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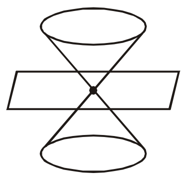
CONIC SECTIONS (PARABOLA, ELLIPSE & HYPERBOLA)

* Point, pair of straight lines, circle, parabola, ellipse and hyperbola are called conic section because they can be obtained when a cone (or double cone) is cut by a plane.

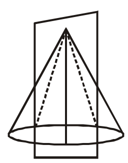
* The mathematicians associated with the study of conics were Euclid, Aristarchus and Apollonius. Most of the objects around us and in space have shape of conic-sections. Hence study of these becomes a very important tool for present knowledge and further exploration.

Section of right circular cone by different planes :

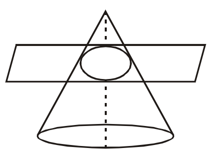
(1) When a double right circular cone is cut by a plane parallel to base at the common vertex, the cutting profile is a point.



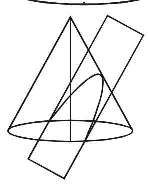
(2) When a right circular cone is cut by any plane through its vertex, the cutting profile is a pair of straight lines through its vertex.



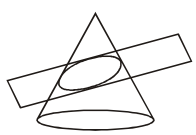
(3) When a right circular cone is cut by a plane parallel to its base the cutting profile is a circle.



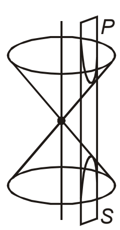
(4) When a right circular cone is cut by a plane parallel to a generator of cone, the cutting profile is a parabola.



(5) When a right circular cone is cut by a plane which is neither parallel to any generator / axis nor parallel to base, the cutting profile is an ellipse.



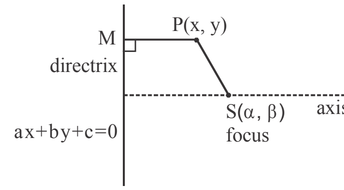
(6) When a double right circular cone is cut by plane, parallel to its common axis, the cut profile is hyperbola



Hence a point, a pair of intersecting straight lines, circle, parabola, ellipse and hyperbola, all are conic-sections. All the conic sections are plane or two dimensional curves.

The conic section is the locus of a point which moves such that the ratio of its distance from a fixed point (focus) to perpendicular distance from a fixed straight line (directrix) is always constant (e). Here e is called eccentricity of conic

i.e., $\dfrac{PS}{PM} = e$



A line through focus and perpendicular to directrix is called axis. The vertex of conic is that point where the curve intersects its axis.

$$\frac{PS}{PM} = e \Rightarrow PS^2 = e^2PM^2$$

$$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = e^2\left(\frac{ax+by+c}{\sqrt{a^2+b^2}}\right)^2$$

Simplification shall lead to the equation of the form
$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

Distinguishing various conics :

* The nature of the conic section depends upon the position of the focus S w.r.t. the directrix & also upon the value of the eccentricity e. Two different cases arise.

Case-I : When The Focus Lies On The Directrix (De-generated conic) :

In this case $\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & the general equation of a conic represents a pair of straight lines if

$e > 1$ i.e. $h^2 > ab$ the lines will be real & distinct, intersecting at S.

$e = 1$ i.e. $h^2 = ab$ the lines will coincident.

$e < 1$ i.e. $h^2 < ab$ the lines will be imaginary.

Case-II : When The Focus Does Not Lie on the Directrix (Non de-generated conic) :

In this case $\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ and conic represent

a parabola	an ellipse	a hyperbola	rectangular hyperbola	Circle
$e = 1$	$0 < e < 1$	$e > 1$	$e = \sqrt{2}$	$e = 0$
$h^2 = ab$	$h^2 < ab$	$h^2 > ab$	$h^2 > ab$; $a + b = 0$	$h = 0$, $a = b$

Note :

(i) For pair of straight lines $e \to \infty$

(ii) All second degree terms in parabola form a perfect square

Definition of various terms related to a conic :

(1) Focus : The fixed point is called a focus of the conic.

(2) Directrix : The fixed line is called a directrix of the conic.

(3) Axis : The line passing through the focus and perpendicular to the directrix is called the axis of the conic.

(4) Vertex : The points of intersection of the conic and the axis are called vertices of the conic.

(5) Centre : The point which bisects every chord of the conic passing through it, is called the centre of the conic.

(6) Latus-rectum : The latus-rectum of a conic is the chord passing through the focus and perpendicular to the axis.

(7) Double ordinate : A chord which is perpendicular to the axis of parabola or parallel to its directrix.

Example 1 :

What conic does $13x^2 - 18xy + 37y^2 + 2x + 14y - 2 = 0$ represent ?

Sol. Compare the given equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\therefore \quad a = 13, h = -9, b = 37, g = 1, f = 7, c = -2$

then $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= (13)(37)(-2) + 2(7)(1)(-9) - 13(7)^2 - 37(1)^2 + 2(-9)^2$

$= -962 - 126 - 637 - 37 + 162 = -1600 \neq 0$

and also $\quad h^2 = (-9)^2 = 81$ and $ab = 13 \times 37 = 481$

Here $\quad h^2 - ab < 0$

So we have $h^2 - ab < 0$ and $\Delta \neq 0$. Hence the given equation represents an ellipse.

Example 2 :

For what value of λ the equation of conic $2xy + 4x - 6y + \lambda = 0$ represents two real intersecting straight lines? if $\lambda = 17$ then this equation represents ?

Sol. Comparing the given equation of conic with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\therefore \quad a = 0, b = 0, h = 1, g = 2, f = -3, c = \lambda$

For two intersecting real lines

$h^2 - ab \geq 0$ and $\Delta = 0$

here $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 0 + 2 \times (-3) \times 2 \times 1 - 0 - 0 - \lambda(1)^2 = -12 - \lambda = 0$

$\lambda = -12$ and $h^2 - ab = 1$

hence for $\lambda = -12$ above equation always represent real intersecting lines.

If $\lambda = 17$ then $\Delta \neq 0$ and $h^2 - ab > 0$

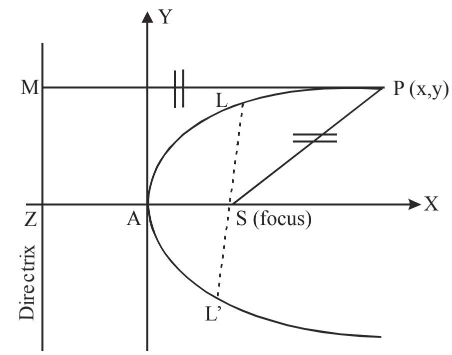
so we have $\Delta \neq 0$ and $h^2 - ab > 0$. Hence the given equation represents a Hyperbola.

PARABOLA

DEFINITION

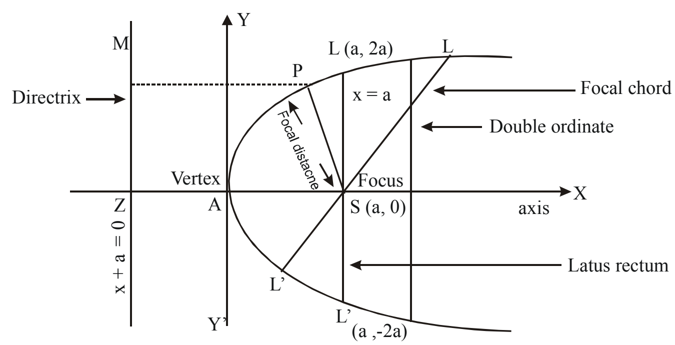
A parabola is the locus of a point which moves in such a way that its distance from a fixed point is equal to its perpendicular distance from a fixed straight line. The fixed point is called the focus of the parabola and the fixed line is called the directrix of the parabola.

Let S be the focus. ZM be the directrix and P be any point on the parabola. Then by definition PS = PM where PM is the length of the perpendicular from P on the directrix ZM.



TERMS RELATED TO PARABOLA

Eccentricity : If P be a point on the parabola and PM and PS are the distances from the directrix and focus S respectively then the ratio PS/PM is called the eccentricity of the Parabola which is denoted by e.



Note : By the definition for the parabola e = 1

If $e > 1 \Rightarrow$ Hyperbola, $e = 0 \Rightarrow$ circle , $e < 1 \Rightarrow$ ellipse

* **Axis:** A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola

* **Vertex :** The point of intersection of a parabola and its axis is called the vertex of the parabola.
It is the middle point of the focus and the point of intersection of axis and directrix

* **Focal Length (Focal distance) :** The distance of any point P(x,y) on the parabola from the focus is called the focal length i.e. the focal distance of P = the perpendicular distance of the point P from the directrix.

* **Double Ordinate :** The chord which is perpendicular to the axis of parabola or parallel to directrix is called double ordinate of the parabola.

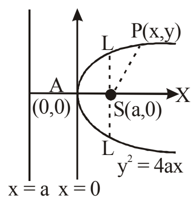
* **Focal Chord :** Any chord of the parabola passing through the focus is called Focal chord.

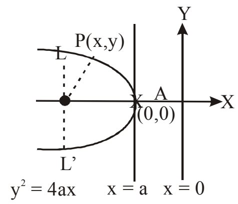
* **Latus Rectum :** If a double ordinate passes through the focus of parabola then it is called as latus rectum.

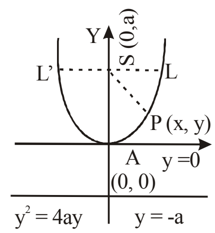
* **Length of latus rectum :** The length of the latus rectum = 2 × perpendicular distance of focus from the directrix.

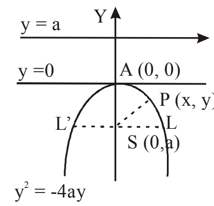
STANDARD FORMS OF EQUATION OF PARABOLA

Shape of Parabola









Standard Equation	$y^2 = 4ax\ (a > 0)$	$y^2 = -4ax\ (a > 0)$	$x^2 = 4ay\ (a > 0)$	$x^2 = -4ay\ (a > 0)$
Vertex	A (0, 0)	A (0, 0)	A (0, 0)	A (0, 0)
Focus	S (a, 0)	S (–a, 0)	S (0, a)	S (0, –a)
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of latus rectum	4a	4a	4a	4a
Extremities of latus rectum	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$
Equation of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$
Equation of tangents at vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Focal distance of a point P (x, y)	$x + a$	$x - a$	$y + a$	$y - a$
Parametric coordinates	$(at^2, 2at)$	$(-at^2, 2at)$	$(2at, at^2)$	$(2at, -at^2)$
Eccentricity (e)	1	1	1	1

Example 3 :

Find the equation of the parabola whose vertex is (–3, 0) and directrix is $x + 5 = 0$.

Sol. A line passing through the vertex (–3, 0) and perpendicular to directrix $x + 5 = 0$ is x-axis which is the axis of the parabola by definition. Let focus of the parabola is (a, 0). Since vertex, is the middle point of Z (–5, 0) and focus S, therefore

$$-3 = \frac{(a-5)}{2} \Rightarrow a = -1$$

$\therefore$ Focus = (–1, 0)

Thus the equation to the parabola is

$$(x+1)^2 + y^2 = (x+5)^2 \Rightarrow y^2 = 8(x+3)$$

Example 4 :

If focus of a parabola is (3, –4) and directrix is $x + y - 2 = 0$, then find its vertex.

Sol. First we find the equation of axis of parabola, which is perpendicular to directrix, So its equation is $x - y + k = 0$. it passes through focus S (3, – 4)

$$\Rightarrow 3 - (-4) + k = 0 \Rightarrow k = -7$$

Let Z is the point of intersection of axis and directrix

Solving equation $x + y - 2 = 0$ and $x - y - 7 = 0$ gives Z (9/2, –5/2)

Vertex A is the mid point of Z and S

$$\Rightarrow A\left(\frac{3+\frac{9}{2}}{2}, \frac{-4-\frac{5}{2}}{2}\right) = A\left(\frac{15}{4}, -\frac{13}{4}\right)$$

REDUCTION TO STANDARD EQUATION

If the equation of a parabola is not in standard form and if it contains second degree term either in y or in x (but not in both) then it can be reduced into standard form. For this we change the given equation into the following forms

$$(y-k)^2 = 4a(x-h) \text{ or } (x-p)^2 = 4b(y-q)$$

And then we compare from the following table for the results related to parabola**.**

Equation of Parabola	Vertex	Axis	Focus	Directrix	Equation of L.R.	Length of L.R.
$(y-k)^2 = 4a(x-h)$	(h, k)	$y = k$	(h + a, k)	$x + a - h = 0$	$x = a + h$	4a
$(x-p)^2 = 4b(y-q)$	(p, q)	$x = p$	(p, b + q)	$y + b - q = 0$	$y = b + q$	4b

Note :

(i) For the parabola $y = Ax^2 + Bx + C$, the length of latus rectum is $\frac{1}{|A|}$ and axis is parallel to y-axis.

If A is positive then it is concave up parbaola, if A is negative then it is concave down parabola.

(ii) For the parabola $x = Ay^2 + By + C$, the length of latus rectum is $\frac{1}{|A|}$ and axis is parallel to x-axis.

Example 5 :

Find the vertex of the parabola $x^2 - 8y - x + 19 = 0$.

Sol. The given equation of Parabola can be written as

$$\left(x - \frac{1}{2}\right)^2 - 8y + 19 - \frac{1}{4} = 0$$

$$\left(x - \frac{1}{2}\right)^2 = 8y - \frac{76-1}{4} \Rightarrow \left(x - \frac{1}{2}\right)^2 = 8\left(y - \frac{75}{32}\right)$$

$$\therefore \text{ Vertex} = \left(\frac{1}{2}, \frac{75}{32}\right)$$

GENERAL EQUATION OF A PARABOLA

If (h, k) be the locus of a parabola and the equation of directrix is $ax + by + c = 0$, then its equation is given by

$$\sqrt{(x-h)^2 + (y-k)^2} = \frac{ax + by + c}{\sqrt{a^2 + b^2}}$$

which gives $(bx - ay)^2 + 2gx + 2fy + d = 0$

where g, f, d are the constants.

Note:

(i) It is a second degree equation in x and y and the terms of second degree forms a perfect square and it can be the at least one linear term.

(ii) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a parabola, if (a) $h^2 = ab$ (b) $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ ($\Delta \neq 0$, $ab - h^2 > 0 \rightarrow$ ellipse ; $\Delta \neq 0$, $ab - h^2 < 0 \rightarrow$ hyperbola

PARAMETRIC EQUATION OF PARABOLA

The parametric equation of parabola $y^2 = 4ax$ are $x = at^2$, $y = 2at$. Hence any point on this parabola is $(at^2, 2at)$ which is called as 't' point.

Note:

(i) Parametric equation of the Parabola $x^2 = 4ay$ is $x = 2at$, $y = at^2$

(ii) Any point on Parabola $y^2 = 4ax$ may also be written as $(a/t^2, 2a/t)$.

(iii) The ends of a double ordinate of a parabola can be taken as $(at^2, 2at)$ and $(at^2, -2at)$.

(iv) Parametric equations of the parabola $(y - h)^2 = 4a(x - k)^2$ is $x - k = at^2$ and $y - h = 2at$.

Example 6 :

Find the parameter 't' of a point (4, – 6) of the parabola $y^2 = 9x$.

Sol. Parametric coordinates of any point on parabola $y^2 = 4ax$ are $(at^2, 2at)$. Here $4a = 9 \Rightarrow a = 9/4$

$\therefore$ y coordinate $2at = -6$ $\quad \therefore 2(9/4)t = -6 \Rightarrow t = -4/3$

EQUATION OF CHORD

Equation of chord joining any two points of a parabola :

Let the points are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ then equation of chord is

$$(y - 2at_1) = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$$

$$\Rightarrow y - 2at_1 = \frac{2}{t_1 + t_2}(x - at_1^2) \Rightarrow (t_1 + t_2)y = 2x + 2at_1t_2$$

Note :

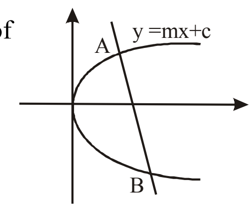
(i) If 't_1' and 't_2' are the Parameters of the ends of a focal chord of the Parabola $y^2 = 4ax$, then $t_1 t_2 = -1$

(ii) If one end of focal chord of parabola is $(at^2, 2at)$, then other end will be $(a/t^2, -2at)$ and length of focal chord$= a(t + 1/t)^2$

(iii) The length of the chord joining two points 't_1' and 't_2' on the parabola $y^2 = 4ax$ is

$$a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4}$$

Length of intercept : The length of intercept made by line $y = mx + c$ between the parabola $y^2 = 4ax$ is



$$AB = \frac{4}{m^2}\sqrt{a(1 + m^2)(a - mc)}$$

Example 7 :

Find the length of intercept by the line $4y = 3x - 48$ on the parabola $y^2 = 64x$.

Sol. $4y = 3x - 48 \Rightarrow m = 3/4, c = -12$; $y^2 = 64x \Rightarrow a = 16$

$$\text{Length of intercept} = \frac{4}{m^2}\sqrt{a(1 + m^2)(a - mc)}$$

$$= \frac{4}{9} \times 16\sqrt{16\left(1 + \frac{9}{16}\right)\left(16 + 12 \times \frac{3}{4}\right)} = \frac{1600}{9}$$

POSITION OF A POINT AND A LINE WRT A PARABOLA

Position of a point with respect to a parabola :

A point (x_1, y_1) lies inside, on or outside of the region of the parabola $y^2 = 4ax$ according as $y_1^2 - 4ax_1 <= \text{or} > 0$

Line and Parabola : The line $y = mx + c$ will intersect a parabola $y^2 = 4ax$ in two real and different, coincident or imaginary point, according as $a - mc >, = < 0$

Example 8 :

For the parabola $y^2 = 8x$, point (2, 5) is

(1) Inside the parabola (2) Focus

(3) Outside the parabola (4) On the parabola

Sol. **(3).** $(y^2 - 8x)_{x=2,\, y=5} = (5)^2 - 8 \times 2 = 9 > 0$

$\Rightarrow$ Point (2, 5) is outside parabola $y^2 = 8x$

TANGENT TO THE PARABOLA

Condition of Tangency : If the line $y = mx + c$ touches a parabola $y^2 = 4ax$ then $c = a/m$

Note :

(i) The line $y = mx + c$ touches parabola $x^2 = 4ay$ if $c = -am^2$

(ii) The line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$ if $a\sin^2\alpha + p\cos\alpha = 0$

(iii) If the equation of parabola is not in standard form, then for condition of tangency, first eliminate one variable quantity (x or y) between equations of straight line and parabola and then apply the condition $B^2 = 4AC$ for the quadratic equation so obtained.

Example 9 :

If the line $2x - 3y = k$ touches the parabola $y^2 = 6x$, then find the value of k.

Sol. Given $x = \dfrac{3y+k}{2}$ .... (1) and $y^2 = 6x$ .... (2)

$\Rightarrow \; y^2 = 6\left(\dfrac{3y+k}{2}\right) \Rightarrow y^2 = 3\,(3y + k)$

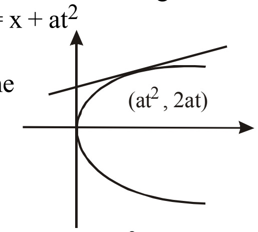
$\Rightarrow \; y^2 - 9y - 3k = 0$ .... (3)

If line (1) touches parabola (2) then roots of quadratic equation (3) is equal $\therefore (-9)^2 = 4 \times 1 \times (-3k) \Rightarrow k = -27/4$

Equation of tangents in different forms :

Point Form : The equation of tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1) is $yy_1 = 2a\,(x + x_1)$ or $T = 0$

Parametric Form: The equation of the tangent to the parabola at t i.e. $(at^2, 2at)$ is $ty = x + at^2$



Slope Form: The equation of the tangent of the parabola $y^2 = 4ax$ is $y = mx + \dfrac{a}{m}$.

Note :

(i) $y = mx + a/m$ is a tangent to the parabola $y^2 = 4ax$ for all value of m and its point of contact is $(a/m^2, 2a/m)$

(ii) $y = mx - am^2$ is a tangent to the parabola $x^2 = 4ay$ for all value of m and its point of contact is $(2am, am^2)$

(iii) Point of intersection of tangents at points t_1 and t_2 of parabola is $[at_1t_2, a\,(t_1 + t_2)]$

(iv) Two perpendicular tangents of a parabola meet on its directrix. So the director circle of a parabola is its directrix or tangents drawn from any point on the directrix are always perpendicular.

(v) The tangents drawn at the end points of a focal chord of a parabola are perpendicular and they meet at the directrix.

Example 10 :

If a tangent to the parabola $y^2 = ax$ makes an angle of 45° with x -axis, then find its point of contact.

Sol. The slope of the tangent = tan 45° = 1 $\therefore$ m = 1 and a ≡ a/4

$\therefore$ Point of contact $= \left(\dfrac{a/4}{1^2}, \dfrac{2.a/4}{1}\right) = \left(\dfrac{a}{4}, \dfrac{a}{2}\right)$

Example 11 :

Find the common tangent of the parabola $x^2 = 4ay$ and $y^2 = 4ax$ $(m > 0)$.

Sol. Equation of tangent for $x^2 = 4ay$ is $y = mx - am^2$

As this line also touches $y^2 = 4ax$

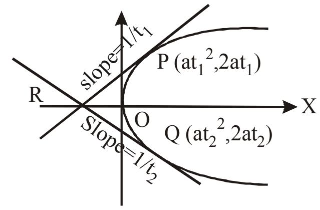
$\therefore -am^2 = a/m$ $(c = a/m)$

$\Rightarrow m^3 = -1 \quad \Rightarrow m = -1$

$\therefore$ equation of tangent $\Rightarrow$ $x + y + a = 0$

POINT OF INTERSECTION OF TANGENTS

The point of intersection of tangents drawn at two different points of contact P $(at_1^2, 2at_1)$ and Q $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is



$R = (at_1t_2, a(t_1+t_2))$

Note :

(i) Angle between tangents at two points P $(at_1^2, 2at_1)$ and Q $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is

$$\theta = \tan^{-1}\left|\frac{t_2 - t_1}{1 + t_1t_2}\right|$$

(ii) The G.M. of the x-coordinates of P and Q (i.e. $\sqrt{at_1^2 \times at_2^2} = at_1t_2$) is the x-coordinate of the point of intersection of tangents at P and Q on the parabola.

(iii) The A.M. of the y-coordinates of P and Q $\left(\text{i.e.} \dfrac{2at_1 + 2at_2}{2} = a(t_1 + t_2)\right)$ is the y-coordinate of the point of intersection of tangents at P and Q on the parabola.

(iv) The orthocentre of the triangle formed by three tangents to the parabola lies on the directrix.

NORMAL TO THE PARABOLA

Equation of Normal :

Point Form : The equation to the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$ is given by

$$y - y_1 = \frac{-y_1}{2a}\,(x - x_1)$$

Parametric Form : The equation to the normal at the point $(at^2, 2at)$ is $y + tx = 2at + at^3$

Slope Form : Equation of normal in terms of slope m is

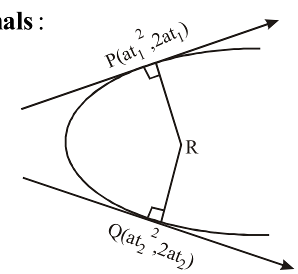
$y = mx - 2am - am^3$

Note :

(i) The foot of the normal is $(am^2, -2am)$

(ii) **Condition for normality** : The line $y = mx + c$ is a normal to the parabola $y^2 = 4ax$ if $c = -2am - am^3$ and $x^2 = 4ay$ if $c = 2a + \dfrac{a}{m^2}$

(iii) **Point of intersection of Normals** : The point of intersection of normals drawn at two different points of contact P $(at_1^2, 2at_1)$ and Q $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is



$R \equiv [2a + a\,(t_1^2 + t_2^2 + t_1t_2) - at_1t_2\,(t_1 + t_2)]$

Example 12 :

Find the equation of normal at the point $(a/m^2, 2a/m)$ on the parabola $y^2 = 4ax$

Sol. The equation of the normal at $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is written as $y + tx = 2at + at^3$

For the given point $t = 1/m$, equation of required normal is

written as $y + \left(\frac{1}{m}\right)x = 2a\left(\frac{1}{m}\right) + a\left(\frac{1}{m}\right)^3$

$\Rightarrow m^3 y = 2am^2 - m^2 x + a$

Properties of normal :

(i) The line $y = mx + c$ will normal to parabola $y^2 = 4ax$ if $c = -2am - am^3$

(ii) If normal passes through point (x_1, y_1) which is not on parabola, then $y_1 = mx_1 - 2am - am^3$

$\Rightarrow am^3 + (2a - x_1)\,m + y_1 = 0$ ....(1)

which gives three values of m

Let three value of m are m_1, m_2 and m_3 then from (1)

$m_1 + m_2 + m_3 = 0$

$m_1m_2 + m_2 m_3 + m_3 m_1 = \frac{2a - x_1}{a}$ and $m_1 m_2 m_3 = -\frac{y_1}{a}$

In General : From a given point three normals can be drawn to the parabola $y^2 = 4ax$. These normal are such that

(a) The algebraic sum of their slopes is zero.

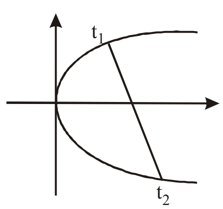
(b) The algebraic sum of the ordinates of their feet is zero.

(c) The centroid of the triangle formed by joining their feet lies on the axis of the parabola.

(d) The centroid of the triangle at least one is real as imaginary normal will always occur in pairs.

(iii) The tangent of one extremity of a focal chord of a parabola is parallel to the normal at the other extremity.

(iv) **Normal Chord :** If the normal at 't_1' meets the parabola $y^2 = 4ax$ again at the point 't_2' then this is called as normal chord. Again for normal chord



$t_2 = -t_1 - \frac{2}{t_1}$

(v) Length of Normal chord is given by a

$$a\,(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4} = \frac{4a(t_1^2 + 1)^{3/2}}{t_1^2}$$

(vi) If two normal drawn at point 't_1' and 't_2' meet on the parabola then $t_1 t_2 = 2$

(vii) The normal at the extremities of the latus rectum of a parabola intersect at right angle on the axis of the parabola

Example 13 :

If two of the normal of the parabola $y^2 = 4x$, that pass through (15, 12) are $4x + y = 72$, $3x - y = 33$, then find the third normal.

Sol. Here, If m_1, m_2, m_3 are slopes of normal, then

$m_1 + m_2 + m_3 = 0$ and $m_1 m_2 m_3 = \frac{y_1}{a}$

$a = 1$ here $m_1 = -4$, $m_2 = 3$ $\therefore -4 + 3 + m_3 = 0 \Rightarrow m_3 = 1$

Also $(-4)(3)(1) = -(12/1)$ is satisfied

But (15, 12) satisfies $y = x - 3$

PAIR OF TANGENTS

If the point (x_1, y_1) is outside the parabola, then two tangents can be drawn. The equation of pair of tangents drawn to the parabola $y^2 = 4ax$ is given by $SS_1 = T^2$

i.e. $(y^2 - 4ax)(y_1{}^2 - 4ax_1) = [yy_1 - 2a(x + x_1)]^2$

Locus of point of Intersection of tangents :

The locus of point of intersection of tangent to the parabola $y^2 = 4ax$ which are having an angle θ between them is given by $y^2 - 4ax = (a + x)^2 \tan^2\theta$

Note :

(i) If $\theta = 0°$ or π then locus is $(y^2 - 4ax) = 0$ which is the given parabola.

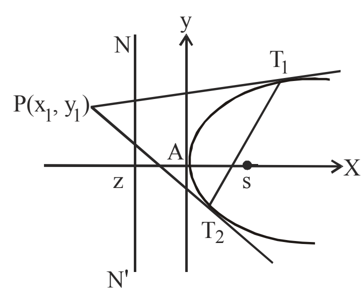
(ii) If $\theta = 90°$, then locus is $x + a = 0$ which is the directrix of the parabola.

CHORD OF CONTACT

The equation of chord of contact of tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is

$yy_1 = 2a\,(x + x_1)$

This equation is same as equation of the tangents at the point (x_1, y_1)



Note :

(i) The chord of contact joining the point of contact of two perpendicular tangents always passes through focus.

(ii) Length of the chord of contact is

$$\frac{1}{a}\sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$$

(iii) Area of triangle formed by tangents drawn from (x_1, y_1) and their chord of contact is $\frac{1}{2a}(y_1{}^2 - 4ax_1)^{3/2}$

Example 14 :

Find the area of triangle made by the chord of contact and tangents drawn from point (4, 6) to the parabola $y^2 = 8x$.

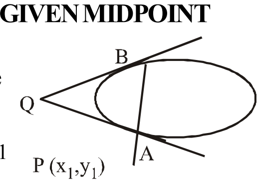
Sol. Area $= \frac{1}{2a}(y_1{}^2 - 4ax_1)^{3/2}$ Here $a = 2$, $(x_1, y_1) = (4, 6)$

$\therefore$ Area of triangle $= \frac{1}{2.2}(36 - 32)^{3/2} = \frac{(4)^{3/2}}{4} = 2$

EQUATION OF THE CHORD WITH GIVEN MIDPOINT

The equation of the chord at the parabola $y^2 = 4ax$ bisected at the point (x_1, y_1) is given by $T = S_1$

i.e. $yy_1 - 2a(x + x_1) = y_1{}^2 - 4ax_1$



POLE & POLAR

The locus of the point of intersection of the tangents to the parabola at the ends of a chord drawn from a fixed point P is called the Polar of point P and the point P is called the pole of the polar.

Equation of Polar : Equation of polar of the point (x_1, y_1) with respect to parabola $y^2 = 4ax$ is same as chord of contact and is given by $yy_1 = 2a(x + x_1)$.

Coordinates of Pole: The pole of the line $\ell x + my + n = 0$ with respect to the parabola $y^2 = 4ax$ is $\left(\frac{n}{\ell}, -\frac{2am}{\ell}\right)$

Conjugate points and conjugate lines :

(i) If the polar of a point A with respect to a parabola passes through another point B then the polar of point B always pass through the point A . The point A and B are called conjugate points with respect to parabola.

$\therefore$ Two points (x_1, y_1) and (x_2, y_2) are conjugate points with respect to parabola $y^2 = 4ax$ if $yy_1 = 2a(x_1 + x_2)$

(ii) If the poles of a line L_1 with respect to a parabola lies on another line L_2 , then the pole of L_2 will always lie on L_1 such lines are known as conjugate lines with respect to the parabola.

$\therefore$ Two lines $\ell_1 x + m_1 y + n_1 = 0$ and $\ell_2 x + m_2 y + n_2 = 0$ are conjugate lines with respect to parabola

$y^2 = 4ax$ if $(\ell_1 n_2 + \ell_2 n_1) = 2am_1 m_2$

Note:

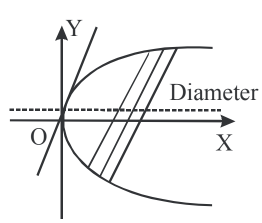
(i) The chord of contact and polar of any point on the directrix always passes through focus.

(ii) The pole of a focal chord lies on directrix and locus of poles of focal chord is a directrix.

(iii) The polars of all points on directrix always pass through a fixed point and this fixed point is focus.

(iv) The polar of focus is directrix and pole of directrix is focus.

DIAMETER OF THE PARABOLA



The locus of the mid points of a system of parallel chords of a parabola is called a diameter of the parabola.

The equation of a system of parallel chord $y = mx + c$ with respect to the parabola $y^2 = 4ax$ is given by $y = \frac{2a}{m}$

Properties of Diameter :

(i) Every diameter of a parabola is parallel to its axis.

(ii) The tangent at the end point of a diameter is parallel to corresponding system of parallel chords.

(iii) The tangents at the ends of any chord meet on the diameter which bisects the chord.

GEOMETRICAL PROPERTIES OF THE PARABOLA

1. The sub tangent at any point on the parabola is twice the abscissa or proportional to square of the ordinate of the point.

2. The sub normal is constant for all points on the parabola and is equal to its semi latusrectum 2a.

3. The semi latus rectum of a parabola is the H. M. between the segments of any focal chord of a parabola i.e. if PQR is a focal chord, then $2a = \frac{2PQ.QR}{PQ + QR}$

4. The tangents at the extremities of any focal chord of a parabola intersect at right angles and their point of intersection lies on directrix i.e. the locus of the point of intersection of perpendicular tangents is directrix.

5. If the tangent and normal at any point P of parabola meet the axes in T and G respectively, then

(a) ST = SG = SP

(b) $\angle$ PSK is a right angle, where K is the point where the tangent at P meets the directrix.

(c) The tangent at P is equally inclined to the axis and the focal distance.

6. The locus of the point of intersection of the tangent at P and perpendicular from the focus on this tangent is the tangent at the vertex of the parabola.

7. If a circle intersect a parabola in four points, then the sum of their ordinates is zero.

8. If vertex and focus of a parabola are on the x-axis and at distance a and a' from origin respectively then equation of parabola $y^2 = 4(a' - a)(x - a)$

9. The area of triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

10. The tangent at any point P on the parabola bisects the angle between the foci chord through P and the perpendicular from P on the directrix.

11. The portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.

TRY IT YOURSELF-1

Q.1 Find the equation of the parabola that satisfies the given conditions: (i) Focus (6, 0); directrix x = – 6

(ii) Vertex (0, 0) ; focus (3, 0)

Q.2 An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

Q.3 If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet passes through a fixed point whose co-ordinates are –

(A) (–2a, 0) (B) (a, 0)

(C) (2a, 0) (D) None

Q.4 The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents

(A) a pair of straight lines (B) an ellipse

(C) a parabola (D) a hyperbola

Q.5 If the line x - 1 = 0 is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of 'k' is :
(A) 1/8 (B) 8
(C) 4 (D) 1/4

Q.6 If x + y = k is normal to $y^2 = 12x$, then 'k' is :
(A) 3 (B) 9
(C) – 9 (D) – 3

Q.7 The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x -axis is
(A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x+3)$
(C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x+1)$

Q.8 The locus of the midpoint of the line segment joining the focus to a moving on the parabola $y^2 = 4ax$ is another parabola with directrix
(A) x = – a (B) x = –a/2
(C) x = 0 (D) x = a/2

Q.9 The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is
(A) π/6 (B) π/4
(C) π/3 (D) π/2

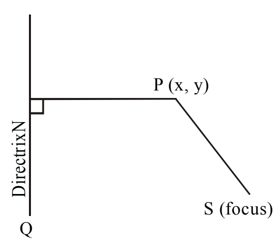
ANSWERS

(1) (i) $y^2 = 24x$, (ii) $y^2 = 12x$ **(2)** 2.23 m (approx.)
(3) (B) **(4)** (C) **(5)** (C)
(6) (B) **(7)** (C) **(8)** (C)
(9) (C)

ELLIPSE

DEFINITION

An ellipse is the locus of a point which moves in a plane so that the ratio of its distance from a fixed point (called focus) and a fixed line (called directrix) is a constant which is less than one. This ratio is called eccentricity and is denoted by e. For an ellipse, e < 1.



Let S be the focus, QN be the directrix and P be any point on the ellipse. Then, by definition $\frac{PS}{PN} = e$ or PS = e PN, e < 1

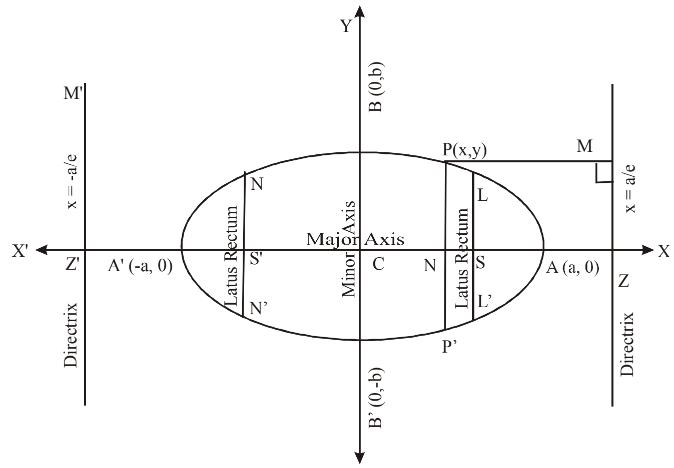
Where PN is the length of the perpendicular from P on the directrix QN.

In other words, an ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (called foci) is constant.

EQUATION OF AN ELLIPSE IN STANDARD FORM :

The **Standard form** of the equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a > b), \text{ where a and b are constants}$$



TERMS RELATED TO AN ELLIPSE

Symmetry :

(a) On replacing y by – y, the above equation remains unchanged. So, the curve is symmetrical about x-axis.

(b) On replacing x by – x, the above equation remains unchanged. So, the curve is symmetrical about y = axis

* **Foci :** S and S' are two foci of the ellipse and their coordinates are (ae,0) and (– ae, 0) respectively, then distance between foci is given by SS' = 2ae.

* **Directrices :** ZM and Z' M' are the two directrices of the ellipse and their equations are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$ respectively, then the distance between directrices is given by

$$ZZ' = \frac{2a}{e}.$$

* **Axes :** The lines AA' and BB' are called the major axis and minor axis respectively of the ellipse
The length of major axis = AA' = 2a
The length of minor axis = BB' = 2b

* **Centre :** The point of intersection C of the axes of the ellipse is called the centre of the ellipse. All chords, passing through C are bisected at C.

* **Vertices :** The end points A (a, 0) and A' (–a, 0) of the major axis are known as the vertices of the ellipse.

* **Focal chord :** A chord of the ellipse passing through its focus is called a focal chord.

* **Ordinate and Double Ordinate :** Let P be a point on the ellipse. From P, draw PN ⊥ AA' (major axis of the ellipse) and produce PN to meet the ellipse at P'. Then PN is called an ordinate & PNP' is called the double ordinate of the point P.

* **Latus Rectum :** If LL' and NN' are the latus rectum of the ellipse, then these lines are ⊥ to the major axis AA' passing through the foci S and S' respectively.

$$L \equiv \left(ae, \frac{b^2}{a}\right), \quad L' \equiv \left(ae, \frac{-b^2}{a}\right),$$

$$N \equiv \left(-ae, \frac{b^2}{a}\right), \quad N' \equiv \left(-ae, \frac{-b^2}{a}\right)$$

Length of latus rectum $= LL' = \dfrac{2b^2}{a} = NN'$

By definition $SP = ePM = e\left(\dfrac{a}{e} - x\right) = a - ex$

and $S'P = e\left(\dfrac{a}{e} + x\right) = a + ex$.

This implies that distances of any point P (x, y) lying on the ellipse from foci are : $(a - ex)$ and $(a + ex)$. In other words.

$SP + S'P = 2a$

i.e. sum of distances of any point P (x, y) lying on the ellipse from foci is constant.

* **Eccentricity** Since, $SP = ePM$, therefore $SP^2 = e^2\,PM^2$

or $(x - ae)^2 + (y - 0)^2 = e^2\left(\dfrac{a}{e} - x\right)^2$

$(x - ae)^2 + y^2 = (a - ex)^2$

$x^2 + a^2e^2 - 2aex + y^2 = a^2 - 2aex + e^2x^2$

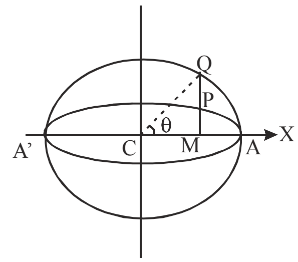
$x^2(1 - e^2) + y^2 = a^2(1 - e^2)$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2(1 - e^2)} = 1$

On comparing with $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, we get

$b^2 = a^2(1 - e^2)$ or $e = \sqrt{1 - \dfrac{b^2}{a^2}}$

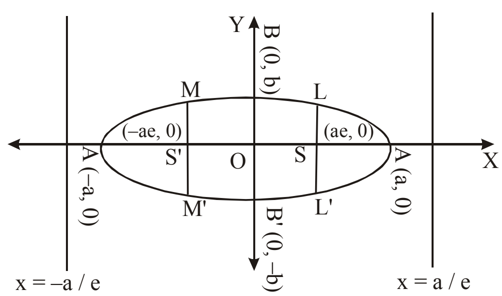
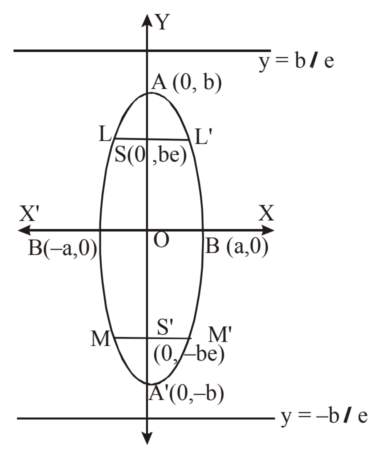
Auxiliary circle : The circle drawn on major axis AA' as diameter is known as the Auxiliary circle.



Let the equation of the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Then the equation of its auxiliary circle is $x^2 + y^2 = a^2$

Let Q be a point on auxiliary circle so that QM, perpendicular to major axis meets the ellipse at P. The points P and Q are called as corresponding point on the ellipse and auxiliary circle respectively. The angle θ is known as eccentric angle of the point P on the ellipse it may be noted that the CQ and CP is inclined at θ with x-axis.

TWO STANDARD FORMS OF THE ELLIPSE

Standard equation	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\ (a > b)$ (Horizontal Form of an Ellipse)	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\ (a < b)$ (Vertical Form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	y = 0	x = 0
Equation of minor axis	x = 0	y = 0
Length of major axis	2a	2b
Length of minor axis	2b	2a
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm \dfrac{a}{e}$	$y = \pm \dfrac{b}{e}$

Eccentricity	$e=\sqrt{\dfrac{a^2-b^2}{a^2}}$	$e=\sqrt{\dfrac{b^2-a^2}{b^2}}$
Length of latus rectum	$2b^2/a$	$2a^2/b$
Ends of latus rectum	$\left(\pm ae, \pm\dfrac{b^2}{a}\right)$	$\left(\pm\dfrac{a^2}{b}, \pm be\right)$
Focal radius	$SP=a-ex_1$ and $S'P=a+ex_1$	$SP=b-ey_1$ and $S'P=b+ey_1$
Sum of focal radii SP + S'P=	2a	2b
Distance between foci	2ae	2be
Distance between directrices	2a/e	2b/e
Tangents at the vertices	$x=\pm a$	$y=\pm b$

Example 15 :

Find the centre, the length of the axes, eccentricity and the .... $12x^2+4y^2+24x-16y+25=0$

Sol. The given equation can be written in the form

$12(x+1)^2+4(y-2)^2=3$ or

$$\frac{(x+1)^2}{1/4}+\frac{(y-2)^2}{3/4}=1 \quad ....(1)$$

Co-ordinates of centre of the ellipse are given by

$x+1=0$ and $y-2=0$

Hence centre of the ellipse is (–1,2)

If a and b be the lengths of the semi major and semi minor axes, then $a^2=3/4$, $b^2=1/4$

$\therefore$ Length of major axis $=2a=\sqrt{3}$

$\therefore$ Length of minor axis $=2b=1$ $\therefore a=\frac{\sqrt{3}}{2},\ b=\frac{1}{2}$

Since $b^2=a^2(1-e^2)$ $\therefore 1/4=3/4(1-e^2)$

$\Rightarrow e=\sqrt{2/3}$ $\qquad \therefore ae=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{\sqrt{3}}=\frac{1}{\sqrt{2}}$,

Co– ordinates of foci are given by $x+1=0$, $y-2=\pm ae$

Thus foci are $\left(-1, 2\pm\frac{1}{\sqrt{2}}\right)$

Example 16 :

The foci of an ellipse are (± 2, 0) and its eccentricity is 1/2, find its equation.

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Then coordinates of foci are (± ae, 0)

$ae=2 \Rightarrow a\times(1/2)=2\Rightarrow a=4 \qquad [\because e=1/2]$

We have $b^2=a^2(1-e^2)$ $\therefore$ $b^2=16\left(1-\frac{1}{4}\right)=12$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$

GENERAL EQUATION OF THE ELLIPSE

The general equation of an ellipse whose focus is (h, k) and the directrix is the line $ax+by+c=0$ and the eccentricity will be e. Then let P (x_1, y_1) be any point on the ellipse which moves such that SP = ePM

$$\Rightarrow (x_1-h)^2+(y_1-k)^2=\frac{e^2(ax_1+by_1+c)^2}{a^2+b^2}$$

Hence the locus of (x_1, y_1) will be given by

$(a^2+b^2)[(x-h)^2+(y-k)^2]=e^2(ax+by+c)^2$

which is the equation of second degree from which we can say that any equation of second degree represent equation of an ellipse.

Note : Condition for second degree in x and y to represent an ellipse is that if $h^2-ab<0$ &

$\Delta=abc+2fgh-af^2-bg^2-ch^2\neq 0$

PARAMETRIC EQUATION OF THE ELLIPSE

The coordinates $x=a\cos\theta$ and $y=b\sin\theta$ satisfy the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for all real values of θ.

Thus $x=a\cos\theta$, $y=b\sin\theta$ are the parametric equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where the parameter $0\le\theta<2\pi$.

Hence the coordinate of any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ may be taken as $(a\cos\theta, b\sin\theta)$. This point is also called the point θ. The angle θ is called the eccentric angle of the point $(a\cos\theta, b\sin\theta)$ on the ellipse.

Equation of Chord : The equation of the chord joining the points $P\equiv(a\cos\theta_1, b\sin\theta_1)$ and $Q\equiv(a\cos\theta_2, b\sin\theta_2)$ is

$$\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

POSITION OF A POINT WITH RESPECT TO AN ELLIPSE

The point P (x_1, y_1) lies outside, on or inside the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ according as $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1>0, =0$ or <0

CONDITION OF TANGENCY AND POINT OF CONTACT

The condition for the line $y=mx+c$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is that $c^2=a^2m^2+b^2$

and the coordinates of the points of contact are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

Note :

(i) $x\cos a + y\sin a = p$ is a tangent if $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$.

(ii) $\ell x + my + n = 0$ is a tangent if $n^2 = a^2\ell^2 + b^2m^2$.

EQUATION OF TANGENT IN DIFFERENT FORMS

(i) Point form : The equation of the tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at the point } (x_1, y_1) \text{ is } \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Note : The equation of tangent at (x_1, y_1) can also be obtained by replacing x^2 by xx_1, y^2 by yy_1 x by $\frac{x+x_1}{2}$, y by $\frac{y+y_1}{2}$, and xy by $\frac{xy_1+x_1y}{2}$. This method is used only when the equation of ellipse is a polynomial of second degree in x and y.

(ii) Parametric form : The equation of the tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at the point } (a\cos\theta, b\sin\theta) \text{ is}$$

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

(iii) Slope form : The equation of tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ in terms of slope 'm' is } y = mx \pm \sqrt{a^2m^2+b^2}$$

The coordinates of the points of contact are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

Example 17 :

Find the equation of tangent to the ellipse $4x^2 + 9y^2 = 36$ at the point $(3, -2)$

Sol. We have $4x^2 + 9y^2 = 36 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$.

This is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Where $a^2 = 9$ and $b^2 = 4$

We, know that the equation of the tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at } (x_1, y_1) \text{ is } \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

So, the equation of the tangent to the given ellipse at

$(3,-2)$ is $\frac{3x}{9}-\frac{2y}{4}=1$ i.e. $\frac{x}{3}-\frac{y}{2}=1$.

Example 18 :

For what value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$.

Sol. $\because$ Equation of ellipse is $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16}+\frac{y^2}{9}=1$

Comparing this with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then we get $a^2 = 16$ and $b^2 = 9$ and comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore$ m = 1 and $c = \lambda$

If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then

$c^2 = a^2m^2 + b^2$

$\Rightarrow \lambda^2 = 16 \times 1^2 + 9 \Rightarrow \lambda^2 = 25 \quad \therefore \lambda = \pm 5$

EQUATION OF NORMAL IN DIFFERENT FORMS

(i) Point form : The equation of the normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at the point } (x_1, y_1) \text{ is } \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

(ii) Parametric form : The equation of the normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at the point } (a\cos\theta, b\sin\theta) \text{ is}$$

$$ax\sec\theta - by\,\text{cosec}\theta = a^2-b^2 \quad \text{or} \quad \frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

(iii) Slope form : The equation of normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ in terms of slope 'm' is } y = mx \pm \frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$$

Note:

(i) Points of contact are $\left(\pm\frac{a^2}{\sqrt{a^2+b^2m^2}}, \pm\frac{mb^2}{\sqrt{a^2+b^2m^2}}\right)$

(ii) Condition for normality : The line $y = mx + c$ is normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $c^2=\frac{m^2(a^2-b^2)^2}{(a^2+b^2m^2)}$

Example 19 :

Find the condition that the line $lx + my = n$ may be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Sol. Equation of normal to the given ellipse at $(a\cos\theta, b\sin\theta)$ is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2 \quad \text{....... (1)}$$

If the line $lx + my = n$ is also normal to the ellipse then there must be a value of θ for which line (1) and line $\ell x + my = n$ are identical. For that value of θ we have

$$\frac{\ell}{\left(\frac{a}{\cos\theta}\right)}=\frac{m}{-\left(\frac{b}{\sin\theta}\right)}=\frac{n}{(a^2-b^2)} \quad \text{....... (2)}$$

or $\frac{1}{a}\cos\theta = \frac{an}{\ell(a^2-b^2)}$ .... (3)

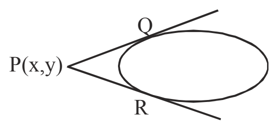
and $\sin\theta = \frac{-bn}{m(a^2-b^2)}$ .... (4)

Squaring and adding (3) and (4),we get

$1 = \frac{n^2}{(a^2-b^2)^2}\left(\frac{a^2}{\ell^2}+\frac{b^2}{m^2}\right)$ which is the required condition.

EQUATION OF THE PAIR OF TANGENTS

The equation of the pair of tangents drawn from a point P (x_1, y_1) to the ellipse



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $SS_1 = T^2$

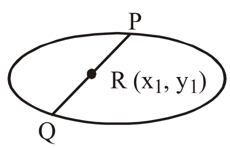
where $S \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}-1,\ S_1 \equiv \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

and $T \equiv \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$

CHORD WITH A GIVEN MID POINT

The equation of the chord of the

ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with P (x_1, y_1)

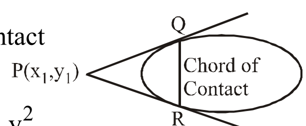


as its middle point is given by $T = S_1$

where $T \equiv \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$ and $S_1 \equiv \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

CHORD OF CONTACT

The equation of chord of contact of tangent drawn from a point

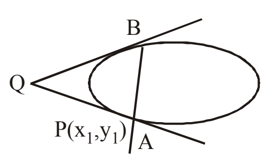


P (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

is $T = 0$ where $T \equiv \frac{xx_1}{a^2}+\frac{yy_1^2}{b^2}-1$

POLE AND POLAR

Let P be a given point. Let a line through P intersect the ellipse at two points A and B. Let the tangents at A &B intersect at Q.



The locus of point Q is a straight line called the polar of point P w.r.t the ellipse and the point P is called the pole of the polar.

The polar of a point P (x_1, y_1) w.r.t the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $T = 0$, where $T \equiv \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$

Note:

(i) Polar of the focus is the directrix

(ii) Any tangent is the polar of its points of contact.

(iii) Pole of a given line $lx + my + n = 0$ w.r.t the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\left(\frac{-a^2\ell}{n},\frac{-b^2m}{n}\right)$

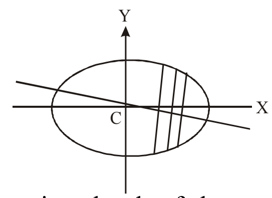
(iv) If the polar of P (x_1, y_1) passes through Q (x_2, y_2) then the polar of Q will pass through P and such points are said to be conjugate points.

(v) If the pole of a line $\ell x + my + n = 0$ lies on the another line $\ell' x + m' y + n' = 0$, then the pole of the second line will lie on the first such lines are said to be conjugate lines.

(vi) The point of intersection of any two lines is the pole of the line joining the pole of the two lines.

DIAMETER OF AN ELLIPSE

The locus of the middle points of a system of a parallel chords of an ellipse is called a diameter of the ellipse.



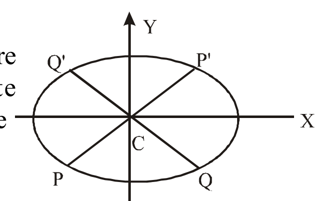
The equation of the diameter bisecting chords of slope m

of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y = \frac{b^2}{a^2m}x$

Note : Diameter of an ellipse always passes through its centre. Thus a diameter of an ellipse is its chord passing through its centre.

CONJUGATE DIAMETERS

The diameters of an ellipse are said to be conjugate diameters if each bisects the chord parallel to the other.



Note :

(i) Major and minor axes of an ellipse is also a pair of conjugate diameters.

(ii) If m_1 and m_2 be the slopes of the conjugate diameters of an

ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $m_1m_2 = \frac{-b^2}{a^2}$

(iii) The eccentric angles of the ends of a pair of conjugate diameters differ by a right angle.

i.e. if PCP' and QCQ' is a pair of conjugate diameters and if eccentric angle of P is θ, then eccentric angles of Q, P', Q' (proceeding in anticlockwise direction) will be

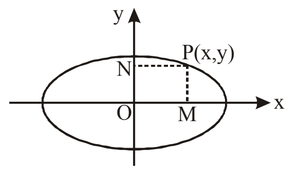
$\theta+\frac{\pi}{2}, \theta+\pi$ and $\theta+\frac{3\pi}{2}$ respectively.

Equation of an ellipse referred to two perpendicular lines :

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given ellipse

Let P(x, y) be any point on the ellipse, then

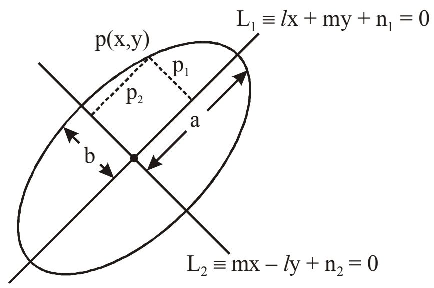
PM = y & PN = x



$\therefore$ above equation can be written as $\frac{PN^2}{a^2}+\frac{PM^2}{b^2}=1$

From above we conclude that if perpendicular distances p_1 & p_2 of a moving point P(x, y) from two mutually perpendicular straight lines $L_1 \equiv \ell x + my + n_1 = 0$ & $L_2 \equiv mx - \ell y + n_2 = 0$ respectively then equation of ellipse in the plane of line will be $\frac{p_2^2}{a^2}+\frac{p_1^2}{b^2}=1$

$$\Rightarrow \frac{\left(\frac{mx-\ell y+n_2}{\sqrt{\ell^2+m^2}}\right)^2}{a^2}+\frac{\left(\frac{\ell x+my+n_1}{\sqrt{\ell^2+m^2}}\right)^2}{b^2}=1$$



Example 20 :

Find the equation the ellipse whose axis are of length 6 & $2\sqrt{6}$ & their equations are $x-3y+3=0$ & $3x+y-1=0$.

Sol. Equation of ellipse will be

$$\frac{\left(\frac{x-3x+3}{\sqrt{10}}\right)^2}{(\sqrt{6})^2}+\frac{\left(\frac{3x+y-1}{\sqrt{10}}\right)^2}{(3)^2}=1$$

TRY IT YOURSELF-2

Q.1 Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Q.2 Find the equation for the ellipse that satisfies the given conditions:
(i) Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$
(ii) Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Q.3 An arch is in the form of a semi-ellipse. It is 8 m wide and 2m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Q.4 If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the midpoint of the intercept made by the tangents between the coordinate axes is

(A) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (B) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

(C) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (D) $\frac{x^2}{4}+\frac{y^2}{2}=1$

Q.5 The minimum area of triangle formed by the tangent to the $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and coordinate axes is

(A) ab sq. units (B) $\frac{a^2+b^2}{2}$ sq. units

(C) $\frac{(a+b)^2}{2}$ sq. units (D) $\frac{a^2+ab+b^2}{3}$ sq. units

Q.6 Find the equation of the common tangent in 1^{st} quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes.

(A) $y=-\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}, \frac{14}{\sqrt{3}}$ (B) $y=\frac{1}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}, \frac{15}{\sqrt{3}}$

(C) $y=\frac{2}{\sqrt{3}}x-4\sqrt{\frac{7}{3}}, \frac{11}{\sqrt{3}}$ (D) $y=\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}, \frac{7}{\sqrt{3}}$

Q.7 Let P (x_1, y_1) and Q (x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2+2\sqrt{3}y=3+\sqrt{3}$ (B) $x^2-2\sqrt{3}y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}y=3+\sqrt{2}$ (D) $x^2-2\sqrt{3}y=3-\sqrt{3}$

Q.8 Tangents are drawn from the point P (3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B. The coordinates of A and B are –

(A) (3, 0) and (0, 2) (B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2) (D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

ANSWERS

(1) Focus : $(-2\sqrt{5}, 0)$, $(2\sqrt{5}, 0)$
Vertices : (–6, 0) and (6, 0) ; $2a = 12$; $2b = 8$
Eccentricity : $\sqrt{5}/3$; Length of latus rectum : 16/3

(2) (i) $\frac{x^2}{25}+\frac{y^2}{9}=1$, (ii) $\frac{x^2}{1}+\frac{y^2}{5}=1$ **(3)** 1.56m

(4) (A) **(5)** (A) **(6)** (A)
(7) (B) **(8)** (D)

HYPERBOLA

A Hyperbola is the locus of a point which moves in a plane so that the ratio of its distance from a fixed point (called focus) and a fixed line (called directrix) is a constant which is greater than one, this ratio is called eccentricity and is denoted by e. For hyperbola e > 1.

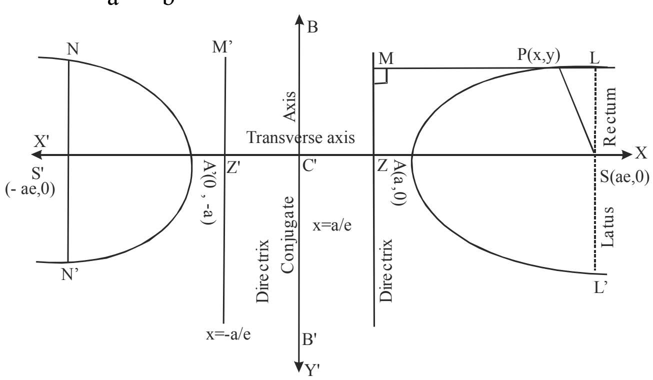
Let S be the focus, QN be the directrix and P be any point on the hyperbola. Then by definition $\frac{PS}{PN}$ or PS=ePN, e >1, where PN is the length of the perpendicular from P on the directrix QN.

In other words a hyperbola is the locus of a point which moves in such a way that the difference of its distances from two fixed points (called foci) is constant.

EQUATION OF HYPERBOLA IN STANDARD FORM

The general form of standard hyperbola is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$ where a and b are constants



TERMS RELATED TO HYPERBOLA

Symmetry : Since only even powers of x and y occur in the above eq, so the curve is symmetrical about both the axes.

Foci : S and S' are the two foci of the hyperbola and their coordinates are (ae, 0) and (–ae, 0) respectively, then distance between foci is given by SS' = 2ae.

Directries : ZM and Z'M' are the two directrices of the hyperbola and their equations are $x=\frac{a}{e}$ and $x=-\frac{a}{e}$ respectively, then the distance between directrices is given by $ZZ'=\frac{2a}{e}$

* **Axes :** The lines AA' and BB' are called the transverse axis and conjugate axis respectively of the hyperbola.
The length of transverse axis = AA' = 2a
The length of conjugate axis = BB' = 2b

* **Centre** The point of intersection C of the axes of hyperbola is called the centre of the hyperbola. All chords passing through C, are bisected at C.

* **Vertices :** The point A ≡ (a, 0) and A' ≡ (–a, 0) where the curve meets the line joining the foci S and S' are called the vertices of the hyperbola.

* **Focal chord :** A chord of the hyperbola passing through its focus is called a focal chord.

* **Focal Distances of a Point** The difference of the focal distances of any point on the hyperbola is constant and equal to the length of the transverse axis of the hyperbola. If P is any hyperbola, then S'P – SP = 2a = Transverse axis.]

* **Latus Rectum** If LL' and NN' are the latus rectum of the hyperbola then these lines are perpendicular to the transverse axis AA', passing through the foci S and S' respectively.

$$L\equiv\left(ae,\frac{b^2}{a}\right),\ L'\equiv\left(ae,\frac{-b^2}{a}\right);\ N\equiv\left(-ae,\frac{b^2}{a}\right),\ N'\equiv\left(-ae,\frac{-b^2}{a}\right)$$

Length of latus rectum = $LL'=\frac{2b^2}{a}=NN'$

* **Eccentricity of the Hyperbola** We know that

$SP = e\,PM$ or $SP^2 = e^2 PM^2$

or $(x-ae)^2+(y-0)^2=e^2\left(x-\frac{a}{e}\right)^2$

$(x-ae)^2+y^2=(ex-a)^2$

$x^2+a^2e^2-2aex+y^2=e^2x^2-2aex+a^2$

$x^2(e^2-1)-y^2=a^2(e^2-1)$

$\frac{x^2}{a^2}-\frac{y^2}{a^2(e^2-1)}=1$. On comparing with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

we get $b^2=a^2(e^2-1)$ or $e=\sqrt{1+\frac{b^2}{a^2}}$

PARAMETRIC EQUATIONS OF THE HYPERBOLA

Since coordinate $x = a\sec\theta$ and $y = b\tan\theta$ satisfy the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, for all real values of θ therefore,

$x = a\sec\theta$,

$y = b\tan\theta$ are the parametric equations of the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, where the parameter $0\le\theta<2\pi$

The coordinates of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ may be taken as $(a\sec\theta, b\tan\theta)$. This point is also called the point 'θ' The angle θ is called the eccentric angle of the point $(a\sec\theta, b\tan\theta)$ on the hyperbola.

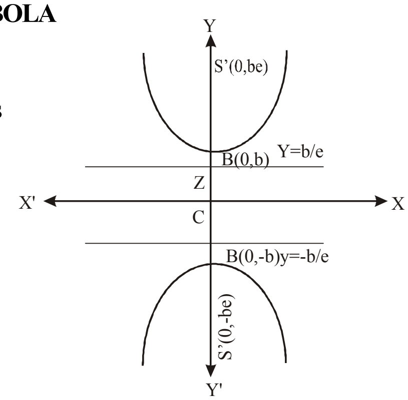
Equation of Chord : The equation of the chord joining the points $P\equiv(a\sec\theta_1, b\tan\theta_1)$ and $Q\equiv(a\sec\theta_2, b\tan\theta_2)$ is

$$\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)$$

or $$\begin{vmatrix} x & y & 1 \\ a\sec\theta_1 & b\tan\theta_1 & 1 \\ a\sec\theta_2 & b\tan\theta_2 & 1 \end{vmatrix}=0$$

CONJUGATE HYPERBOLA

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of a given hyperbola is called the conjugate hyperbola of the given hyperbola.



The conjugate hyperbola of the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ i.e. } \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

is $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ i.e. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

PROPERTIES OF HYPERBOLA AND ITS CONJUGATE

	Hyperbola	Conjugate Hyperbola
Standard equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{-x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Centre	(0, 0)	(0, 0)
Eq. of transverse axis	y = 0	x = 0
Eq. of conjugate axis	x = 0	y = 0
Length of transverse axis	2a	2b
Length of conjugate axis	2b	2a
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm a/e$	$y = \pm b/e$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e=\sqrt{\frac{a^2+b^2}{a^2}}$	$e=\sqrt{\frac{b^2+a^2}{b^2}}$
Length of latus rectum	$2b^2/a$	$2a^2/b$
Parameter Coordinates	$(a\sec\theta, b\tan\theta)$	$(b\sec\theta, a\tan\theta)$
Focal radii	$SP = ex_1 - a$ and $S'P = ex_1 + a$	$SP = ey_1 - b$ and $S'P = ey_1 + b$
Difference of focal radii (S'P – SP)	2a	2b
Tangent of the vertices	$x = \pm a$	$y = \pm b$

Example 21 :

Find the eccentricity of the hyperbola $16x^2 - 32x - 3y^2 + 12y = 44$

Sol. We have, $16(x^2 - 2x) - 3(y^2 - 4y) = 44$

$\Rightarrow 16(x-1)^2 - 3(y-2)^2 = 48$

$$\Rightarrow \frac{(x-1)^2}{3}-\frac{(y-2)^2}{16}=1$$

This equation represents a hyperbola with eccentricity

$$\text{given } e=\sqrt{1+\left(\frac{\text{Conjugate axis}}{\text{Transverse axis}}\right)}=\sqrt{1+\left(\frac{4}{\sqrt{3}}\right)^2}=\sqrt{\frac{19}{3}}$$

Example 22 :

Find the equation of the hyperbola whose directrix is $2x + y = 1$, focus (1, 2) and eccentricity $\sqrt{3}$.

Sol. Let P (x, y) be any point on the hyperbola and PM is perpendicular from P on the directrix,

Then by definition SP = e PM

$\Rightarrow (SP)^2 = e^2 (PM)^2$

$$\Rightarrow (x-1)^2+(y-2)^2=3\left\{\frac{2x+y-1}{\sqrt{4+1}}\right\}^2$$

$\Rightarrow 5(x^2 + y^2 - 2x - 4y + 5) = 3(4x^2 + y^2 + 1 + 4xy - 2y - 4x)$

$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 9y - 22 = 0$

which is the required hyperbola

POSITION OF A POINT WITH RESPECT TO A HYPERBOLA

The point P (x_1, y_1) lies outside,on or inside the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ according as $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1>0=0$ or <0

CONDITION FOR TANGENCY AND POINTS OF CONTACT

The condition for the line $y = mx + c$ to be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is that $c^2 = a^2m^2 - b^2$ and the points of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

Example 23 :

Show that the line $x\cos\alpha + y\sin\alpha = p$ touches the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ If $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$

Sol. The given line is

$x\cos\alpha + y\sin\alpha = p \Rightarrow y\sin\alpha = -x\cos\alpha + p$

$\Rightarrow\ y = -x\cot\alpha + p\,\text{cosec}\,\alpha$

Comparing this lies with $y = mx + c$

$m = -\cot\alpha,\ c = p\,\text{cosec}\,\alpha$

Since the given line touches the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then

$c^2 = a^2m^2 - b^2 \Rightarrow p^2\,\text{cosec}^2\alpha = a^2\cot^2\alpha - b^2$

or $p^2 = a^2\cos^2\alpha - b^2\sin^2\alpha$

EQUATION OF TANGENT IN DIFFERENT FORMS

* **Point Form** The equation of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

Note : The equation of tangent at (x_1, y_1) can also be obtained by replacing x^2 by xx_1, y^2 by yy_1, x by $\frac{x+x_1}{2}$ y by $\frac{y+y_1}{2}$ and xy by $\frac{xy_1+x_1y}{2}$. This method is used only when the equation of hyperbola is a polynomial of second degree in x and y.

* **Parametric Form** The eq^n of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\theta, b\tan\theta)$ is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

* **Slope Form** The equation of tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ in terms of slope 'm' is $y = mx \pm \sqrt{a^2m^2-b^2}$

The points of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

EQUATION OF NORMAL IN DIFFERENT FORMS

Point Form : The equation of the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_1, y_1) is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

Parametric Form : The equation of the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\theta, b\tan\theta)$ is

$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

Slope Form : The equation of normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ in terms of slope 'm' is $y = mx \pm \frac{m(a^2+b^2)}{\sqrt{a^2-b^2m^2}}$

NOTE

(i) Number of Normals : In general, four normals can be drawn to a hyperbola from a point in its plane i.e. there are four points on the hyperbola, the normals at which will pass through a given point. These four points are called the co-normal points

(ii) Tangent drawn at any point bisects the angle between the lines joining the point to the foci, where as normal bisects the supplementary angle between the lines.

EQUATION OF THE PAIR OF TANGENTS

The equation of the pair of tangents drawn from a point P (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $SS_1 = T^2$

where $S \equiv \frac{x^2}{a^2}-\frac{y^2}{b^2}-1$, $S_1 \equiv \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$

and $T \equiv \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$

CHORD WITH A GIVEN MID POINT

The equation of chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with P (x_1, y_1) as its middle point is given by $T = S_1$ where

$T \equiv \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$ and $S_1 \equiv \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$

CHORD OF CONTACT

The equation of chord of contact of tangent drawn from a point P (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $T = 0$

where $T \equiv \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$

Example 24 :

Find the locus of the mid points of the chords of the circle $x^2+y^2=16$, which are tangent to the hyperbola $9x^2-16y^2=144$.

Sol. Any tangent to hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is

$y = mx + \sqrt{(16m^2 - 9)}$ ..(i)

Let (x_1, y_1) be the mid point of the chord of the circle $x^2 + y^2 = 16$, then equation of the chord is

$xx_1 + yy_1 - (x_1^2 + y_1^2) = 0$ ...(ii) $(T = S_1)$

Since (i) and (ii) are same, comparing, we get

$$\frac{m}{x_1} = -\frac{1}{y_1} = \frac{\sqrt{16m^2 - 9}}{-(x_1^2 + y_1^2)}$$

$\Rightarrow m = -\frac{x_1}{y_1}, (x_1^2 + y_1^2)^2 = y_1^2 (15m^2 - 9)$

Eliminating m and generalizing (x_1, y_1) required locus is

$(x^2 + y^2)^2 = 16x^2 - 9y^2$

POLE AND POLAR

The polar of a point P (x_1, y_1) w.r.t. the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is T = 0, where $T \equiv \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$

Note :

(i) Pole of a given line $lx + my + n = 0$ w.r.t the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\left(\frac{-a^2\ell}{n}, \frac{-b^2m}{n}\right)$

(ii) Polar of the focus is the directrix.

(iii) Any tangent is the polar of its point of contact.

(iv) If the polar of P (x_1, y_1) passes through Q (x_2, y_2) then the polar of Q will pass through P and such points are said to the conjugate points.

(v) If the pole of a line $\ell x + my + n = 0$ lies on the another line $\ell' x + m'y + n' = 0$, then the pole of the second line will lie on the first and such lines are said to be conjugate lines.

EQUATION OF A DIAMETER OF A HYPERBOLA

The equation of the diameter bisecting chords of slope m of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = \frac{b^2}{a^2 m}$

CONJUGATE DIAMETERS

Two diameters of a hyperbola are said to be conjugate diameters if each bisects the chord parallel to the other. If m_1 and m_2 be the slopes of the conjugate diameters of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $m_1 m_2 = \frac{b^2}{a^2}$

ASYMPTOTES OF HYPERBOLA

The lines $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ i.e., $y = \pm \frac{bx}{a}$ are called the asymptotes of the hyperbola. The curve comes close to these lines as $x \to \infty$ or $x \to -\infty$ but never meets them. In other words, asymptote to a curve touches the curve at infinity.

Note :

(i) The angle between the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\tan^{-1}(b/a)$.

(ii) A hyperbola and its conjugate hyperbola have the same asymptotes.

(iii) The asymptotes pass through the centre of the hyperbola.

(iv) The bisector of the angle between the asymptotes are the coordinate axes.

(v) The product of the perpendicular from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is a constant equal to $\frac{a^2b^2}{a^2 + b^2}$

(vi) Any line drawn parallel to the asymptote of the hyperbola would meet the curve only at one point.

RECTANGULAR HYPERBOLA

If asymptotes of the standard hyperbola are perpendicular to each other, then it is known as Rectangular Hyperbola.

Then $2\tan^{-1}\frac{b}{a} = \frac{\pi}{2} \Rightarrow b = a$ or $x^2 - y^2 = a^2$

is general form of the equation of the rectangular hyperbola. If we take the coordinate axes along the asymptotes of a rectangle hyperbola then equation of rectangular hyperbola becomes : $xy = c^2$, where c is any constant.

In Parametric form, the equation of rectangular hyperbola

$x = ct, y = c/t$, where t is the parameter

The point (ct, c/t) on the hyperbola $xy = c^2$ is generally referred as the point 't'

Properties of Rectangular Hyperbola, $x^2 - y^2 = t^2$

(i) The equation of asymptotes of the rectangular hyperbola are $y = \pm x$.

(ii) The transverse and conjugate axes of a rectangular hyperbola are equal in length.

(iii) Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$

TRY IT YOURSELF-3

Q.1 Find the coordinates of the foci and the vertices, the eccentricity and the length of the LR of $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Q.2 Find the eq. of the hyperbola satisfying the conditions.

(i) Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

(ii) Foci $(0, \pm 13)$, the conjugate axis is of length 24.

Q.3 If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is –

(A) 1 (B) 5 (C) 7 (D) 9

Q.4 The eq. $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represent a hyperbola
(A) The length of the transverse axes is 4
(B) Length of latus rectum is 9
(C) Equation of directrix is $x = 21/5$ and $x = -11/5$.
(D) None of these

Q.5 For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α'
(A) abscissa of vertices (B) abscissa of foci
(C) eccentricity (D) directrix

Q.6 The line $2x + \sqrt{6}\,y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$ at
(A) $(4, -\sqrt{6})$ (B) $(2, -2\sqrt{6})$
(C) $(-4, \sqrt{6})$ (D) $(-2, 2\sqrt{6})$

Q.7 Let P (6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is –
(A) $\sqrt{5/2}$ (B) $\sqrt{3/2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Q.8 Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are
(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
(C) $(3\sqrt{3}, -2\sqrt{2})$ (D) Both (A) and (B)

ANSWERS

(1) Foci = (± 5, 0) ; vertices = (± 4, 0)
Eccentricity = 5/4 ; Latus rectum = 9/2

(2) (i) $\frac{x^2}{4} - \frac{y^2}{5} = 1$, (ii) $144y^2 - 25x^2 = 3600$

(3) (C) **(4)** (C) **(5)** (B)
(6) (A) **(7)** (B) **(8)** (D)

IMPORTANT POINTS

* In equation, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.
If $A = C \neq 0$ then conic section is Circle.
If $A \neq C$, A and C have the same sign then conic section is Ellipse.
If A and C have opposite signs then conic section is Hyperbola.
If $A = 0$ or $C = 0$ (but not both) then conic section is Parabola.

PARABOLA

* The equation of the parabola with focus at (a, 0) $a > 0$ and directrix $x = -a$ is $y^2 = 4ax$.

* Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the hyperbola.

* Length of the latus rectum of the parabola $y^2 = 4ax$ is 4a.

* **Equation of tangent of standard parabola :**

Equation of Parabolas	Tangent at (x_1, y_1)
$y^2 = 4ax$	$yy_1 = 2a(x + x_1)$
$y^2 = -4ax$	$yy_1 = -2a(x + x_1)$
$x^2 = 4ay$	$xx_1 = 2a(y + y_1)$
$x^2 = -4ay$	$xx_1 = -2a(y + y_1)$

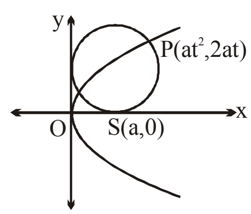
* The equation of tangent, condition of tangency and point of contact in terms of slope (m) of standard parabolas are shown below in the table.

Equation of parabolas	Point of contact in terms of slope (m)	Equation of tangent in terms of slope (m)	Condition of tangency
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$	$c = \frac{a}{m}$
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$	$c = -am^2$

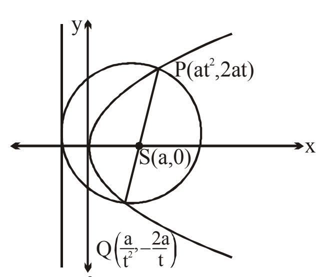
Equation of parabolas	Point of contact in terms of slope (m)	Equation of normals in terms of slope (m)	Condition of normality
$y^2 = 4ax$	$(am^2, -2am)$	$y = mx - 2am - am^3$	$c = -2am - am^3$
$x^2 = 4ay$	$\left(-\frac{2a}{m}, \frac{a}{m^2}\right)$	$y = mx + 2a + \frac{a}{m^2}$	$c = 2a + \frac{a}{m^2}$

Properties of Parabola :

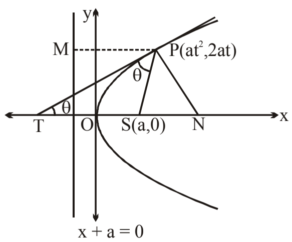
(1) Circle described on the focal length (distance) as diameter touches the tangent at the vertex.



(2) Circle described on the focal chord as diameter touches directrix

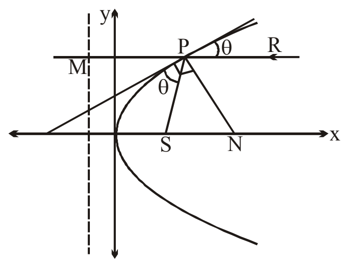


(3) Tangent and Normal at any point P bisect the angle between PS and PM internally and externally. This property leads to the reflection property of parabola.



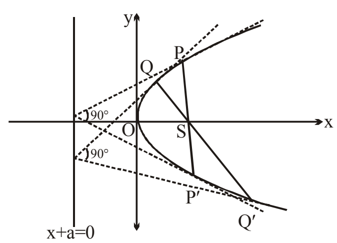
Circle circumscribing the triangle formed by any tangent normal and x-axis, has its centre at focus.

If we extend MP, then from figure ∠RPN = ∠SPN = 90 – θ

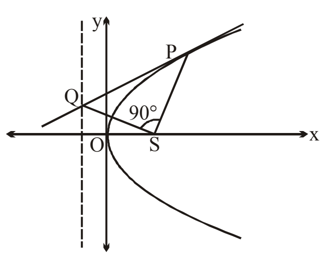


Thus ray parallel axis meet parabola at P and after reflection from P it passes through the focus.

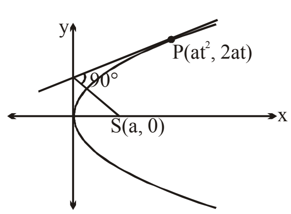
(4) The tangents at the extremities of a focal chord intersect at right angles on the directrix.



(5) The portion of tangent to the parabola intercepted between the directrix & the curve subtends a right angle at the focus.



(6) The foot of the perpendicular from the focus on any tangent to a parabola lies on the tangent at vertex.



(7) **Tangents and Normals** at the extremities of the latus rectum of a parabola $y^2 = 4ax$ **constitute a square**, their points of intersection being (– a, 0) & (3a, 0).

(8) The **circle circumscribing the triangle** formed by **any three tangents** to a parabola passes through the focus.

(9) The **orthocentre** of any triangle formed by three tangents to a parabola $y^2 = 4ax$ lies on the directrix & has the co–ordinates $(-a,\ a(t_1 + t_2 + t_3 + t_1t_2t_3))$.

(10) The area of the triangle formed by three points on a parabola is **twice the area** of the triangle formed by the tangents at these points.

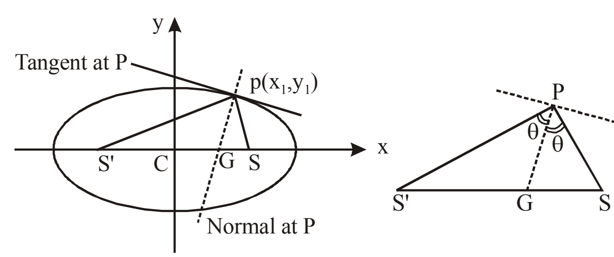
ELLIPSE

	Basic Elements	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	
		a > b	**a < b**
1.	Length of major axis Length of minor axis	2a 2b	2b 2a
2.	Equation of major axis Equation of minor axis	y – k = 0 x – h = 0	x – h = 0 y – k = 0
3.	Centre of ellipse	(h, k)	(h, k)
4.	Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
5.	Foci	(h ± ae, k)	(h, k ± be)
6.	Equation of directrix	$x = h \pm \frac{a}{e}$	$y = k \pm \frac{b}{e}$
7.	Extremities of latus rectum	$\left(h \pm ae,\ k \pm \frac{b^2}{a}\right)$	$\left(h \pm \frac{a^2}{b},\ k \pm be\right)$
8.	Vertices of an ellipse	(h ± a, k)	(h, k ± b)
9.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

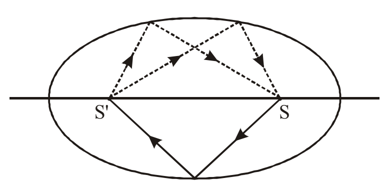
Properties of the ellipse :

(1)

Tangent & normal at any point P bisect the external & internal angles between the focal distances of SP & S'P.

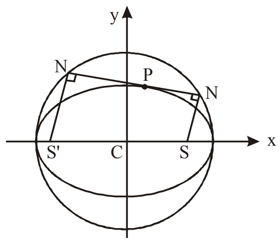


This lead to reflextion property of ellipse.



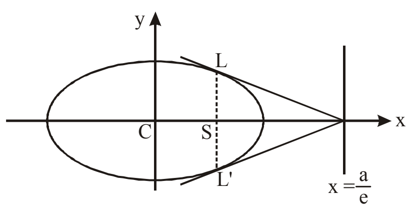
If incoming light ray passes through focus S'(or S), strike the concave side of ellipse the after reflextion it will pass through other focus.

(2) The locus of the point of interseciton of feet of perpendicular from foci on any tangent to an ellipse is the auxiliary circle.



(3) The product of perpendicular distance from the foci to any tangent of an ellipse is equal to square of the semi minor axis.

(4) Tangents at the extremities of latus-rectum of an ellipse intersect on the corresponding directrix.



(5) If P be any point on the ellipse with S & S′ as its foci then $\ell(SP) + \ell(S'P) = 2a$.

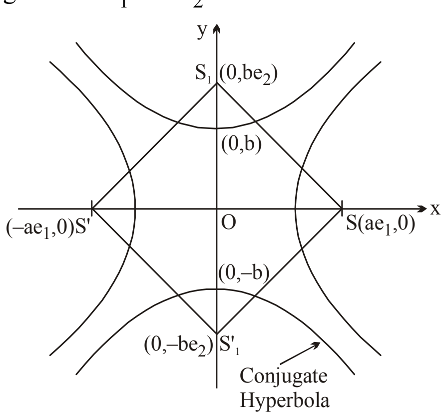
(6) If the normal at any point P on the ellipse with centre C meet the major & minor axes in G & g respectively, & if CF be perpendicular upon this normal, then
(i) $PF. PG = b^2$ (ii) $PF. Pg = a^2$

(7) The portion of the tangent to an ellipse between the point of contact & the directrix subtends a right angle at the corresponding focus.

HYPERBOLA

(1) The foci of a hyperbola and its conjugate are concylic and form the vertices of a square.

(2) If e_1& e_2 are the eccentrcities of the hyperbola & its conjugate then $e_1^{-2} + e_2^{-2} = 1$.



Find all the parameters of a hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 :$$

When equation of the hyperbola is $\dfrac{(x-\alpha)^2}{a^2} - \dfrac{(y-\beta)^2}{b^2} = 1$

This equation is the form of $\dfrac{X^2}{a^2} - \dfrac{Y^2}{b^2} = 1$,

where $X = x - h$ and $Y = y - k$

(1) Length of semi-transverse axis = a, length of semi-conjugate axis = b

(2) Equation of transverse axis is $Y = 0$, i.e, $y - k = 0$
Equation of conjugate axis is $X = 0$, i.e, $x - h = 0$

(3) Coordinates of centre is given by $X = 0$ and $Y = 0$, i.e., $x - h = 0$ and $y - k = 0$
Therefore, centre is (h, k)

(4) Eccentricity of the hyperbola $e = \sqrt{1 + \dfrac{b^2}{a^2}}$

(5) Coordinates of vertices of the hyperbola are given by $X = \pm a, Y = 0$ i.e., $x - h = \pm a, y - k = 0$.
Hence vertices are $(h \pm a, k)$.

(6) Coordinate of foci are given by $X = \pm ae, Y = 0$
i.e., $x - h = \pm ae, y - k = 0$. Hence foci are $(h \pm ae, k)$

(7) Equation of directrices of the hyperbola are

$X = \pm \dfrac{a}{e}$, i.e., $x - h = \pm \dfrac{a}{e}$. Hence directrices are $x = h \pm \dfrac{a}{e}$

(8) Length of latus rectum = $\dfrac{2b^2}{a}$

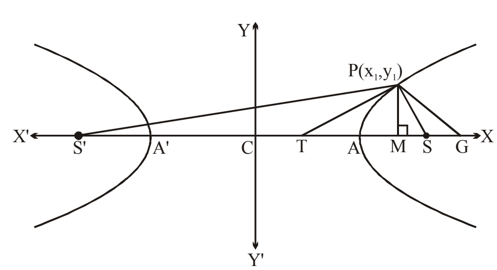
(9) Coordinate of ends of latera recta are given by

$X = ae, Y = \pm \dfrac{b^2}{a}$ i.e. $x - h = \pm ae, y - h = \pm \dfrac{b^2}{a}$

$\therefore$ end if LR is $\left(h \pm ae, k \pm \dfrac{b^2}{a}\right)$

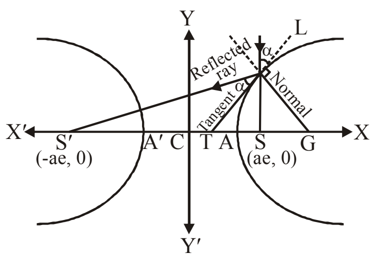
Properties :

(1) The tangent & normal at any point of a hyperbola bisect the angle between the focal radii.



Note : that the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{a^2-k^2}-\frac{y^2}{k^2-b^2}=1\,(a>k>b>0)$ are confocal and therefore orthogonal.

(2) The tangent & normal at any point of a hyperbola bisect the angle between the focal radii. This spells the reflection property of the hyperbola as **"An incoming light ray "** aimed towards one focus is reflected from the outer surface of the hyperbola towards the other focus. It follows that if an ellipse and a hyperbola have the same foci, they cut at right angles at any of their common point.



(3) Locus of the feet of the perpendicular drawn from focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ upon any tangent is its auxiliary circle i.e. $x^2+y^2=a^2$ & the product of the feet of these perpendiculars is b^2 ,i.e., square of semi-conjugate axis. Similar property exists in ellipse also.

(4) The portion of the tangent between the point of contact & the directrix subtends a right angle at the corresponding focus.

(5) The foci of the hyperbola and the points P and Q in which any tangent meets the tangents at the vertices are concyclic with PQ as diameter of the circle.

Properties of rectangular hyperbola $xy=c^2$.

(a) Equation of a chord joining the points P (t_1) & Q(t_2) is $x+t_1t_2y=c\,(t_1+t_2)$ with slope $m=-\frac{1}{t_1t_2}$.

(b) Equation of the tangent at P (x_1, y_1) is $\frac{x}{x_1}+\frac{y}{y_1}=2$ and at P (t) is $\frac{x}{t}+ty=2c$.

(c) Point of intersection of tangents at 't_1' and 't_2' is $\left(\frac{2ct_1t_2}{t_1+t_2},\frac{2c}{t_1+t_2}\right)$

(d) Equation of normal is $y-\frac{c}{t}=t^2(x-ct)$ or $xt^3-yt-ct^4+c=0$

(e) Equation of normal (x_1, y_1) is $xx_1-yy_1=x_1^2-y_1^2$.

(f) Chord with a given middle point as (h, k) is $kx+hy=2hk$.

ADDITIONAL EXAMPLES

Example 1 :

If the focus of a parabola is (1, 0) and its directrix is $x+y=5$, then find its vertex.

Sol. Since axis is a line perpendicular to directrix, so it will be $x-y=k$, it also passes from focus, therefore $k=1$
So equation of axis is $x-y=1$
Solving it with $x+y=5$, we get $Z=(3, 2)$
If vertex is (a, b), then a = 2, b =1. Hence vertex is (2, 1)

Example 2 :

Find the coordinates of an end point of the latus rectum of the parabola $(y-1)^2=4(x+1)$.

Sol. Shifting the origin at (–1, 1) we have

$$\left.\begin{array}{l}x=X-1\\y=Y+1\end{array}\right\}\quad\text{.......(i)}$$

Using (i), the given parabola becomes.
$Y^2=4X$
The coordinates of the endpoints of latus rectum are $(X=1, Y=2)$ and $(X=1, Y=-2)$.
Using (i), the endpoint of the latus rectum are (0,3) & (0,–1).

Example 3 :

Find the equation of the parabola which passes through the point (4,3) and having origin as its vertex and x-axis as its axis

Sol. Since vertex is at (0, 0) and axis is along x-axis, so let the equation of the parabola be $y^2=4ax$
Since it passes through (4, 3), so $9=16a \Rightarrow 4a=9/4$
hence the required equation will be $4y^2=9x$

Example 4 :

Find the slope of tangents drawn from a point (4, 10) to the parabola $y^2=9x$.

Sol. The equation of a tangent of slope m to the parabola $y^2=9x$ is $y=mx+\frac{9}{4m}$

If it passes through (4, 10), then

$$10=4m+\frac{9}{4m}\Rightarrow 16m^2-40m+9=0$$

$\Rightarrow (4m-1)(4m-9) \Rightarrow m=1/4, 9/4$

Example 5 :

Find the locus of the point of intersection of perpendicular tangents to the parabola $x^2 - 8x + 2y + 2 = 0$.

Sol. We know that the locus of the perpendicular tangents of a parabola is its directrix. Now given equation can be written as $(x-4)^2 = -2(y-7)$

Here a = – 1/2, so the required locus i.e. the directrix is

$y - 7 = -(-1/2) \Rightarrow 2y - 15 = 0$

Example 6 :

Tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangent then find the value of tan α.

Sol. Any tangent to $y^2 = 4x$ is $y = mx + 1/m$

If it is drawn from (–2, –1), then

$-1 = -2m + 1/m \Rightarrow 2m^2 - m - 1 = 0$

$_1, m_2$ then $m_1 + m_2 = 1/2, \quad m_1m_2 = -1/2$

$$\tan\alpha = \frac{m_1 - m_2}{1 + m_1m_2} = \frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1 + m_1m_2} = \frac{\sqrt{1/4+2}}{1-1/2} = 3$$

Example 7 :

Find the point of intersection of two tangents drawn at the points where the line 7y– 4x =10 meets the parabola $y^2 = 4x$.

Sol. If (x_1, y_1) is the required point ; then the given line will be the chord of contact of $y^2 = 4x$ with respect to this point and its equation will be $yy_1 = 2(x + x_1)$

Comparing it with ty – 4x = 10, we get

$$\frac{y_1}{7} = \frac{2}{4} = \frac{2x_1}{10} \Rightarrow x_1 = 5/2,\ y_1 = 7/2$$

$\therefore$ required point is (5/2, 7/2)

Example 8 :

A normal is drawn to the parabola $y^2 = 4ax$ at the point $(2a, -2\sqrt{2}a)$ then find the length of the normal chord

Sol. Here comparing $(2a, -2\sqrt{2}a)$ with $(am^2, -2am)$ we get $m = \sqrt{2}$. Now length of normal chord

$$= \frac{4a}{m^2}(1+m^2)^{3/2} = \frac{4a}{2}(1+2)^{3/2} = 2a\,3\sqrt{3} = 6\sqrt{3}a$$

Example 9 :

If the normal to the parabola $y^2 = 4ax$ drawn at (a, 2a) meets the parabola again at the point $(at_2, 2at)$ then find the value of t.

Sol. If t' be the parameter of the given point, then

$2at' = 2a \Rightarrow t' = 1$. Now, $t = -t' - \frac{2}{t'} \Rightarrow t = -1 - \frac{2}{1} = -3$

Example 10 :

Find the length of the chord of parabola $x^2 = 4ay$ passing through the vertex and having slope tanα.

Sol. Let A be the vertex and AP be a chord of $x^2 = 4ay$ such that slope of AP is tan α . Let the coordinates of P be $(2at, at^2)$

Then, Slope of AP = $\frac{at^2}{2at} = \frac{t}{2} \Rightarrow \tan\alpha = \frac{t}{2} \Rightarrow t = 2\tan\alpha$

Now, AP = $\sqrt{(2at-0)^2 + (at^2-0)^2} = at\sqrt{4+t^2}$

$= 2a\tan\alpha\sqrt{4 + 4\tan^2\alpha} = 4a\tan\alpha\sec\alpha$

Example 11 :

If (x_1, y_1) and (x_2, y_2) are extremities of a focal chord of the parabola $y^2 = 4ax$, then find x_1x_2.

Sol. $x = -my - am^3$

Let the given points be $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

Then, $t_1t_2 = -1$,

Now, $x_1x_2 = (at_1^2)(at_2^2) = a^2(t_1t_2)^2$ or $\quad y^2 = a^2(-1)^2 = a^2$

Example 12 :

If LR of an ellipse is half of its minor axis, then find its eccentricity.

Sol. As given $\frac{2b^2}{a} = b \Rightarrow 2b = a \Rightarrow 4b^2 = a^2 \Rightarrow 4a^2(1-e^2) = a^2$

$\Rightarrow 1 - e^2 = 1/4 \quad \therefore e = \sqrt{3}/2$

Example 13 :

Find the equation of tangents to the ellipse $9x^2 + 16y^2 = 144$ which pass through the point (2, 3).

Sol. Ellipse $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Any tangent is $y = mx + \sqrt{16m^2 + 9}$ it passes through (2, 3)

$3 = 2m + \sqrt{16m^2 + 9}$ or $(3 - 2m)^2 = 16m^2 + 9$ or m = 0, – 1

Hence the tangents are y =3, x + y = 5

Example 14 :

Line $\ell x + my + n = 0$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

(1) $a^2\ell^2 + b^2m^2 = n^2$ (2) $a^2\ell^2 - b^2m^2 = n^2$

(3) $a^2b^2 - \ell^2m^2 = n^2$ (4) $a^2b^2 + \ell^2m^2 = n^2$

Sol. (1). $\ell x + my + n = 0 \Rightarrow y = \left(-\frac{\ell}{m}\right)x + \left(-\frac{n}{m}\right)$

$\because$ The line y = mx + c touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$

$\therefore$ Given line will touch the ellipse if

$$\left(-\frac{n}{m}\right)^2 = a^2\left(-\frac{\ell}{m}\right)^2 + b^2 \Rightarrow a^2\ell^2 + b^2m^2 = n^2$$

Example 15 :

Name the conic represented by
$x = 2(\cos t + \sin t)$, $y = 5(\cos t - \sin t)$.

Sol. From given equations, $\frac{x}{2} = \cos t + \sin t$; $\frac{y}{5} = \cos t - \sin t$

Eliminating t from (1) and (2), we have

$$\frac{x^2}{4} + \frac{y^2}{25} = 2 \Rightarrow \frac{x^2}{8} + \frac{y^2}{50} = 1 \text{ which is an ellipse}$$

Example 16 :

The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the real points if

(1) $|t| < 2$ (2) $|t| \le 1$ (3) $|t| > 1$ (4) None of these

Sol. **(2).** Putting $x = at^2$ in the equation of the ellipse, we get

$$\frac{a^2t^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2(1 - t^4)\ ;\ y^2 = b^2(1 - t^2)(1 + t^2)$$

This will give real value of y if $(1 - t^2) \ge 0$; $|t| \le 1$

Example 17 :

Find the centre, the length of the major axes and the eccentricity of the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$

Sol. The given equation can be rewritten as

$2[x^2 - 2x] + 3[y^2 - 4y] + 13 = 0$

or $2(x - 1)^2 + 3(y - 2)^2 = 1$

or $\frac{(x-1)^2}{(1/\sqrt{2})^2} + \frac{(y-2)^2}{(1/\sqrt{3})^2} = 1$, or $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$\therefore$ Centre $X = 0, Y = 0$ i.e., (1, 2)

Length of major axis $= 2a = \sqrt{2}$

and $e = \sqrt{(a^2 - b^2)/a} = 1/\sqrt{3}$

Example 18 :

Find the number of values of c such that the straight line

$y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$.

Sol. We know that the line $y = mx + c$ touches the curve

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$. Here, $a^2 = 4, b^2 = 1, m = 4$

$c^2 = 64 + 1 \Rightarrow c = \pm\sqrt{65}$

Example 19 :

If foci of a hyperbola are foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. If the eccentricity of the hyperbola be 2, then find its equation

Sol. For ellipse $e = 4/5$, so foci $= (\pm 4, 0)$

For hyperbola $e = 2$,

so $a = \frac{ae}{e} = \frac{4}{2} = 2$, $b = 2\sqrt{4-1} = 2\sqrt{3}$

Hence equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Example 20 :

Find the eccentricity of the conjugate hyperbola to the hyperbola $x^2 - 3y^2 = 1$.

Sol. Equation of the conjugate hyperbola to the hyperbola

$$x^2 - 3y^2 = 1 \text{ is } -x^2 + 3y^2 = 1 \Rightarrow -\frac{x^2}{1} + \frac{y^2}{1/3} = 1$$

Here $a^2 = 1, b^2 = 1/3$

eccentricity $e = \sqrt{1 + a^2/b^2} = \sqrt{1+3} = 2$

Example 21 :

Find the distance between directrices of the hyperbola
$x = 8\sec\theta$, $y = 8\tan\theta$

Sol. Hyperbola is $x^2 - y^2 = 64$ which is rectangular hyperbola

So its $e = \sqrt{2}$, Here $a = 8$

Hence distance between directrices $= \frac{2a}{e} = \frac{2(8)}{e} = 8\sqrt{2}$

Example 22 :

Find the product of the length of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes.

Sol. Any point on the given hyperbola is P ($\sqrt{2}\sec\theta, \tan\theta$)

Asymptotes are $x - \sqrt{2}y = 0$, $x + \sqrt{2}y = 0$

Product of perpendiculars from P on these asymptotes

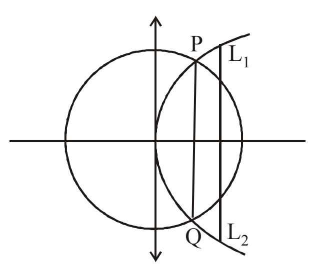
$$= \frac{(\sqrt{2}\sec\theta - \sqrt{2}\tan\theta)(\sqrt{2}\sec\theta + \sqrt{2}\tan\theta)}{1+2}$$

$$= \frac{2\sec^2\theta - 2\tan^2\theta}{3} = \frac{2}{3}$$

Example 23 :

A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus-rectum of the parabola $y^2 = 4ax$. If PQ is the common chord of the circle and the parabola and L_1L_2 is the latusrectum, then find the area of the trapezium PL_1L_2Q

Sol. Centre (0, 0), radius $= \frac{1}{2}.\frac{3}{4}.4a = \frac{3a}{2}$



$\therefore$ Equation of the circle is $4(x^2+y^2)=9a^2$ .... (1)

Equation of the parabola is $y^2=4ax$ .... (2)

Solving (1) and (2)

$$x^2+4ax-\frac{9a^2}{4}=0\ ;$$

$$x=\frac{-4a\pm\sqrt{16a^2+9a^2}}{2}=\frac{-4a\pm 5a}{2}\quad \therefore x=a/2$$

From $x=a/2$, $y^2=4ax=4a(a/2)=2a^2$ $\therefore$ $y=\pm\sqrt{2}a$

$\therefore$ The double ordinate = $2\sqrt{2}a$

$\therefore$ Area of the trap

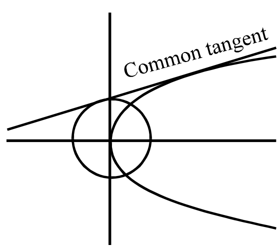
$$=\frac{1}{2}\left(a-\frac{a}{2}\right)(4a+2\sqrt{2}a)=\left(\frac{2+\sqrt{2}}{2}\right)a^2$$

Example 24 :

Find the equations of the straight lines touching both $x^2+y^2=2a^2$ and $y^2=8ax$.

Sol. The given curves are $x^2+y^2=2a^2$ .... (1)

and $y^2=8ax$ .... (2)



The parabola (2) is $y^2=8ax$ or $y^2=4(2a)x$

$\therefore$ Equation of tangent of (2) is $y=mx+\frac{2a}{m}$

or $m^2x-my+2a=0$ ......(3)

It is also tangent of (1), then the length of perpendicular from centre of (1) i.e.

(0, 0) to (3) must be equal to the radius of (1) i..e., $a\sqrt{2}$.

$$\therefore \left|\frac{0-0+2a}{\sqrt{(m^2)^2+(-m)^2}}\right|=a\sqrt{2}\quad\text{or}\quad \frac{4a^2}{m^4+m^2}=2a^2$$

or $m^4+m^2-2=0$ or $(m^2+2)(m^2-1)=0$

$\because m^2+2\neq 0$ (gives the imaginary values)

$\therefore m^2-1=0 \Rightarrow m=\pm 1$. Hence from (3) the required tangents are $x\pm y+2a=0$.

Example 25 :

Find the vertex, axis, directrix, focus, latus rectum and the tangent at vertex for the parabola $9y^2-16x-12y-57=0$

Sol. The given equation can be rewritten as

$$\left(y-\frac{2}{3}\right)^2=\frac{16}{9}\left(x+\frac{61}{16}\right)\text{ which is of the form } Y^2=4AX$$

Hence the vertex is $\left(-\frac{61}{16},\frac{2}{3}\right)$.

The axis is $y-\frac{2}{3}=0\Rightarrow y=\frac{2}{3}$

The directrix is $X+A=0\Rightarrow x+\frac{61}{16}=\frac{4}{9}\Rightarrow x=\frac{-485}{144}$.

Also $\left(-\frac{485}{144},\frac{2}{3}\right)$ is the focus.

Length of the latus rectum = 4A = 16/9.

The tangent at the vertex is $X=0\Rightarrow x=-\frac{61}{16}$.

Example 26 :

Find the equation of the hyperbola whose foci are (8, 3) (0, 3) and eccentricity = 4/3.

Sol. The centre of the hyperbola is the midpoint of the line joining the two foci. So the coordinates of the centre are

$$\left(\frac{8+0}{2},\frac{3+3}{2}\right)\text{ i.e., } (4,\ 3).$$

Let 2a and 2b be the length of transverse and conjugate axes and let e be the eccentricity. Then the equation of the hyperbola is $\frac{(x-4)^2}{a^2}-\frac{(y-3)^2}{b^2}=1$ ... (i)

Now, distance between the two foci = 2ae $\Rightarrow$

$$\sqrt{(8-0)^2+(3-3)^2}=2ae\Rightarrow ae=4\Rightarrow a=3\left(\because e=\frac{4}{3}\right)$$

Now, $b^2=a^2(e^2-1)\Rightarrow b^2=9\left(-1+\frac{16}{9}\right)=7.$

Thus, the equation of the hyperbola is

$\frac{(x-4)^2}{9}-\frac{(y-3)^2}{7}=1$ [Putting the values of a and b in (i)]

$\Rightarrow 7x^2-9y^2-56x+54y-32=0$

QUESTION BANK

CHAPTER 11 : CONIC SECTIONS (PARABOLA, ELLIPSE & HYPERBOLA)

EXERCISE - 1 [LEVEL-1]

PART 1 : PARABOLA

Q.1 The length of the latus rectum of the parabola whose focus is (3, 3) and directrix is $3x - 4y - 2 = 0$ is
(A) 2 (B) 1
(C) 4 (D) None of these

Q.2 The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is
(A) $x = -1$ (B) $x = 1$
(C) $x = -3/2$ (D) $x = 3/2$

Q.3 If the tangents at P and Q on a parabola meet in T, then SP, ST and SQ are in
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.4 The angle between tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x - y - a = 0$, is
(A) $\pi/3$ (B) $\pi/4$
(C) $\pi/6$ (D) $\pi/2$

Q.5 The tangents drawn from the ends of latus rectum of $y^2 = 12x$ meets at
(A) Directrix (B) Vertex
(C) Focus (D) None of these

Q.6 If the tangent and normal at any point P of a parabola meet the axes in T and G respectively, then
(A) $ST \neq SG = SP$ (B) $ST - SG \neq SP$
(C) $ST = SG = SP$ (D) $ST = SG \,.\, SP$

Q.7 Three normals to the parabola $y^2 = x$ are drawn through a point (C, 0), then
(A) $C = 1/4$ (B) $C = 1/2$
(C) $C > 1/2$ (D) None of these

Q.8 The length of the subnormal to the parabola $y^2 = 4ax$ at any point is equal to
(A) $\sqrt{2}a$ (B) $2\sqrt{2}$
(C) $a/\sqrt{2}$ (D) 2a

Q.9 Equation of the parabola, whose vertex and focus are on the x-axis at a distance a and a' from the origin on positive side, is given by –
(A) $y^2 = 4(a' - a)(x - a)$ (B) $y^2 = 4(a' - a)(x - a')$
(C) $y^2 = 4(a - a')(x - a)$ (D) None of these

Q.10 The angle subtended by the double ordinate of length 8a at the vertex of parabola $y^2 = 4ax$ is –
(A) $\pi/4$ (B) $\pi/3$
(C) $\pi/2$ (D) None of these

Q.11 Coordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are –
(A) (0, 2) (B) (2, 1)
(C) (1, 2) (D) (–2, 1)

Q.12 The equation of the tangent to the parabola $y^2 = 4x + 5$ which is parallel to the line $y = 2x + 7$ is –
(A) $y = 2x + 3$ (B) $y = 2x + 4$
(C) $y = 2x$ (D) None of these

Q.13 The locus of the foot of the perpendicular from the focus upon a tangent to the parabola $y^2 = 4ax$ is –
(A) the directrix (B) tangent at the vertex
(C) $x = a$ (D) None of these

Q.14 Chord AB of the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Locus of point of intersection of tangents drawn to the parabola at A and B is given by –
(A) $x + 2a = 0$ (B) $x + a = 0$
(C) $x + 4a = 0$ (D) None of these

Q.15 Equation of common tangent to the parabolas $y^2 = 32x$ and $x^2 = -4y$ is –
(A) $y = 2x + 4$ (B) $y - 2x + 4 = 0$
(C) $x = 2y - 4$ (D) None of these

Q.16 The points on the parabola $y^2 = 12x$ whose focal distance is 4, are
(A) $(2, \sqrt{3}), (2, -\sqrt{3})$ (B) $(1, 2\sqrt{3}), (1, -2\sqrt{3})$
(C) (1, 2) (D) None of these

Q.17 Locus of the poles of focal chords of a parabola is of parabola
(A) The tangent at the vertex (B) The axis
(C) A focal chord (D) The directrix

Q.18 Vertex of the parabola $y^2 + 2y + x = 0$ lies in the quadrant
(A) First (B) Second
(C) Third (D) Fourth

Q.19 The vertex of a parabola is the point (a, b) and latus rectum is of length ℓ. If the axis of the parabola is along the positive direction of y-axis, then its equation is
(A) $(x + a)^2 = \frac{\ell}{2}(2y - 2b)$ (B) $(x - a)^2 = \frac{\ell}{2}(2y - 2b)$
(C) $(x + a)^2 = \frac{\ell}{4}(2y - 2b)$ (D) $(x - a)^2 = \frac{\ell}{8}(2y - 2b)$

Q.20 The focus of the parabola $4y^2 - 6x - 4y = 5$ is
(A) (–8/5, 2) (B) (–5/8, 1/2)
(C) (1/2, 5/8) (D) (5/8, –1/2)

Q.21 The equation of axis of the parabola $2x^2 + 5y - 3x + 4 = 0$ is
(A) $x = 3/4$ (B) $y = 3/4$
(C) $x = -1/2$ (D) $x - 3y = 5$

Q.22 If the line $y = mx + c$ is a tangent to the parabola $y^2 = 4a(x + a)$ then $ma + \frac{a}{m}$ is equal to
(A) c (B) 2c
(C) – c (D) 3c

Q.23 Angle between two curves $y^2 = 4(x+1)$ and $x^2 = 4(y+1)$ is
(A) 0^o (B) 90^o
(C) 60^o (D) 30^o

Q.24 A set of parallel chords of the parabola $y^2 = 4ax$ have their mid-point on
(A) Any straight line through the vertex
(B) Any straight line through the focus
(C) Any straight line parallel to the axis
(D) Another parabola

Q.25 The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible value of the slope of this chord, are
(A) {–1, 1} (B) {–2, 2}
(C) {-2, 1/2} (D) {2, –1/2}

Q.26 If the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola P_2, then vertex of the parabola P_2 is –
(A) $\left(\frac{2}{9}, \frac{-8}{9}\right)$ (B) $\left(\frac{-2}{9}, \frac{8}{9}\right)$ (C) $\left(\frac{2}{9}, \frac{8}{9}\right)$ (D) $\left(\frac{-2}{9}, \frac{-8}{9}\right)$

Q.27 Length of focal chord of $y^2 = 4ax$ inclined at an angle 30° with x-axis is (L.R. is latusrectum)
(A) L.R. (B) 2L.R.
(C) 4L.R. (D) 8L.R.

Q.28 The point (a, 2a) is an interior point of region bounded by the parabola $x^2 = 16y$ and the double ordinate through focus then a belongs to –
(A) $a < 4$ (B) $0 < a < 4$
(C) $0 < a < 2$ (D) $a > 4$

Q.29 A tangent to $y^2 = 16x$ is $y = 4x + 1$. Point on this tangent from which a perpendicular tangent can be drawn to same parabola –
(A) (–4, 0) (B) (–4, –4)
(C) (–4, –15) (D) None of these

Q.30 If line $x + 2y = 2$ intersect parabola $y^2 = 8x$ at points A & B then $\frac{1}{SA} + \frac{1}{SB} =$ {S is focus}
(A) 1/2 (B) 1
(C) 2 (D) 4

Q.31 Focus of parabola $x^2 + 2y + 6x = 0$ is –
(A) (–3, 4) (B) (–3, –4)
(C) (3, –4) (D) (4, –3)

Q.32 The locus of the point of intersection of the tangents drawn at the ends of a focal chord of the parabola $x^2 = -8y$ is –
(A) $x = 2$ (B) $x = -2$
(C) $y = 2$ (D) $y = -2$

Q.33 The condition for the line $y = mx + c$ to be a normal to the parabola $y^2 = 4ax$ is –
(A) $c = -2am - am^3$ (B) $c = -a/m$
(C) $c = a/m$ (D) $c = 2am + am^3$

Q.34 The length of the latus rectum of $3x^2 - 4y + 6x - 3 = 0$ is –
(A) 3 (B) 2
(C) 4/3 (D) 3/4

Q.35 The sum of the reciprocals of focal distances of a focal chord PQ of $y^2 = 4ax$ is –
(A) 1/2a (B) 2a
(C) a (D) 1/a

Q.36 If $x = t^2 + 2$ and $y = 2t$ represent the parametric equation of the parabola –
(A) $(x-2)^2 = 4y$ (B) $x^2 = 4(y-2)$
(C) $(y-2)^2 = 4x$ (D) $y^2 = 4(x-2)$

Q.37 The equation of the tangent to the parabola $y^2 = 4x$ inclined at an angle $\pi/4$ to the +ve direction of x-axis is –
(A) $x + y - 4 = 0$ (B) $x - y + 4 = 0$
(C) $x - y - 1 = 0$ (D) $x - y + 1 = 0$

Q.38 The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of Latus rectum
(A) 18 sq. units (B) 20 sq. units
(C) 17 sq. units (D) 19 sq. units

Q.39 The length of latus rectum of the parabola $4y^2 + 3x + 3y + 1 = 0$ is
(A) 7 (B) 3/4
(C) 4/3 (D) 12

PART 2 : ELLIPSE

Q.40 The eccentric angles of the extremities of latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is-
(A) $\tan^{-1}\left(\frac{\pm ae}{b}\right)$ (B) $\tan^{-1}\left(\frac{\pm be}{a}\right)$
(C) $\tan^{-1}\left(\frac{\pm b}{ae}\right)$ (D) $\tan^{-1}\left(\frac{\pm a}{be}\right)$

Q.41 The line $\ell x + my + n = 0$ cut the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angle differ by $\pi/2$. Then the value of $a^2\ell^2 + b^2m^2$ is-
(A) $2n^2$ (B) $2n$
(C) $2m^2$ (D) $2m$

Q.42 Product of the perpendiculars from the foci upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is-
(A) b (B) a
(C) a^2 (D) b^2

Q.43 The equation of the ellipse which passes through origin and has its foci at the points (1, 0) and (3, 0) is-
(A) $3x^2 + 4y^2 = x$ (B) $3x^2 + y^2 = 12x$
(C) $x^2 + 4y^2 = 12x$ (D) $3x^2 + 4y^2 = 12x$

Q.44 A man running round a racecourse notes that the sum of the distance of two flag posts from him is always 10 meters and the distance between the flag posts is 8 meters. The area of the path he encloses -
(A) 10π (B) 15π
(C) 5π (D) 20π

Q.45 The distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2. Then eccentric angle of the point is
(A) $\pm\pi/2$ (B) $\pm\pi$
(C) $\pi/4, 3\pi/4$ (D) $\pm\pi/4$

Q.46 Find the equation of the ellipse whose eccentricity is 1/2, the focus is (–1, 1) and the directrix is $x - y + 3 = 0$.
(A) $7x^2 + 7y^2 + 10x - 10y + 2xy + 7 = 0$
(B) $5x^2 + 7y^2 + 10x - 12y + 2xy + 7 = 0$
(C) $7x^2 + 7y^2 - 10x + 10y + 2xy + 7 = 0$
(D) $x^2 + 5y^2 + 10x + 10y + 2xy + 7 = 0$

Q.47 Find the equation of the ellipse whose axes are along the coordinate axes, vertices are (± 5,0) and foci at (± 4,0).
(A) $\frac{x^2}{5} + \frac{y^2}{9} = 1$ (B) $\frac{x^2}{25} + \frac{y^2}{3} = 1$
(C) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (D) $\frac{x^2}{15} + \frac{y^2}{12} = 1$

Q.48 Find the centre, the length of the axes and the eccentricity of the ellipse $2x^2+3y^2-4x-12y+13=0$.
(A)(1, 2); $\sqrt{2}$; $2/\sqrt{3}$; $1/\sqrt{3}$ (B) (2, 2); $\sqrt{3}$; $2/\sqrt{3}$; $2/\sqrt{3}$
(C)(1, 1); $\sqrt{2}$; $1/\sqrt{3}$; $1/\sqrt{3}$ (D) (3, 2); $\sqrt{3}$; $4/\sqrt{3}$; $1/\sqrt{3}$

Q.49 P,Q,R be three points α, β, γ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Find the area of the triangle formed by the tangents at P,Q,R.
(A) $ab\ \tan\frac{1}{2}(\beta+\gamma)\tan\frac{1}{2}(\gamma-\alpha)\tan\frac{1}{2}(\alpha+\beta)$
(B) $ab\ \tan\frac{1}{2}(\beta-\gamma)\tan\frac{1}{2}(\gamma-\alpha)\tan\frac{1}{2}(\alpha-\beta)$
(C) $ab\ \tan\frac{1}{2}(\beta+\gamma)\tan\frac{1}{2}(\gamma+\alpha)\tan\frac{1}{2}(\alpha+\beta)$
(D) None of these

Q.50 Find the equations of the tangents to the ellipse $4x^2 + 3y^2 = 5$ which are inclined at an angle of 60° to the axis of x.
(A) $y = \sqrt{3}\,x \pm \sqrt{\frac{65}{12}}$ (B) $y = \sqrt{2}\,x \pm \sqrt{\frac{35}{12}}$
(C) $y = \sqrt{2}\,x \pm \sqrt{\frac{65}{12}}$ (D) None of these

Q.51 The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre (0, 3) is-
(A) 4 (B) 3
(C) $\sqrt{12}$ (D) 7/2

Q.52 The eccentricity of the ellipse represented by the equation $25x^2 + 16y^2 - 150x - 175 = 0$ is-
(A) 2/5 (B) 3/5
(C) 4/5 (D) None of these

Q.53 The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is-
(A) 0 (B) 1
(C) 2 (D) infinite

Q.54 If the chords of contact of tangents from two points (x_1,y_1) and (x_2,y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to-
(A) a^2/b^2 (B) $-b^2/a^2$
(C) $-a^4/b^4$ (D) $-b^4/a^4$

Q.55 If a tangent having slope of –4/3 to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A and B respectively, then the area of Δ OAB is equal to (O is centre of the ellipse)
(A) 12 sq. unit (B) 48 sq. unit
(C) 64 sq. unit (D) 24 sq. unit

Q.56 The equation of an ellipse, whose vertices are (2, – 2), (2, 4) and eccentricity 1/3, is
(A) $\frac{(x-2)^2}{9}+\frac{(y-1)^2}{8}=1$ (B) $\frac{(x-2)^2}{8}+\frac{(y-1)^2}{9}=1$
(C) $\frac{(x+2)^2}{8}+\frac{(y+1)^2}{9}=1$ (D) $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{8}=1$

Q.57 If the normal at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the co-ordinate axes in G and g respectively, then PG : Pg =
(A) a : b (B) $a^2 : b^2$
(C) $b^2 : a^2$ (D) b : a

Q.58 What will be the equation of that chord of ellipse $\frac{x^2}{36}+\frac{y^2}{9}=1$ which passes from the point (2,1) and bisected on the point
(A) $x + y = 2$ (B) $x + y = 3$
(C) $x + 2y = 1$ (D) $x + 2y = 4$

Q.59 The angle between the pair of tangents drawn from the point (1, 2) to the ellipse $3x^2 + 2y^2 = 5$ is
(A) $\tan^{-1}(12/5)$ (B) $\tan^{-1}(6/\sqrt{5})$
(C) $\tan^{-1}(12/\sqrt{5})$ (D) $\tan^{-1}(6/5)$

Q.60 Eccentricity of ellipse whose equation is $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ where t is parameter–
(A) 1/2 (B) $1/\sqrt{3}$
(C) $\sqrt{7}/4$ (D) $2/\sqrt{3}$

Q.61 Eccentricity of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if it passes through point (9, 5) and (12, 4) is –
(A) $\sqrt{3/4}$ (B) $\sqrt{4/5}$
(C) $\sqrt{5/6}$ (D) $\sqrt{6/7}$

Q.62 If Latus rectum of an ellipse is 10 and minor axis is equal to distance between foci, equation of ellipse is –
(A) $5x^2 + 2y^2 = 20$ (B) $x^2 + 2y^2 = 100$
(C) $x^2 + 2y^2 = 50$ (D) None of these

Q.63 Any tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts off intercept h & k on axes then $\frac{a^2}{h^2} + \frac{b^2}{k^2} =$
(A) – 1 (B) 0
(C) 1 (D) None of these

Q.64 Eccentricity of ellipse if minor axis subtend angle 120° at one focus –
(A) 1/2 (B) $1/\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}-1}{\sqrt{3}}$

Q.65 The eccentric angle of the point $(2, \sqrt{3})$ lying on $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is –
(A) $\pi/4$ (B) $\pi/2$
(C) $\pi/3$ (D) $\pi/6$

Q.66 If $x \cos\alpha + y \sin\alpha = 4$ is tangent to $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the value of α is –
(A) $\tan^{-1}(3/\sqrt{7})$ (B) $\tan^{-1}(7/3)$
(C) $\tan^{-1}(\sqrt{3}/7)$ (D) $\tan^{-1}(3/7)$

Q.67 If P is a point on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S and S', then the maximum value of triangle SPS' is –
(A) ab/e (B) abe
(C) abe^2 (D) ab

Q.68 Area of a triangle formed by tangent and normal to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $P\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ with x-axis is –
(A) $\frac{b(a^2+b^2)}{4a}$ (B) $\frac{ab\sqrt{a^2-b^2}}{4}$
(C) $\frac{ab\sqrt{a^2+b^2}}{4}$ (D) 4ab

Q.69 The locus of the point of intersection of perpendicular tangents to the ellipse is called –
(A) director circle (B) hyperbola
(C) ellipse (D) auxiliary circle

Q.70 If the area of the auxillary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is –
(A) $1/\sqrt{3}$ (B) 1/2
(C) $1/\sqrt{2}$ (D) $\sqrt{3}/2$

PART 3 : HYPERBOLA

Q.71 Two straight lines pass through the fixed points (± a,0) and have gradients whose product is k. find the locus of the points of inter-section of the lines
(A) hyperbola (B) parabola
(C) circle (D) None of these

Q.72 The line $x \cos\alpha + y \sin\alpha = p$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2\cos^2\alpha - b^2\sin^2\alpha = ?$
(A) p (B) p^2
(C) p^3 (D) p^4

Q.73 The line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ at the point
(A) (– 5,4/3) (B) (5, – 4/3)
(C) (3, – 1/2) (D) None of these

Q.74 If e and e′ be the eccentricities of a hyperbola and its conjugate then the value of $\frac{1}{e^2} + \frac{1}{e'^2} =$
(A) 0 (B) 1
(C) 2 (D) 4

Q.75 If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is-
(A) 1 (B) 5
(C) 7 (D) 9

Q.76 The equation of the common tangents to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is-
(A) $2x \pm y + 1 = 0$ (B) $x \pm y + 1 = 0$
(C) $x \pm 2y + 1 = 0$ (D) $x \pm y + 2 = 0$

Q.77 The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different values of k is-
(A) Ellipse (B) Parabola
(C) Circle (D) Hyperbola

Q.78 The eccentricity of the hyperbola $\frac{\sqrt{1999}}{3}(x^2 - y^2) = 1$ is
(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) 2 (D) $2\sqrt{2}$

Q.79 For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ which of the following remains constant with change in α
(A) Abscissae of vertices (B) Abscissae of foci
(C) Eccentricity (D) Directrix

Q.80 The locus of the middle points of the chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is
(A) $3x - 4y = 4$ (B) $3y - 4x + 4 = 0$
(C) $4x - 4y = 3$ (D) $3x - 4y = 2$

Q.81 The product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is

(A) $\frac{a^2b^2}{a^2+b^2}$ (B) $\frac{a^2+b^2}{a^2b^2}$

(C) $\frac{ab}{\sqrt{a}+\sqrt{b}}$ (D) $\frac{ab}{a^2+b^2}$

Q.82 Eccentricity of the rectangular hyperbola is

(A) 2 (B) $\sqrt{2}$

(C) 1 (D) $1/\sqrt{2}$

Q.83 The equation of the tangent parallel to $y - x + 5 = 0$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

(A) $x - y - 1 = 0$ (B) $x - y + 2 = 0$
(C) $x + y - 1 = 0$ (D) $x + y + 2 = 0$

Q.84 The area of a triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is-
(A) a^2 (B) $2a^2$
(C) $3a^2$ (D) $4a^2$

Q.85 Find the coordinates of foci, the eccentricity for the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$.
(A) (9,3), (– 1,3), 5/4 (B) (1,3), (– 2,3), 5/4
(C) (3,3), (– 1,3), 1/4 (D) (2,3), (–2, 3), 3/4

Q.86 The line $3x - 4y = 5$ is a tangent to the hyperbola $x^2 - 4y^2 = 5$. The point of contact is
(A) (3, 1) (B) (2, 1/4)
(C) (1, 3) (D) None of these

Q.87 The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represent a hyperbola -
(A) The length of the transverse axes is 4
(B) Length of latus rectum is 9
(C) Equation of directrix is $x = 21/5$ and $x = - 11/5$
(D) None of these

Q.88 The equations to the common tangents to the two hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are

(A) $y = \pm x \pm \sqrt{b^2 - a^2}$ (B) $y = \pm x \pm \sqrt{a^2 - b^2}$

(C) $y = \pm x \pm (a^2 - b^2)$ (D) $y = \pm x \pm \sqrt{a^2 + b^2}$

Q.89 For what value of λ does the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$?
(A) $\pm 2\sqrt{5}$ (B) $\pm 2\sqrt{5}$
(C) $\pm 2\sqrt{5}$ (D) $\pm 2\sqrt{5}$

Q.90 Find the equation of the tangent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.
(A) $x - y \pm 3\sqrt{3} = 0$ (B) $x + y \pm 2\sqrt{3} = 0$
(C) $x + y \pm 5\sqrt{3} = 0$ (D) $x + y \pm 3\sqrt{3} = 0$

Q.91 If centre, vertex and focus of hyperbola are (2, 0), (4, 0) and (8, 0) then length of latus rectum –
(A) 32 (B) 24
(C) 16 (D) None of these

Q.92 Eccentricity of hyperbola $\frac{x^2}{K} + \frac{y^2}{K^2} = 1$ where $K < 0$

(A) $\sqrt{1-K}$ (B) $\sqrt{-K}$

(C) $\sqrt{1+K}$ (D) $\sqrt{1-\frac{1}{K}}$

Q.93 If line $y = 3x + 6$ is tangent to $\frac{x^2}{a^2} - \frac{y^2}{64} = 1$ then eccentricity of hyperbola is –
(A) $\sqrt{2}$ (B) 5/2
(C) 13/5 (D) 3

Q.94 Conjugate hyperbola of hyperbola $2x^2 - 3y^2 = - 6$ is –

(A) $2x^2 + 3y^2 = 6$ (B) $\frac{y^2}{3} - \frac{x^2}{2} = -1$

(C) $\frac{y^2}{2} - \frac{x^2}{3} = -1$ (D) None of these

Q.95 Eccentricity of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if latusrectum subtend 90° at farthest vertex.
(A) $\sqrt{2}$ (B) 2
(C) 3/2 (D) None of these

Q.96 The distance of the focus of $x^2 - y^2 = 4$, from the directrix which is nearer to it, is –
(A) $4\sqrt{2}$ (B) $8\sqrt{2}$
(C) $2\sqrt{2}$ (D) $\sqrt{2}$

Q.97 If the foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ coincide, then value of a is –
(A) 1 (B) 3
(C) $1/\sqrt{3}$ (D) $\sqrt{3}$

Q.98 The equation of a hyperbola whose asymptotes are $3x \pm 5y = 0$ and vertices are $(\pm 5, 0)$ is
(A) $9x^2 - 25y^2 = 225$ (B) $25x^2 - 9y^2 = 225$
(C) $5x^2 - 3y^2 = 225$ (D) $3x^2 - 5y^2 = 25$

Q.99 If $x - y = 1$ is a tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{3} = 1$, the point of contact is –
(A) (5, 4) (B) (4, 3)
(C) (3, 4) (D) (2, 1)

Q.100 The sum of the squares of the eccentricities of the conics $\frac{x^2}{4} + \frac{y^2}{3} = 1$ and $\frac{x^2}{4} - \frac{y^2}{3} = 1$ is –

(A) 2 (B) $\sqrt{7/3}$

(C) $\sqrt{7}$ (D) $\sqrt{3}$

PART 4 : MISCELLANEOUS

Q.101 Parameter 't' of a point (4, – 6) of the parabola $y^2 = 9x$ is
(A) 4/3 (B) – 4/3
(C) – 3/4 (D) – 4/5

Q.102 Which of the following lines, is a normal to the parabola $y^2 = 16x$
(A) $y = x - 11\cos\theta - 3\cos3\theta$
(B) $y = x - 11\cos\theta - \cos3\theta$
(C) $y = (x - 11)\cos\theta + \cos3\theta$
(D) $y = (x - 11)\cos\theta - \cos3\theta$

Q.102 Locus of trisection point of any double ordinate of $y^2 = 4ax$ is
(A) $3y^2 = 4ax$ (B) $y^2 = 6ax$
(C) $9y^2 = 4ax$ (D) None of these

Q.104 If three distinct and real normals can be drawn to $y^2 = 8x$ from the point (a, 0) then
(A) a > 2 (B) a > 4
(C) $a \in (2, 4)$ (D) none of these

Q.105 Equation of parabola having it's focus at S(2, 0) and one extremity of it's latus rectum as (2, 3) is
(A) $y^2 = 4(3 - x)$ (B) $y^2 = 4(1 - x)$
(C) $y^2 = 8(3 - x)$ (D) $y^2 = 8(1 - x)$

Q.106 If e_1 and e_2 are the eccentricities of the conic sections $16x^2 + 9y^2 = 144$ and $9x^2 - 16y^2 = 144$, then
(A) $e_1^2 + e_2^2 = 3$ (B) $e_1^2 + e_2^2 > 3$
(C) $e_1^2 + e_2^2 < 3$ (D) $e_1^2 - e_2^2 = 1$

Q.107 If P (4, 8) and Q are points on the parabola $y^2 = 16x$ and the chord PQ subtends a right angle at the vertex of the parabola, then the co-ordinates of the point of intersection of normal at P and Q is –
(A) (8, 20) (B) (45/4, 3/4)
(C) (60, – 48) (D) (64, – 32)

Q.108 The locus of the mid point of a focal chord of the parabola $y^2 = 4ax$ is
(A) $y^2 = 2ax$ (B) $y^2 = 2a(x + a)$
(C) $y^2 = 2a(x - a)$ (D) None of these

Q.109 If the lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents to the parabola $y^2 = 4x$, then –
(A) $m_1 + m_2 = 0$ (B) $m_1m_2 = 0$
(C) $m_1m_2 = -1$ (D) $m_1 + m_2 = 1$

Q.110 If the normal drawn to parabola $y^2 = 4ax$ at the point $A(at_1^2, 2at_1)$ meets the curve again at $B(at_2^2, 2at_2)$ then
(A) $|t_2| \ge 2\sqrt{2}$ (B) $|t_2| \le 2\sqrt{2}$
(C) $|t_1| \ge 2\sqrt{2}$ (D) $|t_1| \le 2\sqrt{2}$

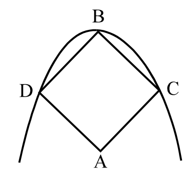
Q.111 Locus of the midpoint of any focal chord of $y^2 = 4ax$ is
(A) $y^2 = a(x - 2a)$ (B) $y^2 = 2a(x - 2a)$
(C) $y^2 = 2a(x - a)$ (D) none of these

Q.112 Slope of the normal chord of $y^2 = 8x$ that gets bisected at (8, 2) is
(A) 1 (B) – 1
(C) 2 (D) – 2

Q.113 Double ordinates AB of the parabola $y^2 = 4ax$ subtends an angle $\pi/2$ at the focus of the parabola then tangents drawn to parabola at A and B will intersect at
(A) (– 4a, 0) (B) (–2a, 0)
(C) (–3a, 0) (D) None of these

Q.114 The angle subtended by double ordinate of length 8a at the vertex of the parabola $y^2 = 4ax$ is
(A) 45° (B) 90°
(C) 60° (D) 30°

Q.115 In the adjacent figure a parabola is drawn to pass through the vertices B, C and D of the square ABCD. If A(2, 1), C (2, 3) then focus of this parabola is -



(A) (1, 11/4) (B) (2, 11/4)
(C) (3, 13/4) (D) (2, 13/4)

Q.116 A point P moves such that the sum of twice its distance from the origin and its distance from the y-axis is a constant equal to 3. P describes-
(A) A circle with its centre at (–1, 0) and radius $2\sqrt{3}$
(B) An ellipse centred at (–1, 0) and of eccentricity 1/2
(C) A hyperbola centred at (1, 0) and of eccentricity 2
(D) None of these

Q.117 Tangents are drawn from the points on the line $x–y–5 = 0$ to $x^2 + 4y^2 = 4$. Then all the chords of contact pass through a fixed point, whose coordinates are
(A) $\left(\frac{4}{5}, -\frac{1}{5}\right)$ (B) $\left(\frac{1}{5}, -\frac{4}{5}\right)$
(C) $\left(\frac{4}{5}, \frac{1}{5}\right)$ (D) $\left(-\frac{4}{5}, \frac{1}{5}\right)$

Q.118 The distance between the directrices of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is
(A) $\frac{9}{\sqrt{5}}$ (B) $\frac{24}{\sqrt{5}}$
(C) $\frac{18}{\sqrt{5}}$ (D) none of these

Q.119 The line $3x + 5y = k$ touches the ellipse $16x^2 + 25y^2 = 400$ if k is
(A) ± 25 (B) $\pm\sqrt{15}$
(C) $\pm\sqrt{5}$ (D) none of these

Q.120 If $\alpha + \beta = 3\pi$ then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through –
(A) focus
(B) centre
(C) one of the end points of the transverse axis
(D) one of the end points of the conjugate axis

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis in P_1 and P_2. If S is the focus of the parabola then $\frac{1}{\ell(SP_1)} + \frac{1}{\ell(SP_2)}$ is equal to

(A) 4/a (B) 2/a
(C) 1/a (D) 1/4a

Q.2 The points of intersection of the two ellipse $x^2 + 2y^2 - 6x - 12y + 23 = 0, 4x^2 + 2y^2 - 20x - 12y + 35 = 0$

(A) Lie on a circle centred at $\left(\frac{8}{3}, 3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(B) Lie on a circle centred at (–8/3, –3) & of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(C) Lie on a circle centred at (8, 9) and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$

(D) Are not concyclic

Q.3 Equation of parabola having the extremities of it's latus rectum as (3, 4) and (4, 3) is

(A) $\left(x-\frac{7}{3}\right)^2 + \left(y-\frac{7}{3}\right)^2 = \left(\frac{x+y-6}{2}\right)^2$

(B) $\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \left(\frac{x+y-8}{2}\right)^2$

(C) $\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \left(\frac{x+y-4}{2}\right)^2$

(D) None of these

Q.4 If two normals drawn from any point to the parabola $y^2 = 4ax$ makes angle α and β with the axis such that $\tan\alpha . \tan\beta = 2$, then locus of this point is

(A) $y^2 = 4ax$ (B) $x^2 = 4ay$
(C) $y^2 = -4ax$ (D) $x^2 = -4ay$

Q.5 If a normal chord of the parabola $y^2 = 4ax$ subtend a right angle at the vertex, its slope is

(A) ± 1 (B) $\pm\sqrt{2}$
(C) $\pm\sqrt{3}$ (D) None of these

Q.6 Length of the shortest normal chord of the parabola $y^2 = 4ax$ is

(A) $a\sqrt{27}$ (B) $3a\sqrt{3}$
(C) $2a\sqrt{27}$ (D) none of these

Q.7 An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having it's focus at 'S'. If chord AB lies towards the left of S, then side length of this triangle is

(A) $3a(2-\sqrt{3})$ (B) $4a(2-\sqrt{3})$
(C) $2(2-\sqrt{3})$ (D) $8a(2-\sqrt{3})$

Q.8 If the locus of middle point of point of contact of tangent drawn to the parabola $y^2 = 8x$ and foot of perpendicular drawn from its focus to the tangent is a conic then length of latusrectum of this conic is –

(A) 9/4 (B) 9
(C) 18 (D) 9/2

Q.9 If the normal at three points P, Q, R of the parabola $y^2 = 4ax$ meet in a point O and S be its focus, then $|SP| . |SQ| . |SR|$ is equal to –

(A) a^3 (B) $a^2(SO)$
(C) $a(SO^2)$ (D) None of these

Q.10 The straight line y = mx + c, touches the parabola $y^2 = 16(x + 4)$ then the set of all the values taken by c is

(A) $(-\infty, -4] \cup [4, \infty)$ (B) $(-\infty, -8] \cup [8, \infty)$
(C) $(-\infty, -12] \cup [12, \infty)$ (D) $(-\infty, -6] \cup [6, \infty)$

Q.11 Minimum area of circle which touches the parabola $y = x^2 + 1$ and $y^2 = x - 1$ –

(A) $\frac{9\pi}{16}$ sq. unit (B) $\frac{9\pi}{32}$ sq. unit
(C) $\frac{9\pi}{8}$ sq. unit (D) $\frac{9\pi}{4}$ sq. unit

Q.12 A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0), (\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is –

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2
(C) $\frac{b}{a}$ (D) $\sqrt{\frac{c}{a}}$

Q.13 A ray of light travels along a line y = 4 and strikes the surface of a curve $y^2 = 4(x + y)$ then equation of the line along reflected ray travel –

(A) x = 0 (B) x = 2
(C) x + y = 4 (D) 2x + y = 4

Q.14 If a normal chord of $y^2 = 4ax$ subtends an angle $\pi/2$ at the vertex of the parabola then it's slope is equal to

(A) ± 1 (B) $\pm\sqrt{2}$
(C) ± 2 (D) none of these

Q.15 An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, such that one vertex of this triangle coincides with the vertex of the parabola. Side length of this triangle is -

(A) $4a\sqrt{3}$ (B) $6a\sqrt{3}$
(C) $2a\sqrt{3}$ (D) $8a\sqrt{3}$

Q.16 If c is the centre and A, B are two points on the conic $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ such that $A\hat{C}B = \frac{\pi}{2}$, then $CA^{-2} + CB^{-2}$ is equal to

(A) $\frac{13}{36}$ (B) $\frac{36}{13}$

(C) $\frac{16}{33}$ (D) $\frac{33}{16}$

Q.17 If $x \cos\alpha + y \sin\alpha = p$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

(A) $p^2 (a^2 \cos^2\alpha + b^2 \sin^2\alpha) = a^2 - b^2$
(B) $p^2 (a^2 \cos^2\alpha + b^2 \sin^2\alpha) = (a^2 - b^2)^2$
(C) $p^2 (a^2 \sec^2\alpha + b^2 \operatorname{cosec}^2\alpha) = a^2 - b^2$
(D) $p^2 (a^2 \sec^2\alpha + b^2 \operatorname{cosec}^2\alpha) = (a^2 - b^2)^2$

Q.18 From the point (15, 12) three normals are drawn to the parabola $y^2 = 4x$, then centroid of triangle formed by three co-normal points is –

(A) (16/3, 0) (B) (4, 0)
(C) (26/3, 0) (D) (6, 0)

Q.19 The tangent and normal at P(t), for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle passing through the points P, T and G is

(A) $\cot^{-1}t$ (B) $\cot^{-1}t^2$
(C) $\tan^{-1}t$ (D) $\tan^{-1}t^2$

Q.20 Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameters intersect in R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangents at P and Q on the parabola and by OR then the value of $\cot\theta_1 + \cot\theta_2 =$

(A) $-2\tan\phi$ (B) $-2\tan(\pi - \phi)$
(C) 0 (D) $2\cot\phi$

Q.21 AB is a chord of the parabola $y^2 = 4ax$ with vertex at A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the x-axis is

(A) a (B) 2a
(C) 4a (D) 8a

Q.22 If $x \cos\alpha + y \sin\alpha = P$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

(A) $a\cos\alpha + b\sin\alpha = p^2$ (B) $a\sin\alpha + b\cos\alpha = P^2$
(C) $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$ (D) $a^2\sin^2\alpha + b^2\cos^2\alpha = P^2$

Q.23 The normal $y = mx - 2am - am^3$ to the parabola $y^2 = 4ax$ subtends a right angle at the origin, then

(A) m = 1 (B) $m = \sqrt{2}$

(C) m = 2 (D) $m = \frac{1}{\sqrt{2}}$

Q.24 If the normal at the point P(θ) to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point Q (2θ), then cos θ is equal to

(A) 2/3 (B) –2/3
(C) 3/2 (D) –3/2

Q.25 Chord of an ellipse are drawn through the positive end of the minor axes. Then their mid point lies on

(A) a circle (B) a parabola
(C) an ellipse (D) a hyperbola

Q.26 If $\tan\theta_1 \tan\theta_2 = -\frac{a^2}{b^2}$, then the chord joining two points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at

(A) Focus (B) Centre
(C) End of the major axes (D) End of minor axes

Q.27 The equation $x^2 + 4y^2 + 2x + 16y + 13 = 0$ represents a ellipse

(A) whose eccentricity is $\sqrt{3}$

(B) whose focus is $(\pm\sqrt{3}, 0)$

(C) whose directrix is $x = \pm\frac{4}{\sqrt{3}} - 1$

(D) None of these

Q.28 The point of the intersection of the tangent at the point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentricity differ by a right angle lies on the ellipse is

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ (B) $\frac{x}{a} + \frac{y}{b} = 2$

(C) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q.29 If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, the locus of P is

(A) a circle (B) a parabola
(C) an ellipse (D) a hyperbola

Q.30 A line segment of length a + b moves in such a way that its ends are always on two fixed perpendicular straight lines. Then the locus of the point on this line which divides it into portions of lengths a and b is

(A) a parabola (B) a circle
(C) an ellipse (D) none of these

Q.31 A chord PQ of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ subtends right angle at its centre. The locus of the point of intersection of tangents drawn at P and Q is

(A) a circle (B) a parabola
(C) an ellipse (D) a hyperbola

Q.32 If the chords of contact of tangent from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to

(A) a^2/b^2 (B) b^2/a^2
(C) $-a^4/b^4$ (D) $-b^4/a^4$

Q.33 The area of the rectangle formed by the perpendiculars from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent and normal at a point whose eccentric angle is $\pi/4$ is

(A) $\frac{(a^2-b^2)ab}{a^2+b^2}$ (B) $\frac{(a^2+b^2)ab}{a^2-b^2}$

(C) $\frac{a^2-b^2}{ab(a^2+b^2)}$ (D) $\frac{a^2+b^2}{ab(a^2-b^2)}$

Q.34 If α, β are eccentric angles of end points of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\tan\alpha/2.\tan\beta/2$ is equal to

(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$

(C) $\frac{e+1}{e-1}$ (D) None of these

Q.35 A tangent to ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ at P meet the line $x = 25/3$ at Q then a circle whose extremities of diameter are P and Q is passes through a fixed point –

(A) (–3, 0) (B) (3, 0)
(C) (5, 0) (D) (4, 0)

Q.36 A parabola is drawn with its vertex at (0, –3), the axis of symmetry along the conjugate axis of the hyperbola $\frac{x^2}{49}-\frac{y^2}{9}=1$, and passes through the two foci of the hyperbola. The coordinates of the focus of the parabola are-

(A) (0, 11/6) (B) (0, –11/6)
(C) (0, 11/12) (D) (0, –11/12)

Q.37 A circle has the same centre as an ellipse and passes through the focii F_1 and F_2 of the ellipse, such that the two curves intersect in 4 points. Let P be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of the triangle PF_1F_2 is 30, then the distance between the focii is –

(A) 11 (B) 12
(C) 13 (D) 15

Q.38 Minimum radius of circle which concentric with ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ so that all normals of ellipse intersect or touch the circle, is–

(A) 1/2 (B) 4
(C) 1 (D) can't be determined

Q.39 Any ordinate MP of an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ meets the auxillary circle in Q, then locus of intersection of normals at P and Q is –

(A) $x^2 + y^2 = 8$ (B) $x^2 + y^2 = 34$
(C) $x^2 + y^2 = 64$ (D) $x^2 + y^2 = 15$

Q.40 If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes through $S \equiv (3, 0)$ and PS = 2 then length of chord PQ is –

(A) 8 (B) 6
(C) 10 (D) 4

Q.41 The locus of the point of intersection of two normals to a parabola which are at right angles to one another is –

(A) $y^2 = a(x - 3a)$ (B) $y^2 = a(x - 2a)$
(C) $y^2 = a(2x - 3a)$ (D) $y^2 = a(x - a)$

Q.42 Let $S \equiv (3, 4)$ and $S' \equiv (9, 12)$ be two foci of an ellipse. If the coordinates of the foot of the perpendicular from focus S to a tangent of the ellipse is (1, – 4) then the eccentricity of the ellipse is –

(A) 5/13 (B) 4/5
(C) 5/7 (D) 7/13

Q.43 The locus of the middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

(A) $3x - 4y = 4$ (B) $3x - 4y + 4 = 0$
(C) $4x - 4y = 3$ (D) $3x - 4y = 2$

Q.44 A point P is taken on the right half of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having its foci as S_1 and S_2. If the internal angle bisector of the angle $\angle S_1PS_2$ cuts the x-axis at the point Q (α, 0) then angle of α is –

(A) [–a, a] (B) [0, a]
(C) (0, a] (D) [–a, 0)

Q.45 Let S be the focus of $y^2 = 4x$ and a point P is moving on the curve such that it's abscissa is increasing at the rate of 4 units/sec, then the rate of increase of projection of SP on $x + y = 1$ when P is at (4, 4) is

(A) $\sqrt{2}$ (B) – 1

(C) $-\sqrt{2}$ (D) $-3/\sqrt{2}$

Q.46 The tangent at a point P on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets one of the directrix in F. If PF subtends an angle θ at the corresponding focus, then θ equals

(A) π/4 (B) π/2
(C) 3π/4 (D) π

Q.47 Radius of the circle that passes through origin and touches the parabola $y^2 = 4x$ at the point (1, 2) is –

(A) $2\sqrt{2}$ (B) $3\sqrt{2}$
(C) 3 (D) $5/\sqrt{2}$

Q.48 If a variable line has its intercepts on the coordinates axes e, e', where $\frac{e}{2}, \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the always touches the circle $x^2 + y^2 = r^2$, where r =

(A) 1 (B) 2
(C) 3 (D) cannot be decided

Q.49 From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If θ_1 & θ_2 are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \pi/4$, then the locus of P is :

(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$
(C) $x - y - 1 = 0$ (D) $x + y + 1 = 0$

Q.50 The equation of the common tangents to the two hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are

(A) $y = \pm x \pm \sqrt{b^2 - a^2}$ (B) $y = \pm x \pm \sqrt{a^2 - b^2}$
(C) $y = \pm x \pm (a^2 - b^2)$ (D) $y = \pm x \pm \sqrt{a^2 + b^2}$

Q.51 If s, s' are the length of the perpendicular on a tangent from the foci, a, a' are those from the vertices is that from the centre and e is the eccentricity of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\frac{ss' - c^2}{aa' - c^2} =$

(A) e (B) 1/e
(C) $1/e^2$ (D) e^2

Q.52 If angle between asymptote's hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 120° and product of perpendicular drawn from focii upon its any tangent is 9, then locus of point of intersection of perpendicular tangents of the hyperbola can be –

(A) $x^2 + y^2 = 6$ (B) $x^2 + y^2 = 9$
(C) $x^2 + y^2 = 3$ (D) $x^2 + y^2 = 18$

Q.53 The locus of the mid point of the chords of the circle $x^2 + y^2 = 16$, which are tangent to the hyperbola $9x^2 - 16y^2 = 144$ is

(A) $x^2 + y^2 = a^2 - b^2$ (B) $(x^2 + y^2) = a^2 - b^2$
(C) $(x^2 + y^2) = a^2 x^2 - b^2 y^2$ (D) $(x^2 + y^2)^2 = a^2 + b^2$

Q.54 The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

(A) $x^2 + 2y^2 - ax = 0$ (B) $2x^2 + y^2 - 2ax = 0$
(C) $2x^2 + 2y^2 - ay = 0$ (D) $2x^2 + y^2 - 2ay = 0$

ASSERTION AND REASON QUESTIONS

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.55 **Statement–1 :** Tangents drawn from ends points of the chord $x + ay - 6 = 0$ of the parabola $y^2 = 24x$ meet on the line $x + 6 = 0$.
Statement–2 : Pair of tangents drawn at the end points of the parabola meets on the directrix of the parabola.

Q.56 **Statement-1 :** Through $(l, \ell + 1)$ there cannot be more than one-normal to the parabola $y^2 = 4x$ if $\ell < 2$.
Statement-2 : The point $(l, \ell + 1)$ lines out side the parabola for all $\ell \neq 1$.

Q.57 **Statement 1 :** Length of focal chord of a parabola $y^2 = 8x$ making an angle of 60° with x-axis is 32.
Statement 2 : Length of focal chord of a parabola $y^2 = 4ax$ making an angle α with x-axis is $4a\text{cosec}^2\,\alpha$.

Q.58 **Statement-1 :** A tangent of the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P & Q. The angle between the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ is $\pi/2$.
Statement-2 : If the two tangents from to the ellipse $x^2/a^2 + y^2/b^2 = 1$ are at right angle, then locus of P is the circle $x^2 + y^2 = a^2 + b^2$.

Q.59 **Statement 1 :** If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$
Statement 2 : Circle drawn with focal radius as diameter touches the auxiliary circle.

Q.60 **Statement–1 :** The equation of the chord of contact of tangents drawn from the point (2, -1) to the hyperbola $16x^2 - 9y^2 = 144$ is $32x + 9y = 144$.
Statement–2 : Pair of tangents drawn from (x_1, y_1) to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $SS_1 = T^2$; $S = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$

Q.61 **Statement 1 :** If a circle cuts a rectangular hyperbola $xy = c^2$ in A, B, C, D and the parameters of these four points be t_1, t_2, t_3 and t_4 respectively then $t_1 t_2 t_3 t_4 = 1$.
Statement 2 : We can take (ct, c/t), $t \neq 0$ as the parametric point on the hyperbola $xy = c^2$.

Q.62 **Statement–1 :** The point (7, –3) lies inside the hyperbola $9x^2 - 4y^2 = 36$ where as the point (2, 7) lies outside this.
Statement–2 : The point (x_1, y_1) lies outside, on or inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according as $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 < \text{or} = \text{or} > 0$

PASSAGE BASED QUESTIONS

Passage 1 (Q.63-Q.65)

From a fixed point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$ chords PQ and PQ' are drawn making equal angle α with the tangent at P.

Q.63 If $Q(at_1^2, 2at_1)$ and $Q'(at_2^2, 2at_2)$, then ordered pair $(t_1 + t_2, t_1 \cdot t_2)$ is

(A) $\left(\frac{2t\left((t^2+2)\tan^2\alpha+1\right)}{1-t^2\tan^2\alpha}, \frac{t^2-(t^2+2)^2\tan^2\alpha}{1-t^2\tan^2\alpha}\right)$

(B) $\left(\frac{2t\left((t^2-2)\tan^2\alpha-1\right)}{1-t^2\tan^2\alpha}, \frac{t^2-(t^2+2)^2\tan^2\alpha}{1-t^2\tan^2\alpha}\right)$

(C) $\left(\frac{2t\left((t^2-2)\tan^2\alpha-1\right)}{1-t^2\tan^2\alpha}, \frac{t^2+(t^2-2)^2\tan^2\alpha}{1-t^2\tan^2\alpha}\right)$

(D) none of these

Q.64 Equation of line QQ' is

(A) $(t^2\tan^2\alpha-1)x-((t^2+2)\tan^2\alpha+1)ty-a((t^2+2)^2\tan^2\alpha-t^2)$

(B) $(t^2\tan^2\alpha+1)x+((t^2-2)\tan^2\alpha-1)ty-a((t^2+2)^2\tan^2\alpha-t^2)$

(C) $(t^2\tan^2\alpha-1)x+((t^2+2)\tan^2\alpha+1)ty+a((t^2+2)^2\tan^2\alpha-t^2)$

(D) none of these

Q.65 Line QQ' for a given point P always passes through a fixed point 'R' regardless of choice of α, then the fixed point R is

(A) $\left(-2a+at^2, -\frac{2a}{t}\right)$ (B) $\left(-2a-at^2, -\frac{2a}{t}\right)$

(C) $\left(2a+at^2, \frac{2a}{t}\right)$ (D) none of these

Passage 2 (Q.66-Q.68)

$y = f(x)$ is a parabola of the form $y = x^2 + ax + 1$, its tangent at the point of intersection of y-axis and parabola also touches the circle $x^2 + y^2 = r^2$. It is known that no point of the parabola is below x-axis.

Q.66 The radius of circle when a attains its maximum value –

(A) $\frac{1}{\sqrt{10}}$ (B) $\frac{1}{\sqrt{5}}$

(C) 1 (D) $\sqrt{5}$

Q.67 The slope of the tangent when radius of the circle is maximum–

(A) 0 (B) 1

(C) – 1 (D) not defined

Q.68 The minimum area bounded by the tangent and the coordinate axes.

(A) 1/4 (B) 1/3

(C) 1/2 (D) 1

Passage 3 (Q.69-Q.71)

Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ and circle $x^2 + y^2 = r^2$. Now any tangent of ellipse will be $y = mx \pm \sqrt{a^2m^2 + b^2}$ and any tangent of circle will be $y = mx \pm r\sqrt{1+m^2}$.

Q.69 The number of common tangents to the ellipse and circle will be

(A) at most 4 (B) exactly 4

(C) at least 4 (D) exactly 2

Q.70 The range of 'r' for which 4 distinct common tangents are possible

(A) [b, a] (B) (b, a)

(C) (b, a] (D) [b, a)

Q.71 The equation of common tangent in 4th quadrant will be

(A) $y = \left(\sqrt{\frac{r^2-b^2}{a^2-r^2}}\right)x + r\sqrt{\frac{a^2-b^2}{a^2-r^2}}$

(B) $y = -\left(\sqrt{\frac{r^2-b^2}{a^2-r^2}}\right)x + r\sqrt{\frac{a^2-b^2}{a^2-r^2}}$

(C) $y = \left(\sqrt{\frac{r^2-b^2}{a^2-r^2}}\right)x - r\sqrt{\frac{a^2-b^2}{a^2-r^2}}$

(D) $y = -\left(\sqrt{\frac{r^2-b^2}{a^2-r^2}}\right)x - r\sqrt{\frac{a^2-b^2}{a^2-r^2}}$

Passage 4 (Q.72-Q.74)

An ellipse whose one focus is (4, 3) passes through (1, 2) and equation of tangent at (1, 2) is $x + y - 3 = 0$. If the abscissa of centre of ellipse is 7.

Q.72 Length of minor axis of ellipse is –

(A) $5\sqrt{10}$ (B) $7\sqrt{2}$

(C) $12\sqrt{2}$ (D) $6\sqrt{2}$

Q.73 Eccentricity of ellipse is –

(A) $\sqrt{\frac{41}{125}}$ (B) $\sqrt{\frac{111}{125}}$

(C) $\sqrt{\frac{89}{125}}$ (D) $\frac{\sqrt{3}}{2}$

Q.74 Equation of auxiliary circle of ellipse is –

(A) $(x-7)^2+(y-15)^2=72$ (B) $(x-7)^2+(y-15)^2=250$

(C) $(x-7)^2+(y-16)^2=250$ (D) $(x-7)^2+(y-14)^2=221$

Passage 5- (Q.75-Q.77)

From a point P three normals are drawn to the parabola $y^2 = 4x$ such that two of them make angles with the abscissa axis, the product of whose tangents is 2. Suppose the locus of the point P is a conic C. Now a circle S = 0 is described on the chord of the conic C as diameter passing through the point (1, 0) and with gradient unity. Suppose (a, b) are the coordinates of the centre of this circle. If L_1 and L_2 are the two asymptotes of the hyperbola with length of its transverse axis 2a and conjugate axis 2b (principal axes of the hyperbola along the coordinate axes) then answer the following questions.

Q.75 Locus of P is a –

(A) circle (B) parabola
(C) ellipse (D) hyperbola

Q.76 Radius of the circle S = 0 is –

(A) 4 (B) 5
(C) $\sqrt{17}$ (D) $\sqrt{23}$

Q.77 The angle $\alpha \in (0, \pi/2)$ between the two asymptotes of the hyperbola lies in the interval –

(A) (0, 15°) (B) (30°, 45°)
(C) (45°, 60°) (D) (60°, 75°)

MATCH THE COLUMN TYPE QUESTIONS

Q.78 For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b), match the items in column I with items in column II.

Column I

(a) Locus equation of point of intersection of perpendicular tangents is
(b) Locus equation of foot of perpendicular from focus upon any tangent is
(c) Locus equation of foot of perpendicular drawn from centre upon any tangent is
(d) Locus equation of mid point of segment OP, where P is foot of perpendicular drawn from centre upon any tangent and O is origin is

Column II

(p) $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$
(q) $4(x^2 + y^2)^2 = (a^2x^2 + b^2y^2)$
(r) $x^2 + y^2 = a^2$
(s) $x^2 + y^2 = a^2 + b^2$

Code :

(A) a-s, b-r, c-p, d-q (B) a-p, b-q, c-r, d-s
(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q

Q.79 Match the column –

Column I | **Column II**

(a) Length of the latusrectum of the conic $25\{(x-2)^2+(y-3)^2\}=(3x+4y-6)^2$ is less than — (p) 4
(b) Two parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$ have a common normal, other than the axis, if $\frac{b}{a-c}$ may be — (q) 5
(c) A tangent to the ellipse $\frac{x^2}{27}+\frac{y^2}{48}=1$ having slope – 4/3 cuts the x and y-axis at the points A and B respectively. If O is the origin then square root of the area of triangle OAB is greater than or equal to — (r) 6
(d) As x range over the interval $(0, \infty)$, the function $f(x)=\sqrt{9x^2+173x+900}-\sqrt{9x^2+77x+900}$ range over (0, M). The possible integral value(s) in the range of f (x) is — (s) 7, (t) 8

Code :

(A) (a) – qrst, (b) – pqrst, (c) – pqr, (d) – pqrst
(B) (a) – pqrst, (b) – rst, (c) – qr, (d) – pqr
(C) (a) – st, (b) – qrst, (c) – pqr, (d) – pqrs
(D) (a) – rst, (b) – pqt, (c) – pqrst, (d) – st

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 An equilateral triangle ABC is inscribed in the parabola $y = x^2$ and one of the side of the equilateral triangle has the gradient 2. If the sum of x-coordinates of the vertices of the triangle is a rational in the form p/q where p and q are coprime, then find the value of (p + q).

Q.2 Eccentricity of the hyperbola conjugate to the hyperbola $\frac{x^2}{4} - \frac{y^2}{12} = 1$ is $A/\sqrt{3}$, find the value of A.

Q.3 The points of contact Q and R of tangent from the point P (2, 3) on the parabola $y^2 = 4x$ are (a_1, a_2) and (b_1, b_2) then find the value of $(a_1 + a_2) + (b_1 + b_2)$.

Q.4 The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is 1/A then find the value of A.

Q.5 A tangent is drawn to the parabola $y^2 = 4x$ at the point 'P' whose abscissa lies in the interval [1, 4]. The maximum possible area of the triangle formed by the tangent at 'P', ordinate of the point 'P' and the x-axis is equal to.

Q.6 For an ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ with vertices A and A', tangent drawn at the point P in the first quadrant meets the y-axis in Q and the chord A'P meets the y-axis in M. If 'O' is the origin then $OQ^2 - MQ^2$ equals to

Q.7 If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angles α and β respectively with the x-axis then $|\tan\alpha(\tan\alpha + \tan\beta)|$ has the value equal to

Q.8 A circle has the same centre as an ellipse & passes through the foci F_1 & F_2 of the ellipse, such that the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the foci is :

Q.9 If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha + y^2 = 25$, then a value of α is π/A. Find the value of A.

Q.10 Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then the product (AB) (CD) is equal to

Q.11 The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Q.12 Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point P (1/2, 2) on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Q.13 Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Q.14 A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta h$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ _______

Q.15 The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is –

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 The length of the latusrectum of the parabola $x^2 - 4x + 8y + 12 = 0$ is – **[AIEEE 2002]**
(A) 4 (B) 6
(C) 8 (D) 10

Q.2 The equation of tangents to the parabola $y^2 = 4ax$ at the ends of its latus rectum is – **[AIEEE 2002]**
(A) $x - y + a = 0$ (B) $x + y + a = 0$
(C) $x + y - a = 0$ (D) Both (A) and (B)

Q.3 If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is- **[AIEEE-2002]**
(A) $e = \frac{1}{\sqrt{2}}$ (B) $e = \frac{1}{\sqrt{3}}$
(C) $e = \frac{1}{\sqrt{4}}$ (D) $e = \frac{1}{\sqrt{6}}$

Q.4 The equation of an ellipse, whose major axis = 8 and eccentricity = 1/2, is **[AIEEE-2002]**
(A) $3x^2 + 4y^2 = 12$ (B) $3x^2 + 4y^2 = 48$
(C) $4x^2 + 3y^2 = 48$ (D) $3x^2 + 9y^2 = 12$

Q.5 The latus rectum of the hyperbola $16x^2 - 9y^2 = 144$ is- **[AIEEE-2002]**
(A) 16/3 (B) 32/3
(C) 8/3 (D) 4/3

Q.6 The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is-
(A) 9 (B) 1 **[AIEEE 2003]**
(C) 5 (D) 7

Q.7 The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then – **[AIEEE 2003]**
(A) $t_2 = t_1 + \frac{2}{t_1}$ (B) $t_2 = -t_1 - \frac{2}{t_1}$
(C) $t_2 = -t_1 + \frac{2}{t_1}$ (D) $t_2 = t_1 - \frac{2}{t_1}$

Q.8 If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then – **[AIEEE 2004]**
(A) $d^2 + (2b + 3c)^2 = 0$ (B) $d^2 + (3b + 2c)^2 = 0$
(C) $d^2 + (2b - 3c)^2 = 0$ (D) $d^2 + (3b - 2c)^2 = 0$

Q.9 The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is x = 4, then the equation of the ellipse is- **[AIEEE 2004]**
(A) $3x^2 + 4y^2 = 1$ (B) $3x^2 + 4y^2 = 12$
(C) $4x^2 + 3y^2 = 12$ (D) $4x^2 + 3y^2 = 1$

Q.10 The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is- **[AIEEE-2005]**
(A) an ellipse (B) a circle
(C) a parabola (D) a hyperbola

Q.11 The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is – **[AIEEE 2006]**
(A) $xy = \frac{3}{4}$ (B) $xy = \frac{35}{16}$
(C) $xy = \frac{64}{105}$ (D) $xy = \frac{105}{64}$

Q.12 In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is – **[AIEEE 2006]**
(A) 1/2 (B) 4/5
(C) $1/\sqrt{5}$ (D) 3/5

Q.13 The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is – **[AIEEE 2007]**
(A) (–1, 1) (B) (0, 2)
(C) (2, 4) (D) (–2, 0)

Q.14 The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a- **[AIEEE-2007]**
(A) ellipse (B) parabola
(C) circle (D) hyperbola

Q.15 For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ which of the following remains constant when α varies ? **[AIEEE-2007]**
(A) Eccentricity (B) Directrix
(C) Abscissae of vertices (D) Abscissae of foci

Q.16 A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at –
(A) (0, 2) (B) (1, 0) **[AIEEE 2008]**
(C) (0, 1) (D) (2, 0)

Q.17 A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is 1/2. Then the length of the semi-major axis is - **[AIEEE 2008]**
(A) 2/3 (B) 4/3
(C) 5/3 (D) 8/3

Q.18 The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is – **[AIEEE 2009]**
(A) $4x^2 + 64y^2 = 48$ (B) $x^2 + 16y^2 = 16$
(C) $x^2 + 12y^2 = 16$ (D) $4x^2 + 48y^2 = 48$

Q.19 If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is – **[AIEEE 2010]**
(A) $2x + 1 = 0$ (B) $x = -1$
(C) $2x - 1 = 0$ (D) $x = 1$

Q.20 Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is : **[AIEEE-2011]**
(A) $3x^2 + 5y^2 - 32 = 0$ (B) $5x^2 + 3y^2 - 48 = 0$
(C) $3x^2 + 5y^2 - 15 = 0$ (D) $5x^2 + 3y^2 - 32 = 0$

Q.21 **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$. **[AIEEE-2012]**
Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.
(A) Statement-1 is false, Statement-2 is true.
(B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(C) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1.
(D) Statement-1 is true, statement-2 is false.

Q.22 An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is **[AIEEE-2012]**
(A) $4x^2 + y^2 = 4$ (B) $x^2 + 4y^2 = 8$
(C) $4x^2 + y^2 = 8$ (D) $x^2 + 4y^2 = 16$

Q.23 The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is – **[JEE MAIN 2013]**
(A) $x^2 + y^2 - 6y - 7 = 0$ (B) $x^2 + y^2 - 6y + 7 = 0$
(C) $x^2 + y^2 - 6y - 5 = 0$ (D) $x^2 + y^2 - 6y + 5 = 0$

Q.24 Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.
Statement-I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$. **[JEE MAIN 2013]**
Statement-II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.
(A) Statement-I is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is true; Statement-II is true; Statement-II is not a correct explanation for Statement-I.
(C) Statement-I is true; Statement-II is false.
(D) Statement-I is false; Statement-II is true.

Q.25 The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is – **[JEE MAIN 2014]**
(A) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ (B) $(x^2 - y^2)^2 = 6x^2 - 2y^2$
(C) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ (D) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

Q.26 The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[JEE MAIN 2014]**
(A) 1/2 (B) 3/2
(C) 1/8 (D) 2/3

Q.27 Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is **[JEE MAIN 2015]**
(A) $y^2 = x$ (B) $y^2 = 2x$
(C) $x^2 = 2y$ (D) $x^2 = y$

Q.28 The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is – **[JEE MAIN 2015]**
(A) 18 (B) 27/2
(C) 27 (D) 27/4

Q.29 Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: **[JEE MAIN 2016]**
(A) $x^2 + y^2 - x + 4y - 12 = 0$ (B) $x^2 + y^2 - (x/4) + 2y - 24 = 0$
(C) $x^2 + y^2 - 4x + 9y + 18 = 0$ (D) $x^2 + y^2 - 4x + 8y + 12 = 0$

Q.30 The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[JEE MAIN 2016]**
(A) $4/\sqrt{3}$ (B) $2/\sqrt{3}$
(C) $\sqrt{3}$ (D) 4/3

Q.31 The centres of those circles which touch the circle, $x^2 + y^2 - 8x - y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE MAIN 2016]**
(A) an ellipse which is not a circle. (B) a hyperbola
(C) a parabola (D) a circle

Q.32 The eccentricity of an ellipse whose centre is at the origin is 1/2. If one of its directrices is $x = -4$, then the equation of the normal to it at (1, 3/2) is : **[JEE MAIN 2017]**
(A) $4x + 2y = 7$ (B) $x + 2y = 4$
(C) $2y - x = 2$ (D) $4x - 2y = 1$

Q.33 A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at (± 2, 0). Then the tangent to this hyperbola at P also passes through the point : **[JEE MAIN 2017]**
(A) $(\sqrt{3}, \sqrt{2})$ (B) $(-\sqrt{2}, -\sqrt{3})$
(C) $(3\sqrt{2}, 2\sqrt{3})$ (D) $(2\sqrt{2}, 3\sqrt{3})$

Q.34 If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is: **[JEE MAIN 2018]**
(A) 85 (B) 95
(C) 195 (D) 185

Q.35 Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T (0, 3) then the area (in sq. units) of ΔPTQ is: **[JEE MAIN 2018]**

(A) $60\sqrt{3}$ (B) $36\sqrt{5}$
(C) $45\sqrt{5}$ (D) $54\sqrt{3}$

Q.36 Tangent and normal are drawn at P (16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is: **[JEE MAIN 2018]**

(A) 3 (B) 4/3
(C) 1/2 (D) 2

Q.37 Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is: **[JEE MAIN 2019 (JAN)]**

(A) $2\sqrt{3}\,y = 12x + 1$ (B) $2\sqrt{3}\,y = -x - 12$
(C) $\sqrt{3}\,y = x + 3$ (D) $\sqrt{3}\,y = 3x + 1$

Q.38 Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ? **[JEE MAIN 2019 (JAN)]**

(A) (4, –4) (B) $(5, 2\sqrt{6})$
(C) (8, 6) (D) $(6, 4\sqrt{2})$

Q.39 Let $0 < \theta < \pi/2$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval : **[JEE MAIN 2019 (JAN)]**

(A) (2, 3] (B) $(3, \infty)$
(C) (3/2, 2] (D) (1, 3/2]

Q.40 If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) 64/17 (B) 2/17
(C) 128/17 (D) 4/17

Q.41 If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is- **[JEE MAIN 2019 (APRIL)]**

(A) $2x - y - 2 = 0$ (B) $3x - 2y = 0$
(C) $2x - 3y + 10 = 0$ (D) $x - 2y + 8 = 0$

Q.42 The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point : **[JEE MAIN 2019 (APRIL)]**

(A) (–1/3, 4/3) (B) (–1/4, 1/2)
(C) (3/4, 7/4) (D) (1/4, 3/4)

Q.43 In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is: **[JEE MAIN 2019 (APRIL)]**

(A) 10 (B) 8
(C) 5 (D) 6

Q.44 If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is **[JEE MAIN 2019 (APRIL)]**

(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{3}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{15}}{2}$

Q.45 If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) $2\sqrt{2} + 1$ (B) $\sqrt{2} - 1$
(C) $\sqrt{2} + 1$ (D) $2\sqrt{2} - 1$

Q.46 The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is : **[JEE MAIN 2019 (APRIL)]**

(A) $4\pi(2 - \sqrt{2})$ (B) $8\pi(3 - 2\sqrt{2})$
(C) $4\pi(3 + \sqrt{2})$ (D) $8\pi(2 - \sqrt{2})$

Q.47 If a directrix of a hyperbola centred at the origin and passing through the point $(4 - 2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then : **[JEE MAIN 2019 (APRIL)]**

(A) $4e^4 - 24e^2 + 35 = 0$ (B) $4e^4 + 8e^2 - 35 = 0$
(C) $4e^4 - 12e^2 - 27 = 0$ (D) $4e^4 - 24e^2 + 27 = 0$

Q.48 If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (3, –9/2), then the length of the latus recturm of the ellipse is : **[JEE MAIN 2019 (APRIL)]**

(A) 9 (B) $8\sqrt{3}$
(C) $12\sqrt{2}$ (D) 5

Q.49 The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P (2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is : **[JEE MAIN 2019 (APRIL)]**

(A) 14/3 (B) 16/3
(C) 68/15 (D) 34/15

Q.50 If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is **[JEE MAIN 2019 (APRIL)]**

(A) (–5/3, 0) (B) (5, 0)
(C) (–5, 0) (D) (5/3, 0)

Q.51 If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q (4, 4) then PQ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) $\frac{\sqrt{221}}{2}$ (B) $\frac{\sqrt{157}}{2}$
(C) $\frac{\sqrt{61}}{2}$ (D) $\frac{5\sqrt{5}}{2}$

Q.52 Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:
[JEE MAIN 2019 (APRIL)]
(A) 5 : 4 (B) 14 : 13
(C) 2 : 1 (D) 13 : 11

Q.53 The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is : **[JEE MAIN 2019 (APRIL)]**
(A) $x + y + 4 = 0$ (B) $x - 2y + 16 = 0$
(C) $2x - y + 2 = 0$ (D) $x - y + 4 = 0$

Q.54 An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ?
[JEE MAIN 2019 (APRIL)]
(A) $(1, 2\sqrt{2})$ (B) $(2, \sqrt{2})$
(C) $(2, 2\sqrt{2})$ (D) $(\sqrt{2}, 2)$

Q.55 If $y = mx + 4$ is common tangent to parabolas $y^2 = 4x$ and $x^2 = 2by$. Then value of b is
[JEE MAIN 2020 (JAN)]
(A) –64 (B) –32
(C) –128 (D) 16

Q.56 If distance between the foci of an ellipse is 6 and distance between its directrices is 12, then length of its latus rectum is **[JEE MAIN 2020 (JAN)]**
(A) 4 (B) $3\sqrt{2}$
(C) 9 (D) $2\sqrt{2}$

Q.57 $3x + 4y = 12\sqrt{2}$ is the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ then the distance between focii of ellipse is-
[JEE MAIN 2020 (JAN)]
(A) $2\sqrt{5}$ (B) $2\sqrt{3}$
(C) $2\sqrt{7}$ (D) 4

Q.58 The locus of a point which divides the line segment joining the point (0,–1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is-
[JEE MAIN 2020 (JAN)]
(A) $9x^2 = 3y + 2$ (B) $9x^2 = 12y + 8$
(C) $9y^2 = 12x + 8$ (D) $9y^2 = 3x + 2$

Q.59 Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet a point P in first quadrant. Normal to the ellipse at P meets x-axis at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and y-axis at $(0, \beta)$, then $|\beta|$ is
[JEE MAIN 2020 (JAN)]
(A) $2/\sqrt{3}$ (B) $2\sqrt{3}/3$
(C) $\sqrt{2}/3$ (D) 2 / 3

Q.60 If a hyperbola has vertices (±6, 0) and P(10, 16) lies on it, then the equation of normal at P is
[JEE MAIN 2020 (JAN)]
(A) $2x + 5y = 100$ (B) $2x + 5y = 10$
(C) $2x - 5y = 100$ (D) $5x + 2y = 100$

Q.61 Let the line $y = mx$ intersects the curve $y^2 = x$ at P and tangent to $y^2 = x$ at P intersects x-axis at Q. If area (ΔOPQ) = 4, find m ($m > 0$). **[JEE MAIN 2020 (JAN)]**

Q.62 If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to : **[JEE MAIN 2020 (JAN)]**
(A) 15 (B) 14
(C) 17 (D) 16

Q.63 The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is **[JEE MAIN 2020 (JAN)]**
(A) $\sqrt{\frac{5}{6}}$ (B) $\frac{1}{2}\sqrt{\frac{11}{3}}$
(C) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (D) $\frac{1}{2}\sqrt{\frac{5}{3}}$

Q.64 If one end of a focal chord AB of the parabola $y^2 = 8x$ is at A (1/2, –2), then the equation of the tangent to it at B is
[JEE MAIN 2020 (JAN)]
(A) $2x + y - 24 = 0$ (B) $x - 2y + 8 = 0$
(C) $2x - y - 24 = 0$ (D) $x + 2y + 8 = 0$

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	D	B	D	A	C	C	D	A	C	B	A	B	C	A	B	D	D	B	B	A	A	B	C	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	C	C	C	A	A	C	A	C	D	D	D	A	B	C	A	D	D	B	C	A	C	A	B	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	A	B	C	C	D	B	C	D	C	C	D	B	C	A	C	A	B	A	A	D	A	B	B	B	C
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	A	D	B	B	A	A	B	A	A	A	A	C	B	A	D	A	D	C	C	B	D	B	A	B	A
Q	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120					
A	B	D	C	B	A	C	C	C	C	A	C	C	A	B	B	B	A	C	A	B					

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	A	B	A	B	C	B	B	C	B	B	D	A	B	D	A	D	C	C	A	C	C	B	B	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	C	A	D	C	C	C	A	A	B	A	C	C	C	C	A	A	A	C	C	B	D	B	C	B
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	D	C	B	A	B	D	A	A	B	A	A	A	C	B	B	A	A	A	B	C	C	C	C	B
Q	76	77	78	79																					
A	A	D	A	A																					

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	14	2	11	3	16	4	2	13	4	65	2	2	4	9	15

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	D	A	B	B	D	B	A	B	D	D	D	D	D	D	B	D	C	B	AB
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	B	D	A	B	C	A	C	C	D	B	C	D	D	B	C	D	C	C	B	B
Q	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	A	C	C	C	C	B	A	A	C	C	D	A	D	D	C	B	C	B	C	A
Q	61	62	63	64																
A	0.5	D	B	B																

CHAPTER- 11 : CONIC SECTIONS (PARABOLA, ELLIPSE & HYPERBOLA SOLUTIONS TO TRY IT YOURSELF

TRY IT YOURSELF-1

(1) (i) The directrix is $x = -6$ i.e., $x = -a$ and focus (6, 0) i.e. (a, 0). So the parabola is of the form $y^2 = 4ax$.
The required equation of parabola is
$y^2 = 4 \times 6x \Rightarrow y^2 = 24x$

(ii) Since the vertex is at (0, 0) and focus is at (3, 0) which lies on x-axis, x-axis is the axis of parabola.
Therefore, the equation of the parabola is
$y^2 = 4ax \Rightarrow y^2 = 4(3)x \Rightarrow y^2 = 12x$

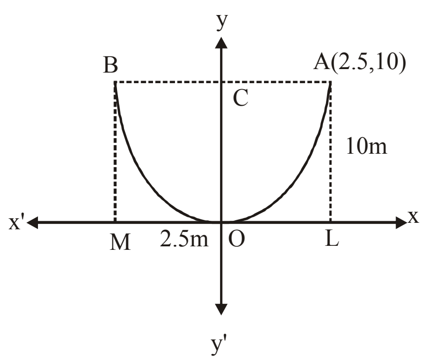
(2) Let the vertex of the parabola be at the origin and axis be along OY. Then, the equations of the parabola is

$x^2 = 4ay$ .... (1)

The co-ordinates of end A of the arc are (2.5, 10) and it lies on the parabola (1)

$\therefore \quad (2.5)^2 = 4a \times 10$ .... (2)

$\Rightarrow \quad a = \dfrac{6.25}{40} = \dfrac{5}{32}$ .... (3)



Putting the value of a from (2) in (1), we get

$$x^2 = 4\left(\frac{5}{32}\right)y$$

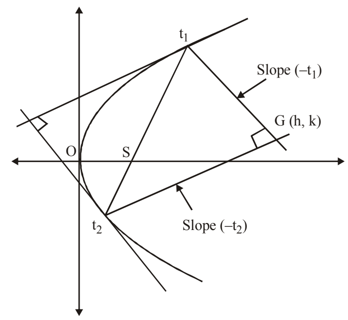
Substituting y = 2 in (3), we get

$$x^2 = \frac{5}{8} \times 2 \Rightarrow x = \frac{\sqrt{5}}{2}m$$

Hence, the width of the arc at a height of 2m from vertex is

$$2 \times \frac{\sqrt{5}}{2}m = \sqrt{5}m = 2.23\,m\,(\text{approx.})$$

(3) **(B).** N : $y + tx = 2at + at^3$; passes through (h, k)



Hence, $at^3 + (2a - h)t + k = 0$

$$t_1t_2t_3 = -\frac{k}{a} \;;\; t_1t_2 = -1$$

Chord joining t_1 and t_2 is
$2x - (t_1 + t_2)y + 2at_1t_2 = 0$
$(2x - 2a) - (t_1 + t_2)y = 0$
$x = a$ & $y = 0$

Alternatively, If the normal intersect at right angles then their corresponding tangent will also intersect at right angles hence the chord joining their feet must be a focal chord.

$\therefore$ It will always pass through (a, 0)

(4) **(C).** $\dfrac{x+y}{2} = t^2 + 1, \; \dfrac{x-y}{2} = t$

Eliminating t, $2(x + y) = (x - y)^2 + 4$
Since 2nd degree terms form a perfect square, it represents a parabola.

(5) **(C).** $y^2 = kx - 8 \Rightarrow y^2 = k\left(x - \dfrac{g}{k}\right)$

Directrix of parabola is $x = \dfrac{8}{k} - \dfrac{k}{4}$

Now, x = 1 also coincides with $x = \dfrac{8}{k} - \dfrac{k}{4}$

Solving, $\dfrac{8}{k} - \dfrac{k}{4} = 1$, we get k = 4

(6) **(B).** $y = mx + c$ is normal to the parabola$y^2 = 4ax$ if $c = -2am - am^3$.
Here, m = –1 and c = k and a = 3
$\therefore \quad c = k = -2(3)(-1) - 3(-1)^3 = 9$

(7) **(C).** Let at pt. (x_1, y_1) of parabola $y^2 = 4x$ equation of tangent is $yy_1 = 2(x + x_1)$
i.e., $2x - yy_1 + 2x_1 = 0$ .... (1)
As it is tangent to the circle $(x - 3)^2 + y^2 = 9$
$\therefore$ Length of $\perp$ from (3, 0) to (1) is 3.

$$\therefore \left|\frac{6 + 2x_1}{\sqrt{4 + y_1^2}}\right| = 3 \Rightarrow 36 + 24x_1 + 4x_1^2 = 9(4 + y_1^2)$$

Also, $y_1^2 = 4x_1$

We get, $4x_1^2 + 24x_1 + 36 = 36 + 36x_1$

$x_1^2 + 6x_1 - 9x_1 = 0$

$x_1 = 0, 3 \Rightarrow y_1 = 0, \pm 2\sqrt{3}$

As tangent is above x-axis, $y_1 \neq 0,\ y_1 \neq -2\sqrt{3}$

$\therefore\ x_1 = 3$ and $y_1 = 2\sqrt{3}$

Equation is $2x - 2\sqrt{3}y + 6 = 0$

(Substituting values in (1))

$\Rightarrow\ \sqrt{3}y = x + 3$

(8) **(C).** If (h, k) is the mid-point of line joining focus (a, 0) and Q $(at^2, 2at)$ on parabola then

$$h = \frac{a + at^2}{2},\quad k = at$$

Eliminating t, we get, $2h = a + a(k^2/a^2)$

$\Rightarrow\ k^2 = a(2h - a)$

$\Rightarrow\ k^2 = 2a(h - a/2)$

$\therefore$ Locus of (h, k) is $(x - a/2) = -a/2$

$\Rightarrow\ x = 0$

(9) **(C).** $y = mx + 1/m$

Above tangent passes through (1, 4).

$\Rightarrow\ 4 = m + 1/m \Rightarrow m^2 - 4m + 1 = 0$

Now, angle between the lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$= \frac{\sqrt{16 - 4}}{1 + 1} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

TRY IT YOURSELF-2

(1) Since, the denominator of $\frac{x^2}{36}$ is larger than the denominator of $\frac{y^2}{16}$, the major axis is along the x-axis.

Comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = 36\ \Rightarrow a = 6$ and $b^2 = 16\ \Rightarrow b = 4$

Also, $c = \sqrt{a^2 - b^2} = \sqrt{36 - 16} = \sqrt{20} = 2\sqrt{5}$

1. Focus : The coordinates of foci (–c, 0) and (c, 0) are $(-2\sqrt{5}, 0), (2\sqrt{5}, 0)$
2. Vertices : Vertices are (–a, 0) and (a, 0) are (–6, 0) and (6, 0).
3. Length of major axis : 2a = 12
4. Length of minor axis : 2b = 8
5. Eccentricity : $e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$
6. Length of latus rectum : $2\ell = \frac{2b^2}{a} = \frac{32}{6} = \frac{16}{3}$

(2) (i) Since the vertices are on x-axis the equation will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, given that a = 5 and c = 4

We know that, $c^2 = a^2 - b^2$ $[\because a = 5, c = 4]$

$\Rightarrow\ b^2 = a^2 - c^2 = 25 - 16$

$\Rightarrow\ b^2 = 9 \Rightarrow b = 3$

Hence, the equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

(ii) Ends of major axis $(0, \pm\sqrt{5})$ lies on y-axis.

So, the equation of ellipse in standard form is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Now ends of major axis $(0, \pm a)$ is $(0, \pm\sqrt{5}) \Rightarrow a = \sqrt{5}$

Ends of minor axis $(\pm b, 0)$ is $(\pm 1, 0) \Rightarrow b = 1$

Thus equation of required ellipse is $\frac{x^2}{1} + \frac{y^2}{5} = 1$

(3) Let ABA' be the given arc such that

AA' = 8m and OB = 2m

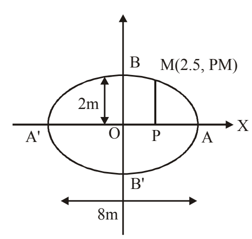
Let the arc be a part of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Then, $AA' = 8m\ \Rightarrow 2a = 8 \Rightarrow a = 4$

and $OB = 2m \Rightarrow b = 2$

So, the equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \qquad \text{.......... (1)}$$



We have to find the height of the arc at point P such that AP = 1.5m. In other words, we have to find the y-coordinate at P.

$\because$ OA = 4m and AP = 1.5m

$\therefore$ OP = 2.5m

Thus, the coordinate of M are $\left(\frac{5}{2}, PM\right)$.

Since, P lies on the ellipse (1). Therefore,

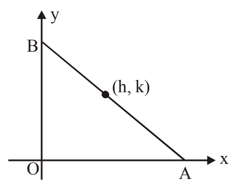
$$\frac{25}{4\times 16}+\frac{PM^2}{4}=1\Rightarrow\frac{PM^2}{4}=1-\frac{25}{64}\Rightarrow\frac{PM^2}{4}=\frac{39}{64}$$

$$\Rightarrow\quad PM=\sqrt{\frac{39}{16}}m=\frac{\sqrt{39}}{4}=1.56m \text{ (approx)}$$

Hence, the height of the arc at a point 1.5m from one end is 1.56m.

(4) **(A).** Any tangent to ellipse

$$\frac{x^2}{2}+\frac{y^2}{1}=1 \text{ is } \frac{x\cos\theta}{\sqrt{2}}+y\sin\theta=1.$$



(Using mid pt. formula)

$\therefore$ $A(\sqrt{2}\sec\theta, 0)$; $B(0, \operatorname{cosec}\theta)$

$\Rightarrow$ $2h=\sqrt{2}\sec\theta$ and $2k=\operatorname{cosec}\theta$

$$\Rightarrow\left(\frac{1}{\sqrt{2}h}\right)^2+\left(\frac{1}{2k}\right)^2=1$$

$$\Rightarrow\frac{1}{2h^2}+\frac{1}{4k^2}=1\Rightarrow\frac{1}{2x^2}+\frac{1}{4y^2}=1$$

(5) **(A).** Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at

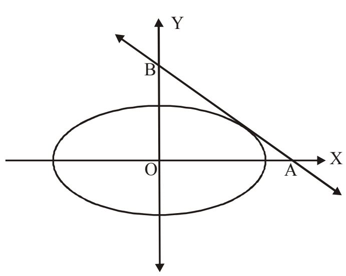
$P(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

It meets co-ordinate axes at

$A(a\sec\theta, 0)$ and $B(0, b\operatorname{cosec}\theta)$

$\therefore$ Area of $\Delta OAB=\frac{1}{2}\times a\sec\theta\times b\operatorname{cosec}\theta$

$$\Rightarrow\quad\Delta=\frac{ab}{\sin 2\theta}$$



For Δ to e min, $\sin 2\theta$ should be maxi. and we know max. value of $\sin 2\theta=1$

$\therefore$ $\Delta_{max}=ab$

(6) **(A).** Let the common tangent to circle, $x^2+y^2=16$ and ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ be $y=mx+\sqrt{25m^2+4}$ .... (1)

As it is tangent to circle $x^2+y^2=16$, we should have

$$\frac{\sqrt{25m^2+4}}{\sqrt{m^2+1}}=4$$

[Using : length of perpendicular from (0, 0) to (1) = 4]

$\Rightarrow$ $25m^2+4=16m^2+16$

$\Rightarrow$ $9m^2=12\Rightarrow m=-2/\sqrt{3}$

[Leaving +ve sign, to consider tangent in I quadrant]

$\therefore$ Equation of common tangent is

$$y=-\frac{2}{\sqrt{3}}x+\sqrt{25\cdot\frac{4}{3}+4}=-\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}$$

This tangent meets the axes at $A(2\sqrt{7}, 0)$ and $B(0, 4, \sqrt{7/3})$

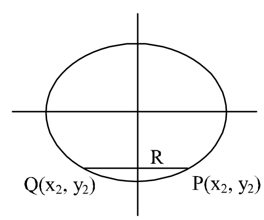
$\therefore$ Length of intercepted portion of tangent between

$$\text{axes}=AB=\sqrt{(2\sqrt{7})^2+\left(4\sqrt{\frac{7}{3}}\right)^2}=\frac{14}{\sqrt{3}}$$

(7) **(B).** $\frac{x^2}{4}+\frac{y^2}{1}=1$; $b^2=a^2(1-e^2)\Rightarrow e=\frac{\sqrt{3}}{2}$

$$\Rightarrow P\left(\sqrt{3},-\frac{1}{2}\right)\text{ and } Q\left(-\sqrt{3},-\frac{1}{2}\right)$$

(given y_1 and y_2 less than 0)



Co-ordinates of mid-point of PQ are $R\equiv(0,-1/2)$

$PQ=2\sqrt{3}$ = length of latusrectum.

$\Rightarrow$ Two parabola are possible whose vertices are

$$\left(0,-\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\text{ and }\left(0,\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

Hence the equations of the parabolas are

$$x^2-2\sqrt{3}y=3+\sqrt{3}\text{ and } x^2+2\sqrt{3}y=3-\sqrt{3}$$

(8) **(D).** $y=mx+\sqrt{9m^2+4}$

$$4-3m=\sqrt{9m^2+4}$$

$$16+9m^2-24m=9m^2+4\Rightarrow m=\frac{12}{24}=\frac{1}{2}$$

Equation is $y-4=\frac{1}{2}(x-3)$

$2y-8=x-3 \Rightarrow x-2y+5=0$

Let $B=(\alpha,\beta) \Rightarrow \frac{x\alpha}{9}+\frac{y\beta}{4}-1=0$

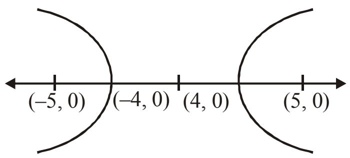
$\Rightarrow \frac{\alpha/9}{1}=\frac{\beta/4}{-2}=\frac{-1}{5} \Rightarrow \alpha=-\frac{9}{5}, \beta=\frac{8}{5}$

$B \equiv (-9/5, 8/5)$

TRY IT YOURSELF-3

(1) Comparing the equation, $\frac{x^2}{16}-\frac{y^2}{9}=1$ .... (1)

with the standard equation, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ .... (2)



Here, $a^2=16 \Rightarrow a=4$ and $b^2=9 \Rightarrow b=3$

Also, $c^2=a^2+b^2=16+9=25 \Rightarrow c=5$

The given equation (1) is in the form of standard equation (2) so its foci and vertices lie on x-axis.

$\therefore$ The coordinates of foci are $(\pm 5, 0)$ and that of vertices are $(\pm 4, 0)$

Eccentricity $= e=\frac{c}{a}=\frac{5}{4}$

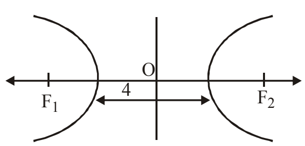
Latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{18}{4}=\frac{9}{2}$

(2) (i) We have, foci $\equiv(\pm c, 0)\equiv(\pm 3, 0) \Rightarrow c=3$

and vertices $(\pm a, 0)\equiv(\pm 2, 0) \Rightarrow a=2$

But, $c^2=a^2+b^2 \Rightarrow 9=4+b^2$

$\Rightarrow b^2=9-4=5 \Rightarrow b^2=5$



Hence, the foci and vertices lie on x-axis, therefore the equation of hyperbola is of the form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{4}-\frac{y^2}{5}=1$$

which is required equation of hyperbola.

Alt. : Foci $(\pm ae, 0)$; $ae=3, e=3/2$

$b=a\sqrt{e^2-1}=\sqrt{5}$

$\therefore$ Required equation is $\frac{x^2}{4}-\frac{y^2}{5}=1$

(ii) Here foci are at $(0, \pm 13) \Rightarrow c=13$

Conjugate axis is of length 24

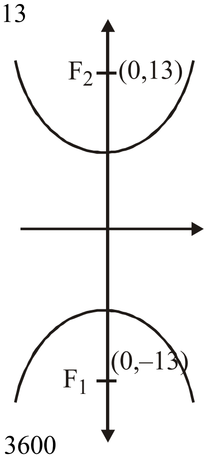
$2b=24 \Rightarrow b=12$

Also, we know that

$c^2=a^2+b^2 \Rightarrow 169=a^2+144$

$\Rightarrow a^2=169-144=25 \Rightarrow a=5$

Here, the foci lie on y-axis, therefore the equation of hyperbola is of the form



$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

i.e., $\frac{y^2}{25}-\frac{x^2}{144}=1 \Rightarrow 144y^2-25x^2=3600$

which is required equation of hyperbola.

(3) **(C).** For hyperbola,

$$\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25} \Rightarrow \frac{x^2}{144/25}-\frac{y^2}{81/25}=1$$

$$e^2=1+\frac{b^2}{a^2}=1+\frac{81}{144}=\frac{225}{144};\ e=\frac{15}{12}=\frac{5}{4}$$

Hence, the foci are

$$(\pm ae, 0)=\left(\pm\frac{12}{5}\cdot\frac{5}{4}, 0\right)=(\pm 3, 0)$$

Now, the foci coincide therefore for ellipse

$$ae=3 \text{ or } a^2e^2=9 \text{ or } a^2\left(1-\frac{b^2}{a^2}\right)=9$$

$a^2-b^2=9$ or $16-b^2=9 \Rightarrow b^2=7$

(4) **(C).** We have, $9x^2-16y^2-18x+32y-151=0$

$9(x^2-2x)-16(y^2-2y)=151$

$9(x^2-2x+1)-16(y^2-2y+1)=144$

$9(x-1)^2-16(y-1)^2=144$

$$\frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$$

Shifting the origin at (1, 1) without rotating the axes

$\frac{X^2}{16}-\frac{Y^2}{9}=1$, where $X=x-1$ and $Y=y-1$

This is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

where $a^2=16$ and $b^2=9$,

The length of the transverse axes $=2a=8$;

$\ell(CA)=2b=6$

The length of the latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

and $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{9}{16}}=\frac{5}{4}$

The equation of the directrix $X = \pm a/e$

$$x - 1 = \pm\frac{16}{5} \Rightarrow x = \pm\frac{16}{5} + 1$$

$$x = \frac{21}{5} \text{ and } x = -\frac{11}{5}$$

$\Rightarrow$ Equation of directrix is $x = 21/5$ and $x = -11/5$.

(5) **(B).** The given equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

$\Rightarrow$ $a = \cos\alpha,\ b = \sin\alpha$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$$

$\Rightarrow$ $ae = 1$

$\therefore$ focii $(\pm 1, 0)$

$\therefore$ focii remain constant with respect to α.

(6) **(A).** Equation of tangent to hyperbola $x^2 - 2y^2 = 4$ at any point $(x1, y1)$ is $xx_1 - 2yy_1 = 4$.

Comparing with $2x + \sqrt{6}y = 2$ or $4x + 2\sqrt{6}y = 4$

$\Rightarrow$ $x_1 = 4$ and $-2y_1 = 2\sqrt{6} \Rightarrow (4, -\sqrt{6})$ is the required point.

(7) **(B).** Equation of normal at P (6, 3)

$$\frac{a^2x}{6} - \frac{b^2y}{3} = a^2 + b^2$$

It passes through (9, 0)

$$\frac{3}{2}a^2 = a^2 + b^2 \Rightarrow \frac{3}{2} = \frac{a^2 + b^2}{a^2} = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$$

(8) **(D).** Let parametric coordinates be P $(3\sec\theta, 2\tan\theta)$

Equation of tangent at point P will be

$$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} = 1$$

$\because$ Tangent is parallel to $2x - y = 1$

$$\Rightarrow \frac{2\sec\theta}{3\tan\theta} = 2 \Rightarrow \sin\theta = \frac{1}{3}$$

$\therefore$ Coordinates are $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

CHAPTER- 11 : CONIC SECTIONS (PARABOLA, ELLIPSE & HYPERBOLA

EXERCISE-1

(1) **(A).** Since the distance between the focus and directrix of the parabola is half of the length of the latus rectum (L.R.). Therefore, L.R. = 2 (Length of the perpendicular from $(3,3)$ on $3x-4y-2=0$) i.e., $\left|\frac{9-12-2}{\sqrt{9+16}}\right|=2.$

(2) **(D).** Here $y^2+4y+4+4x-2=0$

or $(y+2)^2=-4\left(x-\frac{1}{2}\right)$.

Let $y+2=Y, \frac{1}{2}-x=X$. Then the parabola is $Y^2=4X$.

$\therefore$ Directrix is $X+1=0$ or $\frac{1}{2}-x+1=0$. So, $x=\frac{3}{2}$.

(3) **(B).** Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be two points on the parabola $y^2=4ax$. Then the tangents at P and Q intersect at T $\{at_1t_2, a(t_1+t_2)\}$.

Now SP $=\sqrt{(at_1^2-a)^2+(2at_1-0)^2}=a(t_1^2+1)$

$SQ=a(t_2^2+1)$

$ST=\sqrt{(at_1t_2-a)^2+(a(t_1+t_2)-0)^2}=a\sqrt{(1+t_1^2)(1+t_2^2)}$

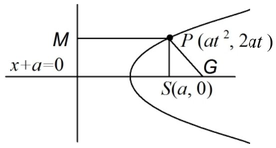
$\therefore ST^2=a^2(1+t_1^2)(1+t_2^2)=SP.SQ$

Hence SP, ST and SQ are in G.P.

(4) **(D).** The co-ordinates of the focus of the parabola $y^2=4ax$ are $(a,0)$. The line $x-y-a=0$ passes through this point. Therefore, it is a focal chord of the parabola. Hence the tangents intersect at right angle.

(5) **(A).** The co-ordinates of ends of LR are $(a, 2a)$ and $(a,-2a)$. In the given parabola, these points are $(3,6)$ and $(3,-6)$ the equation of tangents are $6y=6(x+3)$ and $-6y=6(x+3) \Rightarrow x-y+3=0$ and $x+y+3=0$. The intersection of these tangents are $x=-6$, which is the equation of directrix.

(6) **(C).** Let $P(at^2, 2at)$ be any point on parabola $y^2=4ax$, then equation of tangent and normal at $P(at^2, 2at)$ are $ty=x+at^2$ and $y=-tx+2at+at^3$ respectively.



Since tangent and normal meet its axes in T and G.

$\therefore$ Co-ordinates of T and G are $(-at^2, 0)$ of $(2a+at^2, 0)$ respectively.

$SP=PM=a+at^2$, $SG=VG-VS=2a+at^2-a=a+at^2$

and $ST=VS+VT=a+at^2$. Hence SP = SG = ST.

(7) **(C).** The slope form of the normal to the parabola $y^2=4ax$ is $y=mx-2am-am^3$.

For the given curve $y^2=x$, we will have $4a=1 \Rightarrow a=\frac{1}{4}$

Hence the equation of the normal is $y^2=mx-\frac{1}{2}m-\frac{1}{4}m^3$

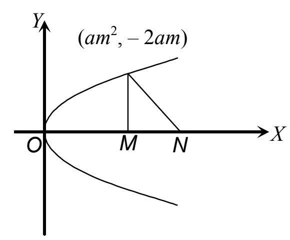
If it passes through $(C,0)$, then

$0=mC-\frac{1}{2}m-\frac{1}{4}m^3 \Rightarrow m=0$

or $C-\frac{1}{2}-\frac{1}{4}m^2=0 \Rightarrow m=\pm 2\sqrt{C-\frac{1}{2}}$

For three normals, value of m should be real, $\therefore C>\frac{1}{2}$.

(8) **(D).** Let co-ordinates of P is $(am^2, -2am)$



2)

Equation of normal at point P is $y=mx-2am-am^3$

This normal cuts x-axis at N

Putting $y=0$; we get, $0=mx-2am-am^3$

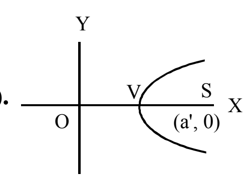
or $mx=am+am^3$ or $x=\frac{m(2a+am^2)}{m}$ or $x=2a+am^2$

So, ON = $2a+am^2$ and $OM=am^2$

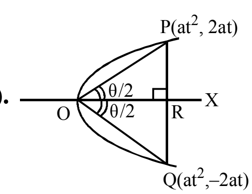
Length of subnormal = MN

$\therefore$ MN = ON – OM $=2a+am^2-am^2=2a$.

(9) **(A).**



(10) **(C).**



PQ = 8a ⇒ 4at = 8a ⇒ t = 2

$\tan(\theta/2) = \frac{PR}{OR} = \frac{2at}{at^2} = \frac{2}{t} = \frac{2}{2} = 1$; $\frac{\theta}{2} = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$

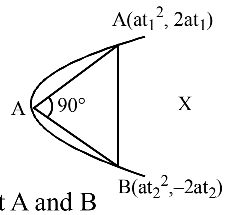
(11) **(B).** Parabola is $(x-2)^2 = 8(y+1)$.

(12) **(A).** Let tangent is $y = 2x + c$
Put in in $y^2 = 4x + 5$
$(2x+1)^2 = 4x + 5$
Discriminant $= 0 \Rightarrow c = 3$
$y = 2x + 3$

(13) **(B).** Direct property

(14) **(C).** Slope of OA, $m_1 = \frac{2}{t_1}$,

Slope of OB, $m_2 = \frac{2}{t_2}$



$m_1m_2 = -1$; $t_1t_2 = -4$
Point of intersection of tangent at A and B
$(at_1t_2, a(t_1+t_2)) \equiv (-4a, a(t_1+t_2))$ lies an $x = -4a$

(15) **(A).** Any tangent to parabola $y^2 = 32$ is

$y = mx + \frac{8}{m}$ also tangent to $x^2 = -4y$ ($y = mx + \frac{8}{m}$,

$c = -am^2$

$\frac{8}{m} = -(-1)m^2 \Rightarrow m^3 = 8 \Rightarrow m = 2$

Tangent $y = 2x + 4$

(16) **(B).** $a = 3 \Rightarrow$ abscissa is $4 - 3 = 1$ and $y^2 = 12, y = \pm 2\sqrt{3}$.

Hence points are $(1, 2\sqrt{3}), (1, -2\sqrt{3})$.

(17) **(D).** It is obvious.

(18) **(D).** Given parabola can be written as $(y+1)^2 = -(x-1)$.
Hence vertex is $(1, -1)$, which lies in IV quadrant.

(19) **(B).** The equation of the parabola referred to its vertex as the origin is $X^2 = lY$, where $x = X + a, y = Y + b$. Therefore the equation of the parabola referred to the point (a,b) as the vertex is

$(x-a)^2 = l(y-b)$ or $(x-a)^2 = \frac{l}{2}(2y - 2b)$.

(20) **(B).** Given equation of parabola written in standard form, we get

$4\left(y - \frac{1}{2}\right)^2 = 6(x+1) \Rightarrow \left(y - \frac{1}{2}\right)^2 = \frac{3}{2}(x+1) \Rightarrow Y^2 = \frac{3}{2}X$

where, $Y = y - \frac{1}{2}$, $X = x + 1$ $\therefore y = Y + \frac{1}{2}$, $x = X - 1$ …..(i)

For focus $X = a,\ Y = 0$

$\because 4a = \frac{3}{2} \Rightarrow a = \frac{3}{8} \Rightarrow x = \frac{3}{8} - 1 = -\frac{5}{8}$

$y = 0 + \frac{1}{2} = \frac{1}{2}$, Focus $= \left(-\frac{5}{8}, \frac{1}{2}\right)$.

(21) **(A).** Given equation of parabola is $2x^2 + 5y - 3x + 4 = 0$

$\Rightarrow x^2 - \frac{3}{2}x = -\frac{5}{2}y - 2 \Rightarrow \left(x - \frac{3}{4}\right)^2 = -\frac{5}{2}y - \frac{23}{16}$

$\therefore$ Equation of axis is, $x - \frac{3}{4} = 0 \Rightarrow x = \frac{3}{4}$.

(22) **(A).** Tangent at $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

Therefore, tangent at $y^2 = 4a(x+a)$ is,

$y = m(x+a) + \frac{a}{m}$ or $y = mx + ma + \frac{a}{m} \Rightarrow ma + \frac{a}{m} = c$.

(23) **(B).** Principal axes of parabolas are x-axis and y-axis, therefore angle between them is 90^o.

(24) **(C).** Let $y = mx + c$ is chord and c is variable

$\Rightarrow x = \left(\frac{y-c}{m}\right)$ by $y^2 = 4ax$

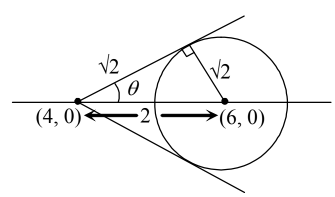
For getting points of intersection,

$y^2 = 4a\left(\frac{y-c}{m}\right) \Rightarrow y^2 - \frac{4ay}{m} + \frac{4ac}{m} = 0$

$\Rightarrow y_1 + y_2 = \frac{4a}{m} \Rightarrow \frac{y_1 + y_2}{2} = \frac{2a}{m}$

which is a constant; independent to c.

(25) **(A).** From diagram, $\theta = 45^o$
$\Rightarrow$ Slope $= \pm 1$.



(26) **(C).** Let P $(t_1^2, 2t_1)$ and Q $(t_2^2, 2t_2)$ be the ends of the chord PQ of the parabola $y^2 = 4x$ ...... (1)

Slope of chord PQ $= \frac{2t_2 - 2t_1}{t_2^2 - t_1^2} = 2 \Rightarrow t_2 + t_1 = 1$...... (2)

If R (x_1, y_1) is a point dividing PQ internally in the ratio

1 : 2, then $x_1 = \frac{1.t_2^2 + 2t_1^2}{1+2}$, $y_1 = \frac{1.2t_2 + 2.2t_1}{1+2}$

$\Rightarrow t_2^2 + 2t_1^2 = 3x_1$ .... (3)
and $t_2 + 2t_1 = (3y_1)/2$ .... (4)

From (2) and (4), we get, $t_1 = \frac{3}{2}y_1 - 1, t_2 = 2 - \frac{3}{2}y_1$

Substituting in eq. (3), we get

$\left(2 - \frac{3}{2}y_1\right)^2 + 2\left(\frac{3}{2}y_1 - 1\right)^2 = 3x_1$

$\Rightarrow \left(\frac{9}{4}\right)y_1^2 - 4y_1 = x_1 - 2$

$$\left(y_1-\frac{8}{9}\right)^2=\left(\frac{4}{9}\right)\left(x_1-\frac{2}{9}\right)$$

$\therefore$ Locus of the point R (x_1, y_1) is

$$\left(y-\frac{8}{9}\right)^2=\left(\frac{4}{9}\right)\left(x-\frac{2}{9}\right)$$

which is parabola having vertex at the point (2/9, 8/9).

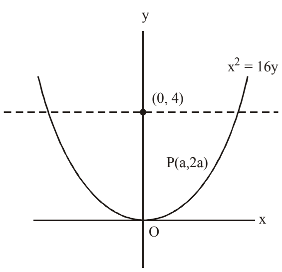
(27) **(C).** Equation of focal chord $(y-0)=\tan 30°\,(x-a)$

$$\Rightarrow\ y=\frac{x}{\sqrt3}-\frac{a}{\sqrt3}\Rightarrow m=\frac{1}{\sqrt3},\ c=-\frac{a}{\sqrt3}$$

$$\text{Length}=\frac{4}{m^2}\sqrt{1+m^2}\ \sqrt{a\,(a-mc)}$$

$$=\frac{4}{1/3}\sqrt{1+\frac{1}{3}}\sqrt{a\left(a+\frac{a}{3}\right)}=12a\left(1+\frac{1}{3}\right)=16a$$

(28) **(C).**



Parabola $x^2=16y$, Axis $\Rightarrow$ y-axis, focus $\Rightarrow$ (0, 4)
It is clear if point P (a, 2a) lies ineterior region than ordinate $\in$ (0, 4)
$\Rightarrow\ 0<2a<4$
$0<a<2$

(29) **(C).** Point of intersection of perpendicular tangents lies on directrix. Hence resultant point is intersection point of tangent $y=4x+1$ and directric $x=-4$
$\Rightarrow\ (-4,-15)$

(30) **(A).** Focus (2, 0)
$\Rightarrow$ line is focal chord and SA, SB are length of segments of focal chord

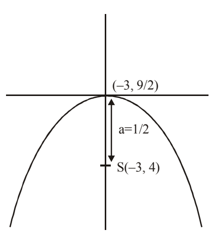
$$\frac{1}{SA}+\frac{1}{SB}=\frac{SA+SB}{SA.SB}=\frac{1}{a}=\frac{1}{2}$$

(31) **(A).** $x^2+6x+9=-2y+9$

$$(x+3)^2=-2\left(y-\frac{9}{2}\right)$$

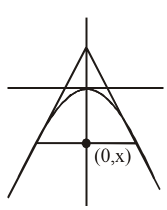
$$4a=2\Rightarrow a=\frac{1}{2}$$

focus $\equiv(-3, 4)$



(32) **(C).** $x^2=-8y$; $4a=-8\Rightarrow a=-2$
$y=-x^2/8$; $y'=-x/4$

(33) **(A).** $mx-y+c=0$ .... (1)
$yy_1=2a\,(x+x_1)$
$2ax-yy_1+2ax_1=0$



(34) **(C).** $3x^2+6x=4y+3\Rightarrow 3\,(x+1)^2=4y+6$

$$\Rightarrow(x+1)^2=\frac{4}{3}\left(y+\frac{6}{4}\right)\qquad \text{LLR}=4/3$$

(35) **(D).** Consider latus rectum, with ends (a, 2a), (a, –2a).

Sum of reciprocals of focal distances is $\frac{1}{2a}+\frac{1}{2a}=\frac{1}{a}$

(36) **(D).** $x=t^2+2,\ y=2t$
Comparing with $x=at^2+h,\ y=2at+k$,
$h=2, k=0, a=1$
$\therefore$ Equation of parabola is, $(y-0)^2=4\,(1)\,(x-2)$
$y^2=4\,(x-2)$

(37) **(D).** $y=mx+c$ be a tangent $\theta=\pi/4\Rightarrow m=1$
$\therefore\ y=1.x+c\ \therefore\ a=1$

Condition is $c=\frac{a}{m}=\frac{1}{1}$

$\therefore\ y=x+1\Rightarrow x-y+1=0$

(38) **(A).** $x^2=12y\Rightarrow 4a=12\Rightarrow a=3$

Area of triangle $=\frac{1}{2}$ (base) (height)

$$\frac{1}{2}(4\,|\,a\,|)\,(|\,a\,|)=\frac{1}{2}(12)\,(3)=18$$

(39) **(B).** $4y^2+3y=-3x-1$

$$\text{LR}=\frac{|\text{coefficient of } x\,|}{\text{coefficient of } y^2}=\frac{3}{4}$$

(40) **(C).** The coordinate of any point on the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angle θ are (a cosθ,b sinθ)

The coordinate of the end point of latus rectum are

$\left(ae,\ \pm\frac{b^2}{a}\right)$ a cos θ = ae and

$$b\sin\theta=\pm\frac{b^2}{a}\ \tan\theta=\pm\frac{b}{ae}\Rightarrow\theta=\tan^{-1}\left(\pm\frac{b}{ae}\right)$$

(41) **(A).** Suppose the line $\ell x+my+n=0$ cuts the ellipse at P (a cos θ, b sin θ) and Q (a cos (π/2 + θ), b sin (π/2 + θ)).
Then these two point lie on the line
$\ell a\cos\theta+mb\sin\theta+n=0$
$-\ell a\sin\theta+mb\cos\theta+n=0$
$\ell a\cos\theta+mb\sin\theta=-n$ ...(i)
$\ell a\sin\theta+mb\cos\theta=-n$ ...(ii)
Square and add the equations (i) and (ii)
$(\ell a\cos\theta+mb\sin\theta)^2+(-\ell a\sin\theta+mb\cos\theta)^2=n^2+n^2$
$\ell^2a^2(\cos^2\theta+\sin^2\theta)+m^2b^2(\sin^2\theta+\cos^2\theta)=n^2+n^2$
$\Rightarrow\ell^2a^2+m^2b^2=2n^2$

(42) **(D).** The equation of any tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } y=mx+\sqrt{a^2m^2+b^2}$$

$\Rightarrow mx - y + \sqrt{a^2m^2 + b^2} = 0$ ...(i)

The two foci of the given ellipse are S(ae, 0) & S′ (–ae, 0). Let p_1 and p_2 be the lengths of perpendicular from S and S′ respectively on (i), Then

p_1 = length of perpendicular from S(ae, 0) on (i)

$$p_1 = \frac{mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$$

p_2 = length of perpendicular from S′(–ae, 0) on (i)

$$p_2 = \frac{-mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$$

$$p_1p_2 = \left(\frac{mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}\right)\left(\frac{-mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}\right)$$

$$= \frac{a^2m^2(1-e^2) + b^2}{1+m^2} \quad \because b^2 = a^2(1-e^2)$$

$$= \frac{m^2b^2 + b^2}{1+m^2} = \frac{b^2(m^2+1)}{m^2+1} = b^2$$

(43) **(D).** Centre being mid point of the foci is

$$\left(\frac{1+3}{2}, 0\right) = (2, 0)$$

Distance between foci 2ae = 2

$ae = 1$ or $a^2 - b^2 = 1$ ...(i)

If the ellipse $\frac{(x-2)^2}{a^2} + \frac{y^2}{b^2} = 1$,

then as it passes from (0, 0)

$\frac{4}{a^2} = 1 \Rightarrow a^2 = 4$; from (i) $b^2 = 3$

Hence $\frac{(x-2)^2}{4} + \frac{y^2}{3} = 1$ or $3x^2 + 4y^2 - 12x = 0$

(44) **(B).** The race course will be an ellipse with the flag posts as its foci. If a and b are the semi major and minor axes of the ellipse, then sum of focal distances 2a = 10 and 2ae=8

$$a = 5, e = 4/5 \;\therefore\; b^2 = a^2(1-e^2) = 25\left(1 - \frac{16}{25}\right) = 9$$

Area of the ellipse = $\pi ab = \pi.5.3 = 15\pi$

(45) **(C).** Any point on the ellipse is

$(\sqrt{6}\cos\phi, \sqrt{2}\sin\phi)$, where ϕ is an eccentric angle.

It's distance from the center (0, 0) is given α

$6\cos^2\phi + 2\sin^2\phi = 4$

or $3\cos^2\phi + \sin^2\phi = 2$

$2\cos^2\phi = 1$

$$\Rightarrow \cos\phi = \pm\frac{1}{\sqrt{2}} \;;\; \phi = \frac{\pi}{4}, \frac{3\pi}{4}$$

(46) **(A).** Let P (x,y) be any point on the ellipse whose focus is S(–1,1) and eccentricity e =1/2. Let PM be perpendicular from P on the directrix. Then,

$$SP = ePM \Rightarrow SP = \frac{1}{2}(PM) \Rightarrow 4(SP)^2 = PM^2$$

$$\Rightarrow 4[(x+1)^2 + (y-1)^2] = \left(\frac{x-y+3}{\sqrt{1^2+(-1)^2}}\right)^2$$

$\Rightarrow 8(x^2 + y^2 + 2x - 2y + 2) = (x - y + 3)^2$

$\Rightarrow 7x^2 + 7y^2 + 10x - 10y + 2xy + 7 = 0$

This is the required equation of the ellipse.

(47) **(C).** Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ...(1)$$

The coordinates of its vertices and foci are (± a, 0) and (± ae,0) respectively.

$\therefore$ a = 5 and ae = 4 $\Rightarrow$ e = 4/5.

Now, $b^2 = a^2(1-e^2) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$.

Substituting the values of a^2 and b^2 in (1) , we get

$\frac{x^2}{25} + \frac{y^2}{9} = 1$, which is the equation of the required ellipse.

(48) **(A).** The given equation can be rewritten as

$2[x^2 - 2x] + 3[y^2 - 4y] + 13 = 0$

or $2(x-1)^2 + 3(y-2)^2 = 1$

or $\frac{(x-1)^2}{(1/\sqrt{2})^2} + \frac{(y-2)^2}{(1/\sqrt{3})^2} = 1$, or $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$\therefore$ Centre X = 0, Y = 0 i.e. (1,2).

Length of major axis = $2a = \sqrt{2}$

Length of minor axis = $2b = 2/\sqrt{3}$ and

$e = \sqrt{(a^2 - b^2)/a} = 1/\sqrt{3}$

(49) **(B).** Equation of tangents at $P(\alpha)$, $Q(\beta)$ are

$$\frac{x\cos\alpha}{a} + \frac{y\sin\alpha}{b} = 1 \text{ and } \frac{x\cos\beta}{a} + \frac{y\sin\beta}{b} = 1,$$

These intersects in (x_1, y_1), then

$x_1 = a\cos 1/2(\alpha+\beta) / \cos 1/2(\alpha-\beta)$;

$y_1 = b\sin 1/2(\alpha+\beta) / \cos 1/2(\alpha-\beta)$.

Similarly, other points of intersections.

(50) **(A).** The slope of the tangent = tan 60º = $\sqrt{3}$

Now, $4x^2 + 3y^2 = 5 \Rightarrow \frac{x^2}{5/4} + \frac{y^2}{5/3} = 1$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = \frac{5}{4}$ and $b^2 = \frac{5}{3}$.

We know that the equations of the tangents of slope m to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by

$y = mx \pm \sqrt{a^2m^2 + b^2}$ and the coordinates of the points of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

Here, $m = \sqrt{3}$, $a^2 = 5/4$ and $b^2 = 5/3$.

So, the equations of the tangents are

$$y = \sqrt{3}\,x \pm \sqrt{\left(\frac{5}{4}\times 3\right) + \frac{5}{3}} \text{ i.e. } y = \sqrt{3}\,x \pm \sqrt{\frac{65}{12}}$$

(51) **(A).** $e = 1 - \frac{b^2}{a^2} = 1 - \frac{9}{16}$ $\quad \therefore e = \frac{\sqrt{7}}{4}$

$\therefore$ Foci are $(\pm ae, 0)$ or $(\pm\sqrt{7}, 0)$.

Centre is $(0,3)$ $\quad \therefore$ Radius $= \sqrt{7+9} = 4$

(52) **(B).** $25(x^2 - 6x + 9) + 16y^2 = 175 + 225$

or $25(x-3)^2 + 16y^2 = 400$ or $\frac{X^2}{16} + \frac{Y^2}{25} = 1$.

Form $\frac{X^2}{b^2} + \frac{Y^2}{a^2} = 1$

$\therefore$ Major axis lies along y- axis.

$\therefore e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{25}$; $\quad \therefore e = \frac{3}{5}$

(53) **(C).** We know that the line $y = mx + c$ touches the curve

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$

Here, $a^2 = 4$, $b^2 = 1$, $m = 4$;

$\therefore c^2 = 64 + 1 \Rightarrow c = \pm\sqrt{65}$

(54) **(C).** The equations of the chords of contact of tangents drawn from (x_1,y_1) and (x_2,y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

are $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ ... (i) ; $\frac{xx_2}{a^2} + \frac{yy_2}{b^2} = 1$ ... (ii)

It is given that (i) and (ii) are at right angles.

$$\therefore \frac{-b^2}{a^2}\frac{x_1}{y_1} \times \frac{-b^2}{a^2}\frac{x_2}{y_2} = -1 \Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{a^4}{b^4}$$

(55) **(D).** Let $P(x_1, y_1)$ be a point on the ellipse

$\frac{x^2}{18} + \frac{y^2}{32} = 1 \Rightarrow \frac{x_1^2}{18} + \frac{y_1^2}{32} = 1$

The equation of the tangent at (x_1, y_1) is $\frac{xx_1}{18} + \frac{yy_1}{32} = 1$.

This meets the axes at $A\left(\frac{18}{x_1}, 0\right)$ and $B\left(0, \frac{32}{y_1}\right)$. It is given that slope of the tangent at (x_1, y_1) is $-\frac{3}{4}$.

Hence $-\frac{x_1}{18}\cdot\frac{32}{y_1} = -\frac{4}{3} \Rightarrow \frac{x_1}{y_1} = \frac{3}{4} \Rightarrow \frac{x_1}{3} = \frac{y_1}{4} = k$ (say)

$\therefore x_1 = 3k,\ y_1 = 4k$

Putting x_1, y_1 in (i), we get $k^2 = 1$.

Now area of $\Delta OAB = \frac{1}{2}OA.OB = \frac{1}{2}\frac{18}{x_1}.\frac{32}{y_1} = \frac{1}{2}\frac{(18)(32)}{(x_1y_1)}$

$= \frac{1}{2}\frac{(18)(32)}{(3k)(4k)} = \frac{24}{k^2} = 24$ sq. unit, $(\because k^2 = 1)$.

(56) **(B).** Given, co-ordinates of the vertices (A and A') of ellipse $\equiv (2,-2)$ and $(2,4)$ and eccentricity $(e) = \frac{1}{3}$. We know that AA' is along a line parallel to y-axis. Therefore mid-point of AA' $(C) = (h,k) = (2,1)$ and distance between $AA'(2b) = 6$ or $b = 3$. We also know that the standard equation of an ellipse at co-ordinates (h, k) is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ and $a^2 = b^2(1-e^2) = 9\left(1 - \frac{1}{9}\right) = 8$.

Therefore equation of the ellipse $\frac{(x-2)^2}{8} + \frac{(y-1)^2}{9} = 1$.

(57) **(C).** Let P $(a\cos\theta, b\sin\theta)$ be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then the equation of the normal at P is

$ax\sec\theta - by\ \text{cosec}\,\theta = a^2 - b^2$.

It meets the co-ordinate axes at

$G\left(\frac{a^2-b^2}{a}\cos\theta, 0\right)$ and $g\left(0, -\frac{a^2-b^2}{b}\sin\theta\right)$.

$$\Rightarrow PG^2 = \left(a\cos\theta - \frac{a^2-b^2}{a}\cos\theta\right)^2 + b^2\sin^2\theta$$

$$= \frac{b^2}{a^2}(b^2\cos^2\theta + a^2\sin^2\theta)$$

and $Pg^2 = \frac{a^2}{b^2}(b^2\cos^2\theta + a^2\sin^2\theta)$, $\therefore PG : Pg = b^2 : a^2$.

(58) **(D).** Let required chord meets to ellipse on the points P and Q, whose co-ordinates are (x_1, y_1) and (x_2, y_2) respectively. Point (2, 1) is mid point of chord PQ

$\therefore\ 2 = \frac{1}{2}(x_1 + x_2)$ or $x_1 + x_2 = 4$

and $1 = \frac{1}{2}(y_1 + y_2)$ or $y_1 + y_2 = 2$

Again point (x_1, y_1) and (x_2, y_2) are situated on ellipse

$\therefore\ \frac{x_1^2}{36} + \frac{y_1^2}{9} = 1$ and $\frac{x_2^2}{36} + \frac{y_2^2}{9} = 1$

(59) **(C).** The combined equation of the pair of tangents drawn from (1,2) to the ellipse $3x^2 + 2y^2 = 5$ is

$(3x^2 + 2y^2 - 5)(3 + 8 - 5) = (3x + 4y - 5)^2$, [using S'=$T^2$]

$\Rightarrow\ 9x^2 - 24xy - 4y^2 + ... = 0$

The angle between the lines given by this equation is

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$, where $a = 9, h = -12, b = -4$

$\Rightarrow\ \tan\theta = 12\sqrt{5} \Rightarrow \theta = \tan^{-1}(12/\sqrt{5})$.

(60) **(C).** Ellipse,

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = (\cos t + \sin t)^2 + (\cos t - \sin t)^2 = 2$

$\Rightarrow \frac{x^2}{18} + \frac{y^2}{32} = 1$; $c = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{18}{32}} = \sqrt{\frac{14}{32}} = \frac{\sqrt{7}}{4}$

(61) **(D).** We have $\frac{81}{a^2} + \frac{25}{b^2} = 1$ .... (1)

$\frac{144}{a^2} + \frac{16}{b^2} = 1$ .... (2)

From eq. (2) – eq. (1)

$\frac{63}{a^2} - \frac{9}{b^2} = 0 \Rightarrow \frac{b^2}{a^2} = \frac{1}{7}$; $e = \sqrt{1 - \frac{1}{7}} = \sqrt{\frac{6}{7}}$

(62) **(B).** $\frac{2b^2}{a} = 10$ & $2b = 2ae \Rightarrow \frac{b^2}{a^2} = e^2 = 1 - e^2$

$\Rightarrow\ e = \frac{1}{\sqrt{2}}$; $\frac{2b^2}{a} = \frac{2a^2(1 - e^2)}{a} = 10$

$\Rightarrow a\left(1 - \frac{1}{2}\right) = 5 \Rightarrow a = 10$ & $b = 5\sqrt{2}$

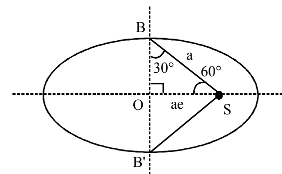
Equation of ellipse $\frac{x^2}{100} + \frac{y^2}{50} = 1 \Rightarrow x^2 + 2y^2 = 100$

(63) **(C).** Any tangent $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

x intercept $h = \frac{a}{\cos\theta}$ and y intercept $k = \frac{b}{\sin\theta}$

$\Rightarrow \frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$

(64) **(A).** In Δ BOS, $\sin 30° = \frac{ae}{a} \Rightarrow e = \frac{1}{2}$



(65) **(C).** $x = 4\cos\theta \Rightarrow \cos\theta = 1/2$

$y = 2\sin\theta \Rightarrow \sin\theta = \sqrt{3}/2 \Rightarrow \theta = \pi/3$

(66) **(A).** $x\cos\alpha + y\sin\alpha = 4$; $y\sin\alpha = -x\cos\alpha + 4$

$y = (-\cot\alpha)\,x + 4\,\text{cosec}\,\alpha$

$\therefore\ m = -\cot\alpha,\ c = 4\,\text{cosec}\,\alpha,\ a^2 = 25,\ b^2 = 9$

$c^2 = a^2m^2 + b^2$

$16\,\text{cosec}^2\alpha = 25\cot^2\alpha + 9$

$16(1 + \cot^2\alpha) = 25\cot^2\alpha + 9$

$7 = 9\cot^2\alpha \Rightarrow \cot\alpha = \frac{\sqrt{7}}{3} \Rightarrow \tan\alpha = \frac{3}{\sqrt{7}}$

$\therefore\ \alpha = \tan^{-1}(3/\sqrt{7})$

(67) **(B).** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; Maximum area $= \frac{1}{2}\,.\,2ae\,.\,b = abe$

(68) **(A).** Put a = b ; $\frac{xx_1}{a^2} + \frac{yy_1}{a^2} = 1$; $\frac{x}{\sqrt{2}a} + \frac{y}{\sqrt{2}a} = 1$

Area $= \frac{1}{2}\sqrt{2}a \times \frac{a}{\sqrt{2}} = \frac{a^2}{2}$

$\therefore$ Option (B), (C) and (D) are incorrect
Hence option (A) is correct.

(69) **(A).** The locus of point of intersection of perpendicular tangents to ellipse is called director circle.

(70) **(D).** Area of auxiliary circle $x^2 + y^2 = a^2$ is πa^2

Area of ellipse = πab

Given, $\pi a^2 = 2\pi ab$; $a = 2b$

$\therefore\ e = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

(71) **(A).** $y = m_1(x - a)$,

$y = m_2(x + a)$ where $m_1m_2 = k$, given

In order to find the locus of their point of intersection we have to eliminate the unknown m_1 and m_2.

Multiplying, we get

$y^2 = m_1m_2(x^2 - a^2)$ or $y^2 = k(x^2 - a^2)$

or $\frac{x^2}{1} - \frac{y^2}{k} = a^2$ which represents a hyperbola.

(72) **(B).** The given line is $x\cos\alpha + y\sin\alpha = p$

$\Rightarrow y\sin\alpha = -x\cos\alpha + p$

$\Rightarrow y = -x \cot\alpha + p \operatorname{cosec}\alpha$

Comparing this line with $y = mx + c$

$m = -\cot\alpha,\ c = p\operatorname{cosec}\alpha$

Since the given line touches the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then $c^2 = a^2m^2 - b^2$

$\Rightarrow p^2 \operatorname{cosec}^2\alpha = a^2\cot^2\alpha - b^2$

or $p^2 = a^2\cos^2\alpha - b^2\sin^2\alpha$

(73) **(B).** We have : m = Slope of the tangent = – 5/12

If a line of slope m is tangent to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the coordinates of the point

of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

Here, $a^2 = 9, b^2 = 1$ and $m = -5/12$

So, points of contact are $\left(\pm 5, \pm\frac{4}{3}\right)$

i.e. (–5, 4/3) and (5, –4/3).

Out of these two points (5, –4/3) lies on the line $5x + 12y = 9$. Hence, (5, –4/3) is the required point.

(74) **(B).** Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$b^2 = a^2(e^2 - 1)$ or $e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}$

Conjugate hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

i.e.Transverse axis is along y-axis and conjugate along x-axis. $a^2 = b^2(e'^2 - 1)$

$e'^2 = \frac{a^2+b^2}{b^2} \quad \therefore \frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2+b^2}{a^2+b^2} = 1$

(75) **(C).** For hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{81}{144} = \frac{225}{144}$

$e = \frac{15}{12} = \frac{5}{4}$ i.e., $e > 1$

Hence the foci are $(\pm ae, 0)\left(\pm\frac{12}{5}.\frac{5}{4}, 0\right) = (\pm 3, 0)$

Now the foci coincide therefore for ellipse

$ae = 3$ or $a^2e^2 = 9$

or $a^2\left(1 - \frac{b^2}{a^2}\right) = 9;\ a^2 - b^2 = 9$

or $16 - b^2 = 9 \Rightarrow b^2 = 7$

(76) **(A).** Parabola $y^2 = 8x \therefore 4a = 8 \Rightarrow a = 2$

Any tangent to the parabola is $y = mx + \frac{2}{m}$ ...(i)

If it is also tangent to the hyperbola $\frac{x^2}{1} - \frac{y^2}{3} = 1$

i.e. $a^2 = 1, b^2 = 3$ then $c^2 = a^2m^2 - b^2 \Rightarrow \left(\frac{2}{m}\right)^2 = 1.m^2 - 3$

or $m^4 - 3m^2 - 4 = 0 \Rightarrow (m^2 - 4)(m^2 + 1) = 0$

$\therefore m = \pm 2$ puting for m in (i), we get the tangents as

$2x \pm y + 1 = 0$

(77) **(D).** $\sqrt{3}x - y = 4\sqrt{3}k$ ...(i)

$K(\sqrt{3}x + y) = 4\sqrt{3}$ ...(ii)

To find the locus of their point of intersection eliminate the variable K between the equations from

(i) $K = \frac{\sqrt{3}x - y}{4\sqrt{3}}$ and putting in (ii), we get

$(\sqrt{3}x - y)(\sqrt{3}x + y) = \sqrt{3}(4)^2$

$3x^2 - y^2 = 48$ or $\frac{x^2}{16} - \frac{y^2}{48} = 1$

Hence the locus is hyperbola

(78) **(B).** Here a = b, so it is a rectangular hyperbola. Hence, eccentricity $e = \sqrt{2}$.

(79) **(B).** $ae = \sqrt{a^2+b^2} = \sqrt{\cos^2\alpha + \sin^2\alpha} = 1$.

(80) **(A).** Let $P(x_1, y_1)$ be the middle point of the chord of the hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$

$\therefore$ Equation of the chord is $T = S_1$

i.e. $3xx_1 - 2yy_1 + 2(x + x_1) - 3(y + y_1) = 0$

or $(3x_1 + 2)x - (2y_1 + 3)y + (2x_1 - 3y_1) = 0$

If this chord is parallel to line $y = 2x$, then

$m_1 = m_2 \Rightarrow -\frac{3x_1 + 2}{-(2y_1 + 3)} = 2 \Rightarrow 3x_1 - 4y_1 = 4$

Hence the locus of the middle point (x_1, y_1) is $3x - 4y = 4$.

(81) **(A).** We know that equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Let (x_1, y_1) be any point on hyperbola,

$\therefore \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$ or $b^2x_1^2 - a^2y_1^2 = a^2b^2$.

We also know that asymptotes of given hyperbola are

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

$\therefore$ Product of $\perp$ from (x_1, y_1) to pair of lines $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

is $\frac{|Ax_1^2+2Hx_1y_1+By_1^2|}{\sqrt{(A-B)^2+4H^2}}$; $\frac{b^2x_1^2-a^2y_1^2}{\sqrt{(b^2+a^2)^2}}=\frac{a^2b^2}{a^2+b^2}$.

(82) **(B).** Eccentricity of rectangular hyperbola is $\sqrt{2}$.

(83) **(A).** Given hyperbola is, $\frac{x^2}{3}-\frac{y^2}{2}=1$ .....(i)

Equation of tangent parallel to $y-x+5=0$ is

$y-x+\lambda=0 \Rightarrow y=x-\lambda$ .....(ii)

If line (ii) is a tangent to hyperbola (i), then

$-\lambda=\pm\sqrt{3\times1-2}$ (from $c=\pm\sqrt{a^2m^2-b^2}$)

$-\lambda=\pm1 \Rightarrow \lambda=-1,+1$.

Put the values of λ in (ii), we get $x-y-1=0$ and $x-y+1=0$ are the required tangents.

(84) **(A).** Any tangent to the hyperbola at P(a sec θ, a tan θ) is

$x\sec\theta-y\tan\theta=a$ ...(i)

Also $x-y=0$ ...(ii)

$x+y=0$ ...(iii)

Solving the above three lines in pairs, we get the point A, B, C as $\left(\frac{a}{\sec\theta-\tan\theta}, \frac{a}{\sec\theta-\tan\theta}\right)$,

$\left(\frac{a}{\sec\theta+\tan\theta}, \frac{-a}{\sec\theta+\tan\theta}\right)$ and (0, 0)

Since the one vertex is the origin therefore the area of the triangle ABC is $\frac{1}{2}(x_1y_2-x_2y_1)$

$=\frac{a^2}{2}\left(\frac{-1}{\sec^2\theta-\tan^2\theta}-\frac{1}{\sec^2\theta-\tan^2\theta}\right)$

$=\frac{a^2}{2}(-2)=-a^2=a^2$

(85) **(A).** Equation can be rewritten as

$\frac{(x-4)^2}{4^2}-\frac{(y-3)^2}{3^2}=1$ so a = 4, b = 3

$b^2=a^2(e^2-1)$ gives e = 5/4

Foci : X = ± ae, Y = 0 gives the foci as (9,3), (– 1,3)

(86) **(A).** Suppose point of contact be (h, k), then tangent is

$hx-4ky-5=0 \equiv 3x-4y-5=0$ or $h=3, k=1$

Hence the point of contact is (3, 1).

(87) **(C).** We have $9x^2-16y^2-18x+32y-151=0$

$9(x^2-2x)-16(y^2-2y)=151$

$9(x^2-2x+1)-16(y^2-2y+1)=144$

$9(x-1)^2-16(y-1)^2=144$

$\frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$

Shifting the origin at (1, 1) without rotating the axes

$\frac{x^2}{16}-\frac{y^2}{9}=1$, where x = X + 1 and y = Y + 1

This is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

where $a^2=16$ and $b^2=9$ so

The length of the transverse axes = 2a = 8

The length of the letus rectum $=\frac{2b^2}{a}=\frac{a}{2}$

The equaiton of the directrix $x=\pm\frac{a}{e}$

$x-1=\pm\frac{16}{5} \Rightarrow x=\pm\frac{16}{5}+1$; $x=\frac{21}{5}$; $x=-\frac{11}{5}$

(88) **(B).** Any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$y=mx\pm\sqrt{a^2m^2-b^2}$ or $y=mx+c$

where $c=\pm\sqrt{a^2m^2-b^2}$

This will touch the hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

if the equation $\frac{(mx+c)^2}{a^2}-\frac{x^2}{b^2}=1$ has equal roots or

$x^2(b^2m^2-a^2)+2b^2mcx+(c^2-a^2)b^2=0$ is an quadratic equation have equal roots

$4b^4m^2c^2=4(b^2m^2-a^2)(c^2-a^2)b^2$

$c^2=a^2-b^2m^2$

$a^2m^2-b^2=a^2-b^2m^2$

$m^2(a^2+b^2)=a^2+b^2 \Rightarrow m=\pm1$

Hence, the equations of the common tangents are

$y=\pm x\pm\sqrt{a^2-b^2}$

(89) **(A).** $\because$ Equation of hyperbola is $16x^2-9y^2=144$

or $\frac{x^2}{9}-\frac{y^2}{16}=1$ comparing this with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

we get $a^2=9$, $b^2=16$ and comparing this line $y=2x+\lambda$ with $m=mx+c$; $m=2$ & $c=\lambda$

If the line $y=2x+\lambda$ touches the hyperbola $16x^2-9y^2=144$

then $c^2=a^2m^2-b^2 \Rightarrow \lambda=9(2)^2-16$

$=36-16=20$; $\therefore \lambda=\pm2\sqrt{5}$

(90) **(D).** Let m be the slope of the tangent since the tangent is perpendicular to the line $x-y+4=0$.

$\therefore m\times1=-1 \Rightarrow m=-1$

since $x^2 - 4y^2 = 36$ or $\frac{x^2}{36} - \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$;

$\therefore\ a^2 = 36\ \&\ b^2 = 9$ so the equation of tangents are

$y = (-1)\,x \pm \sqrt{36\,x\,(-1)^2 - 9}$

$\Rightarrow y = -x \pm \sqrt{27}$ or $x + y \pm 3\sqrt{3} = 0$

(91) **(A).** Distance between centre and vertex = a = 2
Distance between focii and centre = ae = 6 $\Rightarrow$ e = 3
$b^2 = a^2(e^2 - 1) = 4(9 - 1) = 32$

Latus rectum = $\frac{2b^2}{a} = \frac{2 \times 32}{2} = 32$

(92) **(D).** Let $K = -\lambda$ where $\lambda > 0$

Hyperbola $-\frac{x^2}{\lambda} + \frac{y^2}{\lambda^2} = 1$

$\Rightarrow e = \sqrt{1 + \frac{\lambda}{\lambda^2}} = \sqrt{1 + \frac{1}{\lambda}} = \sqrt{1 - \frac{1}{K}}$

(93) **(C).** If line is tangent $c^2 = a^2m^2 - b^2$

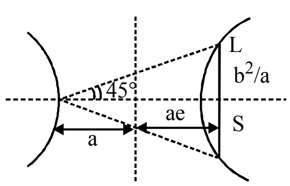
$36 = a^2.9 - 64$; $a^2 = \frac{100}{9}$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{64}{100} \times 9} = \sqrt{1 + \frac{144}{25}} = \frac{13}{5}$

(94) **(C).** Conjugate hyperbola is $2x^2 - 3y^2 = -6$

or $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(95) **(B).**

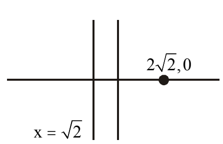


$\tan 45° = \frac{b^2/a}{a + ae}$; $1 = \frac{b^2/a^2}{1 + e} = \frac{e^2 - 1}{e + 1}$

$\Rightarrow 1 = e - 1 \Rightarrow e = 2$

(96) **(D).** $x^2 - y^2 = 2^2$; $(\pm\, ae, 0) = (\pm\, 2\sqrt{2}, 0)$

dir: $x = \pm\, a/e \Rightarrow x = \pm\,\sqrt{2}$



(97) **(B).** Foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is $(\pm\sqrt{12}, 0)$

Foci of $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ is $(\pm\sqrt{a^2 + 3}, 0)$

Given $a^2 + 3 = 12 \Rightarrow a^2 = 9 \Rightarrow a = 3$

(98) **(A).** The asymptotes are $\frac{x}{5} + \frac{y}{3} = 0$ and $\frac{x}{5} - \frac{y}{3} = 0$

The equation of hyperbola is

$\frac{x^2}{5^2} - \frac{y^2}{9} = 1 \Rightarrow 9x^2 - 25y^2 = 225$

(99) **(B).** $y = x - 1$; $m = 1, c = -1, a^2 = 4, b^2 = 3$
Point of Contact

$= \left(-\frac{ma^2}{c}, -\frac{b^2}{c}\right) = \left(\frac{-4}{-1}, \frac{-3}{-1}\right) = (4, 3)$

(100) **(A).** $\frac{x^2}{4} + \frac{y^2}{3} = 1$; $e_1 = \sqrt{\frac{4-3}{4}} = \frac{1}{2}$

$\frac{x^2}{4} - \frac{y^2}{3} = 1$; $e_2 = \sqrt{\frac{4+3}{4}} = \frac{\sqrt{7}}{2}$

$\therefore\ e_1^2 + e_2^2 = \frac{1}{4} + \frac{7}{4} = \frac{8}{4} = 2$

(101) **(B).** Parametric coordinates of any point on parabola
$y^2 = 4\,ax$ are$(at^2, 2at)$
Here $4a = 9 \Rightarrow a = 9/4$
$\therefore$ y coordinate 2 at = – 6
$\therefore\ 2\,(9/4)\,t = -6 \Rightarrow t = -4/3$

(102) **(D).** Here a = 4
Condition of normality $c = -2am - am^3$
(1) and (2) are not clearly the answer as m = 1
for (3), (4) $m = \cos\theta$
$c = -2\,(4)\cos\theta - 4\cos^3\theta = -8\cos\theta - (3\cos\theta + \cos 3\theta)$
$= -11\cos\theta - \cos 3\theta$

(103) **(C).** Let AB be a double ordinate, where $A \equiv (at^2, 2at)$,
$B \equiv (at^2, -at)$. If P(h, k) be it's trisection point then

$3h = 2at^2 + at^2$, $3k = 4at - 2at \Rightarrow t^2 = \frac{h}{a}$, $t = \frac{3k}{2a}$

Thus locus is $\frac{9k^2}{4a^2} = \frac{h}{a}$, i.e., $9y^2 = 4ax$.

(104) **(B).** Equation of normal in terms of m is $y = mx - 4m - 2m^3$.
If it passes through (a, 0) then $am - 4m - 2m^3 = 0$

$\Rightarrow m\,(a - 4 - 2m^2) = 0 \Rightarrow m = 0, m^2 = \frac{a-4}{2}$.

For three distinct normal, $a - 4 > 0 \Rightarrow a > 4$

(105) **(A).** Clearly the other extremity of latus rectum is (2, – 2).

It's axis is x-axis. Corresponding value of $a = \frac{2-0}{2} = 1$.

Hence it's vertex is (1, 0) or (3, 0).
Thus it's equation is $y^2 = 4(x - 1)$ or $y^2 = -4\,(x - 3)$.

(106) **(C).** $e_1^2 = \frac{16-9}{16} = 1 - \frac{9}{16}, e_2^2 = \frac{16+9}{16} = 1 + \frac{9}{16}$

$\Rightarrow e_1^2 + e_2^2 = 2, e_1^2 - e_2^2 < 0.$

(107) **(C).** Let A be the vertex, then slope of AP = 2
let the point Q be $(4t^2, 8t)$, then slope of AQ is 2/t
equation of normal at P is x + y = 12 and equation of normal at Q is $y - 4x + 288 = 0$ point of intersection is $(60, -48)$

(108) **(C).** Let (x_1, y_1) be the mid point of a focal chord.
Then equation of this chord will be
$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$ $[T = S_1]$
Since it passes through the focus (a, 0) so we have
$-2a^2 - 2ax_1 = y_1^2 - 4ax_1$
Hence required locus is $y^2 = 2a(x - a)$

(109) **(C).** Clearly, both the lines passes through (– a, b) which is a point lying on the directrix of the parabola.
Thus, $m_1m_2 = -1$.
Because tangents drawn from any poin on the directrix are always mutually perpendicular.

(110) **(A).** We have $t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_1^2 + t_1t_2 + 2 = 0.$

Since 't_1' is real, thus $t_2^2 - 8 \ge 0 \Rightarrow |t_2| \ge 2\sqrt{2}$.

(111) **(C).** Let the midpoint be P(h, k). Equation of this chord is
$T = S_1$. i.e., $yk - 2a(x + h) = k^2 - 4ah$.
It must pass through (a, 0)
$\Rightarrow 2a(a + h) = k^2 - 4ah.$
Thus required locus is $y^2 = 2ax - 2a^2$.

(112) **(C).** Let AB be the normal chord where $A \equiv (2t_1^2, 4t_1)$,

$B \equiv (2t_2^2, 4t_2)$. It's slope $= \frac{2}{t_1 + t_2}$

We also have $t_2 = -t_1 - \frac{2}{t_1}$ and $16 = 2(t_1^2 + t_2^2)$,

$4 = 4(t_1 + t_2) \Rightarrow t_1 + t_2 = 1.$
Thus slope is 2.

(113) **(A).** Let $A \equiv (at^2, 2at)$, $B \equiv (at^2, -2at)$.

$m_{OA} = \frac{2}{t}, m_{OB} = -\frac{2}{t}$. Thus $\frac{2}{t} \cdot -\frac{2}{t} = -1 \Rightarrow t^2 = 4.$

Thus, tangents will intersect at (– 4a, 0).

(114) **(B).** Let (x_1, y_1) be any point on the parabola $y^2 = 4ax$, then length of double ordinate
$2y_1 = 8a \Rightarrow y_1 = 4a$
$y_1^2 = 4ax_1 \Rightarrow x_1 = 4a$
$\therefore$ Vertices of double ordinate are P (4a, 4a) ; Q (4a, – 4a)
If A is the vertex (0, 0), then Slope of AP = 1 = m_1 ; Slope of AQ = – 1 = m_2
$\therefore m_1 m_2 = -1 \Rightarrow \angle PAQ = 90°$

(115) **(B).** Clearly AC is parallel to y-axis. It's midpoint is (2, 2).
Thus $B \equiv (1, 2)$.
Parabola will be in the form of $(x - 2)^2 = \lambda(y - 3)$.
It passes through (3, 2)
$\Rightarrow 1 = -\lambda$. Thus parabola is $(x - 2)^2 = -1(y - 3)$.

It focus is $x - 2 = 0$. $y - 3 = -\frac{1}{4}$, i.e., $\left(2, \frac{11}{4}\right)$.

(116) **(B).** $2\sqrt{x^2 + y^2} + x = 3 \Rightarrow \frac{(x+1)^2}{4} + \frac{y^2}{3} = 1.$

This represents are ellipse centred at (–1, 0) and of eccentricity $= \sqrt{\frac{4-3}{4}} = \frac{1}{2}$.

(117) **(A).** Any point on the line $x - y - 5 = 0$ will be of the form $(t, t - 5)$. Chord of contact of this point w.r.t curve
$x^2 + 4y^2 = 4$ is $tx + 4(t-5)y - 4 = 0$
or $(-20y - 4) + t(x + 4y) = 0$ which is a family of straight lines, each member of this family pass through the point of intersection of straight lines $-20y - 4 = 0$ and $x + 4y = 0$.

(118) **(C).** $4 = 9(1 - e^2) \Rightarrow e = \sqrt{5}/3$

Distance between the directrices $= \frac{2a}{e} = \frac{2 \times 3 \times 3}{\sqrt{5}} = \frac{18}{\sqrt{5}}$

(119) **(A).** Putting $5y = k - 3x$ in the equation of the ellipse
$16x^2 + 25y^2 = 400$,
we get $16x^2 + (k - 3x)^2 = 400$
$\Rightarrow 25x^2 - 6kx + k^2 - 400 = 0$
Now, $D = 0 \Rightarrow 36k^2 - 100(k^2 - 400) = 0 \Rightarrow k = \pm 25$

(120) **(B).** Equation of chord joining α and β is

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$$

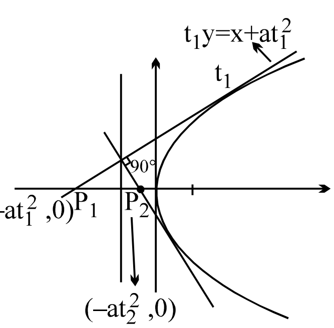
$\therefore \alpha + \beta = 3\pi$

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha-\beta}{2}\right) = 0$$

The above equations passes through centre.

EXERCISE-2

(1) **(C).**



$SP_1 = a(1 + t_1^2)$; $SP_2 = a(1 + t_2^2)$
$\Rightarrow t_1t_2 = -1$

$\frac{1}{SP_1} = \frac{1}{a(1+t^2)}$; $\frac{1}{SP_2} = \frac{t^2}{a(1+t^2)}$ $\therefore \frac{1}{SP_1} + \frac{1}{SP_2} = \frac{1}{a}$

(2) **(A).** If $S_1 = 0$ and $S_2 = 0$ are the equations, $\lambda S_1 + S_2 = 0$ is a second degree curve passing through the points of $S_1 = 0$ and $S_2 = 0$. For it to be a circle choose λ such that the coefficients of x^2 and y^2 are equal : $\lambda + 4 = 2\lambda + 2 \Rightarrow \lambda = 2$.

This gives the equation of the circle as
$6(x^2 + y^2) - 32x - 36y + 81 = 0$.

Centre : $\left(\frac{8}{3}, 3\right)$. Radius : $\sqrt{\frac{64}{9} + 9 - \frac{27}{2}} = \frac{1}{3}\sqrt{\frac{47}{2}}$.

(3) **(B).** Focus is $\left(\frac{7}{2}, \frac{7}{2}\right)$ and it's axis is the line $y = x$.

Corresponding value of 'a' is $\frac{1}{4}\sqrt{(1+1)} = \frac{\sqrt{2}}{4}$.

Let the equation of it's directrix be $y + x + \lambda = 0$.

$\Rightarrow \frac{|3x + 4 + \lambda|}{\sqrt{2}} = 2.\frac{\sqrt{2}}{4} \Rightarrow \lambda = -6, -8$

Thus equation of parabola is

$$\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 6)^2}{2}$$

or $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 8)^2}{2}$.

(4) **(A).** Let the point is (h, k). The equation of any normal to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$

Passes through (h, k)

$k = mk - 2am - am^3$

$am^3 + m(2a - h) + k = 0$ ...(i)

m_1, m_2, m_3 are roots of the equation then

$m_1 m_2 m_3 = -k/a$

but $m_1 m_2 = 2, m_3 = -k/2a$

m_3 is root of (i) $a\left(-\frac{k}{2a}\right)^3 - \frac{k}{2a}(2a - h) + k = 0$

$\Rightarrow k^2 = 4ah$. Thus locus is $y^2 = 4ax$

(5) **(B).** If P $(at_1^2, 2at_1)$ be one end of the normal, the other say Q $(at_2^2, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1}$ ....(1)

Again slope of OP $= \frac{2at_1}{at_1^2} = \frac{2}{t_1}$

Slope of OQ $= \frac{2}{t_2}$ $\therefore \frac{2}{t_1} \times \frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$....(2)

From (1) and (2)

$$-\frac{4}{t_1} = -t_1 - \frac{2}{t_1} \Rightarrow \frac{2}{t_1} = t_1 \Rightarrow t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2}$$

Using (i), the coordinates of the endpoint of the latus rectum are (0, 3) and (0, –1)

(6) **(C).** Let AB be a normal chord where $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$. We have $t_2 = -t_1 - \frac{2}{t_1}$.

$AB^2 = [a^2(t_1 - t_2)^2 (t_1 + t_2)^2 + 4]$

$$= a^2\left(t_1 t_1 + \frac{2}{t_1}\right)^2 \left(\frac{4}{t_1^2} + 4\right) = \frac{16a^2(1 + t_1^2)^3}{t_1^4}$$

$$\Rightarrow \frac{d(AB^2)}{dt_1} = 16a^2\left(\frac{t_1^4[3(1+t_1^2)^2.2t_1] - (1+t_1^2)^3.4t_1^3}{t_1^8}\right)$$

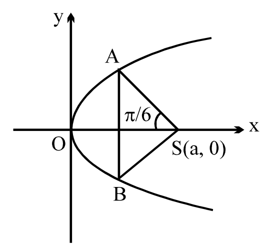
$$= \frac{a^2.32(1+t_1^2)^2}{t_1^5}(t_1^2 - 2)$$

$t_1 = \sqrt{2}$ is indeed the point of minima of AB^2.

Thus $AB_{mini} = \frac{4a}{2}(1+2)^{3/2} = 2a\sqrt{27}$ units.

(7) **(B).** Let $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, -2at_1)$.

We have $m_{AS} = \tan\left(\frac{\pi}{6}\right)$



$\Rightarrow \frac{2at_1}{at_1^2 - a} = -\frac{1}{\sqrt{3}}$

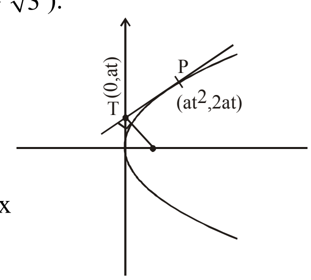
$\Rightarrow t_1^2 + 2\sqrt{3}\,t_1 = -1 = 0$

$\Rightarrow t_1 = -\sqrt{3} \pm 2$

Clearly $t_1 = -\sqrt{3} - 2$ is rejected. Thus $t_1 = (2 - \sqrt{3})$.

Hence $AB = 4at_1 = 4a(2 - \sqrt{3})$.

(8) **(B).** Let middle point of P and T be (h, k)



$\therefore 2h = at^2$

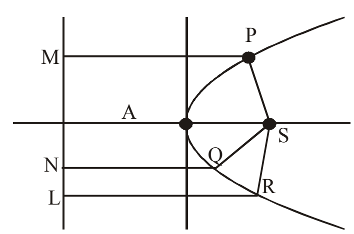
$2k = 3at$

$\therefore 2h = a.\frac{4k^2}{9a^2}$, $2y^2 = 9ax$

$\because a = 2$

then $y^2 = 9x$

(9) **(C).** Let $S \equiv (a, 0)$, $P \equiv (am_1^2, -2am_1)$
$Q \equiv (am_2^2, -2am_2)$, $R \equiv (am_3^2, -2am_3)$



$|SP|.|SQ|.|SR| = |PM|.|QN|.|RL|$
$= a^3(1 + m_1^2)(1 + m_2^2)(1 + m_3^2)$
$= a^3|(1 + (\Sigma m_1)^2 - 2\Sigma m_1 m_2 + (\Sigma m_1 m_2)^2$
$- 2m_1 m_2 m_3 \Sigma m_1 + (m_1 m_2 m3)^2|$

$= a^3 \left| 1 + 0 + \frac{2(h-2a)}{a} + \frac{(h-2a)^2}{a^2} - 0 + \frac{k^2}{a^2} \right|$

$= a\,|k^2 + (h-a)^2| = a\,(SO)^2.$

(10) **(B).** Solving $y = mx + c$ and $y^2 = 16x + 64$ simultaneously,

$(mx+c)^2 = 16x + 64$

$m^2x^2 + 2mcx + c^2 = 16x + 64$

$(2mc - 16)^2 - 4m^2(c^2 - 64) = 0$

$4m^2c^2 - 64\,mc + 256 - 4m^2c^2 + 256\,m^2 = 0$

$4m^2 - mc + 4 = 0$

$c^2 - 64 \ge 0 \qquad \therefore\ |c| \ge 8$

$c \in (-\infty, -8] \cup [8, \infty)$

(11) **(B).** $y = x^2 + 1$; $(\alpha, \alpha^2 + 1)$; $\frac{dy}{dx} = 2x$

$\therefore$ Equation of normal is $y - \alpha^2 - 1 = -\frac{1}{2\alpha}(x - \alpha)$

$2a\alpha - 2\alpha^3 - 2\alpha = -x + \alpha$

i.e., $x + 2\alpha y - 3\alpha - 2\alpha^3 = 0$ .... (1)

Equation of any normal to the parabola $y^2 = (x-1)$ is

$y = -t(x-1) + \frac{1}{2}t + \frac{1}{4}t^3$

i.e., $4y = -4tx + 4t + 2t + t^3$

i.e., $4tx + 4y - 6t - t^3 = 0$ .... (2)

Since (1) and (2) represents the same line

$\therefore \frac{4t}{1} = \frac{4}{2\alpha} = \frac{-6t - t^3}{-3\alpha - 2\alpha^3}$; $t = \frac{1}{2\alpha}$ and $\frac{2}{\alpha} = \frac{\frac{6}{2\alpha} + \frac{1}{8\alpha^3}}{3\alpha + 2\alpha^3}$

i.e. $6\alpha + 4\alpha^3 = 3 + \frac{1}{8\alpha^2}$ i.e.,

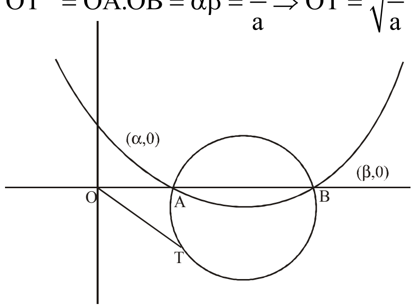
$32\alpha^5 + 48\alpha^3 - 24\alpha^2 - 1 = 0$; $\alpha = 1/2$ and $t = 1$

$\therefore$ The points are $\left(\frac{1}{2}, \frac{5}{4}\right), \left(\frac{5}{4}, \frac{1}{2}\right)$

$\therefore$ Radius $= \frac{1}{2}\sqrt{\left(\frac{1}{2} - \frac{5}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{\frac{9}{16} + \frac{9}{16}} = \frac{3}{8}\sqrt{2}$

$\therefore$ Area $= \pi\frac{9}{32}$

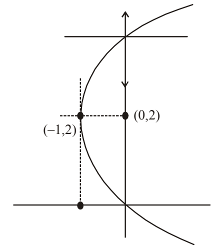
(12) **(D).** $OT^2 = OA.OB = \alpha\beta = \frac{c}{a} \Rightarrow OT = \sqrt{\frac{c}{a}}$



(13) **(A).** Given curve is $y^2 - 4y = 4x$

$(y-2)^2 = 4(x+1)$

Focus : $x + 1 = 1 \Rightarrow x = 0$



$y - 2 = 0 \Rightarrow y = 2$

Point of intersection of the curve and $y = 4$ is (0, 4) from the reflection property of parabola reflected ray passes through the focus.

(14) **(B).** Let AB be a normal chord, where $A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_2^2, 2at_2)$.

We have $t_2 = -t_1 - \frac{2}{t_1}$ and $t_1t_2 = 4$

$\Rightarrow t_1t_2 = -t_1^2 - 2 = -4 \Rightarrow t_1^2 = 2.$

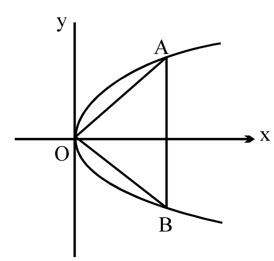
Now slope of chord AB $= \frac{2}{t_1 + t_2} = -t_1 = \pm\sqrt{2}$

(15) **(D).** If triangle OAB is equilateral then OA = OB = AB. Thus AB will be a double ordinate of the parabola.

Thus $\angle AOX = \angle XOB = \pi/6$

Let $A = (at_1^2, 2at_1)$ then $B \equiv (at_1^2, 2at_1)$

$m_{OA} = \frac{2}{t_1} = \frac{1}{\sqrt{3}} \Rightarrow t_1 = 2\sqrt{3}$



$\Rightarrow AB = 4at_1 = 8a\sqrt{3}$ units.

(16) **(A).** The equation can be written as

$4(x-1)^2 + 9(y-2)^2 = 36$ which is an ellipse centred at (1, 2). If CA makes an angle θ with the major axis, then

$A \equiv [1 + CA\cos\theta, 2 + CA\sin\theta]$

$B \equiv \left[1 + CB\cos\left(\frac{\pi}{2} + \theta\right), 2 + CB\sin\left(\frac{\pi}{2} + \theta\right)\right]$

As A and B are points on the conic

$CA^2(4\cos^2\theta + 9\sin^2\theta) = 36$ and

$CB^2(4\sin^2\theta + 9\cos^2\theta) = 36$ giving $CA^{-2} + CB^{-2} = \frac{13}{36}$.

(17) **(D).** Equation of the normal at point 'ϕ' of the ellipse is ax sec ϕ – by cosec ϕ = $a^2 - b^2$ .... (1)

Given line x cosα + y sin α = p .... (2)

Comparing (1) and (2), we have

$$\frac{\cos\alpha}{a\sec\varphi} = \frac{\sin\alpha}{-b\,\text{cosec}\varphi} = \frac{p}{a^2-b^2}$$

$$\Rightarrow \cos\phi = \frac{ap}{(a^2-b^2)\cos\alpha};\ \sin\phi = \frac{-bp}{(a^2-b^2)\sin\alpha}$$

$$1 = \frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha} + \frac{b^2p^2}{(a^2-b^2)\sin^2\alpha}$$

$\Rightarrow p^2 (a^2 \sec^2\alpha + b^2 \text{cosec}^2\alpha) = (a^2 - b^2)^2$

(18) **(C).** Let the equation of any normal be $y = -tx + 2t + t^3$

Since it passes through the points (15, 12)

$\therefore\ 12 = -15t + 2t + t^3$ i.e., $t^3 - 13t - 12 = 0$

$(t + 1)(t^2 - t - 12) = 0$

i.e., $(t + 1)(t - 4)(t + 3) = 0\ \therefore\ t = -1, -3, 4$

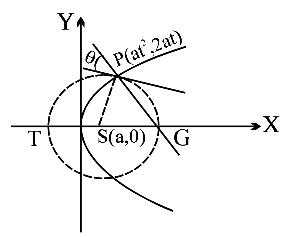
$\therefore$ The points are (1, –2), (9, –6), (16, 8)

$\therefore$ Centroid is (26/3, 0)

(19) **(C).** Slope of tangent $= \frac{1}{t}$ (m_1) at P on parabola

$$\text{slope of PS} = \frac{2at}{a(t^2-1)} = \frac{2t}{t^2-1}$$

$$\therefore \text{ Slope of tangent at P on circle} = \frac{1-t^2}{2t}(m_2)$$



$$\therefore\ \tan\theta = \frac{\frac{1}{t} - \frac{1-t^2}{2t}}{1 + \frac{1-t^2}{2t^2}} = \frac{(2-1+t^2)2t^2}{2t(1+t^2)} = t$$

$\therefore\ \theta = \tan^{-1}t \Rightarrow$ (C)

(20) **(A).** Slope of tangent at P is $1/t_1$ and at

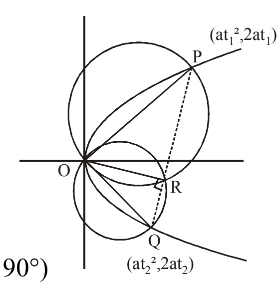
$Q = \frac{1}{t_2} \Rightarrow \cot\theta_1 = t_1$ and $\cot\theta_2 = t_2$

Slope of PQ $= \frac{2}{t_1 + t_2}$

$\Rightarrow$ Slope of OR is

$$-\frac{2}{t_1+t_2} = \tan\phi$$

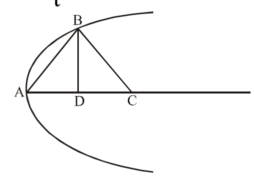
(Note angle in a semicircle is 90°)



$$\Rightarrow \tan\phi = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2) \Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$$

(21) **(C).** Let B be $(at^2, 2at)$.

Slope of AB $= \frac{2}{t}$. Equation of BC is $y - 2at = -\frac{t}{2}(x - at^2)$.



This meets y = 0 at C whose x-coordinate = $4a + at^2$

D = $(at^2, 0)$ $\therefore$ DC = $4a + at^2 - at^2 = 4a$.

(22) **(C).** Given line is x cos α + y sin α = P ....(1)

Any tangent to the ellipse is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$....(2)

Comparing (1) and (2) $\frac{\cos\theta}{a\cos\alpha} = \frac{\sin\theta}{b\sin\alpha} = \frac{1}{P}$

$$\cos\theta = \frac{a\cos\alpha}{P};\ \sin\theta = \frac{b\sin\alpha}{P}$$

Eliminate θ, $\cos^2\theta + \sin^2\theta = \frac{a^2\cos^2\alpha}{P^2} + \frac{b^2\sin^2\alpha}{P^2}$,

or $a^2\cos^2\alpha + b^2\sin^2\alpha = P^2$

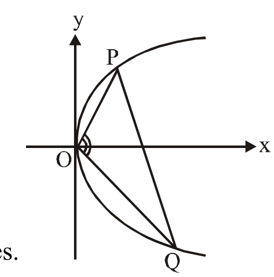
(23) **(B).** Standard equation of the normal at P$(at^2, 2at)$ is $y + xt = 2at + at^3$.

Hence t = – m. This meets the parabola again at Q whose parameter is $-t - \frac{2}{t}$ or $m + \frac{2}{m}$.

$\therefore$ coordinates of Q are $a\left(m + \frac{2}{m}\right)^2, 2a\left(m + \frac{2}{m}\right)$

Since $\angle POQ = \pi/2$, i.e., POQ! = 90º

$$\Rightarrow \left(\frac{2}{m + \frac{2}{m}}\right)\left(\frac{2}{-m}\right) = -1$$



$\Rightarrow m^2 + 2 = 4 \Rightarrow m = \pm\sqrt{2}$

$m = \sqrt{2}$ is one of the choices.

(24) **(B).** Equation of normal at $(a\cos\theta, b\sin\theta)$ is ax sec θ – by cosec θ = $a^2 - b^2$

Now this normal passes through $(a\cos 2\theta, b\sin 2\theta)$,

$$\text{so } a.a\frac{\cos 2\theta}{\cos\theta} - b.b\frac{\sin 2\theta}{\sin\theta} = a^2 - b^2$$

$$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$$

$$\Rightarrow \cos\theta = -\frac{2}{3} \text{ or } \cos\theta = \frac{7}{6} \text{ (Not possible)}$$

(25) **(C).** Let (h, k) be the mid point of a chord passing through the positive end of the minor axis of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Then the equation of the chord, $T = S_1$

$$\frac{hx}{a^2}+\frac{ky}{b^2}-1=\frac{h^2}{a^2}+\frac{k^2}{b^2}-1 \quad \text{or} \quad \frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

This passes through (0,b), therefore $\frac{k}{b}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$

Hence, the locus of (h, k) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{y}{b}$,

which is an ellipse

(26) **(B).** Let P $(a\cos\theta_1, b\sin\theta_1)$ and Q $(a\cos\theta_2, b\sin\theta_2)$ be two points on the ellipse. Then

$$m_1 = \text{Slope of OP (O is centre)} = \frac{b}{a}\tan\theta_1 ;$$

$$m_2 = \text{slope of OQ} = \frac{b}{a}\tan\theta_2$$

$$\therefore m_1 m_2 = \frac{b}{a}\tan\theta_1, \frac{b}{a}\tan\theta_2 = \frac{b^2}{a^2}\tan\theta_1\tan\theta_2$$

$$(\because \tan\theta_1\tan\theta_2 = \frac{-b^2}{a^2}) = \frac{b^2}{a^2}\left(-\frac{a^2}{b^2}\right) = -1$$

$\angle POQ = 90°$. Hence PQ makes a right angle at the centre of the ellipse.

(27) **(C).** We have $x^2+4y^2+2x+16y+13=0$

$\Rightarrow (x^2+2x+1)+4(y^2+2y+4)=4$

$$(x+1)^2+4(y+2)^2=4 \Rightarrow \frac{(x+1)^2}{2^2}+\frac{(y+2)^2}{1^2}=1$$

Shifting the origin at (–1, – 2) without rotating the coordinate axes $\frac{x^2}{2^2}+\frac{y^2}{1^2}=1$ where $x = X-1, y = Y-2$

This is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a = 2, b = 1$

eccentricity of the ellipse $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

Focus of the ellipse (± ae, 0)

$$X = x+1 = \pm 2.\frac{\sqrt{3}}{2} \Rightarrow x = \pm\sqrt{3}-1$$

$\Rightarrow Y = y+2 = 0 \Rightarrow y = -2$

Directrix of the ellipse $X = \pm a/e$

$$\Rightarrow x+1=\frac{2}{\sqrt{3}/2} \;;\; x=\pm\frac{4}{\sqrt{3}}-1$$

(28) **(A).** Let P $(a\cos\theta, b\sin\theta)$ and Q $(a\cos\phi, b\sin\phi)$ be two points of the ellipse such that $\theta-\phi=\frac{\pi}{2}$.

The equation of tangent at P and Q are respectively

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1 \quad ....(i)$$

$$\text{and } \frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi=1 \quad ......(ii)$$

Since $\theta-\phi=\frac{\pi}{2}$, so (i) can be written as

$$-\frac{x}{a}\sin\phi+\frac{y}{b}\cos\phi=1 \quad ....(iii)$$

Squaring (ii) and (iii) and then adding, we get

$$\left(\frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi\right)^2+\left(-\frac{x}{a}\sin\phi+\frac{y}{b}\cos\phi\right)^2=1+1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$$

(29) **(D).** The chord of contact of tangents from $P(x_1, y_1)$ is $yy_1-2ax-2ax_1=0$.

This touches $x^2 = 4by$, the roots of the quadratic in x

$y_1\frac{x^2}{4b}-2ax-2ax_1=0$ are equal.

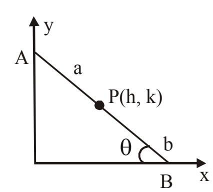
(i.e.) $y_1x^2-8abx-8abx_1=0$ has equal roots.

$64a^2b^2+32aby_1x_1=0$

$\therefore$ locus of (x_1, y_1) is the curve $xy = -2ab$ which is a rectangular hyperbola.

(30) **(C).** $B = ((a+b)\cos\theta, 0)$ $A \equiv (0, (a+b)\sin\theta)$

$$P(h,k) \equiv \left(\frac{a(a+b)\cos\theta+b0}{a+b}; \frac{b(a+b)\sin\theta+0.a}{a+b}\right)$$



i.e., $h = a\cos\theta$; $k = b\sin\theta$

$$\Rightarrow \frac{h^2}{a^2}+\frac{k^2}{b^2}=1,$$

so that locus of (h, k) is an ellipse.

(31) **(C).** Let the point of intersection be R (x_1, y_1).

Then PQ is the chord of contact of the ellipse with respect to R and its equation will be $\frac{xx_1}{9}+\frac{yy_1}{4}=1$

Now combined equation of lines joining P, Q to centre O (0, 0) is [it is obtained making $x^2/9 + y^2/4 = 1$ homogeneous with help of (1)]

$$\frac{x^2}{9}+\frac{y^2}{4}=\left(\frac{xx_1}{9}+\frac{yy_1}{4}-1\right)^2$$

As given OP ⊥ OQ, so coef. of x^2 + coef. of $y^2 = 0$

$$\Rightarrow \left(\frac{x_1^2}{81}-\frac{1}{9}\right)+\left(\frac{y_1^2}{16}-\frac{1}{4}\right)=0$$

Locus of (x_1, y_1) is $\frac{x^2}{81}+\frac{y^2}{16}=\frac{13}{36}$, which is an ellipse.

(32) **(C).** The equation of the chord of contact of tangents drawn from (x_1, y_1) and (x_2, y_2) to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{are} \quad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1 \qquad \text{....(i)}$$

$$\frac{xx_2}{a^2}+\frac{yy_2}{b^2}=1 \qquad \text{...(ii)}$$

It is given that (i) and (ii) are at right angles

$$\frac{-b^2}{a^2}\frac{x_1}{y_1}\times\frac{-b^2}{a^2}\frac{x_2}{y_2}=-1 \Rightarrow \frac{x_1x_2}{y_1y_2}=\frac{-a^4}{b^4}$$

(33) **(A).** Equation of the tangent at $\pi/4$ is

$$\frac{x\left(\frac{1}{\sqrt{2}}\right)}{a}+\frac{y\left(\frac{1}{\sqrt{2}}\right)}{b}=1 \text{ i.e., } \frac{x}{a}+\frac{y}{b}-\sqrt{2}=0 \quad \text{.....(1)}$$

Equation of the normal at $\pi/4$ is

$$\frac{x}{b}-\frac{y}{a}=\frac{a}{b\sqrt{2}}-\frac{b}{a\sqrt{2}} \qquad \text{...........(2)}$$

p_1 = length of the perpendicular from the centre to the

$$\text{tangent} = \left|\frac{-\sqrt{2}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \frac{\sqrt{2}\,ab}{\sqrt{a^2+b^2}}$$

p_2 = length of the perpendicular from the centre to the

$$\text{normal} = \left|\frac{\frac{a}{b\sqrt{2}}-\frac{b}{a\sqrt{2}}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \left|\frac{a^2-b^2}{\sqrt{2}\sqrt{a^2+b^2}}\right|$$

$$\text{Area of the rectangle} = p_1 p_2 = \frac{ab\left(a^2-b^2\right)}{a^2+b^2}$$

(34) **(A).** Equation of line joining points α and β is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$$

If it is a focal chord, then it passes through focus (ae, 0),

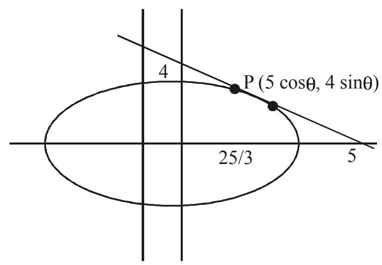
$$\text{so } e\cos\frac{\alpha-\beta}{2}=\cos\frac{\alpha-\beta}{2} \Rightarrow \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}=\frac{e}{1}$$

$$\Rightarrow \frac{\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}}=\frac{e-1}{e+1}$$

$$\Rightarrow \frac{2\sin\alpha/2\,\sin\beta/2}{2\cos\alpha/2\,\cos\beta/2}=\frac{e-1}{e+1} \Rightarrow \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{e-1}{e+1}$$

(35) **(B).** $\frac{x}{5}\cos\theta+\frac{y}{4}\sin\theta=1$

$$Q\left(\frac{25}{3},\frac{4(3-5\cos\theta)}{3\sin\theta}\right),\ P(5\cos\theta, 4\sin\theta)$$



Equation of circle,

$$(x-5\cos\theta)\left(x-\frac{25}{3}\right)+(y-4\sin\theta)\left(y-\frac{4(3-5\cos\theta)}{3\sin\theta}\right)=0$$

$$(x-5\cos\theta)\left(x-\frac{25}{3}\right)+(y-4\sin\theta)\left(y-\frac{4}{\sin\theta}+\frac{20}{3}\frac{\cos\theta}{\sin\theta}\right)=0$$

Now expand this and put the coefficient of $\theta = 0$ then $x = 3, y = 0$.

(36) **(A).** If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the hyperbola, focii are (± ae, 0),

$e=\frac{1}{2}\sqrt{a^2+b^2}$ with (0, –b) as the vertex and the axis of symmetry along the conjugate axis, the equation of the parabola is $x^2 = L(y + b)$.

As it passes through (± ae, 0), $Lb = a^2e^2 = a^2 + b^2$

$$\Rightarrow L=\frac{1}{b}(a^2+b^2)$$

$$\therefore \text{ focus} = \left(0, -b+\frac{a^2+b^2}{4b}\right)=\left(0, -3+\frac{49+9}{12}\right)$$

$$=(0, 11/6)$$

(37) **(C).** Let ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and circle $x^2+y^2=a^2e^2$

radius of circle = ae.

Point of intersection of circle and ellipse $x^2+y^2=a^2e^2$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=a^2e^2$$

$$\left(\frac{a}{e}\sqrt{e^4+e^2-1},\frac{a}{e}(e^2-1)\right)$$

Now area of triangle

$$=\frac{1}{2}\begin{vmatrix}\frac{a}{e}\sqrt{e^4+e^2-1} & \frac{a}{e}(1-e^2) & 1\\ ae & 0 & 1\\ -ae & 0 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left|\frac{a}{e}(1-e^2)(2ae)\right|=30 \text{ or } a^2(1-e^2)=30$$

$$e=\sqrt{1-\frac{30}{a^2}} \;;\; a=\frac{17}{2} \text{ then}$$

$$2ae=2a\sqrt{\frac{a^2-30}{a^2}}=13$$

(38) **(C).** Let a point (5 cos θ, 4 sin θ) then equation of normal at this point 4y cos θ – 5x sin θ + 9 sin θ cos θ = 0

if this normal touches the circle $x^2+y^2=r^2$

then perpendicular distance from centre of the circle = r

$$\left|\frac{9\sin\theta\cos\theta}{\sqrt{16\cos^2\theta+25\sin^2\theta}}\right|=r$$

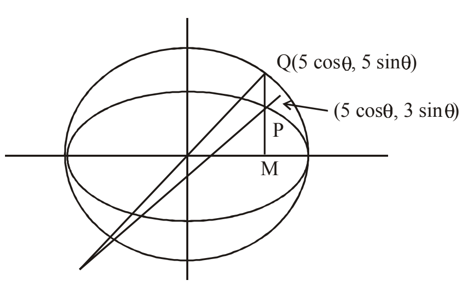
the value of r will be minimum at $\cos 2\theta=\frac{1}{9}$ then $r_{min}=1$

(39) **(C).** Equation of normal to the ellipse at P is

$5x\sec\theta-3y\,\text{cosec}\,\theta=16$ .... (1)

Equation of normal to the circle $x^2+y^2=25$ at point Q is

$y=x\tan\theta$ .... (2)

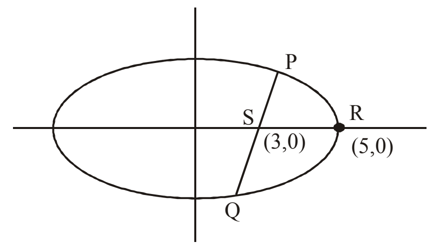


Eliminating θ from (1) and (2).

We get, $x^2+y^2=64$

(40) **(C).** Here SP = 2 and from the figure SR = 2

∴ P and R are the same points



∴ length of focal chord = 2a = 10

(41) **(A).** The equation of the normal to the parabola $y^2=4ax$ is $y=mx-2am-am^3$

It passes through the point (h, k) if $k=mh-2am-am^3$

$\Rightarrow am^3+m(2a-h)+k=0$ ... (1)

Let the roots of the above equation be m_1, m_2 and m_3.

Let the perpendicular normals correspond to the values of m_1 and m_2 so that $m_1 m_2=-1$.

From the equation (1), $m_1 m_2 m_3=-k/a$

Since $m_1 m_2=-1$, $m_3=k/a$

Since m_3 is a root of (1), we have a

$$\left(\frac{k}{a}\right)^3+\frac{k}{a}(2a-h)+k=0\Rightarrow k^2+a(2a-h)+a^2=0$$

$\Rightarrow k^2=a(h-3a)$

Hence the locus of (h, k) is $y^2=a(x-3a)$.

(42) **(A).** SS' = 2ae, where a and e are length of semi-major axis and eccentricity respectively.

$$\sqrt{(9-3)^2+(12-4)^2}=2ae \quad \therefore \; ae=5$$

∴ Centre is mid point of SS'

∴ Center ≡ (6, 8)

Let the equation of auxiliary circle be

$(x-6)^2+(y-8)^2=a^2$

We know that the foot of the perpendicular from the focus on any tangent lies on the auxiliary circle

∴ (1, – 4) lies on auxiliary circle.

i.e. $(1-6)^2+(-4-8)^2=a^2 \Rightarrow a=13 \because ae=5$

$\Rightarrow e=5/13$

(43) **(A).** Let the mid point be (h, k). Equation of a chord whose mid point is (h, k) would be $T=S_1$

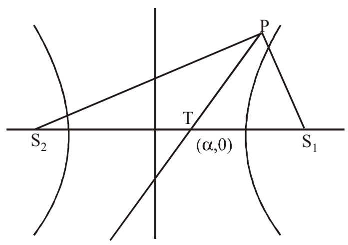
or $3xh-2yk+2(x+h)-3(y+k)=3h^2-2k^2+4h-6k$

$\Rightarrow x(3h+2)-y(2k+3)-(2h+3k)-3h^2+2k^2=0$

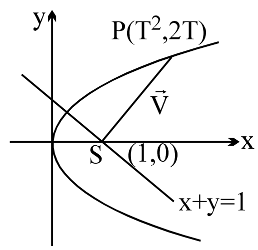
Its slope is $\frac{3h+2}{2k+3}=2$ (given) $\Rightarrow 3h=4k+4$

⇒ Required locus is $3x-4y=4$

(44) **(C).** Angle bisector of focal distances is tangent at that point and whose slope is always greater than the slope of asymptote hence $\alpha\in(0,a]$



(45) **(C).** $\vec{V} = (T^2 - 1)\hat{i} + 2T\hat{j}$



$\vec{n} = \hat{j} - \hat{i}$

direction of $\vec{V}$ on $\vec{n}$

$$y = \frac{\vec{V}\cdot\vec{n}}{|\vec{n}|} = \frac{(1-T^2)+2T}{\sqrt{2}}$$

$$\sqrt{2}\,y - 1 - T^2 + 2T\,;\ \sqrt{2}\frac{dy}{dx} = -2T\frac{dT}{dt} + 2\frac{dT}{dt}$$

Given $\frac{dx}{dt} = u$; but $x = T^2$; $\frac{dx}{dt} = 2T\frac{dT}{dt}$

when P(4, 4) then $T = 2 \Rightarrow u = 2\cdot 2\frac{dT}{dt}$; $\frac{dT}{dt} = 1$

$$\therefore \sqrt{2}\frac{dy}{dt} = -4 + 2 = -2 \quad \Rightarrow \frac{dy}{dt} = -\sqrt{2}$$

(46) **(B).** Let directrix be $x = a/e$ and focus be $S(ae, 0)$.
Let P $(a\sec\theta, b\tan\theta)$ be any point on the curve.

Equation of tangent at P is $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.

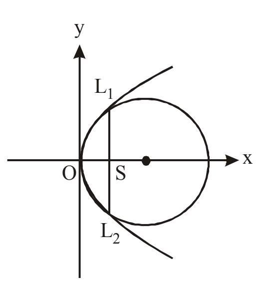
Let F be the intersection point of tangent of directrix,

then $F = \left(a/e, \frac{b(\sec\theta - e)}{e\tan\theta}\right)$

$$\Rightarrow m_{SF} = \frac{b(\sec\theta - e)}{-e\tan\theta\ (a^2-1)}, m_{PS} = \frac{b\tan\theta}{a(\sec\theta - e)}$$

$\Rightarrow m_{SF}.m_{PS} = -1$

(47) **(D).** Equation of tangent of parabola at (1, 2) is
$y\cdot 2 = 2(x+1)$ i.e. $y - x - 1 = 0$
Family of circles touching the parabola at (1, 2) is
$(x-1)^2 + (y-2)^2 + \lambda(y - x - 1) = 0$



Since, it passes through (0, 0) therefore
$1^2 + 4 + \lambda(-1) = 0 \quad \Rightarrow \lambda = 5$
Thus required circle is $x^2 + y^2 - 7x + y = 0$

its radius $= \sqrt{\frac{49}{4} + \frac{1}{4}} = \frac{5}{\sqrt{2}}$

(48) **(B).** Since $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate

$$\therefore \frac{4}{e^2} + \frac{4}{e'^2} = 1 \text{ i.e., } 4 = \frac{e^2e'^2}{e'^2 + e^2}$$

line passing through the points (e, 0) and (0, e')
$e'x + ey - ee' = 0$
it is tangent to the circle $x^2 + y^2 = r^2$

$$\therefore \frac{ee'}{\sqrt{e'^2 + e^2}} = r \quad \therefore r^2 = \frac{e^2e'^2}{e^2 + e'^2} = 4 \quad \therefore r = 2$$

(49) **(C).** $y = mx + \frac{1}{m}$
or $m^2h - mk + 1 = 0$

$$m_1 + m_2 = \frac{k}{h}\ ;\ m_1 m_2 = \frac{1}{h}$$

given $\theta_1 + \theta_2 = \frac{\pi}{4} \Rightarrow \frac{m_1 + m_2}{1 - m_1m_2} \quad \Rightarrow \frac{k}{h} = 1 - \frac{1}{h}$

$\Rightarrow y = x - 1$

(50) **(B).** Any tangent to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m^2 - b^2}$

or $y = mx + c$, where $c = \pm\sqrt{a^2m^2 - b^2}$

This will touch the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

If the equation $\frac{(mx+c)^2}{a^2} - \frac{x^2}{b^2} = 1$ has equal roots

or $x^2(b^2m^2 - a^2) + 2b^2mxc + (c^2 - a^2)b^2 = 0$
is an quadratic equation have equal roots
$4b^4m^2c^2 = 4(b^2m^2 - a^2)(c^2 - a^2)b^2$
$c^2 = a^2 - b^2m^2$
$a^2m^2 - b^2 = a^2 - b^2m^2$
$m^2(a^2 + b^2) = a^2 + b^2 \qquad \Rightarrow m = \pm 1$
Hence, the equation of the common tangents are

$y = \pm x \pm \sqrt{a^2 - b^2}$

(51) **(D).** Let the equation of tangent $y = mx + \sqrt{a^2m^2 + b^2}$
Foci $\equiv (\pm ae, 0)$, vertices $\equiv (\pm a, 0)$, $C \equiv (0, 0)$

$\therefore\ s = \left|\frac{mae+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,\quad s' = \left|\frac{-mae+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|$

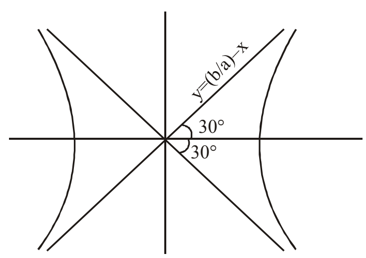
$a = \left|\frac{ma+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,\quad a' = \left|\frac{-ma+\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right|,$

$c = \left|\frac{\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right| \quad \therefore \frac{ss'-c^2}{aa'-c^2} = \frac{\frac{m^2a^2e^2}{1+m^2}}{\frac{m^2a^2}{1+m^2}} = e^2$

(52) **(D).** $b^2 = 9$

$\frac{b}{a} = \tan 30° = \frac{1}{\sqrt{3}} \quad \therefore a^2 = 3b^2 = 27$

$\therefore$ Required locus is director circle of the hyperbola



and which is $x^2 + y^2 = 27 - 9,\ x^2 + y^2 = 18$

If $\frac{b}{a} = \tan 60°$ is taken then $a^2 = \frac{b^2}{3} = \frac{9}{3} = 3$

$\therefore$ Required locus is $-x^2 - y^2 = 3 - 9 = -6$ which is not possible.

(53) **(C).** Let (h, k) be the mid point of the chord of the circle

$x^2 + y^2 = a^2$, so that its equation by $T = S_1$ is

$hx + ky = h^2 + k^2$ or $y = -\frac{h}{k}x + \frac{h^2+k^2}{k}$ i.e.,

the form $y = mx + c$

It will touch the hyperbola if $c^2 = a^2 m^2 - b^2$

$\left(\frac{h^2+k^2}{k}\right)^2 = a^2\left(-\frac{h}{k}\right)^2 - b^2$

$\Rightarrow (h^2 - k^2)^2 = a^2 h^2 - b^2 k^2$

Generalising, the locus of the mid point (h, k) is

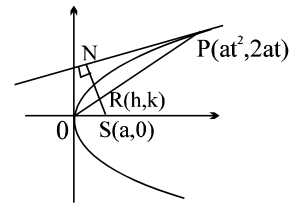
$(x^2 + y^2)^2 = a^2 x^2 - b^2 y^2$

(54) **(B).** $T : ty = x + at^2$ ....(1)

line perpendicular to (1) through (a,0)

$tx + y = ta$ ....(2)

equation of OP : $y - \frac{2}{t}x = 0$ ....(3)



From (2) & (3) eleminating t we get locus

(55) **(A).** For $y^2 = 24x$, focus is (6, 0)

Clearly $x + ay - 6 = 0$ passes through the point (6, 0)

Since we know pair of tangents drawn at the end points of the focal chord of the parabola meets on the directrix of the parabola.

(56) **(B).** Any normal to the parabola $y^2 = 4x$ is $y + tx = 2t + t^3$

It this passes through $(\lambda, \lambda + 1)$

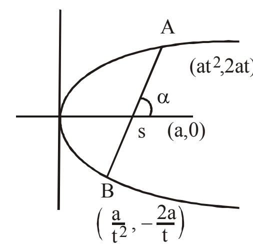
$\Rightarrow t^3 + t(2-\lambda) - \lambda - 1 = 0 = f(t)$ say)

$\lambda < 2$ than $f'(t) = 3t^2 + (2-\lambda) > 0$

$\Rightarrow f(t) = 0$ will have only one real root $\Rightarrow$ A is true

The statement-2 is also true since $(\lambda + 1)^2 > 4\lambda$ is true for all $\lambda \neq 1$. The statement-2 is true but does not follow true statement-2.

(57) **(D).** Let AB be a focal chord.



Slope of AB $= \frac{2t}{t^2-1} = \tan\alpha$

$\Rightarrow \tan\frac{\alpha}{2} = \frac{1}{t} \Rightarrow t = \cot\frac{\alpha}{2}$

Length of AB $= a\left(t + \frac{1}{t}\right)^2 = 4a\,\text{cosec}^2\alpha$

$\Rightarrow$ Statement-2 is correct but statement-1 is false.

(58) **(A).** Equation of PQ (i.e., chord of contact) to the ellipse

$x^2 + 2y^2 = 6\ ;\ \frac{hx}{6} + \frac{ky}{3} = 1$ ... (1)

Any tangent to the ellipse $x^2 + 4y^2 = 4$ is

i.e., $x/2\cos\theta + y\sin\theta = 1$ ... (2)

$\Rightarrow$ (1) & (2) represent the same line $h = 3\cos\theta,\ k = 3\sin\theta$

Locus of R (h, k) is $x^2 + y^2 = 9$

(59) **(A).** The ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\therefore$ Auxiliary circle is $x^2 + y^2 = 9$ and $(-\sqrt{5}, 0)$ and

$(\sqrt{5}, 0)$ are focii. $\therefore$ Statement-1 is true.

Statement-2 is true.

(60) **(B).** Required chord of contact is $32x + 9y = 144$ obtained

from $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

(61) **(A).** Circle is $x^2 + y^2 = a^2$ ..... (i)

Hyperbola is $xy = c^2$ ..... (ii)

Take $P\left(ct, \frac{c}{t}\right)$ any point on (ii)

To find intersection of (i) and (ii), we put P in (i),

$$(ct)^2 + \left(\frac{c}{t}\right)^2 = a^2 \Rightarrow c^2t^4 - a^2t^2 + c^2 = 0$$

$$\Rightarrow c^2t^4 - 0t^3 - a^2t^2 + c^2 = 0 \Rightarrow t^4 - 0t^3 - \frac{a^2}{c^2}t^2 + 1 = 0$$

$$\Rightarrow t_1 + t_2 + t_3 + t_4 = 0,\ t_1 t_2 t_3 t_4 = 1$$

(62) **(A).** $\frac{7^2}{4} - \frac{(-3)^2}{9} - 1 > 0$ and $\frac{2^2}{4} - \frac{7^2}{9} - 1 < 0$

(63) **(A).** $\left|\dfrac{\frac{2}{\lambda + t} - \frac{1}{t}}{1 + \frac{2}{(\lambda + t)t}}\right| = \tan\alpha$

$$\Rightarrow \lambda^2(1 - t^2\tan^2\alpha) - 2\lambda t\left(1 + (t^2+2)\tan^2\alpha\right) + t^2 - (t^2+2)^2\tan^2\alpha$$

$$\Rightarrow t_1 + t_2 = \frac{2(t + \tan^2\alpha\, t^3 + 2t\tan^2\alpha)}{1 - t^2\tan^2\alpha},$$

$$t_1 t_2 = \frac{t^2 - (t^2+2)^2\tan^2\alpha}{1 - t^2\tan^2\alpha}$$

(64) **(C).** $(t_1 + t_2)y = 2x + 2a\, t_1 t_2$ equation of chord QQ'

$$\Rightarrow \frac{2(t + \tan^2\alpha\, t^3 + 2t\tan^2\alpha)}{1 - t^2\tan^2\alpha}$$

$$y = 2x + 2a\frac{(t^2 - (t^2+2)^2\tan^2\alpha\ )}{1 - t^2\tan^2\alpha} \qquad \text{......(i)}$$

(65) **(B).** Rewriting the equation (i), we get

$$2ty - 2x - 2at^2 + \tan^2\alpha((2t^3 + 4t)y + 2a(t^2+2)^2) = 0$$

$L_1 + \mu L_2 = 0$

$L_1 = ty - x - at^2 = 0$

$L_2 = t(t^2+2)y + t^2x + a(t^2+2)^2 = 0$

$$\Rightarrow y = -\frac{2a}{t} \text{ and } x = -2a - at^2$$

(66) (B), (67) (A), (68) (A).

Since no point of the parabola is below x-axis.

$\therefore a^2 - 4 \le 0$

$\therefore$ maximum value of a is = 2

Equation of the parabola, when a = 2 is $y = x^2 + 2x + 1$

it intersect y-axis at (0, 1).

equation of the tangent at (0, 1) is y = 2x + 1

So, y = 2x + 1 touches the circle $x^2 + y^2 = r^2$

$$\therefore r = \frac{1}{\sqrt{5}}$$

Equation of the tangent at (0, 1) to the parabola $y = x^2 + ax + 1$ is

$$\frac{y+1}{2} = \frac{a}{2}(x+0) + 1 \text{ i.e. } ax - y + 1 = 0$$

$$\therefore r = \frac{1}{\sqrt{a^2+1}}$$

radius is maximum when a = 0

$\therefore$ equation of the tangent is y = 1

$\therefore$ slope of the tangent is 0

Equation of tangent is y = ax + 1

Intercepts are –1/a and 1.

$\therefore$ Area of the triangle bounded by tangent and the axes

$$= \frac{1}{2}\left|-\frac{1}{a}.1\right| = \frac{1}{2|a|}$$

it is maximum when a = 2 $\therefore$ minimum area = 1/4

(69) **(A).** For common tangent $a^2m^2 + b^2 = r^2(1 + m^2)$

$$\Rightarrow m^2 = \frac{r^2 - b^2}{a^2 - r^2}$$

since two value of m is possible and for each value of 'm' two tangents are possible.

Hence number of common tangent will be at most 4.

(70) **(B).** For four distinct common tangents $m^2 > 0 \Rightarrow r \in (b, a)$

(71) **(C).** In 4^{th} quadrant slope of common tangent is positive and intercept on y–axis will be negative so equation will

be $y = \left(\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}\right)x - r\sqrt{\frac{a^2 - b^2}{a^2 - r^2}}$

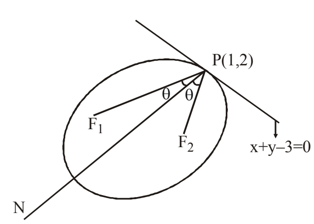
(72) (C), (73) (C), (74) (C).

Let PN is normal to ellipse at point P.

$F_1 \equiv (4, 3), F_2 \equiv (10, \beta)$; Slope PN = 1,

Slope $PF_1 = 1/3$ and Let slope $PF_2 = m$

$$\frac{m-1}{1+m} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \Rightarrow m = 3 \Rightarrow \beta = 29$$



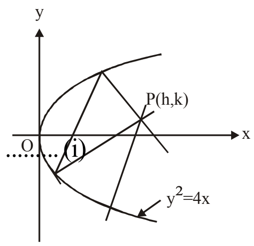
Also, $PF_1 + PF_2 = 2a \Rightarrow a = 5\sqrt{10}$

Also $b^2 = 72 \Rightarrow b = 6\sqrt{2} \Rightarrow 2b = 12\sqrt{2}$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{89}{125}$$

(75) (B), (76) (A), (77) (D).

Equation of a normal $y = mx - 2m - m^3$ passes through (h, k)



$m^3 + (2 - h)\,m + k = 0$

$m_1 m_2 m_3 = -k$

but $m_1 m_2 = 2 \Rightarrow m_3 = -k/2$

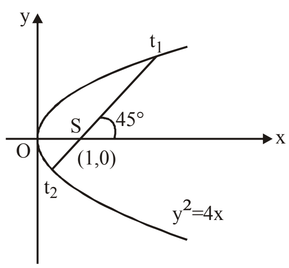
this must satisfy equation (i) ; $\dfrac{k^3}{8} - (2-h)\dfrac{k}{2} + k = 0$

$\Rightarrow k^3 - 4k\,(2 - h) + 8k = 0 \;\; (k \neq 0)$

$\Rightarrow k^2 - 8 - 4h + 8 = 0$

Locus of P is $y^2 = 4x$ which is a parabola.

Now chord passing through (1, 0) is the focal chord.



Given that gradient of focal chord is 1.

$\Rightarrow \dfrac{2}{t_1 + t_2} = 1 \Rightarrow t_1 + t_2 = 2.$ Also $t_1 t_2 = -1$

Equation of circle described on $t_1 t_2$ as diameter is

$(x - t_1^2)(x - t_2^2) + (y - 2t_1)(y - 2t_2) = 0$

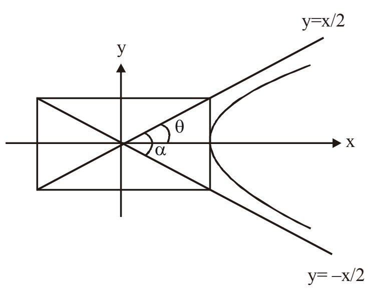
$\Rightarrow x^2 + y^2 - x\,(t_1^2 + t_2^2) + t_1^2 t_2^2 - 2y\,(t_1 + t_2) + 4t_1 t_2 = 0$

$\Rightarrow x^2 + y^2 - x\,(4 + 2) + 1 - 2y\,(2) - 4 = 0$

$\Rightarrow x^2 + y^2 - 6x - 4y - 3 = 0$

Centre a = 3 and b = 2, r = 4

Now the hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$



Asymptotes are $y = \dfrac{2x}{3}$ and $y = -\dfrac{2x}{3}$

Now $\tan\theta = 2/3$, $\therefore\ \alpha = 2\theta$

$\tan\alpha = \dfrac{2.(2/3)}{1-(4/9)}$; $\tan\alpha = \dfrac{12}{5}$; $\alpha = \tan^{-1}\left(\dfrac{12}{5}\right)$

Hence $\alpha \in (60°, 75°)$

(78) **(A).**

(a) Locus will be director circle $x^2 + y^2 = a^2 + b^2$

(b) Locus of foot of perpendicular upon any tangent from focus is auxiliary circle $x^2 + y^2 = a^2$.

(c) Equation of tangent $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ .... (i)

Perpendicular on tangent passing through centre is

$y = \dfrac{a}{b} x \tan\theta$ .... (ii)

$\tan\theta = \dfrac{by}{ax}$; $\sin\theta = \dfrac{by}{\sqrt{a^2x^2 + b^2y^2}}$;

$\cos\theta = \dfrac{ax}{\sqrt{a^2x^2 + b^2y^2}}$

Put value in (i) $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$.

(d) Replace x by 2x and y by 2y in

$(x^2 + y^2)^2 = (a^2x^2 + b^2y^2)$

(79) **(A).**

(a) The conic is a parabola having focus is (2, 3) and Directrix $3x + 4y - 6 = 0$

$\therefore$ Latus rectum = 2 ($\perp$ distance of focus from the directrix)

$= 2\left(\dfrac{6 + 12 - 6}{5}\right) = \dfrac{24}{5} < 5, 6, 7, 8$

(b) Given parabolas : $y^2 = 4ax$ and $y^2 = 4c\,(x - b)$ have common normals

$\Rightarrow y = mx - 2am - am^3$ and $y = m\,(x - b) - 2\,cm - cm^3$ respectively

Normals must be identical

$\Rightarrow\ 1 = \dfrac{2am + am^3}{mb + 2cm + cm^3} \Rightarrow\ m^2 = \dfrac{2a - 2c - b}{c - a}$

$\Rightarrow m = \pm\sqrt{\dfrac{2\,(a-c) - b}{c - a}} \Rightarrow\ -2 + \dfrac{b}{a-c} > 0 \Rightarrow\ \dfrac{b}{a-c} > 2$

(c) Let $(\sqrt{27}\cos\theta, \sqrt{48}\sin\theta)$ be a point on the ellipse

Thus equation of tangent

$\dfrac{\sqrt{27}\cos\theta x}{27} \times \dfrac{\sqrt{48}\sin\theta . y}{48} = 1$

$\therefore$ Slope $- \dfrac{\cos\theta}{\sqrt{27}} \times \dfrac{\sqrt{48}}{\sin\theta} = -\dfrac{4}{3}\cot\theta = -\dfrac{4}{3}$

i.e. $\cot\theta = 1$ i.e. $\theta = \dfrac{\pi}{4}$

$\therefore$ equation of the tangent is $\dfrac{x}{\sqrt{54}} + \dfrac{y}{\sqrt{96}} = 1$

Area of triangle $= \frac{1}{2}3\sqrt{6}\times 4\sqrt{6} = 36$, sq. root $= \sqrt{36} = 6$

(d) Clearly $f(x) > 0 \ \forall \ x \in (0, \infty)$

Now, $f(x) = \dfrac{96x}{\sqrt{9x^2+173x+900}+\sqrt{9x^2+77x+900}}$

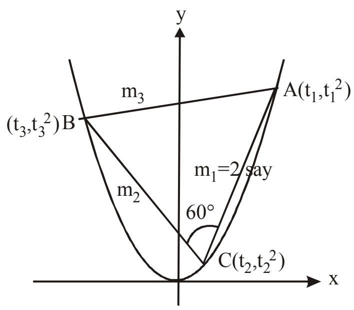
(Rationalise)

$\therefore \lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{96}{\sqrt{9+\frac{173}{x}+\frac{900}{x^2}}+\sqrt{9+\frac{77}{x}+\frac{900}{x^2}}}$

$\therefore R_f = (0, 16)$

EXERCISE-3

(1) **14.** $y = x^2$



To find : $t_1+t_2+t_3 = ?$

$m_1 = \dfrac{t_2^2-t_1^2}{t_2-t_1} = t_2+t_1$

Similarly, $m_2 = t_2+t_3$ and $m_3 = t_3+t_1$

Hence, $\Sigma\, t_i = \dfrac{m_1+m_2+m_3}{2}$

Now, $\tan 60° = \left|\dfrac{m-2}{1+2m}\right| \Rightarrow \pm\sqrt{3}\,(1+2m) = m-2$

Taking +ve sign, we get $\sqrt{3}\,(1+2m) = m-2$

$\Rightarrow m\,(2\sqrt{3}-1) = -(2+\sqrt{3}) \Rightarrow m = \dfrac{-(2+\sqrt{3})}{2\sqrt{3}-1}$

Takin –ve sign, we get $m-2 = -2\sqrt{3}m-\sqrt{3}$

$\Rightarrow m\,(2\sqrt{3}+1) = 2-\sqrt{3} \Rightarrow m = \dfrac{2-\sqrt{3}}{2\sqrt{3}+1}$

$\therefore m_1 = \dfrac{-(2+\sqrt{3})}{2\sqrt{3}-1}$, $m_2 = \dfrac{2-\sqrt{3})}{2\sqrt{3}+1}$ and $m_3 = 2$

$\therefore \sum\limits_{i=1}^{3} t_i = \dfrac{m_1+m_2+m_3}{2} = \dfrac{\frac{-(2+\sqrt{3})}{2\sqrt{3}-1}+\frac{2-\sqrt{3}}{2\sqrt{3}+1}+2}{2}$

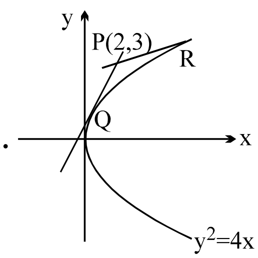
$= \dfrac{\frac{-4\sqrt{3}-2-6-\sqrt{3}+4\sqrt{3}-6-2+\sqrt{3}+22}{11}}{2}$

$= \dfrac{6}{22} = \dfrac{3}{11} = \dfrac{p}{q} \Rightarrow (p+q) = 14$

(2) **2.** $e_1^2 = 1+\dfrac{b^2}{a^2} = 1+\dfrac{12}{4} = 4 \Rightarrow e_1 = 2$

$\dfrac{1}{e_1^2}+\dfrac{1}{e_2^2} = 1$; $\dfrac{1}{e_2^2} = 1-\dfrac{1}{4} = \dfrac{3}{4} \Rightarrow e_2^2 = \dfrac{4}{3} \Rightarrow e_2 = \dfrac{2}{\sqrt{3}}$

(3) **11.**



$\left.\begin{array}{l} t_1t_2 = 2 \\ t_1+t_2 = 3 \end{array}\right] \Rightarrow t_1 = 1$ and $t_2 = 2$

Hence point $(t_1^2, 2t_1)$ and $(t_2^2, 2t_2)$

i.e. (1, 2) and (4, 4)

(4) **3.** $9\,(x-3)^2+9(y-4)^2 = y^2$

$9\,(x-3)^2+8y^2-72y+14y = 0$

$9\,(x-3)^2+8\,(y^2-9y)+144 = 0$

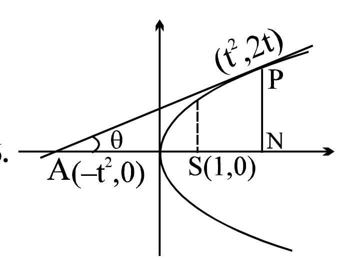
$9\,(x-3)^2+8\left[\left(y-\frac{9}{2}\right)^2-\frac{81}{4}\right]+144 = 0$

$\Rightarrow 9\,(x-3)^2+8\left(y-\frac{9}{2}\right)^2 = 162-144 = 18$

$\Rightarrow \dfrac{9(x-3)^2}{18}+\dfrac{8\left(y-\frac{9}{2}\right)}{18} = 1 \Rightarrow \dfrac{(x-3)^2}{2}+\dfrac{\left(y-\frac{9}{2}\right)}{9/4} = 1$

$e^2 = 1-\dfrac{2\times 4}{9} = \dfrac{1}{9} \quad \therefore e = \dfrac{1}{3}$

(5) **16.**



$T : ty = x+t^2$, $\tan\theta = 1/t$

$A = \frac{1}{2}(AN)(PN) = \frac{1}{2}(2t^2)(2t)$

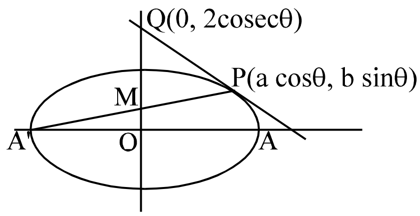
$A = 2t^3 = 2(t^2)^{3/2}$ i.e. $t^2 \in [1, 4]$ & A_{max} occurs when $t^2 = 4 \Rightarrow A_{max} = 16$

(6) **4.** $a = 3$; $b = 2$

$T : \frac{x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1$

$x = 0$; $y = 2\,\text{cosec}\theta$

chord A'P, $y = \frac{2\sin\theta}{3(\cos\theta + 1)}(x + 3)$



put $x = 0$ $y = \frac{2\sin\theta}{1+\cos\theta} = OM$

Now $OQ^2 - MQ^2 = OQ^2 - (OQ - OM)^2$

$= 2(OQ)(OM) - OM^2 = OM\{2(OQ) - (OM)\}$

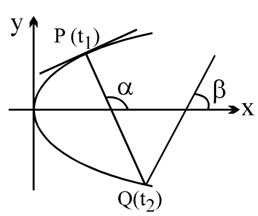
$= \frac{2\sin\theta}{1+\cos\theta}\left[\frac{y}{\sin\theta} - \frac{2\sin\theta}{1+\cos\theta}\right] = 4$

(7) **2.** $\tan\alpha = -t_1$ and $\tan\beta = -t_2$

also $t_2 = -t_1 - \frac{2}{t_1}$

$t_1 t_2 + t_1^2 = -2$

$\tan\alpha\tan\beta + \tan^2\alpha = -2$



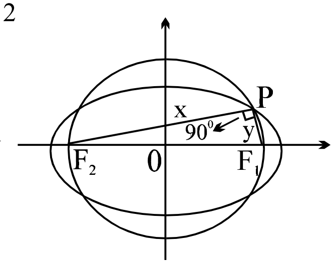
(8) **13.** $x + y = 17$; $xy = 60$,

To find $\sqrt{x^2 + y^2}$

$x^2 + y^2 = (x + y)^2 - 2xy$

$= 289 - 120 = 169$

$\sqrt{x^2 + y^2} = 13$



(9) **4.** $\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1$

$e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{5\cos^2\alpha}{5} = 1 + \cos^2\alpha$

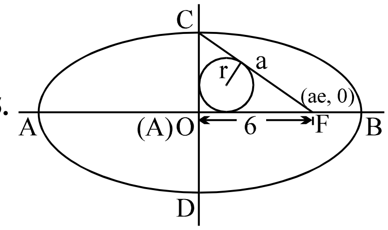
Similar|ly eccentricity of the ellipse

$\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1$ is $e_2^2 = 1 - \frac{25\cos^2\alpha}{25} = \sin^2\alpha$

Put $e_1 = \sqrt{3}\, e_2 \Rightarrow e_1^2 = 3e_2^2$

$\Rightarrow 1 + \cos^2\alpha = 3\sin^2\alpha \Rightarrow 2 = 4\sin^2\alpha \Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$

(10) **65.**



$a^2e^2 = 36 \Rightarrow a^2 - b^2 = 36$ .... (1)

Using $r = (s - a)\tan\frac{A}{2}$ in Δ OCF

$1 = (s - a)\tan 45^\circ$ when $a = CF$

$2 = 2(s - a) = 2s - 2a = 2s - AB = (OF + FC + CO) - AB$

$2 = 6 + \frac{AB}{2} + \frac{CD}{2} - AB$; $\frac{AB - CD}{2} = 4$

$\Rightarrow 2(a - b) = 8 \Rightarrow a - b = 4$ — (2)

From (1) & (2) $a + b = 9 \Rightarrow 2a = 13$; $2b = 5$

$\Rightarrow (AB)(CD) = 65$

(11) **2.** Substituting (a/e, 0) in $y = -2x + 1$

$0 = -\frac{2a}{e} + 1$; $\frac{2a}{e} = 1 \Rightarrow a = \frac{e}{2}$

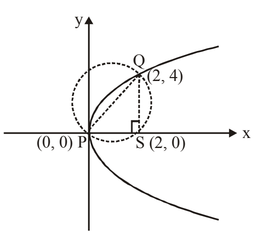
Also, $1 = \sqrt{a^2m^2 - b^2}$

$1 = a^2m^2 - b^2 \Rightarrow 1 = 4a^2 - b^2 \Rightarrow 1 = \frac{4e^2}{4} - b^2$

$\Rightarrow b^2 = e^2 - 1$. Also, $b^2 = a^2(e^2 - 1)$ $\therefore a = 1, e = 2$

(12) **2.** $\because \Delta_2 = \frac{\Delta_1}{2}$ (by property) $\because \frac{\Delta_1}{\Delta_2} = 2$

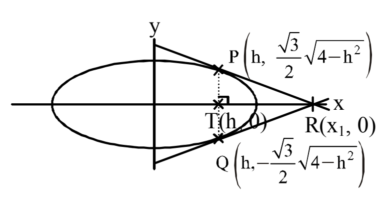
(13) **4.**



Focus of parabola S (2, 0) points of intersection of given curves : (0, 0) and (2, 4).

Area (Δ PSQ) $= \frac{1}{2} \times 2 \times 4 = 4$ sq. units

(14) **9.**



$\frac{x^2}{4}+\frac{y^2}{3}=1$; $y=\frac{\sqrt{3}}{2}\sqrt{4-h^2}$ at $x=h$

Let R $(x_1, 0)$

PQ is chord of contact, so, $\frac{xx_1}{4}=1 \Rightarrow x=\frac{4}{x_1}$

which is equation of PQ, x = h

So, $\frac{4}{x_1}=h \Rightarrow x_1=\frac{4}{h}$

$\Delta(h)$ = area of Δ PQR = $\frac{1}{2}\times PQ \times RT$

$=\frac{1}{2}\times\frac{2\sqrt{3}}{2}\sqrt{4-h^2}\times(x_1-h)=\frac{\sqrt{3}}{2h}(4-h^2)^{3/2}$

$\Delta'(h)=\frac{-\sqrt{3}\,(4+2h^2)}{2h^2}\sqrt{4-h^2}$

which is always decreasing.

so Δ_1 = maximum of $\Delta(h)=\frac{45\sqrt{5}}{8}$ at $h=\frac{1}{2}$

Δ_2 = minimum of $\Delta(h)=\frac{9}{2}$ at h = 1.

So, $\frac{8}{\sqrt{5}}\Delta_1-8\Delta_2=\frac{8}{\sqrt{5}}\times\frac{45\sqrt{5}}{8}-8\cdot\frac{9}{2}=45-36=9$

(15) **15.** $x^2+y^2=2$

Equation of tangent to circle $x^2+y^2=2$ is

$y=mx\pm\sqrt{2}\sqrt{1+m^2}$

Equation of tangent to $y^2=8x$ is $y=mx+\frac{2}{m}$

$\sqrt{2}\,(\sqrt{1+m^2})=\frac{2}{m}$

$2(1+m^2)=\frac{4}{m^2}$

$m^2(1+m^2)=2$

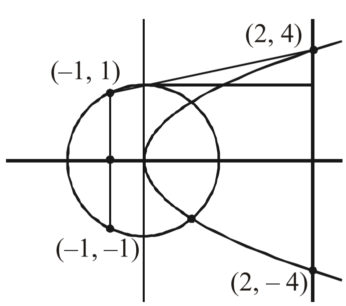
$m^4+m^2-2=0$

$(m^2+2)(m^2-1)=0$

$m=\pm1$

$y=x+2$

$y=-x-2$



Area $=\left(3\times1+\frac{1}{2}\times3\times3\right)\times2=6+9=15$ sq. unit

EXERCISE-4

(1) **(C).** $x^2-4x-8y+12=0$

$x^2-4x+4-8y+8=0$; $(x-2)^2=8y-8$

$(x-2)^2=8(y-1)$

$\Rightarrow X^2=8Y$, where $X=x-2$ and $Y=y-1$

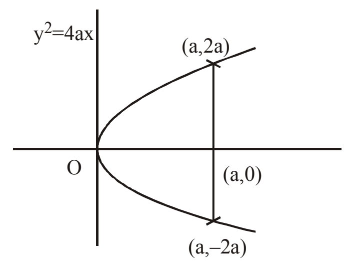
Compare with standard equation $x^2=4ay$

$\because$ length of latus rectum is 4a

$\therefore$ 4a = 8

(2) **(D).** Coordinate of end points of latus rectum is (a, 2a) and (a, –2a).

Tangent at (x_1, y_1) to $y^2=4ax$ is $yy_1=2a(x+x_1)$



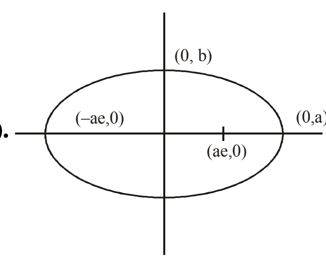
$\therefore$ equation of tangent at (a, 2a) is y.2a = 2a (x + a)

$\Rightarrow y=x+a \Rightarrow x-y+a=0$

and equation of tangent at (a, –2a) is y (–2a) = 2a (x + a)

$\Rightarrow -y=x+a \Rightarrow x+y+a=0$

(3) **(A).**



Distance between foci is 2ae and length of minor axis is 2b. According to question, 2ae = 2b

$\Rightarrow e=\frac{2b}{2a}$ .... (1)

and we know $e=\sqrt{1-\left(\frac{\text{minor axis}}{\text{major axis}}\right)^2}$

$e=\sqrt{1-\{e\}^2}$ (from (1))

$\Rightarrow e^2=1-e^2 \Rightarrow 2e^2=1 \Rightarrow e=\frac{1}{\sqrt{2}}$

(4) **(B).** According to question, $2a=8 \Rightarrow a=4 \Rightarrow a^2=16$ and $e=1/2$ $\therefore$ $b^2=a^2(1-e^2)$

$\Rightarrow b^2=(4)^2(1-(1/2)^2)=16\left(1-\frac{1}{4}\right)=16\times\frac{3}{4}=12$

$\therefore$ equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{12}=1$

$\Rightarrow 3x^2+4y^2=48$

(5) **(B).** Equation of hyperbola is

$$16x^2 - 9y^2 = 144 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1 \quad \text{....... (1)}$$

If we compare equation (1) with standard equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

we get $a^2 = 9$ and $b^2 = 16$

$\Rightarrow a = 3$ and $b = 4$

$$\because \text{ eccentricity } e = \sqrt{1 + \frac{b^2}{a^2}} \; ; \; e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{16}{9} = \frac{25}{9}$$

and length of latus rectum is

$$2a(e^2 - 1) = 2 \times 3\left[\frac{25}{9} - 1\right] = 6 \times \frac{16}{9} = \frac{32}{3}$$

(6) **(D).** Equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

If we compare standard equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow a^2 = 16$. Now eccentricity of ellipise $e = \sqrt{1 - \frac{b^2}{a^2}}$

and foci of ellipse is (ae, 0)

again equation of hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25} \Rightarrow \frac{x^2}{144/25} - \frac{y^2}{81/25} = 1$$

If we compare it with standard equation of hyperbola

$$\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1 \Rightarrow a'^2 = 144/125$$

Now, eccentricity of hyperbola is $e = \sqrt{1 + \frac{b'^2}{a'^2}}$

and foci of hyperbola is (a'e', 0)

According to question ae = a'e'

$$\Rightarrow a^2e^2 = a'^2e'^2 \Rightarrow a^2\left(1 - \frac{b^2}{a^2}\right) = a'^2\left(1 + \frac{b'^2}{a'^2}\right)$$

$$\Rightarrow 16\left(1 - \frac{b^2}{16}\right) = \frac{144}{25}\left(1 + \frac{81}{144}\right)$$

$$\Rightarrow 16 \times \frac{(16 - b^2)}{16} = \frac{144}{25}\left(\frac{144 + 81}{144}\right) \Rightarrow 16 - b^2 = \frac{225}{25}$$

$\Rightarrow 16 - b^2 = 9 \Rightarrow b^2 = 7$

(7) **(B).** Let equation of parabola is $y^2 = 4bx$

$\Rightarrow$ equation of tangent at $(bt_1^2, 2bt_1)$ is

$$y(2bt_1) = 2b(x + bt_1^2) \Rightarrow y = \frac{1}{t_1}x + bt_1$$

$\therefore$ slope of tangent is $1/t_1$

$\therefore$ slope of normal to this point will be, $-t_1$.

$\therefore$ equation of normal to this point is

$(y - 2bt_1) = -t_1(x - bt_1^2)$ ...... (1)

Now this normal passes through $(bt_2^2, 2bt_2)$

$\therefore$ they will satisfy equation of normal i.e. eq^n (1)

$\therefore (2bt_2 - 2bt_1) = -t_1(bt_2^2 - bt_1^2)$

$\Rightarrow 2(t_2 - t_1) = -t_1(t_2^2 - t_1^2)$

$\Rightarrow 2(t_2 - t_1) = -t_1(t_2 + t_1)(t_2 - t_1)$

$$\Rightarrow 2 = -t_1(t_2 + t_1) \Rightarrow t_2 = \frac{-2}{t_1} - t_1$$

(8) **(A).** The given parabolas are $y^2 = 4ax$ and $x^2 = 4ay$ solving these we get A(0, 0), B(4a, 4a) as their point of intersection. Also the line $2bx + 3cy + 4d = 0$ passes through A and B

$\therefore$ d = 0 and 2b . (4a) + 3c. 4a = 0

$\Rightarrow a(2b + 3c) = 0$

$\therefore 2b + 3c = 0 \quad \because a \neq 0$ (given)}

$\therefore d^2 + (2b + 3c)^2 = 0 + 0$

(9) **(B).** Centre of ellipse is (0, 0)

eccentricity e = 1/2

equation of one directrix is x = 4

If we compare of equation of given directrix with equation of directrix of standard ellipse that is x = a/e

$$\Rightarrow \frac{a}{e} = 4 \Rightarrow \frac{a}{1/2} = 4 \Rightarrow a = 2 \; \{\because e = 1/2\}$$

$$\Rightarrow a^2 = 4 \text{ and we know that } e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{1}{4} = 1 - \frac{b^2}{4} \Rightarrow b^2 = 3$$

$\therefore$ equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$$

(10) **(D).** $y = mx + c$ touches $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If $c^2 = a^2m^2 - b^2$

$\therefore y = \alpha x + \beta$ touches if $\beta^2 = a^2\alpha^2 - b^2$

$$\Rightarrow a^2x^2 - y^2 = b^2 \Rightarrow \frac{x^2}{b^2/a^2} - \frac{y^2}{b^2} = 1$$

which is a hyperbola.

(11) **(D).** At a vertex of parabola $y = ax^2 + bx + c$

$$\frac{dy}{dx} = 0 \text{, we are given } y = \frac{1}{3}a^3x^2 + \frac{1}{2}a^2x - 2a \text{ .... (1)}$$

$$\frac{dy}{dx}=\frac{2}{3}a^3x+\frac{a^2}{2}$$

Now, $\frac{dy}{dx}=0 \Rightarrow \frac{2a^3x}{3}+\frac{a^2}{2}=0 \Rightarrow a=\frac{-3}{4x}$

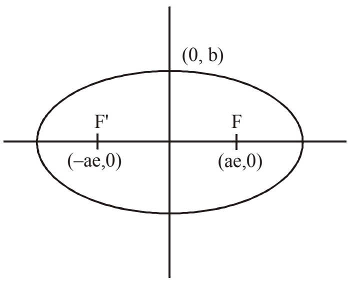
Putting $a=\frac{-3}{4x}$ in (1) we get

$$y=\frac{1}{3}\left(\frac{-3}{4x}\right)^3x^2+\frac{1}{2}\left(\frac{-3}{4x}\right)^2x-2\left(\frac{-3}{4x}\right)$$

$$=\frac{-27x^2}{3\times 64x^3}+\frac{1}{2}\left(\frac{9}{16x^2}\right)x-2\left(\frac{-3}{4x}\right)=\frac{-9}{64x}+\frac{9}{32x}+\frac{3}{2x}$$

$$\Rightarrow xy=\frac{-9}{64}+\frac{9}{32}+\frac{3}{2}=\frac{-9+18+96}{64}=\frac{105}{64}\Rightarrow xy=\frac{105}{64}$$

(12) **(D).**



Given 2ae = 6 and 2b = 8

ae = 3 ⇒ a = 3/e ⇒ b = 4 $\quad \because\ e^2 = 1 - b^2/a^2$

From given data, $e^2=1-\frac{16}{9/e^2}$ $\quad \therefore\ e^2=1-\frac{16e^2}{9}$

$\Rightarrow 9e^2 = 9 - 16e^2 \Rightarrow 25e^2 = 9 \Rightarrow e^2 = 9/25 \Rightarrow e = 3/5$

(13) **(D).** Equation of parabola is $y^2 = 8x$

and equation of one tangent is $y = x + 2$

Let the point on line $y = x + 2$ is (h, k)

∴ they will satisfy equation of line

$\Rightarrow k = h + 2$ .... (1)

Now combine equation of the pair of tangent drawn from (h, k) to parabola is $SS_1 = P^2$

$\Rightarrow (y^2 - 8x)(k^2 - 8h) = [yk - 4(x + h)]^2$

$\Rightarrow y^2(k^2 - 8h) - 8x(k^2 - 8h) = y^2k^2 + 16(x + h)^2 - 8yk(x + h)$

$\Rightarrow y^2(k^2 - 8h) - 8x(k^2 - 8h) = y^2k^2 + 16x^2 + 16h^2 + 32xh - 8yk(x + h)$

$\Rightarrow 16x^2 - y^2(k^2 - 8h) + y^2k^2 + 32xh - 8yk(x + h) + 8x(k^2 - 8h) = 0$

∴ angle between these two tangent is $\pi/2$

∴ Coefficient of x^2 + Coefficient of $y^2 = 0$

$\Rightarrow 16 - (k^2 - 8h) + k^2 = 0 \Rightarrow 16 + 8h = 0 \Rightarrow h = -2$

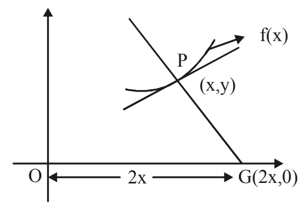
put this value in (1) we get k = 0

∴ (h, k) = (–2, 0)

(14) **(D).** Let equation of normal at point P is

$$(Y-y)=\frac{1}{dy/dx}(X-x)$$

the point G where it meet with X axis Y coordinate will be zero.



$$\therefore\ -y=\frac{-1}{dy/dx}(X-x)\Rightarrow y\frac{dy}{dx}+x=X$$

$$\therefore \text{ Coordinate of G } \left(y\frac{dy}{dx}+x,0\right)$$

According to question, $y\frac{dy}{dx}+x=2x$

$$\Rightarrow y\frac{dy}{dx}+x \Rightarrow\ y\,dy=x\,dx$$

$$\Rightarrow \int ydy=\int xdx \Rightarrow \frac{y^2}{2}=\frac{x^2}{2}+c \Rightarrow \frac{y^2}{2}-\frac{x^2}{2}=c$$

⇒ equation of hyperbola.

(15) **(D).** Given equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1 \qquad .....(1)$$

Standard equation of hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad .....(2)$$

Compare eq. (1) with eq. (2) we get

$a^2 = \cos^2\alpha \Rightarrow a = \cos\alpha$ ..... (3)

$b^2 = \sin^2\alpha \Rightarrow b = \sin\alpha$ ..... (4)

Abscissae of foci is ae

eccenctricity of hyperbola is

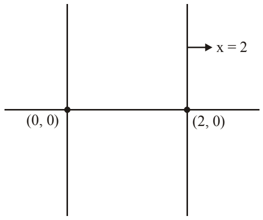
$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{1}{\cos\alpha} \qquad ....(5)$$

∴ Abscissae of foci is ae

from (3) and (5), $\cos\alpha\times\frac{1}{\cos\alpha}=1$

indpendent of α.

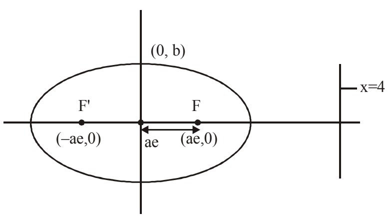
(16) **(B).** According to question focus = (0, 0) and directrix is x = 2



$\therefore$ axis of parabola will be x-axis
because axis passes through focus and perpendicular to directrix. Now vertex is mid point of line joining the focus and intersection of directrix and axis of parabola here intersection point of directrix and axis is (2, 0)

$\therefore$ mid point of focus and (2, 0) is $\left(\frac{0+2}{2}, \frac{0+0}{2}\right) = (1, 0)$

(17) **(D).** Focus is at origin. Equation of directrix is x = 4
eccentricity e = 1/2
$\therefore$ distance between centre and focus F of a ellipse is ae and distance of directrix from centre is a/e



$\therefore$ distance of directrix from focus will be $\frac{a}{e} - ae$

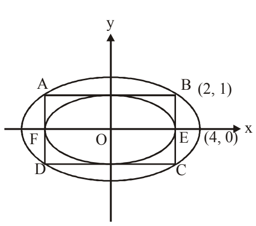
but according $\frac{a}{e} - ae = 4$

$\Rightarrow a\left[\frac{1-e^2}{e}\right] = 4 \Rightarrow a[1-e^2] = 4e$

$\Rightarrow a\left[1-\frac{1}{4}\right] = 4 \times \frac{1}{2}$ $\{\because e = 1/2\}$

$\Rightarrow \frac{3a}{4} = 2 \Rightarrow a = \frac{8}{3} \Rightarrow$ Length of semimajor axis is 8/3

(18) **(C).** Let the new ellipse be $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$



Satisfy (2, 1) we get $b^2 = 16/12$

(19) **(B).** The locus of perpendicular tangents is directrix i.e, x = –a; x = –1

(20) **(AB).** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $\frac{9}{a^2} + \frac{1}{b^2} = 1$ .... (1)

Case 1 : when a > b
$b^2 = a^2(1-e^2) = a^2(1-2/5)$; $5b^2 = 3a^2$ .... (2)
From (1) & (2)

$\frac{9\times 3}{5b^2} + \frac{1}{b^2} = 1 \Rightarrow b^2 = \frac{32}{5}$

$\therefore\ a^2 = \frac{32}{3}$ $\therefore\ \frac{3x^2}{32} + \frac{5y^2}{32} = 1$

$\Rightarrow 3x^2 + 5y^2 - 32 = 0$

Case - 2 : When b > a

$a^2 = b^2(1-e^2) = \frac{3}{5}b^2$ .... (3)

From (1) & (3), $a^2 = \frac{48}{5}, b^2 = 16$

$\therefore\ \frac{5x^2}{48} + \frac{y^2}{16} = 1 \Rightarrow 5x^2 + 3y^2 - 48 = 0$

(21) **(B).** Equation of tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ is

$y = mx \pm \sqrt{2m^2+4}$ .... (1)

Equation of tangent to the parabola $y^2 = 16\sqrt{3}x$ is

$y = mx + \frac{4\sqrt{3}}{m}$ .... (2)

On comparing eq. (1) and (2),

$\frac{4\sqrt{3}}{m} = \pm\sqrt{2m^2+4}$

$\Rightarrow 48 = m^2(2m^2+4) \Rightarrow 2m^4 + 4m^2 - 48 = 0$
$\Rightarrow m^4 + 2m^2 - 24 = 0 \Rightarrow (m^2+6)(m^2-4) = 0$
$\Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

$\Rightarrow$ equation of common tangents are $y = \pm 2x \pm 2\sqrt{3}$
Statement -1 is true.
Statement-2 is obviously true.

(22) **(D).** Length of semi minor axis is = 2
Length of semi major axis is 4

then equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{4} = 1$

$x^2 + 4y^2 = 16$

(23) **(A).** a = 4, b = 3, $e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

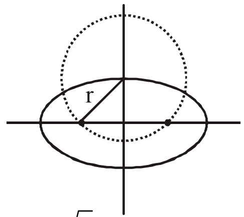
Focii is $(\pm ae, 0) \Rightarrow (\pm\sqrt{7}, 0)$

$r = \sqrt{(ae)^2 + b^2} = \sqrt{7+9} = 4$

Now, equation of circle is

$(x-0)^2 + (y-3)^2 = 16$

$x^2 + y^2 - 6y - 7 = 0$



(24) **(B).** Let common tangent $y = mx + \dfrac{\sqrt{5}}{m}$

$\dfrac{\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}} = \sqrt{\dfrac{5}{2}}$; $m\sqrt{1+m^2} = \sqrt{2}$; $m^2(1+m^2) = 2$

$m^4 + m^2 - 2 = 0$; $(m^2+2)(m^2-1) = 0$; $m = \pm 1$

$y = \pm(x + \sqrt{5})$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement-2.

(25) **(C).** Let the foot of perpendicular be P (h, k)

Equation of tangent with slope m passing P(h, k) is

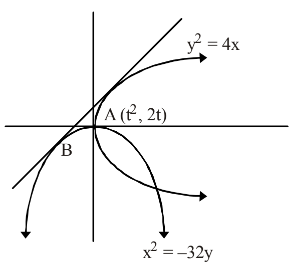
$y = mx \pm \sqrt{6m^2 + 2}$, where $m = -h/k$

$\Rightarrow \sqrt{\dfrac{6h^2}{k^2} + 2} = \dfrac{h^2 + k^2}{k}$

$6h^2 + 2k^2 = (h^2 + k^2)^2$

So required locus is $6x^2 + 2y^2 = (x^2 + y^2)^2$.

(26) **(A).**



Equation of tangent at A $(t^2, 2t)$

$yt = x + t^2$ is tangent to $x^2 + 32y = 0$ at B

$x^2 + 32\left(\dfrac{x}{t} + t\right) = 0 \Rightarrow x^2 + \dfrac{32}{t}x + 32t = 0$

$\Rightarrow \left(\dfrac{32}{t}\right)^2 - 4(32t) = 0 \Rightarrow 32\left(\dfrac{32}{t^2} - 4t\right) = 0$

$\Rightarrow t^3 = 8 \Rightarrow t = 2$

$\Rightarrow$ Slope of tangent is $\dfrac{1}{t} = \dfrac{1}{2}$

(27) **(C).** $x^2 = 8y$

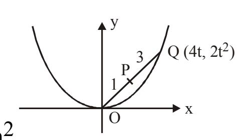
Let Q be $(4t, 2t^2)$

$\therefore$ $P = (t, t^2/2)$

Let P be (h, k)

$\therefore$ $h = t, k = t^2/2$ $\quad \therefore 2k = h^2$

$\therefore$ Locus of (h, k) is $x^2 = 2y$.



(28) **(C).** Ellipse is $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ i.e., $a^2 = 9, b^2 = 5$. So, $e = 2/3$

As, required area $= \dfrac{2a^2}{e} = \dfrac{2 \times 9}{(2/3)} = 27$

(29) **(D).** $(2t^2, 4t)$ $(0, -6)$

$F(t) = 4t^4 + (4t+6)^2 = 4(t^4 + 4t^2 + 9 + 12t)$

$F'(t) = 4(4t^3 + 8t + 12) = 0$

$\Rightarrow F'(t) = t^3 + 2t + 3 = 0$

$t = -1$: $x^2 + y^2 - 4x + 8y + 12 = 0$

(30) **(B).** $\dfrac{2b^2}{a} = 8 \Rightarrow 2b = ae$

$4b^2 = a^2e^2 \Rightarrow 4a^2(e^2 - 1) = a^2e^2 \Rightarrow 3e^2 = 4 \Rightarrow e = 2/\sqrt{3}$

(31) **(C).** $x^2 + y^2 - 8x - y - 4 = 0$; $C \equiv (4, 4)$; $r = 6$

Let centre be (x_1, y_1) ; Radius $= |y_1|$

$C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(x_1-4)^2 + (y_1-4)^2} = 6 + |y_1|$

$\Rightarrow (x_1-4)^2 + (y_1-4)^2 = 36 + y_1^2 + 12|y_1|$

$\Rightarrow x_1^2 - 8x_1 - 8y_1 - 4 = 12|y_1|$; $y_1 > 0$:

$\Rightarrow x_1^2 - 8x_1 - 8y_1 - 4 = 12y_1$

$\Rightarrow x_1^2 - 8x_1 - 4 = 20y_1 \Rightarrow (x_1-4)^2 - 20 = 20y_1$

$\Rightarrow (x_1-4)^2 = 20(y_1+1)$ Parabola

$y_1 < 0 \Rightarrow x_1^2 - 8x_1 - 8y_1 - 4 = -12y_1$

$\Rightarrow x_1^2 - 8x_1 - 4 = -4y_1$

$\Rightarrow (x_1-4)^2 = 20 - 4y_1$

$\Rightarrow (x_1-4)^2 = -4(y-5)$ Parabola

(32) **(D).** $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$; $\dfrac{a}{1/2} = 4 \Rightarrow a = 2$

$e^2 = 1 - \dfrac{b^2}{a^2} \Rightarrow b^2 = 3$; $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

Equation of normal $\dfrac{a^2x}{x_1} - \dfrac{b^2y}{y_1} = a^2 - b^2$

$\dfrac{4x}{1} - \dfrac{3y}{(3/2)} = 1$; $4x - 2y = 1$

(33) **(D).** Clearly, $ae = 2$; For $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{2}{a^2} - \dfrac{3}{a^2(e^2-1)} = 1$; $\dfrac{2}{a^2} - \dfrac{3}{a^2e^2 - a^2} = 1$

$\dfrac{2}{a^2} - \dfrac{3}{4-a^2} = 1$. Solve to get $a^2 = 1, 8$

$a^2 = 8$. Rejected as e can't be less than 1.

$a^2 = 1, b^2 = 3$

$\frac{x^2}{1} - \frac{y^2}{3} = 1$; $x(\sqrt{2}) - y\frac{(\sqrt{3})}{3} = 1$

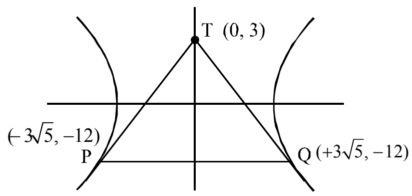
(34) **(B).** $x = \frac{y+7}{2} - 6 \Rightarrow 2x = y + 7 - 12$

$\Rightarrow 2x = y - 5 \Rightarrow 2x - y + 5 = 0$

Also, centre of the circle is (–8, –6) and the radius is $\sqrt{64+36-c}$

$\Rightarrow \left(\frac{-16+6+5}{\sqrt{5}}\right) = \sqrt{100-c} \Rightarrow \sqrt{5} = \sqrt{100-c} \Rightarrow c = 95$

(35) **(C).** Equation of PQ, $4x(0) - 3y = 36$; $y = -12$



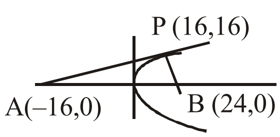
Area of $\Delta TPQ = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$

(36) **(D).** The equation of tangent at P

$y - 16 = \frac{1}{2}(x - 16) \Rightarrow A = (-16, 0)$

The normal is $y - 16 = -2(x - 16)$

$B \equiv (24, 0)$



Since $\angle APB = \pi/2$

$\therefore$ AB is the diameter.

Center of the circle $C \equiv (4, 0)$

Slope of PB $= -2 = m_1$

Slope of CP $= 4/3 = m_2$

$\Rightarrow \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| = 2$

(37) **(C).** Let equation of tangent to the parabola

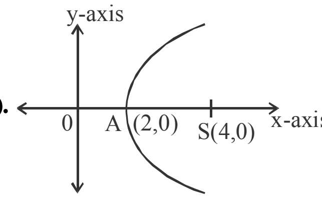
$y^2 = 4x$ is $y = mx + \frac{1}{m}$,

$\Rightarrow m^2x - ym + 1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

$\Rightarrow \frac{|3m^2+1|}{\sqrt{m^4+m^2}} = 3 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$ Tangent are $x + \sqrt{3}\,y + 3 = 0$ and $x - \sqrt{3}\,y + 3 = 0$

(38) **(C).**



Equation of parabola is $y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

(39) **(B).** $\sin^2\theta = \cos^2\theta\,(e^2 - 1)$

$\tan^2\theta = e^2 - 1$

$e = \sqrt{1 + \tan^2\theta} = \sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < 1/2$

$\Rightarrow \theta \in (60^\circ, 90^\circ)$

Now, $\ell(L \times R) = \frac{2b^2}{a} = 2\frac{(1 - \cos^2\theta)}{\cos\theta}$

$= 2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

$\ell(L \times R) \in (3, \infty)$

(40) **(B).** $4a^2 + b^2 = 8$ ....(1)

Also, $\left.\frac{dy}{dx}\right)_{(1,2)} = -\frac{4x}{y} = -2$

$\Rightarrow -\frac{4a}{b} = \frac{1}{2} \Rightarrow b = -8a \Rightarrow b^2 = 64a^2 \Rightarrow 68a^2 = 8$

$\Rightarrow a^2 = 2/17$

(41) **(A).** Let us Suppose equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through (4,6) $\Rightarrow a^2 = 4,\ b^2 = 12$

$\Rightarrow$ Equaiton of tangent $x - \frac{y}{2} = 1 \Rightarrow 2x - y - 2 = 0$

(42) **(C).** Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

By (1) and (2) $\Rightarrow x = 1$ and $y = 2$

Equation of tangent at (1, 2) to $y^2 = 4x$ is $y = x + 1$

(43) **(C).** Let equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$2a - 2b = 10$ ...(1)

$ae = 5\sqrt{3}$ ...(2)

$\frac{2b^2}{a} = ?$; $b^2 = a^2(1 - e^2)$; $b^2 = a^2 - a^2e^2$

$b^2 = a^2 - 25 \times 3$

$\Rightarrow b = 5$ and $a = 10$ $\therefore$ Length of L.R. $= \frac{2(25)}{10} = 5$

(44) **(C).** $\frac{x^2}{24}-\frac{y^2}{18}=1 \Rightarrow a=\sqrt{24}, b=\sqrt{18}$

Parametric normal : $\sqrt{24}\cos\theta \cdot x+\sqrt{18}\cdot y\cot\theta=42$

At $x=0$: $y=\frac{42}{\sqrt{18}}\tan\theta=7\sqrt{3}$ (from given equation)

$\tan\theta=\sqrt{\frac{3}{2}} \Rightarrow \sin\theta=\pm\sqrt{\frac{3}{5}}$

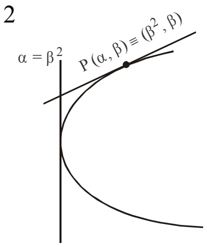
Slope of parametric normal $=\frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta}=m$

$m=-\sqrt{\frac{4}{3}}\sin\theta=-\frac{2}{\sqrt{5}}$ or $\frac{2}{\sqrt{5}}$

(45) **(C).** $T: y(\beta)=\frac{1}{2}(x+\beta^2)$

$2y\beta=x+\beta^2$

$y=\left(\frac{1}{2\beta}\right)x+\frac{\beta}{2}$; $m=\frac{1}{2\beta}$; $C=\frac{\beta}{2}$



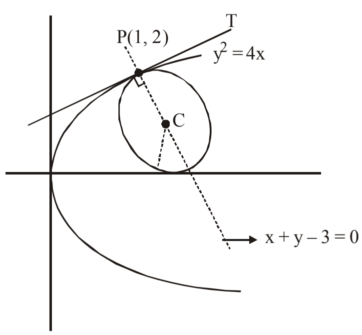
$\frac{\beta}{2}=\pm\sqrt{\frac{1}{4\beta^2}+\frac{1}{2}}$

$\frac{\beta^2}{4}=\frac{1}{4\beta^2}+\frac{1}{2}$; $\frac{\beta^2}{4}=\frac{1+2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4-2\beta^2-1=0$

$(\beta^2-1)^2=2$; $\beta^2-1=\sqrt{2}$; $\beta^2=\sqrt{2}+1$

(46) **(B).**



Equation of circle is

$(x-1)^2+(y-2)^2+\lambda(x-y+1)=0$

$\Rightarrow x^2+y^2+x(\lambda-2)+y(-4-\lambda)+(5+\lambda)=0$

As cirlce touches x axis then $g^2-c=0$

$\frac{(\lambda-2)^2}{4}=(5+\lambda)$; $\lambda^2+4-4\lambda=20+4\lambda$

$\lambda^2-8\lambda-16=0$; $\lambda=\frac{8\pm\sqrt{128}}{2}=4\pm4\sqrt{2}$

Radius $=\left|\frac{(-4-\lambda)}{2}\right|$. Put λ and get least radius.

(47) **(A).** Hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{a}{e}=\frac{4}{\sqrt{5}}$ and $\frac{16}{a^2}-\frac{12}{b^2}=1$; $a^2=\frac{16}{5}e^2$ .... (1)

$\frac{16}{a^2}-\frac{12}{a^2(e^2-1)}=1$ .... (2)

From (1) & (2), $16\left(\frac{5}{16e^2}\right)-\frac{12}{(e^2-1)}\left(\frac{5}{16e^2}\right)=1$

$\Rightarrow 4e^4-24e^2+35=0$

(48) **(A).** Tangent at (3, –9/2) ; $\frac{3x}{a^2}-\frac{9y}{2b^2}=1$

Comparing this with $x-2y=12$; $\frac{3}{a^2}=\frac{9}{4b^2}=\frac{1}{12}$

We get $a=6$ and $b=3\sqrt{3}$; $L(LR)=\frac{2b^2}{a}=9$

(49) **(C).** $3x^2+5y^2=32$

$\left.\frac{dy}{dx}\right|_{(2,2)}=-\frac{3}{5}$; Tangent : $y-2=-\frac{3}{5}(x-2) \Rightarrow Q(16/3, 0)$

Normal : $y-2=\frac{5}{3}(x-2) \Rightarrow R(4/5, 0)$

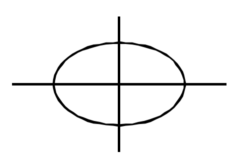
Area is $=\frac{1}{2}(QR)\times 2=QR=68/15$

(50) **(C).** $\frac{x^2}{9}-\frac{y^2}{16}=1$; $a=3, b=4$ & $e=\sqrt{1+\frac{16}{9}}=\frac{5}{3}$

Corresponding focus will be (–ae, 0) i.e., (–5, 0).

(51) **(D).** $3x^2+4y^2=12$

$\frac{x^2}{4}+\frac{y^2}{3}=1$



$x=2\cos\theta, y=\sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3}\sin\theta)$

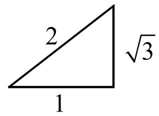
Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$2x\sin\theta-\sqrt{3}\cos\theta\ y=\sin\theta\cos\theta$

Slope $= \frac{2}{\sqrt{3}} \tan\theta = -2 \quad \therefore \tan\theta = -\sqrt{3}$

Equation of tangent is

It passes through (4, 4)



$3x\cos\theta + 2\sqrt{3}\sin\theta\, y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$

$\cos\theta = -\frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$P(-1, 3/2), Q(4, 4)\ ; PQ = \frac{5\sqrt{5}}{2}$

(52) **(A).** Equation of tangents $y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$

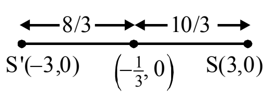
Since they are common tangent

$\frac{3}{m} = \pm\sqrt{m^2 - 8}$ $\qquad \frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^4 - 8m^2 - 9 = 0$ $\qquad e = 3$

$m = \pm 3$ $\qquad ae = 3$

$\therefore\ y = 3x + 1$, $y = -3x - 1$ $> P\left(-\frac{1}{3}, 0\right)$ $\qquad S = (3, 0)$, $S' = (-3, 0)$



(53) **(D).** Tangent to the parabola $y^2 = 16x$ is

$y = mx + \frac{4}{m}$, solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

Condition of common tangent is $D = 0$

$\therefore\ m^3 = 1 \Rightarrow m = 1$

$\therefore$ Equation of common tangent is $y = x + 4$

(54) **(D).** Given that $be = 2$ and $a = 2$ (Here $a < b$)

$a^2 = b^2(1 - e^2)\ ;\ b^2 = 8$

Equation of ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$

(55) **(C).** $y = mx + 4$ ...(i)

$y^2 = 4x$ tangent

$y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{1}{m}$ ...(ii)

From (i) and (ii), $4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$

So line $y = \frac{1}{4}x + 4$ is also tangent to

. . . . $^2 = 2by$, so solve $x^2 = 2b\left(\frac{x+16}{4}\right)$

$\Rightarrow 2x^2 - bx - 16b = 0$

$\Rightarrow D = 0 \Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128, b = 0$ (not possible)

(56) **(B).** $2ae = 6$ and $\frac{2a}{e} = 12 \Rightarrow ae = 3$ and $\frac{a}{e} = 6 \Rightarrow a^2 = 18$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

$\Rightarrow$ L.R. $= \frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}} = 3\sqrt{2}$

(57) **(C).** $3x + 4y = 12\sqrt{2}\ ;\ 4y = -3x + 12\sqrt{2}$

$y = -\frac{3}{4}x + 3\sqrt{2}$

Condition of tangency $c^2 = a^2m^2 + b^2$

$18 = a^2 \times \frac{9}{16} + 9\ ;\ a^2 \times \frac{9}{16} = 9$

$a^2 = 16\ ;\ a = 4$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4} \quad \therefore\ ae = \frac{\sqrt{7}}{4} \times 4 = \sqrt{7}$

$\therefore$ Focus are $(\pm\sqrt{7}, 0)$

$\therefore$ Distance between foci $= 2\sqrt{7}$

(58) **(B).** Let point P be $(2t, t^2)$ and Q be (h, k).

$h = \frac{2t}{3},\ k = \frac{-2 + t^2}{3}$. Hence locus is $3k + 2 = \left(\frac{3h}{2}\right)^2$

$\Rightarrow 9x^2 = 12y + 8$

(59) **(C).** Let P be (x_1, y_1)

Equation of normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$

It passes through

$\left(-\frac{1}{3\sqrt{2}}, 0\right) \Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

So, $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in I[st] quadrant)

So $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

(60) **(A).** Vertex is at $(\pm 6, 0)$ $\therefore$ $a = 6$

Let the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Putting point P (10, 16) on the hyperbola

$\frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow b^2=144$

Hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

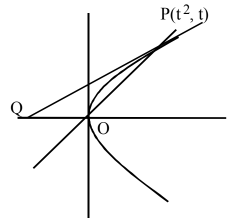
Equation of normal is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

Putting we get $2x+5y=100$

(61) **0.5** $2ty=x+t^2$

$Q(-t^2, 0)$

$\frac{1}{2}\left\|\begin{matrix}0 & 0 & 1\\ t^2 & t & 1\\ -t^2 & 0 & 1\end{matrix}\right\|=4$



$|t|^3=0$

$t=\pm 2\ (t>0)$

$m=1/2$

(62) **(D).** $e_1=\sqrt{1-\frac{4}{18}}=\frac{\sqrt{7}}{3}$; $e_2=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$

(e_1, e_2) lies on $15x^2+3y^2=k$

$15e_1^2+3e_2^2=k \Rightarrow k=16$

(63) **(B).** Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $a>b$

$2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}} \Rightarrow b^2=\frac{4}{3}$

Tangent $y=\frac{-x}{6}+\frac{4}{3}$ compare with

$y=mx\pm\sqrt{a^2m^2+b^2}$

$\Rightarrow\ m=\frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36}+\frac{4}{3}}=\frac{4}{3} \Rightarrow a=4$

$e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{2}\sqrt{\frac{11}{3}}$

(64) **(B).** $y^2=8x$

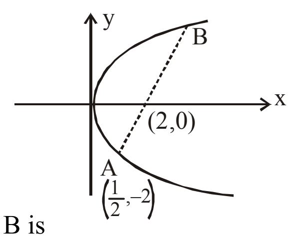
$4t_1=-2 \Rightarrow t_1=-1/2$

$t_1 . t_2=-1$

$t_2=-1/t_1 \Rightarrow t_2=2$

So coordinate of

B is (8, 8)



$\therefore$ Equation of tangent at B is

$8y=4(x+8) \Rightarrow 2y=x+8$