10

SIMPLE HARMONIC MOTION

PERIODIC MOTION

Any motion which repeats itself after regular interval of time is called periodic motion or harmonic motion. The constant interval of time after which the motion is repeated is called time period.

Examples : (i) Motion of planets around the sun.

(ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.

The fixed point about which the body oscillates is called mean position or equilibrium position.

Examples: (i) Vibration of the wire of 'Sitar'.

(ii) Oscillation of the mass suspended from spring.

Note : Every oscillatory motion is periodic but every periodic motion is not oscillatory.

SIMPLE HARMONIC MOTION (S.H.M.)

Simple harmonic motion is the simplest form of vibratory or oscillatory motion.

(i) S.H.M. are of two types :

Linear S.H.M. : When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.

Example : Motion of a mass connected to spring.

Angular S.H.M. : When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.

Example : Motion of a bob of simple pendulum.

(ii) Necessary Condition to execute S.H.M.

In linear S.H.M. : The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position :

$\therefore \quad F \propto x$ or $a \propto -x$

Negative sign shows that direction of force and acceleration is towards equilibrium position and x is displacement of particle from equilibrium position.

In angular S.H.M. : The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

$\therefore \quad \tau \propto -\theta \quad$ or $\quad \alpha \propto -\theta$

EQUATION OF SIMPLE HARMONIC MOTION

In linear S.H.M.

Restoring force acting on the particle, $F = ma = -kx$

$$a = -\frac{kx}{m} \Rightarrow \frac{dv}{dt} = -\frac{kx}{m}$$

$$\Rightarrow v\frac{dv}{dt} = -\frac{kx}{m}\frac{dx}{dt} \qquad \left[\because v = \frac{dx}{dt}\right]$$

$$\int v\,dv = -\int \frac{kx}{m}\,dx$$

$$\Rightarrow \frac{v^2}{2} = -\frac{kx^2}{2m} + C$$

At $x = 0$, $v = v_0$

$$C = \frac{v_0^2}{2} \Rightarrow \frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{v_0^2}{2} \quad \left[\because \omega^2 = \frac{k}{m}\right]$$

$$\Rightarrow v = \sqrt{v_0^2 - \omega^2 x^2}, \quad \frac{dx}{dt} = \sqrt{v_0^2 - \omega^2 x^2}$$

$$\Rightarrow \frac{dx}{\sqrt{v_0^2 - \omega^2 x^2}} = \int dt$$

$$\Rightarrow \frac{1}{\omega}\sin^{-1}\left(\frac{\omega x}{v_0}\right) = t + c_1 \qquad [\because v_0 = \omega A]$$

$\Rightarrow \quad x = A\sin(\omega t + \omega c_1)$

$\because \quad$ At $t = 0$, $x = 0$ and if velocity is in +x direction

$x = A \sin \omega t$

If velocity is in -x direction, $x = -A\sin\omega t$.

In angular SHM :

Restoring torque acting on the particle $\tau = -C\theta$ where C is a constant which can be defined as torque per unit angular displacement.

Mathematically, $I\alpha = -C\theta$, where I is the moment of inertia of the system about the axis of rotation.

$$\Rightarrow I\frac{d^2\theta}{dt^2} + C\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0$$

$$\text{Since, } \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \Rightarrow \omega = \sqrt{\left(\frac{C}{I}\right)}$$

Comparison between linear and angular S.H.M.

S.No.	Linear S.H.M.	Angular S.H.m.
1.	$F \propto -y$ $F = -ky$ where k is restoring force constant	$\tau \propto -\theta$ $\tau = -C\theta$ Where C is restoring torque constant.
2.	$a = -\frac{k}{m}y$	$\alpha = -\frac{C}{I}\theta$
3.	Equation of Motion $\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$	Equation of Motion $\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$
4.	$a = -\omega^2 y$	$\alpha = -\omega^2\theta$
5.	$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi n$	$\omega = \sqrt{\frac{C}{I}} = 2\pi n = \frac{2\pi}{T}$

SOME BASIC TERMS

Mean Position : The point at which the restoring force on the particle is zero and potential energy is minimum.

Restoring Force :
The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force. It always acts in a direction opposite to that of displacement. Displacement is measured from the mean position. It is given by formula, $F = -kx$

Amplitude: The maximum (positive or negative) value of displacement of particle from mean position is define as amplitude.

Time period (T) : The minimum time after which the particle keeps on repeating 'its motion is known as time period. The smallest time taken to complete one oscillation or vibration is also define as time period.

It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.

Oscillation or Vibration : When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.

Frequency (n or f) :
The number of oscillations per second is define as frequency.

It is given by $n = \frac{1}{T}, n = \frac{\omega}{2\pi}$

SI unit : Hertz (Hz), 1 hertz = 1 cycle per second (cycle is a number not a dimensional quantity).
Dimensions: $M^0L^0T^{-1}$.

Phase: Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

* In the equation $x = A\sin(\omega t + \phi)$, $(\omega t + \phi)$ is the phase of the particle.
* The phase angle at time t = 0 is known as initial phase or epoch.
* The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.
* Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$, where n = 0, 1, 2, 3,....
* Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π, i.e., $\Delta\phi = (2n + 1)\pi$, where n = 0, 1, 2, 3,....
* Angular frequency (ω) :The rate of change of phase angle of a particle with respect to time is define as its angular frequency. SI unit: radian/second, $\omega = \sqrt{\frac{k}{m}}$

DISPLACEMENT IN S.H.M.

(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.
(ii) It can be given by relation
$x = A\sin\omega t$ or $x = A\cos\omega t$.
The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

VELOCITY IN S.H.M.

(i) It is define as the time rate of change of the displacement of the particle at the given instant.
(ii) Velocity in S.H.M. is given by

$$\Rightarrow \quad v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \Rightarrow v = A\omega\cos\omega t$$

$$v = \pm A\omega\sqrt{1-\sin^2\omega t}) \Rightarrow v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}}$$

$$= \pm\omega\sqrt{(A^2 - x^2)} \qquad [\because x = A\sin\omega t]$$

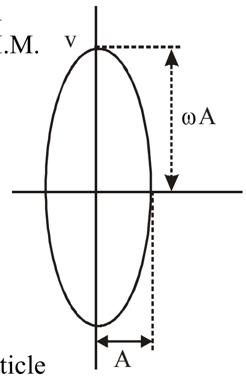
Squaring both the sides

$$v^2 = \omega^2 (A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2$$

$$\Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

This is equation of ellipse. So, curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure. If particle oscillates with unit angular frequency ($\omega = 1$) then curve between v and x will be circular.



($\omega > 1$)

Note :

(i) The direction of velocity of a particle in S.H.M. is either towards or away from the position.

(ii) At mean position ($x = 0$), velocity is maximum ($= A\omega$) and at extreme position ($x = \pm A$), the velocity of particle executing S.H.M. is zero

ACCELERATION IN S.H.M.

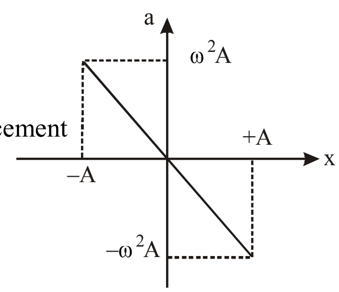
(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t)$

$$a = -\omega^2 A \sin \omega t$$

$$\Rightarrow a = -\omega^2 x$$

(iii) The graph between acceleration and displacement as shown in figure.



Note :

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

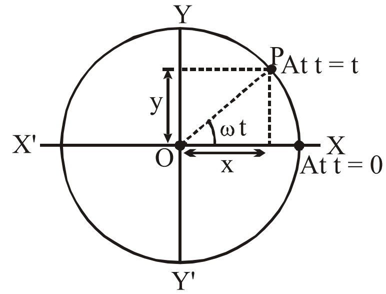
(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position ($= \omega^2 A$) and minimum at mean position ($=$ zero)

GEOMETRICAL MEANING OF S.H.M.

If a particle is moving with. uniform speed along the circumference of a circle then the straight line motion of the foot of perpendicular drawn from the particle on the diameter of the circle is called SHM.

Description of SHM based on Circular motion :

Draw a circle of radius A equal to the amplitude of the particle performing SHM. Suppose particle is moving with constant angular velocity ω along the circle. Perpendicular from particle position on vertical and horizontal diameter shows SHM. After time t radius vector turns by ωt.



$\therefore\ \theta = \omega t \quad \therefore\ x = A \cos \omega t,\ y = A \sin \omega t$

ENERGY OF PARTICLE IN S.H.M.

Potential Energy (U or P.E.)

(i) In terms of displacement

The potential energy is related to force by the relation

$$F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F\,dx$$

for S.H.M. $F = -kx$ so

$$\int dU = -\int (-kx)\,dx = \int kx\,dx \Rightarrow U = \frac{1}{2}kx^2 + C$$

At $x = 0$, $U = U_0 \Rightarrow \quad C = U_0$ so $U = \frac{1}{2}kx^2 + U_0$

where the potential energy at equilibrium position $= U_0$

When $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(ii) In terms of time

Since $X = A \sin(\omega t + \phi)$, $U = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then

$$U = \frac{1}{2}kA^2 \sin^2 \omega t = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

Note:

1. In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position ($x = 0$) and maximum at extreme position ($x = \pm A$)
2. The potential energy is the periodic function of time.

It is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$ and maximum at

$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} ......$

Kinetic Energy (K) :

(i) If mass of the particle executing S.H.M. is m and its velocity is v then kinetic energy at any-instant.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - \omega^2) = \frac{1}{2}k\,(A^2 - x^2)$$

(ii) In terms of time

$v = A\,\omega\cos(\omega t + \phi)$

$$K = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

If initial phase ϕ is zero, $K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

Note:

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum $\left(\frac{1}{2}kA^2\right)$ at mean position (x = 0) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. It is maximum at t = 0, T, 2T, 3T ...and minimum at

$$t = \frac{T}{2}, \frac{3T}{2}, \frac{5T}{2} ....$$

Total energy (E) : Total energy in S.H.M. is given by;

E = potential energy + kinetic energy = U + K

(i) w.r.t. position

$$E = \frac{1}{2}kx^2 + \frac{1}{2}k\,(A^2 - x^2) = \frac{1}{2}kA^2 = \text{constant}$$

(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$= \frac{1}{2}m\omega^2A^2\,(\sin^2\omega t + \cos^2\omega t)$$

$$= \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2 = \text{constant}$$

Note : Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

Average energy in S.H.M.

(i) The time average of PE and KE over one cycle is

(a) $\langle K \rangle_t = \langle \frac{1}{2}m\omega^2A^2\cos^2\omega t \rangle$

$$= \frac{1}{2}m\omega^2A^2 \langle \cos^2\omega t \rangle = \frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right)$$

$$= \frac{1}{4}m\omega^2A^2 = \frac{1}{4}kA^2$$

(b) $\langle PE \rangle_t = \langle \frac{1}{2}m\omega^2A^2\cos^2\omega t \rangle$

$$= \frac{1}{2}m\omega^2A^2 \langle \sin^2\omega t \rangle = \frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right)$$

$$= \frac{1}{4}m\omega^2A^2 = \frac{1}{4}kA^2$$

(c) $\langle TE \rangle_t = \langle \frac{1}{2}m\omega^2A^2 + U_0 \rangle$

$$= \frac{1}{2}m\omega^2A^2 + U_0 = \frac{1}{2}kA^2 + U_0$$

(ii) The position average of P.E. and K.E. between x = – A to x = A

(a) $$\langle K_x \rangle = \frac{\int_{-A}^{A}\frac{1}{2}m\omega^2\,(A^2 - x^2)\,dx}{\int_{-A}^{A}dx} = \frac{1}{3}kA^2$$

(b) $$\langle PE \rangle_x = \frac{\int_{-A}^{A}(PE)\,dx}{\int_{-A}^{A}dx} = \frac{\int_{-A}^{A}\left(U_0 + \frac{1}{2}kA^2\right)dx}{\int_{-A}^{A}dx} = U_0 + \frac{1}{6}kA^2$$

(c) $$\langle TE \rangle_x = \frac{\int_{-A}^{A}(TE)\,dx}{\int_{-A}^{A}dx} = \frac{\int_{-A}^{A}\left(\frac{1}{2}kA^2 + U_0\right)dx}{\int_{-A}^{A}dx} = \frac{1}{2}kA^2 + U_0$$

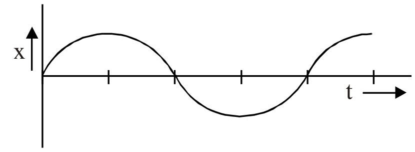
Note :

* Both kinetic energy and potential energy varies periodically but the variation is not simple harmonic.
* The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
* Frequency of total energy is zero

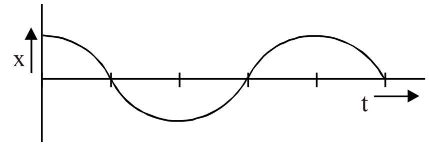
GRAPHICAL REPRESENTATION

Graphical study of displacement, velocity, acceleration and force in S.H.M.

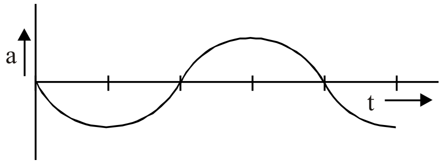
(i) Displacement-Time



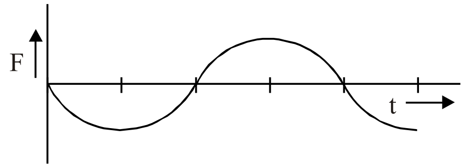
(ii)Velocity- Time



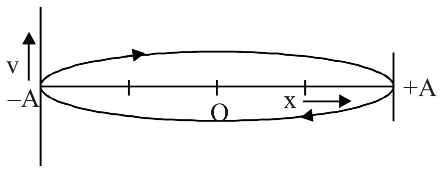
(iii) Acceleration- Time



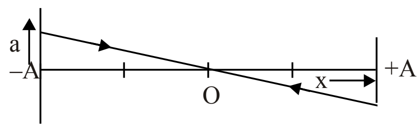
(iv) Force- Time



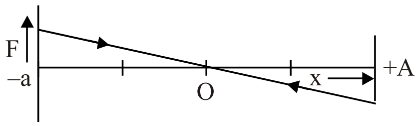
(v)Velocity- Displacement



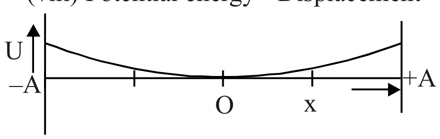
(vi) Acceleration- Displacement



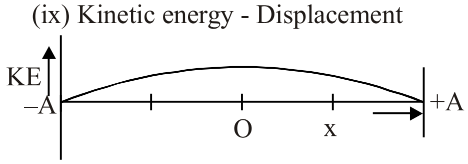
(vii) Force- Displacement



(viii) Potential energy - Displacement



(ix) Kinetic energy - Displacement



Example 1 :

An object performs S.H.M. of amplitude 5cm and time period 4 s. If timing is started when the object is at the centre of the oscillation i.e., x = 0 then calculate.

(i) Frequency of oscillation
(ii) The displacement at 0.5 s
(iii) The maximum acceleration of the object.
(iv) The velocity at a displacement of 3 cm.

Sol. (i) Frequency $f = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

(ii) The displacement equation of object $x = A\sin\omega t$

At t = 0.5 s, $x = 5\sin(2\pi \times 0.25 \times 0.5) = 5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}}$ cm.

(iii) Maximum acceleration

$a_{max} = \omega^2 A = (0.5\pi)^2 \times 5 = 12.3 \text{ cm/s}^2$

(iv) Velocity at x = 3 cm is

$$v = \pm\omega\sqrt{A^2 - x^2} = \pm 0.5\pi\sqrt{5^2 - 3^2} = \pm 6.28 \text{ cm/s}$$

Example 2 :

Amplitude of a harmonic oscillator is A, when velocity of particle is half of maximum velocity, then determine position of particle.

Sol. $v = \omega\sqrt{A^2 - x^2}$ but $v = \frac{v_{max}}{2} = \frac{A\omega}{2}$

$$\Rightarrow \frac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \Rightarrow A^2 = 4[A^2 - x^2]$$

$$\Rightarrow x^2 = \frac{4A^2 - A^2}{4} \Rightarrow x = \frac{\sqrt{3}A}{2}$$

Example 3 :

Which of the following functions represent S.H.M. :

(I) $\sin^2\omega t$ (ii) $\sin 2\omega t$
(iii) $\sin\omega t + 2\cos\omega t$ (iv) $\sin\omega t + \cos 2\omega t$

Sol. A motion will be S.H.M. if acceleration $\propto -y$

(i) $y = \sin^2\omega t \Rightarrow \frac{dy}{dt} = 2(\sin\omega t)(\omega\cos\omega t) = \omega\sin 2\omega t$,

$$\frac{d^2y}{dt^2} = -2\omega^2\cos 2\omega t \Rightarrow \frac{d^2y}{dt^2} \propto y$$

(Oscillatory but not S.H.M)

(ii) As $y = \sin 2\omega t \Rightarrow v = \frac{dy}{dt} = 2\omega\cos 2\omega t$

$$\text{Acceleration} = \frac{d^2y}{dt^2} = -4\omega^2\sin 2\omega t = -4\omega^2 y$$

So, $y = \sin 2\omega t$ represents S.H.M.

(iii) $y = \sin\omega t + 2\cos\omega t$

$$\Rightarrow v = \frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin\omega t.$$

$$\text{acceleration} = \frac{dv}{dt} = -\omega^2\sin\omega t - 2\omega^2\cos\omega t$$

$$= -\omega^2(\sin\omega t + 2\cos\omega t) = -\omega^2 y$$

$\therefore$ The given function represents S.H.M.

(iv) $y = \sin\omega t + \cos 2\omega t$; $\frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin 2\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2 \sin\omega t - 4\omega^2 \cos\omega t = -\omega^2 (\sin\omega t + 4\cos 2\omega t)$$

$$\frac{d^2y}{dt^2} \neq -y \text{ (Oscillatory but S.H.M. not possible)}$$

Example 4 :

In case of simple harmonic motion

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude.

(b) At what displacement the kinetic and potential energies are equal.

Sol. In S.H.M. : Kinetic Energy $K = \frac{1}{2}k(A^2 - x^2)$,

Potential Energy $U = \frac{1}{2}kx^2$, Total Energy (TE) $= \frac{1}{2}kA^2$

(a) Force of Kinetic Energy $f_{K.E.} = \frac{K}{T.E.} = \frac{A^2 - x^2}{A^2}$

Fraction of Potential Energy $f_{p.E.} = \frac{U}{T.E.} = \frac{x^2}{A^2}$

At $\frac{A}{2}$, $f_K = \frac{A^2 - A^2/4}{A^2} = \frac{3}{4}$ and $f_U = \frac{A^2/4}{A^2} = \frac{1}{4}$

(b) $K = U \Rightarrow \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

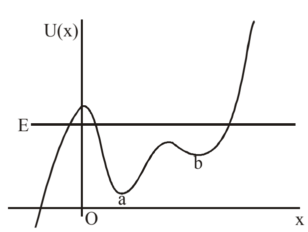
$\Rightarrow 2x^2 = A^2 \Rightarrow x = \pm\frac{A}{\sqrt{2}}$

TRY IT YOURSELF-1

Q.1 The position function for an oscillating body is $x = 20 \sin(0.6t - \pi/2)$. At $t = 0$, the magnitude of the body's acceleration is:

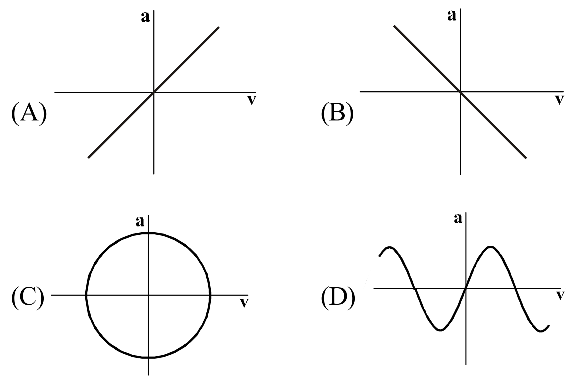
(A) $20\ m/s^2$ (B) $12\ m/s^2$
(C) $7.2\ m/s^2$ (D) None of the above

Q.2 A particle with total mechanical energy E has position $x > 0$ at $t = 0$



(A) escapes to infinity in the – x-direction
(B) approximates simple harmonic motion
(C) oscillates around a
(D) periodically revisits a and b

Q.3 A graph of the acceleration vs. velocity of a body oscillating in simple harmonic motion looks like:



Q.4 Newton's Second Law is applied to a system. After a free body diagram is drawn and the forces summed, the equation $-32x^2 = 2a$ emerges, where x is the position and a is the acceleration of the body at an arbitrary point in time.

(A) This equation does not characterize an oscillatory system.
(B) This equation does characterize an oscillatory system and the motion is simple harmonic in nature.
(C) This equation does characterize an oscillatory system and the motion's frequency is 4 radians per second.
(D) Both Response B and C

Q.5 The position function for an oscillating body is $x = 20 \sin(0.6t - \pi/2)$. The approximate frequency of the motion is:

(A) 20 Hz (B) 1.57 Hz
(C) 0.6 Hz (D) 0.1 Hz

Q.6 A particle of mass 0.8 kg is executing simple harmonic motion with an amplitude of 1.0 metre and periodic time 11/7 sec. Calculate the velocity and the kinetic energy of the particle at the moment when its displacement is 0.6 m

Q.7 A person normally weighing 60 kg stands on a platform which oscillates up and down harmonically at a frequency 2.0 sec^{-1} and an amplitude 5.0 cm. If a machine on the platform gives the person's weight against time, deduce the maximum and minimum reading it will show, take $g = 10\ m/sec^2$.

Q.8 A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} Joule. Obtain the equation of motion of this particle if this initial phase of oscillation is 45º.

Q.9 A particle executing SHM oscillates between two fixed points separated by 20 cm. If its maximum velocity be 30cm/s, find its velocity when its displacement is 5 cm. from its mean position.

Q.10 The total energy of the body executing S.H.M. is E. Then the kinetic energy, when the displacement is half of the amplitude, is:

(A) E/2 (B) E/4
(C) 3E/4 (D) $\sqrt{3/4}\,E$

ANSWERS

(1) (C) **(2)** (D) **(3)** (C)
(4) (A) **(5)** (D) **(6)** 3.2 m/s, 4.1 J.
(7) Maximum reading = 107.3 kg; Min. reading = 12.7 kg
(8) $y = 0.1\sin(\pm 4t + \pi/4)$ metre. **(9)** $15\sqrt{3}$ cm / s
(10) (C)

SPRING SYSTEM

(i) When spring is given small displacement by stretching or compressing it, then restoring elastic force is developed in it because it obeys Hook's law.

$F \propto -x \Rightarrow F = -kx$ Here k is spring constant

(ii) Spring is assumed massless, so restoring elastic force in spring is assumed same everywhere.

(iii) Spring constant (k) depends on length, radius and material of wire used in spring.
for spring $k\ell$ = constant

(iv) When spring is stretched or compressed then work done on it is stored as elastic potential energy.

$$W = \int F\,dx = \int kx\,dx \text{ and } U = W = \frac{1}{2}kx^2$$

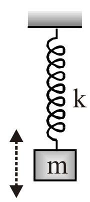
When spring is stretched from ℓ_1 to ℓ_2 then

$$\text{Work done } W = \frac{1}{2}k(\ell_2^2 - \ell_1^2)$$

SPRING PENDULUM

(i) When a small mass is suspended from a massless spring then this arrangement is known as spring pendulum.
For small linear displacement the motion of spring pendulum is simple harmonic.

(ii) For a spring pendulum



$$F = -kx \Rightarrow m\frac{d^2x}{dt} = -kx \quad \left[\because F = ma = m\frac{d^2x}{dt}\right]$$

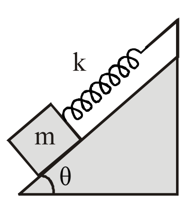
$$\frac{d^2x}{dt} = -\frac{k}{m}x \qquad \because \frac{d^2x}{dt} = -\omega^2 x \Rightarrow \omega^2 = \frac{k}{m}$$

This is standard equation of linear S.H.M.

$$\text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}\ ;\ \text{Frequency } n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

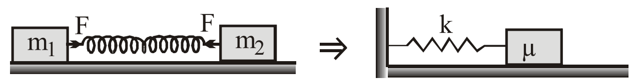
(iii) Time period of a spring pendulum is independent of acceleration due to gravity. This is why a clock based on oscillation of spring pendulum will keep proper time everywhere on a hill or moon or in a satellite or different places of earth.

(iv) If a spring pendulum oscillates in a vertical plane is made to oscillate on a horizontal surface or on an inclined plane then time period will remain unchanged.



(v) By increasing the mass, time period of spring pendulum increases $(T \propto \sqrt{m})$, but by increasing the force constant of spring (k). Its time period decreases $\left(T \propto \frac{1}{\sqrt{k}}\right)$ whereas frequency increases $(n \propto \sqrt{k})$

(vi) If two masses m_1 and m_2 are connected by a spring and made to oscillate then time period $T = 2\pi\sqrt{\frac{\mu}{k}}$



Here, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

(vii) If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m.

$$ky_0 = mg \text{ i.e., } \frac{m}{k} = \frac{y_0}{g}.$$

$$\text{So, time period } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

but remember time period of spring pendulum is independent of acceleration due to gravity.

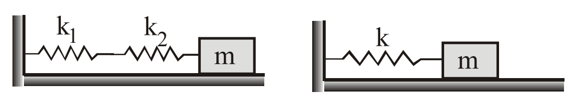
(viii) If two particles are attached with spring in which only one is oscillating

$$\text{Time period} = 2\pi\sqrt{\frac{\text{mass of oscillating particle}}{\text{force constant}}} = 2\pi\sqrt{\frac{m_1}{k}}$$

VARIOUS SPRING ARRANGEMENTS

Series combination of springs

In series combination same restoring force exerts in all springs but extension will be different.



Total displacement $x = x_1 + x_2$
Force acting on both springs $F = -k_1x_1 = -k_2x_2$

$$\because x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2} \therefore x = -\left[\frac{F}{k_1} + \frac{F}{k_2}\right] \text{....... (i)}$$

If equivalent force constant is k_s then $F = -k_sx$
so by eq. (i),

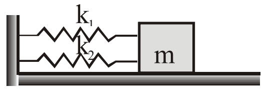
$$-\frac{F}{k_s} = -\frac{F}{k_1} - \frac{F}{k_2} \Rightarrow \frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k_s = \frac{k_1k_2}{k_1 + k_2}$$

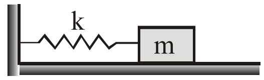
$$\text{Time period } T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$$

$$\text{Frequency } n = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}}\ ;\ \text{Angular frequency, } \omega = \sqrt{\frac{k_s}{m}}$$

Parallel Combination of springs

In parallel combination displacement on each spring is same but restoring force is different.





Force acting on the system

$F = F_1 + F_2 \Rightarrow \quad F = -k_1x - k_2x$ .... (i)

If equivalent force constant is k_p then,

$F = -k_px$, so by equation (i)

$-k_px = -k_1x - k_2x \Rightarrow k_p = k_1 + k_2$

Time period, $T = 2\pi\sqrt{\dfrac{m}{k_p}} = 2\pi\sqrt{\dfrac{m}{k_1+k_2}}$

Frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_p}{m}}$; Angular freq., $\omega = \sqrt{\dfrac{k_1+k_2}{m}}$

Example 5 :

A body of mass m attached to a spring which is oscillating with time period 4 seconds. If the mass of the body is increased by 4 kg, its time period increases by 2 sec. Determine value of initial mass m.

Sol. In Ist case: $T = 2\pi\sqrt{\dfrac{m}{k}} \Rightarrow 4 = 2\pi\sqrt{\dfrac{m}{k}}$ .... (i)

and IInd case : $6 = 2\pi\sqrt{\dfrac{m+4}{k}}$ .... (2)

Divided (i) by (ii), $\dfrac{4}{6} = \sqrt{\dfrac{m}{m+4}} \Rightarrow \dfrac{16}{36} = \dfrac{m}{m+4} \Rightarrow m = 3.2$ kg

Example 6 :

One body is suspended from a spring of length ℓ, spring constant k and has time period T. Now if spring is divided in two equal parts which are joined in parallel and the same body is suspended from this arrangement then determine new time period.

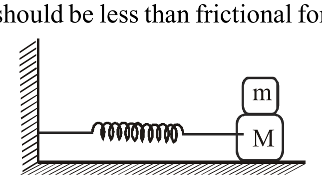
Sol. Spring constant in parallel combination $k' = 2k + 2k = 4k$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{4}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$$

Example 7 :

A block is on a horizontal slab which is moving horizontally and executing S.H.M. The coefficient of static friction between block and slab is μ. If block is not separated from slab then determine angular frequency of oscillation.

Sol. If block is not separated from slab then restoring force due to S.H.M. should be less than frictional force between slab and block.



$F_{restoring} \le F_{friction} \Rightarrow ma_{max.} \le \mu\, mg$

$a_{max.} \le g \Rightarrow \omega^2 A \le mg \Rightarrow \omega \le \sqrt{\dfrac{\mu g}{A}}$

Example 8 :

A block of mass m is suspended from a spring of spring constant k. Find the amplitude of S.H.M.

Sol. Let amplitude of S.H.M. be x_0.

then by work energy theorem W = Δ KE

$$mgx_0 - \frac{1}{2}kx_0^2 = 0 \Rightarrow x_0 = \frac{2mg}{k}$$

SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum.

Expression for time period :

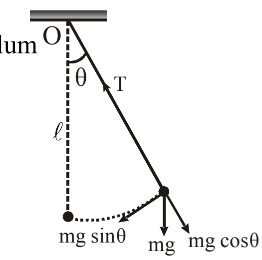
Restoring force acting on pendulum

$F = -mg\sin\theta$

For small angle

$\sin\theta \approx \dfrac{OA}{SA} = \dfrac{y}{\ell}$

$\therefore \quad ma = -mg \times \dfrac{y}{\ell} \Rightarrow a = -\dfrac{g}{\ell}y$



It proves that if displacement is small then simple pendulum performs S.H.M.

$\because \quad |a| = \omega^2 y \Rightarrow \omega^2 = \dfrac{g}{\ell} \Rightarrow \omega = \sqrt{\dfrac{g}{\ell}}$

$\therefore \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{acceleration}}}$

Note :

1. $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ is valid when length of simple pendulum (ℓ) is negligible as compare to radius of earth ($\ell << R$) but if ℓ is comparable to radius of earth then time period

$$T = 2\pi\sqrt{\frac{R_e}{\left[1+\frac{R_e}{\ell}\right]g}}$$

The time period of oscillation of simple pendulum of infinite length ($\ell \to \infty$)

$T = 2\pi\sqrt{\dfrac{R}{g}} \simeq 84.6$ minute $= 1\dfrac{1}{2}$ hour (It is max. time period)

2. If angular amplitude (θ_0) is large ($\theta_0 > 15°$) then time period is given by $T = 2\pi\sqrt{\dfrac{\ell}{g}}\left[1+\dfrac{\theta_0^2}{16}\right]$ here θ_0 is in radian.

3. If a simple pendulum of density ρ is made to oscillate in a liquid of density σ then its time period will increase as compare to that of air and is given by

$$T = 2\pi\sqrt{\frac{\ell}{\left[1-\frac{\sigma}{\rho}\right]g}}$$

Second's pendulum : If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

For second's pendulum, time period $T = 2 = 2\pi\sqrt{\frac{\ell}{g}}$

At the surface of earth $g = 9.8\ m/s^2 \approx \pi^2\ m/s^2$,

Length of second pendulum at the surface of earth $\ell \approx 1$ m.

Example 9 :

A simple pendulum of length L and mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes oscillation in a radial direction about its equilibrium position, then find its time period.

Sol. Centripetal acceleration $a_c = v^2/R$

Acceleration due to gravity = g. So, $g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$

Time period, $T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2+\frac{v^4}{R^2}}}}$

COMPOUND PENDULUM

Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is defined as compound pendulum.

Expression for time period

Torque acting on a body $\tau = -\,mg\ell\sin\theta$

If angle is very small $\sin\theta \approx \theta$

then $\tau = -\,mg\ell\theta$ ...(i) and $\tau = I_s\alpha$ ...(ii)

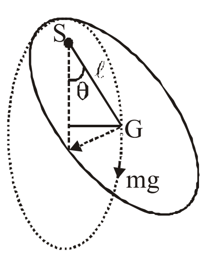
Here m = mass of the body, ℓ = distance between point of suspension and centre of mass

I_s = moment of inertia about horizontal axis passes through point of suspension from equation (i) and (ii)

$I_s\alpha = mg\ell\theta$

$$I_s\frac{d^2\theta}{dt^2} + mg\ell\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{mg\ell}{I_s}\theta = 0 \quad ...(iii)$$



$$\because\ \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad ...(iv)$$

Compare eq. (iii) and (iv), $\omega^2 = \frac{mg\ell}{I_s} \Rightarrow \omega\sqrt{\frac{mg\ell}{I_s}}$

Time period of compound pendulum $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_s}{mg\ell}}$

Applying parallel axis theorem

$I_s = I_{CM} + m\ell^2 \Rightarrow I_s = mK^2 + m\ell^2$

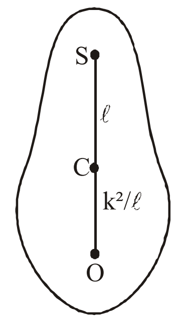
$$\because\ T = 2\pi\sqrt{\frac{I_s}{mg\ell}} = 2\pi\sqrt{\frac{mK^2+m\ell^2}{mg\ell}}\ ;\ T = 2\pi\sqrt{\frac{\frac{K^2}{\ell}+\ell}{g}}$$

$L = \frac{K^2}{\ell} + \ell$ = equivalent length of simple pendulum

= distance between point of suspension and point of oscillation

Here, S = point of suspension,

O = point of oscillation, K = radius of gyration about centre of mass.



Time Period $T = 2\pi\sqrt{\frac{\frac{K^2}{\ell}+\ell}{g}}$

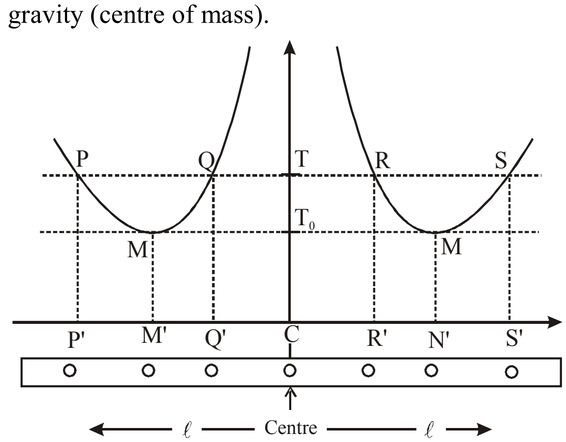
For maximum time period $\ell = 0$

Maximum time period $T_{max} = \infty$

For minimum time period $\frac{dT}{d\ell} = 0$ then $K = \ell$,

$$T_{min} = T = 2\pi\sqrt{\frac{\frac{K^2}{K}+K}{g}} = 2\pi\sqrt{\frac{2K}{g}}$$

Bar pendulum : A bar pendulum is a steel bar of 1 meter length with holes at regular intervals for suspension. The time period is measured for different values of ℓ (distance between S and C). The graph between T and length from one end ℓ is as shown in figure. The time period is infinite when ℓ = 0, i.e., when it is suspended from the centre of gravity (centre of mass).



At four points P, Q, R and S, the time period is the same T

The distance are such that $\frac{PR+QS}{2}=\ell_{eq}=\ell+\frac{K^2}{\ell}$

The time period is minimum when $\ell=K$

The minimum period is $T_0=2\pi\sqrt{\frac{2K}{g}}$

Condition for T minimum : $T^2=\frac{4\pi^2}{g}\left(\frac{K^2}{\ell}+\ell\right)$

diff. w.r.t. to ℓ : $2T\frac{dT}{d\ell}=\left(\frac{4\pi^2}{g}\right)\left[-\frac{K^2}{\ell^2}+1\right]$

$\because\ T\neq 0,$ and with $\frac{dT}{d\ell}=0$; $-\frac{K^2}{\ell^2}+1=0$ or $K^2=\ell^2$

$K=\pm\ell$ then $T_{min}=T_0$

Note :

* There are maximum four points for which time period of compound pendulum is same.
* Minimum time period is obtained at two points
* The point of suspension and point of oscillation are mutually interchangeable.
* Maximum time period will obtain at centre of gravity, which is infinite means compound pendulum will not oscillate at this point.
* Compound pendulum executes angular S.H.M. about its mean position. Here restoring torque is provided by gravitational force.

Example 10 :

A disc is made to oscillate about a horizontal axis passing through mid point of its radius. Determine time period.

Sol. For Disc, $I=MK^2=\frac{MR^2}{2}\Rightarrow K=\frac{R}{\sqrt{2}},\ \ell=\frac{R}{2}$

$$T=2\pi\sqrt{\frac{\ell+\frac{K^2}{\ell}}{g}}=2\pi\sqrt{\frac{3R}{2g}}$$

Example 11 :

A rod with rectangular cross section oscillates about a horizontal axis passing through one of its ends and it behaves like a second's pendulum. Determine its length.

Sol. Because oscillating rod behaves as a second's pendulum so its time period will be 2 second.

$$T=2\pi\sqrt{\frac{\ell+\frac{K^2}{\ell}}{g}}=2s\Rightarrow \ell+\frac{K^2}{\ell}=1\ \text{..... (i)}\ [\because\ \pi^2=g]$$

Assume length of rod is L, because axis passes through one end So $\ell=\frac{L}{2}$ and $K^2=\frac{L^2}{12}$

Putting this values in equation we get

$$\frac{L}{2}+\frac{L^2}{12}\times\frac{2}{L}=1\Rightarrow L=1.5\text{m}$$

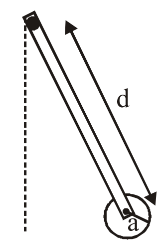
TRY IT YOURSELF-2

Q.1 Suppose the point-like object of a simple pendulum is pulled out at by an angle $\theta_0 << 1$ rad. Is the angular speed of the point-like object equal to the angular frequency of the pendulum?

(A) Yes (B) No

(C) Only at bottom of the swing. (D) Not sure.

Q.2 A physical pendulum consists of a uniform rod of length d and mass m pivoted at one end. A disk of mass m_1 and radius a is fixed to the other end. Suppose the disk is now mounted to the rod by a frictionless bearing so that is perfectly free to spin. Does the period of the pendulum



(A) increase (B) stay the same (C) decrease

Q.3 A block of mass m is attached to a spring with spring constant k is free to slide along a horizontal frictionless surface. At t = 0 the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest. What is the x -component of the velocity of the block when it first comes back to the equilibrium?

(A) $v_x=-x_0\frac{T}{4}$ (B) $v_x=x_0\frac{T}{4}$

(C) $v_x=-\sqrt{\frac{k}{m}}x_0$ (D) $v_x=\sqrt{\frac{k}{m}}x_0$

Q.4 Which of the following statements about a spring-block oscillator in simple harmonic motion about its equilibrium point is false?

(A) The displacement is directly related to the acceleration.

(B) The acceleration and velocity vectors always point in the same direction.

(C) The acceleration vector is always toward the equilibrium point.

(D) The acceleration and displacement vectors always point in opposite directions.

Q.5 A spring oscillates with frequency 1 cycle per second. What approximate length must a simple pendulum have to oscillate with that same frequency?

(A) 25 cm (B) 50 cm.

(C) 67 cm. (D) 90 cm.

Q.6 A pendulum bob's mass is decreased by a factor of 4 while its length is increased by a factor of 4.

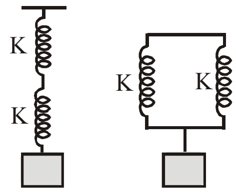
(A) Its frequency will stay the same as will its period.

(B) Its frequency will increase and its period will increase.

(C) Its frequency will decrease and its period will increase.

(D) Its frequency will decrease and its period stays the same.

Q.7 Figure (A) and (B) shows a mass m connected to two identical springs as shown. The ratio of frequency of vibration in case (A) & (B) is



(a) (b)

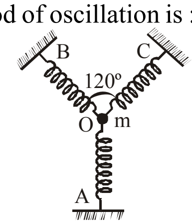
(A) 1 : 1 (B) 1 : 2
(C) 1 : 4 (D) 3 : 1.

Q.8 A pendulum bob carries a +ve charge +q. A positive charge +q is held at the point of support. Then the time period of the bob is

(A) greater than $2\pi\sqrt{L/g}$ (B) less than $2\pi\sqrt{L/g}$

(C) equal to $2\pi\sqrt{L/g}$ (D) equal to $2\pi\sqrt{2L/g}$

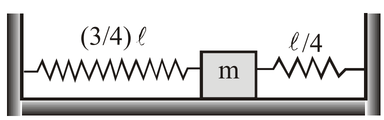
Q.9 A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then time period of oscillation is :



(A) $2\pi\sqrt{\frac{2m}{k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$

(C) $2\pi\sqrt{\frac{2m}{3k}}$ (D) $2\pi\sqrt{\frac{m}{3k}}$

Q.10 A spring of stiffness constant k and natural length ℓ is cut into two parts of length $3\ell/4$ and $\ell/4$ respectively, and an arrangement is made as shown in the figure. If the mass is slightly displaced, find the time period of oscillation.



ANSWERS

(1) (B)	**(2)** (C)	**(3)** (C)
(4) (B)	**(5)** (A)	**(6)** (C)
(7) (B)	**(8)** (C)	**(9)** (C)

(10) $\frac{\pi}{2}\sqrt{\frac{3m}{k}}$

EXAMPLES OF SIMPLE HARMONIC MOTION

1. If a mass m is suspended from a wire of length L, cross section A and young's modulus Y and is pulled along the length of the wire then restoring force will be developed by the elasticity of the wire.

$$Y = \frac{\text{stress}}{\text{strain}} \;;\; Y = \frac{F/A}{\ell/L} = \frac{FL}{\ell A} \Rightarrow F = -\frac{YA}{L}\ell$$

Restoring force is linear so motion is linear simple harmonic

with force constant $k = \frac{YA}{L}$ i.e., $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

2. If the lower surface of a cube of side L and of modulus of rigidity η fixed while fixing a particle of mass m on the upper face, a force parallel to upper face is applied and withdrawn; Here restoring force will be developed due to elasticity of block.

Modulus of rigidity of the block

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} \;;\; \eta = \frac{F}{A\theta} \Rightarrow F = \eta\frac{A}{L}y \quad \left[\because \theta = \frac{y}{L}\right]$$

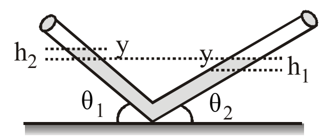
Restoring force is linear so motion will be linear S.H.M.

Force constant (k) = $\eta = \eta\frac{A}{L} = \eta L \qquad [\because A = L^2]$

So, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\eta L}}$

3. Motion of a liquid in a V-shape tube when it is slightly depressed and released

Here cross-section of the tube is uniform and the liquid is incompressible and non viscous. Initially the level of liquid in the two limbs will be at the same height.



If the liquid is pressed by y in one limb, it will rise by y along the length of the tube in the other limb so the restoring force will developed by hydrostatic pressure difference, i.e.,

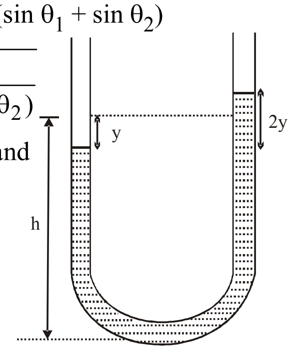
$F = -\Delta P \times A = -(h_1 + h_2)\, gdA$

$\Rightarrow F = -Agd\,(\sin\theta_1 + \sin\theta_2)\, y$

As the restoring force is linear, motion will be linear simple harmonic.

Force constant (k) = $Agd\,(\sin\theta_1 + \sin\theta_2)$

So, $T = 2\pi\sqrt{\frac{m}{Agd\,(\sin\theta_1 + \sin\theta_2)}}$



Note : If the tube is a U-tube and liquid is filled to a height h,

$\theta_2 = \theta_2 = 90°$

and $m = hAd \times 2$

So, time period, $T = 2\pi\sqrt{\frac{h}{g}}$

4. When a partially submerged floating body is slightly pressed and released :

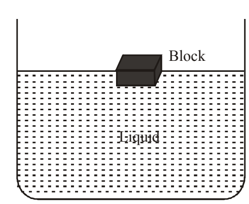
If a body of mass m and cross section A is floating in a liquid of density σ with height h inside the liquid then

$mg = \text{Thrust} = Ah\sigma g$, i.e. $m = Ah\sigma$ .... (i)

Now from this equilibrium position if it is pressed by y, restoring force will developed due to extra thrust i.e.

$F = -A\sigma gy$

As restoring force is linear, motion will be linear simple harmonic with force constant $k = A\sigma g$,



So $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\sigma g}}$

From this expression it is clear that if density of liquid decreases, time period will increase and vice-versa.

And also as from eqn. (i) $m = Ah\sigma$, $T = 2\pi\sqrt{\frac{h}{g}}$

where h is the height of the body inside the liquid.

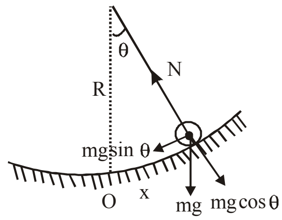
5. Motion of a ball in a bowl : If a small steel ball of mass m is placed at a small distance from O inside a smooth concave surface of radius R and released, it will oscillate about O. The restoring torque here will be due to the force of gravity mg on the ball

i.e., $r = -mg\,(R\sin\theta)$

$= -mgR\theta$ (As θ is small]

Now as restoring torque is angular so motion will be angular simple harmonic.

And as by definition.



$$\tau = I\alpha = mR^2\left[\frac{d^2\theta}{dt^2}\right] \quad [\text{as } I = mR^2 \text{ and } \alpha = \frac{d^2\theta}{dt^2}]$$

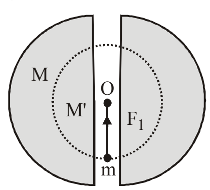
$$\text{So,}\ mR^2\frac{d^2\theta}{dt^2} = -mgR\theta \quad \text{i.e.} \quad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\Rightarrow \omega^2 = \frac{g}{R} \quad \text{so} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

6. Motion of a ball in a tunnel through the earth :

Case I : If the tunnel is along a diameter and a ball is released from the surface and if the ball at any time is at a distance y from the centre of earth. Then the restoring force will act on the ball due to gravitation between ball and earth. But from theory of gravitation we know that force that acts on a particle inside the earth at a distance y from its centre is only due to mass M' of the earth that lies within sphere of radius y. (the portion of the earth that lies out side this sphere does not exert any net force on the particle)

$$\text{so } F = \frac{-GmM'}{y^2}$$



(A)

$$\text{But as } M = \frac{4}{3}\pi R^3\rho \text{ and } M' = \frac{4}{3}\pi y^3\rho$$

$$\text{i.e., } M' = M\left[\frac{y}{R}\right]^3 \ ; \quad F = \frac{-Gm}{y^2}\times M\left[\frac{y^3}{R^3}\right] = -\frac{GMm}{R^3}y$$

Restoring force is linear so the motion is linear SHM with force constant.

$$k = \frac{GMm}{R^3} \quad \text{so} \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$\text{Further more as } g = \frac{GM}{R^2}\ ; \quad T = 2\pi\sqrt{\frac{R}{g}}$$

which is same as that of a simple pendulum of infinite length and is equal to 84.6 minutes.

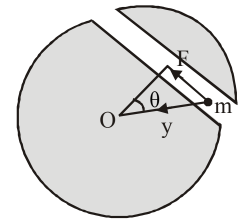
Case II : If the tunnel is along a chord and ball is released from the surface and if the ball at any time is at a distance x from the centre of the tunnel then the restoring force will

$$\text{be } F' = F\sin\theta = \left[-\frac{GMm}{R^3}y\right]\left[\frac{x}{y}\right] = -\frac{GMm}{R^3}x$$

which is again linear with

same force constant $k = \frac{GMm}{R^3}$

so that motion is linear simple harmonic with same time period



(B)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$

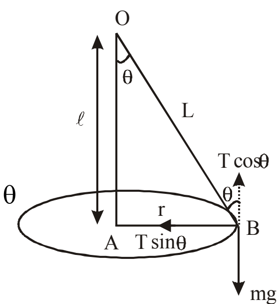
Note : In SHM, $v_{max} = \omega A$

(i) In I case and IInd case time period will be same but v_{max} will be different.

(ii) If ball is dropped from height h it will perform oscillatory motion not SHM [$F \propto \frac{1}{r^2}$ and not $F \propto (-r)$].

7. Conical Pendulum :

It is not example of S.H.M. but example of periodic motion.



$$T = 2\pi\sqrt{\frac{h}{g}} \quad \text{where } h = L\cos\theta$$

$$h = \sqrt{L^2 - r^2}$$

$$\omega = \sqrt{\frac{g}{L\cos\theta}}$$

8. Torsional oscillator : (Angular SHM)

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{where } C = \frac{\eta\pi r^4}{2\ell}$$

η = modulus of elasticity of the wire

r = radius of the wire, L = length of the wire ;

I = Moment of inertia of the disc

9. Oscillation of piston in a gas chamber piston :

$$T = 2\pi\sqrt{\frac{Vm}{A^2E}} \quad \text{where V= volume of cylinder}$$

m = mass of piston, A = area of cylinder ball,

$$E = \text{bulk modulus} = \frac{\Delta P}{-\Delta V/V}$$

For Isothermal process : $E = P$, so $T = 2\pi\sqrt{Vm/PA^2}$

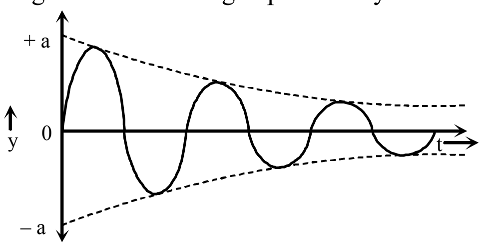
For Adiabatic process: $E = \gamma P$, so $T = 2\pi\sqrt{Vm/\gamma PA^2}$

Damped oscillation :

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation

(ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis etc.

(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially



(iv) The force produces a resistance to the oscillation is called damping force.
If the velocity of oscillator is v then
Dumping force $F_d = -bv$, b = damping constant

(v) Resultant force on a damped oscillator is given by

$$F = F_R + F_d = -Kx - Kv \quad \Rightarrow \frac{md^2x}{dt^2} + b\frac{dx}{dt} + Kx = 0$$

(vi) Displacement of damped oscillator is given by

$$x = x_m e^{-bt/2m}\sin(\omega' t + \phi)$$

where ω' = angular frequency of the damped oscillator

$$= \sqrt{\omega_0^2 - (b/2m)^2}$$

The amplitude decreases continuously with time according to $x = x_m e^{-(b/2m)t}$

(vii) For a damped oscillator if the damping is small then the mechanical energy decreases exponentially with time as

$$E = \frac{1}{2}Kx_m^2 e^{-bt/m}$$

Forced oscillation

(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation

(ii) The amplitude of oscillator decrease due to damping forces but on account of the energy gained from the external source it remains constant.

(iii) **Resonance :** When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(iv) While swinging in a swing if you apply a push periodically by pressing your feet against the ground, you find that not only the oscillations can now be maintained but the amplitude can also be increased. Under this condition the swing has forced or driven oscillation.

(v) In forced oscillation, frequency of damped oscillator is equal to the frequency of external force.

(vi) Suppose an external driving force is represented by
$F(t) = F_0 \cos \omega_d t$
The motion of a particle under combined action of
(a) Restoring force (–Kx)
(b) Damping force (–bv) and
(c) Driving force F(t) is given by

$$ma = -Kx - bv + F_0 \cos\omega_d t$$

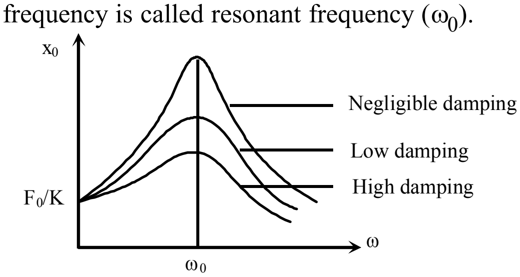
$$\Rightarrow m^2\frac{d^2x}{d^2} + Kx + b\frac{dx}{dt} = F_0 \cos\omega_d t$$

The solution of this equation gives $x = x_0 \sin(\omega_d t + \phi)$

with amplitude $x_0 = \dfrac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega/m)^2}}$

and $\tan\theta = \dfrac{(\omega^2 - \omega_0^2)}{b\omega/m}$, where $\omega_0 = \sqrt{\dfrac{K}{m}}$ = Natural frequency of oscillator.

(vii) **Amplitude resonance :** The amplitude of forced oscillator depends upon the frequency ω_d of external force.
When $\omega = \omega_d$ the amplitude is maximum but not infinite because of presence of damping force. The corresponds frequency is called resonant frequency (ω_0).



(viii) **Energy resonance :** At $\omega = \omega_0$, oscillator absorbs maximum kinetic energy from the driving force system this state is called energy resonance.
At resonance the velocity of a driven oscillator is in phase with the driving term.
The sharpness of the resonance of a driven oscillator depends on the damping.
In the driven oscillator, the power input of the driving term in maximum at resonance.

ADDITIONAL EXAMPLES

Example 1 :
If two S.H.M.'s are represented by equations

$$y_1 = 10\sin\left[3\pi t + \frac{\pi}{4}\right] \text{ and } y_2 = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]$$

then find the ratio of their amplitudes and phase difference in between them.

Sol. As $y_2 = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]$ .... (i)

So, if $5 = A\cos\phi$ and $5\sqrt{3} = A\sin\phi$

Then, $A = \sqrt{5^2 + (5\sqrt{3})^2} = 10$

and $\tan\phi = \frac{5\sqrt{3}}{5} = \sqrt{3}$ so $\phi = \frac{\pi}{3}$

The above equation (i) becomes

$y_2 = A\cos\phi\sin(3\pi t) + A\sin\phi\cos(3\pi t)$

$\Rightarrow$ $y_2 = A\sin(3\pi t + \phi)$

but $y_2 = 10\sin\left[3\pi t + \left(\frac{\pi}{3}\right)\right]$; $\frac{A_1}{A_2} = \frac{10}{10} \Rightarrow A_1 : A_2 = 1 : 1$

Phase difference $= \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

Example 2 :

Periodic time of a simple pendulum is 2 second and it can travel to and fro from equilibrium position upto maximum 5cm. At start the pendulum is at maximum displacement on right side of equilibrium position. Find displacement and time relation.

Sol. Displacement expression for S.H.M., $x = A\sin(\omega t + \phi)$

Time period of simple pendulum

$$T = \frac{2\pi}{\omega} = 2s \quad \therefore \quad \omega = \pi \text{ rad/s}$$

Amplitude of pendulum A = 5 cm $\quad \therefore x = 5\sin(\pi t + \phi)$

Now, at t = 0, displacement x = 5 cm

$\therefore$ $5 = 5\sin(\pi \times 0 + \phi) \Rightarrow \sin\phi = 1 \Rightarrow \phi = \pi/2$

Therefore, $x = 5\sin\pi\left(t + \frac{1}{2}\right)$

$\Rightarrow$ $x = 5\sin\left(\pi t + \frac{\pi}{2}\right) \Rightarrow x = 5\cos\pi t$

Example 3 :

The velocity of a particle in S.H.M. at position x_1 and x_2 are v_1 and v_2 respectively. Determine value of time period and amplitude.

Sol. $v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2(A^2 - x^2)$

At position x_1, $v_1^2 = \omega^2(A^2 - x_1^2)$ .... (i)

At position x_2,' $v_2^2 = \omega^2(A^2 - x_2^2)$ .... (ii)

Subtracting (ii) from (i)

$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)\ ; \quad \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

Time period, $T = 2\pi\omega \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

Dividing (i) by (ii)

$$\frac{v_1^2}{v_2^2} = \frac{A^2 - x_1^2}{A^2 - x_2^2} \Rightarrow v_1^2A^2 - v_1^2x_2^2 = v_2^2A^2 - v_2^2x_1^2$$

So, $A^2(v_1^2 - v_2^2) = v_1^2x_2^2 - v_2^2x_1^2 \Rightarrow A = \sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$

Example 4 :

A particle executing S.H.M. having amplitude 0.01 m and frequency 60 Hz. Determine maximum acceleration of particle.

Sol. Maximum acceleration

$_{max} = \omega^2A = 4\pi^2n^2A = 4\pi^2(60)^2 \times (0.01) = 144\pi^2 \text{ m/s}^2$

Example 5 :

The potential energy of a particle oscillating on x-axis is $U = 20 + (x-2)^2$. Here U is in joules and x in meters. Total mechanical energy of the particle is 36J.

(a) State whether the motion of the particle is simple harmonic or not.

(b) Find the mean position.

(c) Find the maximum kinetic energy of the particle.

Sol. (a) $F = -\frac{dU}{dx} = -2(x-2)$

By assuming $x - 2 = X$, we have $F = -2X$

Since, $F \propto -X$

The motion of the particle is simple harmonic.

(b) The mean position of the particle is

$X = 0 \Rightarrow x - 2 = 0$, which gives x = 2m

(c) Maximum kinetic energy of the particle is,

$K_{max} = E - U_{min} = 36 - 20 = 16$ J

U_{min} is 20J at mean position or at x = 2m.

Example 6 :

Periodic time of oscillation T_1 is obtained when a mass is suspended from a spring if another spring is used with same mass then periodic time of oscillation is T_2. Now if this mass is suspended from series combination of above springs then calculate the time period.

Sol. $T_1 = 2\pi\sqrt{\frac{m}{k_1}} \Rightarrow T_1^2 = 4\pi^2\frac{m}{k_1} \Rightarrow k_1 = \frac{4\pi^2m}{T_1^2}$

$$T_2 = 2\pi\sqrt{\frac{m}{k_2}} \Rightarrow T_2^2 = 4\pi^2\frac{m}{k_2} \Rightarrow k_2 = \frac{4\pi^2m}{T_2^2}$$

Now, $T = 2\pi\sqrt{\frac{m}{k'}}$, where $\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\Rightarrow \quad k' = \frac{k_1k_2}{k_1 + k_2} = \frac{\left(\frac{4\pi^2m}{T_1^2}\right)\left(\frac{4\pi^2m}{T_2^2}\right)}{\frac{4\pi^2m}{T_1^2} + \frac{4\pi^2m}{T_2^2}}$$

$$\Rightarrow \quad k' = \frac{4\pi^2m\left[\frac{4\pi^2m}{T_1^2T_2^2}\right]}{4\pi^2m\left[\frac{1}{T_1^2} + \frac{1}{T_2^2}\right]} = \frac{4\pi^2m}{T_1^2 + T_2^2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{\frac{4\pi^2m}{T_1^2 + T_2^2}}} = \sqrt{T_1^2 + T_2^2}$$

Example 7 :

Infinite spring with force constants k, 2k, 4k, 8k, ... respectively are connected in series. Find the effective force constant of the spring.

Sol. $$\frac{1}{k_{eff}}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+......\infty$$

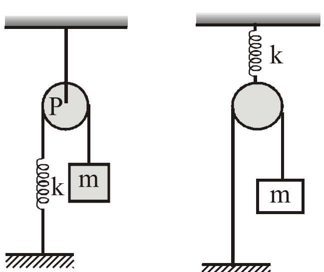
(For infinite G.P. $S_\infty=\frac{a}{1-r}$, where a = First term, r = common ratio)

$$\frac{1}{k_{eff}}=\frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\right]=\frac{1}{k}\left[\frac{1}{1-\frac{1}{2}}\right]=\frac{2}{k}$$

so, $k_{eff}=k/2$

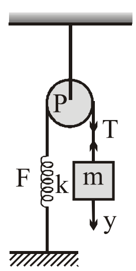
Example 8 :

Figure shows a system consisting of a massless pulley, a spring of force constant k - 4000 N/m and a block of mass m = 1 kg. If the block is slightly displaced vertically down from its equilibrium position and released find the frequency of its vertical oscillation in given cases.



(A) (B)

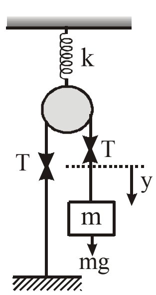
Sol. Case (A) :



As the pulley is fixed and string is inextensible, if mass m is displaced by y the spring will stretch by y. And as there is no mass between string and spring
(as pulley is massless)
F = T = ky i.e., restoring force is linear and so motion of mass m will be linear simple harmonic with frequency

$$n_A=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi\sqrt{\frac{4000}{1}}}\approx 10\text{Hz}$$

Case (B) :



The pulley is movable and string inextensible, so if mass m moves down a distance y, the pulley will move down by(y/2). So the force in the spring F = k (y/2).
Now as pully is massless

F = 2T, i.e., T = F/2 = (k/4)y.
So the restoring force on the mass m

$$T=\frac{1}{4}ky=k'y\Rightarrow k'=\frac{1}{4}k$$

$$\text{So, } n_B=\frac{1}{2\pi}\sqrt{\frac{k'}{m}}=\frac{1}{2\pi}\sqrt{\frac{k}{4\pi}}=\frac{n_A}{2}=5\text{ Hz}$$

Example 9 :

A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to T/2.

Sol. In stationary lift,

$$T=2\pi\sqrt{\frac{\ell}{g}} \qquad .... (i)$$

In accelerated lift,

$$\frac{T}{2}=T'=2\pi\sqrt{\frac{\ell}{g+a}} \qquad .... (ii)$$

Divide (i) by (ii),

$$2=\sqrt{\frac{g+a}{a}}\Rightarrow g+a=4g\Rightarrow a=3g$$

Example 10 :

A liquid of mass m is set into oscillations in a U-tube of cross section A. Its time period recorded is T, where

$T=2\pi\sqrt{\frac{\ell}{2g}}$, here ℓ is the length of liquid column. If the liquid of same mass is set into oscillations in U-tube of cross section A/16 then determine time period of oscillation.

Sol. Mass is constant ⇒ volume × density = constant

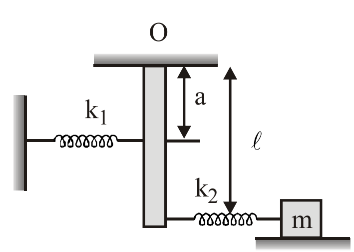
$\Rightarrow V_1d=V_2d$

$$(A\ell)\,d=\left[\frac{A}{16}\ell'\right]d\Rightarrow\ell'=16\ell$$

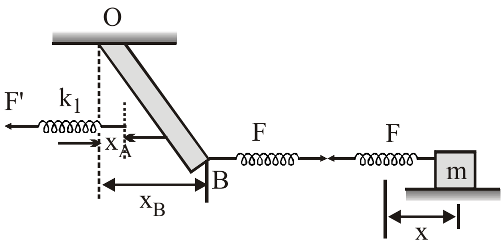
$$\because\ T=2\pi\sqrt{\frac{\ell}{2g}}\ \therefore\ \frac{T'}{T}=\sqrt{\frac{\ell'}{\ell}}=\sqrt{\frac{16\ell}{\ell}}=4\Rightarrow T'=4T$$

Example 11 :

A very light rod of length ℓ pivoted at O is connected with two springs of stiffness k_1 & k_2 at a distance of a & ℓ from the pivot respectively. A block of mass m attached with the spring k_2 is kept on a smooth horizontal surface. Find the angular frequency of small oscillation of the block m.



Sol. Let the block be pulled towards right (figure) through a distance x, then $x = x_B + x_{CB}$ .... (i)



where, x_{CB} = displacement of C (the block) relative to B

Thus $x_{CB} = \dfrac{F}{k_2}$ .... (ii) and $x_B = \left(\dfrac{F'}{k_1}\right)\dfrac{\ell}{a}$ .... (iii)

Torque acting on the rod about point O.

$$\tau_0 = F'a - F\ell \Rightarrow I_0\frac{d^2\theta}{dt^2} = F'a - F\ell$$

Since the rod is very light its moment of inertia I_0 about O is approximately equal to zero

$$\Rightarrow F' = F\left(\frac{\ell}{a}\right) \quad \text{......... (iv)}$$

Using (iii) & (iv), $\Rightarrow$ $x_B = \dfrac{F}{k_1}\left(\dfrac{\ell}{a}\right)^2$ .... (v)

Using (i), (ii) & (v), $x = \dfrac{F}{k_1}\left(\dfrac{\ell}{a}\right)^2 + \dfrac{F}{k_2}$

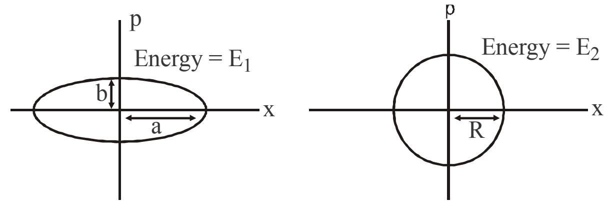
As force F is opposite to displacement x, then

$$\Rightarrow F = -\frac{k_1k_2}{k_2\left(\frac{\ell}{a}\right)^2 + k_2}x \Rightarrow m\omega^2 x = \frac{k_1k_2}{k_2\left(\frac{\ell}{a}\right)^2 + k_1}x$$

$$\Rightarrow \omega = \sqrt{\frac{k_1k_2a^2}{m\,(k_1a^2 + k_2\ell^2)}}$$

Example 12 :

Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in figures. If $\dfrac{a}{b} = n^2$ and $\dfrac{a}{R} = n$, then the correct equation(s) is (are) –



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\dfrac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$ (D) $\dfrac{E_1}{\omega_1} = \dfrac{E_2}{\omega_2}$

Sol. **(BD).** For first oscillator For Second oscillator

$b = ma\omega_1$ $\qquad \dfrac{1}{m\omega_2} = 1$

$\dfrac{a}{b} = \dfrac{1}{m\omega_1} = n^2$ $\qquad \dfrac{\omega_2}{\omega_1} = n^2$ Ans. B

$$E_1 = \frac{1}{2}m\omega_1^2a^2\ ;\ E_2 = \frac{1}{2}m\omega_2^2R^2$$

$$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2}\times n^2 = \frac{\omega_1^2}{\omega_2^2}\times\frac{\omega_2}{\omega_1}$$

$$\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2} \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$ Ans D

QUESTION BANK

CHAPTER 10 : SIMPLE HARMONIC MOTION

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : PARAMETERS RELATED TO SHM

Q.1 In simple harmonic motion, the ratio of acceleration of the particle to its displacement at any time is a measure of
(A) Spring constant (B) Angular frequency
(C) (Angular frequency)^2 (D) Restoring force

Q.2 What is the maximum acceleration of the particle doing the SHM $y\text{ (in cm)} = 2\sin\left[\frac{\pi t}{2}+\phi\right]$
(A) $\frac{\pi}{2}\text{cm/s}^2$ (B) $\frac{\pi^2}{2}\text{cm/s}^2$
(C) $\frac{\pi}{4}\text{cm/s}^2$ (D) $\frac{\pi^2}{4}\text{cm/s}^2$

Q.3 A body of mass 1 kg is executing simple harmonic motion. Its displacement y (cm) at t seconds is given by $y = 6\sin(100t+\pi/4)$. Its maximum kinetic energy is
(A) 6 J (B) 18 J
(C) 24 J (D) 36 J

Q.4 A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy changes into potential energy is –
(A) f/2 (B) f
(C) 2 f (D) 4 f

Q.5 There is a body having mass m and performing S.H.M. with amplitude A. There is a restoring force F = –kx, where x is the displacement. The total energy of body depends upon –
(A) k, x (B) k, A
(C) k, A, x (D) k, A, v

Q.6 The amplitude of a particle executing SHM is made three-fourth keeping its time period constant. Its total energy will be –
(A) E/2 (B) (3/4) E
(C) (9/16) E (D) None of these

Q.7 A body is moving in a room with a velocity of 20 m/s perpendicular to the two walls separated by 5 meters. There is no friction and the collisions with the walls are elastic. The motion of the body is –
(A) Not periodic
(B) Periodic but not simple harmonic
(C) Periodic and simple harmonic
(D) Periodic with variable time period

Q.8 A particle moves such that its acceleration a is given by a = – bx, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is
(A) $2\pi\sqrt{b}$ (B) $\frac{2\pi}{\sqrt{b}}$
(C) $\frac{2\pi}{b}$ (D) $2\sqrt{\frac{\pi}{b}}$

Q.9 The displacement x (in metre) of a particle in, simple harmonic motion is related to time t (in seconds) as $x = 0.01\cos\left(\pi t+\frac{\pi}{4}\right)$. The frequency of the motion will be
(A) 0.5 Hz (B) 1.0 Hz
(C) (π/2) Hz (D) π Hz

Q.10 A simple harmonic wave having an amplitude A and time period T is represented by the equation $y = 5\sin\pi(t+4)\text{m}$. Then the value of amplitude (A) in (m) and time period (T) in second are
(A) A = 10, T = 2 (B) A = 5, T = 1
(C) A = 10, T = 1 (D) A = 5, T = 2

Q.11 If $x = a\sin\left(\omega t+\frac{\pi}{6}\right)$ and $x' = a\cos\omega t$, then what is the phase difference between the two waves –
(A) $\pi/3$ (B) $\pi/6$
(C) $\pi/2$ (D) π

Q.12 Velocity at mean position of a particle executing S.H.M. is v. Velocity of the particle at a distance equal to half of the amplitude is –
(A) 4v (B) 2v
(C) $(\sqrt{3}/2)\text{ v}$ (D) $(\sqrt{3}/4)\text{ v}$

Q.13 Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion –
(A) When v is maximum, a is maximum.
(B) Value of a is zero, whatever may be the value of v.
(C) When v is zero, a is zero.
(D) When v is maximum, a is zero.

Q.14 The motion of a particle varies with time according to the relation $y = A(\sin\omega t+\cos\omega t)$, then
(A) The motion is oscillatory but not S.H.M.
(B) The motion is S.H.M. with amplitude A.
(C) The motion is S.H.M. with amplitude $A\sqrt{2}$.
(D) The motion is S.H.M. with amplitude 2A.

Q.15 For a simple pendulum, graph between velocity (v) & displacement (x)
(A) Parabolic (B) Circular
(C) Elliptical (D) Straight line

Q.16 A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude –
(A) T / 2 (B) T / 4
(C) T / 8 (D) T / 12

Q.17 A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A\sin(\omega t) + B$. The amplitude of the SHM is
(A) A (B) B
(C) A+B (D) $\sqrt{A+B}$

Q.18 A particle executing S.H.M. of amplitude 4cm and T= 4 sec. The time taken by it to move from positive extreme position to half the amplitude is –
(A) 1 sec (B) 1/3 sec
(C) 2/3 sec (D) $\sqrt{3/2}$ sec

Q.19 A simple harmonic oscillator has a period of 0.01 sec and an amplitude of 0.2 m. The magnitude of the velocity in $msec^{-1}$ at the centre of oscillation is
(A) 20π (B) 100
(C) 40π (D) 100π

Q.20 A particle executes S.H.M. with a period of 6 second and amplitude of 3 cm. Its maximum speed in cm/sec is –
(A) $\pi/2$ (B) π
(C) 2π (D) 3π

Q.21 A particle is executing S.H.M. If its amplitude is 2 m and periodic time 2 seconds, then the maximum velocity of the particle will be –
(A) π m/s (B) $\sqrt{2\pi}$ m / s
(C) 2π m/s (D) 4π m/s

Q.22 A particle executing simple harmonic motion with amplitude of 0.1 m. At a certain instant when its displacement is 0.02 m, its acceleration is 0.5 m/s^2. The maximum velocity of the particle is (in m/s)
(A) 0.01 (B) 0.05
(C) 0.5 (D) 0.25

Q.23 The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is 16 cm/sec. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$cm / s, is
(A) $2\sqrt{3}$cm (B) $\sqrt{3}$cm
(C) 1 cm (D) 2 cm

Q.24 The maximum velocity of a simple harmonic motion represented by $y = 3\sin\left(100t + \frac{\pi}{6}\right)$ is given by
(A) 300 (B) $3\pi/6$
(C) 100 (D) $\pi/6$

Q.25 A body executing simple harmonic motion has a maximum acceleration equal to 24 m/sec^2 and maximum velocity equal to 16 m/sec. The amplitude of the SHM
(A) (32/3) m (B) (3/32) m
(C) (1024/9) m (D) (64/9) m

Q.26 For a particle executing simple harmonic motion, which of the following statements is not correct –
(A) The total energy of the particle always remains the same.
(B) The restoring force of always directed towards a fixed point.
(C) The restoring force is maximum at the extreme positions.
(D) The acceleration of the particle is maximum at the equilibrium position.

Q.27 A particle of mass 10 grams is executing simple harmonic motion with an amplitude of 0.5 m and periodic time of (π/5) seconds. The maximum value of the force acting on the particle is –
(A) 25 N (B) 5 N
(C) 2.5 N (D) 0.5 N

Q.28 What is the maximum acceleration of the particle doing the SHM $y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$ where 2 is in cm –
(A) $\frac{\pi}{2}$ cm / s^2 (B) $\frac{\pi^2}{2}$ cm / s^2 (C) $\frac{\pi}{4}$ cm / s^2 (D) $\frac{\pi^2}{4}$ cm / s^2

Q.29 A particle executes linear simple harmonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds
(A) $\frac{1}{2\pi\sqrt{3}}$ (B) $2\pi\sqrt{3}$ (C) $\frac{2\pi}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2\pi}$

Q.30 A particle executes simple harmonic motion along a straight line with an amplitude A. The potential energy is maximum when the displacement is
(A) ± A (B) Zero
(C) ± A/2 (D) $\pm A/\sqrt{2}$

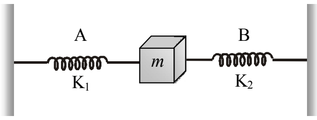
Q.31 For a particle executing simple harmonic motion, the kinetic energy K is given by $K = K_o\cos^2\omega t$. The maximum value of potential energy is
(A) K_0 (B) Zero
(C) $K_0/2$ (D) Not obtainable

PART - 2 : SPRING MASS SYSTEM

Q.32 A spring has a certain mass suspended from it and its period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is
(A) T/2 (B) $T/\sqrt{2}$
(C) $\sqrt{2}T$ (D) 2T

Q.33 A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system if a mass 4 m is suspended from the same spring –
(A) n/4 (B) 4n
(C) n/2 (D) 2n

Q.34 In arrangement given in figure, if the block of mass m is displaced, the frequency is given by –



(A) $\frac{1}{2\pi}\sqrt{\left(\frac{k_1-k_2}{m}\right)}$ (B) $\frac{1}{2\pi}\sqrt{\left(\frac{k_1+k_2}{m}\right)}$

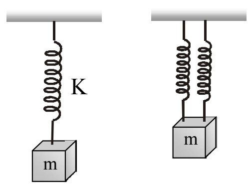
(C) $\frac{1}{2\pi}\sqrt{\left(\frac{m}{k_1+k_2}\right)}$ (D) $\frac{1}{2\pi}\sqrt{\left(\frac{m}{k_1-k_2}\right)}$

Q.35 A particle of mass m is hanging vertically by an ideal spring of force constant K. If the mass is made to oscillate vertically, its total energy is –
(A) Maximum at extreme position
(B) Maximum at mean position
(C) Minimum at mean position
(D) Same at all position

Q.36 Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Then the angular frequency of oscillation of m_2

(A) $\sqrt{\frac{k}{m_1}}$ (B) $\sqrt{\frac{k}{m_2}}$ (C) $\sqrt{\frac{k}{m_1+m_2}}$ (D) $\sqrt{\frac{k}{m_1 m_2}}$

Q.37 A mass m performs oscillations of period T when hanged by spring of force constant K. If spring is cut in two parts and arranged in parallel and same mass is oscillated by them, then the new time period will be

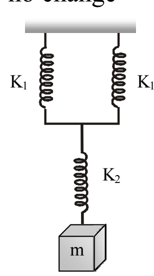


(A) 2T (B) T

(C) $\frac{T}{\sqrt{2}}$ (D) $\frac{T}{2}$

Q.38 If a watch with a wound spring is taken on to the moon,
(A) Runs faster (B) Runs slower
(C) Does not work (D) Shows no change

Q.39 What will be the force constant of the spring system as shown



(A) $\frac{K_1}{2}+K_2$ (B) $\left[\frac{1}{2K_1}+\frac{1}{K_2}\right]^{-1}$

(C) $\frac{1}{2K_1}+\frac{1}{K_2}$ (D) $\left[\frac{2}{K_1}+\frac{1}{K_2}\right]^{-1}$

Q.40 If a spring extends by x on loading, then energy stored by the spring is (if T is the tension in the spring and K is the spring constant)

(A) $\frac{T^2}{2x}$ (B) $\frac{T^2}{2K}$

(C) $\frac{2K}{T^2}$ (D) $\frac{2T^2}{K}$

PART - 3 : PENDULUM

Q.41 The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is
(A) Greater than T (B) Less than T
(C) Equal to T (D) None of these

Q.42 If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day –
(A) 3927 sec (B) 3727 sec
(C) 3427 sec (D) 864 sec

Q.43 The period of simple pendulum is measured as T in a stationary lift. If the lift moves upwards with an acceleration of 5 g, the period will be
(A) The same (B) Increased by 3/5
(C) Decreased by 2/3 times (D) None of the above

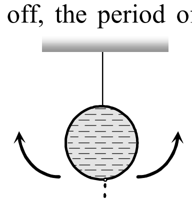
Q.44 The length of a simple pendulum is increased by 1%. Its time period will
(A) Increase by 1% (B) Increase by 0.5%
(C) Decrease by 0.5% (D) Increase by 2%

Q.45 The periodic time of a simple pendulum of length 1 m and amplitude 2 cm is 5 seconds. If the amplitude is made 4 cm, its periodic time in seconds will be
(A) 2.5 (B) 5
(C) 10 (D) $5\sqrt{2}$

Q.46 The ratio of frequencies of two pendulums are 2 : 3, then their length are in ratio –
(A) $\sqrt{2/3}$ (B) $\sqrt{3/2}$
(C) 4/9 (D) 9/4

Q.47 The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)
(A) $1/\sqrt{2}$ sec (B) $2\sqrt{2}$ sec
(C) 2 sec (D) 1/2 sec

Q.48 A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will



(A) Remains unchanged
(B) Increase
(C) Decrease
(D) Become erratic

Q.49 Two pendulums begin to swing simultaneously. If the ratio of the frequency of oscillations of the two is 7 : 8, then the ratio of lengths of the two pendulums will be
(A) 7 : 8 (B) 8 : 7
(C) 49 : 64 (D) 64 : 49

Q.50 In a simple pendulum, the period of oscillation T is related to length of the pendulum ℓ as

(A) $\frac{\ell}{T}$ = constant (B) $\frac{\ell^2}{T}$ = constant

(C) $\frac{\ell}{T^2}$ = constant (D) $\frac{\ell^2}{T^2}$ = constant

Q.51 A simple pendulum of length ℓ has a brass bob attached at its lower end. Its period is T. If a steel bob of same size, having density x times that of brass, replaces the brass bob and its length is changed so that period becomes 2T, then new length is

(A) 2ℓ (B) 4ℓ

(C) $4\ell x$ (D) $4\ell/x$

Q.52 There is a simple pendulum hanging from the ceiling of a lift. When the lift is stand still, the time period of the pendulum is T. If the resultant acceleration becomes g/4 then the new time period of the pendulum is –

(A) 0.8 T (B) 0.25 T

(C) 2 T (D) 4 T

Q.53 The time period of a simple pendulum of length L as measured in an elevator descending with acc. g/3

(A) $2\pi\sqrt{\frac{3L}{g}}$ (B) $\pi\sqrt{\left(\frac{3L}{g}\right)}$

(C) $2\pi\sqrt{\left(\frac{3L}{2g}\right)}$ (D) $2\pi\sqrt{\frac{2L}{3g}}$

PART - 4 : EXAMPLES OF SHM

Q.54 A tunnel has been dug through the centre of the earth and a ball is released in it. It will reach the other end of the tunnel after

(A) 84.6 minutes

(B) 42.3 minutes

(C) 1 day

(D) Will not reach the other end

Q.55 If a hole is bored along the diameter of the earth and a stone is dropped into hole –

(A) The stone reaches the centre of the earth & stops there

(B) The stone reaches the other side of the earth & stops there

(C) The stone executes simple harmonic motion about the centre of the earth

(D) The stone reaches the other side of the earth and escapes into space

Q.56 One wooden cylinder of uniform cross section is floating in water vertically. When it is slightly pressed, it oscillates. If l length of cylinder is drowned in water then its time period.

(A) $T = 2\pi\sqrt{\frac{g}{\ell}}$ (B) $T = 2\pi\sqrt{\frac{m}{k}}$

(C) $T = 2\pi\sqrt{\frac{k}{m}}$ (D) $T = 2\pi\sqrt{\frac{\ell}{g}}$

PART - 5 : FORCED AND DAMPED OSCILLATION

Q.57 Resonance is an example of

(A) forced oscillation (B) damped oscillation

(C) free oscillation (D) none ofthese

Q.58 A particle with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force F sin ωt. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$, then (where ω_0 natural frequency of oscillation of particle)

(A) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$ (B) $\omega_1 = \omega_0$ & $\omega_2 = \omega_0$

(C) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$ (D) $\omega_1 \neq \omega_0$ & $\omega_2 \neq \omega_0$

Q.59 In case of forced oscillations of a body

(A) driving force is constant throughout.

(B) driving force is to be applied only momentarily.

(C) driving force has to be periodic and continuous.

(D) driving force is not required.

Q.60 Amplitude of a wave is represented by $A = \frac{c}{a+b-c}$

Then resonance will occur when

(A) b = –c/2 (B) b = 0 and a = – c

(C) b = –a/2 (D) None of these

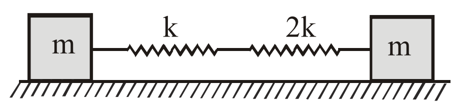
PART - 6 : MISCELLANEOUS

Q.61 A spring mass system performs S.H.M. If the mass is doubled keeping amplitude same, then the total energy of S.H.M. will become –

(A) double (B) half

(C) unchanged (D) 4 times

Q.62 A system is shown in the figure. The time period for small oscillations of the two blocks will be –



(A) $2p\sqrt{\frac{3m}{k}}$ (B) $2p\sqrt{\frac{3m}{2k}}$ (C) $2p\sqrt{\frac{3m}{4k}}$ (D) $2p\sqrt{\frac{3m}{8k}}$

Q.63 A body is executing simple harmonic motion. At a displacement x from mean position, its potential energy is E_1 = 2J and at a displacement y from mean position, its potential energy is E_2 = 8J. The potential energy E at a displacement (x + y) from mean position is –

(A) 10J (B) 14J

(C) 18J (D) 4J

Q.64 A particle is executing simple harmonic motion of amplitude A. At a distance x from the centre, particle receives a blow in the direction of motion which instantaneously doubles the velocity. Its new amplitude will be –

(A) A (B) $\sqrt{A^2 - x^2}$

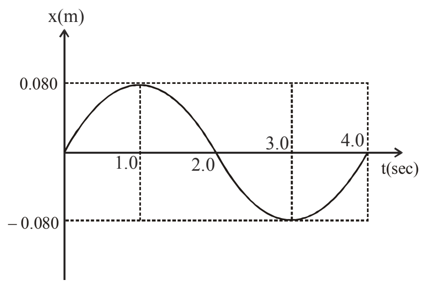
(C) $\sqrt{2A^2 - 3x^2}$ (D) $\sqrt{4A^2 - 3x^2}$

Q.65 A block of 4 kg produces an extension of 0.16 metre in a spring. The block is replaced by a body of mass 0.50 kg. If the spring is stretched and then released the time period of motion will be-
(A) 0.283 sec (B) 0.0283 sec
(C) 2.83 sec (D) 28.3 sec

Q.66 Two linear simple harmonic motions of equal amplitudes 'a' and frequencies ω and 2ω and are impressed on a particle along x and y axis respectively. If the initial phase difference between them is π/2, the resultant trajectory equation of the particle is –
(A) $a^2y^2 = x^2 (a^2 - x^2)$
(B) $a^2y^2 = 2x^2 (a^2 - x^2)$
(C) $a^2y^2 = 4x^2 (a^2 - x^2)$
(D) $a^2y^2 = 8x^2 (a^2 - x^2)$

Q.67 A 1 kg body when suspended from the lower end of a light spring produces a vertical extension of 9.8 cm in it. The time period of the oscillations of the spring will be-
(A) 200π (B) $\frac{2\pi}{100}$ cycles/sec
(C) $\frac{2\pi}{10}$ cycles/sec (D) 20π

Q.68 An object of mass 0.8 kg is attached to one end of a spring and the system is set into simple harmonic motion. The displacement x of the object as a function of time t is shown in the figure. With the aid of the data the magnitude of the acceleration of object at t = 1.0 is –



(A) zero (B) 1.57 m/s^2
(C) 0.197 m/s^2 (D) 0.157 m/s^2

Q.69 What will be the percentage change in the time period of a simple pendulum if its length is increased by 6% -
(A) 3% (B) 9%
(C) 6% (D) 1/9%

Q.70 The height of liquid column in a U tube is 0.3 meter. If the liquid in one of the limbs is depressed and then released, then the time period of liquid column will be-
(A) 1.1 sec (B) 19 sec
(C) 0.11 sec (D) 2 sec

Q.71 A 5 kg. weight is suspended from a spring. The spring stretches by 2 cm/kg. If the spring is stretched and released, its time period will be- (g = 10 m/s^2)
(A) 0.628 sec (B) 6.28 sec
(C) 62.8 sec (D) 0.0628 sec

Q.72 A particle is executing simple harmonic motion along a straight line 8 cm long. While passing through mean position its velocity is 16 cm/s. Its time period will be-
(A) 0.157 sec. (B) 1.57 sec
(C) 15.7 sec (D) 0.0157 sec.

Q.73 The potential energy of a particle executing SHM changes from maximum to minimum in 5s. Then the time period of SHM is –
(A) 5s (B) 10s
(C) 15s (D) 20s

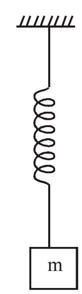
Q.74 A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to T/2 ?
(A) g (B) 2g
(C) 3g (D) – 3g

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 A particle performs S.H.M. on x-axis with amplitude A and time period T. The time taken by the particle to travel a distance A/5 starting from rest is :
(A) $\frac{T}{20}$ (B) $\frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$
(C) $\frac{T}{2\pi}\cos^{-1}\left(\frac{1}{5}\right)$ (D) $\frac{T}{2\pi}\sin^{-1}\left(\frac{1}{5}\right)$

Q.2 A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in figure. The amplitude of the SHM is A spring is never in compressed state during the oscillation. The minimum force exerted by spring is never in compressed state during the oscillation. The minimum force exerted by spring on the block is–



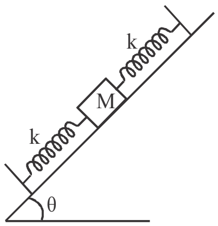
(A) $mg - \frac{4\pi^2}{T^2}mA$ (B) $mg + \frac{4\pi^2}{T^2}mA$
(C) $mg - \frac{\pi^2}{T^2}mA$ (D) $mg + \frac{\pi^2}{T^2}mA$

Q.3 Starting from the mean position body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy –

(A) (1/6) s (B) (1/4) s

(C) (1/3) s (D) (1/12) s

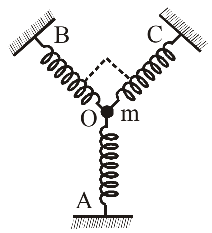
Q.4 On a smooth inclined surface a body of mass M is attached between two spring. The other ends of the springs are fixed to firm supports. If each spring has force constant k, the period of oscillation of the body is



(A) $2\pi\,(M/2k)^{1/2}$ (B) $2\pi\left(\frac{Mg\sin\theta}{2k}\right)$

(C) $2\pi\left(\frac{2M}{k}\right)^{1/2}$ (D) $2\pi\left(\frac{2Mg}{k}\right)^{1/2}$

Q.5 A block is attached to an end of a massless spring whose other end is fixed to ceiling. The block is released at rest when the spring is in its relaxed state. The maximum acceleration of the block during its motion in the vertical plane is (g is acceleration due to gravity).



(A) g (B) 2g (C) g/2

(D) can be determined only if the values of spring and mass of the block are given.

Q.6 A uniform cylinder of length L and mass M having cross-sectional area A is suspended with its vertical length, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density d at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude. If the force constant of the spring is K, the frequency of oscillation of the cylinder is :

(A) $\frac{1}{2}\left(\frac{K-Adg}{M}\right)^{1/2}$ (B) $\frac{1}{2\pi}\left(\frac{K+dgL}{M}\right)^{1/2}$

(C) $\frac{1}{2\pi}\left(\frac{K+Adg}{M}\right)^{1/2}$ (D) $\frac{1}{2\pi}\left(\frac{K-Adg}{Adg}\right)^{1/2}$

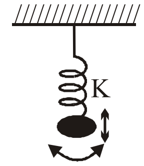
Q.7 A particle is moving on x -axis has potential energy $U = 2 - 20x + 5x^2$ Joules along x-axis. The particle is released at x = –3. The maximum value of x will be –

[x is in meters and U is in joules]

(A) 5 m (B) 3 m

(C) 7 m (D) 8 m

Q.8 A solid disk of radius R is suspended from a spring of linear constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system –

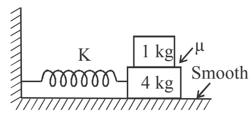


(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$

(C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

Q.9 The friction coefficient between the two blocks of masses 1 kg and 4 kg shown in figure is μ and the horizontal plane surface is smooth. If the system is slight displaced from the mean position and released, it will execute SHM. The maximum amplitude for which the upper block does not slip relative to the lower will be –

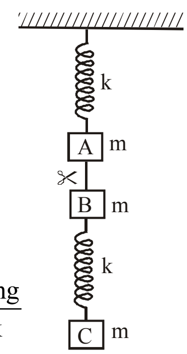
(K is spring constant)



(A) $\frac{5\mu g}{K}$ (B) $\frac{\mu g}{K}$

(C) $\frac{3\mu g}{K}$ (D) $\frac{2\mu g}{K}$

Q.10 The spring block system as shown in figure is in equilibrium. The spring connecting blocks A and B is cut. The mass of all the three blocks is m and spring constant of both the spring is k. The amplitude of resulting oscillation of block A is –



(A) $\frac{mg}{k}$ (B) $\frac{2mg}{k}$

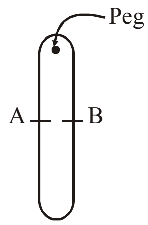
(C) $\frac{3mg}{k}$ (D) $\frac{4mg}{k}$

Q.11 A tunnel is dug along radius of earth that ends at centre. A body is released from the surface along tunnel. The ball will bounce after first collision at centre up to a height of (radius of earth is R and coefficient of restitution is e)

(A) R (B) eR

(C) e^2R (D) $\sqrt{e}\ R$

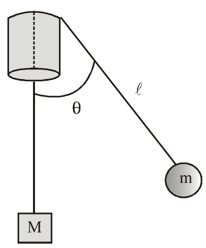
Q.12 A loop consists of two cords of lengths ℓ and 2ℓ, and their masses per unit length are their masses per unit length are μ and 2μ. It is placed in stable equilibrium over a smooth peg as shown in the figure. When slightly displaced, it executes SHM. The period of oscillation is



(A) $2\pi\sqrt{\dfrac{5\ell}{3g}}$ (B) $2\pi\sqrt{\dfrac{5\ell}{g}}$

(C) $2\pi\sqrt{\dfrac{5\ell}{2g}}$ (D) $2\pi\sqrt{\dfrac{3\ell}{g}}$

Q.13 A large mass M hangs stationary at the end of a light string that passes through a smooth fixed tube to a small mass m that moves around in a horizontal circular path. If ℓ is the length of the string from m to the top end of the . . . . . . θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to –



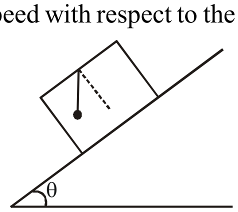
(A) $2\pi\sqrt{\dfrac{\ell}{g\sin\theta}}$ (B) $2\pi\sqrt{\dfrac{\ell m}{gM}}$

(C) $2\pi\sqrt{\dfrac{\ell}{g\cos\theta}}$ (D) $2\pi\sqrt{\dfrac{m\ell}{gM\sin\theta}}$

Q.14 A loaded vertical spring executes simple harmonic oscillations with period of 4 s. The difference between the kinetic energy and potential energy of this system oscillates with a period of :

(A) 8 s (B) 1 s
(C) 2 s (D) 4 s

Q.15 A very heavy box is kept on a frictionless inclined plane inclined at an angle θ from the horizontal. A pendulum of length ℓ is hanging vertically from the roof of the top as shown in the figure. If system is released from the rest, maximum speed with respect to the box achieved by the bob is –



(A) $\sqrt{2g\ell\,(1-\cos\theta)}$ (B) $\sqrt{2g\ell\,\cos\theta\,(1-\cos\theta)}$

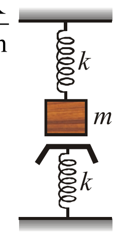
(C) $\sqrt{2g\ell\,\sin\theta\,(1-\cos\theta)}$ (D) $\sqrt{2g\ell}$

Q.16 Two blocks each of mass m, connected by ideal massless spring with force constant K, are placed on smooth horizontal surface. A particle of mass m moving horizontally with velocity v_0 collides one block and gets stuck with it. The system starts oscillation with frequency

(A) $\dfrac{1}{2\pi}\sqrt{\dfrac{2K}{m}}$ (B) $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{2m}}$

(C) $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}}$ (D) $\dfrac{1}{2\pi}\sqrt{\dfrac{3K}{2m}}$

Q.17 A mass m is suspended from a spring of force constant k and just touches another identical spring fixed to the floor as shown in the figure. The time period of small oscillations is



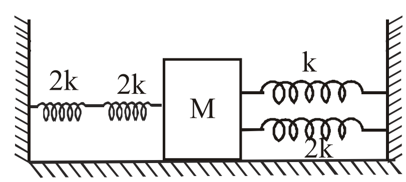
(A) $2\pi\sqrt{\dfrac{m}{k}}$ (B) $\pi\sqrt{\dfrac{m}{k}}+\pi\sqrt{\dfrac{m}{k/2}}$

(C) $\pi\sqrt{\dfrac{m}{3k/2}}$ (D) $\pi\sqrt{\dfrac{m}{k}}+\pi\sqrt{\dfrac{m}{2k}}$

Q.18 A solid sphere of mass 1 kg and diameter 0.3 m is suspended from a wire. If the twisting couple per unit twist for the wire is 6×10^{-3} N-m/radian, then the time period of small oscillations will be-

(A) 0.7 sec (B) 7.7 sec
(C) 77 sec. (D) 777 sec.

Q.19 Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system.



(A) $\dfrac{1}{2p}\sqrt{\dfrac{k}{4M}}$ (B) $\dfrac{1}{2p}\sqrt{\dfrac{4k}{M}}$

(C) $\dfrac{1}{2p}\sqrt{\dfrac{k}{7M}}$ (D) $\dfrac{1}{2p}\sqrt{\dfrac{7k}{M}}$

Q.20 Two oscillating systems; a simple pendulum and a vertical spring-mass-system have same time period of motion on the surface of the Earth. If both are taken to the moon, then–

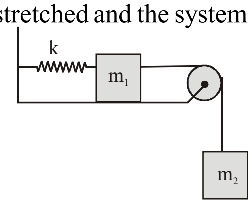
(A) Time period of the simple pendulum will be more than that of the spring-mass system.
(B) Time period of the simple pendulum will be equal is that is of the spring-mass system.
(C) Time period of the simple pendulum will be less than of the spring-mass system.
(D) Nothing can be said definitely without observation.

Q.21 A particle moves simple harmonically along a straight line. It starts from origin without any initial velocity and travels a distance ℓ_1 in 1st second and ℓ_2 in 2nd second in same direction. The amplitude of oscillation is –

(A) $\frac{2\ell_1^2}{3\ell_1 - \ell_2}$ (B) $\frac{3\ell_1^2}{2\ell_1 - \ell_2}$

(C) $\frac{2\ell_2^2}{3\ell_2 - \ell_1}$ (D) $\frac{3\ell_2^2}{3\ell_2 - \ell_1}$

Q.22 m_1 and m_2 are connected with a light inextensible string with m_1 lying on smooth table and m_2 hanging as shown in figure. m_1 is also connected to a light spring which is initially unstretched and the system is released from rest



(A) system performs SHM with angular frequency given by $\sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$

(B) system performs SHM with angular frequency given by $\sqrt{\frac{k}{m_1+m_2}}$

(C) tension in string will be 0 when the system is released.

(D) maximum displacement of m_1 will be $\frac{m_2 g}{k}$

Q.23 Two particles execute SHM on same straight line with same mean position, same time period 6 second and same amplitude 5cm. Both the particles start SHM from their mean position (in same direction) with a time gap of 1 second. Find the maximum separation between the two particles during their motion.

(A) 2 cm. (B) 3 cm.
(C) 4 cm. (D) 5 cm.

Q.24 A simple pendulum with length L and mass M of the bob is vibrating with amplitude a. Then the maximum tension in the string is :

(A) Mg (B) $Mg\left[1+\left(\frac{a}{L}\right)^2\right]$

(C) $Mg\left[1+\frac{a}{L}\right]^2$ (D) $Mg\left[1+\frac{a}{2L}\right]^2$

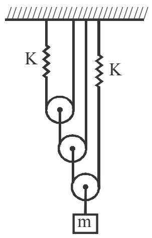
Q.25 A body of mass 0.1 kg is attached to two springs of force constants 6 N/m and 4 N/m and supported by two rigid supports. If the body is displaced along the length of the springs, the frequency of vibrations will be-

(A) 5 vibrations/sec (B) 10 vibrations/sec
(C) $5/\pi$ vibrations/sec (D) $\pi/5$ vibrations/sec

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

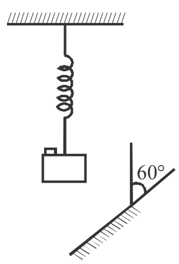
NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Find the time period of vertical oscillations of the mass shown in figure. (Take m = 64 kg, $K = 68\pi^2$ N/m), give answer in seconds.

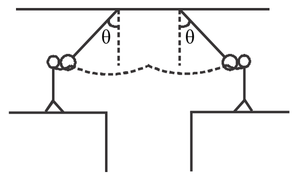


Q.2 A cyclist turns her bicycle upside down to tinker with it. After she gets it upside down, he notices the front wheel executing a show, small-amplitude, back-and-forth rotational motion with a period of 12s. Considering the wheel to be a thin ring of mass 600g and radius 30cm, whose only irregularity is the presence of the small tire valve stem, determine the mass of the valve stem (in gm). Take $\pi^2 = g$ and appropriate approximation.

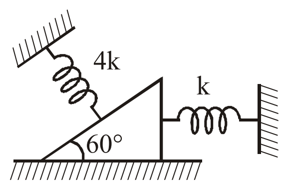
Q.3 An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant k. The block performs simple harmonic motion with amplitude A infront of a plane mirror placed as shown. The maximum speed of insect relative to its image is $A\sqrt{X}\sqrt{\frac{k}{M}}$. Find the value of X.



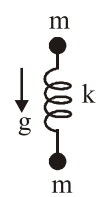
Q.4 Two small circus clowns (each having a mass of 50 kg) swing on two ropes (negligible mass, length 25m) shown in the figure. At the peak of the swing, one grabs the other, and the two swing back to one platform. The time (in sec) for the forward and return motion is (Assume that the angle θ is small)



Q.5 A smooth wedge of mass m and angle of inclination 60° rests unattached between two springs of spring constant k and 4k, on a smooth horizontal plane, both springs in the unextended position. The time period of small oscillations of the wedge is $\pi\left(1+\frac{1}{\sqrt{X}}\right)\sqrt{\frac{m}{k}}$ (Assuming that the springs are constrained to get compressed along their length). Find the value of X.



Q.6 A mass m is hung on an ideal massless spring. Another equal mass is connected to the other end of the spring. The whole system is at rest. At t = 0, m is released and the system falls freely under gravity. Assume that natural length of the spring is L_0, its initial stretched length is L and the acceleration due to gravity is g. The distance between masses as function of time is



$L_0 + (L - L_0)\cos\sqrt{\frac{Xk}{m}}t$. Find the value of X.

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A spring when connected by mass m gives time period 'T'. If spring is cut in n equal parts and each part connected in parallel with same mass. New time- period will be - **[AIEEE-2002]**

(A) nT (B) T/n

(C) $T/\sqrt{n}$ (D) $\sqrt{n}T$

Q.2 A child is sitting on a swing and swinging. If he stands up. The time period of swing will **[AIEEE-2002]**

(A) increase

(B) decrease

(C) remain same

(D) increase if the child is long and decrease if the child is short

Q.3 In a simple harmonic oscillator, at the mean position – **[AIEEE-2002]**

(A) Kinetic energy is minimum, potential energy is maximum.

(B) Both kinetic energy and potential energies are maximum.

(C) Kinetic energy is maximum, potential energy is minimum.

(D) Both kinetic & potential energies are minimum

Q.4 A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. Then the ratio of m/M is – **[AIEEE-2003]**

(A) 25/9 (B) 16/9

(C) 5/3 (D) 3/5

Q.5 The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is – **[AIEEE-2003]**

(A) 21% (B) 42%

(C) 10% (D) 11%

Q.6 Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2 , respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitudes of A and B is **[AIEEE-2003]**

(A) $\frac{k_2}{k_1}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_1}{k_2}$ (D) $\sqrt{\frac{k_1}{k_2}}$

Q.7 The displacement of a particle varies according to the relation x = 4 (cos πt + sin πt). The amplitude of the particle is – **[AIEEE-2003]**

(A) 4 (B) $4\sqrt{2}$

(C) 8 (D) – 4

Q.8 A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as a function of displacement x. Which of the following statement is true ? **[AIEEE-2003]**

(A) T.E. is zero when x = 0

(B) K.E. is maximum when x is maximum

(C) P.E. is maximum when x = 0

(D) K.E. is maximum when x = 0

Q.9 The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. What relationship between t and t_0 is true ? **[AIEEE-2004]**

(A) $t = t_0$ (B) $t = t_0 / 2$

(C) $t = 2t_0$ (D) $t = 4t_0$

Q.10 A particle at the end of a spring executes SHM with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then – **[AIEEE-2004]**

(A) $T = t_1 + t_2$ (B) $T^2 = t_1^2 + t_2^2$

(C) $T^{-1} = t_1^{-1} + t_2^{-1}$ (D) $T^{-2} = t_1^{-2} + t_2^{-2}$

Q.11 The total energy of a particle, executing simple harmonic motion is – **[AIEEE-2004]**

(A) $\propto X$ (B) $\propto X^2$

(C) Independent of X (D) $\propto X^{1/2}$

where x is the displacement from the mean position.

Q.12 A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ω t ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to – **[AIEEE-2004]**

(A) $\frac{m}{(\omega_0^2 - \omega^2)}$ (B) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(C) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (D) $\frac{m}{(\omega_0^2 + \omega^2)}$

Q.13 In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force ; then **[AIEEE-2004]**

(A) $\omega_1 = \omega_2$

(B) $\omega_1 > \omega_2$

(C) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large.

(D) $\omega_1 < \omega_2$

Q.14 The function $\sin^2(\omega t)$ represents **[AIEEE-2005]**

(A) a periodic, but not simple harmonic motion with a period 2π/ω

(B) a periodic, but not simple harmonic motion with a period π/ω

(C) a simple harmonic motion with a period 2π/ω

(D) a simple harmonic motion with a period π/ω

Q.15 If a simple harmonic motion is represented by

$\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[AIEEE-2005]**

(A) $\frac{2\pi}{\alpha}$ (B) $\frac{2\pi}{\sqrt{\alpha}}$ (C) $2\pi\alpha$ (D) $2\pi\sqrt{\alpha}$

Q.16 The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[AIEEE-2005]**

(A) first increases and then decrease to the original value
(B) first decrease and then increase to the original value
(C) remain unchanged
(D) increase towards a saturation value

Q.17 The maximum velocity of a particle, executing simple haromonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is – **[AIEEE 2006]**

(A) 0.1 s (B) 100s
(C) 0.01 s (D) 10 s

Q.18 Starting from the origin a body oscilates simple haromonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy **[AIEEE 2006]**

(A) (1/3)s (B) (1/12) s
(C) (1/6)s (D) (1/4) s

Q.19 A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time– **[AIEEE 2006]**

(A) for an amplitude of g^2/ω^2
(B) at the highest position of the platform
(C) at the mean position of the platform
(D) for an amplitude of g/ω^2

Q.20 The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metres. The time at which the maximum speed first occurs is - **[AIEEE 2007]**

(A) 0.5 s (B) 0.75 s
(C) 0.125 s (D) 0.25 s

Q.21 A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then **[AIEEE 2007]**

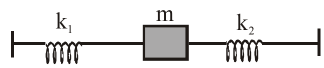
(A) $A = x_0, \delta = -\pi/4$ (B) $A = x_0\omega^2, \delta = \pi/4$
(C) $A = x_0\omega^2, \delta = -\pi/4$ (D) $A = x_0\omega^2, \delta = 3\pi/4$

Q.22 A block of mass 'm' is connected to another block of mass 'M' by a spring (mass less) of spring constant 'k'. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is stretched. Then a constant force 'F' starts acting on the block of mass 'M' to pull it. Find the force on the block of mass 'm' **[AIEEE 2007]**

(A) $\frac{mF}{M}$ (B) $\frac{(M+m)F}{m}$

(C) $\frac{mF}{(m+M)}$ (D) $\frac{MF}{(m+M)}$

Q.23 Two springs, of force constant k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass if f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[AIEEE 2007]**



(A) f/2 (B) f/4
(C) 4f (D) 2f

Q.24 A particle of mass m executes simple harmonic motion with amplitude 'a' and frequency 'v'. The average kinetic energy during its motion from the position of equilibrium to the end is **[AIEEE 2007]**

(A) $\pi^2 m a^2 v^2$ (B) $\frac{1}{4} m a^2 v^2$

(C) $4\pi^2 m a^2 v^2$ (D) $2\pi^2 m a^2 v^2$

Q.25 If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time ? **[AIEEE-2009]**

(A) aT/x (B) $aT + 2\pi v$
(C) aT/v (D) $a^2T^2 + 4\pi^2 v^2$

Q.26 Two particles are executing simple harmonic motion of the same amplitude Aand frequency ω along the x-axis. Their mean position is separated by distance X_0 (X_0 >A). If the maximum separation between them is (X_0 + A), the phase difference between their motion is –

(A) $\pi/2$ (B) $\pi/3$ **[AIEEE-2011]**
(C) $\pi/4$ (D) $\pi/6$

Q.27 A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of (A_1/A_2) is – **[AIEEE-2011]**

(A) $\frac{M}{M+m}$ (B) $\frac{M+m}{M}$

(C) $\left(\frac{M}{M+m}\right)^{1/2}$ (D) $\left(\frac{M+m}{M}\right)^{1/2}$

Q.28 If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds : **[AIEEE-2012]**

(A) $\frac{0.693}{b}$ (B) b

(C) 1/b (D) 2/b

Q.29 The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[JEE MAIN 2013]**

(A) 0.7 (B) 0.81
(C) 0.729 (D) 0.6

Q.30 An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[JEE MAIN 2013]**

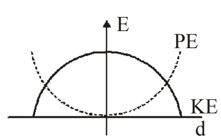
(A) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (B) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

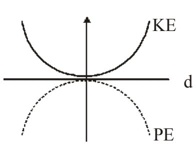
(C) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

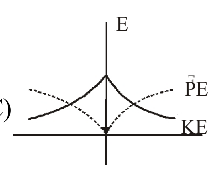
Q.31 A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then – **[JEE MAIN 2014]**

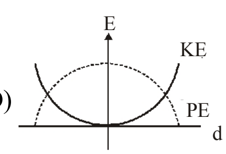
(A) amplitude of motion is 4a
(B) time period of oscillations is 6τ
(C) amplitude of motion is 3a
(D) time period of oscillations is 8τ

Q.32 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (Graphs are schematic and not drawn to scale) **[JEE MAIN 2015]**

(A) 

(B) 

(C) 

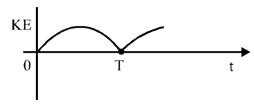
(D) 

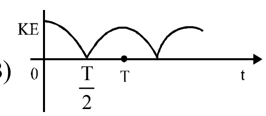
Q.33 A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance 2A/3 from equilibrium position. The new amplitude of the motion is – **[JEE MAIN 2016]**

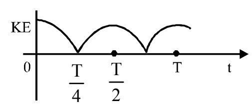
(A) 3A (B) $A\sqrt{3}$

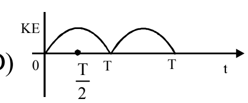
(C) 7A/3 (D) $\frac{A}{3}\sqrt{41}$

Q.34 A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like: **[JEE MAIN 2017]**

(A) 

(B) 

(C) 

(D) 

Q.35 A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE MAIN 2018]**

(A) 2.2 N/m (B) 5.5 N/m
(C) 6.4 N/m (D) 7.1 N/m

Q.36 A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to : **[JEE MAIN 2019 (JAN)]**

(A) 0.17 (B) 0.37
(C) 0.57 (D) 0.77

Q.37 A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1 / 1000 of the original amplitude is close to : **[JEE MAIN 2019 (APRIL)]**

(A) 100 s (B) 20 s
(C) 10 s (D) 50 s

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

Choose one correct response for each question.

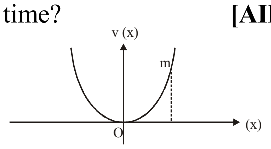
Q.1 The displacement of a particle along the x axis is given by $x = a\sin^2\omega t$. The motion of the particle corresponds to – **[AIPMT (PRE) 2010]**

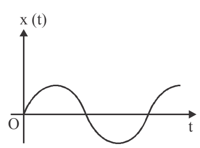
(A) simple harmonic motion of frequency ω/π
(B) simple harmonic motion of frequency $3\omega/2\pi$
(C) non simple harmonic motion
(D) simple harmonic motion of frequency $\omega/2\pi$

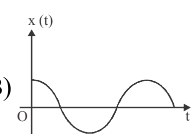
Q.2 The period of oscillation of amass m suspended from a spring of negligible mass is T. If along with it another mass M is also suspended the period of oscillation will now be **[AIPMT (PRE) 2010]**

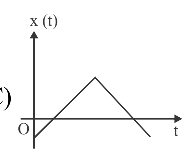
(A) T (B) $T/\sqrt{2}$
(C) 2T (D) $\sqrt{2}T$

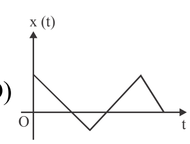
Q.3 A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time? **[AIPMT (PRE) 2011]**



(A)


(B)


(C)


(D)


Q.4 Out of the following functions representing motion of a particle which represents SHM? **[AIPMT (PRE) 2011]**

(i) $y = \sin\omega t - \cos\omega t$ (ii) $y = \sin^3\omega t$

(iii) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ (iv) $y = 1 + \omega t + \omega^2 t^2$

(A) Only (i) and (ii)
(B) Only (i)
(C) Only (iv) does not represent SHM
(D) Only (i) and (iii)

Q.5 Two particle are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is : **[AIPMT (MAINS) 2011]**

(A) 0 (B) $2\pi/3$
(C) π (D) $\pi/6$

Q.6 The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are : **[AIPMT (PRE) 2012]**

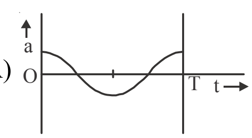
(A) $kgms^{-1}$ (B) $kgms^{-2}$
(C) kgs^{-1} (D) kgs

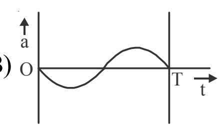
Q.7 The equation of a simple harmonic wave is given by

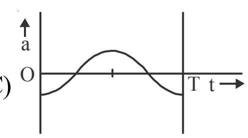
$y = 3\sin\frac{\pi}{2}(50t - x)$, where x and y are in meters and t is in seconds. The ratio of maximum particle velocity to the wave velocity is – **[AIPMT (MAINS) 2012]**

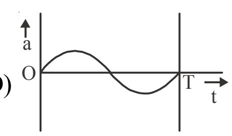
(A) 2π (B) $(3/2)\pi$
(C) 3π (D) $(2/3)\pi$

Q.8 The oscillation of a body on a smooth horizontal surface is represented by the equation, $x = A\cos(\omega t)$, where x = displacement at time t, ω = frequency of oscillation. Which one of the following graphs shows correctly the variation a with t? [**AIPMT 2014]**

(A)


(B)


(C)


(D)


Q.9 When two displacement represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed the motion is **[AIPMT 2015]**

(A) simple harmonic with amplitude (a/b).
(B) simple harmonic with amplitude $\sqrt{a^2 + b^2}$.
(C) simple harmonic with amplitude $\frac{a+b}{2}$.
(D) not a simple harmonic

Q.10 A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are V_1 and V_2 respectively. Its time period is **[AIPMT 2015]**

(A) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ (B) $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$

(C) $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$ (D) $2\pi\sqrt{\frac{x_1^2 - x_2^2}{V_1^2 - V_2^2}}$

Q.11 A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then, its time period of vibration will be **[AIPMT 2015]**

(A) $2\pi\beta/\alpha$ (B) β^2/α^2

(C) α/β (D) β^2/α

Q.12 A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. Value of m in kg is

[NEET 2016 PHASE 2]

(A) 3/4 (B) 4/3

(C) 16/9 (D) 9/16

Q.13 A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. Then they are connected in parallel and force constant is k" . Then k' : k" is – **[NEET 2017]**

(A) 1 : 9 (B) 1 : 11

(C) 1 : 14 (D) 1 : 16

Q.14 A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is **[NEET 2017]**

(A) $\sqrt{5}/2\pi$ (B) $4\pi/\sqrt{5}$

(C) $2\pi/\sqrt{3}$ (D) $\sqrt{5}/\pi$

Q.15 A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m/s^2 at a distance of 5m from the mean position. The time period of oscillation is **[NEET 2018]**

(A) 2 s (B) π s

(C) 2π s (D) 1 s

Q.16 The displacement of a particle executing simple harmonic motion is given by $y = A_0 + A \sin \omega t + B \cos \omega t$

Then the amplitude of its oscillation is given by :

[NEET 2019]

(A) $A_0 + \sqrt{A^2 + B^2}$ (B) $\sqrt{A^2 + B^2}$

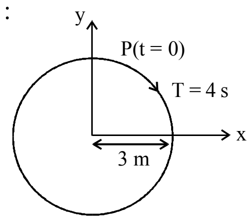
(C) $\sqrt{A_0^2 + (A+B)^2}$ (D) $A + B$

Q.17 Average velocity of a particle executing SHM in one complete vibration is : **[NEET 2019]**

(A) $A\omega / 2$ (B) $A\omega$

(C) $A\omega^2 / 2$ (D) Zero

Q.18 The radius of circle, the period of revolution, initial position and sense of revolution are indicated in the figure. y-projection of the radius vector of rotating particle P is : **[NEET 2019]**



(A) y (t) = –3 cos$2\pi t$, where y in m.

(B) y (t) = 4 sin ($\pi t / 2$), where y in m.

(C) y (t) = 3 cos ($3\pi t / 2$), where y in m.

(D) y (t) = 3 cos ($\pi t / 2$), where y in m.

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	B	B	C	B	C	B	B	A	D	A	C	D	C	C	D	A	C	C	B	C	C	D	A	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	D	B	C	A	A	B	C	B	D	B	D	D	B	B	A	D	D	B	B	D	B	B	D	C
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	
A	B	C	C	B	C	D	A	C	C	B	C	C	C	D	A	C	C	C	A	A	A	B	D	C	

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	A	A	A	C	C	A	A	B	B	C	B	C	B	D	D	B	B	A	A	B	D	B	C

EXERCISE - 3

Q	1	2	3	4	5	6
A	1	5	3	5	3	2

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	B	C	B	C	B	B	D	C	B	C	B	A	B	B	A	C	C	D	A	D	C	D	A	A
Q	26	27	28	29	30	31	32	33	34	35	36	37													
A	B	D	D	C	C	B	A	C	C	D	B	B													

EXERCISE - 5

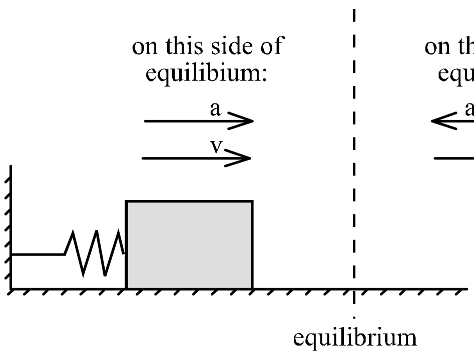
Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	A	D	B	D	B	C	B	C	B	A	A	D	B	B	B	B	D	D

SOLUTIONS
SIMPLE HARMONIC MOTION
TRY IT YOURSELF-1

(1) **(C).** The acceleration function is $a = -\omega^2 A\sin(\omega t + \delta)$.
$a = -(0.6)^2(20)\sin[(0.6(0) - \pi/2) = -7.2\ m/s^2$.
The magnitude is $7.2\ m/s^2$.

(2) **(D).** The range of motion for the particle is limited to the regions in which the kinetic energy is either zero or positive, so the particle is confined to the regions where the potential energy is less than the energy. Hence the particle periodically revisits a and b.

(3) **(C).** If you will remember, the position function for this kind of motion is characterized by a sine wave whereas the velocity function is a cosine and the acceleration function is a negative sine. Put a little differently, the velocity and acceleration are out of phase with one another by a quarter of a cycle. What does this mean?



Consider the sketch. If the body's velocity at the point shown is, say, +2 m/s (note that the body is picking up speed because the acceleration is in the direction of motion), there will be a point on the other side of x = 0 where the body's velocity will again be +2 m/s (in that case, the body will be slowing down because the acceleration will be opposite the direction of motion). That means there are two accelerations to be assigned to +2 m/s. How does one graph such a function? With a circle! In short, graph C does the trick.

(4) **(A).** If this equation had been $-32x = 2a$, we would have had the characteristic equation for an ideal spring (i.e., $-kx = ma$). If that had been the case, a positive displacement x would yield a negative force and a negative displacement x would have yielded a positive force. That is, the force would always have been oriented back toward the equilibrium position. With the expression $-32x^2 = 2a$, the $-x^2$ term guarantees that, no matter what the sign of x,, the net force will always be negative. In short, this force function is not a restoring force and, as a consequence, will not produce oscillatory motion of any type.

(5) **(D).** As the angular frequency is 0.6 radians per second, the frequency is (0.6 rad/sec)/(2π), or approximately 0.1 cycles/second.

(6) We know that, $v = \omega\sqrt{(A^2 - y^2)}$

Further $\omega = \dfrac{2\pi}{T}$

$$\therefore\ v = \frac{2\pi}{T}\sqrt{(A^2 - y^2)} = \frac{2\times 3.14}{(11/7)}\sqrt{[(1.0)^2 - (0.6)^2]}$$
= 3.2 m/sec.

Kinetic energy at this displacement is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\times 0.8\times(3.2)^2 = 4.1\text{ joule.}$$

(7) Acceleration of the platform $a = \omega^2 y$
Maximum acceleration, $a_{max} = \omega^2 A$ (A = Amplitude)
$\therefore\ a_{max} = (2\pi n)^2 A$ (n = frequency)
$= 4(3.14)^2(2)^2 \times 0.05 = 7.88\ m/sec^2$

$$\text{Maximum reading} = \frac{m(g + a_{max})}{g}$$
$$= \frac{60(10 + 7.88)}{10} = 107.3\text{ kg}$$
$$\text{Minimum reading} = \frac{M(g - a_{max})}{g}$$
$$= \frac{60(10 - 7.88)}{10} = 12.7\text{ kg.}$$

(8) The displacement of a particle in S.H.M. is given by :
$y = A\sin(\omega t + \phi)$

$$\text{velocity} = \frac{dy}{dt} = \omega A\cos(\omega t + \phi)$$

The velocity is maximum when the particle passes through the mean position i.e.

$$\left(\frac{dy}{dt}\right)_{max} = \omega A$$

The kinetic energy at this instant is given by

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2_{max} = \frac{1}{2}m\omega^2 A^2 = 8\times 10^{-3}\text{ joule}$$

or $\dfrac{1}{2}\times(0.1)\,\omega^2\times(0.1)^2 = 8\times 10^{-3}$

Solving we get $\omega = \pm 4$
Substituting the values of a, ω and ϕ in the equation of S.H.M., we get
$y = 0.1\sin(\pm 4t + \pi/4)$ metre.

(9) Evidently, the maxi. displacement $= \dfrac{20}{2} = 10$ cm

For a particle executing SHM the speed at a displacement x is given by

$v = \omega\sqrt{A^2 - x^2}$ .... (1)
and $v_{max} = \omega A$ .... (2)

Dividing (2) by (1), we get $\frac{v_{max}}{v} = \frac{A}{\sqrt{A^2 - x^2}}$

$\Rightarrow \frac{v_{max}}{v} = \frac{10}{\sqrt{10^2 - 5^2}} \Rightarrow \frac{10}{5\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\therefore$

$v = \frac{30 \times 3}{2\sqrt{3}} = 15\sqrt{3}$ cm / s

(10) (C)

TRY IT YOURSELF-2

(1) **(B).** The angular frequency is $\omega_0 \equiv 2\pi/T$. For small angle the pendulum approximates a simple harmonic oscillator with $\omega_0 \equiv \sqrt{g/\ell}$ where ℓ is the length of the pendulum. The angular speed is the magnitude of the angular velocity $\omega \equiv d\theta/dt$.
Note that sometimes the symbol ω may be used for both quantities. This is a result of the fact that for uniform circular motion, angular frequency and angular speed are equal because the period $T = 2\pi R/v$ and the speed and angular speed are related by $v = R\omega$.
Therefore $T = 2\pi R/R\omega = 2\pi/\omega$.
So $\omega = \omega_0$ for this special case.

(2) **(C).** When the disk is fixed to the rod, an internal torque will cause the disk to rotate about its center of mass. When the pendulum reaches the bottom of its swing, the decrease in potential energy will be result in an increase in the rotational kinetic energy of both the rod and the disk and the center of mass translation kinetic energy of the rod-disk system. When the disk is mounted on the frictionless bearing there is no internal torque that will make the disk start to rotate about its center of mass when the pendulum is released. Therefore when the pendulum reaches the bottom of its swing, the same decrease in potential energy will be transferred into a larger smaller in rotational kinetic energy of just the rod since the disc is not rotating and a greater increase in the center of mass translation kinetic energy of the rod-disk system. So when the disk bearings are frictionless, the center of mass of the rod-disk system is traveling faster at the bottom of its arc hence will take less time to complete one cycle and so the period is shorter compared to the fixed disk.
You might be tempted to argue that the moment of inertia about the pivot point is the same in both cases, the torque is the same so the period should be the same. But the disk with frictionless bearings is not a rigid body which means that the disk has a different angular acceleration than the rod and hence you must treat each part of the system separately when applying
$\tau_P = I_P\alpha$.

(3) **(C).** The particle starts with potential energy $U_0 = \frac{1}{2}kx_0^2$.
When it first returns to equilibrium it now has only kinetic energy $K_1 = \frac{1}{2}mv_x^2$. Since the energy of the block-spring system is constant, $K_1 = U_0$ and so

$$\frac{1}{2}mv_x^2 = \frac{1}{2}kx_0^2$$

We can solve for the x-component of the velocity

$$v_x = \pm\sqrt{\frac{k}{m}x_0^2}$$

Since the object is moving in the negative x-direction when it first returns to equilibrium so we must take the negative square root,

$$v_e = -\sqrt{\frac{k}{m}x_0^2}$$

(4) **(B).** A, C, and D are all true, but the acceleration and velocity vectors sometimes point in the same direction, and sometimes point in opposite directions.

(5) **(A).** A frequency of 1 cycles/second corresponds to an angular frequency of 2π radians per second. The angular frequency function for a pendulum is $(g/L)^{1/2}$, where L is the pendulum length. Putting it all together, we can write $2\pi = (g/L)^{1/2}$, or $L = g/4\pi^2$, or approximately 0.25 meters.

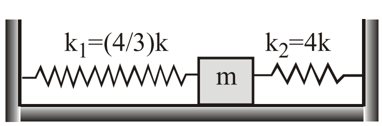
(6) **(C).** The angular frequency of a pendulum is equal to $(g/L)^{1/2}$. This means that changing the mass of the bob does nothing to the angular frequency, the frequency, or the period, but changing the length L does change things. In fact, increasing L decreases the angular frequency and, by extension, the frequency. As the period is the inverse of the frequency, decreasing the frequency increases the period.

(7) (B) **(8)** (C) **(9)** (C)

(10) The stiffness of a spring is inversely proportional to its length. Therefore the stiffness of each part is

$$k_1 = \frac{4}{3}k \text{ and } \quad k_2 = 4k$$

Time period, $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$



or $T = 2\pi\sqrt{\frac{3m}{16k}} = \frac{\pi}{2}\sqrt{\frac{3m}{k}}$

CHAPTER-10 :
SIMPLE HARMONIC MOTION
EXERCISE-1

(1) (C). $a=-\omega^2 x \Rightarrow \left|\frac{a}{x}\right|=\omega^2$

(2) (B). Comparing given equation with standard equation,

$y=A\sin(\omega t+\phi)$, we get, A = 2cm, $\omega=\frac{\pi}{2}$

$$a_{max}=\omega^2 A=\left(\frac{\pi}{2}\right)^2\times 2=\frac{\pi^2}{2}cm/s^2.$$

(3) (B). $A=6cm, \omega=100rad/sec$

$$K_{max}=\frac{1}{2}m\omega^2A^2=\frac{1}{2}\times 1\times(100)^2\times(6\times10^{-2})^2=18\text{ J}$$

(4) (C). In S.H.M., frequency of K.E. and P.E. = 2 × (Frequency of oscillating particle)

(5) (B). Total energy $U=\frac{1}{2}KA^2$

(6) (C). $E\propto(\text{amplitude})^2\Rightarrow\frac{E'}{E}=\frac{\left(\frac{3}{4}A\right)^2}{A^2}\Rightarrow E'=\frac{9}{16}E$

(7) (B). Body collides elastically with walls of room. So, there will be no loss in its energy and it will remain colliding with walls of room, so it's motion will be periodic. There is no change in energy of the body, hence there is no acceleration, so it's motion is not SHM.

(8) (B). In the given case, $\frac{\text{Displacement}}{\text{Acceleration}}=\frac{1}{b}$

$\therefore$ Time period $T=2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}=\frac{2\pi}{\sqrt{b}}$

(9) (A). Comparing given equation with standard equation,

$x=A\cos(\omega t+\phi)$ we get, A = 0.01

and $\omega=\pi\Rightarrow 2\pi n=\pi\Rightarrow n=0.5\,Hz$

(10) (D). $y=5\sin(\pi t+4\pi)$,

Comparing it with standard equation

$$y=A\sin(\omega t+\phi)=A\sin\left(\frac{2\pi t}{T}+\phi\right)$$

$A=5m$ and $\frac{2\pi t}{T}=\pi t\Rightarrow T=2$ sec.

(11) (A). $x=A\sin\left(\omega t+\frac{\pi}{6}\right)$

$$x'=A\cos\omega t=A\sin\left(\omega t+\frac{\pi}{2}\right)$$

$$\therefore\quad\Delta\phi=\left(\omega t+\frac{\pi}{2}\right)-\left(\omega t+\frac{\pi}{6}\right)=\frac{\pi}{3}$$

(12) (C). Velocity in mean position $v=A\omega$,
Velocity at a distance of half amplitude.

$$v'=\omega\sqrt{A^2-y^2}=\omega\sqrt{A^2-\frac{A^2}{4}}=\frac{\sqrt{3}}{2}A\omega=\frac{\sqrt{3}}{2}v$$

(13) (D). In S.H.M., $v=\sqrt{A^2-y^2}$; $a=-\omega^2 y$ when $y=A$

$\Rightarrow\ v_{min}=0$ and $a_{max}=-\omega^2A$

(14) (C). $y=A(\cos\omega t+\sin\omega t)$

$$=A\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\omega t+\frac{1}{\sqrt{2}}\sin\omega t\right]$$

$$=A\sqrt{2}[\sin 45^\circ\cos\omega t+\cos 45^\circ\sin\omega t]$$

$$=A\sqrt{2}\sin(\omega t+45^\circ)$$

$\Rightarrow$ Amplitude $=A\sqrt{2}$

(15) (C). $v=\omega\sqrt{A^2-x^2}$

$$v^2=\omega^2(A^2-x^2)\Rightarrow\frac{x^2}{A^2}+\frac{v^2}{(A\omega)^2}=1$$

Equation represents elliptical curve.

(16) (D). $y=A\sin\omega t=\frac{A\sin 2\pi}{T}t$

$$\Rightarrow\frac{A}{2}=A\sin\frac{2\pi t}{T}\Rightarrow t=\frac{T}{12}.$$

(17) (A). The amplitude is a maximum displacement from the mean position.

(18) (C). Equation of motion $y=a\cos\omega t$

$$\Rightarrow\frac{a}{2}=a\cos\omega t\Rightarrow\cos\omega t=\frac{1}{2}\Rightarrow\omega t=\frac{\pi}{3}$$

$$\Rightarrow\frac{2\pi t}{T}=\frac{\pi}{3}\Rightarrow t=\frac{\frac{\pi}{3}\times T}{2\pi}=\frac{4}{3\times 2}=\frac{2}{3}\text{sec}$$

(19) (C). At centre $v_{max}=a\omega=a.\frac{2\pi}{T}=\frac{0.2\times 2\pi}{0.01}=40\pi$

(20) (B). $v_{max}=a\omega=a\frac{2\pi}{T}=3\times\frac{2\pi}{6}=\pi\,cm/s$

(21) (C). $v_{max}=\omega a=\frac{2\pi}{T}\times a$

$$\Rightarrow v_{max}=\frac{2\times\pi\times 2}{2}=2\pi\ m/s$$

(22) (C). Acceleration $A=\omega^2 y\Rightarrow\omega=\sqrt{\frac{A}{y}}=\sqrt{\frac{0.5}{0.02}}=5$

Maximum velocity $v_{max}=a\omega=0.1\times 5=0.5$

(23) **(D).** At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore\ v = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2cm..$

(24) **(A).** $v_{max} = a\omega = 3 \times 100 = 300$

(25) **(A).** Maximum velocity $= a\omega = 16$
Maximum acceleration $= \omega^2 a = 24$

$\Rightarrow a = \frac{(a\omega)^2}{\omega^2 a} = \frac{16 \times 16}{24} = \frac{32}{3} m$

(26) **(D).** Acceleration $\propto$ – displacement, and direction of acceleration is always directed towards the equilibrium position.

(27) **(D).** Maximum force $= m(a\omega^2) = ma\left(\frac{4\pi^2}{T^2}\right)$

$= 0.5\left(\frac{4\pi^2}{\pi^2/25}\right) \times 0.01 = 0.5N$

(28) **(B).** Comparing given equation with standard equation,

$y = a\sin(\omega t + \phi)$, we get, $a = 2cm$, $\omega = \frac{\pi}{2}$

$\therefore A_{max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2 = \frac{\pi^2}{2} cm/s^2$.

(29) **(C).** Velocity $v = \omega\sqrt{A^2 - x^2}$ and acceleration $= \omega^2 x$

Now given, $\omega^2 x = \omega\sqrt{A^2 - x^2} \Rightarrow \omega^2.1 = \omega\sqrt{2^2 - 1^2}$

$\Rightarrow \omega = \sqrt{3}\ \ \therefore\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

(30) **(A).** P.E. $= \frac{1}{2} m\omega^2 x^2$
It is clear P.E. will be maximum when x will be maximum i.e., at $x = \pm A$

(31) **(A).** Since maximum value of $\cos^2\omega t$ is 1.

$\therefore K_{max} = K_o \cos^2 \omega t = K_o$

Also $K_{max} = PE_{max} = K_o$

(32) **(B).** $T = 2\pi\sqrt{\frac{m}{k}}$. Also spring constant (k) $\propto \frac{1}{\text{Length}\ (\ell)}$,

When the spring is half in length, then k becomes twice $T' = 2\pi\sqrt{\frac{m}{2k}} \Rightarrow \frac{T'}{T} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

(33) **(C).** $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow n \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

$\Rightarrow \frac{n}{n_2} = \sqrt{\frac{4m}{m}} \Rightarrow n_2 = \frac{n}{2}$

(34) **(B).** With respect to the block the springs are connected in parallel combination.

$\therefore$ Combined stiffness $k = k_1 + k_2$ and $n = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

(35) **(D).** In simple harmonic motion, energy changes from kinetic to potential and potential to kinetic but the sum of two always remains constant.

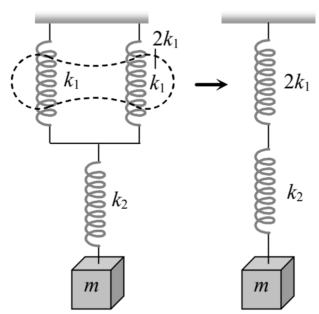
(36) **(B).** $\omega = \sqrt{\frac{k}{m}}$

(37) **(D).** $T \propto \frac{1}{\sqrt{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{k}{4k}} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$

(38) **(D).** The time period of oscillation of a spring does not depend on gravity.

(39) **(B).** In series combination

$\frac{1}{k_S} = \frac{1}{2k_1} + \frac{1}{k_2} \Rightarrow k_S = \left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$



(40) **(B).** $U = \frac{1}{2}Kx^2$ but $T = Kx$

So energy stored $= \frac{1}{2}\frac{(Kx)^2}{K} = \frac{1}{2}\frac{T^2}{K}$

(41) **(A).** Inside the mine g decreases

Hence from $T = 2\pi\sqrt{\frac{\ell}{g}}$; T increase

(42) **(D).** $T \propto \sqrt{\ell} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell} = \frac{0.02}{2} = 0.01$

$\Rightarrow \Delta T = 0.01T$

Loss of time per day $= 0.01 \times 24 \times 60 \times 60 = 864$ sec

(43) **(D).** $\frac{T'}{T} = \sqrt{\frac{g}{g' + a}} = \sqrt{\frac{g}{g + 5g}} = \sqrt{\frac{1}{6}} \Rightarrow T' = \frac{T}{\sqrt{6}}$

(44) **(B).** $T \propto \sqrt{\ell} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell} = \frac{1}{2} \times 1\% = 0.5\%$

(45) **(B).** As periodic time is independent of amplitude.

(46) Frequency $n \propto \frac{1}{\sqrt{\ell}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{\ell_2}{\ell_1}}$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$

(47) **(B).** $g = \frac{GM}{R^2} \Rightarrow \frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}} \Rightarrow T_p = 2\sqrt{2}$ sec

(48) **(B).** When a little mercury is drained off, the position of c.g. of ball falls (w.r.t. fixed and) so that effective length of pendulum increases hence T increase.

(49) **(D).** Suppose at t = 0, pendulums begins to swing simultaneously. Hence, they will again swing simultaneously if $n_1T_1 = n_2T_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$

(50) **(C).** $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{1}{T^2} = \frac{g}{4\pi^2} =$ constant

(51) **(B).** $T \propto \sqrt{\ell}$ Time period depends only on effective length. Density has no effect on time period.
If length made 4 times then time period becomes 2 times.

(52) **(C).** $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\left(\frac{g}{g/4}\right)}$

$\Rightarrow T_2 = 2T_1 = 2T$

(53) **(C).** The effective acceleration in a lift descending with acceleration $\frac{g}{3}$ is $g_{eff} = g - \frac{g}{3} = \frac{2g}{3}$

$\therefore T = 2\pi\sqrt{\left(\frac{L}{g_{eff}}\right)} = 2\pi\sqrt{\left(\frac{L}{2g/3}\right)} = 2\pi\sqrt{\left(\frac{3L}{2g}\right)}$

(54) **(B).** Ball execute S.H.M. inside the tunnel with time period

$T = 2\pi\sqrt{\frac{R}{g}} = 84.63$ min

Hence time to reach the ball from one end to the other end of the tunnel $t = \frac{84.63}{2} = 42.3$ min.

(55) **(C).** The stone execute S.H.M. about centre of earth with time period $T = 2\pi\sqrt{\frac{R}{g}}$; where R = Radius of earth.

(56) $T = 2\pi\sqrt{\frac{m}{APg}}$, Where m is mass of cylinder, A is cross-section area & ρ is density of water. To float, weight of cylinder is equal to buoyancy force. $mg = A\ell\rho g$
where ℓ is length of drowned cylinder in water

$\therefore T = 2\pi\sqrt{\frac{\ell}{g}}$

(57) **(A).** Resonance is an example of forced oscillation.

(58) **(C).** Energy of particle is maximum at resonant frequency i.e., $\omega_2 = \omega_o$. For amplitude resonance (amplitude maximum) frequency of driver force

$\omega = \sqrt{\omega_0^2 - b^2 2m^2} \Rightarrow \omega_1 \neq \omega_0$

(59) **(C).** In case of forced oscillations of a body driving force has to be periodic and continuous.

(60) **(B).** $A = \frac{c}{a+b-c}$; when b = 0, a = c amplitude

$A \to \infty$. This corresponds to resonance.

(61) **(C).** Total energy $= \frac{1}{2}kx^2$

(62) **(C).** Both the spring are in series

$\therefore K_{eq} = \frac{K(2K)}{K+2K} = \frac{2K}{3}$

Time period, $T = 2p\sqrt{\frac{m}{K_{eq}}}$ where $m = \frac{m_1m_2}{m_1+m_2}$

Here $m = \frac{m}{2}$; $T = 2p\sqrt{\frac{m}{2}.\frac{3}{2K}} = 2p\sqrt{\frac{3m}{4K}}$

(63) **(C).** $E_1 = \frac{1}{2}kx^2, E_2 = \frac{1}{2}ky^2,$

$E = \frac{1}{2}k(x+y)^2 = E_1 + E_2 + 2\sqrt{E_1E_2} = 2 + 8 + 2\sqrt{16} = 18J$

(64) **(D).** $\frac{1}{2}KA^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

and $\frac{1}{2}Kx^2 + \frac{1}{2}m(2V)^2 + \frac{1}{2}KA'^2$

$\therefore A' = \sqrt{4A^2 - 3x^2}$

(65) **(A).** According to question

$\because K = \frac{F}{x} = \frac{Mg}{x} = \frac{4 \times 9.8}{0.16} = 245$ N/m

$T = 2\pi\sqrt{\frac{m}{K}} = 2 \times 3.14\sqrt{\frac{0.5}{245}} = 0.283$ sec.

(66) **(C).** Let $x = a\cos(\omega t)$

$\Rightarrow y = a\cos\left(2\omega t + \frac{\pi}{2}\right) \Rightarrow y = -a\sin(2\omega t)$

$\Rightarrow y = -2a\sin(\omega t)\cos(\omega t)$

$\Rightarrow y = -2a\sqrt{1-\frac{x^2}{a^2}}\,\frac{x}{a} \Rightarrow a^2y^2 = 4x^2(a^2-x^2)$

(67) **(C).** $T = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.098}{9.8}} = \frac{2\pi}{10}$

(68) **(C).** Since $x = 0.08\sin\frac{\pi}{2}t$

Thus acceleration of object at t = 1 sec.

$\frac{d^2x}{dt^2} = (0.08)\left(\frac{\pi^2}{4}\right)\sin\left(\frac{\pi}{2}\right) = 0.197\ m/s^2$

(69) **(A).** $\frac{\Delta T}{T}\times 100\% = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100\% = \frac{1}{2}\times 6 = 3\%$

(70) **(A).** $T = 2\pi\sqrt{\frac{h}{g}}$ or $T = 2\times 3.14\times\sqrt{\frac{0.3}{9.8}} = 1.1$ sec

(71) **(A).** $T = 2\pi\sqrt{\frac{\ell}{g}}$ or $T = 2\times 3.14\times\sqrt{\frac{5\times 0.02}{10}} = 0.628$ sec.

(72) **(B).** $\because V_m = \omega a = \frac{2\pi}{T}a;\ T = \frac{2\pi a}{V_m} = \frac{2\times 3.14\times 4}{16} = 1.57\ s$

(73) **(D).** P.E. is maximum at extreme position and minimum at mean position.
Time to go from extreme position to mean position is, $t = T/4$, where T is time period of SHM.

$5s = \frac{T}{4}$ Þ $T = 20s$

(74) **(C).** $T = 2\pi\sqrt{\frac{\ell}{g}}$; $\frac{T}{2} = 2\pi\sqrt{\frac{\ell}{g+a}}$ $\therefore$ $2 = \sqrt{\frac{g+a}{g}}$

or $g + a = 4g$ or $a = 3g$

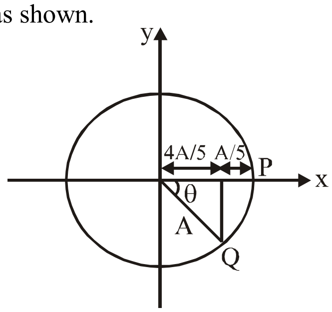
EXERCISE-2

(1) **(B).** Particle is starting from rest, i.e. from one of its extreme position. As particle moves a distance A/5, we can represent it on a circle as shown.

$\cos\theta = \frac{4A/5}{A} = \frac{4}{5}$

$\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$\omega t = \cos^{-1}\left(\frac{4}{5}\right)$



$t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$

Method : As starts from rest i.e. from extreme position

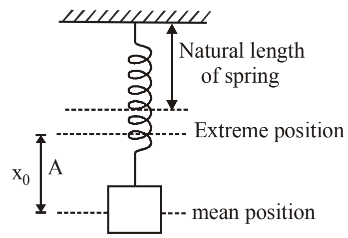
$x = A\sin(\omega t + \phi)$

At $t = 0; x = A \Rightarrow \phi = \pi/2$

$\therefore A - \frac{A}{5} = A\cos\omega t$

$\frac{4}{5} = \cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$; $t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$

(2) **(A).** The spring is never compressed. Hence spring shall exert least force on the block when the block is at top-most position.



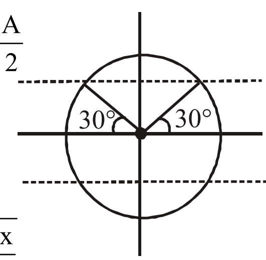
$F_{max} = kx_0 - kA = mg - m\omega^2 A = mg - 4\frac{\pi^2}{T^2}mA$

(3) **(A).** $\frac{1}{2}m\omega^2(A^2-x^2) = \frac{75}{100}\times\frac{1}{2}m\omega^2A^2$

$\Rightarrow A^2 - x^2 = \frac{3}{4}A^2 \Rightarrow x = \frac{A}{2}$

$360° \to 2$ sec.

$30° \to \frac{2}{360}\times 30 = \frac{1}{6}$ sec.

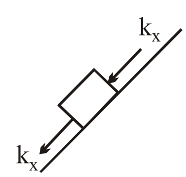


(4) **(A).** Time period $(T) = 2\pi\sqrt{\frac{x}{a}}$

If we displace the block by a distance 'x' upward then the restoring force will be

$F_{restoring} = 2kx \Rightarrow a = \frac{2kx}{M}$

$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{xM}{2kx}} = 2\pi\sqrt{\frac{M}{2k}}$



Alternate : $F_{restoring} = 2kx$

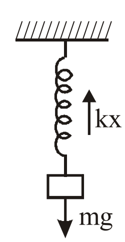
$\Rightarrow T = 2\pi\sqrt{\frac{M}{\text{coeff. of } x}} = 2\pi\sqrt{\frac{M}{2k}}$

(5) **(A).** It will perform SHM

with amplitude $A = \frac{mg}{K}$

Now $a = \omega^2 x$

$\Rightarrow a_{max} = \omega^2 A = \frac{K}{m}\times\frac{mg}{K} = g$



(6) **(C).** Let the spring is further extended by y when the cylinder is given small downward push. Then the restoring forces on the spring are,

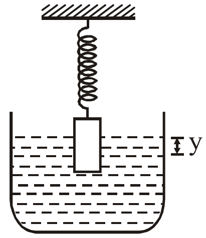
(i) Ky due to elastic properties of spring

(ii) upthrust = y Adg = weight of liquid displaced

$\therefore$ Total restoring force

$= (K + Adg)\,y$

$= M \times a = -(K + Adg)\,y$



Comparing with $a = -\omega^2 y$ we get

$$\omega^2 = \left(\frac{K + Adg}{M}\right) \text{ or } \omega = \sqrt{\frac{K + Adg}{M}}$$

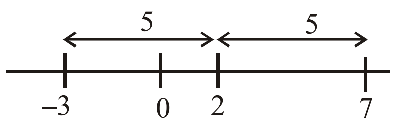
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K + Adg}{M}}$$

(7) **(C).** $U = 2 - 20x + 5x^2$; $F = -\frac{dU}{dx} = 20 - 10x$

At equilibrium position, $F = 0$

$20 - 10x = 0 \Rightarrow x = 2$

Since particle is released at $x = -3$, therefore amplitude of particle is 5.



It will oscillated about $x = 2$ with an amplitude of 5.

$\therefore$ maximum value of x will be 7.

(8) **(A).** $\frac{2c}{mR^2} = \frac{k}{m}$; $R = \sqrt{\frac{2c}{k}}$

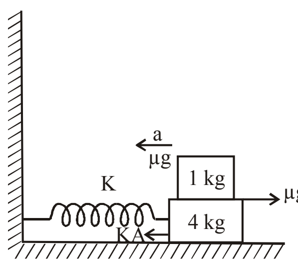
(9) **(A).** For 1 kg : $\mu g = a$

For 4 kg : $kA - \mu g = 4a$

$\Rightarrow \quad kA - \mu g = 4\mu g$

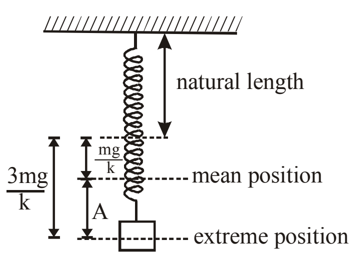
$(\because a = \mu g)$

$$\Rightarrow \quad A = \frac{5\mu g}{k}$$



An extreme position

(10) **(B).**



Just after cutting the string extension in spring

$$= \frac{3mg}{k}.$$

The extension in the spring when block is in mean position $= mg/k$

$$\therefore \quad \text{Amplitude of oscillation} \quad A = \frac{3mg}{k} - \frac{mg}{k} = \frac{2mg}{k}$$

(11) **(B).** As $a = -\frac{GM}{R^3}x = -\omega^2 x$

Velocity at centre before collision

$$(V) = \omega A = \sqrt{\frac{GM}{R^3}} \times R = \sqrt{\frac{GM}{R}}$$

Velocity after collision (V')

$$= e\sqrt{\frac{GM}{R}} = \omega A' \Rightarrow A' = \frac{V'}{\omega} = eR$$

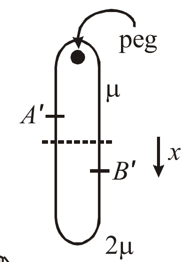
(12) **(C).** If the string is displaced slightly downward by x, we can write, the net (restoring)force

$= (\mu x - 2\mu x)\,2g = -2\mu x g$

$\therefore \quad (5\mu\ell)\cdot\ddot{x} = -2\mu x g$

or, $\quad (5\mu\ell)\cdot\ddot{x} = -2\mu x g$

$$\therefore\ \omega = \sqrt{\frac{2g}{5\ell}} \text{ or } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{5\ell}{g}}$$



(13) **(B).** For M to be stationary

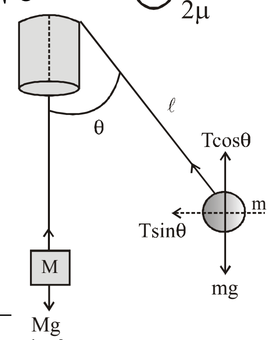
$T = Mg \quad$ ..... (1)

Also for mass m,

$T\cos\theta = mg \quad$ ..... (2)

$$T\sin\theta = \frac{mv^2}{\ell\sin\theta} \quad \text{...... (3)}$$

Dividing (3) by (2)



$$\tan\theta = \frac{v^2}{g\ell\sin\theta} \Rightarrow v = \sqrt{\frac{g\ell}{\cos\theta}}.\sin\theta$$

$$\text{Time period} = \frac{2\pi R}{v} = \frac{2\pi\ell\sin\theta}{\sqrt{\frac{g\ell}{\cos\theta}}.\sin\theta}$$

From (1) and (2) $\cos\theta = \frac{m}{M}$; then time period $= 2\pi\sqrt{\frac{\ell m}{gM}}$

(14) **(C).** $\text{K.E.} = \frac{1}{2}m\omega^2 a^2\cos^2\omega t$; $\text{P.E.} = \frac{1}{2}m\omega^2 a^2\sin^2\omega t$

$$\text{K.E.} - \text{P.E.} = \frac{1}{2}m\omega^2 a^2[\cos^2\omega t - \sin^2\omega t]$$

$$= \frac{1}{2}m\omega^2 a^2.\cos 2\omega t$$

$\therefore \quad$ Angular frequency $= 2\omega$

$$\text{and time period } = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{\pi \times T}{2\pi} = 2s$$

(15) **(B).** As initially string is vertical, angular amplitude will be θ and $g_{eff} = g\cos\theta$

$$\therefore\ v_{max} = \sqrt{2g_{eff}\ell(1 - \cos\theta)} = \sqrt{2g\ell\cos\theta(1 - \cos\theta)}$$

(16) **(D).** Frequency of oscillation is independent of initial velocity but $f = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$; $\mu = \frac{2m \times m}{2m + m} = \frac{2}{3}m$

(17) **(D).** When the spring undergoes displacement in the downward direction it completes one half oscillation while it completes another half oscillation in the upward direction. The total time period is:

$$T = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$$

(18) **(B).** $T = 2\pi\sqrt{\frac{I}{C}}$; $T = 2\pi\sqrt{\frac{2MR^2}{5C}}$

or $T = 2 \times 3.14\sqrt{\frac{2 \times 1 \times (0.15)^2}{5 \times 6 \times 10^{-3}}} = 7.7$ sec

(19) **(B).** Springs on the left of the block are in series, hence their equivalent spring constant is $K_1 = \frac{(2K)(2K)}{2K + 2K} = K$

Springs on the right of the block are in parallel, hence their equivalent spring constant is $K_2 = K + 2K = 3K$

Now again both K_1 and K_2 are in parallel

$\therefore K_{eq} = K_1 + K_2 = K + 3K = 4K$

Hence, frequency is $f = \frac{1}{2p}\sqrt{\frac{K_{eq}}{M}} = \frac{1}{2p}\sqrt{\frac{4K}{M}}$

(20) **(A).** For simple pendulum : $T = 2p\sqrt{\frac{\ell}{g}}$

As g will decrease on moon, time period will increase

For spring mass system : $T = 2p\sqrt{\frac{m}{K}}$

It will not change and remains the same

(21) **(A).** Let the equation of motion be

$x = a\,(1 - \cos\omega t)$ [a = amplitude]

$\ell_1 = a\,(1 - \cos\omega)$

$\ell_1 + \ell_2 = a\,(1 - \cos 2\omega) = 2a\sin^2\omega$

$$\left(1 - \frac{\ell_1}{a}\right)^2 + \left(\frac{\ell_1 + \ell_2}{2a}\right) = 1 \;;\; a = \frac{2\ell_1^2}{3\ell_1 - \ell_2}$$

(22) **(B).** After the system is released, m_2 moves down

The extension in the spring becomes

$\frac{m_2 g}{k}$ $(m_2 g = kx_0)$, which is the new equilibrium position of the system.

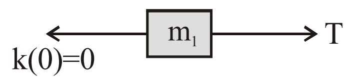
For small 'x', restoring force on the system is $F = kx$

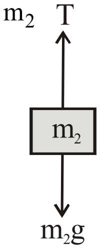
Þ $a = \frac{kx}{m_1 + m_2}$ (For $(m_1 + m_2 +$ spring) system)

Þ $T = 2p\sqrt{\frac{x}{a}} = 2p\sqrt{\frac{x(m_1 + m_2)}{kx}} = 2p\sqrt{\frac{m_1 + m_2}{k}}$

Þ Angular frequency $= w = \frac{2p}{T} = \sqrt{\frac{k}{m_1 + m_2}}$

F.B.D. of m_1 and m_2 just after the system is released :





From above : $T = m_2 g$

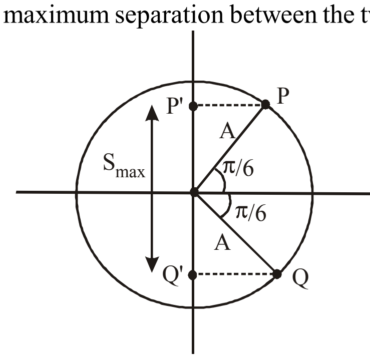
Hence (C) is incorrect.

After $x = \frac{m_2 g}{k}$, m_1 moves towards right till the total kinetic energy acquired does not converted to potential energy. Hence (D) is also incorrect. Hence (B) is the answer

(23) **(D).** Figure shows the mapping of the two SHMs with circular motions having phase difference

$$\phi = \omega t = \frac{2\pi}{6} \times 1 = \frac{\pi}{3}\text{rad}$$

The maximum separation between the two particle is



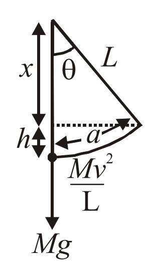
$$S_{max} = 2A\sin\frac{\pi}{6} \text{ or } S_{max} = 2 \times 5 \times \frac{1}{2} = 5\text{cm.}$$

(24) **(B).** Maximum tension in the string is at lowest position.

Therefore, $T = Mg + \frac{Mv^2}{L}$

To find the velocity v at the lowest point of the path, we apply law of conservation of energy i.e.

$\frac{1}{2}Mv^2 = Mgh = MgL(1 - \cos\theta)$



$[\because \quad h = L - x, h = L - L\cos\theta]$

or $v^2 = 2gL(1 - \cos\theta)$

or $v = \sqrt{2gL(1 - \cos\theta)}$

$\therefore \quad T = Mg + 2Mg(1 - \cos\theta)$

$$T = Mg\left[1 + 2 \times 2\sin^2\left(\frac{\theta}{2}\right)\right] \text{ or } T = Mg\left[1 + 4\left(\frac{\theta}{2}\right)^2\right]$$

$[\because \quad \sin(\theta/2) = \theta/2$ for small amplitudes]

$T = Mg[1 + \theta^2]$

From figure $\theta = \frac{a}{L}$ $\therefore$ $T = Mg\left[1+\left(\frac{a}{L}\right)^2\right]$

(25) **(C).** On displacing the body from equilibrium position, one spring gets extended and another one gets contracted. Hence forces due to two springs act in the same direction.

$\therefore$ $F = F_1 + F_2$

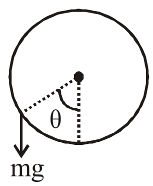
or $K = K_1 + K_2 = 6 + 4 = 10$ N/m

$$n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\times 3.14}\sqrt{\frac{10}{0.1}} = \frac{5}{\pi} \text{ vib/sec.}$$

EXERCISE-3

(1) **1.** $\frac{1}{K_{eq}} = \frac{1}{4K} + \frac{1}{64K}$; $K_{eq} = \frac{64}{17}K$; $T = 2\pi\sqrt{\frac{17m}{64K}} = 1$ sec.

(2) **5.** $\tau = mgR\sin\theta = -I\frac{d^2\theta}{dt^2}$



$\frac{d^2\theta}{dt^2} = -\frac{mgR\theta}{MR^2}$; $T = 2\pi\sqrt{\frac{MR}{mg}}$

Solving for m we get m = 5gm.

(3) **3.** The maximum velocity of the insect is $A\sqrt{\frac{k}{M}}$

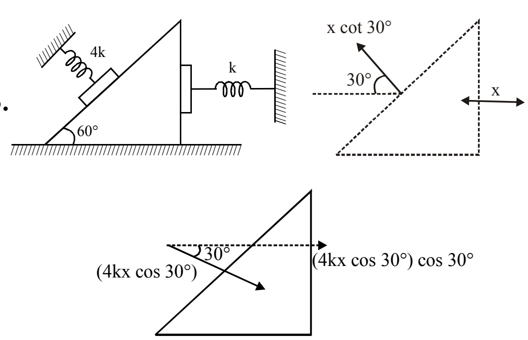
Its component perpendicular to the mirror is

$A\sqrt{\frac{k}{M}}\sin 60°$. Thus maximum relative speed =

$\sqrt{3}A\sqrt{\frac{k}{M}}$

(4) **5.** $T = 2\pi\sqrt{\frac{25}{10}} = \frac{10\pi}{\pi} = 10$ sec. ; T forward $= \frac{T}{2} = 5$ sec.

(5) **3.**



$F = 4kx\cos^2 30°$

$T = \frac{2\pi}{\sqrt{3k}}\sqrt{M}$ $\Rightarrow t_1 = \frac{T_1}{2} = \frac{\pi}{\sqrt{3k}}\sqrt{M}$

$\Rightarrow t_2 = \frac{T_2}{2} = \pi\sqrt{\frac{M}{k}}$; Time period $= t_1 + t_2$

$$= \left[\frac{\pi}{\sqrt{3k}}\sqrt{M} + \pi\sqrt{\frac{M}{k}}\right] = \pi\sqrt{\frac{M}{k}}\left[1+\frac{1}{\sqrt{3}}\right]$$

(6) **2.** In CM frame both the masses execute SHM with $\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{2k}{m}}$

Initially particles are at extreme distance

$= L_0 + (L - L_0)\cos\sqrt{\frac{2k}{m}}\,t$

EXERCISE-4

(1) **(B).** $T = 2\pi\sqrt{\frac{m}{k}}$

If spring is cut in n parts then spring constant of each spring = nk.

and equivalent spring constant due to parallel combination $= n(nk) = n^2k$.

So time period $= 2\pi\sqrt{\frac{m}{n^2k}} = \frac{T}{n}$

(2) **(B).** As the child stands up, distance of c.m. from point of suspension decreases, so.

Time period will decreased.

(3) **(C).** Kinetic energy is maximum, potential energy is minimum.

(4) **(B).** $T = 2\pi\sqrt{\frac{M}{K}}$. Again $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{K}}$

So, $\frac{5}{3}\left(2\pi\sqrt{\frac{M}{K}}\right) = 2\pi\sqrt{\frac{M+m}{K}}$

$\frac{25}{9}M = M + m$; $25M = 9M + 9m$; $\frac{m}{M} = \frac{16}{9}$

(5) **(C).** $T = 2\pi\sqrt{\frac{\ell}{g}}$ Finally $T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$; $T = 1.1\,T$

% increment $= \frac{T' - T}{T} \times 100\% = 10\%$

(6) **(B).** $v_{max} = a\omega$; $a_1\omega_1 = a_2\omega_2$; $\frac{a_1}{a_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{k_1}}$

(7) **(B).** $x = 4\cos\pi t + 4\sin\pi t$

Amplitude $a = \sqrt{a_1^2 + a_2^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

(8) **(D).** Kinetic energy $= \frac{1}{2}m\omega^2(a^2 - x^2)$

(9) In air $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$. In water $t = 2\pi\sqrt{\frac{\ell}{g\left(1-\frac{\rho_w}{\rho}\right)}}$

$\rho_{(w)} = 1000\ kg/m^3$, $\rho = (4/3) \times 1000\ kg/m^3$

So $t = 2\pi\sqrt{\frac{\ell}{g\left[1-\frac{3}{4}\right]}} = 2t_0$

(10) **(B).** $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$; $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

In series combination $T = 2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$

$T = \sqrt{t_1^2 + t_2^2}$

(11) **(C).** Independent of x

(12) **(B).** Time displacement $\propto \frac{1}{m(\omega_0^2 - \omega^2)}$

(13) **(A).** $\omega_1 = \omega_2$

(14) **(B).** $x = \sin^2 \omega t$; $x = \frac{1-\cos 2\omega t}{2}$

Frequency $f = \frac{2\omega}{2\pi} = \frac{\omega}{\pi}$; Time period $= \frac{\pi}{\omega}$

But acceleration is not directly proportional to displacement.

(15) **(B).** $\frac{d^2x}{dt^2} + \alpha x = 0$; Acceleration $= -\alpha x$. So, $\omega = \sqrt{\alpha}$

(16) **(A).** C.M. comes down and then reaches to its initial value.

(17) **(C).** Maximum velocity $v = a\omega$; $T = \frac{2\pi a}{v}$

(18) **(C).** K.E. = 0.75 (T.E.)

$\frac{1}{2}m\omega^2a^2\cos^2\omega t = \frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)$; $\cos\omega t = \frac{\sqrt{3}}{2}$

$\frac{2\pi}{T}t = \frac{\pi}{6}$; $t = \frac{T}{12} = \frac{2}{12} = \frac{1}{6}s$

(19) **(D).** At the highest position particle will contact the surface

$mg = m\omega^2 a$; $a = \frac{g}{\omega^2}$

(20) **(A).** $x = 2 \times 10^{-2} \cos \pi t$

Max. speed will occur when particle reaches to mean position. Time taken $= \frac{T}{4} = 0.5s$

(21) **(D).** $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$ .... (1)

$a = A\cos(\omega t + \delta)$ .... (2)

From equation (1), $v = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$

$a = -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right) = x_0\omega^2\cos\left(\omega t - \frac{\pi}{4} + \pi\right)$

So $a = x_0\omega^2\cos\left(\omega t + \frac{3\pi}{4}\right)$; $\delta = \frac{3\pi}{4}$ and $A = x_0\omega^2$

(22) **(C).** The force on the block of mass m is $\frac{mF}{(M+m)}$

(23) **(D).** $f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$; $f' = \frac{1}{2\pi}\sqrt{\frac{4k_1+4k_2}{m}} = 2f$

(24) **(A).** Average K.E. $k_{avg} = \frac{k_{max}+k_{min}}{2}$

$= \frac{\frac{1}{2}m\omega^2a^2 + 0}{2} = \frac{1}{4}m(2\pi\nu)^2a^2 = \pi^2 m\nu^2 a^2$

(25) **(A).** $x = A\sin(\omega t + \phi)$; $v = A\omega\cos(\omega t + \phi)$

$a = -A\omega^2\sin(\omega t + \phi)$; $aT = -A\omega^2 T\sin(\omega t + \phi)$

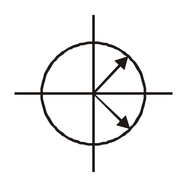
$= -A\omega^2\frac{2\pi}{\omega}\sin(\omega t + \phi) = -2\pi A\omega\sin(\omega t + \phi)$

$\frac{aT}{x} = -2\pi\omega = \text{constant}$

(26) **(B).** $x_1 = A\sin(\omega t + \phi_1)$; $x_2 = A\sin(\omega t + \phi_2)$

$x_1 - x_2 = A\left[2\sin\left(\omega t + \frac{\phi_1+\phi_2}{2}\right)\sin\left(\frac{\phi_1-\phi_2}{2}\right)\right]$

$A = 2A\sin\left(\frac{\phi_1-\phi_2}{2}\right)$



$\frac{\phi_1-\phi_2}{2} = \frac{\pi}{6}$; $\phi_1 = \frac{\pi}{3}$

(27) **(D).** C.O.L.M., $MV_{max} = (m+M)V_{new}$, $V_{max} = A_1\omega_1$

$V_{new} = \frac{MV_{max}}{(m+M)}$; $V_{new} = A_2\omega_2$

$\frac{MA_1}{(m+M)}\sqrt{\frac{K}{M}} = A_2\sqrt{\frac{K}{(m+M)}}$

$A_2 = A_1\sqrt{\frac{M}{(m+M)}}$; $\frac{A_1}{A_2} = \left(\frac{m+M}{M}\right)^{1/2}$

(28) **(D).** $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$; $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

here b is demping coefficient. This has solution of type

$^{\lambda t}$ substituting this $m\lambda^2 + b\lambda + k = 0$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

On solving for x, we get $x = e^{-\frac{b}{2m}t} a\cos(\omega_1 t - \alpha)$

$\omega_1 = \sqrt{\omega_0^2 - \lambda^2}$, where $\omega_0 = \sqrt{\frac{k}{m}}$; $\lambda = +\frac{b}{2}$

So, average life = 2/b

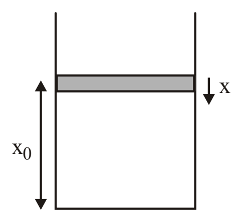
(29) **(C).** $A = A_0 e^{-\frac{bt}{2m}}$

After 5 second, $0.9A_0 = A_0 e^{-\frac{b(5)}{2m}}$ .... (1)

After 10 more second $A_0 = A_0 e^{-\frac{b(15)}{2m}}$ .... (2)

From (i) & (ii), $A = 0.729\, A_0$

(30) **(C).** $\frac{Mg}{A} = P_0$



$Mg = P_0A$ .... (1)

$P_0V_0^{\gamma} = PV^{\gamma}$;

$P_0Ax_0^{\gamma} = PA\,(x_0 - x)^{\gamma}$; $P = \frac{P_0x_0^{\gamma}}{(x_0 - x)^{\gamma}}$

Let piston is displaced by x

$$Mg - \left(\frac{P_0x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right)A = F_{restoring}$$

$$P_0A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right)A = F_{restoring} \quad [x_0 - x \approx x_0]$$

$$F = -\frac{\gamma P_0Ax}{x_0} \quad \therefore\ f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A}{x_0M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

(31) **(B).** $A(1 - \cos\omega\tau) = a$

$A(1 - \cos 2\omega\tau) = 3a$

$\cos\omega\tau = \left(1 - \frac{a}{A}\right)$; $\cos 2\omega\tau = \left(1 - \frac{3a}{A}\right)$

$$2\left(1 - \frac{a}{A}\right)^2 - 1 = 1 - \frac{3a}{A}$$

Solving the equation, $\frac{a}{A} = \frac{1}{2} \Rightarrow A = 2a$

$\cos\omega\tau = 1/2$; $T = 6\tau$

(32) **(A).** $KE = \frac{1}{2}m\omega^2(A^2 - d^2)$; $PE = \frac{1}{2}m\omega^2d^2$

At $d = \pm A$, ; PE = maximum while KE = 0

(33) **(C).** $V = \omega\,(A^2 - x^2)^{1/2}$

At $x = \frac{2A}{3}$, $V_1 = \omega\sqrt{A^2 - \frac{4}{9}A^2} = \omega\left[\frac{5A^2}{9}\right]^{1/2}$

$V_1 = \frac{\omega A}{3}\sqrt{5}$; $V_{new} = 3V_1 = \omega A\sqrt{5}$

Now, $V_{new} = \omega\,[A_{new}^2 - x^2]^{1/2} = \omega A\sqrt{5}$

$A_n{}^2 - x^2 = 5A^2$

$A_n^2 = 5A^2 + \frac{4A^2}{9}$; $A_n = \frac{7A}{3}$

(34) **(C).** K is maximum at mean position and minimum at extreme position and extreme position is reached at T/4.

(35) **(D).** $10^{12} = f = \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi \times 10^{12}$

$\omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2$

$$= \left(\frac{108}{6.023 \times 10^{23}}\right) \times 10^{-3} \times (2\pi \times 10^{12})^2 = 7.1$$

(36) **(B).** Frequency of torsonal oscillations is given by

$$f = \frac{k}{\sqrt{I}}\ ;\ f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}\ ;\ f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$f_2 = 0.8\, f_1$; $m/M = 0.375$

(37) **(B).** $A = A_0e^{-\gamma t}$

$A = A_0/2$ after 10 oscillations

After 2 seconds, $\frac{A_0}{2} = A_0e^{-\gamma(2)}$; $2 = e^{2\gamma}$

$\ln 2 = 2\gamma$; $\gamma = \frac{\ln 2}{2}$ $\because$ $A = A_0e^{-\gamma t}$; $\ln\frac{A_0}{A} = \gamma\, t$

$\ln 1000 = \frac{\ln 2}{2} t \; ; \; 2\left(\frac{3\ln 10}{\ln 2}\right) = t \; ; \; \frac{6\ln 10}{\ln 2} = t$

$t = 19.931$ sec ; $t \approx 20$ sec

EXERCISE-5

(1) **(A).** $x = a\sin^2\omega t = a\left(\frac{1-\cos 2\omega t}{2}\right)$ $(\cos 2\theta = 1 - 2\sin^2\theta)$

Velocity, $u = \frac{dx}{dt} = \frac{2\omega a \sin 2\omega t}{2} = \omega a \sin 2\omega t$

Acceleration, $a = \frac{du}{dt} = 2\omega^2 a \cos 2\omega t$

$a \propto -x$ is satisfied. Hence, the motion of the particle is SHM.

(2) **(D).** Time period of oscillation is $T = 2\pi\sqrt{\frac{M}{k}}$

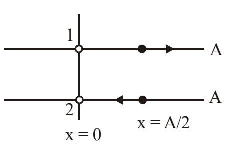
When a another mass M is also suspended with it.

$T' = 2\pi\sqrt{\frac{M+M}{k}} = 2\pi\sqrt{\frac{2M}{k}} = \sqrt{2}T$

(3) **(B).** Motion given here is SHM starting from rest.

(4) **(D).** For SHM, $\frac{d^2y}{dt^2} \propto -y$

(5) **(B).**



$\phi_1 = \frac{\pi}{6} \; ; \; \phi_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Phase difference $= \frac{4\pi}{6} = \frac{2\pi}{3}$

(6) **(C).** $F \propto v \Rightarrow F = kV$

$k = \frac{F}{v} \Rightarrow [k] = \frac{[\text{kgms}^{-2}]}{[\text{ms}^{-1}]} = \text{kg s}^{-1}$

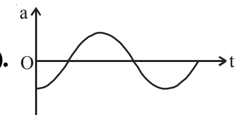
(7) **(B).** $y = 3\sin\frac{\pi}{2}(50t - x) \; ; \; y = 3\sin\left(25\pi t - \frac{\pi}{2}x\right)$

Wave velocity, $v = \frac{\omega}{k} = \frac{25\pi}{\pi/2} = 50$ m/sec.

$v_p = \frac{\partial y}{\partial t} = 75\pi\cos\left(25\pi t - \frac{\pi}{2}x\right)$

$v_{p\,max} = 75\pi$ then $\frac{v_{pmax}}{v} = \frac{75\pi}{50} = \frac{3\pi}{2}$

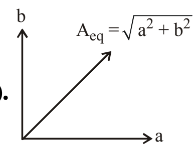
(8) **(C).**



$x = A\cos\omega t \; ; \; v = \frac{dx}{dt} = -A\omega\sin\omega t$

$a = \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t$

(9) **(B).**



$y_{eq} = y_1 + y_2 = a\sin\omega t + b\cos\omega t$

$= a\sin\omega t + b\sin(\omega t + \pi/2)$

(10) **(A).** $V_1^2 = \omega^2(A^2 - x_1^2)$

$V_2^2 = \omega^2(A^2 - x_2^2)$

$\frac{V_1^2}{\omega^2} + x_1^2 = \frac{V_2^2}{\omega^2} + x_2^2 \Rightarrow \frac{V_1^2 - V_2^2}{\omega^2} = x_2^2 - x_1^2$

$\Rightarrow \omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$

(11) **(A).** For S.H.M.

Maximum acceleration $= \omega^2 A = \alpha$

Maximum velocity $= \omega A = \beta$

$\Rightarrow \omega = \frac{\alpha}{\beta} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\beta}{\alpha}$

(12) **(D).** $T_1 = 3 = 2\pi\sqrt{\frac{m}{k}} \; ; \; T_2 = 5 = 2\pi\sqrt{\frac{m+1}{k}}$

Dividing, $\frac{3}{5} = \sqrt{\frac{m}{m+1}} \; ; \; \frac{9}{25} = \frac{m}{m+1}$

$9m + 9 = 25m$; $16m = 9$; $m = 9/16$

(13) **(B).** Length of the spring segments $= \frac{\ell}{6}, \frac{\ell}{3}, \frac{\ell}{2}$

As we know $K \propto \frac{1}{\ell}$

Spring constants for spring segments will be

$K_1 = 6K, K_2 = 3K, K_3 = 2K$

So in parallel combination

$$K'' = K_1 + K_2 + K_3 = 11\,K$$

In series combination $K' = K$ (As it will become original spring).

So $K' : K'' = 1 : 11$

(14) **(B).** Amplitude $A = 3$ cm

When particle is at $x = 2$ cm,

its | velocity | = | acceleration |

i.e., $\omega\sqrt{A^2 - x^2} = \omega^2 x \Rightarrow \omega = \dfrac{\sqrt{A^2 - x^2}}{x}$

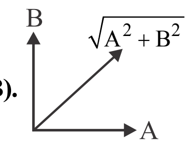
$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{2}{\sqrt{5}}\right) = \frac{4\pi}{\sqrt{5}}$$

(15) **(B).** $|a| = \omega^2 y$; $20 = \omega^2 (5)$

$\omega = 2$ rad / s

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ s}$$

(16) **(B).**



$y = A_0 + A \sin \omega t + B \sin \omega t$

Equate SHM $y' = y - A_0 = A \sin \omega t + B \cos \omega t$

Resultant amplitude

$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ} = \sqrt{A^2 + B^2}$$

(17) **(D).** In one complete vibration, displacement is zero.

So, average velocity in one complete vibration

$$= \frac{\text{Displacement}}{\text{Time interval}} = \frac{y_f - y_i}{T} = 0$$

(18) **(D).** At $t = 0$, y displacement is maximum, so equation will be cosine function. $T = 4$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad / s}$$

$y = a \cos \omega t$; $y = 3 \cos (\pi/2)\, t$