2 + 2^2}} = \frac{\sqrt{4+\lambda^2}}{1+\lambda} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\Rightarrow 2 \left(\frac{x_1^2 + y_1^2 - b^2}{x_1^2 + y_1^2 - b^2}\right) = (x_1^2 + y_1^2 - a^2) + (x_1^2 + y_1^2 - c^2)$
\n $= -1$ cannot be possible in case of circle. So $\lambda = 1$
\nThus, we get the equation of circle.
\n**Example 9:**
\n**Problem 11.** If the straight line ax + by = 2; a, b ≠ 0 touches the circle
\n $\frac{1}{2} + y^2 - 2x = 3$ and is normal to the circle x² + y² - 4y = 6.
\n**Example 9:**
\n \therefore center is (1, 0) and radius is 2 and x² + y² - 4y = 6.
\n \therefore centre it is (0, 2) and radius is $\sqrt{10}$.
\n \therefore (2, 2) and (2, 2) and (3, 3) is 3.00°
\n \therefore (3, 2) and (4) is 1.00°<

Example 9 :

If the centre of a circle which passing through the points of intersection of the circle $x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$ is on the line $y = x$, then find the equation of the circle. rough the points of

y + 4 = 0 and

z = x, then find the

ction of two circles

2x - 4y - 6) = 0

x.

y substituting λ in

y² = 4r² at only one

...(i)

...(ii)

...(ii)
 $\frac{3}{2}x - \frac{k}{2}$ cough the points of
 $2y + 4 = 0$ and
 $z = x$, then find the

costion of two circles
 $2x - 4y - 6 = 0$

x.

y substituting λ in
 $y^2 = 4r^2$ at only one

...(i)

...(i)

...(i)
 $\frac{3}{2}x - \frac{k}{2}$

Sol. Family of circles through points of intersection of two circles is $S_1 + \lambda S_2(\lambda \neq -1)$. $x^2 + y^2 - 6x + 2y + 4 + \lambda (x^2 + y^2 + 2x - 4y - 6) = 0$ Centre is $(3 - \lambda, -1 + 2\lambda)$. It lies on $y = x$.

Therefore, $-1 + 2\lambda = 3 - \lambda \Rightarrow \lambda = 4/3$

Hence equation of circle can be found by substituting λ in the family of circles above.

Example 10 :

The line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one point then find k^2 . .

Sol. Equation of line is $3x - 2y = k$...(i) Circle is $x^2 + y^2 = 4r^2$...(ii)

Equation of line can be written as
$$
y = \frac{3}{2}x - \frac{k}{2}
$$

Here,
$$
c = -\frac{k}{2}
$$
, $m = \frac{3}{2}$

Now the line will meet the circle, if

equation of the circle.
\nFinally of circles through points of intersection of two circles
\nis
$$
S_1 + \lambda S_2(\lambda \neq -1)
$$
.
\n $x^2 + y^2 - 6x + 2y + 4 + \lambda (x^2 + y^2 + 2x - 4y - 6) = 0$
\nCentre is $(3 - \lambda, -1 + 2\lambda)$. It lies on y = x.
\nTherefore, $-1 + 2\lambda = 3 - \lambda \Rightarrow \lambda = 4/3$
\nHence equation of circle can be found by substituting λ in
\nthe family of circles above.
\n**lnpl 10 :**
\nThe line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one
\npoint then find k^2 .
\nEquation of line is $3x - 2y = k$...(i)
\nCricle is $x^2 + y^2 = 4r^2$...(ii)
\nEquation of line can be written as $y = \frac{3}{2}x - \frac{k}{2}$
\nHere, $c = -\frac{k}{2}$, $m = \frac{3}{2}$
\nNow the line will meet the circle, if
\n $c = a\sqrt{1 + m^2} = -\frac{k}{2} = (2r)\sqrt{1 + (\frac{3}{2})^2}$ [from (ii), a = 2r]
\n $= \frac{k^2}{4} = 4r^2 \times \frac{13}{4}$ $\therefore k^2 = 52r^2$
\n**lnpl 11 :**
\nFind the equation of the circle which passes through the
\nintersection of $x^2 + y^2 + 13x - 3y = 0$ and
\n $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on
\n $13x + 30y = 0$
\nThe equation of required circles is $s_1 + \lambda s_2 = 0$
\n $= x^2 (1 + \lambda) + y^2 (1 + \lambda) + x (2 + 13\lambda) - y (\frac{7}{2} + 3\lambda) - \frac{25}{2} = 0$
\nCentre line on $13x + 30y = 0$
\nCentre line on $13x + 30y = 0$

Example 11 :

Find the equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on $13x + 30y = 0$

Sol. The equation of required circles is $s_1 + \lambda s_2 = 0$

$$
= x^{2} (1 + \lambda) + y^{2} (1 + \lambda) + x (2 + 13\lambda) - y \left(\frac{7}{2} + 3\lambda\right) - \frac{25}{2} = 0
$$

$$
\left(\frac{-(2 + 13\lambda)}{2}, \frac{7}{2} + 3\lambda\right)
$$

Centre line on $13x + 30y = 0$

Sol. (3). Let P (x_1, y_1) be the given point and PT₁, PT₂, PT₃ be

$$
\Rightarrow -13\left(\frac{2+13\lambda}{2}\right) + 30\left(\frac{7/2+3\lambda}{2}\right) = 0 \Rightarrow \lambda = 1
$$

$$
2x^2 + 2y^2 + 15x - \frac{13}{2}y - \frac{52}{2} = 0
$$

Example 12 :

Locus of a point which moves such that sum of the square of its distances from the sides of a square of side unity is 9 is

Example 13 :

Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the $point (-1, -1).$

Sol. Let C be the centre of the given circle and C_1 be the $\qquad \qquad \nearrow$ centre of the required circle. C=(2, 3), CP = radius = 5 \therefore C₁P=3 \Rightarrow CC₁=2 The point C_1 divided internally, the line joining C and P in the ratio 2 : 3 \therefore coordinates of C₁ are (4/5, 7/5)

Example 14 :

Find the equation of the circle which is touched by $y = x$, has its centre on the positive direction of the x-axis and S_0 cuts off a chord of length 2 units along the line $\sqrt{3}$ y – x = 0.

Sol. Since the required circle has its centre on X-axis, So, let the coordinates of the centre be $(a, 0)$. The circle touches $y = x$. Therefore, radius = length of the perpendicular from $(a, 0)$

on
$$
x - y = 0 = a / \sqrt{2}
$$

Circle cuts off a chord of length 2 units along $x - \sqrt{3} y = 0$

$$
\left(\frac{a}{\sqrt{2}}\right)^2 = 1^2 + \left(\frac{a - \sqrt{3} \times 0}{\sqrt{1^2 + (\sqrt{3})^2}}\right)^2 \Rightarrow \frac{a^2}{2} = 1 + \frac{a^2}{4} \Rightarrow a = 2 \qquad \Rightarrow \frac{h^2 + h}{\Rightarrow} \frac{h^2}{x^2 + y}
$$

Thus, centre of the circle is at (2, 0) and radius = $\frac{a}{\sqrt{2}} = \sqrt{2}$

So, its equation is
$$
x^2 + y^2 - 4x + 2 = 0
$$

Example 15 :

Find the area of an equilateral triangle inscribed in the circle. $x^2 + y^2 + 2gx + 2fy + c = 0$

Sol. Given cirle is $x^2 + y^2 + 2gx + 2fy + c = 0$...(i) Let O be the centre and ABC be an equilateral triangle inscribed in the circle (i). $O \equiv (-g, -f)$

E
\n
$$
-13\left(\frac{2+13\lambda}{2}\right) + 30\left(\frac{7/2+3\lambda}{2}\right) = 0 \Rightarrow \lambda = 1
$$
\nand OA = OB = OC = $\sqrt{g^2 + f^2 - c}$...(ii)
\n
$$
2x^2 + 2y^2 + 15x - \frac{13}{2}y - \frac{52}{2} = 0
$$
\n12:
\nus of a point which moves such that sum of the square
\ns distances from the sides of a square of side unity is 9
\nStragh² (2) Circle
\n
$$
x^2 + (x-1)^2 + y^2 + (y-1)^2 = 9
$$
\nHence circle.
\n13:
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - 8g - 12g + c = 0
$$
\n
$$
= 0 \text{ and the circle}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + 12g + 12g + c) \text{ sq. units}
$$
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + 1
$$

Example 16 :

Find the value of 'c' for which the power of a point $P(2, 5)$ is negative w.r.t a circle $x^2 + y^2 - 8x - 12y + c = 0$ and the circle neither touches nor intersects the coordinate axis.

Sol.
$$
S = x^2 + y^2 - 8x - 12y + c = 0
$$

\nPower = $2^2 + 5^2 - 8 \times 2 - 12 \times 5 + c < 0$
\n $29 - 16 - 60 + c < 0$; $c < 47$
\nAgain $g^2 - c < 0$ & $f^2 - c < 0$
\n \therefore $16 - c < 0$ & $36 - c < 0$
\nHence $36 < c < 47$

Example 17 :

Find the locus of point "P" which moves such that the angle made by pair of tangents drawn to the circle $x^2 + y^2 = a^2$ is 60°.

(2).
$$
x^2 + (x-1)^2 + y^2 + (y-1)^2 = 9
$$
. Hence circle.
\n**Table 13:**
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\ntouches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the
\npoint $(-1, -1)$.
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. units
\npoint $(-1, -1)$.
\n
$$
= \frac{3\sqrt{3}}{4}(g^2 + f^2 - c)
$$
 sq. $-\frac{1}{2}(2, 5)$ sq. $-\frac{1}{2}(2, -1)$ sq. $-\frac{1}{2}(2, -1)$ sq. $-\frac{1}{2}(2, -1)$ ns. $-\frac{1}{2}(2, -1)$ ms. $-\frac{1}{2}(2, -1)$ ms. $-\frac{1}{2}(2, -1)$ ms. $-\frac{1}{2}(2, -1)$ ms. $-\frac{1}{2}($

Example 18 :

 $\frac{a}{\sqrt{2}} = \sqrt{2}$ Find the locus of middle points of chords of the circle $\frac{1}{2} = \sqrt{2}$ Find the locus of middle points of chords of the circle
 $x^2 + y^2 = r^2$, which subtend right angle at the point (λ , 0).

Sol. Let N (h, k) be the middle point of any chord AB, which subtend a right angle at $P(\lambda, 0)$

Since $\angle APB = 90^\circ$

 \therefore NA = NB = NP (since distances of the vertices from middle point of the hypotenuse are equal)

or
$$
(NA)^2 = (NB)^2 = (h - \lambda)^2 + (k - 0)^2
$$
 ...(i)

But also
$$
\angle
$$
 BNO = 90°
\n∴ $(OB)^2 = (ON)^2 + (NB)^2$
\n⇒ $-(NB)^2 = (ON)^2 - (OB)^2$

- \implies -[(h- λ)² + (k-0)²] = (h² + k²) r² or $2(h^2 + k^2) - 2\lambda h + \lambda^2 - r^2 = 0$
- \therefore Locus of N (h, k) is

$$
2(x^2 + y^2) - 2\lambda x + \lambda^2 - r^2 = 0
$$

Example 19 :

Find the equations to the circles which pass through the point (2, 3) and cut off equal chords of length 6 units along the lines $y - x - 1 = 0$ and $y + x - 5 = 0$.

Sol. The given two lines pas through the point (2, 3) and are inclined at 45º and 135º to the x-axis. The other ends of the chords can easily be calculated as

$$
(2+3\sqrt{2}, 3+3\sqrt{2})
$$
 and $(2-3\sqrt{2}, 3+3\sqrt{2})$.

There is symmetry about the line $x = 2$ and therefore the centres of the circles lie on $x = 2$. As the chords subtend right angles at the centre $2r^2 = 6^2$ gives the radius $r = 3\sqrt{2}$.
The centre is $(2, 3 + 3\sqrt{2})$.

The equations of the two circles are therefore

$$
(x-2)2 + (y-3-3\sqrt{2})2 = 18 \text{ and}
$$

$$
(x-2)2 + (y-3+3\sqrt{2})2 = 18.
$$

- **Q.12** The equation of a circle which passes through the point $(1,-2)$ and $(4,-3)$ and whose centre lies on the line $3x + 4y = 7$ is-(A) $15(x^2+y^2) - 94x + 18y - 55 = 0$
	- (B) $15(x^2+y^2) 94x + 18y + 55 = 0$

 $(C) 15 (x^2 + y^2) + 94 x - 18 y + 55 = 0$

(D) None of these

- **Q.13** The equation of a circle passing through (–4, 3) and touching the lines $x+y = 2$, $x-y = 2$ is – (A) $x^2 + y^2 - 20x - 55 = 0$ (B) $x^2 + y^2 + 20x + 55 = 0$ $(C) x^2 + y^2 - 20 x -$ (D) None of these
- **Q.14** The equation of the circle which touches the axis of y at the origin and passes through $(3,4)$ is – $(A) 4(x^2+y^2) - 25x = 0$ (B) $3(x^2+y^2) - 25x = 0$ $(C) 2(x^2+y^2)-3x=$ $(-) - 3x = 0$ (D) $4(x^2 + y^2) - 25x + 10 = 0$
- **Q.15** The equation of a circle which touches x–axis and the line $4x - 3y + 4 = 0$, its centre lying in the third quadrant and lies on the line $x - y - 1 = 0$, is – $(A) 9 (x^2 + y^2) + 6x + 24y + 1 = 0$ (B) 9 ($x^2 + y^2$) – 6x – 24 y + 1 = 0 $(C) 9(x^2+y^2) -6x+2y+1=0$ (D) None of these ² - 20 x - 55 = 0 (D) None of these

ion of the circle which touches the axis of y at

and passes through (3,4) is -
 y^2 - 25 x = 0 (B) 3 (x² + y²) - 25 x = 0
 (y^2) - 3x = 0 (D) 4 (x² + y²) - 25 x + 10=0

t "y = 2x, -3, 3 - 0

iguation of the circle which touches the axis of y at

igin and passes through (3,4) is -
 $x^2 + y^2$) - 25 x = 0 (B) 3 ($x^2 + y^2$) - 25 x = 0
 $x^2 + y^2$) - 3x = 0 (D) 4 ($x^2 + y^2$) - 25 x = 0

cquation ion of the circle which touches the axis of y at

and passes through (3,4) is-
 $+y^2$) -25×0 (B) 3 ($x^2 + y^2$) -25×10^{-1}
 $(y^2 - y^2) - 3x = 0$ (D) $4(x^2 + y^2) - 25x + 10 = 0$

tion of a circle which touches x-axis and th The motion of the circle which the same of $y = x$,
 $(3x^2 + y^2 - 20)x - 55 = 0$ (B) $x^2 + y^2 + 20x + 55 = 0$
 $(2+x^2 - 20x - 55 = 0$ (D) None of these

equation of the circle which touches the axis of y at
 $y = \frac{x^2 + y^2 - 25x + 0}{(x^2$ e of these

thes the axis of y at
 $\frac{1}{2} + y^2$) – 25 x + 10=0

es x-axis and the

the third quadrant

the third quadrant

ugh the origin and

ugh the origin and

of the axes is –
 y^2 – 5x + 5y = 0
 y^2 + 5x + 5y = vone of these

ouches the axis of y at
 y is $-(x^2+y^2)-25x+10=0$
 $-(x^2+y^2)-25x+10=0$

uuches x-axis and the

g in the third quadrant
 $x = -$

through the origin and
 $x + 5$ of the axes is $-$
 $x^2+y^2-5x+5y=0$

radius is hes the axis of y at
 $\left(-\frac{y^2}{3} + y^2\right) - 25x = 0$
 $\left(-\frac{y^2}{3} - 25x + 10\right) = 0$
 $\left(-\frac{y^2}{3} - 25x + 10\right) = 0$

the third quadrant
 $\left(-\frac{y^2}{3} - 5x + 5y\right) = 0$
 $\left(\frac{y^2}{3} - 5x + 5y\right) = 0$
 $\left(\frac{y^2}{3} - 5x + 5y\right) = 0$ $z^2 + y^2 + 20x + 55 = 0$

None of these

touches the axis of y at

4) is -
 $3(x^2 + y^2) - 25x = 0$
 $4(x^2 + y^2) - 25x + 10 = 0$

touches x-axis and the

ng in the third quadrant

is -

through the origin and
 $x^2 + y^2 - 5x + 5y = 0$

- **Q.16** The equation to a circle passing through the origin and cutting of intercepts each equal to $+5$ of the axes is – $(A) x² + y² + 5x - 3$ $+5x-5y=0$ (B) $x^2+y^2-5x+5y=0$ $-5x-5y=0$ (D) $x^2+y^2+5x+5y=0$
- **Q.17** The equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point $(-1,-1)$ is $-$

(A)
$$
\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{7}{5}\right)^2 = 3^2 (B) \left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2
$$

 $(C) (x-8)² + (y-1)² = 3²$ (D) None of these

Q.18 The equation of a circle which passes through the three points $(3, 0)$ $(1, -6)$, $(4, -1)$ is – (A) 2x² + 2y² + 5x - 11 y + 3 = 0 $(B) x² + y² - 5x + 11y - 3 = 0$ $(C) x^2 + y^2 + 5x - 11 y + 3 = 0$ $(D) 2x^2 + 2y^2 - 5x + 11y - 3 = 0$ (A) $\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{7}{5}\right)^2 = 3^2$ (B) $\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2$

(C) $(x - 8)^2 + (y - 1)^2 = 3^2$ (D) None of these

The equation of a circle which passes through the three

points (3, 0) $(1, -5)$, $(4, -1)$ is (A) $\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{7}{5}\right)^2 = 3^2 (B) \left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = 3^2$

(C) $(x - 8)^2 + (y - 1)^2 = 3^2$ (D) None of these

The equation of a circle which passes through the three

points (3, 0) (1,-6), (4,-1) is-

(A) 2

 $= 16$ Q.19 If (4, -2) is a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, then value of c is - $(A) - 4$ (B) 0

$$
(A) = 4
$$
 (B) 0
(C) 4 (D) 1

Q.20 The abscissa of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$. The radius of the circle with AB as a diameter will be -

(A)
$$
\sqrt{a^2 + b^2 + p^2 + q^2}
$$

\n(B) $\sqrt{b^2 + q^2}$
\n(C) $\sqrt{a^2 + b^2 - p^2 - q^2}$
\n(D) $\sqrt{a^2 + p^2}$

Q.21 Two rods of length a and b slide on the axes in such a way that their ends are always concylic. The locus of centre of the circle passing through the ends is -

(A)
$$
4(x^2 - y^2) = a^2 - b^2
$$

\n(B) $x^2 - y^2 = a^2 - b^2$
\n(C) $x^2 - y^2 = 4(a^2 - b^2)$
\n(D) $x^2 + y^2 = a^2 + b^2$

- **Q.22** Circle $x^2 + y^2 + 6y = 0$ touches (A) y-axis at the origin (B) x-axis at the origin (C) x-axis at the point $(3, 0)$ (D) The line $y + 3 = 0$
- **Q.23** The equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$, is

(A)
$$
x^2 + y^2 + 4x - 10y + 25 = 0
$$

$$
(B) x2 + y2 - 4x - 10y + 25 =
$$

-
-
- **Q.24** The equation of the circle with centre on the x-axis, radius 4 and passing through the origin, is

(A)
$$
x^2 + y^2 + 4x = 0
$$

\n(B) $x^2 + y^2 - 8y = 0$
\n(C) $x^2 + y^2 \pm 8x = 0$
\n(D) $x^2 + y^2 + 8y = 0$

OMM 200032	Cirole $x^2 + y^2 + 6y = 0$ touches	(A) $x + y = 1$	(B) x -axis at the origin	(C) $2x + 2y = 5$
(C) x -axis at the point (3, 0)	(D) The line $y + 3 = 0$	(D) 3	(E) The points of intersection $x + y = 1$	
(A) $x^2 + y^2 + 4x - 10y + 25 = 0$	(D) $x^2 + y^2 - 4x - 10y + 25 = 0$			
(B) $x^2 + y^2 - 4x - 10y + 25 = 0$	(A) $(-2, -6), (-4, 2)$			
(B) $x^2 + y^2 - 4x - 10y + 25 = 0$	(B) $x^2 + y^2 - 4x - 10y + 25 = 0$			
(C) $x^2 + y^2 - 4x - 10y + 25 = 0$	(D) $x^2 + y^2 - 8y = 0$			
(A) $x^2 + y^2 + 4x = 0$	(B) Minimum, if the triangle $x^2 + 4x = 0$			
(B) Minimum, if the triangle $x^2 + 4x = 0$	(C) $x^2 + y^2 + 8y = 0$			
(D) $x^2 + y^2 + 4x = 0$	(D) $x^2 + y^2 - 8y = 0$			
(A) $\left(\frac{1}{4}, 0\right)$ and $\frac{1}{4}$	(B) $\left(-\frac{1}{2}, 0\right)$ and $\frac{1}{2}$	(C) $\left(\frac{1}{2}, 0\right$		

Q.26 The radius of a circle which touches y-axis at (0,3) and cuts intercept of 8 units with x-axis, is –

 $(A) 1$ (B) 3

$$
(C) 2\sqrt{3}
$$
 (D)

Q.28 A circle has radius 3 units and its centre lies on the line $y = x - 1$. Then the equation of this circle if it passes through point $(7, 3)$, is

(A)
$$
x^2 + y^2 - 8x - 6y + 16 = 0
$$

(B)
$$
x^2 + y^2 + 8x + 6y + 16 = 0
$$

$$
(C) x2 + y2 = 8x + 6y + 16 = 0
$$

$$
(C) x2 + y2 = 8x + 6y + 16 = 0
$$

- (D) None of these
- **Q.29** If the coordinates of one end of the diameter of the circle $x^2 + y^2 8x 4y + c = 0$ are (-3, 2), then the coordinates of other end are

(A) (5, 3) (B) (6, 2) (C) (1, –8) (D) (11, 2)

- **Q.30** The equation of the circle whose diameter lies on $2x + 3y = 3$ and $16x - y = 4$ which passes through (4, 6) is (A) $\frac{(A)}{5}$

(A) $\frac{(B)}{2}$

(B) $\frac{(B)}{2}$

(B) $\frac{(B)}{2}$

(B) $\frac{(B)}{2}$

(B) $\frac{(B)}{2}$

(A) $\frac{(B)}{2}$

(C) $2\sqrt{3}$

(C) $2\sqrt{3}$

(C) $2\sqrt{3}$

(C) $\sqrt{10}$

(C) $\sqrt{2} + y^2 + 2x \cos 9 + 2y \sin 0 - 8 = 0$

(C) $x^2 + y^2 +$ (C) 3

(B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(A) 1

(A) 1

(B) 3

(A) $(2\sqrt{3}$

(C) $2\sqrt{3}$

(C) 2 Radius of the circle $x^2 + y^2 + 2x \cos \theta + 2y \sin \theta - 8 = 0$ is (C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(A) 1

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$

A circle has radius 3 umis and its centre lies on the line
 $x = x - 1$. Then the equation of thi (A) 1 (B) 3 (D) $\sqrt{2} + y^2 + 32x + 4y + 235$

(C) $2\sqrt{3}$ (D) $\sqrt{10}$ **0.38** Himses y- $x + 3$ cust be circle to the content of this centre lies on the line

A and B, then equation of circle if it passes
 $y = x - 1$. Then the (C) $2\sqrt{3}$
 Q.28 Acricle has radius 3 units and is centre lies on the line
 $y = x - 1$. Then the equation of this circle if it passes
 $y = x - 1$. Then the equation of this circle if it passes
 $y = x - 1$. Then the cuts con
- (A) $(2, 0)$, $(-3, 0)$ (B) $(3, 0)$, $(4, 0)$ (C) (1, 0), (-1, 0) (D) (1, 0), (2, 0)
- **Q.32** If one end of the diameter is (1, 1) and other end lies on the line $x + y = 3$, then locus of centre of circle is
- (A) $x + y = 1$ (B) 2 $(x y) = 5$ $(C) 2x + 2y = 5$ (D) None of these
- **Q.33** The points of intersection of the line $4x 3y 10 = 0$ and

(A) (–2, –6), (4, 2) (B) (2, 6), (– 4, –2) (C) (–2, 6) (– 4, 2) (D) None of these

- **EXERCULS ANTERENT CONSTRON BANK**

Circle $x^2 + y^2 + 6y = 0$ touches

(A) $x + y = 1$ (B) $2(x y) = 5$

(A) y -axis at the origin (B) x -axis at the origin (C) $2x + 2y = 5$ (D) None of these

(C) x -axis at the point (3, 0) (D COVESTION BANK CULC THE CONTIDENTIAL: MATHERIAL: MATHERI **EXERENT PROBABLE CONSTRON BANK**

CITED **EXERENT CONSTRON BANK**

CITED **EXERENT CONSTRON BANK**

CITED **EXERENT CONSTRON BANK**

CONSTRON BANK

CONSTRON BANK

CONSTRON BANK

CONSTRON BANK

CONSTRON BANK

CONSTRON BANK

CONS **EXERCISE ANTIFIEM ALL:**

Circle $x^2 + y^2 + 6y = 0$ touches

(A) y-axis at the origin (B) x-axis at the origin (C) $2x + 2y = 5$ (D) None

(C) x-axis at the origin (C) $2x + 2y = 5$ (D) None

The equation of the circle which pa Circle $x^2 + y^2 + 6y = 0$ (DUESTION BANK (X) $x+y = 1$ (B) $2(x-y)=5$

(A) y-axis at the origin (B) x-axis at the origin (C) $2x+2y=5$ (D) None of these

(A) y-axis at the origin (E) x-axis at the origin (C) $2x+2y-5$ (D) None o CICE $x^2 + y^2 + 6y = 0$ touches

(A) y axis at the origin (B) x axis at the origin (B) x axis at the origin (B) x axis at the origin (C) x + 2y = 5 (C) x + 2y = 5 (C) None of these

(C) x + 2y = 5 (D) The line y + 3 - 0

Th **EXERCISE THE CONSTRAINS SET SUBSTION BANK**

Q.22 Circle $g^2 - y^2 + 6y = 0$ touches

(A) y-axis at the origin (B) x-axis at the origin (B) x-axis at the origin (D) N-axis at the origin (D) N-axis at the origin (D) N-axis at **COLLESTION BANK** STUDY MATERIAL: MATHEMATIC

at the origin $(A) x + y = 1$ $(B) 2(x-y) = 5$

intervision $(C) 2x + 2y = 5$ (D) None of these

ness through the

es on the straight

the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are

con the st **IVENTION BANK**

(A) $x+y=1$ (B) $2(x-y)=5$

C-axis at the origin

(C) $2x+2y=5$ (D) None of these

the line $y+3=0$

the passes through the

ture lies on the straight

ture lies on the straight

(A) $(-2, 6) (-4, 2)$

(C) $(-2,$ **COLESTION BANK**

(A) $x+y=1$ (B) $2(x-y)=5$

The points of intersection of the line $4x-3y-10=0$ and

the circle $x^2+y^2-2x+4y-20=0$ are

the tire lies on the straight

(A)(-2,-6), (4, 2)

(C)(-x)(-2) (B)(2, 6), (-4, -2)

(C Casis at the origin

(A) $x+y=1$ (B) $2(x-y)=5$

The line $y+3=0$

the since the set of the line 4x-3y-10 = 0 and

the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are

the set of the since the straight

(A) (-2, 6) (-4, 2) (B)) (None of c-axis at the origin

(A) $x + y = 1$ (B) $2(x - y) = 5$

The line $y + 3 = 0$
 C.33 The points of intersection of the line $4x - 3y - 10 = 0$ and

the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are

(A) $(-2, -6)$, $(4, 2)$ (B) $(2, 6)$, $(-4, -$ **STUDY MATERIAL: MATHEMATICS**

(A) $x+y=1$ (B) $2(x-y)=5$

(C) $2x+2y=5$ (D) None of these

The points of intersection of the line $4x-3y-10=0$ and

the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are

(A) $(-2,-6)$, $(4, 2)$ (B) $(2, 6)$, **Q.34** The diameter of a circle is AB and C is another point on circle, then the area of triangle ABC will be (A) Maximum, if the triangle is isosceles (B) Minimum, if the triangle is isosceles (C) Maximum, if the triangle is equilateral (D) None of these
	- **Q.35** The locus of the centre of the circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is - $(A) x² - 10x - 6y + 14 = 0$ (B) $x² - 6x - 10y + 14 = 0$ (C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

 2^{2} and 2 touch each 1 $\qquad \qquad \Omega$ 36 The two circles $x^2 + y^2$ 2 touch each other if - 1) 1 $(A) c = |a|$ $(B) 2c = a$ **Q.36** The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (with $c > 0$)

 (D) None of these

(C) x-axis at the point (3, 0) (D) The line y + 3 = 0
\n2.23 The equation of the circle which passes through the
\npoint z (2, 3) and (4, 5) and the centre less on the straight
\n
$$
y-x+3=0
$$
, is
\n $(x) -2 = 0$ (a)
\n $(x) -2 = 4x-10y+25=0$
\n $(x) -2 = 4x-10y+25=0$
\n $(x) -2 = 4x-10y+16=0$
\n2.24 The equation of the circle with the circle is AB and C is another
\n $(x) -2 = 4x-10y+16=0$
\n $(x) -2 = 4x-10y+16=0$
\n $(x) -2 = 4x-10y+16=0$
\n2.25 The equation of the circle with center of a circle is AB and C is another
\n $(x) -2 = 4x-10y+16=0$
\n2.26 The equation of the circle is the distance of triangle is isosceles
\n $(x) -2 = 4x-10y+16=0$
\n $(x) -2 = 4x-10y+16=0$
\n $(x) -2 = 4x-10y+16=0$
\n2.29 The equation of the circle with the circle of the circle which touches
\n $(x) -2 = 4x^2 + 25x = 0$
\n $(x) -2 = 4x^2 + 4x = 0$
\n $(x) -2 = 4x^2 + 4x = 0$
\n $(x) -2 = 4x^2 + 4x = 0$
\n $(x) -3 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n $(x) -4 = 4x^2 + 4x = 0$
\n<

- **Q.38** If lines $y = x + 3$ cuts the circle $x^2 + y^2 = a^2$ in two points A and B, then equation of circle with AB as diameter is - $(A) x² + y² + 3x - 3y - a² + 9 = 0$ $(B) x^2 + y^2 + 3x - 3y + a^2 + 9 = 0$ $(C) x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$ (D) None of these (A) $x^2 + y^2 + 32x - 4y + 235 = 0$

(B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

A and B, then equation of circle with AB as diameter is-

(A) (B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

A and B, then quation of circle with AB as diameter is-

(A) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(B) (C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

If lines $y = x + 3$ cuts the circle $x^2 + y^2 = a^2$ in two points

A and B, then equation of circle with AB as diameter is

A $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(C) x (D) $x^2 + y^2 + 32x + 4y + 235 = 0$

If lines $y = x + 3$ cuts the circle $x^2 + y^2 = a^2$ in two points

A and B, then equation of circle with AB as diameter is-

(A) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(C) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$

(C
- **Q.39** If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touches each other than α –

(A) 0 (B) 1 (C) –4/3 (D) –3/4

- **Q.40** The centre of the circle $r^2 = 2 4r \cos \theta + 6r \sin \theta$ is $(A)(2, 3)$ (B) (–2, 3) $(C) (-2,-3)$ (D) $(2,-3)$
- **Q.41** The equation of the circle whose radius is 5 and which at the point $(5, 5)$, is

(A)
$$
x^2 + y^2 - 18x - 16y - 120 = 0
$$

(B)
$$
x^2 + y^2 - 18x - 16y + 120 = 0
$$

(C)
$$
x^2 + y^2 + 18x + 16y - 120 = 0
$$

-
- **Q.42** The straight line $2x + 3y k = 0$, $k > 0$ cuts the X- and Yaxes at A and B. The area of \triangle OAB, where O is the origin, is 12 sq. units. The equation of the circle having AB as diameter is –

(A)
$$
x^2 + y^2 - 6x - 4y = 0
$$

\n(B) $x^2 + y^2 + 4x - 6y = 0$
\n(C) $x^2 + y^2 - 6x + 4y = 0$
\n(D) $x^2 + y^2 - 4x - 6y = 0$

- **Q.43** Equation of the circle centered at (4, 3) touching the circle $x^2 + y^2 = 1$ externally, is – (A) $x^2 + y^2 - 8x - 6y + 9 = 0$ (B) $x^2 + y^2 + 8x + 6y + 9 = 0$ **Q.54** The equation of the tangent at the point $(C) x^2 + y^2 + 8x - 6y + 9 = 0$ $(D) x^2 + y^2 - 8x + 6y + 9 = 0$
- **Q.44** The points $(1, 0), (0, 1), (0, 0)$ and $(2k, 3k), k \neq 0$ are concyclic $if k =$

- **Q.45** The number of circles that touch the co-ordinate axes and the line whose slope is –1 and y-intercept is 1, is $(A) 3$ (B) 1 (C) 4 (D) 2
- **Q.46** If $x = 2 + 3 \cos \theta$ and $y = 1 3 \sin \theta$ represent a circle then **Q.55** the centre and radius is

PART 2 : POINT WITH RESPECT TO CIRCLE,
 PART 2 : POINT WITH RESPECT TO CIRCLE,
 Q.56 The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and **TANGENT AND NORMAL**

- **Q47** For what value of m the line $3x + 4y = m$ touches the circle $x^2 + y^2 - 2x - 8 = 0$
	- (A) –18, 12 (B) 18, 12
	- $(C) 18, -12$ $(D) -18, -12$
- **Q.48** The circle S₁ with centre C₁ (a₁, b₁) and radius r₁ touches $\overline{O.5}$ externally the circle S₂ with centre C₂ (a_2 , b_2) and radius r_2 . If the tangent at their common point passes through the origin, then

(A)
$$
(a_1^2 + a_2^2) + (b_1^2 + b_2^2) = r_1^2 + r_2^2
$$

\n(B) $(a_2^2 - a_1^2) + (b_2^2 - b_1^2) = r_2^2 - r_1^2$
\n(C) $(a_1^2 - b_2^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$
\n(D) $(a_1^2 - b_1^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$

- **Q.49** The point from which the tangents to the circles $x^2 + y^2 - 8x + 40 = 0$; $5x^2 + 5y^2 - 25x + 80 = 0$ $x^{2} + y^{2} - 8x + 16y + 160 = 0$ are equal in length is- $(A)(8, 15/2)$ (B) $(-8, 15/2)$ $(C)(8,-15/2)$ (D) None of these (A) $\frac{x^2}{x^2} + y^2 - 2x + 40 = 0$; $5x^2 + 5y^2 - 2x + 80 = 0$

The point from which the tangents to the circles
 $x^2 + y^2 - 8x + 16y + 160 = 0$ are equal in length is

(A) (8, 15/2)

(B)(-8, 15/2)

(C) C(8, -15/2)

(C) C(8, -15/2)
- **Q.50** The total number of common tangents to the two circles $x^{2} + y^{2} - 2x - 6y + 9 = 0$ and $x^{2} + y^{2} + 6x - 2y + 1 = 0$, is - $(A) 1$ (B) 2 (C) 3 (D) 4
- **Q.51** The point P (10, 7) lies outside the circle from the circle is

$$
(A) 5 \t\t (B) \sqrt{ }
$$

$$
(C) \sqrt{5} \tag{D) 15}
$$

Q.52 The equations of the tangents to the circle $x^{2} + y^{2} = 36$ which are inclined at an angle of 45° to the xaxis are

(A)
$$
x + y = \pm \sqrt{6}
$$
 (B) $x = y \pm 3\sqrt{2}$

C)
$$
y = x \pm 6\sqrt{2}
$$
 (D) None of these

Q.53 If the equation of one tangent to the circle with centre at $(2, -1)$ from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is

(A)
$$
3x - y = 0
$$

\n(B) $x + 3y = 0$
\n(C) $x - 3y = 0$
\n(D) $x + 2y = 0$

BANK
\n(A)
$$
3x - y = 0
$$

\n(B) $x + 3y = 0$
\n(C) $x - 3y = 0$
\n(B) $x + 2y = 0$
\n(D) $x + 2y = 0$
\n**Q.54** The equation of the tangent at the point
\n
$$
\left(\frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2}\right) \text{ of the circle } x^2 + y^2 = \frac{a^2b^2}{a^2 + b^2} \text{ is}
$$
\n(A) $\frac{x}{a} + \frac{y}{b} = 1$
\n(B) $\frac{x}{a} + \frac{y}{b} + 1 = 0$
\n(C) $\frac{x}{a} - \frac{y}{b} = 1$
\n(D) $\frac{x}{a} - \frac{y}{b} + 1 = 0$
\n**Q.55** If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same circle, then its radius will be
\n(A) $\frac{\sqrt{3}}{5}$
\n(B) $\frac{17}{6\sqrt{5}}$
\n(C) $\frac{2\sqrt{5}}{3}$
\n(D) $\frac{17}{3\sqrt{5}}$
\n**Q.56** The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and
\n $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other. The equation
\nof their common tangent is
\n(A) $x = 3$
\n(B) $y = 6$
\n(C) $7x - 12y - 21 = 0$
\n(D) $7x + 12y + 21 = 0$
\n(D) $4x + 2y + 2 = 0$ <

Q.55 If $2x-4y=9$ and $6x-12y+7=0$ are the tangents of same circle, then its radius will be

(A)
$$
\frac{\sqrt{3}}{5}
$$
 (B) $\frac{17}{6\sqrt{5}}$
(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

$$
x2 + y2 - 5x + 6y + 15 = 0
$$
 touch each other. The equation
of their common tangent is

- (A) $x = 3$ (B) $y = 6$ (C) $7x - 12y - 21 = 0$ (D) $7x + 12y + 21 = 0$ **Q.57** The area of the triangle formed by the tangent at $(3, 4)$ to the circle $x^2 + y^2 = 25$ and the co-ordinate axes is (A) 24/25 (B) 0 (C) $\frac{x}{a} - \frac{y}{b} = 1$ (D) $\frac{x}{a} - \frac{y}{b} + 1 = 0$

If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same

circle, then its radius will be

(A) $\frac{\sqrt{5}}{5}$ (B) $\frac{17}{6\sqrt{5}}$

(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

- $(D) (24/25)$ **Q.58** Tangents AB and AC are drawn from the point A (0, 1) to

through A, B and C is

(A) $x^2 + y^2 + x + y - 2 = 0$ (B) $x^2 + y^2 - x + y - 2 = 0$

- $(2+3x+4) = 0$. $(3x^2-3x-8) = 0$

(B) 18, 12
 $(4x-3x-8) = 0$ (B) $(4x-3x-8) = 0$ (B) $(4x-3x-8) = 0$ (B) $(4x-3x-2x-4) = 0$ (B) $(4x-3x-2x-4) = 0$ (B) $(4x-3x-2x-5) + 16x^2-5x^2-5x + 80 = 0$
 $(5x^2-3x^2-5x+6x^2-5x^2-5x+80=0$
 $($ (C) (a₁² - b₁² + b₂² + b₂² + b₂² + c₂² + b₂² + c₂² If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same

circle, then its radius will be

(A) $\frac{\sqrt{3}}{5}$ (B) $\frac{17}{6\sqrt{5}}$

(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and
 $x^2 + y$ First Apple 10 $\frac{2\sqrt{5}}{5}$

(C) $\frac{2\sqrt{5}}{5}$ (C) $\frac{17}{5\sqrt{5}}$

(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

(C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{17}{3\sqrt{5}}$

The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touc **Q.59** The equation of pair of tangents drawn from the point $(0,1)$ to the circle $x^2 + y^2 - 2x + 4y = 0$ is – (A) 4x² – 4y² + 6xy + 6x + 8y – 4 = 0 (B) 4x² – 4y² + 6xy – 6x + 8y – 4 = 0 $(C) x^{2} - y^{2} + 3xy - 3x + 2y - 1 = 0$ $(D) x^2 - y^2 + 6xy - 6x + 8y - 4 = 0$ $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other. The equation

of their common tangent is

(A) $x = 3$ (B) $y = 6$ (D) $7x + 12y + 21 = 0$

The area of the triangle formed by the tangent at (3, 4) to

the circle $x^2 + y^2 = 25$ and th of their common tangent is

(A) $x = 3$

(C) $7x - 12y - 21 = 0$ (D) $7x + 12y + 21 = 0$

The area of the triangle formed by the tangent at (3, 4) to

the circle $x^2 + y^2 = 25$ and the co-ordinate axes is

(A) 642425 (B) 0 $- (2$ The area of the triangle formed by the tangent at $(3, 4)$ to

A) $24/25$

(A) $24/25$

(B) 0

(C) $625/24$

(B) 0

(D) $-(24/25)$

Tangents AB and AC are drawn from the point A(0, 1) to

he circle $x^2 + y^2 - 2x + 4y + 1 = 0$. (A) $4x^2 - 4y^2 + 6xy + 6x + 8y - 4 = 0$

(B) $4x^2 - 4y^2 + 6xy + 6x + 8y - 4 = 0$

(B) $4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$

(D) $x^2 - y^2 + 3xy - 3x + 2y - 1 = 0$

(D) $x^2 - y^2 + 6xy - 6x + 8y - 4 = 0$

The the case of the tangents drawn from the poi
	- **Q.60** If the length of the tangents drawn from the point (1,2) to
the circles $x^2 + y^2 + x + y 4 = 0$ and (A) $7/2$ (B) $21/2$ (C) – 21/4 (D) 7/4
- (A) 5 (B) $\sqrt{3}$ **Q.61** Two tangents PQ and PR drawn to the circle centre of the circle is C, then the area of quadrilateral PQCR

(A) 75 sq. units (B) 150 sq. units (C) 15 sq. units (D) None of these

- **Q.62** If the tangent to a circle $x^2 + y^2 = 5$ at point (1, -2) touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$, then its point of contact $(A)(-2, 1)$ $(B)(3, -1)$ $(C)(-1, -3)$ $(D)(5, 0)$
- **Q.63** Length of the tangent drawn from point (1, 5) to the circle $2x^2 + 2y^2 = 3$ is -

(A) 7 (B)
$$
7\sqrt{2}
$$
 (C) $7\sqrt{2}/2$ (D) None

ANGLE OF INTERSECTION

Q.73 The equation of the circle whose centre is (3, –1) and which cuts off a chord of length 6 on the line $2x - 5y + 18 = 0$ is

(A)
$$
(x-3)^2 + (y+1)^2 = 38
$$
 (B) $(x+3)^2 + (y-1)^2 = 38$

(C)
$$
(x-3)^2 + (y+1)^2 = \sqrt{38}
$$
 (D) None of these

Q.74 If the line $x - y + 1 = 0$ is a chord of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ then find the length of this chord $(A) 2$ (B) 3 $(C) 5$ (D) 7

- **Q.75** The pole of the straight line $9x + y 28 = 0$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$, is $(A)(2, 1)$ (B) $(2, -1)$ $(D)(3,1)$ (D) $(3,-1)$
- **Q.76** If the polar of a point (p, q) with respect to the circle $x^{2} + y^{2} = a^{2}$ touches the circle $(x-c)^{2} + (y-d)^{2} = b^{2}$, then (A) $b^2 (p^2 + q^2) = (a^2 - cp - qd)^2$ (B) $b^2 (p^2 + q^2) = (a^2 - cq - dp)^2$ (C) $a^2 (p^2 + q^2) = (b^2 - cp - dq)^2$ (D) None of these **Q.77** From the origin, chords are drawn to the circle The potential that $\sec^2 x$ and $\sec^2 x$ c a b $\sec^2 x$ $\sec^2 x$ $\sec^2 x$ $\sec^2 x$ $\sec^2 x$ $\sec^2 x$ (A) 3 (C) $(3,1)$ (B) $(2,-1)$

(C) $(3,1)$ (B) $(2,-1)$ (A)(2, 1)

(C)(3,1) (B)(2,-1)

(C)(3,1)

If the polar of a point (p, q) with respect to the circle
 $x^2 + y^2 = a^2$ touches the circle $(x-0)^2 + (y-d)^2 = b^2$, then

(A) b² (p² + q²) = (a² - cp - qd)²

(C) a² (p² + (B) (2,-1)

(D) (3,-1)

(D) (3,-1)

ith respect to the circle
 $(x-c)^2 + (y-d)^2 = b^2$, then

and
 $qd)^2$

flaps

rawn to the circle

n of locus of middle points

(B) $x^2 + y^2 = x$

(D) None of these

cle $x^2 + y^2 = 10x$, then the

	- $(x-1)^2 + y^2 = 1$, then equation of locus of middle points of these chords, is - $(A) x² + y² = 1$ $= 1$ (B) $x^2 + y^2 = x$
		- $(C) x^2 + y^2 = y$ (D) None of these
- **Q.78** If $y = 2x$ is a chord of the circle $x^2 + y^2 = 10x$, then the equation of the circle whose diameter is this chord is - (A) $x^2 + y^2 + 2x + 4y = 0$ (B) $x^2 + y^2 + 2x - 4y = 0$ $(C) x^2 + y^2 - 2x - 4y = 0$ (D) None of these

Q.79 The length of the common chord of the circles
$$
(x-a)^2 + y^2 = c^2
$$
 and $x^2 + (y-b)^2 = c^2$ is -

(A)
$$
\sqrt{c^2 + a^2 + b^2}
$$

\n(B) $\sqrt{4c^2 + a^2 + b^2}$
\n(C) $\sqrt{4a^2 - a^2 - b^2}$
\n(D) $\sqrt{a^2 - a^2 - b^2}$

- **Q.80** The angle of intersection of the two circles $x^{2} + y^{2} - 2x - 2y = 0$ and $x^{2} + y^{2} = 4$, is - $(A) 30^{\circ}$ (B) 60[°] $(C) 90^{\circ}$ (D) 45[°]
- **Q.81** If a circle passes through the point (1,2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is -

(A)
$$
x^2 + y^2 - 2x - 6y - 7 = 0
$$
 (B) $x^2 + y^2 - 3x - 8y + 1 = 0$
(C) $2x + 4y - 9 = 0$ (D) $2x + 4y - 1 = 0$

- **Q.82** Circles $x^2 + y^2 = 4$ and $x^2 + y^2 2x 4y + 3 = 0$
	- (A) touch each other externally
	- (B) touch each other internally
	- (C) intersect each other
	- (D) do not intersect
- **Q.83** The circles $x^2 + y^2 + 2x 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other -(A) externally at $(0,1)$ (B) internally at $(0,1)$ (C) externally at $(1,0)$ (D) internally at $(1,0)$ (C) $\sqrt{4c^2 - a^2 - b^2}$ (D) $\sqrt{c^2 - a^2 - b^2}$

The angle of intersection of the two circles
 $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 4$, is -

(A) 30° (B) 60°

(C) 90° (D) 45°

If a circle passes through the point (1,2) and cut is

(A) $x^2 + y^2 - 2x - 6y - 7 = 0$ (B) $x^2 + y^2 - 3x - 8y + 1 = 0$
 Q.82 Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 4y + 3 = 0$

(A) touch each other externally

(B) touch each other reinearily

(C) intersect each other infermally
 Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 4y + 3 = 0$

(A) touch each other externally

(B) touch each other internally

(C) intersect each other internally

(C) intersect each other

(D) do not intersect

The circles $x^2 + y^2$ (A) touch each other externally

(B) touch each other internally

(C) intersect can other

(D) do not intersect

The circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and
 $x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other-

(A) externally at (0,1)
- **Q.84** The angle between the two tangents from the origin to

- **Q.85** Middle point of the chord of the circle $x^2 + y^2 = 25$ intercepted on the line $x - 2y = 2$ is (A) (3/5, 4/5) (B) (-2, -2) (C) $(2/5, -4/5)$ (D) $(8/3, 1/3)$
- A and B, then locus of the centre of the circle drawn on

AB as a diameter is

(A) $x^2 + y^2 - 2ay = 0$

(B) $x^2 + y^2 + ay = 0$

(C) $x^2 + y^2 + ax = 0$

(D) $x^2 + y^2 - ax = 0$

- **Q.87** Chord of contact with respect to point (2, 2) of circle $x^2 + y^2 = 1$ is -
	- (A) $x + y + 1$ (B) $x y = 1/2$
	- (C) $x + y = 1/2$ (D) $x + y = 2$
- **Q.88** If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$, then **PAR** $c + d =$ (A) 40 (B) 50
	- $(C) 60$ (D) 56
- **Q.89** Equation of polar of point (4, 4) with respect to circle $(x-1)^2 + (y-2)^2 = 1$ is (A) $2x + 3y - 8 = 0$ (B) $3x + 2y + 8 = 0$
	- (C) $3x 2y + 8 = 0$ (D) $3x + 2y 8 = 0$
- **Q.90** The locus of the point, the chord of contact of tangents from which to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre is a circle of radius - (A) 2a $(B)a/2$ (C) $\sqrt{2} a$ (D) a^2
- **Q.91** If a chord of the circle $x^2 + y^2 = 8$ makes equal intercepts **Q.1** of length a on the coordinate axes, then-

Q.92 The area of the triangle formed by the tangents from an external point (h, k) to the circle $x^2 + y^2 = a^2$ and the chord of contact, is -

(A)
$$
\frac{1}{2} a \left(\frac{h^2 + k^2 - a^2}{\sqrt{h^2 + k^2}} \right)
$$
 (B) $\frac{a(h^2 + k^2 - a^2)^{3/2}}{2(h^2 + k^2)}$ (C) $\frac{a(h^2 + k^2 - a^2)^{3/2}}{(h^2 + k^2)}$ (D) None of these

- **Q.93** The chord of the circle $x^2 + y^2 4x = 0$ which is bisected $Q.104$ at $(1, 0)$ is perpendicular to the line – (A) $y = x$ (B) $x + y = 0$ $(C) x = 1$ (D) $y = 1$
- **Q.94** Two circles centered at (2, 3) and (5, 6) intersect each other. If the radii are equal, the equation of the common chord is– (A) $x+y+1=0$ (B) $x-y+1=0$

(C)
$$
x+y-8=0
$$

\nD) $x-y-8=0$
\nD) $x-y-8=0$
\nD) $x-y-8=0$

- **Q.95** If $2x^2 + 2y^2 + 4x + 5y + 1 = 0$ and $3x^2 + 3y^2 + 6x 7y + 3k = 0$ are orthogonal, then value of k is $(A) -17/12$ (B) $-12/17$ (C) 12/17 (D) 17/12
- **Q.96** The center of a circle which cuts $x^{2} + y^{2} + 6x - 1 = 0$, $x^{2} + y^{2} - 3y + 2 = 0$ and $x^2 + y^2 + x + y - 3 = 0$ orthogonally is (A) (-1/7, 9/7) (B) (1/7, -9/7) $(C) (-1/7, -9/7)$ (D) $(1/7, 9/7)$
- **Q.97** The number of real circles cutting orthogonally the circle $x^2 + y^2 + 2x - 2y + 7 = 0$ is - $(A) 0$ (B) 1

$$
(C) 2 \t\t (D) infinitely many
$$

Q.98 The length of the chord of the circle $x^2 + y^2 + 3x + 2y - 8 = 0$ intercepted by the y-axis is $(A) 3$ (B) 8

PART 4 : RADICAL AXIS, RADICAL CENTRE, FAMILY OF CIRCLES

- **Q.99** If the point $(2, 0)$, $(0, 1)$, $(4, 5)$ and $(0, c)$ are con-cyclic, then c is equal to $(A) -1, -3/14$ (B) –1, –14/3
	- (C) 14/3, 1 (D) None of these
- **Q.100** If the line $y = x + 3$ meets the circle $x^2 + y^2 = a^2$ at A and B, then the equation of the circle having AB as a diameter will
	- $(A) x² + y² + 3x 3y a² + 9 = 0$
	- $(B) x² + y² + 3x + 3y a² + 9 = 0$
	- $(C) x^2 + y^2 3x + 3y a^2 + 9 = 0$
	- (D) None of these
- **Q.101** The equation of the circle passing through the point of intersection of the circles $x^2 + y^2 = 6$ and $x^{2} + y^{2} - 6x + 8 = 0$, and also through the point (1, 1) is -
	- $-4y+2=0$ (B) $x^2+y^2-3x+1=0$ $(C) x^2 + y^2 - 6x + 4$ (D) None of these
- $\frac{a(h^2 + k^2 a^2)^{3/2}}{(C)x^2 + y^2 13x 8y + 3 = 0}$
 $\frac{(A)}{D}x^2 + y^2 13x + 8y + 3 = 0$ $(x^2 + k^2 - a^2)^{3/2}$ (A) $x^2 + y^2 + 8x + 13y - 3 = 0$ (B) $x^2 + y^2 + 13x - 8y + 3 = 0$ **Q.102** The equation of the circle which passes through points of intersection of circle $x^2 + y^2 + 4x - 5y + 3 = 0$ and $x^{2} + y^{2} + 2x + 3y - 3 = 0$ and point (-3, 2) is equation of the circle passing through the point

section of the circles $x^2 + y^2 = 6$ and
 $y^2 - 6x + 8 = 0$, and also through the point $(1, 1)$ is-
 $x^2 + y^2 - 4y + 2 = 0$ (B) $x^2 + y^2 - 3x + 1 = 0$
 $(2 + y^2 - 6x + 4 = 0$ (D) None section of the circles $x^2 + y^2 - 8$ and $x^2 + y^2 - 8x + 1 = 0$
 $y^2 - 6x + 8 = 0$, and also through the point $(1, 1)$ is-
 $x^2 + y^2 - 4y + 2 = 0$ (B) $x^2 + y^2 - 3x + 1 = 0$
 $(2^2 + y^2 - 6x + 4 = 0$ (D) None of these

equation of the c g through the point of
 $= 6$ and

ugh the point (1, 1) is -
 $x^2 + y^2 - 3x + 1 = 0$

None of these

passes through points
 $4x - 5y + 3 = 0$ and
 $t(-3, 2)$ is -
 $x^2 + y^2 + 13x - 8y + 3 = 0$
 $x^2 + y^2 - 13x + 8y + 3 = 0$

ree circles
 = o and
ugh the point (1, 1) is -
x² + y² - 3x + 1 = 0
None of these
passes through points
 $4x - 5y + 3 = 0$ and
 $t(-3, 2)$ is -
x² + y² + 13x - 8y + 3= 0
x² + y² - 13x + 8y + 3= 0
ree circles
 $1x^2 + (y - b)^2 = a^2$ is $x^2 + y^2 - 4y + 2 = 0$ (B) None of these

equation of the circle which passes through points
 $x^2 + y^2 - 6x + 4 = 0$ (D) None of these

equation of the circle which passes through points

thersection of circle $x^2 + y^2 + 4x - 5y +$ $x^2 + y^2 - 6x + 4 = 0$ (D) None of these

equation of the circle which passes through points

equation of the circle which passes through points
 $x^2 + y^2 + 2x + 3y - 3 = 0$ and point $(-3, 2)$ is-
 $x^2 + y^2 + 8x + 13y - 3 = 0$ (B) $x^2 + y^2 - 3x + 1 = 0$

None of these

passes through points
 $4x - 5y + 3 = 0$ and
 $(-3, 2)$ is $-$
 $x^2 + y^2 + 13x - 8y + 3 = 0$
 $x^2 + y^2 - 13x + 8y + 3 = 0$

ree circles
 $x^2 + (y - b)^2 = a^2$ is $-b/2$, $c/2$

None of these
 f two cir None of these

passes through points
 $4x - 5y + 3 = 0$ and
 $(-3, 2)$ is -
 $x^2 + y^2 + 13x - 8y + 3 = 0$
 $x^2 + y^2 - 13x + 8y + 3 = 0$

ree circles
 $x^2 + (y - b)^2 = a^2$ is -
 $(b/2, c/2)$

None of these
 f two circles
 $+ y - 1 = 0$ is -

- (B) $\frac{1}{2(h^2 + k^2)}$ (C) $x^2 + y^2 13x 8y + 3 = 0$ (D) $x^2 + y^2 13x + 8y + 3 = 0$
Q.103 The radical centre of the of the three circles $x^{2} + y^{2} = a^{2}, (x - c)^{2} + y^{2} = a^{2}$ and $x^{2} + (y - b)^{2} = a^{2}$ is - (A) (a/2, b/2) (B) (b/2, c/2) (C) (c/2, b/2) (D) None of these e of the of the three circles

c)² + y² = a² and x² + (y - b)² = a² is -

(B) (b/2, c/2)

(D) None of these

he radical axis of two circles

be netacled axis of two circles

(B) 3x - y² + y⁻¹ = 0 is -

(B) + 13y - 3 = 0 (B) x² + y² + 13x - 8y + 3=0

x - 8y + 3 = 0 (D) x² + y² - 13x + 8y + 3=0

tre of the of the three circles

core of the of the tree circles

(B) (b/2, c/2)

(D) None of these

fiberadical axis of two
	- **Q.104** The equation of the radical axis of two circles $x^{2} + y^{2} - x + 1 = 0$ and $3(x^{2} + y^{2}) + y - 1 = 0$ is -(A) $3x + y - 4 = 0$ (B) $3x - y - 4 = 0$ (C) $3x-y+4=0$ (D) None of these
	- **Q.105** Find the coordinate of the point from which the tangents to the circles $x^2 + y^2 - 8x + 40 = 0$; $5x^2 + 5y^2 - 25x + 80 = 0$ $x^{2} + y^{2} - 8x + 16y + 160 = 0$ are equal in length.

$$
x^{2} + y^{2} = a^{2}, (x - c)^{2} + y^{2} = a^{2} \text{ and } x^{2} + (y - b)^{2} = a^{2} \text{ is}
$$

\n(A) (a/2, b/2) (B) (b/2, c/2)
\n(C) (c/2, b/2) (D) None of these
\nThe equation of the radical axis of two circles
\n $x^{2} + y^{2} - x + 1 = 0$ and $3(x^{2} + y^{2}) + y - 1 = 0$ is-
\n(A) $3x + y - 4 = 0$ (B) $3x - y - 4 = 0$
\n(C) $3x - y + 4 = 0$ (D) None of these
\nFind the coordinate of the point from which the tangents
\nto the circles $x^{2} + y^{2} - 8x + 40 = 0$; $5x^{2} + 5y^{2} - 25x + 80 = 0$
\n $x^{2} + y^{2} - 8x + 16y + 160 = 0$ are equal in length.
\n(A) $\left(4, -\frac{15}{8}\right)$ (B) $\left(8, -\frac{15}{4}\right)$
\n(C) $\left(8, -\frac{15}{8}\right)$ (D) $\left(2, -\frac{15}{8}\right)$
\nThe equation of circle which passes through the point of intersection of circles $x^{2} + y^{2} - 6x = 0$ and $x^{2} + y^{2} - 6y = 0$
\nand has centre $\left(\frac{3}{2}, \frac{3}{2}\right)$ is –
\n(A) $x^{2} + y^{2} - 3x - 3y = 0$
\n(B) $x^{2} + y^{2} - 3x - 3y + 9 = 0$
\n(C) $x^{2} + y^{2} - 3x - 3y + 9 = 0$
\n(D) $x^{2} + y^{2} - 3x - 3y + 5 = 0$
\n(D) $x^{2} + y^{2} - 3x - 3y + 5 = 0$

Q.106 The equation of circle which passes through the point of intersection of circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 6y = 0$

and has centre $\left(\frac{3}{2}, \frac{3}{2}\right)$ is - $(A) x^2 + y^2 - 3x - 3y = 0$ $(B) x^2 + y^2 - 3x - 3y + 9 = 0$ $(C) x^2 + y^2 - 3x - 3y - 9 = 0$ $(D) x^2 + y^2 - 3x - 3y + 5 = 0$

PART 5 : MISCELLANEOUS

Q.107 Equation of a circle $S(x, y) = 0$, $(S(2, 3) = 16)$ which touches the line $3x + 4y - 7 = 0$ at $(1, 1)$ is given by

(A)
$$
x^2 + y^2 + x + 2y - 5 = 0
$$
 (B) $x^2 + y^2 + 2x + 2y - 6 = 0$
(C) $x^2 + y^2 + 4x - 6y = 0$ (D) none of these Q.115

Q.108 A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is – **Q.109** The pole of the line $3x^2 + y^2 - 2ax = 0$

(A) The pole of the line $x^2 + y^2 + x + 2y - 5 = 0$ (B) $x^2 + y^2 + 2x + 2y - 6 = 0$

(A) $x^2 + y^2 + x + 2y - 5 = 0$ (B) $x^2 + y^2 + 2x + 2y - 6 = 0$

(C) $x^2 + y^2 + 4x - 6y = 0$ (D) none of these

(A)
$$
x^2 + y^2 + ax = 0
$$

\n(B) $x^2 + y^2 + ay = 0$
\n(C) $x^2 + y^2 - ax = 0$
\n(D) $x^2 + y^2 - ay = 0$

 $x^2 + y^2 = c^2$ is –

Answer 9.114 If a chord of the circle $x^2 + y^2$	STUDY M.																																																																							
2 2	10	114	15	20	114	16	24	17																																																																
2 3	24	36	47	48	49	50	60	10	11	12	13																																																													
2 4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	

- **Q.110** If P(2, 8) is an interior point of a circle
	- $x^2 + y^2 2x + 4y p = 0$ which neither touches nor intersects the axes, then set for p is -
	- (A) $p < -1$ (B) $P < -4$ $(C) p > 96$ (D) ϕ
- **Q.111** The number of common tangents that can be drawn to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ and

 $x^2 + y^2 + 2x + 2y + 1 = 0$ is $(A) 1$ (B) 2 (C) 3 (D) 4

Q.112 The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is –

Q.113 The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through a fixed point -

Q.114 If a chord of the circle $x^2 + y^2 = 8$ makes equal to intercepts of length a on the coordinate axes, then

(A)
$$
|a| < 8
$$

(B) $|a| < 4\sqrt{2}$
(C) $|a| < 4$
(D) $|a| > 4$

- **Q.115** The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$ is -
	- $(A) 0$ (B) 1 $(C)-1$ (D) depend on h
- **SCELLANEOUS COUBSTION BANK STUDY MATERIAL:N**
 SCELLANEOUS Q.114 If a chord of the circle $x^2 + y^2 = 8$ makes exp., then
 x, y = 0, (S(2, 3) = 16) which touches
 $5 = 0$ (B) $x^2 + y^2 + 2x + 2y 6 = 0$

(D) none of the **CELLANEOUS 0.114** If a chord of the circle $x^2 + y^2 = 8$ makes equ
 $x = 10$, $y = 0$, $(xz, 3) = 16$) which touches
 $x = 10$, $y = 0$, y **Q.116** Two concentric circles are such that the smaller divides the larger into two regions of equal area. If the radius of the smaller circle is 2, then the length of the tangent from any point P on the larger circle to the smaller circle is –

(A) 1
(B)
$$
\sqrt{2}
$$

(D) None of these

Q.117 The pair of a straight lines joining the origin to the points of inersection of the circles $x^2 + y^2 = a^2$ and

$$
x^{2}+y^{2}+2 (gx+fy)=0
$$
 is
(A) $a^{2}(x^{2}+y^{2})-2(gx+fy)^{2}=0$
(B) $a^{2}(x^{2}+y^{2})-4(gx+fy)^{2}=0$
(C) $a^{2}(x^{2}+y^{2})+4(gx+fy)^{2}=4$
(D) $x^{2}+y^{2}-(gx+fy)^{2}=a^{2}$

Q.118 A circle is drawn touching the x-axis and centre at the point which is the reflection of (a, b) in the line $y - x = 0$. The equation of the circle is-

(A)
$$
x^2 + y^2 - 2bx - 2ay + a^2 = 0
$$

\n(B) $x^2 + y^2 - 2bx - 2ay + b^2 = 0$
\n(C) $x^2 + y^2 - 2ax - 2by + b^2 = 0$
\n(D) $x^2 + y^2 - 2ax - 2by + a^2 = 0$

Q.119 If the straight line $\frac{2x}{a} + \frac{y}{b} = 2\sqrt{2}$ touches the circle $\frac{2x}{a} + \frac{y}{b} = 2\sqrt{2}$ touches the circle

$$
x2 + y2 = 2ab, a, b > 0, then(A) a = b(B) 2a = b(C) a = 2b(D) None of these
$$

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 The common tangents of two circles intersecting orthogonally are perpendicular. If the ratio of their radii

is p then
$$
p + \frac{1}{p} =
$$

(A) 3 (B) 4
(C) 5 (D) 6

- **Q.2** Equation of chord AB of circle $x^2 + y^2 = 2$ passing through $P(2, 2)$ such that PB/PA = 3, is given by- $(A) x = 3y$ $(B) x = y$ (C) $y - 2 = \sqrt{3}$ (x-2) (D) none of these
- **Q.3** The equation of the circle through the point of intersection of $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and touching the line $x + 2y = 0$, is (A) $x^2 + y^2 + x + 2y = 0$ (B) $x^2 + y^2 - y^2 = 0$ $+y^2 - x + 20 = 0$ (C) $x^2 + y^2 - x - 2y = 0$ (D) $2(x^2 + y^2) +y^2$) – x – 2y = 0
- **Q.4** Two circles with radii 'r₁' and 'r₂', r₁ > r₂ ≥ 2, touch each Q.12 other externally. If θ be the angle between the direct common tangents, then

YONE OPTON IS CORREC1
\nThe common tangents of two circles intersecting
\northogonally are perpendicular. If the ratio of their radii
\nis p then
$$
p + \frac{1}{p}
$$

\n(a) 3
\n(b) 4
\n(c) 5
\nEquation of chord AB of circle $x^2 + y^2 = 2$ passing through
\n(a) $x = 3y$
\n(b) 6
\n(c) 5
\n**Equation of chord AB of circle** $x^2 + y^2 = 2$ passing through
\n(a) $x = 3y$
\n(b) $x = y$
\n(c) $y - 2 = \sqrt{3}(x - 2)$
\n(d) $x = 3y$
\n(e) $y - 2 = \sqrt{3}(x - 2)$
\n(f) $y = 2$
\n(g) $x = y$
\n(h) $x = 3y$
\n(i) $y = 2$
\n(j) $y = 2$
\n(k) $x = 3$
\n(l) $y = 2$
\n(m) $y = 2$
\n(e) $y = 2$
\n(f) $y = 2$
\n(g) $y = 2$
\n(h) $y = 2$
\n(i) $y = 2$
\n(j) $y = 2$
\n(k) $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and
\nwhich passes through the point of
\nthe circle through the point of
\naxis through P and touch bo
\nand $x = 2$
\n(c) $(-\infty, -1) \cup (1, \infty) - \{-2, 2\}$
\n(d) $a^2 - 6ab + b^2 = 0$
\n(e) $a^2 - 4ab + b^2 = 0$
\n(f) $a^2 - 6ab + b^2 = 0$
\n(g) $a^2 + y^2 - x - 2y = 0$
\n(h) $a^2 - 6ab + b^2 = 0$
\n(i) $a^2 - 6ab + b^2 = 0$
\n(j) $a^2 - 6ab + b^2 = 0$
\n(k) $a^2 - 6ab +$

Q.5 If the curves $ax^2 + 4xy + 2y^2 + x + y + 5 = 0$ and $ax^2 + 6xy + 5y^2 + 2x + 3y + 8 = 0$ intersect at four concyclic points then the value of a is (A) 4 (B) – 4

 $(D) - 6$

Q.6 Area of triangle formed by common tangents to the circle $x^{2} + y^{2} - 6x = 0$ and $x^{2} + y^{2} + 2x = 0$ is -

$$
(C) 9\sqrt{3} \tag{D}
$$

Q.7 If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 6 units, then minimum distance of T from the director circle of the given circle is –

$$
(A) 6 \t\t (B) 12
$$

(C)
$$
6\sqrt{2}
$$
 (D) $12-4\sqrt{2}$

- **Q.8** A chord AB drawn from the point A (0, 3) at circle $x^2 + 4x + (y-3)^2 = 0$ and it meets to M in such a way that $AM = 2AB$, then the locus of point M will be (A) Straight line (B) Circle (C) Parabola (D) None of these
- **Q.9** A square is inscribed in the circle $x^2 + y^2 2x + 4y + 3 = 0$, whose sides are parallel to coordinate axes. One vertex of the square is –

(A)
$$
(1+\sqrt{2},-2)
$$

\n(B) $(1-\sqrt{2},-2)$
\n(C) $(-2, 1)$
\n(D) $(2,-3)$

(A) $(1+\sqrt{2},-2)$

(B) $(1-\sqrt{2},-2)$

(C) $(-2, 1)$

(C) $(-2, 1)$

(B) $(1-\sqrt{2},-2)$

(C) $(-2, 1)$

(D) $(2, -3)$

Set of values of m for which two points P and Q lie on the

line $y = mx + 8$ so that $\angle APB = \angle AQB = \pi/2$ where
 $A = (-4$ **Q.10** Set of values of m for which two points P and Q lie on the line y = mx + 8 so that \angle APB = \angle AQB = $\pi/2$ where $A \equiv (-4, 0), B \equiv (4, 0)$ is –

WEL-2]
\n(A)
$$
(1+\sqrt{2}, -2)
$$
 (B) $(1-\sqrt{2}, -2)$
\n(C) $(-2, 1)$ (D) $(2, -3)$
\nSet of values of m for which two points P and Q lie on the
\nline y = mx + 8 so that $\angle APB = \angle AQB = \pi/2$ where
\n $A = (-4, 0), B = (4, 0)$ is –
\n(A) $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) - \{-2, 2\}$
\n(B) $[-\sqrt{3}, -\sqrt{3}] - \{-2, 2\}$
\n(C) $(-\infty, -1) \cup (1, \infty) - \{-2, 2\}$
\n(D) $\{-\sqrt{3}, \sqrt{3}\}$
\nP is a point (a, b) in the first quadrant. If the two circles
\nwhich pass through P and touch both the co-ordinate
\naxes cut at right angles, then –
\n(A) $a^2 - 6ab + b^2 = 0$ (B) $a^2 + 2ab - b^2 = 0$
\n(C) $a^2 - 4ab + b^2 = 0$ (D) $a^2 - 8ab + b^2 = 0$
\nThe radical centre of three circles described on the three

Q.11 P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the co-ordinate axes cut at right angles, then –

(A)
$$
a^2 - 6ab + b^2 = 0
$$

\n(B) $a^2 + 2ab - b^2 = 0$
\n(C) $a^2 - 4ab + b^2 = 0$
\n(D) $a^2 - 8ab + b^2 = 0$

Q.12 The radical centre of three circles described on the three sides of a triangle as diameter is (A) the centroid (B) the circumcenter

$$
(C)
$$
 the incentive of the triangle (D) the orthocenter

(B) 6

(B) $(x + 3)$, $y = 2$ passing through

(B) $[-\sqrt{3}, -\sqrt{3}] - (-2, 2)$

(B) $x = y$

(B) $[-\sqrt{3}, -\sqrt{3}] - (-2, 2)$

(B) $x = y$

(B) $[-\sqrt{3}, -\sqrt{3}] - (-2, 2)$

(B) $x = y$

(B) $\sqrt{3}, \sqrt{3}$

(B) $\sqrt{3}, \sqrt{3}$

(B) $\sqrt{3}, \sqrt{3}$

(B) $x^2 +$ ig through (B) $[-\sqrt{3}, -\sqrt{3}] - \{-2, 2\}$

(C) $(-\infty, -1) \cup (1, \infty) - \{-2, 2\}$

e

(D) $\{-\sqrt{3}, \sqrt{3}\}$

point of **Q.11** P is a point (a, b) in the first quadrant. If the $-1 = 0$ and which pass through P and touch both the c

ax ersecting

(C)(-2, 1)

their radii Q.10 Set of values of m for which two points P and Q lie on the

line y = mx + 8 so that \angle APB = \angle AQB = $\pi/2$ where

A = (-4, 0), B = (4, 0) is -

(A) (-∞, -√3) \cup ($\sqrt{3}$, ∞) and **Q.10** Set of values of m for which two points P and Q lie on the

line $y = mx + 8$ so that $\angle APB = \angle AQB = \pi/2$ where
 $A = (-4, 0), B = (4, 0)$ is $-\sqrt{3}, \sqrt{3}$ $-\sqrt{2}$

(A) $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) - \{-2, 2\}$

(B) $[-\sqrt{3}, -\sqrt{3}] - \{-2$ ntersecting (A) $(1+\sqrt{2},-2)$ (B) $(1-\sqrt{2},-2)$

for their radii

(C)(-2, 1)

(D) Set of values of m for which two points P and Q lie on the

line $y = mx + 8$ so that $\angle APB = \angle AQB = \pi/2$ where
 $A = (-4, 0)$, $B = (4, 0)$ is-

(A) $(-\$ ntersecting

(b)(-4)

(c)(-4)

(c)(-4)

(fibeir radii

(c)(-4)

(c)(-4)

(d) Set of values of m for which two points P and Q lie on the

line y = mx + 8 so that $\angle APB = \angle AQB = \pi/2$ where
 $A = (-4, 0), B = (4, 0)$ is \Rightarrow $(A)(\bar{3}, \$ **Q.13** Minimum radius of circle which is orthogonal with both the circles $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is- (A) 4 (B) 3 (C) $\sqrt{15}$ (D) 1

Q.14 If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in A.P., then (A) a, b, c are in G.P. (B) a, b, c are in AP

(C)a², b², c² are in AP
\n**Q.15** If
$$
r_1
$$
 and r_2 are the radii of smallest and largest circles
\nwhich passes through (5, 6) and touches the circle
\n $(x-2)^2 + y^2 = 4$, then r_1r_2 is

(A) 4 / 41 (B) 41 / 4 (C) 5 / 41 (D) 41 / 6

GOUINg the line $x + 2x - 2y = 0$.

(A) $x^2 + y^2 - x + 2y = 0$ (B) $x^2 + y^2 - x + 20 = 0$ (A) $a^2 + 6ab + b^2 = 0$ (B) $a^2 + 2ab - b^2 = 0$

(C) $x^2 + y^2 - x - 2y = 0$ (D) $2(x^2 + y^2) - x - 2y = 0$

(C) $y^2 - 3y = 0$ (D) $z^2 + 3y = 0$

(D) $z^2 + y^2 - 1$ (C) $y^2 + y^2 - x - 2y = 0$ (D) $2(x^2 + y^2) - x - 2y = 0$ (D) $x^2 - 3ab + b^2 - 0$

The radical energy of the radical energy of the energy of the contents of the term of the energy of the term of the energy of the energy of the energy (C) $0 = \sin^{-1} \left(\frac{r_1 - r_2}{r_1 + r_2} \right)$

If the curves $ax^2 + 4xy + 2y^2 + x + y + 5 = 0$ and
 $ax^2 + 6xy + 5y^2 + 2x + 3y + 8 = 0$ interesectation concyclic

(A) Ali Fithe squares of the lengths of the tangents from a point

points then **Q.16** From a point R (5, 8) two tangents RP and RQ are drawn to a given circle $S = 0$ whose radius is 5. If circumference of the triangle PQR is (2, 3), then the equation of circle $S = 0$ is- $(A) x^2 + y^2 + 2x + 4y - 20 = 0$ $(B) x^2 + y^2 + x + 2y - 10 = 0$ (D) 1

the lengths of the tangents from a point
 $z^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$

GP. (B) a, b, c are in AP

in AP (D) a^2 , b^2 , c^2 are in GP

the radii of smallest and largest circles

rough (5, 6) (C) a , b , c act mand
 H if η , and r_2 are the radii of smallest and largest circles

which passes through (5, 6) and touches the circle

(x-2)² + y² = 4, then r_1r_2 is -

(A) 4/4

(C) 5/41 (B) 91/6

From a which passes through (5, 6) and touches the circle
 $(x-2)^2 + y^2 = 4$, then r_1r_2 is $-$
 $(x-2)^2 + y^2 = 4$, then r_1r_2 is $-$
 (x) 4/4/4
 (C) 5/41 (D) 41/6

From a point R (5, 8) two tangents RP and RQ are drawn

to a

(B)
$$
x^2 + y^2 + x + 2y - 10 = 0
$$

\n(C) $x^2 + y^2 - x - 2y - 20 = 0$
\n(D) $x^2 + y^2 - 4x - 6y - 12 = 0$

Q.17 If C₁: $x^2 + y^2 = (3 + 2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents on C_1 , where P is any point on the director circle of C_1 , then the radius of smallest circle which touch C_1 externally and also the two tangents PA and PB is –

(A)
$$
2\sqrt{2}-3
$$

\n(B) $2\sqrt{2}-1$
\n(C) $2\sqrt{2}+1$
\n(D) 1

Q.18 A (1, 0) and B (0, 1) and two fixed points on the circle $x^{2} + y^{2} = 1$. C is a variable point on this circle. As C x^{2} moves, the locus of the orthocentre of the triangle ABC $is -$

(A)
$$
x^2 + y^2 - 2x - 2y + 1 = 0
$$
 (B) $x^2 + y^2 - x - y = 0$
(C) $x^2 + y^2 = 4$ (D) $x^2 + y^2 + 2x - 2y + 1 = 0$ (E)

Q.19 The locus of the middle points of the chords of the circle $x^2 + y^2 = 4a^2$, which subtends a right angle at the centre of the circle is

> $(A) x + y = 2a$ $+ y^2 = 2a^2$ $(C) x^2 + y^2 = a^2$ (D) $x^2 + y^2 = a^2 \sqrt{2}$

Q.20 Circum circle of the quadilateral ABCD ,where $AB = x + y - 10$, $BC = x - 7y + 50 = 0$, $CD = 22x - 4y + 125 = 0$, $DA = 2x - 4y - 5 = 0$, is

EXERCISE SERVE AND CONSTRON BANK

AT A (1, 0) and B (0, 1) and two fixed points on the circle **0.26** Radius (R < 4) of a circle whis

moves, the locus of the orthocentre of the triangle ABC common tangents is tan⁻¹ (2 **Q.21** Suppose $f(x) = x^2 - 3x + 1$. If c_1 and c_2 are the two values **Q.2** of \hat{c} for which the tangent line to the graph of $f(x)$ at the point [c, f(x)] intersects at the point $(-3, 0)$ then $(c_1 + c_2)$ equals – irele of the quadilateral ABCD, where
 $y-10$, BC = $x-7y+50=0$,
 $y^2 = 125$ (B) $2x^2 + 2y^2 = 125$
 $y^2 = 225$ (B) $2x^2 + 2y^2 = 125$

(B) $2x^2 + 2y^2 = 125$

(D) none of these

(K)(-16/5,53/10)

which the tangent line to t y 2a

y 2a

y 2a a

y 2a a

circle of the quadilateral ABCD, where

eircle of the quadilateral ABCD, where

the concurrent. Find the point of

the family and the circle

eircle of the quadilateral ABCD, where

y -2a 25 ($y^2 = a^2$ (D) $x^2 + y^2 = a^2$ (D) $x^2 + y^2 = a^2$ mentro of nearbitrich of the quadilateral ABCD, where
 $y - y = 10$, $BC = x - 7y + 50 = 0$,
 $x - 4y + 125 = 0$, $DA = 2x - 4y - 5 = 0$, is
 $y^2 = 125$ (B) $2x^2 + 2y^2 = 125$
 $x^2 + y^2 = 225$ (

$$
(A) 6 \t\t (B) -6
$$

$$
2\sqrt{19} \qquad \qquad (D) \ -2\sqrt{10}
$$

Q.22 The equation of a tangent from the origin to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ is

(A)
$$
y = 0
$$
 (B) $y = \left(\frac{b^2 - a^2}{2ab}\right)x$

(C)
$$
y = \left(\frac{a^2 - b^2}{2ab}\right) x
$$
 (D) $y = \left(\frac{b^2 - a^2}{ab}\right) x$

- **Q.23** If the line $3x 4y k = 0$ touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ at (a, b), then $k + a + b$ is equal to $(A) 20$ or -28 (B) 22 or -26 (C) –30 or 24 (D) 28 or –20 (A) C

(B)-6

(B)-6

(B)-2 $\sqrt{19}$

Co.30 In a circle with cherch that bisects

The equation of a tangent from the origin to the circle

(P)-2 $\sqrt{10}$

(B) y= $\left(\frac{b^2-a^2}{2ab}\right)$ x

(B) y= $\left(\frac{b^2-a^2}{2ab}\right)$ x

(C) 1003
 If the line 3x-4y-k=0 touches the circle
 $x^2 + y^2 - 4x - 8y - 5 = 0$ at (a, b), then k + a + b is equal to
 $(A) 200r - 28$
 $(A) 200r - 28$
 $(A) 200r - 28$
 $(B) 220r - 26$
 $(C) - 300r 24$
 $(D) 280r - 20$

for all possible real valu
- **Q.24** Two circles of equal radii are inscribed within a regular hexagon, as shown in figure. The sides of the hexagon are of length ℓ , and the circles are tangent at T. The common radius of these circles can be expressed as

is equal to –

Q.25 If (α, β) is a point on the circle whose centre is on the yaxis and which touches $x + y = 0$ at (-2, 2), then the greatest value of β is

(A)
$$
4 - \sqrt{2}
$$

\n(B) 6
\n(C) $4 + 2\sqrt{2}$
\n(D) $4 +$

Q.26 Radius $(R < 4)$ of a circle which touches the circle $x^2 + y^2 = 16$ externally and angle between the direct common tangents is $\tan^{-1}(24/7)$ is –

(A) 3 (B) 2 (C) 1/2 (D) 1

 $+ 2x - 2y + 1 = 0$ Q.27 Consider a family of circles passing through the intersection point of the lines $\sqrt{3} (y-1) = x - 1 \&$

 $= a^2 \sqrt{2}$ member of the family and the circle $x^2 + y^2 + 4x - 6y + 5 = 0$ **EXERCISION BANK COUSSTION BANK CONTRACTION DEVICE STATE STATE STATE STATE STATE STATE STATES (A) A 2** $x^2 + y^2 = 1$ **. C is a variable point on this circle. As C
** $x^2 + y^2 = -1$ **. C is a variable point on this circle. As C
 (x^** $y-1=\sqrt{3}$ (x-1) and having its centre on the acute angle bisector of the given lines. The common chords of each are concurrent. Find the point of concurrency. (A) (1/2, 3/2) (B) (1, 2) $(C)(2, 3)$ (D) (1, 1)

- **Q.28** The centre of the circle passing through the point $(0, 1)$ and touching the curve $y = x^2at (2, 4)$ is: (A) (–16/5, 27/10) (B) (–16/7, 53/10) (C) (-16/5, 53/10) (D) none of these
- **Q.29** Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point at a distance of 4 from the point $(8, 6)$ on the circle, is – $(A) 4x + 3y - 50 = 0$ (B) $4x + 3y - 100 = 0$

$$
(A) 4x + 3y - 30 = 0
$$

(B) 4x + 3y - 100 = 0
(D) None of these

(A) $x^2 + y^2 = 2x - 2y + 1 = 0$ (B) $x^2 + y^2 = x - y = 0$

(C) $x^2 + y^2 = 2x$ (F) $x^2 + y^2 = 2x - 2y + 1 = 0$ (D) in a circle passing through

The loous of the middle points of the chords of the circle intersection point of the lines $y-1 = \sqrt{3} (x-1)$ and having its centre on the acute angle
 $y^2 = 2a^2$
 $y^2 = a^2\sqrt{2}$

member of the given lines. The common chords of each

member of the fainily and the circle $x^2 + y^2 + 4x - 6y + 5 = 0$

where
 $(2)(2,3)$
 chords of the circle

gale at the centre of
 $y^2 = 2a^2$

intersection point of the lines $\sqrt{3} (y-1) = x - 1$ &
 $y^2 = 2a^2$

insector of the given lines. The common chords of each

methor of the given lines. The common cho ggle at the centre of
 $y^2 = 2a^2$
 $y^2 = 2a^2$

bisector of the given lines. The common chords of each
 $y^2 = 2a^2$

bisector of the given lines. The common chords of each

mether of the given lines. The common chords of where

are concurrent. Find the point of concurrency.
 $-5 = 0$, is
 (2) , $2y^2 = 125$
 (2) , (2) , (3)
 $-2y^2 = 125$

and douching the curve $y = x^2$ at $(2, 4)$ is:
 (3) (d) $-(16/5, 53/10)$

of these
 (2) and dou y $2 = a^2\sqrt{2}$ member of the family and the circle $x^2 + y^2 + 4x - 6y + 5=0$

D. where
 $y^2 = a^2\sqrt{2}$ member of the family and the circle $x^2 + y^2 + 4x - 6y + 5=0$

(A)(1/2, 3/2) (B)(1, 1)
 $+2y^2 = 125$ and touching the curve $y^2 = a^2 \sqrt{2}$

are concurrent. Find the point of concurrency.

(A) (1/2, 3/2)

(B) (1, 2)
 $(-5 = 0, \text{ is }$

(A) (1/2, 3/2)

(D) (4, 1)

(D) (4, 1)

(C) and touching the curve $y = x^2at$ (2, 4) is:

e of these

(A) (-16.5, 53 **Q.30** In a circle with centre 'O' PA and PB are two chords. PC is the chord that bisects the angle APB. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R respectively. If QC = 3, QA = 2 and RC = 4, then length of RB equals – (a, o) on ine clicine, is

(A) $4x + 3y - 50 = 0$ (B) $4x + 3y - 100 = 0$

(C) $4x + 3y - 46 = 0$ (D) None of these
 Q.30 In a circle with centre 'O'PA and PB are two chords. PC is

the chord that bisects the angle APB. The tang le passing through the point (0, 1)

e y = x²at (2, 4) is:

(B) (-16/7, 53/10)

(D) none of these

ine meeting the circle $x^2 + y^2 = 100$

int at a distance of 4 from the point

(B) $4x + 3y - 100 = 0$

(D) None of these

O nce of 4 from the point
 $4x + 3y - 100 = 0$

None of these

B are two chords. PC is

PB. The tangent to the

d PB extended at Q and

and RC = 4, then length
 $8/3$
 $3/3$
 $11/3$
 $yx^2 + y^2 - 20x + 64 = 0$

mgth of the shortest None of these
B are two chords. PC is
PB. The tangent to the
1PB extended at Q and
nd RC = 4, then length
3/3
3/3
3/3
3/3
11/3
y x² + y² - 20x +64 = 0
ngth of the shortest line
at P and to C₂ at Q is -
8
4
4
+ ($\sqrt{$

$$
\left(\begin{array}{c}\n\frac{a}{2ab}\n\end{array}\right) x\n\tag{B) 8/3}\n\tag{C) 10/3}\n\tag{D) 11/3
$$

- and $x^2 + y^2 + 30x + 144 = 0$. The length of the shortest line ab \int_0^{∞} segment PQ that is tangent to C₁ at P and to C₂ at Q is – **Q.31** Let C₁ and C₂ are circles defined by $x^2 + y^2 - 20x + 64 = 0$ (A) 15 (B) 18
	- (C) 20 (D) 24
 Q.32 The minimum value of $(x_1 x_2)^2 + (\sqrt{1 x_1^2} (3 x_2))^2$ for all possible real values of x_1 and x_2 is –

(A)
$$
\frac{3}{\sqrt{2}} - 1
$$
 \t\t (B) $\frac{11}{2} - 3\sqrt{2}$

(C)
$$
\frac{3}{\sqrt{2}}
$$
 (D) $\frac{11}{2} + 3\sqrt{2}$

Q.33 In the diagram, DC is a diameter of the large circle centred at A, and AC is a diameter of the smaller circle centred at B. If DE is tangent to the smaller circle at F and $DC = 12$ then the length DE is –

 $Q.43$

Q.44

Q.34 If θ is the angle between the two radius (one to each circle) drawn from one of the point of intersection of two Q.41 circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$, then the length of common chord of two circles is

CE	QUESTION BANK	GE
If θ is the angle between the two radius (one to each circle) drawn from one of the point of intersection of two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$, then the length of common chord of two circles is	Q.41 Statement-1: Number of circles pass circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$, then the length of common chord of two circles is	Statement-2: One and only circle can through three non-collinear points.
(A) $\frac{ab \sin \theta}{\sqrt{a^2 + b^2 - 2ab \cos \theta}}$ (B) $\frac{2ab \sin \theta}{\sqrt{a^2 + b^2 - 2ab \cos \theta}}$ (C) $\frac{2ab}{\sqrt{a^2 + b^2 - 2ab \cos \theta}}$ (D) none of these d) both lying in the first quadrant coordinate axes. In each of the condition of the condition		
A circle is tangent to the y-axis at (0, 2) and cuts the positive x-axis at two distinct points A and B (OB > OA), the coordinate of the point B being (8, 0). The radius of the circle is	I, the ratio of b/a is given in column II. Column I	
(A) $\frac{2ab}{\sqrt{a^2 + b^2 - 2ab \cos \theta}}$ (D) none of these D) 15/4 (E) 2000	II, the ratio of b/a is given in column II. Column I	
1. The ratio of b/a is given in column II. Column I		
2.1. The ratio of b/a is given in column II. Column I		
2.2.1. The ratio of b/a is given in column II. Column I		
3.2.1. The ratio of b/a is given in column II. Column I		
4.3.1. The total of the coordinates of the point B being (8, 0). The radius of the circle is – to 0, 17/4 to 1, 2, 10, 2, 10, 15/4 to 2, 10, 2, 1		

- **Q.35** A circle is tangent to the y-axis at (0, 2) and cuts the positive x-axis at two distinct points A and B ($OB > OA$), the coordinate of the point B being (8, 0). The radius of the circle is –
	- $(A) 9/2$ (B) 15/4

(C) 17/4 (D)
$$
\sqrt{17}
$$

Q.36 Triangle ABC is right angled at A. The circle with centre A and radius AB cuts BC and AC internally at D and E respectively. If $BD = 20$ and $DC = 16$ then the length AC equals –

Q.37 The point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is (A) $(x+g)(x-2)+(y+f)(y-1)=0$

(A)
$$
(x+g)(x-2) + (y+1)(y-1) = 0
$$

(B) $(x+g)(x-2) - (x+f)(x-1) = 0$

(B)
$$
(x+g)(x-2) - (y+f)(y-1) = 0
$$

- (C) $(x-g)(x+2)+(y-f)(y+1)=0$
- (D) none of these
- **Q.38** A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles . The area of the rhombus is
	-
	-

ASSERTION AND REASON QUESTIONS

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
- (C) Statement -1 is True, Statement-2 is False.
- (D) Statement -1 is False, Statement-2 is True.
- (E) Statement -1 is False, Statement-2 is False.
- **Q.39** Tangents are drawn from the origin to the circle $x^2 + y^2 - 2hx - 2hy + h^2 = 0.$

Statement 1: Angle between the tangents is $\pi/2$. **Statement 2:** The given circle is touching the co-ordinate axes.

Q.40 Number of common tangents of $x^2 + y^2 - 2x - 4y - 95 = 0$ and $x^2 + y^2 - 6x - 8y + 16 = 0$ is zero. **Statement 2 :** If $C_1C_2 < |r_1 - r_2|$, then there will be no common tangent. (where C_1 , C_2 are the centre and r_1 , r_2

are radii of circles).

Statement-1 : Number of circles passing through (1, 2), (4, 7) and (3, 0) is one.

2absin through three non-collinear points. **Statement-2 :** One and only circle can be made to pass

MATCH THE COLUMN TYPE QUESTIONS

COUESTION BANK

radius (one to each are radii of circles).

and is (one to each are radii of circles).

a b², then the length (4, 7) and (3, 0) is one.
 Statement-2 : One and only circle can be made to pass
 $2ab\sin\theta$ **COLESTION BANK**

Equity (one to each are radii of circles).

Intersection of two Q.41 Statement-1 : Number of circles passing through (1, 2),
 $\frac{1}{2}$, then the length (4, 7) and (3, 0) is one.

Statement-2 : One and o **Q.42** Consider two circles C_1 of radius a and C_2 of radius b (b>a) both lying in the first quadrant and touching the coordinate axes. In each of the conditions listed in column I, the ratio of b/a is given in column II.

PASSAGE BASED QUESTIONS

- **Passage 1- (Q.45-Q.47)**
	- Let $x^2 + y^2 = 1$ be the equation of the circumcircle of a \triangle ABC. If P(α , β) be a point on the circle but not a vertex of \triangle ABC, perpendiculars PD, PE and PF are drawn to the three sides BC, CA and AB of triangle ABC. X, Y and Z are feet of perpendiculars from A, B and C to the sides BC, CA and AB respectively and H is the orthocentre of \triangle ABC and I, I₁, I₂ and I₃ are incentre and ex-centres of \triangle ABC. Let R, R₁, R₂, R₃ are radii of circumcircle of $\Delta I_1 I_2 I_3$, $\Delta I I_2 I_3$, $\Delta I I_1 I_3$, $\Delta II_1 I_2$. Three circles are given by 2 2 S x y 4 ¹ , 2 2 S (x 4) (y 4) 4 ² , 2 2 S x y 6x 8y 24 0 ³
- **Q.45** Points D, E and F (A) form a right angled Δ (B) form an equilateral Δ (C) form an isosceles Δ (D) are collinear
- **Q.46** I_1 is the orthocentre of $(A) \Delta I_1 I_2 I_3$ $(B) \Delta \Pi_1 I_2$ $(C)\Delta\Pi_1\mathbf{I}_3$ $(D) \Delta H_2 I_3$
- **Q.47** Ex-centred of \triangle XYZ
	- (A) lie inside the \triangle ABC.
	- (B) are the corresponding vertices of the \triangle ABC.
	- (C) may lie inside or outside depending on \triangle ABC is acute or obtuse angled.
	- (D) None of these

Passage 2 : (Q.48-Q.50)

Q.48 Centre of that circle which cuts the circles S_1 , S_2 , S_3 orthogonally is

Q.49 Radius of the circle obtained above is

(A)
$$
4\frac{\sqrt{177}}{7}
$$
 (B) $2\frac{\sqrt{177}}{7}$
(C) $\frac{\sqrt{177}}{7}$ (D) $8\frac{\sqrt{177}}{7}$

Q.50 Point of intersection of direct tangents between S_1 and S_3 always lies on the line (A) $3y - 8x = 0$ (B) $4y + 3x = 0$

(C) $3y+4x=0$ (D) $3y+4x+2=0$

Passage 3 : (Q.51-Q.53)

P is a variable point on the line $L = 0$. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQRS is completed.

Q.51 If $L = 2x + y - 6 = 0$, then the locus of circumcentre of \triangle PQR is –

(A)
$$
2x - y = 4
$$

\n(B) $2x + y = 3$
\n(C) $x - 2y = 4$
\n(D) $x + 2y = 3$

Q.52 If $P \equiv (6, 8)$, then the area of \triangle QRS is –

STUDY MATERIAL: MATHEMATICS
\nIf P = (6, 8), then the area of
$$
\triangle
$$
 QRS is –
\n(A) $\frac{(6)^{3/2}}{25}$ sq. units (B) $\frac{(24)^{3/2}}{25}$ sq. units
\n(C) $\frac{48\sqrt{6}}{25}$ sq. units (D) $\frac{196\sqrt{6}}{25}$ sq. units
\nIf P = (3, 4), then coordinate of S is –
\n(A) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (B) $\left(-\frac{51}{25}, -\frac{68}{25}\right)$
\n(C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$
\n
\nthe 4 : (Q.54-Q.56)
\nConsider the circles : S₁ : x² + y² - 6y + 5 = 0,
\nS₂ : x² + y² - 12x + 35 = 0 and a variable circle
\nS : x² + y² + 2gx + 2fy + c = 0.
\nNumber of common tangents to S₁ and S₂ is –

(C)
$$
\frac{48\sqrt{6}}{25}
$$
 sq. units (D) $\frac{196\sqrt{6}}{25}$ sq. units

Q.53 If $P \equiv (3, 4)$, then coordinate of S is –

(A) 46 63 , 25 25 (B) (C) 46 68 , 25 25 (D)

Passage 4 : (Q.54-Q.56)

Consider the circles : S_1 : $x^2 + y^2 - 6y + 5 = 0$, $S_2: x^2 + y^2 - 12x + 35 = 0$ and a variable circle $S \cdot x^2 + y^2 + 2gx + 2fy + c = 0.$

- **Q.54** Number of common tangents to S_1 and S_2 is $(A) 1$ (B) 2 (C) 3 (D) 4
- **Q.55** Length of a transverse common tangent to S_1 and S_2 is –

(C)
$$
\sqrt{35}
$$
 (D) $11\sqrt{2}$

man equilateral Δ (C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{8}{25}, -\frac{51}{25}\right)$

collinear
 I_1I_2
 Passage 4 : (0.54-0.56)

Sosider the circles $S_1 : x^2 + y^2 - 6y + 5 = 0$,
 $S_2 : x^2 + y^2 - 12x + 35 = 0$ and a variable cir form an equilateral \triangle (C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

are collinear
 $\triangle 11_11_2$
 Passage 4 : (0.54-0.56)

S₂ : $x^2 + y^2 - 12x + 35 = 0$ and a variable circle

so fthe $\triangle ABC$.

S₂ : x form an equilateral Δ (C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

are collinear
 $\Delta 11_11_2$
 Passage 4 : (0.54-0.56)
 $\Delta 11_21_3$
 Passage 4 : (0.54-0.56)
 $\Delta 11_21_3$
 Passage 4 : (0.54-0.5 (C) $\frac{48\sqrt{6}}{25}$ sq. units

If P = (3, 4), then coordinate of S is -

(A) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (B) $\left(-\frac{51}{25}, -\frac{68}{25}\right)$

(C) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

et : (Q.54-Q.56)

Con (C) $\frac{45\sqrt{6}}{25}$ sq. units

If P = (3, 4), then coordinate of S is –

(A) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (B) $\left(-\frac{51}{25}, -\frac{68}{25}\right)$

(C) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (D) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

(C) $\left(-\frac{46}{25}, -\frac{$ **Q.56** If the variable circle $S = 0$ with centre C moves in such a way that it is always touching externally the circles $S_1 = 0$ and $S_2 = 0$ then the locus of the centre C of the variable circle is –

Passage 5 : (Q.57-Q.59)

- collinear
 $\begin{pmatrix}\n1.5 \\
1.2 \\
2.3\n\end{pmatrix}$
 Passage 4 : (0.54-0.56)

Consider the circles : $S_1 : x^2 + y^2 6y + 5 = 0$,
 $S_2 : x^2 + y^2 22x + 35 = 0$ and a variable circle

S: $x^2 + y^2 + 2gx + 25y + c = 0$.

The $\triangle ABC$:
 O.54 Number o 11₁1₂

11₂1₃

Consider the circles : $S_1 : x^2 + y^2 - 6y + 5 = 0$,
 $S_2 : x^2 + y^2 - 12x + 35 = 0$ and a variable circle
 $S_1 : x^2 + y^2 - 12x + 35 = 0$ and a variable circle
 $S_1 : x^2 + y^2 - 12x + 35 = 0$ and a variable circle
 $S_$ 11-12

11-1 are collinear
 $\Delta 11_11_2$
 $\Delta 11_21_3$
 $\Delta 11_21_$ A 11₂1₂

A 11₂¹₂

Consider the circles: S₁: $x^2 + y^2 - 6y + 5 = 0$,

S₂: $x^2 + y^2 - 12x + 35 = 0$ and a variable circle

S: $x^2 + y^2 - 2gx + 2fy + c = 0$.

So fthe A ABC.
 O.54 Number of common tangents to S₁ and S₂ Let $f(x, y) = 0$ be the equation of a circle such that $f(0, y)$ $= 0$ has equal real roots has $f(x, 0) = 0$ has two distinct real roots. Let $g(x, y) = 0$ be the locus of point P from where tangents to circle $f(x, y) = 0$ make angle $\pi/3$ between them and $g(x, y) = x^2 + y^2 - 5x - 4y + c$, $c \in R$.
- (B) $2\frac{\sqrt{177}}{7}$ and $g(x, y) = x^2 + y^2 5x 4y + c$, $c \in K$.
 Q.57 Let Q be a point from where tangents drawn to circle (D) $8\frac{\sqrt{177}}{7}$ points of contact of tangents drawn from Q to circle $g(x, y) = 0$, then area of triangle QAB is – $g(x, y) = 0$ are mutually perpendicular. If A, B are the (A) 25/12 (B) 25/8 S₁ = 0 and S₂ = 0 then the locus of the centre C of the
variable circle is –
(A) a circle
(C) an ellipse (B) a parabola
(C) an ellipse (B) a parabola
(C) an ellipse (B) a hyperbola
e 5 : (Q.57-Q.59)
Let $f(x, y) = 0$ b circle is -

(B) a parabola

(D) a hyperbola

(D) a hyperbola

57-**0.59**)

(D) a hyperbola

(D) a hyperbola

that f (0, y)

(qual real roots has f (x, 0) = 0 has two distinct real

to circle f (x, y) = 0 make angle $\pi/3$ warable circle $S = 0$ win centre C moves in such a
at it is always touching externally the circles
and it is always touching externally the circles
electrice
electrice (B) a parabola
ellipse (D) a hyperbola
ellipse (B) a rabola

le such that f (0, y)

las two distinct real

boint P from where
 $\equiv \pi/3$ between them
 $\equiv R$.

drawn to circle

lar. If A, B are the

om Q to circle

is –

(x, y) = 0 with axis
 $\frac{25}{8} \tan^{-1} \left(\frac{24}{11} \right)$
 the C moves in such a
mally the circles
of the centre C of the
parabola
hyperbola
ircle such that $f(0, y)$
0 has two distinct real
of point P from where
gle $\pi/3$ between them
 $c \in R$.
ts drawn to circle
cular. If A, B ar (C) an ellipse (D) a hyperbola

e 5 : (Q.57-Q.59)

Let $f(x, y) = 0$ be the equation of a circle such that $f(0, y)$

= 0 has equal real roots has $f(x, 0) = 0$ has two distinct real

roots. Let $g(x, y) = 0$ be the locus of point 57-Q.59)

Sy-0 Dbe the equation of a circle such that $f(0, y)$

sy = 0 be the coust of $g(x, 0) = 0$ has two distinct real

tg $(g, y) = 0$ be the locus of point P from where

to circle $f(x, y) = 0$ make angle $\pi/3$ between the ble circle is
 $Q.57-Q.59$
 $Q.59$
 $Q.59$ and red roots has $f(x, 0) = 0$ has t cle such that $f(0, y)$
has two distinct real
point P from where
 $\ln \pi/3$ between them
 $\in \mathbb{R}$.
strawn to circle
lar. If A, B are the
from Q to circle
3 is –
3
 $\frac{25}{8}$
 $f(x, y) = 0$ with axis
 $\frac{25}{8}$ $\tan^{-1}(\frac{24}{11})$ a parabola

a hyperbola

eircle such that $f(0, y)$

= 0 has two distinct real

of point P from where
 $n \neq 0$. Then where
 $f(0, x)$
 $f(0, x)$
 $f(0, x)$
 $f(0, x)$
 $f(0, x) = 0$
 $f(0,$
	- (C) 25/4 (D) 25/2 **Q.58** The area of region bounded by circle $f(x, y) = 0$ with axis in the first quadrant is –

(A)
$$
3 + \frac{25}{8} \left(\pi - \tan^{-1} \frac{1}{2} \right)
$$
 (B) $3 + \frac{25}{8} \tan^{-1} \left(\frac{24}{11} \right)$

C)
$$
3 + \frac{25}{8} \left(2\pi - \tan^{-1} \frac{3}{4} \right)
$$
 (D) $3 + \frac{25}{8} \left(2\pi - \tan^{-1} \frac{24}{7} \right)$

Q.59 The number of points with positive integral coordinates satisfying $f(x, y) > 0$, $g(x, y) < 0$; $y > 3$ and $x < 6$ is – $(A) 7$ (B) 8 $(C) 10$ (D) 11

Passage 6 -(Q.60-Q.62)

A ball is moving around the circle

 $14x^2 + 14y^2 + 216x - 69y + 432 = 0$ in clockwise direction leaves it tangentially at the point $P(-3, 6)$. After getting reflected from a straight line $L = 0$ it passes through the center of the circle. The perpendicular distance of this

straight line L = 0 from the point P is
$$
\frac{11}{13}\sqrt{130}
$$
. You can

assume that the angle of incidence is equal to the angle of reflection.

Q.60 The equation of tangent to the circle at
$$
P
$$
 is

- (A) $2x y + 12 = 0$ (B) $4x + 3y 6 = 0$ (C) $3x - 2y + 21 = 0$ (D) $2x + 5y - 24 = 0$ **Q.61** Radius of the circle is (A) 165/14 (B) 165/46 (C) 165/28 (D) none of these
- straight line L = 0 from the point P is $\frac{11}{12}\sqrt{130}$. You can perpendicular to the line L = 0 is θ , then tan θ is $13 \t\t\t (A) 2/11$ **Q.62** If angle between the tangent at P and the line through 'P' (A) 2/11 (B) 3/11 (C) 4/11 (D) None of these

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

- **Q.1** Two circles each of radius 5 units, touch each other at (1, 2). If the equation of their indirect common tangent is $4x + 3y = 10$ and the equations of two circles are $x^{2} + y^{2} + \alpha x + \beta y - 15 = 0$, $x^{2} + y^{2} + \gamma x + \delta y + 25 = 0$, then find the value of $(\alpha + \beta) - (\gamma + \delta)$.
- **Q.2** If the tangents are drawn from any point on the line $x + y = 3$ to the circle $x^2 + y^2 = 9$, then the chord of contact passes through the point (3, a) then find the value of a.
- **Q.3** As shown in the figure, three circles which have the same radius r, have centres at $(0, 0)$; $(1, 1)$ and $(2, 1)$. If they have a common tangent line, as shown then, their radius

- **Q.4** The circle passing through the distinct points $(1, t)$, $(t, 1)$ $\&$ (t, t) for all values of 't', passes through the point (a, b) . Find the value of $(a + b)$.
- **Q.5** If p_1 and p_2 are the two values of p for which two perpendicular tangents can be drawn from the origin to the circle $x^2 - 6x + y^2 - 2py + 17 = 0$, then find the value Q.13 of $(p_1^2 + p_2^2)$.
- **Q.6** Let $A(-4, 0)$ and $B(4, 0)$. Number of points $C = (x, y)$ on the circle $x^2 + y^2 = 16$ such that the area of the triangle Q.14 whose vertices are A, B and C is a positive integer, is.
- **Q.7** A point moving around a circle $x^2 + y^2 + 8x + 4y 5 = 0$ with centre C broke away from it either at the point A or at the point B on the circle and moved along a tangent to the circle passing through the point $D(3, -3)$. Find the area of the quadrilateral ABCD.
- **Q.8** Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and

centres $(-3, 0)$, $(-1, 0)$, $(1, 0)$ and $(3, 0)$ respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of

this chord can be expressed as \sqrt{x} , find x.

Q.9 Circles A and B are externally tangent to each other and to line *t*. The sum of the radii of the two circles is 12 and the radius of circle A is 3 times that of circle B. The area in between the two circles and its external tangent is

$$
a\sqrt{3}-\frac{b\pi}{2}
$$
 then find the value of $a + b$.

Radius of the circle is

(B) 165/46

(C) 165/28 (D) none of these

If angle between the tangent at P and the line through 'P'

frangle between the tangent at P and the line through 'P'

ceprependicular to the line L = 0 i **Q.10** A circle lying in 1st quadrant touches x and y axis at point P and Q respectively. BC and AD are parallel tangents to the circle with slope -1 . If the points A and B are on the y axis while C and D are on the x-axis and the area of the (C)4/11 (D) None of these
 E BASED QUESTIONS)

centres (-3, 0), (-1, 0), (1, 0) and (3, 0) respectively are

drawn. A chord PQ of the circle is touches the circle B and

passes through the centre of the circle C. If the

circle is –

Q.11 Let W_1 and W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^{2} + y^{2} - 10x - 24y + 153 = 0$ respectively. Let m be the smallest positive value of 'a' for which the line $y = ax$ contains the centre of a circle that is externally tangent to

 W_2 and internally tangent to W_1 . Given that $m^2 = p/q$ where p and q are relatively prime integers, find $(p + q)$. **Q.12** If the tangent at the point P on the circle

 $x^2+y^2+6x+6y=2$ meets the straight line $5x-2y+6=0$ at a point Q on the y-axis, then length of PQ is :

- **Q.13** If one of the diameters of the circle $x^2 + y^2 2x 6y + 6 = 0$ is a chord to the circle with centre $(2, 1)$, then the radius of the circle is
- **Q.14** Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is 2 t W₁ and W₂ denote the circles
 $y + y^2 + 10x - 24y - 87 = 0$ and
 $y + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the
 $y + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the

2 and internally tangent to W₁. Given that
- **Q.15** Two parallel chords of a circle of radius 2 are at a distance

of π/k and $2\pi/k$, where k > 0, then the value of [k] is : [Note : [k] denotes the largest integer less than or equal to k].

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

- **Q.1** The square of the length of tangent from $(3, -4)$ on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ - [AIEEE-2002] $(A) 20$ (B) 30 (C) 40 (D) 50
- **Q.2** Radical axis of the circle $x^2 + y^2 + 6x 2y 9 = 0$ and $x^2 + y^2 - 2x + 9y - 11 = 0$ is – [AIEEE-2002] (A) $8x - 11y + 2 = 0$ (B) $8x + 11y + 2 = 0$ (C) $8x - 11y - 2 = 0$ (D) $8x + 11y - 2 = 0$
- **Q.3** If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then **[AIEEE-2003]** (A) $r > 2$ (B) $2 < r < 8$ (C) $r < 2$ (D) $r = 2$
- **Q.4** The lines $2x 3y = 5$ and $3x 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is **[AIEEE-2003]** $(A) x² + y² - 2x + 2y = 62$ (B) $x² + y² + 2$ $+y^2+2x-2y=62$ $(C) x^2 + y^2 + 2x - 2$ $+2x-2y=47$ (D) $x^2+y^2-2x+2y=47$
- **Q.5** If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is- **[AIEEE-2004]** (A) 2ax + 2by + $(a^2 + b^2 + 4) = 0$
	- (B) $2ax + 2by (a^2 + b^2 + 4) = 0$ (C) 2ax – 2by + $(a^2 + b^2 + 4) = 0$ (D) $2ax - 2by - (a^2 + b^2 + 4) = 0$
- **Q.6** A variable circle passes through the fixed point A(p, q) and touches x- axis. The locus of the other end of the diameter through A is-
[AIEEE-2004] $(A) (x - p)^2 = 4qy$ $= 4$ qy $(B) (x-q)^2 = 4py$ $(C)(y-p)^2 = 4qx$ $= 4ax$ (D) $(y - q)^2 = 4px$
- **Q.7** If the lines $2x + 3y + 1 = 0$ and $3x y 4 = 0$ lie along diameters of a circle of circumference 10π , then the equation of the circle is- **[AIEEE-2004]** $(A) x² + y² - 2x + 2y - 23 = 0$ $(B) x^2 + y^2 - 2x - 2y - 23 = 0$ $(C) x^2 + y^2 + 2x + 2y - 23 = 0$ $(D) x^2 + y^2 + 2x - 2y - 23 = 0$
- **Q.8** The intercept on the line $y = x$ by the circle $x^2 + y^2 2x = 0$ is AB. Equation of the circle on AB as a diameter is: **[AIEEE-2004]**

(A)
$$
x^2 + y^2 - x - y = 0
$$

\n(B) $x^2 + y^2 - x + y = 0$
\n(C) $x^2 + y^2 + x + y = 0$
\nIf the circles $x^2 + y^2 + 2ax + cy + a = 0$ and

Q.9

- $x^{2} + y^{2} 3ax + dy 1 = 0$ intersect in two distinct point P and Q then the line $5x + by - a = 0$ passes through P and Q for - **[AIEEE-2005]** (A) exactly one value of a (B) no value of a (C) infinitely many values of a (D) exactly two values of a
- **Q.10** A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is- **[AIEEE-2005]** (A) an ellipse (B) a circle (C) a hyperbola (D) a parabola
- **Q.11** If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is - **[AIEEE-2005]** (A) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ (B) $2ax + 2by - (a^2 - b^2 + p^2) = 0$ $(C) x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ (D) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
- **Q.12** If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then – **[AIEEE-2005]** (A) 3a² – 10ab + 3b² = 0 $= 0$ (B) $3a^2 - 2ab + 3b^2 = 0$ (C) 3a² + 10ab + 3b² = 0 $= 0$ (D) $3a^2 + 2ab + 3b^2 = 0$
- **Q.13** If the lines $3x 4y 7 = 0$ and $2x 3y 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is– **[AIEEE-2006]** $(A) x² + y² + 2x - 2y - 62 = 0$ $(B) x^2 + y^2 - 2x + 2y - 62 = 0$ $(C) x^2 + y^2 - 2x + 2y - 47 = 0$ $(D) x^2 + y^2 + 2x - 2y - 47 = 0$
- **Q.14** Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $2\pi/3$ at its centre is – **[AIEEE-2006]**

(A)
$$
x^2 + y^2 = 1
$$
 (B) $x^2 + y^2 = \frac{27}{4}$

(C)
$$
x^2 + y^2 = \frac{9}{4}
$$
 (D) $x^2 + y^2 = \frac{3}{2}$

- **Q.15** Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval- **[AIEEE-2007]** (A) 0 < k < 1/2 (B) k \ge 1/2 $(C) - 1/2 \le k \le 1/2$ (D) $k \le 1/2$
- **Q.16** The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is - [AIEEE-2008] $(A)(-3, 4)$ (B) $(-3, -4)$ $(C)(3, 4)$ (D) $(3, -4)$
- **Q.17** If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and $(1, 1)$ for – **[AIEEE-2009]**
	- (A) exactly one value of p (B) all values of p (C) all except one value of $p(D)$ all except two values of p
- **Q.18** The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if – **[AIEEE 2010]** $(A) -35 < m < 15$ (B) $15 < m < 65$ (C) $35 \le m \le 85$ (D) $-85 \le m \le -35$
- **Q.19** The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if : **[AIEEE 2011]** $(A) 2 |a| = c$ (B) | a | = c (C) $a = 2c$ (D) $|a| = 2c$

The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is

Q.27 Three circles of radii a, b, c $(a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then **[JEE MAIN 2019 (JAN)]**

(A)
$$
\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}
$$
 (B) a, b, c are in

(C)
$$
\sqrt{a}
$$
, \sqrt{b} , \sqrt{c} are in A. P. (D) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Q.28 The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is : **[JEE MAIN 2019 (APRIL)]** (A) 320 (B) 160 (C) 105 (D) 210

EDEMADVANCED LEARING:
 Q.29 The tangent and the normal lines at the point ($\sqrt{3}$, 1) to

the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The

area of this triangle (in square units) is :
 [JEE MAIN 2019 (AP the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

[JEE MAIN 2019 (APRIL)]

-
- The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to
the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The
area of this triangle (in square units) is :
[JEE MAIN 2019 (APRIL)]
(A) 1/3 (B) $4/\sqrt{3}$
(C) $1/\sqrt{3}$ The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to
the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The
area of this triangle (in square units) is :
(JEE MAIN 2019 (APRIL)]
(A) 1/3 (B) $4/\sqrt{3}$
(C) $1/\sqrt{3}$ **Q.30** If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is **[JEE MAIN 2019 (APRIL)]** $(A) x² + y² - 2xy =$ $-2xy = 0$ (B) $x^2 + y^2 - 16x^2y^2 = 0$
- $(C) x^2 + y^2 4x^2y^2 = 0$ $= 0$ (D) $x^2 + y^2 - 2x^2y^2 = 0$ **Q.31** The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point : **[JEE MAIN 2019 (APRIL)]** $(A)(-4, 6)$ (B) $(6, -2)$
	- $(C) (-6, 4)$ (D) $(4, -2)$
- **Q.32** If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, (K \in R), intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for : **[JEE MAIN 2019 (APRIL)]** (A) exactly two values of K (A) 1/3 (B) 4/ $\sqrt{3}$ (D) $2/\sqrt{3}$ (D) $2/\sqrt{3}$ (D) $2/\sqrt{3}$

(C) $1/\sqrt{3}$ (D) $2/\sqrt{3}$

Ef a tangent to the circle $x^2 + y^2 = 1$ intersects the

forecordinate axes at distinct points P and Q, then the locus

of the mid-po (C) $1/\sqrt{3}$

(D) $2/\sqrt{3}$

(D) $2/\sqrt{3}$

(D) a tangent to the circle $x^2 + y^2 = 1$ intersects the

focordinate axes at distinct points P and Q, then the locus

of the mid-point of PQ is **[JEEMAIN2019(APRIL)]**

(A) $x^2 + y^$ of the mid-point of PQ is

(A) $x^2 + y^2 - 2xy = 0$ (B) $x^2 + y^2 - 2x^2y^2 = 0$

(C) $x^2 + y^2 - 2x^2y = 0$ (B) $x^2 + y^2 - 2x^2y^2 = 0$

The common tangent to the circles $x^2 + y^2 = 4$ and
 $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes throug (A) $x^2 - 2x^3 - 0$ (D) $x^2 + y^2 - 2x^2y^2 = 0$

(C) $x^2 + y^2 - 4x^2y^2 = 0$ (D) $x^2 + y^2 - 2x^2y^2 = 0$

The common tangent to the circles $x^2 + y^2 = 4$ and
 $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point:

(A) (-4, 6) (
	- (B) exactly one value of K
	- (C) no value of K.
	- (D) infinitely many values of K
- **Q.33** The line $x = y$ touches a circle at the point $(1, 1)$. If the circle also passes through the point $(1, -3)$, then its radius is : **[JEE MAIN 2019 (APRIL)]**
	-
- **Q.34** The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is : **[JEE MAIN 2019 (APRIL)]**

(A)
$$
y = \sqrt{1 + 4x}
$$
, $x \ge 0$
\n(B) $x = \sqrt{1 + 4y}$, $y \ge 0$
\n(C) $x = \sqrt{1 + 2y}$, $y \ge 0$
\n(D) $y = \sqrt{1 + 2x}$, $x \ge 0$

Q.35 If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is

- given by the equation,

is:
 $\frac{1}{2}$ is iven by the equation,

is: **IDEEMAIN2019(APRIL)**

(C) $2\sqrt{2}$ (B) 3

(B) of a circle, s, whose

(C) $2\sqrt{2}$ (B) 2
 IEEMAIN2016 (C) $2\sqrt{2}$
 IEEMAIN2016 (A) $3\sqrt{2}$
 IFERMAIN2016 (A) $3\sqrt{2}$
 IFERMAIN2016 (A) **2.33** The line $x = y$ touches a circle at the point (1, 1). If the

eirdealso passes through the point (1, 3), then its radius

is:

fa circle S, whose

(A) $3\sqrt{2}$ (B) 2

[BEMAIN 2016] (APRIL)]

(C) $2\sqrt{2}$ (D) 2

[BEM **Q.36** A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point : **[JEE MAIN 2019 (APRIL)]** $(A)(3, 10)$ (B) $(2, 3)$
	- $(C)(1, 5)$ (D) (3, 5) **Q.37** Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B.
		- The $(AB)^2$ is equal to : **is JEE MAIN 2020 (JAN)** $(A) 64 / 5$ (B) 24/5 $(C) \frac{8}{5}$ (D) $\frac{8}{13}$

Q.38 If $y = mx + c$ is a tangent to the circle $(x-3)^2 + y^2 = 1$ and also the perpendicular to the tangent

to the circle
$$
x^2 + y^2 = 1
$$
 at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then
\n[**JEE MANY 2020 (JAN)**]
\n(A) $c^2 + 6c + 7 = 0$
\n(B) $c^2 - 6c + 7 = 0$
\n(C) $c^2 + 6c - 7 = 0$
\n(D) $c^2 - 6c - 7 = 0$

QUESTION BANK	STUDY MATERIAL: MATHEMATICS	
0 the circle	Q.39 A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle?	[JEE MAIN 2020 (JAN)]
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(A) $3x - 4y - 24 = 0$	(B) $3x + 4y - 6 = 0$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(A) $3x - 4y - 24 = 0$	(B) $3x + 4y - 6 = 0$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(A) $3x - 4y - 24 = 0$	(B) $3x + 4y - 6 = 0$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(B) $3x - 4y - 24 = 0$	(C) $4x + 3y - 8 = 0$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(D) $4x - 3y + 24 = 0$	(E) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then	(E) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$	(E) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

[JEE MAIN 2020 (JAN)]

ANSWER KEY

CHAPTER- 10 : CIRCLE SOLUTIONS TO TRY IT YOURSELF
TRY IT VOURSELF
 $\text{or } (\sqrt{9+36-k})^2 = 2(\sqrt{9+36-15})^2$ **TRY IT YOURSELF-1**

- **(1)** (C)
- **(2) (C).** Centroid of the triangle coincides with the centre of the circle and the radius of the circle is 2/3 of the length of (7) the median]
- **(3)** The given equation is $x^2 + y^2 4x 8y 45 = 0$ \implies $(x^2-4x)+(y^2-8y)=45$ Adding 4 and 16 to make perfect squares, we get \Rightarrow $(x^2-4x+4)+(y^2-8y+16)=45+4+16$ \implies $(x-2)^2 + (y-4)^2 = 65$ Radius = $\sqrt{65}$.
- **(4) Inside.**

Here the given circle is $x^2 + y^2 = 25$. Its centre O is $(0, 0)$ and radius r is 5. Let P be a point $(-2.5, 3.5)$. $OP² = (-2.5 - 0)² + (3.5 - 0)²$ $OP² = 6.25 + 12.25 = 18.5$

 $OP < r$

Hence, the point $(-2.5, 3.5)$ lies inside the circle, since the distance of the point to the centre of the circle is less than centre the radius of the circle.
Let the equation of the circle $\frac{1}{2}$

 $\overline{\sigma}$

 $P^{\left(-2.5, 3.5\right)}$

One expression of the circle is
$$
x^2 + y^2 + 2gx + 2fy + c = 0
$$
 and $x^2 + y^2 + 2gx + 2fy + c = 0$ and <

$$
h = \frac{7}{2}, k = \frac{-5}{2}
$$

On putting the value of h and k in (2) , we get

$$
\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2 \implies r^2 = \frac{65}{2}
$$

Therefore, the equation of the circle is

$$
\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}
$$

$$
x^2 - 7x + \frac{49}{4} + y^2 + 5y + \frac{25}{4} = \frac{65}{2}
$$

$$
x^2 + y^2 - 7x + 5y - 14 = 0
$$

(6) (A). If area of circle is double then $R' = \sqrt{2}R$ $(R' = radius of new circle)$ then $R'^2 = 2R^2$ If area of circle is double then R' = $\sqrt{2}R$

(R' = radius of new circle)

then R² = 2R²

or $(\sqrt{9+36-k})^2 = 2(\sqrt{9+36-15})^2$

or $45-k=2(30)$

or $k=-15$

Let the equation of tangent is $y = mx$ then
 $\left|\frac{mr-h}{\sqrt{1+ms^2}}\right|$ or $45 - k = 2(30)$ or $k = -15$ **EDENTAD VANISED LEARNING**

is double then R' = $\sqrt{2}R$

(ew circle)

(k)² = $2(\sqrt{9} + 36 - 15)^2$

(30)

n of tangent is y = mx then
 $\frac{2}{(1 + h)^2 - h^2}$
 $\Rightarrow m^2r^2 + h^2 - 2mrh = r^2(1 + m^2)$ area of circle is double then $R' = \sqrt{2}R$
 \therefore $r' = \text{radius of new circle}$
 \therefore $mR'^2 = 2R^2$
 $(\sqrt{9 + 36 - k})^2 = 2(\sqrt{9 + 36 - 15})^2$
 $45 - k = 2(30)$
 $k = -15$
 \therefore the equation of tangent is $y = mx$ then
 $\frac{mr - h}{\sqrt{1 + m^2}} = \sqrt{r^2 + h^2 - h^2}$ SODIMADVANGED LEARNING

The area of circle is double then R' = $\sqrt{2R}$
 $=$ radius of new circle)
 $= R^2 = 2R^2$
 $(\sqrt{9+36-k})^2 = 2(\sqrt{9+36-15})^2$
 $= 45 - k = 2(30)$
 $k = -15$

the equation of tangent is $y = mx$ then
 $\frac{mr - h}{1 + m$ Fire is double then R' = $\sqrt{2R}$

s of new circle)
 $2R^2$
 $36-k)^2 = 2(\sqrt{9+36-15})^2$
 $= 2(30)$

aation of tangent is y = mx then
 $= \sqrt{r^2 + h^2 - h^2}$
 $= r \Rightarrow m^2r^2 + h^2 - 2mrh = r^2 (1 + m^2)$ **EDENTIFYING**

Read of circle is double then R' = $\sqrt{2R}$
 $x' = \text{radius of new circle}$
 $\text{en } R^2 = 2R^2$
 $(\sqrt{9 + 36 - k})^2 = 2(\sqrt{9 + 36 - 15})^2$
 $45 - k = 2(30)$
 $k = -15$

at the equation of tangent is $y = mx$ then
 $\frac{mr - h}{\sqrt{1 + m^2}} = \sqrt{r^2 + h^$ **SOMADVANCED LEARNING**

TODMADVANCED LEARNING
 \therefore The radius of new circle)
 $\ln R^2 = 2R^2$
 $(\sqrt{9 + 36 - k})^2 = 2(\sqrt{9 + 36 - 15})^2$
 $45 - k = 2(30)$
 $k = -15$

the equation of tangent is $y = mx$ then
 $\frac{mr - h}{1 + m^2} = \sqrt{r^2 + h^2 - h^$ a of circle is double then $R' = \sqrt{2}R$

radius of new circle)
 $R^2 = 2R^2$
 $\sqrt{9 + 36 - k}$)² = 2($\sqrt{9 + 36 - 15}$)²

5-k=2(30)
 -15

ne equation of tangent is $y = mx$ then
 $\frac{-h}{m^2}$ = $\sqrt{r^2 + h^2 - h^2}$
 $\frac{-h}{m^2}$ =

(D). Let the equation of tangent is $y = mx$ then

LTIONS TO TRY IT YOURSELE-I
\n(C) Centroid of the triangle coincides with the centre of
\nthe median
\nthe median
\nthe median
\nthe normal
\n
$$
(x^2-4x)+(y^2-8y)=45
$$

\n $\Rightarrow (x^2-4x)+(y^2-8y)=45$
\n $\Rightarrow (x^2-2)^2+(y-4)^2=65$
\n $\Rightarrow (x^2-2)^2+(y-4)^2=65$
\n $\Rightarrow (x^2-2)^2+(y-4)^2=65$
\n $\Rightarrow x^2-2x-1$
\n<

$$
\Rightarrow m = \frac{h^2 - r^2}{2rh} \text{ or one root is } \infty
$$

..
$$
(h^2 - r^2) x - 2rh y = 0, x = 0
$$

\n**(8) (D).** Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
\nIf it is passes through (0, 0) and (1, 0).
\n $1 + 2g = 0$ (1)
\nor $g = -1/2$

If circle touches $x^2 + y^2 = 9$ then distance between $=$ sum of radii or difference of radii.

$$
\therefore \sqrt{g^2 + f^2} = \sqrt{g^2 + f^2} \pm 3 \text{ and } f = \pm \sqrt{2}
$$

\n
$$
\therefore \text{ Centre is } \left(\frac{1}{2}, \pm \sqrt{2}\right)
$$

- **(9) (D).** Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ as it passes through $(-1,0)$ & $(0,2)$
	- $1 2g + c = 0$ and $4 + 4f + c = 0$ Also $f^2 = c$

$$
\Rightarrow f=-2, c=4; g=5/2
$$

\n
$$
\therefore
$$
 Equation of circle is x² + y² + 5x - 4y + 4 =0
\nwhich passes through (-4, 0)

(10) (C). Line $5x - 2y + 6 = 0$ is intersected by tangent at P to circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis at Q (0, 3). In other words tangent passes through (0, 3). \therefore PQ = length of tangent to circle from (0, 3). 0 9 0 18 2 25 5 entre is $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ as

it passes through $(-1,0)$ & $(0,2)$
 $1 - 2g + c = 0$ and $4 + 4f + c = 0$

Also $f^2 = c$
 $f^2 = -2$, $c = 4$; $g = 5/2$

Equation of circle is $x^2 +$ tre is $\left(\frac{1}{2}, \pm \sqrt{2}\right)$

et equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ as

passes through (-1,0) & (0,2)

-2g + c = 0 and 4 + 4 f + c = 0

so $f^2 = c$

-2, c = 4 ; g = 5/2

quation of circle is $x^2 + y^2 + 5x - 4y + 4 = 0$

$$
=\sqrt{0+9+0+18-2}=\sqrt{25}=5
$$

(11) (AC). Equation of circle can be written as $(x-3)^2 + y^2 + \lambda(y) = 0$ $\implies x^2 + y^2 - 6x + \lambda y + 9 = 0.$ Now, $(\text{radius})^2 = 7 + 9 = 16$

$$
\Rightarrow 9 + \frac{\lambda^2}{4} - 9 = 16 \Rightarrow \lambda^2 = 64 \Rightarrow \lambda = \pm 8.
$$

 \therefore Equation is $x^2 + y^2 - 6x \pm 8y + 9 = 0$.

TRY IT YOURSELF-2

(1) Any circle passing through the point of intersection of the given line and circle has the equation $x^2 + y^2 - 9 + \lambda (x + y - 1) = 0$. Its centre = $(-\lambda/2, -\lambda/2)$ The circle is the smallest if $(-\lambda/2, -\lambda/2)$ is on the chord $x + y = 1$.

$$
\Rightarrow -\frac{\lambda}{2} - \frac{\lambda}{2} \Rightarrow \lambda = -1
$$

Putting this value for λ , the equation of the smallest circle is $x^2 + y^2 - 9 - (x + y - 1) = 0$.

(2) (C). Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ then for circles

$$
x^{2}+y^{2}+4x-6y+9=0 \text{ and } x^{2}+y^{2}-4x+6y+4=0
$$

2g(2)+2f(-3)=c+9
2g(-2)+2f(3)=c+4
or eliminating c, we get, 8x-12y-5=0

(3) (C). The given circle is $x^2 + y^2 - 2x - 6y + 6 = 0$

Let AB be one of its diameter which is the chord of other circle with centre at C_1 (2, 1).

Then in
$$
\triangle C_1CB
$$
, $C_1B^2 = CC_1^2 + CB^2$
\n $\Rightarrow r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$
\n $\Rightarrow r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3$

(4) (A). Let mid point be (h, k), then chord of contact : $hx + ky = h^2 + k^2$(i) Let any point on the line $4x - 5y = 20$ be

$$
\left(x_1, \frac{4x_1 - 20}{5}\right)
$$

 \therefore Chord of contact : $5x_1x + (4x_1 - 20)y = 45$ (ii) (i) and (ii) are same

IDENTIFY OURSELE-E-2
\ncircle passing through the point of intersection of
\n
$$
z = \frac{6x}{h} = \frac{4x_1 - 20}{k} = \frac{45}{h^2 + k^2}
$$

\n $z^2 - 9 + \lambda(x + y - 1) = 0$. Its centre = (–λ/2, –λ/2)
\n $z^2 - 9 + \lambda(x + y - 1) = 0$
\n $z = 1$
\n $z = 2$
\n $z =$

Also,
$$
\frac{f^2 - c}{4} = \frac{g^2 - c}{-5} = \frac{2g}{6} = \frac{-1}{1}
$$

$$
f^2 - c = -4, g^2 - c = 5, g = -3
$$

$$
c = 4, f = 0
$$

(6) (A). Equation of circle becomes

with centre C (1, 3) and radius =
$$
\sqrt{1+9-6} = 2
$$
.
\nLet AB be one of its diameter which is the chord of
\nother circle with centre at C₁ (2, 1).
\n $g^2(\ell^2+m^2f^2+1+2fg/m-2g\ell-2mf = (g^2+f^2-c)(\ell^2+m^2)$
\n $g^2(\ell^2+m^2f^2+1+2fg/m-2g\ell-2mf = g^2+\ell^2-c)(\ell^2+m^2)$
\n $g^2(\ell^2+m^2+2f+1+2fg/m-2g\ell-2mf = g^2+\ell^2+g^2m^2+\ell^2\ell^2+fm^2+c\ell^2-cm^2$
\n $= g^2(\ell^2+g^2m^2+\ell^2f^2+1+2fg/m-2g\ell-2mf = g^2+\ell^2+fm^2+c\ell^2-cm^2$
\n $g^2(\ell^2-c)+m^2(g^2-c)+2g\ell+2mf-2g\ell m-1=0$
\nOn comparing we get f = 0
\n $4l^2-5m^2+6\ell+1=0$
\nOn comparing we get f = 0
\nAlso, $\frac{f^2-c}{4}=\frac{g^2-c}{-5}=\frac{2g}{6}=\frac{-1}{1}$
\nf $2-c=-4, g^2-c=5, g=-3$
\n $c=4, f=0$
\n $\Rightarrow r^2=1+4+4 \Rightarrow r^2=9 \Rightarrow r=3$
\nLet mid point be (h, k), then chord of contact:
\n $hx+ky=h^2+k^2$ (i)
\nLet any point on the line $4x-5y=20$ be
\n $kt=3$ (6)
\n $kt=3$ (d) 2, 3, 4, 4, 6
\n $kt=3$ (e) 4. Equation of circle becomes
\n $kt=4x_1-1$ (f) $yt=3$ (g) $z=4$ (h) $2t=4$
\n $2t=5$ (i) and (ii) are same
\n
\nChord of contact: $5x_1x + (4x_1-20)y=45$ (ii)
\n

CHAPTER- 10 : CIRCLE EXERCISE-1

- **(1) (B).** $2g = -2 \implies g = -1$ $2f=4 \implies f=2 \implies$ Centre is $(1, -2)$
- **(2) (D).** First making the coefficient of x^2 and y^2 , 1 by dividing the equation with 2

$$
\Rightarrow x^2 + y^2 + 2x - \frac{3}{2}y + \frac{1}{2} = 0
$$

\n
$$
2g = 2 \Rightarrow g = 1
$$

\n
$$
2f = -\frac{3}{2} \Rightarrow f = -\frac{3}{4}, c = \frac{1}{2}
$$

\n
$$
\Rightarrow r = \sqrt{(1)^2 + (-\frac{3}{4})^2 - \frac{1}{2}} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}
$$

$$
(3) \t\t (B). Centre of circle is $\left(\frac{3}{2}, -4\right)$ \n
$$
\n
$$
(13)
$$

Let the other extremity is (h, k)

$$
\therefore \left(\frac{6+h}{2}\right) = \frac{3}{2}; \left(\frac{-3+k}{2}\right) = -4 \Rightarrow (-3, -5)
$$

(4)
\n
$$
(A) \cdot (x-2)^2 + (y+1)^2 = 3^2
$$
\n
$$
\Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = 9
$$
\n
$$
\Rightarrow x^2 + y^2 - 4x + 2y - 4 = 0
$$

- Equation of circle is $x^2 + y^2 = 53$
- **(6) (C).** $(x-1)(x-3)+(y-2)(y-4)=0$ $\Rightarrow x^2 + y^2 - 4x - 6y + 11 = 0$
- (7) **(A).** Centre $(-1, -2)$, radius $(\sqrt{1^2 + 2^2 1}) = 2$. **(14) (B).** The centre
- **(8) (A).** \because **x** = –7 + 4cos θ , **y** = 3 + 4sin θ or $x + 7 = 4\cos\theta$, $y - 3 = 4\sin\theta$ Squaring and adding $(x + 7)^2 + (y - 3)^2 = 16 (\cos^2 \theta + \sin^2 \theta)$ \Rightarrow $(x+7)^2 + (y-3)^2 = 16$
- **(9) (C).** Here radius of circle $|-2| = 2$ \therefore Equation is $(x+2)^2 + (y+3)^2 = 2^2$ or $x^2 + y^2 + 4x + 6y + 9 = 0$
- **(10) (C).** From figure.

Hence equation is $(x-3)^2 + (y-5)^2 = 5^2$ \Rightarrow x² + y² – 6x – 10y + 9 = 0

(11) (C). The point of intersection of the given lines is $(1,-1)$ which is the centre of the required circle. Also if its radius be r, then as given $\pi r^2 = 154$

CHAPTER 10: CIRCLE
\n**CHAPTER 10: CIRCLE**
\n**EXERCISEI**
\n**10**
\n**21**
$$
2\pi - 3 = -1
$$

\n**22** $2\pi - 1 = 0$
\n**23** $2\pi - 2 = 3$
\n**24** $2\pi - 1 = 0$
\n**25** $2\pi - 1 = 0$
\n**26** $2\pi - 2 = 1$
\n**27** $2\pi - 1 = 0$
\n**28** $2\pi - 2 = 1$
\n**29** $2\pi - 2 = 1$
\n**20** $2\pi - 2 = 1$
\n**21** $2\pi + 2 = 2$
\n**22** $2\pi - 2 = 1$
\n**23** $2\pi + 2 = 2$
\n**24** $2\pi + 2 = 2$
\n**25** $2\pi - 2 = 2$
\n**26** $2\pi - 2 = 2$
\n**27** $2\pi - 2 = 2$
\n**28** $2\pi - 2 = 2$
\n**29** $2\pi - 2 = 2$
\n**20** $2\pi - 2 = 2$
\n**21** $2\pi - 2 = 2$
\n**22** $2\pi - 2 = 2$
\n**23** $2\pi - 2 = 2$
\n**24** $2\pi - 2 = 2$
\n**25** $2\pi - 2 = 2$
\n**26** $2\pi - 2 = 2$
\n**27** $2\pi - 2 = 2$
\n**28** $2\pi - 2 = 2$
\n**29** 2

Hence the equation is

15
$$
(x^2 + y^2) - 94x + 18y + 55 = 0
$$

(13) **(D).** Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Passes through (-4, 3)
\n
$$
25-8g+6f+c=0
$$
 ...(1)
\nTouches both lines

$$
\frac{-g-f-2}{\sqrt{2}} = \sqrt{g^2 + f^2 - c} = \frac{-g+f-2}{\sqrt{2}}
$$

: f=0 :: g²-4g-4-2c=0
Also c = 8g-25

 \therefore g = 10 ± 3 $\sqrt{6}$, f = 0, c = 55 ± 24 $\sqrt{6}$ It is easy to see that the answers given are not near to the

values of g,f,c. Hence none of these is the correct option. **(14) (B).** The centre of the circle lies on x– axis. Let a be the radius of the circle. Then, coordinates of the centre are (a, 0). The circle passes through (3,4). Therefore,

Here
$$
g = \frac{-47}{15}
$$
, $f = \frac{9}{15}$, $c = \frac{55}{15}$
\nHence the equation is
\n $15(x^2 + y^2) - 94x + 18y + 55 = 0$
\n**(13) (D).** Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.
\nPasses through $(-4, 3)$
\n $25-8g + 6f + c = 0$...(1)
\nTouches both lines
\n
$$
\frac{-g - f - 2}{\sqrt{2}} = \sqrt{g^2 + f^2 - c} = \frac{-g + f - 2}{\sqrt{2}}
$$
\n $\therefore f = 0 \therefore g^2 - 4g - 4 - 2c = 0$
\nAlso $c = 8g - 25$
\n $\therefore g = 10 \pm 3\sqrt{6}$, $f = 0$, $c = 55 \pm 24\sqrt{6}$
\nIt is easy to see that the answers given are not near to the values of g, f, c. Hence none of these is the correct option.
\n**(14) (B).** The centre of the circle lies on x– axis. Let a be the radius of the circle. Then, coordinates of the centre are
\n(a, 0). The circle passes through (3, 4). Therefore,
\n
$$
\sqrt{(a-3)^2 + (0-4)^2} = a \Rightarrow -6a + 25 = 0 \Rightarrow a = \frac{25}{6}
$$
\nSo, equation of the circle is $(x-a)^2 + (y-0)^2 = a^2$
\nor, $x^2 + y^2 - 2ax = 0$ or $3(x^2 + y^2) - 25x = 0$
\n**(15) (A).** Let centre be $(-h, -k)$ equation
\n $(x+h)^2 + (y+k)^2 = k^2$
\nAlso $-h + k = 1$
\n $\therefore h = k-1$ radius = k (touches x– axis)
\nTouches the line $4x-3y+4=0$
\n $\left| \frac{-4h - 3(-k) + 4}{5} \right| = k$...(3)
\n $\left| \frac{x-3y+8-8}{5} \right|$

$$
\overrightarrow{Also} - \overrightarrow{h} + \overrightarrow{k} = 1 \qquad \qquad \dots (2)
$$

 \therefore h = k–1 radius = k (touches x– axis) Touches the line $4x-3y+4=0$

Solving (2) and (3),
$$
h = \frac{1}{3}
$$
, $k = \frac{4}{3}$ (21)

Hence the circle is $\left(x + \frac{1}{3}\right)^2 + \left(y + \frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^2$

 \Rightarrow 9 (x² + y²) + 6x + 24 y + 1 = 0

- **(16) (C).** Let the circle cuts the x axis and y– axis at A and B respectively. If O is the origin, then $\angle AOB = 90^\circ$, and $A(5,0)$; $B(0,5)$ is the diameter of the circle. Then using diameter from the equation to the circle, we $get(x-5)(x-0)+(y-0)(y-5)=0$ $\Rightarrow x^2 + y^2 - 5x - 5y = 0$
- **(17) (B).** Let C be the centre of the given circle and C_1 be the centre of the required circle. Now $C = (2,3)$, $C =$ radius = 5 \therefore C₁ P = 3 \Rightarrow CC₁ = 2
	- \therefore The point C₁ divides internally, the line joining C and P in the ratio 2: 3

$$
\therefore \text{ coordinates of C}_1 \text{ are} \left(\frac{2 \times (-1) + 3 \times 2}{2 + 3}, \frac{2 \times (-1) + 3 \times 3}{2 + 3} \right)
$$

Hence (B) is the required circle.

(18) (D). Let the circle be

22) **(B).** Centre is (0, -3) and
$$
R = \sqrt{0^2 + 9 + 0}
$$

\n22) **(B).** Centre is (0, -3) and $R = \sqrt{0^2 + 9 + 0}$
\n33) **(C)** $R = \sqrt{0.3}$
\n34. Hence (B) is the required circle.
\n35. $\sqrt{2} + y^2 + 2gx + 2fy + c = 0$
\n46. $2y - 12f + c = 0$
\n57. $2y^2 + 2gx + 2fy + c = 0$
\n58. $2x^2 + 5y - 10 = 0$
\n59. $5x - 15 = 0$
\n60. $2x - 12f + c = 0$
\n61. $2x - 12f + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n7. $2y^2 + 2gx + 2fy + c = 0$
\n8. $2y^2 + 3x - 2f + c = 0$
\n9. $2y^2 + 3x - 2f + c = 0$
\n10. $2y^2 + 2y^2 - 12f + c = 0$
\n $2y^2 + 3y - 2x + 4y + 12f = 0$
\n $2y^2 + 2y^2 - 5x + 11y - 3 = 0$
\n $2y^2 + 2y^2 - 5x + 11y - 3 = 0$
\n**(A).** Since the first circle is 2x² + y² - 2x + 4y + c = 0
\n**(B)** $2x^2 + y^2 - 2x + 4y + c = 0$
\n**(B)** $2x^2 + y^2 - 2$

$$
\frac{g}{-30+35} = \frac{f}{-21+10} = \frac{1}{5-9} \therefore g = -\frac{5}{4}, f = \frac{11}{4}, c = -\frac{3}{2}
$$
 (25) (A). The circle is x^2

Hence the circle is $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

- **(19) (A).** Since the first circle is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, therefore its equation can be written as $x^2 + y^2 - 2x + 4y + c = 0$ If it passes through $(4,-2)$, then $16 + 4 - 8 - 8 + c = 0$ \Rightarrow c = -4
- **(20) (A).** Let $A = (\alpha, \beta)$; $B = (\gamma, \delta)$. Then $\alpha + \gamma = -2a$, $\alpha \gamma = -b^2$ and $\beta + \delta = -2p$, $\beta \delta = -q^2$ Now equation of the required circle is $(x - \alpha) (x - \gamma) + (y - \beta) (y - \delta) = 0$ \Rightarrow x² + y² - (α + γ) x - (β + δ) + α γ + β δ = 0 \Rightarrow x² + y² + 2ax + 2py - b² - q² = 0 From (1) -(3), -28+ag +121=0
 $\Rightarrow g_1 + 5f = 10$...(5)
 $\Rightarrow g_2 + 3f = 7 = 0$...(5)
 $\Rightarrow g_1 + 5f = 10 = 0$...(5)
 $\Rightarrow g_2 + 5f = 10 = 0$...(5)
 $\Rightarrow g_3 + 5f = 10 = 0$...(5)
 $\Rightarrow g_4 + 5f = 10 = 0$...(6)
 $\Rightarrow g_5 = 3f = 1$
 $\Rightarrow g_6 = 2f + 10 = 5$

4 (21) (A). L $\frac{3}{2}$ are always are $\frac{1}{2}$ **(O.B.- SOLUTIONS)** STUDY MATERIAL: MATHEMATICS
 $\frac{1}{3}$, $k = \frac{4}{3}$ (21) (A). Let a rod AB of length 'a' slides on x-axis and rod CD

of length 'b' slide on y - axis so that ends A, B, C and D

are always concyclic.
 (Q.B.- SOLUTIONS STUDY MATERIAL: MATHEMATICS
 $\frac{1}{3}$, $k = \frac{4}{3}$ (21) (A). Let a rod AB of length 'a' slides on x-axis and rod CD

of length 'b' slide on y - axis so that ends A, B, C and D

are always concyclic.
 (O.B.- SOLUTIONS STUDY MATERIAL: MATHEMATICS
 $h = \frac{1}{3}, k = \frac{4}{3}$ (21) (A). Let a rod AB of length 'a' sides on x-axis and rod CD

of length 'b' slide on y - axis so that ends A, B, C and D
 $\left(x + \frac{1}{3}\right)^2 + \left(y + \frac{4}{3}\$ **(21) (A).** Let a rod AB of length 'a' slides on x-axis and rod CD of length 'b' slide on y - axis so that ends A, B, C and D are always concyclic.

Let equation of circle passing through these ends is $x^2 + y^2 + 2gx + 2fy + c = 0$

3) $\binom{2}{3}$ (3) 3 (3) $\binom{3}{4}$ star and y- axis at star be contened to the circle, we

contened from the contened and C₁ be the contened of the given circle and C₁ be the contened of the given circle and C₁ be t = 0

e x – axis and y – axis at

e equation to the circle.

e equation of the circle.

e equation of the circle, we

e given circle and C₁ be the

e given circle and C₁ be the

six $x^2 + y^2 + 2gx + 2fy + c = 0$

ow C = (2,3 $+\frac{1}{3}$
 $-\frac{1}{3}$
 $-\frac{1}{3}$
 $-\frac{1}{3}$
 $-\frac{1}{3}$
 $-\frac{1}{3}$
 $-\frac{1}{3}$
 Obviously 2 ² g c = a and 2 ² f c = b \therefore 4 (g² – f²) = a² – b² \Rightarrow 4 [(-g)² – (-f)²] = a² – b² therefore locus of centre $(-g, -f)$ is $4(x^2 - y^2) = a^2 - b^2$.

(22) (B). Centre is $(0, -3)$ and $R = \sqrt{0^2 + 9 + 0} = 3$.

(23) (B). First find the centre. Let centre be (h, k), then

$$
\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2} \qquad \dots (i)
$$

and $k - 4h + 3 = 0$ ….(1) From (i), we get $-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$ or $4h + 4k - 28 = 0$ or $h + k - 7 = 0$...(iii) From (iii) and (ii), we get (h, k) as $(2, 5)$. Hence centre is (2, 5) and radius is 2. Now find the equation of circle. Obviously, circle $x^2 + y^2 - 4x - 10y + 25 = 0$ passes through $(2, 3)$ and $(4, 5)$. From (III) and (II), we get (II, *B*) as (2, 5). rence centre is

(2, 5) and radius is 2. Now find the equation of circle.

Obviously, circle $x^2 + y^2 - 4x - 10y + 25 = 0$ passes

through (2, 3) and (4, 5).
 (24) (**C).** As

- $3^{(-1)}$ (b) $^{(-1)}$ **(24) (C).** As the centre may be $(\pm 4, 0)$ and radius = 4.
- $\frac{1}{4}$, $f = \frac{1}{4}$, $c = -\frac{1}{2}$ (25) (A). The circle is $x^2 + y^2 \frac{1}{2}x = 0$. $x^2 + y^2 - \frac{1}{2}x = 0$.

Centre
$$
(-g, -f) = \left(\frac{1}{4}, 0\right)
$$
 and $R = \sqrt{\frac{1}{16} + 0 - 0} = \frac{1}{4}$.

(26) (C). Obviously from figure,

Radius is $r = \sqrt{4^2 + 3^2} = 5$

(28) (A). Let its centre be (h, k) , then $h - k = 1$...(i) **(35)** Also radius $a = 3$ Equation is $(x - h)^2 + (y - k)^2 = 9$ Also it passes through (7, 3)

i.e., $(7-h)^2 + (3-k)^2 = 9$ (ii)

We get h and k from (i) and (ii) solving simultaneously as

(4, 3). Equation is
$$
x^2 + y^2 - 8x - 6y + 16 = 0
$$
.

Since the circle $x^2 + y^2 - 8x - 6y + 16 = 0$ satisfies the given conditions.

- **(29) (D).** Obviously the centre of the circle is (4, 2) which should be the middle point of the ends of diameter. Hence the other end is (11, 2).
- **(30) (A).** Let point (x_1, y_1) on the diameter.

 \Rightarrow 2x₁ + 3y₁ = 3(i)

$$
16x_1 - y_1 = 4 \qquad \qquad \dots (ii)
$$

On solving (i) and (ii), we get centre,

$$
\Rightarrow x_1 = \frac{3}{10}, y_1 = \frac{4}{5}
$$

$$
\Rightarrow \overline{2}
$$

 \therefore Equation of circle,

$$
(x - x_1)^2 + (y - y_1)^2 = r^2 \Rightarrow \left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = r^2 \Rightarrow c = |a| \text{ or } c = 0
$$

$$
\Rightarrow c = |a| \text{ or } c = 0
$$

$$
\Rightarrow c = |a| \quad [\because c > 0]
$$

 \therefore Circle passes through (4, 6).

So,
$$
r^2 = \left(\frac{37}{10}\right)^2 + \left(\frac{26}{5}\right)^2 \Rightarrow r^2 = \frac{4073}{100}
$$
 centre

Required equation of circle is

$$
\left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{4073}{100}
$$

\n
$$
\Rightarrow 5(x^2 + y^2) - 3x - 8y = 200.
$$

- **(31) (D).** Given, equation of circle is $x^2 + y^2 3x 4y + 2 = 0$ \therefore $\frac{4}{2} \left| \frac{x}{2} \right| + \frac{y}{2} \left| \frac{y}{2} \right| + \frac{y}{2}$ and it cuts the x-axis. $x^2 + 0 - 3x + 2 = 0$ or $x^2 + 2x + 2 = 0$ or $x^2 + 7x + 2 = 0$ $x^2 - 3x + 2 = 0$ or $(x - 1)(x - 2) = 0$ or $x = 1, 2$.
Also CP is per Therefore the points are $(1,0)$ and $(2, 0)$.
- **(32) (C).** The other end is $(t, 3 t)$ So the equation of the variable circle is $(x-1)(x-t)+(y-1)(y-3+t)=0$ or $x^2 + y^2 - (1 + t)x - (4 - t)y + 3 = 0$ \therefore The centre (α, β) is given by

$$
\alpha = \frac{1+t}{2}, \beta = \frac{4-t}{2} \implies 2\alpha + 2\beta = 5
$$

Hence, the locus is $2x + 2y = 5$. (38)

(33) (A). Substituting $x = \frac{3y + 10}{4}$ in equation of circle, we

get a quadratic in y. Solving, we get two values of y as 2 and -6 from which we get value of x.

for isosceles Δ .

- (4, 3). Equation is $x^2 + y^2 8x 6y + 16 = 0$. Hence the locus of the centre (h, k) will be **(D).** Let the centre of the required circle $C_1 \equiv (h, k)$. Since it touches y–axis, so its radius $r_1 = h$. For the given circle centre $C_2 \equiv (3, 3)$, radius **S**
 CD). Let the centre of the required circle $C_1 = (h, k)$.

Since it touches y-axis, so its radius $r_1 = h$.

For the given circle centre $C_2 = (3, 3)$, radius
 $r_2 = \sqrt{9 + 9 - 14} = 2$. Since the circle touch externally, $C_1C_2 = r_1 + r_2$ $\Rightarrow (h-3)^2 + (k-3)^2 = (h+2)^2$ \Rightarrow k² - 10h - 6k + 14 = 0. $y^2 - 10x - 6y + 14 = 0$ Let the centre of the required circle C₁ = (h, k).

it touches y-axis, so its radius r₁ = h.

the given circle centre C₂ = (3, 3), radius
 $\overline{9+9-14}$ = 2. Since the circle touch externally, so
 $C_2 = r_1 + r_2$
 -3 **EXERENT OF A CONSTRUMER CONTROVANCE DESCRIPTION**

it to

the centre of the required circle C₁ = (h, k).

it to

the given circle centre C₂ = (3, 3), radius
 $\frac{1}{2} = r_1 + r_2$
 $\frac{1}{2}r_3 + (k-3)^2 = (h+2)^2$
 $-3)^2 + (k-3)^2$. Let the centre of the required circle C₁ = (h, k).

ce it touches y-axis, so its radius r₁ = h, k).

the given circle centre C₂ = (3, 3), radius
 $\sqrt{9+9-14} = 2$. Since the circle touch externally, so
 $1^{\binom{2}{2}}$ Let the centre of the required circle $C_1 = (n, k)$.

e it touches y-axis, so its radius $r_1 = h$.

the given circle centre $C_2 = (3, 3)$, radius $\sqrt{9+9-14} = 2$. Since the circle touch externally, so
 $C_2 = r_1 + r_2$
 $r_2 - 3)^$ EDMANUMEED LEADING

EVERY THE CHARGING CONTROVANCED LEADING

touches y-axis, so its radius r₁ = h.

touches y-axis, so its radius r₁ = h.

e given circle centre C₂ = (3, 3), radius
 $+9-14 = 2$. Since the circle touc
	- **(36) (A).** The centres of the two circles are $C_1(-a/2, 0)$ and

$$
C_2(0, 0)
$$
, and their redii are $\frac{|a|}{2}$ and c.

So, the two circles will touch each other if C_1C_2 = sum or difference of radii

$$
\begin{array}{lll}\n\cdots(i) & \Rightarrow \sqrt{(-a/2 - 0)^2 + (0 - 0)^2} = \left| c \pm \frac{|a|}{2} \right| \\
& \Rightarrow \frac{|a|}{2} = \left| c \pm \frac{|a|}{2} \right| \Rightarrow c \pm \frac{|a|}{2} = \frac{|a|}{2} \\
& \Rightarrow c - \frac{|a|}{2} = \frac{|a|}{2} \& c + \frac{|a|}{2} = \frac{|a|}{2} \\
\Rightarrow c = |a| \text{ or } c = 0 \\
& \Rightarrow c = |a| \quad [\because c > 0]\n\end{array}
$$

 4073 centre $(-8, 12)$ with respect to the line mirror $2 = \left| \frac{37}{10} \right| + \left| \frac{20}{5} \right| \Rightarrow r^2 = \frac{4073}{100}$ **(37) (D).** The centre of the required circle is the image of the $4x + 7y + 13 = 0$ and radius is equal to the radius of the given circle.

> Let (h, k) be the image of the point $(-8, 12)$ with respect to the line mirror. Then the mid-point of the line joining $C(-8, 12)$ and $P(h, k)$ lies on the line mirror.

$$
\therefore 4\left(\frac{h-8}{2}\right) + 7\left(\frac{k+12}{2}\right) + 13 = 0
$$

or $4h + 7k + 78 = 0$ ….(i)

Also CP is perpendicular to $4x + 7y + 13 = 0$

$$
\therefore \quad \frac{k-12}{h+8} \times -\frac{4}{7} = -1 \quad \text{or} \quad 7h-4k+104 = 0 \qquad \qquad \dots \text{(ii)}
$$

Solving (i) and (ii), $h = -16$, $k = -2$.

Thus the centre of the image circle is $(-16, -2)$. The radius of the image circle is same as the radius of $x^{2} + y^{2} + 16x - 24y + 183 = 0$ i.e., 5.

Hence the equation of the required circle is

$$
(x+16)^2 + (y+2)^2 = 5^2
$$
 i.e. $x^2 + y^2 + 32x + 4y + 235 = 0$.

(38) (A). The equation of circle passing through the point of intersection of circle and line can be written as $x^2 + y^2 - a^2 + \lambda(x - y + 3) = 0$

(34) (A). As base is constant and height varies and is maximum
 (34) (A). As base is constant and height varies and is maximum
 (34) (A). As base is constant and height varies and is maximum .: $4\left(\frac{h-8}{2}\right) + 7\left(\frac{k+12}{2}\right) + 13 = 0$

or $4h + 7k + 78 = 0$...(i)

Also CP is perpendicular to $4x + 7y + 13 = 0$

...(i)
 $\frac{k-12}{h+8} \times -\frac{4}{7} = -1$ or $7h - 4k + 104 = 0$...(ii)

Solving (i) and (ii), $h = -16$, $k = -2$.

T mid-point of the line joining
 $3 = 0$...(i)
 $4x + 7y + 13 = 0$...(ii)
 $k = -2$.
 $k = 2$ on the line mirror.
 $3 = 0$ (i)
 $4x + 7y + 13 = 0$ (ii)
 $4x + 104 = 0$ (ii)
 $k = -2$.

e circle is $(-16, -2)$. The ra-

is same as the radius of
 $= 0$ i.e., 5.

equired circle is
 $x^2 + y^2 + 32x + 4y + 235 = 0$.

pass line $y = x + 3$ because this line is a diameter of the circle.

$$
\therefore -\frac{\lambda}{2} - \frac{\lambda}{2} + 3 = 0 \Rightarrow \lambda = 3
$$

Thus equation of required circle is
\n
$$
(x^2 + y^2 - a^2) + 3(x - y + 3) = 0
$$

\n $\Rightarrow x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(39) (C). c_1 (0, 0), $r_1 = 3$ and c_2 (- α , -1), $r_2 = |\alpha|$ Circles touches each other if $c_1c_2 = r_1 \pm r_2$

$$
\sqrt{\alpha^2 + 1} = 3 \pm |\alpha| \; ; \; \alpha^2 + 1 = 9 + \alpha^2 \pm 6 |\alpha|
$$

6 | \alpha | = ±8 ; $\alpha = \pm 4/3$

- **(40) (B).** Put $x = r \cos \theta \& y = r \sin \theta \Rightarrow x^2 + y^2 = 2 4x + 6y$ $\Rightarrow x^2 + y^2 + 4x - 6y - 2 = 0$ \Rightarrow Centre = (-2, 3)
- **(41) (B).** Let the centre of the required circle be (x_1, y_1) and the centre of given circle is $(1, 2)$. Since radii of both (2) . circles are same, therefore, point of contact (5, 5) is the mid point of the line joining the centres of both circles. Hence $x_1 = 9$ and $y_1 = 8$. Hence the required equation
	- is $(x 9)^2 + (y 8)^2 = 25$

 \Rightarrow $x^2 + y^2 - 18x - 16y + 120 = 0$.

The point (5, 5) must satisfy the required circle. Hence the required equation is given by (B).

(42) (A). x- and y- intercepts of $2x + 3y$. $k = 0$ are $k/2$ and $k/3$. \therefore Area of the triangle = $\frac{1}{2} \left(\frac{k}{2} \right) \left(\frac{k}{3} \right) = 12 \Rightarrow k = 12$

> and $2x + 3y - 12 = 0$ is diameter to the circle $x^2 + y^2 - 6x - 4y = 0$

Because it passes through the center
$$
(3, 2)
$$

- **(43) (A).** By inspection
- **(44) (D).** $x(x-1)+y(y-1)=0$ $x^2 + y^2 - x - y = 0$ $(0,1)$ $(0,0)$ (1,0) $4k^2 + 9k^2 - 2k - 3k = 0$ \| $13k^2 - 5k = 0$ $13k = 5 \implies k = 5/13$ **(45) (D).** Equation of line whose slope is –1 and y-intercept 1 is **(47) (a).** A said y intercept by $G = 1$. Area of the triangle $= \frac{1}{2} \left(\frac{k}{2}\right) \left(\frac{k}{3}\right) = 12 \Rightarrow k = 12$. Maximum distance = $10 + 5 = 15$.

and $x^2 + y^2 - x - 0$ is diameter to the circle $x^2 + y^2 - 2 = 0$ (A2) **(C).** $y = mx + c$ and $x \rightarrow y = 0$
 $x^2 + y^2 - 6x - 4y = 0$

Because it passes through the center (3, 2)
 $x^2 + y^2 - 6x - 4y = 0$
 $x(x - 1) + y(y - 1) = 0$
 $x^2 + y^2 - x - y = 0$
 $x^2 + y^2 - x - y = 0$
 $x^2 + y^2 - x - y = 0$
 $x(x - 1) + y(y - 1) = 0$
 $x^2 + y^2 - x - y = 0$
 $x^2 +$ Because it passes through the center (3, 2)
 $2 + y^2 - x - y = 0$
 $x^2 - 2 + y^2 - x - y = 0$
 $3k^2 - 5k = 0$
 $3k^2 - 5k = 5/13$
 $x^2 + 9k^2 - x - y = 0$
 $3k^2 - 5k = 5/13$
 $x^2 - 2k - 3k = 0$
 $3k^2 - 5k = 5/13$
 $x^2 - 2k - 3k = 0$
 $x^2 - 2k - 3k =$
- $y = -x + \phi \Rightarrow x + y 1 = 0$ From the diagram, it is clear two circles can be drawn **(46) (A).** $(x-2)^2 = 9 \cos^2 \theta$ and $(y-1)^2 = 9 \sin^2 \theta$
- \implies $(x-2)^2 + (y-1)^2 = 9$ Centre $(2, 1)$ and $r = 3$
- given line will touch the circle if $p = r$

$$
\Rightarrow \frac{3-m}{\sqrt{9+16}} = 3 \Rightarrow 3-m = \pm 15
$$

(48)
\n**(B).** The two circles are
\n
$$
S_1 = (x - a_1)^2 + (y - b_1^2) = r_1^2
$$
\n...(i)
\n
$$
S_2 = (x - a_2)^2 + (y - b_2^2) = r_2^2
$$
\n...(ii)

The equation of the common tangent of these two circles is given by $S_1 - S_2 = 0$

EXAMPLEMATENATIES
\n
$$
-\frac{\lambda}{2} - \frac{\lambda}{2} + 3 = 0 \Rightarrow \lambda = 3
$$
\nThe equation of the common tangent of these two circles is given by S₁ - S₂ = 0
\nuse equation of required circle is
\n(x² + y² - a²) + 3(x - y + 3) = 0
\nx² + y² + 3x - 3y - a² + 9 = 0
\nr(2² + y² + 3x - 3y - a² + 9 = 0
\nr(2² + y² + 3x - 3y - a² + 9 = 0
\n= 0
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $x^2 + y^2 + 4x - 6y - 2 = 0$
\n $x^2 + 1 = 3 \pm |\alpha|$; $\alpha^2 + 1 = 9 + \alpha^2 \pm 6 |\alpha|$
\n $\alpha| = \pm 8$; $\alpha = \pm 4/3$
\n $\alpha = \pm 4/3$
\n $\alpha = 2/3$
\n $\$

(49) (C). The required point is the radical centre of the three given circles. The radical axes of these three circles taken in pairs are : $3x - 24 = 0$; $16y + 120 = 0$ and $-3x + 16y + 80 = 0$ Solving any two of these three equations, we get

x = 8, y =
$$
-\frac{15}{8}
$$
. Hence, the required point is $\left(8, -\frac{15}{2}\right)$.

x,-1),
$$
r_2 = |\alpha|
$$

\n(a²₂ - a²) + (b² - b²) = r² - r²
\nc² = r₁ ± r₂
\n= 9 + $\alpha^2 \pm 6 |\alpha|$
\n= 9 + $\alpha^2 \pm 6 |\alpha|$
\n(a³₂ - a²) + (b² - b²) = r² - r²
\nand -3x + 16y + 80 = 0
\nand -3x + 16y + 80 = 0
\nSolving any two of these three equations, we get
\n $x = 8$, $y = -\frac{15}{8}$. Hence, the required point is $\left(8, -\frac{15}{2}\right)$.
\n(1, 2). Since radii of both
\nout of contact (5, 5) is the
\nthe centres of both circles.
\n(b) (D). Here c₁ (1,3), r₁ = $\sqrt{1+9-9} = 1$
\n(1, 2) Since radii of both
\nthe entries of both circles.
\n(c) (-3,1), r₂ = $\sqrt{9+1-1} = 3$
\nHence the required equation
\nNow c₁c₂ = $\sqrt{(1+3)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17}$
\nc₁c₂>r₁+r₂
\n= 0.
\nHence the circle
\nby (B).
\nHence the circle
\nby (B).
\nHence the circle
\n $\begin{cases}\n51 \text{ (D)} \cdot r = \sqrt{4+1+20} = 5 \text{ ; } C = (2, 1) \\
\therefore Raximum distance = 10+5=15.\n\end{cases}$
\n= tan 45^o = 1
\nthe center (3, 2)
\n \therefore The equation is $y = x \pm 6\sqrt{2}$.
\n(53) (C). Centre is (2, -1).

Hence the circles are non- intersecting externally. Hence 4 tangents, two direct and two transverse tangents may be drawn.

(51) **(D).**
$$
r = \sqrt{4 + 1 + 20} = 5
$$
 ; $C = (2, 1)$
\n
$$
\therefore CP = \sqrt{(10 - 2)^2 + (7 - 1)^2} = 10
$$
\n
$$
\therefore \text{ Maximum distance} = 10 + 5 = 15.
$$

(52) (C). $y = mx + c$ is a tangent, if $c = \pm a\sqrt{1 + m^2}$, where $m = \tan 45^\circ = 1$

$$
\therefore
$$
 The equation is $y = x \pm 6\sqrt{2}$.

(53) (C). Centre is
$$
(2, -1)
$$
.

Therefore
$$
r = \left| \frac{3(2)-1}{\sqrt{10}} \right| = \frac{5}{\sqrt{10}}
$$

Now draw a perpendicular on $x - 3y = 0$, we get

$$
r = \left| \frac{2 - 3(-1)}{\sqrt{10}} \right| = \frac{5}{\sqrt{10}}.
$$

(54) (A). From formula of tangent at a point,

$$
x \left(\frac{ab^2}{a^2 + b^2} \right) + y \left(\frac{a^2 b}{a^2 + b^2} \right) = \frac{a^2 b^2}{a^2 + b^2} \Rightarrow \frac{x}{a} + \frac{y}{b} = 1.
$$

(55) (B). Since the tangents are parallel, therefore the distance between these two tangents will be its diameter i.e.,

diameter =
$$
\frac{34}{\sqrt{180}} = \frac{17}{3\sqrt{5}}
$$
. Hence, radius = $\frac{17}{6\sqrt{5}}$.

...(ii)

 2^{2}

3

.

- **(56) (A).** Let $S_1 = x^2 + y^2 2x + 6y + 6 = 0$ and $S_2 = x^2 + y^2 - 5x + 6y + 15 = 0$, then common tangent is $S_1 - S_2 = 0$ $\Rightarrow 3x = 9 \Rightarrow x = 3$.
- **(57) (C).** The equation of the tangent at *P*(3, 4) to the

circle $x^2 + y^2 = 25$ is $3x + 4y = 25$, which meets the

co-ordinate axes at
$$
A\left(\frac{25}{3},0\right)
$$
 and $B\left(0,\frac{25}{4}\right)$. If O be the
(63) **(C)**.

origin, then the ΔOAB is a right angled triangle with $OA = 25 / 3$ and $OB = 25 / 4$.

Area of the
$$
\triangle OAB = \frac{1}{2} \times OA \times OB
$$

= $\frac{1}{2} \times \frac{25}{3} \times \frac{25}{4} = \frac{625}{24}$.

(58) (B). Equation of BC (chord of contact) is

 $0.x + 1.y - (x + 0) + 2(y + 1) + 1 = 0$ or $-x + 3y + 3 = 0$ Equation of circle through B and C i.e., intersection of the given circle and chord of contact is

 $3 \t 4 \t 24$

$$
(x2 + y2 - 2x + 4y + 1) + \lambda(-x + 3y + 3) = 0.
$$

It passes through *A*(0, 1), so the equation of the required circle is $x^2 + y^2 - x + y - 2 = 0$. (65)

Aliter :Centre of the required circle is mid-point of *A*(0, 1) and centre of the given circle i.e., $(1, -2)$.

Therefore, centre
$$
\left(\frac{1}{2}, -\frac{1}{2}\right)
$$
 and radius $\sqrt{\frac{5}{2}}$.

Hence the circle is $x^2 + y^2 - x + y - 2 = 0$.

- **(59) (B).** Let $S = x^2 + y^2 2x + 4y$ then $S_1 = 0^2 + 1^2 - 2.0 + 4.1 = 5$ $T = x.0 + y.1 - (x + 0) + 2 (y + 1) = (-x + 3y + 2)$ \therefore The equation of the pair of tangent SS₁ = T² $(x^2 + y^2 - 2x + 4x + 4y)$ 5 = $(-x + 3y + 2)^2$ $\Rightarrow 4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$
- **(60) (C).** Given $\frac{T_1}{T_2} = \frac{4}{3}$, where T₁ and T₂ are the length of $\frac{T_1}{T_2} = \frac{4}{3}$, where T₁ and T₂ are the length of , where T_1 and T_2 are the length of (67)

tangents drawn to the given circle.

$$
\Rightarrow \frac{\sqrt{1+4+1+2-4}}{\sqrt{(1)^2+(2)^2-\frac{1}{3}-\frac{2}{3}+\frac{k}{3}}} = \frac{4}{3} \Rightarrow k = -\frac{21}{4}
$$
 (68) (A). R

(61) (A). Area
$$
PQCR = 2.\Delta PQC = 2 \times \frac{1}{2} L \times r
$$

Where $L =$ length of tangent and $r =$ radius of circle.

 $L = \sqrt{S_1}$ and $r = \sqrt{1 + 4 + 20} = 5$

Hence the required area $= 75$ sq. units

 (63) **(63) (C).** Dividing the equation of the circle by 2, we get (63) $B\left(0, \frac{25}{4}\right)$ If O be the Also then $x = 3$. So the required point is $(3, -1)$ **(62) (B).** Tangent at $(1,-2)$ to $x^2 + y^2 = 5$ is $x - 2y = 5$ To find the point of contact with second circle, we solve this equation with the equation of the second circle, so we have $(2y+5)^2 + y^2 - 8(2y+5) + 6y + 20 = 0$ \Rightarrow 5y² + 10y + 5 = 0 \Rightarrow (y + 1)² = 0 \Rightarrow y = -1 **EXERCISE A SURVEY SET AND REFAINING**
 EXERCISE A 4 + 20 = 5
 EXECUTED A SURVEY SET AND REFAINING
 EXECUTED A SURVEY $y^2 = 5$ **is** $x - 2y = 5$
 EXECUTED and circle, we solve
 equation of the second circle, so
 y^2 **EDENTADYANCED LEARNING**
 $+4+20 = 5$

rea = 75 sq. units

on x² + y² = 5 is x - 2y = 5

contact with second circle, we solve

e equation of the second circle, so
 $y^2 - 8(2y + 5) + 6y + 20 = 0$
 $\Rightarrow (y+1)^2 = 0 \Rightarrow y = -1$

e req **EDENTROVANCED LEARNING**

T + 4 + 20 = 5

area = 75 sq. units

2) to $x^2 + y^2 = 5$ is $x - 2y = 5$

contact with second circle, we solve

the equation of the second circle, so
 $y^2 - 8(2y + 5) + 6y + 20 = 0$
 $\Rightarrow (y + 1)^2 = 0 \Rightarrow y = -1$ S

L = $\sqrt{S_1}$ and $r = \sqrt{1 + 4 + 20} = 5$

Hence the required area = 75 sq. units
 (B). Tangent at (1,-2) to $x^2 + y^2 = 5$ is $x - 2y = 5$

To find the point of contact with second circle, we solve

this equation with the e **EDENTROVANCED LEARNING**
 $\overline{0} = 5$

5 sq. units
 $y^2 = 5$ is $x - 2y = 5$

with second circle, we solve

tion of the second circle, so
 $y + 5$) + 6y + 20 = 0
 1)² = 0 \Rightarrow y = -1

ired point is (3, -1)

f the circle **SPON ADVANCED LEARNING**

5

units

= 5 is x - 2y = 5

second circle, we solve

of the second circle, so

5) + 6y + 20 = 0

= 0 \Rightarrow y = -1

point is (3, -1)

circle by 2, we get

+ (5)² - $\frac{3}{2}$

 $\sqrt{3}$; and $r = \sqrt{1 + 4 + 20} = 5$

ce the required area = 75 sq. units

Tangent at (1,-2) to $x^2 + y^2 = 5$ is $x - 2y = 5$

rnd the point of contact with second circle, we solve

equation with the equation of the second circl $\sqrt{S_1}$ and $r = \sqrt{1 + 4 + 20} = 5$
the required area = 75 sq. units
angent at (1,-2) to $x^2 + y^2 = 5$ is $x - 2y = 5$
d the point of contact with second circle, we solve
quation with the equation of the second circle, so
ve (To find the point of contact with second circle, we solve
this equation with the equation of the second circle, so
we have $(2y+5)^2 + y^2 - 8(2y+5) + 6y + 20 = 0$
 $\Rightarrow 5y^2 + 10y + 5 = 0 \Rightarrow (y+1)^2 = 0 \Rightarrow y = -1$
Also then $x = 3$. So the r

$$
x^2 + y^2 = \frac{3}{2} \implies (x^2 + y^2 - \frac{3}{2}) = 0
$$

: length of the tangent =
$$
\sqrt{(1)^2 + (5)^2 - \frac{3}{2}}
$$

$$
=\sqrt{26-\frac{3}{2}}=\sqrt{\frac{49}{2}}=\sqrt{\frac{7}{2}}=\frac{7\sqrt{2}}{2}
$$

(64) (A). Let centre is
$$
(4 + 2B, B)
$$
.

$$
r = \left| \frac{8 + 4B - B + 1}{\sqrt{5}} \right|^2 = (2B + 2)^2 + (5 - B)^2
$$
; B = 1

(65) (B). The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and

$$
3x - 4y - \frac{7}{2} = 0 \text{ and so it is equal to}
$$

$$
\frac{4}{\sqrt{9 + 16}} + \frac{7/2}{\sqrt{9 + 16}} = \frac{3}{2}.
$$
Hence radius is $\frac{3}{4}$.

uation of BC (chord of contact) is
\n
$$
y-(x+0)+2(y+1)+1=0 \text{ or } -x+3y+3=0
$$
\n
$$
y-(x+0)+2(y+1)+1=0 \text{ or } -x+3y+3=0
$$
\n
$$
z^2-2x+4y+1)+\lambda(-x+3y+3)=0.
$$
\n
$$
z^2-2x+4y+1+2
$$
\n
$$
z^2-2x+4y+2+2=0.
$$
\n
$$
z^2-2x+4y+2=0.
$$
\n
$$
z^2-2x+4y+2=
$$

Least distance = $10 - r = 10 - 5 = 5$

(69) **(D).**
$$
3x + 4y - k = 0
$$
 touches
 $x^2 + y^2 - 16x = 0$
condition is
 $(8,0)$

centre to $3x + 4y - k = 0$

condition is
\n
$$
\perp
$$
 r distance from
\n \therefore Fhe
\n= radius of the circle

8

$$
\left| \frac{3(8) + 4(0) - k}{\sqrt{9 + 16}} \right| = 8
$$
; Centre = (8, 0) (74)

 \therefore 24 – k = 40 or 24 – k = – 40 \Rightarrow k = –16 ; k = 64 **(70) (B).** Let the equation of tangent be $y + 4 = m(x + 5) \implies mx - y + (5m - 4) = 0$

Clearly C = (-2, -3),
$$
r = \sqrt{4+9-8} = \sqrt{5}
$$

Since (1) is a tangent,

$$
\left| \frac{m(-2) + 3 + 5m - 4}{\sqrt{m^2 + 1}} \right| = \sqrt{5}
$$

 \cdots perpendicular distance from center = radius)

23.49-14
\n
$$
x^2+y^2-16x = 0
$$

\n $x^2+y^2-16x = 0$
\n $x^2-16x = 0$
\n x^2-16x

(71) (B). AB = length of Tangent to the circle from B.

$$
AB = \sqrt{x^2 + y^2 - \frac{3}{2}x + 2y} = \sqrt{4 + 1 - 3 + 2} = 2 \text{ units}
$$
\n(72) (A). $C = (3, 4)$ (77)

$$
r = \left| \frac{5(3) + 12(4) - 11}{\sqrt{25 + 144}} \right| = \left| \frac{15 + 48 - 11}{\sqrt{169}} \right| = \left| \frac{52}{13} \right| = 4
$$

$$
A = \pi r^2 = 16 \pi \text{ units}^2
$$

(73) (A). Let AB $(= 6)$ be the chord intercepted by the line $2x - 5y + 18 = 0$ from the circle and let CD be the perpendicular drawn from centre $(3, -1)$ to the chord AB.

i.e.
$$
AD = 3
$$
, $CD = \frac{2.3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

 \Box **EXERIBING**
 $x + y^2 - 16x = 0$ touches
 $x + 4y - k = 0$ touches
 $x + 4y^2 - 16x = 0$

Tradition is

Tradition is

Tradition is

Tradition is

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 3$

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 3$

Hence required e **a**
 $\begin{array}{c|c|c|c|c} \hline \textbf{Q} & \textbf{Q} & \textbf{B} & \textbf{S} & \$ **Q.B.- SOLUTIONS**

 $y-k=0$ touches
 $-16x=0$

on is
 $\tance from 63x+4y-k=0$
 $\frac{44(0)-k}{9+16}$ = 8; Centre = (8, 0)
 $k = 40$ or $24-k=-40 \Rightarrow k=-16$; $k=64$
 $m(x+5) \Rightarrow mx-y+(5m-4)=0$
 $x=40$ **CLE. SOLUTIONS** STUDY MATERIAL: MATHEMATICS
 $x^2 + y^2 - 16x = 0$
 $x^2 + y^2 - 16x = 0$

condition is
 $\pm r$ distance from
 $\pm \pi$ distance from
 $\pm \pi$ distance $\pm \pi$ or $\pm \pi$ or $\pm \pi$ or $\pm \pi$ or $\pm \pi$
 $\pm \pi$ distance **EXECUTE ANTIFIER ALL SET UDV MATERIAL:**
 $x + 4y - k = 0$ fouches
 $x + 4y - k = 0$ fouches

and tion is
 $x + 4y - k = 0$ found to its

c. $x + 4y - k = 0$

c. $x + 4y - k = 0$

and to $x + 4y - k = 0$
 $\frac{3(8) + 4(0) - k}{2} = 8$
 $\frac{3(8) + 4(0)$ m 1 **(O.B.- SOLUTIONS**) STUDY MATERIAL

strained $4y - k = 0$ touches

listic in the set of the circle

is the set of $3x + 4y - k = 0$

is the set of $3x + 4y - k = 0$

is the circle
 $x + 40 - k$
 $y^2 - 16x = 0$
 $y^2 - 16x = 0$
 $y^2 - 16x =$ Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$ Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$. (74) **(A).** Centre of the circle = $(1, -2)$ S STUDY MATERIAL: MATHEMATICS

i.e. $AD = 3$, $CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$.

(A). Centre of the circle = (1, -2)

Radius = Here $p = \frac{1}{\sqrt{2}} = 2\sqrt{2}$ STUDY MATERIAL: MATHEMATICS
 $D = 3$, $CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

fore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

required equation is $(x - 3)^2 + (y + 1)^2 = 38$.

Sentre of the circle = $(1, -2)$
 $s = \sqrt{1 + 4 + 4} = 3$
 $p = \frac{1 + 2 +$ STUDY MATERIAL: MATHEMATICS
 $B, CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$
 $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

aired equation is $(x - 3)^2 + (y + 1)^2 = 38$.

of the circle = (1, -2)
 $\sqrt{1 + 4 + 4} = 3$
 $\frac{+2 + 1}{\sqrt{2}} = 2\sqrt{2}$

chord = $2\$ STUDY MATERIAL: MATHEMATICS

= 3, $CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

re, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

equired equation is $(x - 3)^2 + (y + 1)^2 = 38$.

attre of the circle = (1, -2)
 $\sqrt{1 + 4 + 4} = 3$
 $= \frac{1 + 2 + 1}{\sqrt{2}} = 2\$ **ITIONS**

STUDY MATERIAL: MATHEMATICS

i.e. $AD = 3$, $CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$.
 (74) (A). Centre of the circle = $(1,$ UDY MATERIAL: MATHEMATICS
 $2 \cdot 3 - 5(-1) + 18 = \sqrt{29}$
 $\sqrt{2^2 + 5^2}$
 $2^2 + (\sqrt{29})^2 = 38$

atation is $(x - 3)^2 + (y + 1)^2 = 38$.
 $x = 3$
 $2\sqrt{2}$
 $= 2\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2$.

the pole
 $yy_1 - \frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7 =$ **SITERIAL: MATHEMATICS**
 $\frac{1)+18}{5^2} = \sqrt{29}$
 $(y-3)^2 + (y+1)^2 = 38$.
 (-2)
 $\frac{-p^2}{5^2} = 2\sqrt{9-8} = 2$.
 $(x + x_1) + \frac{5}{2}(y + y_1) - 7 = 0$
 $\frac{-3x_1 + 5y_1 - 14}{1} = 0$
 $\frac{5y_1 - 14}{2} = k$ say .e. $AD = 3$, $CD = \frac{1}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$.
 A). Centre of the circle $= (1, -2)$

dadius = $\sqrt{1 + 4 + 4} = 3$

Here $p = \frac{1 + 2 + 1}{\sqrt{2}} = 2\$ prefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

ace required equation is $(x - 3)^2 + (y + 1)^2 = 38$.

Centre of the circle = $(1, -2)$

iius = $\sqrt{1 + 4 + 4} = 3$

e $p = \frac{1 + 2 + 1}{\sqrt{2}} = 2\sqrt{2}$

ngth of chord = $2\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2$.

Le $AD = 3, CD = \frac{2.3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

efore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

be required equation is $(x - 3)^2 + (y + 1)^2 = 38$.

Centre of the circle = $(1, -2)$

us = $\sqrt{1 + 4 + 4} = 3$
 $p = \frac{1 + 2 + 1}{\sqrt{2}} = 2\sqrt{2}$

gth of ch = 3, $CD = \frac{2 \cdot 3 - 5(-1) + 18}{\sqrt{2^2 + 5^2}} = \sqrt{29}$

e, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

equired equation is $(x - 3)^2 + (y + 1)^2 = 38$.

tre of the circle = (1, -2)
 $\sqrt{1 + 4 + 4} = 3$
 $\frac{1 + 2 + 1}{\sqrt{2}} = 2\sqrt{2}$

of chord = $2\sqrt{a^2 - p^2} =$ state, $(a - 3 + (62) - 36)$
 $= 98$
 $= 24$
 $= 36$
 $= 24$
 $= 2$ quation is $(x - 3)^2 + (y + 1)^2 = 38$.

eircle = (1, -2)
 $\frac{1}{4} = 3$

= $2\sqrt{2}$

= = $2\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2$.

e the pole
 $2yy_1 - \frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7 = 0$
 $y_1 + 5y_1 + \frac{-3x_1 + 5y_1 - 14}{1} = 0$

iven line

= $\frac{$ the deptation is $(x - 3)^2 + (y + 1)^2 = 38$.

e of the circle = (1,-2)
 $\sqrt{1 + 4 + 4} = 3$
 $\frac{1 + 2 + 1}{\sqrt{2}} = 2\sqrt{2}$

chord = $= 2\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2$.
 $\frac{1}{1}, y_1$ be the pole
 $2xx_1 + 2yy_1 - \frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7$ = (1, -2)

 $\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2$.

ole
 $-\frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7 = 0$
 $y + \frac{-3x_1 + 5y_1 - 14}{1} = 0$

me

me
 $\frac{x_1 + 5y_1 - 14}{-28} = k$ say
 $\frac{5}{-4}$
 $\left(\frac{k-5}{4}\right) - 14 = -28 k$
 $\frac{5}{-8} = -112 k$
 $0 \Rightarrow k = 1$

Length of chord =
$$
2\sqrt{a^2 - p^2} = 2\sqrt{9 - 8} = 2
$$
.
(D). Let (x_1, y_1) be the pole

Radius =
$$
\sqrt{1+4+4} = 3
$$

\nHere $p = \frac{1+2+1}{\sqrt{2}} = 2\sqrt{2}$
\nLength of chord = = $2\sqrt{a^2 - p^2} = 2\sqrt{9-8} = 2$.
\n(D). Let (x_1, y_1) be the pole
\n \therefore Polar $2xx_1 + 2yy_1 - \frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7 = 0$
\nor $(4x_1 - 3)x + (4y_1 + 5)y + \frac{-3x_1 + 5y_1 - 14}{1} = 0$
\nComparing with given line
\n
$$
\frac{4x_1 - 3}{9} = \frac{4y_1 + 5}{1} = \frac{-3x_1 + 5y_1 - 14}{-28} = k \text{ say}
$$
\n $\therefore x_1 = \frac{9k + 3}{4}, y_1 = \frac{k - 5}{4}$
\nHence $-3(\frac{9k + 3}{4}) + 5(\frac{k - 5}{4}) - 14 = -28k$
\n $\Rightarrow -27k - 9 + 5k - 25 - 56 = -112k$
\n $\Rightarrow (-27 + 5 + 112)k = 90 \Rightarrow k = 1$
\nPole is $x = \frac{9 + 3}{4} = 3, y = \frac{1 - 5}{4} = -1$ $\therefore (3, -1)$
\n(A). The polar of point (p, q) with respect to the circle
\n $x^2 + y^2 = a^2$ is $px + qy = a^2$
\nThis line touches $(x - c)^2 + (y - d)^2 = b^2$
\n $\left| \frac{c_1 + d_0}{x^2 + 1} \right|$

or
$$
(4x_1-3)x + (4y_1 + 5)y + \frac{-3x_1 + 3y_1 - 14}{1} = 0
$$

Comparing with given line

(A). Centre of the circle = (1, -2)
\nRadius =
$$
\sqrt{1+4+4} = 3
$$

\nHere $p = \frac{1+2+1}{\sqrt{2}} = 2\sqrt{2}$
\nLength of chord = = $2\sqrt{a^2 - p^2} = 2\sqrt{9-8} = 2$.
\n(D). Let (x_1, y_1) be the pole
\n \therefore Polar $2xx_1 + 2yy_1 - \frac{3}{2}(x + x_1) + \frac{5}{2}(y + y_1) - 7 = 0$
\nor $(4x_1 - 3) x + (4y_1 + 5)y + \frac{-3x_1 + 5y_1 - 14}{1} = 0$
\nComparing with given line
\n
$$
\frac{4x_1 - 3}{9} = \frac{4y_1 + 5}{1} = \frac{-3x_1 + 5y_1 - 14}{-28} = k \text{ say}
$$

\n $\therefore x_1 = \frac{9k+3}{4}, y_1 = \frac{k-5}{4}$
\nHence -3 $\left(\frac{9k+3}{4}\right) + 5 \left(\frac{k-5}{4}\right) - 14 = -28$ k
\n⇒ -27 k-9+5k-25-56 = -112 k
\n⇒ (-27+5+112) k = 90 ⇒ k = 1
\nPole is $x = \frac{9+3}{4} = 3, y = \frac{1-5}{4} = -1$ ∴ (3, -1)
\n(A). The polar of point (p, q) with respect to the circle
\n $x^2 + y^2 = a^2$ is $px + qy = a^2$
\nThis line touches $(x-c)^2 + (y-d)^2 = b^2$
\n $\therefore \frac{cp + dq - a^2}{\sqrt{p^2 + q^2}} = b$
\n⇒ (a²- cp - dq)² = b² (p² + q²).
\n(B). Here equation of the given circle is $x^2 + y^2 - 2x = 0$
\nThis clearly passes through origin
\nHence if (x_1, y_1) be midpoint of the chord then its equation

$$
x_1 = \frac{9k+3}{4}, y_1 = \frac{k-5}{4}
$$

Hence $-3(\frac{9k+3}{4})+5(\frac{k-5}{4})-14=-28k$
\n⇒ $-27k-9+5k-25-56=-112k$
\n⇒ $(-27+5+112)k=90 \Rightarrow k=1$
\nPole is $x = \frac{9+3}{4} = 3, y = \frac{1-5}{4} = -1$ ∴ $(3, -1)$
\n**(A).** The polar of point (p, q) with respect to the circle $x^2 + y^2 = a^2$ is $px + qy = a^2$
\nThis line touches $(x-c)^2 + (y-d)^2 = b^2$
\n∴ $\left| \frac{cp+dq-a^2}{\sqrt{p^2+q^2}} \right| = b$
\n⇒ $(a^2 - cp - dq)^2 = b^2 (p^2 + q^2)$.
\n**(B).** Here equation of the given circle is $x^2 + y^2 - 2x = 0$
\nThis clearly passes through origin
\nHence if (x_1, y_1) be midpoint of the chord then its equation

Pole is
$$
x = \frac{9+3}{4} = 3
$$
, $y = \frac{1-5}{4} = -1$ $\therefore (3,-1)$

(76) (A). The polar of point (p, q) with respect to the circle
\n
$$
x^2 + y^2 = a^2
$$
 is $px + qy = a^2$
\nThis line touches $(x - c)^2 + (y - d)^2 = b^2$

$$
\therefore \left| \frac{\text{cp} + \text{dq} - a^2}{\sqrt{\text{p}^2 + \text{q}^2}} \right| = b
$$

⇒ $\lim_{\rho \to 0} 2 \ln \left| \frac{3}{\rho + 4} - \frac{1}{2} \right| \le 1/2$

⇒ $2m^2 - 4m + m - 2 = 0 \Rightarrow 2m(m - 2) + (m - 2) = 0$

∴ Equations of tangents are
 $y + 4 = -\frac{1}{2}(x + 5) \Rightarrow x + 2y + 13 = 0$
 $x + 4 = -\frac{1}{2}(x + 5) \Rightarrow x + 2y + 13 = 0$

AB = length of Tangent to 1-12

1-12

1-12

1-12

1-14

1-14

1-14

1-14

1-14

1-14

1-14

1-14

2-0

2x-y+13=0

2x-y+13=0

2x-y+13=0

2

2x-y-3=0

2x-y-3= +1|
 $9 = \frac{9k+3}{1} - \frac{28}{28}$
 $\frac{1}{2}$ (m+1 m-2=0 ⇒ 2m(m-2)+(m-2)=0
 $\sinh 0$ m+m-2=0 ⇒ 2m(m-2)+(m-2)=0
 $2k + 5$) ⇒ 2x – y + 6=0
 $\frac{1}{2}$ (x + 5) ⇒ x + 2y + 13 = 0
 $\frac{1}{2}$ (x + 5) ⇒ x + 2y + 13 = 0
 $\frac{4}{2}$ (x + r (nm -2 = 0 = 2m² - 3m - 2= 0

2m² - 4m² - 6m - 4 = 0 ⇒ 2m(m - 2) + 0

= - 2m² - 4m² - 5m - 2= 0

Equations of tangents are

Fractions of tangents are
 $y+4=2(x+5)$ ⇒ $x+2y+13=0$

= - 2x k

= - 2x k

= - 2x k
 ring with given line
 $\frac{3}{5} = \frac{4y_1 + 5}{1} = \frac{-3x_1 + 5y_1 - 14}{-28} = k$ say
 $\frac{9k + 3}{4}$, $y_1 = \frac{k - 5}{4}$
 $-3(\frac{9k + 3}{4}) + 5(\frac{k - 5}{4}) - 14 = -28 k$
 $\frac{k - 9 + 5k - 25 - 56 = -112 k}{k + 5 + 112}$ $k = 90 \Rightarrow k = 1$
 $x = \frac{9 + 3}{4} = 3$, $y =$ \Rightarrow $(a^2 - cp - dq)^2 = b^2 (p^2 + q^2)$. (77) **(B).** Here equation of the given circle is $x^2 + y^2 - 2x = 0$ This clearly passes through origin Hence if (x_1, y_1) be midpoint of the chord then its equation is given by $T = S_1$ $xx_1 + yy_1 - (x + x_1) = x_1^2 + y_1^2 - 2x_1$ or $xx_1 + yy_1 - x = x_1^2 + y_1^2 - x_1$ This passes through the origin (0, 0) \therefore $x_1^2 + y_1^2 - x_1 = 0$ \therefore Locus reqd. is $x^2 + y^2 = x$ **(78) (C).** Here equation of the circle $(x^2 + y^2 - 10x) + \lambda(y - 2x) = 0$ Y_{\star} Now centre C $(5 + \lambda, -\lambda/2)$ lies on the Chord again

 \overline{O} \overline{C} $\overline{$

11E
\n
$$
\frac{-\lambda}{2} = 2(5 + \lambda) \Rightarrow \frac{-5\lambda}{2} = 10
$$
\n
$$
\therefore \frac{-\lambda}{2} = 2(5 + \lambda) \Rightarrow \frac{-5\lambda}{2} = 10
$$
\n
$$
\therefore \lambda = -4
$$
\nHence $x^2 + y^2 = 10x - 4y + 8x = 0$
\n $x^2 + y^2 = 2x - 4y = 0$
\n(C). The equation of the common chord is
\n
$$
[(x-a)^2 + y^2 - c^2] - [x^2 + (y-b)^2 - c^2] = 0
$$
\n
$$
\Rightarrow 2ax - 2by - a^2 + b^2 = 0 \qquad ...(1)
$$
\nNow $p =$ length of perpendicular from (a, 0) on (1)
\n
$$
= \frac{2a^2 - a^2 + b^2}{\sqrt{4a^2 + 4b^2}} = \frac{1}{2}\sqrt{a^2 + b^2}
$$
\n
$$
= 2\sqrt{c^2 - p^2} = 2\sqrt{c^2 - \frac{a^2 + b^2}{4}} = \sqrt{4c^2 - a^2 - b^2}
$$
\n(B) Here is the intersection point of chord and found by solving the equation of circle with of given line, therefore, the points of intersection point of chord and (24, 7)
\n
$$
= 2\sqrt{c^2 - p^2} = 2\sqrt{c^2 - \frac{a^2 + b^2}{4}} = \sqrt{4c^2 - a^2 - b^2}
$$
\n(B) Here is the intersection point of chord and found by solving the equation of circle with the equation of the line.

(79) (C). The equation of the common chord is $[(x-a)^2 + y^2 - c^2] - [x^2 + (y-b)^2 - c^2] = 0$ \Rightarrow 2ax – 2by – $a^2 + b^2 = 0$...(1) Now $p =$ length of perpendicular from $(a, 0)$ on (1)

$$
=\frac{2a^2-a^2+b^2}{\sqrt{4a^2+4b^2}}=\frac{1}{2}\sqrt{a^2+b^2}
$$
\n(85)

 \therefore length of common chord

$$
=2\sqrt{c^2-p^2}=2\sqrt{c^2-\frac{a^2+b^2}{4}}=\sqrt{4c^2-a^2-b^2}
$$
 (–4,–5) and (5 '5').

(80) (D). Here circles are $x^2 + y^2 - 2x - 2y = 0$...(1) $x^2 + y^2 = 4$ $= 4$...(2)

> Now c₁ (1, 1), r₁ = $\sqrt{1^2 + 1^2} = \sqrt{2}$ (86) (D). $c_2(0, 0), r_2 = 2$ If θ is the angle of intersection then

Hence
$$
x^2 + y^2 = 1
$$
 or $x = 4y + 8x = 0$
\n (79) (C). The equation of the common chord is
\n $(x-4)^2 + y^2 = 2x - 4y = 0$
\n $(x-4)^2 + y^2 = 2x - 4y = 0$
\n $(x-4)^2 + y^2 = 2x - 2y = 4x + 4y^2 = 0$
\nNow p = length of perpendicular from (a, 0) on (1)
\n $= \frac{2a^2 - a^2 + b^2}{\sqrt{4a^2 + b^2}} = \frac{1}{2}\sqrt{a^2 + b^2}$
\n $= 2\sqrt{c^2 - p^2} = 2\sqrt{c^2 - \frac{a^2 + b^2}{4}} = \sqrt{4c^2 - a^2 - b^2}$
\n \therefore length of common chord
\n $x^2 + y^2 = 2x - 2y = 0$
\n $x^2 + y^2 = 2x - 2y = 0$
\n $x^2 + y^2 = 4$
\n $x^2 + y^2 =$

$$
C_1 C_2 = \sqrt{1+4} = \sqrt{5} = 2.23.
$$

Also r₁ = 2, r₂ = $\sqrt{1+4-3} = \sqrt{2} = 1.41$
∴ r₁ - r₂ < C₁C₂ < r₁ + r₂

 (82)

 \Rightarrow circles intersect each other. **(83) (A).** The centres of the two circles are $C_1(-1, 1)$ and $C_2(1, 1)$ and both have radii equal to 1. We have: $C_1C_2 = 2$ and sum of the radii = 2 So, the two circles touch each other externally. The equation of the common tangent is obtained by subtracting the two equations. The equation of the common tangent is $4x = 0 \implies x = 0$. Putting $x = 0$ in the equation of the either circle, we get $y^2 - 2y + 1 = 0 \implies (y - 1)^2 = 0 \implies y = 1.$ Hence, the points where the two circles touch is $(0,1)$.

(84) (D). Any line through $(0, 0)$ be $y - mx = 0$ and it is a tangent to circle $(x - 7)^2 + (y + 1)^2 = 25$, if

$$
\frac{-1-7m}{\sqrt{1+m^2}} = 5 \Rightarrow m = \frac{3}{4}, -\frac{4}{3}.
$$

Therefore, the product of both the slopes is -1 .

i.e.,
$$
\frac{3}{4} \times -\frac{4}{3} = -1
$$
.

Hence the angle between the two tangents is $\pi/2$.

(Q.B.- SOLUTIONS

(**84) (D).** Any line through (0, 0)

tangent to circle $(x - 7)^2 + (y - 8x) = 0$
 $\sqrt{1 + m^2} = 5 \Rightarrow m = \frac{3}{4}, -\frac{4}{3}$.

(38) Therefore, the product of bot
 $y-b)^2-c^2 = 0$

...(1)

i.e., $\frac{3}{4} \times -\frac{4}{3} = -1$. **(O.B.- SOLUTIONS)**
 $\Rightarrow \frac{-5\lambda}{2} = 10$
 $x-4y+8x = 0$
 $-4y = 0$
 $x^2 + (y+1)^2 = 25$, if
 $x^2 + (y-1)^2 = 25$
 $y^2 = 0$ **(85) (C).** Here the intersection point of chord and circle can be found by solving the equation of circle with the equation of given line, therefore, the points of intersection are

$$
(-4,-3) \text{ and } \left(\frac{24}{5},\frac{7}{5}\right).
$$

 Hence the midpoint is 5 4 , ⁵ 2 2 5 7 3 , ² 5 ²⁴ ⁴ .

(86) (D). Let chord AB is $y = mx$ (i) Equation of CM, $x + my = \lambda$ It is passing through (a, 0)

$$
∴ x + my = a
$$
(ii)

From (i) and (ii), $x + y \cdot \frac{y}{x} = a \implies x^2 + y^2 = ax$ $x + y \cdot \frac{y}{x} = a \implies x^2 + y^2 = ax$

$$
\Rightarrow x^2 + y^2 - ax = 0
$$
 is the locus of the centre of the circle.
(87) (C). T = 0 \Rightarrow 2x + 2y = 1

- \Rightarrow x + y = 1/2 **(88) (B).** The common chord of the given circles is $6x^2 + 14y + c + d = 0$ Since $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$. So, (i) passes through the centre of the second circle i.e. $(1, -4)$. \therefore 6 – 56 + c + d = 0 \Rightarrow c + d = 50
- **(89) (D).** $(x-1)^2 + (y-2)^2 = 1$; $x^2 + y^2 2x 4y + 4 = 0$ equation of polar of point (4, 4) is $4x+4y-(x+4)-2(y+4)+4=0$ \Rightarrow 3x + 2y - 8 = 0
- **(90) (C).** Let P (h, k) be the point. Then, the chord of contact of tangents drawn from P to the circle $x^2 + y^2 = a^2$ is $hx + ky = a^2$.

$$
x^{2} + y^{2} = a^{2}
$$
 is
$$
hx + ky = a^{2}
$$
. The combined equation of the lines joining the (centre) origin to the points of intersection of the circle
$$
x^{2} + y^{2} = a^{2}
$$
 and the chord of contact of tangents drawn.

from P (h, k) is a homogeneous equation of second degree given by

$$
x^2 + y^2 = a^2 \left(\frac{hx + ky}{a^2}\right)^2
$$
 or $a^2 (x^2 + y^2) = (hx + ky)^2$

The lines given by the above equation will be perpendicular if coeff. of x^2 + coeff. of y^2 = 0 \Rightarrow h² - a² + k² - a² = 0 h² + k² = 2a² So, locus of (h,k) is $x^2 + y^2 = 2a^2$. .

Clearly, it is a circle of radius $\sqrt{2}$ a.

(91) (C). Since the chord makes equal intercepts of length a on the coordinate axes.

So, its equation can be written as $x \pm y = \pm a$. This line meets the given circle at two distinct points. So, length of the perpendicular from the centre $(0, 0)$ of
the given girels must be less than the radius. the given circle must be less than the radius. written as $x \pm y = \pm a$.

(98) (D), $x^2 + y^2 + 3x + 2y - 8 = 0$

Intercept made by y-axis = 2x

dicular from the centre (0, 0) of

less than the radius.

(99) (C). Circle with (2, 0), (0, 1) as e

(x - 2)x + (y - 1)y = 0 and li Conclude the distinct points.

Let $\begin{aligned}\n\text{where at two distinct points.} \\
\text{and } \text{from the centre } (0, 0) \text{ of } \\
\text{in the radius.} \\
\text{where at two distinct points.} \\
\text{where at two distinct points.} \\
\text{(or -2)}\nx + (y - 1)y &= 0 \text{ and line } \\
\text{equated to circle } x^2 + y^2 = a^2, \\
\text{equized to circle } x^2 + y^2 = a^2, \\
\text{Equation 2: } x(x - 2) + y(y - 1) + \lambda(2y + x - 2) \\
\text{Equation 3$ + y² = 2a².

radius $\sqrt{2}$ a.

radius $\sqrt{2}$ a.

radius $\sqrt{2}$ a.

written as x ± y = ±a.

(98) (b) x² + y² + 3x + 2y - 8 = 0

written as x ± y = ±a.

(98) (b) x² + y² + 3x + 2y - 8 = 0

en circle at two d tius $\sqrt{2}$ at $\sqrt{2}$ condinate axes.

quation can be written as $x \pm y = \pm a$.

(98) (D). $x^2 + y^2 + 3x + 2y - 8 = 0$

eneets the given circle at two distinct points.

Intercept made by y -axis = 2 \sqrt{a}
 \sqrt{a} is $y - 0 = \left(\frac{-1}{2}\right)(x - 2)$ or $2y +$

i.e.
$$
\left| \frac{\pm a}{\sqrt{2}} \right| < \sqrt{8} \Rightarrow a^2 < 16 \Rightarrow |a| < 4.
$$

(92) (C). Here area of \triangle PQR is required Now chord of contact w.r. to circle $x^2 + y^2 = a^2$, , and point (h, k) hx + ky – $a^2 = 0$

Perp. from (h, k), PN =

\n
$$
\frac{h^2 + k^2 - a^2}{\sqrt{h^2 + k^2}}
$$
\nAlso length QR

\nIt passes

$$
=2\sqrt{a^{2}-\frac{(a^{2})^{2}}{h^{2}-k^{2}}}=\frac{2a\sqrt{h^{2}+k^{2}-a^{2}}}{\sqrt{h^{2}+k^{2}}}\qquad \qquad \frac{c(c-1)}{\Rightarrow 3c^{2}}
$$

Now end of contact w.1. to circle
$$
x^2 + y^2 - a^2
$$
,
\nand point (h, k) $hx + ky - a^2 = 0$
\n $h^2 + k^2 - a^2$
\nAlso length QR
\n $= 2\sqrt{a^2 - \frac{(a^2)^2}{h^2 + k^2}}$
\n $\therefore \Delta PQR = \frac{1}{2}(QR)(PN)$
\n $= \frac{1}{2} 2a \sqrt{\frac{h^2 + k^2 - a^2}{h^2 + k^2}}$
\n $= \frac{a}{2} (h^2 + k^2 - a^2)$
\n $= \frac{1}{2} (h^2 + k^2 - a^2)$
\n $= \frac{1}{2} (2R)(PN)$
\n $= \frac{1}{2} (2R)(PN)$
\n $= \frac{1}{2} (2R)(4P)$
\n $= \frac{a}{2} (2R)(4P)$

$$
\frac{1}{2} \text{erp. from (h, k), PN} = \frac{h^2 + k^2 - a^2}{h^2 + k^2}
$$
\n
$$
= 2\sqrt{a^2 - \frac{(a^2)^2}{h^2 + k^2}} = \frac{2a\sqrt{h^2 + k^2 - a^2}}{\sqrt{h^2 + k^2}}
$$
\n
$$
= \frac{1}{2} 2a \sqrt{\frac{h^2 + k^2 - a^2}{h^2 + k^2}} = \frac{(h^2 + k^2 - a^2)}{\sqrt{h^2 + k^2}}
$$
\n(100) (A). Let the equation of the region is $(x^2 + y^2 - a^2) + \lambda(y - x - 3) = 0$
\n
$$
= \frac{1}{2} 2a \sqrt{\frac{h^2 + k^2 - a^2}{h^2 + k^2}} = \frac{(h^2 + k^2 - a^2)}{\sqrt{h^2 + k^2}}
$$
\n(100) (A). Let the equation of the region is $(x^2 + y^2 - a^2) + \lambda(y - x - 3) = 0$ since its centre $(\lambda/2, -\lambda/2)$ lie $2a^2 - 17c + 14 = 0$ or $c = \frac{1}{3}$
\n
$$
= \frac{a (h^2 + k^2 - a^2)^{3/2}}{(h^2 + k^2)}
$$
\n(101) (B). Let the equation of the region $(x^2 + y^2 - 3x - 3y - a^2 + 9 = 0$ $(x^2 + y^2 - 5x + 8) + (x^2 + y^2 - 1)(x^2 + 3x - 3y - a^2 + 9) = 0$
\n
$$
= \frac{a (h^2 + k^2 - a^2)^{3/2}}{(h^2 + k^2)}
$$
\n(101) (B). Let the equation of the region is $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$ $(x^2 + y^2 - 6x + 8) + (x^2 + y^2 - 1)(x^2 + 9) = 0$
\n
$$
= \frac{a (x^2 + y^2 - 6x + 8) + (x^2 + y^2 - 1)(x^2 + 9) - 2}{x^2 + y^2 + 4x - 5y
$$

$$
(x-5)^2 + (y-6)^2 = r^2
$$
 (2)
\nRadical axis is, Eq. (1) – Eq. (2)
\n
$$
-4x + 10x - 6y + 12y + 4 + 9 - 25 - 36 = 0
$$
\n
$$
6x + 6y - 48 = 0; x + y - 8 = 0
$$
\n(95) (A). $2g_1g_2 + 2f_1f_2 = c_1 + c_2$
\n
$$
\Rightarrow 2(1) + \frac{5}{2} \left(-\frac{7}{6}\right) = \frac{1}{2} + k \Rightarrow k = -\frac{17}{12}
$$

Q.B. SOLUTIONS STUDY MATERIAL: MATHEMATICS
\n
$$
\left(\frac{hx+ky}{a^2}\right)^2 \text{ or } a^2(x^2+y^2) = (hx+ky)^2
$$
\n
$$
\left(\frac{bx+ky}{a^2}\right)^2 \text{ or } a^2(x^2+y^2) = (hx+ky)^2
$$
\n
$$
\left(\frac{5x^2+9^2}{2}-8^2=0 \Rightarrow x-4y+5=0 \mid \therefore x=5-4(9/7)=1/7
$$
\n
$$
= -6x-24y+30=0 \Rightarrow x+3y=3=0
$$
\n
$$
= -6x-24y+30=0 \Rightarrow (x,y) = (-1/7,9/7)
$$
\n
$$
= -6x-24y+30=0 \Rightarrow (x,y) = (-1/7,9/7)
$$
\n
$$
= -21y+27=0
$$
\n
$$
= -21y+27
$$

 \therefore Number of real circles cutting orthogonally given imaginary circle is zero.

(98) (D). $x^2 + y^2 + 3x + 2y - 8 = 0$

Intercept made by y-axis =
$$
2\sqrt{f^2 - C} = 2\sqrt{(1) + 8} = 6
$$

(C). Circle with $(2, 0)$, $(0, 1)$ as end points of diameter is $(x - 2)x + (y - 1)y = 0$ and line through these two points

is
$$
y-0 = \left(\frac{-1}{2}\right)(x-2)
$$
 or $2y + x - 2 = 0$

Family of circles through these two points are

$$
x(x-2) + y(y-1) + \lambda(2y + x - 2) = 0.
$$

It means through (4, 5).

It passes through (4, 5).

i.e.,
$$
4(2) + 5(4) + \lambda(10 + 4 - 2) = 0 \implies \lambda = \frac{-7}{3}
$$
.

Hence equation of circle is

$$
x(x-1) + y(y-1) - \frac{7}{3}(2y + x - 2) = 0
$$

It passes through (0, c), therefore

$$
c(c-1) - \frac{7}{3}(2c-2) = 0
$$

$$
\Rightarrow 3c^2 - 17c + 14 = 0
$$
 or $c = \frac{14}{3}$ and 1.

- ⇒ | a | < 4.

required

o circle x² + y² = a²,
 $x(x-2) + y(y-1) + \lambda(2y + x 2) = 0$
 $x(x-2) + y(y-1) + \lambda(2y + x 2) = 0$
 $x(x-2) + y(y-1) + \lambda(2y + x 2) = 0$

It passes through (4, 5).

i.e., 4(2) + 5(4) + $\lambda(10 + 4 2) = 0$

Hence equa $\sqrt{8}$ \Rightarrow $a^2 < 16 \Rightarrow |a| < 4$.

Fig. $y - 0 = (\frac{-1}{2})(x - 2)$ or $2y + x$

Fig. $y - 0 = (\frac{-1}{2})(x - 2)$ or $2y + x$

Family of circles through these
 $x(x - 2) + y(y - 1) + \lambda(2y + x - 2)$

It passes through (4, 5).

Fig. $x(x - 2) + y(y - 1) + \lambda(2y + x$ 3. Is equation can be written as $x \pm y = 4$.

In this line meets the given circle at two distinct points.

a length of the perpendicular from the centre (0, 0) of

a $\left[\frac{1}{\sqrt{2}}\right] < \sqrt{8} = a^2 < 16 > |a| < 4$.

But there are o Here per of the perpendicular from the centre (0, 0) of

of the perpendicular from the centre (0, 0) of

circle must be less than the radius.
 $\sqrt{8} \Rightarrow a^2 < 16 \Rightarrow |a| < 4$.

area of $\triangle POR$ is required
 $\frac{a^2}{2} + b^2 = a^2$,
 transition of the state of the direct three periodic div y-axis = $2\sqrt{f^2 - C} = 2\sqrt{(1)}$

line can be entre (0, 0) of (99) (C). Circle with (2, 0), (0, 1) as end points of area of $\triangle PQR$ is required

(a) do foontaat w.r. to circle $x^2 + y^2 = a^2$,

(b, k) hx + ky = $a^2 = 0$
 $x(x - 2) + y(y - 1) + \lambda(2y + x - 2) = 0$
 $x(x - 1) + y(y - 1) - \frac{7}{3}(2y + x - 2) = 0$

Hence equation of circle is
 $x(x - 1) + y(y - 1) - \frac{7}{3}($ of contact w.r. to circle $x^2 + y^2 = a^2$,
 $x(x - 1) + y(y - 1) = \lambda(2y + x - 2)$
 $x(x - 2) + y(y - 1) = \lambda(2y - 17c + 14 = 0 \text{ or } c = \frac{1}{3}$
 $x^2 - k^2 = \frac{2a\sqrt{h^2 + k^2 - a^2}}{\sqrt{h^2 + k^2}}$

(100) (A). Let the equation of the require
 $x^2 + y^2 = 2$
 $\sqrt{8}$

a of $\triangle PQR$ is $y-0 = \left(\frac{-1}{2}\right)(x-2)$ or $2y + x$

f contact w.r. to circle $x^2 + y^2 = a^2$,
 $x(x-2) + y(y-1) + \lambda(2y + x - 2)$
 $x(x-2) + y(y-1) + \lambda(2y + x - 2)$
 $x(x-2) + y(y-1) + \lambda(2y + x - 2)$

It passes through (4, 5).
 $\frac{c}{\sqrt{h^2 + k$ It passes through (4, 5).
 $\frac{k^2 - a^2}{a^2 + k^2}$
 $\frac{k^2 - a^2}{b^2 + k^2}$
 $\frac{k^2 - a^2}{b^2 + k^2}$
 $\frac{2 + k^2 - a^2}{b^2 + k^2}$
 \frac 1.e., $4(2) + 5(4) + \lambda(10 + 4 - 2) = 0$

Hence equation of circle is
 $x(x - 1) + y(y - 1) - \frac{7}{3}(2y + x - 2) = 0$
 $\frac{1}{2}x^2$

1. It passes through (0, c), therefore
 $c(c - 1) - \frac{7}{3}(2c - 2) = 0$
 $\frac{1}{2} + k^2$
 $\Rightarrow 3c^2 - 17c + 14 = 0$ or is required

t. to circle $x^2 + y^2 = a^2$,
 $x(x - 2) + y(x - 1) + \lambda(2y + x - 2) = 0$.
 $x(x - 2) + y(x - 1) + \lambda(2y + x - 2) = 0$.
 $x(x - 2) + y(x - 1) + \lambda(2y + x - 2) = 0$.
 $x(x - 1) + y(x - 1) - \frac{7}{3}(2y + x - 2) = 0$
 $x(x - 1) + y(y - 1) - \frac{7}{3}(2y + x - 2) = 0$
 $x(x - 1) +$ circle $x^2 + y^2 = a^2$,
 $x(x-2)+y(y-1)+ \lambda(2y+x-2) = 0$.

It passes through these two points are
 $\frac{x}{a(x-1)}$
 \therefore Al(2) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 5(4) + 7(4) + 7(4) + 7(4) + 7(4) + $\frac{1}{2}$ and $\frac{1}{2}$ $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$ **(100) (A).** Let the equation of the required circle be $(x^2+y^2-a^2)+\lambda (y-x-3)=0$ since its centre $(\lambda/2, -\lambda/2)$ lies on the given line, so we have $-\lambda/2 = \lambda/2 + 3 = -3$ Putting this value of in (A) we get the reqd. eqn. as
	- **(101) (B).** Let the equation of the required circle be $(x^2+y^2-6x+8)+(x^2+y^2-6)=0$ Since it passes through (1, 1), so we have $1+1-6+8+\lambda(1+1-6)=0=1$

$$
\therefore
$$
 the required equation is $x^2 + y^2 - 3x + 1 = 0$

33.30.
$$
x = \frac{1}{2} (QR) (PN)
$$

\n $(100) (A). Let the equation of the required circle be\n $(x^2 + y^2 - a^2) + \lambda (y-x-3) = 0$
\nsince its centre $(\lambda/2, -\lambda/2)$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave – $\lambda/2 - \lambda/2$ lies on the given line, so we
\nhave its value of in (A) we get the read, eqn. as
\n $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$
\n $(x^2 + y^2 - 6x + 8) + (x^2 + y^2 - 6) = 0$
\n $(x^2 + y^2 - 6x + 8) + (x^2 + y^2 - 6) = 0$
\n $(x^2 + y^2 - 4x + 5y + 3x - 3y - a^2 + 9) = 0$
\n $x^2 + y^2 + 4x - 5y + 3x^2 + 4x - 5y + 3x^2 + 3y - 3) = 0$
\n $x^2 + y^2 + 4x - 5y + 3x + \lambda(x^2 + y^2 - 3x + 1 = 0$
\n $x^2 + y^2 + 4x - 5y + 3x + \lambda(x^2 + y^2 - 3x + 1 = 0$
\n $x^2 + y^2 + 4x - 5y + 3x + \lambda(x^2 + y^2 + 2x + 3y - 3) = 0$
\n $x^2 + y^2 + 4x - 5y + 3x + \lambda(x^2 + y^2 + 2x + 3y - 3) = 0$
\n $x^2 + y^2 + 4x - 5y + 3x + \lambda(x^2 + y^2 + 2x$$

or $x = c/2$

Also the radical axis of first and third circle is given by (x² + y²) – (x² + y² – 2by + b²) = 0 or y = b/2 their radical centre = (c/2, b/2) **(104) (A).** The given equations may be written as 3x² + 3y² – 3x + 3 = 0 3x² + 3y² + y – 1 = 0 Now required equation is given by S – S' = 0 – 3x + 3 – y +1 = 0 3x + y – 4 = 0 **(105) (C).** The required point is the radical centre of the three given circles. The radical axes of these three circles taken in pairs are 3x – 24 = 0 16y + 120 = 0 and –3x + 16y + 80 = 0 Solving any two of these three equations, we get x = 8, y = – 15/8 Hence, the required point is (8, –15/8) **(106) (A).** x² + y² – 6x + (x² + y² – 6y) = 0 (1 +) x² + (1 +) y² – 6x – 6y = 0 – 6 6 x y 0 1 1 3 3 , 1 1 3 3 3 3 , 1 2 1 2 = 1

$$
x^{2} + y^{2} - \frac{6}{1+\lambda}x - \frac{6\lambda}{1+\lambda}y = 0
$$
\n(117) **(B).** $x^{2} + y^{2} + 2g$
\n $x^{2} + y^{2} - a^{2} = 0$
\nCentre $\left(\frac{3}{1+\lambda}, \frac{3\lambda}{1+\lambda}\right) \Rightarrow \frac{3}{1+\lambda} = \frac{3}{2}, \frac{3\lambda}{1+\lambda} = \frac{3}{2} \Rightarrow \lambda = 1$

- **(107) (A).** Any circle which touches $3x + 4y 7 = 0$ at $(1, 1)$ will be of the form $S(x, y) = (x-1)^2 + (y-1)^2 + \lambda (3x + 4y - 7) = 0$ Since $S(2, 3) = 16 \Rightarrow \lambda = 1$, so required circle will be $x^2 + y^2 + x + 2y - 5 = 0$.
- **(108) (C).** Let (h, k) be the centre of the required circle. Then (h,k) being the mid-point of the chord of the given circle, its equation is $hx + ky - a(x + h) = h^2 + k^2 - 2ah$ Since it passes through the origin, we have $- ah = h^2 + k^2 - 2ah \implies h^2 + k^2 - ah = 0$ Hence locus of (h, k) is $x^2 + y^2 - ax = 0$ (1+x) $x^2 + y^2 - \frac{6}{1+x}x - 6x - 6xy = 0$
 $x^2 + y^2 - \frac{6}{1+x}x - \frac{6x}{1+x}y = 0$

Centre $(\frac{3}{1+x}, \frac{3x}{1+x}) \Rightarrow \frac{3}{1+x} = \frac{3}{2}, \frac{3\lambda}{1+x} = \frac{3}{2} \Rightarrow \lambda = 1$

(4). Any circle which touches $3x + 4y - 7 = 0$ at (1,1) will

be of the form
 $\frac{6}{b} \times x - \frac{6\lambda}{1+\lambda} y = 0$ (117) (B), $x^2 + y^2 + 2gx + 2fy = 0$
 $\frac{3\lambda}{x^2 + y^2 - a^2 = 0}$

Equation of common chord is 2
 $\frac{1}{x}, \frac{3\lambda}{1+\lambda} \Rightarrow \frac{3}{1+\lambda} = \frac{3}{2}, \frac{3\lambda}{1+\lambda} = \frac{3}{2} \Rightarrow \lambda = 1$

Equation of common chord is 2

Equa re $\left(\frac{3}{1+\lambda}, \frac{3\lambda}{1+\lambda}\right) \Rightarrow \frac{3}{1+\lambda} = \frac{3}{2}, \frac{3\lambda}{1+\lambda} = \frac{3}{2} \Rightarrow \lambda = 1$

Any circle which touches $3x + 4y - 7 = 0$ at $(1, 1)$ will

The form
 $x^2 + y^2 - a^2\left(\frac{2gx}{x}\right)$
 $y = (x - 1)^2 + (y - 1)^2 + \lambda(3x + 4y - 7) = 0$
 $y = (x - 1)^2 + ($ The $(1+x)^{1} + \lambda^{1} - 1 + \lambda^{2} - 2 + \lambda^{2} - 2 = 0$

Solution the form

of the form

of the form
 $x, y = (x-1)^{2} + (y-1)^{2} + \lambda (3x+4y-7)=0$

of the form
 $x, y = (x-1)^{2} + (y-1)^{2} + \lambda (3x+4y-7)=0$
 $y = (x-1)^{2} + (y-1)^{2} + \lambda (3x+4y-7)=0$
 $y = (x \frac{3}{1+\lambda} \cdot \frac{3\lambda}{1+\lambda} \Rightarrow \frac{3}{1+\lambda} = \frac{3}{2}, \frac{3\lambda}{1+\lambda} = \frac{3}{2} \Rightarrow \lambda = 1$

Since which touches $3x + 4y - 7 = 0$ at (1, 1) will

form

(xn -1)²+(y-1)²+ (3x + 4y - 7) = 0

(xn -1)²+ (y -1)² + (3x + 4y - 7) = 0

(xn -2)

(b S(x, y) =(x -1)² + λ (3x + 4y - 7) = 0

S(x, y) =(x -1)² + λ (2x + y²) - 1 (g) = λ = 1,so required circle will be

S(x, y) =(x -1)² + x + y² - 3 = 0.

(C). Let (h, k) be the centre of the required circle. Th Since $S(z, z) = 16 - 2 - 1$, a required circle intercept = 2 (119) (CD, Condition of the circle is $(x - b)^2 + (y - a)^2 = a$

(C). Let (h, k) be the centre of the required circle. Then

(c), Let (h, k) be the centre of the celorated c

(109) (A). Let the pole is (h, k) Hence polar of this pole is $xh + yk - c^2 = 0$ (1)

but polar is
$$
\frac{x}{a} + \frac{y}{b} = 0
$$
(2)

comparing the coefficient of x and y

$$
\frac{h}{(1/a)} = \frac{k}{(1/b)} = \frac{-c^2}{-1} \implies h = \frac{c^2}{a}, k = \frac{c^2}{b}
$$

- **(110) (D).** For internal point p $(2, 8)$ 4 + 64 4 + 32 p < 0
- **(111) (C).** The two circles are $x^2 + y^2 4x 6y 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ Centre : C₁ = (2, 3), C₂ = (-1, -1) radii : r₁ = 4, r₂ = 1 We have $C_1 C_2 = 5 = r_1 + r_2$, therefore there are 3 common (2) tangents to the given circles.

(112) (C).
$$
x^2 + y^2 + 4x - 6y + 9 = 0
$$

\t $x^2 + y^2 - 5x + 4y - 2 = 0$
\t $9x - 10y + 11 = 0$

(113) (C). The chord of contact of tangents from (α, β) is $\alpha x + \beta y = 1$ (1)

Hence, (1) passes through $\left(\frac{1}{2}, \frac{1}{4}\right)$.

1 1 **SODWADVANCED LEARNING**
 SODWADVANCED LEARNING
 ngents from (α, β) **is**

.......(1)
 $\left(\frac{1}{2}, \frac{1}{4}\right)$

ual intercepts of length a

sequation can be written

as the given circle at two

from the centre $(0, 0)$ of
 1170NS

(113) (C). The chord of contact of tangents from (α, β) is
 $\alpha x + \beta y = 1$ (1)

Hence, (1) passes through $\left(\frac{1}{2}, \frac{1}{4}\right)$.

(114) (C). Since the chord makes equal intercepts of length a

on the coordina on the coordinates axes. So, its equation can be written as $x \pm y = \pm a$. This line meets the given circle at two distinct points. **(114) (C).** Since the chord makes equal intercepts of length a

on the coordinates axes. So, its equation can be written

as $x \pm y = \pm a$. This line meets the given circle at two

distinct points.

So, length of the perp **EXERCISE ANTIFICATE SET ASSESS ANDENSIGNATIES**

The chord of contact of tangents from (α, β) is
 $+\beta y = 1$

nec, (1) passes through $\left(\frac{1}{2}, \frac{1}{4}\right)$.

Since the chord makes equal intercepts of length a

e coordinate **EDENTIFY**
 EDENTADVANCED LEARNING

to f tangents from (α, β) is

.......(1)

ugh $\left(\frac{1}{2}, \frac{1}{4}\right)$.

kes equal intercepts of length a

So, its equation can be written

e meets the given circle at two

dicular from or ancidency (2 4)

ord makes equal intercepts of length a

s axes. So, its equation can be written

this line meets the given circle at two

perpendicular from the centre (0, 0) of

nust be less than the radius. i.e.
 $<$ $\left(\frac{1}{2}, \frac{1}{4}\right)$.

unal intercepts of length a

is equation can be written

ts the given circle at two

r from the centre (0, 0) of

an the radius. i.e.
 $(4.4.$

tt at (h, h) to $x^2 + y^2 = a^2$ is

of the tangent = -h/ through $\left(\frac{1}{2}, \frac{1}{4}\right)$.

makes equal intercepts of length a

sxes. So, its equation can be written

line meets the given circle at two

pendicular from the centre (0, 0) of

be less than the radius. i.e.
 $6 \Rightarrow |a| < 4$

So, length of the perpendicular from the centre $(0, 0)$ of the given circle must be less than the radius. i.e.

$$
\frac{\pm a}{\sqrt{2}} \leq \sqrt{8} \Rightarrow a^2 < 16 \Rightarrow |a| < 4 \, .
$$

- **(115) (C).** The equation of the tangent at (h, h) to $x^2 + y^2 = a^2$ is $hx + hy = a^2$. Therefore slope of the tangent $= -h/h = -1$
- Note P lies on the director circle of radius r_1 \Rightarrow L = r₁ = 2 cm. erpendicular from the centre (0, 0) of

sist be less than the radius. i.e.
 $2 \cdot 16 \Rightarrow |a| < 4$.
 $2 \cdot 16 \Rightarrow |a| < 4$.
 $r_1^2 \Rightarrow 2r_1^2 = r_2^2 \Rightarrow r_2 = \sqrt{2}r_1$

director circle of radius r_1
 $r_2 f y = 0$

on chord is $2gx + 2fy + a^2 =$

(117) **(B).**
$$
x^2 + y^2 + 2gx + 2fy = 0
$$

\n $x^2 + y^2 - a^2 = 0$
\nEquation of common chord is $2gx + 2fy + a^2 = 0$

Homogenization $x^2 + y^2$ 2 a^2) $-$ 0 \Rightarrow a² (x² + y²) – 4 (gx + fy)² = 0

- **(118) (B).** The reflection of (a, b) in $y x = 0$ is (b, a) so that the equation of the circle is $(x - b)^2 + (y - a)^2 = a^2$ as it touches the x-axis. 4.4. Intertual the Natural Heronics Acts

of the tangent = -h/h = -1
 $r_2^2 \Rightarrow r_2 = \sqrt{2}r_1$

cle of radius r_1

2gx + 2fy + a² = 0
 $\left(\frac{2gx + 2fy}{a^2}\right)^2 = 0$

= 0

y - x = 0 is (b, a) so that the

b)² + (y - a)² = 54.

t at (h, h) to $x^2 + y^2 = a^2$ is

of the tangent = -h/h = -1
 $r_2^2 \Rightarrow r_2 = \sqrt{2}r_1$

cle of radius r_1

2gx + 2fy + a² = 0
 $\left(\frac{2gx + 2fy}{a^2}\right)^2 = 0$

= 0

y - x = 0 is (b, a) so that the

b)² + (y - a)² = a²
- **(119) (C).** Condition for tangency is

$$
c2 = a2 (1 + m2) \Rightarrow 8b2 = 2ab \left(1 + \frac{4b2}{a2}\right)
$$

\Rightarrow 4b² - 4ab + a² = 0 \Rightarrow a = 2b

EXERCISE-2

(1) (B). Angle between direct common tangents

2 2 c c h , k a b ⁼ ¹ 1 2 r ~ r 2sin 90 d 1 2 r ~ r ¹ ^d ² 2 (r1–r²)² = d²(1) ^r² 1 2 2 1 r r ⁴ r r p 4 y 2 x 2 sin cos

circles are orthogonal $\Rightarrow d^2 = r_1^2 + r_1^2$ (2)

From (1) and (2), we get $2(r_1-r_2)^2 = r_1^2 + r_2^2$

$$
\Rightarrow r_1^2 + r_2^2 = 4r_1r_2 \Rightarrow \frac{r_1}{r_2} + \frac{r_2}{r_1} = 4 \Rightarrow p + \frac{1}{p} = 4
$$

(2) (B). Any line passing through (2, 2) will be of the form

$$
\frac{y-2}{\sin \theta} = \frac{x-2}{\cos \theta} = r
$$

223

When this line cuts the circle $x^2 + y^2 = 2$, $(\cos\theta + 2)^2 + (2 + \sin\theta)^2 = 2$ \Rightarrow r² + 4 (sin θ + cos θ) r + 6 = 0 2 \sim 2 $1 \t 1 \t 2$ PB r₂ **(Q.B.- SOLUTIONS** STUDY

hen this line cuts the circle
 $x^2 + y^2 = 2$, $(\cos\theta + 2)^2 + (2 + \sin\theta)^2 = 2$
 $r^2 + 4(\sin\theta + \cos\theta) r + 6 = 0$
 $\frac{PB}{PA} = \frac{r_2}{r_1}$, now if $r_1 = \alpha$, $r_2 = 3\alpha$,
 \therefore coordinate of A are ($\theta = \pi/4$.

requi $=\alpha, r_2 = 3\alpha,$ then $4\alpha = -4 \left(\sin \theta + \cos \theta \right), 3\alpha^2 = 6 \Rightarrow \sin 2\theta = 1$ $\Rightarrow \theta = \pi/4$. So required chord will be $y-2=1$ $(x-2) \Rightarrow y=x$. **(3) (C).** Family of circles is $x^2 + y^2 - 2x - 4y + 1 + \lambda (x^2 + y^2 - 1) = 0$ $(1 + \lambda) x^2 + (1 + \lambda) y^2 - 2x - 4y + (1 - \lambda) = 0$ $x^2 + y^2 - \frac{1}{1}$ **CORE SOLUTIONS**

SUBSIDING THE SOLUTIONS STUDY MATERIAL: MATI

In this line cuts the circle
 $+\gamma^2 = 2$, (rcos0+2)²+(2+ rsin0)²=2
 $\frac{B}{A} = \frac{F_2}{T_1}$, now if $r_1 = \alpha$, $r_2 = 3\alpha$,
 $4\alpha = -4$ (sinθ + cos6), $3\alpha^2 = 6$ (**Q.B. SOLUTIONS** STUDY MATERIAL:M
 $x = \frac{1}{1 + \lambda} y + \frac{1 - \lambda}{1 + \lambda} = 0$
 $\frac{1}{1 + \lambda}$ and radius

(**8)** $\left(\frac{4}{5}\right) - \frac{4}{5}$
 $\left(\frac$ a **(O.B. SOLUTIONS)** STUDY MATERIAL: MATI

ine cuts the circle
 $\frac{1}{1+\lambda}$, $\frac{2}{1+\lambda}$

and + cos(b) $\frac{1}{1+\lambda}$
 $\frac{1}{1+\lambda}$
 (CB. SOLUTIONS) STUDY MATERIAL: MATHE

(rcosθ + 2)² + (2 + rsinθ)² = 2

(rcosθ + 2)² + (2 + rsinθ)² = 2

now if r₁ = α, r₂ = 3α,

(sinθ + cosθ), 3α² = 6 ⇒ sin2θ = 1

hord will be y-2 = 1 (x - 2) ⇒ y = x.
 Centre is $\left[\frac{1}{1+\lambda}, \frac{2}{1+\lambda}\right]$ and radius **(O.B. SOLUTIONS** STUDY MATER)
 $\arctan 90^\circ + \arctan 90^\$ Colored and the circle
 $22, (\text{rcos}\theta + 2)^2 + (2 + \text{rsin}\theta)^2 = 2$
 $\theta + \cos\theta) r + 6 = 0$
 $\cos\theta + \cos\theta + 2 = 3\alpha$
 $\cos\theta + 2\theta + 6 = 0$
 $\cos\theta + 2\theta + 1 + 2\theta + 2 = 1$
 COLE SOLUTIONS STUDY MATERIAL

line cuts the circle
 $\frac{2}{2}$, $(\cos\theta + 2)^2 + (2 + \sin\theta)^2 = 2$
 $\frac{2}{\sin\theta + \cos\theta} + \frac{2}{\theta - 1} = 0$
 $\frac{1}{\cos\theta} + \frac{1}{\cos\theta} + \frac{1}{\cos\theta} = 1$
 $\frac{1}{\cos\theta} + \frac{1}{\cos\theta} + \frac{1}{\cos\theta} = 1$
 $\frac{1}{\cos\theta} + \frac{1}{$ **O.B. SOLUTIONS** STUDY MATERIAL

line cuts the circle
 $\frac{1}{2}$, $(\cos\theta + 2)^2 + (2 + \sin\theta)^2 = 2$
 $\Rightarrow (\cos\theta) + 6 = 0$
 $\Rightarrow (\sin\theta + \cos\theta) + 6 = 0$
 $\Rightarrow (\sin\theta + \cos\theta) + 6 = 0$
 $\Rightarrow (\sin\theta + \cos\theta) + 6 = 0$
 $\Rightarrow (\sin\theta + \cos\theta) + 6 = 0$
 $\Rightarrow (\sin\theta + \cos\theta) + 6 = 0$
 $\$ Alternative states and the context of the n this line cuts the circle (see the circle of Act -3.6)
 $\frac{3}{2} = \frac{F_2}{F_1}$, now if $r_1 = \alpha$, $r_2 = 3\alpha$,
 $4\alpha = -4$ (sinθ + cosθ), $3\alpha^2 = 6 \Rightarrow \sin 2\theta = 1$
 $\Rightarrow \pi/4$
 \Rightarrow **EXERENT ALTERNAL: MATTER

EXERENT ALTERNAL: MATTER

The initial interest is the circle
** $\frac{PB}{\sqrt{2}} + \frac{r_2}{r_1} = 2$ **,** $(r \cos \theta + 2)^2 + (2 + r \sin \theta)^2 = 2$ **
** $\frac{PB}{\sqrt{2}} - \frac{r_1}{r_1}$ **, now if** $r_1 = \alpha$ **,** $r_2 = 3\alpha$ **,
 r_1 = 4 \times (-4 \sin \theta + \cos ** i.e. , 2 2 = 2
 $\frac{1}{4}$, now if r₁ = α, r₂ = 3α,
 $\frac{1}{4}$, now if r₁ = α, r₂ = 3α,
 $\frac{1}{4}$, now if r₁ = α, r₂ = 3 = 3(x-2) = 3(x-2) = 3(x-2) = 3(x-3)

and radius
 $\left(\frac{1}{2} + \frac{2}{\lambda} + \frac{2}{1+\lambda}\right)$ and radius
 \left 4α = -4 (sinθ + cosθ), 3α² = 6 ⇒ sin2θ = 1

and + cosθ), 3α² = 6 ⇒ sin2θ = 1

we have sin θ = 1/2 ∴ θ = 3

and the y-2 = 1 (x - 2) ⇒ y = x.

analy of circles is
 (7) (D). (x - 1)² + (y + 2)² = 16
 $(y^2 - 2x - 4y$ We have sin $\theta = 1/2$ $\therefore \theta = 3$

dechord will be $y-2=1$ $(x-2) \Rightarrow y=x$.

Area $=3 \times 3 \tan 30^\circ = 3\sqrt{3}$
 $y = 2x-4y+1+\lambda (x^2+y^2-1)=0$
 $-\frac{2}{1+\lambda}x-\frac{4}{1+\lambda}y+\frac{1-\lambda}{1+\lambda}=0$
 $-\frac{2}{1+\lambda}x-\frac{4}{1+\lambda}y+\frac{1-\lambda}{1+\lambda}=0$
 $\left[\frac{1}{1+\lambda},\frac{2}{1+\lambda}\$ = $\frac{1}{2}$, now if r₁ = α, r₂ = 3α,

α = -4 (sinθ + cosθ), 3α² = 6 ⇒ sin2θ = 1

we have sin θ = 1/2 . (exaction of Aarc (-3, 0)

with and cost of Aarc (-3, 0)

with and cost of Aarc (-3, 0)

with and cost of Aarc So required chord will be $y-2 = 1(x-2) \Rightarrow y = x$.

Area = 3 $\times 3 \tan 30^\circ = 3 \tan 3x$
 $x^2 + y^2 - 2x - 4y + 1 + \lambda (x^2 + y^2 - 1) = 0$
 $(x-1)^2 + (y-2)^2 = 32$
 $(1 + \lambda)x^2 + (1 + \lambda)y^2 - 2x - 4y + (1 - \lambda) = 0$
 $x^2 + y^2 - \frac{2}{1 + \lambda}x + \frac{1}{1 + \lambda} + \frac{1}{1 + \lambda} =$

Since it touches the line $x + 2y = 0$, hence Radius = Perpendicular from centre to the line.

$$
= \sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2} - \left(\frac{1-\lambda}{1+\lambda}\right) = \frac{\sqrt{4+\lambda^2}}{1+\lambda}
$$

\nSince it touches the line $x + 2y = 0$, hence
\nRadius = Perpendicular from centre to the line.
\ni.e.
$$
\frac{\left|\frac{1}{1+\lambda} + 2\frac{2}{1+\lambda}\right|}{\sqrt{1^2 + 2^2}} = \frac{\sqrt{4+\lambda^2}}{1+\lambda}
$$

\n \therefore vertices
\n $\Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$
\n $\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$
\nThus, we get the equation of circle.
\n(4) **(B).** $\sin \alpha = \frac{r_1 - r_2}{r_1 + r_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{r_1 - r_2}{r_1 + r_2}\right)$
\n(10) **(A).** Since *A* is the circle with the circle with the circle of the circle.

$$
\Rightarrow \sqrt{5} = \sqrt{4 + \lambda^2} \Rightarrow \lambda = \pm 1
$$

$$
\lambda = -1
$$
 cannot be possible in case c

of circle. So $\lambda = 1$ Thus, we get the equation of circle.

(4) **(B).**
$$
\sin \alpha = \frac{r_1 - r_2}{r_1 + r_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{r_1 - r_2}{r_1 + r_2} \right)
$$

(5) (B). Any second degree curve passing through the intersection of the given curves is

 $ax^{2} + 4xy + 2y^{2} + x + y + 5$ $+ \lambda (ax^2 + 6xy + 5y^2 + 2x + 3y + 8) = 0$ (11) If it is a circle, then coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$ $a(1 + \lambda) = 2 + 5\lambda$ and $4 + 6\lambda = 0$ cannot be possible in case of circle. So $\lambda = 1$
 $(2,-1)$ and $(2,-3)$
 $\alpha = \frac{r_1 - r_2}{r_1 + r_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{r_1 - r_2}{r_1 + r_2} \right)$
 $\alpha = \frac{r_1 - r_2}{r_1 + r_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{r_1 - r_2}{r_1 + r_2} \right)$
 \Rightarrow the circle whose di $\sqrt{4 + \lambda^2}$ ⇒ $\lambda = \pm 1$

annot be possible in case of circle. So $\lambda = 1$
 $\epsilon = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$ ⇒ $\theta = 2 \sin^{-1} \left(\frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2} \right)$
 $\epsilon = \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$ ⇒ $\theta = 2 \sin^{-1} \left(\frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2} \right)$
 where $\frac{F_1 - F_2}{F_1 + F_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{F_1 - F_2}{F_1 + F_2} \right)$
 $= \frac{F_1 - F_2}{F_1 + F_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{F_1 - F_2}{F_1 + F_2} \right)$
 $= \frac{F_1 - F_2}{F_1 + F_2} \Rightarrow \theta = 2 \sin^{-1} \left(\frac{F_1 - F_2}{F_1 + F_2} \right)$
 $= \frac{F_1 - F_2}{F_1 + F_2} \Rightarrow \theta = 2 \sin^{-1}$

$$
\Rightarrow
$$
 a = $\frac{2+5\lambda}{1+\lambda}$ and $\lambda = -\frac{2}{3} \Rightarrow$ a = $\frac{2-(10/3)}{1-(2/3)} = -4$.

(6) (A). A divides C_1C_2 externaly in the ratio 1 : 3.

$$
A(-3,0) \xrightarrow{\theta} \underbrace{(C_1(-1,0)) \begin{pmatrix} 3 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}}_{(2,0,0)}
$$

 \therefore coordinate of A are $(-3, 0)$ We have $\sin \theta = 1/2$: $\theta = 30^{\circ}$

 1 r r r r r r r r Area = 3 × 3 tan 30° = 3 3 **(7) (D).** (x – 1)² + (y + 2)² = 16 (x – 1)² + (y – 2)² = 32 OS = 4 2 O 4 4 S P Q director circle of given circle 12 Required distance T TS = OT – SO TS = 12 4 2 **(8) (B).** 2 2 h h k 3 4 3 0 2 2 2 ^x ^M ⁰ or x² + y² + 8x – 6y + 9 = 0 2 2 h 8h (k 3) 4 4 4 = 0 B A Y (-2,3) (0,3) 1 2 cos 45 , 2 2 sin (2,–1)

or
$$
x^2 + y^2 + 8x - 6y + 9 = 0
$$

\nThis is a circle.
\n(9) **(D).** Centre (1, -2), radius $\sqrt{2}$

(8) **(B)**
$$
\left(\frac{\pi}{2}\right) + 4\left(\frac{\pi}{2}\right) + 4\left(\frac{\pi}{2}\
$$

(10) (A). Since \angle APB = \angle AQB = $\frac{\pi}{2}$ so y = mx + 8 intersect so $y = mx + 8$ intersect

the circle whose diameter is AB. Equation of circle is $x^2 + y^2 = 16$ $CD < 4$

If the line passing throw the point $A(-4, 0)$, $B(4, 0)$, then \angle APB = \angle AQB = π / 2 does not formed. \therefore m $\neq \pm 2$

 $P \swarrow$ \overline{B} $\mathcal V$

 $\nu \sim$

 $C \quad / \quad$

A

(11) (C). Equation of the two circles be $(x - r)^2 + (y - r)^2 = r^2$ i.e., $x^2 + y^2 - 2rx - 2ry + r^2 = 0$, where $r = r_1$ and r_2 . Condition of orthogonality gives

1
$$
1 \div 2 \cos 45^\circ, -2 \pm \sqrt{2} \sin \frac{\pi}{4}
$$

\n= $(1 \pm 1, -2 \pm 1) = (0, -1)$ and $(0, -3)$
\n= $(1 \pm 1, -2 \pm 1) = (0, -1)$ and $(0, -3)$
\n10 (A). Since $\angle APB = \angle AQB = \frac{\pi}{2}$ so $y = mx + 8$ intersect
\nthe circle whose diameter is AB.
\nEquation of circle is $x^2 + y^2 = 16$
\nCD < 4
\n $\Rightarrow \frac{8}{\sqrt{1 + m^2}} < 4 \Rightarrow 1 + m^2 > 4$
\n $\Rightarrow m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$
\ncurve passing through the
\n $\angle APB = \angle AQB = \pi/2$ does not formed.
\n $\therefore m \ne \pm 2$
\n $+ 6xy + 5y^2 + 2x + 3y + 8 = 0$
\n11 (C). Equation of the two circles be $(x - r)^2 + (y - r)^2 = r^2$
\n $+ 6xy + 5y^2 + 2x + 3y + 8 = 0$
\n $\Rightarrow 0$
\n $\Rightarrow 0$
\n $\Rightarrow 1 = \frac{2 - (10/3)}{1 - (2/3)} = -4$
\n $\Rightarrow a^2 + b^2 - 2rx - 2ry + r^2 = 0$, where $r = r_1$ and r_2 .
\n11 (C). Equation of the two circles be $(x - r)^2 + (y - r)^2 = r^2$
\nCondition of orthogonality gives
\n $\lambda = 0$
\n $\Rightarrow a^2 + b^2 - 2ra - 2rb + r^2 = 0$
\n $r_1 + r_2 = 2 (a + b)$ and $r_1r_2 = a^2 + b^2$
\n $\Rightarrow a^2 + b^2$
\n $\Rightarrow a^2 + b^2 - 2ra - 2rb + r^2 = 0$
\n $r_1 + r_2 = 2 (a + b)$ and $r_1r_2 = a^2 + b^2$

P

 $R \swarrow$

A

Q ν

$$
\therefore 4(a^{2} + b^{2}) = 4(a + b)^{2} - 2(a^{2} + b^{2})
$$

i.e., a²-4ab+b² = 0

(12) (D)

$$
\begin{array}{c}\n\mathbb{R} \\
\hline\n\mathbb{R} \\
\hline\n\mathbb{R}\n\end{array}
$$

(13) (C). x 2 + y² – 12x + 35 = 0 (1) and x 2 + y² + 4x + 3 = 0 (2) Equation of radical axis of (1) and (2) is $-16x + 32 = 0$ i.e., $x = 2$ It intersect the line joining the centers i.e., $y = 0$ at the point $(2, 0)$ **EXECUTE:**
 $\therefore 4(a^2 + b^2) = 4(a + b)^2 - 2(a^2 + b^2)$
 $\therefore a^2 - 4ab + b^2 = 0$
 $\therefore a^2 - 4ab + b^2 = 0$
 $\therefore a^2 + 3ab + b^2 = 0$
 $\therefore a^2 + 3ab + b^2 = 0$
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXEC

$$
\therefore \text{ required radius} = \sqrt{4 - 24 + 35} = \sqrt{15}
$$

(14) Let $P(x_1, y_1)$ be the given point and PT_1, PT_2, PT_3 be the lengths of the tangents from P to the circles $x^{2} + y^{2} = a^{2}$, $x^{2} + y^{2} = b^{2}$ and $x^{2} + y^{2} = c^{2}$ respectively.

Let r be required radius
\n
$$
3\sqrt{2} + 4 = 3 + 2\sqrt{2} + r + r\sqrt{2}
$$

\n $\sqrt{2} + 1 = r(1 + \sqrt{2}) \Rightarrow r = 1$
\n $\sqrt{2} + 1 = r(1 + \sqrt{2}) \Rightarrow r = 1$
\nand $x^2 + y^2 - 12x + 35 = 0$ (1)
\nEquation of radical axis of (1) and (2) is
\nEquation of radical axis of (1) and (2) is
\n $x = 16x + 32 = 0$ i.e., $x = 2$
\nIt intersect the line joining the centers i.e., $y = 0$
\nLet P (x₁, y₁) be the given point and PT₁, PT₂, PT₃ be
\nthe length of 2.
\n $x^2 + y^2 = a^2, x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ respectively.
\nThen, PT₁ = $\sqrt{x_1^2 + y_1^2 - a^2}$, PT₂ = $\sqrt{x_1^2 + y_1^2 - b^2}$ and
\n $2x^2 + y^2 - 2x - 2y + 1 = 0$
\n $\Rightarrow 2 \text{Pr} 2 = \text{Pr} 2 + r^2 = \text{Pr} 2$
\n $\Rightarrow 2 \text{Pr} 2 = \text{Pr} 2 + r^2 = \text{Pr} 2$
\n $\Rightarrow 2 \text{Pr} 2 = \text{Pr} 2 + r^2 = 4$
\n $\Rightarrow 2 \text{Pr} 2 = \text{Pr} 2 + \text{Pr} 2 = 3$
\n $\Rightarrow 2 \text{Pr} 2 = \text{Pr} 2 + \text{Pr} 2 = 4$
\n $\Rightarrow 2 \text{Pr} 3 = 3\sqrt{5}$
\n $\Rightarrow 2 \text{Pr} 3 = 3\sqrt{5}$
\n $\Rightarrow 2 \text{Pr} 4 = 3\sqrt{5}$
\n $\Rightarrow 2 \text{Pr} 3 = 3\sqrt{5}$
\n $\Rightarrow 2 \text{Pr} 4 = 3\sqrt{5}$
\n $\Rightarrow 2$

$$
\Rightarrow 2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2
$$
 are in A.P.

(15) (B). $(x-2)^2 + b^2 = 4$ centre is (2, 0) and radius 2. Distance between $(2, 0)$ and $(5, 6)$ is

$$
\sqrt{9+36} = \sqrt{45} = 3\sqrt{5}
$$
\n
$$
(5,6)
$$
\n
$$
(2,0)
$$

$$
\therefore r_1 r_2 = \frac{3\sqrt{5} - 2}{2} \cdot \frac{3\sqrt{5} + 2}{2} = \frac{45 - 4}{4} = \frac{41}{4}
$$

(16) (A). Let C be the centre of the given circle. Then circumcircle of the \triangle RPQ passes through C. \therefore (2, 3) is the mid point of RC

- \therefore Coordinates of C are $(-1, -2)$
- \therefore Equation of the circle is $x^2 + y^2 + 2x + 4y 20 = 0$

 2 2 2 2 2 4 (a b) 4 (a b) 2 (a b) **(17) (D).** AQ = 3 + 2 $\sqrt{2}$

PQ = 3 $\sqrt{2} + 4$

Let r be required radius
 $3\sqrt{2} + 4 = 3 + 2\sqrt{2} + r + r\sqrt{2}$
 $\sqrt{2} + 1 = r(1 + \sqrt{2}) \Rightarrow r = 1$ Let r be required radius S

S

(D). AQ = 3 + 2 $\sqrt{2}$

PQ = $3\sqrt{2} + 4$

Let r be required radius
 $3\sqrt{2} + 4 = 3 + 2\sqrt{2} + r + r\sqrt{2}$
 $\sqrt{2} + 1 = r(1 + \sqrt{2}) \Rightarrow r = 1$

(A). Let C (cos θ , sin θ), H (h, k) is the orthocentre of the

AABC

SOM ADVANCED LEADERS

PQ = 3 $\sqrt{2}$ + 4

PQ = 3 $\sqrt{2}$ + 4
 $\sqrt{2}$ + 4 = 3 + 2 $\sqrt{2}$ + r + r $\sqrt{2}$
 $\frac{1}{2}$ + 1 = r(1+ $\sqrt{2}$) \Rightarrow r = 1
 C). Let C (cos θ , sin θ), H (h, k) is the orthocentre of the

(18) (A). Let C (cos θ , sin θ), H (h, k) is the orthocentre of the $\triangle ABC$

 $h = 1 + \cos \theta$, $k = 1 + \sin \theta$

 $x^2 + y^2 - 2x - 2y + 1 = 0$ **(19) (B).** Since \angle AOB = 90° $PA = PB = PO = AO \cos 45^\circ$

 $=\frac{2a}{\sqrt{2}}=a\sqrt{2}$ Since OP = $a\sqrt{2}$, locus of P is the circle with O as origin

and radius a $\sqrt{2}$ and its equation is $x^2 + y^2 = 2a^2$.

(20) **(B).** Equation of circum circle be L_1 . $L_3 + \lambda L_2 L_4 = 0$ For circle a = b, h = 0. Put λ and find circle $2x^2 + 2y^2 = 125$

(21) (B). $\frac{dy}{dx}\bigg|_P = \frac{f(c)}{c+3}$

$$
(2c-3)(c+3) = c2 - 3c + 1
$$

2c²+3c-9 = c² - 3c + 1

225

(22) (B). $y = mx$ is a tangent to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ if "p=r", (i.e.) $\left| \frac{1}{\sqrt{1+m^2}} \right| = \sqrt{a^2 + b^2 - b^2}$ $-ma$ $\qquad \qquad$

$$
\left|\sqrt{1+m^2}\right|^{3/2} \text{ and } \left|\sqrt{1+m^2}\right|^{3/2} \text{ (20) (b).} \le 1
$$

$$
\therefore b^2 - 2abm = a^2 \text{ or } m = \frac{b^2 - a^2}{2ab}
$$

$$
\tan 2\alpha
$$

Equation of the tangent is $y = \left(\frac{}{}{}_{2ab} \right) x$. $\frac{4-R}{2a}$

Also $x = 0$ is a tangent, since $y^2 - 2by + b^2$ is a perfect square.

(23) (A). Perpendicular distance from centre upon line equal to radius

$$
\Rightarrow (x-2)^2 + (y-4)^2 = 25
$$

\Rightarrow 4y-16 = 3x-6 \pm 25 \Rightarrow K = -35, K = +15

Slope of tangent =
$$
\frac{3}{4}
$$
 $\Rightarrow \frac{b-4}{a-2} \cdot \frac{3}{4} = -1$

$$
x_0 + 2a\cos a + b + k = 0
$$

\n
$$
x_1 + 2b - 2b = 0
$$

\n
$$
x_2 + 2b - 2b = 0
$$

\n
$$
x_3 + 2b - 16 = 3x - 6 + 25
$$

\n
$$
x_4 = 2
$$

\n
$$
x_5 = 2
$$

\n
$$
x_6 = 2
$$

\n
$$
x_7 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_8 = 2
$$

\n
$$
x_9 = 2
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_2 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_2 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_2 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_2 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_3 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_4 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_5 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_6 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_7 = \sqrt{3} + \sqrt{3} - 1 = 0
$$

\n
$$
x_8 = \sqrt{3} - \sqrt{3} = -1
$$

\n
$$
x_9 = \sqrt{3} - \sqrt{3} = -1
$$

\n
$$
x_1 = \sqrt{3} - \sqrt{3} = 1
$$

\n
$$
x_2 = \sqrt{3}
$$

$$
\Rightarrow x = 2 \pm 5 \left(-\frac{3}{5} \right), y = 4 \pm 5 \left(\frac{4}{5} \right)
$$
 given

$$
\frac{x}{5} = \frac{36}{5} = \frac{36}{5} = \frac{15 \times 15}{5} = \frac{36}{5}
$$

$$
\Rightarrow a+b+K \Rightarrow -1+8-35=-28 \text{ and } 5+15=20
$$

(24) **(B).**
$$
\sin 60^\circ = \frac{r}{1-r} = \frac{\sqrt{3}}{2}
$$

 $2r = \sqrt{3} - \sqrt{3}r$;
 $\sqrt{3}$

$$
\Rightarrow a = 2, b = -3 \Rightarrow (a + b) = -1
$$

(25) (C). A = (-2, 2)
Equation of AC is, y, 2 = 1 (y + 2)

Equation of AC is $y-2=1(x+2)$ i.e. $x - y + 4 = 0$. Hence $C = (0, 4)$

For any point (α, β) on this circle β is maximum when (α, β) corresponds to point B and then

$$
\beta = OB = OC + CB = 4 + 2\sqrt{2}
$$

$$
(\text{Q.B. SOLUTIONS}) \qquad \text{STUDY MATERIAL: MATHEMATICS} \text{By + b2 = 0} \text{ b-ma} \text{ b-mb} \text{ is a perfect}
$$
\n
$$
\left.\begin{array}{l}\n\text{B = OB = OC + CB = 4 + 2\sqrt{2}} \\
\hline\n\frac{b-m}{1+m^2} \\
\frac{b-m}{2ab} \\
\frac{b-m}{
$$

$$
\tan 2\alpha = \frac{24}{7} \quad ; \quad \therefore \quad \tan \alpha = \frac{3}{4} \quad \therefore \quad \sin \alpha = \frac{3}{5}
$$

 R = 1

(27) (A). The given lines
$$
\sqrt{3}(y-1) = x-1
$$
(1)

$$
y-1 = \sqrt{3}(x-1)
$$
(2)

intersect at the point (1, 1).

Rewriting the equation of the given lines such that their constant terms are both positive, we have

$$
x - \sqrt{3} y + \sqrt{3} - 1 = 0
$$
(3)

and
$$
-\sqrt{3}x+y+\sqrt{3}-1=0
$$
(4)

Here, we have

(product of coeff.'s of x) + (product of coeff.'s of y)

$$
= -\sqrt{3} - \sqrt{3} = -ve
$$
 quantity

which implies that the acute angle between the given lines contains the origin.

Therefore, equation of the acute angle bisector of the given lines is

$$
\frac{x-\sqrt{3}y+\sqrt{3}-1}{2} = +\frac{-\sqrt{3}x+y+\sqrt{3}-1}{2}
$$
 i.e. y = x

 $\frac{4-R}{4+R} = \frac{3}{5}$ $\therefore \frac{R}{4} = \frac{5-3}{5+3} = \frac{2}{8} = \frac{1}{4}$ $\therefore R = 1$

D. The given lines $\sqrt{3}(y-1) = x-1$ (1)
 $y-1 = \sqrt{3}(x-1)$ (2)

ersect at the point (1, 1).

writing the equation of the given lines such $n2\alpha = \frac{1}{7}$; $\therefore \tan \alpha = \frac{1}{4}$ $\therefore \sin \alpha = \frac{1}{5}$
 $\frac{4-R}{4+R} = \frac{3}{5}$ $\therefore \frac{R}{4} = \frac{5-3}{5+3} = \frac{2}{8} = \frac{1}{4}$ $\therefore R = 1$

The given lines $\sqrt{3}(y-1) = x-1$ (1)
 $-1 = \sqrt{3}(x-1)$

sect at the point (1, 1).

The give $\frac{R}{4} = \frac{5-3}{5+3} = \frac{2}{8} = \frac{1}{4}$... $R = 1$
 $\sqrt{3} (y-1) = x-1$ (1)
 $\dots(2)$
 $t(1, 1)$.

(1)(2)
 $t(1, 1)$.

(1)(3)
 $\sqrt{3} - 1 = 0$ (3)
 $\sqrt{3} - 1 = 0$ (4)
 $\sqrt{3} - 1 = 0$ (4)
 $\sqrt{$ $\alpha \alpha = \frac{3}{4}$ \therefore $\sin \alpha = \frac{3}{5}$
 $\frac{5-3}{5+3} = \frac{2}{8} = \frac{1}{4}$ \therefore R = 1
 $(y-1) = x-1$ (1)

......(2)

1).

If the given lines such that their

positive, we have

......(3)
 $1=0$ (4)

+ (product of coef Any point on the above bisector can be chosen as (α, α) and equation of any circle passing through (1, 1) and having centre at (α, α) is

⇒ (x-2)² + (y-4)² = 25
\n⇒4y-16 = 3x-6±25
\nSlope of tangent =
$$
\frac{3}{4}
$$
 ⇒ $\frac{b-4}{a-2} \cdot \frac{3}{4} = -1$
\nSlope of tangent = $\frac{3}{4}$ ⇒ $\frac{b-4}{a-2} \cdot \frac{3}{4} = -1$
\n $\frac{3x+4y+k=0}{4x-6+2} = 4$
\n $\frac{3x+4y-k}{4x-k} = 4$
\n $\frac{3x+4y-k}{4x-k} = 4$
\n $\frac{3x-4y-k}{4x-k} = 4$
\n $\frac{3x-4}{4x-k} = 4$
\n $\frac{3x-4}{4} = -1$
\n $\frac{3x-4}{4$

$$
4x - 6y + 7 = 0
$$
(9)

and $x+y-2=0$ (10) Solving equation (9), (10) gives the coordinates of the

fixed point as $(1/2, 3/2)$.

(28) (C). Let centre of circle be P (h, k) . So, that $PA^2 = PB^2$ where $A = (2, 4)$ and $B = (0, 1)$

and (slope of OA) (slope of tangent at A) $= -1$

$$
\Rightarrow h^{2} + (k - 1)^{2} = (h - 2)^{2} + (k - 4)^{2}
$$

or 4h + 6k - 19 = 0(1)

 $X \sim$

1.13
\n1.130 slope of OA –
$$
\frac{k-4}{h-2}
$$
 and slope of tangent at (2, 4) to (32) (A). $y_1 = \sqrt{1-x_1^2}$ and $y_2 - 3 = k$
\n $y_2 - x^2 is 4$
\n $x_1^2 + y_1^2 = 1$ and $y_2 + x_2 = 3$
\n $x_2^2 + y_1^2 = 1$ and $y_2 + x_2 = 3$
\n $x_2^2 + y_1^2 = 1$ and $y_2 + x_2 = 3$
\n $x_2^2 + y_1^2 = 1$ and $y_2^2 + y_2^2 = 1$
\n $x_1^2 - y_1^2 = 1$
\n $x_2^2 - y_1^2 = 1$ and $y_2^2 + y_2^2 = 3$
\n $x_1^2 - y_1^2 = 10$
\n x_1

(37) (A). Equation of required circle is

$$
(x-2)(x+g)+(y-1)(y+f)=0
$$

(38) (A) Area =
$$
4.\left(\frac{1}{2} \times 2 \times 2\sqrt{3}\right) = 8\sqrt{3}
$$
 square units

- **(39) (A).** The centre of circle is (h, h) and radius = h \Rightarrow The circle is touching the co-ordinate axes.
- **(40) (A).** $C_1(1, 2)$, $r_1 = 10$ $C_2(3, 4)$, $r_2 = 3$
	- \therefore the statement is true

(41) **(D).** Slope of line joining its (1, 2) & (4, 7) =
$$
\frac{7-2}{-4-1} = -1
$$

Slope of line joining points $(1, 2)$ & $(3, 0) = \frac{1}{3-1} = -1$ $\left(\frac{b}{2}\right)^2 - 4\left(\frac{b}{2}\right) + 1 = 0$

 \therefore points are collinear

 \therefore no circle can be drawn

(42) (A).

Equation of circle touching the coordinates axes and centre (r, r) in the first quadrant is $x^2 + y^2 - 2xr - 2yr + r^2 = 0$

For r = a or b
\nHence C₁:
$$
x^2 + y^2 - 2ax - 2ay + a^2
$$
(1)
\nCentre (a, a), radius = a, a > 0
\nC₂: $x^2 + y^2 - 2bx - 2by + b^2$ (2)
\nCentre (b, b), radius b, b > 0
\n(a) C₁ and C₂ touch each other radical axis between (1)
\nand (2) is (1) – (2) = 0
\n2 (b – a) x + 2 (b – a) y – (b² – a²) = 0
\n2 x + 2y – (b + a) = 0(3)
\nIf it touches both C₁ and C₂ then perpendicular from
\n(a, a) = radius 'a'

STUDY MATERIAL: MATHEMATICS
\n
$$
\left| \frac{2a + 2a - (b + a)}{\sqrt{8}} \right| = a
$$
\n...(4)
\n
$$
|3a - b| = 2\sqrt{2} a
$$
\n...(5)
\n(5)
\n(6, 0) gives – ve sign with (3).
\n20.11 (0, 0) gives – ve sign with (3).

now origin and (a, a) must lie on the same side of (3)

EXECUTIONS
 EXECUTIONS EXECUTIONS

STUDY MATERIAL: MATHEM

of required circle is
 $\frac{2a + 2a - (b + a)}{\sqrt{8}} = a$ (4)
 $\frac{1}{36} - b| = 2\sqrt{2} a$ (5)

now origin and (a, a) must lie on the same side of

but (0, 0) gives - ve sign with (3).
 $\left(\$ **STUDY MATERIAL: MATHEMATICS**
 $\frac{2a + 2a - (b + a)}{\sqrt{8}}$ = a(4)
 $3a - b$ = $2\sqrt{2}$ a(5)

w origin and (a, a) must lie on the same side of (3)

(0, 0) gives – ve sign with (3).

nec (a, a) should also give the same but $(0, 0)$ gives – ve sign with (3) . hence (a, a) should also give the same sign i.e. $4a - b - a < 0 \Rightarrow 3a - b < 0$ Hence (5) becomes **STUDY MATERIAL: MATHEMATICS**
 $\left|\frac{2a+2a-(b+a)}{\sqrt{8}}\right| = a$ (4)
 $|3a-b| = 2\sqrt{2} a$ (5)

wo origin and (a, a) must lie on the same side of (3)

tt (0, 0) gives – ve sign with (3).

mee (a, a) should also give the same **MATERIAL: MATHEMATICS**

....(4)

....(5)

t lie on the same side of (3)

with (3).

give the same sign
 $> 0 < 0$
 $3 + 2\sqrt{2}$

ad C₂ touch each other externally

r centre = sum of their radius

(a + b)
 $a^2 + b^2 - 6ab = 0$ MATERIAL: MATHEMATICS

....(4)

....(5)

ust lie on the same side of (3)

n with (3).

9 give the same sign
 $-b < 0$

= $3 + 2\sqrt{2}$

and C₂ touch each other externally

eir centre = sum of their radius

= (a + b)

$$
b - 3a = 2\sqrt{2}a \implies \frac{b}{a} = 3 + 2\sqrt{2}
$$

Alternativly: (A) As C_1 and C_2 touch each other externally so, distance between their centre = sum of their radius

EXAMPLE 1.1.1
\n(A). Equation of required circles
\n(A). Equation of required circles
\n(A) Equation of required circles
\n(A) Area =
$$
4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0
$$

\n(A) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n(A) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n(A) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n(B) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n(C) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n5.3 a 22 $\sqrt{2}$ A Hermit's (A) A's C, and C₂ (such each other externally)
\n(A) Area = $4(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3}) = 0$
\n5.4 a 3*u* - 2*u* = 4*u* + 3*u* = 2*u* = 2*u*
\n5.5 a 22 $\sqrt{2}$ b) 2.00
\n6.60.101, the correct circle is 6, b) and radius = h
\n6.11.11
\n6.12.12
\n6.13.12
\n7.13.13
\n8.14
\n8.15
\n9.16
\n10.17.18
\n11.19
\n12.10
\n13.10
\n14.11
\n15.11
\n16.12, 17, -10
\n17.13
\n18.15
\n19.16
\n10.17
\n11.18
\n12.19
\n13.10
\n14.11
\n15.11
\n16.12, 17, -19, 15
\n17.13
\n18.16
\n19.17
\n10.18
\n11.19
\n12.10
\n13.10
\n14.

(c) If common chord is longest then (3) must pass through the centre (a, a) of C_1 . i.e. $4a - b - a = 0$

$$
\begin{array}{c}\n\cdot \text{a} & \circ \\
\text{b}\n\end{array}
$$

$$
3a = b \Longrightarrow \frac{b}{a} = 3 \Longrightarrow q
$$

(d) If C_2 passes through the centre of C_1 then (a, a) must satisfy (2)

i.e.
$$
a^2 + a^2 - 2b(2a) + b^2 = 0
$$

\n $\Rightarrow 2a^2 - 4ab + b^2 = 0$

$$
\left(\frac{\mathbf{b}}{\mathbf{a}}\right)^2 - 4\left(\frac{\mathbf{b}}{\mathbf{a}}\right) + 2 = 0
$$

LE	Q.B.- SOLUTIONS	DEFing	
Put $\frac{b}{a} = t$; $t^2 - 4t + 2 = 0$ $\Rightarrow (t-2)^2 = 4-2 = 2 \Rightarrow t-2 = \sqrt{2} \text{ or } -\sqrt{2}$ $\Rightarrow t = 2 + \sqrt{2}, t \neq 2 - \sqrt{2} \text{ (as } t > 1) \Rightarrow p$ \n <th>Example from ed by these three point has rational coordinates, which is not so.</th> \n	Example from ed by these three point has rational coordinates, which is not so.		
$t = 2 + \sqrt{2}, t \neq 2 - \sqrt{2} \text{ (as } t > 1) \Rightarrow p$ \Rightarrow	$(d) Let (h, k) be the centre, then$ $ h = k and h + k - 4 = \sqrt{2} h $		
(a) Greatest distance is $AD = C_1C_2 + AC_1 + DC_2 = 5 + 1 + 3 = 9$ \Rightarrow	C	C	C
11 gives two different values of (h, k)	C as $2 : \text{If } h = -k, \text{ then } -4 = \sqrt{2} h \text{ i.e. } h = \pm 2\sqrt{2}$ \Rightarrow		

(43) (D).

(a) Greatest distance is AD = C1C² + AC¹ + DC² = 5 + 1 + 3 = 9

9=3
$$
\lambda
$$
 ⇒ λ =3
(b) x^2 = 200 ; $2x^2$ = 4 r^2 ; $2r^2$ = 200 ⇒ r = 10

(c)
$$
y = cos^4x - 6cos^2x + 5cos^2x = t
$$

\n $y = t^2 - 6t + 5$ $0 \le t \le 1$

(d) Distance between $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{8}{3}, \frac{8}{3}\right)$ is $\frac{7}{3}\sqrt{2}$

$$
\therefore \quad \frac{\ell}{\sqrt{2}} = 7
$$

(44) (C).

(a) Centre and radius of the circle $x^2 + y^2 + 14x - 4y + 28 = 0$ are (-7, 2), 5 respectively Centre and radius of the circle. $x^{2} + y^{2} - 14x + 4y - 28 = 0$ are $(7, -2)$, 9 \therefore length of direct common tangent

$$
= \sqrt{(7+7)^2 + (-2-2)^2 - (9-5)^2} = 14
$$

(b) the line is $mx - y + 2 - m = 0$

 $\left|\frac{12}{2} \right|$ < 5 which is true for all real values of m $\left|\frac{m}{r+1}\right| < 5$ which is true for all real values of m

(c)
$$
x^2 + (y - \sqrt{2})^2 = r^2
$$
 i.e., $x^2 + y^2 - 2\sqrt{2}y + 2 = r^2$
\n \therefore for $y = 0$, we have $x^2 + 2 = r^2$

 \therefore if r is rational and r² > 2, then there are 2 points on the circle which have rational co-ordinates.

further if there are three point, then circumcentre of the triangle fromed by these three point has rational coordinates, which is not so. **SOMANGED LEARNING**

There if there are three point, then circumcentre of the

iangle fromed by these three point has rational

oordinates, which is not so.

maximum number of points is 2.
 $|h| = |k|$ and $|h + k - 4| = \sqrt{2} |h|$

- \therefore maximum number of points is 2.
- (d) Let (h, k) be the centre, then

$$
|h| = |k|
$$
 and $|h + k - 4| = \sqrt{2} |h|$

Case - 1 : If h = k, then $|2h-4| = \sqrt{2} |h|$ i.e. $2h-4 = \pm \sqrt{2}h$ It gives two different values of (h, k)

S

S

further if there are three point, then circumcentre of the

triangle fromed by these three point has rational

coordinates, which is not so.
 \therefore maximum number of points is 2.
 (3) Let (h, k) be the centre, the it a gain gives two different points (h, k) thus there are 4 different circles. gle fromed by these three point has rational
dinates, which is not so.
aximum number of points is 2.
et (h, k) be the centre, then
 $= |k|$ and $|h + k - 4| = \sqrt{2} |h|$
-1: If h=k, then $|2h-4| = \sqrt{2} |h|$
-1: If h=k, then $|2h-4| =$ linates, which is not so.

et (h, k) be the centre, then
 $= |k|$ and $|h + k - 4| = \sqrt{2} |h|$
 $= 1$: If $h = k$, then $|2h - 4| = \sqrt{2} |h|$
 $= 1$: If $h = k$, then $|2h - 4| = \sqrt{2} |h|$ i.e. $2h - 4 = \pm \sqrt{2}h$

es two different values of ther if there are three point, then circumcentre of the
nigle fromed by these three point has rational
rdinates, which is not so.
maximum number of points is 2.
Let (h, k) be the centre, then
 $1 = |k|$ and $|h + k - 4| = \sqrt{2} |h$ ther if there are three point, then circumcentre of the
ngle fromed by these three point has rational
ratinates, which is not so.
maximum number of points is 2.
Let (h, k) be the centre, then
 $1 = |k|$ and $|h + k - 4| = \sqrt{2} |h|$ e-1: If h=k, then $|2h - 2 + 3| \rightarrow 2 + h|$

e-1: If h=k, then $|2h - 4| = \sqrt{2} |h|$ i.e. $2h - 4 = \pm \sqrt{2}h$

ives two different values of (h, k)

e2: If h = – k, then $|-4| = \sqrt{2} |h|$ i.e. $h = \pm 2\sqrt{2}$

gain gives two different poin h=k, then $|2h-4| = \sqrt{2}$ |h|i.e. $2h-4 = \pm \sqrt{2}h$

o different values of (h, k)

h = - k, then $|-4| = \sqrt{2}$ |h | i.e. $h = \pm 2\sqrt{2}$

ves two different points (h, k) thus there are 4

rcles.

f perpendicular are collinear.
 k | and | h + k - 4 | = $\sqrt{2}$ | h |

If h = k, then | 2h - 4 | = $\sqrt{2}$ | h | i.e. 2h - 4 = $\pm \sqrt{2}$ h

wo different values of (h, k)

If h = - k, then $|-4| = \sqrt{2}$ | h | i.e. $h = \pm 2\sqrt{2}$

gives two different points = |k | and | h + k - 4 | = $\sqrt{2}$ | h |

-1 : If h = k, then | 2h - 4 | = $\sqrt{2}$ | h | i.e. 2h - 4 = $\pm \sqrt{2}$ h

es two different values of (h, k)

2 : If h = - k, then $|-4| = \sqrt{2}$ | h | i.e. h = $\pm 2\sqrt{2}$

ain gives

- **(45) (D).** Feet of perpendicular are collinear.
- **(46) (D).** I_1 is the orthocentre of ΔI $I_2 I_3$ by property of triangle.
- **(47) (B).** As \triangle XYZ is pedal triangle of \triangle ABC, ex-centers of \triangle XYZ lie on vertices of \triangle ABC.
- **(48) (D).** $S_1 S_2 = 0 \Rightarrow x + y = 4$ (Radical axis) $S_1 - S_3 = 0 \Rightarrow 3x - 4y = 14$ (Radical axis) Radical centre = intersection point of radical axis

$$
\Rightarrow \left(\frac{30}{7}, \frac{-2}{7}\right)
$$

(49) (B). Radius of circle is nothing but length of tangent from radical centre to any of the given circle.

$$
\Rightarrow r = \sqrt{\left(\frac{30}{7}\right)^2 + \left(\frac{-2}{7}\right)^2 - 4} = 2\frac{\sqrt{177}}{7}
$$

(50) (C). Point of intersection of direct tangent always lie on the line joining there centre \Rightarrow (0, 0) and (3, -4)

 \Rightarrow line is 3y + 4x = 0

$$
(51) (B), (52) (D), (53) (B).
$$

 $\frac{7}{2}\sqrt{2}$: PQ = PR i.e. parallelogram PQRS is a rhombus

- \therefore Mid point of QR = Midpoint of PS and QR \perp PS
- \therefore S is the mirror image of P w.r.t. QR
- \therefore L = 2x + y = 6, Let P = (k, 6 2k)

$$
\because \angle PQO = \angle PRO = \frac{\pi}{2}
$$

 \therefore OP is diameter of circumcircle PQR,

then centre is
$$
\left(\frac{k}{2}, 3-k\right)
$$

EXAMPLE A INATERAL: MATHEMATICS				
\n $x = \frac{k}{2} \Rightarrow k = 2x$ \n $y = 3 - k \therefore 2x + y = 3.$ \n	\n The sum of QR is 6x + 8y = 4 ⇒ 3x + 4y - 2 = 0\n $y = 3 - k \therefore 2x + y = 3.$ \n	\n Note that radius of g(x, y) = twice the radius of f(x, y) = 0\n $y = 3 - k \therefore 2x + y = 3.$ \n	\n Note that radius of g(x, y) = twice the radius of f(x, y) = 0\n $y = 3 - k \therefore 2x + y = 3.$ \n	\n Note that radius of g(x, y) = twice the radius of f(x, y) = 0\n $y = 3 - k \therefore 2x + y = 3.$ \n
\n $PM = \frac{48}{5} \text{ and } PQ = \sqrt{96}$ \n $PM = \frac{48}{5} \text{ and } PQ = \sqrt{96}$ \n	\n The sum of g(x, y) = 5\n $P = \frac{59}{4} \text{ and } PQ = \frac{196\sqrt{6}}{25}.$ \n			
\n $PM = \sqrt{96 - \frac{(48)^2}{25}} = \sqrt{\frac{96}{25}}.$ \n	\n The sum of QR is 3x + 4y = 4\n $P = \frac{196\sqrt{6}}{25} \text{ sq. units}.$ \n	\n The sum of QR is 3x + 4y = 4\n $P = \frac{196\sqrt{6}}{25} \text{ sq. units}.$ \n	\n The sum of g(x, y) is $x^2 + y^2 - 5x - 4y - \frac{59}{4} = 0$ \n $P = \frac{39}{4} = \frac{91}{4} = \frac{-2(3 \times 3 + 4 \times 4 - 4)}{3^2 + 4^2} = -\frac{42}{$	

$$
\because S \text{ is mirror image of P w.r.t. eq. (i)}
$$

then
$$
\frac{x_1 - 3}{3} = \frac{y_1 - 4}{4} = \frac{-2(3 \times 3 + 4 \times 4 - 4)}{3^2 + 4^2} = -\frac{42}{25}
$$
 Equation of equation

$$
x_1 = -\frac{51}{25}, y_1 = -\frac{68}{25}
$$
; $S = \left(-\frac{51}{25}, -\frac{68}{25}\right)$

(54) (D), (55) (A), (56) (D).

 $r_1 = 2, r_2 = 1, C_1 = (0, 3), C_2 = (6, 0)$ Clearly the circle with centre C_1 and C_2 are separated $CC_1 = r + r_1$; $CC_2 = r + r_2$ $CC_1 - CC_2 = r_1 - r_2 = constant$

(57) (D), (58) (D), (59) (D).

Given $f(x, y) = 0$ is circle. As $f(0, y)$ has equal roots hence $f(x, y) = 0$ touches the y-axis and as $f(x, 0) = 0$ has two distinct real roots hence $f(x, y) = 0$ cuts the x-axis in two distinct points. Hence $f(x, y) = 0$ will be as shown now, given $g(x, y) = x^2 + y^2 - 5x - 4y + c$

centre =
$$
\left(\frac{5}{2}, 2\right)
$$
, radius = $\sqrt{\frac{25}{4} + 4 - c}$

STUDY MATERIAL: MATHEMATICS
 $\left(\frac{5}{2}, 2\right)$, radius = $\sqrt{\frac{25}{4} + 4 - c}$

radius of g (x, y) = twice the radius of f (x, y) = 0

s clear from the adjacent figure r = 5/2

of g (x, y) = 5

59 **TERIAL: MATHEMATICS**
 $\frac{25}{4} + 4 - c$

vice the radius of $f(x, y) = 0$

cent figure $r = 5/2$
 $= -\frac{59}{4}$ RIAL: MATHEMATICS

+4-c

e the radius of $f(x, y) = 0$

at figure $r = 5/2$ Note that radius of $g(x, y)$ = twice the radius of $f(x, y) = 0$ but as it is clear from the adjacent figure $r = 5/2$ \therefore radius of $g(x, y) = 5$

EXAMPLE 13.24
\n
$$
x = \frac{1}{2} \Rightarrow k = 2x
$$
\n
$$
y = 3 + 2x \Rightarrow y = 3.
$$
\n
$$
y = 4 \Rightarrow k = 2x
$$
\n
$$
y = 6 \Rightarrow k = 2x \Rightarrow y = 3.
$$
\n
$$
x = \frac{10}{2} \text{ rad} \times 10^{-1} \text{ cm}^2/\sqrt{10} = 600
$$
\n
$$
y = 10 \text{ rad} \times 10^{-1} \text{ cm}^2/\sqrt{10} = 1000 \text{ cm}^2
$$

Area of region inside $f(x, y) = 0$ above the x-axis is

$$
\begin{aligned} \n\text{x-axis} &= \frac{1}{2} \left(\frac{5}{2} \right)^2 \left(2\pi - \tan^{-1} \left(\frac{27}{4} \right) \right) + \frac{1}{2} \times 3 \times 2 \\ \n&= 3 + \frac{25}{8} \left(2\pi - \tan^{-1} \left(\frac{27}{4} \right) \right) \n\end{aligned}
$$

(11) Points satisfying the conditions are $(1, 5)$ $(1, 6)$, $(2, 5)$, $(2, 6)$ $(3, 5)$, $(3, 6)$, $(4, 5)$, $(4, 6)$, $(5, 4)$, $(5, 5)$, $(5, 6)$.

(60) **(B).** 14. x. (-3) + 14. y. 6 + 108(x-3) -
$$
\frac{69}{2}
$$
(y+6) + 432=0
\n
$$
\Rightarrow x(108-42) + y\left(84 - \frac{69}{2}\right) + (432 - 531) = 0
$$
\n
$$
\Rightarrow 4x + 3y - 6 = 0
$$
\n(2)

(61) (C).
$$
g = \frac{216}{28}
$$
, $f = -\frac{69}{28}$, $c = \frac{432}{14}$

$$
radius = \sqrt{g^2 + f^2 - c} = \frac{165}{28}
$$

Let PT =
$$
\ell
$$
, tan 2 θ = $\frac{165}{28 \ell}$ (i)

$$
\cos \theta = \frac{11\sqrt{130}}{13 \ell} \qquad \qquad \dots \dots \dots \dots (ii)
$$

Dividing (i) by (ii)
$$
\frac{\tan 2\theta}{\cos \theta} = \frac{15.13}{28.\sqrt{13}.\sqrt{10}}
$$

$$
\sin \theta = \frac{-56\sqrt{10} \pm 74\sqrt{10}}{60\sqrt{13}}
$$
 (only positive value is possible)

$$
\Rightarrow \tan \theta = \frac{3}{11}
$$

EXERCISE-3

EXERICISE-3
 EXERICISE-3
 EXERICISE-3
 EXERICISE-3
 $(x-1)^2 + (y-2)^2 + \lambda (4x + 4y - 10) = 0$
 \therefore comparing twith $x^2 + y^2 + (4\lambda - 2)x + (3\lambda - 4)y + 5 = 10\lambda - 0$
 \therefore comparing twith $x^2 + y^2 + (x + 2y - 15) = 0$, we get
 $\alpha = 4\lambda$ **EXERCISE-3**

EXERCISE-3

EXERCISE-3

Comparing the state of common tangent is $4x + 3y = 10$
 $(x, 2)$
 $(x, 3)$
 $(x, 4)$
 $(x, 5)$
 $(x, 6)$
 $(x$ **EXERCISE-3**
 $\begin{pmatrix}\n\frac{1}{32} \\
\frac{1}{32} \\
\$ **(1) 28.** Equation of common tangent is $4x + 3y = 10$ \therefore equation of a circle is $(x-1)^2 + (y-2)^2 + \lambda (4x+4y-10) = 0$ i.e. $x^2 + y^2 + (4\lambda - 2)x + (3\lambda - 4)y + 5 - 10\lambda = 0$ Comparing it with $x^{2} + y^{2} + \alpha x + \beta y - 15 = 0$, we get $\alpha = 4\lambda - 2$, $\beta = 3\lambda - 4$ and $15 = 10\lambda - 5$: $\alpha = 6$, $\beta = 2$ $\lambda-2$) $x + (3\lambda - 4) y + 5 - 10\lambda = 0$

with
 $-8y - 15 = 0$, we get
 $-3\lambda - 4$ and $15 = 10\lambda - 5$ $\therefore \alpha = 6, \beta = 2$
 \therefore
 $\lambda^2 + y^2 + \gamma x + 8y + 25 = 0$, we get
 $x^2 + y^2 + \gamma x + 8y + 25 = 0$, we get
 $-3\lambda - 4$ and $25 = 5 - 10\lambda$
 -10
 Erricle is
 $y^2 + \lambda (4x + 4y - 10) = 0$
 $-2) x + (3\lambda - 4) y + 5 - 10\lambda = 0$

h
 $3x - 15 = 0$, we get
 $3\lambda - 4$ and $15 = 10\lambda - 5$ $\therefore \alpha = 6, \beta = 2$
 $x^2 + y^2 + \gamma x + 8y + 25 = 0$, we get
 $x^2 + y^2 + \gamma x + 8y + 25 = 0$, we get
 $x^2 - 4$ and $25 =$

$$
\left(\bigvee(1,2)\right)
$$

 $\frac{69}{(y+6)+432=0}$ $\gamma = 4\lambda - 2, \delta = 3\lambda - 4$ and $25 = 5 - 10\lambda$ $\therefore \gamma = -10, \delta = -10$ Comparing with $x^2 + y^2 + \gamma x + \delta y + 25 = 0$, we get Thus $\alpha + \beta - (\gamma + \delta) = 28$

- $=\frac{432}{14}$ of α . **(2) 3.** Let $(\alpha, 3 - \alpha)$ be any point on $x + y = 3$ \therefore equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$ i.e., $\alpha(x-y) + 3y - 9 = 0$ \therefore the chord passes through the point (3, 3) for all values $of \alpha$. mparing with $x^2 + y^2 + \gamma x + \delta y + 25 = 0$, we get
 $=4\lambda - 2, \delta = 3\lambda - 4$ and $25 = 5 - 10\lambda$
 $= -10, \delta = -10$
 $\alpha + \beta - (\gamma + \delta) = 28$
 αt ($\alpha, 3 - \alpha$) be any point on $x + y = 3$
 $\alpha(x - y) + 3y - 9 = 0$

equation of chord of octorate is paring with $x^2 + y^2 + \gamma x + \delta y + 25 = 0$, we get
 $= -4\lambda - 2$, $\delta = 3\lambda - 4$ and $25 = 5 - 10\lambda$
 $= -10$, $\delta = -10$

s α + β – (γ + δ) = 28

equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
 $\alpha (x - y) + 3y - 9 = 0$
 $\alpha (x - y) + 3y - 9 =$ fing with $x^2 + y^2 + \gamma x + \delta y + 25 = 0$, we get
 $x-2, \delta = 3\lambda - 4$ and $25 = 5-10\lambda$
 $+8 - (\gamma + \delta) = 28$

(a, 3 − α) be any point on x + y = 3

(a, 3 − α) be any point on x + y = 3

(a, 3 − α) be any point on x + y = 3
 $x - y$) + 3 $\gamma = -10, \delta = -10$

hus $\alpha + \beta - (\gamma + \delta) = 28$

Let $(\alpha, 3 - \alpha)$ be any point on $x + y = 3$

equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
 $\therefore \alpha (x - y) + 3y - 9 = 0$

the chord passes through the point (3, 3) for all values
 α .
 = 4) - 2, δ = 3) - 4 and 25 = 5 - 10).

= -10, δ = -10

= -10, δ = -10

= -10, δ = -10

equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$

equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
 $\alpha (x - y) + 3y - 9 = 0$
 $\left[\frac{1}{x-y} + 3y - 9 = 0\right]$
 $\left[\frac$ x, 3 – α) be any point on $x + y = 3$
ation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
ation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
ond passes through the point (3, 3) for all values
tion of line joining origin and centre o equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$

equation of chord of contact is $\alpha x + (3 - \alpha) y = 9$
 $\alpha (x-y) + 3y - 9 = 0$

he chord passes through the point (3, 3) for all values
 x .
 α
 α (aution of line joining ori
	- **(3) 5.** Equation of line joining origin and centre of circle

$$
C_2 = (2, 1)
$$
 is, $y = \frac{x}{2} \Rightarrow x - 2y = 0$

Let equation of common tangent is $x - 2y + c = 0$ (1)

$$
\therefore
$$
 Perpendicular distance from (0, 0) on this line
= perpendicular distance from (1, 1)

$$
\left| \frac{c}{\sqrt{5}} \right| = \left| \frac{c-1}{\sqrt{5}} \right| \Rightarrow c = 1 - c \Rightarrow c = \frac{1}{2}
$$

Equation of common tangent is

$$
x-2y+\frac{1}{2}=0 \implies 2x-4y+1=0
$$
(2)

perpendicular from $(2, 1)$ on the line (2)

Alternative sol 1 : P is the mid point of C_1C_2 \therefore P (3/2, 1)

232 hence eq. of the common tangent is 1 3 y 1 x 2 2 2x – 4y + 1 = 0 now proceed Alternative sol 2 : sin = 2r as (PC² = 1/2) 2 1 sin as (CC 5) ⁵ . Hence, ¹ 2r ⁵ 1 r 2 5 **(4) 2.** Equation of circle is x² + y² + 2gx +2fy + c = 0 (1 , t) 1 + t² + 2g + 2ft + c = 0 (t, t) t² + t² + 2gt + 2ft + c = 0 (t, 1) 1 + t² + 2gt + 2f + c = 0 subtract 1 + 2g – t² – 2gt = 0 1 – t² + 2g(1 – t) = 0 (1 – t)(1 + t + 2g) = 0 t = 1 one point (t, t) passes through (1, 1) **(5) 50.** The equation of given circle is S(x, y) = x² + y² – 6x – 2py + 17 = 0 (x – 3)² + (y – p)² = (p² – 8) S (0, 0) = 17 > 0 (0, 0) lies outside the circle. Equation of director circle of S = 0 will be (x – 3)² + (y – p)² = 2(p² – 8). Tangents drawn from (0, 0) to S = 0 are perpendicular to each other (0, 0) must lie on director circle. (0 – 3)² + (0 – p)² = 2 (p² – 8) p² = 25 p = ± 5 Hence p¹ 2 + p² ² = (5)² + (–5)² = 25 + 25 = 50 **(6) 62.** ^A ^B C (4cos , 4sin) (4,0) ¹ A 8 4sin |16sin | ² Now sin can be 1 2 15 , , 16 16 16 i.e. 15 points in each quadrant 60 + 2 more with sin = 1 total = 62 **(7) 25 .** ^D 45° 45° (–3,3) L =5 ^T L =5 ^T A B r = 5 r = 5 C (–4,–2) L S 5 ¹

Area of quadrilateral ABCD

STUDY MATERIAL: MATHEMATICS
\nArea of quadrilateral ABCD
\n= 2 Area of
$$
\triangle ACD = 2\left(\frac{1}{2} \times 5 \times 5\right) = 25
$$
 sq. units
\n3. Triangles BCM and OCN are similar
\nnow let ON = p. N will be mid point of chord PQ

(8) 63. Triangles BCM and OCN are similar now let $ON = p$. N will be mid point of chord PQ

Now R =
$$
2\sqrt{r^2 - p^2}
$$

for large circle = $2\sqrt{16 - (1/4)} = \sqrt{63}$
Alternatively: Equation of large circle as $x^2 + y^2 = 16$
now C = (1, 0) with slope PQ = $-\frac{1}{\sqrt{3}}$ (think!)
equation of PQ: $\sqrt{3}y + x = 1$

P (from origin) =
$$
\frac{1}{2}
$$
 \Rightarrow result

(9) 69. Let r be the radius of circle A
and R be the radius of circle B

$$
\therefore
$$
 r + R = 12 and r = 3R
 \therefore 4R = 12; \therefore R = 3 and r = 9

Area of trapezium ABCD = $\frac{1}{2}(3+9)\sqrt{(12)^2-6^2}$ $2^{(3/2)}\sqrt{(12)}$ - 0 (3 + 9) $\sqrt{(12)^2 - 6^2}$
 $= 6\sqrt{108} = 36\sqrt{3}$
 $= \frac{27\pi}{2}$ R

F = 9

(1988)
 $\frac{1}{3}$
 $+ 9$) $\sqrt{(12)^2 - 6^2}$

= $6\sqrt{108} = 36\sqrt{3}$
 $\frac{27\pi}{2}$
 3π Area of arc ADC = $\frac{1}{2} \times 81 \times \frac{\pi}{3} = \frac{27\pi}{2}$ 1 27 ⁸¹ s of circle A

circle B

and r = 3R

and r = 9

(a)

BCD = $\frac{1}{2}(3+9)\sqrt{(12)^2-6^2}$

= $6\sqrt{108} = 36\sqrt{3}$

 $\frac{1}{2} \times 81 \times \frac{\pi}{3} = \frac{27\pi}{2}$
 $\frac{1}{2} \times 9 \times \frac{2\pi}{3} = 3\pi$
 $\sqrt{3} - (\frac{27\pi}{2} + 3\pi) = 36\sqrt{3} - \frac{33\pi$ > result

of circle A

ircle B
 $r = 3R$

and $r = 9$
 $\frac{9}{30}$
 $\frac{12}{3}$
 $\frac{3}{3}$
 $\frac{27\pi}{3} = 3\pi$
 $\frac{27\pi}{3} = 3\$ Area of arc BCE = $\frac{1}{2} \times 9 \times \frac{1}{2}$ f circle B

and r = 3R

and r = 9

and r = 9

BCD = $\frac{1}{2}(3+9)\sqrt{(12)^2-6^2}$

= $6\sqrt{108} = 36\sqrt{3}$
 $\frac{1}{2} \times 81 \times \frac{\pi}{3} = \frac{27\pi}{2}$
 $\frac{1}{2} \times 9 \times \frac{2\pi}{3} = 3\pi$
 $\sqrt{3} - (\frac{27\pi}{2} + 3\pi) = 36\sqrt{3} - \frac{33\pi}{2}$
 nd r=3K

and r=9

and r=9

BCD = $\frac{1}{2}(3+9)\sqrt{(12)^2-6^2}$

= $6\sqrt{108} = 36\sqrt{3}$
 $\frac{1}{2} \times 81 \times \frac{\pi}{3} = \frac{27\pi}{2}$
 $\frac{1}{2} \times 9 \times \frac{2\pi}{3} = 3\pi$
 $\sqrt{3} - (\frac{27\pi}{2} + 3\pi) = 36\sqrt{3} - \frac{33\pi}{2}$
 \therefore a + b = 69 π and π ∴ required area = $36\sqrt{3} - \left(\frac{27\pi}{2} + 3\pi\right) = 36\sqrt{3} - \frac{33\pi}{2}$ 27 33 36 3 3 36 3 E

(a) $\sqrt{3}$ B

(b)

(b)

(c)

(c)
 $\frac{1}{2}$ (3+9) $\sqrt{(12)^2 - 6^2}$

= $6\sqrt{108} = 36\sqrt{3}$
 $\frac{\pi}{3} = \frac{27\pi}{2}$
 $\frac{27\pi}{2}$
 $\frac{17\pi}{2} + 3\pi$ = $36\sqrt{3} - \frac{33\pi}{2}$

69 = 3 and r = 9

A
 $\frac{A}{3}$
 $\frac{3}{3}$

D
 $\frac{3}{3}$
 $\frac{3}{1}$
 $\frac{1}{3}$
 \therefore a = 36, b = 33 \therefore a + b = 69

let C(h, k) = c(h, ah)
\n
$$
CC_1^2 = (16 - r)^2
$$
\n⇒ (h + 5)² + (12 - ah)² = (16 - r)²
\n
$$
CC_2^2 = (4 + r)^2
$$
\n⇒ (h - 5)² + (12 - ah)² = (4 + r)²
\nBy subtraction, 20h = 240 - 40r
\n⇒ h = 12 - 2r ⇒ 12r = 72 - 6h ...(1)
\nBy addition
\n
$$
2[h^2 + 25 + a^2h^2 - 24ah + 144] = 272 - 24r + 2r^2
$$
\n
$$
h^2(1 + a^2) - 24ah + 169 = 136 - 12r + r^2 = 136 + (6h - 72)
$$
\n
$$
+ \left(\frac{12 - h}{2}\right)^2
$$
 [using (1)] Let $\frac{\pi}{k}$ = 14.162 + 23, 24th + 169 = 176.164 + 6th + 16th + 12h

 \Rightarrow 4 [h²(1 + a²) – 24ah + 169] = 4[64 + 6h] + (12 – h)² $= 256 + 144 + h²$ \Rightarrow h² (3 + 4a²) – 96ah + 105 · 4 – 36 · 4 = 0 \Rightarrow h² (3 + 4a²) – 96ah + 69 · 4 = 0; for 'h' to be real D ≥ 0 $\Rightarrow (96a)^2 - 4 \cdot 4 \cdot 69 (3 + 4a^2) \ge 0$ \Rightarrow 576a² – 69.3 – 276a² ≥ 0

$$
300a2 \ge 207 \Rightarrow a2 \ge \frac{69}{100}; \text{ hence } m \text{ (smallest)} = \frac{13}{10}
$$

So, m² =
$$
\frac{69}{100}
$$
 : p+q=169

(12) 5. Line $5x - 2y + 6 = 0$ is intersected by tangent at P to circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis at Q (0, 3). In other words tangent passes through (0, 3). \therefore PQ = length of tangent to circle from (0, 3). **EXERCISE ARRIVATE:**
 EXERCISE ARRIVATE:
 $7 \Rightarrow a^2 \ge \frac{69}{100}$; hence m (smallest) = $\frac{13}{10}$
 $\therefore p+q = 169$
 $-2y+6 = 0$ is intersected by tangent at P to
 $+y^2+6x+6y-2 = 0$ on y-axis at Q (0, 3).

words tangent passe **SODIMAD VANCED LEARNING**
 $0a^2 \ge 207 \Rightarrow a^2 \ge \frac{69}{100}$; hence m (smallest) = $\frac{13}{10}$
 $m^2 = \frac{69}{100}$ $\therefore p+q = 169$

Line $5x - 2y + 6 = 0$ is intersected by tangent at P to

circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis a

$$
=\sqrt{0+9+0+18-2}=\sqrt{25}=5
$$

(13) 3. The given circle is
$$
x^2 + y^2 - 2x - 6y + 6 = 0
$$

Let AB be one of its diameter which is the chord of other circle with centre at C_1 (2, 1).

Then in
$$
\triangle C_1CB
$$
, $C_1B^2 = CC_1^2 + CB^2$
\n $\Rightarrow r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$
\n $\Rightarrow r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3$

Then in
$$
2C_1CB
$$
, $C_1B^- = C_1^- + C_2$
\n⇒ $r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$
\n⇒ $r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow r = 3$
\n
$$
D \xrightarrow{\text{th}} 2a - h
$$
\n
$$
D \xrightarrow{\text{th}} 2a - h
$$
\n
$$
D \xrightarrow{\text{th}} 2a - h
$$
\nLet AB = a and AD = 2h
\nIn triangle BCL, $a^2 + 4h^2 = (3a - 2h)^2$; $a = 3h/2$
\n $\frac{1}{2} \times 3a \times 2h = 18 \Rightarrow h = 2$; Radius = 2 unit.
\n2. $2 \cos \frac{\pi}{2k} + 2 \cos \frac{\pi}{k} = \sqrt{3} + 1$
\n $\cos \frac{\pi}{2k} + \cos \frac{\pi}{k} = \frac{\sqrt{3} + 1}{2}$
\nLet $\frac{\pi}{k} = 0$, $\cos \theta + \cos \frac{\theta}{2} = \frac{\sqrt{3} + 1}{2}$
\n $\Rightarrow 2 \cos^2 \frac{\theta}{2} - 1 + \cos \frac{\theta}{2} = \frac{\sqrt{3} + 1}{2}$
\n $\cos \frac{\theta}{2} = t$; $2t^2 + t - \frac{\sqrt{3} + 3}{2} = 0$

15) 2.
$$
2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1
$$

$$
\cos\frac{\pi}{2k} + \cos\frac{\pi}{k} = \frac{\sqrt{3}+1}{2}
$$

Let
$$
\frac{\pi}{k} = 0
$$
, $\cos \theta + \cos \frac{\theta}{2} = \frac{\sqrt{3} + 1}{2}$
2 cos² θ 1 cos² θ $\sqrt{3} + 1$

$$
2 \t 2 \t 2
$$

$$
\cos\frac{\theta}{2} = t \text{ ; } 2t^2 + t - \frac{\sqrt{3} + 3}{2} = 0
$$

PROOF MATERIAL
\n
$$
t = \frac{-1 \pm \sqrt{1 + 4(3 + \sqrt{3})}}{4}
$$

\n $t = \frac{-1 \pm \sqrt{1 + 4(3 + \sqrt{3})}}{4}$
\n $= \frac{-1 \pm (2\sqrt{3} + 1)}{4} = \frac{-2 - 2\sqrt{3}}{4}, \frac{\sqrt{3}}{2}$
\n $\therefore t \in [-1, 1]$, $\cos \frac{\theta}{2} = \frac{\sqrt{3}}{2}$; $\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3$
\n $\therefore t \in [-1, 1]$, $\cos \frac{\theta}{2} = \frac{\sqrt{3}}{2}$; $\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3$
\n \therefore Locus is a² + b² + 2a² + b² +

EXERCISE-4

(1) (C). Length of tangent from any point (x¹ , y¹) to the circle is S¹ . Length of tangent from (3, – 4) on the circle x 2 + y² – 4x – 6y + 3 = 0 is 2 2 3 (4) 4 (3) 6 (4) 3 ⁼ 9 16 12 24 3 40 and is square is 40 **(2) (A).** Given equation of circle are x 2 + y² + 6x – 2y – 9 = 0 and x² + y² – 2x + 9y – 11 = 0 equation of radical axis is S¹ – S² = 0 8x – 11y + 2 = 0 **(3) (B).** Given equation of two circle are (x – 1)² + (y – 3)² = r² (1) Coordinate of centre is (1, 3) and radius is r and x² + y² – 8x + 2y + 8 = 0 (2) Coordinate of centre is (4, –1) radius = 16 1 8 3 Now, 2 2 C C (1 4) [3 (1)] 5 1 2 If circle intersect in two distinct points then C1C² < r¹ + r² and C1C² > r¹ – r² 5 < r + 3 and 5 > r – 3 2 < r and 8 > r 2 < r < 8 **(4) (D).** If lines 2x – 3y = 5 and 3x – 4y = 7 are diameter of a circle then their intersection point will be centre of circle. Intersection point of these two lines is (1, –1) Coordinate of centre of circle is (1, –1) Now let radius of circle is r Area is r² = 154 (given Area = 154 sq² unit) ² 154 154 r 7 ²² r² = 7 × 7 r = 7 unit equation of circle will be (x – 1)² + [y – (–1)]² = 7² x 2 – 2x + 1 + y² + 1 + 2y = 49 x² + y² – 2x + 2y = 47 **(5) (B).** Let the equation of circle whose centre is (– g, – f) is x² + y² + 2gx + 2fy + c = 0 (1) this circle passes through (a, b) a² + b² + 2ag + 2bf + c = 0 (2) Now circle (1) cut circle x² + y² – 4 = 0

MIS

IT $\frac{1 \pm \sqrt{1 + 4(3 + \sqrt{3})}}{4}$
 $\frac{1 \pm (2\sqrt{3} + 1)}{4} = \frac{-2 - 2\sqrt{3}}{4}, \frac{\sqrt{3}}{2}$

The oriental control of the c MINE (Q.B.- SOLUTIONS
 $-1 \pm \sqrt{1+4(3+\sqrt{3})}$
 $+ \frac{2g(9)+2f(0) = c-$
 $\frac{2g(g_2+2f_1f_2 = c_1+c_2) \Rightarrow c-4 = 0}{2}$

Put this value in (2) we get $a^2 + b^2 +$
 \Rightarrow for locus of centre replace (-g, -f)
 $\Rightarrow g = -x$ and $f = -y$ **EXERCISE 4**

EXERCISE **EXERCISE A**

EXERCISE **EXERCISE A**

EXERCISE **EXERCISE**

EXERCISE THE CONDUCTIONS

COLUTIONS

COLUTIONS

Orthogonally :. 2g(0)+2f(0)=c-4

orthogonally :. 2g(0)+2f(0)=c-4

(if two circle cuts cuts c **(Q.B.- SOLUTIONS**

STUDY MATERIAL: MATHEMATI
 $+4(3+\sqrt{3})$
 $+4(3+\sqrt{3})$
 $+3(3+\sqrt{3})$
 $+1 = -2-2\sqrt{3}$, $\frac{\sqrt{3}}{2}$
 $+1 = \frac{-2-2\sqrt{3}}{2}$, $\frac{\sqrt{3}}{2}$
 $+1 = \frac{-2-2\sqrt{3}}{2}$, $\frac{\sqrt{3}}{2}$
 $+1 = \frac{-2-2\sqrt{3}}{2}$, $\frac{\sqrt{3}}{2}$
 MING
 $\frac{-1 \pm \sqrt{1+4(3+\sqrt{3})}}{4}$
 $\frac{-1 \pm (2\sqrt{3}+1)}{4} = \frac{-2-2\sqrt{3}}{4}, \frac{\sqrt{3}}{2}$
 $\frac{1}{2} = \frac{\sqrt{3}}{2}; \frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3$
 $\frac{\pi}{2}$
 $\frac{1}{2}$

(b) CPthogonally :. 2g(0) + 2f(0) = c - 4

(if two circle cuts orthogonally **EXERCISE-4**

EXERCISE-4

C.B. solutions
 $\therefore t \in [-1, 1], \cos \frac{\theta}{2} = \frac{\sqrt{3}}{2}$; $\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3$

EXERCISE-4

C.B. Length of tangent from any point (x_1, y_1) to the circle
 $\frac{\sinh(x_1, y_1)}{\cosh(x_1, y_1)}$ to the circle c **(Q.B.- SOLUTIONS** Orthogonally \therefore 2g(0) + 2f (0) = c - 4

(if two circle cuts orthogonally then condition is
 $2g_{1}g_{2} + 2f_{1}f_{2} = c_{1} + c_{2}$) \Rightarrow c - 4 = 0 \Rightarrow c - 4

Put this value in (2) we get $a^{2} + b^{2} + 2ga + 2$ **(O.B.- SOLUTIONS** STUDY MATERIAL: MATHEN

Orthogonally \therefore 2g(0) + 2f(0) = c-4

{if two circle cuts orthogonally then condition is
 $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ } $\Rightarrow c - 4 = 0 \Rightarrow c = 4$

Put this value in (2) we get $a^2 + b^2 +$ **(Q.B.- SOLUTIONS** Orthogonally \therefore 2g(0) + 2f (0) = c - 4

{if two circle cuts orthogonally then condition is
 $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ } $\Rightarrow c - 4 = 0 \Rightarrow c = 4$

Put this value in (2) we get $a^2 + b^2 + 2ga + 2bf + 4 = 0$
 \Rightarrow f Orthogonally \therefore 2g(0) + 2f(0) = c – 4 {if two circle cuts orthogonally then condition is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ \Rightarrow $c - 4 = 0 \Rightarrow c = 4$ Put this value in (2) we get $a^2 + b^2 + 2ga + 2bf + 4 = 0$ \Rightarrow for locus of centre replace (–g, –f) with (x, y) \Rightarrow g = – x and f = – y \therefore Locus is $a^2 + b^2 - 2ax - 2by + 4 = 0$ \Rightarrow 2ax + 2by – (a² + b² + 4) = 0 onally \therefore 2g(0) + 2f (0) = c-4

circle cuts orthogonally then condition is
 $2f_1 f_2 = c_1 + c_2$ } $\Rightarrow c - 4 = 0 \Rightarrow c = 4$

value in (2) we get $a^2 + b^2 + 2ga + 2bf + 4 = 0$

ocus of centre replace (-g, -f) with (x, y)

- x and f = -y + 2f (0) = c - 4

ggonally then condition is
 \Rightarrow c - 4 = 0 \Rightarrow c = 4

gget $a^2 + b^2 + 2ga + 2bf + 4 = 0$

place (-g, -f) with (x, y)

x - 2by + 4 = 0

which touches x axis is

2 where let h, k a re coordinate of

- 2ky = 0 .

(6) (A). Equation of circle which touches x axis is $(x-h)^2 + (y-k)^2 = k^2$ where let h, k a re coordinate of centre $\Rightarrow (x-h)^2 + y^2 - 2ky = 0$ (1)

 \therefore (p, q) lies on circle \therefore (p-h)² + q² - 2kq = 0 (2) .. $(p-h)^2 + q^2 - 2kq = 0$ (2)
Let coordinate of other end of diameter α, β

$$
\therefore h = \frac{\alpha + p}{2} \text{ and } k = \frac{\beta + q}{2}
$$

Put this in (2) we get

2 2 (p) q p q 2 q 0 2 2 2 ^p 2 2 q q q 0 2 2 (p) q 0 4 (p –)² = 4q

.. Locus of
$$
(\alpha, \beta)
$$
 is $(p - x)^2 = 4yq \Rightarrow (x - p)^2 = 4qy$
\n**(7) (A).** If lines $2x + 3y - 1 = 0$ and $3x - y - 4 = 0$ are diameter
\nof circle then their intersection point will be centre of
\ncircle.

$$
∴ their interpretation is (1, -1)
$$
\n∴ coordinate of circle is (1, -1)
\nNow circumcircle of circle is 2πr = 10π (given)
\n⇒ r = 5 unit
\n∴ equation of circle is (x – 1)² + [y – (–1)²] = 25
\n
$$
x^2 + 1 - 2x + y^2 + 1 + 2y = 25
$$
\n⇒
$$
x^2 + y^2 - 2x + 2y - 23 = 0
$$
\n
$$
x^2 + y^2 - 2x + 2y - 23 = 0
$$
\n(A).
\n
\nGiven circle is $x^2 + y^2 - 2x = 0$ (1)
\nGiven line is y = x (2)

Putting $y = x$ in (1) we get $2x^2 - 2x = 0 \Rightarrow x = 0, 1$

(8) (A).

From (1) , $y = 0$, 1 \therefore Intersection points are $(0, 0)$ and $(1, 1)$ Let A $(0, 0)$ and B $(1, 1)$ equation of required circle is $(x-0)(x-1)+(y-0)(y-1)=0$ $\Rightarrow x^2 + y^2 - x - y = 0$ **(9) (B).** Given equation of circles are $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ and their intersection point are P and Q \therefore equation of line passing through P and Q is $S_1-S_2=0 \Rightarrow 5ax + (c-d)y + a+1=0$ \Rightarrow 5x + $\left(\frac{c-d}{a}\right)$ y + $\frac{a+1}{a}$ = 0 **(O.B.- SOLUTIONS**

Suppose $y=0, 1$

contain points are $(0, 0)$ and $(1, 1)$

(12) **(D)**. Equation of pair of lines is
 $\cos^2 x^2 + 2(a + b) \sin x + b y^2 = 0$

o) and B $(1, 1)$
 $\cos x + y - y = 0$
 $\cos x + y + z = 0$
 $\cos^2 x + 2x + y + b = 0$
 \cos = 0, 1 (12) (13) Equation of pair of lines is

a and B (1, 1) (12) (13). Equation of pair of lines is

a and B (1, 1)

and B (1, 1)

a and B (1, 1)
 \therefore x - y = 0

equation of circles are
 $y^2 + 2ax + cy + a = 0$
 $y^2 + 2ax + cy$ **(Q.B.- SOLUTIONS**

Section points are (0, 0) and (1, 1)

(12) (D). Equation of pair of lines is
 $ax^2 + 2(a + b) xy + by^2 = 0$
 $x^2 + 2x + y + b^2 = 0$

of required circle is
 $\Rightarrow x^2 + 2x + 2y + a = 0$
 $\Rightarrow x^2 + 2ax + cy + a = 0$
 $\Rightarrow x^2 + 2ax + cy + a =$ Given line is $5x + by - a = 0$ (2) Comine eq. (1) and (2) we get **ED**
 ED

From (1), y=0, 1
 \therefore Intersection points are (0, 0) and (1, 1)
 \therefore Intersection points are (0, 0) and (1, 1)
 \therefore Intersection of required circle is
 $(x-0)(x-1)+(y-0)(y-1)=0$
 $\Rightarrow x^2+y^2-x-y=0$
 B). Given eq $\frac{1}{a} = -a \Rightarrow a+1=-a^2 \Rightarrow a^2+a+1=0$ { : $\frac{+1}{2} = -a \implies a + 1 = -a^2 \implies a^2 + a + 1 = 0 \{ \therefore d \text{ is } c > 0 \}$ **(1)**, $y = 0, 1$

(a) $y = 0$, $y = 0$

(a) $y = 0$, $y = 0$

(a) $y = 0$
 $y = 0$

(a) which is not possible. **(10) (D).** $(0, 3)$ 2 \mathbb{R} Q \mathbb{R} (h, k) \mathbf{k} P $\left\langle \right\rangle$ R \ $k\sum_{l}$ $PR = k$ and $QR = 2$ \Rightarrow PR + QR = k + 2 Let centre of circle which touches x axis (h, k) \therefore equation of this circle is $(x-h)^2 + (y-k)^2 = k^2$ $=k²$ Now, $PQ = PR + RQ$ $\therefore PQ^2 = (PR + RQ)^2$ \Rightarrow $(h-0)^2 + (k-3)^2 = (k+2)^2$ \Rightarrow h² + k² - 6k + 9 = k² + 4 + 4k \Rightarrow h² - 10k + \Rightarrow h² + k² – 6k + 9 = k² + 4 + 4k \Rightarrow h² – 10k + 5 = 0 \therefore Locus of (h, k) is $x^2 - 10y + 5 = 0$ $x^2 = 10y - 5$ equation of parabola **(11) (D).** Let the equation of circle whose centre is $(-g - f)$ is $x^2 + y^2 + 2gx + 2fy + c = 0$ (1) \therefore this circle passes through (a, b) \therefore $a^2 + b^2 + 2ag + 2bf + c = 0$ (2) Now circle (1) cut circle $x^2 + y^2 - p^2 = 0$ Orthogonally, $2g(0) + 2f(0) = c - p^2 \Rightarrow c - p^2 = 0$

 \Rightarrow c = p² {if two circle cuts orthogonally then condition is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ \Rightarrow $c - 4 = 0 \Rightarrow c = 4$ Put this value in (2) we get $a^2 + b^2 + 2ga + 2bf + p^2 = 0$

$$
\Rightarrow \text{ for locus of centre replace } (-g, -f) \text{ with } (x, y) \n\Rightarrow g = -x \text{ and } f = -y \n\therefore \text{ Locus is } a^2 + b^2 - 2ax - 2by + p^2 = 0 \n\Rightarrow 2ax + 2by - (a^2 + b^2 + p^2) = 0
$$

(12) (D). Equation of pair of lines is $ax^2 + 2(a + b)xy + by^2 = 0$

 \therefore Area of one sector is thrice of the area of other section \therefore 40 = $\pi \Rightarrow \theta = \pi/4$

Angle between lines is given by

² 2 h ab tan | a b | tan tan 1 and h a b 4 ² 2 (a b) ab 1 | a b | (a + b)² = 4 [(a + b)² – ab] ⁼ 154 154 ⁷ ²²

$$
\Rightarrow 3a^2 + 3b^2 + 2ab = 0
$$

(13) (C). If lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are diameter of a circle then their intersection point will be centre of circle. \therefore Intersection point of these two lines is (1, 1) \therefore Coordinate of centre of circles $(1, -1)$ Now let radius of circle r

Area is
$$
\pi r^2 = 154 \Rightarrow
$$
 given area = 154 sq² unit

$$
\Rightarrow r^2 = \frac{154}{\pi} = \frac{154}{22} \times 7 \Rightarrow r^2 = 7 \times 7 \Rightarrow r = 7 \text{ unit}
$$

Equation of circle will be $(x - 1)^2 + 5(x - (-1))^2 = 7$

 \therefore equation of circle will be $(x-1)^2 + [(y-(-1))^2] = 7^2$ \Rightarrow x² - 2x + 1 + y² + 1 + 2y = 49 \Rightarrow x² + y² - 2x + 2y = 47

(14) (C). Let M (p, q) be the mid point of chord AB of circle subtending an angle of $2\pi/3$ at centre as Δ AOB is an isoscleles triangle $OM \perp AB$

of a circle then their intersection point will be centre of
\ncircle. ∴ Intersection point of these two lines is (1, 1)
\n∴ Coordinate of centre of circles (1, -1)
\nNow let radius of circle r
\nArea is
$$
\pi r^2 = 154 \Rightarrow
$$
 given area = 154 sq² unit
\n $\Rightarrow r^2 = \frac{154}{\pi} = \frac{154}{22} \times 7 \Rightarrow r^2 = 7 \times 7 \Rightarrow r = 7$ unit
\n∴ equation of circle will be $(x-1)^2 + [(y-(-1))^2 = 7^2$
\n $\Rightarrow x^2 - 2x + 1 + y^2 + 1 + 2y = 49 \Rightarrow x^2 + y^2 - 2x + 2y = 47$
\n(C). Let M (p, q) be the mid point of chord AB of circle
\nsubtending an angle of 2π/3 at centre as Δ AOB is an
\nisosceleles triangle OM ⊥ AB
\n \Rightarrow AM = $\sqrt{9 - (p^2 + q^2)}$
\n \Rightarrow AM = $\sqrt{9 - (p^2 + q^2)}$
\n \Rightarrow AM = $\sqrt{9 - (p^2 + q^2)}$
\nBy law of cosine
\n $\cos \frac{2\pi}{3} = \frac{OA^2 + OB^2 - AB^2}{2(OA)(OB)}$
\n $\Rightarrow -\frac{1}{2} = \frac{9 + 9 - 4(9 - (p^2 + q^2)}{2 \times 3 \times 3}$
\n $\Rightarrow -9 = 18 - 36 + 4p^2 + 4q^2 \Rightarrow p^2 + q^2 = \frac{9}{4}$
\nthus required locus is $x^2 + y^2 = \frac{9}{4}$

4

$$
(15) \t(B). \tA\n\begin{matrix}\n\text{a.1,1} \\
\text{b.2,2} \\
\text{c.3,3}\n\end{matrix}
$$

Circle touches the x-axis and coordinate of centre is (h,k) : radius will be k Now equation of circle will be $(x-h)^2 + (y-k)^2 = k^2$ (1) \therefore Circle passes through (-1, 1), \therefore it will satisfy (1) $(-1 - h)^2 + (1 - k)^2 = k^2$ \Rightarrow 1 + h² + 2h + 1 + k² – 2k = k² \Rightarrow 1 + h² + 2h = 2k - 1 (2) From the figure we see that $k > 0$ and **AB 2k** 2 (1) $\frac{(-1)^2 + (1-k)^2 = k^2}{k}$

(and the control of the control o

$$
\Rightarrow 1 + h^{2} + 2h + 1 + k^{2} - 2k = k^{2}
$$
\n
$$
\Rightarrow 1 + h^{2} + 2h = 2k - 1 \qquad(2)
$$
\nFrom the figure we see that k > 0 and
\n
$$
AB \leq 2k \Rightarrow \sqrt{(h+1)^{2} + (0-1)^{2}} \leq 2k \qquad 0 = 1 - 6h + 9
$$
\n
$$
\Rightarrow h^{2} + 1 + 2h + 1 \leq 4k^{2} \Rightarrow h^{2} + 1 + 2h \leq 4k^{2} - 1 \qquad 6h = 10; h = 5/3
$$
\nFrom (2), $2k - 1 \leq 4k^{2} - 1$
\n
$$
\Rightarrow 4k^{2} - 2k \geq 0 \Rightarrow 2k (k - 1) \geq 0 \qquad (21) \qquad (C). Let the equation of c is $(x-3)^{2} + (y-0)^{2} + \lambda$.\n
$$
\therefore k \geq 1/2
$$
\n(B). Given equation of circle is $x^{2} + y^{2} + 2x + 4y - 3 = 0$
\n
$$
\therefore \text{ Coordinate of centre of circle is } (-1, -2)
$$
\n
$$
\Rightarrow \text{d. } 1, 0)
$$
\n
$$
\text{Let coordinate of point diametrically opposite to point P}
$$
\n
$$
\therefore \frac{h + 1}{2} = -1 \Rightarrow h + 1 = -2 \Rightarrow h = -3
$$
\nand $\frac{k + 0}{2} = -2 \Rightarrow k = -4$
\n
$$
\therefore (h, k) = (-3, -4)
$$
\n
$$
\therefore (h, k) = (-3, -4)
$$
\n
$$
\therefore \frac{1 + y}{2} = -1 \Rightarrow h + 1 = -2 \Rightarrow h = -3
$$
\n
$$
\therefore \frac{1}{2} = -1 \Rightarrow h + 1 = -2 \Rightarrow h = -3
$$
\n
$$
\therefore \frac{k + 1}{2} = -2 \Rightarrow k = -4
$$
\n
$$
\therefore (22) \qquad (D). (0, y) \qquad \qquad \therefore \frac{1 + y}{2} = -3
$$
$$

(16) (B). Given equation of circle is $x^2 + y^2 + 2x + 4y - 3 = 0$ \therefore Coordinate of centre of circle is $(-1, -2)$

Let coordinate of point diameterically opposite to point P $(1, 0)$ is (h, k)

$$
\frac{1}{2} + 1 + 12 + 2h = 2k - 1
$$
\nFrom the figure we see that k > 0 and
\nAB $\leq 2k \Rightarrow \sqrt{(h+1)^2 + (0-1)^2} \leq 2k$
\n
$$
\Rightarrow h^2 + 1 + 2h + 1 \leq 4k^2 \Rightarrow h^2 + 1 + 2h \leq 4k^2 - 1
$$
\n
$$
\Rightarrow 4k^2 - 2k - 1 \leq k^2 - 2k + 1 = 2k \leq k^2 - 1
$$
\n
$$
\Rightarrow 4k^2 - 2k = 2k^2 - 2k + 1 = 2k \leq k^2 - 1
$$
\n
$$
\Rightarrow 4k^2 - 2k = 2k^2 - 2k + 1 = 2k \leq k^2 - 1
$$
\n
$$
\Rightarrow 4k^2 - 2k = 2k^2 - 2k + 1 = 2k \leq k^2 - 1
$$
\n(10) (B), Given equation of circle is $x^2 + y^2 + 2x + 4y - 3 = 0$
\n
$$
\therefore \text{ Coordinate of the circle is } (-1, -2)
$$
\n
$$
\therefore \text{ Coordinate of the circle is } (-1, -2)
$$
\n
$$
\therefore \text{ Coordinate of the circle is } (-1, -2)
$$
\n
$$
\therefore \text{ (1, 0) is (h, k)}
$$
\nAs it passes through (1, -2)
\n
$$
\therefore \text{ (1, 0) is (h, k)}
$$
\nAs it passes through (1, -2)
\n
$$
\therefore \text{ (2, 1, 0) is (h, k)}
$$
\n
$$
\therefore \text{ Equation of circle is } (x - 3)^2 + y^2 - 8 = 0
$$
\n
$$
\therefore \text{ (3, 0) is the circle of } x^2 + 3x + 7y + 2p - 5 + \lambda(x^2 + y^2 + 2x + 2y - p^2) = 0
$$
\n
$$
\text{pass through (1, 1)}
$$
\n
$$
\text{ (1, 1, 1)}
$$
\n
$$
\text{ (1, 2, 2)} \text{ (1, 0) is (h, k)}
$$
\n
$$
\text{where } (-2, 4), \text{ Radius } = \sqrt{
$$

points.

 \Rightarrow length of perpendicular from centre \leq radius

$$
\Rightarrow \frac{|6-16-m|}{5} < 5 \Rightarrow |10+m| < 25
$$

\n
$$
\Rightarrow -25 < m+10 < 25 \Rightarrow -35 < m < 15.
$$

\n**(19) (B).** $x^2 + y^2 = ax$ (1)
\n
$$
\Rightarrow
$$
 centre $2 \cdot \left(-\frac{a}{2} \cdot 0\right)$ and radius $x = \frac{|a|}{2}$

 (18)

 \Rightarrow centre c₁ $\left(-\frac{a}{2}, 0\right)$ and radius r₁ = $\left|\frac{a}{2}\right|$ a_{α} $|a|$ $=\frac{a}{2}$ $\frac{x^{2}+b}{2}$

 $x^2 + y^2 = c^2$ (2) \Rightarrow centre c₂ (0, 0) and radius r₂ = c

both touch each other if

STUDY MATERIAL: MATHEMATICS
\n
$$
2 + y^2 = c^2
$$
(2)
\n \Rightarrow centre c₂ (0, 0) and radius r₂ = c
\n $\text{coth touch each other if}$
\n
$$
\frac{1^2}{4} = \left(\pm \frac{a}{2} \pm c\right)^2 \Rightarrow \frac{a^2}{4} = \frac{a^2}{4} \pm |a|c + c^2 \Rightarrow |a| = c
$$
\n(1, 2h)
\n(1, h)

UTIONS STUDY MATERIAL: MATHEMATICS
\n
$$
x^2 + y^2 = c^2
$$
(2)
\n \Rightarrow centre c₂ (0, 0) and radius r₂ = c
\nboth touch each other if
\n
$$
\frac{a^2}{4} = \left(\pm \frac{a}{2} \pm c\right)^2 \Rightarrow \frac{a^2}{4} = \frac{a^2}{4} \pm |a| c + c^2 \Rightarrow |a| = c
$$
\n(1, 2h)
\n(1, 2h)
\n(20) (A).

h² = (1-2)² + (h-3)²
\n0 = 1-6h+9
\n6h = 10; h = 5/3
\nNow, diameter is 2h = 10/3
\n(21) (C). Let the equation of circle be
\n
$$
(x-3)^2 + (y-0)^2 + \lambda y = 0
$$

As it passes through
$$
(1, -2)
$$

\n $\therefore (1-3)^2 + (-2)^2 + \lambda (-2) = 0 \Rightarrow \lambda = 4$
\n \therefore Equation of circle is $(x-3)^2 + y^2 - 8 = 0$
\nSo, $(5, -2)$ satisfies equation of circle.

(22) **(D).** (0,y)
$$
\left(\begin{array}{c} (1,1) \\ 1+y \\ 1-y \end{array}\right)
$$

According to the figure
\n
$$
(1+y)^2 = (1-y)^2 + 1
$$
 (y>0)
\n
$$
\Rightarrow y = 1/4
$$

(23) (B). After solving equation (i) $\&$ (ii) $2x-3y+4=0$...(i) $2x-4y+6=0$...(ii) $x = 1$ and $y = 2$ Slope of $AB \times$ Slope of $MN = -1$

As it passes through (1, -2)
\n
$$
\therefore (1-3)^2 + (-2)^2 + \lambda (-2) = 0 \Rightarrow \lambda = 4
$$
\n
$$
\therefore
$$
 Equation of circle is $(x-3)^2 + y^2 - 8 = 0$
\nSo, (5, -2) satisfies equation of circle.
\n
$$
(1, 1)
$$
\n
$$
(0,y) \times \frac{1}{\sqrt{1-y}}
$$
\n
$$
(1, 1)
$$
\nAccording to the figure
\n
$$
(1 + y)^2 = (1 - y)^2 + 1 \qquad (y > 0)
$$
\n
$$
y = 1/4
$$
\nAfter solving equation (i) & (ii)
\n
$$
2x - 3y + 4 = 0 \qquad ...(i) \qquad 2x - 4y + 6 = 0 \qquad ...(ii)
$$
\n
$$
x = 1
$$
 and $y = 2$
\nSlope of AB × Slope of MN = -1
\n
$$
\frac{b+3}{a-2} - 2\frac{b+3}{a+2} - 1 = -1 \qquad (2, 3)
$$
\n
$$
\frac{b+3}{a-2} - 2\frac{a+2}{a+2} - 1 = -1 \qquad (2, 3)
$$
\n
$$
(y-3)(y-1) = -(x-2)x
$$
\n
$$
y^2 - 4y + 3 = -x^2 + 2x
$$
\n
$$
x^2 + y^2 - 2x - 4y + 3 = 0
$$
\nCircle of radius = $\sqrt{2}$.

2 Circle of radius = $\sqrt{2}$.

(28) **(D).**
$$
p = \frac{n}{\sqrt{2}}
$$
, but $\frac{n}{\sqrt{2}} < 4 \implies n = 1, 2, 3, 4, 5$

 (center) = (2, 3), 2 2 r 2 3 12 5 (center) (–3, –9), r 9 81 26 64 8 1 1 16 1 17 2 2 ; 4 r r 2 4 4 (2 1) 2 2 2 2 (b a) (b a) (a c) (a c) **(28) (D).** n n p , but 4 2 2 n = 1, 2, 3, 4, 5. ² n 2 16 ² ² 64 2n (say) **(29) (D).** P(3, 1) – Given x² + y² = 4 Equation of tangent 3x y 4 ...(1) Equation of normal x 3y 0 ...(2) Coordinate of ⁴ T , 0 3 Area of triangle = 2 3 **(30) (C).** Let the mid point be S (h, k), P(2h, 0) and Q (0, 2k) Equation of PQ : x y ¹ 2h 2k PQ is tangent to circle at R(say) OR = 1 2 2 1 1 1 1 2h 2k 2 2 1 1 ¹ 4h 4k x 2 + y² – 4x2y 2 = 0 P Q O R S(h,k) **Aliter :** Tangent to circle x cos + y sin = 1 P : (sec , 0) ; Q : (0, cosec) 2h = sec 1 1 cos & sin 2h 2k 2 2 1 1 ¹ (2x) (2y) **(31) (B).** Circle touches internally C¹ (0, 0); r¹ = 2 C² : (–3, –4); r² = 7 C1C² = | r¹ – r² | S¹ – S² = 0 eqn. of common tangent 6x + 8y – 20 = 0 3x + 4y = 10 (6, –2) satisfy it 10 k

(32) (C). Equation of common chord

$$
4kx + \frac{1}{2}y + k + \frac{1}{2} = 0
$$
(1)

and given line is $4x + 5y - k = 0$ (2) On comparing (1) & (2) , we get

$$
k = \frac{1}{10} = \frac{k + \frac{1}{2}}{-k}
$$
 \Rightarrow No real value of k exist

 $\Rightarrow \frac{|100 \text{ o} + \text{k}|}{10} = 1$

 $\Rightarrow k = 36$ or $16 \Rightarrow k_{max} = 36$

EXAMPLEMATEMALIMTHEMATEN
\n**(33)** (C) **Equation of circle can be written as**
\n
$$
(x-1)^2 + (y-1)^2 + \lambda (x - y) = 0
$$

\nIt passes through (1, -3)
\n $\Rightarrow x^2 + (y-1)^2 = 4(x - y) = 0$
\n $\Rightarrow x^2 + (y-1)^2 = 4(x - y) = 0$
\n $\Rightarrow x^2 + y^2 = 6x + 2y + 2 = 0$
\n**(34)** (D)
\n**(35)** (A) Slope of tangent to x² + y² = 1 at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
\n $\Rightarrow r = 2\sqrt{2}$
\n**(36)** (A) Slope of tangent to x² + y² = 1 at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
\n $\sqrt[3]{a+b}$
\n \sqrt

238