10

VECTOR & 3-DIMENSIONAL GEOMETRY

VECTOR

SCALAR QUANTITY & VECTOR QUANTITY

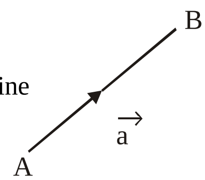
Scalar Quantity : A quantity which has only magnitude and not related to any direction is called a scalar quantity. For example Mass,Lenght ,Time ,Temperature,Area Volume, Speed ,Density Work etc.

Vector Quantity : A quantity which has magnitude and also a direction in space is called a vector quantity .

For example , Displacement ,Velocity, Acceleration, Force, Torque etc.

REPRESENTATION OF VECTORS :

Vectors are represented by directed line segments. A vector $\vec{a}$ is represented by the directed line segment $\overrightarrow{AB}$.



The magnitude of vector $\vec{a}$ is equal to AB and the direction of vector $\vec{a}$ is along the line from A to B.

TYPE OF VECTORS

(1) Null vector or zero vector : If the initial and terminal points of a vector coincide then it is called a zero vector .It is denoted by $\vec{0}$ or **0**. Its magnitude is zero and direction indeterminate.

(2) Unit vector : A vector whose magnitude is of unit length along any vector $\vec{a}$ is called a unit vector in the direction of $\vec{a}$ and is denoted by $\hat{a}$.

Note : (a) $|\hat{a}| = 1$

(b) Two unit vectors may not be equal unless they have the same direction.

(c) Unit vectors parallel to x- axis ,y-axis and z-axis are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

(3) Reciprocal vector : A vector whose direction is same as that of a given vector $\vec{a}$ but its magnitude is the reciprocal of the magnitude of the given vector $\vec{a}$ is called the reciprocal of $\vec{a}$ and is denoted by a^{-1}.

Thus if $\vec{a} = a.\hat{a}$ then $a^{-1} = \frac{1}{a}\hat{a}$

(4) Equal vector : Two non zero vectors are said to be equal vectors if their magnitude are equal and directions are same i.e. they act parallel to each other in the same direction.

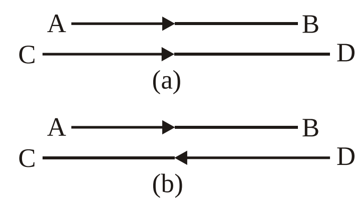
(5) Negative vector : The negative of a vector is defined as the vector having the same magnitude but opposite direction.

For example if $\vec{a} = \overrightarrow{PQ}$, then the negative of $\vec{a}$ is the vector $\overrightarrow{QP}$ and is denoted as $-\vec{a}$

(6) Collinear vector : Two or more non zero vectors are said to be collinear vectors if these are parallel to the same line.

(7) Like and unlike vector : Collinear vectors having the same direction are known as like vectors while those having opposite direction are known as unlike vectors.

For example, the vectors given by figure (a) are like and given by figure (b) are unlike vectors



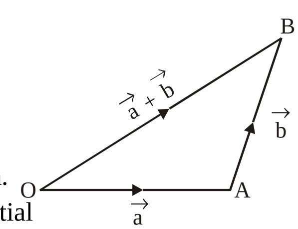
(8) Coplanar vector : Two or more non-zero vectors are said to be coplanar vectors if these are parallel to the same plane.

(9) Localised vector and free vector : A vector drawn parallel to a given vector through a specified point as the initial point, is known as a localised vector. If the initial point of a vector is not specified it is said to be a free vector.

(10) Position vector : Let O be the origin and let A be a point such that $\overrightarrow{OA} = \vec{a}$ then, we say that the position vector of A is $\vec{a}$.

ADDITION OF VECTORS :

(A) Let $\vec{a}$ and $\vec{b}$ be any two vectors. From the terminal point of $\vec{a}$, vector $\vec{b}$ is drawn. Then, the vector from the initial point O of $\vec{a}$ to the terminal point B of $\vec{b}$ is called the sum of vectors $\vec{a}$ and $\vec{b}$ and is denoted by $\vec{a}+\vec{b}$. This is called the triangle law of addition of vectors.



(B) The vectors are also added by using the following method. Let $\vec{a}$ and $\vec{b}$ be any two vectors. From the initial point of $\vec{a}$, vector $\vec{b}$ is drawn. Let O be their common initial point. If A and B be respectively the terminal points of $\vec{a}$ and $\vec{b}$, then parallelogram OACB is completed with OA and OB as adjacent sides. The vector $\overrightarrow{OC}$ is defined as the sum of $\vec{a}$ and $\vec{b}$. This is called the parallelogram law of addition of vectors.

(a) Properties of Vector Addition :

(i) Vector addition is commutative, i.e. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$.

(ii) Vector addition is associative,

i.e. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$

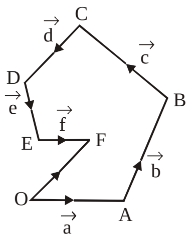
(iii) $\vec{O}+\vec{a}=\vec{a}+\vec{O}=\vec{a}$. So, the zero vector is additive identity.

(iv) $\vec{a}+(-\vec{a})=\vec{O}=(-\vec{a})+\vec{a}$. So, the additive inverse of $\vec{a}$ is $-\vec{a}$.

(b) Addition of any Number of Vectors :

To find the sum of any number of vectors we represent the vectors by directed line segment with the terminal point of the previous vector as the initial point of the next vector. Then the line segment joining the initial point of the first vector to the terminal point of the last vector will represent the sum of the vectors :

Thus if, $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{AB}=\vec{b}$, $\overrightarrow{BC}=\vec{c}$, $\overrightarrow{CD}=\vec{d}$, $\overrightarrow{DE}=\vec{e}$ and $\overrightarrow{EF}=\vec{f}$ then



$\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}$

$=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EF}=\overrightarrow{OF}$

If the terminal point F of the last vector coincide with initial point of the first vector then

$\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}$

$=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EO}=\vec{O}$,

i.e. the sum of vectors is zero or null vector in this case.

Example 1 :

If C is the middle point of AB and P is any point outside AB, then –

(1) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (2) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$

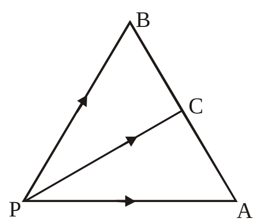
(3) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$ (4) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$

Sol. **(2).** $\because \dfrac{AC}{CB}=\dfrac{1}{1}$

$\Rightarrow \overrightarrow{AC}=\overrightarrow{CB}$

$\Rightarrow \overrightarrow{AP}+\overrightarrow{PC}=\overrightarrow{CP}+\overrightarrow{PB}$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$



Example 2 :

ABCD is a parallelogram whose diagonals meet at P. If O is a fixed point, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equals

(1) $\overrightarrow{OP}$ (2) $2\overrightarrow{OP}$

(3) $3\overrightarrow{OP}$ (4) $4\overrightarrow{OP}$

Sol. **(4).** Since, P bisects both the diagonal AC and BD, so

$\therefore \overrightarrow{OA}+\overrightarrow{OC}=2\overrightarrow{OP}$ and $\overrightarrow{OB}+\overrightarrow{OD}=2\overrightarrow{OP}$

$\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\overrightarrow{OP}$

Example 3 :

If $A\equiv(2i+3j)$, $B\equiv(pi+9j)$ and $C\equiv(i-j)$ are collinear, then the find the value of p.

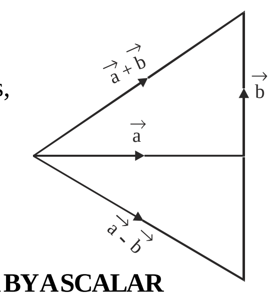
Sol. $\overrightarrow{AB}=(p-2)i+6j$, $\overrightarrow{AC}=i-4j$

Now A,B,C are collinear $\Leftrightarrow \overrightarrow{AB}\parallel\overrightarrow{AC}$

$$\Leftrightarrow \frac{p-2}{-1}=\frac{6}{-4} \Rightarrow p=-7/2$$

DIFFERENCE OF VECTORS

If $\vec{a}$ and $\vec{b}$ be any two vectors, then their difference $\vec{a}-\vec{b}$ is defined as $\vec{a}+(-\vec{b})$.



MULTIPLICATION OF A VECTOR BY A SCALAR

If $\vec{a}$ be any vector and m any scalar, then the multiplication of $\vec{a}$ by m is defined as a vector having magnitude $|m||\vec{a}|$ and direction same as of $\vec{a}$, if m is positive and reversed if m is negative. The product of $\vec{a}$ and m is denoted by m. $\vec{a}$. If m = 0, then m$\vec{a}$ is the zero vector,

For example, if $\vec{a}=\overrightarrow{AB}$ then $|2\vec{a}|=|2||\vec{a}|=2|\vec{a}|$ and direction same as that of $\vec{a}$.

The magnitude of the vectors $|-3\vec{a}|=3|\vec{a}|$ and direction opposite as that of $\vec{a}$.

IMPORTANT PROPERTIES AND FORMULAE

1. **(a)** Triangle law of vector addition $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

(b) Parallelogram law of vector addition : If ABCD is a parallelogram, then $\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$

(c) If $\vec{r_1}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r_2}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r_1}+\vec{r_2}=(x_1+x_2)\hat{i}+(y_1+y_2)\hat{j}+(z_1+z_2)\hat{k}$

and $\vec{r_1}=\vec{r_2} \Leftrightarrow x_1=x_2, y_1=y_2, z_1=z_2$

2. **(a)** $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{a}=m\vec{b}$ for some non-zero scalar m.

(b) $\hat{a}=\dfrac{\vec{a}}{|\vec{a}|}$ or $\vec{a}=|\vec{a}|\hat{a}$

(c) Associative law : $m(n\vec{a})=(mn)\vec{a}=n(m\vec{a})$

(d) Distributive laws : $(m+n)\vec{a}=m\vec{a}+n\vec{a}$ and $n(\vec{a}+\vec{b})=n\vec{a}+n\vec{b}$

(e) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then $m\vec{r}=mx\hat{i}+my\hat{j}+mz\hat{k}$

(f) $\vec{r}$, $\vec{a}$, $\vec{b}$ are coplanar if and only if $\vec{r} = x\vec{a} + y\vec{b}$ for some scalars x and y

3. **(a)** If the position vectors of the points A and B be $\vec{a}$ and $\vec{b}$ then,

(i) The position vectors of the points dividing the line AB in the ratio m :n internally and externally are $\dfrac{m\vec{b}+n\vec{a}}{m+n}$ and $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

(ii) Position vector of the middle point of AB is given by $\dfrac{1}{2}(\vec{a}+\vec{b})$

(iii) $\overrightarrow{AB} = \vec{b} - \vec{a}$

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $|\vec{r}| = \sqrt{x^2+y^2+z^2}$

(c) The points A,B,C will be collinear if and only if $\overrightarrow{AB} = m\,\overrightarrow{AC}$, for some non zero scalar m.

(d) Given vectors $x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$, $x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$, $x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, will be coplanar if and only if

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

(e) Method to prove four points to be coplanar : To prove that the four points A,B,C and D are coplanar.Find the vector $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ and then prove them to be coplanar by the method of coplanarity i.e. one of them is a linear combination of the other two.

(f) $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

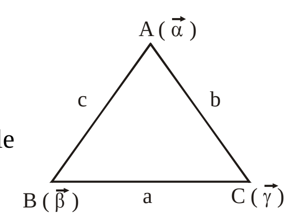
$|\vec{a}+\vec{b}| \ge |\vec{a}| - |\vec{b}|$

$|\vec{a}-\vec{b}| \le |\vec{a}| + |\vec{b}|$

$|\vec{a}-\vec{b}| \ge |\vec{a}| - |\vec{b}|$

(g) **Centroid, Incentre, Circumcentre and Orthocentre:**

Consider triangle



* Position vector of the centroid of a triangle ABC

$= \dfrac{\vec{\alpha}+\vec{\beta}+\vec{\gamma}}{3}$ (Concurrency of medians)

* p.v. of incentre of the $\Delta = \dfrac{a\vec{\alpha}+b\vec{\beta}+c\vec{\gamma}}{a+b+c}$

(Concurrency of internal angle bisectors)
Excentres of the Δ are

$\dfrac{-a\vec{\alpha}+b\vec{\beta}+c\vec{\gamma}}{-a+b+c}$, $\dfrac{a\vec{\alpha}-b\vec{\beta}+c\vec{\gamma}}{a-b+c}$ and $\dfrac{a\vec{\alpha}+b\vec{\beta}-c\vec{\gamma}}{a+b-c}$

* p.v. of circumcentre of the Δ

$= \dfrac{\vec{\alpha}\sin 2A + \vec{\beta}\sin 2B + \vec{\gamma}\sin 2C}{\Sigma \sin 2A}$

(Concurrency of perpendicular bisectors of sides)

* p.v. of orthocentre of the Δ

$= \dfrac{\vec{a}\tan A + \vec{b}\tan B + \vec{c}\tan C}{\Sigma \tan A}$ (Concurrency of altitudes)

Example 4 :

If vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar, then find the value of a .

Sol. If given vectors are coplanar, then there exists two scalar quantities x and y such that

$2i - j + k = x(i + 2j - 3k) + y(3i + aj + 5k)$ ....(1)

Comparing coefficient of i, j and k on both sides of (1) we get $x + 3y = 2$, $2x + ay = -1$, $-3x + 5y = 1$ ......(2)

Solving first and third equations, we get $x = \dfrac{1}{2}, y = \dfrac{1}{2}$

Since the vectors are coplanar, therefore these values of x and y will satisfy the equation

$2x + ay = -1 \quad \therefore 2 \times \dfrac{1}{2} + a \times \dfrac{1}{2} = -1 \Rightarrow a = -4$

SCALAR PRODUCT OR DOT PRODUCT

(a) $\vec{a}\,.\,\vec{b} = ab\cos\theta$, where $0 \le \theta \le \pi$

(b) $\vec{a}\,.\,\vec{b} = a$ (Projection of $\vec{b}$ along $\vec{a}$)

(c) Projection of $\vec{b}$ along $\vec{a} = \dfrac{\vec{b}.\vec{a}}{|\vec{a}|}$

(d) The vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by $\vec{a} \times \vec{b}$

The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by $\hat{n} = \dfrac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

(e) $\vec{a}\,.\,\vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$

(f) Component of a vector $\vec{r}$ in the direction of $\vec{a}$ and perpendicular to $\vec{a}$ are $\left(\dfrac{\vec{r}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}$ and $\vec{r} - \left\{\dfrac{(\vec{r}.\vec{a})}{|\vec{a}|^2}\right\}\vec{a}$ respectively.

(g) If $\vec{a}$ and $\vec{b}$ are the non-zero vectors, then

$\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

(h) $\cos\theta = \hat{a}.\hat{b} = \dfrac{\vec{a}.\vec{b}}{|a|.|b|}$

(i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{i}.\hat{k} = 0$

(j) If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$

i.e. if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

(ii) $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$

(iii) $\vec{a}$ and $\vec{b}$ will be perpendicular if and only if $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(iv) $\vec{a}$ and $\vec{b}$ will be parallel if and only if $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

Example 5 :

Find the projection of vectors i + j + k on the vector $i - j + k$.

Sol. Projection $= \dfrac{(i+j+k).(i-j+k)}{|i-j+k|} = \dfrac{1-1+1}{\sqrt{1+1+1}} = \dfrac{1}{\sqrt{3}}$

Example 6 :

Find the angle between the vectors

$4\hat{i} + \hat{j} + 3\hat{k}$ and $2\hat{i} + 2\hat{j} - \hat{k}$.

Sol. Let the required angle is θ.

$$\therefore \quad \theta = \cos^{-1}\left(\frac{4\cdot 2 + 1\cdot 2 + 3(-1)}{\sqrt{16+1+9}\sqrt{4+4+1}}\right) = \cos^{-1}\left(\frac{7}{3\sqrt{26}}\right)$$

Example 7 :

Find the vector components of a vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ along and perpendicular to non-zero vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Sol. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$

Now, vector component of $\vec{a}$ along $\vec{b}$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\vec{b} = \frac{4+3+12}{9}(2\hat{i} + \hat{j} + 2\hat{k}) = \frac{19}{9}(2\hat{i} + \hat{j} + 2\hat{k})$$

and vector component of $\vec{a}$ perpendiclar to $\vec{b}$.

$$= \vec{a} - \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = (2\hat{i} + 3\hat{j} + 6\hat{k}) - \frac{19}{9}(2\hat{i} + \hat{j} + 2\hat{k})$$

$$= \frac{1}{9}(-20\hat{i} + 8\hat{j} + 16\hat{k})$$

TRY IT YOURSELF-1

Q.1 Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b} = \sqrt{6}$.

Q.2 Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

Q.3 Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$ having the same magnitude and such that the angle between them is 60° and their scalar product is 1/2.

Q.4 If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to c, then find the value of λ.

Q.5 If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2) respectively, then find ∠ ABC.

Q.6 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 45° (B) 60°
(C) $\cos^{-1}(1/3)$ (D) $\cos^{-1}(2/7)$

Q.7 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $1/\sqrt{3}$, is given by –

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$
(C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Q.8 Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2.

Q.9 The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal one. Find the value of λ.

ANSWERS

(1) π/4 **(2)** 0 **(3)** 1, 1

(4) 8 **(5)** $\cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$ **(6)** (B)

(7) (C) **(8)** $3\vec{a} + 5\vec{b}$ **(9)** 1

VECTOR OR CROSS PRODUCT OF TWO VECTORS

1. The product of vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$.

$\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|\sin\theta)\hat{n}$

2. $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

3. If $\vec{a} = \vec{b}$ or if $\vec{a}$ is parallel to $\vec{b}$, then sinθ = 0 and so $\vec{a} \times \vec{b} = 0$

4. **Distributive laws:** $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

and $(\vec{b}+\vec{c})\times\vec{a}=\vec{b}\times\vec{a}+\vec{c}\times\vec{a}$

5. The vector product of a vector $\vec{a}$ with itself is a null vector, i.e. $\vec{a}\times\vec{a}=\vec{0}$

6. if $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then

(i) $\vec{a}\times\vec{b}=(a_2b_3-a_3b_2)\hat{i}+(a_3b_1-a_1b_3)\hat{j}+(a_1b_2-a_2b_1)\hat{k}$

(ii) $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}$

(iii) $\sin^2\theta=\dfrac{(a_2b_3-a_3b_2)^2+(a_3b_1-a_1b_3)^2+(a_1b_2-a_2b_1)^2}{(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)}$

7. If two vectors $\vec{a}$ and $\vec{b}$ are parallel, then $\theta=0$ or π i.e. $\sin\theta=0$ in both cases

$\because (a_1b_2-a_2b_1)^2+(a_2b_3-a_3b_2)^2+(a_3b_1-a_1b_3)^2=0$

$\Rightarrow a_1b_2-a_2b_1=0,\ a_2b_3-a_3b_2=0,\ a_3b_1-a_1b_3=0$

$\Rightarrow \dfrac{a_1}{b_1}=\dfrac{a_2}{b_2},\ \dfrac{a_2}{b_2}=\dfrac{a_3}{b_3},\ \dfrac{a_3}{b_3}=\dfrac{a_1}{b_1}\ \Rightarrow \dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}$

Thus, two vectors $\vec{a}$ and $\vec{b}$ are parallel if their corresponding co,ponents are proportional.

8. Area of the parallelogram ABCD

$=|\overrightarrow{AB}\times\overrightarrow{AD}|$ or $\dfrac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$

9. Area of the triangle ABC $=\dfrac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Example 8 :

If angle between i – 2j + 3k and 2i + j + k is θ then find the value of sin θ.

Sol. We know that $\sin\theta=\dfrac{|a\times b|}{|a||b|}$

Now $a\times b=-5i+5j+5k$

$\therefore |a\times b|=\sqrt{(5)^2+(5)^2+(5)^2}=\sqrt{75}=5\sqrt{3}$

$|a|=\sqrt{1+4+9},\ |b|=\sqrt{4+1+1}$

$\therefore \sin\theta=\dfrac{5\sqrt{3}}{\sqrt{1+4+9}\sqrt{4+1+1}}=\dfrac{5\sqrt{3}}{\sqrt{14}\sqrt{6}}=\dfrac{5}{\sqrt{28}}=\dfrac{5}{2\sqrt{7}}$

Example 9 :

Find the area of a parallelogram whose two adjacent sides are represented by a = 3i + j + 2k and b = 2i – 2j + 4k .

Sol. Area of parallelogram = |a × b|

Now $a\times b=\begin{vmatrix}i&j&k\\3&1&2\\2&-2&4\end{vmatrix}=8i-8j-8k$

$\therefore$ Area $=|8i-8j-8k|=8\sqrt{3}$ units

SCALAR TRIPLE PRODUCT

1. If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$, then

$(\vec{a}\times\vec{b}).\vec{c}=[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}$

2. [a b c] = volume of the parallelopiped whose coterminous edges are formed by $\vec{a},\vec{b},\vec{c}$

3. [a b c] = [b c a] = [c a b] but [a b c] = – [a c b] etc. i.e. change of any two vector in scalar triple product changes the sign of the scalar triple product.

4. If any two of the vectors $\vec{a},\vec{b},\vec{c}$ are equal, then [a b c] = 0

5. The position of dots and cross in a scalar triple product can be interchanged. Hence $(\vec{a}\times\vec{b}).\vec{c}=\vec{a}.(\vec{b}\times\vec{c})$

6. The value of a scalar triple product is zero, if two of its vectors are parallel.

7. $\vec{a},\vec{b},\vec{c}$ are coplanar if and only if $[\vec{a}\ \vec{b}\ \vec{c}]=0$

8. Four points A, B, C, D with position vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively are coplanar if and only if $[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}]=0$

i.e. if and only if $[\vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\ \ \vec{d}-\vec{a}]=0$

9. Volume of a tetrahedron with three coterminous edges

$\vec{a},\vec{b},\vec{c}=\dfrac{1}{6}\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$.

If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of vertices A, B, C, D of a tetrahedron ABCD, then its volume

$$=\begin{cases}\dfrac{1}{6}\left|[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}]\right|\\ \text{or}\\ \dfrac{1}{6}\left|[\vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\ \ \vec{d}-\vec{a}]\right|\end{cases}$$

10. Volume of prism on a triangular base with three coterminous edges $\vec{a},\vec{b},\vec{c}=\dfrac{1}{2}\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$

Example 10 :

If $a = 2i - 3j$, $b = i + j - k$ and $c = 3i - k$ represent three coterminous edges of a parallelopiped, then find the volume of that parallelopiped.

Sol. Volume = $[\,abc\,] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = -2 + 9 - 3 = 4$

Example 11 :

If the vertices of any tetrahedron be $a = j + 2k$, $b = 3i + k$, $c = 4i + 3j + 6k$ and $d = 2i + 3j + 2k$ then find its volume.

Sol. Let the p.v. of the vertices A,B,C,D with respect to O are a,b,c and d respectively then

$\overline{AB} = b - a = 3i - j - k$, $\overline{AC} = 4i + 2j + 4k$ & $\overline{AD} = 2i + 2j$

Now volume of tetrahedron

$$= \frac{1}{6}\left[\,\overline{AD}\ \overline{AC}\ \overline{AD}\,\right] = \frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = -6$$

$\therefore$ Required volume = 6 units

VECTOR TRIPLE PRODUCT

The vector triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$ is defined as the vector product of two vectors $\vec{a}$ and $\vec{b} \times \vec{c}$. It is denoted by $\vec{a} \times (\vec{b} \times \vec{c})$.

$(\vec{a} \times \vec{b}) \times \vec{c}$ is a vector which is coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{c}$.

Hence $(\vec{a} \times \vec{b}) \times \vec{c} = x\vec{a} + y\vec{b}$ ....(1)

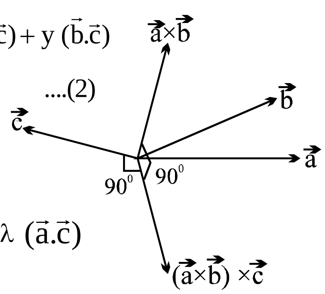
[linear combination of $\vec{a}$ and $\vec{b}$)

$\vec{c} \cdot (\vec{a} \times \vec{b}) \times \vec{c} = x\,(\vec{a}.\vec{c}) + y\,(\vec{b}.\vec{c})$

$0 = x\,(\vec{a}.\vec{c}) + y\,(\vec{b}.\vec{c})$ ....(2)

$\frac{x}{\vec{b} \cdot \vec{c}} = -\frac{y}{\vec{a} \cdot \vec{c}} = \lambda$

$x = \lambda(\vec{b}.\vec{c})$ and $y = -\lambda\,(\vec{a}.\vec{c})$



Substituting the values of x and y in $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda(\vec{b}.\vec{c})\vec{a} - \lambda(\vec{a}.\vec{c})\vec{b}$

This is an identity and must be true for all values of $\vec{a}, \vec{b}, \vec{c}$

Put $\vec{a} = \hat{i}$; $\vec{b} = \hat{j}$ and $\vec{c} = \hat{i}$

$(\hat{i} \times \hat{j}) \times \hat{i} = \lambda\,(\hat{j}.\hat{i})\hat{i} - \lambda\,(\hat{i}.\hat{i})\,\hat{j}$

$\hat{j} = -\lambda\,\hat{j} \Rightarrow \lambda = -1$

Hence $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\,\vec{a}$

Properties :

1. Expansion formula for vector triple product is given by

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$(\vec{b} \times \vec{c}) \times \vec{a} = (\vec{b}.\vec{a})\vec{c} - (\vec{c}.\vec{a})\vec{b}$

2. $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

Note that if $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors then $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ will also be non coplanar vectors.

3. Vector triple product is a vector quantity.

4. $\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

5. Unit vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{c}$ is $\pm \frac{(\vec{a} \times \vec{b}) \times \vec{c}}{|(\vec{a} \times \vec{b}) \times \vec{c}\,|}$

Scalar Product of four Vector :

$$(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = (\vec{a}.\vec{c})(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})(\vec{b}.\vec{c}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix}$$

Proof : $\underbrace{(\vec{a} \times \vec{b})}_{\vec{u}} \cdot (\vec{c} \times \vec{d}) = \vec{u} \cdot (\vec{c} \times \vec{d}) = (\vec{u} \times \vec{c}) \cdot \vec{d}$

(Dot and Cross are interchangeable in STP)

$\left((\vec{a} \times \vec{b}) \times \vec{c}\right) \cdot \vec{d} = \left((\vec{a} \times \vec{c})\vec{b} - (\vec{b} \times \vec{c})\vec{a}\right) \cdot \vec{d}$

$= \left((\vec{a} \times \vec{c})(\vec{b} \times \vec{d}) - (\vec{b} \times \vec{c})(\vec{a} \times \vec{d})\right) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}$

$(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b}) = (\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

$= (\vec{a})^2 = (\vec{b})^2 = (\vec{a} \cdot \vec{b})^2$ which is Lagrange's identity.

Example 12 :

$i \times (j \times k) + j \times (k \times i) + k\,(i \times j)$ equals

(1) i (2) j

(3) k (4) 0

Sol. (4). $i \times (j \times k) + j \times (k \times i) + k \times (i \times j)$

$= i \times i + j \times j + k \times k = 0 + 0 + 0 = 0$

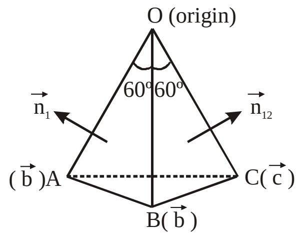
Example 13 :

Prove that acute angle between the two plane faces of a regular tetrahedron is $\cos^{-1}\frac{1}{3}$.

Sol. Let edge length of regular tetrahedron = 1

$\vec{n}_1$ = normal vector to plane OAB = $\vec{a}\times\vec{b}$

$\vec{n}_2$ = normal vector to plane OBC = $\vec{b}\times\vec{c}$



Acute angle between plane focus OAB & OBC is given as

$$\cos\theta=\frac{|\vec{n}_1\,\vec{n}_2|}{|\vec{n}_1|\,|\vec{n}_2|}=\frac{|(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})|}{|(\vec{a}\times\vec{b})|\;|(\vec{b}\times\vec{c})|}=\frac{\begin{vmatrix}\vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\end{vmatrix}}{\sin 60\cdot\sin 60^\circ}$$

$$=\frac{\begin{vmatrix}\cos 60^\circ & \cos 60^\circ\\ \cos 0^\circ & \cos 60^\circ\end{vmatrix}}{3/4}=\frac{\left|\frac{1}{4}-\frac{1}{2}\right|}{\frac{3}{4}}=\frac{1}{3}\Rightarrow\theta=\cos^{-1}\left(\frac{1}{3}\right)$$

RECIPROCAL SYSTEM OF VECTORS

(a) If $\vec{a},\vec{b},\vec{c}$ be any three non coplanar vectors so that $[\vec{a},\vec{b},\vec{c}]\neq 0$ then the three vectors $\vec{a}',\vec{b}',\vec{c}'$ defined by the equations $\vec{a}'=\frac{\vec{b}\times\vec{c}}{[\vec{a},\vec{b},\vec{c}]}$, $\vec{b}'=\frac{\vec{c}\times\vec{a}}{[\vec{a},\vec{b},\vec{c}]}$, $\vec{c}'=\frac{\vec{a}\times\vec{b}}{[\vec{a},\vec{b},\vec{c}]}$ are called the reciprocal system of vectors to the given vectors $\vec{a},\vec{b},\vec{c}$.

(b) **Properties:**

(i) $\vec{a}.\vec{a}'=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'=1$

(ii) The scalar product of any vector of one system with a vector of the other system which does not correspond to it, is zero i.e.

$\vec{a}.\vec{b}'=\vec{a}.\vec{c}'=\vec{b}.\vec{a}'=\vec{b}.\vec{c}'=\vec{c}.\vec{a}'=\vec{c}.\vec{b}'=0$

(iii) $[\vec{a}\ \vec{b}\ \vec{c}][\vec{a}'\ \vec{b}'\ \vec{c}']=1$

(iv) $\hat{i}'=\hat{i}$, $\hat{j}'=\hat{j}$, $\hat{k}'=\hat{k}$

(v) If $\{\vec{a}',\vec{b}',\vec{c}'\}$ is reciprocal system of $\{\vec{a},\vec{b},\vec{c}\}$ and $\vec{r}$ is any vector, then

$\vec{r}=(\vec{r}.\vec{a})\vec{a}'+(\vec{r}.\vec{b})\vec{b}'+(\vec{r}.\vec{c})\vec{c}'$

$\vec{r}=(\vec{r}.\vec{a}')\vec{a}+(\vec{r}.\vec{b}')\vec{b}'+(\vec{r}.\vec{c}')\vec{c}$

Vector Product of Four Vector :

$\vec{V}=(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$

$=\vec{u}\times(\vec{c}\times\vec{d})=[\vec{a}\,\vec{b}\,\vec{d}]\vec{c}-[\vec{a}\,\vec{b}\,\vec{c}]\vec{d}$ ....(1) (where $\vec{u}=\vec{a}\times\vec{b}$)

again $\vec{V}=(\vec{a}\times\vec{b})\times\underbrace{(\vec{c}\times\vec{d})}_{\vec{V}}=(\vec{a}.\vec{v})\vec{b}-(\vec{b}.\vec{v})\vec{a}$

$==[\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}$ ....(2)

From (1) and (2),

$[\vec{a}\,\vec{b}\,\vec{d}]\vec{c}-[\vec{a}\,\vec{b}\,\vec{c}]\vec{d}=[\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}$ ....(3)

Note that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=0$ $\Rightarrow$ planes containing the vectors $\vec{a}$ & $\vec{b}$ and $\vec{c}$ & $\vec{d}$ are parallel.

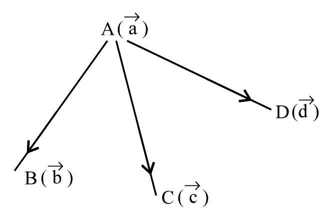
$|||^{ly}$ $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=0\Rightarrow$ the two planes are perpendicular.

(i) Equation (3) is suggestive that if $\vec{a},\vec{b},\vec{c},\vec{d}$ are four vectors no 3 three of them are coplanar then each one of them can be expressed as a linear combination of other.

(ii) If $\vec{a},\vec{b},\vec{c},\vec{d}$ are p.v.'s of four points then these four points are in the same plane if $[\vec{a}\,\vec{b}\,\vec{d}]-[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{a}\,\vec{c}\,\vec{d}]-[\vec{b}\,\vec{c}\,\vec{d}]$

Condition for coplanarity of four points :

4 points with pv's $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar iff $\exists$ scalars x, y, z and t not all simultaneously zero and satisfying $x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$ where $x+y+z+t=0$.



Case I : Let the four points A, B, C, D are in the same plane

$\Rightarrow$ the vectors $\vec{b}-\vec{a},\vec{c}-\vec{a}$ and $\vec{d}-\vec{a}$ are in the same plane.

hence $\vec{d}-\vec{a}=\ell(\vec{b}-\vec{a})+m(\vec{c}-\vec{a})$

or $\underbrace{(\ell+m-1)}_{x}\vec{a}-\underbrace{\ell}_{y}\vec{b}-\underbrace{m}_{z}\vec{c}+\underbrace{1}_{t}\vec{d}=0$

$\Rightarrow$ $x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$ where, $x+y+z+t=0$ and x, y, z, t not all simultaneous zero.

Case II : Let $x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$ where $x+y+z+t=0$ and not all simultaneously zero

Let $t\neq 0$ $(-y-z-t)\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$

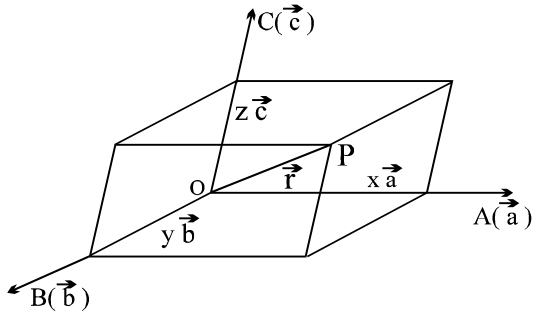
[putting $x=-y-z-t$]

$(\vec{d}-\vec{a})\ t+y(\vec{b}-\vec{a})+z(\vec{c}-a)=0$

$\Rightarrow \quad \vec{d}-\vec{a},\ \vec{b}-\vec{a}$ and $\vec{c}-a$ are coplanar

$\Rightarrow$ Points A, B, C, D are coplanar

Theorem in space : If $\vec{a},\vec{b},\vec{c}$ are 3 non zero non coplanar vectors then any vector $\vec{r}$ can be expressed as a linear combination : $\vec{r}=x\vec{a}+y\vec{b}+z\vec{c}$ of $\vec{a},\vec{b},\vec{c}$



Examples 14 :

Express the non coplanar vectors $\vec{a},\vec{b},\vec{c}$ in terms of $\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}$.

Sol. Since $[\vec{a}\ \vec{b}\ \vec{c}]^2=[(\vec{a}\times\vec{b})\ (\vec{b}\times\vec{c})\ (\vec{c}\times\vec{a})]$

$\therefore$ If $\vec{a},\vec{b},\vec{c}$ are non-coplanar

$\Rightarrow \quad \vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ are also non coplanar.

$\vec{a}=x\,(\vec{a}\times\vec{b})+y\,(\vec{b}\times\vec{c})+z\,(\vec{c}\times\vec{a})$

Taking dot product with $\vec{a}$

$$\vec{a}^2=y\,[\vec{a}\ \vec{b}\ \vec{c}]\Rightarrow y=\frac{(\vec{a})^2}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

Taking dot product with $\vec{b}$

$$\vec{a}\cdot\vec{b}=z\,[\vec{b}\ \vec{c}\ \vec{a}]\Rightarrow z=\frac{\vec{a}\cdot\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

Similarly taking dot product with $\vec{c}$

$$\vec{a}\cdot\vec{c}=x\,[\vec{b}\ \vec{c}\ \vec{a}]\Rightarrow x=\frac{\vec{a}\cdot\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$$\therefore \quad \vec{a}=\frac{(\vec{a}\cdot\vec{c})\,(\vec{a}\times\vec{b})+(\vec{a})^2\,(\vec{b}\times\vec{c})+(\vec{a}\cdot\vec{b})\,(\vec{c}\times\vec{a})}{[\vec{a}\,\vec{b}\,\vec{c}]}$$

APPLICATION OF VECTOR IN MECHANICS

(a) Work done by a force = (Force) . (Displacement)

(b) Moment of a force $\vec{F}$ about a point O = $\overrightarrow{OP}\times\vec{F}$, where P is any point on the line of action of the force $\vec{F}$

(c) Moment of the couple $(\vec{F},r)=\vec{r}\times\vec{F}$

Example 15 :

Find the work done by a force represented by 4i + j – 3k which displaces a particle from the point A (i + 2j + 3k) to the point B (5i + 4j + k)

Sol. Here F = 4i + j – 3k

$d=\overrightarrow{AB}=(5-1)\,i+(4-2)\,j+(1-3)\,k=4i+2j-2k$

Work done by the given force = F.d

$=(4i+j-3k).(4i+2j-2k)=16+2+6=24$ units

Example 16 :

Find the moment of the force 3i + k passing through the point A (2i – j + 3k) about the point O (i + 2j + k)

Sol. Here $r=\overrightarrow{OA}=i-3j+2k$

$\therefore$ Moment of force F at A with respect to O

$=r\times F=(i-3j+4k)\ x\ (3i+k)$

$$=\begin{vmatrix} i & j & k\\ 1 & -3 & 4\\ 3 & 0 & 1\end{vmatrix}=-3i+11j+9k$$

VECTOR EQUATION OF A STRAIGHT LINE

(a) Vector equation of a straight line passing through a point $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r}=\vec{a}+t\vec{b}$ where t is an arbitrary constant.

(b) Vector equation of a straight line passing through two points $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+t(\vec{b}-\vec{a})$

(c) (i) Vector equation of internal bisectors of angle between two straight lines : $\vec{r}=t\left(\frac{\vec{a}}{a}+\frac{\vec{b}}{b}\right)$

(ii) Equation of external bisector : $\vec{r}=t\left(\frac{\vec{a}}{a}-\frac{\vec{b}}{b}\right)$

Corollary : If the lines intersect at point having position vector $\vec{\alpha}$, then the above equations becomes

$\vec{r}=\vec{\alpha}+t\,(\hat{a}+\hat{b})$ and $\vec{r}=\vec{\alpha}+t\,(\hat{a}-\hat{b})$ respectively.

Example 17 :

Find the vector equation of the line through the point $2\hat{i}+\hat{j}-3\hat{k}$ and parallel to the vector $\hat{i}+2\hat{j}+\hat{k}$.

Sol. Let the given point be A $(\vec{a})$ and given vector be $\vec{b}$ and O be the origin.

Then, $\vec{a}=\overrightarrow{OA}=2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

Now, vector equation of the line through A and parallel to $\vec{b}$ is $\vec{r}=\vec{a}+t\,\vec{b}$, where t is a scalar.

or $\quad \vec{r}=2\hat{i}+\hat{j}-3\hat{k}+t\,(\hat{i}+2\hat{j}+\hat{k})$

TRY IT YOURSELF-2

Q.1 Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ & $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Q.2 Find the volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$

Q.3 Let $\vec{a}=-\hat{i}-\hat{k}, b=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is

Q.4 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]=$

(A) 0 (B) 1

(C) $-\sqrt{3}$ (D) $\sqrt{3}$

Q.5 Let $\vec{a}=\vec{i}-\vec{k}$, $\vec{b}=x\vec{i}+\vec{j}+(1-x)\vec{k}$ and $\vec{c}=y\vec{i}+x\vec{j}+(1+x-y)\vec{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

(A) only x (B) only y

(C) Neither on x Nor on y (D) both x and y

Q.6 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors, then

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed

(A) 4 (B) 9

(C) 8 (D) 6

Q.7 The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is –

(A) –3 (B) 3

(C) $1/\sqrt{3}$ (D) $\sqrt{3}$

Q.8 The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a},\hat{b},\hat{c}$ such that $\hat{a}.\hat{b}=\hat{b}.\hat{c}=\hat{c}.\hat{a}=\frac{1}{2}$. The volume of the parallelopiped is –

(A) $1/\sqrt{2}$ (B) $1/2\sqrt{2}$

(C) $\sqrt{3}/2$ (D) $1/\sqrt{3}$

Q.9 Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is –

(A) 5 (B) 20

(C) 10 (D) 30

ANSWERS

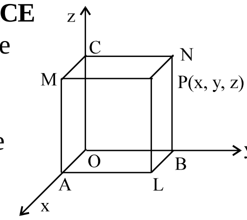
(1) $15\sqrt{2}$ sq. units **(2)** π cubic units **(3)** 9

(4) (A) **(5)** (C) **(6)** (B)

(7) (C) **(8)** (A) **(9)** (C)

THREE DIMENSIONAL COORDINATE GEOMETRY

COORDINATES OF A POINT IN SPACE



Consider a point P in space whose position is given by triad (x, y, z) where x, y, z are perpendicular distance from YZ-plane, ZX-plane and XY-plane respectively.

If we assume $\hat{i}, \hat{j}, \hat{k}$ unit vectors along OX, OY, OZ respectively, then position vector of point P is $x\hat{i}+y\hat{j}+z\hat{k}$ or simply (x, y, z).

Any point on -

- x-axis = $\{(x, y, z) \mid y = z = 0\}$
- y-axis = $\{(x, y, z) \mid x = z = 0\}$
- z-axis = $\{(x, y, z) \mid x = y = 0\}$
- xy plane = $\{(x, y, z) \mid z = 0\}$
- yz plane = $\{(x, y, z) \mid x = 0\}$
- zx plane = $\{(x, y, z) \mid y = 0\}$
- OP = $\sqrt{x^2+y^2+z^2}$

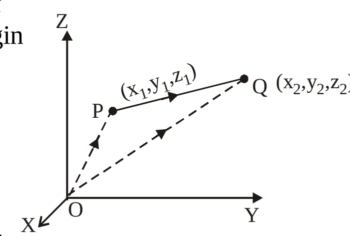
DISTANCE BETWEEN TWO POINTS

If P (x_1, y_1, z_1) and Q (x_2, y_2, y_2) are two points, then distance between them PQ

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

In particular distance of a point (x, y, z) from origin

$$=\sqrt{x^2+y^2+z^2}$$



Note :

(a) The selection of origin and co-ordinate axes are completely arbitrary.

(b) Distance between two points remains invariant under any co-ordinate system i.e. does not depend upon selection of the system.

Distance of a point from coordinate axes:

Let P(x, y, z) be any point in the space. Let PA, PB and PC be the perpendicular drawn from P to the axes OX, OY and OZ resp. Then

PA = $\sqrt{(y^2+z^2)}$; PB = $\sqrt{(x^2+z^2)}$; PC = $\sqrt{(x^2+y^2)}$

Example 18 :

Find the distance between the points P(3,4, 5) & Q (–1, 2, – 3)

Sol. $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here, $(x_1, y_1, z_1) \equiv (3, 4, 5)$ and $(x_2, y_2, z_2) \equiv (-1, 2, -3)$

$PQ=\sqrt{(-1-3)^2+(2-4)^2+(-3-5)^2}=\sqrt{16+4+64}$

$=\sqrt{84}=2\sqrt{21}$

Example 19 :

Show that the points A (0, 7, 10), B (–1, 6, 6) and C (– 4, 9, 6) are vertices of an isosceles right-angled triangle.

Sol. $AB^2 = (-1-0)^2 + (6-7)^2 + (6-10)^2 = 18$

$BC^2 = (-1+4)^2 + (6-9)^2 + (6-6)^2 = 18$

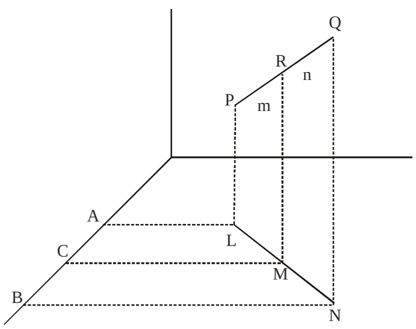
$CA^2 = (0+4)^2 + (7-9)^2 + (10-6)^2 = 36$

Since AB = BC and $AB^2 + BC^2 = AC^2$. Hence A, B and C are vertices of an isosceles right-angled triangle.

SECTION FORMULA

To find the co-ordinates of a point which divides the join of points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) in the ratio m : n.

(A) Internal division : Let the point R (x, y, z) divide the join of points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n. Draw perpendiculars PL, QN and RM from the points P, Q and R on xy plane. Again draw perpendiculars AL, MC and NB from L, M, N to x-axis.



Now, from the ratio obtained by the intersection of a transversal to parallel lines.

$$\frac{PQ}{RQ} = \frac{m}{n} \text{ and } \frac{PR}{RQ} = \frac{LM}{MN} = \frac{AC}{CB}$$

$$\frac{m}{n} = \frac{x - x_1}{x_2 - x} \Rightarrow (x_2 - x)\,m = n\,(x - x_1)$$

$$\Rightarrow x = \frac{mx_2 + nx_1}{m+n}$$

Similarly, $y = \dfrac{my_2 + ny_1}{m+n}$ and $z = \dfrac{mz_2 + nz_1}{m+n}$

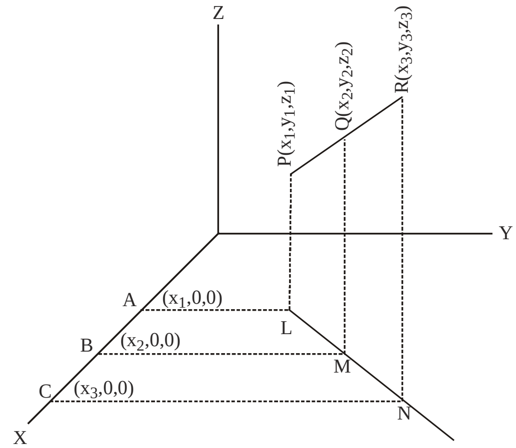
Hence, the co-ordinates of R are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

(B) External division : If the point, dividing the line segment joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) in the ratio of m : n lies outside on the join of PQ then such a division is known as external division.

Draw perpendiculars PL, QM and RN from the points P, Q and R on XY plane and draw perpendiculars AL, BM and CN from L, M, N on x-axis as done in the internal division.

Now, $\dfrac{PR}{QM} = \dfrac{m}{n}$; $\dfrac{PR}{QM} = \dfrac{LN}{ML} = \dfrac{AC}{BC} = \dfrac{m}{n}$



$$\Rightarrow \frac{x - x_1}{x - x_2} = \frac{m}{n} \Rightarrow x = \frac{mx_2 - nx_1}{m-n}$$

Similarly $y = \dfrac{my_2 - ny_1}{m-n}$ and $z = \dfrac{mz_2 - nz_1}{m-n}$

Hence, in the case of external division co-ordinates of R are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

1. Instead of ratio m : n we can take $\dfrac{m}{n} : 1$ or λ : 1. In such situation the co-ordinates of R are

   (a) For internal division $\left(\dfrac{\lambda x_2 + x_1}{\lambda+1}, \dfrac{\lambda y_2 + y_1}{\lambda+1}, \dfrac{\lambda z_2 + z_1}{\lambda+1}\right)$

   (b) For external division $\left(\dfrac{\lambda x_2 - x_1}{\lambda-1}, \dfrac{\lambda y_2 - y_1}{\lambda-1}, \dfrac{\lambda z_2 - z_1}{\lambda-1}\right)$

2. If in a division in the ratio λ : 1, the λ > 0 then the division is internal and if λ < 0 the division is external.

3. Every point lying on the join of P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) divides it in some ratio i.e. λ has same value for every point on the line PQ. Conversely, for every value of λ (≠ –1) there is a point on the line PQ. Hence, co-ordinates of any point lying on the line PQ are

$$\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}, \frac{\lambda z_2 + z_1}{\lambda+1}\right)$$

Centroid of a Triangle : If (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) be the vertices of a triangle, then the centroid of the triangle is $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}, \dfrac{z_1 + z_2 + z_3}{3}\right)$

Division by Co ordinate Planes : The ratios in which the line segment PQ joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is divided by coordinate planes are as follows.

(i) by yz – plane : $\dfrac{x_1}{x_2}$ ratio (ii) by zx – plane : $\dfrac{y_1}{y_2}$ ratio

(iii) by xy – plane : $\frac{z_1}{z_2}$ ratio

Centroid of a Tetrahedron : If (x_r, y_r, z_r) r = 1, 2, 3, 4 are vertices of a tetrahedron, then coordinates of its centroid are $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$

Example 20 :

Find the ratio in which the planes (1) XY (2) YZ divide the line joining the points P (–2, 4, 7) and Q (3, –5, 8).

Sol. (1) Let the XY plane divide the line joining the points P (–2, 4, 7) and Q (3, – 5, 8) in the ratio λ : 1 at point R.

Then the co-ordinates of R are $\left(\frac{3\lambda-2}{\lambda+1}, \frac{-5\lambda+4}{\lambda+1}, \frac{8\lambda+7}{\lambda+1}\right)$.

Since, the point R lies in the XY plane, its z-co-ordinate = 0

$$\frac{8\lambda+7}{\lambda+1} = 0 \Rightarrow 8\lambda+7 = 0 \text{ or } \lambda = -\frac{7}{8}$$

Hence, the required ratio is 7 : 8. Also, here λ < 0, so the XY plane divides the line segment PQ externally.

Here, we put x = 0 in the co-ordinates of R, since x = 0 for YZ plane. Hence, $\frac{3\lambda-2}{\lambda+1} = 0 \Rightarrow 3\lambda = 2$ or $\lambda = \frac{2}{3}$

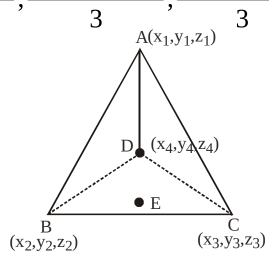
Example 21 :

If the vertices of a tetrahedron are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4). Prove that its centroid is

$$\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right).$$

Sol. Centroid of the Δ BDC is

$$\left(\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right)$$



Now, the centroid G of the tetrahedron ABCD divide the line segment AE in the ratio of 3 : 1.

Hence, the co-ordinates of G are

$$\left(\frac{1.x_1+3.\frac{(x_2+x_3+x_4)}{3}}{1+3}, \frac{1.y_1+3.\frac{(y_2+y_3+y_4)}{3}}{1+3}, \frac{1.z_1+3.\frac{(z_2+z_3+z_4)}{3}}{1+3}\right)$$

or $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$

Example 22 :

Find the coordinates of the point which divides the line segment joining the points (1, –2, 3) and (3, 4,–5) in the ratio 2 : 3. (i) internally, and (ii) externally.

Sol. (i) Let P (x, y, z) be the point which divides line segment joining A(1, – 2, 3) and B (3, 4, –5) internally in the ratio 2 : 3. Therefore,

$$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5}, \quad y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5},$$

$z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$. The required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$

(ii) Let P (x, y, z) be the point which divides segment joining A (1, –2, 3) and B (3, 4, –5) externally in the ratio 2 : 3. Then

$$x = \frac{2(3)+(-3)1}{2+(-3)} = -3, \quad y = \frac{2(4)+(-3)(-2)}{2+(-3)} = -14,$$

$$z = \frac{2(-5)+(-3)(3)}{2+(-3)} = 19$$

Therefore, the required point is (–3, –14, 19).

Example 23 :

Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, – 8) is divided by the YZ-plane.

Sol. Let YZ-plane divides the line segment joining A (4, 8, 10) and B (6, 10, – 8) at P (x, y, z) in the ratio k : 1. Then the coordinates of P are $\left(\frac{4+6k}{k+1}, \frac{8+10k}{k+1}, \frac{10-8k}{k+1}\right)$

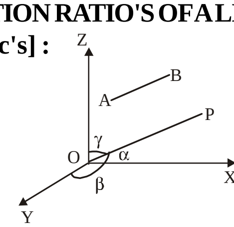
Since P lies on the YZ-plane, its x-coordinate is zero,

i.e., $\frac{4+6k}{k+1} = 0$ or $k = -\frac{2}{3}$

Therefore, YZ-plane divides AB externally in the ratio 2 : 3.

DIRECTION COSINES & DIRECTION RATIO'S OF A LINE

1. Direction cosines of a line [Dc's] :



The cosines of the angle made by a line with coordinate axes are called the direction cosines of that line.

Let α, β, γ be the angles made by a line AB with coordinate axes then cos α, cos β, cos γ are the direction cosines of AB which are generally denoted by ℓ, m, n. Hence ℓ = cos α, m = cos β, n = cos γ

Note: –1 < cos x < 1 ∀ n ∈ R, hence values of ℓ, m, n are such real numbers which are not less than –1 and not greater than 1. Hence DC's ∈ [–1, 1]

2. Direction cosines of coordinate axes :

x-axis makes 0º, 90º and 90º angles with three coordinates axes, so its direction cosines are cos 0º, cos 90º, cos 90º, i.e. 1, 0, 0.

Similarly direction cosines of y-axis and z-axis are 0, 1, 0 and 0, 0, 1 respectively . Hence

dc's of x-axis = 1, 0, 0 ; dc's of y-axis = 0, 1, 0

dc's of z-axis = 0, 0, 1

Note :
(i) The direction cosines of a line parallel to any coordinates axis are equal to the direction cosines of the corresponding axis.
(ii) Relation between dc's : $\ell^2 + m^2 + n^2 = 1$

3. Direction ratios of a line [DR's] :
Three numbers which are proportional to the direction cosines of a line are called the direction ratios of that line. If a, b, c are such numbers which are proportional to the direction cosines ℓ, m, n of a line then a, b, c are direction ratios of the line. Hence, a, b, c dr's $\Leftrightarrow \frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$

Further we may observe that in above case, a, b, c dr's $\Leftrightarrow \frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$

or $\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c} = \pm\frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}} = \pm\frac{1}{\sqrt{a^2+b^2+c^2}}$

$\Rightarrow \ell = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \ m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}},$

$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

Note :
(i) Numbers of dr's are not unique whereas numbers of dc's are unique.
(ii) $a^2 + b^2 + c^2 \neq 1$.

4. Direction cosines of a line joining two points :
Let $P \equiv (x_1, y_1, z_1)$ and $Q \equiv (x_2, y_2, z_2)$; then
(i) dr's of PQ : $(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

(ii) dc's of PQ : $\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$

i.e. $\frac{x_2-x_1}{\sqrt{\Sigma(x_2-x_1)^2}}, \frac{y_2-y_1}{\sqrt{\Sigma(x_2-x_1)^2}}, \frac{z_2-z_1}{\sqrt{\Sigma(x_2-x_1)^2}}$

Example 24 :
Find the direction ratios and direction cosines of the line joining the points A(6, –7, –1) and B (2, –3, 1).

Sol. Direction ratios of AB are (4, – 4, –2) = (2, –2, –1)

$a^2 + b^2 + c^2 = 9$. Direction cosines are $\left(\pm\frac{2}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}\right)$.

Example 25 :
If 3, – 4, 12 are direction ratios of a straight line, then find its direction cosines.

Sol. $\frac{3}{\sqrt{3^2+(-4)^2+12^2}}, \frac{-4}{\sqrt{3^2+(-4)^2+12^2}}, \frac{12}{\sqrt{3^2+(-4)^2+12^2}}$

i.e. $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$

Example 26 :
If a line makes angles α, β, γ with OX, OY, OZ respectively, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Sol. Let l, m, n be the d.c.'s of the given line, then l = cos α, m = cos β, n = cos γ

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

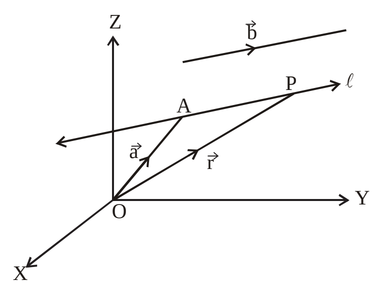
$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

EQUATION OF A LINE IN SPACE

A line is uniquely determined if (i) it passes through a given point and has given direction, or (ii) it passes through two given points.

1. Equation of a line through a given point and parallel to a given vector $\vec{b}$: Let $\vec{r}$ be the position vector of an arbitrary point P on the line (figure).



$\overrightarrow{AP}$ is parallel to the vector $\vec{b}$, i.e., $\overrightarrow{AP} = \lambda\vec{b}$, where λ is some real number. But $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$ i.e., $\lambda\vec{b} = \vec{r} - \vec{a}$

Conversely, for each value of the parameter λ, this equation gives the position vector of a point P on the line. Hnece, the vector equation of the line is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$ .... (1)

Cartesian form from vector form :
Let the coordinates of the given point A be (x_1, y_1, z_1) and the direction ratios of the line be a, b, c.
Consider the coordinates of any point P be (x, y, z). Then

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}; \ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ & $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

Substituting these values in (1) and equating the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ we get

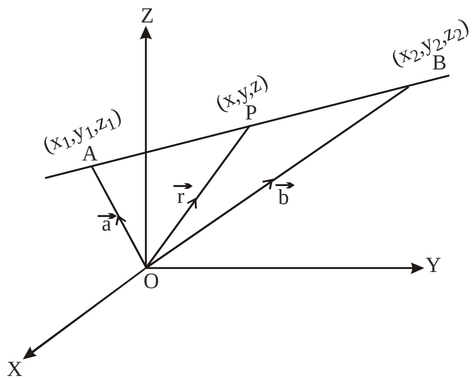
$x = x_1 + \lambda a; \ y = y_1 + \lambda b; \ z = z_1 + \lambda c$ .... (2)

These are parametric equations of the line. Eliminating the parameter λ from (2), we get

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ .... (3)

This is the Cartesian equation of the line.

2. Equation of a line passing through two given points
Let $\vec{a}$ and $\vec{b}$ be the position vectors of two points A (x_1, y_1, z_1) and B (x_2, y_2, z_2), respectively that are lying on a line figure.



Let $\vec{r}$ be the position vector of an arbitrary point P(x, y, z), then P is a point on the line if and only if $\overrightarrow{AP} = \vec{r} - \vec{a}$ and $\overrightarrow{AB} = \vec{b} - \vec{a}$ are collinear vectors. Therefore, P is on the line if and only if $\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$

or $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}),\ \lambda \in R$ ....(1)

This is the vector equation of the line.

Cartesian form from vector form :

We have $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},\ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Substituting these values in (1), we get

$$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}]$$

Equating the like coefficients of $\hat{i}, \hat{j}, \hat{k}$ we get

$$x = x_1 + \lambda(x_2 - x_1);\ y = y_1 + \lambda(y_2 - y_1);\ z = z_1 + \lambda(z_2 - z_1)$$

On eliminating λ, we obtain $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

which is the equation of the line in Cartesian form.

ANGLE BETWEEN TWO LINES

Case-I : When dc's of the lines are given

If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are dc's of given two lines, then the angle θ between them is given by-

(i) $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$ (ii) $\sin\theta$

$$= \sqrt{(\ell_1m_2 - \ell_2m_1)^2 + (m_1n_2 - m_2n_1)^2 + (n_1\ell_2 - n_2\ell_1)^2}$$

The value of $\sin\theta$ can easily be obtained by the following

form : $\sin\theta = \sqrt{\begin{vmatrix}\ell_1 & m_1\\ \ell_2 & m_2\end{vmatrix}^2 + \begin{vmatrix}m_1 & n_1\\ m_2 & n_2\end{vmatrix}^2 + \begin{vmatrix}n_1 & \ell_1\\ n_2 & \ell_2\end{vmatrix}^2}$

Case-II : When dr's of the lines are given

If a_1, b_1, c_1 and a_2, b_2, c_2 are dr's of given two lines, then the angle θ between them is given by.

(i) $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

(ii) $\sin\theta = \frac{\sqrt{\Sigma(a_1b_2 - a_2b_1)^2}}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Conditions of Parallelism and Perpendicularity of Two Lines : If two lines are parallel then angle between them is 0° and if they are perpendicular then angle between them is 90°. In these cases using above formulae for $\sin\theta$ and $\cos\theta$ respectively, we shall get the following conditions.

Case-I : When dc's of two lines AB and CD, say ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are known.

$AB \,||\, CD \Leftrightarrow \ell_1 = \ell_2,\ m_1 = m_2,\ n_1 = n_2$

$AB \perp CD \Leftrightarrow \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

Case-II : When dr's of two lines AB and CD, say a_1, b_1, c_1 and a_2, b_2, c_2 are known

$$AB \,||\, CD \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$AB \perp CD \Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Note: If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are the direction cosines of two concurrent lines then the d.c.'s of the lines bisecting the angles between them are proportional to $\ell_1 \pm \ell_2$, $m_1 \pm m_2$, $n_1 \pm n_2$.

SKEW LINES

The straight lines which are not parallel and non-coplanar i.e. non-intersecting are called skew lines.

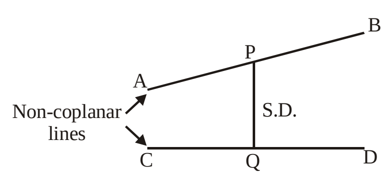
Shortest Distance (S.D.) between two skew straight lines: Shortest distance between two skew lines is perpendicular to both.

(i) If the equation are in cartesian form :

Suppose the equation of the lines are

$$\frac{x - \alpha}{\ell} = \frac{y - \beta}{m} = \frac{z - \gamma}{n} \quad ....(1)$$

and $\frac{x - \alpha'}{\ell'} = \frac{y - \beta'}{m'} = \frac{z - \gamma'}{n'}$ ....(2)



Then shortest distance between them is given by

$$S.D. = \begin{vmatrix}\alpha - \alpha' & \beta - \beta' & \gamma - \gamma'\\ \ell & m & n\\ \ell' & m' & n'\end{vmatrix} \div \sqrt{\{\sum(mn' - m'n)^2\}}$$

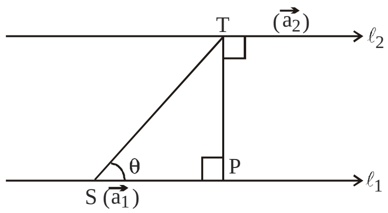
If the lines intersect, the S.D. between them is zero.

$$\text{Therefore } \begin{vmatrix}\alpha - \alpha' & \beta - \beta' & \gamma - \gamma'\\ \ell & m & n\\ \ell' & m' & n'\end{vmatrix} = 0$$

(ii) If the equation are in vector form

Suppose the equation of the lines are $\vec{r} = \vec{a}_1 - \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 - \lambda\vec{b}_2$. Then shortest distance between them is given by S.D. = $\left|\dfrac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

Distance between parallel line (vector approach)



If two lines ℓ_1 and ℓ_2 are parallel, then they are coplanar. Let the lines be given by

$\vec{r} = \vec{a}_1 + \lambda\vec{b}$ .... (1) and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ .... (2)

where $\vec{a}_1$ is the position vector of a point S on ℓ_1 and $\vec{a}_2$ is the position vector of a point T on ℓ_2 figure.

As ℓ_1, ℓ_2 are coplanar, if the foot of the perpendicular from T on the line ℓ_1 is P, then the distance between the lines ℓ_1 and ℓ_2 = |TP|. Let θ be the angle between the vectors $\overrightarrow{ST}$ and $\vec{b}$. Then $\vec{b} \times \overrightarrow{ST} = (|\vec{b}||\overrightarrow{ST}|\sin\theta)\,\hat{n}$ .... (3)

where $\hat{n}$ is the unit vector perpendicular to the plane of the lines ℓ_1 and ℓ_2. But $\overrightarrow{ST} = \vec{a}_2 - \vec{a}_1$

Therefore, from (3) we get $\vec{b} \times (\vec{a}_2 - \vec{a}_1) = |\vec{b}|\,PT\,\hat{n}$ (since PT = ST sin θ)

i.e., $|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = |\vec{b}|\,PT.1$ (as $|\hat{n}| = 1$)

Hence, the distance between the given parallel lines is

$$d = |\overrightarrow{PT}| = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$$

Example 27 :

Find the shortest distance between lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda\,(2\hat{i} + \hat{j} + 2\hat{k})$ and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\,(2\hat{i} + \hat{j} + 2\hat{k})$.

Sol. Here lines are passing through the points $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, respectively, and parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$.

Hence, the distance between the lines using the formula,

$$\frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 1 & -3 & -2 \end{vmatrix}}{3}$$

$$= \frac{|4\hat{i} - 6\hat{j} - 7\hat{k}|}{3} = \frac{\sqrt{16 + 36 + 49}}{3} = \frac{\sqrt{101}}{3}$$

Example 28 :

The equations of a line are 6x – 2 = 3y + 1 = 2z – 2. Find its direction ratios and its equation in symmetric form.

Sol. The given line is 6x – 2 = 3y + 1 = 2z – 2

$$6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z - 1) \Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z - 1}{3}$$

[We make the coefficients of x, y and z as unity]

This equation is in symmetric form.

Thus the direction ratios of the line are 1, 2 and 3 and this line passes through the point (1/3, –1/3, 1).

Example 29 :

The Cartesian equation of a line is $\dfrac{x+3}{2} = \dfrac{y-5}{4} = \dfrac{z+6}{2}$.

Find the vector equation for the line.

Sol. Comparing the given equation with the standard form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

We observe that $x_1 = -3, y_1 = 5, z_1 = -6; a = 2, b = 4, c = 2$.

Thus, the required line passes through the point (– 3, 5, – 6) and is parallel to the vector $2\hat{i} + 4\hat{j} + 2\hat{k}$.

Let $\vec{r}$ be the position vector of any point on the line, then the vector equation of the line is given by

$$\vec{r} = (-3\hat{i} + 5\hat{j} - 6\hat{k}) + \lambda\,(2\hat{i} + 4\hat{j} + 2\hat{k}).$$

Example 30 :

Show that the lines whose direction ratios are –2, –3, 1 and 2, –2, –2 are perpendicular

Sol. We have $a_1a_2 + b_1b_2 + c_1c_2 = (-2)(2) + (-3)(-2) + 1(-2)$
$= -4 + 6 - 2 = 0$

∴ The given lines are perpendicular.

Example 31 :

Find the shortest distance between the lines

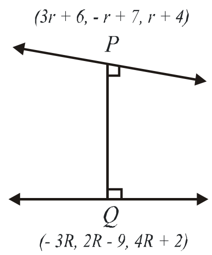
$\dfrac{x-6}{3} = \dfrac{y-7}{-1} = \dfrac{z-4}{1}$ and $\dfrac{x}{-3} = \dfrac{y+9}{2} = \dfrac{z-2}{4}$.

Also find the equation of the line of the shortest distance.

Sol. The coordinates of any point P on the first line are (3r + 6, – r + 7, r + 4) and those of Q on the second line are (–3R, 2R– 9, 4R + 2) where r is proportional to the distance of P from the point (6, 7, 4) and R to the distance of Q from the point (0, - 9, 2). The d.c.s' of PQ are proportional to – 3R – 3r – 6, 2R + r – 16, 4R – r – 2. The points P and Q will be nearest to each other if PQ is at right angles to both the lines. Therefore, we have

$3(-3R - 3r - 6) + (-1)(2R + r - 16) + 1(4R - r - 2) = 0$

and $-3(-3R - 3r - 6) + 2(2R + r - 16) + 4(4R - r - 2) = 0$



[Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$]
i.e., $7R + 11r + 4 = 0$ and
$29R + 7r - 22 = 0$
Solving these, we get r = – 1, R = 1.
P and Q are the points (3, 8, 3) and (–3, –7, 6) respectively.
The shortest distance

$$PQ = \sqrt{(-3-3)^2 + (-7-8)^2 + (6-3)^2} = \sqrt{270} \text{ units.}$$

The equation of PQ is

$$\frac{x-3}{-3-3} = \frac{y-8}{5} = \frac{z-3}{-1} \Rightarrow \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Example 32 :

Find the angle between lines $2x = 3y = -z$ and $6x = -y = -4z$.

Sol. Equation of given lines is symmetric form are

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \quad ......(1); \quad \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3} \quad .....(2)$$

If θ be the angle between these lines, then

$$\cos\theta = \frac{3(2) + 2(-12) + (-6)(-3)}{\sqrt{9+4+36}\sqrt{4+144+9}} = 0 \Rightarrow \theta = 90^\circ$$

Example 33 :

Find the shortest distance between the two lines whose vector equations are given by :

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\,(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ and}$$

$$\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \mu\,(3\hat{i} + 4\hat{j} + 5\hat{k})$$

Sol. If the equations of the lines are $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$, then shortest distance 'd' between them is

given by $d = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$ .... (1)

Here, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k},\ \vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} + 2\hat{j} + 2\hat{k}$ ...... (2)

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = (15+16)\,\hat{i} - (10-12)\,\hat{j} + (8-9)\,\hat{k}$$

$= \hat{i} + 2\hat{j} - \hat{k}$ ...... (3)

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$ ...... (4)

and $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_2 \times \vec{b}_1) = (\hat{i} + 2\hat{j} + 2\hat{k})\cdot(-\hat{i} + 2\hat{j} - \hat{k})$

$= 1 \times (-1) + 2 \times 2 + 2 \times (-1) = 1$ ...... (5)

Substituting the values from (4) and (5) in eq. (1), we get

$$d = \left| \frac{1}{\sqrt{6}} \right| = \frac{1}{\sqrt{6}}$$

TRY IT YOURSELF-3

Q.1 If a line has the direction ratios – 18, 12, – 4, then what are its direction cosines ?

Q.2 Find the angle between the lines whose direction ratios are a, b, c and b – c, c – a, a – b.

Q.3 If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (– 4, 3, – 6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Q.4 Find the angle between the pair of lines:

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

Q.5 Find the values of p so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \text{ are at}$$

right angles.

Q.6 Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Q.7 If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

Q.8 Find the shortest distance between lines

$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{i} - 2\hat{j} + 2\hat{k})$ and

$\vec{r} = -4\hat{i} - \hat{k} + \mu\,(3\hat{i} - 2\hat{j} - 2\hat{k})$.

Q.9 Find the angle between the pairs of lines:

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\,(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu\,(\hat{i} + 2\hat{j} + 2\hat{k})$

ANSWERS

(1) $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ **(2)** $\pi/2$ **(3)** 0°

(4) $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$ **(5)** $\frac{70}{11}$ **(6)** $2\sqrt{29}$

(7) –10/7 **(8)** 9 **(9)** $\cos^{-1}\left(\frac{19}{21}\right)$

THE PLANE

Consider the locus of a point P(x, y, z). If x, y, z are allowed to vary without any restriction for their different combinations, we have a set of points, the surface on which these points lie, is called the locus of P. It may be a plane or any curved surface. If Q be any other point on it's locus and all points of the straight line PQ lie on it is a plane.

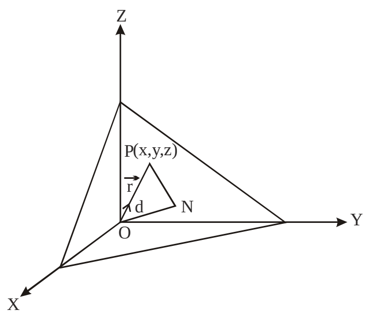
In other words if the straight line PQ, however small and in whatever direction it may be, lies completely on the locus, it is otherwise any curved surface.

Equations of a Plane in normal form :

Consider a plane whose perpendicular distance from the origin is d ($d \neq 0$). Fig. If $\overrightarrow{ON}$ is the normal from the origin to the plane, and $\hat{n}$ is the unit normal vector along $\overrightarrow{ON}$. Then $\overrightarrow{ON} = d\hat{n}$. Let P be any point on the plane. Therefore, $\overrightarrow{NP}$ is perpendicular to $\overrightarrow{ON}$. Therefore, $\overrightarrow{NP} \cdot \overrightarrow{ON} = 0$ ... (1)

Let $\vec{r}$ be the position vector of the point P, then

$$\overrightarrow{NP} = \vec{r} - d\,\hat{n} \quad (\text{as } \overrightarrow{ON} + \overrightarrow{NP} = \overrightarrow{OP})$$



Therefore, (1) becomes

$$(\vec{r} - d\,\hat{n}).d\,\hat{n} = 0 \quad \text{or} \quad (\vec{r} - d\,\hat{n}).\,\hat{n} = 0 \qquad (d \neq 0)$$

or $\vec{r}\hat{n} - d\hat{n}\,.\,\hat{n} = 0$ i.e., $\vec{r}.\hat{n} = d$ (as $\hat{n}.\hat{n} = 1$) .... (2)

This is the vector form of the equation of the planes.

Cartesian form :

Equation (2) gives the vector equation of a plane, where $\hat{n}$ is the unit vector normal to the plane.

Let P(x, y, z) be any point on the plane. Then

$$\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Let ℓ, m, n be the direction cosines of $\hat{n}$. Then

$$\hat{n} = \ell\hat{i} + m\hat{j} + n\hat{k}$$

Therefore, (2) gives $(x\hat{i} + y\hat{j} + z\hat{k}).(\ell\hat{i} + m\hat{j} + n\hat{k}) = d$

i.e., $\ell x + my + nz = d$. This is the Cartesian equation of the plane in the normal form.

Plane Parallel to the Coordinate Planes :

(i) Equation of y-z plane is x = 0

(ii) Equation of z-x plane is y = 0

(iii) Equation of x-y plane is z = 0

(iv) Equation of the plane parallel to x-y plane at a distance c (if c > 0, towards positive z-axis) is z = c. Similarly, planes parallel to y–z plane and z–x plane are respectively x = c and y = c.

Division by Coordinate Planes :

The ratios in which the line segment PQ joining P (x_1, y_1, z_1) and Q(x_2, y_2, z_2) is divided by coordinate planes are as follows.

(i) by yz - plane : $-\dfrac{x_1}{x_2}$ ratio

(ii) by zx - plane : $-\dfrac{y_1}{y_2}$ ratio

(iii) by xy - plane : $-\dfrac{z_1}{z_2}$ ratio

Equation of Planes Parallel to the Axes :

If a = 0, the plane is parallel to x-axis i.e. equation of the plane parallel to x-axis is by + bz + d = 0

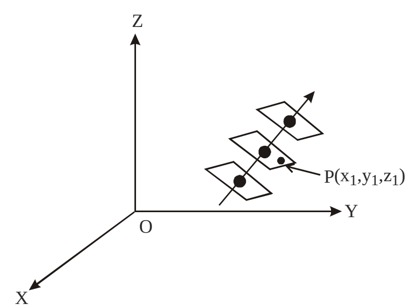
Similarly, equation of plane || to y-axis and || to z-axis are ax + cz + d = 0 and ax + by + d = 0, respectively

Equation of a plane perpendicular to a given vector and passing through a given point :

In the space, there can be many planes that are perpendicular to the given vector, but through a given point P(x_1, y_1, z_1), only one such plane exists (Fig.).

Let a plane pass through a point A with position vector $\vec{a}$ and perpendicular to the vector $\vec{N}$.

Let $\vec{r}$ be the position vector of any point P(x, y, z) in the plane. (Fig).



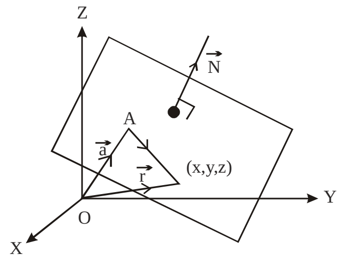
Then the point P lies in the plane if and only if $\overrightarrow{AP}$ is perpendicular to $\vec{N}$. i.e., $\overrightarrow{AP}.\vec{N} = 0$.

But $\overrightarrow{AP} = \vec{r} - \vec{a}$. Therefore, $(\vec{r} - \vec{a}).\vec{N} = 0$ ....(1)

This is the vector equation of the plane.

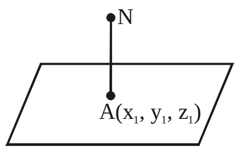
Cartesian form : Let the given point A be (x_1, y_1, z_1), P be (x, y, z) and direction ratios of $\vec{N}$ are A, B and C. Then,

$$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k},\ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \text{ and } \vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$$



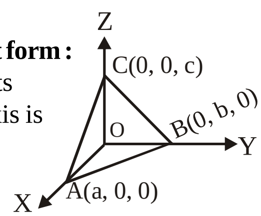
Now, $(\vec{r}-a).\vec{N}=0$, So

$[(x-x_1)\,\hat{i}+(y-y_1)\,\hat{j}+(z-z_1)\,\hat{k}].(A\hat{i}+B\hat{j}+C\hat{k})=0$



i.e., $A(x-x_1)+B(y-y_1)+C(z-z_1)=0$
Equation of plane passing through origin is
$Ax+By+Cz=0$

Equation of a Plane in intercept form :
Equation of the plane which cuts off intercepts a, b, c, from the axis is



$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Proof : Equation of plane passing through three points A (a, 0, 0), B (0, b, 0) and C (0, 0, c) will be

$$\begin{vmatrix} x-a & y-0 & z-0 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}=0$$

$\Rightarrow (x-a)\,bc - y\,(-ac-0) + z\,(0+ab) = 0$
$\Rightarrow xbc + yac + zab = abc$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Note : Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right|$

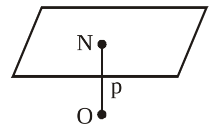
$$=\frac{1}{2}\left|(b\hat{j}-a\hat{i})\times(c\hat{k}-b\hat{j})\right|=\frac{1}{2}\left|bc\hat{i}+ac\hat{j}+ab\hat{k}\right|$$

$$=\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}=\sqrt{\left(\frac{ab}{2}\right)^2+\left(\frac{bc}{2}\right)^2+\left(\frac{ca}{2}\right)^2}$$

$\therefore$ Area of ΔABC

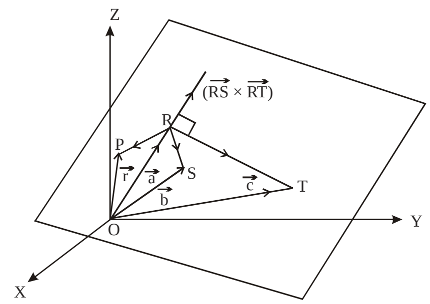
$$=\sqrt{(\text{area of } \Delta OAB)^2+(\text{area of } \Delta OBC)^2+(\text{area of } \Delta OCA)^2}$$

Equation of a Plane in Normal Form :
If the length of the perpendicular distance of the plane from the origin is p and direction cosines of this perpendicular are (ℓ, m, n), then the equation of the plane is $\ell x+my+nz=p$.



In solving problems of plane, first consider its normal. In the equation ax + by + cz + d = 0, a, b, c are the direction ratios of the normal of the plane.

Equation of a plane passing through three non collinear points
Let R, S and T be three non collinear points on the plane with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively (Fig.).



The vectors $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are in the given plane. Therefore, the vector $\overrightarrow{RS}\times\overrightarrow{RT}$ is perpendicular to the plane containing points R, S and T.
Let $\vec{r}$ be the position vector of any point P in the plane.
Therefore, the equation of the plane passing through R and perpendicular to the vector $\overrightarrow{RS}\times\overrightarrow{RT}$ is

$$(\vec{r}-\vec{a}).(\overrightarrow{RS}\times\overrightarrow{RT})=0$$

or $\quad (\vec{r}-\vec{a}).[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0 \qquad$ ....(1)

This is the equation of the plane in vector form passing through three noncollinear points.

Cartesian form : Let (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) be the coordinates of the points R, S and T respectively. Let (x, y, z) be the coordinates of any point P on the plane with position vector $\vec{r}$. Then

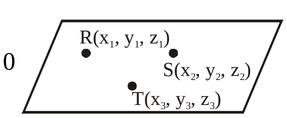
$$\overrightarrow{RP}=(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k},$$

$$\overrightarrow{RS}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k},$$

$$\overrightarrow{RT}=(x_3-x_1)\hat{i}+(y_3-y_1)\hat{j}+(z_3-z_1)\hat{k}$$

Substituting these values in equation (1) of the vector form & expressing it in the form of a determinant, we have

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_{11} \end{vmatrix}=0$$



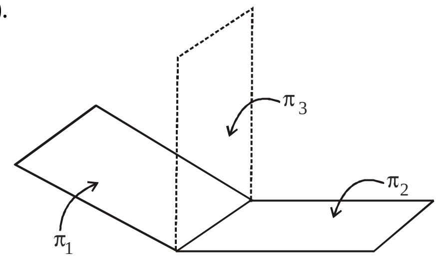
which is the equation of the plane in Cartesian form passing through three non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3)

Planes parallel to a given Plane :

Equation of a plane parallel to the plane ax + by + cz +d= 0 is ax + by + cz + d' = 0 d' is to be found by other given function.

Plane passing through the intersection of two given planes :

Let π_1 and π_2 be two planes with equations $\vec{r}.\hat{n}_1 = d_1$ and $\vec{r}.\hat{n}_2 = d_2$ respectively. The position vector of any point on the line of intersection must satisfy both the equations (Fig).



If $\vec{t}$ is the position vector of a point on the line, then

$$\vec{t}.\hat{n}_1 = d_1 \text{ and } \vec{t}.\hat{n}_2 = d_2$$

Therefore, for all real values of λ, we have

$$\vec{t}.(\hat{n}_1 + \lambda\hat{n}_2) = d_1 + \lambda d_2$$

Since $\vec{t}$ is arbitrary, it satisfies for any point on the line.

Hence, the equation $\vec{r}.(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2$ represents a plane π_3 which is such that if any vector $\vec{r}$ satisfies both the equations π_1 and π_2, it also satisfies the equation π_3 i.e., any plane passing through the intersection of the planes $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$

has the equation $\vec{r}.(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2$ .... (1)

Cartesian form : In Cartesian system, let

$\vec{n}_1 = A_1\hat{i} + B_1\hat{j} + C_1\hat{k}$, $\vec{n}_2 = A_2\hat{i} + B_2\hat{j} + C_2\hat{k}$,

$r = x\hat{i} + y\hat{j} + z\hat{k}$. Then (1) becomes

$$x\,(A_1 + \lambda A_2) + y\,(B_1 + \lambda B_2) + z\,(C_1 + \lambda C_2) = d_1 + \lambda d_2$$

or $(A_1x + B_1y + C_1z - d_1) + \lambda\,(A_2x + B_2y + C_2z - d_2) = 0$ .... (2)

which is the required Cartesian form of the equation of the plane passing through the intersection of the given planes for each value of λ.

Condition of coplanarity of four points

Four points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ will be coplanar if a plane through any three of them passes through the fourth also i.e. if

$$\begin{vmatrix} x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Angle between two planes :

The angle between two planes is defined as the angle between their normals. If θ is an angle between the two planes, then so is 180 – θ. We shall take the acute angle as the angles between two planes.

If $\vec{n}_1$ and $\vec{n}_2$ are normals to the planes and θ be the angle between the planes $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$

Then θ is the angle between the normals to the planes drawn from some common point.

We have, $\cos\theta = \left|\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$

Cartesian form : Consider two planes ax + by + cz + d = 0 and a'x + b'y + cz' + d' = 0. Angle between these planes is the angle between their normals.

$$\cos\theta = \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}$$

∴ Planes are perpendicular if aa' + bb' + cc' = 0 and they are parallel if a/a' = b/b' = c/c'.

Example 34 :

Find the equation of the plane through the points A(2, 2, –1), B(3, 4, 2) and C(7, 0, 6)

Sol. The general equation of a plane through (2, 2, –1) is

$a\,(x - 2) + b(y - 2) + c(z + 1) = 0$ ...(i)

It will pass through B(3, 4, 2) and C(7, 0, 6) if

$a\,(3 - 2) + b\,(4 - 2) + c\,(2 + 1) = 0$

or, $a + 2b + 3c = 0$ ...(ii)

& $a\,(7 - 2) + b\,(0 - 2) + c\,(6 + 1) = 0$

or, $5a - 2b + 7c = 0$ ...(iii)

Solving (ii) and (iii) by cross multiplication, we get

$$\frac{a}{14 + 6} = \frac{b}{15 - 7} = \frac{c}{-2 - 10} \text{ or, } \frac{a}{5} = \frac{b}{2} = \frac{c}{-3} = \lambda \text{ (say)}$$

⇒ $a = 5\lambda, b = 2\lambda, c = -3\lambda$

Substituting the values of a, b and c in (i), we get

$5\lambda(x - 2) + 2\lambda(y - 2) - 3\lambda(z + 1) = 0$

or, $5(x - 2) + 2(y - 2) - 3(z + 1) = 0$

⇒ $5x + 2y - 3z = 17$, which is the required equation of the plane.

Example 35 :

Find the vector equations of the plane passing through the points R (2, 5, – 3), S (– 2, – 3, 5) and T(5, 3,– 3).

Sol. Let $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$, $\vec{b} = -2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{c} = 5\hat{i} + 3\hat{j} - 3\hat{k}$

Then the vector equation of the plane passing through $\vec{a}, \vec{b}$ and $\vec{c}$ and is given by

$$(\vec{r} - \vec{a}).(\overrightarrow{RS} \times \overrightarrow{RT}) = 0 \text{ or } (\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

i.e., $[\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})].[(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j})] = 0$

Example 36 :

Find the angle between the planes x + y + 2z = 9 and 2x – y + z = 15.

Sol. The angle between x + y + 2z = 9 & 2x – y + z = 15 is given

by $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$$\cos\theta = \frac{(1)(2)+1(-1)+(2)(1)}{\sqrt{1^2+1^2+2^2}\sqrt{2^2+(-1)^2+1^2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Example 37 :

Find the angle between the two planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7 using vector method.

Sol. The angle between two planes is the angle between their normals. From the equation of the planes, the normal vectors are $\vec{N}_1 = 2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{N}_2 = 3\hat{i}-6\hat{j}-2\hat{k}$

$$\cos\theta = \left|\frac{\vec{N}_1.\vec{N}_2}{|\vec{N}_1||\vec{N}_2|}\right| = \left|\frac{(2\hat{i}+\hat{j}-2\hat{k}).(3\hat{i}-6\hat{j}-2\hat{k})}{\sqrt{4+1+4}\sqrt{9+36+4}}\right| = \left(\frac{4}{21}\right).$$

Hence, $\theta = \cos^{-1}\left(\frac{4}{21}\right)$.

Example 38 :

Find the ratio in which the line joining the points (3, 5, –7) and (–2, 1, 8) is divided by yz-plane.

Sol. Let the line joining the points (3, 5, –7) and (–2, 1, 8) divides yz-plane in the ratio λ : 1, then coordinates of the dividing point will be $\left(\dfrac{-2\lambda+3}{\lambda+1}, \dfrac{\lambda+5}{\lambda+1}, \dfrac{8\lambda-7}{\lambda+1}\right)$

Now above points lies on the yz-plane, so its x-coordinate should be zero i.e. $\dfrac{-2\lambda+3}{\lambda+1} = 0 \Rightarrow \lambda = \dfrac{3}{2}$

Hence yz-plane divides line joining the given points in the ratio $\frac{3}{2}$: 1 or 3 : 2.

Example 39 :

Express the equation of a plane

$\vec{r} = \hat{i} - 2\hat{j} + \lambda(2\hat{i} - \hat{j} + 3\hat{k}) + \mu(3\hat{i} + 4\hat{j} - \hat{k})$ in

(a) cartesian form. (b) Scalar product form.

Sol. (a) Clearly plane is passing through the point $\hat{i} - 2\hat{j}$ and parallel to vectors $2\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

∴ Equation of plane is

$$\begin{vmatrix} x-1 & y+2 & z-0 \\ 2 & -1 & 3 \\ 3 & 4 & -1 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(-11) - (y+2)(-11) + z(11) = 0$

$\Rightarrow x - 1 - y - 2 - z = 0 \Rightarrow x - y - z = 3$

$\Rightarrow x(1) + y(-1) + z(-1) = 3$

(b) Therefore equation of plane is scalar product form is $\vec{r}\cdot(\hat{i} - \hat{j} - \hat{k}) = 3$.

COPLANARITY

Let the two lines be

$$\frac{x-\alpha_1}{\ell_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1} \quad \text{..........(1) and}$$

$$\frac{x-\alpha_2}{\ell_2} = \frac{y-\beta_2}{m_2} = \frac{z-\gamma_2}{n_2} \quad \text{..........(2)}$$

These lines will coplanar if

$$\begin{vmatrix} \alpha_2-\alpha_1 & \beta_2-\beta_1 & \gamma_2-\gamma_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

The plane containing the two lines is

$$\begin{vmatrix} x-\alpha_1 & y-\beta_1 & z-\gamma_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

Condition of coplanarity if both lines are in general form:

Let the lines be ax + by + cz + d = 0 = a'x + b'y + c'z + d' and αx + βy + γz + δ = 0 = α'x + β'y + γ'z + δ'

These are coplanar if $\begin{vmatrix} a & b & c & d \\ a' & b' & c' & d' \\ \alpha & \beta & \gamma & \delta \\ \alpha' & \beta' & \gamma' & \delta' \end{vmatrix} = 0$

Coplanarity of two lines in vector form :

Let the given lines be

$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ .... (1) and

$\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ .... (2)

The line (1) passes through the point, say A, with position vector $\vec{a}_1$ and is parallel to $\vec{b}_1$.

The line (2) passes through the point, say B with position vector $\vec{a}_2$ and is parallel to $\vec{b}_2$. Thus, $\overrightarrow{AB} = \vec{a}_2 - \vec{a}_1$

The given lines are coplanar if and only if $\overrightarrow{AB}$ is perpendicular to $\vec{b}_1 \times \vec{b}_2$.

i.e. $\overrightarrow{AB}.(\vec{b}_1 \times \vec{b}_2) = 0$ or $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$

Example 40 :

Show that the lines $\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}$,

$3x+2y+z-2=0=x-3y+2z-13$ are coplanar and find the equation to the plane in which they lie.

Sol. The general equation of the plane through the second line is $3x+2y+z-2+k(x-3y+2z-13)=0$

$\Leftrightarrow$ $x(3+k)+y(2-3k)+z(1+2k)-2-13k=0$;

k being the parameter

This contains the first line only if

$3(3+k)+(2-3k)-2(1+2k)=0 \Rightarrow k=9/4$

Hence the equation of the plane which contains the two lines is $21x-19y+22z-125=0$

This plane clearly passes through the point (–5, –4, 7).

POINT WITH RESPECT TO PLANE

Position of Two Points w.r.t. a Plane :

Two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on the same or opposite sides of a plane ax + by + cz = 0 according to $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ are of same or opposite signs.

The plane divides the line joining the points P and Q externally or internally according to P and Q are lying on same or opposite sides of the plane.

Distance of a Point from a Plane

Vector form : Consider a point P with position vector $\vec{a}$ and a plane π_1 whose equation is $\vec{r}.\hat{n}=d$ then distance is

$|d-\vec{a}.\hat{n}|$

which is the length of the perpendicular from a point to the given plane.

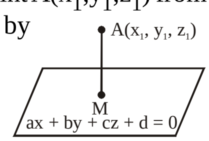
1. If the equation of the plane π_2 is in the form $\vec{r}.\vec{N}=d$, where $\vec{N}$ is normal to the plane, then the perpendicular distance is $\frac{|\vec{a}.\vec{N}-d|}{|\vec{N}|}$

2. The length of the perpendicular from origin O to the plane $\vec{r}.\vec{N}=d$ is $\frac{|d|}{|\vec{N}|}$ (since $\vec{a}=0$)

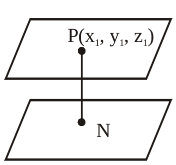
Cartesian form :

Perpendicular distance p, of the point $A(x_1,y_1,z_1)$ from the plane ax + by + cz + d = 0 is given by

$$p=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{(a^2+b^2+c^2)}}$$



Distance between two parallel planes



Let $a_1x+b_1y+c_1z+d_1=0$ and $a_1x+b_1y+c_1z+d_2=0$ be two parallel planes.

Then the distance between them can be obtained by taking any point $P(x_1, y_1, z_1)$ on any one of the given planes and finding the perpendicular distance from $P(x_1, y_1, z_1)$ on the other.

Note :

(i) Planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are

(a) parallel but not identical if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\neq\frac{d_1}{d_2}$

(b) perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

(c) identical if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$

(ii) The equation of a plane parallel to the plane ax + by + cz + d is ax + by + cz + k = 0, where k is an arbitrary constant and is determined by the given condition.

(iii) Distance between two parallel planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$ is equal to $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$.

(iv) 3 planes $a_rx+b_ry+c_rz=d_r$ r = 1, 2, 3

(a) Can intersect at a point ≡ system of equations in 3 variables having unique solution.

(b) Can intersect coaxially ≡ system of equations in 3 variables having infinite solutions.

(c) May not have a common point ≡ system of equations in 3 variables having no solution.

Example 41 :

Find the distance of the point (2, 1, 0) from the plane $2x+y+2z+5=0$

Sol. Required distance = $\left|\frac{2\times2+1+2\times0+5}{\sqrt{2^2+1^2+2^2}}\right|=\frac{10}{3}$

Example 42:

Find the distance between the parallel planes $2x-y+2z+3=0$ and $4x-2y+4z+5=0$.

Sol. Let $P(x_1, y_1, z_1)$ be any point on $2x-y+2z+3=0$, then, $2x_1-y_1+2z_1+3=0$ ...(i)

The length of the perpendicular from $P(x_1, y_1, z_1)$ to $4x-2y+4z+5=0$ is

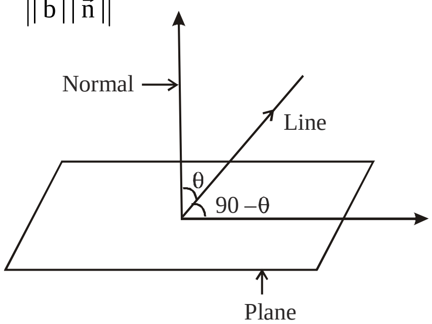
$$=\left|\frac{4x_1-2y_1+4z_1+5}{\sqrt{4^2+(-2)^2+4^2}}\right|=\left|\frac{2(2x_1-y_1+2z_1)+5}{\sqrt{36}}\right|$$

$$=\left|\frac{2(-3)+5}{6}\right|=\frac{1}{6} \quad \text{[Using (i)]}$$

ANGLE BETWEEN A LINE AND A PLANE

Vector form : If the equation of the line is $\vec{r} = \vec{a} + \lambda\vec{b}$ and the equation of the plane is $\vec{r}.\vec{n} = d$. Then the angle θ between the line & the normal to the plane is

$$\cos\theta = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$$



and so the angle φ between the line and the plane is given by 90 – θ, i.e.,

$\sin(90 - \theta) = \cos\theta$

i.e. $\sin\phi = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$ or $\phi = \sin^{-1}\left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$

Cartesian form : Let equations of the line and plane be

$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ and $ax + by + cz + d = 0$

respectively and θ be the angle which line makes with the plane. Then (π/2 – θ) is the angle between the line and the normal to the plane.

So $\sin\theta = \frac{a\ell + bm + cn}{\sqrt{(a^2 + b^2 + c^2}\sqrt{\ell^2 + m^2 + n^2}}$

Line is parallel to plane : If θ = 0 i.e $a\ell + bm + cn = 0$

Line is { to the plane : If line is parallel to the normal of the plane i.e. if $\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$

Condition in order that the line may lie on the given plane:

The line $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ will lie on the plane

$Ax + By + Cz + D = 0$ if

(i) $A\ell + Bm + Cn = 0$ and **(ii)** $Ax_1 + By_1 + Cz_1 + D = 0$

Family of Plane :

Equation of plane passing through the line of intersection of two planes u = 0 and v = 0 is u + λv = 0.

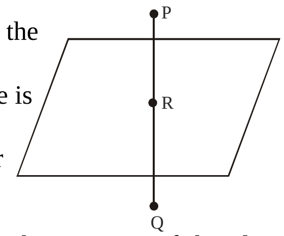
Image of point in a plane :

In order to find the image of a point P (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$, assume it as a mirror and find the image of the point. Let Q (x_2, y_2, z_2) be the image of the point P (x_1, y_1, z_1) in the plane then

(i) line PQ is perpendicular to the plane.

(ii) the plane passes through the middle point of PQ.

Hence equation of the line is



$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{z_2 - z_1}{c} = r$$

and the middle point satisfies the equation of the plane

i.e., $a\left(\frac{x_2 + x_1}{2}\right) + b\left(\frac{y_2 + y_1}{2}\right) + c\left(\frac{z_2 + z_1}{2}\right) + d = 0$

The co-ordinates of Q can be obtained by solving these equations.

Example 43 :

Find the angle between the plane 2x + y – 3z + 4 + 0 and the straight line having direction ratios 3, 2, 4.

Sol. Required angle is given by

$$\sin\theta = \frac{3.2 + 2.1 + 4(-3)}{\sqrt{2^2 + 1^2 + (-3)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$= \frac{6 + 2 - 12}{\sqrt{14}\sqrt{29}} = \frac{-4}{\sqrt{406}} \quad \Rightarrow \theta = \sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$$

Note : If $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represents a pair of plane, then the angle between the planes is given

by $\tan\theta = \frac{2\sqrt{f^2 + g^2 + h^2 - ab - bc - ca}}{a + b + c}$

TRY IT YOURSELF-4

Q.1 Find the perpendicular distance of the point (2, 3, – 5) from the plane x+ 2y – 2z = 9

Q.2 Find dc's of the normal to the plane **r**.(2**i** – 3**j** + 6**k**) + 2 = 0 and its distance from the origin.

Q.3 Find the coordinates of the foot of the perpendicular drawn from the origin 2x + 3y + 4z – 12 = 0

Q.4 Find the intercepts cut off by the plane 2x + y – z = 5.

Q.5 Find the equation of the plane through the intersection of the planes 3x – y + 2z – 4 = 0 and x + y + z – 2 = 0 and the point (2, 2, 1).

Q.6 Distance between the two planes: 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(A) 2 units (B) 4 units

(C) 8 units (D) $\frac{2}{\sqrt{29}}$ units.

Q.7 The planes: 2x – y + 4z = 5 and 5x – 2.5y + 10z = 6 are –

(A) Perpendicular (B) Parallel

(C) intersect y-axis (D) passes through (0, 0, 5/4)

Q.8 A variable plane at a distance of one unit from the origin cuts the coordinates axes at A, B and C. If the centroid D(x, y, z) of triangle ABC satisfies the relation

$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, the value of k is

(A) 3 (B) 1
(C) 1/3 (D) 9

Q.9 Find equation of plane passing through line $2x+y+z-3=0$, $3x+y+z=5$, whose perpendicular distance from, (2, 1, –1) is $1/\sqrt{6}$.

(A) $62x+29y+19z-105=0$
(B) $62x+29y-19z-105=0$
(C) $62x+19y+19z-95=0$
(D) $52x-29y+19z-105=0$

Q.10 A plane passes through (1, –2, 1) and is perpendicular to two planes $2x-2y+z=0$, $x-y+2z=4$. The distance of the plane from the point (1, 2, 2) is

(A) 0 (B) 1
(C) $\sqrt{2}$ (D) $2\sqrt{2}$

ANSWERS

(1) 3 **(2)** (2/7, –3/7, 6/7) ; 2/7
(3) $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$ **(4)** $\frac{5}{2}, 5$ and –5
(5) $7x-5y+4z-8=0$ **(6)** (D) **(7)** (B)
(8) (D) **(9)** (A) **(10)** (D)

IMPORTANT POINTS

1. Given vectors $x_1\vec{a}+y_1\vec{b}+z_1\vec{c}$, $x_2\vec{a}+y_2\vec{b}+z_2\vec{c}$, $x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors, will be coplanar if and only if $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}=0$

2. **Scalar triple product :**

(a) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then

$$(\vec{a}\times\vec{b}).\vec{c}=[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) [a b c] = volume of the parallelopiped whose coterminous edges are formed by $\vec{a},\vec{b},\vec{c}$

(c) $\vec{a},\vec{b},\vec{c}$ are coplanar if and only if $[\vec{a}\ \vec{b}\ \vec{c}]=0$

(d) Four points A, B, C, D with position vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively are coplanar if and only if $[\vec{AB}\ \vec{AC}\ \vec{AD}]=0$ i.e. if and only if $[\vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\ \ \vec{d}-\vec{a}]=0$

(e) Volume of a tetrahedron with three coterminous edges $\vec{a},\vec{b},\vec{c}=\frac{1}{6}\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$

(f) Volume of prism on a triangular base with three coterminous edges $\vec{a},\vec{b},\vec{c}=\frac{1}{2}\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$

3. **Lagrange's identity :**

$$(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=\begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix}=(\vec{a}.\vec{c})(\vec{b}.\vec{d})-(\vec{a}.\vec{d})(\vec{b}.\vec{c})$$

4. **Reciprocal system of vectors :**

If $\vec{a},\vec{b},\vec{c}$ be any three non coplanar vectors so that $[\vec{a}\ \vec{b}\ \vec{c}]\neq 0$ then the three vectors $\vec{a}'\vec{b}'\vec{c}'$ defined by the equations $\vec{a}'=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{b}'=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{c}'=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ are called the reciprocal system of vectors to the given vectors $\vec{a},\vec{b},\vec{c}$.

5. If $\vec{r}$ and $\vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $|\vec{r}+b\vec{s}|$ is minimum, then the value of $|b\vec{s}|^2+|\vec{r}+b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

6. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of the vertices of a cyclic quadrilateral ABCD then

$$\frac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})}+\frac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}\times\vec{b}|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})}=0$$

7. If $\vec{A},\vec{B}\ \&\ \vec{C}$ are vectors such that $|\vec{B}|=|\vec{C}|$, then $[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})]\times(\vec{B}\times\vec{C}).(\vec{B}+\vec{C})=0$

8. If $p\vec{x}+(\vec{x}\times\vec{a})=\vec{b}$; $(p\neq 0)$ then $\vec{x}=\frac{p^2\vec{b}+(\vec{b}.\vec{a})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+\vec{a}^2)}$

9. The coordinates of the origin O are (0,0,0). The coordinates of any point on the x-axis will be as (x,0,0) and the coordinates of any point in the YZ-plane will be as (0, y, z).

10. The equation of the x-axis is y = 0, z = 0. Similarly, the equation of y-axis is x = 0, z = 0 and that of z-axis is x = 0, y = 0.

11. If ax + by + cz + d = 0 be the plane, then the points (x_1, y_1, z_1) and (x_2, y_2, z_2) lie on the same side or opposite side according as $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}>0$ or <0.

12. If ℓ, m, n are the direction cosines and a, b, c are the direction ratios of a line then

$$\ell = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

13. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then $\cos\theta = |\ell_1\ell_2 + m_1m_2 + n_1n_2|$.

14. Equation of a line through a point (x_1, y_1, z_1) and having direction cosines ℓ, m, n is $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

15. Shortest distance between $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

16. The equation of a plane through a point whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $(\vec{r}-\vec{a}).\vec{N} = 0$

17. Vector equation of a plane that passes through the intersection of planes $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$ is $\vec{r}.(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2$, where λ is any nonzero constant.

18. The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r}.\hat{n} = d$ is $|d - \vec{a}.\hat{n}|$.

19. Skew line : Two straight lines are said to be skew lines if they are neither parallel nor intersecting.

Shortest distance : $\frac{\Sigma(x_2-x_1)(m_1n_2-m_2n_1)}{\sqrt{\Sigma(m_1n_2-m_2n_1)^2}}$

ADDITIONAL EXAMPLES

Example 1 :

Determine whether each statement is true or false

(a) Two lines parallel to a third line are parallel.
(b) Two lines perpendicular to a third line are parallel.
(c) Two planes parallel to a third plane are parallel.
(d) Two planes perpendicular to a third plane are parallel.
(e) Two lines parallel to a plane are parallel.
(f) Two lines perpendicular to a plane are parallel.
(g) Two planes parallel to a line are parallel.
(h) Two planes perpendicular to a line are parallel.
(i) Two planes either intersect or are parallel.
(j) Two lines either intersect or are parallel.
(k) A plane and a line either intersect or are parallel.

Ans. (a) True, (b) False, (c) True, (d) False, (e) False, (f) True, (g) False, (h) True, (i) True, (j) False, (k) True

Example 2 :

If a.b = a.c and a × b = a × c, then correct statement is -

(A) a || (b – c) (B) a ⊥ (b – c)
(C) a = 0 or b = c (D) None of these

Sol. **(C).** a. b = a.c ⇒ a.(b – c) = 0

⇒ a = 0 or b – c = 0 or a ⊥ (b – c)

⇒ a = 0 or b = c or a ⊥ (b – c) ....(1)

Also a × b = a × c ⇒ a × (b – c) = 0

⇒ a = 0 or b – c = 0 or a || (b – c)

⇒ a = 0 or b = c or a || (b – c) ....(2)

Observing to (1) and (2) we find that a = 0 or b = c

Example 3 :

If vectors ai + j + k, i + bj + k and i + j + ck (a ≠ b ≠ c ≠ 1) are coplanar, then find $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$.

Sol. Since vectors are coplanar

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 0 & 1-b & c-1 \end{vmatrix} = 0$$

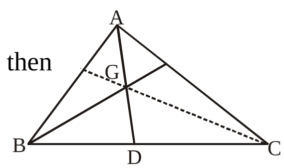
[Using $R_2 - R_1, R_3 - R_2$]

⇒ a (b – 1) (c – 1) – (1 – a) {(c – 1) – (1 –b)} = 0
⇒ a (1 – b) (1 – c) + (1 – a) (1 – c) + (1 – a) (1 – b) = 0
⇒ (a – 1 + 1) (1 – b) (1 – c) + (1– a) (1 – c) + (1 – a) (1–b)=0
⇒ (1 – b) (1 – c) + (1 – a) (1 – c) + (1 – a) (1 – b) = (1 – a) (1 – b) (1 – c)

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Example 4 :

If G is the centroid of triangle ABC then find the value of $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$.



Sol. If D is middle point of side BC then

$$\overrightarrow{GD} = \frac{1}{2}(\overrightarrow{GB} + \overrightarrow{GC})$$

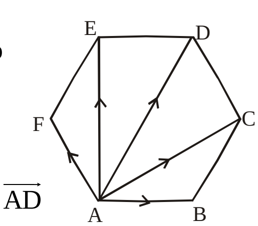
∵ G divides AD in the ratio of 2 : 1

∴ $\overrightarrow{AG} = 2\,\overrightarrow{GD}$

⇒ $-\overrightarrow{GA} = \overrightarrow{GB} + \overrightarrow{GC}$ ⇒ $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$

Example 5 :

If ABCDEF is a regular hexagon and $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = k\,\overrightarrow{AD}$, then find the value of k.



Sol. ∵ $\overrightarrow{AB} = \overrightarrow{ED}$ and $\overrightarrow{AF} = \overrightarrow{CD}$, So

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD}$

$= (\overrightarrow{AC} + \overrightarrow{CD}) + (\overrightarrow{AE} + \overrightarrow{ED}) + \overrightarrow{AD}$

$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD}$ ∴ k = 3

Example 6 :

Find the length of diagonal AC of a parallellogram ABCD whose two adjacent sides AB and AD are represented respectively by 2i + 4j – 5k and i + 2j + 3k.

Sol. $\because \ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD} = 3i + 6j - 2k$

$\therefore$ Length of the diagonal $\overrightarrow{AC} = |\overrightarrow{AC}|$

$= \sqrt{3^2 + 6^2 + (-2)^2} = 7$

Example 7 :

Find the equation of the plane through point (4, 2, 4) and perpendicular to planes 2x + 5y + 4z + 1 = 0 and $4x + 7y + 6z + 2 = 0$.

Sol. Let equation of the plane be

$a(x-4) + b(y-2) + c(z-4) = 0$ .... (1)

It is perpendicular to given two planes, So we have

$2a + 5b + 4c = 0$ ....(2)

and $4a + 7b + 6c = 0$ ....(3)

from (2) & (3) $\Rightarrow \frac{a}{2} = \frac{b}{4} = \frac{c}{-6} \Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{-3}$

Putting these proportional values of a, b, c in (1), required equation is $x + 2y - 3z + 4 = 0$.

Example 8 :

If vectors a = i – j + k, b = i + 2j – k and c = 3i + pj + 5k are coplaner, then find the value of p.

Sol. a, b, c are coplanar ⇒ [a b c] = 0

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & p & 5 \end{vmatrix} = 0$$

$\Rightarrow (10 + p + 3) - (6 - 5 - p) = 0 \Rightarrow p = -6$

Example 9 :

Let a, b, c such that |a| = 1, |b| = 1 and |c| = 2 and if a × (a × c) + b = 0 then find acute angle between a and c.

Sol. If angle between a and c is θ then -

$a.c = |a||c|\cos\theta = 1.2\cos\theta = 2\cos\theta$

but a × (a × c) + b = 0

$\Rightarrow$ (a. c) a – (a.a) c + b = 0

$\Rightarrow$ (2 cosθ).a – 1. c = – b

$\Rightarrow [(2\cos\theta)\,a - c]^2 = [-b]^2$

$\Rightarrow 4\cos^2\theta\,|a|^2 - 2.(2\cos\theta)\,a.c + |c|^2 = |b|^2$

$\Rightarrow 4\cos^2\theta - 4\cos\theta\,(2\cos\theta) + 4 = 1$

$\Rightarrow 4(1 - \cos^2\theta) = 1$ $[\because |a| = 1, |b| = 1]$

$\Rightarrow \sin\theta = 1/2 \Rightarrow \theta = \pi/6$

Example 10 :

If θ be the angle between vectors a = i + 2j + 3k and b = 3i + 2j + k , then find cosθ.

Sol. $\cos\theta = \frac{a.b}{|a||b|} = \frac{3+4+3}{\sqrt{14}\sqrt{14}} = \frac{5}{7}$

Example 11 :

If ℓi + mj + nk is a unit vector which is perpendicular to vectors 2i – j + k and 3i + 4j – k then find the value of | ℓ |.

Sol. Unit vector perpendicular to vectors

$2i - j + k$ and $3i + 4j - k$ is $= \frac{(2i - j + k) \times (3i + 4j - k)}{|(2i - j + k) \times (3i + 4j - k)|}$

$= \frac{i(1-4) - j(-2-3) + k(8+3)}{\sqrt{9+25+121}} = \frac{-3i + 5j + 11k}{\sqrt{155}}$

$\therefore \ |\ell| = \left|\frac{-3}{\sqrt{155}}\right| = \frac{3}{\sqrt{155}}$

Example 12 :

Find the equation of set of points P such that $PA^2 + PB^2 = 2k^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively.

Sol. Let the coordinates of point P be (x, y, z).

Here $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$,

$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

By the given condition $PA^2 + PB^2 = 2k^2$, we have

$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2$

i.e., $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$.

Example 13 :

Find the ratio in which the yz–plane divides the line joining the points (3, 5, –7) and (–2, 1, 8). Find also the coordinates of the point of division.

Sol. Let the yz–plane divide the line joining the given points in the ratio $m_1 : m_2$. Then the coordinates of the point of division are $\left(\frac{-2m_1 + 3m_2}{m_1 + m_2}, \frac{m_1 + 5m_2}{m_1 + m_2}, \frac{8m_1 - 7m_2}{m_1 + m_2}\right)$.

Since this point lies on the yz–plane, its x-coordinates is zero. Therefore $-2m_1 + 3m_2 = 0$, i.e. $m_1 : m_2 = 3 : 2$

The other coordinates of the point of division are now

$y = \frac{m_1 + 5m_2}{m_1 + m_2} = \frac{3 + 2.5}{3 + 5} = \frac{13}{5}$

and $z = \frac{8m_1 - 7m_2}{m_1 + m_2} = \frac{3.8 - 2.7}{3 + 2} = 2$

Example 14 :

Using section formula, prove that the three points (– 4, 6, 10), (2, 4, 6) and (14, 0, –2) are collinear.

Sol. Let A (– 4, 6, 10), B (2, 4, 6) and C(14, 0, – 2) be the given points. Let the point P divides AB in the ratio k : 1. Then coordinates of the point P are

$\left(\frac{2k-4}{k+1}, \frac{4k+6}{k+1}, \frac{6k+10}{k+1}\right)$

Let us examine whether for some value of k, the point P coincides with point C.

On putting $\frac{2k-4}{k+1}=14$, we get $k=-\frac{3}{2}$

When $k=-\frac{3}{2}$ then $\frac{4k+6}{k+1}=\frac{4(-3/2)+6}{-\frac{3}{2}+1}=0$ and

$$\frac{6k+10}{k+1}=\frac{6(-3/2)+10}{-\frac{3}{2}+1}=-2$$

Therefore, C (14, 0, –2) is a point which divides AB externally in the ratio 3 : 2 and is same as P. Hence A, B, C are collinear.

Example 15 :

Find the locus of a point, which moves in such a way that its distance from the origin is thrice the distance from xy-plane.

Sol. Let the point be P (x, y, z), then its distance from origin is

$OP=\sqrt{x^2+y^2+z^2}$

The distance of P from xy-plane is z

According to question, $\sqrt{x^2+y^2+z^2}=3z$

Thus the required locus is $x^2+y^2-8z^2=0$

Example 16 :

Find the projection of the line segment joining the points (–1, 0, 3) and (2, 5, 1) on the line whose direction ratios are 6, 2, 3.

Sol. The direction cosines ℓ, m, n of the line are given by

$$\frac{\ell}{6}=\frac{m}{2}=\frac{n}{3}=\frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{6^2+2^2+3^2}}=\frac{1}{\sqrt{49}}=\frac{1}{7}$$

$$\therefore\ \ell=\frac{6}{7},\ m=\frac{2}{7},\ n=\frac{3}{7}$$

The required projection is given by

$$=\ell(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)$$

$$=\frac{6}{7}[2-(-1)]+\frac{2}{7}(5-0)+\frac{3}{7}(1-3)$$

$$=\frac{6}{7}\times 3+\frac{2}{7}\times 5+\frac{3}{7}\times -2$$

$$=\frac{18}{7}+\frac{10}{7}-\frac{6}{7}=\frac{18+10-6}{7}=\frac{22}{7}$$

Example 17 :

Find the equation of the plane which bisects the line segment joining points (2, 3, 4) & (6, 7, 8) perpendicularly.

Sol. Mid-point of the line segment = (4, 5, 6) and its dr;s are 6 – 2, 7 – 3, 8 – 4 i.e., 1, 1, 1 Required plane passes through (4, 5, 6) and dr's of its normal are 1, 1, 1; hence its equation will be $1(x-4)+1(y-5)+1(z-6)=0$

$\Rightarrow x+y+z-15=0$

Example 18 :

Find the point of intersection of the line

$\frac{x-6}{-1}=\frac{y+1}{0}=\frac{z+3}{4}$ and the plane x + y – z = 3.

Sol. Any point on the line is (– r + 6, –1, 4r –3)

If given line meets given plane at this point, then

$(-r+6)-1-(4r-3)=3 \Rightarrow r=1$

$\therefore$ Required point of intersection = (5, –1, 1)

Example 19 :

A plane meets coordinate axes at points A,B,C. If a,b,c be centroid of Δ ABC, then find the equation of this plane.

Sol. Let $A\equiv(p,0,0)$, $B\equiv(0,q,0)$, $C\equiv(0,0,r)$.Then equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=1$ .... (1)

As given centroid ΔABC = (a, b, c).Hence

$$\frac{p+0+0}{3}=a,\ \frac{0+q+0}{3}=b,\ \frac{0+0+r}{3}=r$$

$\Rightarrow$ p = 3a, q = 3b, r = 3c

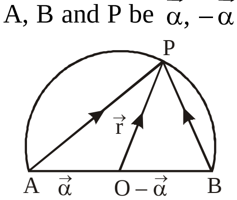
Puting value of p, q, r in (1) required equation is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$$

Example 20 :

Prove that the angle in a semi circle is a right angle.

Sol. Let O be the centre of the semi circle with AOB as its diameter. Let P be a point on the semi-circle, so that ∠APB is an angle in the semi circle. Join OP. Let O be taken as origin. Let the position vectors of A, B and P be $\vec{\alpha}$, $-\vec{\alpha}$ and $\vec{r}$ respectively.



Clearly, OA = OB = OP

Now $\overrightarrow{AP}=(\vec{r}-\vec{\alpha})$

and $\overrightarrow{BP}=(\vec{r}+\vec{\alpha})$

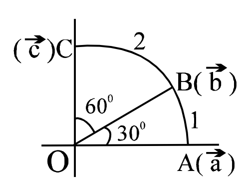
$\therefore\ \overrightarrow{AP}\cdot\overrightarrow{BP}=(\vec{r}-\vec{\alpha})\cdot(\vec{r}+\vec{\alpha})=r^2-\alpha^2=OP^2-OA^2=0$

[$\because$ OP = OA]

$\therefore$ AP ⊥ BP i.e. ∠APB = 90°.

Example 21 :

Arc AC of the quadrant of a circle with centre as origin and radius unity subtends a right angle at the origin. Point B divides the arc AC in the ratio 1 : 2. Express the vector $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.



Sol. $\because\ \vec{c} = x\vec{a} + y\vec{b}$ …(i)

Taking dot product with $\vec{a}$ in (i)

$$\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{a} + y\vec{a}\cdot\vec{b}\ ;\ 0 = x + y\frac{\sqrt{3}}{2} \quad \text{…(ii)}$$

Taking dot product with $\vec{c}$ in (i)

$$\vec{c}\cdot\vec{c} = x\,\vec{a}\cdot\vec{c} + y\,\vec{b}\cdot\vec{c}\ ;\ 1 = 0 + \frac{y}{2} \Rightarrow y = 2$$

from (ii), $x = -\sqrt{3}$ $\quad\therefore\ \vec{c} = -\sqrt{3}\,\vec{a} + 2\vec{b}$

Example 22 :

AD, BE and CF are the medians of a triangle ABC intersecting in G. Show that

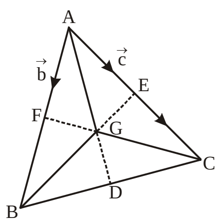
$$\Delta\,AGB = \Delta\,BGC = \Delta\,CGA = \frac{1}{3}\Delta\,ABC.$$

Sol. Let $\vec{b}, \vec{c}$ be the position vectors of B and C with respect to A as the origin of reference.

Therefore, the position vectors of D, E, F are

$\frac{1}{2}(\vec{b}+\vec{c}), \frac{1}{2}\vec{c}, \frac{1}{2}\vec{b}$ respectively.

Also the position vector of the point G, the centroid, is



$$\frac{1}{3}(0+\vec{b}+\vec{c}) = \frac{1}{3}(\vec{b}+\vec{c})$$

Area of Δ AGB = $\frac{1}{2}(\overrightarrow{AB}\times\overrightarrow{AG})$

$$= \frac{1}{2}\left|\vec{b}\times\frac{1}{3}(\vec{b}+\vec{c})\right| = \frac{1}{6}\left|\vec{b}\times\vec{b}+\vec{b}\times\vec{c}\right|$$

$$= \frac{1}{6}\left|\vec{b}\times\vec{c}\right| = \frac{1}{3}\text{ area of }\Delta\,ABC.$$

Similarly, we can show that area of Δ BGC

$$= \frac{1}{3}\text{ area of }\Delta\,ABC$$

and area of Δ CGA = $\frac{1}{3}$ area of Δ ABC

Example 23 :

Find the equation of the plane through (2, 3, – 4), (1, – 1, 3) and parallel to x-axis.

Sol. The equation of the plane passing through (2, 3, – 4) is a

$$(x-2) + b\,(y-3) + c\,(z+4) = 0 \quad \text{....(1)}$$

Since (1, – 1, 3) lies on it, we have

$$a + 4b - 7c = 0 \quad \text{....(2)}$$

Since required plane is parallel to x-axis i.e. perpendicular to YZ plane i.e.

$$1\cdot a + 0\cdot b + 0\cdot c = 0 \Rightarrow a = 0 \Rightarrow 4b - 7c = 0 \Rightarrow \frac{b}{7} = \frac{c}{4}$$

$\therefore$ Equation of required plane is 7y + 4z = 5.

Example 24 :

Two planes are given by equations x + 2y – 3z = 0 and 2x + y + z + 3 = 0. Find

(i) DC's of their normals and the acute angle between them.

(ii) DC's of their line of intersection.

(iii) Equation of the plane perpendicular to both of them through the point (2, 2, 1).

Sol. $\vec{n}_1 = \hat{i} + 2\hat{j} - 3\hat{k}\ ,\ \vec{n}_2 = 2\hat{i} + \hat{j} + \hat{k}$

(i) $$\cos\theta = \frac{\vec{n}_1 - \vec{n}_2}{|\vec{n}_1|\ |\vec{n}_2|} = \frac{1}{2\sqrt{21}}$$

(ii) $$\vec{n}_1\times\vec{n}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 1 & 1\end{vmatrix} = 5\hat{i} - 7\hat{j} - 3\hat{k}$$

DC's of line of intersection of the plane

$$\left(\pm\frac{5}{\sqrt{83}},\ \mp\frac{7}{\sqrt{83}},\ \mp\frac{3}{\sqrt{83}}\right)$$

(iii) $(\vec{r} - (2\hat{i} + 2\hat{j} + \hat{k}))\cdot(\vec{n}_1\times\vec{n}_2) = 0$

$\Rightarrow\ 5\,(x-2) - 7\,(y-2) - 3\,(z-1) = 0$

$\Rightarrow\ 5x - 7y - 3z + 7 = 0$

Example 25 :

Let equation of plane be $\vec{r}\cdot(6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$ then find perpendicular distance of plane from origin and also find direction cosines of this perpendicular.

Sol. Plane is $\vec{r}\cdot(6\hat{i} - 3\hat{j} - 2\hat{k}) = -1$

$$\Rightarrow\ \vec{r}\cdot\left(\frac{-6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right) = \frac{1}{7}$$

Perpendicular distance from origin = 1/6

and dcs of perpendicular = $\left(\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}\right)$.

Example 26 :

Find the vector equation of plane which is at a distance of 8 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Sol. Here, d = 8 and $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\therefore\ \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$$

Hence, the required equation of plane is, $\vec{r}.\hat{n} = d$

$$\Rightarrow\ \vec{r}\left(\frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right) = 8 \text{ or } \vec{r}\cdot(2\hat{i} + \hat{j} + 2\hat{k}) = 24$$

Example 27 :

Find the angle between the planes $2x - y + z = 11$ and $x + y + 2z = 3$.

Sol. $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}$

$$\Rightarrow \cos\theta = \frac{2\cdot 1 + (-1)\cdot 1 + 1\cdot 2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}} = \frac{1}{2} \Rightarrow \theta = \pi/3$$

Example 28 :

Find the equation of plane passing through the intersection of planes $2x - 4y + 3z + 5 = 0$, $x + y + z = 6$ and parallel to straight line having direction cosines $(1, -1, -1)$.

Sol. Equation of required plane be

$(2x - 4y + 3z + 5) + \lambda(x + y - z - 6) = 0$

i.e. $(2 + \lambda)x + (-4 + \lambda)y + z(3 - \lambda) + (5 - 6\lambda) = 0$

This plane is parallel to a straight line. So, $al + bm + cn = 0$

$1(2 + \lambda) + (-1)(-4 + \lambda) + (-1)(3 - \lambda) = 0$ i.e. $\lambda = -3$

$\therefore$ Equation of required plane is $-x - 7y + 6z + 23 = 0$.

i.e. $x + 7y - 6z - 23 = 0$.

Example 29 :

Find the angle between the line $\dfrac{x+1}{3} = \dfrac{y-1}{2} = \dfrac{z-2}{4}$ and the plane $2x + y - 3z + 4 = 0$

Sol. The given line is parallel to the vector $\vec{b} = 3\hat{i} + 2\hat{j} + 4\hat{k}$

and the given plane is normal to the vector $\vec{n} = 2\hat{i} + \hat{j} - 3\hat{k}$

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|} = \frac{(3\hat{i} + 2\hat{j} + 4\hat{k}).(2\hat{i} + \hat{j} - 3\hat{k})}{\sqrt{3^2 + 2^2 + 4^2}\sqrt{2^2 + 1^2 + 3}}$$

$$= \frac{6 + 2 - 12}{\sqrt{29}\sqrt{14}} = \frac{-4}{\sqrt{406}} \quad \therefore \theta = \sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$$

Example 30 :

Find the angle between the line $x - 2y + z = 0 = x + 2y - 2z$ and $x + 2y + z = 0 = 3x + 9y + 5z$.

Sol. Let a_1, b_1, c_1 be the direction ratios of the line $x - 2y + z = 0$ and $x + 2y - 2z = 0$. Since it lies in both the planes, therefore, it is $\perp$ to the normals to the two planes.

$\therefore$ $a_1 - 2b_1 + c_1 = 0$

$a_1 + 2b_1 - 2c_1 = 0$

Solving these two equations by cross-multiplication, we have $\dfrac{a_1}{4-2} = \dfrac{b_1}{1+2} = \dfrac{c_1}{2+2}$ or $\dfrac{a_1}{2} = \dfrac{b_1}{3} = \dfrac{c_1}{4}$

Let a_2, b_2, c_2 be the direction ratios of the line

$x + 2y + z = 0 = 3x + 9y + 5z$.

$a_2 + 2b_2 + c_2 = 0$

$3a_2 + 9b_2 + 5c_2 = 0$

$\Rightarrow \dfrac{a_2}{10-9} = \dfrac{b_2}{3-5} = \dfrac{c_2}{9-6}$ or $\dfrac{a_2}{1} = \dfrac{b_2}{-2} = \dfrac{c_2}{3}$

Let θ be the angle between the given lines. Then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(1)(2) + (-2)(3) + (3)(4)}{\sqrt{2^2 + 3^2 + 4^2}\sqrt{1^2 + (-2)^2 + (3)^2}}$$

$$= \frac{2 - 6 + 12}{\sqrt{29}\sqrt{14}} = \frac{8}{\sqrt{406}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{\sqrt{406}}\right)$$

Example 31 :

Find the coordinates of the point where the line joining the points $(2, -3, 1)$ and $(3, -4, -5)$ cuts the plane $2x + y + z = 7$.

Sol. The direction ratios of the line are

$3 - 2, -4 - (-3), -5 - 1$ i.e. $1, -1, -6$

Hence equation of the line joining the given points is

$$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = r \text{ (say)}$$

Coordinates of any point on this line are

$(r + 2, -r - 3, -6r + 1)$

If this point lies on the given plane $2x + y + z = 7$, then

$2(r + 2) + (-r - 3) + (-6r + 1) = 7 \Rightarrow r = -1$

Coordinates of the point are

$(-1 + 2, -(-1) - 3, -6(-1) + 1)$ i.e. $(1, -2, 7)$.

Example 32 :

Find the equation of the plane which contains the two parallel lines $\dfrac{x+1}{3} = \dfrac{y-2}{2} = \dfrac{z}{1}$ and $\dfrac{x-3}{3} = \dfrac{y+4}{2} = \dfrac{z-1}{1}$.

Sol. The equations of the two parallel lines are

$$\frac{x+1}{3} = \frac{y-2}{2} = \frac{z-0}{1} \qquad ....(1)$$

and $$\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1} \qquad ....(2)$$

the equation of any plane through the line (1) is

$a(x + 1) + b(y - 2) + cz = 0$ ..... (3)

where $3a + 2b + c = 0$ ....(4)

The line (2) will also lie on the plane (3) if the point $(3, -4, 1)$ lies on the plane (3), and for this we have

$a(3 + 1) + b(-4 - 2) + c = 0$ or $4a - 6b + c = 0$ ....(5)

Solving (4) and (5), we get $\dfrac{a}{8} = \dfrac{b}{1} = \dfrac{c}{-26}$

Putting the values of a, b, c in (3), the required equation of the plane is $8x + y - 26z + 6 = 0$.

QUESTION BANK | CHAPTER 10 : VECTOR & 3-DIMENSIONAL GEOMETRY

EXERCISE - 1 [LEVEL-1]

PART - 1 - VECTOR

Q.1 If i, j, k are unit vectors along the positive direction of X, Y and Z-axes, then a false statement in the following is –

(A) $\Sigma\, \hat{i} \times (\hat{j} + \hat{k}) = \vec{0}$ (B) $\Sigma\, \hat{i} \times (\hat{j} \times \hat{k}) = \vec{0}$

(C) $\Sigma\, \hat{i}\,.\,(\hat{j} \times \hat{k}) = 0$ (D) $\Sigma\, \hat{i}\,.\,(\hat{j} + \hat{k}) = 0$

Q.2 $\vec{a}, \vec{b}$ and $\vec{c}$ are nonzero coplanar vectors, then

$[2\vec{a} - \vec{b} \quad 3\vec{b} - \vec{c} \quad 4\vec{c} - \vec{a}] =$

(A) 25 (B) 0
(C) 27 (D) 9

Q.3 A space vector makes the angles 150° and 60° with the positive direction of X- and Y-axes. The angle made by the vector with the positive direction of Z-axis is –
(A) 90° (B) 60°
(C) 180° (D) 120°

Q.4 If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $3\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + \vec{c}.\vec{a} =$

(A) –1 (B) 1
(C) –3 (D) 3

Q.5 If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ 0 then angle between $\vec{a}$ & $\vec{b}$ is –

(A) π (B) 2π/3
(C) π/3 (D) π/2

Q.6 If $\vec{a}, \vec{b}$ & $\vec{c}$ are noncoplanar, then the value of

$\vec{a}.\left\{\dfrac{\vec{b} \times \vec{c}}{3\vec{b}.(\vec{c} \times \vec{a})}\right\} - \vec{b}.\left\{\dfrac{\vec{c} \times \vec{a}}{3\vec{c}.(\vec{a} \times \vec{b})}\right\}$ is –

(A) 1/6 (B) –1/6
(C) –1/3 (D) –1/2

Q.7 If $2\hat{i} + 3\hat{j}, \hat{i} + \hat{j} + \hat{k}$ and $\lambda\hat{i} + 4\hat{j} + 2\hat{k}$ taken in an order are conterminous edges of a parallelepiped of volume 2 Cu units, then value of λ is –
(A) 4 (B) 3
(C) 2 (D) – 4

Q.8 A unit vector perpendicular to both $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$ is –

(A) $\dfrac{3\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ (B) $2\hat{i} + \hat{j} + \hat{k}$

(C) $\dfrac{(2\hat{i} - \hat{j} - \hat{k})}{\sqrt{6}}$ (D) $(2\hat{i} - \hat{j} + \hat{k})$

Q.9 If $\vec{a}, \vec{b}$ & $\vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ are vectors defined by $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ and $\vec{r} = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, then the value of

$(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r} =$

(A) 3 (B) 0
(C) 1 (D) 2

Q.10 If $(\vec{a} \times \vec{b})^2 + (\vec{a}\,.\,\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}| =$

(A) 12 (B) 16
(C) 8 (D) 3

Q.11 If $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} - 3\hat{j} + \hat{k}$ are adjacent sides of a parallelogram, then the lengths of its diagonals are –

(A) $\sqrt{21}, \sqrt{13}$ (B) $\sqrt{3}, \sqrt{14}$

(C) $\sqrt{13}, \sqrt{14}$ (D) $\sqrt{21}, \sqrt{3}$

Q.12 If the volume of the parallelepiped formed by three non-coplanar vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 4 cubic units, then

$[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] =$

(A) 8 (B) 64
(C) 16 (D) 4

Q.13 If $\vec{a} = (1, 2, 3)$, $\vec{b} = (2, –1, 1)$, $\vec{c} = (3, 2, 1)$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha\,\vec{a} + \beta\,\vec{b} + \gamma\,\vec{c}$, then –

(A) $\alpha = 1, \beta = 10, \gamma = 3$ (B) $\alpha = 0, \beta = 10, \gamma = –3$
(C) $\alpha + \beta + \gamma = 8$ (D) $\alpha = \beta = \gamma = 0$

Q.14 If $\vec{a} \perp \vec{b}$ & $(\vec{a} + \vec{b}) \perp (\vec{a} + m\,\vec{b})$, then m = ?

(A) –1 (B) 1

(C) $\dfrac{-|\vec{a}|^2}{|\vec{b}|^2}$ (D) 0

Q.15 If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = ?$

(A) 3/2 (B) –3/2
(C) 2/3 (D) 1/2

Q.16 If $\vec{a}$ is vector perpendicular to both $\vec{b}$ and $\vec{c}$, then –

(A) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ (B) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{0}$

(C) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{0}$ (D) $\vec{a} + (\vec{b} + \vec{c}) = \vec{0}$

Q.17 The angle between two diagonals of a cube is
(A) $\cos^{-1}(1/3)$ (B) 30
(C) $\cos^{-1}(1/\sqrt{3})$ (D) 45°

Q.18 The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is
(A) 3 (B) $\sqrt{2}$
(C) 4 (D) $\sqrt{3}$

Q.19 If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3$, then the value of $|\vec{a}+\vec{b}|$ is
(A) equal to 1 (B) greater than 1
(C) equal to 0 (D) less than 1

Q.20 The value of $[\vec{a}-\vec{b}\;\; \vec{b}-\vec{c}\;\; \vec{c}-\vec{a}]$ is equal to –
(A) 0 (B) 1
(C) $2[\vec{a}\;\vec{b}\;\vec{c}]$ (D) 2

Q.21 If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, |\vec{b}|=5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then the area of the triangle formed by these two vectors as two sides is
(A) 15/2 (B) 15
(C) 15/4 (D) $15\sqrt{3}/2$

Q.22 Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ if $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|=$
(A) 7 (B) 14
(C) $\sqrt{7}$ (D) 21

Q.23 If direction cosines of a vector of magnitude 3 are $\frac{2}{3},-\frac{a}{3},\frac{2}{3}$ and a > 0, then vector is –
(A) $2\hat{i}+\hat{j}+\hat{k}$ (B) $2\hat{i}-\hat{j}+2\hat{k}$
(C) $\hat{i}-2\hat{j}+2\hat{k}$ (D) $\hat{i}+2\hat{j}+2\hat{k}$

PART - 2 - BASICS OF 3-D GEOMETRY

Q.24 (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an –
(A) isosceles triangle. (B) right angled triangle
(C) equilateral triangle (D) none of these

Q.25 Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).
(A) $x+z=0$ (B) $2x-y=0$
(C) $x-2z=0$ (D) $3x-z=0$

Q.26 Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio 2 : 3 internally
(A) $\left(\frac{4}{5},\frac{4}{5},\frac{27}{5}\right)$ (B) $\left(\frac{-4}{5},\frac{1}{5},\frac{27}{5}\right)$
(C) $\left(\frac{-4}{5},\frac{1}{5},\frac{6}{5}\right)$ (D) $\left(\frac{-3}{5},\frac{2}{5},\frac{27}{5}\right)$

Q.27 Given that P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.
(A) 1 : 2 internally (B) 1 : 3 internally
(C) 1 : 2 externally (D) 2 : 3 internally

Q.28 Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).
(A) 1 : 3 (B) 4 : 3
(C) 5 : 2 (D) 2 : 3

Q.29 Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).
(A) (6, 4, 2) ; (8, –10, 2) (B) (3, –4, –2) ; (8, –10, –2)
(C) (6, –4, –2) ; (8, –10, 2) (D) (3, –1, –2) ; (4, –5, 2)

Q.30 Three vertices of a parallelogram ABCD are A(3, – 1, 2), B (1, 2, – 4) and C (– 1, 1, 2). Find the coordinates of the fourth vertex.
(A) (1, –2, 8) (B) (1, 2, 5)
(C) (2, –3, 8) (D) (3, –5, 6)

Q.31 If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (– 4, 3b, –10) and R (8, 14, 2c), then find the values of a, b and c.
(A) 2, 16/3, 3 (B) –2, –16/3, 2
(C) 2, –14/3, 2 (D) –2, 19/3, 3

Q.32 Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, –2, 5).
(A) (0, 2, 0) ; (0, –6, 0) (B) (1, 2, 0) ; (0, –3, 0)
(C) (1, 2, 3) ; (1, –6, 0) (D) (0, 3, 0) ; (1, –5, 0)

Q.33 A point R with x-coordinate 4 lies on the line segment joining the points P(2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.
(A) (4, 2, 5) (B) (3, –2, 6)
(C) (4, –2, 6) (D) (1, –1, 4)

Q.34 If A and B be the point (3, 4, 5) and (–1, 3, –7) respectively. Find the equation of the set of points P such that $PA^2 + PB^2 = k^2$ where k is a constant.
(A) $x^2+y^2+z^2-2x-7y+2z=\frac{k^2-109}{2}$
(B) $x^2-y^2+z^2-2x-6y+2z=\frac{k^2-103}{2}$
(C) $x^2+2y^2+z^2-x-7y+z=\frac{k^2-109}{2}$
(D) None of these

Q.35 The distance of the point (4, 3, 5) from the y-axis is –
(A) $\sqrt{34}$ (B) 5
(C) $\sqrt{41}$ (D) $\sqrt{15}$

PART - 3 - LINE IN SPACE

Q.36 Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$.
(A) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda\,(3\hat{i}+2\hat{j}-2\hat{k})$
(B) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda\,(3\hat{i}+2\hat{j}+2\hat{k})$
(C) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda\,(-3\hat{i}-2\hat{j}-2\hat{k})$
(D) $(\hat{i}-2\hat{j}+3\hat{k})+\lambda\,(3\hat{i}-2\hat{j}-2\hat{k})$

Q.37 Find the cartesian equation of the line which passes through the point (– 2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

(A) $\frac{x+2}{3}=\frac{y+4}{5}=\frac{z+5}{3}$ (B) $\frac{x-2}{5}=\frac{y-4}{3}=\frac{z+5}{6}$

(C) $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$ (D) $\frac{x+2}{3}=\frac{y+4}{4}=\frac{z-5}{5}$

Q.38 Find the values of p so that the lines

$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$

are at right angles.

(A) 70/11 (B) 70/9
(C) 11/70 (D) 9/70

Q.39 Equation of the straight line in the plane $\vec{r}.\vec{n}=d$ which is parallel to $\vec{r}=a+\lambda\vec{b}$ and passes through the foot of perpendicular drawn from the point $P(\vec{a})$ to the plane $\vec{r}.\vec{n}=d$.

(A) $\vec{r}=\vec{a}+\left(\frac{d-\vec{a}.\vec{n}}{n^2}\right)+\vec{n}+\lambda\vec{b}$

(B) $\vec{r}=\vec{a}+\left(\frac{d-\vec{a}.\vec{n}}{n}\right)+\vec{n}+\lambda\vec{b}$

(C) $\vec{r}=\vec{a}+\left(\frac{\vec{a}.\vec{n}-d}{n^2}\right)+\vec{n}+\lambda\vec{b}$

(D) $\vec{r}=\vec{a}+\left(\frac{\vec{a}.\vec{n}-d}{n}\right)+\vec{n}+\lambda\vec{b}$

Q.40 Shortest distance between lines

$\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2}$ and $\frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}$ is

(A) 108 (B) 9
(C) 27 (D) None of these

Q.41 The equation of straight line $3x+2y-z-4=0$; $4x+y-2z+3=0$ in the symmetrical form is

(A) $\frac{x-2}{3}=\frac{y-5}{2}=\frac{z}{5}$ (B) $\frac{x+2}{3}=\frac{y-5}{-2}=\frac{z}{5}$

(C) $\frac{x+2}{3}=\frac{y-5}{2}=\frac{z}{5}$ (D) None of these

Q.42 The angle between the lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ & $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ is

(A) $\cos^{-1}(1/5)$ (B) $\cos^{-1}(1/3)$
(C) $\cos^{-1}(1/2)$ (D) $\cos^{-1}(1/4)$

Q.43 The angle between the straight lines

$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is

(A) 45° (B) 30°
(C) 60° (D) 90°

Q.44 A line with direction cosines proportional to 2, 1, 2 meets each of the lines given by the equation x = y + 2 = z ; x + 2 = 2y = 2z. The coordinates of the points of intersection are –

(A) (6, 4, 6) , (2, 4, 2) (B) (6, 6, 6) , (2, 6, 2)
(C) (6, 4, 6) , (2, 2, 0) (D) None of these

PART - 4 - PLANE

Q.45 Equation of the plane perpendicular to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and passing through the point (2, 3, 4) is –

(A) $2x+3y+z=17$ (B) $x+2y+3z=9$
(C) $3x+2y+z=16$ (D) $x+2y+3z=20$

Q.46 The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane

(A) $2x+3y+4z=0$ (B) $3x+4y+5z=7$
(C) $2x+y-2z=0$ (D) $x+y+z=2$

Q.47 The equation to the plane through the point (2, 3, 1) and (4, –5, 3) and parallel to x-axis is –

(A) $x+y+4z=7$ (B) $x+4z=7$
(C) $x-4z=7$ (D) $y+4z=7$

Q.48 The equation of a plane passing through the line of intersection of the planes 2x + 3y – 4z = 1 and $3x-y+z+2=0$ is $(2x+3y-4z-1)+\lambda(3x-y+z+2)=0$, then find the value of λ if it is perpendicular to the plane $2x+3y-4z=0$

(A) 1/2 (B) 29/2
(C) 29 (D) –1/2

Q.49 If (2, 3, – 1) is the foot of the perpendicular from (4, 2, 1) to a plane, then find the equation of the plane –

(A) $2x-y-2z-3=0$ (B) $2x+y-2z-9=0$
(C) $2x+y+2z-5=0$ (D) $2x-y+2z+1=0$

Q.50 If M denotes the mid-point of the line joining $A(4\hat{i}+5\hat{j}-10\hat{k})$ & $B(-\hat{i}+2\hat{j}+\hat{k})$, then the equation of the plane through M and perpendicular to AB is –

(A) $\vec{r}.(-5\hat{i}-3\hat{j}+11\hat{k})+\frac{135}{2}=0$

(B) $\vec{r}.\left(\frac{3}{2}\hat{i}+\frac{7}{2}\hat{j}-\frac{9}{2}\hat{k}\right)+\frac{135}{2}=0$

(C) $\vec{r}.(4\hat{i}+5\hat{j}-10\hat{k})+4=0$

(D) $\vec{r}.(-\hat{i}+2\hat{j}+\hat{k})+4=0$

Q.51 The plane 2x – 3y + 6z – 11 = 0 makes an angle $\sin^{-1}(a)$ with the x-axis. Then the value of a is –

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{3}{7}$ (D) $\frac{2}{7}$

Q.52 Image point of (5, 4, 6) in the plane x + y + 2z – 15 = 0 is
(A) (3, 2, 2) (B) (2, 3, 2)
(C) (2, 2, 3) (D) (– 5, –4, –6)

Q.53 The plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ cuts the axes in A, B, C then the area of the Δ ABC is
(A) $\sqrt{29}$ (B) $\sqrt{41}$
(C) $\sqrt{61}$ (D) None of these

Q.54 The angle between the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ and the plane $2x+y-3z+4=0$, is
(A) $\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (B) $\sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$
(C) $\sin^{-1}\left(\frac{4}{14\sqrt{229}}\right)$ (D) None of these

Q.55 The equation of the plane passing through the intersection of the planes x + y + z = 6 and 2x + 3y + 4z + 5 = 0 the point (1, 1, 1) is
(A) $20x+23y+26z-69=0$ (B) $20x+23y+26z+69=0$
(C) $23x+20y+26z-69=0$ (D) None of these

Q.56 The equation of the plane through (2, 3, 4) and parallel to the plane x + 2y + 4z = 5 is
(A) x + 2y + 4z = 10 (B) x + 2y + 4z = 3
(C) x + y + 2z = 2 (D) x + 2y + 4z = 24

Q.57 The equation of the plane which is parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and passes through the points (0, 0, 0) and (3, –1, 2) is
(A) x + 19y + 11z = 0 (B) x – 19y – 11z = 0
(C) x – 19y + 11z = 0 (D) None of these

Q.58 The angle between the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ and the plane 2x + y – 3z + 4 = 0, is
(A) $\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (B) $\sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$
(C) $\sin^{-1}\left(\frac{4}{14\sqrt{229}}\right)$ (D) None of these

Q.59 The line $\frac{x-1}{2}=-(y+1)=\frac{z}{3}$ and the plane 3x + 2y – z = 5 intersect in a point. The co-ordinates of the point are
(A) (1, 1, 0) (B) (9, –5, 12)
(C) (2, 0, 1) (D) (–9, 5, –12)

Q.60 The planes bx – ay = n ; cy – bz = ℓ ; az – cx = m intersect in a line then the value of aℓ + bm + cn is equal to
(A) 1 (B) 2
(C) 3 (D) None

PART - 5 - MISCELLANEOUS

Q.61 What is the value of linear velocity, if $\vec{\omega}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{r}=5\hat{i}-6\hat{j}+6\hat{k}$
(A) $-18\hat{i}-13\hat{j}+2\hat{k}$ (B) $6\hat{i}+2\hat{j}-3\hat{k}$
(C) $4\hat{i}-13\hat{j}+6\hat{k}$ (D) $6\hat{i}-2\hat{j}+8\hat{k}$

Q.62 With respect to a rectangular cartesian coordinate system, three vectors are expressed as :
$\vec{a}=4\hat{i}-\hat{j}, \vec{b}=-3\hat{i}+2\hat{j}$ and $\vec{c}=-\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the X, Y and Z-axis respectively. The unit vector $\hat{r}$ along the direction of sum of these vector is –
(A) $\hat{r}=\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (B) $\hat{r}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}-\hat{k})$
(C) $\hat{r}=\frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$ (D) $\hat{r}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

Q.63 If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors then the value of the scalar $((\vec{a}\times b)\times\vec{a}).(\vec{b}\times\vec{a})\times\vec{b})$ equals –
(A) $|\vec{b}|^2|\vec{a}\times\vec{b}|^2$ (B) $\vec{a}^2\,|\vec{a}\times\vec{b}|^2$
(C) $-(\vec{a}.b)\,|\vec{a}\times\vec{b}|^2$ (D) $(\vec{a}.\vec{b})\,|\vec{a}\times\vec{b}|^2$

Q.64 If $|\vec{a}|=2$ and $|\vec{b}|=3$ then the value of the scalar $(\vec{a}\times\vec{b})\times\vec{a}.\vec{b}+(\vec{a}.\vec{b})^2$ is equal to –
(A) 36 (B) 18
(C) 12 (D) 6

Q.65 If $\vec{u}$ and $\vec{v}$ are the two vectors such that $|\vec{u}|=3$; $|\vec{v}|=2$ and $|\vec{u}\times\vec{v}|=6$ then the correct statement is –
(A) $\vec{u}\wedge\vec{v}\in(0,90°)$ (B) $\vec{u}\wedge\vec{v}\in(90°,180°)$
(C) $\vec{u}\wedge\vec{v}=90°$ (D) $(\vec{u}\times\vec{v})\times\vec{u}=6\vec{v}$

Q.66 Given $\vec{a},\vec{b},\vec{c}$ are vector such that $[\vec{a},\vec{b},\vec{c}]=\frac{1}{3}$. If the vector $\vec{V}$ can be expressed as linear combination of $\vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ and $\vec{a}\times\vec{b}$ as
$\vec{V}=x\,(\vec{b}\times\vec{c})+y\,(\vec{c}\times\vec{a})+z\,(\vec{a}\times\vec{b})$ then (x + y + z) has the value equal to –
(A) $\vec{V}.(\vec{a}+\vec{b}+\vec{c})$ (B) $2\vec{V}.(\vec{a}+\vec{b}+\vec{c})$
(C) $3\vec{V}.(\vec{a}+\vec{b}+\vec{c})$ (D) None of these

Q.67 Determine the value of c so that for all real x, the vector $cx\vec{i}-6\vec{j}-3\vec{k}$ and $x\vec{i}+2\vec{j}-2cx\vec{k}$ make an obtuse angle with one another.
(A) – 4/3 < c < 0 (B) – 1/3 < c < 0.
(C) – 2/3 < c < 0. (D) – 2 < c < 0.

Q.68 The number of vectors of unit length perpendicular to the vectors $\vec{a}$ = (1, 1, 0) and $\vec{b}$ = (0, 1, 1) is
(A) One (B) Two
(C) Three (D) Infinite

Q.69 The three vectors $\vec{i}+\vec{j}$, $\vec{j}+\vec{k}$, $\vec{k}+\vec{i}$ taken two at a time form three planes . The three unit vectors drawn perpendicular to these three planes form a parallelepiped of volume :
(A) 1/3 (B) 4
(C) $3\sqrt{3}/4$ (D) $4/3\sqrt{3}$

Q.70 The points with position vectors $60\vec{i}+3\vec{j}$, $40\vec{i}-8\vec{j}$, $a\vec{i}-52\vec{j}$ are collinear if-
(A) a = – 40 (B) a = 40
(C) a = 20 (D) None of these

Q.71 Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.
(A) $\frac{1}{\sqrt{3}}(\vec{i}+\vec{j}+\vec{k})$ (B) $\frac{1}{3}(2\vec{i}+\vec{j}+\vec{k})$
(C) $\frac{1}{2}(\vec{i}+2\vec{j}+\vec{k})$ (D) $\frac{1}{\sqrt{2}}(\vec{i}+\vec{j}+2\vec{k})$

Q.72 The volume of the parallelopiped whose sides are given by $\overrightarrow{OA}=2\vec{i}-3\vec{j}$, $\overrightarrow{OB}=\vec{i}+\vec{j}-\vec{k}$ and $\overrightarrow{OC}=3\vec{i}-\vec{k}$ is-
(A) 2/7 (B) 4/13
(C) 4 (D) None of these

Q.73 If $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vectors and form a relation $(\vec{a}\times\vec{b}).\vec{c}=|\vec{a}||\vec{b}||\vec{c}|$, then the angle between $\vec{a}$ and $\vec{b}$ is-
(A) π/2 (B) 0
(B) π (D) π/3

Q.74 If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar unit vectors and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is-
(A) π/4 (B) 2π/2
(C) 3π/4 (D) π/2

Q.75 If $\vec{a}=\vec{i}+2\vec{j}+\vec{k}, \vec{b}=2\vec{i}+3\vec{j}+\vec{k}, \vec{c}=3\vec{i}+\vec{j}+2\vec{k}$ and $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=-3(\vec{i}-\vec{k})$, then the number triple (α, β, γ) is-
(A) (2, –1, –1) (B) (–2, 1, 1)
(C) (–2, –1, 1) (D) (2, 1, –1)

Q.76 Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ then the value of the expression $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$ is equal to-
(A) 0 (B) 1
(C) 2 (D) 3

Q.77 The vectors $x\vec{i}-\vec{j}-3\vec{k}$, $\vec{i}+x\vec{j}+2\vec{k}$ and $3\vec{i}-2\vec{j}+x\vec{k}$ are coplanar
(A) For no real value of x (B) For one real value of x
(C) For two real values of x (D) For three real values of x

Q.78 The two vectors $(x^2-1)\vec{i}+(x+2)\vec{j}+x^2\vec{k}$ and $2\vec{i}-x\vec{j}+3\vec{k}$ are orthogonal –
(A) For no real value of x (B) For x = – 1
(C) For x = 1/2 (D) For x = –1/2 and x = 1

Q.79 Let $\vec{a}=2\vec{i}+\vec{j}-2\vec{k}$ and $\vec{b}=\vec{i}+\vec{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|.|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ and 30º, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ is equal to-
(A) 2/3 (B) 3/2
(C) 2 (D) 3

Q.80 The triple scalar product $(\vec{a}+\vec{b}-\vec{c}, \vec{b}+\vec{c}-a, \vec{c}+\vec{a}-\vec{b})$ is equal to
(A) 0 (B) $(\vec{a}\ \vec{b}\ \vec{c})$
(C) 2 $(\vec{a}\ \vec{b}\ \vec{c})$ (D) 4 $(\vec{a}\ \vec{b}\ \vec{c})$

Q.81 Let a, b, c be distinct non-negative numbers. If the vectors $a\vec{i}+a\vec{j}+c\vec{k}$, $\vec{i}+\vec{k}$ and $c\vec{i}+c\vec{j}+b\vec{k}$ lie in a plane, then c is-
(A) The A.M. of a and b (B) The G.M. of a and b
(C) The H.M. of a and b (D) Equal to zero.

Q.82 Let $\vec{a},\vec{b},\vec{c}$ such that $|\vec{a}|=1, |\vec{b}|=1$ and $+|\vec{c}|=2$ and if $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=0$ then acute angle between $\vec{a}$ and $\vec{c}$ is
(A) π/3 (B) π/4
(C) π/6 (D) None of these

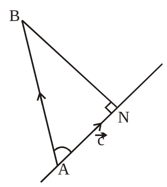
Q.83 If $\vec{a}=\vec{i}+\vec{j}+\vec{k}$, $\vec{b}=4\vec{i}+3\vec{j}+4\vec{k}$ and $\vec{c}=\vec{i}+\alpha\vec{j}+\beta\vec{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then-
(A) α = 1, β = – 1 (B) α = 1, β = ± 1
(C) α = – 1, β = ± 1 (D) α = ± 1, β = 1

Q.84 In a Δ ABC, let M be the mid point of segment AB and let D be the foot of the bisector of ∠C . Then find the ratio of $\frac{\text{Area } \Delta\ \text{CDM}}{\text{Area } \Delta\ \text{ABC}}$.
(A) $\frac{\sin A - \sin B}{2(\sin A + \sin B)}$ (B) $\frac{\sin A + \sin B}{2(\sin A + \sin B)}$
(C) $\frac{\sin A - \sin B}{2(\sin A - \sin B)}$ (D) $\frac{\sin A - \sin B}{(\sin A + \sin B)}$

Q.85 The resultant vector of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. On reversing the direction of $\vec{Q}$ the resultant vector becomes $\vec{S}$. Find the value of $R^2 + S^2$
(A) $2(P^2 + Q^2)$ (B) $2(P^2 - Q^2)$
(C) $P^2 + Q^2$ (D) $P^2 - Q^2$

Q.86 If $\vec{a} = \alpha\vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} + \alpha\vec{j} - \vec{k}$, $\vec{c} = \vec{i} + 2\vec{j} + \vec{k}$ and $[\vec{a}\ \vec{b}\ \vec{c}] = 6$, then α is either
(A) 8 or –3 (B) –8 or 3
(C) –8 or –3 (D) 8 or 3

Q.87 Find the distance of the point $B(\vec{i} + 2\vec{j} + 3\vec{k})$ from the line which is passing through $A(4\vec{i} + 2\vec{j} + 2\vec{k})$ and which is parallel to the vector $\vec{c} = 2\vec{i} + 3\vec{j} + 6\vec{k}$.



(A) $\sqrt{10}$ (B) $\sqrt{5}$
(C) $\sqrt{2}$ (D) $\sqrt{7}$

Q.88 If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$, $\vec{c} = \vec{i} + \vec{j} - \vec{k}$, $\vec{d} = \vec{i} - \vec{j} - \vec{k}$, then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is a vector which is orthogonal to both
(A) $\vec{i}$ and $\vec{j}$ (B) $\vec{j}$ and $\vec{k}$
(C) $\vec{k}$ and $\vec{i}$ (D) $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$

Q.89 A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is –
(A) $(-\vec{j} + \vec{k})$ (B) $\frac{1}{\sqrt{2}}(-\vec{j} + \vec{k})$
(C) $\frac{1}{3}(-\vec{j} + \vec{k})$ (D) $\frac{1}{\sqrt{3}}(-\vec{j} + \vec{k})$

Q.90 OABCDE is a regular hexagon of side 2 units in the xy-plane, O being the origin and OA taken along the x-axis. A point P is taken on a line parallel to z-axis through the centre of the hexagon at a distance of 3 units from O. The vector $\overrightarrow{AP}$ is
(A) $-\vec{i} + 3\vec{j} + \sqrt{5}\vec{k}$ (B) $\vec{i} - \sqrt{3}\vec{j} + 5\vec{k}$
(C) $-\vec{i} + \sqrt{3}\vec{j} + \sqrt{5}\vec{k}$ (D) $\vec{i} + \sqrt{3}\vec{j} + \sqrt{5}\vec{k}$

Q.91 If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} + \vec{j} - \vec{k}$ then the vectors $\vec{a} + 2\vec{b} + \mathrm{j}$ and $2\vec{a} + \vec{c} - \vec{k}$ are inclined at an angle
(A) $\pi/2$ (B) $\pi/3$
(C) $\pi/4$ (D) $\pi/6$

Q.92 Let $\vec{a} = \vec{i} + \vec{j}$, $\vec{b} = \vec{j} + \vec{k}$ and $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors $\vec{i} - 2\vec{j} + \vec{k}$, $3\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{c}$ are coplanar then α/β is
(A) 1 (B) 2
(C) 3 (D) – 3

Q.93 If $\vec{u}$ and $\vec{v}$ are non-zero vectors such that none of them can be expressed as a scalar multiple of the other and satisfy $(2\cos\theta + 1)\,\vec{u} + (\sqrt{3}\cot\theta + 1)\,\vec{v} = 0$ then the most general values of θ are – (where $n \in I$)
(A) $n\pi + \frac{2\pi}{3}$ (B) $2n\pi + \frac{2\pi}{3}$
(C) $n\pi + \frac{5\pi}{6}$ (D) $2n\pi \pm \frac{5\pi}{6}$

Q.94 Given $|\vec{p}| = 2, |\vec{q}| = 3$ and $\vec{p}.\vec{q} = 0$. If $(\vec{p} \times (\vec{p} \times (\vec{p} \times (p \times \vec{q}))))$ then the vector $\vec{V}$ is –
(A) collinear with $\vec{p}$ (B) $\vec{V} = 16\vec{p}$
(C) $\vec{V} = 48\vec{q}$ (D) $\vec{V} = 16\,\vec{q}$

Q.95 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero noncoplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors defined as $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$, $\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$. If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is V_1 and that of the parallelopiped determined by $\vec{p}, \vec{q}$ and $\vec{r}$ is V_2 then $V_2 = KV_1$ implies that K is equal to –
(A) 13 (B) 15
(C) 25 (D) 23

Q.96 If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar then the box product $[(2\vec{a} + \vec{b} - \vec{c})\ (\lambda^2\vec{a} + 3\vec{b} - 3\vec{c})\ (3\vec{a} + 2\vec{b} - 2\vec{c})] = 0$ for
(A) no value of λ
(B) exactly one value of λ
(C) exactly 2 values of λ
(D) all values of λ

Q.97 The equation of the parallel plane lying midway between the parallel planes $2x - 3y + 6z - 7 = 0$ and $2x - 3y + 6z + 7 = 0$ is –
(A) $2x - 3y + 6z + 1 = 0$ (B) $2x - 3y + 6z - 1 = 0$
(C) $2x - 3y + 6z = 0$ (D) None of these

Q.98 If a point moves so that the sum of the square of its distances from the six faces of a cube having length of each edge 2 units is 46 units, then the distance of the point from point (1, 1, 1) is –
(A) a variable
(B) a constant equal to $2\sqrt{5}$ units
(C) a constant equal to $\sqrt{29}$ units
(D) cannot be determined

Q.99 The equation of the plane through the point (–1, 2, 0) and parallel to the lines $\frac{x}{3}=\frac{y+1}{0}=\frac{z-2}{-1}$ & $\frac{z-2}{-1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ is–

(A) $2x+3y+6z-4=0$ (B) $x-2y+3z+5=0$
(C) $x+y-3z+1=0$ (D) $x+2y+3z-3=0$

Q.100 A line segment has length 63 and direction ratios are 3, –2, 6. If the line makes an obtuse angle with x–axis, the components of the line vector are

(A) 27, – 18, 54 (B) – 27, 18, 54
(C) – 27, 18, –54 (D) 27, – 18, – 54

Q.101 A line makes the same angle θ, with each of the x and z–axis. If the angle β, which it makes with y–axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equal –

(A) 3/5 (B) 1/5
(C) 3/2 (D) 2/5

Q.102 If the foot of the perpendicular from the origin to a plane is P (a, b, c) the equation of the plane is –

(A) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$ (B) $ax+by+cz=3$
(C) $ax+by+cz=a^2+b^2+c^2$ (D) $ax+by+cz=a+b+c$

Q.103 Equation of a plane bisecting the angles between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is

(A) $5x-y-4z-45=0$ (B) $5x-y-4z-3=0$
(C) $23x+13y+32z-45=0$ (D) $23x-13y+32z+5=0$

Q.104 Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is –

(A) $4x+3y+5z=25$ (B) $4x+3y+5z=50$
(C) $3x+4y+5z=49$ (D) $x+7y-5z=2$

Q.105 If a plane meets the co–ordinate axes in A, B, C such that the centroid of the triangle is the point$(1, r, r^2)$, then equation of the plane is

(A) $x+ry+r^2z=3r^2$ (B) $r^2x+ry+z=3r^2$
(C) $x+ry+r^2z=3$ (D) $r^2x+ry+z=3$

Q.106 Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is –

(A) 7/2 (B) 5/2
(C) 3/2 (D) 9/2

Q.107 Gives the line L : $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-3}{-1}$ and the plane $\pi : x-2y=0$. Of the following assertions, the only one that is always true is –

(A) L is $\perp$ to π (B) L lies in π
(C) L is parallel to π (D) None of these

Q.108 Let $A(\vec{a})$ and $B(\vec{b})$ be points on the two skew lines $\vec{r}=\vec{a}+\lambda\vec{p}$ and $\vec{r}=\vec{b}+u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of 1/2 units. If an angle between AB and the line of shortest distance is 60°, then AB =

(A) 1/2 (B) 2
(C) 1 (D) $\lambda \in R-\{0\}$

Q.109 The Cartesian equation of the plane passing through the line of intersection of the planes $r.(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $r.(\hat{i}-\hat{j})+4=0$ and perpendicular to the plane $r(2\hat{i}-\hat{j}+\hat{k})+8=0$ is

(A) $3x-4y+4z=5$ (B) $x-2y+4z=3$
(C) $5x-2y-12z+47=0$ (D) $2x+3y+4=0$

Q.110 Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$.

(A) $\sin^{-1}\left(\frac{8}{21}\right)$ (B) $\sin^{-1}\left(\frac{5}{21}\right)$
(C) $\sin^{-1}\left(\frac{7}{21}\right)$ (D) $\sin^{-1}\left(\frac{1}{21}\right)$

Q.111 The image of the point P (1, 3, 4) in the plane $2x-y+z+3=0$ is

(A) (3, 5, –2) (B) (–3, 5, 2)
(C) (3, –5, 2) (D) (3, 5, 2)

Q.112 The volume of the tetrahedron included between the plane $3x+4y-5z-60=0$ and the coordinate planes is

(A) 60 (B) 600
(C) 720 (D) none of these.

Q.113 Two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=z$ intersect at a point if k is equal to –

(A) 2/9 (B) 1/2
(C) 9/2 (D) 1/6

Q.114 The coordinates of a point of the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) nearer the origin are

(A) (9, –13, 4)
(B) $\left(8\sqrt{14}+1,\ -12\sqrt{14}-1,\ 4\sqrt{14}\right)$
(C) (–7, 11, –4)
(D) $\left(-8\sqrt{14}+1,\ 12\sqrt{14}-1,\ -a4\sqrt{14}\right)$

Q.115 If $\vec{a},\vec{b},\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is –

(A) $\theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$ (B) $\theta=\sin^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$
(C) $\theta=\cos^{-1}\left(\frac{\sin\alpha/2}{\sin\alpha}\right)$ (D) $\theta=\sin^{-1}\left(\frac{\sin\alpha/2}{\sin\alpha}\right)$

Q.116 Find the distance between the lines ℓ_1 and ℓ_2 given by $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\,(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu\,(2\hat{i} + 3\hat{j} + 6\hat{k})$.

(A) $\frac{\sqrt{193}}{7}$ (B) $\frac{\sqrt{293}}{7}$ (C) $\frac{\sqrt{93}}{7}$ (D) $\frac{\sqrt{113}}{7}$

Q.117 A line passes through the points (6, –7, –1) and (2, –3, 1). The direction cosines of the line so directed that the angle made by it with the positive direction of x–axis is acute, are

(A) $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (B) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (C) $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ (D) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

Q.118 Equation of the plane containing the lines $\vec{r} = (1,1,0) + t\,(1,-1,2)$, $\vec{r} = (2,0,2) + s\,(-1,1,0)$ is –

(A) $x + 3y + z = 4$ (B) $x + y - 2 = 0$
(C) $5x - 3y - 4z = 2$ (D) None of these

Q.119 Which one of the following lines is parallel to the line L : (x, y, z) = (1, 0, –2) + t (–1, 3, 0), t ∈ R

(A) $\frac{x+1}{3} = \frac{z-3}{2}, y = 3$ (B) $1 - x = \frac{y}{3} = z + 2$
(C) $1 - x = \frac{y}{3}, z = 5$ (D) $x + 1 = \frac{y}{3}, z = 2$

Q.120 If $P_1 : \vec{r}.\vec{n}_1 - d_1 = 0$, $P_2 : \vec{r}.\vec{n}_2 - d_2 = 0$ and $P_3 : \vec{r}.\vec{n}_3 - d_3 = 0$ are three planes and $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are three non–coplanar vectors then, the three lines $P_1 = P_2 = 0$, $P_2 = P_3 = 0$ and $P_1 = P_3 = 0$ is –

(A) parallel lines (B) coplanar lines
(C) coincident lines (D) concurrent lines

Q.121 If P (x, y, z) is a point on the line segment joining Q (2, 3, 4) and R (3, 5, 6) such that the projections of the vector $\overrightarrow{OP}$ on the axes are $\frac{13}{5}, \frac{21}{5}, \frac{26}{5}$ respectively. The P divides QR in the ratio –

(A) 2 : 3 (B) 3 : 1
(C) 1 : 3 (D) 3 : 2

Q.122 If α, β, γ are the angle which a directed line makes with the positive directions of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(A) 1 (B) 2
(C) 3 (D) None of these

Q.123 Number of unit vectors perpendicular to the line $\vec{r} = \hat{i} - \hat{j} + 3\hat{k} + \lambda\,(\hat{i} + 4\hat{j} + 2\hat{k})$ is –

(A) 1 (B) 2
(C) 3 (D) infinitely many

Q.124 Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z–axis respectively.

(A) $6x + 2y + 3z = 12$ (B) $x + 4y + 3z = 12$
(C) $6x + 4y + 3z = 12$ (D) $2x + 2y + 3z = 12$

Q.125 If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, then the value of k is –

(A) 4 (B) – 7
(C) 7 (D) No real value

Q.126 The vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6) is –

(A) $-\hat{i} + 2\hat{k} + \lambda\,(4\hat{i} + 4\hat{j} + 4\hat{k})$ (B) $-\hat{i} + 2\hat{k} + \lambda\,(4\hat{i} - 4\hat{j} + 4\hat{k})$
(C) $\hat{i} - 2\hat{k} + \lambda\,(4\hat{i} - 4\hat{j} + 4\hat{k})$ (D) $\hat{i} - 2\hat{k} + \lambda\,(4\hat{i} - 4\hat{j} - 4\hat{k})$

Q.127 The distance of the plane 2x – 3y + 4z – 6 = 0 from the origin is –

(A) $\frac{6}{\sqrt{29}}$ (B) $\frac{1}{\sqrt{29}}$ (C) $\frac{3}{\sqrt{29}}$ (D) $\frac{2}{\sqrt{29}}$

Q.128 The angle between the two planes 3x – 6y + 2z = 7 and 2x + 2y – 2z = 5.

(A) $\cos^{-1}\left(\frac{5\sqrt{3}}{21}\right)$ (B) $\cos^{-1}\left(\frac{\sqrt{3}}{21}\right)$
(C) $\cos^{-1}\left(\frac{2\sqrt{3}}{21}\right)$ (D) $\cos^{-1}\left(\frac{2\sqrt{3}}{15}\right)$

Q.129 Find the distance of a point (2, 5, – 3) from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$

(A) 1/7 (B) 13/7
(C) 2/7 (D) 15/7

Q.130 The coordinates of the foot of the perpendicular from the point (2, 6, 3) to the line $\frac{x}{2} = \frac{y-1}{2} = \frac{z-2}{3}$ is –

(A) (2, 3, 5) (B) (–2, –1, –1)
(C) (0, –3, 2) (D) (–2, –3, –1)

Q.131 The line of intersection of planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$ and $\vec{r}.(3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$ is –

(A) equally inclined with all the three axis
(B) equally inclined with x-axis and z-axis only
(C) equally inclined with x-axis and y-axis only
(D) equally inclined with y-axis and z-axis only

Q.132 If θ is the angle between the lines in which the planes 3x – 7y – 5z = 1 and 5x – 13y + 3z + 2 = 0 cuts the plane 8x – 11y + 2z = 0, find sinθ.

(A) 1/2 (B) 1
(C) $1/\sqrt{2}$ (D) None of these

EXERCISE - 2 [LEVEL-2]

Q.1 If the moduli of the vectors $\vec{a}, \vec{b}, \vec{c}$ are 3, 4, 5 respectively and $\vec{a}$ and $\vec{b}+\vec{c}$, $\vec{b}$ and $\vec{c}+\vec{a}$, $\vec{c}$ and $\vec{a}+\vec{b}$ are mutually perpendicular, then the modulus of $\vec{a}+\vec{b}+\vec{c}$ is

(A) $\sqrt{12}$ (B) 12
(C) $5\sqrt{2}$ (D) 50

Q.2 Let a = 2i – j + k, b = i + 2j – k and c = i + j + 2k be three vectors. A vector in the plane of b and c whose projection on a is $\sqrt{2/3}$ will be –

(A) 2i + 3j – 3k (B) 2i + 3j + k
(C) – 2i – j + 5k (D) 2i + j + 5k

Q.3 For any two vectors a and b, $(a \times b)^2$ equals

(A) $a^2b^2 - (a.b)^2$ (B) $a^2 + b^2$
(C) $a^2 - b^2$ (D) None of these

Q.4 Let, $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ & $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$. If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 =$

(A) 0 (B) 1
(C) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$ (D) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

Q.5 The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are –

(A) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ & $\vec{a} - \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (B) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$ & $\vec{a} + \left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

(C) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{a} - \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{a}$ (D) None of these

Q.6 Given $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$. A unit vector perpendicular to both $(\vec{a}+\vec{b})$ & $(\vec{b}+\vec{c})$ is

(A) $\hat{i}$ (B) $\hat{k}$
(C) $\hat{j}$ (D) $\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

Q.7 $\hat{i} \times (\vec{x} \times \hat{i}) + \hat{j} \times (\vec{x} \times \hat{j}) + \hat{k} \times (\vec{x} \times \hat{k})$ is equal to –

(A) $\vec{0}$ (B) $\vec{x}$
(C) $2\vec{x}$ (D) 0

Q.8 The vectors $(2\hat{i} - m\hat{j} + 3m\hat{k})$ & $\{(1+m)\hat{i} - 2m\hat{j} + \hat{k}\}$ include an acute angle for –

(A) all values of m (B) m < – 2 or m > – 1/2
(C) m = – 1/2 (D) $m \in \left[-2, -\frac{1}{2}\right]$

Q.9 The distance between the line $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is –

(A) $\frac{10}{3\sqrt{3}}$ (B) $\frac{10}{3}$ (C) 10/9 (D) None of these

Q.10 If $\vec{a}, \vec{b}$ & $\vec{c}$ are three non-coplanar non-zero vectors, then $(\vec{a}.\vec{a})(\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})(\vec{c} \times \vec{a}) + (\vec{a}.\vec{c})(\vec{a} \times \vec{b})$ is equal to –

(A) $[\vec{b}\ \vec{c}\ \vec{a}]\vec{a}$ (B) $[\vec{c}\ \vec{a}\ \vec{b}]\vec{b}$
(C) $[\vec{a}\ \vec{b}\ \vec{c}]\vec{c}$ (D) None of these

Q.11 Let $\vec{a}$ & $\vec{b}$ be unit vectors inclined at an angle α to each other, then $|\vec{a}+\vec{b}| < 1$ if –

(A) $\alpha = \frac{\pi}{2}$ (B) $\alpha < \frac{\pi}{3}$ (C) $\alpha > \frac{2\pi}{3}$ (D) $\frac{\pi}{3} < \alpha < \frac{2\pi}{3}$

Q.12 The position vector of coplanar points A, B, C, D are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively, in such a way that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c}) = (\vec{b}-\vec{d}).(\vec{c}-\vec{a}) = 0$, then the point D of the triangle ABC is

(A) Incentre (B) Circumcentre
(C) Orthocentre (D) None of these

Q.13 $\vec{a}, \vec{b}$ & $\vec{c}$ are three non-zero, non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are three other vectors such that $p = \frac{b \times c}{a.b \times c}$, $q = \frac{c \times a}{a.b \times c}$, $r = \frac{a \times b}{a.b \times c}$. Then [p q r] =

(A) $\vec{a}.\vec{b} \times \vec{c}$ (B) $\frac{1}{a.b \times c}$
(C) 0 (D) None of these

Q.14 Choose a point Q on the plane 2x – 2y + z = 1. If $\vec{N}$ is a vector normal to this plane then the value of t for which the point with position vector $(\overrightarrow{OQ} + t\vec{N})$ lies on the plane 2x – 2y + z = 3, is (O is origin)

(A) 1/3 (B) 2/3
(C) 1 (D) 2/9

Q.15 Let $V = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is

(A) – 1 (B) $\sqrt{10} + \sqrt{6}$
(C) $\sqrt{59}$ (D) $\sqrt{60}$

Q.16 A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The angle between **a** and the vector $i-2j+2k$ is

(A) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (B) $\frac{2\pi}{4}$ or $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ (D) None of these

Q.17 The perimeter of the triangle whose vertices have the position vectors $(i+j+k), (5i+3j-3k)$ and $(2i+5j+9k)$, is given by

(A) $15+\sqrt{157}$ (B) $15-\sqrt{157}$

(C) $\sqrt{15}-\sqrt{157}$ (D) $\sqrt{15}+\sqrt{157}$

Q.18 The direction cosines of the resultant of the vectors $(\hat{i}+\hat{j}+\hat{k}), (-\hat{i}+\hat{j}+\hat{k}), (\hat{i}-\hat{j}+\hat{k})$, and $(\hat{i}+\hat{j}-\hat{k})$ are

(A) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$ (B) $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(C) $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$ (D) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Q.19 The point of intersection of the lines

$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ is

(A) $\left(\frac{1}{2}, \frac{1}{2}, -\frac{3}{2}\right)$ (B) $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ (D) $\left(-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

Q.20 The equation of the plane passing through the points (0, 1, 2) and (–1, 0, 3) and perpendicular to the plane $2x+3y+z=5$ is

(A) $3x-4y+18z+32=0$

(B) $3x+4y-18z+32=0$

(C) $4x+3y-17z+31=0$

(D) $4x-3y+z+1=0$

Q.21 The equation of line of intersection of the planes $4x+4y-5z=12, 8x+12y-13z=32$ can be written as

(A) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ (B) $\frac{x}{2}=\frac{y}{3}=\frac{z-2}{4}$

(C) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$ (D) $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z}{4}$

Q.22 A plane is at unit distance from origin. It cut co-ordinates axes at P, Q, R respectively. If the locus of centroid of the Δ PQR is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then k =

(A) 3 (B) 9
(C) 2 (D) 1

Q.23 The equation of the plane in which the lines

$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ lie, is

(A) $17x-47y-24z+172=0$ (B) $17x+47y-24z+172=0$
(C) $17x+47y+24z+172=0$ (D) $17x-47y+24z+172=0$

Q.24 The volume of the tetrahedron whose vertices are with position vectors $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+7\hat{k}$, $5\hat{i}-\hat{j}+\lambda\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic unit. If λ equals –

(A) –3 (B) 3
(C) 7 (D) –1

Q.25 If $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ is perpendicular to both $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ (where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors) and

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^{x^2+x+3}=1$; $\forall x \in R$ then the angle between $\vec{a}$ & $\vec{b}$ is –

(A) $\pi/3$ (B) $\pi/6$
(C) $\pi/2$ (D) $\pi/4$

Q.26 If $3\vec{a}+4\vec{b}+5\vec{c}=0$ then $(\vec{a}\times\vec{b})\times[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]=$

(A) A vector perpendicular to plane of $\vec{a}, \vec{b}, \vec{c}$
(B) zero
(C) 12
(D) None of these

Q.27 The volume of the tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ is –

(A) 6 (B) 18
(C) 36 (D) 9

Q.28 The set of values of m for which the vectors $\hat{i}+\hat{j}+m\hat{k}$, $\hat{i}+\hat{j}+(m+1)\hat{k}$ and $\hat{i}-\hat{j}+m\hat{k}$ are non-coplanar is –

(A) ϕ (B) $R-\{1\}$
(C) $R-\{2\}$ (D) R

Q.29 If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors and if

$\Delta = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$ then

(A) $\Delta=0$ (B) $\Delta=1$
(C) Δ = any non zero value (D) None

Q.30 If shortest distance between lines $\frac{x}{a}-\frac{z}{c}=1, y=0$ and $\frac{y}{b}+\frac{z}{c}=1, x=0$ is 2, then perpendicular distance of origin from the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is –

(A) 1/4 (B) 1/2
(C) 3/4 (D) 1

Q.31 If $\vec{a}, \vec{b}, \vec{c}$ are coplanar unit vectors then the scalar triple product $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$

(A) 2 (B) 0
(C) 9 (D) 4

Q.32 Image point of (5, 4, 6) in the plane $x + y + 2z - 15 = 0$ is

(A) (3, 2, 2) (B) (2, 3, 2)
(C) (2, 2, 3) (D) (–5, –4, –6)

Q.33 The equation of the plane which bisects the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$ which contains the origin is

(A) $33x - 13y + 32z + 45 = 0$ (B) $x - 3y + z - 5 = 0$
(C) $33x + 13y + 32z + 45 = 0$ (D) None of these

Q.34 If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, then the value of

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta =$

(A) 1 (B) 4/3
(C) Variable (D) None of these

Q.35 The distance of the point (–1, –5, –10) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 5$, is

(A) 10 (B) 11
(C) 12 (D) 13

Q.36 The equation of the plane through the point (2, –1, –3) and parallel to the lines $\frac{x-1}{3}=\frac{y+2}{2}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is

(A) $8x + 14y + 13z + 37 = 0$ (B) $8x - 14y + 13z + 37 = 0$
(C) $8x + 14y - 13z + 37 = 0$ (D) $8x + 14y + 13z - 37 = 0$

Q.37 If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{2}$ intersect, then the value of k is

(A) 3/2 (B) 9/2
(C) –2/9 (D) –3/2

Q.38 If $\vec{b}$ and $\vec{c}$ are any two non-collinear orthogonal unit vectors and $\vec{a}$ is any vector, then

$(\vec{a}.\vec{b})\,\vec{b}+(\vec{a}.\vec{c})\,\vec{c}+\frac{\vec{a}.\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|^2}\,\vec{b}\times\vec{c}=$

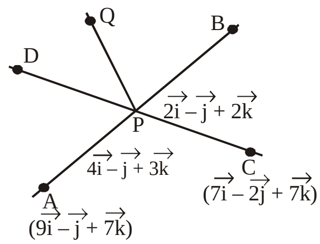
(A) $\vec{a}$ (B) $\vec{b}$
(C) $\vec{c}$ (D) None of these

Q.39 The vector $-\vec{i}+\vec{j}-\vec{k}$ bisects the angle between the vector $\vec{c}$ & $3\vec{i}+4\vec{j}$. Determine the unit vector along $\vec{c}$.

(A) $-\frac{1}{15}(11\vec{i}+10\vec{j}+2\vec{k})$ (B) $-\frac{1}{13}(11\vec{i}+10\vec{j}+2\vec{k})$

(C) $-\frac{1}{15}(9\vec{i}+8\vec{j}+2\vec{k})$ (D) $-\frac{1}{17}(11\vec{i}+10\vec{j}+\vec{k})$

Q.40 The position vectors of two points A and C are $9\vec{i}-\vec{j}+7\vec{k}$ and $7\vec{i}-2\vec{j}+7\vec{k}$ respectively. The point of intersection of vectors $\overrightarrow{AB}=4\vec{i}-\vec{j}+3\vec{k}$ and $\overrightarrow{CD}=2\vec{i}-\vec{j}+2\vec{k}$ is P. If vector $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ and PQ = 15 units, find the position vector of Q.



(A) $2(\vec{i}+3\vec{j}-3\vec{k})$ (B) $(\vec{i}-3\vec{j}-3\vec{k})$

(C) $5(2\vec{i}-3\vec{j}+3\vec{k})$ (D) $3(\vec{i}-3\vec{j}-3\vec{k})$

Q.41 Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4, 5 respectively. Let A be perpendicular to $\vec{B}+C$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is

(A) $\sqrt{2}$ (B) $3\sqrt{2}$
(C) $5\sqrt{2}$ (D) $5/\sqrt{2}$

Q.42 If $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar, then the value of $\left(\vec{A}+\vec{B}+\vec{C}\right).\left\{\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right\}$ will be-

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}]$

(C) $2[\vec{A}\ \vec{B}\ \vec{C}]$ (D) $-[\vec{A}\ \vec{B}\ \vec{C}]$

Q.43 If $\vec{A}$ = (1, 1, 1), $\vec{C}$ = (0, 1, –1) are given vectors, then a vector $\vec{B}$ satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}.\vec{B}=3$ is –

(A) $\frac{5}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}$ (B) $\frac{1}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}$

(C) $\frac{5}{3}\vec{i}-\frac{2}{3}\vec{j}-\frac{2}{3}\vec{k}$ (D) $\vec{B}=\frac{5}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}$

Q.44 Find the area of the triangle with vertices (1, 1, 2), (2, 3, 5) and (1, 5, 5).

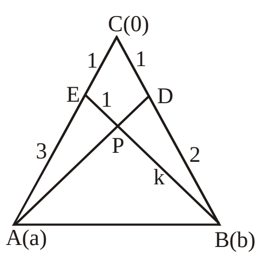
(A) $\frac{1}{3}\sqrt{61}$ sq. units
(B) $\frac{1}{3}\sqrt{51}$ sq. units
(C) $\frac{1}{4}\sqrt{45}$ sq. units
(D) $\frac{1}{2}\sqrt{61}$ sq. units

Q.45 Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$, $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ and $\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$ is equal to-

(A) 0
(B) 1
(C) $\frac{1}{4}$ $(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
(D) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

Q.46 In a triangle ABC, D and E are points on BC and AC respectively such that BD = 2DC and AE = 3EC. Let P be the point of intersection of AC and BE. Find $\frac{BP}{PE}$ by using vector methods.



(A) 1 : 3
(B) 2 : 3
(C) 8 : 3
(D) 4 : 1

Q.47 OA, OB, OC are the sides of a rectangular parallelopiped whose diagonals are OO', AA', BB' and CC'. D is the centre of the rectangle AC'O'B' and D' is the centre of the rectangle O'A'CB'. If the sides OA, OB, OC are in the ratio 1 : 2 : 3, the angle DOD' is equal to-

(A) $\cos^{-1}\frac{24}{\sqrt{697}}$
(B) $\cos^{-1}\frac{22}{\sqrt{619}}$
(C) $\sin^{-1}\frac{24}{\sqrt{697}}$
(D) $\sin^{-1}\frac{22}{\sqrt{691}}$

Q.48 Let $\vec{a}$ & $\vec{b}$ are two vectors making angle θ with each other, then unit vectors along bisector of $\vec{a}$ & $\vec{b}$ is :

(A) $\pm\frac{\hat{a} + \hat{b}}{2}$
(B) $\pm\frac{\hat{a} + \hat{b}}{2\cos\theta}$
(C) $\pm\frac{\hat{a} + \hat{b}}{2\cos\theta/2}$
(D) $(\hat{a} + \hat{b})/|\hat{a} + \hat{b}|$

Q.49 If the non zero vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other then the solution of the equation, $\vec{r} \times \vec{a} = \vec{b}$ is :

(A) $\vec{r} = x\vec{a} + \frac{1}{\vec{a}\,.\,\vec{a}}(\vec{a} \times \vec{b})$
(B) $\vec{r} = x\vec{b} - \frac{1}{\vec{b}\,.\,\vec{b}}(\vec{a} \times \vec{b})$
(C) $\vec{r} = x\left(\vec{a} \times \vec{b}\right)$
(D) none of these

Q.50 Consider the following statements :
(a) Let f : R → R be given by
$f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_3)(x - a_1)$
with a_1, a_2, a_3 being real numbers. Then $f(x) \geq 0$ for all real numbers x if and only if $a_1 = a_2 = a_3$.
(b) $e^x > x + 1$ holds for all non-zero real numbers
(c) The number of cubic polynomials f (x) with positive integral coefficients such that f (1) = 9 is 56.
(d) If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are distinct vectors relations $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then $\vec{a}.\vec{b} + \vec{c}.\vec{d} = \vec{a}.\vec{c} = \vec{b}.\vec{d}$

Correct statements are –
(A) abcd
(B) abc
(C) bcd
(D) cd

Q.51 Which of the following statement (s) hold good –

(A) If $\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{b} = \vec{c}$ $(\vec{a} \neq 0)$
(B) If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{b} = \vec{c}$ $(\vec{a} \neq 0)$
(C) If $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{b} = \vec{c}$ $(\vec{a} \neq 0)$
(D) If $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are non-coplanar vectors and

$\vec{k}_1 = \frac{\vec{v}_2 \times \vec{v}_3}{\vec{v}_1.(\vec{v}_2 \times \vec{v}_3)}$; $\vec{k}_2 = \frac{\vec{v}_3 \times \vec{v}_1}{\vec{v}_1.(\vec{v}_2 \times \vec{v}_3)}$ and

$\vec{k}_3 = \frac{\vec{v}_1 \times \vec{v}_2}{\vec{v}_1.(\vec{v}_2 + \vec{v}_3)}$ then $\vec{k}_1.(\vec{k}_2 \times \vec{k}_3) = \frac{1}{\vec{v}_1.(\vec{v}_2 \times \vec{v}_3)}$

Q.52 The coordinates of the point where the line through the points A (3, 4, 1) and B(5, 1, 6) crosses the XY–plane is

(A) $\left(\frac{1}{5}, \frac{23}{5}, 0\right)$
(B) $\left(\frac{1}{5}, \frac{3}{5}, 0\right)$
(C) $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$
(D) $\left(\frac{2}{5}, \frac{3}{5}, 0\right)$

Q.53 Which of the following are equations for the plane passing through the points P (1, 1, –1), Q (3, 0, 2) & R (–2, 1, 0)

(A) $(2\hat{i} - 3\hat{j} + 3\hat{k}) \cdot [(x + 2)\,\hat{i} + (y - 1)\hat{j} + z\hat{k}] = 0$
(B) $x = 3 - t,\ y = -11t,\ z = 2 - 3t$
(C) $(x + 2) + 11\,(y - 1) = 3z$
(D) $(2\hat{i} - \hat{j} + 3\hat{k}) \times (-3\hat{i} + \hat{k}).[(x + 2)\,\hat{i} + (y - 1)\hat{j} + z\hat{k}] = 0$

Q.54 Find the distance from the line $x = 2 + t$, $y = 1 + t$, $-\frac{1}{2}-\frac{1}{2}t$ to the plane $x + 2y + 6z = 10$.

(A) $\frac{9}{\sqrt{41}}$ (B) $\frac{3}{\sqrt{41}}$ (C) $\frac{7}{\sqrt{41}}$ (D) $\frac{11}{\sqrt{41}}$

Q.55 In a 3D rectangular coordinate system with origin 'O', point A, B and C are on the x, y and z axes respectively. If the area of the triangles OAB, OAC and OBC are 4, 6 and 12 respectively then the area of the triangle ABC equals

(A) 14 (B) 16
(C) 18 (D) 22

Q.56 L_1 and L_2 are two lines whose vector equations are :

$L_1 : \vec{r} = \lambda\,[(\cos\theta + \sqrt{3})\,\hat{i} + (\sqrt{2}\sin\theta)\,\hat{j} + (\cos\theta - \sqrt{3})\,\hat{k}]$,

$L_2 : \vec{r} = \mu(a\hat{i} + b\hat{j} + c\hat{k})$, where λ and μ are scalars and α is the acute angle between L_1 and L_2. If the angle α is the independent of θ then the value of α is –

(A) $\pi/6$ (B) $\pi/4$
(C) $\pi/3$ (D) $\pi/2$

Directions : Assertion-Reason type questions.

(A) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement -1
(B) Statement-1 is True, Statement -2 is true; statement-2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.57 Let the equation of plane

$\vec{r} = \hat{i} + \hat{j} + \lambda\,(\hat{i} + \hat{j} + \hat{k}) + \mu\,(\hat{i} - 2\hat{j} + 3\hat{k})$ and line

$\vec{v} = 2\hat{i} - \hat{j} + 3\hat{k} + t\,(\hat{i} + \hat{j} + \hat{k})$

Statement –1 : Line lies in the plane.
Statement –2 : System of linear equations is λ, μ and t has infinite solution.

Q.58 **Statement 1 :** Let $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are position vectors of four points A, B, C and D $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$ then points A, B, C and D are coplanar.
Statement 2 : Three non-zero, linearly dependent co-initial vectors $(\overrightarrow{PQ}, \overrightarrow{PR}$ & $\overrightarrow{PS})$ are coplanar.

Q.59 If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are position vector of four distinct coplanar points then $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = \vec{0}$ such that at least one of x, y, z, w is non-zero & x+ y + z + w= 0
Statement 2 : The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, respectively. If the points A, B, C and D lie in a plane, then λ is 0.

Q.60 **Statement 1 :** If $\vec{a}$ & $\vec{b}$ are unit vectors and θ is the angle between them, then $\sin\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{2}$.
Statement 2 : The number of vectors of unit length perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j}$ & $\vec{b} = \hat{j} + \hat{k}$ is two.

Q.61 **Statement 1 :** Line $\frac{x-1}{3} = \frac{y-2}{11} = \frac{z+1}{11}$ lies in the plane $11x - 3z - 14 = 0$.
Statement 2 : A straight line lies in a plane if the line is parallel to the plane and a point of the line lies in the plane.

Q.62 **Statement 1 :** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (2\hat{i} - \hat{j} + 3\hat{k}) = 3\hat{i} + \hat{k}$ represents a straight line.
Statement 2 : If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (\hat{i} + 2\hat{j} - 3\hat{k}) = 2\hat{i} - \hat{j}$ represent a straight line.

Q.63 **Statement 1 :** Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$ then OABC is a tetrahedron.
Statement 2 : Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar then OABC is a tetrahedron, where O is the origin.

Q.64 **Statement 1 :** Let $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are position vectors of four points A, B, C and D $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$ then points A, B, C and D are coplanar.
Statement 2 : Three non-zero, linearly dependent co-initial vectors $(\overrightarrow{PQ}, \overrightarrow{PR}$ & $\overrightarrow{PS})$ are coplanar.

Q.65 **Statement 1 :** Let θ be the angle between the line $\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and the plane $x + y - z = 5$.
Then $\theta = \sin^{-1}\frac{1}{\sqrt{51}}$
Statement 2 : Angle between a straight line and a plane is the complement of angle between the line and normal to the plane.

Q.66 **Statement 1 :** A point on the straight line $2x + 3y - 4z = 5$ and $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number.
Statement 2 : If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number.

Passage (Q.67-Q.69)

Let $\vec{a} = 2\hat{i}+3\hat{j}-6\hat{k}$, $\vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$ and $\vec{c} = -2\hat{i}+3\hat{j}+6\hat{k}$ - Let $\vec{a}_1$ be projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$, then –

Q.67 $\vec{a}_2 =$

(A) $\frac{943}{49}(2\hat{i}-3\hat{j}-6\hat{k})$ (B) $\frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

(C) $\frac{943}{49}(-2\hat{i}+3\hat{j}+6\hat{k})$ (D) $\frac{943}{49^2}(-2\hat{i}+3\hat{j}+6\hat{k})$

Q.68 $\vec{a}_1\vec{b} =$

(A) – 41 (B) –41/7
(C) 41 (D) 287

Q.69 Which of the following is true –

(A) $\vec{a}$ and $\vec{a}_2$ are collinear
(B) $\vec{a}_1$ and $\vec{c}$ are collinear
(C) $\vec{a}, \vec{a}_1$ and $\vec{b}$ are coplanar
(D) $\vec{a}, \vec{a}_1$ and $\vec{a}_2$ coplanar

Passage (Q.70-Q.72)

Let P denotes the plane consisting of all points that are equidistant from the points A (– 4, 2, 1) and B (2, – 4, 3) and Q be the plane, x – y + cz = 1 where c ∈ R.

Q.70 The plane P is parallel to plane Q

(A) for no value of c (B) if c = 3
(C) if c = 1/3 (D) if c = 1

Q.71 If the angle between the the planes P and Q is 45° then the product of all possible value of c is –

(A) –17 (B) –2
(C) 17 (D) 24/17

Q.72 If the line L with equation $\frac{x-1}{1} = \frac{y+2}{3} = \frac{z-7}{-1}$ intersects the plane P at the point R (x_0, y_0, z_0) then the sum $(x_0 + y_0 + z_0)$ has the value equal to –

(A) 12 (B) – 15
(C) 13 (D) – 11

Passage (Q.73-Q.75)

Vertices of a parallelogram taken in order are A (2, –1, 4), B (1, 0, –1), C (1, 2, 3) and D.

Q.73 The distance between the parallel lines AB and CD is –

(A) $\sqrt{6}$ (B) $3\sqrt{6/5}$
(C) $2\sqrt{2}$ (D) 3

Q.74 Distance of the point P (8, 2, –12) from the plane of the parallelogram is –

(A) $\frac{4\sqrt{6}}{9}$ (B) $\frac{32\sqrt{6}}{9}$

(C) $\frac{16\sqrt{6}}{9}$ (D) None of these

Q.75 The areas of the orthogonal projections of the parallelogram on the three coordinates planes xy, yz and zx respectively.

(A) 14, 4, 2 (B) 2, 4, 14
(C) 4, 2, 14 (D) 2, 14, 4

Passage (Q.76-Q.78)

A plane Π is determined by three points A (0, 0, 1), B (2, 0, 0) and C (0, 3, 0) and a line L whose vector equation is $\vec{r} = \frac{8}{3}\hat{i}+\hat{k}+\lambda\,(2\hat{i}-2\hat{j}+\hat{k})$. The line L intersects the plane Π at P.

Q.76 Distance of the point P from the origin is –

(A) $\frac{3\sqrt{10}}{2}$ (B) $\frac{2\sqrt{10}}{3}$

(C) $\sqrt{10}$ (D) $2\sqrt{10}$

Q.77 A unit vector along the line of intersection of the plane Π and the plane 2x + y – 2z = 5, is –

(A) $\frac{10\hat{i}-18\hat{j}+\hat{k}}{5\sqrt{17}}$ (B) $\frac{10\hat{i}+18\hat{j}+\hat{k}}{5\sqrt{17}}$

(C) $\frac{10\hat{i}-18\hat{j}-\hat{k}}{5\sqrt{15}}$ (D) $\pm\left(\frac{10\hat{i}+18\hat{j}-\hat{k}}{5\sqrt{15}}\right)$

Q.78 Acute angle between the plane Π and the given L, is

(A) $\sin^{-1}(8/21)$ (B) $\cos^{-1}(8/21)$
(C) $\tan^{-1}(8/19)$ (D) None of these

Passage (Q.79-Q.81)

Consider the lines represented paramatrically as :
$L_1 : x = 1 - 2t\,;\ y = t\,;\ z = -1 + t\,;$
$L_2 : x = 4 + s\,;\ y = 5 + 4s\,;\ z = -2 - s.$
Find :

Q.79 Acute angle between the lines L_1 and L_2, is –

(A) $\cos^{-1}\left(\frac{1}{18}\right)$ (B) $\cos^{-1}\left(\frac{1}{3\sqrt{6}}\right)$

(C) $\cos^{-1}\left(\frac{1}{6\sqrt{3}}\right)$ (D) $\cos^{-1}\left(\frac{1}{3\sqrt{2}}\right)$

Q.80 Equation of a plane P containing the line L_2 and parallel to the line L_1, is –

(A) 5x + y + 9z – 7 = 0 (B) 2x – 3y + 4z – 15 = 0
(C) 5x – y + 9z + 3 = 0 (D) 9x – 5y – z – 13 = 0

Q.81 Distance between the plane P and the line line L_1 is –

(A) $\frac{17}{\sqrt{29}}$ (B) $\frac{3}{\sqrt{87}}$

(C) $\frac{11}{\sqrt{107}}$ (D) $\frac{1}{\sqrt{107}}$

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.82 Given $\vec{A} = 2\hat{i}+3\hat{j}+6\hat{k}, \vec{B} = \hat{i}+\hat{j}-2\hat{k}$ and $\vec{C} = \hat{i}+2\hat{j}+\hat{k}$. Compute the value of $|\vec{A}\times(\vec{A}\times(\vec{A}\times\vec{B})).\vec{C}|$.

Q.83 If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to $x(\vec{b}\times\vec{c})$ then find the value of x.

Q.84 If $\vec{a}'=\hat{i}+\hat{j}, \vec{b}'=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}'=2\hat{i}+\hat{j}-\hat{k}$. Then altitude of the parallelepiped formed by the vectors $\vec{a},\vec{b},\vec{c}$ having base formed by $\vec{b}$ & $\vec{c}$ is $\frac{1}{\sqrt{x}}$ then find the value of x.
($\vec{a},\vec{b},\vec{c}$ and $\vec{a}',\vec{b}',\vec{c}'$ are reciprocal systems of vectors)

Q.85 If $\vec{u}$ and $\vec{v}$ are two unit vectors such that $\vec{u}\times\vec{v}+\vec{u}=\vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$ then value of $[\vec{u}\ \vec{v}\ \vec{w}]$ is –

Q.86 Let $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{B}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{C}=\hat{j}+\hat{k}$. If the vector $\vec{B}\times\vec{C}$ can be expressed as a linear combination $\vec{B}\times\vec{C}=x\vec{A}+y\vec{B}+zC$ where x, y, z are scalars, then find the value of (100x + 10y + 8z).

Q.87 If $\vec{a}$ & $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is

Q.88 Let $\vec{a}=-\hat{i}-\hat{k}, b=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is

Q.89 If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vector satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$ then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Q.90 Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is –

Q.91 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\pi/3$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is –

Q.92 Let $A_1, A_2, \ldots, A_n$ (n > 2) be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ a be the position vector of the point A_k, k = 1, 2, ... , n. If $\left|\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right|=\left|\sum_{k=1}^{n-1}(\vec{a}_k\cdot\vec{a}_{k+1})\right|$ then the minimum value of n is

Q.93 The reflection of the point P (1, 0, 0) in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is (A, – 8, – 4) then find the value of A.

Q.94 If plane 2x + 3y + 6z + k = 0 is tangent to the sphere $x^2+y^2+z^2+2x-2y+2z-6=0$, then find a positive value of k/13.

Q.95 The equation of the line passing through the point (1, 4, 3) which is perpendicular to both of the lines $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$ is $\frac{x-A}{-10}=\frac{y-4}{16}=\frac{z-3}{1}$. Find the value of A.

Q.96 In the above question, if one of the point (x_1, y_1, z_1) lies on the line such that the square of whose distance from (1, 4, 3) is 357. Find the value of $(x_1+y_1+z_1)$.

Q.97 In the above question, if another point (x_2, y_2, z_2) lies on the line such that the square of whose distance from (1, 4, 3) is 357. Find the value of $(x_2+y_2+z_2)$.

Q.98 The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane 2x – 4y + z = 7, is

Q.99 A variable plane at a distance of one unit from the origin cuts the coordinates axes at A, B and C. If the centroid D(x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}$ = k, the value of k is

Q.100 If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then |d| is :

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 If $\vec{a}, \vec{b}, \vec{c}$ are three non zero vectors out of which two are not collinear. If $\vec{a} + 2\vec{b}$ and $\vec{c}$; $\vec{b} + 3\vec{c}$ and $\vec{a}$ are collinear then $\vec{a} + 2\vec{b} + 6\vec{c}$ is – **[AIEEE 2002]**
(A) Parallel to $\vec{c}$ (B) Parallel to $\vec{a}$
(C) Parallel to $\vec{b}$ (D) 0

Q.2 If $[\vec{a}\ \vec{b}\ \vec{c}] = 4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] =$ **[AIEEE 2002]**
(A) 4 (B) 2
(C) 8 (D) 16

Q.3 If $\vec{c} = 2\lambda(\vec{a} \times \vec{b}) + 3\mu(\vec{b} \times \vec{a})$; $\vec{a} \times \vec{b} \neq 0$, $\vec{c} \cdot (\vec{a} \times \vec{b}) = 0$ then- **[AIEEE-2002]**
(A) $\lambda = 3\mu$ (B) $2\lambda = 3\mu$
(C) $\lambda + \mu = 0$ (D) None of these

Q.4 If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$, then orthogonal Component of $\vec{a}$ on $\vec{b}$ is- **[AIEEE-2002]**
(A) $3\hat{i} - 3\hat{j} + \hat{k}$ (B) $\frac{9(5\hat{i} - 3\hat{j} + \hat{k})}{35}$
(C) $\frac{(5\hat{i} - 3\hat{j} + \hat{k})}{35}$ (D) $9(5\hat{i} - 3\hat{j} + \hat{k})$

Q.5 A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is- **[AIEEE- 2002]**
(A) $\frac{4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$ (B) $\frac{2\hat{i} - 6\hat{j} - 3\hat{k}}{7}$
(C) $\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$ (D) $\frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$

Q.6 If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ & $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = **[AIEEE 2002]**
(A) 5/7 (B) 7/5
(C) –7/10 (D) –10/7

Q.7 The angle between the lines, whose direction ratios are 1, 1, 2 and $\sqrt{3} - 1, -\sqrt{3} - 1, 4$, is- **[AIEEE 2002]**
(A) 45° (B) 30°
(C) 60° (D) 90°

Q.8 The acute angle between the planes 2x – y + z = 6 and x + y + 2z = 3 is- **[AIEEE 2002]**
(A) 30° (B) 45°
(C) 60° (D) 75°

Q.9 The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ & $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if – **[AIEEE 2003]**
(A) k = 3 or – 3 (B) k = 0 or – 1
(C) k = 1 or – 1 (D) k = 0 or – 3

Q.10 A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1), B (2, 1, 3) and C(–1, 1, 2). Then the angle between the faces OAB and ABC will be- **[AIEEE 2003]**
(A) 90° (B) $\cos^{-1}(19/35)$
(C) $\cos^{-1}(17/31)$ (D) 30°

Q.11 Two systems of rectangular axes have the same origin. If a plane makes intercepts a, b, c and a′, b′, c′ on the two systems of axes respectively, then **[AIEEE-2003]**
(A) $a^2 + b^2 + c^2 = a'^2 + b'^2 + c'^2$
(B) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a'} + \frac{1}{b'} + \frac{1}{c'}$
(C) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$
(D) $\frac{1}{a^2 - a'^2} + \frac{1}{b^2 - b'^2} + \frac{1}{c^2 - c'^2} = 0$

Q.12 Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to- **[AIEEE 2003]**
(A) 3 (B) 0
(C) 1 (D) 2

Q.13 A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is-
(A) 50 units (B) 20 units **[AIEEE 2003]**
(C) 30 units (D) 40 units

Q.14 The vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ & $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[AIEEE 2003]**
(A) $\sqrt{288}$ (B) $\sqrt{18}$
(C) $\sqrt{72}$ (D) $\sqrt{33}$

Q.15 $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ is equal to- **[AIEEE 2003]**
(A) 1 (B) 0
(C) – 7 (D) 7

Q.16 Consider points A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then ABCD is a- **[AIEEE 2003]**
(A) parallelogram but not a rhombus
(B) square
(C) rhombus
(D) None of these

Q.17 If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{u}-\vec{w})$ equals **[AIEEE 2003]**
(A) $3\vec{u}.\vec{v}\times\vec{w}$ (B) 0
(C) $\vec{u}.\vec{v}\times\vec{w}$ (D) $\vec{u}.\vec{w}\times\vec{v}$

Q.18 If $\vec{a}, \vec{b}, \vec{c}$ are non- coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non- coplanar for **[AIEEE 2004]**
(A) all values of λ
(B) all except one value of λ
(C) all except two values of λ
(D) no value of λ

Q.19 Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$, $\vec{v}$ and $\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals- **[AIEEE 2004]**
(A) 2 (B) $\sqrt{7}$
(C) $\sqrt{14}$ (D) 14

Q.20 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non- zero vectors such that
$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$ If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals- **[AIEEE 2004]**
(A) 1/3 (B) $\sqrt{2}/3$
(C) 2/3 (D) $2\sqrt{2}/3$

Q.21 A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y- axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals- **[AIEEE 2004]**
(A) 2/3 (B) 1/5
(C) 3/5 (D) 2/5

Q.22 Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[AIEEE 2004]**
(A) 3/2 (B) 5/2
(C) 7/2 (D) 9/2

Q.23 A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by- **[AIEEE 2004]**
(A) (3a, 3a, 3a), (a, a, a) (B) (3a, 2a, 3a), (a, a, a)
(C) (3a, 2a, 3a), (a, a, 2a) (D) (2a, 3a, 3a), (2a, a, a)

Q.24 If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = t/2$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively are coplanar then λ equals- **[AIEEE 2004]**
(A) – 2 (B) – 1
(C) – 1/2 (D) 0

Q.25 If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\,z + 4 = 0$ is such that $\sin\theta = 1/3$ the value of λ is – **[AIEEE-2005]**
(A) 5/3 (B) –3/5
(C) 3/4 (D) – 4/3

Q.26 The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is- **[AIEEE-2005]**
(A) 0º (B) 90º
(C) 45º (D) 30º

Q.27 The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is - **[AIEEE-2005]**
(A) 10/9 (B) $\frac{10}{3\sqrt{3}}$
(C) 3/10 (D) 10/3

Q.28 For any vector $\vec{a}$, $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$ is equal to
(A) $|\vec{a}|^2$ (B) $2|\vec{a}|^2$ **[AIEEE- 2005]**
(C) $3|\vec{a}|^2$ (D) None of these

Q.29 If C is the mid point of AB and P is any point outside AB, then - **[AIEEE-2005]**
(A) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$
(B) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$
(C) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$
(D) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

Q.30 If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $[\lambda(\vec{a}+\vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}] = [\vec{a}\ \vec{b}+\vec{c}\ \vec{b}]$ for - **[AIEEE-2005]**
(A) exactly one value of λ (B) no value of λ
(C) exactly three values of λ (D) exactly two values of λ

Q.31 If $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{a}.\vec{b} \neq 0$] $\vec{b}.\vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are– **[AIEEE 2006]**
(A) inclined at an angle of $\pi/6$ between them
(B) perpendicular
(C) parallel
(D) inclined at an angle of $\pi/3$ between them

Q.32 ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ with magnitudes 1/AB and 1/AC respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is **[AIEEE 2006]**
(A) $\frac{(AB)(AC)}{AB+AC}$ (B) $\frac{1}{AB} + \frac{1}{AC}$
(C) $\frac{1}{AD}$ (D) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$

Q.33 The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$] $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right-angled triangle with C = π/2 are – **[AIEEE 2006]**
(A) –2 and –1 (B) –2 and 1
(C) 2 and –1 (D) 2 and 1

Q.34 The two lines x = ay + b, z = cy + d; and x = a′y + b′, z = c′y + d′ are perpendicular to each other if **[AIEEE-2006]**
(A) aa′ + cc′ = 1 (B) $\frac{a}{a'}+\frac{c}{c'}=-1$
(C) $\frac{a}{a'}+\frac{c}{c'}=1$ (D) aa′+ cc′= – 1

Q.35 The image of the point (–1, 3, 4) in the plane x – 2y = 0 is **[AIEEE 2006]**
(A) (15, 11, 4) (B) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$
(C) (8, 4, 4) (D) None of these

Q.36 Let L be the line of intersection of the planes 2x+ 3y+z= 1 and x + 3y + 2z = 2. If L makes an angle α with the positive x-axis, then cos α equals- **[AIEEE 2007]**
(A) $1/\sqrt{3}$ (B) 1/2
(C) 1 (D) $1/\sqrt{2}$

Q.37 If a line makes an angle of π/4 with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is-
(A) π/6 (B) π/3 **[AIEEE-2007]**
(C) π/4 (D) π/2

Q.38 If $\vec{u}$ and $\vec{v}$ are unit vectors and θ is the acute angle between them, then $2\vec{u}\times 3\vec{v}$ is a unit vector for –
(A) Exactly two values of θ **[AIEEE 2007]**
(B) More than two values of θ
(C) No value of θ
(D) Exactly one value of θ

Q.39 Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$] $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ & $\vec{b}$, then x equals - **[AIEEE 2007]**
(A) 0 (B) 1
(C) –4 (D) –2

Q.40 The non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is
(A) π/4 (B) π/2 **[AIEEE 2008]**
(C) π (D) 0

Q.41 The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ & $\vec{c}=\hat{j}+\hat{k}$ & bisects the angle between $\vec{b}$ & $\vec{c}$. Then which one of the following gives possible values of α & β ? **[AIEEE 2008]**
(A) α = 1, β = 2 (B) α = 2, β = 1
(C) α = 1, β = 1 (D) α = 2, β = 2

Q.42 If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to- **[AIEEE-2008]**
(A) 5 (B) 2
(C) –2 (D) –5

Q.43 The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0,\frac{17}{2},\frac{-13}{2}\right)$. Then **[AIEEE-2008]**
(A) a = 4, b = 6 (B) a = 6, b = 4
(C) a = 8, b = 2 (D) a = 2, b = 8

Q.44 If $\vec{u},\vec{v},\vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality
$[3\vec{u}\ p\vec{v}\ p\vec{w}]-[p\vec{v}\ \vec{w}\ q\vec{u}]-[2\vec{w}\ q\vec{v}\ q\vec{u}]=0$
holds for : **[AIEEE 2009]**
(A) exactly two values of (p,q)
(B) more than two but not all values of (p, q)
(C) all values of (p, q)
(D) exactly one value of (p, q)

Q.45 Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane x + 3y – αz + β = 0. then (α, β) equals : **[AIEEE-2009]**
(A) (–6, 7) (B) (5, –15)
(C) (–5, 5) (D) (6, –17)

Q.46 The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[AIEEE-2009]**
(A) $\frac{6}{5},\frac{-3}{5},\frac{2}{5}$ (B) $\frac{6}{7},\frac{-3}{7},\frac{2}{7}$
(C) $\frac{-6}{7},\frac{-3}{7},\frac{2}{7}$ (D) 6, –3, 2

Q.47 Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}=3$ is – **[AIEEE 2010]**
(A) $2\hat{i}-\hat{j}+2\hat{k}$ (B) $\hat{i}-\hat{j}-2\hat{k}$
(C) $\hat{i}+\hat{j}-2\hat{k}$ (D) $-\hat{i}+\hat{j}-2\hat{k}$

Q.48 If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k},\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) =
(A) (2, –3) (B) (–2, 3) **[AIEEE 2010]**
(C) (3, –2) (D) (–3, 2)

Q.49 **Statement-1:** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5. **[AIEEE 2010]**
Statement-2: The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is false.
(C) Statement-1 is false, Statement-2 is true.
(D) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1.

Q.50 A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals – **[AIEEE 2010]**
(A) 45° (B) 60°
(C) 75° (D) 30°

Q.51 If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane x + 2y + 3z = 4 is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals –
(A) 2/3 (B) 3/2 **[AIEEE 2011]**
(C) 2/5 (D) 5/3

Q.52 If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$ then the vlaue of $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times [(\vec{a} + 2\vec{b})]$ is – **[AIEEE 2011]**
(A) – 5 (B) –3
(C) 5 (D) 3

Q.53 **Statement-1 :** The point A (1, 0, 7) is the mirror image of the point B (1, 6, 3) in the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement-2 : The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B (1, 6, 3). **[AIEEE 2011]**
(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.54 The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ & $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to – **[AIEEE 2011]**
(A) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{d}}\right)\vec{c}$ (B) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$
(C) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (D) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Q.55 Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is **[AIEEE 2012]**
(A) π/6 (B) π/2
(C) π/3 (D) π/4

Q.56 A equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is **[AIEEE 2012]**
(A) x – 2y + 2z – 3 = 0 (B) x – 2y + 2z + 1 = 0
(C) x – 2y + 2z – 1 = 0 (D) x – 2y + 2z + 5 = 0

Q.57 If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersection, then k is equal to – **[AIEEE 2012]**
(A) –1 (B) 2/9
(C) 9/2 (D) 0

Q.58 Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and ∠ BAD be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE 2012]**
(A) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (B) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}.\vec{q}}{(\vec{p}.\vec{p})}\right)\vec{p}$
(C) $\vec{r} = \vec{q} - \left(\frac{\vec{p}.\vec{q}}{(\vec{p}.\vec{p})}\right)\vec{p}$ (D) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

Q.59 Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is – **[JEE MAIN 2013]**
(A) 3/2 (B) 5/2
(C) 7/2 (D) 9/2

Q.60 If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ & $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have – **[JEE MAIN 2013]**
(A) any value (B) exactly one value
(C) exactly two values (D) exactly three values

Q.61 If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is – **[JEE MAIN 2013]**
(A) $\sqrt{18}$ (B) $\sqrt{72}$
(C) $\sqrt{33}$ (D) $\sqrt{45}$

Q.62 If $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}] = \lambda [\vec{a} \;\; \vec{b} \;\; \vec{c}]^2$, then λ is equal to **[JEE MAIN 2014]**
(A) 2 (B) 3
(C) 0 (D) 1

Q.63 The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane 2x – y + z + 3 = 0 is the line – **[JEE MAIN 2014]**
(A) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$ (B) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$
(C) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$ (D) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

Q.64 The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is – **[JEE MAIN 2014]**

(A) $\pi/3$ (B) $\pi/4$
(C) $\pi/6$ (D) $\pi/2$

Q.65 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ & $\vec{c}$, then a value of $\sin\theta$ is **[JEE MAIN 2015]**

(A) $\frac{-\sqrt{2}}{3}$ (B) $\frac{2}{3}$ (C) $\frac{-2\sqrt{3}}{3}$ (D) $\frac{2\sqrt{2}}{3}$

Q.66 The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is – **[JEE MAIN 2015]**

(A) 8 (B) $3\sqrt{21}$
(C) 13 (D) $2\sqrt{14}$

Q.67 The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is **[JEE MAIN 2015]**

(A) $x + 3y + 6z = -7$ (B) $x + 3y + 6z = 7$
(C) $2x + 6y + 12z = -13$ (D) $2x + 6y + 12z = 13$

Q.68 If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $\ell x + my - z = 9$, then $\ell^2 + m^2$ is equal to: **[JEE MAIN 2016]**

(A) 18 (B) 5
(C) 2 (D) 26

Q.69 Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is – **[JEE MAIN 2016]**

(A) $\pi/2$ (B) $2\pi/3$
(C) $5\pi/6$ (D) $3\pi/4$

Q.70 The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[JEE MAIN 2016]**

(A) $10\sqrt{3}$ (B) $10/\sqrt{3}$
(C) 20/3 (D) $3\sqrt{10}$

Q.71 Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} \cdot \vec{c}$ is equal to :

(A) 5 (B) 1/8 **[JEE MAIN 2017]**
(C) 25/8 (D) 2

Q.72 If the image of the point P (1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to : **[JEE MAIN 2017]**

(A) $\sqrt{42}$ (B) $6\sqrt{5}$ (C) $3\sqrt{5}$ (D) $2\sqrt{42}$

Q.73 The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ is – **[JEE MAIN 2017]**

(A) $5/\sqrt{83}$ (B) $10/\sqrt{74}$
(C) $20/\sqrt{74}$ (D) $10/\sqrt{83}$

Q.74 If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is: **[JEE MAIN 2018]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{4\sqrt{2}}$ (D) $\frac{1}{3\sqrt{2}}$

Q.75 Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to

(A) 256 (B) 84 **[JEE MAIN 2018]**
(C) 336 (D) 315

Q.76 length of the projection of the line segment joining the points (5, –1, 4) & (4, –1,3) on the plane, $x + y + z = 7$ is: **[JEE MAIN 2018]**

(A) 1/3 (B) $\sqrt{2/3}$
(C) $2/\sqrt{3}$ (D) 2/3

Q.77 The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point :**[JEE MAIN 2019 (JAN)]**

(A) (–3, 0, –1) (B) (3, 3, –1)
(C) (3, 2, 1) (D) (–3, 1, 1)

Q.78 Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$ then $|\vec{c}|^2$ is equal to **[JEE MAIN 2019 (JAN)]**

(A) 19/2 (B) 8
(C) 17/2 (D) 9

Q.79 The equation of the line passing through (– 4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is: **[JEE MAIN 2019 (JAN)]**

(A) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$ (B) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(C) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$ (D) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Q.80 The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is **[JEE MAIN 2019 (APRIL)]**

(A) less than 2
(B) greater than 3 but less than 4
(C) greater than 4
(D) greater than 2 but less than 3

Q.81 The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is **[JEE MAIN 2019 (APRIL)]**

(A) $\sqrt{3}/2$ (B) $\sqrt{3/2}$
(C) $\sqrt{6}$ (D) $3\sqrt{6}$

Q.82 The equation of a plane containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point (1, 1, 0) is : **[JEE MAIN 2019 (APRIL)]**

(A) $x+3y+z=4$ (B) $x-y-z=0$
(C) $x-3y-2z=-2$ (D) $2x-z=2$

Q.83 Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if :**[JEE MAIN 2019 (APRIL)]**

(A) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (B) $0<r\le\sqrt{\frac{3}{2}}$
(C) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$ (D) $r\ge 5\sqrt{\frac{3}{2}}$

Q.84 The vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$ is : **[JEE MAIN 2019 (APRIL)]**

(A) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (B) $\vec{r}\cdot(\hat{i}-\hat{k})-2=0$
(C) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$ (D) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

Q.85 If a point R (4, y, z) lies on the line segment joining the points P (2, –3, 4) and Q (8, 0, 10), then the distance of R from the origin is : **[JEE MAIN 2019 (APRIL)]**

(A) $2\sqrt{14}$ (B) 6
(C) $\sqrt{53}$ (D) $2\sqrt{21}$

Q.86 Let $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to **[JEE MAIN 2019 (APRIL)]**

(A) $-3\hat{i}+9\hat{j}+5\hat{k}$ (B) $3\hat{i}-9\hat{j}-5\hat{k}$
(C) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$ (D) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$

Q.87 A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle π/4 with the plane $y-z+5=0$, also passes through the point **[JEE MAIN 2019 (APRIL)]**

(A) $(-\sqrt{2},1,-4)$ (B) $(\sqrt{2},1,4)$
(C) $(\sqrt{2},-1,4)$ (D) $(-\sqrt{2},-1,-4)$

Q.88 If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x+2y+3z=15$ at a point P, then the distance of P from the origin is **[JEE MAIN 2019 (APRIL)]**

(A) 9/2 (B) $2\sqrt{5}$
(C) $\sqrt{5}/2$ (D) 7/2

Q.89 The vertices B and C of a Δ ABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is **[JEE MAIN 2019 (APRIL)]**

(A) $2\sqrt{34}$ (B) $\sqrt{34}$
(C) 6 (D) $5\sqrt{17}$

Q.90 If a unit vector $\vec{a}$ makes angles π/3 with $\hat{i}$, π/4 with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$, then a value of θ is **[JEE MAIN 2019 (APRIL)]**

(A) 5π/12 (B) 5π/6
(C) 2π/3 (D) π/4

Q.91 If the length of the perpendicular from the point (β, 0, β) (β ≠ 0) to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) –1 (B) 2
(C) –2 (D) 1

Q.92 If Q (0, –1, –3) is the image of the point P in the plane $3x-y+4z=2$ and R is the point (3, –1, –2), then the area (in sq. units) of Δ PQR is **[JEE MAIN 2019 (APRIL)]**

(A) $\frac{\sqrt{65}}{2}$ (B) $\frac{\sqrt{91}}{4}$
(C) $2\sqrt{13}$ (D) $\frac{\sqrt{91}}{2}$

Q.93 The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is **[JEE MAIN 2019 (APRIL)]**

(A1) 7 (B) $4\sqrt{3}$ (C) $2\sqrt{13}$ (D) 6

Q.94 A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are **[JEE MAIN 2019 (APRIL)]**

(A) (2, 0, 1) (B) (4, 0, –1)
(C) (–1, 0, 4) (D) (1, 0, 2)

Q.95 If the plane $2x - y + 2z + 3 = 0$ has the distances 1/3 & 2/3 units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) 15 (B) 5
(C) 13 (D) 9

Q.96 If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the palne $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) $2\sqrt{14}$ (B) $\sqrt{14}$
(C) $2\sqrt{7}$ (D) 14

Q.97 Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is **[JEE MAIN 2019 (APRIL)]**

(A) $4(2\hat{i} + 2\hat{j} - \hat{k})$ (B) $4(-2\hat{i} - 2\hat{j} + \hat{k})$
(C) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (D) $4(2\hat{i} + 2\hat{j} + \hat{k})$

Q.98 The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is **[JEE MAIN 2019 (APRIL)]**

(A) $\sqrt{3}$ (B) $1/\sqrt{3}$
(C) 1/3 (D) 3

Q.99 A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point: **[JEE MAIN 2019 (APRIL)]**

(A) (2, 4, 1) (B) (2, –4, 1)
(C) (1, 4, –1) (D) (1, –4, 1)

Q.100 Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is : **[JEE MAIN 2020 (JAN)]**

(A) (6, 5, –2) (B) (4, 3, 2)
(C) (3, 4, –2) (D) (6, 5, 2)

Q.101 If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ and $\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$ then $(\lambda, \vec{d}) =$ **[JEE MAIN 2020 (JAN)]**

(A) $\left(\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (B) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$
(C) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ (D) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$

Q.102 If Q (5/3, 7/3, 17/3) is foot of perpendicular drawn from P (1, 0, 3) on a line L and if line L is passing through (α, 7, 1), then value of α is **[JEE MAIN 2020 (JAN)]**

Q.103 The shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} \text{ is –}$$

[JEE MAIN 2020 (JAN)]

(A) $\frac{7}{2}\sqrt{30}$ (B) $3\sqrt{30}$
(C) 3 (D) $2\sqrt{30}$

Q.104 If volume of parallelopiped whose coterminous edges are $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{w} = \hat{i} + \hat{j} + 3\hat{k}$ is 1 cubic unit then cosine of angle between $\vec{u}$ and $\vec{v}$ is – **[JEE MAIN 2020 (JAN)]**

(A) $\frac{7}{3\sqrt{10}}$ (B) $\frac{7}{6\sqrt{3}}$
(C) $\frac{5}{3\sqrt{3}}$ (D) 5/7

Q.105 Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ is non-zero vector and $\vec{b}\times\vec{c} = \vec{b}\times\vec{a}$, $\vec{a}\cdot\vec{c} = 0$ find $\vec{b}\cdot\vec{c} = 0$. **[JEE MAIN 2020 (JAN)]**

(A) 1/2 (B) 1/3
(C) –1/2 (D) –1/3

Q.106 Image of (1, 2, 3) w.r.t a plane is $\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$ then which of the following points lie on the plane **[JEE MAIN 2020 (JAN)]**

(A) (–1, 1, –1) (B) (–1, –1, –1)
(C) (–1, –1, 1) (D) (1, 1, –1)

Q.107 $\lambda x + 2y + 2z = 5$
$2\lambda x + 3y + 5z = 8$
$4x + \lambda y + 6z = 10$
for the system of equation check the correct option. **[JEE MAIN 2020 (JAN)]**

(A) Infinite solutions when $\lambda = 8$
(B) Infinite solutions when $\lambda = 2$
(C) no solutions when $\lambda = 8$
(D) no solutions when $\lambda = 2$

Q.108 If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in R)$ are coplanar and $3(\vec{p}\cdot\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$, then the value of λ is ___. **[JEE MAIN 2020 (JAN)]**

Q.109 The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is———. **[JEE MAIN 2020 (JAN)]**

Q.110 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5, \vec{b}\cdot\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/3$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to __. **[JEE MAIN 2020 (JAN)]**

Q.111 If the di stance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

[JEE MAIN 2020 (JAN)]

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A	C	B	A	C	B	B	A	C	A	D	A	C	B	C	B	B	A	D	B	A	C	C	B	A	C	B	A
Q	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
A	D	C	A	B	A	C	A	C	A	C	A	A	B	B	A	D	D	D	C	D	C	D	A	D	A	C	B
Q	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81
A	A	D	B	B	B	D	A	A	C	A	C	C	A	B	D	A	A	C	A	C	A	D	B	D	B	D	B
Q	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
A	C	D	A	A	B	A	C	B	C	B	D	B	D	B	D	C	B	D	C	A	C	B	B	B	A	B	B
Q	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132			
A	C	A	B	B	C	C	A	B	A	B	C	D	D	B	D	C	C	A	A	A	B	A	B	B			

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A	C	C	A	C	A	B	C	B	A	A	D	C	B	D	C	A	A	D	C	D	C	B	A	C	C	B	C
Q	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
A	A	A	D	B	A	D	B	D	A	B	A	A	D	C	D	A	D	C	C	A	C	A	B	C	C	D	A
Q	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81
A	A	A	C	A	A	B	A	D	D	A	D	B	B	A	C	C	B	A	C	B	D	B	A	A	C	A	C
Q	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100								
A	343	6	2	1	101	5	9	3	10	4	8	5	2	1	15	1	7	9	6								

EXERCISE-3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	D	D	B	B	C	D	C	C	D	B	C	A	D	D	C	D	C	C	C	D	C	C	B	A	A	B	B	B	A	B
Q	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	C	C	D	D	D	A	D	D	D	C	C	D	B	D	A	B	D	D	A	B	A	A	B	D	C	A	C	B	C	C
Q	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
A	C	D	A	A	D	C	B	C	C	A	D	D	D	D	C	B	C	A	B	B	B	B	D	C	A	C	B	A	B	C
Q	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111									
A	A	D	A	A	C	A	C	A	B	A	D	4	B	2	C	D	D	1	8	30	3									

CHAPTER- 10: VECTORS & 3-DIMENSIONAL GEOMETRY

SOLUTIONS TO TRY IT YOURSELF

TRY IT YOURSELF-1

(1) Here $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}.2} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

(2) Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$ be two given vectors.

The projection of vector $\vec{a}$ on the vector $\vec{b}$ is given by

$$\frac{1}{|\vec{b}|}(\vec{a}.\vec{b}) = \frac{(\hat{i}-\hat{j}).(\hat{i}+\hat{j})}{\sqrt{(1)^2+(1)^2}} = \frac{1\times1-1\times1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

(3) Given $|\vec{a}| = |\vec{b}|$ and $\vec{a}.\vec{b} = \frac{1}{2}$

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\ |\vec{b}|}\ ;\ \cos 60° = \frac{\frac{1}{2}}{|\vec{a}|.|\vec{a}|}\quad [\because |\vec{a}| = |\vec{b}|]$$

$$\frac{1}{2} = \frac{1}{2|\vec{a}|^2} \Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1$$

Thus, $|\vec{a}| = |\vec{b}| = 1$

(4) Given $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

Now, $\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$

$$= 2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$$
$$= (2-\lambda)\,\hat{i} + (2+2\lambda)\,\hat{j} + (3+\lambda)\,\hat{k}$$

Since $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then $(\vec{a} + \lambda\vec{b}).\vec{c} = 0$

$\Rightarrow\ [(2-\lambda)\,\hat{i} + (2+2\lambda)\,\hat{j} + (3+\lambda)\,\hat{k}\,].(3\hat{i} + \hat{j}) = 0$
$\Rightarrow\ (2-\lambda)\times 3 + (2+2\lambda)\times 1 + (3+\lambda)\times 0 = 0$
$\Rightarrow\ 6 - 3\lambda + 2 + 2\lambda = 0$
$\Rightarrow\ -\lambda + 8 = 0 \Rightarrow\ \ \lambda = 8$

(5) Position vector of point $A = \hat{i} + 2\hat{j} + 3\hat{k}$

Position vector of point $B = -\hat{i}$

Position vector of point $C = \hat{j} + 2\hat{k}$

Now, $\overrightarrow{BA}$ = P.V. of A – P.V. of B

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (-\hat{i}) = \hat{i} + 2\hat{j} + 3\hat{k} + \hat{i} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{BC}$ = P.V. of C – P.V. of B $= (\hat{j} + 2\hat{k}) - (-\hat{i}) = \hat{i} + \hat{j} + 2\hat{k}$

Now, $\overrightarrow{BA}.\overrightarrow{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}).(\hat{i} + \hat{j} + 2\hat{k})$
$= 2\times1 + 2\times1 + 3\times2 = 2 + 2 + 6 = 10$

$$|\overrightarrow{BA}| = \sqrt{(2)^2 + (2)^2 + (3)^2} = \sqrt{4+4+9} = \sqrt{17}$$

$$|\overrightarrow{BC}| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\therefore\ \cos\angle ABC = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{10}{\sqrt{17}.\sqrt{6}} = \frac{10}{\sqrt{102}}$$

$$\therefore\ \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

(6) **(B).** Given that $\vec{a}$ and $\vec{b}$ are two unit vectors.

$\therefore\ \ |\vec{a}| = 1$ and $|\vec{b}| = 1$

Also given that $(\vec{a} + 2\vec{b}) \perp (5\vec{a} - 4\vec{b})$

$\Rightarrow\ (\vec{a} + 2\vec{b})\,.\,(5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow\ 5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}.\vec{b} + 10\vec{b}.\vec{a} = 0$

$\Rightarrow\ 5 - 8 + 6\vec{a}.\vec{b} = 0\ \Rightarrow\ \ 6|\vec{a}||\vec{b}|\cos\theta = 3$

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow\ \cos\theta = 1/2 \Rightarrow \theta = 60°$

(7) **(C).** Let $\vec{v} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow\ \vec{v} = (\lambda+\mu)\,\hat{i} + (\lambda-\mu)\,\hat{j} + (\lambda+\mu)\,\hat{k}$

Now, $\vec{v}.\hat{c} = \frac{1}{\sqrt{3}} \Rightarrow \frac{(\lambda+\mu) - (\lambda-\mu) - (\lambda+\mu)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \mu - \lambda = 1 \Rightarrow \mu = \lambda + 1$

$\therefore\ \vec{v} = (2\lambda+1)\,\hat{i} - \hat{j} + (2\lambda+1)\,\hat{k}$

For $\lambda = 1$, $\vec{v} = 3\hat{i} - \hat{j} + 3\hat{k}$

(8) Here position vector of point P = $2\vec{a} + \vec{b}$

Position vector of point Q = $\vec{a} - 3\vec{b}$

Now point R divides PQ in the ratio 1 : 2 externally

$\therefore$ Position vector of point

$$R = \frac{(\vec{a} - 3\vec{b})\times 1 - (2\vec{a} + \vec{b})\times 2}{1-2}$$

$$= \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

(9) Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} + \vec{c} = 2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k} = (2+\lambda)\,\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore\ |\vec{b}+\vec{c}| = \sqrt{(2+\lambda)^2+(6)^2+(-2)^2}$

$= \sqrt{4+\lambda^2+4\lambda+36+4} = \sqrt{\lambda^2+4\lambda+44}$

The unit vector parallel to vector $(\vec{b}+\vec{c})$ is

$$\frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = \frac{(2+\lambda)\,\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

It is given that $\vec{a}.\dfrac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$

$\therefore\ (\hat{i}+\hat{j}+\hat{k}).\dfrac{(2+\lambda)\,\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow\ \dfrac{2+\lambda+6-2}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow\ \lambda+6 = \sqrt{\lambda^2+4\lambda+44}$

$\Rightarrow\ (\lambda+6)^2 = \lambda^2+4\lambda+44$

$\Rightarrow\ \lambda^2+36+12\lambda = \lambda^2+4\lambda+44$

$\Rightarrow\ 12\lambda-4\lambda = 44-36 \Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$

TRY IT YOURSELF-2

(1) Here adjacent sides of parallelogram are

$\vec{a} = \hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}-7\hat{j}+\hat{k}$

$$\therefore\ \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$= (-1+21)\,\hat{i}-(1-6)\,\hat{j}+(-7+2)\,\hat{k} = 20\hat{i}+5\hat{j}-5\hat{k}$

$\therefore\ |\vec{a}\times\vec{b}| = \sqrt{(20)^2+(5)^2+(-5)^2} = \sqrt{400+25+25}$

$= \sqrt{450} = 15\sqrt{2}$

$\therefore$ Area of parallelogram $|\vec{a}\times b| = 15\sqrt{2}$ sq. units

(2) $V = \left\|\begin{matrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{matrix}\right\| = \pi$ cubic units

(3) **9.** $(\vec{r}-\vec{c})\times\vec{b} = 0$

$\vec{r}-\vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c}+\lambda\vec{b}\,;\ \lambda \in R \qquad \because\ \vec{r}.\vec{a} = 0$

$\Rightarrow\ (\vec{c}+\lambda\vec{b}).\vec{a} = 0$

$\Rightarrow\ ((\hat{i}+2\hat{j}+3\hat{k})+\lambda\,(-\hat{i}+\hat{j})).(-\hat{i}-\hat{k}) = 0$

$\Rightarrow\ ((1-\lambda)\,\hat{i}+(2+\lambda)\,\hat{j}+3\hat{k}).(-\hat{i}-\hat{k}) = 0$

$\Rightarrow\ \lambda-1-3 = 0 \Rightarrow \lambda = 4$

So, $\vec{r}.\vec{b} = (-3\hat{i}+6\hat{j}+3\hat{k}).(-\hat{i}+\hat{j}) = 3+6 = 9$

(4) **(A).** $\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors,

$2\vec{a}-\vec{b},\ 2\vec{b}-\vec{c}$ and $2\vec{c}-\vec{a}$ are also coplanar vectors.

Being linear combination of $\vec{a}, \vec{b}, \vec{c}$

Thus, $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}] = 0$

(5) **(C).** $\vec{a} = \hat{i}-\hat{k},\ \vec{b} = x\hat{i}+\hat{j}+(1-x)\,\hat{k}$

$\vec{c} = y\hat{i}+x\hat{j}+(1+x-y)\,\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= 1\,(1+x-y-x+x^2)-1\,(x^2-y)$

$= 1 \quad \therefore$ depends neither on x nor on y.

(6) **(B).** $\hat{a}, \hat{b}, \hat{c}$ are unit vectors.

$\therefore\ \hat{a}.\hat{a} = \hat{b}.\hat{b} = \hat{c}.\hat{c} = 1$

Now, $x = |\hat{a}-\hat{b}|^2+|\hat{b}-\hat{c}|^2+|\hat{c}-\hat{a}|^2$

$= \hat{a}.\hat{a}+\hat{b}.\hat{b}-2\hat{a}.\hat{b}+\hat{b}.\hat{b}+\hat{c}.\hat{c}-2\hat{b}.\hat{c}+\hat{c}.\hat{c}+\hat{a}.\hat{a}-2\hat{c}.\hat{a}$

$= 6-2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})$ .... (1)

Also, $|\hat{a}+\hat{b}+\hat{c}| \ge 0$

$\Rightarrow\ \hat{a}.\hat{a}+\hat{b}.\hat{b}+\hat{c}.\hat{c}-2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \ge 0$

$\Rightarrow\ 3+2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \ge 0 \Rightarrow 2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \ge -3$

$\Rightarrow\ -2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \le 3 \Rightarrow 6-2\,(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \le 9$ .... (2)

From eq. (1) and (2), $x \le 9 \quad \therefore$ x does not exceed 9.

(7) **(C).** Volume of parallelopiped formed by

$\vec{u} = \hat{i}+a\hat{j}+\hat{k},\ \vec{v} = \hat{j}+a\hat{k},\ \vec{w} = a\hat{i}+\hat{k}$ is

$$V = [\vec{u}\ \ \vec{v}\ \ \vec{w}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$= 1\,(1-0)-a\,(0-a^2)+1\,(0-a) = 1+a^3-a$

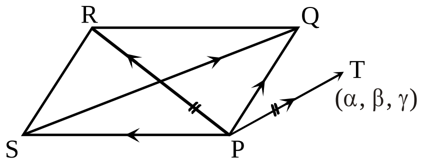
For V to be min $\dfrac{dV}{da} = 0 \Rightarrow 3a^2-1 = 0$

$\Rightarrow\ a = \pm 1/\sqrt{3}$ But $a > 0 \Rightarrow a = 1/\sqrt{3}$

(8) **(A).** Volume $= |\hat{a}.(\hat{b}\times\hat{c})| = \sqrt{\begin{vmatrix} \hat{a}.\hat{a} & \hat{a}.\hat{b} & \hat{a}.\hat{c} \\ \hat{b}.\hat{a} & \hat{b}.\hat{b} & \hat{b}.\hat{c} \\ \hat{c}.\hat{a} & \hat{c}.\hat{b} & \hat{c}.\hat{c} \end{vmatrix}}$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}$$

(9) **(C).**



Area of base (PQRS)

$$= \frac{1}{2}|\overrightarrow{PR} \times \overrightarrow{SQ}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & -4 \end{vmatrix}$$

$$= \frac{1}{2}|-10\hat{i}+10\hat{j}-10\hat{k}| = 5|\hat{i}-\hat{j}+\hat{k}| = 5\sqrt{3}$$

Height = proj. of PT on

$$\hat{i}-\hat{j}+\hat{k} = \left|\frac{1-2+3}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

$$\text{Volume} = (5\sqrt{3})\left(\frac{2}{\sqrt{3}}\right) = 10 \text{ cu. units}$$

TRY IT YOURSELF-3

(1) Given a = –18, b = 12 and c = – 4
Direction cosines of a line are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\therefore \frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

$$\Rightarrow \frac{-18}{\sqrt{324+144+16}}, \frac{12}{\sqrt{324+144+16}}, \frac{-4}{\sqrt{324+144+16}}$$

$$\Rightarrow \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} \Rightarrow \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

(2) Let ℓ_1 and ℓ_2 be two lines having direction ratios a, b, c and b – c, c – a, a – b respectively.
Let θ be the angle between ℓ_1 and ℓ_2 then

$$\cos\theta = \left|\frac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right|$$

$$= \left|\frac{ab-ac+bc-ab+ac-bc}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right| = 0$$

$$\therefore \cos\theta = 0 = \cos\frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2}$$

(3) Here coordinates of the points A, B, C and D are (1, 2, 3), (4, 5, 7), (– 4, 3, – 6) and (2, 9, 2) respectively.
∴ Direction ratios of line AB are (4 – 1), (5 – 2), (7 – 3) i.e., 3, 3, 4.
Direction ratios of line CD are (2 + 4), (9 – 3), (2 + 6) i.e., 6, 6, 8
Let θ be the angle between AB and CD then

$$\cos\theta = \left|\frac{3\times6+3\times6+4\times8}{\sqrt{(3)^2+(3)^2+(4)^2}\sqrt{(6)^2+(6)^2+(8)^2}}\right|$$

$$= \left|\frac{18+18+32}{\sqrt{9+9+6}\sqrt{36+36+64}}\right| = \frac{68}{\sqrt{34\times136}} = \frac{68}{68} = 1$$

∴ cos θ = 1 = cos 0 ⇒ θ = 0°

(4) Here the equation of given lines are

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

∴ Direction ratios of two lines are (2, 5, –3) and (–1, 8, 4).
Let θ be the angle between two given lines then

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\therefore \cos\theta = \frac{2\times(-1)+5\times8+(-3)\times4}{\sqrt{(2)^2+(5)^2+(-3)^2}\sqrt{(-1)^2+(8)^2+(4)^2}}$$

$$= \frac{-2+40-12}{\sqrt{4+25+9}\sqrt{1+64+16}} = \frac{26}{\sqrt{38}\sqrt{81}} = \frac{26}{9\sqrt{38}}$$

$$\therefore \cos\theta = \frac{26}{9\sqrt{38}} \Rightarrow \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

(5) Here the equation of given lines are :

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2} \text{ and } \frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$$

∴ Direction ratios of two lines are

$$\left(-3, \frac{2p}{7}, 2\right) \text{ and } \left(\frac{-3p}{7}, 1, -5\right)$$

We know that two lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$$\therefore -3\times\frac{-3p}{7}+\frac{2p}{7}\times1+2\times-5 = 0$$

$$\Rightarrow \frac{9p}{7}+\frac{2p}{7}-10 = 0 \Rightarrow 9p+2p-70 = 0$$

$$\Rightarrow 11p = 70 \Rightarrow p = \frac{70}{11}$$

(6) The equations of given lines are

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \quad ....(1)$$

and $$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \quad ....(2)$$

Now, the line (i) passes through the point (–1, –1, –1) and has direction ratios 7, –6, 1.

$\therefore \quad \vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\lambda\,(7\hat{i}-6\hat{j}+\hat{k}) \quad ....(3)$

The line (ii) passes through the point (3, 5, 7) and has direction ratios 1, –2, 1.

$\therefore \quad \vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\mu\,(\hat{i}-2\hat{j}+\hat{k}) \quad ....(4)$

Comparing equations (3) and (4) with

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$, we have

$\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}$ and $\vec{b}_2=7\hat{i}-6\hat{j}+\hat{k}$

$\vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$ and $\vec{b}_2=\hat{i}-2\hat{j}+\hat{k}$

$\therefore \quad \vec{a}_2-\vec{a}_1=(3\hat{i}+5\hat{j}+7\hat{k})-(-\hat{i}-\hat{j}-\hat{k})$

$=3\hat{i}+5\hat{j}+7\hat{k}+\hat{i}+\hat{j}+\hat{k}=4\hat{i}+6\hat{j}+8\hat{k}$

$$\therefore \quad \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$$

$=(-6+2)\,\hat{i}-(7-1)\,\hat{j}+(-14+6)\,\hat{k}=-4\hat{i}-6\hat{j}-8\hat{k}$

$\therefore \quad |\vec{b}_1\times\vec{b}_2|=\sqrt{(-4)^2+(-6)^2+(-8)^2}$

$=\sqrt{16+36+64}=\sqrt{116}=2\sqrt{29}$

Also, $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(4\hat{i}+6\hat{j}+8\hat{k}).(-4\hat{i}-6\hat{j}-8\hat{k})$

$=4\times(-4)+6\times(-6)+8\times(-8)=-16-36-64=-116$

$\therefore$ Shortest distance between given lines

$$=\frac{|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$$

$$=\frac{|-116|}{2\sqrt{29}}=\frac{116}{2\sqrt{29}}=\frac{58}{\sqrt{29}}=2\sqrt{29}$$

(7) Here the given lines are

$$\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2} \text{ and } \frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$$

The direction ratios of two lines are –3, 2k, 2 and 3k, 1, –5.

We know that two lines are perpendicular to each other iff

$a_1a_2+b_1b_2+c_1c_2=0$

$\therefore \quad -3\times 3k+2k\times 1+2\times -5=0 \Rightarrow -9k+2k-10=0$

$\Rightarrow \quad -7k-10=0 \Rightarrow \quad k=-10/7$

(8) The given equations are

$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\,(\hat{i}-2\hat{j}+2\hat{k})$ and

$\vec{r}=-4\hat{i}-\hat{k}+\mu\,(3\hat{i}-2\hat{j}-2\hat{k})$

Comparing the given lines with $\vec{r}_1=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}_2=\vec{a}_2+\mu\vec{b}_2$

$\vec{a}_1=6\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}_1=\hat{i}-2\hat{j}+2\hat{k}$

$\vec{a}_2=-4\hat{i}-\hat{k}$ and $\vec{b}_2=3\hat{i}-2\hat{j}-2\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=(-4\hat{i}-\hat{k})-(6\hat{i}+2\hat{j}+2\hat{k})$

$=-4\hat{i}-\hat{k}-6\hat{i}-2\hat{j}-2\hat{k}=-10\hat{i}-2\hat{j}-3\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}$$

$=(4+4)\,\hat{i}-(-2-6)\,\hat{j}+(-2+6)\,k=8\hat{i}+8\hat{j}+4\hat{k}$

$\therefore \quad |\vec{b}_1\times\vec{b}_2|=\sqrt{(8)^2+(8)^2+(4)^2}=\sqrt{64+64+16}=\sqrt{144}=12$

Also, $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(-10\hat{i}-2\hat{j}-3\hat{k}).(8\hat{i}+8\hat{j}+4\hat{k})$

$=10\times 8-2\times 8-3\times 4=-80-16-12=-108$

$\therefore$ Shortest distance between given lines

$$=\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{|\vec{b}_1\times\vec{b}_2|}=\frac{|-108|}{12}=\frac{108}{12}=9$$

(9) Here equations of given lines are

$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\,(3\hat{i}+2\hat{j}+6\hat{k})$ and

$\vec{r}=7\hat{i}-6\hat{k}+\mu\,(\hat{i}+2\hat{j}+2\hat{k})$

The given lines are parallel to the vectors

$\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$

Let θ be the angle between two given lines then

$$\cos\theta=\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

$$=\frac{(3\hat{i}+2\hat{j}+6\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{(3)^2+(2)^2+(6)^2}.\sqrt{(1)^2+(2)^2+(2)^2}}$$

$$=\frac{3\times1+2\times2+6\times2}{\sqrt{9+4+36}\,\sqrt{1+4+4}}=\frac{3+4+12}{7\times3}=\frac{19}{21}$$

$$\therefore \quad \cos\theta=\frac{19}{21}\Rightarrow\theta=\cos^{-1}\left(\frac{19}{21}\right)$$

TRY IT YOURSELF-4

(1) Required distance $=\dfrac{|2+2(3)-2(-5)-9|}{\sqrt{1+4+4}}=\dfrac{9}{3}=3$

(2) Here $\hat{n}=\dfrac{2i-3j+6k}{7}$

So dc's of the normal = 2/7, – 3/7, 6/7.

Also distance of the plane from the origin

$$\frac{|d|}{|n|}=\frac{2}{7}$$

(3) Let N be the foot of the perpendicular from point O on the plane $2x + 3y + 4z - 12 = 0$.

$\therefore$ Direction ratios of the normal to the given plane are 2, 3, 4.

ON is the normal to the plane and its direction ratios are 2, 3, 4.

$\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = k$

$\therefore x = 2k, y = 3k, z = 4k$

Since N (2k, 3k, 4k) lies on the given plane.

$\therefore 2 \times 2k + 3 \times 3k + 4 \times 4k - 12 = 0$

$\Rightarrow 4k + 9k + 16k = 12$

$\Rightarrow 29k = 12 \Rightarrow k = \frac{12}{29}$

$\therefore x = 2 \times \frac{12}{29} = \frac{24}{29}, \; y = 3 \times \frac{12}{29} = \frac{36}{29},$

$z = 4 \times \frac{12}{29} = \frac{48}{29}.$

$\therefore$ Coordinate of the foot of perpendicular from the origin are $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$.

(4) The equation of the given plane is

$2x + y - z = 5$

$\therefore \frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1 \Rightarrow \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

which is of the form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where a, b, c are intercepts on x, y and z axis. Thus intercepts on x, y and z axis are $\frac{5}{2}$, 5 and –5 respectively.

(5) The given planes are $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$

The equation of the plane through the intersection of given planes is $3x - y + 2z - 4 + \lambda (x + y + z - 2) = 0$ .... (1)

Since the plane (1) passes through the point (2, 2, 1), then

$3 \times 2 - 2 + 2 \times 1 - 4 + \lambda (2 + 2 + 1 - 2) = 0$

$\Rightarrow 6 - 2 + 2 - 4 + \lambda (3) = 0 \Rightarrow \lambda = -2/3$

Putting value of λ in (1), we have

$3x - y + 2z - 4 - \frac{2}{3}(x + y + z - 2) = 0$

$\Rightarrow 9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$

$\Rightarrow 7x - 5y + 4z - 8 = 0$

which is required equation of the plane.

(6) **(D).** Here equation of two planes are

$2x + 3y + 4z = 4$ .... (1)

$4x + 6y + 8z = 12$ .... (2)

Now, equation of plane (2) can be written as

$2x + 3y + 4z = 6$ .... (3)

Now plane (1) and (3) are parallel to each other

$\therefore$ Distance between two planes $= \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

$= \frac{|4-6|}{\sqrt{(2)^2 + (3)^2 + (4)^2}} = \frac{|-2|}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}}$

(7) **(B).** We know that two planes $a_1x + b_1y + c_1d = d_1$ and $a_2x + b_2y + c_2d = d_2$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, equation of given planes are $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$

$\therefore \frac{2}{5} = \frac{-1}{-2.5} = \frac{4}{10} \Rightarrow \frac{2}{5} = \frac{2}{5} = \frac{2}{5}$, which is true.

(8) **(D).** Let the equation of variable plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

which meets the axes at A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

$\therefore$ Centroid of Δ ABC is (a/3, b/3, c/3) and it satisfies the relatiton

$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k \Rightarrow \frac{9}{a^2} + \frac{9}{b^2} + \frac{9}{c^2} = k$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{k}{9}$ .... (1)

Also given that the distance of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ from (0, 0, 0) is 1 unit.

$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 1 \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1$ .... (2)

From (1) and (2), we get $\frac{k}{9} = 1$ i.e. $k = 9$

(9) **(A).** The given line is

$2x - y + z - 3 = 0 = 3x + y + z - 5$

which is intersection line of two planes

$2x - y + z - 3 = 0$ .... (1)

and $3x + y + z - 5 = 0$ .... (2)

Any plane containing this line will be the plane passing through the intersection of two planes (1) and (2).

Thus the plane containing given line can be written as

$(2x - y + z - 3) + \lambda (3x + y + z - 5) = 0$

$(3\lambda + 2) x + (\lambda - 1) y + (\lambda + 1) z + (-5\lambda - 3) = 0$

As its distance from the pt. (2, 1, –1) is $1/\sqrt{6}$.

$\therefore \left| \frac{(3\lambda+2)\cdot 1 + (\lambda-1)\cdot 1 + (\lambda+1)(-1) + (-5\lambda-3)}{\sqrt{(3\lambda+2)^2 + (\lambda-1)^2 + (\lambda+1)^2}} \right| = \frac{1}{\sqrt{6}}$

$\Rightarrow \left| \frac{\lambda - 1}{\sqrt{11\lambda^2 + 12\lambda + 6}} \right| = \frac{1}{\sqrt{6}}$

Squaring both sides, we get

$$\frac{(\lambda-1)^2}{11\lambda^2+12\lambda+6}=\frac{1}{6}$$

$\Rightarrow$ $6\lambda^2-12\lambda+6-11\lambda^2-12\lambda-6=0$

$\Rightarrow$ $5\lambda^2+24\lambda=0$

$\Rightarrow$ $\lambda(5\lambda+24)=0$

$\Rightarrow$ $\lambda=0 \text{ or } -24/5$

$\therefore$ The required equations of planes are

$2x-y+z-3=0$

and $\left[3\left(\frac{-24}{5}\right)+2\right]x+\left[-\frac{24}{5}-1\right]y$

$$+\left[-\frac{24}{5}+1\right]z-5\left(\frac{-24}{5}\right)-3=0$$

or $-62x-29y-19z+105=0$

or $62x+29y+19z-105=0$

(10) **(D).** The equation of plane through the point (1, –2, 1) and perpendicular to the planes 2x – 2y + z = 0 and

x – y + 2z = 4 is given by

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0 \Rightarrow x+y+1=0$$

It's distance from the point (1, 2, 2) is $\left|\frac{1+2+1}{\sqrt{2}}\right|=2\sqrt{2}$.

CHAPTER- 10 :
VECTORS & 3-DIMENSIONAL GEOMETRY
EXERCISE-1

(1) **(C).** We have, $\Sigma\, \hat{i}\,.\,(\hat{j}\times\hat{k}) = \Sigma\, \hat{i}\,.\,\hat{i} = \Sigma\, 1 = 3$

(2) **(B).** Since three vectors are coplanar their scalar triple product is zero. $\vec{a}.(\vec{b}\times\vec{c}) = 0$

$(2\vec{a}-\vec{b}).[(3\vec{b}-\vec{c})\times(4\vec{c}-\vec{a})]$

$= 12\vec{a}.(\vec{b}\times\vec{c}) - 0 - 0 + 0 - \vec{a}.(\vec{b}\times\vec{c}) = 0$

(3) **(A).** Let $\alpha = 150°, \beta = 60°$
We have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = 90°$

(4) **(C).** $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, implies

$\vec{a}+\vec{b} = -\vec{c} \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = \vec{c}.\vec{c}$; $\vec{a}.\vec{a}+\vec{b}.\vec{b}+2\vec{a}.\vec{b} = \vec{c}.\vec{c}$

$[\vec{a}.\vec{a} = \vec{b}.\vec{b} = \vec{c}.\vec{c} = 1$ because they are unit vectors]

implies $\vec{a}.\vec{b} = -\frac{1}{2} = \vec{b}.\vec{c} = \vec{a}.\vec{c}$

Therefore given expression is –3.

(5) **(B).** $|\vec{a}+\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow |\vec{a}| + |\vec{b}|^2 + 2\vec{a}\vec{b} = 1$

$\vec{a}.\vec{b} = -\frac{1}{2} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

(6) **(B).** $3\left\{\frac{\vec{a}.(\vec{b}\times\vec{c})}{\vec{a}.(\vec{b}\times\vec{c})}\right\} - \frac{1}{2}\left\{\frac{\vec{b}.(\vec{c}\times\vec{a})}{\vec{b}.(\vec{c}\times\vec{a})}\right\} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

(7) **(A).** $\begin{vmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ \lambda & 4 & 2 \end{vmatrix} = 2$

$2(2-4) - 3(2-\lambda) = 2$
$-4 - 6 + 3\lambda = 2 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$

(8) **(C).** $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} - \hat{k}$; $\hat{n} = \frac{(2\hat{i}-\hat{j}-\hat{k})}{\sqrt{6}}$

(9) **(A).** $\frac{(\vec{a}+\vec{b}).(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{(\vec{b}+\vec{c}).(\vec{c}\times\vec{a})}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{(\vec{c}+\vec{a}).(\vec{a}\times\vec{b})}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$= (1+0) + (1+0) + (1+0) = 3$

(10) **(D).** $|\vec{a}\times\vec{b}| = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2$

$(\vec{a}\times\vec{b})^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

$144 = 16|\vec{b}|^2 \therefore |\vec{b}| = \frac{144}{16} = \frac{(12)^2}{(4)^2} = 3^2 \quad \therefore\ |\vec{b}| = 3$

(11) **(A).** Let $\vec{a} = \hat{i}+\hat{j}-\hat{k}, \vec{b} = 2\hat{i}-3\hat{j}+\hat{k}$

$\vec{a}+\vec{b} = 3\hat{i}-2\hat{j}$; $\vec{a}-\vec{b} = -\hat{i}+4\hat{j}-2\hat{k}$

Clearly, $\vec{a}+\vec{b}, \vec{a}-\vec{b}$ are the diagonals

$|\vec{a}+\vec{b}| = \sqrt{9+4} = \sqrt{13}$

$|\vec{a}-\vec{b}| = \sqrt{1+16+4} = \sqrt{21}$

(12) **(C).** $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$

Given $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 4$ $\quad\therefore$ Reqd. $= 4^2 = 16$

(13) **(B).** $(\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c} = \alpha\,\vec{a} + \beta\,\vec{b} + \gamma\,\vec{c}$

$\alpha = 0$; $\beta = \vec{a}\cdot\vec{c} = 3+4+3 = 10$

$\gamma = -(\vec{a}\cdot\vec{b}) = -(2-2+3) = -3$

(14) **(C).** $\vec{a}\cdot\vec{b} = 0$

$(\vec{a}+\vec{b})\cdot(\vec{a}+m\,\vec{b}) = \vec{a}\cdot\vec{a} + m\,(\vec{a}\cdot\vec{b}) + (\vec{b}\cdot\vec{a}) + m\,(\vec{b}\cdot\vec{b})$

$0 = |\vec{a}|^2 + 0 + 0 + m|\vec{b}|^2 \Rightarrow m = -\frac{|\vec{a}|^2}{|\vec{b}|^2}$

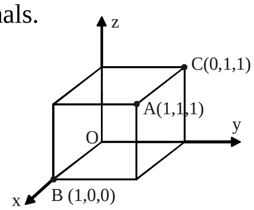
(15) **(B).** $|\vec{a}+\vec{b}+\vec{c}| = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2GE = 0$; $GE = -3/2$

(16) **(B).** $\vec{a} = i, \vec{b} = j, \vec{c} = k$; $\hat{i}\times(\hat{j}\times\hat{k}) = \hat{i}\times\hat{i} = 0$

Other options will be not correct

(17) **(A).** Consider a diagonal with each side 1.
Now BC and OA are diagonals.
Angle between diagonals
= Angle b/w $\overrightarrow{OA}$ & $\overrightarrow{BC}$.



$\overrightarrow{OA} = (1, 1, 1) - (0, 0, 0)$
$= (1, 1, 1)$

$\overrightarrow{BC} = (0, 1, 1) - (1, 0, 0) = (-1, 1, 1)$

Now $\cos\theta = \frac{1(-1) + 1(1) + (1)(1)}{\sqrt{1^2+1^2+1^2}\ \sqrt{(-1)^2+1^2+1^2}} = \frac{1}{3}$

(18) **(D).** Area $= |\vec{a}\times\vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -\hat{i}+\hat{j}+\hat{k}$

$\therefore$ Area $= \sqrt{1+1+1} = \sqrt{3}$

(19) **(B).** $|\vec{a}| = |\vec{b}| = 1, \theta = \pi/3$

$|\vec{a}+\vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 + 2(|\vec{a}|\cdot|\vec{b}|\cos\theta)$
$= 1 + 1 + 2\times1\times1\times1/2 = 3$

$|\vec{a}+\vec{b}| = \sqrt{3} \quad\therefore\ |\vec{a}+\vec{b}| > 1$

(20) **(A).** $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = (\vec{a}-\vec{b})\,[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})]$

$= (\vec{a}-\vec{b})\,[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a}]$

$= (\vec{a}-\vec{b})\,[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}+\vec{c}\times\vec{a}]$ $\quad [\because\ \vec{c}\times\vec{c}=0]$

$\vec{a}\cdot(\vec{b}\times\vec{c})-\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})$

$-\vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{b}\times\vec{a})-\vec{b}\cdot(\vec{c}\times\vec{a})$

$= [\vec{a}\ \vec{b}\ \vec{c}]-0+0-0+0-[\vec{a}\ \vec{b}]=0$

(21) **(C).** Required area $= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|\vec{a}||\vec{b}|\sin\theta$

$= \frac{1}{2}\times 3\times 5\times\frac{1}{2} = \frac{15}{4}$

(22) **(C).** $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}\cdot\vec{b}|$

$\Rightarrow\ 7 = 4 + |\vec{b}|^2 - 2|\vec{b}|^2 \Rightarrow |\vec{b}|^2 = 7\ \ \therefore\ |\vec{b}| = \sqrt{7}$

(23) **(B).** $4 + a^2 + 4 = 9 \Rightarrow a = 1,\ \because\ a > 0$

(24) **(A).** Let A (0, 7, –10), B (1, 6, – 6) and C (4, 9, – 6) be three vertices of triangle ABC. Then

$AB = \sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}$

$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}$

$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(4-0)^2+(9-7)^2+(-6+10)^2}$

$= \sqrt{16+4+16} = \sqrt{36} = 6$

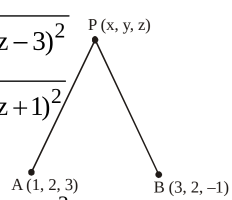
Now, AB = BC

Thus, ABC is isosceles triangle.

(25) **(C).** Let a point P (x, y, z) be equidistance from the points A (1, 2, 3), B (3, 2, –1).

$PA = \sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

$PB = \sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$



According to the question,

PA = PB

$\Rightarrow\ (x-1)^2+(z-3)^2 = (x-3)^2+(z+1)^2$

Simplifying, $4x-8z=0 \Rightarrow x-2z=0$

(26) **(B).** Let P (x, y, z) be any point which divides the line segment joining points A (–2, 3, 5) and B (1, – 4, 6) in the ratio 2 : 3 internally.

Then, $x = \frac{2\times1+3\times-2}{2+3} = \frac{2-6}{5} = \frac{-4}{5}$

$y = \frac{2\times-4+3\times3}{2+3} = \frac{-8+9}{5} = \frac{1}{5}$

$z = \frac{2\times6+3\times5}{2+3} = \frac{12+15}{5} = \frac{27}{5}$

$\therefore$ Coordinate of P are $\left(\frac{-4}{5},\frac{1}{5},\frac{27}{5}\right)$

(27) **(A).** Let Q (5, 4, – 6) divides the segment joining points P (3, 2, – 4), and R (9, 8, –10) in the ratio k : 1 internally.

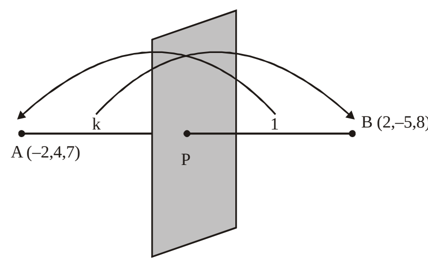
$\therefore$ Coordinates of Q are $\left(\frac{9k+3}{k+1},\frac{8k+2}{k+1},\frac{-10k-4}{k+1}\right)$

But it is given that coordinates of Q is (5, 4, –6)

$\therefore\ \frac{9k+3}{k+1} = 5 \Rightarrow 9k+3 = 5k+5 \Rightarrow 4k = 2 \Rightarrow k = 1/2$

Thus Q divides the line segment joining points P and R in the ratio $\frac{1}{2}:1$ i.e., 1 : 2 internally.

(28) **(D).** Let YZ-plane divide the join of A (–2, 4, 7) and B (3, –5, 8) at P (x, y, z) in the ratio k : 1, then the coordinate of P are $\left(\frac{3k-2}{k+1},\frac{-5k+4}{k+1},\frac{8k+7}{k+1}\right)$



Since, P lies on the yz-plane, its x-coordinate is zero

i.e., $\frac{3k-2}{k+1} = 0 \Rightarrow \quad 3k-2=0 \Rightarrow k = 2/3$

Therefore, y z-plane divides AB in the ratio 2 : 3.

(29) **(C).** Let A and B be the points which trisect PQ.

Then, AP = AB = QB. Therefore, A divides PQ in the ratio 1 : 2 and B divides it in the ratio 2 : 1.

As A divides PQ in the ratio 1 : 2, so co-ordinates of

A are $\left[\frac{1(10)+2(4)}{1+2},\frac{1(-16)+2(2)}{1+2},\frac{1(6)+2(-6)}{1+2}\right]$

$= (6, -4, -2)$

Since, B divides PQ in the ratio 2 : 1, co-ordinates of

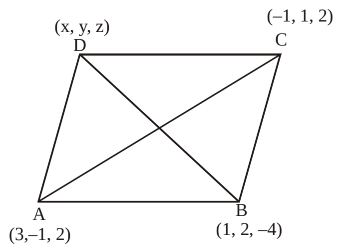
B are $\left[\frac{2(10)+1(4)}{2+1},\frac{2(-16)+1(2)}{1+2},\frac{2(6)+1(-6)}{1+2}\right]$

$= (8, -10, 2)$

Hence, the co-ordinates of A and B are (6, –4, –2) and (8, –10, 2) respectively.

(30) **(A).** Let the co-ordinates of the vertex D be (x, y, z).

We know that the diagonals of a parallelogram bisect each other. Therefore, the middle point of AC is the same as the middle point of BD.



$\therefore \quad \frac{1+x}{2} = \frac{3-1}{2} \Rightarrow 1 + x = 3 - 1 \Rightarrow \quad x = 1$

and $\frac{2+y}{2} = \frac{-1+1}{2} \Rightarrow 2 + y = 0 \quad \Rightarrow \quad y = -2$

and $\frac{-4+z}{2} = \frac{2+2}{2} \Rightarrow -4 + z = 4 \Rightarrow z = 8$

Hence, the fourth vertex D is the point (1, –2, 8)

(31) **(B).** Here, P (2a, 2, 6), Q (– 4, 3b, –10) and R (8, 14, 2c) are vertices of triangle PQR.

$\therefore$ Coordinates of centroid of Δ PQR is

$$\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$$

But it is given that coordinates of centroid is (0, 0, 0)

$\therefore \quad \frac{2a+4}{3} = 0 \Rightarrow 2a + 4 = 0 \Rightarrow a = -2$

$\frac{3b+16}{3} = 0 \Rightarrow 3b + 16 = 0 \Rightarrow b = -16/3$

$\frac{2c-4}{3} = 0 \Rightarrow 2c - 4 = 0 \Rightarrow c = 2$

(32) **(A).** Let Q (0, y, 0) be any point on y-axis.

Then, $\quad PQ = \sqrt{(0-3)^2 + (y+2)^2 + (0-5)^2}$

$= \sqrt{9 + y^2 + 4 + 4y + 25} = \sqrt{y^2 + 4y + 38}$

But $\sqrt{y^2 + 4y + 38} = 5\sqrt{2}$

Squaring both sides, we have

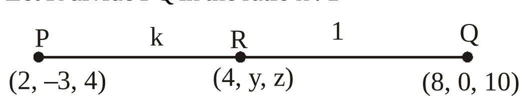
$y^2 + 4y + 38 = 50 \Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow \quad (y - 2)(y + 6) = 0 \Rightarrow y = 2, -6$

Thus, coordinates of point Q are (0, 2, 0) and (0, –6, 0).

(33) **(C).** Let the coordinate of R be (4, y, z).

Let R divide PQ in the ratio k : 1



$\therefore \quad$ R is $\left(\frac{8k+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{10k+4}{k+1}\right)$

But x coordinate of R is 4.

So, $\frac{8k+2}{k+1} = 4 \Rightarrow 8k + 2 = 4k + 4 \Rightarrow 4k = 2 \Rightarrow k = \frac{1}{2}$

$$y = \frac{-3}{k+1} = \frac{-3}{\frac{1}{2}+1} = \frac{-3 \times 2}{3} = -2$$

$$z = \frac{10k+4}{k+1} = \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} = \frac{9 \times 2}{3} = 6$$

Coordinates of R are (4, –2, 6)

(34) **(A).** Let P (x, y, z) be any point

Then, $PA = \sqrt{(x-3)^2 + (y-4)^2 + (z-5)^2}$

$= \sqrt{x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z}$

$PB = \sqrt{(x+1)^2 + (y-3)^2 + (z+7)^2}$

$= \sqrt{x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 - 14z}$

Now, $PA^2 + PB^2 = k^2$

$\therefore \quad x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z = k^2$

$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow \quad 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$\Rightarrow \quad x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$

(35) **(C).** Distance from y-axis is $\sqrt{x^2 + z^2}$

$= \sqrt{4^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41}$

(36) **(A).** Let $\vec{a}$ be the position vector of the point (1, 2, 3).

Then $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Now the equation of the line passing through the point having position vector $\vec{a}$ and parallel to vector

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \quad \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\,(3\hat{i} + 2\hat{j} - 2\hat{k})$

(37) **(C).** The equation of given line is $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

The direction ratios of the given line are 3, 5, 6.

Since the required line is parallel to given line, so the direction ratios of required line are proportional i.e., 3, 5, 6. Now, the equation of the line passing through point (–2, 4, –5) and having direction ratios 3, 5, 6 is

$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$, which is equation of required line.

(38) **(A).** Here the equation of given lines are :

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

$\Rightarrow \quad \dfrac{x-1}{-3} = \dfrac{y-2}{2p/7} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{-3p/7} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$

$\therefore$ Direction ratios of two lines are

$\left(-3, \dfrac{2p}{7}, 2\right)$ and $\left(\dfrac{-3p}{7}, 1, -5\right)$

We know that two lines are perpendicular if

$a_1a_2 + b_1b_2 + c_1c_2 = 0.$

$\therefore \quad -3 \times \dfrac{-3p}{7} + \dfrac{2p}{7} \times 1 + 2 \times -5 = 0$

$\Rightarrow \quad \dfrac{9p}{7} + \dfrac{2p}{7} - 10 = 0 \Rightarrow 9p + 2p - 70 = 0$

$\Rightarrow \quad 11p = 70 \Rightarrow p = 70/11$

(39) **(A).** Foot of the perpendicular from point $A(\vec{a})$ on the plane $\vec{r}.\vec{n} = d$ is $\vec{r} = \vec{a} + \dfrac{(\vec{a}.\vec{n} - d)}{|\vec{n}|^2}\hat{n}$

$\therefore$ Equation of line parallel $\vec{r} = a + \lambda\vec{b}$ in the plane $\vec{r}.\vec{n} = d$ is given by $\vec{r} = \vec{a} + \left(\dfrac{d - \vec{a}.\vec{n}}{n^2}\right) + \vec{n} + \lambda\vec{b}$

(40) **(B).** Use formula,

$$\text{S.D.} = \frac{\begin{vmatrix} 10 & 2 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}}{\sqrt{8^2 + 8^2 + 4^2}} = \frac{108}{12} = 9.$$

(41) **(B).** Since any point on line $\dfrac{x+2}{3} = \dfrac{y-5}{-2} = \dfrac{z}{5}$ is $(3r-2, -2r+5, 5r)$ which satisfies both the plane. Hence the result.

(42) **(A).** $^{-} = \cos^{-1}\left(\dfrac{3+0-5}{\sqrt{1+1}\sqrt{9+16+25}}\right)$

$= \cos^{-1}\left(\dfrac{-2}{\pm 10}\right) = \cos^{-1}\left(\dfrac{1}{5}\right).$

(43) **(D).** $^{-} = \cos^{-1}\dfrac{(2+10-12)}{\sqrt{4+25+16}\sqrt{1+4+9}} = \cos^{-1}(0)$

$\Rightarrow {}^{-} = 90^o.$

(44) **(D).** General points of 2 lines

P (r, r – 2, r) Q (2K – 2, K, K)

Dr's of PQ are (r – 2K + 2, r – K – 2, r – K)

$\dfrac{r-2K+2}{2} = \dfrac{r-K-2}{1} = \dfrac{r-K}{2}$

r = 6 : K = 2 put in P and Q.

(45) **(D).** Since plane is perpendicular to $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$

Direction ratio of normal to the plane is 1, 2, 3.

$\therefore$ Eq is $1x + 2y + 3z + d = 0.$

Passes through the point (2, 3, 4)

$\therefore \quad 2 + 6 + 12 + d = 0 \Rightarrow d = -20$

$\therefore$ Eq. is $x + 2y + 3z = 20$

(46) **(C).** D.R of line = 3, 4, 5

Line and plane are parallel

$\therefore$ Normal to plane and line are perpendicular

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ For plane $2x + y - 2z = 0$

$3(2) + 4(1) - 2(5) = 0 \quad \therefore \; 2x + y - 2z = 0$

(47) **(D).** Any plane through point

A (2, 3, 1) is $a(x-2) + b(y-3) + c(z-1) = 0$ .... (1)

It passes through point B (4, – 5, 3)

Then, $a(4-2) + b(-5-3) + c(3-1) = 0$

$\Rightarrow 2a - 8b + 2c = 0$

$\Rightarrow a - 4b + c = 0$ .... (2)

$\because$ Plane (1) is parallel to x-axis whose DC's are 1, 0, 0

Then, $a(1) + b(0) + c(0) = 0$ .... (3)

By (2) and (3), find the values of a, b and c & put in eq. (1)

(48) **(C).**

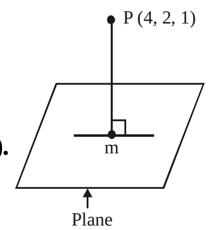
Let $(2x + 3y - 4z - 1) + \lambda(3x - y + z + 2) = 0$ .... (1)

and $2x + 3y - 4z = 0$ .... (2)

$\therefore$ plane (1) $\perp$ plane (2)

$\Rightarrow 2(2 + 3\lambda) + 3(3 - \lambda) - 4(\lambda - 4) = 0 ; \lambda = 29$

(49) **(D).**



where $m(x_1, y_1, z_1) = (2, 3, -1)$

Let, m (2, 3, – 1) is the foot of the perpendicular from the point P. The line PM is the normal to the plane.

$\therefore$ DR's of the normal are (4 – 2), (2 – 3), (1 + 1)

= 2, – 1, 2 = a, b, c

$\therefore$ Required palne,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$2(x-2) + (-1)(y-3) + 2(z+1) = 0$

$2x - y + 2z + 1 = 0$

(50) **(A).** $\because$ M is the mid point of AB

$\therefore M = \left\{\dfrac{(4\hat{i} + 5\hat{j} - 10\hat{k}) + (-\hat{i} + 2\hat{j} + \hat{k})}{2}\right\}$

$= \dfrac{1}{2}(3\hat{i} + 7\hat{j} - 9\hat{k}) = \vec{a}$

$\vec{n} = \overrightarrow{AB}$ = normal vector of the plane

$\because$ Plane is perpendicular to the line AB

$\Rightarrow \vec{n} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-5\hat{i} - 3\hat{j} + 11\hat{k})$

$\Rightarrow$ Equation of plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$, where $\vec{a} = \overrightarrow{OM}$

(51) **(D).** $\because$ DR's of the normal of the plane = 2, –3, 6

& DR's of x-axis is 1, 0, 0

& If θ is the angle between the plane

& x-axis, then $\sin\theta = \dfrac{2(1)+(-3)(0)+(6)(0)}{\sqrt{2^2+(-3)^2+6^2}\sqrt{1}} = \dfrac{2}{7}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{2}{7}\right) \quad \therefore a = \dfrac{2}{7}$

(52) **(A).** From option (A) , midpoint of (3, 2, 2) and (5, 4, 6) satisfies the equation of given plane.

(53) **(C).** Area of $\Delta ABC = \dfrac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$

$\Delta = \dfrac{1}{2}\sqrt{4\times 9+9\times 16+16\times 4} = \dfrac{1}{2}\sqrt{244} = \sqrt{61}$.

(54) **(B).** $\theta = \sin^{-1}\left(\dfrac{6+2-12}{\sqrt{14}.\sqrt{29}}\right) \Rightarrow \theta = \sin^{-1}\left(\dfrac{-4}{\sqrt{406}}\right)$.

(55) **(A).** $(x+y+z-6)+ a\,(2x+3y+4z+5)=0 \Rightarrow a = \dfrac{3}{14}$

$\Rightarrow 20x+23y+26z-69=0$.

(56) **(D).** The plane will be $x+2y+4z = 2\times 1+3\times 2+4\times 4$ or $x+2y+4z=24$.

(57) **(B).** Direction ratios of line a = 1, b = – 4, c = 7

$1\times 1+19\times 4-11\times 7 = 1+76-77 = 0$

It passes through given points, $\therefore x-19y-11z=0$.

(58) **(B).** $- = \sin^{-1}\left(\dfrac{6+2-12}{\sqrt{14}\,.\sqrt{29}}\right) \Rightarrow - = \sin^{-1}\left(\dfrac{-4}{\sqrt{406}}\right)$.

(59) **(B).** Since the only point (9, – 5, 12) satisfies the line and the plane.

(60) **(D).** $bx-ay=n \qquad \dfrac{bx-n}{a} = y = \dfrac{bz+\ell}{c}$

$cy-bz=\ell \qquad \dfrac{x-n/b}{a/b} = \dfrac{y}{1} = \dfrac{z+\ell/b}{c/b}$

$az-cx=n \qquad \therefore a\left(-\dfrac{\ell}{b}\right)-c\left(\dfrac{n}{b}\right) = m$

(61) **(A).** Angular velocity $\vec{\omega} = 3\hat{i}-4\hat{j}+\hat{k}$ and distance from centre $\vec{r} = 5\hat{i}-6\hat{j}+6\hat{k}$.

We know that linear velocity $(\vec{v}) = \vec{\omega}\times\vec{r}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i}-13\hat{j}+2\hat{k}$$

(62) **(A).** $\vec{r} = \vec{a}+\vec{b}+\vec{c} = 4\hat{i}-\hat{j}-3\hat{i}+2\hat{j}-\hat{k} = \hat{i}+\hat{j}-\hat{k}$

$\hat{r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{1^2+1^2+(-1)^2}} = \dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

(63) **(C).** We have $((\vec{a}\times\vec{b})\times\vec{a}).((\vec{b}\times\vec{a})\times\vec{b})$

$= ((\vec{a}.\vec{a})\,\vec{b}-(\vec{b}.\vec{a})\,\vec{a}) . ((\vec{b}.\vec{b})\,\vec{a}-(\vec{a}.\vec{b})\,\vec{b})$

$= (\vec{a}^2\vec{b}^2)\,\vec{a}.\vec{b}-\vec{a}^2\vec{b}^2(\vec{a}.\vec{b})-(\vec{a}.\vec{b})\,\vec{a}^2\vec{b}^2+(\vec{a}.\vec{b})^3$

$= -\,[(\vec{a}.\vec{b})\,\vec{a}^2\vec{b}^2-(\vec{a}.\vec{b})^3] = -(\vec{a}.\vec{b})\,[\vec{a}^2\vec{b}^2-(\vec{a}.\vec{b})^2]$

$= -(\vec{a}.\vec{b})\,|\vec{a}\times\vec{b}|^2$

(64) **(A).**

$(\vec{a}\times\vec{b})\times\vec{a}.\vec{b}+(\vec{a}.\vec{b})^2 = ((\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}).\vec{b}+(\vec{a}.\vec{b})^2$
$= (\vec{a}.\vec{a})\,(\vec{b}.\vec{b})-(\vec{a}.\vec{b})^2+(\vec{a}.\vec{b})^2$

$= \vec{a}^2.\vec{b}^2 = (4)\,(9) = 36$

(65) **(C).** $|\vec{u}\times\vec{v}|^2 = \vec{u}^2\vec{v}^2-(\vec{u}.\vec{v})^2$; $36 = (9)\,(4)-(\vec{u}.\vec{v})^2$

$\Rightarrow \vec{u}.\vec{v} = 0 \Rightarrow \vec{u}$ and $\vec{v}$ are orthogonal.

Also $(\vec{u}\times\vec{v})\times\vec{u} = (\vec{u}.\vec{u})\,\vec{v}-(\vec{v}.\vec{u})\,\vec{u} = 9\vec{v}$

$\Rightarrow$ (D) is incorrect.

(66) **(C).** $\vec{V}.\vec{a} = x\,[\vec{a}\;\vec{b}\;\vec{c}] = \dfrac{x}{3}$ .... (1)

Similarly, $\vec{V}.\vec{b} = y\,[\vec{a}\;\vec{b}\;\vec{c}] = \dfrac{y}{3}$ .... (2) and

$\vec{V}.\vec{c} = z\,[\vec{a}\;\vec{b}\;\vec{c}] = \dfrac{z}{3}$ .... (3)

eq. (1) + eq. (2) + eq. (3)

$\dfrac{x+y+z}{3} = \vec{V}.(\vec{a}+\vec{b}+\vec{c})$

$x+y+z = 3\vec{V}.(\vec{a}+\vec{b}+\vec{c})$

(67) **(A).** We know that $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$.

If the vectors make obtuse angle with each other, then $\cos\theta < 0$.

Let $\vec{a} = cx\vec{i}-6\vec{j}-3\vec{k}$ and $\vec{b} = x\vec{i}+2\vec{j}-2cx\vec{k}$

Then $\vec{a}.\vec{b} = cx^2-12+6cx$

Since the vectors $\vec{a}$ and $\vec{b}$ make an obtuse angle with each other, therefore

$cx^2+6cx-12<0 \Rightarrow -cx^2-6cx+12>0$

This is possible if (–c) > 0 i.e. c < 0 and

$(6c)^2-4\times(-c)\,(12)<0$

$\Rightarrow 36c^2+48c<0 \Rightarrow 12c(3c+4)<0 \Rightarrow -\dfrac{4}{3}<c<0$.

(68) **(B).** The vector perpendicular to $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \vec{i}(1-0)-\vec{j}(1-0)+\vec{k}(1-0)$$

$= \vec{i}-\vec{j}+\vec{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{1+1+1} = \sqrt{3}$.

Hence the unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is

$\pm \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \pm \frac{1}{\sqrt{3}} (\vec{i} - \vec{j} + \vec{k})$.

Hence the number of such vectors is 2.

(69) **(D)** $\left(\vec{i} + \vec{j}\right) \times \left(\vec{j} + \vec{k}\right) = \vec{i} - \vec{j} + \vec{k}$

$\Rightarrow$ unit vector perpendicular as to the plane of

$\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$ is $\frac{1}{\sqrt{3}}\left(\vec{i} - \vec{j} + \vec{k}\right)$

Similarly other two unit vectors are

$\frac{1}{\sqrt{3}}\left(\vec{i} + \vec{j} - \vec{k}\right)$ and $\frac{1}{\sqrt{3}}\left(-\vec{i} + \vec{j} + \vec{k}\right)$

$$\Rightarrow v = \left[\vec{n}_1\ \vec{n}_2\ \vec{n}_3\right] = \frac{1}{3\sqrt{3}}\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{4}{3\sqrt{3}}$$

(70) **(A).** Let $\vec{r}_1 = 60\vec{i} + 3\vec{j}$, $r_2 = 40\vec{i} - 8\vec{j}$ and $\vec{r}_3 = a\vec{i} - 52\vec{j}$

If $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are collinear, then there exists scalar t such that $\vec{r}_3 = \vec{r}_1 + t\left(\vec{r}_2 - \vec{r}_1\right)$; by using $\vec{r} = \vec{a} + t\left(\vec{b} - \vec{a}\right)$

i.e. $a\vec{i} - 52\vec{j} = \left(60\vec{i} + 3\vec{j}\right) + t(-20\vec{i} - 11\vec{j})$

Hence equating the coefficients of $\vec{i}$ and $\vec{j}$, we get

$a = 60 - 20t$ .... (1)

and $-52 = 3 - 11t$ .... (2)

Now (2) $\Rightarrow 11t = 55 \therefore t = 5$

$\therefore$ from (1), $a = 60 - 100 = -40$.

(71) **(A).** We have point P is (1, 2, 3) and point Q is (4, 5, 6).

$\overrightarrow{PQ} = \vec{Q} - \vec{P} = (4,5,6) - (1,2,3)$

$= (4-1), (5-2), (6-3) = (3, 3, 3) = 3\vec{i} + 3\vec{j} + 3\vec{k}$

unit vector $PQ = \frac{3\vec{i} + 3\vec{j} + 3\vec{k}}{\sqrt{3^2 + 3^2 + 3^2}} = \frac{3(\vec{i} + \vec{j} + \vec{k})}{\sqrt{27}}$

$= \frac{3(\vec{i} + \vec{j} + \vec{k})}{3\sqrt{3}} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$

(72) **(C).** Let $\overrightarrow{OA} = \vec{a} = 2\vec{i} - 3\vec{j}$, $\overrightarrow{OB} = \vec{b} = \vec{i} + \vec{j} - \vec{k}$,

$\overrightarrow{OC} = \vec{c} = 3\vec{i} - \vec{k}$

Then the volume of the parallelopiped = $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

Now, $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$

$= 2(-1) + 3(-1+3) + 0(0-3) = -2 + 6 + 0 = 4$.

(73) **(A).** Let the angle between $\vec{a}$ and $\vec{b}$ be θ and let the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ be ϕ.

Then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\ \hat{n}$

$\Rightarrow (\vec{a} \times \vec{b}).\vec{c} = \{|\vec{a}||\vec{b}| \sin\theta\ \hat{n}\}.\vec{c}$

$= |\vec{a}||\vec{b}||\vec{c}| \sin\theta \cos\phi$

$\Rightarrow |\vec{a}||\vec{b}||\vec{c}| = |\vec{a}||\vec{b}||\vec{c}| \sin\theta \cos\phi$. from the given condition. $\Rightarrow \sin\theta \cos\phi = 1$.

This is possible only when $\sin\theta$ and $\cos\phi = 1$.

$\Rightarrow \theta = \pi/2$.

(74) **(C).** We have, $\frac{\vec{b} + \vec{c}}{\sqrt{2}} = \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c}).\vec{b} - (\vec{a}.\vec{b}).\vec{c}$

$\Rightarrow \left(\vec{a}.\vec{c} - \frac{1}{\sqrt{2}}\right).\vec{b} - \left(\vec{a}.\vec{b} + \frac{1}{\sqrt{2}}\right)\vec{c} = 0$

Since $\vec{b}$ and $\vec{c}$ are non-coplanar, therefore

$\vec{a}.\vec{c} - \frac{1}{\sqrt{2}} = 0 \Rightarrow \vec{a}.\vec{c} = \frac{1}{\sqrt{2}}$

and $\vec{a}.\vec{b} + \frac{1}{\sqrt{2}} = 0 \Rightarrow \vec{a}.\vec{b} = -\frac{1}{\sqrt{2}}$

Hence the angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{4}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{3\pi}{4}$.

(75) **(A).** Equating the components in

$\alpha(\vec{i} + 2\vec{j} + 3\vec{k}) + \beta(2\vec{i} + 3\vec{j} + \vec{k}) + \gamma(3\vec{i} + \vec{j} + 2\vec{k})$

$= -3(\vec{i} - \vec{k})$

$\alpha + 2\beta + 3\gamma = -3,\ 2\alpha + 3\beta + \gamma = 0,\ 3\alpha + \beta + 2\gamma = 3$.

$\Rightarrow \alpha + \beta + \gamma = 0 \Rightarrow \alpha + 2\beta = 0$

$\Rightarrow \gamma = -1, \beta = -1, \alpha = 2$.

(76) **(D).** We have $\left(\vec{a} + \vec{b}\right).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$

$= \left(\vec{a}.\vec{p} + \vec{b}.\vec{p}\right) + \left(\vec{b}.\vec{q} + \vec{c}.\vec{q}\right) + \left(\vec{c}.\vec{r} + \vec{a}.\vec{r}\right)$

$$= \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{b}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{b}\ \vec{c}\ \vec{a}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{c}\ \vec{c}\ \vec{a}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{c}\ \vec{a}\ \vec{b}]}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{[\vec{a}\ \vec{a}\ \vec{b}]}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$= 1 + 0 + 1 + 0 + 1 + 0 = 3$.

(77) **(B).** For coplanarity $\begin{vmatrix} x & -1 & -3 \\ 1 & x & 2 \\ 3 & -2 & x \end{vmatrix} = x^3 + 14x = 0$

This has only real root x = 0

(78) **(D).** For orthogonality, the scalare product = 0.

$2(x^2 - 1) - x(x+2) + 3x^2 = 2(2x+1)(x-1) = 0$.

(79) **(B).** We have, $|\vec{a}| = \sqrt{2^2+1+2^2} = 3$

$|\vec{b}| = \sqrt{1+1+0} = \sqrt{2}$

Now $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = \vec{i}(0+2) - \vec{j}(0+2) + \vec{k}(2-1)$

$= 2\vec{i} - 2\vec{j} + \vec{k}$

$\therefore \left|\vec{a}\times\vec{b}\right| = \sqrt{2^2+2^2+1+3}$

Given : $|\vec{c}-\vec{a}| = 2\sqrt{2} \Rightarrow (\vec{c}-\vec{a})^2 = 8$

$\Rightarrow c^2 - 2\vec{c}.\vec{a} + a^2 = 8 \Rightarrow c^2 - 2c + 9 = 8$

$\Rightarrow c^2 - 2c + 1 = 0 \quad \Rightarrow (c-1)^2 = 0 \Rightarrow c = 1$ i.e. $|\vec{c}| = 1$

Hence, $|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}|\ |\vec{c}|\sin 30^\circ = 3\times 1 . \frac{1}{2} = \frac{3}{2}$.

(80) **(D).** $(\vec{b}+\vec{c}+\vec{a})\times(\vec{c}+\vec{a}-\vec{b}) = 2\vec{b}\times\vec{c} + 2\vec{c}\times\vec{a}$

Triple scalar product

$= 2(\vec{a}+\vec{b}-\vec{c}).(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}) = 4(\vec{a}\ \vec{b}\ \vec{c})$

(81) **(B).** Since the given vectors are coplanar, therefore

$$\Delta \equiv \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$\Rightarrow a(0-c) - a(b-c) + c(c-0) = 0$
$\Rightarrow -ac - ab + ac + c^2 = 0 \Rightarrow c^2 = ab$
$\Rightarrow$ c is the G.M. of a and b.

(82) **(C).** If angle between a and c is θ then -
a.c = |a| |c| cos θ = 1.2 cos θ = 2 cos θ
but a × (a × c) + b = 0
$\Rightarrow$ (a. c) a – (a.a) c + b = 0 $\Rightarrow$ (2 cosθ).a – 1. c = – b
$\Rightarrow [(2\cos\theta)\,a - c]^2 = [-b]^2$
$\Rightarrow 4\cos^2\theta\,|a|^2 - 2.(2\cos\theta)\,a.c + |c|^2 = |b|^2$
$\Rightarrow 4\cos^2\theta - 4\cos\theta\,(2\cos\theta) + 4 = 1$
$\Rightarrow 4(1-\cos^2\theta) = 1$ $\quad [\because |a| = 1, |b| = 1]$
$\Rightarrow \sin\theta = 1/2 \Rightarrow \theta = \pi/6$

(83) **(D).** If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c} = p\vec{a} + q\vec{b}$

i.e. $\vec{i} + \alpha\vec{j} + \beta\vec{k} = p\left(\vec{i}+\vec{j}+\vec{k}\right) + q\left(4\vec{i}+3\vec{j}+4\vec{k}\right)$

Equating the coefficient $\vec{i}, \vec{j}, \vec{k}$, we get 1 = p + 4q,
$\alpha = p + 3q$ and $\beta = p + 4q$.
From first and third, $\beta = 1$.
Now $|\vec{c}| = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$
$\Rightarrow \alpha^2 = 1 \quad \therefore \alpha = \pm 1$.
Hence , $\alpha = \pm 1, \beta = 1$.

(84) **(A).** $\frac{\text{Area } \Delta \text{ CDM}}{\text{Area } \Delta \text{ CBA}} = \frac{1}{2(a+b)} \frac{\left(a\vec{b}+b\vec{a}\right)\times\left(\vec{a}+\vec{b}\right)}{\left|\vec{b}\times\vec{a}\right|}$

$= \frac{1}{2(a+b)} \frac{(a-b)\left|b\times\vec{a}\right|}{\left|b\times\vec{a}\right|}$

$\frac{a-b}{2(a+b)} = \frac{\sin A - \sin B}{2(\sin A + \sin B)}$

(85) **(A).** The resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$, and the resultant of $\vec{P}$ and $-\vec{Q}$ is $\vec{S}$, that is $\vec{R} = \vec{P}+\vec{Q}$ and $\vec{S} = \vec{P}-\vec{Q}$

Hence $\vec{R}.\vec{R} = (\vec{P}+\vec{Q}).(\vec{P}+\vec{Q})$

or $R^2 = \vec{P}.\vec{P} + \vec{P}.\vec{Q} + \vec{Q}.\vec{P} + \vec{Q}.\vec{Q}$ .... (1)

Similarly, $\vec{S}.\vec{S} = (\vec{P}-\vec{Q}).(\vec{P}-\vec{Q})$

or $S^2 = \vec{P}.\vec{P} - \vec{P}.\vec{Q} - \vec{Q}.\vec{P} + \vec{Q}.\vec{Q}$ .... (2)

Adding eq. (1) and (2), we get

$R^2 + S^2 = 2(\vec{P}.\vec{P} + \vec{Q}.\vec{Q}) = 2(P^2 + Q^2)$

(86) **(B).** $(\vec{a}\ \vec{b}\ \vec{c}) = \begin{vmatrix} \alpha & 2 & -3 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} \alpha+3 & 8 & 0 \\ 3 & \alpha+2 & 0 \\ 1 & 2 & 1 \end{vmatrix}$

$= \alpha^2 + 5\alpha - 18 = 6$
$\Rightarrow \alpha^2 + 5\alpha - 24 \equiv (\alpha+8)(\alpha-3) = 0 \Rightarrow \alpha = -8, 3$

(87) **(A).** Here $\overrightarrow{AB} = (\vec{i}+2\vec{j}+3\vec{k}) - (4\vec{i}+2\vec{j}+2\vec{k}) = -3i + k$

Draw BN $\perp$ AN. Then AN is the projection of $\overrightarrow{AB}$ along $\vec{c}$ and is therefore equal to

AB.cos θ = 1 . AB cos θ = $\frac{\vec{c}}{|\vec{c}|}.\overrightarrow{AB}$

$= \frac{2\vec{i}+3\vec{j}+6\vec{k}}{\sqrt{4+9+36}} . \left(-3\vec{i}+\vec{k}\right) = \frac{-6+6}{7} = 0$

$\therefore BN^2 = AB^2 - AN^2 = 10 - 0 = 10 \;\therefore BN = \sqrt{10}$.

(88) **(C).** $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = (\vec{a}\ \vec{b}\ \vec{d})\,\vec{c} - (\vec{a}\ \vec{b}\ \vec{c})\,\vec{d}$

$[\vec{a}\ \vec{b}\ \vec{d}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 4, \ [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 4$

The cross product = $4(\vec{c}-\vec{d}) = 8\vec{j}$ which is orthogonal to both $\vec{i}$ and $\vec{k}$.

(89) **(B).** A vector coplanar with the given vectors can be expressed as $\vec{r} = (\vec{i}+\vec{j}+2\vec{k}) + \lambda\,(\vec{i}+2\vec{j}+\vec{k})$

$= (1+\lambda)\vec{i} + (1+2\lambda)\vec{j} + (2+\lambda)\vec{k}$ .... (1)

Where λ is a scalar.
If this vector is perpendicular to

$\vec{a} = \vec{i} + \vec{j} + \vec{k}$, then $\vec{r}.\vec{a} = 0$

$\Rightarrow (1+\lambda).1 + (1+2\lambda).1 + (2+\lambda).1 = 0$

$\Rightarrow 4 + 4\lambda = 0 \Rightarrow \lambda = -1.$

Hence putting $\lambda = -1$ in (1), we get $\vec{r} = -\vec{j} + \vec{k}$ which is a vector coplanar with the given vectors and $\perp$ to $\vec{i} + \vec{j} + \vec{k}$.

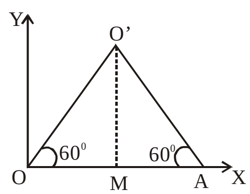
Hence the required unit vector is $\frac{\vec{r}}{|\vec{r}|} = \frac{1}{\sqrt{2}}(-\vec{j} + \vec{k})$

(90) **(C).** If O' is the centre of the hexagon. OAO' is an equilateral triangle as the interior angles of a regular hexagon measure 120° each. If M is the mid-point OA,

$OM = 1,\ MO' = \sqrt{3}\ ;\ O'P = \sqrt{9-4} = \sqrt{5}$

so that $\overrightarrow{OP} = \vec{i} + \sqrt{3}\,\vec{j} + \sqrt{5}\,\vec{k}$. If follows that

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = -\vec{i} + \sqrt{3}\,\vec{j} + \sqrt{5}\,\vec{k}$.



(91) **(B).** $\vec{a} + 2\vec{b} + \vec{j} = 3(\vec{i} + \vec{k})$, $2\vec{a} + \vec{c} - \vec{k} = 3(\vec{i} + \vec{j})$ are inclined at an angle $\cos^{-1}\frac{9}{18} = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$.

(92) **(D).** $\begin{vmatrix} \alpha & \alpha+\beta & \beta \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{vmatrix} \alpha & \alpha+\beta & \beta \\ 1 & -2 & 1 \\ 4 & 0 & 0 \end{vmatrix} = 4(\alpha + 3\beta) = 0$

gives $\frac{\alpha}{\beta} = -3.$

(93) **(B).** $\cos\theta = -\frac{1}{2}$ and $\cot\theta = -\frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{2\pi}{3}$

$\therefore$ most general values = $2n\pi + \frac{2\pi}{3}$

(94) **(D).** $\vec{p} \times (\vec{p} \times \vec{q}) = (\vec{p}.\vec{q})\vec{p} - (\vec{p}.\vec{p})\vec{q} = -4\vec{q}$

$\therefore\ \vec{V} = -4\vec{p} \times (\vec{p} \times \vec{q}) = -4\,[(\vec{p}.\vec{q})\vec{p} - (\vec{p}.\vec{p})\vec{q}]$

$= +(4)\,(4)\,\vec{q} = 16\vec{q}$

(95) **(B).** Given $[\vec{a}\ \vec{b}\ \vec{c}] = V_1$

$[\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & 1 \\ 1 & -4 & 2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$= [\,1(-4+4) - 1(6-1) - 2(-12+2)]\ V_1$

$V_2 = (-5+20)\ V_1 = 15V_1 \Rightarrow K = 15$

(96) **(D).** $\left.\begin{matrix} \vec{V}_1 = 2\vec{a} + \vec{b} - \vec{c} \\ \vec{V}_2 = \lambda^2\vec{a} + 3\vec{b} - 3\vec{c} \\ \vec{V}_3 = 3\vec{a} + 2\vec{b} - 2\vec{c} \end{matrix}\right\},$

where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar.

$\therefore\ [\vec{V}_1\ \vec{V}_1\ \vec{V}_1] = \begin{vmatrix} 2 & 1 & -1 \\ \lambda^2 & 3 & -3 \\ 3 & 2 & -2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}] = 0$

but $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ \lambda^2 & 3 & -3 \\ 3 & 2 & -2 \end{vmatrix} = 0$ which is true $\forall\ \lambda \in R$

(97) **(C).** The equation of plane is of the form
$2x - 3y + 6z + k = 0$

Given planes are $\frac{x}{7/2} + \frac{y}{-7/3} + \frac{z}{7/6} = 1$ and

$\frac{x}{-7/2} + \frac{y}{7/3} + \frac{z}{-7/6} = 1$

Given plane lies midway between the 2 planes
It passes through $7/2 - 7/2,\ 7/3 - 7/3,\ 7/6 - 7/6 \equiv (0, 0, 0)$
$\therefore$ Equation of plane is $2x - 3y + 6z = 0$

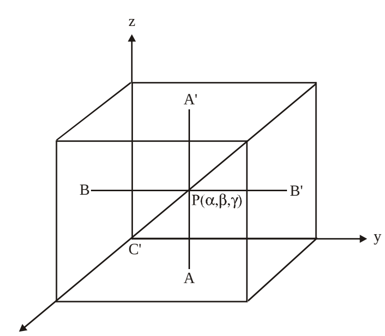
(98) **(B).** A º (a, b, 0), A¢º (a, b, 2), B º (a, 0, g),
B¢º (a, 2, g), C º (0, b, g), C¢º (2, b, g)

$\therefore$ According to condition given

$a^2 + b^2 + g^2 + (a-2)^2 + (b-2)^2 + (g-2)^2 = 46$

$2a^2 - 4a + 4 + 2b^2 - 4b + 4 + 2g^2 - 4g + 4 = 46$

$\therefore\ a^2 - 2a + 2 + b^2 - 2b + 2 + g^2 - 2g + 2 = 23$



$\therefore\ (a-1)^2 + (b-1)^2 + (g-1)^2 = 20$

$\therefore$ Distance = $\sqrt{20} = 2\sqrt{5}$

(99) **(D).** Let $\vec{a} = 3\hat{i} - \hat{k}$ be vector parallel to 1st line and $\vec{b} = -\hat{i} + 2\hat{j} - \hat{k}$ be vector parallel to 2nd line

$$\therefore \ \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ -1 & 2 & -1 \end{vmatrix}$$

$= \hat{i}(0+2) - \hat{j}(-3-1) + \hat{k}(6) = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore$ Equation of plane is of the form $2x + 4y + 6z + k = 0$

$\therefore$ It passes through $(-1, 2, 0)$

$\therefore$ $-2 + 8 + k = 0 \Rightarrow k = -6$

$\therefore$ Equation is $2x + 4y + 6z = 6$, or $x + 2y + 3z = 3$

(100) **(C).** Let the components of the line vector be a, b, c.

Then $a^2 + b^2 + c^2 = (63)^2$ ...... (i)

Also $\frac{a}{3} = \frac{b}{-2} = \frac{c}{6} = \lambda$ (say), then $a = 3\lambda$, $b = -2\lambda$ and $c = 6\lambda$ and from (i) we have

$9\lambda^2 + 4\lambda^2 + 36\lambda^2 = (63)^2 \Rightarrow 49\lambda^2 = (63)^2$

$\Rightarrow \lambda = \pm\frac{63}{7} = \pm 9$

Since $a = 3\lambda < 0$ as the line makes an obtuse angle with x-axis, $\lambda = -9$ and the required components are $-27, 18, -54$.

(101) **(A).** Here, $\ell = \cos\theta, m = \cos\beta, n = \cos\theta$

Now, $\ell^2 + m^2 + n^2 = 1 \Rightarrow 2\cos^2\theta + \cos^2\beta = 1$

$\Rightarrow 2\cos^2\theta = \sin^2\beta$ .... (1)

Given : $\sin^2\beta = 3\sin^2\theta$

$\Rightarrow 2\cos^2\theta = 3\sin^2\theta \Rightarrow 5\cos^2\theta = 3$

(102) **(C).** Direction ratios of OP are (a, b, c)

$\therefore$ equation of the plane is

$a(x-a) + b(y-b) + c(z-c) = 0$

$ax + by + cz = a^2 + b^2 + c^2$

(103) **(B).** Equations of the planes bisecting the angles between the given planes are

$$\frac{2x - y + 2z + 3}{\sqrt{2^2 + (-1)^2 + 2^2}} = \pm\frac{3x - 2y + 6z + 8}{\sqrt{3^2 + (-2)^2 + 6^2}}$$

$\Rightarrow 7(2x - y + 2z + 3) = \pm 3(3x - 2y + 6z + 8)$

$\Rightarrow 5x - y - 4z - 3 = 0$ taking the positive sign, and

$23x - 13y + 32z + 45 = 0$ taking the negative sign.

(104) **(B).** Let a point $(3\lambda + 1, \lambda + 2, 2\lambda + 3)$ of the first line also lies on the second line.

Then, $\frac{3\lambda + 1 - 3}{1} = \frac{\lambda + 2 - 1}{2} = \frac{2\lambda + 3 - 2}{3} \Rightarrow \lambda = 1$

Hence, the point of intersection P of the two lines is (4, 3, 5)

Equation of plane perpendicular to OP where O is (0, 0, 0) and passing through P is $4x + 3y + 5z = 50$

(105) **(B).** Let an equation of the required plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

This meets the coordinates axes in A (a, 0, 0), B(0, b, 0) and C(0, 0, c).

So that the coordinates of the centroid of the triangle ABC are $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, r, r^2)$ (given) $\Rightarrow a = 3, b = 3r$, $c = 3r^2$. Hence the required equation of the plane is

$\frac{x}{3} + \frac{y}{3r} = 1$ or $r^2x + ry + z = 3r^2$

(106) **(A).** Given planes are $4x + 2y + 4z - 16 = 0$ .... (1)

and $4x + 2y + 4z + 5 = 0$ .... (2)

Distance between planes (1) and (2) is

$$\left|\frac{-16 - 5}{\sqrt{16 + 4 + 16}}\right| = \frac{21}{6} = \frac{7}{2}$$

(107) **(B).** Since $3(1) + 2(-2) + (-1)(-1) = 3 - 4 + 1 = 0$

$\therefore$ Given line is $\perp$ to the normal to the plane i.e., given line is parallel to the given plane.

Also, (1, –1, 3) lies on the plane $x - 2y - z = 0$ if

$1 - 2(-1) - 3 = 0$ i.e., $1 + 2 - 3 = 0$

which is true $\therefore$ L lies in plane π.

(108) **(B).** $1 = \left|(\vec{b} - \vec{a}).\frac{(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right| \Rightarrow |\vec{b} - \vec{a}|.|\vec{p} \times \vec{q}|.\cos 60° = \frac{1}{2}$

$\Rightarrow AB.\frac{1}{2}.\frac{1}{2}. = \frac{1}{2} \Rightarrow AB = 2$

(109) **(C).** Equation of any plane passing through the intersection of the planes $r.(2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and

$r.(\hat{i} - \hat{j}) + 4 = 0$ is $2x - 3y + 4z - 1 + \lambda(x - y + 4) = 0$

or $(2 + \lambda)x - (3 + \lambda)y + 4z + 4\lambda - 1 = 0$

This plane is perpendicular to the plane

$r.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$ if $2(2 + \lambda) + (3 + \lambda) + 4 = 0$

$11 + 3\lambda = 0 \Rightarrow \lambda = -11/3$

Hence the required equation of the plane is

$3(2x - 3y + 4z - 1) - 11(x - y + 4) = 0$

$\Rightarrow 5x - 2y - 12z + 47 = 0$

(110) **(A).** Let θ be the angle between the line and the normal to the plane. Converting the given equations into vector form, we have

$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r}.(10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$

Here, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$

$$\sin\phi = \left|\frac{(2\hat{i} + 3\hat{j} + 6\hat{k}).(10\hat{i} + 2\hat{j} - 11\hat{k})}{\sqrt{2^2 + 3^2 + 6^2}\sqrt{10^2 + 2^2 + 11^2}}\right|$$

$$= \left|\frac{-40}{7 \times 15}\right| = \left|\frac{-8}{21}\right| = \frac{8}{21} \text{ or } \phi = \sin^{-1}\left(\frac{8}{21}\right)$$

(111) **(B).** Let image of the point P(1, 3, 4) in the given plane be the point Q. The equation of the line through P and norma to the given plane is $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}$

Since this line passes through Q, so let the coordinates of Q be $(2r+1, -r+3, r+4)$.

The coordinates of the mid-point of PQ are

$$\left(r+1, -\frac{r}{2}+3, \frac{r}{2}+4\right).$$

This point lies on the given plane. Therefore, r = –2.

Hence (b) is the correct answer because the coordinates of Q are (–3, 5, 2).

(112) **(B).** Equation of the given plane can be written as

$$\frac{x}{20}+\frac{y}{15}+\frac{z}{-12}=1$$

which meets the coordinates axes in points A(20, 0, 0), B(0, 15, 0) and C(0, 0, - 12) and the coordinates of the origin are (0, 0, 0).

∴ the volume of the tetrahedron OABC is

$$\frac{1}{6}\begin{vmatrix}20 & 0 & 0\\ 0 & 15 & 0\\ 0 & 0 & -12\end{vmatrix}=\left|\frac{1}{6}\times 20\times 15\times(-12)\right|=600$$

(113) **(C).** $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=r\,(\text{say})$

$\Rightarrow x=2r+1,\ y=3r-1,\ z=4r+1$

As the two given lines intersect,

$$\frac{2r+1-3}{1}=\frac{3r-1-k}{2}=\frac{4r+1}{1}$$

From some value of r

we have $2r-2=4r+1 \Rightarrow r=-3/2$

Also, $3r-1-k=8r+2$

$$\Rightarrow k=-5r-3=\frac{15}{2}-3=\frac{9}{2}$$

(114) **(C).** The coordinates of any point on the given line are $(2r+1, -3r-1, r)$

The distance of this point from the point (1, –1, 0) is given to be $4\sqrt{14}$

$\Rightarrow (2r)^2+(-3r)^2+(r)^2=(4\sqrt{14})^2$

$\Rightarrow 14r^2=16\times 14 \quad = r=\pm 4$

So the coordinates of the required point are $(9, -13, 4)$ or $(-7, 11, -4)$

Out of which nearer the origin is $(-7, 11, -4)$

(115) **(A).** Let θ be the required angle then θ will be the angle between $\vec{a}$ and $\vec{b}+\vec{c}$

($\vec{b}+\vec{c}$ lies along the angular bisector of $\vec{a}$ and $\vec{b}$)

$$\cos\theta=\frac{\vec{a}.(\vec{b}+\vec{c})}{(\vec{a})|\vec{b}+\vec{c}|}=\frac{2\cos\alpha}{\sqrt{2+2\cos\lambda}}=\frac{\cos\alpha}{\cos(\alpha/2)}$$

$$\theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$$

(116) **(B).** The two lines are parallel. We have

$\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k},\ \vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

Therefore, the distance between the lines is given by

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|=\left|\frac{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 2 & 1 & -1\end{vmatrix}}{\sqrt{4+9+36}}\right|$$

$$=\frac{|-9\hat{i}+14\hat{j}-4\hat{k}|}{\sqrt{49}}=\frac{\sqrt{293}}{\sqrt{49}}=\frac{\sqrt{293}}{7}$$

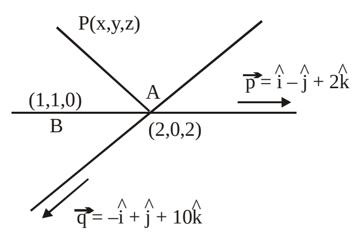
(117) **(A).** Let ℓ, m, n be the DC of the given the line. Then as it makes an acute angle with x-axis, therefore ℓ > 0.

The line passes through (6, –7, –1) and (2, –3, 1), therefore its DR are

$6-2, -7+3, -1-1$ or $4, -4, -2$ or $2, -2, -1$

DC of the given line are $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

(118) **(B).**



Lines intersect at (2, 0, 2) equation of the plane is

$$\begin{vmatrix}x-2 & y & z-2\\ -1 & 1 & 0\\ 1 & -1 & 2\end{vmatrix}=0 \Rightarrow x+y-2=0$$

(119) **(C).** L is $\frac{x-1}{-1}=\frac{y-0}{3}=\frac{z+2}{0}$

here L is along the vector $-\hat{i}+3\hat{j}$

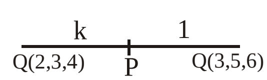
only in (C) the line is $\frac{x-1}{-1}=\frac{y-0}{3}$; z = 5

which is ∥ to the vector $-\hat{i}+3\hat{j}$

(120) **(D).** $P_1=P_2=0,\ P_2=P_3=0$ and $P_3=P_1=0$ are lines of intersection of the three planes P_1, P_2 and P_3.

As $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are non-coplanar, planes P_1, P_2 and P_3 will intersect at unique point. So the given lives will pass through a fixed point.

(121) **(D).** Any point P is $\frac{3k+2}{k+1}, \frac{5k+3}{k+1}, \frac{6k+4}{k+1}$



Hence, $\frac{3k+2}{k+1} = \frac{13}{5} \Rightarrow 15k + 10 = 13k + 13$

$\Rightarrow 2k = 3 \Rightarrow k = 3/2$

Similarly with other two the ratio is 3 : 2.

(122) **(B).** The DC of the line are

$\ell = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$.

Since $\ell^2 + m^2 + n^2 = 1\ \therefore\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(123) **(D).** Number of unit vectors perpendicular to a line in space is infinitely.

(124) **(C).** Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \text{....... (1)}$$

Here a = 2, b = 3, c = 4.

Substituting the values of a, b and c in (1), we get the required equation of the

plane as $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ or $6x + 4y + 3z = 12$.

(125) **(C).** The point (4, 2, k) on the line also lies on the plane $2x - 4y + z = 7$.

So, $8 - 8 + k = 7 \Rightarrow k = 7$

(126) **(A).** Let $\vec{a}$ and $\vec{b}$ be the position vectors of the point A (– 1, 0, 2) and B (3, 4, 6).

Then, $\vec{a} = -\hat{i} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$.

Therefore, $\vec{b} - \vec{a} = 4\hat{i} + 4\hat{j} + 4\hat{k}$

Let $\vec{r}$ be the position vector of any point on the line. Then the vector equation of the line is

$\vec{r} = -\hat{i} + 2\hat{k} + \lambda\,(4\hat{i} + 4\hat{j} + 4\hat{k})$

(127) **(A).** Since the direction ratios of the normal to the plane are 2, –3, 4; the direction cosines of it are

$$\frac{2}{\sqrt{2^2 + (-3)^2 + 4^2}}, \frac{-3}{\sqrt{2^2 + (-3)^2 + 4^2}}, \frac{4}{\sqrt{2^2 + (-3)^2 + 4^2}}$$

i.e., $\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$

Hence, dividing the equation $2x - 3y + 4z - 6 = 0$

i.e., $2x - 3y + 4z = 6$ throughout by $\sqrt{29}$, we get

$$\frac{2}{\sqrt{29}}x + \frac{-3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}z = \frac{6}{\sqrt{29}}$$

This is of the form $\ell x + my + nz = d$, where d is the distance of the plane from the origin. So, the distance of

the plane from the origin is $\frac{6}{\sqrt{29}}$.

(128) **(A).** Comparing the given equations of the planes with the equations $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$

We get $A_1 = 3, B_1 = -6, C_1 = 2,\ A_2 = 2, B_2 = 2, C_2 = -2$

$$\cos\theta = \left|\frac{3\times 2 + (-6)(2) + (2)(-2)}{\sqrt{(3^2 + (-6)^2 + (-2)^2)}\sqrt{(2^2 + 2^2 + (-2)^2)}}\right|$$

$$= \left|\frac{-10}{7\times 2\sqrt{3}}\right| = \frac{5}{7\sqrt{3}} = \frac{5\sqrt{3}}{21};\quad \theta = \cos^{-1}\left(\frac{5\sqrt{3}}{21}\right)$$

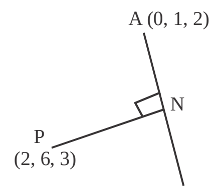
(129) **(B).** Here, $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$, $\vec{N} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ and d = 4

Therefore, the distance of the point (2, 5, – 3) from the

given plane is $\frac{|(2\hat{i} + 5\hat{j} - 3\hat{k}).(6\hat{i} - 3\hat{j} + 2\hat{k}) - 4|}{|6\hat{i} - 3\hat{j} + 2\hat{k}|}$

$$= \frac{|12 - 15 - 6 - 4|}{\sqrt{36 + 9 + 4}} = \frac{13}{7}$$

(130) **(A).** N, the foot of the perpendicular from P can be taken as $(2\lambda, 1 + 2\lambda,\ 2 + 3\lambda)$



Direction ratios of PN are

$(2\lambda - 2), (1 + 2\lambda - 6),\ (2 + 3\lambda - 3)$

Hence $(2\lambda - 2)\times 2 + (2\lambda - 5)2 + (3\lambda - 1)\,3 = 0$

$\therefore \lambda = 1.$ Hence $N = (2, 3, 5)$

(131) **(B).** Vector along line of intersection of planes is

$(\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} + 3\hat{j} + 3\hat{k})$

(132) **(B).** Vector $\vec{v}_1$ along the line of intersection of $3x - 7y - 5z = 1$ and $8x - 11y + 2z = 0$ is given by

$$\vec{v}_1 = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -7 & -5 \\ 8 & -11 & 2 \end{vmatrix} = -23(3\hat{i} + 2\hat{j} - \hat{k})$$

|||ly vector $\vec{v}_2$ along the line of intersection of the planes $5x - 13y + 3z = 0$ and $8x - 11y + 2z = 0$ is

$$\vec{v}_2 = \vec{n}_3 \times \vec{n}_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -13 & 3 \\ 8 & -11 & 2 \end{vmatrix} = 7(\hat{i} + 2\hat{j} + 7\hat{k})$$

now $\vec{v}_1 \cdot \vec{v}_2 = 0 \Rightarrow$ angle is 90° $\Rightarrow$ sin 90° = 1

EXERCISE-2

(1) **(C).** According to the given condition,

$\mathbf{a}.(\mathbf{b}+\mathbf{c})=0$ .....(i)

$\mathbf{b}.(\mathbf{c}+\mathbf{a})=0$ .....(ii)

$\mathbf{c}.(\mathbf{a}+\mathbf{b})=0$ .....(iii)

Now adding (i), (ii) and (iii), we get

$2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=0$, $\{\because \mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{a} \text{ etc.}\}$

Hence, $|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=a^2+b^2+c^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})$

$=3^2+4^2+5^2$

$\Rightarrow |\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{50}=5\sqrt{2}$.

(2) **(C).** Let the required vector r = b + tc

$\Rightarrow r=(1+t)\,i+(2+t)\,j-(1+2t)\,k$

Also projection of r on a = $\sqrt{2/3}$

$\Rightarrow \dfrac{r.a}{|a|}=\sqrt{2/3} \Rightarrow \dfrac{2(1+t)-(2+t)-(1+2t)}{\sqrt{6}}=\sqrt{\dfrac{2}{3}}$

$\Rightarrow -t-1=2 \Rightarrow t=-3 \quad \therefore r=-2i-j+5k$

(3) **(A).** $(a\times b)^2=|a\times b|^2=(ab\sin\theta)^2$

$=a^2b^2\sin^2\theta=a^2b^2(1-\cos^2\theta)$

$=a^2b^2-(ab\cos\theta)^2=a^2b^2-(a.b)^2$

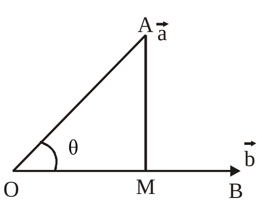
(4) **(C).** $\because \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2=[\vec{a}\,\vec{b}\,\vec{c}]^2$

$=\{(\vec{a}\times\vec{b}).\vec{c}\}^2=|\vec{a}\times\vec{b}|^2\,|\vec{c}|^2\cos 0°$

$\because (\vec{a}\times\vec{b})$ is parallel to $\vec{c}$

$=|\vec{a}|^2|\vec{b}|^2\sin^2\dfrac{\pi}{6}\;(\because|\vec{c}|=1)=\dfrac{1}{4}|\vec{a}|^2|\vec{b}|^2$

(5) **(A).**



$\overrightarrow{OM}$ = component of $\vec{a}$ along $\vec{b}$

$\overrightarrow{MA}$ = component of $\vec{a}$ perpendicular to $\vec{b}$

Δ OMA $\Rightarrow \cos\theta=\dfrac{OM}{OA}$

$\Rightarrow OM=|\overrightarrow{OM}|=|\overrightarrow{OA}|\cos\theta=|\vec{a}|\cos\theta$

$\because \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta=|\vec{b}|(OM)$

$\therefore \overrightarrow{OM}=|\overrightarrow{OM}|\,\hat{b}=\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\dfrac{\vec{b}}{|\vec{b}|}=\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

$\overrightarrow{OM}+\overrightarrow{MA}=\overrightarrow{OA} \quad \therefore \overrightarrow{MA}=\overrightarrow{OA}-\overrightarrow{OM}$

(6) **(B).** $\vec{r}=\pm\left\{(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right\}$ and unit vector $\hat{r}=\dfrac{\vec{r}}{|\vec{r}|}$

where, $\vec{r}$ is a vector which is perpendicular to $(\vec{a}+\vec{b})\,\&\,(\vec{b}+\vec{c})$ both.

(7) **(C).** Let $\vec{x}=x_1\hat{i}+x_2\hat{j}+x_3\hat{k}$

$\therefore \vec{x}\times\hat{i}=x_1(\hat{i}\times\hat{i})+x_2(\hat{j}\times\hat{i})+x_3(\hat{k}\times\hat{i})$

$=-x_2\hat{k}+x_3\hat{j}$

$\Rightarrow \hat{i}\times(\vec{x}\times\hat{i})=-x_2(\hat{i}\times\hat{k})+x_3(\hat{i}\times\hat{j})$

$=x_2\hat{j}+x_3\hat{k}$ .... (1)

Similarly, $\hat{j}\times(\vec{x}\times\hat{j})=x_1\hat{i}+x_3\hat{k}$ .... (2)

$\hat{k}\times(\vec{x}\times\hat{k})=x_1\hat{i}+x_2\hat{j}$ .... (3)

By adding eq. (1) + (2) + (3)

LHS = $2\,(x_1\hat{i}+x_2\hat{j}\times x_3\hat{k})=2\vec{x}$

(8) **(B).** $\vec{a}=2\hat{i}-m\hat{j}+3m\hat{k}\;\&\;\vec{b}=(1+m)\,\hat{i}-2m\hat{j}+\hat{k}$ and if angle between $\vec{a}$ and $\vec{b}$ is an acute, then $\vec{a}.\vec{b}>0$

$\Rightarrow 2(1+m)+2m^2+3m>0 \Rightarrow 2m^2+5m+2>0$

$\Rightarrow 2m^2+4m+m+2>0 \Rightarrow (2m+1)(m+2)>0$

$\Rightarrow m<-2$ or $m>-1/2$

(9) **(A).** Given line is,

$\vec{r}=(2\hat{i}-2\hat{j}+3\hat{k})+\lambda\,(\hat{i}-\hat{j}+4\hat{k})$ .... (1)

and $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ .... (2)

By (1) $\Rightarrow \begin{cases}\vec{a}=(2\hat{i}-2\hat{j}+3\hat{k})\\ \vec{b}=(\hat{i}-\hat{j}+4\hat{k})\end{cases}$

By (2) $\Rightarrow \vec{n}=(\hat{i}+5\hat{j}+\hat{k}) \quad \because \vec{b}.\vec{n}=0$

Therefore, the line is parallel to the plane. Thus, the distance between the line and the plane is equal to the length of the perpendicular from a point $\vec{a}=(2\hat{i}-2\hat{j}+3\hat{k})$ on the line to the given plane.

Hence, the required distance

$$=\left|\dfrac{(2\hat{i}-2\hat{j}+3\hat{k}).(\hat{i}+5\hat{j}+\hat{k})-5}{\sqrt{1+5^2+1}}\right|=\dfrac{10}{3\sqrt{3}}$$

(10) **(A).** Since, $\vec{a},\vec{b}\,\&\,\vec{c}$ are non-coplanar

$\therefore \vec{b}\times\vec{c},\,\vec{c}\times\vec{a},\,\vec{a}\times\vec{b}$ are along non-coplanar.

So, any vector can be expressed as a linear combination of these vectors.

Let, $\vec{r}=\lambda\,(\vec{b}\times\vec{c})+\mu\,(\vec{c}\times\vec{a})+\gamma\,(\vec{a}\times\vec{b})$

$\therefore \vec{a}.\vec{r}=\lambda\,[\vec{a}\,\vec{b}\,\vec{c}]+\mu\,(0)+\gamma\,(0) \Rightarrow \lambda=\dfrac{\vec{a}.\vec{r}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

Similarly, $\mu = \dfrac{\vec{b}.\vec{r}}{[\vec{a}\;\vec{b}\;\vec{c}]}$ & $\gamma = \dfrac{\vec{c}.\vec{r}}{[\vec{a}\;\vec{b}\;\vec{c}]}$

If $\vec{r} = \vec{a}$ then $\lambda\,(\vec{b}\times\vec{c}) + \mu\,(\vec{c}\times\vec{a}) + \gamma\,(\vec{a}\times\vec{b}) = \vec{a}$

& $(\vec{a}.\vec{a})\,(\vec{b}\times\vec{c}) + (\vec{b}.\vec{a}) + (\vec{c}.\vec{a})\,(\vec{a}\times\vec{b}) = \vec{a}\,[\vec{a}\;\vec{b}\;\vec{c}]$

(11) **(D).** $\because\; |\vec{a}+\vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 + 2\,(\vec{a}.\vec{b})$

$= 1 + 1 + 2\cos\alpha$

$= 2\,(1+\cos\alpha)$

$= 2\,(2\cos^2\alpha/2)$

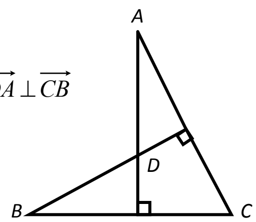
$\therefore\; |\vec{a}+\vec{b}| = 2\,|\cos\dfrac{\alpha}{2}| < 1 \Rightarrow |\cos\dfrac{\alpha}{2}| < \dfrac{1}{2}$

$\Rightarrow\; \dfrac{\pi}{3} < \alpha < \dfrac{2\pi}{3}$

(12) **(C).** $\overrightarrow{DA}.\overrightarrow{CB} = \overrightarrow{DB}.\overrightarrow{AC} = 0 \Rightarrow \overrightarrow{DA}\perp\overrightarrow{CB}$

and $\overrightarrow{DB}\perp\overrightarrow{AC}$.

Hence the point D is orthocentre of ΔABC.



(13) **(B).** $[\mathbf{p}\;\mathbf{q}\;\mathbf{r}] = \mathbf{p}.(\mathbf{q}\times\mathbf{r}) = \dfrac{(\mathbf{b}\times\mathbf{c}).[(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$\dfrac{(\mathbf{b}\times\mathbf{c}).[\{(\mathbf{c}\times\mathbf{a}).\mathbf{b}\}\mathbf{a} - \{(\mathbf{c}\times\mathbf{a}).\mathbf{a}\}\mathbf{b}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$= \dfrac{(\mathbf{b}\times\mathbf{c})[\{(\mathbf{c}\times\mathbf{a}).\mathbf{b}\}\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$, $\{\because (\mathbf{c}\times\mathbf{a}).\mathbf{a} = 0\}$

$= \dfrac{(\mathbf{b}\times\mathbf{c})[\{(\mathbf{a}.(\mathbf{b}\times\mathbf{c})\}\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3} = \dfrac{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]\,[(\mathbf{b}\times\mathbf{c}).\mathbf{a}]}{[\mathbf{a}.(\mathbf{b}\times\mathbf{c})]^3}$

$= \dfrac{1}{\mathbf{a}.\mathbf{b}\times\mathbf{c}}$.

(14) **(D).** Let the point Q be x_1, y_1, z_1 ; $\vec{N} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\overrightarrow{OQ} + t\,\vec{N} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + t\,(2\hat{i} - 2\hat{j} + \hat{k})$

$= (x_1 + 2t)\,\hat{i} + (y_1 - 2t)\,\hat{j} + (z_1 + t)\,\hat{k}$

Now, $x_1 + 2t,\; y_1 - 2t,\; z_1 + t$ lies on $2x - 3y + z = 3$

$2\,(x_1 + 2t) - 2\,(y_1 - 2t) + z_1 + t = 3$

$\underbrace{2x_1 - 2y_1 + z_1} + 4t + 4t + t = 3 \;\Rightarrow\; 1 + 4t + 4t + t = 3$

$9t = 2 \;\Rightarrow\; t = 2/9$

(15) **(C).** $\mathbf{V}\times\mathbf{W} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3\mathbf{i} - 7\mathbf{j} - \mathbf{k}$

But $\mathbf{U}$ is a unit vector, $\therefore\; \mathbf{U} = \dfrac{3\mathbf{i} - 7\mathbf{j} - \mathbf{k}}{\sqrt{59}}$

Hence, $[\mathbf{U}\;\mathbf{V}\;\mathbf{W}] = \dfrac{3^2 + 7^2 + 1^2}{\sqrt{59}} = \sqrt{59}$.

(16) **(A).** Equation of the plane containing $\mathbf{i}$ and $\mathbf{i}+\mathbf{j}$ is

$|\;\mathbf{r}-\mathbf{i}\;\;\mathbf{i}\;\;\mathbf{i}+\mathbf{j}| = 0 \;\Rightarrow (\mathbf{r}-\mathbf{i}).[\mathbf{i}\times(\mathbf{i}+\mathbf{j})] = 0$

$\Rightarrow [(x-1)\mathbf{i} + y\mathbf{j} + z\mathbf{k}].\,\mathbf{k} = 0 \;\Rightarrow z = 0$ .....(i)

Equation of the plane containing $\mathbf{i}-\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$ is

$\Rightarrow [\mathbf{r} - (\mathbf{i}-\mathbf{j})\;\;\mathbf{i}-\mathbf{j}\;\;\mathbf{i}+\mathbf{k}] = 0$

$\Rightarrow (\mathbf{r} - \mathbf{i} + \mathbf{j}).[(\mathbf{i}-\mathbf{j})\times(\mathbf{i}+\mathbf{k})] = 0$

$\Rightarrow [(x-1)\mathbf{i} + (y+1)\mathbf{j} + z\mathbf{k}].(-\mathbf{i} - \mathbf{j} + \mathbf{k}) = 0$

$\Rightarrow x + y - z = 0$ …..(ii)

Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$. Since $\mathbf{a}$ is parallel to (i) and (ii)

$a_3 = 0$ and $a_1 + a_2 - a_3 = 0 \Rightarrow a_1 = -a_2$

Thus a vector in the direction of $\mathbf{a}$ is $\mathbf{u} = \mathbf{i} - \mathbf{j}$.

If ¯ is the angle between $\mathbf{a}$ and $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, then

cos ¯ $= \pm\dfrac{(1)(1) + (-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}} = \pm\dfrac{3}{(\sqrt{2})(3)}$

$\Rightarrow$ cos ¯ $= \pm\dfrac{1}{\sqrt{2}} \Rightarrow$ ¯ $= \dfrac{®}{4}$ or $\dfrac{3®}{4}$.

(17) **(A).** $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \Rightarrow |\,\mathbf{a}| = \sqrt{16 + 16 + 4} = 6$

$\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} + 12\mathbf{k} \Rightarrow |\,\mathbf{b}| = \sqrt{144 + 4 + 9} = \sqrt{157}$

$\mathbf{c} = -\mathbf{i} - 4\mathbf{j} - 8\mathbf{k} \Rightarrow |\,\mathbf{c}| = \sqrt{64 + 16 + 1} = 9$

Hence perimeter is $15 + \sqrt{157}$.

(18) **(D).** Resultant vector $= 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

Direction cosines are $\left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$.

(19) **(C).** Let $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7} =$ ª .....(i)

Then $x = 3$ª -1, $y = 5$ª $- 3, z = 7$ª $- 5$

General point on this line is (3ª − 1, 5ª − 3, 7ª − 5)

Again let $\dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5} =$ « .....(ii)

Then $x =$ « $+ 2, y = 3$« $+ 4, z = 5$« $+ 6$

A general point on this line is (« + 2, 3« + 4, 5« + 6)

For intersection, they have a common point, for some values of ª and «, we must have (3ª − 1) = (« + 2), (5ª − 3) = (3« + 4), (7ª − 5) = (5« + 6)

From first two we have, « = 3ª − 3 ……(iii)

and 3« = 5ª − 7 …..(iv)

From (iii), put the values of « in (iv),

we have 3(3ª − 3) = 5ª − 7

$\Rightarrow$ 9ª − 9 = 5ª − 7 or 4ª = 2 or ª $= \dfrac{1}{2}$

Put ª $= \dfrac{1}{2}$ in (iii), we get « $= -\dfrac{3}{2}$ (Putting ª $= \dfrac{1}{2}$)

The required point of intersection is

$\left[\frac{3}{2}-1\right],\left[\frac{5}{2}-3\right],\left[\frac{7}{2}-5\right]=\left[\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right]$.

(20) **(D).** Equation of any plane passing through (0, 1, 2) is

$a(x-0)+b(y-1)+c(z-2)=0$ ......(i)

Plane (i) passes through (–1, 0, 3), then

$a(-1-0)+b(0-1)+c(3-2)=0$

$\Rightarrow -a-b+c=0 \Rightarrow a+b-c=0$ .....(ii)

Plane (i) is perpendicular to the plane $2x+3y+z=5$, then $2a+3b+c=0$ ......(iii)

Solving (ii) and (iii), we get $a=-4k, b=3k, c=-k$

Putting these values in (i),

$-4k(x)+3k(y-1)-k(z-2)=0$

$\Rightarrow -4x+3y-3-z+2=0$

$\Rightarrow -4x+3y-z-1=0 \Rightarrow 4x-3y+z+1=0$.

(21) **(C).** Let l, m, n are the d.c's of the line. As line is present on both planes, so this line should be perpendicular on the normal of both plane.

So, $4l+4m-5n=0$ and $8l+12m-13n=0$

$\therefore$ So, $\frac{l}{b_1c_2-b_2c_1}=\frac{m}{c_1a_2-c_2a_1}=\frac{n}{a_1b_2-a_2b_1}$

$\Rightarrow \frac{l}{4(-13)-12(-5)}=\frac{m}{-5(8)-(-13)(4)}=\frac{n}{4(12)-8(4)}$

$\Rightarrow \frac{l}{8}=\frac{m}{12}=\frac{n}{16}$

As l, m and n may not be simultaneously zero.

So, taking $a_1b_2-a_2b_1\neq 0$ and $4l+4m=12$

$8l+12m=32$

Suppose point (x,y) lies in this plane.

So, putting (x,y) in place of (l,m)

$4x_1+4y_1=12$ and $8x_1+12y_1=32$

On solving, $x_1=1, y_1=2$

So, equation of line is,

$\frac{x-1}{8}=\frac{y-2}{12}=\frac{z}{16}$ or $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$.

(22) **(B).** Let the equation of plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$,

where $=\frac{1}{\sqrt{\sum\left(\frac{1}{a^2}\right)}}$ or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$ ......(i)

Now according to equation, $x=\frac{a}{3}, y=\frac{b}{3}, z=\frac{c}{3}$

Put the values of x, y, z in (i), we get the locus of the centroid of the triangle $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$, i.e., $k=9$.

(23) **(A).** The equation of plane, in which the line $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ lies, is

$A(x-5)+B(y-7)+C(z+3)=0$ ......(i)

Where $4A+4B-5C=0$ ......(ii)

Also, since line $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ lies in this plane

$\therefore$ $7A+B+3C=0$ ......(iii)

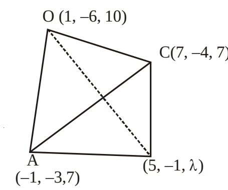
By (ii) and (iii), we get $\frac{A}{17}=\frac{B}{-47}=\frac{C}{-24}$

$\therefore$ The required plane is

$17(x-5)-47(y-7)+(-24)(z+3)=0$

$\Rightarrow$ $17x-47y-24z+172=0$.

(24)



Volume of tetrahedron $=\frac{1}{6}[\overrightarrow{OA}\ \overrightarrow{OC}\ \overrightarrow{OC}]=11 \Rightarrow \lambda=7$

(25) **(C).** $[\vec{a}\ \vec{b}\ \vec{c}]^{x^2+x+3}=1$ but $x^2+x+3\neq 0$ because $D<0$

$\therefore$ $[\vec{a}\ \vec{b}\ \vec{c}]=1$ $\therefore$ $a\perp b\perp c$

(26) **(B).** $3\vec{a}+4\vec{b}+5\vec{c}=0$ $\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplanar.

$\therefore$ $\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$ are collinear.

(27) **(C).** $\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]=3$ (Given) ; $[\vec{a}\ \vec{b}\ \vec{c}]=18$

Vol. of || pipe $=[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$

$=2\times 18=36$

(28) **(A).** $\begin{vmatrix}1 & 1 & m\\ 1 & 1 & m+1\\ 1 & -1 & m\end{vmatrix}=0$

$1(m+m+1)-1(m-m-1)+m(-1-1)=0$

$2m+1+1-2m=0$; $2=0$

coplanar for no value of m.

(29) **(A).** $\vec{a},\vec{b},\vec{c}$ are coplanar therefore,

$x\vec{a}+y\vec{b}+z\vec{c}=0$ where $x, y, z\neq 0$

$x\vec{a}.\vec{a}+y\vec{a}.\vec{b}+z\vec{a}.\vec{c}=0$

$x\vec{a}.\vec{c}+y\vec{b}.\vec{c}+z\vec{c}.\vec{c}=0$

$\therefore$ $\Delta=\begin{vmatrix}\vec{a} & \vec{b} & \vec{c}\\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c}\end{vmatrix}=0$

(30) **(D).** $\frac{x-a}{a}=\frac{y}{0}=\frac{z}{c}$ and $\frac{x}{0}=\frac{y-b}{b}=\frac{z}{-c}$

$$\text{S.D.}=\left|\frac{(a_2-a_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$2=\left|\frac{(a\hat{i}-b\hat{j}).(-bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}\right|$$

$$=\frac{2abc}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

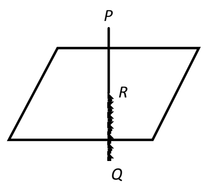
$\therefore$ distance = 1

(31) **(B).** Since $\vec{a},\vec{b},\vec{c}$ are coplanar

$\therefore$ $2\vec{a}-\vec{b},\ 2\vec{b}-\vec{c}$ and $2\vec{c}-\vec{a}$ are also coplanar.

$\therefore$ $[2\vec{a}-\vec{b},\ 2\vec{b}-\vec{c},\ 2\vec{c}-\vec{a}]=0$

(32) **(A).** Let Q be image of the point $P(5,4,6)$ in the given plane, then PQ is normal to the plane. The direction ratios of PQ are 1,1,2.
Since PQ passes through (5,4, 6) and has direction ratio 1,1,2;
Therefore, equation of PQ is



$\frac{x-5}{1}=\frac{y-4}{1}=\frac{z-6}{2}=r$, (say)

$\therefore$ $x=r+5, y=r+4, z=2r+6$

So, co-ordinates of Q be $(r+5, r+4, 2r+6)$

Let R be the mid point of PQ then co-ordinates of R are

$\left(\frac{r+5+5}{2},\frac{r+4+4}{2},\frac{2r+6+6}{2}\right)$ i.e., $\left(\frac{r+10}{2},\frac{r+8}{2},\frac{2r+12}{2}\right)$

Since R lies on the plane

$\therefore$ $\frac{r+10}{2}+\frac{r+8}{2}+2\left(\frac{2r+12}{2}\right)-15=0$

$\Rightarrow$ $r+10+r+8+4r+24-30=0 \Rightarrow 6r+12=0 \Rightarrow r=-2$
So, co-ordinates of Q are (3, 2, 2).
From option (A) , midpoint of (3, 2, 2) and (5, 4, 6) satisfies the equation of given plane.

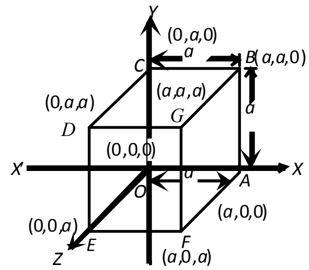
(33) **(D).** Equation of plane bisecting the angle containing origin is (making constant term of same sign)

$$\frac{-3x+6y-2z-5}{\sqrt{3^2+6^2+2^2}}=+\left[\frac{4x-12y+3z-3}{\sqrt{4^2+12^2+3^2}}\right]$$

or $\frac{-3x+6y-2z-5}{7}=\frac{4x-12y+3z-3}{13}$

or $67x-162y+47z+44=0$.

(34) **(B).** Let side of the cube = a
Then OG, BE and AD, CF will be four diagonals.
d.r.'s of OG = a, a, a = 1, 1, 1
d.r.'s of BE = –a, –a, a = 1, 1, –1
d.r.'s of AD = –a, a, a = –1, 1, 1
d.r.'s of CF = a, –a, a = 1, –1, 1
Let d.r.'s of line be l, m, n.
Therefore angle between line and diagonal



$\cos\sim=\frac{l+m+n}{\sqrt{3}}$, $\cos\ =\frac{l+m-n}{\sqrt{3}}$,

$\cos¥=\frac{-l+m+n}{\sqrt{3}}$, $\cos¢=\frac{l-m+n}{\sqrt{3}}$

$\Rightarrow$ $\cos^2\sim+\cos^2\ +\cos^2¥+\cos^2¢$

$=\frac{1}{3}[(l+m+n)^2+(l+m-n)^2+(-l+m+n)^2+(l-m+n)^2]$

= 4/3.

(35) **(D).** Any point on the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=t$ is $(3t+2, 4t-1, 12t+2)$. This lies on $x-y+z=5$

$\therefore$ $3t+2-4t+1+12t+2=5$ i.e., $11t=0 \Rightarrow t=0$

$\therefore$ Point is $(2,-1,2)$. Its distance from $(-1,-5,-10)$ is,

$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}=\sqrt{9+16+144}=13$.

(36) **(A).** Equation of plane passing through the point (2,–1,–3)
Also, $A(x-2)+B(y+1)+C(z+3)=0$
Also, $3A+2B-4C=0$ and $2A-3B+2C=0$

$\therefore$ $\frac{A}{-8}=\frac{B}{-14}=\frac{C}{-13}=k$, (Let)

So, $A=-8k, B=-14k, C=-13k$
Equation of required plane is,
$-k[8(x-2)+14(y+1)+13(z+3)]=0$
i.e., $8x+14y+13z+37=0$.

(37) **(B).** $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=\lambda$

$\Rightarrow$ $x=2\lambda+1, y=3\lambda-1$ and $z=4\lambda+1$

$\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=\mu$

$\Rightarrow$ $x=3+\mu, y=k+2\mu$ and $z=\mu$
Since above lines intersect
$2\lambda+1=3+\mu$ .... (1) $3\lambda-1=2\mu+k$ .... (2)
$\mu=4\lambda+1$ .... (3)
Solving eq. (1) and (3) and putting the value of λ and μ in (2) $\Rightarrow k=9/2$

(38) **(A).** Let $\vec{a} = x\vec{b} \times y\vec{c} + z\dfrac{\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|}$ .... (1)

{As we know any vector in the space can be represented as linear combination of three non-zero non-coplanar vectors}

$$\left.\begin{array}{l} \vec{a}.\vec{b} = x + 0 + 0 \\ \vec{a}.\vec{c} = 0 + y + 0 \\ \vec{a}.\vec{b} \times \vec{c} = 0 + 0 + z \end{array}\right\} \quad \text{.......... (2) as } \vec{b} \perp \vec{c}$$

Eq. (2) in eq. (1)

$$\vec{a} = (\vec{a}.\vec{b})\,\vec{b} + (\vec{a}.\vec{c})\,\vec{c} + \frac{\vec{a}.\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|^2}\vec{b} \times \vec{c}$$

(39) **(A).** Let $\hat{c}$ be the unit vector along $\vec{c}$.
Then from the equation of the bisector,

$$\hat{c} + \frac{3\vec{i} + 4\vec{j}}{\sqrt{3^2 + 4^2}} = \lambda(-\vec{i} + \vec{j} - \vec{k}) \text{ for some non-zero } \lambda.$$

$$\Rightarrow \hat{c} + \frac{1}{5}\left(3\vec{i} + 4\vec{j}\right) = \lambda(-\vec{i} + \vec{j} - \vec{k})$$

$$\Rightarrow \hat{c} = \left(-\lambda - \frac{3}{5}\right)\vec{i} + \left(\lambda - \frac{4}{5}\right)\vec{j} - \lambda\,\vec{k} \text{ .... (1)}$$

Using $\hat{c}.\hat{c} = 1$, we get

$$\left(\lambda + \frac{3}{5}\right)^2 + \left(\lambda - \frac{4}{5}\right)^2 + \lambda^2 = 1$$

$$\Rightarrow \lambda^2 + \frac{6}{5}\lambda + \frac{9}{25} + \lambda^2 - \frac{8}{5}\lambda + \frac{16}{25} + \lambda^2 - 1 = 0.$$

$$\Rightarrow 3\lambda^2 + \lambda\left(\frac{6}{5} - \frac{8}{5}\right) + \left(\frac{9}{25} + \frac{16}{25} - 1\right) = 0.$$

$$\Rightarrow 3\lambda^2 - \lambda.\frac{2}{5} + 0 = 0 \Rightarrow \lambda\left(3\lambda - \frac{2}{5}\right) = 0 \;\therefore\; \lambda = 0 \text{ or } \frac{2}{15}$$

As $\lambda = 0$ is not admissible, therefore putting $\lambda = \dfrac{2}{15}$ in (1), we get

$$\hat{c} = \left(-\frac{2}{15} - \frac{3}{5}\right)\vec{i} + \left(\frac{2}{15} - \frac{4}{5}\right)\vec{j} - \frac{2}{15}\vec{k}$$

$$= -\frac{11}{15}\vec{i} - \frac{10}{15}\vec{j} - \frac{2}{15}\vec{k} = -\frac{1}{15}(11\vec{i} + 10\vec{j} + 2\vec{k})$$

(40) **(D).** The vector normal to the plane of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is

$$\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 3 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= \vec{i}(-2+3) - \vec{j}(8-6) + \vec{k}(-4+2) = \vec{i} - 2\vec{j} - 2\vec{k}$$

$\therefore$ The magnitude of this vector is $\sqrt{1+4+4} = 3$.

Hence the vector normal to the plane of $(\overrightarrow{AB}, \overrightarrow{CD})$ of magnitude = 15 units will be $5(\vec{i} - 2\vec{j} - 2\vec{k})$.

Now to find out the position vector of P, we need to find the equations of AB and CD.

Now, the equation of $\overrightarrow{AB}$ is (by using $\vec{r} = \vec{a} + t\vec{b}$)

$$\vec{r} = 9\vec{i} - \vec{j} + 7\vec{k} + t(4\vec{i} - \vec{j} + 3\vec{k})$$

$$= \vec{i}(9 + 4i) + \vec{j}(-t - 1) + \vec{k}(3t + 7) \text{ .... (1)}$$

Similarly the equation of $\overrightarrow{CD}$ is

$$\vec{r} = 7\vec{i} - 2\vec{j} + 7\vec{k} + s(2\vec{i} - \vec{j} + 2\vec{k})$$

$$= \vec{i}(7 + 2s) + \vec{j}(-s - 2) + \vec{k}(2s + 7) \text{ .... (2)}$$

Therefore for the point of intersection P, we shall have
$9 + 4t = 7 + 2s$, $i + 1 = s + 2$ and $3t + 7 = 2s + 7$.
Now solving them, we get $t = -2$, $s = -3$.
Hence putting $t = -2$ in (1) or putting $s = -3$ in (2), we find that the position vector of P is $\vec{i} + \vec{j} + \vec{k}$

Let O be the origin of reference.

Now, the position vector of Q i.e. $\overrightarrow{OQ}$ is

$$\overrightarrow{OP} + \overrightarrow{OQ} = \vec{i} + \vec{j} + \vec{k} + 5(\vec{i} - 2\vec{j} - 2\vec{k})$$
$$= 6\vec{i} - 9\vec{j} - 9\vec{k} = 3(\vec{i} - 3\vec{j} - 3\vec{k})$$

(41) **(C).** Given : $\vec{A}.(\vec{B} + \vec{C}) = 0$

$\Rightarrow \vec{A}.\vec{B} + \vec{A}.\vec{C} = 0$ .... (1)

$\vec{B}.(\vec{C} + \vec{A}) = 0 \Rightarrow \vec{B}.\vec{C} + \vec{A}.\vec{B} = 0$ .... (2)

$\vec{C}.(\vec{A} + \vec{B}) = 0 \Rightarrow \vec{C}.\vec{A} + \vec{C}.\vec{B} = 0$ .... (3)

Adding, $2(\vec{A}.\vec{B} + \vec{B}.\vec{C} + \vec{C}.\vec{A}) = 0$

$\Rightarrow \vec{A}.\vec{B} + \vec{B}.\vec{C} + \vec{C}.\vec{A} = 0$ .... (4)

Subtracting (1), (2) and (3) successively from (4) we get

$\vec{B}.\vec{C} = 0$, $\vec{C}.\vec{A} = 0$, $\vec{A}.\vec{B} = 0$.

Thus $\vec{A}, \vec{B}, \vec{C}$ are mutually perpendicular vectors.

Hence $\vec{A}, \vec{B}, \vec{C}$ are the edges of a rectangular parallelopiped & $\left|\vec{A} + \vec{B} + \vec{C}\right|$ is the length of its diagonal.

Given : $\left|\vec{A}\right| = 3$, $\left|\vec{B}\right| = 4$, $\left|\vec{C}\right| = 5$.

$\therefore$ Length of the diagonal of the rectangular parallelopiped is $\sqrt{3^2 + 4^2 + 5^2} = 5\sqrt{2}$.

$\therefore \left|\vec{A} + \vec{B} + \vec{C}\right| = 5\sqrt{2}$.

(42) **(D).** We have, $\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)$

$$= \vec{A}\times\vec{A}+\vec{A}\times\vec{C}+\vec{B}\times\vec{A}+\vec{B}\times\vec{C}$$

$$= 0+\vec{A}\times\vec{C}-\vec{A}\times\vec{B}+\vec{B}\times\vec{C}$$

$$= \vec{B}\times\vec{C}-\vec{C}\times\vec{A}-\vec{A}\times\vec{B}$$

$$\therefore \left(\vec{A}+\vec{B}+\vec{C}\right).\left\{(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})\right\}$$

$$= \left(\vec{A}+\vec{B}+\vec{C}\right).\left[\vec{B}\times\vec{C}-\vec{C}\times\vec{A}-\vec{A}\times\vec{B}\right]$$

$$= \vec{A}.\left(\vec{B}\times\vec{C}\right)-\vec{A}.\left(\vec{C}\times\vec{A}\right)-\vec{A}.\left(\vec{A}\times\vec{B}\right)$$

$$+\vec{B}.\left(\vec{B}\times\vec{C}\right)-\vec{B}.\left(\vec{C}\times\vec{A}\right)-\vec{B}.\left(\vec{A}\times\vec{B}\right)$$

$$+\vec{C}.\left(\vec{B}\times\vec{C}\right)-\vec{C}.\left(\vec{C}\times\vec{A}\right)-\vec{C}.\left(\vec{A}\times\vec{B}\right)$$

$$= \vec{A}.(\vec{B}\times\vec{C})-0-0-0-\vec{B}.(\vec{C}\times\vec{A})-0+0-\vec{C}.\left(\vec{A}\times\vec{B}\right)$$

$$= \vec{A}.\left(\vec{B}\times\vec{C}\right)-\vec{A}.\left(\vec{B}\times\vec{C}\right)-\vec{A}.\left(\vec{B}\times\vec{C}\right)$$

$$= -\vec{A}.\left(\vec{B}\times\vec{C}\right) = -\left[\vec{A}\ \vec{B}\ \vec{C}\right]$$

(43) **(A).** Let $\vec{B} = (b_1, b_2, b_3)$

Then $\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= \vec{i}(b_3-b_2)-\vec{j}(b_3-b_1)+\vec{k}(b_2-b_1)$$

Since, $\vec{A}\times\vec{B} = \vec{C} = (0, 1, -1)$

Therefore $b_3 - b_2 = 0, b_1 - b_3 = 1,$
$b_2 - b_1 = -1.$

Solving these equations, we get
$b_1 = 1 + b_3$ and $b_2 = b_3$.

Also, $\vec{A}.\vec{B} = 3 \Rightarrow b_1 + b_2 + b_3 = 3.$

Hence from (1), $1 + b_3 + b_3 + b_3 = 3$
$\Rightarrow 3b_3 = 2 \Rightarrow b_3 = 2/3.$

$\therefore\ b_1 = 1 + b_3 = 1 + \frac{2}{3} = \frac{5}{3}$ and $b_2 = b_3 = \frac{2}{3}$.

Hence $\vec{B} = \frac{5}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}$

(44) **(D).** Let O be the origin and P (1, 1, 2), Q (2, 3, 5) and R (1, 5, 5) be the given points, then

$\overrightarrow{OP} = \vec{i}+\vec{j}+2\vec{k}$, $\overrightarrow{OQ} = 2\vec{i}+3\vec{j}+5\vec{k}$ and

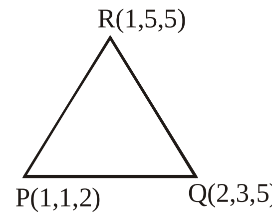
$\overrightarrow{OR} = \vec{i}+5\vec{j}+5\vec{k}$

Now, $\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = (2\vec{i}+3\vec{j}+5\vec{k})-(\vec{i}+\vec{j}+2\vec{k})$

$$= (\vec{i}+2\vec{j}+3\vec{k})$$

$\overrightarrow{PR} = \overrightarrow{OR}-\overrightarrow{OP} = (\vec{i}+5\vec{j}+5\vec{k})-(\vec{i}+\vec{j}+2\vec{k})$

$$= 0\vec{i}+4\vec{j}+3\vec{k} = 4\vec{j}+3\vec{k}$$



$$\therefore \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= (6-12)\,\vec{i}-(3-0)\,\vec{j}+(4-0)\vec{k} = -6\vec{i}-3\vec{j}+4\vec{k}$$

Hence, area of Δ PQR $= \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$$= \frac{1}{2}|(-6\vec{i}-3\vec{j}+4\vec{k})| = \frac{1}{2}\sqrt{36+9+16}$$

$$= \frac{1}{2}\sqrt{61}\text{ sq. units}$$

(45) **(C).** Let $|\vec{a}| = a, |\vec{b}| = b$ and $|\vec{c}| = c = 1$.

Now the volume of the parallelopiped whose coterminal edges are $\vec{a}, \vec{b}, \vec{c}$ is given by

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\therefore \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 \quad \text{............ (1)}$$

Now $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$.

$$\therefore\ \vec{c} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} \Rightarrow \vec{c}.\vec{c} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}.\vec{c}$$

$$\Rightarrow 1 = \frac{\vec{a}.\vec{b}\times\vec{c}}{|\vec{a}\times\vec{b}|} \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\times\vec{b}] \quad \text{.................(2)}$$

But $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= \vec{i}(a_2b_3-a_3b_2)-\vec{j}(a_1b_3-a_3b_1)+\vec{k}(a_1b_2-a_2b_1)$$

$$= |\vec{a}\times\vec{b}|$$

$$= \sqrt{(a_2b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+(a_1b_2-a_2b_1)^2} \quad \text{...(3)}$$

Now, from (1) and (2), $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \left|\vec{a}\times\vec{b}\right|^2$

$= (a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2$
$= (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
$- (a_1b_1 + a_2b_2 + a_3b_3)^2$ ....(4)

But the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$

Therefore $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\frac{\pi}{6}$

$\Rightarrow a_1b_1 + a_2b_2 + a_3b_3 =$

$\sqrt{a_1^2 + a_2^2 + a_3^2}\ \sqrt{b_1^2 + b_2^2 + b_3^2}\ .\ \frac{\sqrt{3}}{2}$

$\Rightarrow (a_1b_1 + a_2b_2 + a_3b_3)^2$

$= (a_1{}^2 + a_2{}^2 + a_3{}^2)(b_1{}^2 + b_2{}^2 + b_3{}^2)\ \frac{3}{4}$

Hence from (4),

$[\vec{a}\ \vec{b}\ \vec{c}]^2 = (a_1{}^2 + a_2{}^2 + a_3{}^2)(b_1{}^2 + b_2{}^2 + b_3{}^2)(1 - 3/4)$

$= \frac{1}{4}\Sigma a_1^2\, \Sigma b_1^2$

(46) **(C).** Take C as the origin. Let the position vectors of A and B w.r.t. C be $\vec{a}$ and $\vec{b}$ respectively.

Then the equation of AD and BE are

$\vec{r} = \vec{a} + t\left(\frac{\vec{b}}{3} - \vec{a}\right)$ .... (1)

and $\vec{r} = \vec{b} + s\left(\frac{\vec{a}}{4} - \vec{b}\right)$ .... (2)

If they intersect at P, we must have identical values of $\vec{r}$ from (1) and (2).

Comparing the coefficients of $\vec{a}$ and $\vec{b}$ from (1) and (2),

we get $1 - t = \frac{s}{4}, \frac{t}{3} = 1 - s.$

Solving them, we get $t = \frac{9}{11}$ and $s = \frac{8}{11}$.

Putting for t or s in (1) and (2), we find the position vector

of P as $\frac{2\vec{a} + 3\vec{b}}{11}$ .... (3)

Let P divide BE in the ratio of k : 1.

Then the position vector of P will be

$\frac{k\frac{\vec{a}}{4} + 1\vec{b}}{k+1}$ .... (4)

Comparing (3) and (4), we get

$\frac{k\frac{\vec{a}}{4} + \vec{b}}{k+1} = \frac{2\vec{a} + 3\vec{b}}{11} \Rightarrow \frac{k}{4(k+1)} = \frac{2}{11}$ and $\frac{1}{k+1} = \frac{3}{11}$.

From the second equation, we have

$3k + 3 = 11 \Rightarrow 3k = 8.\ \therefore\ k = 8/3.$

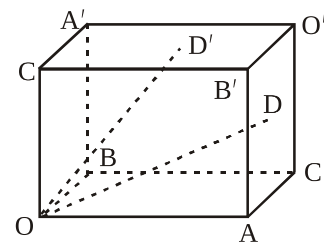
This also satisfies the first equation.

Hence the required ratio is 8 : 3.

(47) **(A).** If OA = 2, then OB = 4 and OC = 6 units

$\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC'} + \frac{1}{2}\overrightarrow{AB'} = 2\vec{i} + 2\vec{j} + 3\vec{k}$

$\overrightarrow{OD'} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CB'} + \frac{1}{2}\overrightarrow{B'O'} = \vec{i} + 2\vec{j} + 6\vec{k}$



$\cos D\hat{O}D' = \frac{\overrightarrow{OD}.\overrightarrow{OD'}}{|\overrightarrow{OD}||\overrightarrow{OD'}|} = \frac{24}{\sqrt{697}}$.

(48) **(C).** Now in ΔABC

$\frac{BD}{DC} = \frac{a}{b}\ \therefore BD = ak, DC = bk\ \ \therefore BC = (a+b)k$

$(BC)^2 = (AB)_2 + (AC)^2 - 2AB\,.\,AC \cos\theta$

$\Rightarrow (a+b)^2 k^2 = a^2 + b^2 - 2ab\cos\theta$

$\therefore k^2 = \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}$

In ΔADC and ΔABD

$\cos\frac{\theta}{2} = \frac{b^2 + (AD)^2 - b^2k^2}{2b\,AD} = \frac{a^2 + (AD)^2 - a^2k^2}{2a\,AD}$

$\Rightarrow (AD)^2 = ab(1-k)^2$

$= ab\left\{1 - \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}\right\} = \frac{4a^2b^2\cos^2\frac{\theta}{2}}{(a+b)^2}$

$\therefore AD = \frac{2ab\cos\frac{\theta}{2}}{(a+b)}$

$\therefore \overrightarrow{AD} = \pm\frac{(\vec{a}b + \vec{b}a)}{(a+b)} = \pm\frac{ab}{(a+b)}\left(\frac{\vec{a}}{a} + \frac{\vec{b}}{b}\right)$

$= \pm\frac{ab}{(a+b)}(\hat{a} + \hat{b})\quad \therefore \hat{AD} = \frac{\overrightarrow{AD}}{AD} = \pm\frac{(\hat{a} + \hat{b})}{2\cos\frac{\theta}{2}}$

(49) **(A).** Since $\vec{a}, \vec{b}$ and are non coplanar

$\therefore \vec{r} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$ ...... (1)

for some scalars x, y, z

Now $\vec{b} = \vec{r} \times \vec{a}$

$\therefore\ \vec{b} = \left\{x\vec{a} + y\vec{b} + z\left(\vec{a}\times\vec{b}\right)\right\}\times\vec{a}$

$= x\left(\vec{a}\times\vec{a}\right) + y\left(\vec{b}\times\vec{a}\right) + z\left\{\left(\vec{a}\times\vec{b}\right)\times\vec{a}\right\}$

$= 0 + y\left(\vec{b}\times\vec{a}\right) + z\left\{\left(\vec{a}\,.\,\vec{a}\right)\vec{b} - \left(\vec{a}\,.\,\vec{b}\right)\times\vec{a}\right\}$

$\therefore\ \vec{b} = y\left(\vec{b}\times\vec{a}\right) + z\left(\vec{a}\,.\,\vec{a}\right)\vec{b}$ $\quad\left\{\because\ \vec{a}\,.\,\vec{b} = 0\right\}$

Comparing the coefficients, we get

$y = 0$ and $z = \dfrac{1}{\left(\vec{a}\,.\,\vec{a}\right)}$

Putting the values of y and z in (1), we get

$\vec{r} = x\vec{a} + \dfrac{1}{\left(\vec{a}\,.\,\vec{a}\right)}\left(\vec{a}\times\vec{b}\right)$

(50) **(B).** (a) $a_1 = a_2 = a_3 \Rightarrow f(x) = 3(x - a_1)^2 \geq 0$
$f(x) = 3x^2 - 2(a_1 + a_2 + a_3)x + a_1a_2 + a_2a_3 + a_3a_1 \geq 0$
for all $x \in R$
$\Rightarrow (a_1 + a_2 + a_3)^2 - 3(a_1a_2 + a_2a_3 + a_3a_1) \leq 0$
$\Rightarrow (a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 \leq 0$
$\Rightarrow a_1 = a_2 = a_3$
$\therefore$ Statement (a) is correct
(b) : Let $f(x) = e^x - x - 1$
$f'(x) = e^x - 1$
$\therefore\ f'(x) > 0$ for all $x > 0$ $f'(x) < 0$ for all $x < 0$
$\therefore\ f(x) > 0$ for all $x \neq 0$
$\therefore$ Statement (b) is correct
(c) Let $f(x) = ax^3 + bx^2 + cd + d$
$f(1) = a + b + c + d = 9$
Number of +ve integral solutions $= {}^{9-1}C_{4-1} = {}^8C_3 = 56$
$\therefore$ Statement (c) is correct.
(d) $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$

$\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

$\vec{a}\times(\vec{b}-\vec{c}) = (\vec{c}-\vec{b})\times\vec{d}$

$(\vec{a}-\vec{d})\times(\vec{c}-\vec{b}) = 0 \Rightarrow (\vec{a}-\vec{d}).(\vec{c}-\vec{b}) \neq 0$

(51) **(C).** (A) $\vec{a}.(\vec{b}-\vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or angle between $\vec{a}$ and $\vec{b}-\vec{c}$ is 90°
$\Rightarrow$ (A) is not correct.

(B) $\vec{a}\times(\vec{b}-\vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a}$ and $\vec{b}-\vec{c}$ are collinear
$\Rightarrow$ (B) is not correct.

(C) $\vec{a}.(\vec{b}-\vec{c}) = 0$ and $\vec{a}\times(\vec{b}-\vec{c}) = 0$

$\Rightarrow \left.\begin{array}{l}\text{either } \vec{b} = \vec{c} \\ \vec{a}\wedge(\vec{b}-\vec{c}) = 90^\circ\end{array}\right]$ and $\begin{array}{l}\vec{b} = \vec{c} \\ \vec{a}\wedge(\vec{b}-\vec{c}) = 0\end{array}$;

$\vec{a}$ and $\vec{b}-\vec{c}$ are collinear. Hence, $\vec{b} = \vec{c}$
(D) D is true, refer reciprocal system of vectors.

(52) **(C).** The vector equation of the line through the points A and B is $\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda\,[(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k}]$

i.e., $\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda\,(2\hat{i} - 3\hat{j} + 5\hat{k})$ .... (1)

Let P be the point where the line AB crosses the XY-plane. Then the position vector of the point P is of the form $x\hat{i} + y\hat{j}$.

This point must satisfy the equation (1).

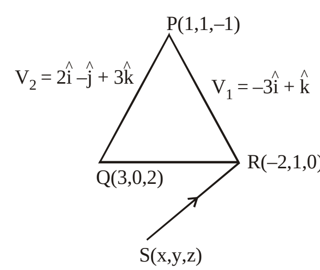
i.e., $x\hat{i} + y\hat{j} = (3 + 2\lambda)\hat{i} + (4 - 3\lambda)\hat{j} + (1 + 5\lambda)\hat{k}$

Equating the like coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$, we have
$x = 3 + 3\lambda$; $y = 4 - 3\lambda$; $0 = 1 + 5\lambda$

Solving the above equations, we get $x = \dfrac{13}{5}$ and $y = \dfrac{23}{5}$

The coordinates of the required point are $\left(\dfrac{13}{5}, \dfrac{23}{5}, 0\right)$

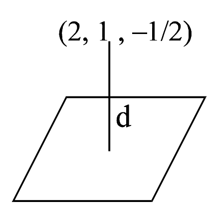
(53) **(D).** $\vec{V}_1, \vec{V}_2, \overrightarrow{RS}$ are in the same plane.
$\therefore$
$(2\hat{i} - 3\hat{j} + 3\hat{k})\times(-3\hat{i} + \hat{k}).[(x+2)\,\hat{i} + (y-1)\hat{j} + z\hat{k}] = 0 \Rightarrow$ (D)



Actual plane is $x + 11y + 3z = 6$.

(54) **(A).** The line is $\dfrac{x-2}{1} = \dfrac{y-1}{1} = \dfrac{z+\frac{1}{2}}{-\frac{1}{2}} = t$ ....(1)

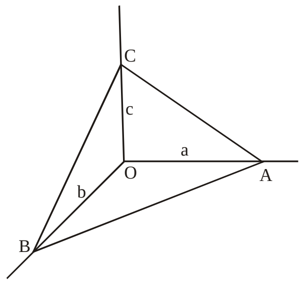
line passes through $2\hat{i} + \hat{j} - \dfrac{1}{2}\hat{k}$ and is parallel to the vector $\vec{V} = \hat{i} + \hat{j} - \dfrac{1}{2}\hat{k}$



vector normal to the plane $x + 2y + 6z = 10$, is
$\vec{n} = \hat{i} + 2\hat{j} + 6\hat{k}$
$\therefore\ \vec{V}.\vec{n} = 1 + 2 - 3 \Rightarrow$ line (1) is | | to the plane

$$\therefore d = \left|\frac{2+2-3-10}{\sqrt{1+4+36}}\right| = \frac{9}{\sqrt{41}}$$

(55) **(A).**



ab = 8
bc = 12
ca = 24

$$\text{Area} = \frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2} = \frac{1}{2}\sqrt{64+144+576}$$

$$= \sqrt{16+36+144} = \sqrt{196} = 14$$

(56) **(A).** Both the lines pass through origin
Line L_1 is parallel to the vector

$$V_1 = (\cos\theta+\sqrt{3})\,\hat{i} + (\sqrt{2}\sin\theta)\,\hat{j} + (\cos\theta-\sqrt{3})\,\hat{k}$$

and L_2 is parallel to the vector

$$\vec{V}_2 = a\hat{i}+b\hat{j}+c\hat{k} \quad \therefore \cos\alpha = \frac{\vec{V}_1.\vec{V}_2}{|\vec{V}_1||\vec{V}_2|}$$

$$= \frac{a(\cos\theta+\sqrt{3})+(b\sqrt{2})\sin\theta+c(\cos\theta-\sqrt{3})}{\sqrt{a^2+b^2+c^2}\sqrt{(\cos\theta+\sqrt{3})^2+2\sin^2\theta+(\cos\theta-\sqrt{3})^2}}$$

$$= \frac{(a+c)(\cos\theta+b\sqrt{2}\sin\theta+(a-c)\sqrt{3}}{\sqrt{a^2+b^2+c^2}\sqrt{2+6}}$$

In order that cos α is independent of θ
a + c = 0 and b = 0

$$\therefore \cos\alpha = \frac{2a\sqrt{3}}{a\sqrt{2}.2\sqrt{2}} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{6}$$

(57) **(C).** ∵ P lies on $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1 \Rightarrow \frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c} = 1$

The Dr's of normal to the plane perpendicular to OP are the same as the DC's of OP which are proportional to α, β, γ also, this plane passes through P.
⇒ Its equation will be of the form

$$\alpha(x-\alpha)+\beta(y-\beta)+\gamma(z-\gamma) = 0$$

$$\Rightarrow \alpha x+\beta y+\gamma z = \alpha^2+\beta^2+\gamma^2$$

This meets yz plane in A

$$\Rightarrow A = \left(\frac{\alpha^2+\beta^2+\gamma^2}{\alpha}, 0, 0\right)$$

(58) $1+\mu+\lambda = 2+t \Rightarrow \lambda+\mu-t = 1$
$\Rightarrow 1+\lambda-2\mu = -1+t \Rightarrow \lambda-2\mu-t = -2$
$\lambda+3\mu = 3+t \Rightarrow \lambda+3\mu-t = 3$
$\Rightarrow \lambda = t$ and $\mu = 1$
⇒ Infinite solutions
⇒ Plane and line intersect at infinite number of points
⇒ Line completely lies in plane.

(59) **(A).** $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d} = (2\vec{a}-2\vec{b})+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})$

$$= -2\overrightarrow{AB}+5\overrightarrow{AC}-6\overrightarrow{AD} = \vec{0}$$

∴ $\overrightarrow{AB}, \overrightarrow{AC}$ & $\overrightarrow{AD}$ are linearly dependent, hence by statement-2, the statement-1 is true.

(60) **(B).** S-1 : $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta$

$$= 2(1-\cos\theta) \quad (\because |\vec{a}| = |\vec{b}| = 1)$$

$$= 2.2\sin^2\frac{\theta}{2}$$

$$\therefore \sin\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{2}$$

S-2 : Numberof vectors of unit length perpendicular to the vectors $\vec{a}$ & $\vec{b}$ are two, i.e. $\pm\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

(61) **(A).** S-1 : (1, 2, –1) is a point on the line and 11 + 3– 14 = 0
∴ The point lies on the plane 11x – 3z – 14 = 0
Further 3 × 11 + 11 (–3) = 0
∴ The line lies in the plane.
S-2 : Trivially true.

(62) **(D).** S-2 : $\vec{r}\times(\hat{i}+2\hat{j}-3\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 1 & 2 & -3\end{vmatrix}$

$$\hat{i}(-3y-2z)-\hat{j}(-3x-z)+\hat{k}(2x-y)$$

∴ – 3y – 2z = 2, 3x + z = –1, 2x – y = 0
i.e., –6x – 2z = 2, 3x + z = – 1
∴ Straight line 2x – y = 0, 3x + z = – 1

S-1 : $\vec{r}\times(2\hat{i}-\hat{j}+3\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 2 & -1 & 3\end{vmatrix}$

$$= \hat{i}(3y+z)-\hat{j}(3x-2z)+\hat{k}(-x-2y)$$

∴ 3y + z = 3, 3x – 2z = 0, – x – 2y = 1
3x – 2 (3 – 3y) = 0 ⇒ 3x + 6y = 6 ⇒ x + 2y = 2
Now, x + 2y = –1, x + 2y = 2 are parallel planes

∴ $\vec{r}\times(2\hat{i}-\hat{j}+3\hat{k}) = 3\hat{i}+\hat{k}$ is not a straight line

(63) **(D).** Statement 1 is false and statement 2 is true.
Since $\vec{a}.(\vec{b}\times\vec{c}) = 0$ ∴ $\vec{a}, \vec{b}$, $\vec{c}$ are coplanar.

(64) (A).

$3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=(2\vec{a}-2\vec{b})+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})$

$=-2\overrightarrow{AB}+5\overrightarrow{AC}-6\overrightarrow{AD}=\vec{0}$

$\therefore$ $\overrightarrow{AB},\overrightarrow{AC}$ & $\overrightarrow{AD}$ are linearly dependent, hence by statement-2, the statement-1 is true.

(65) (D). $\sin\theta=\left|\dfrac{2-3+2}{\sqrt{4+9+4}\,\sqrt{3}}\right|=\dfrac{1}{\sqrt{51}}$

Statement 1 is true, statement 2 is true by definition.

(66) (B). Statement 1 : $3y-4z=5-2k$; $-2y+4z=7-3k$

$\therefore$ $x=k, y=12-5k, z=\dfrac{31-13k}{4}$ is a point on the line for all real values of k.

Statement is true.

Statement 2 : Direction ratios of the straight line are $<bc'-kbc, kac-ac', 0>$

Direction ratios of normal to be plane $<0, 0, 1>$

Now $0\times(bc'-kbc)+0\times(kac-ac')+1\times 0=0$

$\therefore$ The straight line is parallel to the plane.

$\therefore$ Statement is true but does not explain statement-1.

(67) (B), (68) (A), (69) (C).

(i) $\vec{a}_1=\left[(2\hat{i}+3\hat{j}-6\hat{k}).\dfrac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right]\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

$=\dfrac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k})$

$\vec{a}_2=\dfrac{-41}{49}\left(2\hat{i}-3\hat{j}+6\hat{k}.\right)\left(\dfrac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}\right)\dfrac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}$

$=\dfrac{-41}{(49)^2}(-4-9+36)\,(-2\hat{i}+3\hat{j}+6\hat{k})$

$=\dfrac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

(ii) $\vec{a}_1.\vec{b}=\dfrac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})=-41$

(iii) $\vec{a},\vec{a}_1$ and $\vec{b}$ are coplanar, because $\vec{a}_1$ and $\vec{b}$ are collinear.

(70) (C), (71) (B), (72) (A).

(i) Plane P will contain the point M (–1, –1, 2) and dr's of its normal will be proportional to (3, –3, 1)

$\therefore$ equation of the plane P is

$3(x+1)-3(y+1)+1(z-2)=0$

$P: 3x-3y+z-2=0$ ....(1)

$Q: x-y+cz-1=0$ ....(2)

If P is parallel to Q then $\dfrac{3}{1}=\dfrac{1}{c}\Rightarrow c=\dfrac{1}{3}$

(ii) $\vec{n}_1=3\hat{i}-3\hat{j}+\hat{k}$ and $\vec{n}_2=\hat{i}-\hat{j}+c\hat{k}$

now $\cos 45°=\left|\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1|.|\vec{n}_2|}\right|=\left|\dfrac{3+3+c}{\sqrt{19}\,\sqrt{2+c^2}}\right|$

$\therefore 2(6+c)^2=19(2+c^2)$

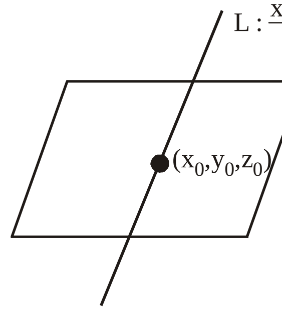
$2(36+c^2+12c)=38+19c^2$

$17c^2-24c-34=0$

product = – 2 Ans.

(iii) $x=t+1$; $y=3t-2$ and $z=-t+7$

They must satisfy then equation of plane 'P'



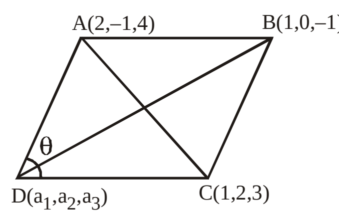
$3(t+1)-3(3t-2)+(7-t)=2$

$-7t+16=2\Rightarrow t=2$

hence $x_0=3; y_0=4; z_0=5$

$\Rightarrow(x_0+y_0+z_0)=3+4+5=12$

(73) (C). $a_1+1=3\Rightarrow a_1=2$; $a_2+0=1\Rightarrow a_2=1$



$a_3-1=7\Rightarrow a_3=8$

D (2, 1, 8)

$\vec{d}=\left|\dfrac{(\overrightarrow{AB})\times(\overrightarrow{AD})}{|\overrightarrow{AB}|}\right|$;

$\overrightarrow{AB}=\hat{i}-\hat{j}+5\hat{k},\ \overrightarrow{AD}=0\hat{i}+2\hat{j}+4\hat{k}$

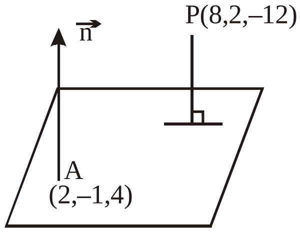
$\overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&5\\0&2&4\end{vmatrix}$

$=(-4-10)\,\hat{i}-(4)\hat{j}+(2)\hat{k}=-14\hat{i}-4\hat{j}+2\hat{k}$

$=-2(7\hat{i}+2\hat{j}-\hat{k})=2\sqrt{2}$

(74) **(B).** $\vec{n} = 7\hat{i} + 2\hat{j} - \hat{k}$ is normal to plane

$\therefore P = (8, 2, -12)$



$\overrightarrow{AP} = 6\hat{i} + 3\hat{j} - 16\hat{k}$

$\therefore$ Distance

$$d = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \left|\frac{42+6+16}{\sqrt{49+4+1}}\right| = \frac{64}{\sqrt{54}} = \frac{64}{3\sqrt{6}} = \frac{64\sqrt{6}}{18} = \frac{32\sqrt{6}}{9}$$

(75) **(D).** Vector normal to the plane in RHS

$\overrightarrow{AD} \times \overrightarrow{AB} = +2\,(7\hat{i} + 2\hat{j} - \hat{k})$

Projection of xy = 2, yz = 14, zx = 4

(76) (B), (77) (A), (78) (A).

Equation of the plane through (0, 0, 1) ; (2, 0, 0) ; (0, 3, 0)

is $\frac{x}{2} + \frac{y}{3} + \frac{z}{1} = 1 \Rightarrow 3x + 2y + 6z = 6$ ..... (1)

Equation of L is $\vec{r} = \frac{8}{3}\hat{i} + 0\hat{j} + \hat{k} + \lambda\,(2\hat{i} - 2\hat{j} + \hat{k})$ ..... (2)

p.v. of any point on (1) is $\left(2\lambda + \frac{8}{3}\right); -2\lambda, \lambda + 1$

This lies on 3x + 2y + 6z = 6

$3\left(2\lambda + \frac{8}{3}\right) - 4\lambda + 6\,(\lambda + 1) = 6$; $8\lambda = -8 \Rightarrow \lambda = -1.$

Hence P is (2/3, 2, 0)

Distance of origin then P is $\sqrt{\frac{4}{9} + 4} = \frac{2\sqrt{10}}{3}$

Vector perpendicular to given plane is $\vec{n}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

Vector perpendicular to plane 2x + y – 2z = 5 is

$\vec{n}_2 = 2\hat{i} + \hat{j} - 2\hat{k}$

A vector along the line of intersection of both these plane

is $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 6 \\ 2 & 1 & -2 \end{vmatrix}$

$= \hat{i}\,(-4-6) - \hat{j}\,(-6-12) + \hat{k}\,(3-4) = -10\hat{i} + 18\hat{j} - \hat{k}$

Hence a unit vector along the line of intersection is

$$\pm\frac{-10\hat{i} + 18\hat{j} - \hat{k}}{5\sqrt{17}}$$

Vector along the line $\vec{V} = 2\hat{i} - 2\hat{j} + \hat{k}$

Vector along the line $\vec{n} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\therefore \sin\theta = \frac{6-4+6}{\sqrt{9}.\sqrt{49}} = \frac{8}{21}\ ;\ \theta = \sin^{-1}\left(\frac{8}{21}\right)$$

(79) (C), (80) (A), (81) (C).

$$L_1 : \frac{x-1}{-2} = \frac{y-0}{1} = \frac{z+1}{1}\ ;\ L_2 : \frac{x-4}{1} = \frac{y-5}{4} = \frac{z+2}{-1}$$

$\vec{V}_1 = -2\hat{i} + \hat{j} + \hat{k},\ \vec{V}_2 = \hat{i} + 4\hat{j} - \hat{k}$

$$\cos\theta = \left|\frac{-2+4-1}{\sqrt{6}.\sqrt{18}}\right| = \frac{1}{6\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6\sqrt{3}}\right) \Rightarrow (C)$$

Equation of the plane containing the line L_2 is

$A(x-4) + B\,(y-5) + C(z+2) = 0$ ....(1)

where A + 4B – C = 0

since (1) is parallel to L_1

hence – 2A + B + C = 0

$$\therefore \frac{A}{4+1} = \frac{B}{2-1} = \frac{C}{1+8} \Rightarrow A = 5k;\ B = k;\ C = 9k$$

hence equation of plane P

$5(x-4) + y - 5 + 9(z+2) = 0 \Rightarrow 5x + y + 9z - 7 = 0 \Rightarrow (A)$

Distance between P and L_1 is

$$d = \left|\frac{5+0-9-7}{\sqrt{25+1+81}}\right| = \frac{11}{\sqrt{107}}$$

(82) **343.** $\vec{V} = \vec{A} \times ((\vec{A}.\vec{B})\vec{A} - (\vec{A}.\vec{A})\vec{B}).\vec{C}$

$$= \left(\underbrace{\vec{A} \times (\vec{A}.\vec{B})\vec{A}}_{\text{zero}} - (\vec{A}.\vec{A})\vec{A} \times \vec{B})\right).\vec{C} = -|\vec{A}|^2\,[\vec{A}\ \ \vec{B}\ \ \vec{C}]$$

Now, $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$$[\vec{A}\ \ \vec{B}\ \ \vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 2\,(1+4) - 1\,(3-12) + 1\,(-6-6)$

$= 10 + 9 - 12 = 7$

$\therefore \left|-|\vec{A}|^2\,[\vec{A}\ \ \vec{B}\ \ \vec{C}]\right| = 49 \times 7 = 343$

(83) **6.** $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} + 3\vec{c} \times \vec{b} = \vec{0}$

i.e. $\vec{a} \times \vec{b} = 3\vec{b} \times \vec{c}$

$\vec{a} \times \vec{c} + 2\vec{b} \times \vec{c} = \vec{0}$ i.e. $2\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$\therefore \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 3\vec{b} \times \vec{c} + \vec{b} \times \vec{c} + 2\vec{b} \times \vec{c} = 6\,(\vec{b} \times \vec{c})$

(84) **2.**

Volume of the parallelopiped formed by $\vec{a}', \vec{b}', \vec{c}'$ is 4.

$\therefore$ Volume of the parallelopiped formed by $\vec{a}, \vec{b}, \vec{c}$ is 1/4

$\vec{b}\times\vec{c} = \frac{(\vec{c}'\times\vec{a}')\times\vec{c}}{4} = \frac{1}{4}\vec{a}'$

$\therefore \quad \vec{b}\times\vec{c} = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$

$\therefore$ Length of altitude $= \frac{1}{4}\times 2\sqrt{2} = \frac{1}{\sqrt{2}}$

(85) **1.** $(\vec{u}\times\vec{v}+\vec{u})\times\vec{u} = \vec{w}\times\vec{u} = \vec{v}$

$\Rightarrow (\vec{u}\times\vec{v})\times\vec{u} = \vec{v} \Rightarrow \vec{v}-(\vec{u}.\vec{v})\,\vec{u} = \vec{v}$

$\Rightarrow \vec{u}.\vec{v} = 0$ .... (i)

$[\vec{u}\ \vec{v}\ \vec{w}] = \vec{u}.(\vec{v}\times\vec{w}) = \vec{u}.\{\vec{v}\times(\vec{u}\times\vec{v}+\vec{u})\}$

$= \vec{u}\{\vec{v}^2\vec{u}-(\vec{u}.\vec{v})\,\vec{v}+\vec{v}\times\vec{u}\} = 1$

(86) **101.** We have $\vec{B}\times\vec{C} = x\vec{A}+y\vec{B}+zC$ .... (1)

Dot with $\vec{B}\times\vec{C}$ gives $(\vec{B}\times\vec{C}).(\vec{B}\times\vec{C}) = x\,[\vec{A}\ \vec{B}\ \vec{C}]$

$\Rightarrow x = \frac{(\vec{B}\times\vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]} = \frac{\vec{B}^2\vec{C}^2-(\vec{B}.\vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]}$

|||ly Dot with $\vec{C}\times\vec{A}$ gives $y = \frac{(\vec{B}\times\vec{C}).(\vec{C}\times\vec{A})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

and dot with $\vec{A}\times\vec{B}$ gives $\frac{(\vec{B}\times\vec{C}).(\vec{A}\times\vec{B})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

Now $[\vec{A}\ \vec{B}\ \vec{C}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 1\,(1+1)-2\,(-2-3) = 12$

We have $\vec{B}^2\vec{C}^2-(\vec{B}.\vec{C})^2 = (6)\,(2)-(0) = 12$

$\therefore \quad x = \frac{12}{12} = 1$

$y = \frac{\begin{vmatrix} \vec{B}.\vec{C} & \vec{B}.\vec{A} \\ \vec{C}.\vec{C} & \vec{C}.\vec{A} \end{vmatrix}}{12} = \frac{\begin{vmatrix} 0 & -3 \\ 2 & 1 \end{vmatrix}}{12} = \frac{0-(-6)}{12} = \frac{1}{2}$

$z = \frac{\begin{vmatrix} \vec{B}.\vec{A} & \vec{B}.\vec{B} \\ \vec{C}.\vec{A} & \vec{C}.\vec{B} \end{vmatrix}}{12} = \frac{\begin{vmatrix} -3 & 6 \\ +1 & 0 \end{vmatrix}}{12} = \frac{0-(6)}{12} = \frac{-1}{2}$

Hence, $100x+10y+8z = 100+5-4 = 101$

(87) $E = (2\vec{a}+\vec{b}).[2\,|\vec{b}|^2\,\vec{a}-2\,(\vec{a}.\vec{b})\,\vec{b}-(\vec{a}.\vec{b})\,\vec{a}+|\vec{a}|^2+\vec{b}]$

$\vec{a}.\vec{b} = \frac{2-2}{\sqrt{70}} = 0$; $|\vec{a}| = 1, |\vec{b}| = 1$; $\vec{a}.\vec{b} = 0$

$E = (2\vec{a}+\vec{b}).[2\,|\vec{b}|\,\vec{a}+|\vec{a}|^2\,\vec{b}]$

$= 4\,|\vec{a}|^2\,|\vec{b}|^2+|\vec{a}|^2\,(\vec{a}.\vec{b})+2\,|\vec{b}|^2\,(\vec{b}.\vec{a})+|\vec{a}|^2|\vec{b}|^2$

$= 5\,|\vec{a}|^2\,|\vec{b}|^2 = 5$

(88) **9.** $(\vec{r}-\vec{c})\times\vec{b} = 0$

$\vec{r}-\vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c}+\lambda\vec{b}\,;\ \lambda\in R \quad \because\ \vec{r}.\vec{a} = 0$

$\Rightarrow \quad (\vec{c}+\lambda\vec{b}).\vec{a} = 0$

$\Rightarrow \quad ((\hat{i}+2\hat{j}+3\hat{k})+\lambda\,(-\hat{i}+\hat{j})).(-\hat{i}-\hat{k}) = 0$

$\Rightarrow \quad ((1-\lambda)\,\hat{i}+(2+\lambda)\,\hat{j}+3\hat{k}).(-\hat{i}-\hat{k}) = 0$

$\Rightarrow \quad \lambda-1-3 = 0 \Rightarrow \lambda = 4$

So, $\vec{r}.\vec{b} = (-3\hat{i}+6\hat{j}+3\hat{k}).(-\hat{i}+\hat{j}) = 3+6 = 9$

(89) **3.** $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2 = 9$

$\Rightarrow \quad 6-2\Sigma\,\vec{a}.\vec{b} = 9 \Rightarrow \Sigma\,\vec{a}.\vec{b} = -\frac{3}{2}$ .... (1)

$|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0 \Rightarrow \Sigma\vec{a}^2+2\Sigma\,\vec{a}.\vec{b} \geq 0$

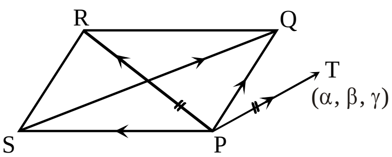
$\Rightarrow \quad \Sigma\,\vec{a}.\vec{b} \geq -\frac{3}{2}$

For equality $|\vec{a}+\vec{b}+\vec{c}| = 0 \Rightarrow \vec{a}+\vec{b}+\vec{c} = 0$

$5\vec{b}+5\vec{c} = -5\vec{a}$; $2\vec{a}+5\vec{b}+5\vec{c} = -3\vec{a}$

$|2\vec{a}+5\vec{b}+5\vec{c}| = 3\,|\vec{a}| = 3$

(90) **10.**



Area of base (PQRS)

$= \frac{1}{2}|\overrightarrow{PR}\times\overrightarrow{SQ}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & -4 \end{vmatrix}$

$= \frac{1}{2}|-10\hat{i}+10\hat{j}-10\hat{k}| = 5\,|\hat{i}-\hat{j}+\hat{k}| = 5\sqrt{3}$

Height = proj. of PT on

$\hat{i}-\hat{j}+\hat{k} = \left|\frac{1-2+3}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

Volume $= (5\sqrt{3})\left(\frac{2}{\sqrt{3}}\right) = 10$ cu. units

(91) **4.** $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$,

Now, $[\vec{a}\ \vec{b}\ \ \vec{c}]^2=\begin{vmatrix}1 & 1/2 & 1/2\\ 1/2 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix}$

$$=1\left(1-\frac{1}{4}\right)-\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)+\frac{1}{2}\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{3}{4}-\frac{1}{8}-\frac{1}{8}=\frac{1}{2}$$

$\therefore\ [\vec{a}\ \vec{b}\ \ \vec{c}]=\pm\frac{1}{\sqrt{2}}$

Now, $\vec{a}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c})=p+\frac{q}{2}+\frac{r}{2}$

$\pm\frac{1}{\sqrt{2}}=p+\frac{q}{2}+\frac{r}{2}$

$2p+q+r=\pm\sqrt{2}$ .... (1)

$\vec{b}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c})=\frac{p}{2}+q+\frac{r}{2}$

$\Rightarrow\ p+2q+r=0$ .... (2)

$\vec{c}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c})=\frac{p}{2}+\frac{q}{2}+r$

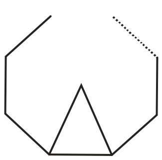
$p+q+2r=\pm\sqrt{2}$ .... (3)

Now, $p=r=-q$

$p=r=\pm\frac{1}{\sqrt{2}},\ \ q=\mp\frac{1}{\sqrt{2}}$

$\frac{p^2+2q^2+r^2}{q^2}=4$

(92) **8.**



$$(n-1)\cdot R^2\sin\left(\frac{2\pi}{n}\right)=(n-1)\,R^2\cos\frac{2\pi}{n}$$

$\Rightarrow\ \tan\left(\frac{2\pi}{n}\right)=1\Rightarrow n=8$

(93) **5.** Let reflection of P (1, 0, 0) in the line

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ be (α,β,γ) then $\left(\frac{\alpha+1}{2},\frac{\beta}{2},\frac{\gamma}{2}\right)$

lies on the line.

and $(\alpha-1)\,\hat{i}+\beta\hat{j}+\gamma\hat{k}$ is perpendicular to $2\hat{i}-3\hat{j}+8\hat{k}$

$\therefore\ \frac{\frac{\alpha+1}{2}-1}{2}=\frac{\frac{\beta}{2}+1}{-3}=\frac{\frac{\gamma}{2}+10}{8}=\lambda$ and

$2(\alpha-1)-3(\beta)+\gamma(8)=0\ \Rightarrow\ \alpha=5,\beta=-8,\gamma=-4$

(94) **2.** Centre and radius of the sphere are (–1, 1, –1) and 3 respectively.

Distance of (–1, 1, –1) from the plane is $\left|\frac{-2+3-6+k}{\sqrt{4+9+36}}\right|$

Since the plane is tangent to the sphere

$\therefore\ \left|\frac{k-5}{7}\right|=3$ is $|k-5|=21\ \therefore\ k=-16, 26$

(95) 1, (96) 15, (97) 1.

Equation of the line passing through (1, 4, 3)

$\frac{x-1}{a}=\frac{y-4}{b}=\frac{z-3}{c}$ ....(1)

since (1) is perpendicular to $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$

and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$

hence $2a+b+4c=0$ and $3a+2b-2c=0$

$\therefore\ \frac{a}{-2-8}=\frac{b}{12+4}=\frac{c}{4-3}\Rightarrow\frac{a}{-10}=\frac{b}{16}=\frac{c}{1}$

hence the equation of the lines is

$\frac{x-1}{-10}=\frac{y-4}{16}=\frac{z-3}{1}$ .... (2)

Now any point P on (2) can be taken as

$1-10\lambda$; $16\lambda+4$; $\lambda+3$

distance of P from Q (1, 4, 3)

$(10\lambda)^2+(16\lambda)^2+\lambda^2=357$

$(100+256+1)\lambda^2=357$

$\lambda=1$ or -1

Hence Q is (–9, 20, 4) or (11, –12, 2)

(98) **7.** As the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, the point (4, 2, k) through which line passes must also lie on the given plane and hence,

$2\times4-4\times2+k=7\Rightarrow k=7$

(99) **9.** Let the equation of variable plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

which meets the axes at A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

$\therefore$ Centroid of Δ ABC is (a/3, b/3, c/3)

and it satisfies the relatiton

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k\ \Rightarrow\ \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{k}{9}$ .... (1)

Also given that the distance of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ from (0, 0, 0) is 1 unit.

$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$ .... (2)

From (1) and (2), we get $\frac{k}{9}=1$ i.e. $k=9$

(100) **6.** $2\ell+3m+4n=0$; $3\ell+4m+5n=0$

$\frac{\ell}{-1}=\frac{m}{2}=\frac{n}{-1}$

Equation of plane will be

$a(x-1)+b(y-2)+c(z-3)=0$

$-1(x-1)+2(y-2)-1(z-3)=0$

$-x+1+2y-4-z+3=0$

$-x+2y-z=0$; $x-2y+z=0$

$\frac{|d|}{\sqrt{6}}=\sqrt{6} \Rightarrow d=6$

EXERCISE-3

(1) **(D).** $\because \vec{a}+2\vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a}+2\vec{b}=\lambda\vec{c}$ .... (1)

where λ is scalar and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

$\therefore \vec{b}+3\vec{c}=\mu\vec{a}$ .... (2), where μ is scalar

Now, (1) – (2) gives

$\vec{a}+2\vec{b}-2(\vec{b}+3\vec{c})=\lambda\vec{c}-2\mu\vec{a} \Rightarrow \vec{a}-6\vec{c}=\lambda\vec{c}-2\mu\vec{a}$

On comparing, $1=-2\mu \Rightarrow \mu=-1/2$ and $-6=\lambda \Rightarrow \lambda=-6$

Put value of λ in (1) we get

$\vec{a}+2\vec{b}=-6\vec{c} \Rightarrow \vec{a}+2\vec{b}+6\vec{c}=0$

(2) **(D).** $[\vec{a}\ \vec{b}\ \vec{c}]=4$ .... (1)

$\because [\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$

$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=(4)^2$ {from (1)}

$=16$

(3) **(B).** $\vec{c}=2\lambda(\vec{a}\times\vec{b})+3\mu(\vec{b}\times\vec{a})$

$\vec{c}=2\lambda(\vec{a}\times\vec{b})-3\mu(\vec{a}\times\vec{b})=(2\lambda-3\mu)(\vec{a}\times\vec{b})$

$\Rightarrow \vec{c}.(\vec{a}\times\vec{b})=(2\lambda-3\mu)(\vec{a}\times\vec{b}).(\vec{a}\times\vec{b})$

$\Rightarrow 0=(2\lambda-3\mu)|\vec{a}\times\vec{b}|^2$

$\{\because \vec{c}.(\vec{a}\times\vec{b})=0$ (given) and $\vec{a}\times\vec{a}=|a|^2\}$

$\Rightarrow 2\lambda-3\mu=0$ $\{\because (\vec{a}\times\vec{b})\neq 0$ (given)$\}$

$\Rightarrow 2\lambda=3\mu$

(4) **(B).** $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=5\hat{i}-3\hat{j}+\hat{k}$

Projection of vector $\vec{a}$ on $\vec{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$=\frac{(2\hat{i}+\hat{j}+2\hat{k}).(5\hat{i}-3\hat{j}+\hat{k})}{|5\hat{i}-3\hat{j}+\hat{k}|}=\frac{9}{\sqrt{35}}$

The orthogonal projection of $\vec{a}$ on $\vec{b}$

$=\frac{(\vec{a}.\vec{b})}{|\vec{b}|^2}\vec{b}=\frac{9(5\hat{i}-3\hat{j}+\hat{k})}{(\sqrt{35})^2}=\frac{9(5\hat{i}-3\hat{j}+\hat{k})}{35}$

(5) **(C).** $\vec{a}=2\hat{i}-6\hat{j}-3\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}-\hat{k}$

Unit vector perpendicular to the plane

$\vec{a}$ & $\vec{b}$ is $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$=\frac{15\hat{i}-10\hat{j}+30\hat{k}}{|15\hat{i}-10\hat{j}+30\hat{k}|}=\frac{5[3\hat{i}-2\hat{j}+6\hat{k}]}{5\times 7}=\frac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$

(6) **(D).** $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$

and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are perpendicular to each other.

$\therefore -3(3k)+2k(1)+2(-5)=0$

$\Rightarrow -9k+2k-10=0$

$\Rightarrow 7k=-10 \Rightarrow k=-10/7$

Reason : If line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are perpendicular than

$a_1a_2+b_1b_2+c_1c_2=0$.

(7) **(C).** d.R's of given lines are 1, 1, 2 and $\sqrt{3}-1, -\sqrt{3}-1, 4$

Now angle between is

$\cos\theta=\frac{1(\sqrt{3}-1)+1(-\sqrt{3}-1)+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}$

$=\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16}}$

$=\frac{6}{\sqrt{6}\sqrt{24}}=\frac{6}{\sqrt{6}\,2\sqrt{6}}$

$\cos\theta=\frac{1}{2} \Rightarrow \theta=60°$

Reason : If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines then angle between them is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

(8) **(C).** Equation of planes are

$2x - y + z = 6$ and $x + y + 2z = 3$

Now the angle them is

$$\cos\theta = \frac{2.1 - 1.1 + 1.2}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+1^2+2^2}} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$\Rightarrow \theta = 60°$

Reason : Angle between two plane as $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

(9) **(D).**

Lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar.

If $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

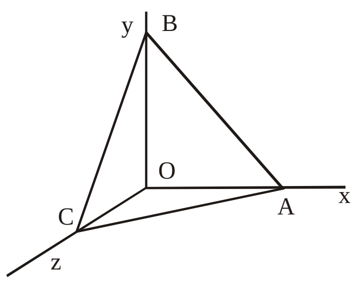
$$\Rightarrow \begin{vmatrix} (2-1) & (3-4) & (4-5) \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 + 2k) - 1(-k^2 - 1) - 1(2 - k) = 0$

$\Rightarrow 1 + 2k + k^2 + 1 - 2 + k = 0$

$\Rightarrow k^2 + 3k = 0 \Rightarrow k(k+3) = 0 \Rightarrow k = 0, -3$

(10) **(B).** O (0, 0, 0), A (1, 2, 1), B (2, 1, 3), C (–1, 1, 2)



Peperndicular to face OAB is

$\overrightarrow{OA} \times \overrightarrow{OB} = (\hat{i} + 2\hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 3\hat{k}) = 5\hat{i} - \hat{j} - 3\hat{k}$

Vector perpendicular to face ABC

$\overrightarrow{AB} \times \overrightarrow{AC} = (\hat{i} - \hat{j} + 2\hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 5\hat{j} - 3\hat{k}$

$\because$ Angle between face equal to angle between their normals

$$\therefore \cos\theta = \frac{5\times1 + (-1)(-5) + (-3)(-3)}{\sqrt{5^2+(-1)^2+(-3)^2}\sqrt{1^2+(-5)^2+(-3)^2}}$$

$$= \frac{5+5+9}{\sqrt{35}\sqrt{35}} = \frac{19}{35} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

(11) **(C).** Equation of plane which cuts intercept for a, b, and c length at respectively coordinate axis is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .... (1)

Let it's distance from origin is p

$$\therefore p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

Now origin equation of place which cuts intercept of length a', b', c' at respectively axis is

$\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1$ .... (2)

distance of thise place from origin is let p'

$$\Rightarrow p' = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

Plane (1) and plane (2) is same, $p = p'$

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$

(12) **(A).** $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$, $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let $\hat{n} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\hat{n}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$ .... (1)

$\vec{u}.\hat{n} = 0 \Rightarrow x + y = 0$ .... (2)

and $\vec{v}.\hat{n} = 0 \Rightarrow x - y = 0 \Rightarrow x = y$ .... (3)

From (3) we put value in (2)

we get $x + x = 0 \Rightarrow 2x = 0 \Rightarrow x = 0 = y$ [from (3)]

Now from (1), $x^2 + y^2 + z^2 = 1 \Rightarrow z^2 = 1 \Rightarrow z = \pm 1 \therefore \hat{n} = \pm\hat{k}$

Now, $\vec{w}.\hat{n} = (\hat{i} + 2\hat{j} + 3\hat{k}).(\pm\hat{k}) = \pm 3$

$\Rightarrow |\vec{w}.\hat{n}| = 3$

(13) **(D).** $\vec{F}_1 = 4\hat{i} + \hat{j} - 3\hat{k}$, $\vec{F}_2 = 3\hat{i} + \hat{j} - \hat{k}$

Resultant force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7\hat{i} + 2\hat{j} - 4\hat{k}$

Particle displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to $5\hat{i} + 4\hat{j} + \hat{k}$

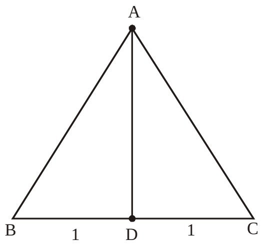
$\therefore$ Displacement $\vec{d} = (5\hat{i}+4\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$

$= 4\hat{i}+2\hat{j}-2\hat{k}$

Now work done $W = \vec{F}.\vec{d}$

$= (7\hat{i}+2\hat{j}-4\hat{k}).(4\hat{i}+2\hat{j}-2\hat{k}) = 28+4+8 = 40$

(14) **(D).**



$\because$ AD is median $\quad \therefore$ D is mid point of BC

$\therefore \dfrac{BD}{DC} = 1$; $\overrightarrow{AB} = 3\hat{i}+4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$

Let w.r.t. A position vector of B is $3\hat{i}+4\hat{k}$ and position vector of C is $5\hat{i}-2\hat{j}+4\hat{k}$
and D is the mid point of BC

$\therefore$ Position vector of D is $= \dfrac{3\hat{i}+4\hat{k}+5\hat{i}-2\hat{j}+4\hat{k}}{2}$

$= \dfrac{8\hat{i}-2\hat{j}+8\hat{k}}{2} = 4\hat{i}-\hat{j}+4\hat{k} \Rightarrow \overrightarrow{AD} = 4\hat{i}-\hat{j}+4\hat{k}$

$|\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$

(15) **(C).** $\vec{a}+\vec{b}+\vec{c} = 0$

$|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$

$\vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 0 \Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow 1^2+2^2+3^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = \dfrac{-14}{2} = -7$

(16) **(D).** Position vector of A is $7\hat{i}-4\hat{j}+7\hat{k}$

Position vector of B is $\hat{i}-6\hat{j}+10\hat{k}$

Position vector of C is $-\hat{i}-3\hat{j}+4\hat{k}$

Position vector of D is $5\hat{i}-\hat{j}+5\hat{k}$

$\Rightarrow \overrightarrow{AB} = (\hat{i}-6\hat{j}+10\hat{k})-(7\hat{i}-4\hat{j}+7\hat{k}) = -6\hat{i}-2\hat{j}+3\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = 7$

$\overrightarrow{BC} = (-\hat{i}-3\hat{j}+4\hat{k})-(\hat{i}-6\hat{j}+10\hat{k}) = -2\hat{i}+3\hat{j}-6\hat{k}$

$\Rightarrow |\overrightarrow{BC}| = 7$

$\overrightarrow{CD} = (5\hat{i}-\hat{j}+5\hat{k})-(-\hat{i}-3\hat{j}+4\hat{k}) = 6\hat{i}+2\hat{j}+\hat{k}$

$\Rightarrow |\overrightarrow{CD}| = \sqrt{41}$

$\overrightarrow{DA} = (7\hat{i}-4\hat{j}+7\hat{k})-(5\hat{i}-\hat{j}+5\hat{k}) = 2\hat{i}-3\hat{j}+2\hat{k}$

$\Rightarrow |\overrightarrow{DA}| = \sqrt{17}$

$\because AB = BC \neq CD \neq DA$

(17) **(C).** $\vec{u}, \vec{v}$ & $\vec{w}$ are non-coplanar vector

$\therefore [\vec{u}, \vec{v}\ \vec{w}] \neq 0$ .... (1)

Now, $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$= (\vec{u}+\vec{v}-\vec{w}).\{\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w}\}$

$= (\vec{u}+\vec{v}-\vec{w}).\{\vec{u}\times\vec{v}+\vec{w}\times\vec{u}+\vec{v}\times\vec{w}\}$ $\{\because \vec{a}\times\vec{a} = 0\}$

$= \vec{u}.(\vec{u}\times\vec{v})+\vec{u}.(\vec{w}\times\vec{u})+\vec{u}.(\vec{v}\times\vec{w})$

$+ \vec{v}.(\vec{u}\times\vec{v})+\vec{v}.(\vec{w}\times\vec{u})+\vec{v}.(\vec{v}\times\vec{w})$

$- \vec{w}.(\vec{u}\times\vec{v})-\vec{w}.(\vec{w}\times\vec{u})-\vec{w}.(\vec{v}\times\vec{w})$

$= [\vec{u}\ \vec{v}\ \vec{w}]+[\vec{u}\ \vec{v}\ \vec{w}]-[\vec{u}\ \vec{v}\ \vec{w}] = [\vec{u}\ \vec{v}\ \vec{w}] = \vec{u}\ (\vec{v}\times\vec{w})$

$\{\because [\vec{a}\ \ \vec{a}\ \ \vec{b}] = 0$ $\because$ if two vectors are same then scalar triple product is zero.}

(18) **(C).** $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector.

Now, $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\ \vec{c}$ are coplanar then

$\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix} = 0 \Rightarrow \lambda(2\lambda-1) = 0 \Rightarrow \lambda = 0, 1/2$

Hence given vectors are non-coplanar except for two values of λ i.e. $\lambda = 0, 1/2$

(19) **(C).** $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$

By given condition $\dfrac{\vec{v}.\vec{u}}{|\vec{u}|} = \dfrac{\vec{w}.\vec{u}}{|\vec{u}|}$

$\Rightarrow \vec{v}.\vec{u} = \vec{w}.\vec{u}$ .... (1)

and also, $\vec{v}.\vec{w} = 0\ (\because \vec{v}\perp\vec{w})$ .... (2)

Now, $|\vec{u}-\vec{v}+\vec{w}|^2 = |\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2$

$-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w} = |\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2$

$\{\because \vec{v}.\vec{u} = \vec{w}.\vec{u}$ and $\vec{v}.\vec{w} = 0$ from (1) & (2)}

$= 1^2+2^2+3^2 = 14$

$|\vec{u}-\vec{v}+\vec{w}| = \sqrt{14}$

(20) **(D).** $(\vec{a}\times\vec{b})\times\vec{c} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}|$

$\Rightarrow -[\vec{c}\times(\vec{a}\times\vec{b})] = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}|$

$\Rightarrow -[(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}]=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}|$

$\Rightarrow (\vec{c}.\vec{a})\,\vec{b}-(\vec{c}.\vec{b})\,\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}|$

$\therefore$ On comparing $\vec{a}.\vec{c}=0 \Rightarrow \vec{a}\perp\vec{c}$ and $-(\vec{c}.\vec{b})=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta = -1/3$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

(21) **(C).** $\because$ If a line makes α, β, γ angle with x, y and z axis respectively then $\cos\alpha, \cos\beta, \cos\gamma$ are called direction cosines of the line and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
But according to question $a = \gamma = \theta$
$\therefore \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$
$2\cos^2\theta + \cos^2\beta = 1$
$2\cos^2\theta = 1 - \cos^2\beta$
$2\cos^2\theta = \sin^2\beta$ .... (1)
and $\sin^2\beta = 3\sin^2\theta$ .... (2) (given)
From (1) & (2)
$2\cos^2\theta = 3\sin^2\theta$
$\Rightarrow 2\cos^2\theta = 3(1-\cos^2\theta) \Rightarrow 5\cos^2\theta = 3 \Rightarrow \cos^2\theta = 3/5$

(22) **(C).** Parallel lines are
$2x+y+2z-8=0$ .... (1)
$4x+2y+4z+5=0$
$\Rightarrow 2x+y+2z+5/2=0$ .... (2)
$\Rightarrow$ Distance between parallel lines is

$$d = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right| = \left|\frac{(5/2)-(-8)}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{21}{2\times 3}\right| = \frac{7}{2}$$

$\because$ If equation of parallel lines are $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$

then distance between them is $\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$

(23) **(B).** Let the equation of line AB be

$$\frac{x-0}{1}=\frac{y+a}{1}=\frac{z-0}{1}=k \text{ (say)}$$

Coordinate of a point C on AB is $(k, k-a, k)$ and other line is PQ and equation of PQ is

$$\frac{x+a}{2}=\frac{y-0}{1}=\frac{z-0}{1}=(\lambda) \text{ (say)}$$

$\therefore$ Coordinate of a point D on PQ is $(2\lambda-a, \lambda, \lambda)$
Now direction ratio of CD is
$(2\lambda-a-k, \lambda-k+a, \lambda-k)$
CD cuts AB & PQ at C and D respectively but D.R. of CD is 2, 1, 2 given $\therefore \frac{2\lambda-a-k}{2}=\frac{\lambda-k+a}{1}=\frac{\lambda-k}{2}$

On solving first and second fraction

$$\frac{2\lambda-a-k}{2}=\frac{\lambda-k+a}{1}$$

$2\lambda-a-k=2\lambda-2k+2a \Rightarrow k=3a$
and on solving second and third

$\frac{\lambda-k+a}{1}=\frac{\lambda-k}{2} \Rightarrow 2\lambda-2k+2a=\lambda-k$

$\Rightarrow k=\lambda+2a \Rightarrow \lambda=k-2a \Rightarrow \lambda=a$
$\therefore$ Coordinate of C = (3a, 2a, 3a)
and coordinate of D = (a, a, a)

(24) **(A).** Straight line $x=1+s, y=-3-\lambda s, z=1+\lambda s$ can be written as

$$\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s \qquad \text{.......... (1)}$$

and another line $x=t/2, y=1+t, z=2-t$

can be written as $\frac{x-0}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$ .... (2)

Now, eq. (1) and (2) are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1-0 & -3-1 & 1-2 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix}=0$$

$\Rightarrow 1(\lambda-\lambda)-4(\lambda/2+1)-1(1+\lambda/2)=0$
$\Rightarrow 0-2\lambda-4-1-\lambda/2=0 \Rightarrow -5\lambda/2=5 \Rightarrow \lambda=-2$

(25) **(A).** Equation of line is $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ .... (1)

and equation of plane is $2x-y+\sqrt{\lambda}z+4=0$ .... (2)

$\because$ Angle between line and plane is $\Rightarrow$ and $\sin\theta=\frac{1}{2}$ .... (3)

If equation of line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

and equation of plane $ax+by+cz+d=0$ and θ is angle between them, then

$$\sin\theta=\frac{aa_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\sqrt{a_1^2+b_1^2+c_1^2}}$$

Now from eq. (1) and eq. (2)

$$\sin\theta=\frac{1.2+(-1)(2)+(\sqrt{\lambda})^2}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+(-1)^2+(\sqrt{\lambda})^2}}$$

$\Rightarrow \frac{2-2+\sqrt{\lambda}\times(2)}{3\sqrt{5+\lambda}}=\frac{1}{3}$ {from (3) $\sin\theta=1/3$}

$\Rightarrow \frac{2\sqrt{\lambda}}{\sqrt{5+\lambda}}=1 \Rightarrow 4\lambda=5+\lambda \Rightarrow 3\lambda=5 \Rightarrow \lambda=5/3$

(26) **(B).** The given lines can be written as

$$\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1} \quad \text{......... (1)}$$

and $$\frac{x}{1/6} = \frac{y}{-1} = \frac{z}{-1/4} \quad \text{......... (2)}$$

$\therefore$ Angle beween them is

$$\cos\theta = \frac{(1/2)\times(1/6)+1/3(-1)+(-1)(-1/4)}{\sqrt{(1/2)^2+(1/3)^2+(-1)^2+(-1)^2+(-1/4)^2}}$$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

(27) **(B).** Equation of line is $\vec{r} = 2\hat{i}-2\hat{j}+3\hat{k}+\lambda\,(\hat{i}-\hat{j}+4\hat{k})$

and equation of plane is $\vec{r}.(\hat{i}+5\hat{j}+\hat{k}) = 5$ is –

$\because$ Vector parallel to line is $\hat{i}-\hat{j}+4\hat{k}$ and vector normal to plane is $\hat{i}+5\hat{j}+\hat{k}$

and $(\hat{i}-\hat{j}+4\hat{k}).(\hat{i}+5\hat{j}+\hat{k}) = 1-5+4 = 0$

$\therefore$ Line is parallel to the plane and point whose position vector is $2\hat{i}-2\hat{j}+3\hat{k}$ lies on line

$\therefore$ perpendicular distance of plane from this point is

$$= \left|\frac{(2\hat{i}-2\hat{j}+3\hat{k}).(\hat{i}+5\hat{j}+\hat{k})-5}{|\hat{i}+5\hat{j}+\hat{k}|}\right|$$

$$= \left|\frac{2-10+3-5}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}$$

(28) **(B).** Let $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{a_1^2+a_2^2+a_3^2}$

$\Rightarrow |\vec{a}|^2 = a_1^2+a_2^2+a_3^2$

Now, $\vec{a}\times\hat{i} = -a_2\hat{k}+a_3\hat{j}$

$\Rightarrow |\vec{a}\times\hat{i}| = \sqrt{a_2^2+a_3^2} = |\vec{a}\times\hat{i}|^2 = a_2^2+a_3^2$

$\vec{a}\times\hat{j} = a_1\hat{k}-a_3\hat{j}$

$|\vec{a}\times\hat{j}| = \sqrt{a_1^2+a_3^2} \Rightarrow |\vec{a}\times\hat{j}|^2 = a_1^2+a_3^2$

and $\vec{a}\times\hat{k} = -a_1\hat{j}+a_2\hat{i}$

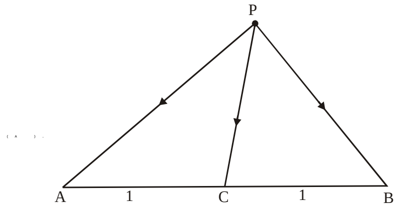
$\Rightarrow |\vec{a}\times\hat{k}| = \sqrt{a_1^2+a_2^2} = |\vec{a}\times\hat{k}|^2 = a_1^2+a_2^2$

$\Rightarrow |\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$

$= a_2^2+a_3^2+a_1^2+a_3^2+a_1^2+a_2^2$

$= 2(a_1^2+a_2^2+a_3^2) = 2|\vec{a}|^2$

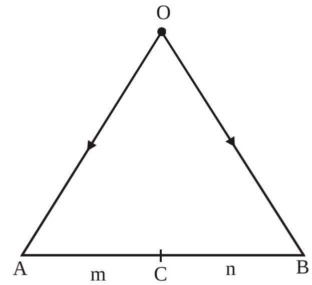
(29)



$\therefore$ Let P is the reference point

$(\overrightarrow{PA})+(\overrightarrow{PB}) = (1+1)\,\overrightarrow{PC} \Rightarrow \overrightarrow{PA}+\overrightarrow{PB} = 2\,\overrightarrow{PC}$

$\because$ from $m-n$ theorem



$n\,(\overrightarrow{OA})+m\,(\overrightarrow{OB}) = (m+n)\,\overrightarrow{OC}$

Here, $m:n = 1:1$

(30) **(B).** $[\lambda\,(\vec{a}+\vec{b})\;\lambda^2\,\vec{b}\;\lambda\,\vec{c}] = [\vec{a}\;\;\vec{b}+\vec{c}\;\;\vec{b}]$

$\lambda^4[\vec{a}+\vec{b}\;\vec{b}\;\vec{c}] = [\vec{a}\;\vec{b}\;\vec{b}]+[\vec{a}\;\;\vec{c}\;\;\vec{b}]$

$\lambda^4\,\{[\vec{a}\;\vec{b}\;\vec{c}]+[\vec{b}\;\;\vec{b}\;\;\vec{c}]\} = [\vec{a}\;\;\vec{c}\;\;\vec{b}]$

$\lambda^4\{[\vec{a}\;\vec{b}\;\vec{c}]+0\} = -[\vec{a}\;\;\vec{b}\;\;\vec{c}]$

$\lambda^4\,[\vec{a}\;\vec{b}\;\vec{c}]+[\vec{a}\;\;\vec{b}\;\;\vec{c}] = 0$

$[\vec{a}\;\vec{b}\;\vec{c}]\,(\lambda^4+1) = 0$

$[\vec{a}\;\vec{b}\;\vec{c}] \neq 0$ then $\lambda^4+1 = 0$ $\{\because \vec{a}\;\vec{b}\;\vec{c}$ are non-coplanar$\}$

$\Rightarrow$ No real value of λ exist

(31) **(C).** $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow\; -(\vec{c}\times(\vec{a}\times\vec{b})) = \vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow\; -[(\vec{c}.\vec{b})\,\vec{a}-(\vec{c}.\vec{a})\vec{b}] = (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

$\Rightarrow\; (\vec{a}.\vec{c})\vec{b}-(\vec{c}.\vec{b})\,\vec{a} = (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

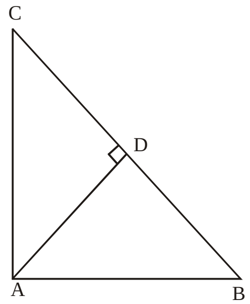
$\Rightarrow\; -(\vec{c}.\vec{b})\,\vec{a} = -\,(\vec{a}.\vec{b})\vec{c}$

$\Rightarrow\; (\vec{b}.\vec{c})\,\vec{a} = (\vec{a}.\vec{b})\vec{c} \;\Rightarrow\; \vec{a} = \left(\frac{\vec{a}.\vec{b}}{\vec{b}.\vec{c}}\right)\vec{c}$

$\because\; \vec{b}.\vec{c}$ & $\vec{a}.\vec{b}$ are scalar and $\vec{b}.\vec{c} \neq 0$ & $\vec{a}.\vec{b} \neq 0$

$\Rightarrow \vec{a} \parallel \vec{c}$ $\{\because$ if two vector $\vec{a}$ & $\vec{b}$ are parallel then $\vec{a} = \lambda\,\vec{b}$ where λ is scalar$\}$

(32) **(C).**



From ΔABC,
$BC^2 = AB^2 + AC^2$ .... (1)

and area of $\Delta\ \frac{1}{2} AB \times AC = \frac{1}{2} BC \times AD$

$\Rightarrow AB \times AC = BC \times AD$ .... (2)

Magnitude of two forces are $\frac{1}{AB}$ & $\frac{1}{AC}$

$\therefore$ their resultant is $= \sqrt{\frac{1}{(AB)^2} + \frac{1}{(AC)^2}}$

$= \sqrt{\frac{(AB)^2 + (AC)^2}{(AB)^2 (AC)^2}} = \sqrt{\frac{BC^2}{(AB)^2 (AC)^2}}$ {from (1)}

$= \frac{BC}{AB \times AC} = \frac{BC}{BC \times AD} = \frac{1}{AD}$ {from (2)}

(33) **(D).** Position vector A, B and C respectively are

$2\hat{i} - \hat{j} + \hat{k},\ \hat{i} - 3\hat{j} - 5\hat{k},\ a\hat{i} - 3\hat{j} + \hat{k}$

Now, $\overrightarrow{AC} = (a-2)\,\hat{i} - 2\hat{j},\ \overrightarrow{BC} = (a-1)\hat{i} + 6\hat{k}$

$\because \angle C = \pi/2$

$\therefore \overrightarrow{AC} \perp \overrightarrow{BC} \Rightarrow \overrightarrow{AC}.\overrightarrow{BC} = 0$

$\Rightarrow ((a-2)\,\hat{i} - 2\hat{j}).((a-1)\,\hat{i} + 6\hat{k}) = 0$

$\Rightarrow (a-2)\,(a-1) = 0 \Rightarrow a = 1, 2$

(34) **(D).** The equation of given lines are $x = ay + b,\ z = cy + d$

and $x = a'y + b',\ z = c'y + d'$

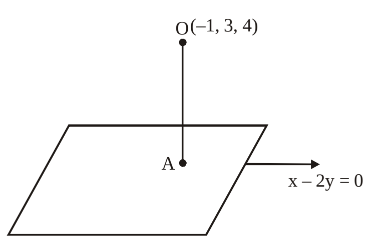
These equations can be rewritten as

$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$ and $\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

These lines will be perpendicular if $aa' + cc' + 1 = 0$

(35) **(D).** $\because$ Equation of plane is $x - 2y = 0$

where z coordinate is absent.



$\therefore$ z coordinate of each point on the plane will be zero and z coordinate of image of (–1, 3, 4) on plane $x - 2y = 0$ will zero. $\therefore$ Option eq. (A), (B) and (C) do not match.

(36) **(A).** Let direction ratios of line of intersection of plane $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ are a, b and c.

$\because$ Normal to the planes are also perpendicular to line of intersection $\therefore$ $2a + 3b + c = 0$

$a + 3b + 2c = 0$

$\frac{a}{6-3} = \frac{b}{1-4} = \frac{c}{6-3} \Rightarrow \frac{a}{3} = \frac{b}{-3} = \frac{c}{3}$

$\Rightarrow \frac{a}{1} = \frac{b}{-1} = \frac{c}{1} = \lambda$ (say)

If ℓ, m and n are d.c. of that line then

$\cos\alpha = \ell = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$

$= \frac{\lambda}{\sqrt{\lambda^2 + (-\lambda)^2 + \lambda^2}} = \frac{\lambda}{\lambda\sqrt{3}} = \frac{1}{\sqrt{3}}$

(37) **(D).** A line makes an angle of $\pi/4$ with the positive direction of each of x axis and y axis.

Let it makes γ angle with positive direction of z-axis.

Now, if ℓ, m and n are d.c.'s of line

$\therefore\ \ell = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$; $m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$; $n = \cos\gamma$

$\ell^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{2} + n^2 = 1 \Rightarrow n^2 = 0$

$\Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = \pi/2$

(38) **(D).** $\hat{u}$ & $\hat{v}$ are unit vector

θ is acute angle between them.

According to question

$|2\hat{u} \times 3\hat{v}| = 1 \Rightarrow |2\hat{u}||3\hat{v}|\sin\theta = 1$

$\Rightarrow 2 \times 3 \sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{6}$ $\because$ θ is acute

$\therefore$ only one value of θ is possible.

(39) **(D).** $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{c} = x\hat{i} + (x-2)\,\hat{j} - \hat{k}$

$\because$ vector $\vec{c}$ lie in plane of $\vec{a}$ & $\vec{b}$

$\therefore$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$

$\Rightarrow 1\,(1 - 2\,(x-2)) + 1\,(2x+1) + 1\,(x - 2 + x) = 0$

$\Rightarrow 1 - 2x + 4 + 2x + 1 + 2x - 2 = 0$

$\Rightarrow 2x + 4 = 0$

$\Rightarrow 2x = -4 \Rightarrow x = -2$

(40) **(C).** $\because \vec{a} = 8\vec{b}$

$\therefore$ $\vec{a}$ & $\vec{b}$ are like vector

angle between then is 0° .... (1)

and $\vec{c} = -7\vec{b}$

$\Rightarrow$ $\vec{b}$ & $\vec{c}$ are unlike vector

$\therefore$ angle between $\vec{b}$ & $\vec{c}$ are π. .... (2)

From (1) and (2) angle between $\vec{a}$ & $\vec{c}$ is π.

(41) **(C).** $\vec{a} = \alpha\,\hat{i} + 2\hat{j} + \beta\,\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{j} + \hat{k}$

$\because$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\therefore \begin{vmatrix} \alpha & 2 & \beta \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha\ (1-0) + 2\,(0-1) + \beta\,(1-0) = 0$

$\Rightarrow \alpha\ -2 + \beta = 0$

$\Rightarrow \alpha\ + \beta = 2$ .... (1)

$\because$ Only third option satisfy the (i) $\therefore$ Ans is (C)

but $\vec{a}$ bisects the angle between $\vec{b}$ & $\vec{c}$

$\therefore$ angle between $\vec{a}, \vec{b}$ & $\vec{a}, \vec{c}$ are equal.

{if angle between $\vec{a}$ & $\vec{b}$ is θ then $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$ }

$$\therefore \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|} \Rightarrow \frac{\alpha+2}{|\vec{a}|\sqrt{2}} = \frac{2+\beta}{|\vec{a}|\sqrt{2}}$$

$\{\because |\vec{b}| = |\vec{c}| = \sqrt{2}\}$

$\Rightarrow \alpha + 2 = \beta + 2 \Rightarrow \alpha = \beta$ .... (2)

Put this in (1) we get $\alpha = \beta = 1$

(42) **(D).** Equation of lines are

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}\ ;\ \frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$$

$\because$ Lines intersect at a point

$\Rightarrow$ they lies in a plane

$$\begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $(4-3k) + 1\,(9-2k) - 2\,(k^2-6) = 0$

$\Rightarrow$ $-2k^2 - 5k + 25 = 0 \Rightarrow$ $2k^2 + 5k - 25 = 0$

$\Rightarrow$ $(k+5)\,(2k-5) = 0 \Rightarrow$ $k = -5, 5/2$

$\therefore$ $k = -5$ ($\because$ k is integer)

(43) **(B).** Equation of line passing through (5, 1, a) and (3, b, 1)

is $\dfrac{x-5}{3-5} = \dfrac{y-1}{b-1} = \dfrac{z-a}{1-a} = \lambda$ (say)

$$\frac{x-5}{-2} = \frac{y-1}{b-1} = \frac{z-a}{1-a} = \lambda$$

Let coordinate of any point on this line is

$(-2\lambda + 5, \lambda\,(b-1) + 1, \lambda\,(1-a) + a)$

Let this point lie on yz and this represents (0, 17/2, –13/2)

$\therefore$ $-2\lambda + 5 = 0 \Rightarrow \lambda = 5/2$ .... (1)

and $\lambda\,(b-1) + 1 = 17/2$

$\Rightarrow \dfrac{5}{2}(b-1) + 1 = 17/2$ (from (1))

$= \dfrac{5b-5+2}{2} = \dfrac{17}{2}$

$\Rightarrow 5b = 20 \Rightarrow b = 4$ and $\lambda\,(1-a) + a = \dfrac{-13}{2}$

$\Rightarrow$ $\dfrac{5}{2}(1-a) + a = \dfrac{-13}{2}$ $\Rightarrow$ $\dfrac{5-5a+2a}{2} = \dfrac{-13}{2}$

$\Rightarrow$ $3a = -18 \Rightarrow a = 6$

(44) **(D).** $(3p^2 - pq + 2q^2)\,[\vec{u}\ \vec{v}\ \vec{w}] = 0$

$\Rightarrow 3p^2 - pq + 2q^2 = 0$ for real p

$D \geq 0$

$\Rightarrow q^2 - 4 \times 2 \times 3q^2 \geq 0 \Rightarrow -23q^2 \geq 0 \Rightarrow q = 0$

$\Rightarrow p = 0$

So exactly one value of (p, q)

(45) **(A).** (2, 1, –2) lies on $x + 3y - \alpha z + \beta = 0 \Rightarrow 2\alpha + \beta = -5$

Also, $3 - 15 - 2\alpha = 0 \Rightarrow \alpha = -6, \beta = 7$

$(\alpha, \beta) \equiv (-6, 7)$

(46) **(B).** Direction ratios are: 6, –3, 2

Direction cosines are: $\dfrac{6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$

(47) **(D).** $\vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow$ $\vec{b}.\vec{c} = 0 \Rightarrow$ $(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}).(\hat{i} - \hat{j} - \hat{k}) = 0$

$b_1 - b_2 - b_3 = 0$ and $\vec{a}.\vec{b} = 3 \Rightarrow b_2 - b_3 = 3$

$b_1 = b_2 + b_3 = 3 + 2b_3$

$\vec{b} = (3 + 2b_3)\,\hat{i} + (3 + b_3)\,\hat{j} + b_3\hat{k}$

(48) **(D).** $\vec{a}.\vec{b} = 0, \vec{b}.\vec{c} = 0, \vec{c}.\vec{a} = 0 \Rightarrow 2\lambda + 4 + \mu = 0$

$\lambda - 1 + 2\mu = 0$

Solving we get, $\lambda = -3, \mu = 2$

(49) **(A).** A(3, 1, 6); B = (1, 3, 4)

Mid-point of AB = (2, 2, 5) lies on the plane

and d.r's of AB = (2, –2, 2)

d.r's of normal to plane = (1, –1, 1).

AB is perpendicular bisector

$\therefore$ A is image of B

Statement-2 is correct but it is not correct explanation.

(50) **(B).** $\ell = \cos 45° = \dfrac{1}{\sqrt{2}}$; $m = \cos 120° = -\dfrac{1}{2}$

$n = \cos\theta$

where θ is the angle which line makes with positive z-axis.

Now, $\ell^2 + m^2 + n^2 = 1$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2} \text{ (}\theta \text{ Being acute)} \Rightarrow \theta = \frac{\pi}{3}$$

(51) (A). $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ .... (1)

$x + 2y + 3z = 4$ .... (2)

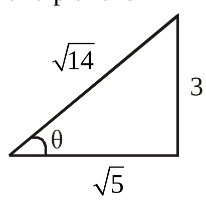
Angle between the line and plane is

$$\cos(90-\theta) = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\sin\theta = \frac{1+4+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}} = \frac{5+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}} \text{ .... (3)}$$

But given that angle between line and plane is

$$\theta = \cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$$



$$\Rightarrow \sin\theta = \frac{3}{\sqrt{14}}$$

From (3), $\frac{3}{\sqrt{14}} = \frac{5+3\lambda}{\sqrt{14}\times\sqrt{5+\lambda^2}}$

$\Rightarrow 9(5+\lambda^2) = 25 + 9\lambda^2 + 30\lambda \Rightarrow 30\lambda = 20 \Rightarrow \lambda = 2/3$

(52) (A). $(2\vec{a} - \vec{b}).[(\vec{a}\times\vec{b})\times[(\vec{a}+2\vec{b})]$

$= -(2\vec{a}-\vec{b}).[(\vec{a}+2\vec{b})\times(\vec{a}\times\vec{b})]$

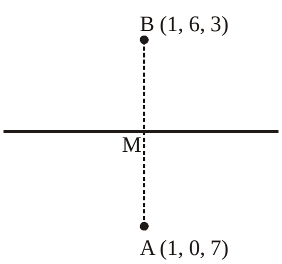
$= -(2\vec{a}-\vec{b}).[((\vec{a}+2\vec{b}).\vec{b})\,\vec{a} - ((\vec{a}+2\vec{b}).\vec{a})\,\vec{b}]$

$= -(2\vec{a}-\vec{b}).[(\vec{a}.\vec{b}) + 2\vec{b}.\vec{b})\,\vec{a} - (\vec{a}.\vec{a} + 2\vec{b}).\vec{a})\,\vec{b})]$

$= -(2\vec{a}-\vec{b}).[0 + 2\vec{a} - (0+\vec{b})] = -(2\vec{a}-\vec{b}).(2\vec{a}-\vec{b})$

$= -(2\vec{a}-\vec{b})^2 = -4\vec{a}^2 + 4\vec{a}.\vec{b} - \vec{b}^2 = -4 + 0 - 1 = -5$

(53) (B).



Mid-point of AB ≡ M (1, 3, 5)

M lies on line

Direction ratios of AB is < 0, 6, – 4 >

Direction ratios of given line is < 1, 2, 3 >

As AB is perpendicular to line

$\therefore\ 0.1 + 6.2 - 4.3 = 0$

(54) (D). $\vec{a}.\vec{b} \neq 0,\ \vec{b}\times\vec{c} = \vec{b}\times\vec{d},\ \vec{a}.\vec{d} = 0$

$(\vec{b}\times\vec{c})\times\vec{a} = (\vec{b}\times\vec{d})\times\vec{a}$

$(\vec{b}.\vec{a})\,\vec{c} - (\vec{c}.\vec{a})\,\vec{b} = (\vec{b}.\vec{a})\,\vec{d} - (\vec{d}.\vec{a})\,\vec{b}$

$$\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$$

(55) (C). $\vec{c} = \hat{a} + 2\hat{b}$; $\vec{d} = 5\hat{a} - 4\hat{b}$; $\vec{c}.\vec{d} = 0$

$\Rightarrow (\hat{a}+2\hat{b}).(5\hat{a}-4\hat{b}) = 0 \Rightarrow 5 + 6\hat{a}.\hat{b} - 8 = 0$

$$\Rightarrow \hat{a}.\hat{b} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

(56) (A). Equation of parallel plane $x - 2y + 2z + d = 0$

Now, $\left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right| = 1$; $d = \pm 3$

So, equation required plane $x - 2y + 2z \pm 3 = 0$

(57) (C). $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

$$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

$\vec{a}\,(1,-1,1),\ \vec{b}\,(2,3,4)$; $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{c}\,(3,k,0),\ \vec{d}\,(1,2,1)$; $\vec{r} = \vec{c} + \mu\,\vec{d}$

These lines will intersect if lines are coplanar.

$\vec{a}-\vec{c},\ \vec{b}\ \&\ \vec{d}$ are coplanar

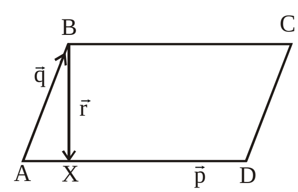
$\therefore\ [\vec{a}-\vec{c}, \vec{b}, \vec{d}\,] = 0$

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow 2(-5) - (k+1)(-2) - 1(1) = 0$$

$\Rightarrow 2(k+1) = 11 \Rightarrow k = 9/2$

(58) (B). $\overrightarrow{AX} = \frac{\vec{p}.\vec{q}}{|\vec{p}|}\frac{\vec{p}}{|\vec{p}|} = \frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

$$\overrightarrow{BX} = \overrightarrow{BA} + \overrightarrow{AX} = -\vec{q} + \frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$$



(59) (C). $2x + y + 2z - 8 = 0$ ...(P_1)

$2x + y + 2z + \frac{5}{2} = 0$ ...(P2)

Distance between P_1 and P_2 = $\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right| = \frac{7}{2}$

(60) **(C).** $[a-c, b, d] = 0$

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \; ; \; \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

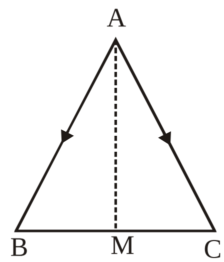
$\Rightarrow \; 1(1+2k) + (1+k^2) - (2-k) = 0$

$\Rightarrow \; k^2 + 2k + k = 0 \Rightarrow k^2 + 3k = 0 \Rightarrow \quad k = 0, -3$

Note : If 0 appears in the denominator, then the correct way of representing the equation of straight line is

$$\frac{x-2}{1} = \frac{y-3}{1}; \; z = 4$$

(61) **(C).** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$



$\Rightarrow \; \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$\Rightarrow \; \overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2}$

$\Rightarrow \; \overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\Rightarrow \; \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$

$\Rightarrow \; \overrightarrow{AM} + \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k} = \sqrt{33}$

(62) **(D).** $[\vec{a}\times\vec{b} \;\; \vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a}] = [\vec{a} \;\; \vec{b} \;\; \vec{c}]^2$

$\lambda = 1$

(63) **(A).** Line is parallel to plane.
Image of (1, 3, 4) is (–3, 5, 2)

(64) **(A).** $\ell = -m - n$

$m^2 + n^2 = (m+n)^2$

$\Rightarrow \; mn = 0$

So, possibilities are

$$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \text{ or } \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \; \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$.

(65) **(D).** $(\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{b}\cdot\vec{c})\,\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\therefore \; -(\vec{b}\cdot\vec{c}) = \frac{1}{3}|\vec{b}||\vec{c}| \; \therefore \; \cos\theta = -\frac{1}{3} \quad \therefore \; \sin\theta = \frac{2\sqrt{2}}{3}$

(66) **(C).** $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda$; $P(3\lambda+2, 4\lambda-1, 12\lambda+2)$

Lies on plane $x - y + z = 16$

Then, $3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 16$

$11\lambda + 5 = 16$; $\lambda = 1$; P (5, 3, 14)

Distace = $\sqrt{16+9+144} = \sqrt{169} = 13$

(67) **(B).** Reqd. plane is $(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

It is parallel to $x + 3y + 6z = 1$

$\therefore \; \frac{2+\lambda}{1} = \frac{-5+\lambda}{3} = \frac{1+4\lambda}{6}$. Solving $\lambda = -11/2$

$\therefore$ Plane is $(2x - 5y + z - 3) - \frac{11}{2}(x + y + 4z - 5) = 0$

$\therefore \; x + 3y + 6z - 7 = 0$

(68) **(C).** $3\ell - 2m = 5$ .... (1)

$2\ell - m - 3 = 0$

$2\ell - m = 3$ .... (2)

$4\ell - 2m = 6$ .... (3)

Eq. (3) – eq. (1), $\ell = 1, m = -1, \; \ell^2 + m^2 = 2$

(69) **(C).** $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$(\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

$\vec{a}\cdot\vec{c} = \frac{\sqrt{3}}{2}$ and $\vec{a}\cdot\vec{b} = -\frac{\sqrt{3}}{2}$

Angle between $\vec{a}$ and $\vec{c} = 30°$; $\vec{a}$ & $\vec{b} = 150° = \frac{5\pi}{6}$

(70) **(A).** Equation of line: $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

Any point is $(\lambda+1, \lambda-5, \lambda+9)$

It lies on plane

$\Rightarrow \; (\lambda+1) - (\lambda-5) + (\lambda+9) = 5$

$\Rightarrow \; \lambda + 1 - \lambda + 5 + \lambda + 9 = 5 \Rightarrow \lambda + 10 = 0 \Rightarrow \lambda = -10$

Point is (–9, –15, –1) another is (1, –5, 9)

Distance = $\sqrt{100+100+100} = 10\sqrt{3}$

(71) **(D).** $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$; $\vec{b} = \hat{i} + \hat{j}$.

$|\vec{c} - \vec{a}| = 3, |(\vec{a}\times\vec{b})\times\vec{c}| = 3$

$|\vec{a}\times\vec{b}||\vec{c}|\sin\theta = 3$

$|\vec{a}\times\vec{b}||\vec{c}| = 6$ .... (1)

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} \equiv \hat{i}(2) - \hat{j}(2) + \hat{k}(1) = 2\hat{i} - 2\hat{j} + \hat{k}$$

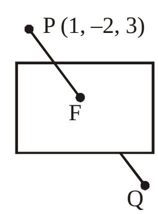
$|\vec{a}\times\vec{b}| = 3$

From eq. (1), $3\cdot|\vec{c}| = 6$; $|\vec{c}| = 2$

Now, $|\vec{c} - \vec{a}| = 3$; $|\vec{c}| + |\vec{a}| - 2\vec{a}\cdot\vec{c} = 9$

$4 + 9 - 2\vec{a}\cdot\vec{c} = 9$; $\vec{a}\cdot\vec{c} = 2$

(72) **(D).** Line PQ : $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5}$



Let $F(\lambda+1, 4\lambda-2, 5\lambda+3)$

F lines on the plane

$2(\lambda+1)+3(4\lambda-2)$
$\quad -4(5\lambda+3)+22=0$
$-6\lambda+6=0 \Rightarrow \lambda=1$
F (2, 2, 8)
PQ = 2, PF = $2\sqrt{42}$

(73) **(D).** Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$
$\Rightarrow 5x+7y+3z+5=0$

$$\text{Distance} = \frac{5+21-21+5}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}}$$

(74) **(D).** $(2+\lambda)x-(2+\lambda)y+(3+\lambda)z-2+\lambda=0$
$(1+3\mu)x+(2-\mu)y+(2\mu-1)z-3-\mu=0$

$$\frac{2+\lambda}{1+3\mu} = \frac{-(2+\lambda)}{2-\mu}$$

$\Rightarrow \mu-2=1+3\mu \Rightarrow 2\mu=-3 \Rightarrow \mu=-3/2$
So the equation of plane is
$7x-7y+8z+3=0$
Now, distance from origin equal to

$$\left|\frac{3}{\sqrt{7^2+7^2+8^2}}\right| = \frac{1}{3\sqrt{2}}$$

(75) **(C).** $\vec{u}\cdot(\vec{a}\times\vec{b})=0$; $\vec{u}\cdot\vec{a}=0$ and $\vec{u}\cdot\vec{b}=24$

Let $\vec{b}=(\vec{b}\cdot\hat{a})\,\hat{a}+(\vec{b}\cdot\hat{u})\,\hat{u}$

$$|\vec{b}|^2=(\vec{b}\cdot\hat{a})^2+(\vec{b}\cdot\hat{u})^2$$

$$|\vec{b}|^2=(\vec{b}\cdot\hat{a})^2+\frac{(\vec{b}\cdot\vec{u})^2}{|\vec{u}|^2}$$

$$2=\frac{2}{7}+\frac{(24)^2}{|\vec{u}|^2} \Rightarrow |\vec{u}|^2=336$$

(76) **(B).** $\dfrac{x-5}{1}=\dfrac{y+1}{1}=\dfrac{z-4}{1}=\lambda$

P $(\lambda+5, \lambda-1, \lambda+4)$
P is foot of perpendicular from A to plane
$3\lambda+8=7$; $\lambda=-1/3$

$$P\left(\frac{14}{3}, \frac{-4}{3}, \frac{11}{3}\right)$$

$$\frac{x-4}{1}=\frac{y+1}{1}=\frac{z-3}{1}$$

Q $(\lambda+4, \lambda-1, \lambda+3)$
Q is foot of perpendicular from B to plane
$3\lambda+6=7$; $\lambda=1/3$

$$Q\left(\frac{13}{3}, \frac{-2}{3}, \frac{10}{3}\right) \; ; \; PQ=\frac{\sqrt{1+4+1}}{3}=\frac{\sqrt{6}}{3}=\sqrt{\frac{2}{3}}$$

(77) **(C).** Equation of plane
$(x+y+z-1)+\lambda(2x+3y-z+4)=0$
$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$
dr's of normal of the plane are
$1+2\lambda, 1+3\lambda, 1-\lambda$
Since plane is parallel to y - axis, $1+3\lambda=0$
$\Rightarrow \lambda=-1/3$
So the equation of plane is $x+4z-7=0$
Point (3, 2, 1) satisfies this equation.

(78) **(A).** $\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$(\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$(\vec{a}\cdot\vec{a})\,\vec{c}-(\vec{c}\cdot\vec{a})\,\vec{a}=\vec{a}\times\vec{b}$

$2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

$$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$$

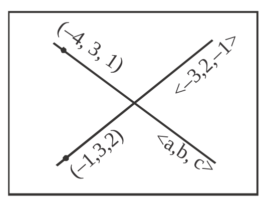
So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}=3\hat{i}-5\hat{j}+2\hat{k}$

$$\vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$$

$$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$$

$$|\vec{c}|^2=\frac{19}{2}$$

(79) **(B).**



Normal vector of plane containing two intersecting lines is parallel to vector.

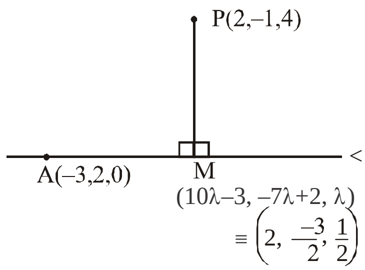
$$(\vec{V}_1)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}=-2\hat{i}+6\hat{k}$$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix}=3\hat{i}-\hat{j}+\hat{k}$$

$\therefore$ Required equation of line is $\dfrac{x+4}{3}=\dfrac{y-3}{-1}=\dfrac{z-1}{1}$

(80) (B).



$\overrightarrow{MP}\cdot(10\hat{i}-7\hat{j}+\hat{k})=0 \Rightarrow \lambda = 1/2$

∴ Length of perpendicular

$(=PM)=\sqrt{0+\frac{1}{4}+\frac{49}{4}}=\sqrt{\frac{50}{4}}=\sqrt{\frac{25}{2}}=\frac{5}{\sqrt{2}}$

which is greater than 3 but less than 4.

(81) (B). Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is parallel to vector

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

∴ Required magnitude of projection

$$=\frac{|(2\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+\hat{k}|}=\frac{|2-6+1|}{|\sqrt{6}|}=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

(82) (B). The required plane is $(2x-y-4)+\lambda(y+2z-4)=0$ it passes through (1, 1, 0)

$\Rightarrow (2-1-4)+\lambda(1-4)=0 \Rightarrow -3-3\lambda=0 \Rightarrow \lambda=-1$

$\Rightarrow x-y-z=0$

(83) (D). $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}=(2+x)\hat{i}+(x-3)\hat{j}-5\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{4+x^2+4x+x^2+9-6x+25}$

$=\sqrt{2x^2-2x+38}$

$\Rightarrow |\vec{a}\times\vec{b}|\ge\sqrt{\frac{75}{2}} \Rightarrow |\vec{a}\times\vec{b}|\ge 5\sqrt{\frac{3}{2}}$

(84) (C). Let the plane be

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$\Rightarrow (2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0$

$\perp$ to the plane $x-y+z=0 \Rightarrow \lambda=-1/3$

$\Rightarrow$ The required plane is $x-z+2=0$

(85) (A). $\frac{4}{2}=\frac{-y}{y+3}=\frac{10-z}{z-4} \Rightarrow z=6 \;\&\; y=-2$

$\Rightarrow$ R (4, –2, 6)

Distance from origin $=\sqrt{16+4+36}=2\sqrt{14}$

(86) (C). $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$.

$\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$; $\vec{\beta}_1=\lambda(3\hat{i}+\hat{j}), \vec{\beta}_2=\lambda(3\hat{i}+\hat{j})-2\hat{i}+\hat{j}-3\hat{k}$

$\vec{\beta}_2\cdot\vec{\alpha}=0$

$(3\lambda-2).3+(\lambda+1)=0$

$9\lambda-6+\lambda+1=0 \Rightarrow \lambda=1/2$

$\vec{\beta}_1=\frac{3}{2}\hat{i}+\frac{1}{2}\hat{j}$; $\vec{\beta}_2=-\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3k$

$$\vec{\beta}_1\times\vec{\beta}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{3}{2} & \frac{1}{2} & 0\\ -\frac{1}{2} & \frac{3}{2} & -3\end{vmatrix}$$

$=\hat{i}\left(-\frac{3}{2}-0\right)-\hat{j}\left(-\frac{9}{2}-0\right)+\hat{k}\left(\frac{9}{4}+\frac{1}{4}\right)$

$=-\frac{3}{2}\hat{i}+\frac{9}{2}\hat{j}+\frac{5}{2}\hat{k}=\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$

Aliter : $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2 \Rightarrow \vec{\beta}\cdot\hat{\alpha}=\vec{\beta}_1\cdot\hat{\alpha}=|\vec{\beta}_1|$

$\Rightarrow \vec{\beta}_1=(\vec{\beta}\cdot\hat{\alpha})\,\hat{\alpha} \Rightarrow \vec{\beta}_2=(\vec{\beta}\cdot\hat{\alpha})\,\hat{\alpha}-\vec{\beta}$

$\Rightarrow \vec{\beta}_1\times\vec{\beta}_2=-(\vec{\beta}\cdot\hat{\alpha})\,\hat{\alpha}\times\vec{\beta}$

$=\frac{-5}{10}(3\hat{i}+\hat{j})\times(2\hat{i}-\hat{j}+3\hat{k})=\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$

(87) (B). Let ax + by + cz = 1 be the equation of the plane

$\Rightarrow 0-b+0=1 \Rightarrow b=-1$

$0+0+c=1 \Rightarrow c=1$

$\cos\theta=\left|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}\right|$

$\frac{1}{\sqrt{2}}=\frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$

$\Rightarrow a^2+2=4 \Rightarrow a=\pm\sqrt{2} \Rightarrow \pm\sqrt{2}x-y+z=1$

Now for –sign $-\sqrt{2}\cdot\sqrt{2}-1+4=1$

(88) (A). Any point on the given line can be

$(1+2\lambda, -1+3\lambda, 2+4\lambda)$; $\lambda\in R$

Put in plane $1+2\lambda+(-2+6\lambda)+(6+12\lambda)=15$

$20\lambda+5=15$; $20\lambda=10$; $\lambda=1/2$

∴ Point (2, 1/2, 4)

Distance from origin

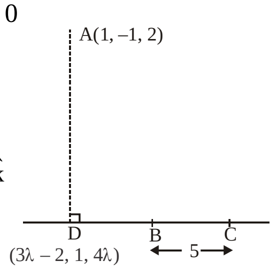
$=\sqrt{4+\frac{1}{4}+16}=\frac{\sqrt{16+1+64}}{2}=\frac{\sqrt{81}}{2}=\frac{9}{2}$

(89) **(B).** $\overrightarrow{AD}\cdot(3\hat{i}+4\hat{k})=0$

$3(3\lambda-3)+0+4(4\lambda-2)=0$

$(9\lambda-9)+(16\lambda-8)=0$

$25l=17 \Rightarrow \lambda=17/25$



$$\overrightarrow{AD}=\left(\frac{51}{25}-3\right)\hat{i}+2\hat{j}+\left(\frac{68}{25}-2\right)\hat{k}$$

$$=\frac{24}{25}\hat{i}+2\hat{j}+\frac{18}{25}\hat{k}$$

$$|\overrightarrow{AD}|=\sqrt{\frac{576}{625}+4+\frac{324}{625}}$$

$$=\sqrt{\frac{900}{625}+4}=\sqrt{\frac{3400}{625}}=\sqrt{34}\times\frac{10}{25}=\frac{2}{5}\sqrt{34}$$

$$\text{Area of }\Delta=\frac{1}{2}\times5\times\frac{2\sqrt{34}}{5}=\sqrt{34}$$

(90) **(C).** $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1\ ;\quad \cos^2\gamma=1-\frac{3}{4}=\frac{1}{4}$$

$$\cos^2\gamma=\pm\frac{1}{2}\Rightarrow\gamma=\frac{\pi}{3}\ \text{or}\ \frac{2\pi}{3}$$

(91) **(A).** One of the point on line is P(0, 1, –1) and given point is Q (β, 0, β). So, $\overrightarrow{PQ}=\beta\hat{i}-\hat{j}+(\beta+1)\hat{k}$

$$\text{Hence, }\beta^2+1+(\beta+1)^2-\frac{(\beta-\beta-1)^2}{2}=\frac{3}{2}$$

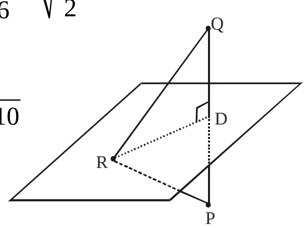
$2\beta^2+2\beta=0\ ;\ \beta=0,-1\ ;\ \beta=-1\ (\text{as }\beta\neq0)$

(92) **(D).** R lies on the plane.

$$DQ=\frac{|1-12-2|}{\sqrt{9+1+16}}=\frac{13}{\sqrt{26}}=\sqrt{\frac{13}{2}}$$

$\Rightarrow\ PQ=\sqrt{26}$

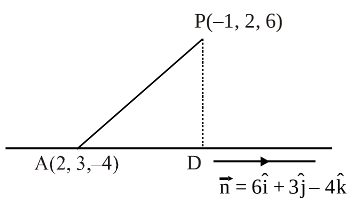
Now, $RQ=\sqrt{9+1}=\sqrt{10}$



$$\Rightarrow\quad RD=\sqrt{10-\frac{13}{2}}=\sqrt{\frac{7}{2}}$$

$$\text{ar}(\Delta PQR)=\frac{1}{2}\times26\times\sqrt{\frac{7}{2}}=\frac{\sqrt{91}}{2}$$

(93) **(A).**



$$AD=\left|\frac{\overrightarrow{AP}\cdot\hat{n}}{|\hat{n}|}\right|=\sqrt{61}\ ;\ PD=\sqrt{AP^2-AD^2}=\sqrt{110-61}=7$$

(94) **(A).** Let point P on the line is (2λ +1, – λ –1, λ)

foot of perpendicular Q is given by

$$\frac{x-2\lambda-1}{1}=\frac{y+\lambda+1}{1}=\frac{z-\lambda}{1}=\frac{-(2\lambda-3)}{3}$$

Q lies on $x+y+z=3$ & $x-y+z=3$

$\Rightarrow\ x+z=3$ & $y=0$

$$y=0\Rightarrow\lambda+1=\frac{-2\lambda+3}{3}\Rightarrow\lambda=0$$

$\Rightarrow$ Q is (2, 0, 1)

(95) **(C).** $4x-2y+4z+6=0$

$$\frac{|\lambda-6|}{\sqrt{16+4+16}}=\left|\frac{\lambda-6}{6}\right|=\frac{1}{3}\ ;\ |\lambda-6|=2\ ;\ \lambda=8,4$$

$$\frac{|\mu-3|}{\sqrt{4+4+1}}=\frac{2}{3}\ ;\ |\mu-3|=2\ ;\ \mu=5,1$$

∴ Maximum value of (μ + λ) = 13.

(96) **(A).** $\dfrac{x-2}{3}=\dfrac{y+1}{2}=\dfrac{z-1}{-1}=\lambda$

$x=3\lambda+2,\ y=2\lambda-1,\ z=-\lambda+1$

Intersection with plane $2x+3y-z+13=0$

$2(3\lambda+2)+3(2\lambda-1)-(-\lambda+1)+13=0$

$13\lambda+13=0$ C ; $\lambda=-1$

∴ P (–1, –3, 2)

Intersection with plane $3x+y+4z=16$

$3(3\lambda+2)+(2\lambda-1)+4(-\lambda+1)=16\ ;\lambda=1$

Q (5, 1, 0)

$$PQ=\sqrt{6^2+4^2+2^2}=\sqrt{56}=2\sqrt{14}$$

(97) **(C).** $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=2(\vec{b}\times\vec{a})$

$$=2\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-2\\3&2&2\end{vmatrix}=2(8\hat{i}-8\hat{j}+4\hat{k})$$

$$\text{Required vector }=\pm12\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}=\pm4(2\hat{i}-2\hat{j}-\hat{k})$$

(98) **(A).** Perpendicular vector to the plane

$$\vec{n}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-1\\-1&1&-2\end{vmatrix}=-3\hat{i}+3\hat{j}+3\hat{k}$$

Equationh of plane $-3(x-1)+3(y-1)+3z=0$

$\Rightarrow\ x-y-z=0$

$$d_{(2,1,4)}=\frac{|2-1-4|}{\sqrt{1^2+1^2+1^2}}=\sqrt{3}$$

(99) **(B).** Equation of bisector of angle

$$\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}$$

(+) gives $x-3y=2$

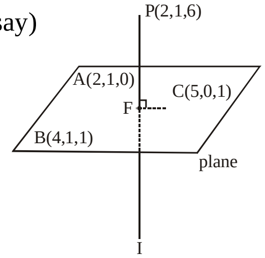
(–) gives $3x+y+4z=6$

(100) (A). Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0 \Rightarrow x+y-2z=3$$

Let I and F are respectively image and foot of perpendicular of point P in the plane.
Eq^n of line PI

$$\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda \text{ (say)}$$



Let I $(\lambda+2, \lambda+1, -2\lambda+6)$

$$\Rightarrow F\left(2+\frac{\lambda}{2}, 1+\frac{\lambda}{2}, -\lambda+6\right)$$

F lies in the plane

$$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$$

$$\Rightarrow \lambda = 4 \Rightarrow I(6, 5, -2)$$

(101) (D). $|\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$$3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

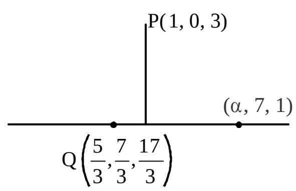
$$(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$$

$$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+(-\vec{a}-\vec{b})\times\vec{a}$$

$$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}\times\vec{a}\times\vec{b}$$

$$\vec{d} = 3(\vec{a}\times\vec{b})$$

(102) 4. Since PQ is perpendicular to L, therefore



$$\left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0 \Rightarrow \frac{2\alpha}{3}=\frac{24}{9} \Rightarrow \alpha = 4$$

(103) (B). Shortest distance

$$= \frac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{11\times 29-49}} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$$

(104) (2). $\pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \Rightarrow -\lambda+3 = \pm 1$

$\lambda = 2$ or $\lambda = 4$.

For $\lambda = 4$, $\cos\theta = \dfrac{2+1+4}{\sqrt{6}\sqrt{18}} = \dfrac{7}{6\sqrt{3}}$

(105) (C). $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{a})$

$$-(\vec{a}\cdot\vec{b})\,\vec{c} = (\vec{a}\cdot\vec{a})\,\vec{b}-(\vec{a}\cdot\vec{b})\,\vec{a}$$

$$-4\vec{c} = 6(\hat{i}-\hat{j}+\hat{k})-4(\hat{i}-2\hat{j}+\hat{k})$$

$$-4\vec{c} = 2\hat{i}+2\hat{j}+2\hat{k}$$

$$\vec{c} = -\frac{1}{2}(\hat{i}+\hat{j}+\hat{k}) \;\; ; \;\; \vec{b}\cdot\vec{c} = -\frac{1}{2}$$

(106) (D). d.r of normal to the plane

$$\frac{10}{3}, \frac{10}{3}, \frac{10}{3}$$

1, 1, 1

Midpoint of P and Q is $\left(\dfrac{-2}{3}, \dfrac{1}{3}, \dfrac{4}{3}\right)$

Equation of plane x + y + z = 1

(107) (D). $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$; $D = (\lambda+8)(2-\lambda)$

for $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$$= 5[18-10]-2[48-50]+2(16-30) = 40+4-28 \neq 0$$

No solutions for $\lambda = 2$

(108) (1.00)

$\vec{p} = (a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q} = a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and

$\vec{r} = a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $\quad\because$ $\vec{p}, \vec{q}, \vec{r}$ are coplanar

$$\Rightarrow [\vec{p}\;\vec{q}\;\vec{r}] = 0$$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

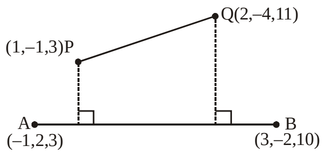
$$\Rightarrow 3a+1=0 \Rightarrow a = -1/3$$

$$\vec{p}\cdot\vec{q} = -\frac{1}{3}, \; \vec{r}\cdot\vec{q} = -\frac{1}{3}$$

$$|\vec{r}|^2 = |\vec{q}|^2 = \frac{2}{3} \qquad \therefore \qquad 3(\vec{p}\cdot\vec{q})^2-\lambda[\vec{r}\times\vec{q}]^2 = 0$$

$$\Rightarrow \lambda = \frac{3(\vec{p}\cdot\vec{q})^2}{[\vec{r}\times\vec{q}]^2} = \frac{3(\vec{p}\cdot\vec{q})^2}{|\vec{r}|^2|\vec{q}|^2-(\vec{r}\cdot\vec{q})^2} = 1.00$$

(109) (8.00)



Projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB} = \left|\dfrac{\overrightarrow{PQ}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}\right|$

$$= \left|\frac{(\hat{i}-3\hat{j}+8\hat{k})\cdot(4\hat{i}-4\hat{j}+7\hat{k})}{9}\right| = 8$$

(110) (30). $\vec{b}\cdot\vec{c} = 10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3} = 10 \Rightarrow |\vec{c}| = 4$

$$|\vec{a}\times(\vec{b}\times\vec{c})| = |\vec{a}||\vec{b}\times\vec{c}| = \sqrt{3}\cdot 5\cdot 4\sin\frac{\pi}{4} = 30$$

(111) (3). If $\lambda = -7$, then planes will be parallel &

distance between them will be $\dfrac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \approx -7$, then planes will be intersecting and distance between them will be 0.