11

PROPERTIES OF MATTER

ELASTICITY

The three states of matter i.e. solid, liquid and gas can be explained on the basis of interatomic and intermolecular forces. A solid has a definite shape, size and volume but a liquid possesses a definite volume only and not a definite shape, whereas a gas has neither a definite shape, nor a definite volume. The three states of matter differ from each other due to the following two factors.

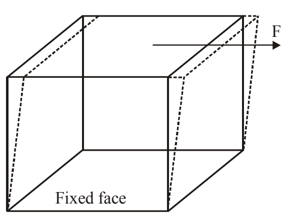
1. The different magnitude of the interatomic and intermolecular forces.
2. The extent of random thermal motion of the atoms and molecules of a substance (which depends upon temperature).

Elasticity : The property of the body to regain its original configuration (length, volume or shape) when the deforming forces are removed is called elasticity.

SOME TERMS RELATED TO ELASTICITY

(a) Deforming Force : If a force applied on a body produces a change in the normal positions of the molecules of the body, which results in a change in the configuration of the body either in length, volume or shape, then the force applied in called deforming force. Thus a deforming force is one which when applied changes the configuration on the body.

(b) Perfectly Elastic Body : A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body, quartz and phosphor bronze force, however small the deforming force may be is called perfectly plastic body.



(c) Perfectly Plastic Body : A body which does not regain its original configuration at all on the removal of deforming force, however small the deforming force may be is called perfectly plastic body.

STRESS

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play, which tends to bring the body back to its initial configuration. This internal restoring force acting per unit area of a deformed body is called stress i.e., Stress = Restoring force/ area

Example 1 :

A human bone is subjected to a compressive force of 5.0×10^5 N/m^2. The bone is 25 cm long and has an approximate cross sectional area 4.0 cm^2. If the ultimate compressive strength of the bone is 1.70×10^8 N/m^2, will the bone be compressed or will it break under this force ?

Sol. $\text{Stress} = \frac{F}{A} = \frac{5 \times 10^5}{4 \times 10^{-4}} = 12.5 \times 10^8 \text{ N/m}^2$

Since this stress exceeds the ultimate compressive stress of the lone, the bone will break.

STRAIN

When a deforming force is applied on a body, there is a change in the configuration of the body. The body is said to be strained or deformed. The ratio of change in configuration to the original configuration is called strain.

i.e. $\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$

Strain being the ratio of two like quantities has no units and dimensions.

Example 2 :

A brass wire 0.75 cm long is stretched by 0.001 cm. Find the strain of the wire.

Sol. $\text{Strain} = \Delta L/L = 0.001/0.75 = 1.33 \times 10^{-3}$

ELASTIC LIMIT

Elastic limit is the upper limit of deforming force up to which, f deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body loses its property of elasticity and gets permanently deformed.

Example 3 :

The elastic limit of brass is 3.5×10^{10} N/m^2. Find the maximum load that can be applied to a brass wire of 0.75 mm diameter without exceeding the elastic limit.

Sol. $\text{Stress} = \frac{F}{A}$; for elastic limit, stress $= 3.5 \times 10^{10}$ N/m^2

Thus $F = (\pi r^2) \times \text{stress}$

$$= 3.14 \times \left(\frac{75}{2} \times 10^{-3}\right)^2 \times 3.5 \times 10^{10}$$

$$= 3.14 \times \frac{(0.75)^2}{4} \times 3.5 \times 10^4 = 1.55 \times 10^4 \text{ N}$$

HOOKE'S LAW

Hooke's law states that the extension produced in the wire is directly proportional to the load applied with in elastic limit i.e. with in elastic limit, extension $\propto$ load applied.

MODULUS OF ELASTICITY

According to Hook's law, within elastic limit,

stress $\propto$ strain

or stress = E × strain or $\frac{\text{stress}}{\text{strain}} = E = \text{a constant}$,

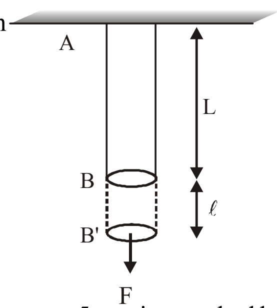
Thus modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of the stress to the corresponding strain produced, within the elastic limit.

TYPES OF MODULUS OF ELASTICITY

Corresponding to three types of strain, there are three types of modulus of elasticity as described below:

(a) Young's Modulus of elasticity (Y).

It is defined as the ratio of normal stress to the longitudinal strain with in the elastic limit.



Thus $Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$

$$\therefore \quad Y = \frac{F/\pi r^2}{\ell/L} = \frac{MgL}{\pi r^2 \ell}$$

Example 4 :

A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel of wire is 2.4×10^{12} dyne/cm^2.

Sol. Here, $\ell = 4\,\text{m} = 400$ cm, $2r = 5$ mm or $r = 2.5$ mm $= 0.25$ cm

$f = 5$ kg-wt $= 5000$ g-wt $= 5000 \times 980$ dyne

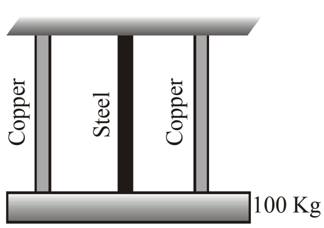
$\Delta\ell = ?$, $Y = 2.4 \times 10^{-12}$ dyne/ cm^2

As $Y = \frac{F}{\pi r^2} \times \frac{\ell}{\Delta\ell}$

$$\Delta\ell = \frac{F\ell}{\pi r^2 y} = \frac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}} = 0.0041 \text{ cm.}$$

Example 5 :

A heavy plank of mass 100 kg hangs on three vertical wires of equal length arranged symmetrically (see figure). All the wires have the same cross section. The middle wire is of steel and the other two are of copper. The modulus of elasticity of steel is assumed to be double that of copper. Determine the tensions in the wire.



Sol. Let tension in copper and steel wires are T_{cu} and T_{st}. Assuming same strain (due to heavy plank)

$$Y_{steel} = \frac{T_{st}L}{A(\Delta L)} \text{ and } Y_{cu} = \frac{T_{cu}L}{A(\Delta L)} \text{ or } \frac{T_{st}}{T_{cu}} = \frac{Y_{steel}}{Y_{cu}} = 2$$

or $T_{st} = 2T_{cu}$

Further, $T_{st} + 2T_{cu} = 100$ g (given)

or $2T_{st} = 2T_{cu} = 100$ g or $T_{cu} = 25$ g

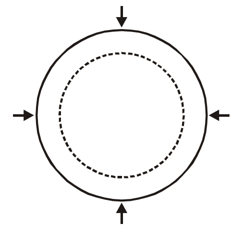
And therefore, $T_{st} = 2T_{cu} = 50$ g

Thus T_{cu} is 25 kgf T_{steel} is 50 kg f,

that is $T_{cu} = 245$ N and T_{steel} is 490 N.

(b) Bulk Modulus of elasticity (K).

It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit.



Thus, $K = \frac{\text{normal stress}}{\text{volumetric strain}}$

$$\therefore \quad K = \frac{F/a}{\Delta v/v} = \frac{Fv}{a\,\Delta v}$$

If p is the increase in pressure applied on the spherical body then F/a = P

Example 6 :

A liter of glycerine contracts 0.21 cm^3/Nm^2, What is the bulk modulus of glycerine ?

Sol. Bulk modulus $= \frac{p}{\left(\frac{-\Delta V}{V}\right)}$

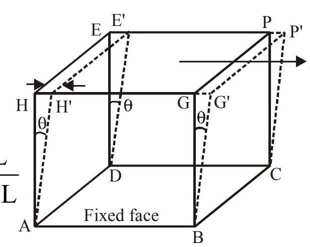
$-\Delta V = 0.21\ \text{cm}^3 = 0.21 \times 10^{-6}\ \text{m}^3$; $V = 1$ litre $= 10^{-3}\ \text{m}^3$

$$K = \frac{9.8 \times 10^5}{0.21 \times 10^{-6}} \times 10^{-3}$$

$= 4.7 \times 10^9$ N/m^2 N/m^2 $= 0.47 \times 10^{10}$ N/m^2

(c) Modulus of rigidity (η)

It is defined as the ratio of tangential stress to the shearing strain, with the elastic limit. It is also called shear modulus of rigidity.



$$\eta = \frac{\text{tangential stress}}{\text{shearing strain}}$$

$$\therefore \quad \eta = \frac{F/a}{\theta} \Rightarrow \frac{F}{a\theta} = \frac{FL}{a\Delta L}$$

Example 7 :

A copper block, 7.50 cm on a side, is subjected to a tangential force of 3.5×10^3 N. Find the angle of shear

Sol. The tangential stress $= \frac{F}{A} = \frac{3.5 \times 10^3}{(7.5 \times 10^{-2})^2}$

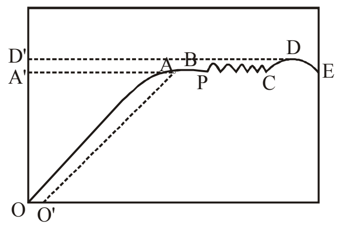
The shear modulus (or modulus of rigidity) of copper is

$\eta = 4.2 \times 10^{10}$ N/m^2.

Angle of shear = Shearing strain

$$= \frac{\text{stress}}{\eta} = \frac{3.5 \times 10^3}{(7.5)^2 \times 10^{-4} \times 4.2 \times 10^{10}} = 1.48 \times 10^{-5}\ \text{rad}$$

STRESS STRAIN RELATIONSHIP IN A WIRE



AO = elastic range, P = Yield point, OD' = breaking stress or tensile stress, E = breaking point, OO_1 = Permanent set.

WORK DONE IN A STRETCHED WIRE

Elastic potential energy U stored in the wire.

$\therefore\quad U = \frac{1}{2} F \times \ell = \frac{1}{2}$ (stress) × (strain) × volume of the wire

$\therefore\quad$ elastic potential energy per unit volume of the wire

$$U = \frac{1}{2}\ (\text{stress}) \times (\text{strain})$$

$$= \frac{1}{2}\ (\text{Young's modulus} \times \text{strain}) \times \text{strain}$$

($\because$ Young's modulus = stress/strain)

$$\therefore\quad U = \frac{1}{2}\ (\text{Young's modulus}) \times (\text{strain})^2$$

Example 8 :

A steel wire of 4.0 m in length is stretched through 2.0 mm. The cross-sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is $2.0 \times 10^{11}\ N/m^2$ find (i) the energy density of wire (ii) the elastic potential energy stored in the wire.

Sol. Here, $\ell = 4.0$ m

$\Delta\ell = 2 \times 10^{-3}$ m; $a = 2.0 \times 10^{-6}\ m^2$; $y = 2.0 \times 10^{11}\ N/m^2$

(i) The energy density of stretched wire

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times y \times (\text{strain})^2$$

$= (1/2) \times 2.0 \times 10^{11} \times (2 \times 10^{-3})/4)^2$

$= 0.25 \times 10^5 = 2.5 \times 10^4\ J/m^3$.

(ii) Elastic potential energy = energy density × volume

$= 2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0\ J = 20 \times 10^{-2} = 0.20$ J

POISSON'S RATIO

$$\sigma = \frac{\text{lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta R / R}{\ell / L}$$

Poisson's Ratio is a dimensional less and unit less quantity.

Theoretical value of Poisson's Ratio : $-1 < \sigma < 1/2$

Practical value of Poisson's Ratio : $\sigma < 1/2$

Example 9 :

The Poisson's ratio for a material is 0.1. If longitudinal strain of a rod of this material is 1×10^{-1}, then the percentage change in the volume of the rod will be (calculate).

Sol. For a rod $V = \pi r^2 L$

Therefore, $\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$. Now $s = 0.1$; $\frac{\Delta L}{L} = 10^{-3}$

Thus from definition of Poisson's ratio,

$$s = \frac{(-\Delta r / r)}{\Delta L / L} \quad \text{or} \quad \frac{\Delta V}{V} = -0.1 \times 10^{-3}$$

$$\text{Thus } \frac{\Delta V}{V} = -0.2 \times 10^{-3} + 1 \times 10^{-3} = 0.8 \times 10^{-3}$$

or percentage change = 0.8 × 100 = 0.08%

THERMAL STRESS

When a rods is rigidly fixed at its two ends and its temperature is changed, then a thermal stress is set up in the rod, which is given by

$$\text{Thermal stress} = \frac{\text{force}}{\text{area of cross section}} = \frac{F}{A} = Y\,\alpha\,\Delta\theta.$$

where α = coefficient of linear expansion of the rod and $\Delta\theta$ = change in temperature.

Example 10 :

Two rods of different materials having coefficients of linear expansions α_1 and α_2 and Young's modulus of elasticity Y_1 and Y_2 respectively, are fixed between two rigid massive walls. The rods area heated such that they undergo the same increase in temperature without bending. If $\alpha_1 : \alpha_2 = 2 : 3$, then the thermal stress developed in the two rods will be equal if $Y_1 : Y_2$ is equal to -

(1) 2 : 3 (2) 3 : 2

(3) 1 : 1 (4) 4 : 9

Sol. **(2).** The thermal stress in a rod is stress = $Y\,\alpha\,(\Delta T)$

Thus for same rise of temperature (ΔT), the thermal stress will be equal if $Y_1\alpha_1 = Y_2\alpha_2$ or $Y_1 : Y_2 = \alpha_2 : \alpha_1 = 3 : 2$

INTERATOMIC FORCE CONSTANT

$k = Yr_0$, where r_0 = normal distance between the atoms

Example 11 :

If the interatomic spacing in a steel wire is 3.0 A° and $Y_{steel} = 20 \times 10^{10}\ N/m^2$, then find (i) Interatomic force constant in Newton/A°, and (ii) Increase in interatomic spacing for a stress of $2 \times 10\ N/m^2$.

Sol. (i) The interatomic force constant

$k = Yr_0 = 20 \times 10^{10} \times 10^{-10}$ N/m = 60 N/m

$$= \frac{60}{10^{10}\ \text{Å}} = 6 \times 10^{-9}\ \text{N/Å}$$

(ii) Interatomic force

$F = \text{stress} \times r_0^2 = 2 \times 10^9 \times (3 \times 10^{-10})^2$

Using $F = k\Delta r$, we get increase in interatomic spacing

$\Delta r = F/k = 3 \times 10^{-12}$ m = 0.03 Å

RELATION BETWEEN Y, K, η AND σ

(i) $Y = 3K(1-2\sigma)$ (ii) $Y = 2\eta(1+\sigma)$

(iii) $\sigma = \frac{3K-2\eta}{2\eta+6K}$ (iv) $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$

RELATION BETWEEN ANGLE OF SHEAR AND ANGLE OF TWIST

In case of a rod of length ℓ and radius r fixed at one end, angle of shear ϕ is related the to angle of twist θ by the relation, $r\theta = \ell\phi$

Example 12 :

The radii of two rods of the same length and same material are in the ratio $r_1 : r_2$. If these rods are twisted by applying the same torsional torque, then the ratio of the angle of twist produced in the two rods will be -

Sol. Torsional torque required for θ twist is

$$\tau = C\theta = \frac{\eta \pi r^4}{2\ell}\theta \text{ Thus for same } \tau, \ell \text{ and } \eta,\ \theta \propto \frac{1}{r^4}$$

$$\therefore\ \theta_1 : \theta_2 = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{2}{1}\right)^4 = 16:1$$

APPLICATIONS OF ELASTICITY

(1) The metallic parts of the machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed.

(2) The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

(3) The bridges are designed in such a way that they do not bend much or break under the load of heavy traffic, force of strongly blowing wind and its own weight. So they are made in shape of I.

TRY IT YOURSELF-1

Q.1 A material breaks up under a stress of 20×10^5 N/m^2. If the density of the material is 2.5×10^3 kg/m^3, calculate the length may break under its own weight (take g = 10 m/s^2).

Q.2 Modulus of rigidity of ideal liquids is
(A) infinity. (B) zero. (C) unity.
(D) some finite small non-zero constant value.

Q.3 The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will
(A) be double (B) be half
(C) be four times (D) remain same

Q.4 The temperature of a wire is doubled. The Young's modulus of elasticity
(A) will also double. (B) will become four times.
(C) will remain same. (D) will decrease.

Q.5 Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.
(A) Both the rods will elongate but there shall be no perceptible change in shape.
(B) The steel rod will elongate and change shape but the rubber rod will only elongate.
(C) The steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse.
(D) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.

Q.6 A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.
(A) Tensile stress at any cross section A of the wire is F/A.
(B) Tensile stress at any cross section is zero.
(C) Tensile stress at any cross section A of the wire is 2F/A.
(D) Tension at any cross section A of the wire is F.

Q.7 For an ideal liquid
(A) the bulk modulus is infinite.
(B) the bulk modulus is zero.
(C) the shear modulus is infinite.
(D) the shear modulus is zero.

Q.8 A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1cm. The two wires will have
(A) the same stress. (B) different stress.
(C) the same strain. (D) different strain.

Q.9 Is stress a vector quantity?

Q.10 What is the Young's modulus for a perfect rigid body?

ANSWERS

(1) 80 meter.	**(2)** (B)	**(3)** (D)
(4) (D)	**(5)** (D)	**(6)** (AD)
(7) (AD)	**(8)** (AD)	**(9)** No
(10) Infinite		

SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called surface tension.

INTERMOLECULAR FORCES

The force which acts between the atoms or the molecules of different substances is called intermolecular force. This force is of two types.

(a) **Cohesive force :** The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) **Adhesive force :** The force acting between different types of molecules or molecules of different substance is called adhesive force.

Intermolecular forces are different from the gravitational forces not obey the inverse-square law.

The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases. Molecular range depends on the nature of the substance.

Examples:

1. Water wets glass surface but mercury does not. Because when water comes in contact with glass the adhesive force acts between water and glass molecules. As adhesive force is greater than the cohesive force of water molecules, the water molecules, cling with glass surface and surface become wet. In case of mercury adhesive force is less than that of cohesive force and Hg-molecules do not cling with glass surface and surface does not wet with Hg-molecules.
2. **Oil-water:** Since Cohesive force of water >Adhesive force oil-water> Cohesive force of oil.
(i) If water drop is poured on the surface of oil, it contracts in the form of globule.
(ii) If oil drop is poured on the surface of water it spreads to a larger area in the form of thin film.
3. **Ink-paper :** Since adhesive force between ink-paper> cohesive force on ink, the ink sticks to the paper.

EXPLANATION OF SURFACE TENSION

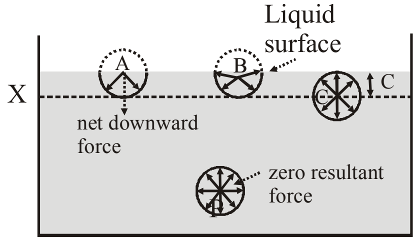
Laplace firstly explained the phenomenon of surface tension on the basis of intermolecular forces. According to him surface tension is a molecular phenomenon and its root causes are electromagnetic forces. He explained the cause of surface tension as :

If the distance between two molecules is less than the molecular range C ($\approx 10^{-9}$ m) then they attract each other, but if the distance is more than this then the attraction becomes negligible.

If a sphere of radius C with a molecule at centre is drawn, then only those molecules which are enclosed within this sphere can attract or be attracted by the molecule at the centre of the sphere. This sphere is called sphere of molecular activity or sphere of influence.

In order to understand the tension acting at the free surface of liquid, let us consider four liquid molecules like A, B, C and D along with their spheres of molecular activity.

(a) According to figure D sphere is completely inside liquid. So molecule is attracted equally in all directions and hence resultant force is equal to zero.



(b) According to figure sphere of molecule C is just below the liquid surface. So resultant force is equal to zero.

(c) The molecule B which is a little below the liquid surface is more attracted downwards due to excess of molecules downwards. Hence the resultant force is acting downwards.

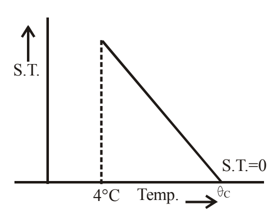
(d) Molecule A is situated at surface so that its sphere of molecular activity is half outside the liquid and half inside. Only down portion has liquid molecules. Hence it experiences a maximum downward force. Thus all the molecules situated between the surface and a plane XY, distant C below the surface, experience a resultant downward cohesive force.

When the surface area of liquid is increased molecules from the interior of the liquid rise to the surface. As these molecules reach near the surface, work is done against the downward cohesive force. This work is stored in the molecules in the form of potential energy. Thus the potential energy of the molecules lying in the surface is greater than that of the molecules in the interior of the liquid. A system is in stable equilibrium when its potential energy is minimum.

Hence in order to have minimum potential energy the liquid surface tends to have minimum number of molecules in it. In other words the surfaces tends to contract to a minimum possible area. This tendency is exhibited as surface tension.

DEPENDENCY OF SURFACE TENSION

1. **On Cohesive Force:** Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.
2. **On Impurities:** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g. (a) On mixing detergent in water its surface tension decreases. (b) Surface tension of water is more than (alcohol + water) mixture.
3. **On Temperature:** On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero. Surface tension of water is maximum at 4°C.

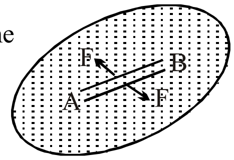


4. **On Contamination :** The dust particles or lubricating materials on the liquid surface decreases its surface tension.
5. **On Electrification :** The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.

DEFINITION OF SURFACE TENSION

Surface tension can be defined in the form of an imaginary line on the surface or by relating it to the work done. The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension. Let an imaginary line AB be drawn in any direction on a liquid surface.

The surface on either side of this line exerts a pulling force, which is perpendicular to line AB. If force is F and length of AB is L then $T = \dfrac{F}{L}$



SI units : N/m and J/m^2 ; CGS units : dyne/cm and erg/cm^2

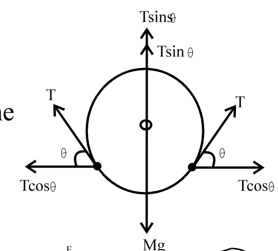
Dimensions : $M^1L^0T^{-2}$

Examples :

1. When any needle floats on the liquid surface then

$2T\ell \sin\theta = mg$

Ex. A mosquito sitting on a liquid surface.

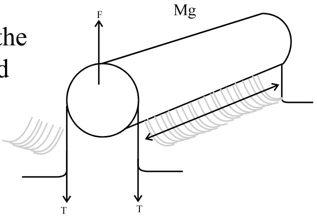


2. If the needle is lifted from the liquid surface then required excess force will be

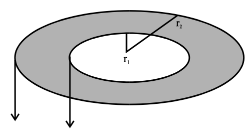
$F_{excess} = 2T\ell$

Minimum force required

$F_{min} = mg + 2T\ell$



3. Required excess force for a circular thick ring (or hollow disc) having internal and external radii r_1 & r_2 is dipped in and taken out from liquid

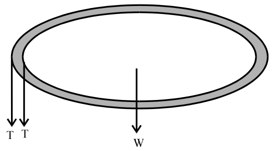


$$F_{excess} = F_1 + F_2 = T(2\pi r_1) + T(2\pi r_2)$$

$$= 2\pi T(r_1 + r_2)$$

4. Required excess force for a ring $(r_1 = r_2 = r)$

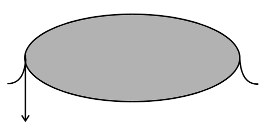
$F_{excess} = 2\pi T(r + r) = 4\pi rT$



5. Required excess force for a circular disc

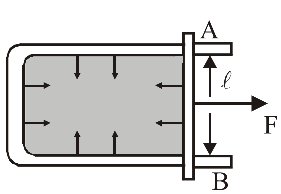
$(r_1 = 0,\ r_2 = r)$

$F_{excess} = 2\pi rT$



SURFACE ENERGY

According to molecular theory of surface tension the molecule in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called Surface energy.



Let a liquid film be formed on a wire frame and a straight wire of length ℓ can slide on this wire frame as shown in figure. The film has two surfaces and both the surface are in contact with the sliding wire and hence, exert force of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T\ell$. Thus, net force on the wire due to both the surface is $2T\ell$. Apply an external force F equal and opposite to keep the wire in equilibrium. Thus, $F = 2T\ell$

Now, suppose the wire is moved through a small distance dx, the work done by the force is, $dW = F\,dx = (2T\ell)\,dx$

But $(2\ell)(dx)$ is the total increase in area of both the surface of the film. Let it be dA. Then, $dW = T\,dA \Rightarrow T = \dfrac{dW}{dA}$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the new surface. $T = \dfrac{dU}{dA}$ [as $dW = dU$]

Special Cases

* Work done (surface energy) in formation of a drop of radius r = Work done against surface tension

W = Surface tension T × change in area

$\Delta A = T \times 4\pi r^2 = 4\pi r^2 T$

* Work done (surface energy) in formation of a soap bubble of radius r :

$W = T \times \Delta A = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$

[$\because$ soap bubble has two surfaces]

Example 13 :

The length of a needle floating on water is 2.5 cm. Calculate the added force required to pull the needle out of water. [$T = 7.2 \times 10^{-2}$ N/m]

Sol. The force of surface tension $F = T \times 2\ell$

($\because$ Two free surfaces are there)

$\Rightarrow\ F = 7.2 \times 10^{-2} \times 2 \times 2.5 \times 10^{-2} = 3.6 \times 10^{-3}$ N

Example 14 :

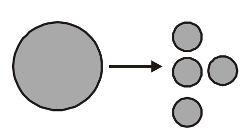
A paper disc of radius R from which a hole of radius r is cut out, is floating in a liquid of surface tension, T. What will be force on the disc due to surface tension?

Sol. $T = \dfrac{F}{L} = \dfrac{F}{2\pi(R + r)} \quad \therefore F = 2\pi(R + r)T$

SPLITTING OF BIGGER DROP INTO SMALLER DROPLETS

If bigger drop is splitted into smaller droplets then in this process volume of liquid always remain conserved. Let bigger drop has radius R. It is splitted into n smaller drops of radius r then by conservation of volume

(i) $\dfrac{4}{3}\pi R^3 = n\left[\dfrac{4}{3}\pi r^3\right]$



$$\Rightarrow \quad n = \left[\frac{R}{r}\right]^3 \Rightarrow r = \frac{R}{n^{1/3}}$$

(ii) Initial surface area = $4\pi R^2$ and final surface area = $n(4\pi r^2)$
Therefore initial surface energy $E_i = 4\pi r^2 T$
and final surface energy $E_f = n(4\pi r^2 T)$
Change in area $\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi(nr^2 - R^2)$
Therefore the amount of surface energy absorbed i.e.
$\Delta E = E_f - E_i = 4\pi T(nr^2 - R^2)$
$\therefore$ Magnitude of work done against surface tension i.e.
$W = 4\pi(nr^2 - R^2)T = 4\pi T(nr^2 - R^2) = 4\pi R^2 T(n^{1/3} - 1)$

$$= 4\pi R^2 T\left[\frac{R}{r} - 1\right] = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

In this process temperature of system decreases as energy gets absorbed in increasing surface area.

$$W = Jms\,\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow \quad \Delta\theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

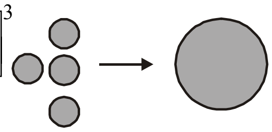
where ρ = liquid density, s = liquid's specific heat
Thus in this process area increasing, surface energy increasing, internal energy decreasing, temperature decreasing, and energy is absorbed.

FORMATION OF BIGGER DROP BY A NUMBER OF SMALLER DROPLETS

When n smaller droplets each of radius r are combined to form a bigger drop of radius R, then

(i) Volume of bigger drop = volume of n smaller droplets

$$\Rightarrow \frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow n = \left[\frac{R}{r}\right]^3$$



$$\Rightarrow R = n^{1/3} r$$

(ii) Energy is released because the surface area and hence the surface energy reduces.
Released energy = $4\pi T(nr^2 - R^2)$. This is equal to work done i.e. $W = 4\pi T(nr^2 - R^2)$

(iii) Increase in temperature of the bigger drop

$$\Delta\theta = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

where ρ = density of liquid, s = specific heat
In this process area decreases , surface energy decreases , internal energy increases , temperature increases and energy is released.

Example 15 :
A big drop is formed by coalescing 1000 small droplets of water .What will be the change in surface energy. What will be the ratio between the total surface energy of the droplets and the surface energy of the big drop?

Sol. By conservation of volume

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{10}$$

Surface energy of 1000 droplets

$$= 1000 \times T \times 4\pi\left[\frac{R}{10}\right]^2 = 10(T \times 4\pi R^2)$$

Surface energy of the big drop = $T \times 4\pi R^2$
Surface energy will decrease in the process of formation of bigger drop, hence energy is released and temperature increases.

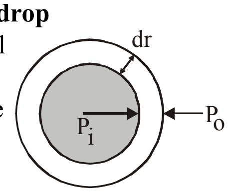
$$\therefore \quad \frac{\text{total energy of 1000 droplets}}{\text{surface energy of big drop}} = \frac{10(T \times 4\pi R^2)}{T \times 4\pi R^2} = \frac{10}{1}$$

EXCESS PRESSURE INSIDE A CURVED LIQUID SURFACE

The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore pressure difference exists across two sides of a curved surface. This pressure difference is called excess pressure.

Case I : Excess pressure inside the drop

Let a drop of radius r having internal and external pressure P_i and P_0 respectively, so that excess pressure
$P_{ex} = (P_i - P_0)$

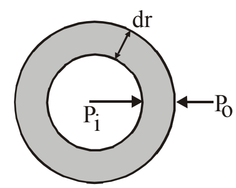


If the radius of the drop is changed from r to (r + dr) then
Work done = F dr = (PA) dr = P . $4\pi r^2$ dr
Change in surface area == $4\pi(r + dr)^2 - 4\pi r^2 = 8\pi r\,dr$
So by definition of surface energy

$$T = \frac{W}{\Delta A} = \frac{4\pi r^2 P dr}{8\pi r dr} \quad \Rightarrow \quad P_{ex} = (P_i - P_0) = \frac{2T}{r}$$

Case II : Excess pressure inside soap bubble:

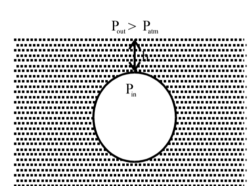
Since the soap bubble has two surfaces. The excess pressure will get double as compared to a drop



$$P_i - P' = \frac{2T}{r} \quad P' - P_0 = \frac{2T}{r}$$

$$\Rightarrow \quad \text{excess pressure} = P_i - P_0 = \frac{4T}{r}$$

Case III : Excess Pressure Inside the cavity or air bubble in liquid :



$$P_{excess} = P_{in} - P_{out} = hdg + \frac{2T}{R}$$

$$\Rightarrow \quad P_{inside} = P_{atm} + hdg + \frac{2T}{R}$$

Example 16 :

Calculate the excess pressure within a bubble of air of radius 0.1 mm in water. If the bubble had been formed 10 cm below the water surface on a day when the atmospheric pressure was 1.013×10^5 Pa, then what would have been the total pressure inside the bubble?

Surface tension of water = 73×10^{-3} N/m

Sol. Excess pressure $P_{excess} = \frac{2T}{r} = \frac{2 \times 73 \times 10^{-3}}{0.1 \times 10^{-3}} = 1460$ Pa

The pressure at a depth d, in liquid is P = hdg. Therefore, the total pressure inside the air bubble is

$$P_{in} = P_{atm} + hdg + \frac{2T}{r}$$

$$= (1.013 \times 10^5) + (10 \times 10^{-2} \times 10^3 \times 9.8) + 1460$$

$$= 101300 + 980 + 1460 = 103740 = 1.037 \times 10^5 \text{ Pa}$$

ANGLE OF CONTACT (θ)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact.

The angle of contact depends the nature of the solid and liquid in contact.

Effect of Temperature on angle of contact: On increasing θ_C increases $\left[\because \cos\theta_C \propto \frac{1}{T}\right]$ and θ_C decrease. So on increasing temperature, θ_C decreases.

Effect of Impurities on angle of contact: (a) Solute impurities increase surface tension, so cos θ_C decreases and angle of contact increases. (b) Partially solute impurities decrease surface tension, so angle of contact θ_C decreases.

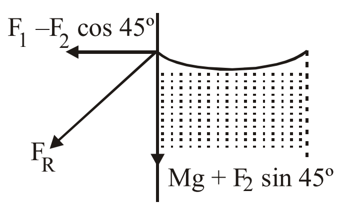
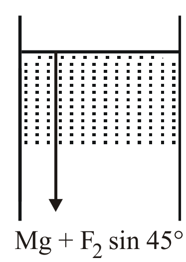
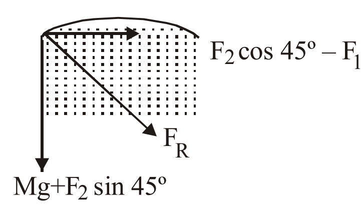
Effect of Water Proofing Agent : Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

Table of angle of contact of various solid-liquid pairs

Solid - Liquid Pair	**θ°C**
Glass -Normal water	8°
Glass -Distilled water	0°
Glass - Alcohol	0°
Glass - Mercury	135°
Paraffin wax - Water	108°
Silver - Water	90°

Shape of Liquid Surface : When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as meniscus. The cohesive force acts at an angle 45° from liquid surface whereas the adhesive force acts at right angles to the solid surface. The relation between the shape of liquid surface, cohesive/adhesive forces, angle of contact, etc are summarised in the table below :

Relation between cohesive and adhesive force	 $F_A > \frac{F_C}{\sqrt{2}}$	 $F_A = \frac{F_C}{\sqrt{2}}$	 $F_A < \frac{F_C}{\sqrt{2}}$
Shape of meniscus	concave	Plane	Convex
Angle of contact	$\theta_C < 90°$ (Acute angle)	$\theta_C = 90°$ (Right angle)	$\theta_C > 90°$ (Obtuse angle)
Level of liquid	Liquid rises up	Liquid neither rises nor falls	Liquid falls
Wetting property	Liquids wets the solid surface	Liquid does not wet the solid surface	Liquid does not wet the solid surface
Example	Glass-Water	Silver-water	Glass-mercury

CAPILLARY TUBE

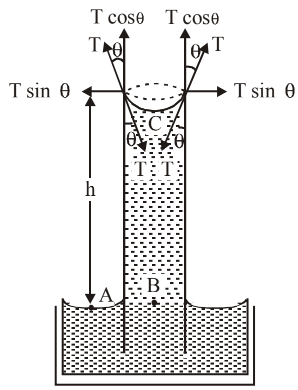
A glass tube with fine bore and open at both ends is known as capillary tube. The property by virtue of which a liquid rise or depress in a capillary tube is known as capillarity.

Rise or fall of liquid in tubes of narrow bore (capillary tube) is called capillary action.

To calculate capillary height :

(a) Pressure balance method:

When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. The liquid continues to rise in the capillary tube until the weight of the liquid column becomes equal to force due to surface tension.



Let the radius of the meniscus is R and the radius of the capillary tube is r. The angle of contact is S, surface tension is T, density of liquid is ρ and the liquid rises to a height h. Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$P_A = P_B \Rightarrow P_A = P_C + \rho gh$

$\Rightarrow P_A - P_C = \rho\, gh \qquad (\because P_B = P_C + \rho gh)$

Now, the point C is on the curved meniscus which has P_A and P_C as the two pressures on its concave and convex sides respectively.

$$\Rightarrow P_A - P_C = \frac{2T}{R} = \frac{2T}{r/\cos\theta}$$

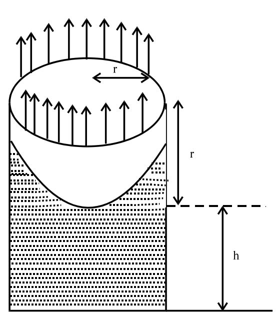
$$\Rightarrow \frac{2T}{r/\cos\theta} = \rho gh \Rightarrow 2T\cos\theta = r\rho\, gh \;\Rightarrow h = \frac{2T\cos\theta}{r\rho g}$$

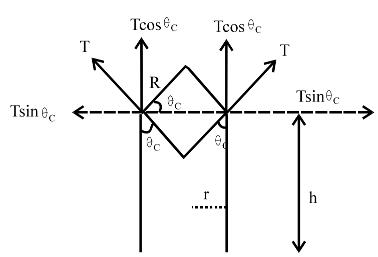
Zurin's Law : The height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube, if T, θ, ρ and g are constant $h \propto \frac{1}{r}$ or rh=const. It implies that liquid will rise more in capillary tube of less radius & vice versa.

(b) Force balance method:

Force due to surface tension

$F_{ST} = (2\pi r)\, T\cos\theta_C$





real volume of water column

$$V_{real} = \pi r^2(h+r) - \frac{2\pi}{3}r^3 \quad = \pi r^2\left[h + r - \frac{2}{3}r\right]$$

$$V_{real} = \pi r^2\left(h + \frac{r}{3}\right)$$

In Balance condition:

Weight of water column = Force due to ST.

$\Rightarrow V_R.d.g. = F_{ST}$

$\pi r^2 (h + r/3)\, dg = (2\pi r)\, T\cos\theta_C$

$$\Rightarrow T = \frac{r\left(h + \frac{r}{3}\right) dg}{2\cos\theta_C}$$

where r /3 = Capillarity correction

Jeggers' Method of determining surface tension :

Surface tension is calculated by

$$T = \frac{rg}{2}(H\rho - hd)$$

H = difference of liquid levels in manometer

ρ = Density of liquid in manometer

h = length of immersed portion of capillary

d = density of given liquid

r = radius of capillary

Jaeger's method is better than the capillary rise method because :

(i) Bubble is formed inside the liquid, so contamination due to surface impurities is minimized. With every new bubble, a new surface is formed.

(ii) There is no need of knowing the angle of contact.

(iii) Variation of surface tension with temperature can be studied easily.

(iv) Variation of surface tension with the addition of soluble impurity can be studied for different concentrations of impurity.

Example 17 :

Calculate the height to which water will rise in a capillary tube of diameter 1×10^{-3} m.

[Given: surface tension of water is 0.072 N m^{-1}, angle of contact is 0°, g = 9.8 m s^{-2} and density of water = 1000 kg m^{-3}]

Sol. Height of capillary rise

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.072 \times \cos 0°}{5 \times 10^{-4} \times 1000 \times 9.8} \text{m} = 2.94 \times 10^{-2}\,\text{m}$$

TRY IT YOURSELF-2

Q.1 The angle of contact at the interface of water-glass is 0°, Ethylalcohol-glass is 0°, Mercury-glass is 140° and Methyliodide glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is

(A) water (B) ethylalcohol

(C) mercury (D) methyliodide.

Q.2 For a surface molecule

(A) the net force on it is zero.

(B) there is a net downward force.

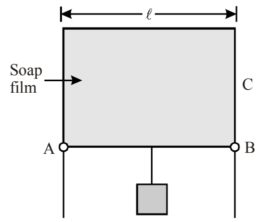
(C) potential energy is less than that of a molecule inside.

(D)potential energy is more than that of a molecule inside.

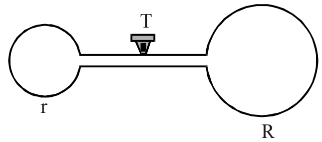
Q.3 Is surface tension a vector?

Q.4 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^5 Pa.

Q.5 A film of soap solution (surface tension T) is formed on a wire frame C, between C and straight wire AB which is free to slide along the frame. What should be the value of mass m, for AB to remain in equilibrium in the position shown ?



Q.6 Two soap bubbles of different size are formed at two ends of a tube as shown in fig. If the stop cock is opened then which bulb shinks ?



Q.7 Two circular plates of radius 5 cm each, have a 0.01 mm thick water film between them. Then the force required to separate these plate (S.T. of water = 73 dyne/cm). ?

Q.8 An air bubble of radius r in water is at a depth h below the water surface at some instant. If P is atmospheric pressure and d and T are the density and surface tension of water respectively, the pressure inside the bubble will be :

(A) $P + hdg - \frac{4T}{r}$ (B) $P + hdg + \frac{2T}{r}$

(C) $P + hdg - \frac{2T}{r}$ (D) $P + hdg + \frac{4T}{r}$

Q.9 A capillary tube of radius 0.20 mm is dipped vertically in water. The height of the water column raised in the tube, will be (surface tension of water = 0.075 N/m and density of water = 1000 kg/m^3.
Taking g = 10 m/s² & contact angle 0°).
(A) 7.5 cm. (B) 6 cm.
(C) 5 cm. (D) 3 cm.

Q.10 Water rises in a capillary tube to a height of 2.0 cm. In another capillary tube whose radius is one third of it, how much the water will rise ?
(A) 5 cm (B) 3 cm
(C) 6 cm (D) 9 cm

ANSWERS

(1) (C) **(2)** (AD) **(3)** No

(4) 1.0131×10^5 Nm^{-2}. **(5)** $\frac{T(2\ell)}{g}$

(6) smaller bubble shrinks **(7)** 115 newton

(8) (B) **(9)** (A) **(10)** (C)

VISCOSITY

Motion of Liquids & Viscosity : The motion of liquids are of following four types :

(i) Streamline Motion : In a liquid in motion, if liquid particles preceding or succeeding a liquid particle follow the same path, then the path is called streamline; and then the motion of the liquid is called streamline motion. This type of motion takes place in non-viscous liquids having very small speed.

(ii) Steady State Motion : In a liquid in motion, when liquid particles, crossing a point, cross it with same velocity, then the motion of the liquid is called steady state motion. This type of motion takes place in non-viscous liquids having very small speed.

(iii) Laminar Motion : Viscous liquids flow, in bounded region or in a pipe, in layers and when viscous liquid is in motion, different layers have different velocities. The layers in contact with the fixed surface has least velocity and the velocity of other parallel layers increases uniformly and continuously with the distance from the fixed surface to the free surface of the liquid. This is called laminar motion of the liquid.

(iv) Turbulent Motion : When the velocity of a liquid is irregular, haphazard and large, the motion of the liquid is called turbulent motion.

The cause of different velocities of different layers of viscous liquid is that a layer below opposes the motion of layers above it. The opposing viscous force per unit area between two layers has been found to be directly proportional to their velocity gradient i.e.

$$\frac{F}{A} \propto \frac{dv}{dx} \Rightarrow \frac{F}{A} = -\eta \frac{dv}{dx}$$

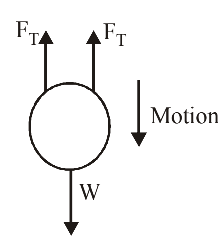
Dimension of η are ML^{-1}.
C.G.S. unit of η is poise while S.I. unit is Pa.s or Poiseuille; and poise is (1/10) of Pa.s. (or Poiseuile)
The cause of negative sign is because the direction of viscous force F is in a direction opposite to that of flow of liquid. And η is called coefficient of viscosity
Stoke's Law : $F = 6 \pi \eta r v$
where F = upward viscus drag force

Importance of stoke's law

(1) This law is used in the determination of electronic charge with the help of milikan's experiment.
(2) This law accounts the formation of clouds.
(3) This law accounts why the speed of rain drops is less then that of a body falling freely with a constant velocity from the height of clouds.
(4) This law helps a man coming down with the help of a parachute.

TERMINAL VELOCITY

It is maximum constant velocity acquired by the body while falling freely in a viscous medium.
Three types of forces acts on it.



$F_T + F_V = W$ .... (1)

Where, F_T = upward buoyancy force = $\frac{4}{3}\pi r^3 \sigma g$

F_V = upward viscous darg = $6\pi\eta r v$
W = weight of the body acting vertically downward

$= \frac{4}{3}\pi r^3 \rho g$; Here ρ = density of the body

σ = density of the liquid
By putting these values in equation (1) on solving it we get

$$v = \frac{2r^2(\rho-\sigma)g}{9\eta}$$

Effect on viscosity : (1) Effect of temperature : On increasing temperature viscosity of a liquid decreases.
(2) Effect of pressure :On increasing pressure viscosity of a liquid increases but viscosity of water decreases
Critical velocity : The critical velocity is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.

$$V_C = \frac{K\eta}{\rho r}$$

Reynold Number : Reynold number is pure number which determines the nature of flow of liquid through a pipe.

$$N_R = \frac{\rho D V_C}{\eta}$$

Example 18 :
A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg m^{-1} s^{-1}. What will be the terminal velocity of the drop. Density of air can be neglected.

Sol. $$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} = \frac{2\times\left[\frac{15\times10^{-4}}{1000}\right]^2\times10^3\times9.8}{9\times1.8\times10^{-5}}$$

$= 2.72 \times 10^{-4}$ m/sec.

TRY IT YOURSELF-3

Q.1 With increase in temperature, the viscosity of
(A) gases decreases. (B) liquids increases.
(C) gases increases. (D) liquids decreases.

Q.2 What is the largest average velocity of blood low in an artery of radius 2×10^{-3} m if the flow must remain laminar? (Take viscosity of blood to be 2.084×10^{-3} pas)

Q.3 A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Q.4 In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa s. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.

Q.5 Viscosity is the property of a liquid due to which it :
(A) occupies minimum surface area
(B) opposes relative motion between its adjacent layers
(C) becomes spherical in shape
(D) tends to regain its deformed position

Q.6 Consider the following statements :
(i) Young's modulus is numerically equal to the stress which will double the length of a wire.
(ii) Viscosity of gases is greater than that of liquids.
(iii) The surface tension of a liquid decreases due to the presence of insoluble contamination.
The number of above statements that are true is –
(A) one (B) two
(C) three (D) zero

Q.7 A space 2.5 cm. wide between two large plane surfaces is filled with oil. Force required to drag a very thin plate of area 0.5m^2 just midway the surfaces at a speed of 0.5 m/sec. is 1N. The coefficient of viscosity in kg-sec/m^2 is –
(A) 5×10^{-2} (B) 2.5×10^{-2}
(C) 1×10^{-2} (D) 7.5×10^{-2}

ANSWERS

(1) (CD) **(2)** 0.98 m/s
(3) Rate of heat loss $\propto r^5$ **(4)** 3.93×10^{-19} N
(5) (B) **(6)** (B) **(7)** (B)

HYDRO-STATICS

Fluids are the substances that can flow. Therefore liquids and gases both are fluids. Study of a fluid at rest is called fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics of hydrodynamics. Both combined are called fluid mechanics.
The intermolecular force in liquids are comparatively weaker than in solids. Therefore, their shape can be changed easily. Thus liquids assume the shape of the container. But their volume (or density) cannot be changed so easily. Liquids are incompressible and have free surface of their own. The intermolecular forces are weakest in gases, so their shape and size can be changed easily. Gases are compressible and
occupy all the space of the container.
The main characteristic properties of liquid are :
(1) Density (2) Specific weight
(3) Relative density (4) Specific gravity
(5) Pressure (6) Compressibility
(7) Viscosity (8) Surface tension
(9) Capillarity

DENSITY (ρ)
Mass per unit volume is defined as density. So density at a point of a fluid is represented as $\rho = \lim_{\Delta V\to 0}\frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

Density is a positive scalar quantity.
SI unit : kg/m^3 ; CGS unit : g/cc ; Dimensions : [ML^{-3}]

Note :

* For a solid body volume and density will be same as that of its constituent substance of equal mass i.e. if $M_{body} = M_{sub}$ then $V_{body} = V_{sub}$ and $\rho_{body} = \rho_{sub}$.
  But for a hollow body or body with air gaps $M_{body} = M_{sub}$ and $V_{body} > V_{sub}$ then P$\rho_{body} < \rho_{sub}$
* If m_1 mass of liquid of density ρ_1 and m_2 mass of liquid of density ρ_2 are mixed then

$$M_{mix} = m_1 + m_2 \text{ and } V_{mix} = V_1 + V_2 = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

$$\rho_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

If same masses are mixed i.e. $m_1 + m_2 = m$ then

$$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \text{ (Harmonic mean of individual densities)}$$

* If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed then

$$V_{mix} = V_1 + V_2 \text{ and } M_{mix} = m_1 + m_2 = \rho_1 V_1 + \rho_2 V_2$$

$$\therefore \quad \rho_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

* If same volumes are mixed i.e. $V_1 = V_2 = V$ then $\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$

(Arithmetic mean of individual densities)

SPECIFIC WEIGHT OR WEIGHT DENSITY (ω)

It is defined as the ratio of the weight of the fluid to its volume or the weight acting per unit volume of the fluid.

$$\omega = \frac{\text{weight (W)}}{\text{volume (V)}} = \frac{mg}{V} = \left[\frac{m}{V}\right]g = \rho g$$

S I Unit: N/m^3 Dimension: $[ML^{-2}T^{-2}]$

Specific weight of pure water at 4°C is 9.81 kN/m^3

RELATIVE DENSITY

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D).} = \frac{\text{density of given liquid}}{\text{density of pure water at 4°C}}$$

Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity.

Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

SPECIFIC GRAVITY

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

Specific gravity

$$= \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at 4°C } (9.81 \text{ kN/m}^3)}$$

$$= \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

Example 19 :

In an experiment the weight of 2.5 m^3 of a certain liquid was found to be 18.75 kN. Then find the specific weight of given liquid.

Sol. Specific weight $= \frac{\text{Weight}}{\text{Volume}} = \frac{18.75}{2.5} = 7.5 \text{ kN/m}^3$

PRESSURE

If a uniform force is exerted normal to an area (A), then pressure (P) is defined as the normal force (F) per unit area

i.e. $P = \frac{F}{A}$

SI unit : pascal (Pa), 1 Pa = 1 N/m^2 ; Dimension: $[ML^{-1}T^{-2}]$

Practical units: atmospheric pressure (atm), bar and torr

1 atm = 1.01325×10^5 Pa = 1.01325 bar = 760 torr = 760mm of Hg column pressure

1 bar = 10^5 Pa

1 torr = pressure exerted by 1 mm of mercury column=133Pa.

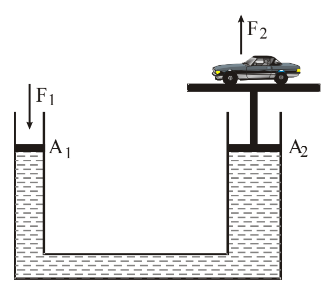
PASCAL'S LAW

Pascal's law is stated in following ways

* The pressure in a fluid at rest is same at all the points if gravity is ignored.
* A liquid exerts equal pressures in all directions.
* If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

Applications of pascal's law hydraulic jacks, lifts, presses, brakes, etc.

For the hydraulic lift :



Pressure applied $= \frac{F_1}{A_1}$

Pressure transmitted $= \frac{F_2}{A_2}$

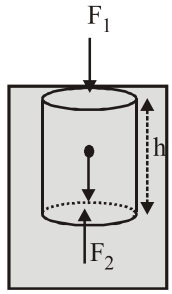
$\because$ Pressures $\frac{F_1}{A_2} = \frac{F_2}{A_2}$

Upward force on A_2 is

$$F_2 = \frac{F_1}{A_1} A_2 = \frac{A_2}{A_1} \times F_1$$

PRESSURE EXERTED BY A UQUID (EFFECT OF GRAVITY)

Consider a vessel containing liquid. As the liquid is in equilibrium, so every volume element of the fluid is also in equilibrium. Consider one volume element in the form of a cylindrical column of liquid of height h and of area of cross section A. The various forces acting on the cylindrical column of liquid are :



(i) Force, $F_1 = P_1A$ acting vertically downward on the top face of the column. P_1 is the pressure of the liquid on the top face of the column and is known as atmospheric pressure.

(ii) Force, $F_2 = P_2A$ acting vertically upward at the bottom face of the cylindrical column. P_2 is the pressure of the liquid on the bottom face of the column.

(iii) Weight, W = mg of the cylindrical column of the liquid acting vertically downward. Since the cylindrical column of the liquid is in equilibrium, so the net force acting on the column is zero. i.e. $F_1 + W - F_2 = 0 \Rightarrow P_1A + mg - P_2A = 0$

$$\Rightarrow P_1A + mg = P_2A \qquad \therefore P_2 = P_1 + \frac{mg}{A} \qquad ......(i)$$

Mass of the cylindrical column of the liquid

m = volume × density of the liquid

= Area of cross section × height × density = Ahρ

$$\therefore \text{ equation (i) becomes } P_2 = P_1 + \frac{Ah\rho g}{A}$$

$$P_2 = P_1 + h\rho g \qquad ....(ii)$$

P_2 is the absolute pressure at depth h below the free surface of the liquid. Equation (ii), shows that the absolute pressure at depth h is greater than the atmospheric pressure (P_1) by an amount equal to hρg.

$(P_2 - P_1) = h\rho g$ which is the difference of pressure between two points separated by a depth h.

Example 20 :

In a car lift, compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1 ? What is the pressure necessary to accomplish this task ?

Sol. Since pressure is transmitted undiminished throughout the fluid (Pascal's law)

$$F_1 = \frac{A_1}{A_2}F_2 = \frac{\pi\,(5\times5)}{\pi\,(15\times15)}(1350\times9.81) \approx 1.5\times10^3\,\text{N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.5\times10^3}{\pi\,(5\times10^{-2}\,\text{m})^2} = 1.9\times10^5\,\text{Pa}$$

TYPES OF PRESSURE

Pressure is of three types

(i) Atmospheric pressure (P_0)

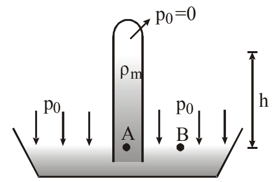
(ii) Gauge Pressure (P_{gauge})

(iii) Absolute Pressure (P_{abs})

Atmospheric pressure : Force exerted by air column on unit cross-section area of sea level is called atmospheric pressure (P_0) $\quad P_0 = \frac{F}{A} = 101.3\ \text{kN/m}^2$

$$\therefore \quad P_0 = 1.013\times10^5\ \text{N/m}^2$$

Barometer is used to measure atmospheric pressure.



Which was discovered by Torricelli.

Atmospheric pressure varies from place to place and at a particular place from time to time.

Gauge Pressure: Excess Pressure (P- P_{atm}) measured with the help of pressure measuring instrument called Gauge pressure. $\quad P_{gauge} = h\rho g$ or $P_{gauge} \propto h$.

Gauge pressure is always measured with help of "manometer".

Absolute Pressure : Sum of atmospheric and Gauge pressure is called absolute pressure.

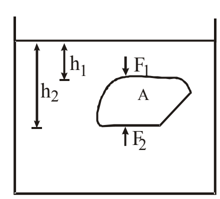
$$P_{abs} = P_{atm} + P_{gauge} \Rightarrow \quad P_{abs} = P_0 + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.

BUOYANCY AND ARCHIMEDE'S PRINCIPLE

Buoyant Force: Buoyant force is nothing but the force on the solid object due to pressure by the liquid. If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it. This phenomenon of force exerted by fluid on the body is called buoyancy and force is called buoyant force or upthrust.

Archimede's Principle : It states that the buoyant force on a body that is partially or totally immersed in a liquid is equal to the weight of the fluid displaced by it.



Now consider a body immersed in a liquid of density σ

Top surface of the body experiences a downward force

$$F_1 = A.P_1 = A\,[h_1.\sigma.g + P_0] \qquad ....(i)$$

Lower face of the body will experiences a upward force

$$F_2 = A.P_2 = A\,[h_2.\sigma.g + P_0] \qquad ....(ii)$$

As $h_2 > h_1$ so F_2 is greater than F_1

so net upward force $F = F_2 - F_1 = A\sigma g\,[h_2 - h_1]$

$\therefore \quad F = A\sigma gL = V\sigma g \qquad [\because\ V = AL]$

Note :

* Buoyant force act vertically upward through the centre of gravity (C.G.) of the displaced fluid. This point is called centre of buoyancy (C.B.). Thus centre of buoyancy is the point through which the force of buoyancy is supposed to act.
* Buoyant force or upthrust does not depend upon the characteristics of the body such as its mass, size, density, etc. But it depends upon the volume of the body inside the liquid. $Th \propto V_{in}$
* It depends upon the nature of the fluid as it is proportional to the density of the fluid. $Th \propto \sigma$
  This is the reason that upthrust on a fully submerged body is more in sea water than in pure water ($\because \sigma_{sea} > \sigma_{pure}$)
* It depends upon the effective acceleration.
  If a lift is accelerated downwards with acceleration a (a< g) then $Th = V_{in}\,\sigma\,(g - a)$
  If a lift is accelerated downwards with a = g then
  $Th = V_{in}\,\sigma\,(g - g) = 0$
  If a lift is accelerated upward with acceleration a then
  $Th = V_{in}\,\sigma\,(g + a)$
* Due to upthrust the weight of the body decreases
  $W_{App} = W - Th$ (W is the true weight of the body)
  Decrease in weight $= W - WA_{pp} = Th =$ Weight of the fluid displaced
* Using Archimede's principle we can determine relative density (R D) of a body as

$$R.D. = \frac{\text{density of body}}{\text{density of pure water at } 4°C}$$

$$= \frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{weight of body}}{\text{thrust due to water}} = \frac{\text{weight of body}}{\text{loss of weight in water}}$$

$$= \frac{\text{weight of body in air}}{\text{weight in air} - \text{weight in water}}$$

* If a body is weighed in air (W_A) in water (W_w) and in a liquid (W_L), then Specific gravity of oil

$$= \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}} = \frac{W_A - W_L}{W_A - W_w}$$

Example 21 :

A body weighs 160 g in air, 130 g in water and 136 g in oil. What is the specific gravity of oil ?

Sol. Specific gravity of oil

$$= \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}} = \frac{160 - 136}{160 - 130} = \frac{24}{30} = \frac{8}{10} = 0.8$$

FLOATATION

When a body of density (ρ) and volume (V) is completely immersed in a liquid of density (σ), the forces acting on the body are :

(i) Weight of the body $W = Mg = V\rho g$ directed vertically downwards through C.G. of the body.

(ii) Buoyant force or Upthrust $Th = V\sigma g$ directed vertically upwards through C.B.

The apparent weight W_{App} is equal to $W - Th$.

The following three cases are possible:

Case I : Density of the body is greater than that of liquid ($\rho > \sigma$). In this case $W > Th$

So the body will sink to the bottom of the liquid.

$W_{App} = W - Th = V\rho g - V\sigma g = V\rho g\,(1 - \sigma/\rho) = W\,(1 - \sigma/\rho)$

Case II : Density of the body is equal to the density of liquid ($\rho = \sigma$). Iri this case $W = Th$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium. $W_{App} = W - Th = 0$

Case III : Density of the body is lesser than that of liquid ($\rho < \sigma$). In this case $W < Th$

So the body will float partially submerged in the liquid. In this case the body will move up and the volume of liquid displaced by the body (V_{in}) will be less than the volume of body V, so as to make Th equal to W

$\therefore\ W_{App} = W - Th = 0$

The above three cases constitute the law of floatation which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

Note :

* A body will float only if its density is lower or equal to the density of the liquid i.e. $\rho \le \sigma$
* When a body floats its weight is equal to the upthrust i.e.
  $W = Th$ or $V\rho g = V_{in}\sigma g$ or $V\rho = V_{in}\sigma$
* In case of floating as W = Th, the apparent weight of the floating body will be zero.
* In case of W = Th, the equilibrium of floating body does not depend upon variations in g though both thrust and weight depends upon g.
* The weight of the plastic bag full of atmospheric air is same as that of empty bag because the additional upthrust is equal to the weight of the air enclosed.

Example 22 :

An iceberg is floating partially immersed in sea-water. The density of sea-water is 1.03 gm/cm^3 and that of ice is 0.92 gm/cm^3. What is the fraction of the total volume of the iceberg above the level of sea-water?

Sol. In case of floatation weight = upthrust i.e. $mg = V_{in}\sigma g$

$$\Rightarrow V\rho g = V_{in}\sigma g \Rightarrow \quad V_{in} = \frac{\rho}{\sigma}V$$

$$V_{out} = V - V_{in} = V\left[1 - \frac{\rho}{\sigma}\right] \text{ and } f_{out} = \frac{V_{out}}{V} = \left[1 - \frac{\rho}{\sigma}\right]$$

$$= \left[1 - \frac{0.92}{1.03}\right] = \frac{0.11}{1.03} = 0.106 = 10.6\%$$

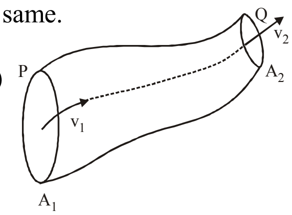
HYDRODYNAMICS

Principle of continuity :

When incompressible, nonviscous liquid flows in non-uniform tube then in streamline flow product of area and velocity at any section remain same.

$m_1 = m_2$

i.e. $v_1 A_1 \rho_1 = v_2 A_2 \rho_2$ .... (1)



As we have considered the fluid incompressible thus,

$v_1 A_1 = v_2 A_2$ ....(2)

Since $\rho_1 = \rho_2$

Equations (1) and (2) ar said to be as equation of continuity.

Example 23 :

A liquid is flowing through a non-sectional tube with its axis horizontally. If two points X and Y on the axis of tube has a sectional area 2.0 cm^3 and 25 mm^2 respectively then find the flow velocity at Y when the flow velocity at X is 10 m/s.

Sol. According to principle of continuity $v_x A_x = v_y A_y$

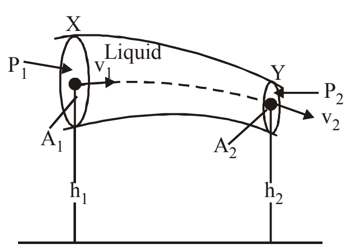
therefore, $v_y = \dfrac{v_x A_x}{A_y} = \dfrac{10(m/s) \times 2(cm^2)}{25 \times 10^{-2}(cm^2)} = 80$ m/s

Therefore, the flow velocity at y is 80 m/s.

BERNOULLI'S PRINCIPLE

When incompressible non-viscous liquid flow from one position to other in streamline path then in its path at every point sum of pressure energy, kinetic energy and potential energy per unit volume remains constant.

$$\therefore \quad P + \rho g h + \frac{\rho v^2}{2} = \text{constant}$$



Where $\quad$ P = Pressure energy per unit volume

$\rho g h$ = potential energy per unit volume and

$\frac{1}{2}\rho v^2$ = kinetic energy per unit volume.

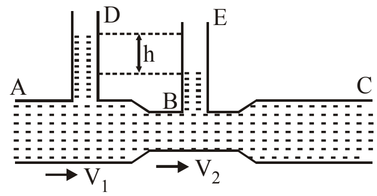
The above equation is known as Bermoulli's equation.

APPLICATION OF PRINCIPLE OF CONTINUITY AND BERNOULLI'S PRINCIPLE

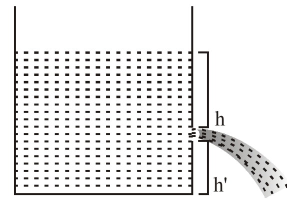
Venturimeter

The device is used for the measurement of rate of flow of fluid through a tube. The working of venturimeter is based on Bernoulli's principle.

$$V = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$



Torricelli's Theorem : Velocity with which the liquid flows out of are orifice is equal to that which a freely falling body would acquire in falling through a vertical distances equal to the depth of orifice below the free surface of liquid.



$v = \sqrt{2gh}$; Horizontal range $= 2\sqrt{hh'}$

Example 24 :

The ratio of the radius of the tube of a venturimeter is η ($\eta > 1$). The ratio of the densities of the liquid in the manometer and the moving fluid is η_1. If the difference in heights of the liquid column in the manometer is h, find the minimum speed of flow of the fluid.

Sol. The speed of flow is minimum when the cross-sectional area of the tube is maximum. The equation for minimum speed flow of the fluid in a venturimeter is given as

$$v_1 = A_2 \sqrt{\frac{2\rho_0 g h}{\rho(A_1^2 - A_2^2)}} = \sqrt{\frac{2\rho_0/gh}{\left(\frac{A_1}{A^2}\right)^2 - 1}}$$

Putting $\rho_0/\rho = \eta_1$,

$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \eta^2, \text{ We obtain } Q = \sqrt{\frac{2\eta_1 g h}{\eta^4 - 1}}$$

TRY IT YOURSELF-4

Q.1 An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is

(A) 9 : 4 $\quad$ (B) 3 : 2

(C) $\sqrt{3} : \sqrt{2}$ $\quad$ (D) $\sqrt{2} : \sqrt{3}$

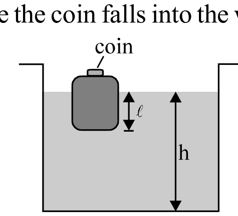
Q.2 A wooden block with a coin placed on its top, floats in water as shown in Figure. The distance *l* and h are shown in the figure. After some time the coin falls into the water. Then

(A) *l* decreases.

(B) h decreases.

(C) *l* increases.

(D) h increase



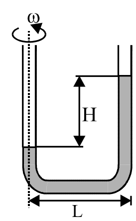
Q.3 A hydraulic automobile lift is designed to lift car with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?

Q.4 An iron casting containing a number of cavities weighs 6000 N in air and 4000 N in water. What is the total volume of all the cavities in the casting ? The density of iron (that is, a sample with no cavities) is 7.87 g/cm^3.

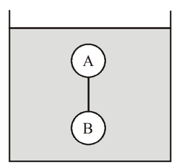
Q.5 A boy carries a fish in one hand and a bucket of water in the other hand; if he places the fish in the bucket, the weight now carried by him :
(A) is less than before (B) is more than before
(C) is the same as before (D) depends upon his speed.

Q.6 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to
$L/\sqrt{2\pi}$ (B) $2\pi L$
(C) L (D) $L/2\pi$

Q.7 A U-shaped tube contains a liquid of density ρ and it is rotated about the line as shown in the figure. Find the difference in the levels of liquid column.



Q.8 Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string.
The arrangement is possible only if
(A) $d_A < d_F$
(B) $d_B > d_F$
(C) $d_A > d_F$
(D) $d_A + d_B = 2d_F$



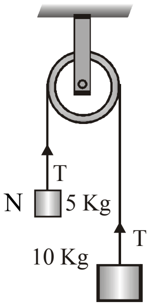
Q.9 A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is
(A) more than half-filled if ρ_C is less than 0.5
(B) more than half-filled if ρ_C is more than 1.0
(C) half-filled if ρ_C is more than 0.5
(D) less than half-filled if ρ_C is less than 0.5

ANSWERS

(1) (A) **(2)** (AB) **(3)** $6.92 \times 10^5\ Nm^{-2}$
(4) $0.127\ m^3$. **(5)** (C) **(6)** (A)
(7) $H = \dfrac{L^2\omega^2}{2g}$ **(8)** (ABD) **(9)** (A)

ADDITIONAL EXAMPLES

Example 1 :
Two blocks of masses 5 kg and 10 kg are connected by a metal wire going over a smooth pulley as shown if fig. The breaking stress of the metal wire is 2×10^9 N/m^2. If $g = 10\ m/s^2$, then the minimum radius of the wire which will not break should be



Sol. This is Atwood's machine.
Let the tension in the wire is T and acceleration of masses a.
Then, $T - 5g = 5a$ and $10g - T = 10a$

This gives $a = \dfrac{1}{3}g$ and $T = \dfrac{20}{3}g = \dfrac{200}{3}N$

$$\text{Stress} = \frac{T}{\text{area}} = \frac{200}{3 \times 3.14 \times r^2} = 2 \times 10^9$$

Thus the minimum radius r should be

$$r = \sqrt{\frac{200}{3 \times 3.14 \times 2 \times 10^9}} = 1.03 \times 10^{-4}\ \text{meter} = 0.103\ \text{mm}$$

Example 2 :
Calculate the work done against surface tension in blowing a soap bubble from a radius 10 cm to 20 cm if the surface tension of soap solution is 25×10^{-3} N/m. Then compare it with liquid drop of same radius.

Sol. (i) For soap bubble : Extension in area

$$= 2 \times 4\pi r_2^2 - 2 \times 4\pi r_1^2 = 8\pi\,[(0.2)^2 - (0.1)^2] = 0.24\,\pi\,m^2$$

Work done W_1 = surface tension x extension in area
$= 25 \times 10^{-3} \times 0.24\pi = 6\pi \times 10^{-3}$ J

(ii) For Liquid Drop: in case of liquid drop only one free surface, so extension in area will be half of soap bubble

$$\therefore\ W_2 = \frac{W_1}{2} = 3\pi \times 10^{-3}\,J$$

Example 3 :
A water drop of radius 1 mm is broken into 10^6 identical drops. Surface tension of water is 72 dynes/cm. Find the energy spent in this process.

Sol. As volume of water remains constant, so

$$\frac{4}{3}\pi R^2 = n\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{n^{1/3}}$$

Increase in surface area : $\Delta A = n(4\pi r^2) - 4\pi R^2$

$$= 4\pi\,(n^{1/3} - 1)\,R^2 = 4\pi\,(100 - 1)\,10^{-6}$$

$\therefore$ Energy spent $= T\Delta A = 4\pi \times 99 \times 10^{-6} \times 72 \times 10^{-3}$
$= 89.5 \times 10^{-6}$ J

Example 4 :
Two separate air bubbles (r_1 = 0.002 cm, r_2 = 0.004 cm) formed of same liquid T = 0.07 N/m come together to form a double bubble. Find the radius and sense of curvature of the internal film surface common to both the bubbles.

Sol. Radius of curvature of the common surface

$$r = \frac{r_1 r_2}{r_2 - r_1} = \frac{0.002 \times 0.004}{0.004 - 0.002} = 0.004 \text{ cm.}$$

As the excess pressure is always towards concave surface and pressure in smaller bubble is greater than larger bubble, the common surface is concave towards the centre of the smaller bubble.

Example 5 :

The limbs of a manometer consist of uniform capillary tubes of radii 1.4×10^{-4} m. Find out the correct pressure difference if the level of the liquid in narrower tube stands 0.2 m above that in the broader tube. (Density of liquid: 10^3 kg/m^3, Surface tension: 72×10^{-3} N/m)

Sol. If P_1 and P_2 are the pressures in the broader and narrower tubes of radii r_1 and r_2 respectively, the pressure just below the meniscus in the respective tubes will be

$$P_1 - \frac{2T}{r_1} \text{ and } P_2 - \frac{2T}{r_2} \text{ So that } \left[P_1 - \frac{2T}{r_1}\right] - \left[P_2 - \frac{2T}{r_2}\right] = h\rho g$$

or $$P_1 - P_2 = h\rho g - 2T\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$P_1 - P_2 = 0.2 \times 10^3 \times 9.8$$

$$-2 \times 72 \times 10^{-3}\left[\frac{1}{7.2 \times 10^{-4}} - \frac{1}{14 \times 10^{-4}}\right]$$

$$= 1960 - 97 = 1863 \text{ Pa}$$

Example 6 :

Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made one third of its previous value then what is the new value of the capillary rise?

Sol. Since $h = \frac{2T\cos\theta}{r\rho g}$ and for the same liquid and capillaries

of difference radii $h_1 r_1 = h_2 r_2 \;\therefore\; \frac{h_2}{h_1} = \frac{r_1}{r_2} = \frac{1}{(1/3)} = 3$

hence $h_2 = 3h_1 = 3 \times 20\text{mm} = 60\text{mm}$.

Example 7 :

A plate of area 2m^2 is made to move horizontally with a speed of 2m/s by applying a horizontal tangential force over the free surface of a liquid. If the depth of the liquid is 1m, coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate.

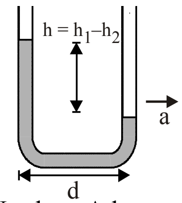
Sol. Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{2-0}{1-0} = 2s^{-1}$

Viscous force $|F| = \eta A \frac{\Delta v}{\Delta y} = (0.01 \times 10^{-1})(2)(2) = 4 \times 10^{-3}$N

So, to keep the plate moving a force of 4×10^{-3} N must be applied.

Example 8 :

Figure shows a U-tube of uniform cross-sectional area A, accelerated with acceleration a as shown. If d is the separation between the limbs, then what is the difference in the levels of the liquid in the U-tube.



Sol. Mass of liquid in horizontal portion of U-tube = Adρ

Pseudo force on this mass = Adρa

Force due to pressure difference in the two limbs

$$= (h_1\rho g - h_2\rho g)\, A$$

Equating both the forces $(h_1 - h_2)\,\rho g A = Ad\rho a$

$$\Rightarrow (h_1 - h_2) = \frac{Ad\rho a}{\rho g A} = \frac{ad}{g}$$

Example 9 :

Water flows through a tunnel a reservoir of dam towards the turbine installed in the power plant. The is situated h m below the reservoir. If the ratio of cross-sectional areas of the tunnel at the reservoir and power station is η, find the speed of the water entering into the turbine.

Sol. Applying Bernoulli's theorem at reservoir and power plant for the flowing water, we obtain

$$P_0 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g h_2 + \rho v_2^2$$

$$\Rightarrow v_2^2 = v_1^2 + 2g\,(h_1 - h_2).$$

Putting $(h_1 - h_2) = h_1$ we obtain, $v_2 = \sqrt{h_1 + h_2}$ ...(1)

Equation of continuity yields, $A_1 v_1 = A_2 v_2$ ...(2)

Eliminating v_1 from equation (1) and (2), we obtain

$$v_2 = \sqrt{\left(\frac{A_2}{A_1}v_2\right)^2 + 2gh} \quad \Rightarrow v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Putting $A_1/A_2 = \eta$, we obtain, $v_2 = \eta\sqrt{\frac{2gh}{\eta^2 - 1}}$

Example 10 :

Air flows horizontally with a speed v = 106 Ckm/hr. A house has plane roof of area A = 20m^2. Find the magnitude of aerodynamic lift of the roof.

Sol. Air flows just above the roof and there is no air flow just below the roof inside the room. Therefore $v_1 = 0$ and $v_2 = v$. Applying Bermoulli's theorem at the points inside and outside the roof, we obtain.

$(1/2)\,\rho v_1^2 + \rho g h_1 + P_1 = (1/2)\,\rho v_2^2 + \rho g h_2 + P_2$.

Since $h_1 = h_2 = h$, $v_1 = 0$ and $v_2 = v_1$

$P_1 = P_2 + 1/2\,\rho v^2$; $P_1 - P_2 = \Delta P = 1/2\,\rho v^2$.

Since the area of the roof is A, the aerodynamic exerted on it = F = (ΔP) A ⇒ F = 1/2 ρAv^2

where ρ = density of air = 1.3 kg/m^3

A = 20 m^2, v = 29.44 m/sec.

$\Rightarrow F = \{1/2 \times 1.3 \times 20 \times (29.44)^2\}$ N $= 1.127 \times 10^4$ N.

QUESTION BANK

CHAPTER 11 : PROPERTIES OF MATTER

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : ELASTICITY

Q.1 A wire of length L and area of cross-section A, is stretched by a load. The elongation produced in the wire is ℓ. If Y is the Young's modulus of the material of the wire, then the force constant of the wire is –
(A) YL/A (B) $Y\ell/A$
(C) YA/L (D) YA/ℓ

Q.2 Length of wire is doubled, when 20 x 10^8 N/m^2 stress is applied on it. Modulus of elasticity in N/m^2 will be
(A) 20×10^8 (B) 20×10^9
(C) 20×10^{10} (D) 10×10^8

Q.3 The work done in increasing the length of a one meter long wire of cross-sectional area 1mm^2 through 1mm will be
($Y = 2 \times 10^{11}$ N/m^2)
(A) 250 J (B) 10 J
(C) 5 J (D) 0.1 J

Q.4 A brass rod is to support a load of 400 N. If its elastic limit is 4.0×10^8 N/m^2 its minimum diameter must be -
(A) 1.13 mm (B) 2.26 mm
(C) 3.71 mm (D) 4.52 mm

Q.5 A 4.0 m long copper wire of cross sectional area 1.2 cm^2 is stretched by a force of 4.8×10^3 N stress will be -
(A) 4.0×10^7 N/mm^2 (B) 4.0×10^7 KN/m^2
(C) 4.0×10^7 N/m^2 (D) None

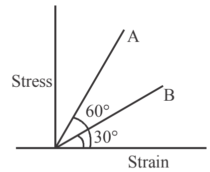
Q.6 A copper rod 2m long is stretched by 1mm. Strain will be -
(A) 10^{-4}, volumetric (B) 5×10^{-4}, volumetric
(C) 5×10^{-4}, longitudinal (D) 5×10^{-3}, volumetric

Q.7 A metallic wire is suspended by suspending weight to it. If S is longitudinal strain and Y its young's modulus of elasticity. Potential energy per unit volume will be
(A) $\frac{1}{2}Y^2S^2$ (B) $\frac{1}{2}Y^2S$
(C) $\frac{1}{2}YS^2$ (D) $2YS^2$

Q.8 The ratio of radii of two wires of same material is 2 : 1. If these wires are stretched by equal force, then the ratio of stresses produced in them will be
(A) 1 : 2 (B) 2 : 1
(C) 1 : 4 (D) 4 : 1

Q.9 According to Hook's law of elasticity, if stress is increased, the ratio of stress to strain –
(A) Increases (B) Decreases
(C) Becomes zero (D) Remains constant

Q.10 The stress versus strain graphs for wires of two materials A and B as shown in the figure If Y_A and Y_B are the Young modulus of the materials, then –



(A) $Y_B = 2Y_A$ (B) $Y_A = Y_B$
(C) $Y_B = 3Y_A$ (D) $Y_A = 3Y_B$

Q.11 A composite wire of uniform diameter 3.0 nm consist of a copper wire of length 2.2 m and a steel wire of length 1.6m stretches under a load by 0.7 mm.
Calculate the load, given that the Young's modulus for copper is 1.1×10^{11} Nm^{-2} and that for steel is 2×10^{11} Nm^{-2}.
(A) 1.8×10^2 N (B) 0.8×10^2 N
(C) 3.8×10^2 N (D) 5.8×10^2 N

Q.12 The relation between modulus of rigidity (η) and Young's modulus (Y) is –
(A) $\eta \approx Y/2$ (B) $\eta \approx Y$
(C) $\eta \approx Y/3$ (D) $\eta \approx Y/4$

Q.13 The Young's modulus of a wire of length L and radius r is Y. If the length is reduced to L/2 and radius is r/2, then the Young's modulus will be –
(A) Y/2 (B) Y
(C) 2Y (D) 4Y

PART - 2 : SURFACE TENSION

Q.14 A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in the capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will be
(A) M (B) 2M
(C) M/2 (D) 4M

Q.15 Mercury does not wet glass, wood or iron because
(A) Cohesive force is less than adhesive force
(B) Cohesive force is greater than adhesive force
(C) Angle of contact is less than 90°
(D) Cohesive force is equal to adhesive force

Q.16 Cohesive force is experienced between
(A) Magnetic substances
(B) Molecules of different substances
(C) Molecules of same substances
(D) None of these

Q.17 Consider a liquid contained in a vessel. The liquid solid adhesive force is very weak as compared to the cohesive force in the liquid. The shape of the liquid surface near the solid shall be –
(A) Horizontal (B) Almost vertical
(C) Concave (D) Convex

Q.18 Radius of a soap bubble is increased from R to 2R work done in this process in terms of surface tension (T) is
(A) $24\pi R^2T$ (B) $48\pi R^2T$
(C) $12\pi R^2T$ (D) $36\pi R^2T$

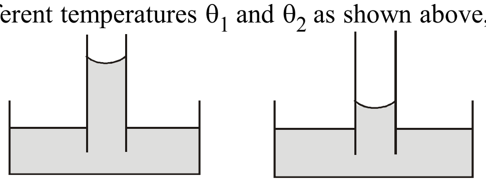
Q.19 For which of the two pairs, the angle of contact is same
(A) Water and glass; glass and mercury.
(B) Pure water and glass; glass and alcohol.
(C) Silver and water; mercury and glass.
(D) Silver and chromium; water and chromium.

Q.20 A capillary tube of radius r is dipped in a liquid of density ρ and surface tension T. If the angle of contact is θ, the pressure difference between the two surfaces in the beaker and the capillary
(A) $\frac{T}{r}\cos\theta$ (B) $\frac{2T}{r}\cos\theta$ (C) $\frac{T}{r\cos\theta}$ (D) $\frac{2T}{r\cos\theta}$

Q.21 Excess pressure of one soap bubble is four times more than the other. Then the ratio of volume of first bubble to another one is
(A) 1 : 64 (B) 1 : 4
(C) 64 : 1 (D) 1 : 2

Q.22 The ratio of radii of two soap bubbles is 1 :4 the ratio of excess pressures in them will be -
(A) 4 : 1 (B) 1 : 4
(C) 1 : 16 (D) 16 : 1

Q.23 A student observes surface tension of water at two different temperatures θ_1 and θ_2 as shown above, then



(θ_1) (θ_2)

(A) $\theta_1 = \theta_2$ (B) $\theta_1 > \theta_2$
(C) $\theta_1 < \theta_2$ (D) data insufficient

Q.24 A drop of water of volume V is pressed between the two glass plates so as to spread to an area A. If T is the surface tension, the normal force required to separate the glass plate is –
(A) $\frac{TA^2}{V}$ (B) $\frac{TA^2}{2V}$
(C) $\frac{2TA^2}{V}$ (D) $\frac{4TA^2}{V}$

Q.25 When a drop of water splits up is to number of drops –
(A) area increases (B) volume increases
(C) energy is absorbed (D) both (A) and (C)

PART - 3 : VISCOSITY

Q.26 The onset of turbulance in a liquid is determined by
(A) Pascal's law (B) Reynolds number
(C) Torricell's law (D) Bernoulli's principle

Q.27 The viscous force acting on a solid ball moving in air with terminal velocity v is directly proportional to
(A) $\sqrt{v}$ (B) v
(C) $1/\sqrt{v}$ (D) v^2

Q.28 The velocity of a small ball of mass M and density d_1, when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, then find viscous force acting on the ball.
(A) $Mg\left(1-\frac{d_2}{d_1}\right)$ (B) $Mg\left(1+\frac{d_2}{d_1}\right)$
(C) $2Mg\left(1-\frac{d_2}{d_1}\right)$ (D) None of these

Q.29 With increase in temperature the viscosity of
(A) liquids increases and of gases decreases.
(B) liquids decreases and of gases increases.
(C) both liquids and gases increases.
(D) both liquids and gases decreases.

Q.30 For turbulent flow, the value of Reynolds number is –
(A) $R_e < 2000$ (B) $R_e > 2000$
(C) $1000 < R_e < 2000$ (D) $R_e = 1000$

Q.31 When the flow parameters of any given instant remain same at every point, then flow is said to
(A) laminar (B) steady state
(C) turbulent (D) quasistatic

Q.32 The terminal velocity of a liquid drop of radius 'r' falling through air is v. If two such drops are combined to form a bigger drop, the terminal velocity with which the bigger drop falls through air is (ignore any buoyant force due to air)
(A) $\sqrt{2}$ v (B) 2v
(C) $\sqrt[3]{4}$ v (D) $\sqrt[3]{2}$ v

PART - 4 : HYDROSTATICS AND HYDRODYNAMICS

Q.33 Bernoulli's equation for steady, non-viscous, incompressible flow expresses the –
(A) conservation of linear momentum
(B) conservation of angular momentum
(C) conservation of energy
(D) conservation of mass

Q.34 Two pieces of metal when immersed in a liquid have equal upthrust on them; then
(A) Both pieces must have equal weights.
(B)Both pieces must have equal densities.
(C) Both pieces must have equal volumes.
(D) Both are floating to the same depth.

Q.35 Water enters through end A with speed v_1 and leaves through end B with speed v_2 of a cylindrical tube AB. The tube is always completely filled with water. In case I tube is horizontal and in case II it is vertical with end A upwards and in case III it is vertical with end B upwards. We have $v_1 = v_2$ for
(A) Case I (B) Case II
(C) Case III (D) Each case

Q.36 Spheres of iron and lead having same mass are completely immersed in water. Density of lead is more than that of iron. Apparent loss of weight is W_1 for iron sphere and W_2 for lead sphere. Then W_1/W_2 is –
(A) between 0 and 1 (B) = 0
(C) > 1 (D) = 1

PART - 5 : MISCELLANEOUS

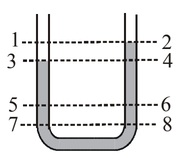
Q.37 Two rods of different materials having coefficients of linear expansions α_1 and α_2 and Young's modulus of elasticity Y_1 and Y_2 respectively, are fixed between two rigid massive walls. The rods area heated such that they undergo the same increase in temperature without bending. If $\alpha_1 : \alpha_2 = 2 : 3$, then the thermal stress developed in the two rods will be equal if $Y_1 : Y_2$ is equal to-
(A) 2 : 3 (B) 3 : 2
(C) 1 : 1 (D) 4 : 9

Q.38 The elastic energy stored in a wire of Young's modulus Y is -
(A) $Y \times \dfrac{\text{strain}^2}{\text{volume}}$ (B)stress× strain × volume
(C) $\dfrac{\text{strain}^2 \times \text{volume}}{2Y}$ (D) $\dfrac{1}{2}$ Y×stress×strain× vol.

Q.39 An open capillary tube is lowered in a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube Δh = 4.6mm. What is the radius of curvature of the mercury meniscus in the capillary tube? Surface tension of mercury is 0.46 N/m, density of mercury is 13.6 gm/cc.
(A) 1/340 m (B) 1/680 m
(C) 1/1020 m (D) Information insufficient

Q.40 A uniform rod of mass m, length L, area of cross-section A and Young's modulus Y hangs from a rigid support. Its elongation under its own weight will be :
(A) zero (B) mgL/2YA
(C) mgL/YA (D) 2mgL/YA

Q.41 In U-tube, three immiscible liquids are placed as shown in figure. The pair of points having same pressure is –

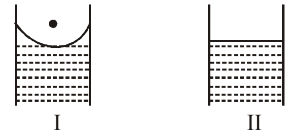


(A) 1-2 (B) 3-4
(C) 5-6 (D) 7-8

Q.42 A steel wire is 4.0m long and 2mm in diameter. How much is it elongated by a suspended body of mass 20 kg ? Young's modulus for steel is 1,96,000 Mpa.
(A) 1.273 mm (B) 1.73 mm
(C) 0.123 mm (D) 2.124 mm

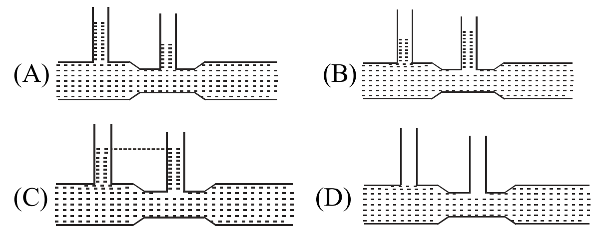
Q.43 Four uniform wires of the same material are stretched by the same force. The dimensions of wire are as given below. The one which has the minimum elongation has
(A) radius 3mm, length 3m (B) radius 0.5mm, length 0.5m
(C) radius 2mm, length 2m (D) radius 3mm, length 2m

Q.44 Shape of the meniscus formed by two liquids when capillaries are dipped in them are shown. In I it is hemispherical where as in II it is flat. Pick correct statement regarding contact angle formed by the liquids in both situations –



(A) It is 180° in I and 90° in II
(B) It is 0° in I and 90° in II
(C) It is 90° in I and 0° in II
(D) It is greater than 90° in I and equal to 90° in II

Q.45 For a fluid which is flowing steadily, the level in the vertical tubes is best represented by –

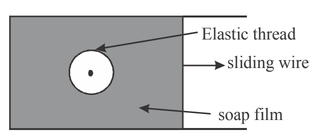


Q.46 A block of silver of mass 4 kg. hanging from a string is immersed in a liquid of relative density 0.72. If relative density of silver is 10, then tension in the string will be
(A) 37.12 N (B) 42 N
(C) 73 N (D) 21 N

Q.47 A block of iron is kept at the bottom of a bucket full of water at 2°C. The water exerts buoyant force on the block. If the temperature of water is increased by 1°C the temperature of iron block also increases by 1°C. The buoyant force on the block by water –
(A) will increase
(B) will decrease
(C) will not change
(D) many decrease or increase depending on the values of their coefficient of expansion

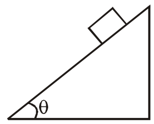
Q.48 The coefficient of viscosity η of a liquid is defined as the tangential force on a layer in that liquid per unit area per unit velocity gradient across it. Then a sphere of radius 'a', moving through it under a constant force F attains a constant velocity 'V' given by – (where K is a numerical constant)
(A) KFaη (B) $K\dfrac{F}{a}\eta$ (C) $K\dfrac{F}{a\eta}$ (D) $K\eta\dfrac{a}{F}$

Q.49 The figure shows a soap film in which a closed elastic thread is lying. The film inside the thread is pricked. Now the sliding wire is moved out so that the surface area increases. The radius circle of the circle formed by elastic thread will



(A) increase (B) decrease
(C) remains same (D) data insufficient

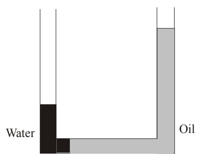
Q.50 A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be –

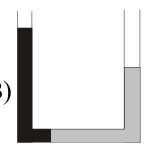


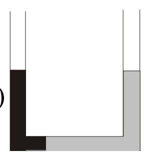
(A) $\eta = \dfrac{\rho a g t \sin\theta}{v}$ (B) $\dfrac{\rho g t \sin\theta}{a v}$

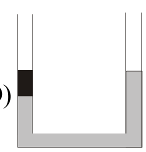
(C) $\dfrac{v}{\rho a g t \sin\theta}$ (D) None of these

Q.51 A U shaped tube of constant cross-sectional area is filled with equal masses of oil and water. These do not mix and stay in the left and right parts of the tube respectively. The water has twice the density of the oil. The diagram best representing the case is (water in the darker shade on the left)

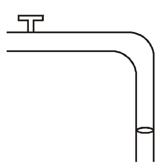
(A) 

(B) 

(C) 

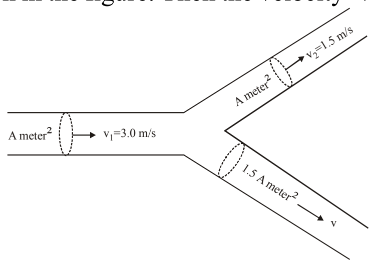
(D) 

Q.52 A steady stream of water falls straight down from a pipe as shown. Assume the flow is incompressible then –



(A) the pressure in the water is higher at lower points in the stream.
(B) the pressure in the water is lower at lower points in the stream.
(C) the pressure in the water is the same at all points in the stream.
(D) pressure variation will depend upon density and exit speed of the water.

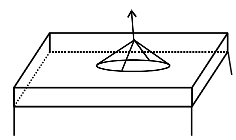
Q.53 An incompressible liquid flows through a horizontal tube as shown in the figure. Then the velocity 'v' of the fluid is–



(A) 3.0 m/s (B) 1.5 m/s
(C) 1.0 m/s (D) 2.25 m/s

Q.54 A mosquito with 8 legs stands on water surface and each leg makes depression of radius 'a'. If the surface tension and angle of contact are 'T' and zero respectively, then the weight of mosquito is –
(A) 8Ta (B) 16πTa
(C) $\dfrac{Ta}{8}$ (D) $\dfrac{Ta}{16\pi}$

Q.55 The surface tension of water is 75 dyne/cm. Find the minimum vertical force required to pull a thin wire ring up (refer figure) if it is initially resting on a horizontal water surface. The circumference of the ring is 20 cm and its weight is 0.1 N :

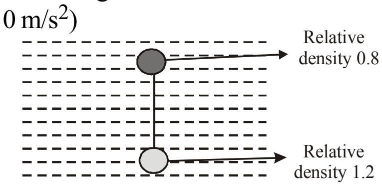


(A) 0.125 N (B) 0.225 N
(C) 0.115 N (D) 0.130 N

Q.56 An iceberg is floating in ocean. What fraction of its volume is above the water ? (Given : density of ice = 900kg/m^3 and density of ocean water = 1030 kg/m^3)

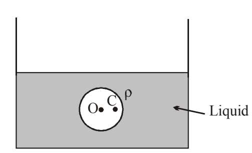
(A) $\dfrac{90}{103}$ (B) $\dfrac{13}{103}$ (C) $\dfrac{10}{103}$ (D) $\dfrac{1}{103}$

Q.57 Two spheres of volume 250cc each but of relative densities 0.8 and 1.2 are connected by a string and the combination is immersed in a liquid in vertical position as shown in figure. The tension in the string is ($g = 10\ m/s^2$)



(A) 5.0N (B) 0.5N
(C) 1.0N (D) 2.0N

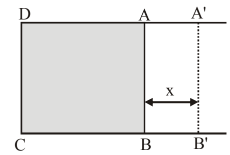
Q.58 An nonhomogeneous small sphere having average density same as that of the liquid. It is released from rest, in the position as shown in figure. C being its centre of mass and O being the centre of sphere.



(A) O moves up (B) O moves down
(C) C moves left (D) None of these

Q.59 A sealed glass bulb containing mercury (incompletely filled) just floats in water at 4°C. If the water and bulb are (i) cooled to 2°C and (ii) warmed to 8°C, the bulb –
(A) (i) sinks and (ii) sinks (B) (i) sinks and (ii) floats
(C) (i) floats and (ii) floats (D) (i) floats and (ii) sinks

Q.60 The figure shows a soap film formed between a rectangular frame in which side AB is movable over DA and CB. Now the side AB is moved towards right to a new position A'B' so that surface area of the film increases. In the position AB and A'B' force applied by the soap film on the movable side is F and F' respectively

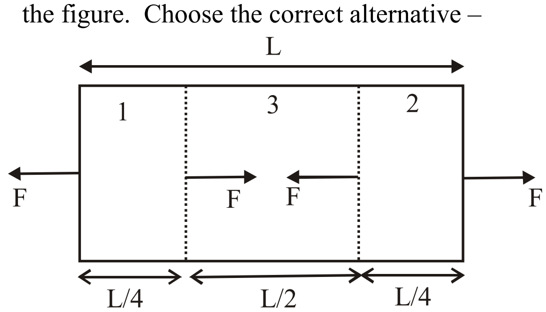


(A) F > F' (B) F < F'
(C) F = F' (D) F' = xF

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 External forces acting on a rod of length L, cross-sectional area A and Young's modulus Y are as shown in the figure. Choose the correct alternative –



(A) there will be a change in length only in parts 2 & 3.
(B) no change in length of rod is observed
(C) the net elongation of the whole rod is $\frac{FL}{2AY}$
(D) the net compression of the whole rod is $\frac{FL}{AY}$

Q.2 A steel wire of length 4.7 m and cross section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross section 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper ?
(A) 1.79 (B) 0.79
(C) 2.12 (D) 3.14

Q.3 A thin steel ring of inner radius r and cross-sectional area A is fitted on to a wooden disc of radius R (R > r). If Young's modulus be Y, then tension in the steel ring is –
(A) $AY\left(\frac{R}{r}\right)$ (B) $AY\left(\frac{R-r}{r}\right)$
(C) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (D) $\frac{Yr}{AR}$

Q.4 If the work done in stretching a wire by 1mm is 2J, the work necessary for stretching another wire of the same material but double the radius and half the length y 1mm is –
(A) 16 J (B) 8J
(C) 4J (D) (1/4) J

Q.5 The length of elastic string, obeying Hooke's law is ℓ_1 metres when the tension 4N and ℓ_2 metres when the tension is 5N. The length in metres when the tension is 9N is –
(A) $5\ell_1 - 4\ell_2$ (B) $5\ell_2 - 4\ell_1$
(C) $9\ell_1 - 8\ell_2$ (D) $9\ell_2 - 8\ell_1$

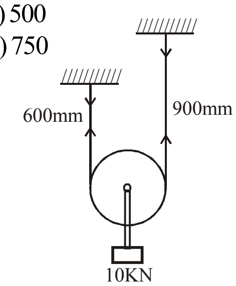
Q.6 A platform is suspended by four wires at its corners. The wires are 3m long and have a diameter of 2.0mm. Young's modulus for the material of the wires is 1,80,000 MPa. How far will the platform drop (due to elongation of the wires) if a 50 kg load is placed at the centre of the platform ?
(A) 0.25 mm (B) 0.65 mm
(C) 1.65 mm (D) 0.35 mm

Q.7 What is the minimum diameter of a brass rod if it is to support a 400N load without exceeding the elastic limit ? Assume that the stress for the elastic limit is 379 MPa.
(A) 1.16mm (B) 2.32mm
(C) 0.16mm (D) 1.35mm

Q.8 The pressure in an explosion chamber is 345 MPa. What would be the percent change in volume of a piece of copper subjected to this pressure ? The bulk modulus for copper is 138 Gpa (= 138×10^9 Pa)
(A) 0.1% (B) 0.5%
(C) 0.25% (D) 0.2%

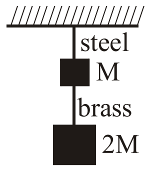
Q.9 A power cable of copper is just stretched (initial tension zero) straight between two fixed towers. If the temperature decreases, the cable tends to contract. The amount of contraction for a free copper cable of rod is 0.0002% per degree Celsius. Estimate what temperature decrease (in °C) will cause the cable to snap. Assume that the cable obeys Hooke's law until it reaches its breaking point, which for copper occurs at a tensile stress of 2.2×10^8 N/m^2. Ignore the weight of the cable and the sag and stress produced by the weight.
[Young's modulus for copper is 1.1×10^{11}N/m^2]
(A) 1000 (B) 500
(C) 1500 (D) 750

Q.10 A load of 10kN is supported from a pulley which in turn is supported by a rope of sectional area 1×10^3 mm^2 and modulus of elasticity 10^3 N mm^{-2}, as shown in figure. Neglecting the friction at the pulley, determine the deflection of the load.



(A) 2.75 mm (B) 3.75 mm
(C) 5.25 mm (D) 6.50 mm

Q.11 If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is–



(A) $2a^2cb$ (B) $3a/2b^2c$
(C) $2ac/b^2$ (D) $3c/2ab^2$

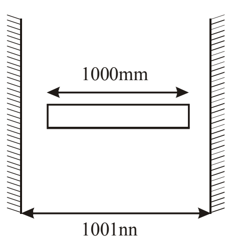
Q.12 Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 2:3. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of :

(A) $\sqrt{3}:\sqrt{2}$ (B) $3:2$
(C) $3\sqrt{3}:2\sqrt{2}$ (D) $9:4$.

Q.13 Four identical hollow cylindrical columns of steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 and 60 cm. respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Nm^{-2}.
(A) 1.2×10^{-7} (B) 7.2×10^{-7}
(C) 6.2×10^{-3} (D) 7.2×10^{-5}

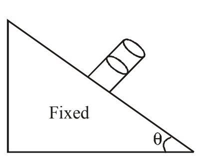
Q.14 The edges of an aluminium cube are 10 cm. long. One face of the cube is firmly fixed to a vertical wall. A mass of 100kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25×10^9 Nm^{-2}. What is the vertical deflection of this face ? (in m)
(A) 1.2×10^{-7} (B) 7.2×10^{-7}
(C) 6.2×10^{-3} (D) 4×10^{-7}

Q.15 A rod of length 1000mm and coefficient of linear expansion $\alpha = 10^{-4}$ per degree is placed symmetrically between fixed walls separated by 1001 mm. The Young's modulus of the rod is 10^{11} N/m^2. If the temperature is increased by 20°C, then the stress developed in the rod is (in N/m^2)



(A) 10 (B) 10^8
(C) 2×10^8 (D) cannot be calculated

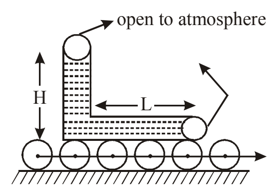
Q.16 A cylindrical vessel filled with water is released on an inclined surface of angle θ as shown in figure. The friction coefficient of surface with vessel is μ (< tan θ). Then the constant angle made by the surface of water with the incline will be –



(A) $\tan^{-1}\mu$ (B) $\theta - \tan^{-1}\mu$
(C) $\theta + \tan^{-1}\mu$ (D) $\cot^{-1}\mu$

Q.17 An open tank 10m long and 2m deep is filled up to 1.5 height of oil of specific gravity 0.82. The tank is uniformly accelerated along its length from rest to a speed of 20 m/sec horizontally. The shortest time in which the speed may be attained without spilling any oil is
(A) 20 sec. (B) 18 sec $[g = 10\ m/sec^2]$
(C) 10 sec (D) 5 sec.

Q.18 A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in figure. Assuming no sliding between any surfaces, the value of acceleration of the cylinders for which liquid will not come out of the tube from anywhere is given by –



(A) $\frac{gH}{2L}$ (B) $\frac{gH}{L}$ (C) $\frac{2gH}{L}$ (D) $\frac{gH}{\sqrt{2}L}$

Q.19 A broad vessel with water stands on a smooth surface. The level of the water in the vessel is h. The vessel together with the water weighs G. The side wall of the vessel has at the bottom a plugged hole (with rounded edges) with an area A. At what coefficient of friction between the bottom and the surface will the vessel begin to move if the plug is removed ?

(A) $\frac{\rho g h A}{G}$ (B) $\frac{2\rho g h A}{G}$ (C) $\frac{\rho g h A}{2G}$ (D) $\frac{2\rho g h A}{3G}$

Q.20 An isolated and charged spherical soap bubble has a radius r and the pressure inside is atmospheric. If T is the surface tension of soap solution, then charge on drop is –

(A) $2\sqrt{\frac{2rT}{\varepsilon_0}}$ (B) $8\pi r\sqrt{2rT\varepsilon_0}$

(C) $8\pi r\sqrt{rT\varepsilon_0}$ (D) $8\pi r\sqrt{\frac{2rT}{\varepsilon_0}}$

Q.21 A balloon of volume V, contains a gas whose density is to that of the air at the earth's surface as 1 : 15. If the envelope of the balloon be of weight w but of negligible volume, find the acceleration with which it will begin to ascend.

(A) $\left(\frac{7Vg\sigma - w}{Vg\sigma + w}\right) \times g$ (B) $\left(\frac{2Vg\sigma - w}{Vg\sigma + w}\right) \times g$

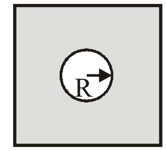
(C) $\left(\frac{14Vg\sigma - w}{Vg\sigma + w}\right) \times g$ (D) $\left(\frac{14Vg\sigma + w}{Vg\sigma - w}\right) \times g$

Q.22 A vertical capillary is brought in contact with the water surface what amount of heat is liberated while the water rises along the capillary ? The wetting is assumed to be complete and the surface tension is S –

(A) $\frac{2\pi S^2}{\rho g}$ (B) $\frac{\pi S^2}{\rho g}$

(C) zero (D) $\frac{4\pi S^2}{\rho g}$

Q.23 There is a rectangular wire frame having a thin film of soap solution. A massless thin wire of radius R and area of cross section A is placed on the surface of film, and inside portion of the film is pricked. If surface tension of soap solution is S and Young's modulus of wire is Y then change in radius of the wire is –



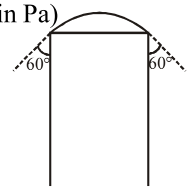
(A) $\frac{SR^2}{AY}$ (B) $\frac{2SR^2}{AY}$ (C) $\frac{SR^2}{3AY}$ (D) None

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

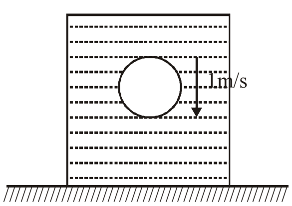
NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 A sphere of radius 10cm. and density 500 kg/m^3 is under water of density 1000 kg/m^3. The acceleration of the sphere is 9.80m/s^2 upward. Viscosity of water is 1.0 centipoise. If g = 9.81 m/s^2, the velocity of the sphere is (in m/s)

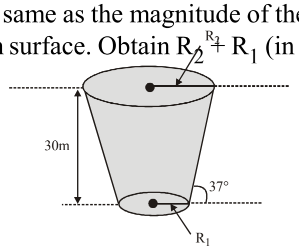
Q.2 A soap bubble is being blown on a tube of radius 1 cm. The surface tension of the soap solution is 0.05 N/m and the bubble makes an angle of 60° with the tube as shown. The excess of pressure over the atmospheric pressure in the tube is (in Pa)



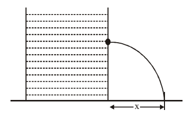
Q.3 A metal sphere of volume 1m^3 is sinking at a speed of 1 m/s in a closed container filled with a liquid of density 2 kg/m^3. The momentum of the liquid will be (in kg-m/s)



Q.4 The figure shows a pond full of water having the shape of a truncated cone. The depth of the pond is 30m. The atmospheric pressure above the pond is 1.0 × 10^5 Pa. The circular top surface (radius = R_2) and circular bottom surface (radius = R_1) of the pond are both parallel to the ground. The magnitude of the force acting on the top surface is the same as the magnitude of the force acting on the bottom surface. Obtain $R_2 + R_1$ (in m).



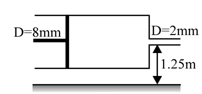
Q.5 Water is filled in a fixed container filled upto a height of 13m from ground. A sniper fires and makes a small hole in the side at a height of 9m from the ground. How far dows the water jet strike the ground (in m) ?



Q.6 A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9 × 10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is n×10^9 Nm^{-2}, the value of n is :

Q.7 Steel wire of length L at 40°C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5} /°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of m in kg is nearly.

Q.8 Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25m/s. Find the horizontal range of water stream on the ground. (g = 10 m/s^2)



Q.9 Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2cm and 4cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Q.10 A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.
[Take atmospheric pressure = 1.0 × 10^5 N/m^2 , density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension]

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is –

(A) 10 m/s (B) 20 m/s **[AIEEE-2002]**

(C) 25.5 m/s (D) 5 m/s

Q.2 A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is – **[AIEEE-2003]**

(A) 10 J (B) 20 J

(C) 0.1 J (D) 0.2J

Q.3 A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is – **[AIEEE-2003]**

(A) 18.75 N-m (B) 25.00 N-m

(C) 6.25 N-m (D) 12.50 N-m

Q.4 A wire fixed at the upper end stretched by length ℓ by applying a force F. The work done in stretching is – **[AIEEE-2004]**

(A) $2F\ell$ (B) $F\ell$

(C) $\frac{F}{2\ell}$ (D) $\frac{F\ell}{2}$

Q.5 Which one of the following represents the correct dimensions of the coefficient of viscosity ? **[AIEEE-2004]**

(A) $ML^{-1}T^{-1}$ (B) MLT^{-1}

(C) $ML^{-1}T^{-2}$ (D) $ML^{-2}T^{-2}$

Q.6 Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is – **[AIEEE-2004]**

(A) inversely proportional to both radius 'R' and velocity 'v'

(B) directly proportional to both radius 'R' and velocity 'v'

(C) directly proportional to 'R' but inversely proportional to 'v'

(D) inversely proportional to 'R' but directly proportional to velocity 'v'

Q.7 If two soap bubbles of different radii are connected by a tube – **[AIEEE-2004]**

(A) air flows from the smaller bubble to the bigger

(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(C) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(D) there is no flow of air

Q.8 If 'S' is stress and 'Y' is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(A) $2S^2Y$ (B) $S^2/2Y$ **[AIEEE-2005]**

(C) $2Y/S^2$ (D) $S/2Y$

Q.9 A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be **[AIEEE-2005]**

(A) 8 cm (B) 10 cm (C) 4 cm (D) 20 cm

Q.10 A wire elongates by ℓ mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)– **[AIEEE-2006]**

(A) zero (B) $\ell/2$

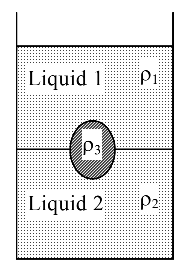
(C) ℓ (D) 2ℓ

Q.11 If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid–

(A) 0.1 m/s (B) 0.2 m/s **[AIEEE-2006]**

(C) 0.4 m/s (D) 0.133 m/s

Q.12 A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3 **[AIEEE-2008]**



(A) $\rho_1 > \rho_3 > \rho_2$ (B) $\rho_1 < \rho_2 < \rho_3$

(C) $\rho_1 < \rho_3 < \rho_2$ (D) $\rho_3 < \rho_1 < \rho_2$

Q.13 A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed υ, i.e., $F_{viscous} = -k\upsilon^2$ (k > 0). The terminal speed of the ball is **[AIEEE-2008]**

(A) $\frac{Vg\rho_1}{k}$ (B) $\sqrt{\frac{Vg\rho_1}{k}}$

(C) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (D) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

Q.14 Two wires are made of the same material and have the same volume. Howerver wire 1 has cross-section area A and wire 2 has cross-section area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount ?

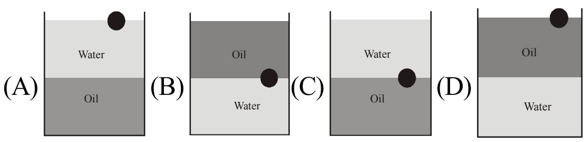
(A) F (B) 4 F **[AIEEE-2009]**

(C) 6 F (D) 9 F

Q.15 A ball is made of a material of density ρ where

$\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

[AIEEE 2010]



Q.16 Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly. (Surface tension of soap solution = 0.03 Nm^{-1}) **[AIEEE 2011]**

(A) 4π mJ (B) 0.2π mJ

(C) 2π mJ (D) 0.4π mJ

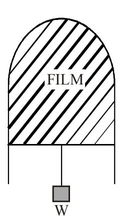
Q.17 Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

[AIEEE 2011]

(A) 5.0×10^{-3} m (B) 7.5×10^{-3} m

(C) 9.6×10^{-3} m (D) 3.6×10^{-3} m

Q.18 A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is – **[AIEEE 2012]**



(A) 0.0125 Nm^{-1} (B) 0.1 Nm^{-1}

(C) 0.05 Nm^{-1} (D) 0.025 Nm^{-1}

Q.19 Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible ? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization

[JEE MAIN 2013]

(A) $\rho L/T$ (B) $\sqrt{T/\rho L}$

(C) $T/\rho L$ (D) $2T/\rho L$

Q.20 The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

[JEE MAIN 2014]

(A) 2.2×10^7 Pa (B) 2.2×10^6 Pa

(C) 2.2×10^8 Pa (D) 2.2×10^9 Pa

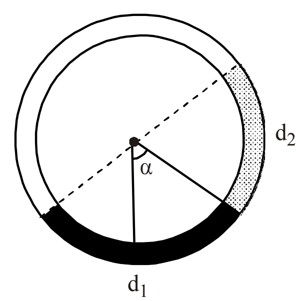
Q.21 An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

[JEE MAIN 2014]

(A) 38 cm (B) 6 cm

(C) 16 cm (D) 22 cm

Q.22 There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. ratio d_1/d_2 is – **[JEE MAIN 2014]**



(A) $\dfrac{1+\tan\alpha}{1-\tan\alpha}$

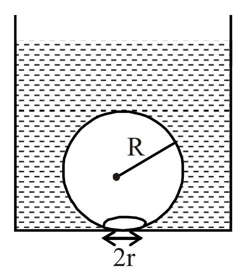
(B) $\dfrac{1+\sin\alpha}{1-\cos\alpha}$

(C) $\dfrac{1+\sin\alpha}{1-\sin\alpha}$

(D) $\dfrac{1+\cos\alpha}{1-\cos\alpha}$

Q.23 On heating water, bubbles being formed at the bottom of the vessel detach and rise.

Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel.



If r < < R, and the surface tension of water is T, value of r just before bubbles detach is : (density of water is ρ_w) **[JEE MAIN 2014]**

(A) $R^2\sqrt{\dfrac{\rho_w g}{T}}$ (B) $R^2\sqrt{\dfrac{3\rho_w g}{T}}$

(C) $R^2\sqrt{\dfrac{2\rho_w g}{3T}}$ (D) $R^2\sqrt{\dfrac{\rho_w g}{6T}}$

Q.24 A pendulum made of a uniform wire of crosssectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then (1/Y) is equal to – (g = gravitational acceleration) **[JEE MAIN 2015]**

(A) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$ (B) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

(C) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (D) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

Q.25 A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : **[JEE MAIN 2017]**

(A) 1/9 (B) 81
(C) 1/81 (D) 9

Q.26 A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, (dr / r) is –

(A) $\frac{mg}{3Ka}$ (B) $\frac{mg}{Ka}$ **[JEE MAIN 2018]**

(C) $\frac{Ka}{mg}$ (D) $\frac{Ka}{3mg}$

Q.27 A rod of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is : **[JEE MAIN 2019 (JAN)]**

(A) $\frac{F}{2A\alpha\Delta T}$ (B) $\frac{F}{A\alpha(\Delta T - 273)}$

(C) $\frac{F}{A\alpha\Delta T}$ (D) $\frac{2F}{A\alpha\Delta T}$

Q.28 Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas)

[JEE MAIN 2019 (APRIL)]

(A) 10^6 (B) 10^3
(C) 10^4 (D) 10^2

Q.29 A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02kg on it and stretches the cord by 20cm by applying a constant force. When released, the stone flies off with a velocity of 20m/s. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to: **[JEE MAIN 2019 (APRIL)]**

(A) 10^4 Nm^{-2} (B) 10^8 Nm^{-2}
(C) 10^6 Nm^{-2} (D) 10^3 Nm^{-2}

Q.30 A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given that g = 3.1 π ms^{-2}, what will be the tensile stress that would be developed in the wire ? **[JEE MAIN 2019 (APRIL)]**

(A) 4.8×10^6 Nm^{-2} (B) 5.2×10^6 Nm^{-2}
(C) 6.2×10^6 Nm^{-2} (D) 3.1×10^6 Nm^{-2}

Q.31 Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :- **[JEE MAIN 2019 (APRIL)]**

(A) 1.9 mm (B) 1.7 mm
(C) 1.5 mm (D) 1.3 mm

Q.32 An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is: **[JEE MAIN 2020 (JAN)]**

(A) 81 / 256 (B) 9 / 16
(C) 3 / 4 (D) 3 / 16

Q.33 Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is : **[JEE MAIN 2020 (JAN)]**

(A) $\rho_0 / 5$ (B) $\rho_0 / 3$
(C) $2\rho_0 / 3$ (D) $2\rho_0 / 5$

Q.34 A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to

[JEE MAIN 2020 (JAN)]

(A) 1.01 (B) 1.04
(C) 1.03 (D) 1.26

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

Q.1 The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? **[NEET 2013]**
(A) length = 300cm, diameter = 3mm
(B) length = 50 cm, diameter = 0.5 mm
(C) length = 100 cm, diameter = 1mm
(D) length = 200 cm, diameter = 2mm

Q.2 Wettability of a surface by a liquid depends primarily on
(A) angle of contact between surface & liquid.
(B) viscosity **[NEET 2013]**
(C) surface tension
(D) density

Q.3 Copper of fixed volume V is drawn into wire of length ℓ. When this wire is subjected to a constant force F, the extension produced in the wire is $\Delta\ell$. Which of the following graphs is a straight line? **[AIPMT 2014]**
(A) $\Delta\ell$ versus $1/\ell$ (B) $\Delta\ell$ versus ℓ^2
(C) $\Delta\ell$ versus $1/\ell^2$ (D) $\Delta\ell$ versus ℓ

Q.4 A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If 'T' is the surface tension of the liquid, then –
(A) Energy = 4VT $\left(\frac{1}{r}-\frac{1}{R}\right)$ is released **[AIPMT 2014]**
(B) Energy = 3VT $\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed
(C) Energy = 3VT $\left(\frac{1}{r}-\frac{1}{R}\right)$ is released
(D) Energy is neither released nor absorbed.

Q.5 The approximate depth of an ocean is 2700m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean? **[AIPMT 2015]**
(A) 1.0×10^{-2} (B) 1.2×10^{-2}
(C) 1.4×10^{-2} (D) 0.8×10^{-2}

Q.6 A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be : ($P_{air} = 1.2\ kg/m^3$) **[AIPMT 2015]**
(A) 4.8×10^5 N, upwards (B) 2.4×10^5 N, upwards
(C) 2.4×10^5 N, downwards (D) 4.8×10^5 N, downwards

Q.7 The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is V, the speed of the ejection of the liquid through the holes is : **[RE-AIPMT 2015]**
(A) $\frac{V^2R}{nr}$ (B) $\frac{VR^2}{n^2r^2}$ (C) $\frac{VR^2}{nr^2}$ (D) $\frac{VR^2}{n^3r^2}$

Q.8 The Young's modulus of steel is twice that of brass. Two wires of same lenght and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of **[RE-AIPMT 2015]**
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 1

Q.9 Water rises to height 'h' in capillary tube. If the length of capillary tube above the surface of water is made less than 'h', then - **[RE-AIPMT 2015]**
(A) water does not rise at all.
(B) water rises upto the tip of capillary tube and then starts overflowing like a fountain.
(C) water rises upto the top of capillary tube and stays there without overflowing.
(D) water rises upto a point a little below the top and stays there.

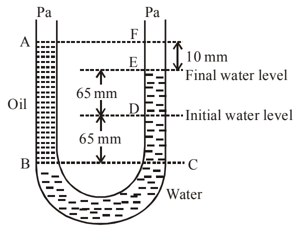
Q.10 Two non-mixing liquids of densities ρ and $n\rho$ $(n > 1)$ are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL $(p < 1)$ in the denser liquid. The density d is equal to **[NEET 2016 PHASE 1]**
(A) $\{1 + (n + 1)p\}\ \rho$ (B) $\{2 + (n + 1)p\}\ \rho$
(C) $\{2 + (n - 1)p\}\ \rho$ (D) $\{1 + (n - 1)p\}\ \rho$

Q.11 A rectangular film of liquid is extended from (4 cm × 2 cm) to (5 cm × 4 cm). If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is **[NEET 2016 PHASE 2]**
(A) 0.250 Nm^{-1} (B) 0.125 Nm^{-1}
(C) 0.2 Nm^{-1} (D) 8.0 Nm^{-1}

Q.12 Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1 , θ_2 and θ_3 obey **[NEET 2016 PHASE 2]**
(A) $\pi/2 > \theta_1 > \theta_2 > \theta_3 \geq 0$ (B) $0 \leq \theta_1 < \theta_2 < \theta_3 < \pi/2$
(C) $\pi/2 < \theta_1 < \theta_2 < \theta_3 < \pi$ (D) $\pi > \theta_1 > \theta_2 > \theta_3 > \pi/2$

Q.13 The bulk modulus of a spherical object is 'B'. If it is subjected to uniform pressure 'p', the fractional decrease in radius is **[NEET 2017]**
(A) B/3p (B) 3p/B
(C) p/3B (D) p/B

Q.14 A U tube with both ends open to the atmosphere, is partially filled with water. Oil, which is immiscible with water, is poured into one side until it stands at a distance of 10 mm above the water level on the other side. Meanwhile the water rises by 65 mm from its original level (see diagram). The density of the oil is **[NEET 2017]**



(A) 425 kg m^{-3} (B) 800 kg m^{-3}
(C) 928 kg m^{-3} (4) 650 kg m^{-3}

Q.15 A small sphere of radius 'r' falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to **[NEET 2018]**
(A) r^5 (B) r^2
(C) r^3 (4) r^4

Q.16 Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area 3A. If the length of the first wire is increased by $\Delta\ell$ on applying a force F, how much force is needed to stretch the second wire by the same amount? **[NEET 2018]**
(A) 4 F (B) 6 F (C) 9 F (D) F

Q.17 A soap bubble, having radius of 1 mm, is blown from a detergent solution having a surface tension of 2.5×10^{-2} N/m. The pressure inside the bubble equals at a point Z_0 below the free surface of water in a container. Taking g = 10 m/s^2, density of water = 10^3 kg/m^3, the value of Z_0 is : **[NEET 2019]**
(A) 100 cm (B) 10 cm (C) 1 cm (D) 0.5 cm

Q.18 A small hole of area of cross-section 2 mm^2 is present near the bottom of a fully filled open tank of height 2 m. Taking g = 10 m/s^2, the rate of flow of water through the open hole would be nearly **[NEET 2019]**
(A) 12.6×10^{-6} m^3/s (B) 8.9×10^{-6} m^3/s
(C) 2.23×10^{-6} m^3/s (D) 6.4×10^{-6} m^3/s

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	C	A	D	A	C	C	C	C	D	D	A	C	B	B	B	C	D	A	B	B	A	A	C	C	D	B	B	A	B	B
Q	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	B	C	C	C	D	C	B	C	B	B	D	A	D	B	A	A	A	C	C	A	A	C	C	B	D	B	B	A	A	C

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
A	C	A	B	A	B	B	A	C	A	B	B	A	B	D	B	A	A	A	B	B	C	A	B

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10
A	11	10	2	120	12	7	3	2	6	6

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	A	D	C	B	A	B	D	C	A	C	D	D	B	D	D	D	D	C	C	A	C	D	D
Q	26	27	28	29	30	31	32	33	34																
A	A	C	C	C	D	B	B	D	A																

EXERCISE - 5

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	B	A	B	C	B	B	C	C	C	D	B	B	C	C	A	C	C	A

PROPERTIES OF MATTER

TRY IT YOURSELF-1

(1) Let L, A and ρ be length, area of cross section and density of the material of the wire. Then weight of the wire is

The stress due to this weight is

$$\frac{W}{A} = L\rho g$$

This is equal to the breaking stress (given).

Therefore,

$L\rho g = 20 \times 10^5$

or $L = \frac{20 \times 10^5}{2.5 \times 10^3 \times 10} = \frac{200}{2.5} = 80$ meter.

(2) (B)

(3) (D)

(4) (D)

(5) (D)

(6) (AD)

(7) (AD)

(8) (AD)

(9) No

(10) Infinite

TRY IT YOURSELF-2

(1) (C)

(2) (AD)

(3) No

(4) Here $R = 3.00\text{ mm} = 3 \times 10^{-3}$ m, $T = 4.65 \times 10^{-1}\text{ Nm}^{-1}$.

Pressure inside the drop = atmospheric pressure + excess pressure

$$= P + \frac{2T}{R} = 1.01 \times 10^5 + \frac{2 \times 4.65 \times 10^{-1}}{3.00 \times 10^{-3}}$$

$= 1.01 \times 10^{15} + 2 \times 1.55 \times 10^2 = 1.0131 \times 10^5\text{ NM}^{-2}$.

(5) There are two free surfaces of the film (film has two sides). Since by definition T, the surface tension is force per unit length, the upward pull on the wire AB is $F = T(2\ell)$

For equilibrium this must be balanced by weight mg. Thus

$mg = T(2\ell)$ or $m = \frac{T(2\ell)}{g}$

(6) When the stop cock T is opened, then air will rush from higher pressure region. For the smaller bubble, radius is less i.e. $r < R$.

Therefore $P_{excess} = \frac{4T}{r}$, is more for the smaller bubble.

Therefore, when valve T is opened, then air rush from smaller to the bigger bubble. The smaller bubble shrinks and the bigger bubble expands.

(7) The force required to separate plate of area A each, separated by a distance d, and a water film of surface tension T is

$$F = \frac{2AT}{d} = \frac{2 \times \pi \times (0.05)^2 \times 73 \times 10^{-3}}{0.01 \times 10^{-3}}$$

$= 36.5\,\pi \approx 115$ newton

(8) (B)

(9) **(A).** We have,

$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2 \times 0.075\text{ N/m} \times 1}{(0.20 \times 10^{-3}\text{ m}) \times (1000\text{kg/m}^3)(10\text{m/s}^2)}$$

$= 0.075\text{m} = 7.5\text{cm}$

(10) (C) $h \propto 1/r$

TRY IT YOURSELF-3

(1) (CD)

(2) Here, $r = 2 \times 10^{-3}$ m, $\rho = 1.06 \times 10^3\text{ kg m}^{-3}$.

For laminar flow $R_e = 2000$

$$R_e = \frac{\rho VD}{\eta}$$

$$\therefore\ V = \frac{\eta R_e}{\rho D} = \frac{2000 \times 2.084 \times 10^{-3}}{(1.06 \times 10^3) \times 4 \times 10^{-3}} = 0.98\text{ ms}^{-1}$$

(3) Rate of heat loss = power = F × v = $6\pi\eta r v \times v = 6\pi\eta r v^2 =$

$$6\pi\eta r\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$$

Rate of heat loss $\propto r^5$

(4) Here, $r = 2.0 \times 10^{-5}$ m, $\rho = 1.2 \times 10^3\text{ kg m}^{-3}$, $\eta = 1.8 \times 10–5\text{ Nm}^{-2}$.

From formula, terminal velocity

$$V = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

$$\Rightarrow V = \frac{2 \times (2 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}} = 5.8 \times 10^{-2}\text{ms}^{-1}$$

Now viscous force on the drop

$F = 6\pi\eta rv$

$$\Rightarrow F = 6 \times \frac{22}{7} \times (1.8 \times 10^{-5}) \times (2 \times 10^{-5}) \times 5.8 \times 10^{-2}$$

$= 3.93 \times 10^{-19}$ N

(5) (B)

(6) (B)

(7) (B)

TRY IT YOURSELF-4

(1) (A)

(2) (AB)

ℓ will decrease because the block moves up, h will decrease because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water $V_1 < V_2$.

(3) Here mass of car = 3000 kg.

Area of cross section of larger piston

$= 425\ cm^2 = 425 \times 10^{-4}\ m^2$.

$\therefore$ The maximum pressure that the smaller piston would have to bear

$$= \frac{\text{weight of car}}{\text{area of cross-section}} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5\ Nm^{-2}$$

(4) The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} in the casting : $V_{cav} = V_{cast} - V_{iron}$.

The volume of the iron is given by $V_{iron} = W/g\rho_{iron}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water can be used to find the volume of the casting. It is less than the actual weight W because the water pushes up on it with a force of $g\rho_w V_{cast}$.

That is, $W_{eff} = W - g\rho_w V_{cast}$.

Thus $V_{cast} = (W - Weff) / g\rho_w$ and

$$V_{cav} = \frac{W - W_{eff}}{g\rho_w} - \frac{W}{g\rho_{iron}}$$

$$= \frac{6000N - 4000N}{(9.8m/s^2)(998kg/m^3)} - \frac{6000N}{(9.8m/s^2)(7.87 \times 10^3 kg/m^3)}$$

$= 0.127\ m^3$.

(5) (C)

(6) **(A).** Velocity of efflux at a depth h is given by $v = \sqrt{2gh}$.

Volume of water flowing out per second from both the holes are equal.

$\therefore\ a_1 v_1 = a_2 v_2$

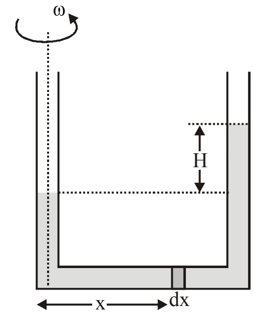
or $(L^2) = \sqrt{2g(y)} = \pi R^2 \sqrt{2g(4y)}$ or $R = \dfrac{L}{\sqrt{2\pi}}$

(7)

$$\Delta PA = \int_0^L dmx\omega^2$$

$$\rho gHA = \frac{\omega^2 L^2 \rho A}{2}$$

$$H = \frac{L^2\omega^2}{2g}$$



(8) **(ABD).**

For equilibrium,

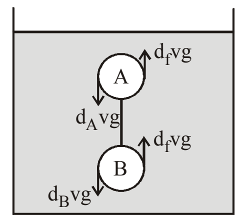
$d_A vg + d_B vg = d_F vg + d_F vg$

$$d_F = \frac{d_A + d_B}{2}$$

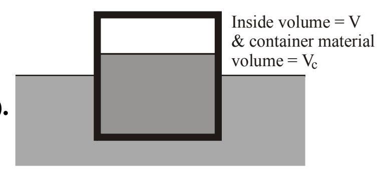
$\Rightarrow$ Option (D) is correct

To keep the string tight

$d_B > d_F$ and $d_A < d_F$



(9) **(A).**



For floating : $m_c g + m_w g = F_B$

$$\rho_C V_C g + 1 V_w g = 1\left(\frac{V}{2} + \frac{V_C}{2}\right) g$$

$$\Rightarrow V_w = \frac{V}{2} + V_C\left(\frac{1}{2} - \rho_C\right)$$

$$\text{if } \left(\rho_C < \frac{1}{2}\right) \text{ then } V_w > \frac{V}{2}$$

CHAPTER-11 : PROPERTIES OF MATTER EXERCISE-1

(1) (C). $Y = \frac{F/A}{\ell/L}$ $\therefore$ $F = \frac{YA\ell}{L}$. Force constant $= \frac{F}{\ell} = \frac{YA}{L}$

(2) (A). $Y = \frac{MgL}{\pi r^2 \Delta\ell}$ but $Mg/\pi r^2 = 20 \times 10^8$ & $\Delta\ell = L$ then

$Y = 20 \times 10^8\ N/m^2$

(3) (D). Work done on the wire

$$W = \frac{1}{2} F \times \ell = \frac{1}{2} \times \text{stress} \times \text{volume} \times \text{strain}$$

$$W = \frac{1}{2} \times Y \times \text{strain}^2 \times \text{volume}$$

$$W = \frac{1}{2} \times Y \times \frac{\Delta\ell^2}{L^2} \times AL = \frac{YA\Delta L^2}{2L}$$

$$W = \frac{2 \times 10^{11} \times 10^{-6} \times 10^{-6}}{2 \times 1} = 0.1\ J$$

(4) (A). Limiting stress $= 4.0 \times 10^8\ N/m^2$

$$\frac{F}{A} = \frac{400}{A} = 4.0 \times 10^8 \text{ or } A = 10^{-6}\ m^2$$

$$\therefore D = \left(\frac{4A}{\pi}\right)^{1/2} = \left(\frac{4 \times 10^{-6}}{\pi}\right)^{1/2} = 1.13 \times 10^{-3}\ m = 1.13mm$$

(5) (C). Stress $= \frac{F}{A} = \frac{4.8 \times 10^3\ N}{1.2 \times 10^{-4}\ m^2} = 4.0 \times 10^7\ N/m^2$

(6) (C). Strain $= \frac{\Delta\ell}{\ell} = \frac{1 \times 10^{-3}}{2} = 5 \times 10^{-4}$, longitudinal

(7) (C). Potential energy per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times (YS)S = \frac{1}{2} YS^2$$

(Stress $= Y \times$ strain $= Y \times S$)

(8) (C). Stress $= \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$ $\therefore$ Stress, $S \propto \frac{1}{r^2}$

$$\therefore \left(\frac{S_1}{S_2}\right) = \left(\frac{r_2}{r_1}\right)^2 \quad \text{Given } \frac{r_1}{r_2} = \frac{2}{1} \quad \therefore \frac{S_1}{S_2} = \frac{1}{4}$$

(9) (D). $Y = \frac{\text{Stress}}{\text{Strain}} = \text{Constant}$

It depends only on nature of material.

(10) (D). $\frac{Y_A}{Y_B} = \frac{\tan 60°}{\tan 30°} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$ or $Y_A = 3Y_B$

(11) (A). Here, $r = \frac{3.0}{2} = 1.5\ mm = 1.5 \times 10^{-3}\ m$

$L_c = 2.2\ m$, $L_s = 1.6\ m$

$$\Delta L_c + \Delta L_s = 0.7 \times 10^{-3}\ m \quad \text{.......... (1)}$$

$Y_c = 1.1 \times 10^{11}\ Nm^{-2}$, $Y_s = 2 \times 10^{11}\ Nm^{-2}$.

$$Y = \frac{MgL}{\pi r^2 \Delta L} \quad \therefore \Delta L_c = \frac{FL_c}{Y_c \pi r_c^2}$$

$$\Rightarrow \Delta L_c = \frac{F \times 2.2}{1.1 \times 10^{11} \times 3.14 \times (1.5 \times 10^{-3})^2} \quad \text{.......... (2)}$$

$\therefore$ Using eq. (i), (ii) and (iii)

$$F = \left[\frac{2.2}{1.1 \times 10^{11} \times 2.25 \times 10^5} + \frac{16}{2 \times 3.14 \times 2.25 \times 10^5}\right]$$

$= 0.7 \times 10^{-3}$ or $F = 1.8 \times 10^2\ N$

(12) (C). Shear modulus (or modulus of rigidity) is generally less than Young's modulus. For most materials $\eta \approx Y/3$.

(13) (B). Young's modulus depends upon the nature of the material and not on geometrical dimensions.

(14) (B). Mass of liquid in capillary tube $M = \pi R^2 H \times \rho$

$$\therefore M \propto R^2 \times \left(\frac{1}{R}\right) \quad (\text{As } H \propto 1/R) \therefore M \propto R.$$

If radius becomes double then mass will becomes twice.

(15) (B). Mercury does not wet glass, wood or iron because cohesive force is greater than adhesive force.

(16) (C). The cohesive force is the force of attraction between the molecules of same substance

(17) (D). Cohesive force > Adhesive force, so shape of liquid surface near the solid would be convex.

For example mercury surface in glass capillary is convex.

(18) (A). $W = 8\pi T(R_2^2 - R_1^2) = 8\pi T[(2R)^2 - R^2] = 24\pi R^2 T$

(19) (B). Both liquids water and alcohol have same nature (i.e. wet the solid). Hence angle of contact for both is acute.

(20) (B). $T = \frac{rhdg}{2\cos\theta} \Rightarrow$ Pressure difference $= hdg = \frac{2T}{r}\cos\theta$

(21) (A). $\Delta P = \frac{4T}{r} \Rightarrow \frac{\Delta P_1}{\Delta P_2} = 4 \therefore \frac{r_2}{r_1} = 4$; $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{64}$

(22) (A). $\frac{P_1}{P_2} = \frac{r_2}{r_1} = \frac{4}{1}$

(23) (C). Temperature $\uparrow$, surface tension $\downarrow$, height $\downarrow$

So, $\theta_2 > \theta_1$.

(24) **(C).** Since $P_0 - P_{in} = T\left[\frac{1}{r} - \frac{1}{R}\right]$,

r = radius of meniscus = $\frac{d}{2}$, $R >> d$

$\Delta P = \frac{T}{r}$, $F = (\Delta P)\, A = \frac{TA}{r} = \frac{2TA^2}{V}$

(25) **(D).** When a drop splits up in the number of tiny drops, then then surface area increases, volume remains constant. In this process energy is absorbed.

(26) **(B).** The onset of turbulence in a liquid is determined a dimensionless parameter called Reynolds number.

(27) **(B).** Since $F = 6\pi\eta r v$ so $F \propto v$

(28) **(A).** Since Effective force $= V(d_1 - d_2)\,g$ but

$\frac{M}{d_1} = V$, effective force will be $Mg\left(1 - \frac{d_2}{d_1}\right)$

(29) **(B).** With increase in temperature, the viscosity of liquids decreases and that of gases increases.

(30) **(B).** For turbulent flow, the value of Reynolds number is $R_e > 2000$.

(31) **(B).** When the flow parameters of any given instant remain same at every point, then flow is said to be steady state.

(32) **(C).** Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)\,g}{\eta}$

When, the two drops of same radius r coalesce then radius of new drop is R.

$\therefore\ \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}\cdot r$

Critical velocity $\propto r^2$ $\therefore\ \frac{v}{v_1} = \frac{r^2}{2^{2/3}\cdot r^2}$ $\Rightarrow$ $v_1 = \sqrt[3]{4}\cdot v$

(33) **(C).** Bernoulli's equation for steady, non-viscous, incompressible flow expresses the conservation of energy.

(34) **(C).** Since, up thrust $(F) = V\sigma g$ i.e. $F \propto V$

(35) **(D).** This happens in accordance with equation of continuity and this equation was derived on the principle of conservation of mass and it is true in every case, either tube remain horizontal or vertical.

(36) **(C).** $R.D = \frac{W\,(air)}{W(lost)}$; $RD_{(I)} = \frac{W}{W_1}$; $RD_{(L)} = \frac{W}{W_2}$

$\frac{RD_{(I)}}{RD_{(L)}} = \frac{W_2}{W_1}$; $\frac{W_1}{W_2} = \frac{RD\,(L)}{RD\,(I)} > 1$

(37) **(B).** The thermal stress in a rod is stress $= Y\,\alpha\,(\Delta T)$

Thus for same rise of temperature (ΔT), the thermal stress will be equal if

$Y_1\alpha_1 = Y_2\alpha_2$ or $Y_1 : Y_2 = \alpha_2 : \alpha_1 = 3:2$

(38) **(C).** The elastic energy stored in a wire of Young's modulus $Y = \frac{\text{strain}^2 \times \text{volume}}{2Y}$

(39) **(B).** $\Delta h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{R\rho g} \Rightarrow R = \frac{2S}{\Delta h \rho g}$

(40) **(B).** Elongation of thin rod under its own weight is mgL/2YA.

(41) **(D).** As fluid above two points or below it are of same nature, the pressure at these two points are same.

(42) **(A).** Let ΔL be the elongation. Then, by Hooke's law,

$\frac{F}{A} = Y\frac{\Delta L}{L}$ where Y is Young's modulus.

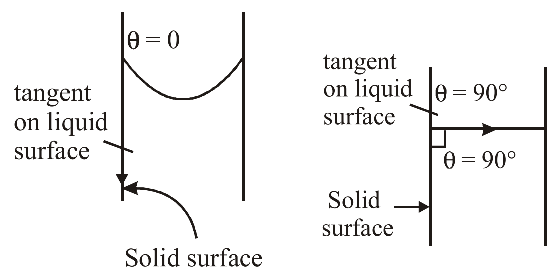
The elongation is

$$\Delta L = \frac{1}{Y}\frac{F}{A}L = \frac{mgL}{YA} = \frac{(20)\,(9.8)\,(4.0)}{(196\times10^9)\,\pi\,(0.001)^2}$$

$= 1.273 \times 10^{-3}$ m $= 1.273$ mm

(43) **(D).** $\Delta\ell = \frac{F\ell}{\pi r^2 y} \Rightarrow \Delta\ell \propto \frac{\ell}{r^2}$. Only option radius 3mm, length 2m is satisfying the given condition.

(44) **(B).** For hemispherical shape   For flat surface



(45) **(A).** From continuity equation, velocity at cross- section (1) is more than that at cross-section (2). Hence $P_1 < P_2$.

(46) **(A).** Let ρ_s, ρ_L be the density of silver and liquid. Also m and V be the mass and volume of silver block.

$\therefore$ Tension in string = mg – bouyant force

$T = \rho_s Vg - \rho_L Vg = (\rho_s - \rho_L)Vg$ Also $V = \frac{m}{\rho_s}$

$\therefore\ T = \left(\frac{\rho_s - \rho_s}{\rho_s}\right) mg = \frac{(10 - 0.72)\times10^3}{10\times10^3} \times 4\times10 = 37.12N$

(47) **(A).** Increasing the temperature of water from 2°C to 3°C increases its density while decreases the density of iron. Hence the bouyant force increases.

(48) **(C).** By dimensional analysis, (C) is the only correct answer.

(49) **(C).** The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

(50) **(A).** Viscous force = mg sin ρ

or $\eta a^2\frac{v}{t} = a^3\rho g\sin\theta$ or $\eta = \frac{t\rho g\sin\theta . a}{v}$

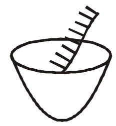
(51) **(A).** As masses are same interface cannot lie in vertical limb. At rest pressure at bottom level is same in both limbs. $\rho_w g y_w = \rho_0 g y_0 \Rightarrow y_w = \frac{\rho_0}{\rho_w} y_0 \Rightarrow y_w < y_0$

(52) **(C).** Pressure at all points in stream will be atmospheric.

(53) **(C).** $A_1v_1 = A_2v_2 + A_3v_3$

$A \times 3 = A \times 1.5 + 1.5A \times v_3 \Rightarrow v_3 = 1$ m/s

(54) **(B).** Figure shows one of the legs of the mosquito landing upon the water surface.



Therefore, $T = 2\pi a \times 8 = W$ = weight of the mosquito.

(55) **(D).** $F_{req} = mg + 2\,[T\,(2\,\pi R)]$ $[T = 75 \times 10^{-3}$ N/m$]$

$= 0.1 + 2\,[75 \times 10^{-3}\,(0.2)] = 0.130$ N

(56) **(B).** Let V be the volume of iceberg and let x be the fraction of volume above water. Using law of floatation, weight of floating body = weight of liquid displaced by part of the floating body inside the liquid. Therefore,

$V\rho_{ice}g = (1 - x)\,V\rho_{water}g.$

Using the value of ρ_{ice} and ρ_{water}, we get x = (13/103).

(57) **(B).** $T + 0.8 \times 250 \times 10^{-3} g = 250\, d_\ell g$

$T + 250\, d_\ell g = 1.2 \times 250 \times 10^{-3} g$

Solving, $T = 0.5$ N

(58) **(A).** As net force on the sphere is zero therefore centre of mass does not move. But as there is a torque on sphere due to mg and buoyant force in clockwise direction there sphere rotates clockwise and hence equilibrium is achieved when O comes directly above C.

(59) **(A).** Density is maximum at 4°C.

(60) **(C).** Force by soap film, on the side is independent of surface area.

EXERCISE-2

(1) **(C).** Net elongation $= \frac{FL}{AY} - \frac{F\,(L/2)}{AY} = \frac{FL}{2AY}$

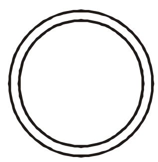
(2) **(A).** Here $L_s = 4.7$ m, $L_c = 3.5$ m, $a_s = 3.0 \times 10^{-5}$ m^2, $a_c = 4.0 \times 10^{-5}$ m^2

$\Delta L_s = \Delta L_c$ $\quad F_s = F_c$

$$Y_s = \frac{F_s L_s}{a_s \Delta L_s} \quad \text{and} \quad Y_c = \frac{F_c L_c}{a_c \Delta L_c}$$

$$\frac{Y_s}{Y_c} = \frac{L_s}{a_s} \cdot \frac{a_c}{L_c} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4 \times 10^{-5}}{3.5} = 1.79$$

(3) **(B).** Strain $= \frac{\Delta\ell}{\ell} = \left(\frac{2\pi R - 2\pi r}{2\pi r}\right)$



Strain $= \left(\frac{R - r}{r}\right)$

$$Y = \frac{R/A}{\Delta\ell/\ell}, \quad Y\frac{\Delta\ell}{\ell}A = F \;;\; F = AY\left(\frac{R - r}{r}\right)$$

(4) **(A).** Work $W = \frac{1}{2}F\Delta\ell$

Since $\Delta\ell$ is the same in both the cases, $W \propto F$

Now, $F = \frac{YA\Delta\ell}{\ell} = \frac{Y\pi r^2 \Delta\ell}{\ell}$

$$\therefore \frac{F_1}{F_2} = \frac{r_1^2 \ell_2}{\ell_1 r_2^2} = \left(\frac{r_1}{r_2}\right)^2 \cdot \frac{\ell_2}{\ell_1} = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$$

$\therefore F_2 = 8F_1 \Rightarrow W_2 = 8W_1$

Work necessary in the second case = 8 × 2 = 16J

(5) **(B).** Let ℓ_0 be the unstretched length and ℓ_3 be the length under a tension of 9N.

$$\text{Then,} \quad Y = \frac{4\ell_0}{A\,(\ell_1 - \ell_0)} = \frac{5\ell_0}{A\,(\ell_2 - \ell_0)} = \frac{9\ell_0}{A\,(\ell_3 - \ell_0)}$$

These give $\frac{4}{\ell_1 - \ell_0} = \frac{5}{\ell_2 - \ell_0} \Rightarrow \ell_0 = 5\ell_1 - 4\ell_2$

Further, $\frac{4}{\ell_1 - \ell_0} = \frac{9}{\ell_2 - \ell_0}$

Substituting the value of ℓ_0 and solving, we get

$\ell_3 = 5\ell_2 - 4\ell_1$

(6) **(B).** $\Delta L = \frac{LF}{AY}$, where L = 3m,

$A = \pi\,(1.0 \times 10^{-3}\text{ m})^2 = 3.14 \times 10^{-6}\text{ m}^2$,

and since each wire supports one-quarter of the load,

$$F = \frac{(50\text{ kg})\,(9.8\text{ m/s}^2)}{4} = 123\text{ N}$$

$$\Delta L = \frac{(3\text{m})\,(123\text{N})}{(3.14 \times 10^{-6}\text{m}^2)\,(1.8 \times 10^{11}\text{N/m}^2)}$$

$= 65 \times 10^{-5}$ m or 0.65 mm

(7) **(A).** To find the minimum diameter, and hence minimum cross-sectional area, we assume that the force F = 400 N brings us to the elastic limit. Then from the stress, $F/A = 379 \times 10^6$ Pa, we get

$$A = \frac{400\text{N}}{379 \times 10^6\text{ Pa}} = 1.0554 \times 10^{-6}\text{m}^2 \;;\; A = \frac{\pi D^2}{4}$$

$$D^2 = \frac{4A}{\pi} = \frac{4\,(1.0554 \times 10^{-6}\text{m}^2)}{\pi} = 1.344 \times 10^{-6}$$

and $D = \sqrt{1.344 \times 10^{-6}\text{m}^2} = 1.16 \times 10^{-3}$ m $= 1.16$ mm

(8) **(C).** The bulk modulus is defined as $B = -\frac{\Delta p}{\Delta V/V}$, where the minus sign is inserted because ΔV is negative when Δp is positive.

$$100\left|\frac{\Delta V}{V}\right| = 100\frac{\Delta p}{B} = 100\,\frac{345 \times 10^6}{138 \times 10^9} = 0.25\%$$

(9) **(A).** Decrease in temperature would cause shrinking of wire, as wire is attached at 2 ends, this would result in tension (stress) in wire $a = 2 \times 10^{-6}$

$$\frac{F}{A} = \frac{\Delta\ell}{\ell} \; ; \; \frac{\Delta\ell}{\ell} = \frac{2.2\times10^{8}}{1.1\times10^{11}} = 2\times10^{-3}$$

$$\frac{\Delta\ell}{\ell} = \alpha\Delta T = (2\times10^{-3}) = (2\times10^{-6})\,(\Delta T)$$

$\Delta\ell = 1000°C$

(10) **(B).** If T be the tension in the rope, then T = 10kN

T = 5 kN.

$\therefore$ Longitudinal stress in the rope

$$\sigma = \frac{T}{A} = \frac{5kN}{10^3\,mm^2} = 5\ Nmm^{-2}$$

$\therefore$ Extension in the rope

$$= \frac{\text{Stress}}{Y}\times L = \frac{5\ Nmm^{-2}}{10^3\ Nmm^{-2}}\times 1500mm = 7.5mm$$

$\therefore$ Deflection of the load $\delta = \dfrac{7.5}{2} = 3.75mm$

(11) **(B).** $$\frac{\Delta\ell_{steel}}{\Delta\ell_{brass}} = \frac{\dfrac{3Mg}{A_sY_s}\ell_s}{\dfrac{2Mg}{A_bY_b}\ell_b} = \frac{3}{2}\frac{\ell_s}{\ell_b}\frac{A_bY_b}{A_sY_s} = \frac{3}{2}\frac{a}{b^2c}$$

(12) **(A).** Young's modulus $Y = \dfrac{F}{A}.\dfrac{L}{\ell}$

Force constant $k = \dfrac{F}{l} = \dfrac{YA}{L}$

Where ℓ is the extension in the spring of original length L and cross-sectional area A when a force F = Mg is applied.

Now, the time period of vertical oscillations is given

by : $T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{ML}{YA}}$; $\dfrac{T_1}{T_2} = \sqrt{\dfrac{Y_2}{Y_1}} = \sqrt{\dfrac{3}{2}}$

(13) **(B).** Here, M = 50000 kg

Area of cross sectional of 4 cylindrical columns

$a = 4\pi\,(0.6^2 - 0.3^2)$; $Y = 2.0 \times 10^{11}\ Nm^{-2}$.

$$\therefore\ Y = \frac{MgL}{a\Delta L} \quad \therefore \frac{\Delta L}{L} = \frac{Mg}{aY}$$

$$= \frac{50000\times9.8}{4\times3.14\times(0.6^2-0.3^2)\times2\times10^{11}} = 7.2\times10^{-7}$$

(14) **(D).** Here, $a = (0.1)^2 = 0.01\ m^2$, L = 10 cm = 0.10 m, m = 100 kg

$\therefore F = 100 \times 10 = 10^3\ N$, $\eta = 25 \times 10^9\ Nm^{-2}$. $\Delta x = ?$

From formula, $\eta = \dfrac{F.L}{\Delta x / L}$ or $\Delta x = \dfrac{F.L}{a.\eta}$

$$\Delta x = \frac{10^3\times0.1}{0.01\times25\times10^9} = 4\times10^{-7}\ m$$

(15) **(B).** The change in length of rod due to increase in temperature in absence of walls is

$\Delta\ell = \ell \propto \Delta T = 1000 \times 10^{-4} \times 20mm = 2mm.$

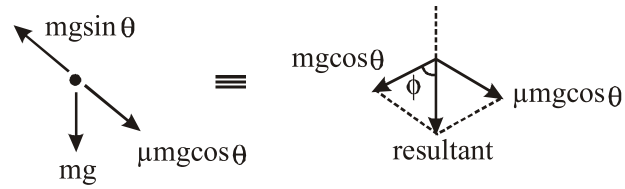
But the rod can expand upto 1001mm only.

At that temperature its natural length = 1002 mm.

$\therefore$ Compression = 1mm

Mechanical stress = $Y\dfrac{\Delta\ell}{\ell} = 10^{11}\times\dfrac{1}{1000} = 10^8\ N/m^2$

(16) **(A).** Figure shows forces acting on a particle on the surface, with respect to vessel.



(mg sin θ and μ mg cos θ are pseudo forces).

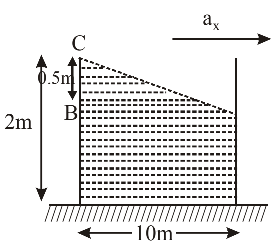
$\tan\phi = \mu$ $\therefore \phi = \tan^{-1}\mu$

ϕ is angle between normal to the inclined surface and the resultant force.

The same angle will be forced between the surface of water and the inclined surface.

($\because$ free surface is $\perp$ to the resultant force acting on it)

(17) **(A).** $v = u + a_x \Rightarrow a_x = \dfrac{v}{t}$



$\tan\theta = \dfrac{a_x}{g} = \dfrac{v}{tg} = \dfrac{0.5}{5}$

(in triangle ABC)

$\Rightarrow t = \dfrac{10\times20}{10} = 20$ sec.

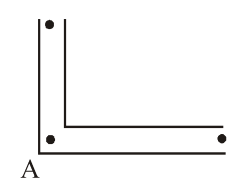
(18) **(A).** No sliding $\Rightarrow$ pure rolling

Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

$P_A = P_{atm} + \rho(2a)L$ (From horizontal limb)

Also, $P_A = P_{atm} + \rho gh$ (From vertical lmb)

$\Rightarrow a = \dfrac{gH}{2L}$



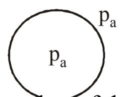
(19) **(B).** The velocity of water outflow from a hole is $v = \sqrt{2gh}$. The impulse of the force acting from the side of the vessel on the outflowing water $F\Delta t = \Delta mv$, where $\Delta m = \rho Av\Delta t$ is the mass of the water flowing out during the time Δt. Hence, $F = \rho v^2A = 2\rho ghA$. The pressure at the bottom $p = \rho gh$ and therefore F = 2pA. The same force acts on the vessel from the side of the stream. Thus, the water acts on the wall with the hole with a force 2pA smaller than that acting on the opposite wall, and net with a force smaller by pA as might be expected. This is due to a reduction in the pressure acting on the wall with the hole, since the water flows faster at this wall.

The vessel will begin to move if $\mu G < 2pA$ or $\mu < \dfrac{2\rho g h A}{G}$

($\mu \rightarrow$ coefficient of friction)

(20) **(B).** Inside pressure must be 4T / r greater than outside pressure in bubble.
This excess pressure is provided by charge on bubble.

$$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}\ ;\ \ \frac{4T}{r} = \frac{Q^2}{16\pi^2 r^2 \times 2\varepsilon_0}\ \left[\sigma = \frac{Q}{4\pi r^2}\right]$$



$$Q = 8\pi r\sqrt{2rT\varepsilon_0}$$

(21) **(C).** Let σ be the density of the gas, then that of the air is 15σ. Then the weight of the balloon = weight of the gas + weight of the envelope = $Vg\sigma + w$.
If f be the required acceleration of the balloon acting vertically upward and then from "mass × acceleration = forces acting in the sense of acceleration" we get

$$\frac{(Vg\sigma + w)}{g} \times a = \text{force of buoyance} - \text{wt. of the balloon}$$

with gas $= V\,15\sigma g - (Vg\sigma + w)$ or $a = \left(\dfrac{14Vg\sigma - w}{Vg\sigma + w}\right) \times g$

(22) **(A).** $W_{ST} = \pi r^2 h\rho g\ ,\ \ W_g = \dfrac{\pi r^2 h\rho g h}{2}$

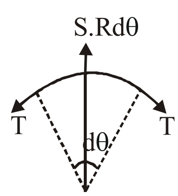
$$\text{Heat} = Q = W_{ST} - W_g = \frac{\pi r^2 \rho g h^2}{2}$$

and $\ h = \dfrac{2S}{r\rho g} \Rightarrow Q = \dfrac{2\pi S^2}{\rho g}$

(23) **(B).** Consider an element of wire forming angle θ at the centre $\ 2T\sin\dfrac{d\theta}{2} = 2 \times [S \times R \times d\theta]\ \ \therefore\ T = 2SR$

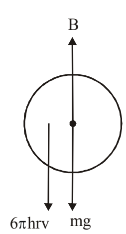
$$Y = \frac{F\ell}{A\Delta\ell}\ ;$$

$$\frac{YA \times \Delta R}{R} = 2SR\ \ ;\ \Delta R = \frac{2SR^2}{YA}$$



EXERCISE-3

(1) **11.** $100 \times \dfrac{4}{3}\pi r^3 g - 500 \times \dfrac{4}{3}\pi r^3 g - 6\pi\eta r v = 500 \times \dfrac{4}{3}\pi r^3 a$



$$\frac{0.2}{3} = 6 \times 1 \times 10^{-3} v\ \ ;\ \ \frac{200}{18} = v \Rightarrow v = 11 m/s$$

(2) **10.** $R = 2$ cm. ; $\Delta P = \dfrac{4S}{R} = \dfrac{4 \times 0.05}{2 \times 10^{-2}} = 10Pa$

(3) **2.** When the sphere moves through the liquid, we can assume that a sphere of the liquid of same volume is moving in vertically upward direction.

$\Rightarrow\ P_{Total} = v\rho_b V - v\rho_\ell V\ \ \because\ P_{Total}$ should be zero

Momentum of the liquid $= v\rho_1 V = (1)(2)(1) = 2$ kg ms^{-1}

(4) **120.** As force equal $P_A a_A = P_B a_B$, where P_A = pressure at top, a_A = area of top surface, P_B = pressure at bottom, a_B = area of bottom surface

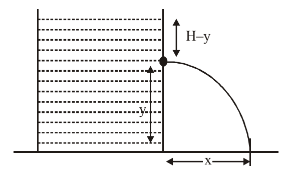
$P_B = P_0 + h\rho g = 1 \times 10^5 + (30)(1000)(10) = 4 \times 10^5$

$(1 \times 10^5)(\pi R_2^2) = 4 \times 10^5 (\pi R_1^2)$

$$\frac{R_1^2}{R_2^2} = \frac{1}{4} \Rightarrow R_1 = \frac{1}{2}R_2 \ \text{ and}$$

$$R_2 - R_1 = \frac{4}{3}h \Rightarrow R_2 - R_1 = 40$$

(5) **12.** $y = x\tan 0 - \dfrac{1}{2}\dfrac{gx^2}{(\sqrt{2g(H-y)})^2\cos^2 0}$



$$\Rightarrow -9 = -\frac{1}{2} \times \frac{gx^2}{2g \times 24} \Rightarrow x^2 = 144 \Rightarrow x = 12m$$

(6) **4.** $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{yA/\ell}{m}} = \sqrt{\dfrac{yA}{\ell m}}$

$$\Rightarrow \sqrt{\frac{(n \times 10^9) \times (4.9 \times 10^{-7})}{1 \times 0.1}} = 140 \Rightarrow n = 4$$

(7) **3.** Change in length $\Delta L = \dfrac{mgL}{YA} = L\alpha\Delta T$; $m = 3kg$

(8) **2.** AV = Constant
$D^2 V = d^2 v$

$$v = \frac{D^2}{d^2}V = \left(\frac{8}{2}\right)^2 \times 0.25 = 16 \times 0.25 = 4\ m/s$$

$$x = v\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2 \times 1.25}{10}} = 4 \times \frac{1}{2} = 2m$$

(9) **6.** $P_A = P_0 + \dfrac{4T}{r_A} \Rightarrow P_A = 8 + \dfrac{4 \times 0.04}{0.02}$; $P_A = 16\ N/m^2$

$$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04} = 12\ N/m^2$$

For bubble A, PV = nRT

$(16)\frac{4}{3}\pi\,(0.02)^3 = n_A RT$ .... (1)

For bubble B, $(12)\left(\frac{4}{3}\pi\,(0.04)^3\right) = n_B RT$ .... (2)

Dividing eq. (1) and (2), $\frac{n_A}{n_B} = \frac{1}{6}\,;\ \frac{n_B}{n_A} = 6$

(10) **6.** $P + 200 \times 10^{-3} \times 1000 \times 10 = P_0$ ....(1)

$P_0\,(500 - H) = P\,(300\text{ mm})$

$\Rightarrow\ P = \frac{P_0\,(500 - H)\text{ mm}}{300\text{mm}}$ ....(2)

From (1) and (2)

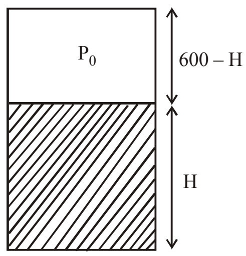
$\frac{P_0\,(500 - H)}{300} + 2000 = P_0$

$\frac{10^5\,(500 - H)}{300} + 2000 = 10^5$

$\Rightarrow\ 5 \times 10^7 - H \times 10^5 + 6 = 300$

$\Rightarrow\ H = 206\text{ mm}$

Fall in height = 6mm. **Ans : 6**



EXERCISE-4

(1) **(B).** Velocity of efflux = $v = \sqrt{2gh}$

$v = \sqrt{2 \times 10 \times 20} = 20$ m/sec.

(2) **(C).** Stored energy

$= \frac{1}{2} \times \text{force} \times \text{extension} = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1$ J

(3) **(A).** Work done $W = 1/2\ kx^2$

$W_1 = (1/2) \times 5 \times 10^3 \times (5 \times 10^{-2})^2$

$W_2 = \frac{1}{2} \times 5 \times 10^3\,(10 \times 10^{-2})^2$

$\Delta W = W_2 - W_1 = \frac{1}{2} \times 5 \times 10^3 \times 10^{-4}\,(100 - 25)$

$= \frac{5 \times 10^{-1}}{2} \times 75 = 18.75$ N-m

(4) **(D).** Work done $= \frac{1}{2} \times \text{force} \times \text{extension} = \frac{1}{2} \times F \times \ell$

(5) **(C).** $F = \eta A \frac{\Delta v}{\Delta x}\ ;\ \eta = \frac{F}{A} \times \frac{\Delta x}{\Delta v} = \frac{M^1L^1T^{-2}}{L^2} \times \frac{L}{LT^{-1}}$

(6) **(B).** $F = 6\pi\eta\, rv$

(7) **(A).** Smaller bubble have big pressure $\left(P = \frac{4T}{r}\right)$.

So air flow high pressure to lower pressure.

(8) **(B).** Stored energy $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain} = \frac{\text{stress}}{Y}$

$\frac{\text{stored energy}}{\text{volume}} = \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$

(9) **(D).** When g = 0 then water rises up to maximum height.

(10) **(C).** Tension is same in both cases, so streching will also be same.

(11) **(A).** Terminal speed $v \propto (d_0 - d_\ell)$

$\frac{v}{0.2} = \frac{10.5 - 1.5}{19.5 - 1.5} \Rightarrow \frac{v}{0.2} = \frac{9}{18} \Rightarrow v = 0.1$ m/sec

(12) **(C).** Solid ball is floating in liquid 2 so $\rho_3 < \rho_2$, and liquid 1 is floating on liquid 2. So $\rho_1 < \rho_2$.

(13) **(D).** Weight Buoyancy = viscous force

$V \times \rho_1 \times g - V \times \rho_2 \times g = kv^2$

$v = \sqrt{\frac{V_g\,(\rho_1 - \rho_2)}{k}}$

(14) **(D).** $Y = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}} \Rightarrow Y\frac{\Delta\ell}{\ell} = \frac{F}{A}\ ;\ \Delta\ell = \frac{F\ell}{AY}\ ;\ \frac{F_1\ell_1}{A_1Y_1} = \frac{F_2\ell_2}{A_2Y_2}$

$\Rightarrow\ \frac{F\,(\ell)}{AY} = \frac{F'\,(\ell/3)}{3AY} \Rightarrow F' = 9F$

(15) **(B).** $r_{oil} < r < r_{water}$. Oil is the least dense of them so it should settle at the top with water at the base. Now the ball is denser than oil but less denser than water. So, it will sink through oil but will not sink in water. So it will stay at the oil-water interface.

(16) **(D).** $W = T\,\Delta A = 0.03\,(2 \times 4\pi \times (5^2 - 3^2)\,10^{-4}$

$= 24\pi\,(16) \times 10^{-6} = 0.384\,\pi \times 10^{-3}$ Joule $= 0.4\pi$ mJ

(17) **(D).** Diameter $= 8 \times 10^{-3}$ m ; $v = 0.4$ m/s

$v = \sqrt{u^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2} = 2$ m/s

$A_1v_1 = A_2v_2$

$\pi\left(\frac{8 \times 10^{-3}}{4}\right)^2 \times 0.4 = \pi \times \frac{d^2}{4} \times 2\ ;\ d = 3.6 \times 10^{-3}$ m

(18) **(D).** $2TL = mg;\ T = \frac{mg}{2L} = \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}} = \frac{1.5}{600}$

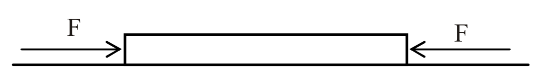
$= 0.025$ N/m

(19) **(D).** When radius is decrease by dr

Decrease in surface energy = Heat reqd. for vap.

$(4\pi r\,dr) \times T \times 2 = 4\pi r^2 dr\,\rho = L \Rightarrow r = \frac{2T}{\rho L}$

(20) **(C).** $0.10 \times 1.1 \times 10^{-5} \times 100 = \frac{F/A}{2 \times 10^{11}} \times 0.10$

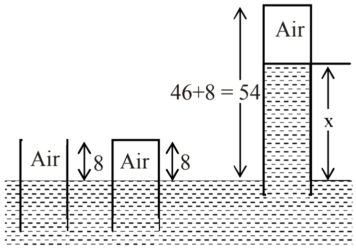


$\therefore\ \frac{F}{A} = \text{Pressure} = 1.1 \times 10^{-5} \times 100 \times 2 \times 10^{11}$

$= 2.2 \times 10^8$ Pa

(21) **(C).** $(76)(8) = (54 - x)(76 - x) \Rightarrow x = 38$ cm

Length of air column = 54 – 38 = 16 cm



(22) **(A).** $P_A = P_B$

$P_0 + d_1 gR(\cos\alpha - \sin\alpha) = P_0 + d_2 gR(\cos\alpha + \sin\alpha)$

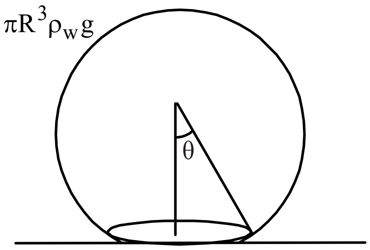
$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

(23) **(C).** $(2\pi rT)\sin\theta = \frac{4}{3}\pi R^3 \rho_w g$

$$T \times \frac{r}{R} \times 2\pi r = \frac{4}{3}\pi R^3 \rho_w g$$

$$r^2 = \frac{2}{3}\frac{R^4 \rho_w g}{T}$$

$$r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$



(24) **(D).** $T = 2\pi\sqrt{\frac{\ell}{g}}$; $T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$

$$Y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{Mg\ell}{AY} \Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

(25) **(D).** Stress $= \frac{\text{Weight}}{\text{Area}} = \frac{(9)^3 \times W_0}{(9)^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$

As volume increases by $(9)^3$ times and area increases by $(9)^2$ times.

(26) **(A).** $B = \left|\frac{\Delta P}{\Delta V / V}\right| \quad \therefore \frac{\Delta V}{V} = \frac{\Delta P}{B}$

Also, $\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$ (As $\Delta V = 4\pi r^2 \Delta r$)

$$\frac{\Delta r}{r} = \left(\frac{\Delta P}{3B}\right) = \frac{mg}{3Ka}$$

(27) **(C).** Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta\ell/\ell} = \frac{F}{A(\alpha\Delta T)}$

(28) **(C).** Reynolds Number $= \frac{\rho v d}{\eta}$

Volume flow rate $= v \times \pi r^2$

$$v = \frac{100 \times 10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}} = \frac{2}{3\pi} m/s$$

Reynolds Number $= \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi} = 2 \times 10^4$

Order $= 10^4$

(29) **(C).** Energy of catapult $= \frac{1}{2} \times \left(\frac{\Delta\ell}{\ell}\right)^2 \times Y \times A \times \ell$

= Kinetic energy of the ball $= \frac{1}{2}mv^2$

$$\frac{1}{2} \times \left(\frac{20}{42}\right)^2 \times Y \times \pi \times 9 \times 10^{-6} \times 42 \times 10^{-2}$$

$$= \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2 \ ; \ Y = 3 \times 10^6\, Nm^{-2}$$

(30) **(D).** Tensile stress in wire will be $= \frac{\text{Tensile force}}{\text{Cross section Area}}$

$$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} Nm^{-2} = 3.1 \times 10^6\, Nm^{-2}$$

(31) **(B).** Given $\frac{Y_A}{Y_B} = \frac{7}{4}$; $L_A = 2m$, $A_A = \pi R^2$

$L_B = 1.5$ m, $A_B = \pi(2mm)^2$

$$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$$

Given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B} ; \quad \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2mm)^2 \cdot Y_B}{1.5}$$

R = 1.74 mm

(32) **(B).** Using equation of continuity, $A_1V_1 = A_2V_2$

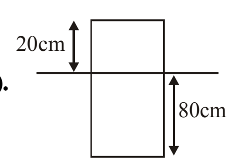
$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

(33) **(D).** In case of minimum density of liqued, sphere will be floating while completely submerged. So, mg = B.

$$m = \int_0^R \rho(4\pi r^2 dr) = \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr \ ; \ B = \frac{4}{3}\pi R^3 \rho_\ell g$$

On solving, $\rho_\ell = \frac{2\rho_0}{5}$

(34) **(A).**



$m = \rho_0 A(80)$ ....(i)

$m = \rho A\ (79)$ ....(ii)

EXERCISE-5

(1) **(B).** $Y = \dfrac{F/A}{\Delta\ell/\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{YA} = \dfrac{F\ell}{Y\pi r^2} \Rightarrow \Delta\ell \propto \dfrac{\ell}{r^2}$

which is maximum for $\ell = 50$ cm & diameter = 0.5 mm

(2) **(A).** The wettability of a surface by a liquid depends primarily on angle of contact between the surface and the liquid.

(3) **(B).** $V = A\ell,\ Y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{AY} = \dfrac{F\ell^2}{VY} \Rightarrow \Delta\ell \propto \ell^2$

(4) **(C).** Energy released = $(A_f - A_i)\,T$

$A_f = 4\pi R^2 = \dfrac{3V}{R};\quad A_i = n \times 4\pi r^2 = \dfrac{V}{\frac{4}{3}\pi r^3} 4\pi r^2 = \dfrac{3V}{r}$

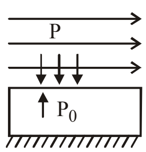
$\Rightarrow$ Energy released = $3VT\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

(5) **(B).** $B = \dfrac{\Delta P}{\Delta V/V}\ ;\ K = \dfrac{1}{B} = \dfrac{\Delta V/V}{\Delta P}$

So, $\dfrac{\Delta V}{V} = K\Delta P$

$= 45.4 \times 10^{-11} \times 10^3 \times 10 \times 2700 = 1.2258 \times 10^{-2}$

(6) **(B).** $P + \dfrac{1}{2}\rho v^2 = P_0 + 0$



$\Delta P = \dfrac{1}{2}\rho v^2$

$F_{near} = \dfrac{1}{2} \times 1.2 \times 40 \times 40 \times 250\ N = 2.4 \times 10^5\ N$

(7) **(C).** 

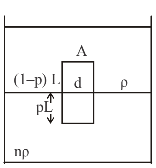
Av = constant

$\pi R^2 V = n\pi r^2 v_1 \Rightarrow v_1 = \dfrac{VR^2}{nr^2}$

(8) **(C).** $Y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{AY}\ ;\ (\Delta\ell)_{steel} = (\Delta\ell)_{brass}$

$\dfrac{W_s\ell}{AY_s} = \dfrac{W_B\ell}{AY_B} \Rightarrow \dfrac{W_s}{W_B} = \dfrac{Y_s}{Y_B} = 2$

(9) **(C).** Water will not overflow but will change its radius of curvature.

(10) **(D).** 

Weight of cylinder = $Th_1 + Th_2$

$ALdg = (1 - p)\,LA\rho g + (pLA)\,n\rho g$

$\Rightarrow\ d = (1 - p)\,\rho + pn\rho = \rho - p\rho + np\rho$

$= \rho + (n - 1)\,p\rho = \rho\,[1 + (n - 1)\,p]$

(11) **(B).** $W = 2\,(A_f - A_i)\,T$

$T = \dfrac{W}{(A_f - A_i) \times 2} = \dfrac{3 \times 10^{-4}\,J}{2\,[5 \times 4 \times 10^{-4} - 4 \times 2 \times 10^{-4}]}$

$= 0.125\ Nm^{-1}$

(12) **(B).** $h = \dfrac{2T\cos\theta}{r\rho g}$

$\rho \propto \cos\theta$ (as T, h and r are constants)

$\rho \uparrow \Rightarrow \theta \downarrow;\quad \theta_1 < \theta_2 < \theta_3$

Its rise so $0 \le \theta_1 < \theta_2 < \theta_3 < \pi/2$

(13) **(C).** $B = \dfrac{\Delta P}{-\frac{\Delta V}{V}},\ \dfrac{\Delta V}{V} = \dfrac{3\Delta R}{R};\ B = \dfrac{\Delta P}{-\frac{3\Delta R}{R}} \Rightarrow -\dfrac{\Delta R}{R} = \dfrac{P}{3B}\ (\Delta P = P)$

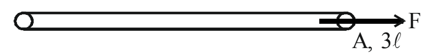
(14) **(C).** $\rho_0 g \times 140 \times 10^{-3} = \rho_w g \times 130 \times 10^{-3}$

$\rho_0 = \dfrac{130}{140} \times 10^3 \approx 928\ kg/m^3$

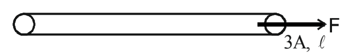
(15) **(A).** Power $= 6\pi\eta r V_T . V_T = 6\pi\eta r V_T^2\ ;\ V_T \propto r^2$

$\Rightarrow$ Power $\propto r^5$

(16) **(C).** For wire 1, $\Delta\ell = \left(\dfrac{F}{AY}\right) 3\ell$ .... (i)



For wire 2, $\dfrac{F'}{3A} = Y\dfrac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \left(\dfrac{F'}{3AY}\right)\ell$ .... (ii)



From equation (i) & (ii),

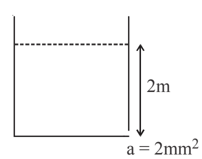
$\Delta\ell = \left(\dfrac{F}{AY}\right) 3\ell = \left(\dfrac{F}{3AY}\right)\ell \Rightarrow F' = 9F$

(17) **(C).** Excess pressure = 4T/R

Gauge pressure $= \rho g Z_0\ ;\ P_0 + \dfrac{4T}{R} = P_0 + \rho g Z_0$

$Z_0 = \dfrac{4T}{R \times \rho g} = \dfrac{4 \times 2.5 \times 10^{-2}}{10^{-3} \times 1000 \times 10}\ m = 1\ cm$

(18) **(A).** Rate of flow liquid



$Q = au = a\sqrt{2gh}$

$= 2 \times 10^{-6}\ m^2 \times \sqrt{2 \times 10 \times 2}\ m/s$

$= 2 \times 2 \times 3.14 \times 10^{-6}\ m^3/s$

$= 12.56 \times 10^{-6}\ m^3/s = 12.6 \times 10^{-6} m^3/s$