11

PROBABILITY

INTRODUCTION

There are various phenomenon in nature, leading to an outcome, which cannot be predicted a priori e.g. in tossing of a coin, a head or a tail may result. Probability theory aims at measuring the uncertainties of such outcomes.

BASIC TERMS

1. An Experiment : An action or operation resulting in two or more outcomes is called an experiment

Ex. (i) Tossing of a coin is an experiment. There are two possible outcomes head or tail. (ii) Drawing a card from a pack of 52 cards is an experiment There are 52 possible outcomes.

2. Sample space : The set of all possible outcomes of an experiment is called the sample space, denoted by S.

An element of S is called a sample point.

Ex. (i) In the experiment of tossing of a coin, the sample space has two points corresponding to head (H) and Tail (T) i.e. S {H,T.}

Ex. (ii) When we throw a die then any one of the numbers 1, 2, 3, 4, 5 and 6 will come up. So, the sample space.

$S = \{1, 2, 3, 4, 5, 6\}$

3. Event: Any subset of sample space is an event.

Ex. (i) If the experiment is done throwing a die which has faces numbered 1 to 6, then $S = \{1, 2, 3, 4, 5, 6\}$

Then $A = \{1, 3, 5\}$, B $\{2, 4, 6\}$, the null set ϕ and S itself are some events with respect to S. The null set ϕ is called the impossible event or null event.

Ex.(ii) Getting 7 when a die is thrown is called a null event. The entire sample space is called the certain event.

4. Complement of an Event : The complement of an event A, denoted by $\overline{A}$, A' or A^c , is the set of all sample points of the space other then the sample points in A.

Ex. In the experiment of casting a fair die.

$S = \{1, 2, 3, 4, 5, 6\}$

If $A = \{1, 3, 5, 6\}$ then $A^c = \{2, 4\}$

$A \cup A^c = S$, $A \cap A^c = \phi$

5. Simple Event: An event is called a simple event if it is a singleton subset of the sample space S.

Ex. (i) When a coin is tossed, sample space $S = \{H, T\}$

Let $A = \{H\}$ = the event of occurrence of head.

and $B = \{T\}$ = the event of occurrence of tail.

Here A and B are simple events.

Ex. (ii) When a die is thrown, sample space

$S = \{1, 2, 3, 4, 5, 6\}$

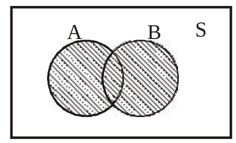
Let $A = \{5\}$ = the event of occurrence of 5

$B = \{2\}$ = the event of occurrence of 2

Here A and B are simple events

6. Compound Event : It is the joint occurrence of two or more simple events. **Ex.** The event of at least one head appears when two fair coins are tossed is a compound event i.e. $A = \{HT, TH, HH\}$

7. The Event 'A or B' : Union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both.

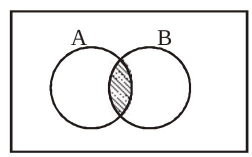


A Ć B

When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

Therefore Event 'A or B' $= A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

8. The Event 'A and B' : We know that intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B. i.e., which belong to both 'A and B'.



$A \cap B$

If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'. Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

For example, in the experiment of 'throwing a die twice' Let A be the event 'score on the first throw is six' and B is the event 'sum of two scores is atleast 11' then

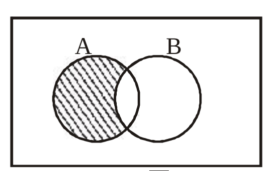
$A = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5) (6, 6)\}$

and $B = \{(5, 6), (6, 5), (6, 6)\}$

so $A \cap B = \{(6, 5), (6, 6)\}$

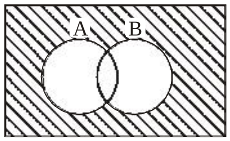
Note that the set $A \cap B = \{(6, 5), (6, 6)\}$ may represent the event ' the score on the first throw is six and the sum of the scores is atleast 11.'

9. The Event 'A but not B' : We know that A – B is the set of all those elements which are in A but not in B. Therefore, the set A – B may denote the event 'A but not B'. We know that $A - B = A \cap B'$



$A \cup \overline{B}$

10. **The Event 'neither A nor B' :** The set of the elements which are neither in set A nor in set B. i.e. $S - (A \cup B)$ and which is denoted on $\overline{A} \cap \overline{B}$.



$\overline{A} \cap \overline{B}$

11. **Equally Likely Events :** A number of simple events are said to be equally likely if there is no reason for one event to occur in preference to any other event.

Ex. In drawing a card from a well shuffled pack, there are 52 outcomes which are equally likely.

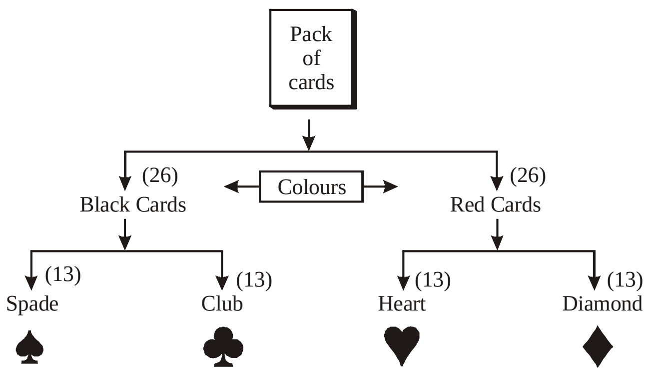
NOTE

Designation of Cards :

Colours : There are two colours. Red & Black

Suits : There are four (4) suits (types).

Each suit contains 13 cards



Recognition of Cards :

	K King	Q Queen	J Jack	A Ace
♥	1	1	1	1
♦	1	1	1	1
♠	1	1	1	1
♣	1	1	1	1
	4	4	4	4

(i) **Face Cards :** Face cards contain 12 cards all of K, Q and J having designed a figure of a person.
i.e., Face cards = 4 + 4 + 4 = 12,

(ii) **Honours Cards :** It contains all face cards and also a card marked A. i.e. Honours cards = (4 + 4 + 4) + 4 = 16 cards.

(iii) **Knave Cards :** (10, J, Q) = 4 + 4 + 4 = 12 cards

12. **Exhaustive Events:** All the possible outcomes taken together in which an experiment can result are said to be exhaustive.

Ex. A card is drawn from well shuffled pack.

(i) The following events are exhaustive.
(a) The card is black (b) The card is red

(ii) The following events are not exhaustive
(a) The card is heart (b) The card is diamond

If A and B are exhaustive events of the sample space S, then $A \cup B = S$.

In general if $E_1, E_2, E_3, \ldots\ldots\ldots E_n$ are the exhaustive events of the sample space then $E_1 \cup E_2 \cup E_3 \ldots\ldots\ldots\ldots \cup E_n = S$

13. **Mutually Exclusive Events:** If two events cannot occur simultaneously then they are mutually exclusive. If A and B are mutually exclusive, then $A \cap B = \phi$

Ex. In drawing a card from a well shuffled pack, the following events:

A = the card is a spade

B = the card is a heart are mutually exclusive.

(i) In a single throw of a coin either the head or the tail will appear and not both.

(ii) In a throw of a cubic die either an odd number or an even number will turn up and not both.

Following events are not mutually exclusive.

(a) The card is a heart (b) The card is a king

The card can be king of heart.

14. **Dependent and Independent Events :**

Independent events : Events A and B are said to be independent if occurrences or non-occurrence of one does not affect the probability of occurrence or non-occurrence of the other.

(i) Two people holding a normal dice and the other a coin, throw them once, then getting a 6 on normal dice and getting a head on the coin are the examples of events which are independent.

(ii) From an urn containing 2R, 3G and 4W balls, a ball is drawn its colour is noted, the ball is replaced in the urn and another ball is drawn. Getting a red and a red ball on both the occasion are the examples of events which are independent.

(iii) Similar example can be given in playing cards 'getting an ace' and 'an ace' in two successive draws from a well shuffled pack of 52 cards when the first drawn card is replaced in the pack before the second is drawn. If it is not replaced, the events become dependent or contingent.

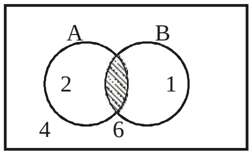
Note : Dependent/Independent events come from two different experiments while mutually exclusive events come from the same experiment.

Example 1 :

Consider the experiment of rolling a die. Let A be the event 'getting a prime number', B be the event 'getting an odd number'. Write the sets representing the events

(i) A or B (ii) A and B (iii) A but not B (iv) 'not A'.

Sol. Here S = {1, 2, 3, 4, 5, 6}, A = {2, 3, 5}



(i) 'A or B' = $A \cup B$ = {1, 2, 3, 5}
(ii) 'A and B' = $A \cap B$ = {3, 5}
(iii) 'A but not B' = A – B = {2}
(iv) 'not A' = A' = {1, 4, 6}

Note :

$$\left.\begin{array}{l}\text{(i) } \overline{A \cup B} = \bar{A} \cap \bar{B} \\ \text{(ii) } \overline{A \cap B} = \bar{A} \cup \bar{B}\end{array}\right\}\text{ De Morgan's Law}$$

MATHEMATICAL DEFINITION OF PROBABILITY

Let the outcomes of an experiment consists of n exhaustive mutually exclusive and equally likely cases. Then the sample spaces S has n sample points. If an event A consists of m sample points, $(0 \le m \le n)$, then the probability of event A, denoted by P(A) is defined to be $\frac{m}{n}$ i.e. $P(A) = \frac{m}{n}$

Let S = $a_1, a_2 ....a_n$ be the sample space

$P(S) = \frac{n}{n} = 1$ corresponding to the certain event.

$P(\phi) = \frac{0}{n} = 0$ corresponding to the null event ϕ or impossible event.

If $A_i = \{a_i\}$, i = 1, 2 ....n then A_i is the event corresponding to a single sample point a_i then $P(A_i) = \frac{1}{n}$

$0 \le P(A) \le 1$

$\because \; 0 \le m \le n \Rightarrow 0 \le \frac{m}{n} \le 1 \Rightarrow 0 \le P(A) \le 1$

$P(A') = 1 - P(A)$

If the event A has m elements, then A' has (n – m) elements in S.

$\therefore \; P(A') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$

Example 2 :

Two dice are thrown at a time. Find the probability of the following:

(i) These numbers shown are equal
(ii) The difference of numbers shown is 1

Sol. The sample space in a throw of two dice

s = {1, 2, 3, 4, 5, 6} × {1, 2, 3, 4, 5, 6}

$\therefore$ Total no. of cases n (s) = 6 × 6 = 36.

(i) Here E_1 = the event of showing equal number on both dice = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}

$\therefore \; n(E_1) = 6 \; \therefore P(E_1) = \frac{n(E_1)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(ii) Here E_2 = the event of showing numbers whose difference is 1.

= {(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)}

$\therefore \; n(E_2) = 10$

$\therefore \; p(E_2) = \frac{n(E_2)}{n(s)} = \frac{10}{36} = \frac{5}{18}$

Example 3 :

If three cards are drawn from a pack of 52 cards, what is the chance that all will be queen ?

Sol. If the sample space be s then n (s) = the total number of . . . . $^{52}C_3$

Now, if A = the event of drawing three queens then $n(A) = {}^4C_3$

$\therefore \; P(E) = \frac{n(A)}{n(s)} = \frac{^4C_3}{^{52}C_3} = \frac{4}{\frac{52 \times 51 \times 50}{3 \times 2}} = \frac{1}{5525}$

Example 4 :

Words are formed with the letters of the word PEACE Find the probability that 2 E' s come together

Sol. Total number of words which can be formed with the letters

P. E. A. C. E = $\frac{5!}{2!} = 60$

Number of words in which 2 E's come together = 4! = 24

$\therefore$ Reqd prob. = $\frac{24}{60} = \frac{2}{5}$

Example 5 :

A bag contains 5 red and 4 green balls. For balls are drawn at random then find the probability that two balls are of red and two balls are of green colour.

Sol. n (s) = the total number of ways of drawing 4 balls out of total 9 balls = $^9C_4$

If A_1 = the event of drawing 2 red balls out of 5 red balls then $n(A_1) = {}^5C_2$

A_2 = the event of drawing 2 green balls out of 4 green balls then $n(A_2) = {}^4C_2$

$\therefore \; n(A) = n(A_1).\, n(A_2) = {}^5C_2 \times {}^4C_2$

$\therefore \; P(A) = \frac{n(A)}{n(s)} = \frac{^5C_2 \times {}^4C_2}{^9C_4} = \frac{\frac{5 \times 4 \times 4 \times 3}{2 \times 2}}{\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}} = \frac{10}{21}$

Example 6 :

An old man while dialing a seven digit telephone number, after having dialed the first five digits, suddenly forgets the last two. But he remembered that the last two digits were different. On this assumption he randomly dials the last two digits. What is the probability that the correct telephone number is dialed.

Sol. Note that total number of ways in which the last two digits (different) can be dialed is 10 × 9 = 90.

Out of these 90 EL/ME/ and exhaustive outcomes only one of them favours happening of the event "correct telephone is dialed". Hence $P(E) = \frac{1}{90}$.

What the probability would have been if he did not even remember the last two digits were different:
Here $n(S) = 10 \times 10 = 100$

Hence $P(E) = \frac{1}{100}$.

Example 7 :
4 Apples and 3 Oranges are randomly placed in a line. Find the chances that the extreme fruits are both oranges.

Sol. $n(S) = \frac{7!}{4! \cdot 3!}$; $n(A) = \frac{5!}{4!} \Rightarrow P = \frac{5!}{4!} \cdot \frac{4!3!}{7!} = \frac{1}{7}$

Note whether fruits the same species are different or alike that probability of the required event remains the same.

Example 8 :
Two natural numbers are randomly selected from the set of first 20 natural numbers. Find the probability that (A) their sum is odd (B) sum is even (C) selected pair is twin prime.

Sol. $S = \{1, 2, 3, ..., 19, 20\}$; $n(S) = {}^{20}C_2$

$n(A) = {}^{10}C_1 . {}^{10}C_1 = 100 \Rightarrow P(A) = \frac{100}{190} = \frac{10}{19}$

(sum odd $\Rightarrow$ one odd and one even)

$n(B) = {}^{10}C_2 + {}^{10}C_2 = 2.{}^{10}C_2 = 90 \Rightarrow P(B) = \frac{90}{190} = \frac{9}{19}$

(sum even $\Rightarrow$ both odd or both even)
$n(C) = \{(3, 5), (5, 7), (11, 13), (17, 19)\}$

$\Rightarrow P(C) = \frac{4}{190} = \frac{2}{95}$

Example 9 :
What is the chance that the fourth power of an integer chosen randomly ends in the digit six.

Sol. Any integer randomly selected can end in 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These are EL/ME and Exhaustive cases. Out of these 10 case only four cases, when the integer ends in 2, 4, 6 and 8 favours happening of the required event. Hence

$P(\text{required event}) = \frac{4}{10} = 40\%$

It will be incorrect to think this problem as :
4th power of an integer can end in 0, 1, 5 and 6. Hence the probability $= \frac{1}{4}$ which is wrong. Note that four events are ME and exhaustive but not equally likely hence the definition of probability can not be based on them.

In factor 4th power of an integer.
Ending in '0' is favoured by only 1 case {0}
Ending in '1' is favoured by only 4 cases {1, 3, 7, 9}
Ending in '5' is favoured by only 1 case {5}
Ending in '6' is favoured by only 4 cases {2, 4, 6, 8}

$\Rightarrow P(0) = \frac{1}{10}$; $P(1) = \frac{4}{10}$; $P(5) = \frac{1}{10}$; $P(6) = \frac{4}{10}$

Example 10 :
Pair of dice has been rolled/thrown/cast once. Find the probability that atleast one of the dice shows up the face one.

Sol. There are four Mutually Exclusive and Exhaustive cases
E_1 : 1^{st} dice only shows up the face one.
E_2 : 2^{nd} dice only shows up the face one.
E_3 : both dice shows up the face one.
E_4 : None of the dice shows up the face one.
Out of these, first 3 cases favours happening of the required event. Hence

$P(\text{required event}) = 1 - P(E_4) = 1 - \frac{5 \times 5}{36} = \frac{11}{36}$

Note that E_1, E_2, E_3, E_4 are not equally likely.

Example 11 :
A leap year is selected at random. Find the probability that it has
(A) 53 Sundays (B) 53 Sundays and Mondays
(C) 53 Sundays or 53 Mondays.

Sol. Leap year means which is divisible by 4 if it not a century year. If it is a century year it must be divisible by 400 as well. A leap year has 366 days out of this 364 days are consumed for 52 weeks i.e. 52 times
S, M, T, W, Th, F and Sat. For remaining 2 days of the leap year can begin with SM, MT, TW, W Th., Th. F, F Sat and Sat Sun.

$\Rightarrow P(A) = \frac{2}{7}$; $P(B) = \frac{1}{7}$; $P(C) = \frac{3}{7}$

Example 12 :
A card is drawn randomly from a well shuffled pack of 52 cards. The probability that the drawn card is "neither a heart nor a face card".

Sol. Note that there are 22 cards which either H or Face cards (All K, Q and J) hence

$P(\text{either a H or Face card}) = \frac{22}{52} = \frac{11}{26}$

$\therefore P(\text{neither a H nor FC}) = 1 - \frac{11}{26} = \frac{15}{26}$

It is to be noted that
$P(\text{not } A \text{ or } \overline{A} \text{ or } A^c) = 1 - P(A)$
Note that A and A^c makes an event a sure event and probability of a sure event is one.

ODDS AGAINST AND ODDS IN FAVOUR OF AN EVENT

Let there be m + n equally likely, mutually exclusive and exhaustive cases out of which an event A can occur in m cases and does not occur in n cases. Then by definition of

probability of occurrences = $\frac{m}{m+n}$

The probability of non- occurrence = $\frac{n}{m+n}$

$\therefore \quad P(A) : P(A') = m : n$

Thus the odd in favour of occurrences of the event A are defined by m : n i.e. P (A) : P (A') ; and the odds against the occurrence of the event A are defined by n : m i.e. P(A') : P (A).

Note : If $P(A) = \frac{a}{b}$ then

(i) odds in favour of event A = a : b – a.

(ii) odds against of event = b – a: a.

Example 13 :

In a single cast with two dice find the odds against drawing 7

Sol. E = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$$\therefore \quad P(E) = \frac{6}{6\times 6} = \frac{1}{6}.$$

So, the odds against drawing

$$7 = \frac{P(\bar{E})}{P(E)} = \frac{1-\frac{1}{6}}{\frac{1}{6}} = \frac{5}{1} = 5:1$$

Example 14 :

Find the odds in favors of getting a king when a card is drawn from a well shuffled pack of 52 cards.

Sol. $\frac{{}^4C_1}{{}^{48}C_1} = \frac{4}{48} = \frac{1}{12}$

Example 15 :

5 different marbles are placed in 5 different boxes randomly. Find the odds in favour that exactly two boxes remain empty. Given each box can hold any number of marbles.

Sol. n (S) = 5^5; For computing favourable outcomes.

2 boxes which are remain empty, can be selected in ${}^5C_2$ ways and 5 marbles can be placed in the remaining 3 boxes

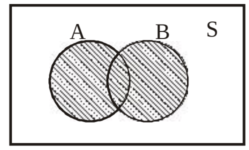
in groups of 221 or 311 in $3! \times \left[\frac{5!}{2!\,2!\,2!} + \frac{5!}{3!\,2!}\right] = 150$ ways

$$\therefore \quad P(E) = {}^5C_2 \times \frac{150}{5^5} = \frac{12}{25}$$

Hence, odds in favour of event E = 12 : 13

ADDITION THEOREM ON PROBABILITY

If A and B are two events associated with an experiment then $P(A \cup B)$ is called the sum of the probabilities of all the sample points in $A \cup B$ or probability of occurrence of atleast one of the events from A and B and the expression for $P(A\cup B)$ is called the addition theorem on probability.



From the Venn diagram it is clear that

P(Occurence atleast one of the events from A and B)

P (A or B or both) or P (A + B)

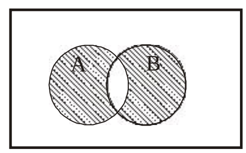
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(\bar{A} \cap \bar{B}) = P(B) + (A \cap \bar{B})$

$= P(A \cap \bar{B}) + P(AB) + P(\bar{A} \cap B)$

$= 1 - P(\bar{A} \cap \bar{B}) = 1 - P(\overline{A \cap B})$

P(occurrence of exactly one of the events) or P(A or B but not both) $\begin{cases} P(A \cap \bar{B}) + P(\bar{A} \cap B) \\ P(A) + P(B) - P(A \cap B) \end{cases}$



General form of addition theorem :

For n events $A_1, A_2, A_3, \ldots\ldots A_n$ in S, we have

$P(A_1 \cup A_2 \cup A_3 \cup A_4 \ldots\ldots \cup A_n)$

$$= \sum_{i=1}^{n} P(A_i) - \sum_{i<j} A(A_i \cap A_j) +$$

$$\sum_{i<j<k} P(A_i \cap A_j \cap A_k) \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap A_3 \ldots \cap A_n)$$

Special Addition rule :

Generalizing if $A_1, A_2, A_3, \ldots\ldots, A_n$ are n mutually exclusive events then

$P(A_1 \cup A_2 \cup \ldots\ldots \cup A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$.

If A is any event is S, then

$P(A) = 1 - P(A) \quad \because A \cup A' = s$ and $A \cap A' = \phi$.

Note :

(i) If A and B are mutually exclusive events then –
$P(A \cup B) = P(A) + P(B) \qquad \{\because P(A \cap B) = 0\}$

(ii) If A and B are exhaustive events then $P(A \cup B) = 1$

(iii) $P(A \cup B) = 1 - P(\overline{A \cup B})$

(iv) Since the probability of an event is a non-negative number, it follows that $P(A \cup B) \le P(A) + P(B)$

(v) For three events A, B and C in S we have
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

(vi) **De Morgan's law :**
If A and B are two subsets of a universal set U, then
(a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$

(vii) **Distributive laws :**
(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example 16 :
A bag contains 6 white, 5 black and 4 red balls, Find the probability of getting either a white or a black ball in a single draw.

Sol. Let A = event that we get a white ball,
B = event that we get a black ball
So, the events are mutually exclusive

$$P(A) = \frac{^6C_1}{^{15}C_1},\ P(B) = \frac{^5C_1}{^{15}C_1}$$

So, $P(A + B) = P(A) + P(B) = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$

Example 17 :
One digit is selected from 20 positive integers. What is the probability that it is divisible by 3 or 4.

Sol. Let A = event that selected number is divisible by 3
B = event that selected number is divisible by 4
Here, the events are not mutually exclusive then

$$P(A) = \frac{6}{20},\ P(B) = \frac{5}{20},\ P(AB) = \frac{1}{20}$$

$$\therefore\ P(A + B) = P(A) + P(B) - P(AB)$$

$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}$$

Example 18 :
Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that
(a) Both Anil and Ashima will not qualify the examination
(b) Atleast one of them will not qualify the examination
(c) Only one of them will qualify the examination.

Sol. Let E and F denote the events that Anil and Ashima will qualify the examination, respectively. Given that
$P(E) = 0.05,\ P(F) = 0.10$ and $P(E \cap F) = 0.02$

(a) The event 'both Anil and Ashima will not qualify the examination' may be expressed as $E' \cap F'$.
Since, E' is not E, i.e., Anil will not qualify the examination and F' is 'not F, i.e., Ashima will not qualify the examination.
Also, $E' \cap F' = (E \cup F)'$ (By Demorgan's Law)
Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
or $P(E \cup F) = 0.05 + 0.10 - 0.02 = 0.13$
Therefore $P(E' \cap F') = P(E \cup F)' = 1 - P(E \cup F)$
$= 1 - 0.13 = 0.87$

(b) P(atleast one of them will not qualify)
= 1 – P (both of them will qualify)
$= 1 - 0.02 = 0.98$

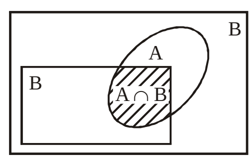
(c) The event only one of them will qualify the examination is same as the event either (Anil will qualify, and Ashima will not qualify) or (Anil will not qualify and Ashima will qualify) i.e., $E \cap F'$ or $E' \cap F$, where $E \cap F'$ and $E' \cap F$ are mutually exclusive.
Therefore, P(only one of them will qualify)
$= P(E \cap F'$ or $E' \cap F)$
$= P(E \cap F') + P(E' \cap F) = P(E) - P(E \cap F) + P(F) - P(E \cap F)$
$= 0.05 - 0.02 + 0.10 - 0.02 = 0.11$

Example 19 :
A and B are any two events such that P(A) = 0.3, P(B) = 0.1 and $P(A \cap B) = 0.16$. Find the probability that exactly one of the events happens.

Sol. Exactly one of the events happens = $P(A \cap B')$ or $P(A' \cap B)$
$P(A \cap B') + P(A' \cap B) = P(A) + P(B) - 2P(A \cap B)$
$= 0.3 + 0.1 - 2 \times 0.16 = 0.08$

CONDITIONAL PROBABILITY

Let A and B be any two events associated with a random experiment. The probability of occurrence of event A when the event B has already occurred is called the conditional probability of A when B is given and is denoted as P(A/B). The conditional probability P(A/B) is meaningful only when $P(B) \neq 0$, i.e., when B is not an impossible event.



$P\left(\frac{A}{B}\right)$ = Probability of occurrence of event A when the event B as already occurred

$$= \frac{\text{Number of cases favourable to B which are also favourable to A}}{\text{Number of cases favourable to B}}$$

$$\therefore\ P\left(\frac{A}{B}\right) = \frac{\text{Number of cases favourable to } A \cap B}{\text{Number of cases favourable to B}}$$

$$\text{Also,}\ P\left(\frac{A}{B}\right) = \frac{\dfrac{\text{Number of cases favourable to } A \cap B}{\text{Number of cases in the sample space}}}{\dfrac{\text{Number of cases favourable to B}}{\text{Number of cases in the sample space}}}$$

$$\therefore\ P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)},\ \text{provided } P(B) \neq 0.$$

Similarly, we have

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)},\ \text{provided } P(A) \neq 0.$$

Example 20 :

Two dice are thrown. Find the probability that the numbers appeared has a sum of 8 if it is known that the second dice always exhibits 4.

Sol. Let A be the event of occurrence of 4 always on the second die = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)} $\therefore$ n (A) = 6 and B be the event of occurrences of such numbers on both dice whose sum is 8 = {(4, 4)}

Thus, $A \cap B = A \cap \{(4, 4)\} = \{(4, 4)\}$

$$\therefore n(A \cap B) = 1 \quad \therefore \quad P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6}$$

Example 21 :

A coin is tossed thrice. If E be the event of showing at least two heads and F the event of showing head in the first throw, then find P (E/F).

Sol. There are following 8 outcomes of three throws :

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Also $P(E \cap F) = \frac{3}{8}$ and $P(F) = \frac{4}{8}$

$$\therefore \text{Reqd prob.} = P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{8}}{\frac{4}{8}} = \frac{3}{4}$$

Example 22 :

In a class, 30% of the students failed in Physics, 25% failed in Mathematics and 15% failed in both Physics and Mathematics. If a student is selected at random failed in Mathematics, find the probability that he failed in Physics also.

Sol. Let A be the event "failed in Physics" and B be the event "failed in Mathematics". We want to find $P\left(\frac{A}{B}\right)$.

It is given that $P(A) = \frac{30}{100}$ and $P(B) = \frac{25}{100}$

Also, $P(A \cap B) = \frac{15}{100}$

$$\text{Therefore } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{15/100}{25/100} = \frac{15}{25} = \frac{3}{5}.$$

Example 23 :

Let A and B be two events such that P(A) = 0.3, P(B) = 0.6 and $P\left(\frac{B}{A}\right) = 0.5$. Then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ equals

(A) 3/4 (B) 5/8

(C) 9/40 (D) 1/4

Sol. $P(A \cap B) = P(A)\,P(B/A) = (0.3)(0.5) = 0.15$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.15 = 0.75$

Also, $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= \frac{1 - 0.75}{1 - 0.6} = \frac{0.25}{0.4} = \frac{250}{400} = \frac{5}{8}$$

MULTIPLICATION THEOREM

Let A and B be two events associated with a sample space S. Clearly, the set $A \cap B$ denotes the event that both A and B have occurred. In other words, $A \cap B$ denotes the simultaneous occurrence of the events E and F. The event $A \cap B$ is also written as AB.

We know that the conditional probability of event A given that B has occurred is denoted by $P(A \mid B)$ and is given by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

From this result, we can write

$$P(A \cap B) = P(B) \cdot P(A \mid B) \quad \ldots(i)$$

Also, we know that

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

or $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$ (since $A \cap B = B \cap A$)

Thus, $P(A \cap B) = P(A) \cdot P(B \mid A)$ …(ii)

Combining (i) and (ii), we find that

$$P(A \cap B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$$

provided $P(A) \neq 0$ and $P(B) \neq 0$

The above result is known as the multiplication rule of probability.

Note : If A & B are independent events then

$P\left(\frac{A}{B}\right) = P(A)$ and $P\left(\frac{B}{A}\right) = P(B)$ and in this case multiplication theorem $P(A \cap B) = P(A) \cdot P(B)$.

Theorem : Let A and B be events associated with a random experiment. If A and B are independent, then show that the events (i) $\bar{A}$, B (ii) A, $\bar{B}$ (iii) $\bar{A}, \bar{B}$ are also independent.

Proof : The events A and B are independent .

$\therefore \quad P(A \cap B) = P(A)\,P(B)$ ……(i)

(i) $(A \cap B) \cap (\bar{A} \cap B) = (A \cap \bar{A}) \cap (B \cap B) = \phi \cap B = \phi$

and $(A \cap B) \cup (\bar{A} \cap B) = (A \cup \bar{A}) \cap B = S \cap B = B$

$\therefore$ The events $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive and their union is B.

$\therefore$ By addition theorem, we have

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) \quad \ldots\ldots(i)$$

$\Rightarrow P(\bar{A} \cap B) = P(B) - P(A \cap B) = P(B) - P(A)\,P(B)$

$= (1 - P(A))\,P(B) = P(\bar{A})\,P(B)$ (Using (i))

$\therefore \quad P(\bar{A} \cap B) = P(\bar{A})\,P(B)$ i.e., $\bar{A}$ and B are independent.

(ii) $(A \cap B) \cap (A \cap \bar{B}) = (A \cap A) \cap (B \cap \bar{B}) = A \cap \phi = \phi$

and $(A \cap B) \cup (A \cap \bar{B}) = A \cap (B \cup \bar{B}) = A \cap S = A$

$\therefore$ The events $A \cap B$ and $A \cap \bar{B}$ are mutually exclusive and their union is A.

$\therefore$ By addition theorem, we have

$P(A) = P(A \cap B) + P(A \cap \bar{B})$ ……(i)

$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B)$

$= P(A)(1 - P(B)) = P(A)P(\bar{B})$

(Using (i))

$\therefore P(A \cap \bar{B}) = P(A)P(\bar{B})$ i.e., A and $\bar{B}$ are independent.

(iii) $(\bar{A} \cap B) \cap (\bar{A} \cap \bar{B}) = (\bar{A} \cap \bar{A}) \cap (B \cap \bar{B}) = \bar{A} \cap \phi = \phi$

and $(\bar{A} \cap B) \cup (\bar{A} \cap \bar{B}) = \bar{A} \cap (B \cup \bar{B}) = \bar{A} \cap S = \bar{A}$

$\therefore$ The events $\bar{A} \cap B$ and $\bar{A} \cap (B \cap \bar{B})$ are mutually exclusive and their union is $\bar{A}$.

$\therefore$ By addition theorem, we have

$P(\bar{A}) = P(\bar{A} \cap B) + P(\bar{A} \cap \bar{B})$ ……(i)

$\Rightarrow P(\bar{A} \cap \bar{B}) = P(\bar{A}) - P(\bar{A} \cap B) = P(\bar{A}) - P(\bar{A})P(B)$

$= P(\bar{A})(1 - P(B)) = P(\bar{A})P(\bar{B})$ (Using (i))

$\therefore P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

i.e., $\bar{A}$ and $\bar{B}$ are independent.

Probability of at least one of the n Independent events

If p_1, p_2, p_3, ... p_n are the probabilities of n independent events A_1, A_2, A_3 ...... A_n then the probability of happening of at least one of these event is

$1 - [(1 - p_1)(1 - p_2) \ldots\ldots (1 - p_n)]$

$P(A_1 + A_2 + A_3 + \ldots + A_n)$

$= 1 - P(\bar{A}_1)P(\bar{A}_2)P(\bar{A}_3)..P(\bar{A}_n)$

TOTAL PROBABILITY THEOREM

Let E_1, E_2, ... E_n be n mutually exclusive and exhaustive events, with non-zero probabilities of a random experiment. If A be any arbitrary event of the sample space of the above random experiment with P (A) > 0, then

$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \ldots + P(E_n)P(A/E_n)$

Note : In practical problems, it is found convenient to write as follows :

$P(A) = P(E_1A \text{ or } E_2A \text{ or } \ldots\ldots E_nA)$

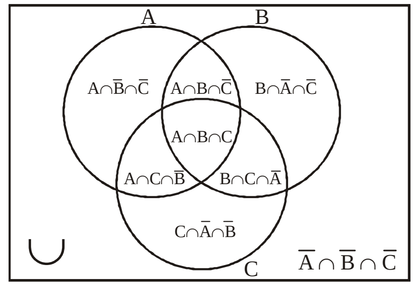
$= P(E_1A) + P(E_2A) + \ldots\ldots + P(E_nA)$

$$P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)\left(\frac{A}{E_2}\right).$$

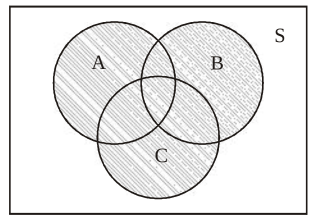
THREE EVENTS ASSOCIATED WITH AN EXPERIMENTAL PERFORMANCE

The addition theorem can be extended when three events are associated with the experiment.

If A, B and C are three events then $P(A \cup B \cup C)$ denotes the sum of probabilities of all the sample points in $(A \cup B \cup C)$ or probability of occurrence of atleast one of the events.

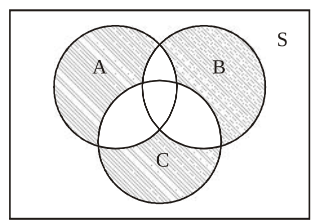


(i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$



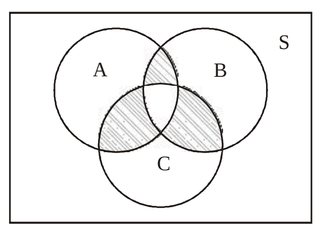
$(A \cup B \cup C)$

(ii) P (occurrence of exactly one of the events) =
$P(A) + P(B) + P(C) - 2[P(A \cap B) + P(B \cap C) + P(C \cap A)] + 3P(A \cap B \cap C)$



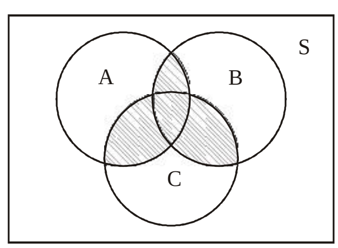
$(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

(iii) P (occurrence of exactly two of the events)
$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$



$(A \cap B \cap \bar{C}) \cup (\bar{A} \cap B \cap C) \cup (A \cap \bar{B} \cap C)$

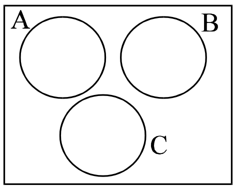
(iv) P (occurrence of atleast two of the events) =
$P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

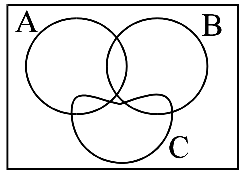


$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (A \cap B \cap C)$

Note :

(a) If A, B, C are three pair wise mutually exclusive $\Rightarrow$ they are mutually exclusive however if A, B, C are mutually exclusive $\not\Rightarrow$ they are pair wise mutually exclusive





ME $\not\Rightarrow$ pair wise ME   Pair wise ME $\Rightarrow$ ME

(b) If A,B,C are pair wise independent $\not\Rightarrow$ they are independent. Infact for 3 events A, B and C to be independent they must be
(i) pair wise
(ii) mutually independent , mathematically
$P(A \cap B) = P(A) . P(B)$; $P(B \cap C) = P(B) . P(C)$
$P(C \cap A) = P(C) . P(A)$
and $P(A \cap B \cap C) = P(A) . P(B) . P(C)$
for n independent events, the total number of conditions would be ${}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - n - 1$

Example 24 :

A bag contains 4 red and 4 blue balls. Four balls are drawn one by one from the bag, then find the probability that the drawn balls are in alternate colour.

Sol. E_1 : Event that first drawn ball is red, second is blue and so on.
E_2: Event that first drawn ball is blue, second is red and so on.

$$\therefore\ P(E_1) = \frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5} \text{ and } P(E_2) = \frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$$

$$P(E) = P(E_1) + P(E_2) = 2 \times \frac{4}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{3}{5} = \frac{6}{35}$$

Example 25 :

Let A, B, C be 3 independent events such that

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(C) = \frac{1}{4}.$$

Then find probability of exactly 2 events occurring out of 3 events.

Sol. P (exactly two of A, B, C occur)
$= P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$
$= P(B). P(C) + P(C). P(A) + P(A). P(B) - 3P(A). P(B). P(C)$

$$= \frac{1}{2}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{2}-3\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{4} = \frac{1}{4}$$

Example 26 :

A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn one by one. What is the probability that first ball is white and second ball is blue when first drawn ball is not replaced in the bag ?

Sol. Let A be the event of drawing first ball white and B be the event of drawing second ball blue.

Here A and B are dependent events.

$$P(A) = \frac{6}{16}, P\left(\frac{B}{A}\right) = \frac{7}{15}$$

$$P(AB) = P(A). P\left(\frac{B}{A}\right) = \frac{6}{16}\times\frac{7}{15} = \frac{7}{40}$$

Example 27 :

Three coins are tossed together, What is the probability that first shows head, second shows tail and third shows head ?

Sol. Let A, B, C denote three given component events which are mutually independent.

$$\text{So, } P(ABC) = P(A). P(B). P(C) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

Example 28 :

A problem of mathematics is given to three students A, B, and C, Whose chances of solving it are 1/2, 1/3, 1/4 respectively. Then find the probability that the problem is solved.

Sol. Obviously the events of solving the problem by A, B and C are independent. Therefore required probability

$$= 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\right] = 1 - \frac{1}{2}.\frac{2}{3}.\frac{3}{2} = \frac{3}{4}$$

Example 29 :

A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent?

Sol. We know that the sample space is $S = \{1, 2, 3, 4, 5, 6\}$
Now, $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

$$\text{Then, } P(E) = \frac{2}{6} = \frac{1}{3}, P(F) = \frac{3}{6} = \frac{1}{2} \text{ and } P(E \cap F) = \frac{1}{6}$$

Clearly, $P(E \cap F) = P(E) \cdot P(F)$
Hence, E and F are independent events.

Example 30 :

Three coins are tossed simultaneously. Consider the event E 'three heads or three tails', F 'at least two heads' and G 'at most two heads'. Of the pairs (E, F), (E, G) and (F, G), which are independent? which are dependent?

Sol. The sample space of the experiment is given by
$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$
Clearly $E = \{HHH, TTT\}$, $F = \{HHH, HHT, HTH, THH\}$
and $G = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$
Also, $E \cap F = \{HHH\}$, $E \cap G = \{TTT\}$,
$F \cap G = \{HHT, HTH, THH\}$

$$\text{Therefore. } P(E) = \frac{2}{8} = \frac{1}{4}, P(F) = \frac{4}{8} = \frac{1}{2}, P(G) = \frac{7}{8}$$

$$\text{and } \quad P(E \cap F) = \frac{1}{8}, P(E \cap G) = \frac{1}{8}, P(F \cap G) = \frac{3}{8}$$

Also $P(E)\cdot P(F)=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$, $P(E)\cdot P(G)=\frac{1}{4}\times\frac{7}{8}=\frac{7}{32}$

and $P(F)\cdot P(G)=\frac{1}{2}\times\frac{7}{8}=\frac{7}{16}$

Thus $P(E\cap F)=P(E)\cdot P(F)$
$P(E\cap G)\neq P(E)\cdot P(G)$
and $P(F\cap G)\neq P(F)\cdot P(G)$

Hence, the events (E and F) are independent, and the events (E and G) and (F and G) are dependent.

Example 31 :

A pair of fair dice is thrown. Find the probability that either of the dice shows 2 if the sum is 6.

Sol. The sample space of the experiment "throwing a pair of fair dice" consists of $36(=6\times 6)$ ordered pair (a, b), where a and b can be any integers from 1 to 6. Let A be the event "2 appears on either of the dice" and B be the event "sum is 6". We want to find $P\left(\frac{A}{B}\right)$. Note that

$A=[(2,b)\,|\,1\le b\le 6]\cup[(a,2)\,|\,1\le a\le 6]$
and $B=[(1,5),(2,4),(3,3),(4,2),(5,1)]$
Also, $A\cap B=[(2,4),(4,2)]$

Therefore, $P(B)=\frac{5}{36}$ and $P(A\cap B)=\frac{2}{36}$

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{2/36}{5/36}=\frac{2}{5}$$

Example 32 :

A jar contains 10 white balls and 6 blue balls, all are of equal size. Two balls are drawn without replacement. Find the probability that the second ball is white if it is known that the first is white.

Sol. Let E_1 be the event " the first ball drawn is white" and E_2 be the event" the second ball drawn is white again. Then

$$P(E_1)=\frac{10}{16}$$

since 10 out of 10 + 6 balls are white. But, after one ball is chosen, there remain 9 white balls and 6 blue balls. Therefore the required probability is

$$P\left(\frac{E_2}{E_1}\right)=\frac{P(E_1\cap E_2)}{P(E_1)}=\frac{\frac{10}{16}\cdot\frac{9}{15}}{\frac{10}{16}}=\frac{9}{15}=\frac{3}{5}$$

Example 33 :

There are four machines and it is known that exactly two of them are faulty. They are tested one by one, in a random order till both the faulty machines are identified. The probability that only two tests are needed is

(A) 1/3 (B) 1/6
(C) 1/2 (D) 1/4

Sol. The procedure ends in first two tests if either both are faulty or both are good. Therefore the probability is

$$=P(G\cap G)+P(F\cap F)=P(G)\cdot P\left(\frac{G}{G}\right)+P(F)\cdot P\left(\frac{F}{F}\right)$$

$$=\frac{2}{4}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{3}$$

Example 34 :

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Sol. Consider E_1 and E_2 be events that red ball is drawn in first draw and black ball is drawn in first draw respectively.
Consider A be the event that ball drawn in second draw is red. There are 5 red and 5 black balls in the urn.

$$\Rightarrow P(E_1)=\frac{5}{10}=\frac{1}{2} \text{ and } P(E_2)=\frac{5}{10}=\frac{1}{2}$$

When 2 additional balls of red colour are put in the urn there are 7 red and 5 black balls in the urn.
$\therefore P(A/E_1)=7/12$
When 2 additional balls of black colour are put in the urn there are 5 red and 7 black balls in the urn.
$\therefore P(A/E_2)=5/12$
Reqd. probability $P(A)=P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)$

$$=\frac{1}{2}\times\frac{7}{12}+\frac{1}{2}\times\frac{5}{12}=\frac{7}{24}+\frac{5}{24}=\frac{1}{2}$$

Example 35 :

A box contains three coins, one coin is fair, one coin is two-headed, and one coin is weighted so that the probability of head appearing is 1/3. A coin is selected at random and tossed. Find the probability that (i) head (ii) tail appears.

Sol. Let E_1, E_2 and E_3 be the events of selecting at random first coin, second coin and third coin respectively.
$\therefore P(E_1)=1/3$, $P(E_2)=1/3$ and $P(E_3)=1/3$
Let H and T be events of getting head and tail respectively.

$$\therefore P\left(\frac{H}{E_1}\right)=\frac{1}{2},\ P\left(\frac{T}{E_1}\right)=\frac{1}{2} \quad (\because \text{ First coin is fair})$$

$$P\left(\frac{H}{E_2}\right)=I,\ P\left(\frac{T}{E_2}\right)=0$$

($\because$ Second coin is two-headed)

(i) P (getting head) = $P(H)=P(E_1H \text{ or } E_2H \text{ or } E_3H)$
$=P(E_1H)+P(E_2H)+P(E_3H)$

$$=P(E_1)P\left(\frac{H}{E_1}\right)+P(E_2)P\left(\frac{H}{E_2}\right)+P(E_3)P\left(\frac{H}{E_3}\right)$$

$$=\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{3}=\frac{11}{18}$$

(ii) P(getting tail) = P (T)

$= P(E_1T \text{ or } E_3T) = P(E_1T) + P(E_3T)$

$= P(E_1)P\left(\frac{T}{E_1}\right) + P(E_3)P\left(\frac{T}{E_3}\right)$

$\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{2}{3}=\frac{7}{18}$

Example 36 :

A, B and C are three newspapers from a city. 25% of the population reads A, 20% reads B, 15% reads C, 16% reads both A and B, 10% reads both B and C, 8% reads both A and C and 4% reads all the three. Find the percentage of the population who read atleast one of A, B and C.

Sol. We are given that

$$P(A) = \frac{25}{100}, P(B) = \frac{20}{100}, P(C) = \frac{15}{100}$$

$$P(A \cap B) = \frac{16}{100}, P(B \cap C) = \frac{10}{100}, P(C \cap A) = \frac{8}{100}$$

and $P(A \cap B \cap C) = \frac{4}{100}$

We have to find $P(A \cup B \cup C)$. We can use the formula

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{1}{100}(25 + 20 + 15 - 16 - 10 - 8 + 4) = \frac{30}{100}$$

Thus 30% of the people read atleast one of the newspapers.

TRY IT YOURSELF-1

Q.1 If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

(A) 0.62 (B) 0.32
(C) 0.12 (D) 0.42

Q.2 If from each of the 3 boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls , one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is:

(A) 13/32 (B) 1/4
(C) 1/32 (D) 3/16

Q.3 7 white balls and 3 black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals:

(A) 1/2 (B) 7/15
(C) 2/15 (D) 1/3

Q.4 There are 4 machines and it is known that exactly 2 of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only 2 testes are needed is

(A) 1/3 (B) 1/6
(C) 1/2 (D) 1/4

Q.5 An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(A) $\frac{m}{m+n}$ (B) $\frac{m}{m-n}$
(C) $\frac{m+n}{m}$ (D) None of these

Q.6 If A and B are events such that P(A/B) = P(B/A), then

(A) $A \subset B$ but $A \neq B$ (B) $A = B$
(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

Q.7 A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Q.8 Given that the events A and B are such that P(A) = 1/2, $P(A \cup B) = 3/5$ and P (B) = p. Find p if they are (i) mutually exclusive (ii) independent.

Q.9 In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English news papers.
(b) If she reads Hindi news paper, find the probability that she reads English news paper.
(c) If she reads English news paper, find the probability that she reads Hindi news paper.

ANSWERS

(1) (A) **(2)** (A) **(3)** (B)
(4) (B) **(5)** (A) **(6)** (D)
(7) 44/91 **(8)** (i) 1/10, (ii) 1/5
(9) (a) 1/5, (b) 1/3, (c) 1/2

BAYE'S THEOREM

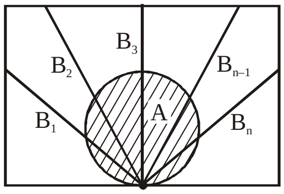
If an event A can occur only with one of the n pairwise mutually exclusive and exhaustive events $B_1, B_2, ... B_n$ & if the conditional probabilities of the event.
$P(A/B_1), P(A/B_2) ....P(A/B_n)$ are known then,

$$P(B_1/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

Proof : The event A occurs with one of the 'n' mutually exclusive exhaustive events $B_1, B_2, B_3, \ldots\ldots, B_n$
$A = AB_1 + AB_2 + AB_3 + \ldots\ldots + AB_n$

$$P(A) = P(AB_1) + P(AB_2) + \ldots\ldots + P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

Note : A = event what we have,
B_1 = event what we want,
$B_1, B_2, \ldots, B_n$ are alternative events.



Now, $P(AB_i) = P(A)\cdot P\left(\frac{B_i}{A}\right) = P(B_i)\cdot P\left(\frac{A}{B_i}\right)$

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i)\,P\left(\frac{A}{B_i}\right)}{P(A)} = \frac{P(B_i)\cdot P\left(\frac{A}{B_i}\right)}{\sum_{i=1}^{n} P(AB_i)} = \frac{P(B_i)\cdot P\left(\frac{A}{B_i}\right)}{\sum_{i=1}^{n} P(B_i)\cdot P\left(\frac{A}{B_i}\right)}$$

Example 37 :

A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag B.

Sol. Let E_1 = The event of ball being drawn from bag A

E_2 = The event of ball being drawn from bag B

E = The event of ball being red

Since, both the bags are equally likely to be selected,

$$P(E_1) = P(E_2) = \frac{1}{2} \text{ and } P\left(\frac{E}{E_1}\right) = \frac{3}{5} \text{ and } P\left(\frac{E}{E_2}\right) = \frac{5}{9}$$

$\therefore$ Required probability

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\,P\left(\frac{E}{E_2}\right)}{P(E_1)\,P\left(\frac{E}{E_1}\right) + P(E_2)\,P\left(\frac{E}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}$$

Example 38 :

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Sol. Consider E_1 and E_2 be the events that a student residue in hostel and a day scholar respectively. Consider A be the event that a student attain A grade.

$$\therefore P(E_1) = 60\% = \frac{60}{100} \text{ and } P(E_2) = 40\% = \frac{40}{100}$$

Also, $P(A/E_1) = 30\% = \frac{30}{100}$

and $P(A/E_2) = 20\% = \frac{20}{100}$

Using Bayes' theorem

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$$

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\frac{60}{100}\times\frac{30}{100}+\frac{40}{100}\times\frac{20}{100}} = \frac{\frac{18}{100}}{\frac{18}{100}+\frac{8}{100}} = \frac{\frac{18}{100}}{\frac{26}{100}} = \frac{18}{100}\times\frac{100}{26}$$

$$= \frac{18}{26} = \frac{9}{13}$$

Example 39 :

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Sol. Consider E_1 and E_2 be the events that the girl gets 5 to 6 on throwing a die and 1, 2, 3, or 4 on throwing a die respectively.

Consider A be the event that exactly one head appears on the coin.

$$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Now probability of getting exactly one head on tossing a coin three times

$\therefore \quad P(A/E_1) = P(HTT) + P(THT) + P(TTH)$

$$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{3}{8}$$

Probability of getting exactly on head on tossing a coin once. $P(A/E_2) = 1/2$

By Bayes' theorem,

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{3+8}{24}} = \frac{\frac{1}{3}}{\frac{11}{24}} = \frac{1}{3}\times\frac{24}{11} = \frac{8}{11}$$

Example 40 :

A letter is to come from either LONDON or CLIFTON. The postal mark on the letter legibly shows consecutive letters "ON". The probability that the letter has come from LONDON is

(A) 12/17 (B) 13/17

(C) 5/17 (D) 4/17

Sol. Let the events be defined as

E_1 : Letter coming from LONDON

E_2 : Letter coming from CLIFTON

E_3 : Two consecutive letters ON.

The word LONDON contains 5 types of consecutive letters (LO, ON, ND, DO, ON) of which there are two ON's. The word CLIFTON contains 6 types of consecutive letters (CL, LI, IF, FT, TO, ON) of which there is one "ON". Now

$$P(E_1) = \frac{1}{2} = P(E_2) \Rightarrow P\left(\frac{E_3}{E_2}\right) = \frac{2}{5} \text{ and } P\left(\frac{E_3}{E_2}\right) = \frac{1}{6}$$

By Bayes' theorem

$$P\left(\frac{E_1}{E_3}\right) = \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}} = \frac{12}{17}$$

Example 41 :

In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolt. A bolts is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

Sol. Let events B_1, B_2, B_3 be the following

B_1 : the bolt is manufactured by machine A

B_2 : the bolt is manufactured by machine B

B_3 : the bolt is manufactured by machine C

Clearly, B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space. Let the event E be 'the bolt is defective'.

The event E occurs with B_1 or with B_2 or with B_3 . Given that, $P(B_1) = 25\% = 0.25$, $P(B_2) = 0.35$ and $P(B_3) = 0.40$

Again $P(E \mid B_1)$ = Probability that the bolt drawn is defective given that it is manufactured by machine $A = 5\% = 0.05$

Similarly, $P(E \mid B_2) = 0.04$, $P(E \mid B_3) = 0.02$

Hence, by Bayes' Theorem, we have

$$P(B_2 \mid E) = \frac{P(B_2)\ P(E \mid B_2)}{P(B_1)P(E \mid B_1) + P(B_2)\ P(E \mid B_2) + P(B_3)\ P(E \mid B_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 \times 0.35 \times 0.04 + 0.40 \times 0.02}$$

$$= \frac{0.0140}{0.0345} = \frac{28}{69}$$

BOOLE'S INEQUALITY

For any two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\therefore\ P(A \cup B) \le P(A) + P(B) \quad \{\because P(A \cap B) \ge 0)\}$

For any three events A, B, C

$$P(A \cup B \cup C) \le P(A) + P(B) + P(C)$$

In general for any n events $A_1, A_2, \ldots\ldots A_n$

$$P(A_1 \cup A_2 \cup \ldots\ldots \cup A_n) \le P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$$

Example 42 :

If two events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$, then prove that $P(A \cup B) \ge 2/3$.

Sol. $P(A \cup B) = P(A) + P(B) - P(AB)$ and clearly $P(AB) \le 1/3$

Hence , $P(A \cup B) \ge P(B) \Rightarrow P(A \cup B) \ge 2/3$

GEOMETRICAL APPLICATIONS

The following statements are axiomatic :

(i) If a point is taken at random on a given straight line segment AB, the chance that it fails on a particular segment PQ of the line segment is PQ/AB i.e.

$$\text{probability} = \frac{\text{favourable length}}{\text{total length}}$$

(ii) If a point is taken at random on the area S which includes an area σ, the chance that the point falls on σ is σ/S.

i.e. $\frac{\text{favourable area}}{\text{total area}}$

NOTE

* The sample space of some random experiments have infinite sample points, and hence the events also have infinite sample points in favour of their occurrence.

In some cases the probability is calculated by relating the sample space or the sets representing the events, with lengths or areas of geometrical figures etc.

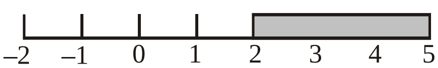
Example 43 :

If a be chosen at random in the interval [0, 5], show that the probability that the equation $4x^2 + 4ax + (a + 2) = 0$, to have real roots is 3/5.

Sol. Equation $4x^2 + 4ax + (a + 2) = 0$ will have equal roots, if its discriminant $16a^2 - 16(a + 2) \ge 0$

i.e. $a^2 - a - 2 \ge 0$ or $(a - 2)(a + 1) \ge 0$

$\therefore\ a \ge 2$ or $a \le -1$



But in the interval [0, 5], $a > 2 \Rightarrow a \in [2, 5]$

$$\therefore \text{ probability} = \frac{\text{length of interval } [2,5]}{\text{length of interval } [0,5]} = \frac{3}{5}$$

Example 44 :

A line is divided at random into three parts. What is the probability that they form the sides of a triangle ?

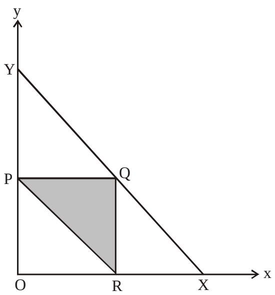
Sol. Let AP = x, BQ = y and AB = a.

Since the sum of two sides of a triangle is greater than the third side, AP must be less than $\frac{a}{2}$ i.e. $x < \frac{a}{2}$.

Similarly, $y < \frac{a}{2}$ and $a - (x + y) < \frac{a}{2}$ or $x + y > \frac{a}{2}$ ...... (i)

For all possible cases of dividing the line,

$0 < x < a.\ 0 < y < a$ and $x + y < a$ ..... (ii)



Condition (ii) corresponds to the triangular region OXY and condition (i) corresponds to the triangular region PQR.

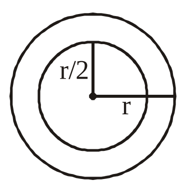
$$p = \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta OXY} = \frac{\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}{\frac{1}{2}(a)\,(a)} = \frac{1}{4}$$

Example 45 :

A point is taken inside a circle of radius find the probability that the point is closer to the centre as a circumference.

Sol. $n\,(s) = \pi r^2$

$$n\,(A) = \pi\left(\frac{r}{2}\right)^2$$

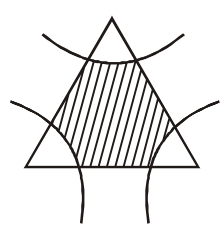


$$P = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

Example 46 :

A point is selected random inside a equilateral triangle whose length of side is 3. Find the probability that its distance from any corner is greater than 1.

Sol. Area of sector $= \frac{r^2\theta}{2}$



$$n\,(s) = \frac{\sqrt{3}}{4}\cdot 9$$

$$n(A) = \frac{\sqrt{3}}{4}\cdot 9 - 3\,\frac{1\cdot 1}{2}\cdot\frac{\pi}{3}$$

$$\therefore\ P(A) = \frac{\frac{\sqrt{3}}{4}\cdot 9 - \frac{\pi}{2}}{\frac{\sqrt{3}}{4}\cdot 9} = 1 - \frac{2\pi}{9\sqrt{3}}$$

MATHEMATICAL EXPECTATION (PRACTICAL USE OF PROBABILITY IN DAY TO DAY LIFE):

It is worthwhile indicating that if 'P' represents a person's chance of success in any venture and 'M 'the sum of money which he will receive in case of success, then the sum of money denoted by $'P\cdot M'$ is called his expectation.

Example 47 :

Two players of equal skill A and B are playing a game. They leave off playing (due to some force majeure conditions) when A wants 3 points and B wants 2 to win. If the prize money is Rs.16000/-. How can the referee divide the money in a fair way.

Sol. Since, A wins if he scores 3 points before B scores 2.

Probability of A's scoring a point = Probability of B's scoring at point = 1/2

Hence, required probability that A succeeds

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{5}{16}$$

Probability that B succeeds $= 1 - \frac{5}{16} = \frac{11}{16}$

$$\therefore\ \text{A's expectation} = \frac{5}{16}\times 16000 = 5000$$

$$\text{B's expectation} = \frac{11}{16}\times 16000 = 11000$$

COINCIDENCE TESTIMONY

If p_1 and p_2 are the probabilities of speaking the truth of two independent witnesses A and B who give the same statement.

P (their combined statement is true)

$$= P(H_1 / H_1 \cup H_2) = \frac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)}$$

where H_1 means both speaks the truth and H_2 means both speaks false.

In this case it has been assumed that we have no knowledge of the event except the statement made by A and B.

However if p is the probability of the happening of the event before their statement then

P (their combined statement is true)

$$= \frac{p\,p_1 p_2}{p\,p_1 p_2 + (1-p)(1-p_1)(1-p_2)}$$

Here it has been assumed that the statement given by all the independent witnesses can be given in two ways only, so that if all the witnesses tell falsehoods they agree in telling the same falsehood.

If this is not the case and c is the chance of their coincidence testimony then the

Probability that the statement is true $= P\,p_1\,p_2$

Probability that the statement is false

$$= (1-p)\,.\,c\,(1-p_1)\,(1-p_2)$$

However chance of coincidence testimony is taken only if the joint statement is not contradicted by any witness.

Example 48 :

A speaks the truth 3 out of 4 times, and B 5 out of 6 times. What is the probability that they will contradict each other in stating the same fact.

Sol. $P(A) = \frac{3}{4}$; $P(B) = \frac{5}{6}$

$$P(\text{contradict}) = \frac{3}{4}\cdot\frac{1}{6}+\frac{5}{6}\cdot\frac{1}{4} = \frac{8}{24} = \frac{1}{3}$$

Example 49 :

A speaks truth 3 times out of 4, and B 7 times out of 10. They both assert that a white ball has been drawn from a bag containing 6 balls of different colours; find the probability of the truth of their assertion.

P(A) = 3/4; P (B)=7/10

Sol. There are 2 hypothesis

(i) Their coincidence testimony is true (ii) it is false

H_1:white ball is actually drawn & both speaks the truth

$$P(H_1) = \frac{1}{6}\cdot\frac{3}{4}\cdot\frac{7}{10}$$

H_2 : (white has not been withdrawn) and (their statement coincides) and they both speaks false

$$P(H_2) = \frac{5}{6}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{4}\times\frac{3}{10}$$

Let E : their assertion is true

$$\therefore P(E) = P\left(\frac{H_1}{H_1 \cup H_2}\right) = \frac{\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{7}{10}}{\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{7}{10}+\frac{5}{6}\left(\frac{1}{25}\right)\cdot\frac{1}{4}\cdot\frac{3}{10}} = \frac{35}{36}$$

PROBABILITIES THROUGH STATISTICAL (STOCHASTIC) TREE DIAGRAM

Example 50 :

A : box contains three coins A, B and C

A : Normal coin; B : Double Headed (DH) coin ;

C : a weighted coin so that P (H) = 1/3

A coin is randomly selected & tossed

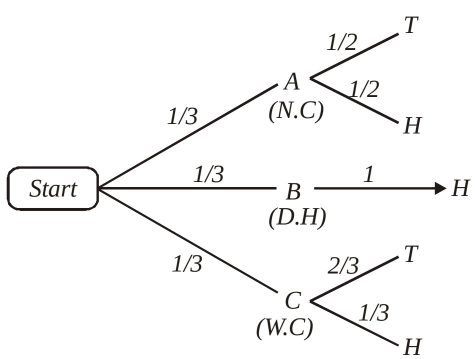
(A) Find the probability that head appears.

(B) If head appear find the probability that it is a normal coin P (A/H)

(C) Find the probability that tail appears.

(D) If tail appears, find the probability that it is a weighted coin P (C/T)

Sol.



(A) $P(H) = \frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{1}{3} = \frac{11}{18}$

(B) $P\left(\frac{A}{H}\right) = \frac{P(A\cap B)}{P(H)} = \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{11}{18}} = \frac{3}{11}$

(C) $P(T) = \frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot 0+\frac{1}{3}\cdot\frac{2}{3} = \frac{7}{18}$

or $1 - P(H) = 1-\frac{11}{18} = \frac{7}{18}$

(D) $P\left(\frac{C}{T}\right) = \frac{P(C\cap T)}{P(T)} = \frac{\frac{1}{3}\cdot\frac{2}{3}}{\frac{7}{18}} = \frac{4}{7}$

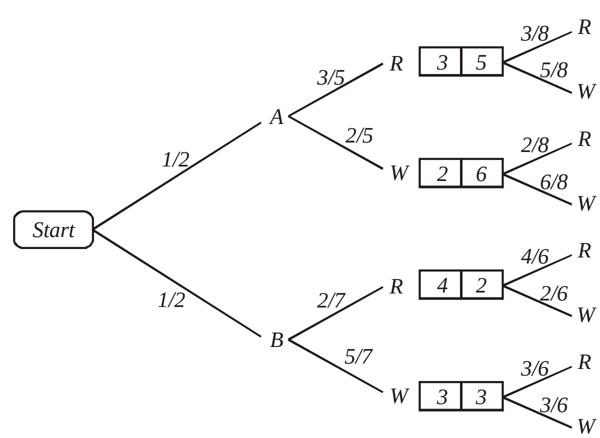
Example 51 :

Let the contents of the two boxes A and B with respect to number of R and W marbles is as given below:

Bag	R	W
A	3	2
B	2	5

A bag is selected at random; a marble is drawn and put into the other box; then a marble is drawn from the second box. Find the probability the both marbles drawn the of same colour.

Sol.



$$P(E) = \frac{1}{2}\cdot\frac{3}{5}\cdot\frac{3}{8}+\frac{1}{2}\cdot\frac{2}{5}\cdot\frac{6}{8}+\frac{1}{2}\cdot\frac{2}{7}\cdot\frac{4}{6}+\frac{1}{2}\cdot\frac{5}{7}\cdot\frac{3}{6} = \frac{901}{1680}$$

PROBABILITY REGARDING N LETTERS AND THEIR ENVELOPES

If n letters corresponding to n envelopes are placed in the envelopes at random, then

Probability that all letters are in right envelopes = $\frac{1}{n!}$

Probability that all letters are not in right envelopes

$= 1 - \frac{1}{n!}$

Probability that no letters is in right envelopes

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....+ (-1)^n \frac{1}{n!}$

Probability that exactly r letters are in right envelopes

$= \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - .....+ (-1)^{n-r} \frac{1}{(n-r)!}\right]$

Example 52 :

There are four letters and four envelopes, the letters are placed into the envelopes at random, find the probability that all letters are placed in the wrong envelopes.

Sol. We know from the above given formula that probability that no letter is in right envelope out of n letters and n envelopes is given by $\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....(-1)^n \frac{1}{n!}\right]$

Since all 4 letters are to be placed in wrong envelopes then

required probability $= \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \frac{3}{8}$

PROBABILITY DISTRIBUTION

A probability distribution spells out how a total probability of 1 is distributed over several values of a random variable.

The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0, \sum_{i=n}^{n} p_i = 1, i = 1, 2, ..., n$

The real numbers $x_1, x_2, ..., x_n$ are the possible values of the random variable X and p_i (i = 1,2,..., n) is the probability of the random variable X taking the value x_i i.e., $P(X = x_i) = p_i$

Mean of a random variable :

Mean is a measure of location or central tendency in the sense that it roughly locates a middle or average value of the random variable.

The mean of a random variable X is also called the expectation of X, denoted by E(X).

Thus, $E(X) = \mu = \sum_{i=n}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + ...+ x_n p_n$.

Variance of a random variable :

The variance is a measure of the spread or scatter in data. Let X be a random variable whose possible values $x_1, x_2, ..., x_n$ occur with probabilities $p(x_1), p(x_2), ..., p(x_n)$ respectively.

Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or σ_x^2 is defined as

$$\sigma_x^2 = \text{Var}(X) = \sum_{i=n}^{n} (x_i - \mu)^2 p(x_i)$$

or equivalently, $\sigma_x^2 = E(X - \mu)^2$

Standard deviation: The non-negative number

$\sigma_x = \sqrt{\text{Var}(X)} = \sqrt{\sum_{i=n}^{n} (x_i - \mu)^2 p(x_i)}$ is called the standard deviation of the random variable X.

Another formula to find the variance of a random variable.

We know that,

$$\text{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i) = \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2\mu x_i)\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) + \sum_{i=1}^{n} \mu^2 p(x_i) - \sum_{i=1}^{n} 2\mu x_i p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) + \mu^2 \sum_{i=1}^{n} p(x_i) - 2\mu \sum_{i=1}^{n} x_i p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) + \mu^2 - 2\mu^2 \left[\text{since } \sum_{i=1}^{n} p(x_i) = 1 \text{ and } \mu = \sum_{i=1}^{n} x_i p(x_i)\right]$$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) - \mu^2$$

$$\text{or } \text{Var}(X) = \sum_{i=1}^{n} x_i^2 p(x_i) - \left(\sum_{i=1}^{n} x_i^2 p(x_i)\right)^2$$

$$\text{or } \text{Var}(X) = E(X^2) - [E(X)]^2, \text{where } E(X^2) = \sum_{i=1}^{n} x_i^2 p(x_i)$$

Example 53 :

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance & standard deviation of X.

Sol. Here total students = 15

The ages of students in ascending order are :

14, 14, 15, 16, 16, 17, 17, 17, 18, 19, 19, 20, 20, 20, 21

Now, $P(X=14)=\frac{2}{15}$; $P(X=15)=\frac{1}{15}$; $P(X=16)=\frac{2}{15}$

$P(X=17)=\frac{3}{15}=\frac{1}{5}$; $P(X=18)=\frac{1}{15}$; $P(X=19)=\frac{2}{15}$

$P(X=20)=\frac{3}{15}=\frac{1}{5}$; $P(X=21)=\frac{1}{15}$

Thus the required probability distribution is

X :	14	15	16	17	18	19	20	21
P (X) :	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

Now,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
14	$\frac{2}{15}$	$\frac{28}{15}$	$\frac{392}{15}$
15	$\frac{1}{15}$	1	15
16	$\frac{2}{15}$	$\frac{32}{15}$	$\frac{512}{15}$
17	$\frac{1}{5}$	$\frac{17}{5}$	$\frac{289}{5}$
18	$\frac{1}{15}$	$\frac{18}{15}$	$\frac{324}{15}$
19	$\frac{2}{15}$	$\frac{38}{15}$	$\frac{722}{15}$
20	$\frac{1}{5}$	4	80
21	$\frac{1}{15}$	$\frac{21}{15}$	$\frac{441}{15}$
	1	$\frac{263}{15}$	$\frac{4683}{15}$

Mean, $(\mu)=\Sigma\, p_i x_i=\frac{263}{15}=17.53$

Variance $(\sigma^2)=\Sigma\, p_i x_i^2-(\Sigma\, p_i x_i)^2$

$$=\frac{4683}{15}-\left(\frac{263}{15}\right)^2=\frac{4683}{15}-\frac{69169}{225}$$

$$=\frac{70245-69169}{225}=\frac{1076}{225}=4.78$$

Standard deviation $=\sqrt{\text{Variance}}=\sqrt{4.78}=2.19$

Example 54 :

Two bad eggs are accidently mixed with 10 good eggs 3 eggs are drawn simultaneously from the basket. Find the mean and variance of the number of bad eggs drawn.

Sol. At $x=0$, $P(0)=\frac{{}^{10}C_3}{{}^{12}C_3}=\frac{10\cdot 9\cdot 8}{12\cdot 11\cdot 10}=\frac{6}{11}$

At $x=1$, $P(1)=\frac{{}^{2}C_1\times{}^{10}C_2}{{}^{12}C_3}=\frac{9}{22}$

At $x=2$, $P(2)=\frac{{}^{2}C_2\cdot{}^{10}C_1}{{}^{12}C_3}=\frac{10\cdot 6}{12\cdot 11\cdot 10}=\frac{1}{22}$

x_i	p_i	$p_i x_i$
0	$\frac{6}{11}$	0
1	$\frac{9}{22}$	$\frac{9}{22}$
2	$\frac{1}{22}$	$\frac{2}{22}$

$$\mu=\sum p_i x_i=\frac{11}{22}=\frac{1}{2}$$

$$\therefore\ \sum p_i x_i^2=0+\frac{9}{22}+\frac{4}{22}$$

$$\sigma^2=\sum p_i x_i^2-\mu^2=\frac{13}{22}-\frac{1}{4}=\frac{15}{44}$$

BINOMIAL DISTRIBUTION FOR REPEATED TRIALS

Binomal experiment :

1. The same experiment is repeated several times.
2. There are only two possible outcomes, success or failure.
3. The repeated trials are independent so that the probability of each outcome remains the same for each trial.

Binomial probability : Let an experiment has n-independent trials, and each of the trial has two possible outcomes (i) success (ii) failure

If probability of getting success, P(S) = p and probability getting failure, P(F) = q such that p + q = 1.

Then, P(r successes) = ${}^nC_r\, p^r\, q^{n-r}$

Proof : Consider the compound event where r successes are in succession and (n – r) failures are in succession.

$$P\left(\underbrace{SSS\ldots\ldots S}_{r}\ \underbrace{FFF\ldots\ldots F}_{(n-r)}\right)$$

$$=\underbrace{P(S).P(S)\ldots\ldots P(S)}_{r\text{ times}}\ \underbrace{P(F).P(F)\ldots\ldots P(F)}_{(n-r)\text{ times}}=P^r\,.\,q^{n-r}$$

But these r successes and (n – r) failures can be arranged in $\frac{n!}{r!\,(n-r)!}={}^nC_r$ ways and in each arrangement the probability will be $p^r\,.\,q^{n-r}$

Hence total pr. = $P(r) = {}^nC_r\, p^r\, q^{n-r}$ ......(1)

Recurrence relation

$p(r+1) = {}^nC_{r+1}\, p^{r+1} \,.\, q^{n-r-1}$

$$\therefore \quad \frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1}}{{}^nC_r}\frac{p}{q} = \frac{n-r}{r+1}\frac{p}{1-p}$$

$$\therefore \quad P(r+1) = \frac{n-r}{r+1}\cdot\frac{p}{1-p}P(r) \qquad ......(2)$$

Equation (2) is used for completely the probabilities of P (1); P (2); P (3); ...... etc. once P (0) is determined.

Note :

* The mean the variance and the standard deviation of binomial distribution are np, npq, $\sqrt{npq}$.

Example 55 :

Find the probability of getting exactly 7 heads in 8 tosses of a fair coin.

Sol. The probability of success, getting a head in a single toss, is 1/2, so the probability of failure, getting a tail, is 1/2.

$$P\text{ (7 heads in 8 tosses)} = \binom{8}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^1 = 8\left(\frac{1}{2}\right)^8 = .3125$$

Example 56 :

The advertising agency that handles the Diet Supercola account believes that 40% of all consumers prefer this product over its competitors. Suppose a random sample of 6 people is chosen. Assume that all responses are independent of each other. Find the probability of the following. (a) Exactly 4 of the 6 people prefer Diet Supercola. (b) None of the 6 people prefers Diet Supercola.

Sol. (a) We can think of the 6 responses as 6 independent trials. A success occurs if a person prefers Diet Supercola. Then this is a binomial experiment with

p = P (success) = P(prefer Diet Supercola) = .4.

The sample is made up of 6 people, so n = 6.

To find the probability that exactly 4 people prefer this drink, we let x = 4 and use the result in the box.

$$P\text{ (exactly 4)} = \binom{6}{4}(.4)^4(1-.4)^{6-4} = 15(.4)^4(6)^2$$

$$= 15\,(.0256)(.36) = .13824$$

(b) Let x = 0

$$P\text{ (exactly 0)} = \binom{6}{0}(.4)^0(1-.4)^6 = 1(1)(.6)6 \approx 0.0467$$

Example 57 :

A pair of dice is thrown 6 times, getting a doublet is considered a success. Compute the probability of

(i) no success

(ii) exactly one success

(iii) at least one success

(iv) at most one success

Sol. Total sample spaces are = 36

In which six doublets then

$$p = \frac{3}{36} = \frac{1}{6}; \qquad q = 1 - \frac{1}{6} = \frac{5}{6}$$

(i) No success for r = 0

$$\therefore \quad p(0) = {}^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6 = \left(\frac{5}{6}\right)^6$$

(ii) Exactly one success for r = 1

$$\therefore \quad p(1) = {}^6C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^5$$

(iii) For at least one success for r = 1, 2, 3, 4, 5, 6.

$$\therefore \quad \sum_{r=1}^{6} {}^6C_r p^r q^{6-r} = {}^6C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5$$

$$+ {}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4 + {}^6C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 + {}^6C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2$$

$$+ {}^6C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^1 + {}^6C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^0$$

(iv) For at most one success for r = 0, 1

$$\sum_{r=0}^{1} {}^6C_r\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^{6-r} = {}^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6 + {}^6C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5$$

Example 58 :

The mean and variance of a binomial distribution are 4 and 3 respectively. Find the probability of getting exactly six successes in this distribution.

Sol. Let n and p be the parameters of the binomial variate.

Then, np = 4, and npq = 3

$$\Rightarrow q = \frac{npq}{np} = \frac{3}{4} \Rightarrow p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \; np = 4 \Rightarrow n \times \frac{1}{4} = 4 \Rightarrow n = 16$$

So, required probability

$$= {}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{16-6} = {}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$$

Example 59 :

A pair of dice is thrown 5 times if getting a doublet is considered as a success then find the mean & variance of the successes.

Sol. Here, n = 5 and favourable sample space are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ p = 1/6 and q = 5/6

Mean = μ = np = 5/6

Variance = σ^2 = npq = 25/36

Example 60 :

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Sol. Let p be the probability of success.

Then $p = 5\% = \frac{5}{100} = \frac{1}{20}$ and $q = 1 - \frac{1}{20} = \frac{19}{20}$, n = 10

The probability of getting r successes is given by

$$P(X=r) = {}^{10}C_r \left(\frac{19}{20}\right)^{10-r} \cdot \left(\frac{1}{20}\right)^r, \text{ where } r = 0, 1, 2, \ldots, 10$$

$$P(X \le 1) = P(X=0) + P(X=1)$$

$$= {}^{10}C_0 \left(\frac{19}{20}\right)^{10} \cdot \left(\frac{1}{20}\right)^0 + {}^{10}C_1 \left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)^1$$

$$= 1 \times \left(\frac{19}{20}\right)^{10} \times 1 + 10 \times \left(\frac{19}{20}\right)^9 \cdot \frac{1}{20}$$

$$= \left(\frac{19}{20}\right)^{10} \left(\frac{19}{20} + \frac{10}{20}\right) = \left(\frac{19}{20}\right)^9 \times \frac{29}{20}$$

Example 61 :

The probability that a student is not a swimmer is 1/5. Then the probability that out of five students, four are swimmers is –

(A) ${}^5C_4 \left(\frac{4}{5}\right)^4 \cdot \frac{1}{5}$ (B) $\left(\frac{4}{5}\right)^4 \cdot \frac{1}{5}$

(C) ${}^5C_1 \frac{1}{5} \cdot \left(\frac{4}{5}\right)^4$ (D) None of these

Sol. **(A).** If p be the probability of success.

Then $p = 1 - \frac{1}{5} = \frac{4}{5}$ and $q = \frac{1}{5}$, n = 5

The probability of getting r successes is given by

$$P(X=r) = {}^5C_r \left(\frac{4}{5}\right)^r \cdot \left(\frac{1}{5}\right)^{5-r}$$

where r = 0, 1, 2, ... 5

$$P(X=4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)^1 = {}^5C_4 \left(\frac{4}{5}\right)^4 \cdot \frac{1}{5}$$

TRY IT YOURSELF-2

Q.1 In answering a question on a multiple choice test, a student either knows the answer or guesses. Consider 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?

Q.2 There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?

Q.3 A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Q.4 A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Q.5 Probability that A speaks truth is 4/5 . A coin is tossed. A reports that a head appears. The probability that actually there was head is –

(A) 4/5 (B) 1/2

(C) 1/5 (D) 2/5

Q.6 A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P (X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine : (i) k (ii) $P(X < 3)$

Q.7 Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Q.8 A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Q.9 In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is–

(A) 10^{-1} (B) $(1/2)^5$

(C) $(9/10)^5$ (D) 9/10

ANSWERS

(1) 12/13	**(2)** 4/9	**(3)** 1/4
(4) 5/34	**(5)** (A)	**(6)** (i)1/10, (ii) 3/10
(7) 1/3	**(8)** 25/216	**(9)** (C)

IMPORTANT POINTS

* Probability of an event: For a finite sample space with equally likely outcomes Probability of an event

$P(A) = \frac{n(A)}{n(S)}$, where n (A) = number of elements in the set A, n (S) = number of elements in the set S.

* f A and B are any two events, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ equivalently,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
* If A and B are mutually exclusive, then
$P(A \text{ or } B) = P(A) + P(B)$.
* If A is any event, then P (not A) = 1 – P(A)
* The conditional probability of an event E, given the occurrence of the event F is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

* Theorem of total probability : Let $\{E_1, E_2, ..., E_n\}$ be a partition of a sample space and suppose that each of $E_1, E_2, ..., E_n$ has nonzero probability. Let A be any event associated with S, then
$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + ... + P(E_n)P(A|E_n)$$
* Bayes' theorem If $E_1, E_2, ..., E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, ..., E_n$ are pairwise disjoint and $E1 \cup E_2 \cup ... \cup E_n = S$ and A be any event with nonzero probability, then

$$P(E_i | A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)}$$

* Let X be a random variable whose possible values $x_1, x_2, x_3, ..., x_n$ occur with probabilities $p_1, p_2, p_3, ... p_n$ respectively. The mean of X, denoted by μ, is the number

$$\sum_{i=1}^{n} x_i p_i$$

The mean of a random variable X is also called the expectation of X, denoted by E (X).
* Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :
(a) There should be a finite number of trials. (b) The trials should be independent. (c) Each trial has exactly two outcomes : success or failure. (d) The probability of success remains the same in each trial.
For Binomial distribution B (n, p), $P(X = x) = {}^nC_x q^{n-x} p^x$, $x = 0, 1, ..., n$ $(q = 1 - p)$
* Let A and B be two events, then $P(A) + P(\bar{A}) = 1$

$P(A + B) = 1 - (\bar{A}\bar{B})$

$$P(A/B) = \frac{P(AB)}{P(B)}$$

$P(A + B) = P(AB) + P(\bar{A}B) + P(A\bar{B})$

$A \subset B \Rightarrow P(A) \leq P(B)$

$P(\bar{A}B) = P(B) - P(AB)$

$P(AB) \leq P(A)P(B) \leq P(A + B) \leq P(A) + P(B)$

$P(AB) = P(A) + P(B) - P(A + B)$

$P(\text{Exactly one event}) = P(A\bar{B}) + P(\bar{A}B)$
$= P(A) + P(B) - 2P(AB)$
$= P(A + B) - P(AB)$

$P(\text{neither A nor B}) = P(\bar{A}\bar{B}) = 1 - P(A + B)$

$P(\bar{A} + \bar{B}) = 1 - P(AB)$

* Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = 2^n
* Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times) = 6^n
* **Playing Cards :**
Total Cards : 52(26 red. 26 black)
Four suits : Heart, Diamond, Spade, Club - 13 cards each
Court Cards : 12 (4 kings, 4 queens, 4 jacks)
Honour Cards : 16 (4 aces, 4 kings, 4 queens, 4 jacks)
* **Expectation :** If there are n possibilities $A_1, A_2, ... A_n$ in an experiment having the probabilities $p_1, p_2, ... p_n$ respectively. If value $M_1, M_2, ...M_n$ are associated with the respective possibility. Then the expected value of the experiment is given by $\sum_{r=1}^{n} p_i.M_i$

ADDITIONAL EXAMPLES

Example 1 :
A bag contains 5 brown and 4 white socks. A man pulls out 2 socks. Then find the probability that they are of the same colour.

Sol. Let $A \equiv$ event of two socks being brown.
$B \equiv$ event of two socks being white.

Then $P(A) = \frac{{}^5C_2}{{}^9C_2} = \frac{5.4}{9.8} = \frac{5}{18}$, $P(B) = \frac{{}^4C_2}{{}^9C_2} = \frac{4.3}{9.8} = \frac{3}{18}$

Now since A and B are mutually exclusive events, so required probability

$$= P(A + B) = P(A) + P(B) = \frac{5}{18} + \frac{3}{18} = \frac{4}{9}$$

Example 2 :
Two dice are thrown together. Find the probability that atleast one will show its digit greater than 3.

Sol. Total exhaustive cases = $6^2 = 36$
Out of these cases following 9 pairs are not favourable
(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

$$\therefore \text{Required probability} = 1 - \frac{9}{36} = \frac{3}{4}$$

Example 3 :

Four persons are selected at random out of 3 men, 2 women and 4 children. Find the probability that there are exactly 2 children in the selection.

Sol. Total number of ways in which 4 persons can be selected out of 3 + 2 + 4 = 9 persons = ${}^9C_4 = 126$.

Number of ways in which a selection of 4 contains exactly 2 children = ${}^4C_2 \times {}^5C_2 = 60$.

$$\therefore \text{ Required probability} = \frac{60}{126} = \frac{10}{21}$$

Example 4 :

The probability that an anti aircraft gun can hit an enemy plane at the first, second and third shot are 0.6, 0.7 and 0.1 respectively. Find the probability that the gun hits the plane.

Sol. Let the events of hitting the enemy plane at the first, second and third shot are respectively A, B and C. Then as given P (A) = 0.6, P (B) = 0.7, P (C) = 0.1

Since events A, B, C are independent, so

Required probability

$$= P(A + B + C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$$

$$= 1 - (1 - 0.6)(1 - 0.7)(1 - 0.1)$$

$$= 1 - (0.4)(0.3)(0.9) = 1 - 0.108 = 0.892$$

Example 5 :

A purse contains 4 copper and 3 silver coins and another purse contains 6 copper and 2 silver coins. One coin is drawn from any one of these two purses. Find the probability that it is a copper coin.

Sol. Let A ≡ event of selecting first purse

B ≡ event of selecting second purse

C ≡ event of drawing a copper coin from first purse

D ≡ event of drawing a copper coin from second purse

Then given event has two disjoint cases : AC and BD.

$\therefore$ Reqd. probability = P (AC + BD)

$$= P(AC) + P(BD) = P(A)\,P(C) + P(B)\,P(D)$$

$$= \frac{1}{2}\cdot\frac{4}{7} + \frac{1}{2}\cdot\frac{6}{8} = \frac{37}{56}$$

Example 6 :

A box contains 3 white and 2 red balls. If first drawing ball is not replaced then find the probability that the second drawing ball will be red.

Sol. Let A ≡ the event that drawing ball is white

B ≡ the event that drawing ball is red

There are two mutually exclusive cases of the required event: WR and RR

$$\text{Now, } P(WR) = P(W)\,P\left(\frac{R}{W}\right) = \frac{3}{5}\cdot\frac{2}{4} = \frac{6}{20}$$

$$P(RR) = P(R)\,P\left(\frac{R}{R}\right) = \frac{2}{5}\cdot\frac{1}{4} = \frac{2}{20}$$

Reqd. probability = P (WR + RR)

$$= P(WR) + P(RR) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

Example 7 :

Two numbers are selected at random from 40 consecutive natural numbers. Find the probability that the sum of the selected numbers is odd.

Sol. Total number of selection of 2 numbers from 40 natural numbers = ${}^{40}C_2$

Now, since the sum of two natural numbers is odd if one of them is even and the other is odd. Also among 40 consecutive natural numbers 20 are even and 20 are odd. Hence number of ways of selection of one even and one odd number = ${}^{20}C_1 \times {}^{20}C_1$

$$\therefore \text{ Reqd. probability} = \frac{{}^{20}C_1 \times {}^{20}C_1}{{}^{40}C_2} = \frac{20 \times 20 \times 2}{40 \times 39} = \frac{20}{39}$$

Example 8 :

It has been found that is A and B play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. Find the probability that they will win alternately.

Sol. The probability of A winning in a game = $P(A) = \frac{6}{12} = \frac{1}{2}$

The probability of B winning in a game = $P(B) = \frac{4}{12} = \frac{1}{3}$

$$\therefore \text{ Reqd. probability } = P(A \cap B \cap A) + P(B \cap A \cap B)$$

$$= P(A).P(B).P(A) + P(B).P(A).P(B)$$

$$= \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3} = \frac{5}{36}$$

Example 9 :

A coin is tossed 7 times. Each time a man calls head. Find the probability that he wins the toss on more occasions.

Sol. The man has to win at least 4 times.

$$\therefore \text{ Reqd. probability} = {}^7C_4\left(\frac{1}{2}\right)^4.\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5.\left(\frac{1}{2}\right)^2$$

$$+ {}^7C_6\left(\frac{1}{2}\right)^6.\frac{1}{2} + {}^7C_7\left(\frac{1}{2}\right)^7$$

$$= ({}^7C_4 + {}^7C_5 + {}^7C_7 + {}^7C_7)\,.\,\frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}$$

Example 10 :

If a fair coin is tossed 15 times, what is the probability of getting head as many times in the first ten throws as in the last five ?

Sol. In the last five throws there can be 0, 1, 2, 3, 4 or 5 heads and the same should be the case in the first ten throws.

Hence the favourable number of cases.

$${}^5C_0\,{}^{10}C_0 + {}^5C_1\,{}^{10}C_1 + {}^5C_2\,{}^{10}C_2 + {}^5C_3\,{}^{10}C_3 + {}^5C_4\,{}^{10}C_4 + {}^5C_5\,{}^{10}C_5$$

$$= 1 + 50 + 450 + 1200 + 1050 + 252 = 3003$$

And the total number of ways $n = 2^{15} = 32768$

Hence the required probability $= \frac{m}{n} = \frac{3003}{32768}$

Example 11 :

Two friends Ashok and Baldev have equal number of sons. There are 3 tickets for a cricket match which are to be distributed among the sons. The probability that 2 tickets go to the sons of the one and one ticket go to the sons of the other is 6/7. Find how many sons each of the two friends have.

Sol. Let each of them have n sons each. Hence we have to distribute 3 tickets amongst the sons of Ashok and Baldev, in such a manner that one ticket goes to the sons of one and two tickets to the sons of other.

1 to Ashok's sons and 2 to Baldev's son + 2 to Ashok's son and 1 to Baldev's son = Total number of ways of distributing the ticket as per directions.

${}^nC_1 . {}^nC_2 + {}^nC_2 . {}^nC_1 = 2 . {}^nC_1 . {}^nC_2 = m$

But in all 3 tickets are to be distributed amongst 2n sons of both. Hence total number of ways is ${}^{2n}C_3 = n$

Hence required probability $= \frac{m}{n} = \frac{6}{7}$ given

$$\therefore\ 2.\frac{{}^nC_1.{}^nC_2}{{}^{2n}C_3} = \frac{6}{7};\quad 7.\frac{n\,(n-1)}{2}.n = 3.\frac{2n(2n-1)(2n-2)}{6}$$

Cancel n (n – 1) from both sides

or $7n = 4\,(2n-1)$ or $n = 4$

Example 12 :

If the integers m and n are chosen at random between 1 and 100, then find the probability that a number of the form 7^m+7^n is divisible by 5.

Sol. $7^2 = 49$, $7^4 = 49 \times 49 = 2401$ $\therefore$ 7^6, 7s, 7^{14} all end with 9

$7^8, 7^{12}, 7^{16}$ all end with 1

Now $7^m + 7^n$ will be divisible by 5 if one ends with 9 and other ends with 1 so that the sum has 0 in the end.

$\therefore$ m can be 2, 6, 10, 14, ... 98 (25)

n can be 4, 8, 12, 16, ... 100 (25)

But m and n can interchange $p = \frac{2 \times 25 \times 25}{100 \times 100} = \frac{1}{8}$

Example 13 :

Two sets of candidates are competing for the positions on the board of directors of a company. The probabilities that the first and second sets will win are 0.6 and 0.4 respectively. If the first set wins the probability of introducing a new product is 0.8 and the corresponding probability if the second set wins is 0.3. What is the probability that the new product will be introduced ?

Sol. Let $P(A_1)$ = probability that the first set wins = 0.6

$P(A_2)$ = probability that the second set wins = 0.4

P (B) = probability that a new product is introduced

$\therefore\ P(B) = P(B \cap A_1) + P(B \cap A_2)$

$= P(A_1)\,P(B\,|\,A_1) + P(A_2)\,.\,P(B\,|\,A_2)$ .... (1)

Now as per hypothesis

$P(B\,|\,A_1) = 0.8,\ P(B\,|\,A_2) = 0.3$

Hence from (1), $P(B) = 0.6 \times 0.8 + 0.4 \times 0.3 = 0.60$

Example 14 :

There is 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 days ? Given that there is at least one rainy day, what is the probability that there are at least two rainy days?

Sol. $p(r) = \frac{3}{10}$, so $p(\overline{r}) = \frac{7}{10}$

The probability of at least one rainy day in 7 days

$$P(A) = 1 - \left(\frac{7}{10}\right)^7$$

Now the probability that at least two rainy days in 7 days

$$P(B) = 1 - \left(\frac{7}{10}\right)^7 - {}^7C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^6$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1-\left(\frac{7}{10}\right)^7 - {}^7C_1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^6}{1-\left(\frac{7}{10}\right)^7}$$

Example 15 :

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that (i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is a spade?

Sol. Let p be the probability of success.

Then $p = \frac{13}{52} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$, $n = 5$

The probability of getting r successes is given by

$$P(X = r) = {}^5C_r \left(\frac{3}{4}\right)^{5r} . \left(\frac{1}{4}\right)^r \quad \text{where } r = 0, 1, 2, \ldots\ldots, 5$$

(i) $P(X = 5) = {}^5C_5\left(\frac{3}{4}\right)^0 . \left(\frac{1}{4}\right)^5 = 1 \times 1 \times \frac{1}{1024} = \frac{1}{1024}$

(ii) $P(X = 3) = {}^5C_3\left(\frac{3}{4}\right)^2 . \left(\frac{1}{4}\right)^3 = 10 \times \frac{9}{16} \times \frac{1}{64} = \frac{45}{512}$

(iii) $P(X = 0) = {}^5C_0\left(\frac{3}{4}\right)^5 . \left(\frac{1}{4}\right)^0 = 1 \times \frac{243}{1024} \times 1 = \frac{243}{1024}$

Example 16 :

A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Sol. Let p be the probability of success.

Then $p = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$, $n = 4$

The probability of getting r successes is given by

$$P(X=r) = {}^4C_r \left(\frac{9}{10}\right)^{4-r} \cdot \left(\frac{1}{10}\right)^r \quad \text{where } r = 0, 1, 2, 3, 4$$

$$P(X=0) = {}^4C_0 \left(\frac{9}{10}\right)^4 \cdot \left(\frac{1}{10}\right)^0 = 1 \times \left(\frac{9}{10}\right)^4 \times 1 = \left(\frac{9}{10}\right)^4$$

Example 17 :

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Sol. Let p be the probability of success.

The $p = 10\% = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$, $n = 12$

The probability of getting r successes is given by

$$P(X=r) = {}^{12}C_r \left(\frac{9}{10}\right)^{12-r} \cdot \left(\frac{1}{10}\right)^r, \text{ where } r = 0, 1, 2, \ldots\ldots 12$$

$$P(X=9) = {}^{12}C_9 \left(\frac{9}{10}\right)^3 \cdot \left(\frac{1}{10}\right)^9 = 220 \times \frac{(9)^3}{(10)^{12}} = 22 \times \frac{(9)^3}{(10)^{11}}$$

Example 18 :

In a hurdle race, a man has to clear 9 hurdles. Probability that he clears a hurdle 2/3 and the probability that he knocks down the hurdle is 1/3. Find the probability that he knocks down fewer than 2 hurdles.

Sol. For probability that he knocks down fewer than two hurdles for r = 0, 1 ; where p = 1/3, q = 2/3

$$\sum_{r=0}^{1} {}^9C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{9-r} = {}^9C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^9 + {}^9C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^8$$

Example 19 :

A drunkard takes a step forward or backward. The probability that he takes a step forward is 0.4. Find the probability that at the end of 11 steps he is one step away from the starting point.

Sol. At the end of 11 steps he is one step away from the starting point by two ways

(i) Man has taken 6 steps forward and 5 steps backward

(ii) Man has taken 6 steps backward and 6 steps forward

Here, p = probability of forward = 2/5

q = probability of backward = 3/5

$$\therefore \text{ Probability} = {}^{11}C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^5 + {}^{11}C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^6$$

Example 20 :

There are two bags. The first bag contains 5 white and 3 black balls and the second bag contains 3 white and 5 black balls. Two balls are drawn at random from the first bag and are put into the second bag, without noting their colours. Then two balls are drawn from the second bag. Find the probability that the balls drawn are white and black.

Sol.

Bag-I	Bag-II
5 White 3 Black	3 White 5 Black

Let E_1, E_2 and E_3 be the events of transferring 2 white, 1 white and 1 black, 2 black balls respectively from the first bag to the second bag.

$$\therefore \; P(E_1) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28} = \frac{5}{14}$$

$$P(E_2) = \frac{{}^5C_1 \times {}^3C_1}{{}^8C_2} = \frac{5 \times 3}{28} = \frac{15}{28}; \; P(E_3) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$$

Let A be the event of drawing one white and one black ball from the second bag.

$P(A) = P(E_1A \text{ or } E_2A \text{ or } E_3A)$

$= P(E_1A) + P(E_2A) + P(E_3A)$

$$= P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)$$

$$= \frac{5}{14} \times \frac{{}^5C_1 \times {}^5C_1}{{}^{10}C_2} + \frac{15}{28} \times \frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} + \frac{3}{28} \times \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2}$$

$$= \frac{5}{14} \times \frac{5}{9} + \frac{15}{28} \times \frac{8}{15} + \frac{3}{28} \times \frac{7}{15} = \frac{673}{12600}$$

Example 21 :

Two machines A and B produce respectively 60% and 40% of the total numbers of items of a factory. The percentages of defective output of these machines are respectively 2% and 5%. If an item is selected at random, what is the probability that the item is (i) defective (ii) non-defective?

Sol. Let E_1, E_2 be the events of drawing an item produced by machine A and machine B respectively. Let A be the event of selecting a defective item.

$\bar{A}$ represent the event of selecting a non-defective item.

We have, $P(E_1) = 60\%$; $P(E_2) = 40\%$

$P(A/E_1)$= Probability that an item produced A is defective = 2%

$P(A/E_3)$ = Probability that an item produced by B is defective = 5%

(i) P (selected item is defective)

$= P(A) = P(E_1 A \text{ or } E_2 A) = P(E_1 A) + P(E_2 A)$

$= P(E_1)\, P(A/E_1) + P(E_2)\,(A/E_2)$

$= (60\%)\,(2\%) + (40\%)\,(5\%)$

$$= \frac{60}{100}\times\frac{2}{100}+\frac{40}{100}\times\frac{5}{100}=\frac{320}{1000}=0.032$$

(ii) P (selected item is non-defective)

$= P(\bar{A}) = P(E_1\bar{A} \text{ or } E_2\bar{A}) = P(E_1\bar{A}) + P(E_2\bar{A})$

$$= P(E_1)\,P\left(\frac{\bar{A}}{E_1}\right) + P(E_2)\,P\left(\frac{\bar{A}}{E_2}\right)$$

$= (60\%)\,(98\%) + (40\%)\,(95\%)$

$$= \frac{60}{100}\times\frac{98}{100}+\frac{40}{100}\times\frac{95}{100}=\frac{9680}{10000}=0.968$$

Example 22 :

In a test, an examinee either guesses or copies or knows the answer for a multiple choice question having FOUR choices of which exactly one is correct. The probability that he makes a guess is 1/3 and the probability for copying is 1/6. The probability that his answer is correct, given that he copied it is 1/8. The probability that he knew the answer, given that his answer is correct is

(A) 5 / 29 (B) 9 / 29

(C) 24 / 29 (D) 20 / 29

Sol. Let the events be defined as

E_1 : Guessing

E_2 : Copying

E_3 : Knowing

E : Correct answer

By hypothesis,

$$P(E_1)=\frac{1}{3},\ P(E_2)=\frac{1}{6},\ P(E_3)=1-\frac{1}{3}-\frac{1}{6}=\frac{1}{2}$$

$$P\left(\frac{E}{E_1}\right)=\frac{1}{4}$$ (out of four choices only one is correct)

$$P\left(\frac{E}{E_2}\right)=\frac{1}{8}\ ;\ P\left(\frac{E}{E_3}\right)=1$$

Therefore by Bayes' theorem

$$P\left(\frac{E_3}{E}\right)=\frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$=\frac{\frac{1}{2}\times 1}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}\times 1}=\frac{24}{29}$$

Example 23 :

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus scooter or by other means of transport are respectively $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4},\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

Sol. Let E be the event that the doctor visits the patient late and let T_1, T_2, T_3, T_4 be the events that the doctor comes by train, bus, scooter, and other means of transport respectively. Then

$$P(T_1)=\frac{3}{10},\ P(T_2)=\frac{1}{5},\ P(T_3)=\frac{1}{10}\ \text{and}\ P(T_4)=\frac{2}{5}$$

(given)

$P(E\,|\,T_1)$ = Probability that the doctor arriving late comes by train = 1/4

Similarly, $P(E\,|\,T_2)=\frac{1}{3}$, $P(E|T_3)=\frac{1}{12}$ and $P(E\,|\,T_4)=0$,

since he is not late if he comes by other means by other means of transport. Therefore, by Bayes' Theorem, we have $P(T_1\,|E)$ = Probability that the doctor arriving late comes by train

$$=\frac{P(T_1)\ P(E|T_1)}{P(T_1)\ P(E|T_1)+P(T_2)\ P(E|T_2)+P(T_3)\ P(E|T_3)+P(T_4)\ P(E|T_4)}$$

$$=\frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}=\frac{3}{40}\times\frac{120}{8}=\frac{1}{2}$$

Hence, the required probability is 1/2.

Example 24 :

Suppose that the reliability of a HIV test is specified as follows :

Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ive but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?

Sol. Let E denote the event that the person selected is actually having HIV and A the event that the person's HIV test is diagnosed as +ve. We need to find P(E|A). Also E' denotes the event that the person selected is actually not having HIV.

Clearly, {E, E'} is a partition of the sample space of all people in the population. We are given that

$$P(E) = 0.1\% = \frac{0.1}{100} = 0.001$$

$P(E') = 1 - P(E) = 0.999$

P (A|E) = P (Person tested as HIV +ve given that he/she is actually having HIV) = 90% = $\frac{90}{100} = 0.9$

and P(A|E') = P(Person tested as HIV +ve given that he/she is actually not having HIV) = 1% = $\frac{1}{100} = 0.01$

Now, by Bayes' theorem

$$P(E|A) = \frac{P(E)\ P(A|E)}{P(E)\ P(A|E) + P(E')\ P(A|E')}$$

$$= \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01} = \frac{90}{1089}$$

Thus, the probability that a person selected at random is actually having HIV given that he/she is tested HIV +ve is $\frac{90}{1089}$.

Example 25 :

A bag contains 4 balls of unknown colours. A ball is drawn at random from it and is found to be white. The probability that all the balls in the bag are white is

(A) 4/5 (B) 1/5

(C) 3/5 (D) 2/5

Sol. Let W_j (j = 1, 2, 3, 4) denote 1, 2, 3 and 4 white balls are in the bag. Let W be the ball drawn is white.

Then, $P(W_1) = P(W_2) = P(W_3) = P(W_4) = 1/4$

$$P\left(\frac{W}{W_1}\right) = \frac{1}{4},\ P\left(\frac{W}{W_2}\right) = \frac{2}{4},\ P\left(\frac{W}{W_3}\right) = \frac{3}{4},\ P\left(\frac{W}{W_4}\right) = 1$$

Therefore by Bayes' theorem

$$P\left(\frac{W}{W_4}\right) = \frac{P(W_4)\,P\left(\frac{W}{W_4}\right)}{\sum_{j=1}^{4} P(W_j)\ P\left(\frac{W}{W_j}\right)} = \frac{\frac{1}{4} \times 1}{\frac{1}{4}\left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4}\right)} = \frac{4}{10} = \frac{2}{5}$$

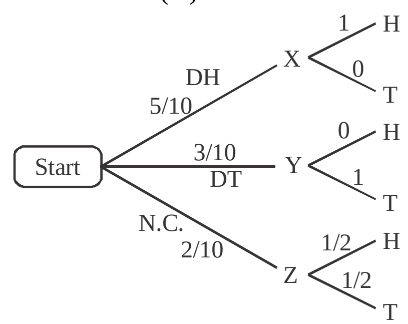
Example 26 :

A box contains 10 coins

- 5 coins DH denoted by say X
- 3 coins DT denoted by say Y
- 2 coins normal denoted by Z

A coin is dream at random from the box and tossed, fall headwise. Find the probability that it was a normal coin. (DH : Double Head)

Sol. $$P\left(\frac{Z}{H}\right) = \frac{P(H \cap Z)}{P(H)}$$



$$P(H) = \frac{5}{10} \cdot 1 + \frac{2-1}{10-2} = \frac{6}{10} = \frac{\frac{2}{10} \cdot \frac{1}{2}}{\frac{6}{10}} = \frac{1}{6}$$

Example 27 :

A sphere of radius r is circumscribed about a cube. Find the probability that a point lies in the sphere but out side the cube.

Sol. $$P = \frac{\text{fav. volume}}{\text{total volume}} = \frac{\frac{4\pi}{3} r^3 - \left(\frac{2r}{\sqrt{3}}\right)^3}{\frac{4}{3}\pi r^3} = 1 - \frac{2}{\sqrt{3}\pi}$$

Example 28 :

Three balls are drawn are by one without replacement from a bag containing 5 white and 4 red balls. Find the probability distribution of the number of red balls drawn.

Sol. Let x denote the discrete random variable "number of red balls" ∴ The possible values of x are 0, 1, 2, 3.

5 White 4 Red

Let R_i be the event of drawing a red ball from the bag in the ith draw, i = 1, 2, 3.

$$P(x=0) = P(\bar{R}_1\bar{R}_2\bar{R}_3) = P(\bar{R}_1)\ P\left(\frac{\bar{R}_2}{\bar{R}_1}\right)\ P\left(\frac{\bar{R}_3}{\bar{R}_1\bar{R}_2}\right)$$

$$= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{60}{504} = \frac{5}{42}$$

$$P(x=1) = P(R_1\bar{R}_2\bar{R}_3 \text{ or } \bar{R}_1R_2\bar{R}_3 \text{ or } \bar{R}_1\bar{R}_2R_3)$$

$$= P(R_1)\,P\left(\frac{\bar{R}_2}{R_1}\right)\ P\left(\frac{\bar{R}_3}{R_1\bar{R}_2}\right) + \frac{P(\bar{R}_1)}{P\left(\frac{R_2}{\bar{R}_1}\right)P\left(\frac{\bar{R}_3}{\bar{R}_1R_2}\right)}$$

$$+ P(\bar{R}_1)\ P\left(\frac{\bar{R}_2}{\bar{R}_1}\right)\ P\left(\frac{R_3}{\bar{R}_1\bar{R}_2}\right)$$

$$= \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} = \frac{240}{504} = \frac{10}{21}$$

$P(x=2) = P(R_1 R_2 \bar{R}_3 \text{ or } R_1 \bar{R}_2 R_3 \text{ or } \bar{R}_1 R_2 R_3)$

$$= P(R_1) P\left(\frac{R_2}{R_1}\right) P\left(\frac{\bar{R}_3}{R_1 R_2}\right) + P(R_1) P\left(\frac{\bar{R}_2}{R_1}\right) P\left(\frac{R_3}{R_1 \bar{R}_2}\right)$$

$$+ P(\bar{R}_1) P\left(\frac{R_2}{\bar{R}_1}\right) P\left(\frac{R_3}{\bar{R}_1 R_2}\right)$$

$$= \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{180}{504} = \frac{5}{14}$$

$$P(x=3) = P(R_1 R_2 R_3) = R(R_1) P\left(\frac{R_2}{R_1}\right) P\left(\frac{R_3}{R_1 R_2}\right)$$

$$= \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{24}{504} = \frac{1}{21}$$

The required Probability distribution is

x	0	1	2	3
P(x)	$\frac{5}{42}$	$\frac{10}{21}$	$\frac{5}{14}$	$\frac{1}{21}$

Example 29 :

If difference between mean & variance of a BPD is 1 and difference between squares is 11 then find the probability of getting exactly 3 successes.

Sol. Given $np - npq = 1$ ……(i)

$n^2p^2 - n^2p^2q^2 = 11$ ……(ii)

$\therefore$ $np + npq = 11$

$npq = 5;\ q = 5/6;\ p = 1/6;\ n = 36$

$\therefore$ Required probability $= {}^{36}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{33}$

Example 30 :

If X follows a binomial distribution with mean 3 and variance (3/2), find (i) $P(X \geq 1)$ (ii) $P(X \leq 5)$.

Sol. We know that mean = np and variance = npq

$\therefore$ $np = 3$ and $npq = 3/2 \Rightarrow 3q = 3/2 \Rightarrow q = 1/2$

$\therefore$ $p = (1-q) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$

Now, $np = 3$ and $p = 1/2 \Rightarrow n \times (1/2) = 3 \Rightarrow n = 6$

So, the binomial distribution is given by

$${}^nC_r \cdot p^r \cdot q^{(n-r)} = {}^6C_r \cdot \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{(6-r)}$$

$$= {}^6C_r \left(\frac{1}{2}\right)^6$$

(i) $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^6C_0 \cdot \left(\frac{1}{2}\right)^6 = \left(1 - \frac{1}{64}\right) = \frac{63}{64}$$

(ii) $P(X \leq 5) = 1 - P(X = 6)$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 = \left(1 - \frac{1}{64}\right) = \frac{63}{64}$$

Example 31 :

If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.

Sol. We know that

mean = up and variance = npq

It is being given that $n = 5$ and mean + variance = 1.8

$\therefore$ $np + npq = 1.8$, where $n = 5$

$\Leftrightarrow 5p + 5pq = 1.8$

$\Leftrightarrow p + p(1-p) = 0.36$ $[\because q = (1-p)]$

$\Leftrightarrow p^2 - 2p + 0.36 = 0$

$\Leftrightarrow 100p^2 - 200p + 36 = 0$

$\Leftrightarrow 25p^2 - 50p + 9 = 0$

$\Leftrightarrow 25p^2 - 45p - 5p + 9 = 0$

$\Leftrightarrow 5p(5p - 9) - (5p - 9) = 0$

$\Leftrightarrow (5p - 9)(5p - 1) = 0$

$\Leftrightarrow p = \frac{1}{5} = 0.2$ $[\because$ p cannot exceed 1]

Thus, $n = 5$, $p = 0.2$, and $q = (1-p) = (1-0.2) = 0.8$

Let X denote the binomial variate. Then, the required distribution is

$$P(X = r) = {}^nC_r \cdot p^r \cdot q^{(n-r)} = {}^5C_r \cdot (0.2)^r \cdot (0.8)^{(5-r)}$$

where $r = 0, 1, 2, 3, 4, 5$.

QUESTION BANK

CHAPTER 11 : PROBABILITY

EXERCISE - 1 [LEVEL-1]

Q.1 Two dice are thrown simultaneously, the probability of obtaining a total score of 5 is
(A) 1/12 (B) 1/6
(C) 1/18 (D) 1/9

Q.2 Suppose that A, B, C are events such that
P (A) = P (B) = P (C) = 1/4, P (AB) = P (CB)=0, P(AC)=1/8
then P (A + B) =
(A) 0.125 (B) 0.25
(C) 0.375 (D) 0.5

Q.3 A card is drawn at random from a pack of cards. The probability of this card being a red or a queen is
(A) 1/13 (B) 1/26
(C) 1/2 (D) 7/13

Q.4 If A and B an two events such that

$P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{3}$, then

P (A) =
(A) 1/4 (B) 1/3
(C) 1/2 (D) 2/3

Q.5 If P(A) = 1/5, P (B) = 1/2 and A and B are mutually exclusive then P (A ∪ B) equals-
(A) 1/6 (B) 1/10
(C) 7/10 (D) 1/4

Q.6 One card is drawn from a pack of playing cards, then the probability that it is a card of king is-
(A) 1/12 (B) 1/13
(C) 1/2 (D) 1/4

Q.7 If P (A) =3/8 , then find the odds in against of A -
(A) 3 : 5 (B) 4 : 5
(C) 3 : 4 (D) 5 : 3

Q.8 An integer is chosen at random from the numbers 1, 2, ... 25 the probability that the chosen number is divisible by 3 or 4, is –
(A) 2/25 (B) 11/25
(C) 12/25 (D) 14/25

Q.9 Two squares are chosen at random on a chess-board. The probability that they have a side in common, is
(A) 1/9 (B) 2/7
(C) 1/18 (D) None of these

Q.10 The probability that a certain beginner at golf gets a good shot if he uses the correct club is 1/3 and the probability of a good shot with an incorrect club is 1/4. In his bag are 5 different clubs, only one of which is correct for the shot in question. If he chooses a club at random and takes a stroke, then the probability that he gets a good shot, is
(A) 1/3 (B) 1/12
(C) 4/15 (D) 7/12

Q.11 Two cards are drawn one by one at random from a pack of 52 cards. The probability that both of them are king, is
(A) 2/13 (B) 1/169
(C) 1/221 (D) 30/221

Q.12 Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to
(A) 1/27 (B) 1/9
(C) 4/27 (D) 1/6

Q.13 In a simultaneous throw of three coins, what is the probability of getting at least 2 tails
(A) 1/8 (B) 1/4
(C) 1/2 (D) None of these

Q.14 A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is 1/4 and that of the woman's selection is 1/3. What is the probability that none of them will be selected
(A) 1/2 (B) 1/12
(C) 1/4 (D) None of these

Q.15 If A and B are two events such that $A \subseteq B$, then
P (B/A) =
(A) 0 (B) 1
(C) 1/2 (D) 1/3

Q.16 A bag contains 5 blue, 4 white, and 4 red balls. Three balls are drawn at random then find the probability that all the drawn balls are blue.
(A) 3/143 (B) 1/143
(C) 5/143 (D) 9/143

Q.17 The letters of the word 'SHANU' are written in a row randomly. Then find the probability that vowels occupies the even places.
(A) 7/10 (B) 3/10
(C) 1/21 (D) 1/10

Q.18 One number is selected from first 20 positive integers. What is the probability that it is divisible by 3 or 4.
(A) 1/5 (B) 1/2
(C) 3/16 (D) 1/9

Q.19 If the probability for A to fail in an examination is 0.2 and that of B to fail is 0.3, then the probability that either A or B fails is-
(A) 0.5 (B) 0.44
(C) 0.56 (D) None of these

Q.20 If two dice are thrown together then what is the probability that the sum of their numbers is greater than 9.
(A) 1/2 (B) 1/4
(C) 1/6 (D) 2/6

Q.21 A card is drawn at random from a pack of card. What is the probability that the drawn card is neither a heart nor a king
(A) 4/13 (B) 9/13
(C) 1/4 (D) 13/26

Q.22 X speaks truth in 60% and Y in 50% of the cases. The probability that they contradict each other narrating the same incident is –
(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3

Q.23 In a class of 125 students 70 passed in Mathematics, 55 in Statistics and 30 in both. The probability that a student selected at random from the class has passed in only one subject is –
(A) 13/25 (B) 3/25
(C) 17/25 (D) 8/25

Q.24 In a college, 25% of the boys and 10% of the girls offer Mathematics. The girls constitute 60% of the total number of students. If a student is selected at random and is found to be studying Mathematics, the probability that the student is a girl, is
(A) 1/6 (B) 3/8
(C) 5/8 (D) 5/6

Q.25 A box containing 4 white pens and 2 black pens. Another box containing 3 white pens and 5 black pens. If one pen is selected from each box, then the probability that both the pens are white is equal to
(A) 1/2 (B) 1/3
(C) 1/4 (D) 1/5

Q.26 A basket contains 5 apples and 7 oranges and another basket contains 4 apples and 8 oranges. One fruit is picked out from each basket. Find the probability that the fruits are both apples or both oranges
(A) $\frac{24}{144}$ (B) $\frac{56}{144}$
(C) $\frac{68}{144}$ (D) $\frac{76}{144}$

Q.27 A bag contains tickets numbered from 1 to 20. Two tickets are drawn. The probability that both the numbers are prime, is
(A) 14/95 (B) 7/95
(C) 1/95 (D) None of these

Q.28 Three mangoes and three apples are in a box. If two fruits are chosen at random, the probability that one is a mango and the other is an apple is
(A) 2/3 (B) 3/5
(C) 1/3 (D) None of these

Q.29 5 cards are drawn from a pack of 52 cards what is the probability that these 5 will contain just one king?
(A) $\frac{1243}{10829}$ (B) $\frac{1243}{8829}$
(C) $\frac{3243}{10829}$ (D) $\frac{1243}{10829}$

Q.30 A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all, is
(A) 1/2 (B) 5/9
(C) 4/9 (D) 2/9

Q.31 A bag contains 3 white and 7 red balls. If a ball is drawn at random, then what is the probability that the drawn ball is either white or red
(A) 0 (B) 3/10
(C) 7/10 (D) 10/10

Q.32 If two cards are drawn from a pack of cards then the probability of getting at least one Ace is -
(A) 1/5 (B) 33/221
(C) 3/16 (D) 1/9

Q.33 A card is drawn from a pack of playing cards. Find the probability that the drawn card is a court card when it is black.
(A) 3/26 (B) 3/13
(C) 1/2 (D) None of these

Q.34 If A and B are two events such that P(A) = 1/2, P(B)= 1/3 and $P(A \cup B) = 7/12$ then P (A/B) equals-
(A) 3/4 (B) 1/4
(C) 1/2 (D) None of these

Q.35 Two dice are thrown. Then the probability that the numbers appeared has a sum 8 if it is known that the second die always exhibits 4, is-
(A) 1/3 (B) 2/3
(C) 1/6 (D) 1/2

Q.36 Two cards are drawn one by one from a pack of 52 cards. If the first card is not replaced in the pack , then what is the probability that first card is that of a king and second card is that of a queen?
(A) 4/664 (B) 5/663
(C) 6/663 (D) 4/663

Q.37 For any two events A and B, $P\left(\frac{A}{A \cup B}\right)$ equals-
(A) $\frac{P(\overline{A})}{P(A \cup B)}$ (B) $\frac{P(\overline{B})}{P(A \cup B)}$
(C) $\frac{P(A)}{P(A \cup B)}$ (D) $\frac{P(A \cap B)}{P(A \cup B)}$

Q.38 A person can kill a bird with probability 3/4. He tries 5 times. What is the probability that he may not kill the bird
(A) $\frac{243}{1024}$ (B) $\frac{781}{1024}$
(C) $\frac{1}{1024}$ (D) $\frac{1023}{1024}$

Q.39 Six cards are drawn simultaneously from a pack of playing cards. What is the probability that 3 will be red and 3 black
(A) ${}^{26}C_6$ (B) $\frac{{}^{26}C_3}{{}^{52}C_6}$
(C) $\frac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6}$ (D) 1/2

Q.40 A box contains 15 tickets numbered 1, 2, ... 15. Seven tickets are drawn at random one after the other with replacement. The probability that the greatest number on a drawn ticket is 9, is
(A) $(9/10)^6$ (B) $(8/15)^7$
(C) $(3/5)^7$ (D) None of these

Q.41 If a party of n persons sit at a round table, then the odds against two specified individuals sitting next to each other are

(A) 2 : (n – 3) (B) (n – 3) : 2
(C) (n – 2) : 2 (D) 2 : (n – 2)

Q.42 Let E and F be two independent events. The probability that both E and F happens is 1/12 and the probability that neither E nor F happens is 1/2, then

(A) $P(E)=\frac{1}{3},\ P(F)=\frac{1}{4}$ (B) $P(E)=\frac{1}{2},\ P(F)=\frac{1}{6}$

(C) $P(E)=\frac{1}{6},\ P(F)=\frac{1}{2}$ (D) None of these

Q.43 If the probabilities of boy and girl to be born are same, then in a 4 children family the probability of being at least one girl, is

(A) 14/16 (B) 15/16
(C) 1/8 (D) 3/8

Q.44 A man and his wife appear for an interview for two posts. The probability of the husband's selection is 1/7 and that of the wife's selection is 1/5. What is the probability that only one of them will be selected

(A) 1/7 (B) 2/7
(C) 3/7 (D) None of these

Q.45 Seven chits are numbered 1 to 7. Three are drawn one by one with replacement. The probability that the least number on any selected chit is 5, is

(A) $1-\left(\frac{2}{7}\right)^4$ (B) $4\left(\frac{2}{7}\right)^4$

(C) $\left(\frac{3}{7}\right)^3$ (D) None of these

Q.46 'A' draws two cards with replacement from a pack of 52 cards and 'B' throws a pair of dice what is the chance that 'A' gets both cards of same suit and 'B' gets total of 6

(A) 1/144 (B) 1/4
(C) 5/144 (D) 7/144

Q.47 A and B are two events such that P (A) $\neq$ 0, P (B/A) if

(i) A is a subset of B
(ii) $A \cap B = \phi$ are respectively

(A) 1, 1 (B) 0 and 1
(C) 0, 0 (D) 1, 0

Q.48 A box contains 6 red marbles numbers from 1 through 6 and 4 white marbles 12 through 15. Find the probability that a marble drawn 'at random' is white and odd numbered

(A) 5 (B) 1/5
(C) 6 (D) 1/6

Q.49 There are eighty cards numbered 1 to 80, two cards are selected randomly. The probability that both the cards have the numbers divisible by 4, is –

(A) $\frac{21}{316}$ (B) $\frac{1}{4}$

(C) $\frac{19}{316}$ (D) $\frac{2}{3}$

Q.50 A and B draw two cards each, one after another, from a pack of well– shuffled pack of 52 cards. The probability that all the four cards drawn are of the same suit is –

(A) $\frac{44}{85\times 49}$ (B) $\frac{11}{85\times 49}$

(C) $\frac{13\times 24}{17\times 25\times 49}$ (D) $\frac{1}{4}$

Q.51 Cards are drawn one after the other from a well shuffled pack of 52 playing cards until 2 aces are obtained for the first time, the probability that 18 draws are required for this

(A) 3/34 (B) 17/455

(C) $\frac{561}{15925}$ (D) $\frac{3}{4}$

Q.52 If three cards are drawn from a bag containing 6n cars numbered 0, 1, 2, 3, ... 6n – 1 then probability that all the cards are multiple of 3 is –

(A) $\frac{3}{(6n-1)(6n-2)}$ (B) $\frac{(2n-1)(2n-2)(2n-3)}{12(6n-1)(3n-1)}$

(C) $\frac{3n}{(6n-1)(6n-2)}$ (D) $\frac{3n-2}{4(6n-1)}$

Q.53 If three cards are drawn from a bag containing 6n cars numbered 0, 1, 2, 3, ... 6n – 1 then probability of all the cards are even numbered

(A) $\frac{3}{(6n-1)(6n-2)}$ (B) $\frac{(2n-1)(2n-2)(2n-3)}{12(6n-1)(3n-1)}$

(C) $\frac{3n}{(6n-1)(6n-2)}$ (D) $\frac{3n-2}{4(6n-1)}$

Q.54 A fair coin is tossed repeatedly. The probability of getting a result in the fifth toss different from those obtained in the first four tosses is 1/4x. Find the value of x

(A) 1 (B) 2
(C) 3 (D) 4

Q.55 Three people each flip two fair coins. The probability that exactly two of the people flipped one head and one tail, is–

(A) 1/2 (B) 3/8
(C) 5/8 (D) 3/4

Q.56 A is a 3×3 matrix wit entries from the set {–1, 0, 1}. Then the probability that A is neither symmetric nor skew-symmetric is –

(A) $\frac{3^9+3^6-3^3+1}{3^9}$ (B) $\frac{3^9-3^6-3^3+1}{3^9}$

(C) $\frac{3^9-3^6+3^3+1}{3^9}$ (D) $\frac{1}{2}$

Q.57 4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is
(A) 4/35 (B) 1/70
(C) 2/35 (D) 1/35

Q.58 A bag contains 3 red and 3 white balls. Two balls are drawn one by one. The probability that they are of different colours is.
(A) 3/10 (B) 2/5
(C) 3/5 (D) None of these

Q.59 A 5 digit number is formed by using the digits 0, 1, 2, 3, 5 & 5 without repetition. The probability that the number is divisible by 6 is –
(A) 0.08 (B) 0.17
(C) 0.18 (D) 0.36

Q.60 The probability that at least one of the events A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5 then $P(A') + P(B')$ is -
(A) 2/5 (B) 4/5
(C) 6/5 (D) 7/5

Q.61 3 integers are chosen at random from the set of first 20 natural numbers. The chance that their product is a multiple of 3, is –
(A) 194/285 (B) 1/57
(C) 13/19 (D) 3/4

Q.62 Two dice are thrown together. The probability that at least one will show its digit greater than 3 is
(A) 1/4 (B) 3/4
(C) 1/2 (D) 1/8

Q.63 If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$, then choose the incorrect statement
(A) E_1 and E_2 are independent
(B) E_1 and E_2 are exhaustive
(C) E_2 is twice as likely to occur as E_1
(D) Probabilities of the events $E_1 \cap E_2$, E_1 & E_2 are in G.P.

Q.64 The probability that atleast one of the events A and B happens is 0.6. If probability of their simultaneous happening is 0.2, then $P(\bar{A}) + P(\bar{B})$ is
(A) 0.4 (B) 0.8
(C) 1.2 (D) 1.4

Q.65 If $a, b \in N$ then the probability that $a^2 + b^2$ is divisible by 5, is
(A) 9/25 (B) 7/18
(C) 1/10 (D) 17/81

Q.66 n books are to be arranged on a shelf. These include m volumes of a science book (m > n). The probability that in any arrangement, the volumes of science books are in ascending order is
(A) $\frac{1}{n!}$ (B) $\frac{1}{(n-m)!}$
(C) $\frac{1}{m!}$ (D) $\frac{m!}{n!}$

Q.67 The probability that the number formed by taking all the digits 1, 2, 3, 4, 5 is divisible by 4 is -
(A) 1/5 (B) 1/4
(C) 1/3 (D) None of these

Q.68 If A and B are two independent events with P(A) = 0.6, P(B) = 0.3, then $P(A' \cap B')$ is equal to
(A) 0.18 (B) 0.28
(C) 0.82 (D) 0.72

Q.69 The odds against A solving a certain problem are 3 to 2 and the odds in favour of B solving the same problem are 2 to 1. The probability that the problem will be solved if they both try, is
(A) 2/5 (B) 11/15
(C) 4/5 (D) 2/3

Q.70 Four balls are drawn at random from a bag containing 5 white, 4 green and 3 black balls. The probability that exactly two of them are white is-
(A) 14/33 (B) 7/16
(C) 18/33 (D) 9/16

Q.71 Five fair coins are tossed. It p is the probability that not more than two heads appear and q is the probability that not less than three heads appear, then
(A) p > q (B) p = q
(C) p < q (D) pq = 1

Q.72 In shuffling a pack of cards three are accidentally dropped. The probability that the missing cards are of distinct colours is
(A) $\frac{169}{425}$ (B) $\frac{165}{429}$ (C) $\frac{162}{459}$ (D) $\frac{164}{529}$

Q.73 All the letters of the word HAMSANANDI are placed at random in a row. The probability that the word ANAND occurs without getting split is-
(A) 1/42 (B) 1/60
(C) 1/420 (D) None of these

Q.74 Events A, B, C satisfy P (A) = 0.3, P (B) = 0.3, P (C) = 0.5. Events A and B are mutually exclusive. Events A and C are independent and P (B/C) = 0.2. $P(A \cup B \cup C)$ equals
(A) 0.95 (B) 0.85
(C) 0.75 (D) 0.65

Q.75 A is a set containing n elements. A subset P_1 of A is chosen at random. The set A is reconstructed by replacing the elements of P_1. A subset P_2 is again chosen at random. The probability that $P_1 \cup P_2$ contains exactly one element, is
(A) $3n / 4^n$ (B) $3^n / 4^n$
(C) 3/4 (D) none of these

Q.76 If the letter of the word SUCCESS are arranged, then the probability that similar letters occurs together is -
(A) 4/35 (B) 2/35
(C) 1/35 (D) 3/35

Q.77 Two cards are selected at random from a deck of 52 playing cards. The probability that both the cards are greater than 2 but less than 9 is
(A) $\frac{46}{221}$ (B) $\frac{63}{221}$
(C) $\frac{81}{221}$ (D) $\frac{93}{221}$

Q.78 India and Pakistan play a 5 match test series of hockey, the probability that India wins at least three matches is -
(A) 1/2 (B) 3/5
(C) 4/5 (D) none of these

Q.79 The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target at least three times is
(A) $\frac{291}{364}$ (B) $\frac{371}{461}$ (C) $\frac{471}{502}$ (D) $\frac{459}{512}$

Q.80 A die is loaded such that the probability of throwing the number i is proportional to its reciprocal. The probability that 3 appears in a single throw is-
(A) 3/22 (B) 3/11
(C) 9/22 (D) None of these

Q.81 A man draws a card from a pack of 52 cards and then replace it. After shuffling the pack, he again draws a card. This he repeats a number of times. The probability that he will draw a heart for the first time in the third draw is -
(A) $\frac{9}{64}$ (B) $\frac{27}{64}$
(C) $\frac{1}{4}\times\frac{^{39}C_2}{^{52}C_2}$ (D) None of these

Q.82 If two events A and B are such that $P(A') = 0.3$, P(B) = 0.4 and $(A \cap B') = 0.5$, then $P\left(\frac{B}{A \cup B'}\right) =$
(A) 1/4 (B) 1/5
(C) 3/5 (D) 2/5

Q.83 Out of all the arrangements that can be made taking 5 letters at a time of the word BRILLIANT one is chosen at random. The probability that this will have 5 distinct letters is
(A) $\frac{257}{502}$ (B) $\frac{252}{507}$
(C) $\frac{522}{705}$ (D) $\frac{255}{702}$

Q.84 The probability that in a group of N (< 365) people, at least two will have the same birthday is
(A) $1-\frac{(365)!}{(365-N)!(365)!}$ (B) $\frac{(365)^N(365)!}{(365-N)!}-1$
(C) $1-\frac{(365)^N(365)!}{(365+N)!}$ (D) none of these

Q.85 Let E and F be two independent events such that P(E) > P(F). The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2, then
(A) P(E) = 1/3, P(F) = 1/4 (B) P(E) = 1/2, P(F) = 1/4
(C) P(E) = 1, P(F) = 1/12 (D) none of these

Q.86 If the letters of INTERMEDIATE are arranged, then the probability no two E's occur together is -
(A) 7/11 (B) 5/11
(C) 2/11 (D) 6/11

Q.87 The probability that two integers chosen at random and their product will have the same last digit is
(A) 3/10 (B) 1/25
(C) 4/15 (D) 7/15

Q.88 A six digit number is formed with the digits 0, 1, 2, 3, 4, 7 without repetition. Then the probability that it is divisible by 4 is
(A) 12/25 (B) 6/25
(C) 3/25 (D) 21/100

Q.89 Out of 20 consecutive numbers, three are chosen at random. The probability that their sum is odd is the same as that their sum is even.
(A) 1/2 (B) 1/4
(C) 1/3 (D) 1/8

Q.90 If n integers taken at random are multiplied together, then the probability that the least digit of the product is 2, 4, 6, 8 is
(A) $\frac{2^n}{5^n}$ (B) $\frac{4^n-2^n}{5^n}$
(C) $\frac{4^n}{5^n}$ (D) $\frac{8^n-4^n}{5^n}$

Q.91 A letter is taken from the word ASSISTANT and another from the word STATISTICS. What is the probability that both the letters are the same ?
(A) 1/45 (B) 17/70
(C) 19/90 (D) 13/90

Q.92 A company has two plants to manufacture televisions. Plant I manufacture 70% of televisions and plant II manufacture 30%. At plant I, 80% of the televisions are rated as of standard quality and at plant II, 90% of the televisions are rated as of standard quality. A television is chosen at random and is found to be of standard quality. The probability that it has come from plant II is
(A) 17/50 (B) 27/83
(C) 3/5 (D) none of these

Q.93 If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^n + 7^m$ is divisible by 5 equals
(A) 1/4 (B) 1/2
(C) 1/8 (D) none of these

Q.94 A die is thrown 7 times. The chance that an odd number turns up at least 4 times, is
(A) 1/4 (B) 1/2
(C) 1/8 (D) none of these

Q.95 A pair of unbiased dice are rolled together till a sum of 'either 5 or 7' is obtained. The probability that 5 comes before 7 is –
(A) 2/5 (B) 3/5
(D) 4/5 (D) None of these

Q.96 A box contains 6 red, 5 blue and 4 white marbles. Four marbles are chosen at random without replacement. The probability that there is atleast one marble of each colour among the four chosen, is –
(A) 48/91 (B) 44/91
(C) 88/91 (D) 24/91

Q.97 One percent of the population suffers from a certain disease. There is blood test for this disease, and it is 99% accurate, in other words, the probability that it gives the correct answer is 0.99, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is –
(A) 0.99% (B) 25%
(C) 50% (D) 75%

Q.98 40 slips are placed into a hat each bearing a number 1, 2, 3, 4 ,5, 6, 7, 8, 9, 10, with each number entered on four slips are drawn from the hat at random and without replacement. Let P_1 be the probability that all four slips bear the same number and P_2 be the probability that two of the slips bear a number 'a' and the other two bear the number 'b' ($a \neq b$). The ratio P_2/P_1 equals –
(A) 162 (B) 180
(C) 324 (D) 360

Q.99 If m/n, in lowest terms, be the probability that a randomly chosen positive divisor of 1099 is an integral multiple of 10^{88} then (m + n) is equal to –
(A) 634 (B) 643
(C) 632 (D) 692

Q.100 An object moves 8cm in a straight line from A to B, turns at an angle α, measured in radians chosen at random from the interval $(0, \pi)$ and moves 5 cm. in a straight line to C. The probability that AC < 7 is –
(A) 1/6 (B) 1/4
(C) 1/3 (D) 1/2

EXERCISE - 2 [LEVEL-2]

Q.1 A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective is
(A) 9/10 (B) $(1/10)^5$
(C) $(9/10)^5$ (D) $(1/2)^5$

Q.2 A man takes a step forward with probability 0.4 and one step backward with probability 0.6, then the probability that at the end of eleven steps he is one step away from the starting point is –
(A) ${}^{11}C_5 \times (0.48)^5$ (B) ${}^{11}C_5 \times (0.24)^5$
(C) ${}^{11}C_5 \times (0.12)^5$ (D) ${}^{11}C_6 \times (0.72)^6$

Q.3 The probability distribution of x is

x	0	1	2	3
P (x)	0.2	k	k	2k

. Find the value of k.
(A) 0.2 (B) 0.3
(C) 0.4 (D) 0.1

Q.4 Two numbers are selected at random from 1, 2, 3 ......100 and are multiplied, then the probability correct to two places of decimals that the product thus obtained is divisible by 3, is
(A) 0.55 (B) 0.44
(C) 0.22 (D) 0.33

Q.5 The probability of solving a question by three students are $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ respectively. Probability of question is being solved will be
(A) 33/48 (B) 35/48
(C) 31/48 (D) 37/48

Q.6 In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is
(A) 37/40 (B) 1/37
(C) 36/37 (D) 1/9

Q.7 Find the probability that a leap year will not have 53 Mondays.
(A) 1/7 (B) 5/7
(C) 4/7 (D) 1/2

Q.8 Two dice are thrown together 4 times. The probability that both dice will show same numbers twice is-
(A) 1/3 (B) 25/36
(C) 25/216 (D) None of these

Q.9 If two events A and B are such that
$P(\bar{A}) = 0.3, P(B) = 0.4$ & $P(A \cap \bar{B}) = 0.5$ then $P\left(\frac{B}{A \cup \bar{B}}\right) =$
(A) 0.9 (B) 0.5
(C) 0.6 (D) 0.25

Q.10 Bag A contains 4 green and 3 red balls and Bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is noted it is green. The probability that it comes from Bag B is –
(A) 2/7 (B) 2/3
(C) 3/7 (D) 1/3

Q.11 A rifle man is firing at a distant target and has only 10% chance of hitting it. The minimum number of rounds he must fire in order to have 50% chance of hitting at least once is –
(A) 7 (B) 8
(C) 9 (D) 6

Q.12 A man alternately tosses a coin and throws a dice beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is
(A) 3/4 (B) 1/2
(C) 1/3 (D) None of these

Q.13 Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is
(A) 1/2 (B) 7/18
(C) 3/4 (D) 5/12

Q.14 Three six faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $k\,(3 \le k \le 8)$, is

(A) $\frac{(k-1)(k-2)}{432}$ (B) $\frac{k(k-1)}{432}$

(C) $\frac{k^2}{432}$ (D) None of these

Q.15 Out of 21 tickets marked with numbers from 1 to 21, three are drawn at random. The chance that the numbers on them are in A.P., is

(A) 9/1330 (B) 9/133
(C) 10/133 (D) None of these

Q.16 Three squares of a chess board are chosen at random, the probability that two are of one colour and one of another is

(A) 16/21 (B) 8/21
(C) 32/12 (D) None of these

Q.17 A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is

(A) 1/5 (B) 2/5
(C) 4/5 (D) None of theses

Q.18 A student appears for test I, II and III. The student is successful if he passes either in tests I and II or test I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then

(A) p = 1, q = 0
(B) p = 2/3, q = 1/2
(C) There are infinitely many values of p and q
(D) All of the above

Q.19 For the three events A, B and C, P (exactly one of the events A or B occurs)= P (exactly one of the events B or C occurs)= P (exactly one of the events C or A occurs)= p and P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events A, B and C occurring is

(A) $\frac{3p+2p^2}{2}$ (B) $\frac{p+3p^2}{4}$

(C) $\frac{p+3p^2}{2}$ (D) $\frac{3p+2p^2}{4}$

Q.20 If $P(B)=\frac{3}{4},\ P(A\cap B\cap \bar{C})=\frac{1}{3}$ and

$P(\bar{A}\cap B\cap \bar{C})=\frac{1}{3}$, then $P(B\cap C)$ is

(A) 1/12 (B) 1/6
(C) 1/15 (D) 1/9

Q.21 A box contains 100 tickets numbered 1, 2 ...... 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability

(A) 1/8 (B) 13/15
(C) 1/7 (D) None of these

Q.22 A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing cards is black, is

(A) 1/3 (B) 2/3

(C) 1/2 (D) $\frac{^{25}C_{13}}{^{51}C_{13}}$

Q.23 One hundred identical coins each with probability p of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

(A) 1/2 (B) 49/101
(C) 50/101 (D) 51/101

Q.24 Suppose X follows a binomial distribution with parameters n and p, where $0 < p < 1$. If $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then

(A) p = 1/2 (B) p = 1/3
(C) p = 1/4 (D) None of these

Q.25 A locker can be opened by dialing a fixed three digit code (between 000 and 999). A stranger who does not know the code tries to open the locker by dialing three digits at random. The probability that the stranger succeeds at the k^{th} trial is

(A) $\frac{k}{999}$ (B) $\frac{k}{1000}$

(C) $\frac{k-1}{1000}$ (D) None of these

Q.26 For a post three persons A, B & C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is thrice that of C. Then the odds in favour of B to be selected is –

(A) 3 : 7 (B) 7 : 3
(C) 1 : 4 (D) 4 : 3

Q.27 Twelve coupons are numbered from 1 to 12. Six coupons are selected at random one at a time with replacement. The probability that the largest number appearing on selected coupon is less than or equal to 8, is –

(A) $(7/12)^6$ (B) $(2/3)^6$
(C) (1/33) (D) None of these

Q.28 Two fair and ordinary dice are rolled simultaneously 4 times, the probability that both dice will show same digit exactly twice, is equal to –

(A) 25/216 (B) 25/36
(C) 25/108 (D) 25/72

Q.29 There are 8 different coloured balls and corresponding 8 coloured bags. The balls are placed in the bags, each one in one bag. The probability that 5 of the balls are placed in the respective coloured bags is –

(A) 1/20 (B) 1/360
(C) 1/720 (D) 1/50

Q.30 A pack of playing cards was found to contain only 51 cards. Now 13 cards are drawn and found to be red, then the probability that the missing card is black, is –
(A) 1/3 (B) 2/3
(C) 1/2 (D) $\frac{^{25}C_{13}}{^{51}C_{13}}$

Q.31 A die is rolled three times, the probability of getting a larger number than the previous number is –
(A) 5/216 (B) 5/54
(C) 1/6 (D) 5/36

Q.32 Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the three critics. The probability that majority are in favour of the book is
(A) $\frac{35}{49}$ (B) $\frac{125}{343}$
(C) $\frac{164}{343}$ (D) $\frac{209}{343}$

Q.33 Consider the Cartesian plane R^2 and let X denotes the subset of points for which both coordinates are integers. A coin of diameter 1/2 is tossed randomly into the plane. The probability p that the coin covers a point of X satisfies–
(A) $p = \frac{\pi}{16}$ (B) $p > \frac{\pi}{3}$
(C) $p < \frac{\pi}{30}$ (D) $p = \frac{1}{4}$

Q.34 There are 8 girls among whom two are sisters, all of them are to sit on a round table. The probability that the two sisters do not sit together is 5/A then find the value of A.
(A) 7 (B) 8
(C) 6 (D) 9

Q.35 Lot A consists of 1 maths book and 5 physics books, lot B consists of 2 maths books and 4 physics books and lot C has 3 maths and 3 physics books. A mixed lot M is formed by taking 5 from lot A, 3 from lot B and 2 from lot C. The probability that an book randomly chosen from the mixed lot M is maths, is –
(A) 17/60 (B) 15/60
(C) 13/60 (D) 19/60

Q.36 There are n bags containing three balls each. Two balls are drawn from each bag and found to be white. If probability that at least one bag contains all white balls is greater than 19/20, then n cannot be –
(A) 2 (B) 3
(C) 4 (D) 5

Q.37 A natural number less than 10^7 is selected. The probability that is of the form 3^n is $\frac{p}{10^q - 1}$. Find the value of p/q. [Given $n \in N$, $\log_{10}3 = 0.477$]
(A) 1 (B) 2
(C) 3 (D) 4

Q.38 An artillery target may be either at point A with probability 8/9 or at point B with probability 1/9. We have 21 shells each of which can be fixed either at point A or B. Each shell may hit the target independently of the other shell with probability 1/2. How many shells must be fired at point A to hit the target with maximum probability?
(A) 12 (B) 14
(C) 16 (D) 18

Q.39 Find the minimum number of tosses of a pair of dice, so that the probability of getting the sum of the numbers on the dice equal to 7 on atleast one toss, is greater than 0.95.
(Given $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$).
(A) 17 (B) 18
(C) 19 (D) 20

Q.40 There are four six faced dice such that each of two dice bears the numbers 0, 1, 2, 3, 4 and 5 and the other two dice are ordinary dice bearing numbers 1, 2, 3, 4, 5 and 6. If all the four dice are thrown, find the probability that the total of numbers coming up on all the dice is 10.
(A) $\frac{125}{1296}$ (B) $\frac{120}{1296}$
(C) $\frac{130}{1296}$ (D) $\frac{140}{1296}$

Q.41 A bag contains a large number of white and black marbles in equal proportions. Two samples of 5 marbles are selected (with replacement) at random. The probability that the first sample contains exactly 1 black marble, and the second sample contains exactly 3 black marbles, is
(A) $\frac{15}{1024}$ (B) $\frac{15}{32}$
(C) $\frac{25}{512}$ (D) $\frac{35}{256}$

Q.42 Two players A and B toss 4 coins and 3 coins respectively. The probability that both of them get the same number of heads is
(A) $\frac{35}{256}$ (B) $\frac{35}{128}$
(C) $\frac{1}{16}$ (D) $\frac{15}{128}$

Q.43 n different books ($n \geq 3$) are put at random in a shelf. Among these books there is a particular book 'A' and a particular book B. The probability that there are exactly 'r' books between A and B is –
(A) $\frac{2}{n(n-1)}$ (B) $\frac{2(n-r-1)}{n(n-1)}$
(C) $\frac{2(n-r-2)}{n(n-1)}$ (D) $\frac{(n-r)}{n(n-1)}$

Q.44 An ant is situated at the vertex A of the triangle ABC. Every movement of the ant consists of moving to one of the other two adjacent vertices from the vertex where it is situated. The probability of going to any of the other two adjacent vertices of the triangle is equal. The probability that at the end of the fourth movement the ant will be back to the vertex A, is –

(A) 4/16 (B) 6/16
(C) 7/16 (D) 8/16

Q.45 Suppose families always have one, two or three children, with probabilities $\frac{1}{4},\frac{1}{2}$ and $\frac{1}{4}$ respectively. Assume everyone eventually gets married and has children, the probability of a couple having exactly four grandchildren is –

(A) 27/128 (B) 37/128
(C) 25/128 (D) 20/128

Q.46 Lot A consists of 1 defective and 5 good articles, lot B consists of 2 defective and 4 good articles and lot C has 3 defective and 3 good articles. A mixed lot M is formed by taking 5 from lot A, 3 from lot B and 2 from lot C. The probability that an article randomly chosen from the mixed lot M is defective, is –

(A) 17/60 (B) 15/60
(C) 13/60 (D) 19/60

Directions : Assertion-Reason type questions.

(A) Statement- 1 is True, Statement-2 is true, statement-2 is a correct explanation for Statement -1
(B) Statement-1 is True, Statement-2 is true ; statement-2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.47 Let A and B be two independent events.

Statement–1 : If P(A) = 0.3 and $P\left(A\cup\bar{B}\right)=0.8$ then P(B) is 2/7.

Statement–2 : $P(\bar{E})=1-P(E)$ where E is any event.

Q.48 **Statement-1:** If A and B be two events such that $P(A)=0.3$ and $P(A\cup B)=0.8$ also A and B are independent events P(B) is 0.5.

Statement-2: If A & B are two independent events then $P(A\cap B)=P(A).P(B)$.

Q.49 Let A and B are two events such that P(A) = 3/5 and P(B)= 2/3, then

Statement–1 : $\frac{4}{15}\le P\left(A\cap B\right)\le\frac{3}{5}$.

Statement–2 : $\frac{2}{5}\le P\left(\frac{A}{B}\right)\le\frac{9}{10}$

Q.50 A fair die is rolled once.

Statement–1 : The probability of getting a composite number is 1/3

Statement–2 : There are three possibilities for the obtained number (i) the number is a prime number (ii) the number is a composite number (iii) the number is 1, and hence probability of getting a prime number = 1/3

Q.51 **Statement-1:** The probability of being at least one white ball selected from two balls drawn simultaneously from the bag containing 7 black & 4 white balls is 34/35.

Statement-2: Sample space = ${}^{11}C_2=55$, Number of fav. Cases = ${}^{4}C_1\times{}^{7}C_1+{}^{4}C_2\times{}^{7}C_0$

Q.52 A fair dice is closed twice. Let E denotes the event that the sum of the numbers appearing on two rolls equals 5 and F denotes the event that an even number comes up in the first roll.

Statement 1 : Event E and F are independent.

Statement 2 : For two independent event E and F defined on S. $P(E\cap F)=P(E).P(F)$

Q.53 **Statement-1:** If $P(A/B)\ge P(A)$ then $P(B/A)\ge P(B)$

Statement-2:: $P(A/B)=\frac{P(A\cap B)}{P(B)}$

Q.54 **Statement-1:** If A, B, C be three mutually independent events then A and $B\cup C$ are also independent events.

Statement-2: Two events A and B are independent if and only if $P(A\cap B)=P(A)P(B)$.

Q.55 **Statement-1:** If P(A) = 0.25, P(B) = 0.50 and $P(A\cup B)=0.14$ then the probability that neither A nor B occurs is 0.39.

Statement-2: $\overline{(A\cup B)}=\bar{A}\cup\bar{B}$

Q.56 **Statement-1:** Balls are drawn one by one without replacement from a bag containing a white and b black balls, then probability that white balls will be first to exhaust is a/a+b.

Statement-2: Balls are drawn one by one without replacement from a bag containing a white and b black balls then probability that third drawn ball is white is a/a+b.

Q.57 **Statement–1 :** Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices equilateral equals to 3/10.

Statement–2 : A die is rolled three times. The probability of getting a large number than the previous number is 5/64.

Q.58 **Statement-1:** The probability of occurrence of a doublet when two identical dies are thrown is 2/7.

Statement-2: When two identical dies are thrown then the total number of cases are 21 in place of 36 (when two distinct dies are thrown) because the cases like (5, 6). (6, 5) are considered to be same.

Q.59 **Statement-1:** Out of 5 tickets consecutively numbers, three are drawn at random, the chance that the numbers on them are in A.P. is 2/15.

Statement-2: Out of (2n + 1) tickets consecutively numbered, three are drawn at random, the chance that the numbers on them are in A.P. is $\frac{3n}{4n^2-1}$.

Q.60 **Statement-1:** If the odds against an event is 2/3 then the probability of occurring of an event is 3/5.
Statement-2: For two events A and B
$P(A' \cap B') = -1\ P(A \cup B)$

Q.61 Statement-1: A, B, C are events such that P(A) = 0.3, P(B) = 0.4 P(C) = 0.8, $P(A\cap B) = 0.08$, $P(A\cap C) = 0.28$, $P(A\cap B\cap C) = 0.09$ then $P(B \cap C) \in (0.23, 0.48)$.
Statement-2: $0.75 \le P(A \cup B \cup C) \le 1$.

Q.62 Let A and B be two independent events of a random experiment.
Statement–1 : $P(A \cap B) = P(A).\ P(B)$
Statement–2 : Probability of occurrence of A is independent of occurrence or non–occurrence of B.

Passage (Q.63-Q.65)

A dart board is a square piece of dimension 5m × 5m. The board has two concentric circles of radius 1m and 2m respectively, drawn with the centre of the board as the centre of the circles also. Gagan throws a dart at the dart board. The probability of Gagan missing the dart board is 0.25.

Q.63 What is the probability that Gagan will hit the board within the space enclosed by the inner cycle –
(A) 3/4 (B) 33 / 350
(C) 66 / 350 (D) 99 / 350

Q.64 What is the probability that Gagan will hit the board in space between the boundaries of the two circles –
(A) 3/4 (B) 33 / 350
(C) 66 / 350 (D) 99 / 350

Q.65 What is the probability that Gagan will hit the board outside both the circles –
(A) 3/4 (B) 33 / 350
(C) 261/ 700 (D) 99 / 350

Passage (Q.66-Q.68)

A multiple choice test question has five alternative answers, of which only one is correct. If a student has done his home work, then he is sure to identify the correct answer, otherwise, he chooses an answer at random.
Let E : denotes the event that a student does his homework with P (E) = p and F : denotes the event that he answer the question correctly.

Q.66 If p = 0.75 the value of P (E/F) equals –
(A) 8/16 (B) 10/16
(C) 12/16 (D) 15/16

Q.67 The relation $P(E/F) \ge P(E)$ holds good for –
(A) All values of p in [0, 1]
(B) all values of p in (0, 1) only
(C) all values of p in [0.5, 1] only
(D) no value of p

Q.68 Suppose that each question has n alternative answers of which only one is correct, and p is fixed but not equal to 0 or 1 then P (E/F)
(A) decreases an n increases for all $p \in (0, 1)$
(B) increases an n increases for all $p \in (0, 1)$
(C) remains constant for all $p \in (0, 1)$
(D) decreases if $p \in (0, 0.5)$ and increases if $p \in (0.5, 1)$ as n increases.

Passage (Q.69-Q.71)

Urn-I contains 5 red balls and 1 blue ball, Urn-II contains 2 red balls and 4 blue balls.
A fair die is tossed. If it results in an even number, balls are repeatedly by drawn one at at time with replacement from urn-I. If it is an odd number, balls are repeatedly by drawn one at a time with replacement from urn-II. Given that the first two draws both have resulted in a blue ball.

Q.69 Conditional probability that the first two draws have resulted in blue balls given urn-II is used is –
(A) 1/2 (B) 4/9
(C) 1/3 (D) None of these

Q.70 If the probability that the urn-I is being used is p, and q is the corresponding figure for urn-II then –
(A) q = 16p (B) q = 4p
(C) q = 2p (D) q = 3p

Q.71 The probability of getting a red ball in the third draw, is –
(A) 1/3 (B) 1/2
(C) 37/102 (D) 41/102

Passage (Q.72-Q.74)

A jar contains 2n throughly mixed balls, n white and n black balls. n persons each of whom draw 2 balls simultaneously from the bag without replacement.

Q.72 If the probabiltiy that each of the n person draw both balls of different colours is 8/35, then the value of n equals–
(A) 3 (B) 4
(C) 5 (D) 6

Q.73 If n = 4 the probability that each of the 4 persons draw balls of the same colour, is equal to –
(A) 1/35 (B) 2/35
(C) 3/35 (D) 4/35

Q.74 If n = 7 then the probability that each of the 7 persons draw balls of same colour, lies in the interval –
(A) [0, 0.1] (B) (0.1, 0.2]
(C) (0.2, 0.3] (D) (0.3, 1]

Passage (Q.75-Q.77)

Suppose that the event 'A' has the probability p of at least one occurence in n independent trials.

Q.75 The probability of event A on a single trial, is –
(A) $(1-p)^n$ (B) $(1-p)^{1/n}$
(C) $1-(1-p)^{1/n}$ (D) $1-p^{1/n}$

Q.76 The probability that A occurs at most once, is –

(A) $n\,(1-p)^{\frac{n-1}{n}}$

(B) $n\,(1-p)^{1/n} - (n-1)\,p$

(C) $n\,(1-p)^{\frac{1}{n-1}} - (n-1)\,(1-p)$

(D) $n\,(1-p)^{\frac{n-1}{n}} - (n-1)\,(1-p)$

Q.77 The probability of event A on a single trial if p = 65/81 and n = 4, is –
(A) 1/3 (B) 2/3
(C) 1/4 (D) 1/2

Passage (Q.78-Q.80)

A bag contains 6 ball of 3 different colours namely White, Green and Red, atleast one ball of each different colour. Assume all possible probability distributions are equally likely.

Q.78 The probability that the bag contains 2 balls of each colour, is –
(A) 1/3 (B) 1/5
(C) 1/10 (D) 1/4

Q.79 Three balls are picked up at random from the bag and found to be one of each different colour. The probability that the bag contained 4 red balls is –
(A) 1/14 (B) 2/14
(C) 3/14 (D) 4/14

Q.80 Three balls are picked at random from the bag and found to be one of each different colour. The probability that the bag contained equal number of white and green balls, is –
(A) 4/14 (B) 3/14
(C) 2/14 (D) 5/14

Passage (Q.81-Q.83)

There are four boxes A_1, A_2, A_3 and A_4. Box A_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selection of box A_i is $\frac{i}{10}$ and then a card is drawn. Let E_i represents the event that a card with number i is drawn.

Q.81 P (E_1) is equal to –
(A) 1/5 (B) 1/10
(C) 2/5 (D) 1/4

Q.82 P (A_3/E_2) is equal to –
(A) 1/4 (B) 1/3
(C) 1/2 (D) 2/3

Q.83 Expectation of the number on the card is –
(A) 2 (B) 2.5
(C) 3 (D) 3.5

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.84 An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. The probability that the third ball drawn is black is 23/A. Find the value of A.

Q.85 The probability that $\sin^{-1}(\sin x) + \cos^{-1}(\cos y)$ is an integer if x, y ∈ {1, 2, 3, 4} is A/16. Find the value of A.

Q.86 A 13 digit number is choose at random then probability that the selected number is a palindrome (when read in reverse order forms the same number) is $A/10^6$. Find the value of A.

Q.87 Two integers x and y are chosen (with replacement) from the set {0, 1, 2, ..., 10}. If P be the probability that |x – y| is less than or equal to 5, then find the value of 121P.

Q.88 P (A) = 3/7, P (B') = 1/2, P (A' ∩ B') = 1/14 then P (A ∩ B) is equal to (where A', B' are complement of A, B)

Q.89 Square is selected with all their vertices belonging to point (x_i, y_j) where i, j ∈ (1, 2, ... 14, 15}.
If $x_{i+1} - x_i = y_{j+1} - y_j = 1$ unit, ∀ i, j ∈ (1, 2, ... 13, 14} then probability that length of side of selected square equals to integer is A/840. Find the value of A.

Q.90 Mr. A makes a bet with Mr. B that in a single throw with two dice he will throw a total of seven before B throws four. Each of them has a pair of dice and they throw simultaneously until ine of them wins, equal throws being disregarded. Probability that B wins, is A/3. Find the value of A.

Q.91 A fair coin is tossed until a head or five tails occur. If the probability that the coin is tossed for a maximum number of times can be expressed as a rational p/q (in the lowest form), then (p + q) equals –

Q.92 All the face cards from a pack of 52 playing cards are removed. From the remaining pack half of the cards are randomly removed without looking at them and then randomly drawn two cards simultaneously from the remaining. If the probability that two cards drawn are both aces is $\frac{p({}^{38}C_{20})}{{}^{40}C_{20} \cdot {}^{20}C_2}$, find p.

Q.93 Of the three independent events E_1, E_2, and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ = ______

Q.94 Two number is selected randomly from the set S = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4 is (4/X). Find the value of X.

Q.95 If P(B) = 3/4, $P(A \cap B \cap \bar{C}) = 1/3$ and $P(\bar{A} \cap B \cap \bar{C}) = 1/3$ and , then $P(B \cap C)$ is (1/X).
Find the value of X.

Q.96 If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three are divisible by both 2 and 3 is (X / 1155). Find the value of X.

Q.97 A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is (X/11). Find the value of X.

EXERCISE - 3 [PREVIOUS YEARS JEE MAIN QUESTIONS]

Q.1 If the probability of solving a problem by three students are 1/2, 2/3 and 1/4 then probability that the problem will be solved- **[AIEEE 2002]**
(A) 1/2 (B) 3/4
(C) 7/8 (D) 1/8

Q.2 If $P(A \cup B) = 3/4$ and $P(\bar{A}) = 2/3$ then $P(\bar{A} \cap B)$ equals
(A) 1/12 (B) 7/12 **[AIEEE 2002]**
(C) 5/12 (D) 1/2

Q.3 A pair of dice is thrown. If 5 appears on at least one of the dice, then the probability that the sum is 10 or greater, is-
(A) 11/36 (B) 2/9 **[AIEEE 2002]**
(C) 3/11 (D) 1/12

Q.4 Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval-**[AIEEE 2003]**
(A) [0, 1] (B) $\left[\frac{1}{3}, \frac{1}{2}\right]$
(C) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (D) $\left[\frac{1}{3}, \frac{13}{3}\right]$

Q.5 Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is- **[AIEEE 2003]**
(A) 2/5 (B) 4/5
(C) 3/5 (D) 1/5

Q.6 The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact is-
(A) 3/20 (B) 1/5 **[AIEEE 2004]**
(C) 7/20 (D) 4/5

Q.7 A random variable X has the probability distribution : **[AIEEE 2004]**

X :	1	2	3	4	5	6	7	8
p(X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F= {X < 4}, the probability $P(E \cup F)$ is-
(A) 0.87 (B) 0.77
(C) 0.35 (D) 0.50

Q.8 The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is- **[AIEEE 2004]**
(A) $\frac{37}{256}$ (B) $\frac{219}{256}$
(C) $\frac{128}{256}$ (D) $\frac{28}{256}$

Q.9 Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that alll the three apply for the same house is - **[AIEEE-2005]**
(A) 2/9 (B) 1/9
(C) 8/9 (D) 7/9

Q.10 Let A and B be two events such that $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$ and $P(\bar{A}) = 1/4$, where $\bar{A}$ stands for complement of event A. Then events A and B are
(A) equally likely and mutually exclusive **[AIEEE-2005]**
(B) equally likely but not independent
(C) independent but not equally likely
(D) mutually exclusive and independent

Q.11 At a telephone enquiry system the number of phone cells regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is
(A) $6/5^e$ (B) 5/6 **[AIEEE 2006]**
(C) 6/55 (D) $6/e^5$

Q.12 A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is-
(A) 1/729 (B) 8/9 **[AIEEE 2007]**
(C) 8/729 (D) 8/243

Q.13 Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is- **[AIEEE 2007]**
(A) 0.06 (B) 0.14
(C) 0.2 (D) 0.7

Q.14 A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is
(A) 0 (B) 1 **[AIEEE 2008]**
(C) 2/5 (D) 3/5

Q.15 It is given that the events A and B are such that P(A) = 1/4, P(A|B) = 1/2 and P(B|A) = 2/3. Then P(B) is
(A) 1/3 (B) 2.3 **[AIEEE 2008]**
(C) 1/2 (D) 1/6

Q.16 In a binomial distribution B (n, p = 1/4), if the probability of at least one success is greater than or equal to 9/10, then n is greater than : **[AIEEE 2009]**
(A) $\frac{1}{\log_{10}^{4} + \log_{10}^{3}}$ (B) $\frac{9}{\log_{10}^{4} - \log_{10}^{3}}$
(C) $\frac{4}{\log_{10}^{4} - \log_{10}^{3}}$ (D) $\frac{1}{\log_{10}^{4} - \log_{10}^{3}}$

Q.17 One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals : **[AIEEE 2009]**
(A) 1/7 (B) 5/14
(C) 1/50 (D) 1/14

Q.18 Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}. **[AIEEE 2010]**

Statement-1: The probability that the chosen numbers when arranged in some order will form an AP is 1/85.

Statement-2: If the four chosen numbers from an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is false.
(C) Statement-1 is false, Statement-2 is true.
(D) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1.

Q.19 An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour is – **[AIEEE 2010]**
(A) 2/7 (B) 1/21
(C) 2/23 (D) 1/3

Q.20 Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to 31/32, then p lies in the interval **[AIEEE 2011]**
(A) $\left(\frac{1}{2},\frac{3}{4}\right]$ (B) $\left(\frac{3}{4},\frac{11}{12}\right]$
(C) $\left[0,\frac{1}{2}\right]$ (D) $\left(\frac{11}{12},1\right]$

Q.21 If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is : **[AIEEE 2011]**
(A) $P(C|D) = P(C)$ (B) $P(C|D) \geq P(C)$
(C) $P(C|D) < P(C)$ (D) $P(C|D) = P(D)/P(C)$

Q.22 Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is – **[AIEEE 2012]**
(A) 3/8 (B) 1/5
(C) 1/4 (D) 2/5

Q.23 A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is – **[JEE MAIN 2013]**
(A) $17/3^5$ (B) $13/3^5$
(C) $11/3^5$ (D) $10/3^5$

Q.24 Let A and B be two events such that

$P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are – **[JEE MAIN 2014]**
(A) mutually exclusive and independent .
(B) equally likely but not independent.
(C) independent but not equally likely.
(D) independent and equally likely.

Q.25 If 12 identical balls are to be placed in 3 identical boxes, then the probability that one the boxes contains exactly 3 balls is **[JEE MAIN 2015]**
(A) $55\,(2/3)^{10}$ (B) $220\,(1/3)^{12}$
(C) $22\,(1/3)^{11}$ (D) $(55/3)\,(2/3)^{11}$

Q.26 Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ?
(A) E_2 and E_3 are independent **[JEE MAIN 2016]**
(B) E_1 and E_3 are independent
(C) E_1, E_2 and E_3 are independent
(D) E_1 and E_2 are independent

Q.27 A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :
(A) 4 (B) 6/25 **[JEE MAIN 2017]**
(C) 12/5 (D) 6

Q.28 If two different numbers are taken from the set {0, 1, 2, 3, ……., 10} ; then the probability that their sum as well as absolute difference are both multiple of 4, is :
(A) 14/45 (B) 7/55 **[JEE MAIN 2017]**
(C) 6/55 (D) 12/55

Q.29 For three events A, B and C, P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of C or A occurs) = 1/4 and P (All the three events occur simultaneously) = 1/16 . Then the probability that at least one of the events occurs, is : **[JEE MAIN 2017]**
(A) 7/64 (B) 3/16
(C) 7/32 (D) 7/16

Q.30 A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is: **[JEE MAIN 2018]**
(A) 1/5 (B) 3/4
(C) 3/10 (D) 2/5

Q.31 Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P (X = 1) + P(X = 2) equals **[JEE MAIN 2019 (JAN)]**
(A) 52/169 (B) 25/169
(C) 49/169 (D) 24/169

Q.32 Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ? **[JEE MAIN 2019 (APRIL)]**
(A) $P(A|B) = 1$ (B) $P(A|B) = P(B) - P(A)$
(C) $P(A|B) \leq P(A)$ (D) $P(A|B) \geq P(A)$

Q.33 The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is : **[JEE MAIN 2019 (APRIL)]**
(A) 5 (B) 3
(C) 2 (D) 4

Q.34 Four persons can hit a target correctly with probabilities $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is **[JEE MAIN 2019 (APRIL)]**
(A) 25 / 192 (B) 1 / 192
(C) 25 / 32 (D) 7 / 32

Q.35 Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is : **[JEE MAIN 2019 (APRIL)]**
(A) 1/11 (B) 1/17
(C) 1/10 (D) 1/12

Q.36 Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is : **[JEE MAIN 2019 (APRIL)]**
(A) 5 (B) 6
(C) 7 (D) 8

Q.37 If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : **[JEE MAIN 2019 (APRIL)]**
(A) 3/10 (B) 1/10
(C) 3/20 (D) 1/5

Q.38 Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(x \le 2) = k/2^{16}$, then k is equal to **[JEE MAIN 2019 (APRIL)]**
(A) 17 (B) 1
(C) 121 (D) 137

Q.39 For and initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is 4/5, then the probability that he is unable to solve less than two problems is : **[JEE MAIN 2019 (APRIL)]**
(A) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (B) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$
(C) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (D) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

Q.40 A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/ loss (in Rs.) of the person is : **[JEE MAIN 2019 (APRIL)]**
(A) 2 gain (B) 1/2 loss
(C) 1/4 loss (D) 1/2 gain

Q.41 An unbiased coin is thrown 5 times. Let X be a random variable and k be the value assigned to X for k = 3, 4, 5 times Head occurs consecutively and otherwise the value of X is assigned –1. What is value of expectation. **[JEE MAIN 2020 (JAN)]**
(A) 1/8 (B) –1/8
(C) 3/8 (D) –3/8

Q.42 In a workshop, there are five machines and the probability of any one of them to be out of service on a day is (1/4). If the probability that at most two machines will be out of service on the same day is $(3/4)^3$ k, then k is equal to : **[JEE MAIN 2020 (JAN)]**
(A) 17 / 2 (B) 4
(C) 17 / 8 (D) 17 / 4

Q.43 Let P (A) = 1/3, P (B) = 1/6, where A and B are independent events then **[JEE MAIN 2020 (JAN)]**
(A) P (A/B) = 1/6 (B) P (A/B') = 1/3
(C) P (A/B') = 2/3 (D) P (A/B) = 5/6

Q.44 Let A and B are two events such that P(exactly one) = 2/5, $P(A \cup B) = 1/2$ then $P(A \cap B) =$ **[JEE MAIN 2020 (JAN)]**
(A) 1/10 (B) 2/9
(C) 1/8 (D) 1/12

Q.45 In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is : **[JEE MAIN 2020 (JAN)]**
(A) 11/16 (B) 13/16
(C) 9/16 (D) 15/16

Q.46 A random variable X has the following probability distribution :

X :	1	2	3	4	5
P(X) :	K^2	2K	K	2K	$5K^2$

Then P (X > 2) is equal to : **[JEE MAIN 2020 (JAN)]**
(A) 7/12 (B) 23/36
(C) 1/36 (D) 1/6

Q.47 If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is : **[JEE MAIN 2020 (JAN)]**
(A) $\frac{945}{2^{11}}$ (B) $\frac{965}{2^{11}}$
(C) $\frac{945}{2^{10}}$ (D) $\frac{17 \times 945}{2^{15}}$

ANSWER KEY

EXERCISE - 1																									
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	D	D	D	C	C	B	D	C	C	C	C	D	C	A	B	C	D	B	B	C	B	C	A	B	C
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	D	A	B	C	C	D	B	B	A	C	D	C	C	C	C	B	A	B	B	C	C	D	B	C	A
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
A	C	B	D	D	B	B	D	C	C	C	A	B	B	C	A	C	A	B	C	A	B	A	C	B	A
Q	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
A	B	A	A	D	D	A	A	B	A	B	D	B	B	A	B	C	B	A	B	A	A	C	A	A	C

EXERCISE - 2																									
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	C	B	A	A	A	B	B	C	D	C	A	A	D	A	C	A	D	D	A	A	B	B	D	A	B
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	A	B	A	B	B	B	D	A	A	A	A	B	A	A	A	C	B	B	B	A	A	A	D	A	C
Q	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
A	A	A	A	A	C	D	D	D	D	B	A	A	B	D	C	D	A	B	B	A	C	B	C	A	C
Q	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**			
A	D	A	C	A	B	C	B	A	7	3	1	91	0	229	1	17	6	6	5	12	4	5			

EXERCISE-3																									
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
A	C	C	C	B	A	C	B	D	B	C	D	D	B	B	A	D	D	B	A	C	B	B	C	C	D
Q	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**			
A	C	C	C	D	D	B	D	D	C	A	C	B	D	B	B	A	C	B	A	A	B	D			

CHAPTER -11PROBABILITY

SOLUTIONS TO TRY IT YOURSELF

TRY IT YOURSELF-1

(1) **(A).** The required probability = 1 – (probability of the event that the roots of $x^2 + px + q = 0$ are non-real).
The roots of $x^2 + px + q = 0$ will be non-real if and only if $p^2 - 4q < 0$ i.e. if $p^2 < 4q$.
We enumerate the possible values of p and q, for which this can happen in the following table.

q	p	No. of pairs of p,q
1	1,	1
2	1,2	2
3	1,2,3	3
4	1,2,3	3
5	1,2,3,4	4
6	1,2,3,4	4
7	1,2,3,4,5	5
8	1,2,3,4,5	5
9	1,2,3,4,5	5
10	1,2,3,4,5,6	6

Thus, the number of possible pairs = 38.
Also, the total number of possible pairs is $10 \times 10 = 100$.

$\therefore$ The required probability $= 1 - \frac{38}{100} = 1 - 0.38 = 0.62$

(2) **(A).** P (2 white and 1 black)

$= P(W_1W_2B_3 \text{ or } W_1B_2W_3 \text{ or } B_1W_2W_3)$
$= P(W_1W_2B_3) + P(W_1B_2W_3) + P(B_1W_2W_3)$
$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$$= \frac{3}{4}\cdot\frac{2}{4}\cdot\frac{3}{4} + \frac{3}{4}\cdot\frac{2}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{2}{4}\cdot\frac{1}{4} = \frac{1}{32}(9+3+1) = \frac{13}{32}$$

(3) **(B).** The no. of ways of placing 3 black balls without any restriction is ${}^{10}C_3$. Since we have total 10 places of putting 10 balls in a row and firstly we will put 3 black balls. Now the no. of ways in which no two black balls put together is equal to the no. of ways of choosing 3 placed marked out of eight places.
– W – W – W – W – W – W – W –
This can be done is ${}^{8}C_3$ ways. Thus, probability of the required event $= \frac{{}^{8}C_3}{{}^{10}C_3} = \frac{8\times7\times6}{10\times9\times8} = \frac{7}{15}$

(4) **(B).** The probability that only two tests are needed
= (probability that the first machine tested is faulty)
× (probability that the second machine tested is faulty given the first machine tested is faulty)

$$= \frac{2}{4}\times\frac{1}{3} = \frac{1}{6}.$$

(5) **(A).** Let W_1 (B_1) be the even that a white (a black) ball is drawn in the first draw and let W be the event that white ball is drawn in the second draw. Then,
$P(W) = P(B_1)P(W/B_1) + P(W_1)P(W/W_1)$

$$= \frac{n}{m+n}\frac{m}{m+n+k} + \frac{m}{m+n}\frac{m+k}{m+n+k}$$

$$= \frac{m(n+m+k)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

(6) **(D).** Given, P (A/B) = P (B/A)

$$\therefore \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A) = P(B)$$

(7) The box contains 15 oranges out of which 12 are good and 3 are bad. In the first draw, one orange is drawn out of 12 good oranges.

$\therefore \quad P(A) = \frac{12}{15}$

After first draw, there are 14 oranges left.
In the second draw, one orange is drawn out of 11 good oranges. $\therefore \quad P(B/A) = \frac{11}{14}$

After second draw, there are 13 oranges left.In the third draw, one orange is drawn out of 10 good oranges.

$\therefore \quad P(C/AB) = \frac{10}{13}$

$\therefore \quad P(A\cap B\cap C) = P(A).P(B/A).P(C/AB)$

$$= \frac{12}{15}\times\frac{11}{14}\times\frac{10}{13} = \frac{44}{91}$$

(8) Given, $P(A) = \frac{1}{2}$, $P(A\cup B) = \frac{3}{5}$ and $P(B) = p$

(i) A and B are mutually exclusive
$\therefore \quad P(A\cap B) = 0$
Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\therefore \quad \frac{3}{5} = \frac{1}{2} + p - 0 \Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$

(ii) A and B are independent
$\therefore \quad P(A\cap B) = P(A)\times P(B)$
Now, $P(A\cup B) = P(A) + P(B) - P(A)\times P(B)$

$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}\times p$

$\Rightarrow \quad \frac{3}{5} - \frac{1}{2} = p\left(1 - \frac{1}{2}\right) \Rightarrow \frac{1}{10} = p\times\frac{1}{2} \Rightarrow p = \frac{1}{5}$

(9) Consider A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper.

$\therefore \quad P(A) = \frac{60}{100} = 0.6,\ P(B) = \frac{40}{100} = 0.4$

and $P(A\cap B) = \frac{20}{100} = 0.2$

(a) Now $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.4 - 0.2 = 0.8$

Probability that that she reads neither Hindi nor English news paper.

$$= 1 - P(A \cup B) = 1 - 0.8 = 0.2 = \frac{1}{5}$$

(b) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$$

(c) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$$

TRY IT YOURSELF-2

(1) Consider E_1 and E_2 be the events that a student knows the answer or guesses the answer respectively.
Consider A be the event that answer is correct.
$\therefore$ $P(E_1) = 3/4$ and $P(E_2) = 1/4$
Since the student knows the answer
$\therefore$ $P(A/E_1) = 1$ and $P(A/E_2) = 1/4$
By Bayes' theorem

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{\frac{3}{4}\times 1}{\frac{3}{4}\times 1 + \frac{1}{4}\times\frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{3}{4}\times\frac{16}{13} = \frac{12}{13}$$

(2) Consider E_1, E_2 and E_3 be the events of selecting a coin and let A be the event of getting a head.
There are three coins and each coin is equally likely to be selected.
$\therefore$ $P(E_1) = P(E_2) = P(E_3) = 1/3$
Since the first coin has two heads
$\therefore$ $P(A/E_1) = 1$
Second coin is biased

$$\therefore \quad P(A/E_2) = 75\% = \frac{75}{100} = \frac{3}{4}$$

Second coin is unbiased $\therefore$ $P(A/E_3) = 1/2$
By Bayes' theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}} = \frac{\frac{1}{3}}{\frac{4+3+2}{12}}$$

$$= \frac{\frac{1}{3}}{\frac{9}{12}} = \frac{1}{3}\times\frac{12}{9} = \frac{4}{9}$$

(3) Consider E_1 and E_2 be the events that item drawn is produced by machine A and machine B respectively.
Consider A be the event that item drawn is defective.

$$\therefore \quad P(E_1) = 60\% = \frac{60}{100} \text{ and } P(E_2) = 40\% = \frac{40}{100}$$

It is given that

$$P(A/E_1) = 2\% = \frac{2}{100} \text{ and } P(A/E_2) = 1\% = \frac{1}{100}$$

By Bayes' theorem,

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{40}{100}\times\frac{1}{100}}{\frac{60}{100}\times\frac{2}{100} + \frac{40}{100}\times\frac{1}{100}} = \frac{\frac{4}{1000}}{\frac{12}{1000}+\frac{4}{1000}}$$

$$= \frac{\frac{4}{100}}{\frac{16}{1000}} = \frac{4}{1000}\times\frac{1000}{16} = \frac{1}{4}$$

(4) Consider E_1, E_2 and E_3 be the events that the item is manufactured by operator A, operator B and operator C respectively.
Consider A be the event that the defective item is produced.

$$\therefore \quad P(E_1) = 50\% = \frac{50}{100},\ P(E_2) = 30\% = \frac{30}{100} \text{ and}$$

$$P(E_3) = 20\% = \frac{20}{100}$$

It is given that

$$P(A/E_1) = 1\% = \frac{1}{100},\ P(A/E_2) = 5\% = \frac{5}{100} \text{ and}$$

$$P(A/E_3) = 7\% = \frac{7}{100}$$

By Bayes' theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100} + \frac{30}{100}\times\frac{5}{100} + \frac{20}{100}\times\frac{7}{100}} = \frac{\frac{5}{1000}}{\frac{5}{1000}+\frac{15}{1000}+\frac{14}{1000}}$$

$$= \frac{\frac{5}{1000}}{\frac{34}{1000}} = \frac{5}{1000}\times\frac{1000}{34} = \frac{5}{34}$$

(5) **(A).** Consider E_1 and E_2 be the events that A speaks truth and A does not speak truth respectively.

Consider A be the event that a head appears.

$\therefore \quad P(E_1) = \frac{4}{5}$ and $P(E_2) = 1 - P(E_1) = 1 - \frac{4}{5} = \frac{1}{5}$

It is given that $P(A/E_1) = \frac{1}{2}$ and $P(A/E_2) = \frac{1}{2}$

By Bayes' theorem,

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{4}{5}\times\frac{1}{2}}{\frac{4}{5}\times\frac{1}{2}+\frac{1}{5}\times\frac{1}{2}} = \frac{\frac{4}{10}}{\frac{4}{10}+\frac{1}{10}} = \frac{\frac{4}{10}}{\frac{5}{10}} = \frac{4}{10}\times\frac{10}{5} = \frac{4}{5}$$

(6) (i) $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$[\because \; P_1 + P_2 + P_3 + \ldots\ldots P_n = 1]$

$\Rightarrow \; 10k^2 + 9K - 1 = 0 \;\Rightarrow\; 10k^2 + 10k - k - 1 = 0$

$\Rightarrow \; 10k(k+1) - 1(k+1) = 0 \;\Rightarrow (k+1)(10k-1) = 0$

$\Rightarrow \; k + 1 = 0$ or $10k - 1 = 0$

$\Rightarrow \; k = -1$ or $k = 1/10$

$k = -1$ is not possible

$\therefore \; k = 1/10$

(ii) $P(X < 3) = 0 + k + 2k = 3k = 3\times\frac{1}{10} = \frac{3}{10}$

(7) If X be the random variable which denote the number of sixes on two dice then X may have values 0, 1 or 2.

Consider P (A) be probability of getting six on die.

$\therefore \quad P(A) = \frac{1}{6}$

Consider P (B) be the probability of not getting six on die.

$\therefore \quad P(B) = 1 - \frac{1}{6} = \frac{5}{6}$

Now $P(X = 0)$ = Probability of getting no six on any die

$$= P(B)\times P(B) = \frac{5}{6}\times\frac{5}{6} = \frac{25}{36}$$

$P(X = 1)$ = Probability of getting one six on any one die

$$= P(A)\,P(B) + P(B)\,P(A)$$

$$= \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6} = \frac{5}{36}+\frac{5}{36} = \frac{10}{36} = \frac{5}{18}$$

Now $P(X = 2)$ = Probability of getting six on both die

$$= P(A)\,P(A) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$$

Thus the required probability distribution is

X :	0	1	2
P(X) :	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

Now, E (X)

$$= 0\times\frac{25}{36} + 1\times\frac{5}{18} + 2\times\frac{1}{36} = 0 + \frac{5}{18} + \frac{1}{18} = \frac{6}{18} = \frac{1}{3}$$

(8) Let p be the probability of success.

Then, $p = \frac{6}{36} = \frac{1}{6}$ and $q = 1 - \frac{1}{6} = \frac{5}{6}$, $n = 8$

The probability of getting r successes is given by

$$P(X = r) = {}^4C_r\left(\frac{5}{6}\right)^{4-r}.\left(\frac{1}{6}\right)^r \text{ where } r = 0, 1, 2, 3, 4$$

$$P(X = 2) = {}^4C_2\left(\frac{5}{6}\right)^2.\left(\frac{1}{6}\right)^2 = 6\times\frac{25}{36}\times\frac{1}{36} = \frac{25}{216}$$

(9) **(C).** Let p be the probability of success.

The $p = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$, $n = 5$

The probability of getting r successes is given by

$$P(X = r) = {}^5C_r\left(\frac{9}{10}\right)^{5-r}.\left(\frac{1}{10}\right)^r \text{ where } r = 0, 1, 2, \ldots\ldots 5$$

$$P(X = 0) = {}^5C_0\left(\frac{9}{10}\right)^5.\left(\frac{1}{10}\right)^0$$

$$= 1\times\left(\frac{9}{10}\right)^5\times 1 = \left(\frac{9}{10}\right)^5$$

CHAPTER -11 : PROBABILITY
EXERCISE-1

(1) **(D).** n (S) = 36
Favourable outcomes : (1, 4), (4, 1), (3, 2), (2, 3)
Required probability = $\frac{4}{36} = \frac{1}{9}$

(2) **(D).** $P(A+B) = P(A) + P(B) - P(AB) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}.$

(3) **(D).** Required probability is
$P(\text{Red} + \text{Queen}) - P(\text{Red} \cap \text{Queen})$
$= P(\text{Red}) + P(\text{Queen}) - P(\text{Red} \cap \text{Queen})$
$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}.$

(4) **(C).** $P(A) = P(A \cap B) + P(A \cup B) - P(B)$
$= \frac{1}{3} + \frac{5}{6} - \frac{2}{3} = \frac{3}{6} = \frac{1}{2}.$

(5) **(C).** From formula, $P(A \cup B) = P(A) + P(B)$
$= \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$

(6) **(B).** Probability of one card to be king
$p = \frac{4}{52} = \frac{1}{13}$ ($\because$ favourable cases = 4, Total cases = 52)

(7) **(D).** $P(A) = \frac{3}{8} \Rightarrow P(\overline{A}) = 1 - \frac{3}{8} = \frac{5}{8}$
$\therefore$ odds in against of A = $\frac{P(\overline{A})}{P(A)} = \frac{5}{3} = 5:3$

(8) **(C).** $P(3 \cup 4) = P(3) + P(4) - P(3 \cap 4)$
$= \frac{8}{25} + \frac{6}{25} - \frac{2}{25} = \frac{12}{25}$

(9) **(C).** The number of ways of chosing the first square is 64 and that for the second square is 63. Therefore the number of ways of chosing the first and second square is $64 \times 63 = 4032$. Now we proceed to find the number of favourable ways. If the first happens to be any of the four squares in the corner, the second square can be chosen in two ways. If the first square happens to be any of the 24 square on either side of the chess board, the second square can be chosen in 3 ways. If the first square happens to be any of the 36 remaining squares, the second square can be chosen in 4 ways.
Therefore the number of favourable ways is
$(4)(2) + (24)(3) + (36)(4) = 224$
Hence the required probability $= \frac{224}{4032} = \frac{1}{18}.$

(10) **(C).** Required probability = probability of right club and good shot or probability of wrong club and good shot
$= \frac{1}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{1}{4} = \frac{4}{15}.$

(11) **(C).** Probability of first card to be a king = $\frac{4}{52}$
and probability of also second to be a king $= \frac{3}{51}$
Hence required probability $= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}.$

(12) **(D).** Total no. of ways placing 3 letters in three envelops = 3!, out of these ways only one way is correct.
Hence the required probability $= \frac{1}{3!} = \frac{1}{6}.$

(13) **(C).** Total ways are 8 and favourable ways are 4
$S = \{HHH, HHT \, ... \, TTT\}.$
Hence probability $= \frac{4}{8} = \frac{1}{2}.$

(14) **(A).** Let E_1 be the event that man will be selected and E_2 the event that woman will be selected. Then
$P(E_1) = \frac{1}{4}$ so $P(\overline{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$ and $P(E_2) = \frac{1}{3}$
So $P(\overline{E}_2) = \frac{2}{3}$. Clearly E_1 & E_2 are independent events.
So, $P(\overline{E}_1 \cap \overline{E}_2) = P(\overline{E}_1) \times P(\overline{E}_2) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

(15) **(B).** Since $A \subseteq B \Rightarrow A \cap B = B \cap A = A$
Hence $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1.$

(16) **(C).** If the sample space be s then n(s) = the total number of ways of drawn 3 balls out of total 13 balls = ${}^{13}C_3$
If A = the event of drawing three blue balls then
n (A) = ${}^{5}C_3$
$\therefore P(E) = \frac{n(A)}{n(s)} = \frac{{}^{5}C_3}{{}^{13}C_3} = \frac{\frac{5 \times 4 \times 3}{3 \times 2 \times 1}}{\frac{13 \times 12 \times 11}{3 \times 2 \times 1}}$
$= \frac{5 \times 4 \times 3}{13 \times 12 \times 11} = \frac{5}{143}$

(17) **(D).** The total no. of ways to arrange 5 letters at 5 places
n (s) = 5!
In the five letter word, two place are even (second and fourth) and there are two vowels A and U in the give word. So we have to arrange 2 vowel at 2 even places and 3 consonants at remaining 3 places.
$\therefore$ n (A) = 2!. 3!
$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{2!.3!}{5!} = \frac{1}{10}$

(18) **(B).** Let A = event that selected number is divisible by 3
B = event that selected number is divisible by 4
Here the events are not mutually exclusive.

then $P(A) = \frac{6}{20}$, $P(B) = \frac{5}{20}$, $P(AB) = \frac{1}{20}$

$\therefore P(A + B) = P(A) + P(B) - P(AB)$

$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}$$

(19) **(B).** Let A be event for A to fail and B be the event for B to fail, then P (A) = 0.2 and P(B) = 0.3
Since A and B are independent events,
$\therefore P(AB) = P(A)\,P(B)$
$\therefore$ Required probability = P(A+ B)
$= P(A) + P(B) - P(AB) = P(A) + P(B) - P(A)\,P(B)$
$= 0.2 + 0.3 - 0.2 \times 0.3 = 0.5 - 0.06 = 0.44$

(20) **(C).** The sum of the numbers greater than 9 may be 10,11 and 12. If these events be A, B, C respectively, then
P (A) = 3/36 [$\because$ favourable cases are (6, 4), (5, 5), (4, 6)]
P (B) = 2/36 [$\because$ favourable cases are (6, 5), (5, 6)]
P (C) = 1/36 $\because$ favourable case is (6, 6)]
Now since A, B, C are mutually exclusive,
so P (A + B + C) = P(A) + P(B) + P (C)

$$= \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{1}{6}$$

(21) **(B).** $P(\overline{King} \cap \overline{Heart}) = 1 - P(King \cup Heart)$

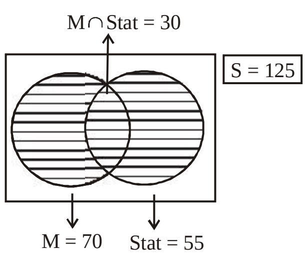
$= 1 - [P(K) + P(H) - P(K \cap H)$

$$= 1 - \left[\frac{4}{52} + \frac{13}{52} - \frac{1}{52}\right] = 1 - \frac{16}{52} = \frac{36}{52} = \frac{9}{13}$$

(22) **(C).** $P(X).P(\bar{Y}) + P(\bar{X}).P(Y)$

$$= \frac{60}{100} \times \frac{50}{100} + \frac{40}{100} \times \frac{50}{100} = \frac{1}{2}$$

(23) **(A).**



$P(M) + P(Stat) - 2P(M \cap Stat)$

$$= \frac{70}{125} + \frac{55}{125} - 2\frac{30}{125} = \frac{60}{125} = \frac{13}{25}$$

(24) **(B).** Let 100 students studying in which 60 % girls and 40% boys.
Boys = 40, Girls = 60

25% of boys offer Maths = $\frac{25}{100} \times 40 = 10$ Boys

10% of girls offer Maths = $\frac{10}{100} \times 60 = 6$ Girls

It means, 16 students offer Maths.

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$.

(25) **(C).** Total number of pens in first bag = 4 + 2 = 6
and total number of pens in second bag = 3 + 5 = 8.
The probability of selecting a white pen from first bag $= \frac{4}{6} = \frac{2}{3}$ and probability of selecting a white pen from second bag = 3/8.
$\therefore$ Required probability that both the pens are white $= \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$.

(26) **(D).** Required probability

$$= \frac{{}^5C_1\,{}^4C_1}{{}^{12}C_1\,{}^{12}C_1} + \frac{{}^7C_1\,{}^8C_1}{{}^{12}C_1\,{}^{12}C_1} = \frac{20 + 56}{144} = \frac{76}{144}.$$

(27) **(A).** There are 8 prime numbers 2, 3, 5, 7, 11, 13, 17, 19.

Hence required probability $= \frac{{}^8C_2}{{}^{20}C_2} = \frac{8.7}{20.19} = \frac{14}{95}$.

(28) **(B).** Required probability $= \frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} = \frac{3 \times 3}{15} = \frac{3}{5}$.

(29) **(C).** The total ways of drawing 5 out of 52 cards i.e., the number of elements in sample space S . n (s) = ${}^{52}C_5$.
Out of 52 cards, there are 4 kings and 48 other cards.
Out of 4 kings the total number of ways of drawing 1 king = ${}^4C_1$.
So if E_1 = the event of having one King $n(E_1) = {}^4C_1$.
Remaining 4 can be drawn out of 48 in ${}^{48}C_4$ ways.
So, if E_2= the event of having any 4 from the remaining cards. $n(E_2) = {}^{48}C_4$
Let E = the event of drawing, 1 king and 4 other cards.
then $n(E) = n(E_1)\,.\,n(E_2) = {}^4C_1 \times {}^{48}C_4$

$$\therefore P(E_1) = \frac{n(E)}{n(s)} = \frac{{}^4C_1 \times {}^{48}C_4}{{}^{52}C_5}$$

$$= \frac{4!}{1!\,3!} \times \frac{48!}{4!\,44!} \times \frac{5!\,47!}{52!} = \frac{3243}{10829}$$

(30) **(C).** Required probability $= \frac{{}^7C_3}{{}^9C_5} + \frac{{}^7C_5}{{}^9C_5} = \frac{56}{126} = \frac{4}{9}$.

(31) **(D).** Required probability $= \frac{{}^3C_1 + {}^7C_1}{{}^{10}C_1} = \frac{10}{10}$.

(32) **(B).** The Total number of exhaustive cases of drawing two cards = ${}^{52}C_2$
Now, to get at least one Ace out of two drawn cards , one card of Ace and second card of others or both cards can be of Ace.
If these events are denoted by A and B respectively then
$P(A) = ({}^4C_1 \times {}^{48}C_1)/\,{}^{52}C_2 = 32/221$,
$P(B) = {}^4C_2/{}^{52}C_2 = 1/221$
$\because P(A + B) = P(A) + P(B)$

$$\therefore P(A + B) = \frac{33}{221} + \frac{1}{221} = \frac{33}{221}$$

(33) **(B).** Let A = event that drawn card is a court card i.e. a card of king, queen or jack.
B = Event that drawn card is black.

then $P(B) = \frac{26}{52}$, $P(AB) = \frac{6}{52}$

$\therefore P\left(\frac{A}{B}\right) = \frac{P(AB)}{P(B)} = \frac{6/52}{26/52} = \frac{6}{26} = \frac{3}{13}$

(34) **(A).** $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\therefore 7/12 = 1/2 + 1/3 - P(A \cap B)$
$\Rightarrow P(A \cap B) = 1/4$

Now, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

(35) **(C).** Let A = the event of occurrence of 4 always on the second die = {(1,4), (2,4), (3,4), (4,4), (5,4), (6,4)}
$\therefore n(A) = 6$
and B = the event of occurrence of such numbers on both dice whose sum is 8 = {(4,4)}
Thus $A \cap B = A \cap \{(4,4)\} = \{(4,4)\}$

$\therefore n(A \cap B) = 1 \quad \therefore P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6}$

(36) **(D).** Let A $\equiv$ first card is that of a king
B $\equiv$ second card is that of a queen

that $P(A) = \frac{4}{52} = \frac{1}{13}$, $P(B/A) = \frac{4}{51}$

$\therefore P(AB) = P(A)\,P(B/A) = \frac{1}{13}.\frac{4}{51} = \frac{4}{663}$

(37) **(C).** Here $P\left(\frac{A}{A \cup B}\right) = \frac{P(A \cap A \cup B)}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$

(38) **(C).** Person has to miss all times probability of it will be

$\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$.

(39) **(C).** Required probability $= \frac{{}^{26}C_3 \,.\, {}^{26}C_3}{{}^{52}C_6}$.

(40) **(C).** On trial, n = 15 since any of the 15 numbers can be on the selected coin and m = 9 since the largest number is 9 and so it can be 1 or 2 or 3....... or 9.

We have required probability $= \left(\frac{9}{15}\right)^7 = \left(\frac{3}{5}\right)^7$.

(41) **(B).** The total number of ways in which n persons can sit at a round table $= (n-1)!$.

$\therefore$ Favourable number of cases $= 2!\,(n-2)!$

Thus the required probability $= \frac{2!\,(n-2)!}{(n-1)!} = \frac{2}{n-1}$

Hence the odds against are $(1-p):p$ or $(n-3):2$.

(42) **(A).** We are given $P(E \cap F) = \frac{1}{12}$ and $P(\bar{E} \cap \bar{F}) = \frac{1}{2}$

$\Rightarrow P(E).P(F) = \frac{1}{12}$ .....(i) and $P(\bar{E}).P(\bar{F}) = \frac{1}{2}$ .....(ii)

$\Rightarrow \{(1-P(E)\}\ \{(1-P)(F)\} = \frac{1}{2}$

$\Rightarrow 1 + P(E)P(F) - P(E) - P(F) = \frac{1}{2}$

$\Rightarrow 1 + \frac{1}{12} - [P(E) + P(F)] = \frac{1}{2} \Rightarrow P(E) + P(F) = \frac{7}{12}$ .....(iii)

On solving (i) and (iii), we get

$P(E) = \frac{1}{3}, \frac{1}{4}$ and $P(F) = \frac{1}{4}, \frac{1}{3}$.

(43) **(B).** Required probability is

$1 - P(\text{no girl}) = 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$.

(44) **(B).** The probability of husband is not selected

$= 1 - \frac{1}{7} = \frac{6}{7}$

The probability that wife is not selected $= 1 - \frac{1}{5} = \frac{4}{5}$

The probability that only husband selected $= \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

The probability that only wife selected $= \frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$

Hence required probability $= \frac{6}{35} + \frac{4}{35} = \frac{10}{35} = \frac{2}{7}$.

(45) **(C).** $P(5 \text{ or } 6 \text{ or } 7)$ in one draw $= \frac{3}{7}$

Probability that in each of 3 draws, the chits bear 5 or 6 or 7 $= \left(\frac{3}{7}\right)^3$.

(46) **(C).** The probaility of drawing two cards of same suit

$= \frac{13}{52} \times \frac{13}{52}$ and it can be of any suit out of 4.

So $P(A) = \frac{4 \times 13 \times 13}{52 \times 52} = \frac{1}{4}$ and $P(B) = \frac{5}{36}$

Thus $P(A \cap B) = P(A).P(B) = \frac{1}{4} \times \frac{5}{36} = \frac{5}{144}$.

(47) **(D).** $P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} \quad \therefore \quad P(B\,|\,A) = 0$

$\therefore$ Since $P(A) \neq 0$, $P(A) = 1$ (Inspection)
$\therefore$ $P(A) = 1$, $P(A \cap B) = 0 \qquad \because A \cap B = \phi$

(48) **(B).** n (S) = 6 + 4 = 10
There are only two white odd numbered marbles

$\therefore$ n (E) = 2 $\qquad \therefore P(E) = \frac{2}{10} = \frac{1}{5}$

(49) **(C).** Total number of cards which divisible by 4 = 20

i.e. (4, 8, 12, ... 80) numbered
Total number of ways of selecting 2
Cards out of 80 = ${}^{80}C_2$.
Number of favourable cases = ${}^{20}C_2$.

$\therefore$ Required probability = $\frac{{}^{20}C_2}{{}^{80}C_2} = \frac{20\times19}{80\times79} = \frac{19}{316}$

(50) **(A).** The probability of the four cards being spades

$= \frac{{}^{13}C_2}{{}^{52}C_2}\times\frac{{}^{11}C_2}{{}^{50}C_2}$

Similarly for other suits

The required probability $= 4\times\frac{{}^{13}C_2\times{}^{11}C_2}{{}^{52}C_2\times{}^{50}C_2} = \frac{44}{85\times49}$

(51) **(C).** Required probability

$= \frac{17!}{16!1!}\frac{{}^4C_1\times{}^{48}C_{16}}{{}^{52}C_{17}}.\frac{{}^3C_1}{{}^{35}C_1} = \frac{561}{15925}$

(52) **(B).** P (All cards are multiple of 3)

$= \frac{{}^{2n-1}C_3}{{}^{6n}C_3} = \frac{(2n-1)(2n-2)(2n-3)}{12n(6n-1)(3n-1)}$

(53) **(D).** P (All cards are even numbered)

$= \frac{{}^{3n}C_3}{{}^{6n}C_3} = \frac{(3n-2)}{4(6n-1)}$

(54) **(D).** Required probability = P(HHHHT) + P(TTTTH)

$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{16}$

(55) **(B).** n = 3, P (success) = P (HT or TH) = $\frac{1}{2} \Rightarrow p = q = \frac{1}{2}$

and r = 2

$P(r=2) = {}^3C_2\left(\frac{1}{2}\right)^2.\ \frac{1}{2} = \frac{3}{8}$

(56) **(B).** Total number of matrices = 3^9
Number of symmetric matrices = 3^6
Number of skew-symmetric matrices = 3^3
The zero matrix is both symmetric and skew symmetric

and therefore the required probability = $\frac{3^9-3^6-3^3+1}{3^9}$

(57) **(D).** n(S) = 7!, n(E) = (3!) × (4!)

$\therefore P(E) = \frac{(3!)\times(4!)}{7!} = \frac{6}{7\times6\times5} = \frac{1}{15} = \frac{1}{35}$

(58) **(C).** Let A ≡ event that drawn ball is red
B ≡ event that drawn ball is white
Then AB and BA are two disjoint cases of the given event.

$\therefore$ P (AB + BA) = P(AB) + P (BA)

$= P(A)\,P\left(\frac{B}{A}\right) + P(B)\,P\left(\frac{A}{B}\right) = \frac{3}{6}.\frac{3}{5}+\frac{3}{6}.\frac{3}{5} = \frac{3}{5}$

(59) **(C).** n (S) = Total number of numbers = $5\times{}^5C_4\times4! = 5(5!)$
Five digit numbers divisible by 6 are formed by using the numbers 0, 1, 2, 4 and 5 or 1, 2, 3, 4, and 5.
$\therefore$ number of such numbers
= n (E) = 2 (4)! + 2 × 3 × 3! + 4 × 3! = 108

(60) **(C).** Here, $P(A\cup B) = \frac{3}{5}$ and $P(A\cap B) = \frac{1}{5}$.

So, from the addition theorem.

$\frac{3}{5} = P(A)+P(B)-\frac{1}{5}$ or $\frac{4}{5} = 1-P(A')+1-P(B')$

$\therefore P(A')+P(B') = 2-\frac{4}{5} = \frac{6}{5}$

(61) **(A).** Total number of ways of selecting 3 integers from 20 natural numbers = ${}^{20}C_3$ = 1140
their product is a multiple of 3 means, at least one number is divisible by 3.
The numbers which are divisible by 3 are 3, 6, 9, 12, 15, 18 and the number of ways of selecting atleast one of them

is ${}^6C_1\times{}^{14}C_2+{}^6C_2\times{}^{14}C_1+{}^6C_3 = 776$

Probability = $\frac{776}{1140} = \frac{194}{285}$

(62) **(B).** Total exhaustive cases = 6^2 = 36
Out of these cases following 9 pairs are not favourable
(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

$\therefore$ reqd. prob. = $1-\frac{9}{36} = \frac{3}{4}$

(63) **(B).** $P(E_2/E_1) = \frac{P(E_1\cap E_2)}{P(E_1)}$

$\frac{1}{2} = \frac{P(E_1\cap E_2)}{1/4}$

$\Rightarrow P(E_1\cap E_2) = 1/8 = P(E_2).P(E_1/E_2) = P(E_2).\ 1/4$
$\Rightarrow P(E_2) = 1/2$
Since $P(E_1\cap E_2) = 1/8 = P(E_1).P(E_2)$
$\Rightarrow$ events are independent

Also $P(E_1\cup E_2) = \frac{1}{2}+\frac{1}{4}-\frac{1}{8} = \frac{5}{8}$

$\Rightarrow$ E_1 & E_2 are non exhaustive

(64) **(C).** As given P(A + B) = 0.6 and P(AB) = 0.2
P(A + B) = P(A) + P(B) – P(AB)
$\Rightarrow$ 0.6 = P(A) + P(B) – 0.2
$\Rightarrow$ P(A) + P(B) = – 0.8
$\Rightarrow$ [1 – P(A)] + [1 – P(B) = 2 – 0.8 = 1.2
$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$

(65) **(A).** Square of a number ends in 0, 1, 4, 5, 6 and 9 favourable ordered pairs of (a^2, b^2) can be $(0, 0)$; $(5, 5)$; $(1, 4)$, $(4, 1)$; $(1, 9)$, $(9, 1)$; $(4, 6)$, $(6, 4)$; $(6, 9)$, $(9, 6)$ and $P(0) = 1/10 = P(5)$;
$P(1) = P(4) = P(6) = P(9) = 2/10$

(66) **(C)** Corresponding to each arrangement of $(n - m)$ other books there is a unique arrangement of the m volumes of the science book in ascending order and m! arrangements of the m volumes in random order. $\therefore\ p = 1/m!$

(67) **(A).** Total number of numbers formed with the given digits $= 5! = 120$

We know that a number is divisible by 4 if the number formed by its first two digits (from right) is divisible by 4
Here such numbers are 12, 24, 32 and 52.
So total number of numbers formed by given 5 digits which are divisible by $4 = 4\,(3!) = 24$

$\therefore$ reqd. prob. $= \dfrac{24}{120} = \dfrac{1}{4}$

(68) **(B).** The independence of two events A and B implies the independence of their complements and
$A\ P(A' \cap B') = P(A').\,P(B') = 4.0 \times 0.7 = 0.28.$

(69) **(C).** Probability of A solving the problem = 2/5.
Probability of B solving the problem = 2/3
As A's solving the problem and B's solving the problem are two independent events, required probability

$$= \frac{2}{5} + \frac{2}{3} - \frac{2}{5} \times \frac{2}{3} = \frac{4}{5}.$$

(70) **(A).** Total number of balls = 12.

Required probability $= \dfrac{{}^5C_2\ {}^7C_2}{{}^{12}C_2} = \dfrac{14}{33}.$

(71) **(B).** Probability that not more than two heads or two tails appear $= \left(\dfrac{1}{2}\right)^5 + {}^5C_1\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)^4 + {}^5C_2\left(\dfrac{1}{2}\right)^2\left(\dfrac{1}{2}\right)^3 = \dfrac{1}{2}$

q = Probability that not less than three heads appear
= Probability that not more than two tails appear = p

(72) **(A).** The first card can be one of the 4 colours, the second can be one of the three and the third can be one of the two. The required probability is therefore

$$4 \times \frac{13}{52} \times 3 \times \frac{13}{51} \times 2 \times \frac{13}{50} = \frac{169}{425}.$$

(73) **(C).** The total number of arrangements

$$= \frac{10!}{3!\,2!}\ (\text{A's} - 3,\ \text{N's} - 2)$$

The number of arrangements in which ANAND occurs without any split = 6!

$\Rightarrow$ Probability $= \dfrac{6!\,3!\,2!}{10!} = \dfrac{1}{420}.$

(74) **(B).** $P(B/C) = \dfrac{P(B \cap C)}{P(C)}$ $\therefore\ P(B \cap C) = 0.2\ P(C) = 0.1$

$P(A \cup B \cup C) = \Sigma\,P(A) - \Sigma\,P(A \cap B) + P(A \cap B \cap C)$
$= 1.1 - [0 + 0.1 + (0.5)(0.3)] + 0 = 1.1 - 0.25 = 0.85$

(75) **(A).** Any element of A has four possibilities **:** element belongs to (i) both P_1 and P_2 (ii) neither P_1 nor P_2 (iii) P_1 but not to P_2 (iv) P_2 but not to P_1. Thus $n(S) = 4^n$. For the favourable cases, we choose one element in n ways and this element has three choices as (i), (iii) and (iv), while the remaining n – 1 elements have one choice each, namely (ii). Hence required probability $= \dfrac{3n}{4^n}$.

(76) **(B).** Success $\to$ 3S, 2C, 1U, 1E

Total arrangements $\to \dfrac{7!}{3!2!} = \dfrac{2}{35}$

Now, similar letters are together (consider 3S one letter and 2C's one letter) = 4! (favourable cases)

$\therefore$ probability $= \dfrac{4!\,3!\,2!}{7!} = \dfrac{2}{35}$

(77) **(A).** The cards are of four colours and the number of cards of given description is 24.

The probability $= \dfrac{24}{52}.\dfrac{23}{51} = \dfrac{46}{221}.$

(78) **(A).** India win atleast three matches

$$= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^5 (16) = \left(\frac{1}{2}\right)$$

(79) **(D).** $P = \dfrac{3}{4}, q = \dfrac{1}{4}$, n = 5

Required probability

$$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4.\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5 = \frac{459}{512}$$

(80) **(D).** $P(i) = \dfrac{k}{i} \Rightarrow 1 = \sum_{i=1}^{6} P(i) = k\sum_{i=1}^{6}\dfrac{1}{i} = k\dfrac{49}{20}$

$\Rightarrow k = \dfrac{20}{49}$. $P(3) = \dfrac{20}{147}$

(81) **(A).** Let E = the event of drawing a heart.

Clearly, $P(E) = \dfrac{{}^{13}C_1}{{}^{52}C_1} = \dfrac{1}{4}$

$\therefore$ the required probability

$$= P(\bar{E}\,\bar{E}\,E) = P(\bar{E})\,.\,P(\bar{E})\,.\,P(E) = \frac{3}{4}.\frac{3}{4}.\frac{1}{4} = \frac{9}{64}$$

(82) **(A).** $P(B / A \cup B') = \dfrac{P(B \cap (A \cup B'))}{P(A \cup B')}$

$$= -\frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')} = \frac{P(A) - P(A \cap B')}{0.7 + 0.6 - 0.5}$$

$$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$$

(83) **(B).** The given word is made up of the letters II, ii, brant.
Possibilities of arrangements :
Umber of arrangements :

(i) two alike + two alike + one different $1 \times 5 \times \frac{5!}{(2!)^2} = 150$

(ii) two alike + three different $2 \times {}^6C_3 \times \frac{5!}{2!} = 2400$

(iii) all are different ${}^7C_5 \times 5! = 2520$
Total number of different arrangements = 5070.

The required probability = $\frac{2520}{5070} = \frac{252}{507}$.

(84) **(A).** Let $\bar{A}$ be the event of different birthdays. Each can have birthday in 365 ways, so N persons can have their birthdays in 365^N ways. Number of ways in which all have different birthdays = ${}^{365}P_N$

$$\therefore P(A) = 1 - P(\bar{A}) = 1 - \frac{{}^{365}P_N}{(365)^N} = 1 - \frac{(365)!}{(365)^N (365-N)!}$$

(85) **(B).** $P(E \cap F) = P(E)\,P(F) = \frac{1}{12}$ . . . (i)

$P(E^c \cap F^c) = P(E^c).P(F^c) = \frac{1}{2}$

$\Rightarrow (1-P(E))(1-P(F)) = \frac{1}{2}$ . . . (ii)

Solving (i) and (ii), we get

$P(E) = \frac{1}{2}$ & $P(F) = \frac{1}{4}$, as $P(E) > P(F)$.

(86) **(D).** $I \to 2, N \to 1, T \to 2, E \to 3, R \to 1, M \to 1, D \to 1, A \to 1$ [3 E's Rest 9]

First arrange rest of letters = $\frac{9!}{2!2!}$

Now 3E's can be placed into place in ${}^{10}C_3$ ways so

favourable cases = $\frac{9!}{2!\,2!} \times {}^{10}C_3 = 3 \times 10!$

Total cases = $\frac{12!}{2!\,2!\,3!}$;

Probability = $\frac{3 \times 10! \times 2! \times 2! \times 3!}{12!} = \frac{6}{11}$

(87) **(B).** The condition implies that the last digit in both the integers should be 0, 1, 5 or 6 and the probability -

$4.\left(\frac{1}{10}\right)^2 = \frac{1}{25}$.

(88) **(B).** If the 6 digit number is to be divisible by 4, the last two digits have to be one of the following pairs
04, 12, 20, 24, 32, 40, 72.

$\therefore$ the number of favourable ways
= 4! + 3.3.2.1 + 4! + 3.3.2.1 + 3.3.2.1 + 4! + 3.3.2.1 = 144.
Total number of numbers that can be formed = 6! – 5! = 5! (6 – 1) = 600

Hence the probability = $\frac{144}{600} = \frac{6}{25}$.

(89) **(A).** Out of 20 consecutive numbers 10 will be odd and 10 even. Let O be the event that the sum of three randomly chosen numbers is odd.
The cases favourable for the event to occur are that all the three numbers should be odd or one of them should be odd and the other two should be even. Thus

$$P(0) = \frac{10}{20} \cdot \frac{9}{19}\,\frac{8}{18} + {}^3C_1\,\frac{10}{20}\,\frac{10}{19} \cdot \frac{9}{18} = \frac{2}{19} + \frac{15}{38} = \frac{1}{2}$$

Probability that the sum is even = $1 - \frac{1}{2} = \frac{1}{2}$

(90) **(B).** If the least digit in the product is to be 2, 4, 6, 8, the last digit in all the n numbers should not be 0 and 5 and the last digit of all numbers should not be selected exclusively from the set of numbers {1, 3, 7, 9}
$\therefore$ favourable number of cases = $8^n - 4^n$
But generally the last digit can be any one of 0, 1,2,3,..., 9.
Hence the total number of ways = 10^n

$\therefore$ the required probability = $\frac{8^n - 4^n}{10^n} = \frac{4^n - 2^n}{5^n}$.

(91) **(C).** The word ASSISTANT contains two A's, one I, one N, three S's and two T's whereas the word STATISTICS contains one A, one C, two I's, three S's and three T's.
$\therefore$ total number of ways of choosing one letter from each word = ${}^9C_1 \,.\, {}^{10}C_1 = 90$.
Common letters are A, I, S, T.
$\therefore$ the number of favourable cases
$= {}^2C_1 \,.\, {}^1C_1 + {}^1C_1 \,.\, {}^2C_1 + {}^3C_1 \,.\, {}^3C_1 + {}^2C_1 \,.\, {}^3C_1$
= 2 + 2 + 9 + 6 = 19
Hence the required probability = 19/90.

(92) **(B).** Let E be the event that a television chosen randomly is of standard quality. We have to find

$$P(II/E) = \frac{P(E/II).P(II)}{P(E/I).P(I) + P(E/II).P(II)}$$

$$= \frac{(9/10)(3/10)}{(4/5)(7/10) + (9/10)(3/10)} = \frac{27}{83}$$

(93) **(A).** We observe that $7^1, 7^2, 7^3$ and 7^4 ends in 7, 9, 3 and 1 respectively. Thus 7^l ends in 7, 9, 3 or 1 according as l is of the form 4k + 1, 4k + 2, 4k – 1 or 4k respectively. If S is the sample space, then n (S) = $(100)^2$. $7^m + 7^n$ is divisible by 5 if (i) m is of the form 4k + 1 and n is of the form 4k– 1 or (ii) m is of the form 4k + 2 and n is of the form 4k or (iii) m is of the form 4k – 1 and n is of the form 4k + 1 or (iv) m is of the form 4k and n is of the form 4k + 1.
Thus number of favourable ordered pairs (m, n)
= 4 × 25 × 25. Hence required probability is 1/4.

(94) **(B).** For at least 4 successes, required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2$$

$$+ {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1}{2}^7\right) = \frac{1}{2}.$$

(95) **(A).** Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs.

We have, $P(A) = \frac{4}{36} = \frac{1}{9}$, $P(B) = \frac{6}{36} = \frac{1}{6}$ and

$P(C) = \frac{26}{36} = \frac{13}{18}$

Thus P (A occurs before B)

$$= \frac{1}{9} + \left(\frac{13}{18}\right)\times\frac{1}{9} + \left(\frac{13}{18}\right)^2\times\frac{1}{9} + ... = \frac{1/9}{1-13/18} = \frac{2}{5}$$

[sum of an infinite G.P.]

(96) **(A).** P(E) = P(R R B W or B B R W or W W R B)

$n(E) = {}^6C_2 \cdot {}^5C_1 \cdot {}^4C_1 + {}^5C_2 \cdot {}^6C_1 \cdot {}^4C_1 + {}^4C_2 \cdot {}^6C_1 \cdot {}^5C_1$

$n(S) = {}^{15}C_4$

Box → 6R, 5B, 4W

$$\therefore\ P(E) = \frac{720\cdot 4!}{15\cdot 14\cdot 13\cdot 12} = \frac{48}{91}$$

(97) **(C).** A: blood result says positive about the disease

B_1: Person suffers from the disease $= \frac{1}{100}$,

B_2 : person does not suffer $= \frac{99}{100}$

$P(A/B_1) = \frac{99}{100}$, $P(A/B_2) = \frac{1}{100}$

$$P(B_1/A) = \frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1) + P(B_2).P(A/B_2)}$$

$$= \frac{\frac{1}{100}\cdot\frac{99}{100}}{\frac{1}{100}\cdot\frac{99}{100} + \frac{99}{100}\cdot\frac{1}{100}} = \frac{99}{2\cdot 99} = \frac{1}{2} = 50\%$$

(98) **(A).** $P_1 = \frac{{}^{10}C_1 \cdot {}^4C_4}{{}^{40}C_4}$ (Selecting one denomination out of 10 and taking all 4)

1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
10	10	101	10

$P_2 = \frac{{}^{10}C_2 \cdot {}^4C_2 \cdot {}^4C_2}{{}^{40}C_4}$ (Selecting two denomination, and taking 2 from each)

$$\frac{P_2}{P_1} = \frac{45\cdot 6\cdot 6}{10} = 9\cdot 3\cdot 6 = 162$$

(99) **(A).** $N = 10^{99} = 2^{99}.5^{99}$

$\therefore$ number of divisors of N = (100) (100) = 104

now $10^{88} = 2^{88}\,.\,5^{88}$

Hence divisors which are integral multiple of $2^{88}.5^{88}$ must be of the form of $2^a.\,5^b$ where $88 \le a, b \le 99$.

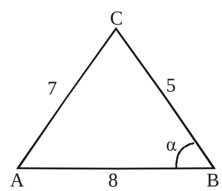
There there are 12×12 ways to choose a and b and hence there are 12×12 divisors which are integral multiple if $2^{88}.5^{88}$

Hence, $p = \frac{144}{10000} = \frac{9}{625}$ $\quad\therefore\ m + n = 634$

(100) **(C).**

If AC = 7, then $\cos\alpha = \frac{8^2+5^2-7^2}{2\times 8\times 5} = \frac{64+25-49}{2\times 40}$

Hence, $\cos\alpha = \frac{40}{2\times 40} = \frac{1}{2} \Rightarrow \alpha = 60°$



Now, $AC < 7 \Rightarrow \alpha \in (0, 60°)$. Hence, $p = \frac{60}{80} = \frac{1}{3}$

EXERCISE-2

(1) **(C).** $p = \frac{10}{100} = 0.1$, $q = 0.9$, $n = 5$

$\therefore$ $p(x) = {}^5C_x (0.1)^x (0.9)^{5-x}$

$p(0) = {}^5C_0 (0.1)^0 (0.9)^5 = (9/10)^5$

(2) **(B).** Let p = probability of taking a step forward

= " probability of failure"

x – the number of successes. It is a binomial variations with n = 11, b = 0.4 and q = 0.6

Required = P (x = 5 or x = 6) = P (x = 5) + P (x = 6)

$= {}^{11}C_5\, p^5 q^6 + {}^{11}C_6\, p^6 q^5$; ${}^{11}C_5 = {}^{11}C_6$

$= {}^{11}C_5\, p^5 q^5 (p + q) = {}^{11}C_5 (pq)^5 = {}^{11}C_5 (0.24)^5$

Aliter : To be 1 step away after 11 steps, he should be in the original position after 10 steps, which means he has taken 5 steps forward and 5 steps backward. The probability is $(0.4)^5 (0.6)^5 .\, {}^{11}C_5$.

(3) **(A).** $\Sigma\, p(x) = 1 \Rightarrow 4k + 0.2 = 1 \Rightarrow 4k = 0.8\ \therefore\ k = 0.2$

(4) **(A).** Total number of cases obtained by taking multiplication of only two numbers out of $100 = {}^{100}C_2$.

Out of hundred $(1, 2, ..., 100)$ given numbers, there are the numbers $3, 6, 9, 12, ..., 99,$ which are 33 in number such that when any one of these is multiplied with any one of remaining 67 numbers or any two of these 33 are multiplied, then the resulting products is divisible by 3. Then the number of numbers which are the products of two of the given number are divisible by $3 = {}^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2$. Hence the required probability

$$= \frac{{}^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2}{{}^{100}C_2} = \frac{2739}{4950} = 0.55.$$

(5) **(A).** (i) This question can also be solved by one student
(ii) This question can be solved by two students simultaneously.
(ii) This question can be solved by three students all together. $P(A) = \frac{1}{2},\ P(B) = \frac{1}{4}, P(C) = \frac{1}{6}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$-[P(A).P(B) + P(B).P(C) + P(C).P(A)] + [P(A).P(B).P(C)]$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \left[\frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2}\right] + \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6}\right] = \frac{33}{48}.$$

(6) **(B).** We define the following events :
A_1 : He knows the answer.
A_2 : He does not know the answer.
E : He gets the correct answer.

Then $P(A_1) = \frac{9}{10},\ P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}, P\left(\frac{E}{A_1}\right) = 1,$

$P\left(\frac{E}{A_2}\right) = \frac{1}{4}$

∴ Required probability

$$= P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)} = \frac{1}{37}.$$

(7) **(B).** First, we will find the probability of leap year containing 53 Mondays.
In a leap year there are 366 days, in which there are 52 weeks and 2 days, so it is certain that there are 52 Mondays. Now, there shall be 53 Mondays, if out of additional two days one is a Monday. Taking all possible cases for the additional two consecutive days, the sample space is S = {(Sunday, Monday), (Monday, Tuesday) (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday).
The event of being one Monday out of two consecutive day. E = (Sunday, Monday), (Monday, Tuesday)

$$\therefore n(s) = 7, n(E) = 2 \quad \therefore P(E) = \frac{n(E)}{n(s)} = \frac{2}{7}$$

but $P(\bar{E}) = 1 - P(E)$ = the probability that a leap year will not have 53 Monday.

$\therefore P(\bar{E}) = 1 - 2/7 = 5/7$

(8) **(C).** The probability of showing same number by both dice p = 6/36 = 1/6
In binomial distribution here n = 4, r = 2, p = 1/6, q = 5/6
$\therefore$ Required probability $= {}^nC_r\, q^{n-r} p^r = {}^4C_2\, (5/6)^2\, (1/6)^2$

$$= 6\left(\frac{25}{36}\right)\left(\frac{1}{36}\right) = \frac{25}{216}$$

(9) **(D).** $P\left(\frac{B}{A \cup \bar{B}}\right) = \frac{P[B \cap (A \cup \bar{B})]}{P(A \cup \bar{B})}$

$$= \frac{P[B \cap A) \cup (B \cap \bar{B})]}{P(A) + P(\bar{B}) - P(A \cap \bar{B})} \quad \begin{cases} P(A \cap \bar{B}) = 0.5 \\ P(A) - P(A \cap B) = 0.5 \\ P(A \cap B) = 0.7 - 0.5 = 0.2 \end{cases}$$

$$= \frac{P(A \cap B)}{P(A) + P(\bar{B}) - P(A \cap \bar{B})} = \frac{0.2}{0.7 + 0.6 - 0.5}$$

$$= \frac{0.2}{0.8} = \frac{1}{4} = 0.25$$

(10) **(C).** $P\left(\frac{\text{Bag} - B}{\text{Green}}\right)$

$$= \frac{P(\text{Bag} - B) \times P\left(\frac{\text{green}}{\text{Bag} - B}\right)}{P(\text{Bag} - B) \times P\left(\frac{\text{green}}{\text{Bag} - B}\right) + P(\text{Bag} - A) \times P\left(\frac{\text{green}}{\text{Bag} - A}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{4}{7}} = \frac{3}{7}$$

(11) **(A).** $P(\text{Hit}) = \frac{1}{10} = p,\ P(\overline{\text{Hit}}) = \frac{9}{10} = q$

$P(\text{at least 1 time hit}) \geq \frac{1}{2}$ (50%)

$$1 - P(\text{No hit}) \geq \frac{1}{2} \Rightarrow 1 - {}^nC_0\left(\frac{9}{10}\right)^n \geq \frac{1}{2}$$

$$\frac{1}{2} \geq \left(\frac{9}{10}\right)^n \Rightarrow n = 7, 8, 9, \ldots\ldots \Rightarrow n = (\text{min.}) = 7$$

(12) **(A).** Probability of getting head = 1/2 and probability of throwing 5 or 6 with a dice $= \frac{2}{6} = \frac{1}{3}$. He starts with a coin and alternately tosses the coin and throws the dice and he will win if he get a head before he get 5 or 6.

$$\therefore \text{Probability} = \frac{1}{2} + \left(\frac{1}{2} \cdot \frac{2}{3}\right) \cdot \frac{1}{2} + \left(\frac{1}{2} \cdot \frac{2}{3}\right) \cdot \left(\frac{1}{2} \cdot \frac{2}{3}\right) \times \frac{1}{2} + ..$$

$$= \frac{1}{2}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \ldots\ldots\right] = \frac{1}{2} \cdot \frac{1}{1 - (1/3)} = \frac{3}{4}.$$

(13) Consider two events :

$A_i \rightarrow$ getting a number i on first die.

$B_i \rightarrow$ getting a number more than i on second die.

The required probability

$= P(A_1 \cap B_1) + P(A_2 \cap B_2) + P(A_3 \cap B_3) + P(A_4 \cap B_4)$

$+ P(A_5 \cap B_5)$

$= \sum_{i=1}^{5} P(A_i \cap B_i) = \sum_{i=1}^{5} P(A_i)P(B_i)$

$(\because A_i, B_i$ are independent)

$= \frac{1}{6}(P(B_1) + P(B_2) + ... + P(B_5))$

$= \frac{1}{6}\left(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}\right) = \frac{5}{12}$.

(14) **(A).** The total number of cases $= 6 \times 6 \times 6 = 216$

The number of favourable ways

= Coefficient of x^k in $(x + x^2 + ... + x^6)^3$

= Coefficient of x^{k-3} in $(1 - x^6)(1 - x)^{-3}$

= Coefficient of x^{k-3} in $(1 - x)^{-3}$, $\{0 \le k - 3 \le 5\}$

= Coefficient of x^{k-3} in $(1 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + ...)$

$= {}^{k-1}C_2 = \frac{(k-1)(k-2)}{2}$

Thus the probability of the required event is

$\frac{(k-1)(k-2)}{432}$.

(15) **(C).** Total number of ways $= {}^{21}C_3 = 1330$. If common difference of the AP is to be 1 then the possible groups are $1, 2, 3;\ 2, 3, 4; .... 19, 20, 21$.

If the common difference is 2, then possible groups are $1, 3, 5;\ 2, 4, 6; .... 17, 19, 21$.

Proceeding in the same way if the common difference is 10 then the possible group is 1, 10, 21. Thus if the common difference of the $A.P.$ is to be ≥ 11, obviously there is no favourable case.

Hence total number of favourable cases are

$= 19 + 17 + 15 + ... + 3 + 1 = 100$

Hence required probability $= \frac{100}{1330} = \frac{10}{133}$.

(16) **(A).** Three squares can be chosen out of 64 squares in $^{64}C_3$ ways. Two squares of one colour and one another colour can be chosen in two mutually exclusive ways : (i) Two white and one black and (ii) Two black and one white. Thus the favourable number of cases

$= {}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2$.

Hence the required probability $= \frac{2\left({}^{32}C_1 . {}^{32}C_2\right)}{{}^{64}C_3} = \frac{16}{21}$.

(17) **(D).** Let A denotes the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs, we have

$P(A) = \frac{4}{36}, P(B) = \frac{6}{36}$ and $P(C) = \frac{26}{36} = \frac{13}{18}$

Thus $P(A$ occurs before $B)$

$= P[A$ or $(C \cap A)$ or $(C \cap C \cap A)$ or ....]

$= P(A) + P(C \cap A) + P(C \cap C \cap A) + ...$

$= P(A) + P(C).P(A) + P(C)^2 P(A) + ...$

$= \frac{P(A)}{1 - P(C)}$, [by G.P.] $= \frac{\frac{1}{9}}{1 - \frac{13}{18}} = \frac{2}{3}$.

(18) **(D).** Let A, B and C be the events that the student is successful in test I, II and III respectively, then

P (the student is successful)

$P[(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A \cap B \cap C)]$

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A \cap B \cap C)$

$P(A).P(B).P(C') + P(A)P(B')P(C) + P(A)P(B)P(C)$

$\{\because A, B, C$ are independen t$\}$

$= pq\left(1 - \frac{1}{2}\right) + p(1 - q)\left(\frac{1}{2}\right) + pq\left(\frac{1}{2}\right) = \frac{1}{2}p(1 + q)$

$\Rightarrow \frac{1}{2} = \frac{1}{2}p(1 + q) \Rightarrow p(1 + q) = 1$

This equation has infinitely many values of p and q

(19) **(A).** We know that P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B)$

Therefore, $P(A) + P(B) - 2P(A \cap B) = p$ .....(i)

Similarly, $P(B) + P(C) - 2P(B \cap C) = p$ .....(ii)

and $P(C) + P(A) - 2P(C \cap A) = p$ .....(iii)

Adding (i), (ii) and (iii), we get

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3p}{2}$ .....(iv)

We are also given that $P(A \cap B \cap C) = p^2$ .....(v)

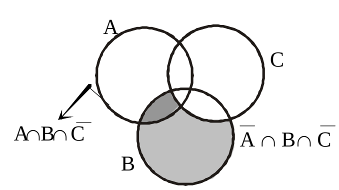
Now, P (at least one of A, B and C)

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3p}{2} + p^2$, [By (iv) and (v)].

(20) **(A).** From Venn diagram, we can see that



$P(B\cap C)=P(B)-P(A\cap B\cap \overline{C})-P(\overline{A}\cap B\cap \overline{C})$

$=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{1}{12}.$

(21) **(B).** Let A be the event that the maximum number on the two chosen tickets is not more than 10 i.e., the number on them ≤ 10 and B be the event that the maximum number on them is 5, i.e., the number on them is ≥ 5 we have to find $P(B/A)$.

Now $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{n(A\cap B)}{n(A)}$

Now the number of ways of getting a number r on the two tickets is the coefficient of x^r in the expansion of

$(x^1+x^2+x^3+........+x^{100})^2=x^2(1+x+.......+x^{99})^2$

$=x^2\left(\frac{1-x^{100}}{1-x}\right)^2=x^2(1-2x^{100}+x^{200})(1-x)^{-2}$

$=x^2(1-2x^{100}+x^{200})(1+2x+3x^2+....+(r+1)x^r+.....)$

Thus coefficient of $x^2=1$, of $x^3=2$, of $x^4=3......$ of x^{10} is 9.

Hence $n(A)=1+2+3+4+5+6+7+8+9=45$

and $n(A\cap B)=4+5+6+7+8+9=39$

[Note that in finding $n(A)$ we have to add the coefficients of $x^2, x^3, ...x^{10}$ and in $n(A\cap B)$ we add the coefficient of $x^5, x^6, ... x^{10}$]

Hence required probability $=\frac{39}{45}=\frac{13}{15}.$

(22) **(B).** Let A_1 be the event that the black card is lost, A_2 be the event that red card is lost and let E be the event that first 13 cards examined are red.

Then the required probability $=P\left(\frac{A_1}{E}\right).$

We have $P(A_1)=P(A_2)=\frac{1}{2}$; as black and red cards were intially equal in number.

Also, $P\left(\frac{E}{A_1}\right)=\frac{^{26}C_{13}}{^{51}C_{13}}$ and $P\left(\frac{E}{A_2}\right)=\frac{^{25}C_{13}}{^{51}C_{13}}$

The required probability $=P\left(\frac{A_1}{E}\right)$

$=\frac{P(E/A_1)P(A_1)}{P(E/A_1)P(A_1)+P(E/A_2)P(A_2)}$

$=\frac{\frac{1}{2}\cdot\frac{^{26}C_{13}}{^{51}C_{13}}}{\frac{1}{2}\cdot\frac{^{26}C_{13}}{^{51}C_{13}}+\frac{1}{2}\cdot\frac{^{25}C_{13}}{^{51}C_{13}}}=\frac{2}{3}.$

(23) **(D).** We have $^{100}C_{50}\,p^{50}(1-p)^{50}={}^{100}C_{51}(1-p)^{49}$

or $\frac{1-p}{p}=\frac{100\,!}{51\,!\,.\,49\,!}\times\frac{50\,!\,.\,50\,!}{100\,!}=\frac{50}{51}$

or $51-51p=50p \Rightarrow p=\frac{51}{101}.$

(24) **(A).** $\frac{P(X=r)}{P(X=n-r)}=\frac{^nC_r p^r(1-p)^{n-r}}{^nC_{n-r}p^{n-r}(1-p)^r}=\left(\frac{1}{p}-1\right)^{n-2r}$

Note that $\frac{1}{p}-1>0.$

Therefore the ratio will be independent of n and r, if $\frac{1}{p}-1=1$ or $p=\frac{1}{2}.$

(25) **(B).** Let A denote the event that the stranger succeeds at the k^{th} trial. Then

$P(A')=\frac{999}{1000}\times\frac{998}{999}\times.....\times\frac{1000-k+1}{1000-k+2}\times\frac{1000-k}{1000-k+1}$

$\Rightarrow P(A')=\frac{1000-k}{1000}\Rightarrow P(A)=1-\frac{1000-k}{1000}=\frac{k}{1000}.$

(26) **(A).** Let P (C) = x, then P (B) = 3x, P (A) = 6x
Since A, B, C are mutually exclusive and exhaustive events. $P(A\cup B\cup C)=P(A)+P(B)+P(C)$

$1=6x+3x+x\Rightarrow x=1/10$; $P(B)=\frac{3}{10}\left(\frac{m}{n}\right)$

Odds in favour of B $=\frac{m}{n-m}=\frac{3}{10-3}=\frac{3}{7}$

(27) **(B).** Total coupons = 12
$1\le$ selected coupon no. ≤ 8

Probability of one selected coupon $=\frac{8}{12}=\frac{2}{3}$

Required probability $=\frac{2}{3}\times\frac{2}{3}\times$.........6 times $=\left(\frac{2}{3}\right)^6$

(28) **(A).** Probability of same digit on both dice is $=\frac{6}{36}=\frac{1}{6}$

$\therefore p=\frac{1}{6}, q=1-P=\frac{5}{6}$

$P(X=2)={}^nC_2(p)^2(q)^{n-2}$

$={}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2=\frac{4\times3}{2\times1}\times\frac{1}{36}\times\frac{25}{36}=\frac{25}{216}$

(29) **(B).** Total number of ways $={}^8P_8=8!$
Number of favourable cases when 5 balls are placed in respectively bags is ${}^8C_3.3!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right\}$

Required probability $= \dfrac{{}^8C_3 . 3!\left\{\dfrac{1}{2!} - \dfrac{1}{3!}\right\}}{8!} = \dfrac{1}{360}$

(30) **(B).** The probability of missing red card is

$$P(E_1) = \frac{{}^{26}C_1}{{}^{52}C_1} = \frac{1}{2}$$

$\therefore$ The probability of missing black card is

$$P(E_2) = \frac{{}^{26}C_1}{{}^{52}C_1} = \frac{1}{2}$$

Let A be the event of drawn 13 red cards

$$P\left(\frac{A}{E_1}\right) = \frac{{}^{25}C_{13}}{{}^{51}C_{13}},\quad P\left(\frac{A}{E_2}\right) = \frac{{}^{26}C_{13}}{{}^{51}C_{13}}$$

Required probability

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2}\times\frac{{}^{25}C_{13}}{{}^{51}C_{13}} + \frac{1}{2}\frac{{}^{26}C_{13}}{{}^{51}C_{13}}} = \frac{{}^{26}C_{13}}{{}^{25}C_{13} + {}^{26}C_{13}} = \frac{26}{13+26} = \frac{2}{3}$$

(31) **(B).** Total number of cases = 6 × 6 × 6 = 216
Let the second number is i (clearly 1 < i < 6), then first number can be chosen in i – 1, ways and third number can be chosen in 6 – i ways.
Hence number of ways = (i – 1) (6 – i).
$\therefore$ i can take values 2 to 5.
$\therefore$ Favourable no. of cases

$$= \sum_{i=2}^{5}(i-1)(6-1) = 1\times4 + 2\times3 + 3\times2 + 4\times1 = 20$$

$\therefore$ Required probability $= \dfrac{20}{216} = \dfrac{5}{54}$

(32) **(D).** Probability that the first critic favours the book,

$$P(E_1) = \frac{5}{5+2} = \frac{5}{7}$$

Probability that the second critic favours the book,

$$P(E_2) = \frac{4}{4+3} = \frac{4}{7}$$

Probability that the third critic favours the book,

$$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Majority will be in favour if at least two critics favour the book = $P(E_1 \cap E_2 \cap \bar{E}_3) + P(E_1 \cap \bar{E}_2 \cap E_3)$

$+ P(\bar{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$

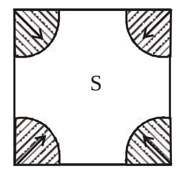
$= P(E_1)P(E_2)P(\bar{E}_3) + P(E_1)P(\bar{E}_2)P(E_3)$

$+ P(\bar{E}_1 \cap E_2 \cap E_3) + P(E_1)P(E_2)P(E_3)$

$$= \frac{5}{7}\times\frac{4}{7}\times\left(1-\frac{3}{7}\right) + \frac{5}{7}\times\left(1-\frac{4}{7}\right)\times\frac{3}{7} + \left(1-\frac{5}{7}\right)$$

$$\times\frac{4}{7}\times\frac{3}{7} + \frac{5}{7}\times\frac{4}{7}\times\frac{3}{7} = \frac{209}{343}$$

(33) **(A).** Let S denote the set of points inside a square with corners (x, y), (x, y + 1), (x + 1, y), (x + 1, y + 1), x and y are integers. Clearly each of the four points belong to the set X. Let P deonote the set of points in S with distance less than 1/4. From any corner point P consists of four quarter circles each of radius 1/4.
A coin, whose centre falls in S, will cover a point of X if and only if its centre falls in P.
Hence, the required probability,



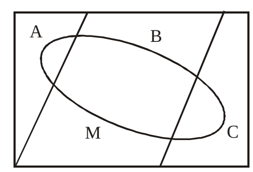
$$p = \frac{\text{area of P}}{\text{area of S}} = \frac{\pi(1/4)^2}{1\times1} = \frac{\pi}{16}$$

(34) **(A).** Total number of ways = 7!
Favourable number of ways = 7! – 2.(6!)

$$\therefore \text{Probability} = \frac{7! - 2(6!)}{7!} = 1 - \frac{2}{7} = \frac{5}{7}$$

(35) **(A).** $P(A) = \dfrac{5}{10}$, $P(B) = \dfrac{3}{10}$,

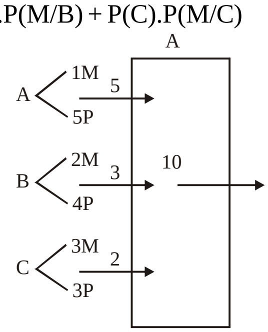
$P(C) = \dfrac{2}{10}$



P(M) = P (A) . P (M/A) + P (B).P(M/B) + P(C).P(M/C)

$$= \frac{5}{10}\cdot\frac{1}{6} + \frac{3}{10}\cdot\frac{2}{6} + \frac{2}{10}\cdot\frac{3}{6}$$

$$= \frac{5+6+6}{60} = \frac{17}{60}$$



(36) **(A).** Let us consider one bag out of n bags
E_1 : event of two white balls
E_2 : event of three white balls
W : event of drawing two balls, found to be white

$$P(E_1) = P(E_2) = \frac{1}{2},\quad P\left(\frac{W}{E_1}\right) = \frac{{}^2C_2}{{}^3C_2} = \frac{1}{3}$$

$$P\left(\frac{W}{E_2}\right) = 1,\quad P\left(\frac{E_2}{W}\right) = \frac{\frac{1}{2}.1}{\frac{1}{2}.\frac{1}{3} + \frac{1}{2}.1} = \frac{3}{4}$$

Probability that bags contains not all white balls is $1-\frac{3}{4}=\frac{1}{4}$

Probability that atleast bag contain not white balls

$$1-\left(\frac{1}{4}\right)^n > \frac{19}{20} \Rightarrow \frac{1}{20} > \frac{1}{4^n} \Rightarrow 4^n > 20 \Rightarrow n > 2$$

(37) **(B).** $1 < 3^n < 10^7$; $0 < n < \frac{7}{\log_{10} 3} \Rightarrow 0 < n \le 14$

Favourable numbers = 14 ; Reqd. probability = $\frac{14}{10^7 - 1}$

(38) **(A).** Let E denote the event that the target is hit when x shells are fired at point A.

Let E_1 (E_2) denote the event that the artillery target is at point A (B). We have $P(E_1) = \frac{8}{9}$, $P(E_2) = \frac{1}{9}$

$$\Rightarrow P\left(\frac{E}{E_1}\right) = 1-\left(\frac{1}{2}\right)^x \text{ and } P\left(\frac{E}{E_2}\right) = 1-\left(\frac{1}{2}\right)^{21-x}$$

$$\text{Now } P(E) = \frac{8}{9}\left[1-\left(\frac{1}{2}\right)^x\right]+\frac{1}{9}\left[1-\left(\frac{1}{2}\right)^{21-x}\right]$$

$$\Rightarrow \frac{d}{dx}(P(E)) = \frac{8}{9}\left(\frac{1}{2}\right)^x \ell n\,2 + \frac{1}{9}\left(-\left(\frac{1}{2}\right)^{21-x} \ell n 2\right)$$

Now we must have $\frac{d}{dx}(P(E)) = 0$

$\Rightarrow x = 12$, also $\frac{d^2}{dx^2}(P(E)) < 0$

Hence P(E) is maximum, when x = 12.

(39) **(A).** n (S) = 36

Let E be the event of getting the sum of digits on the dice equal to 7, then n(E) = 6.

$P(E) = \frac{6}{36} = \frac{1}{6} = p$, then $P(E') = q = \frac{5}{6}$

probability of not throwing the sum 7 in first m trails = q^m

$\therefore$ P(at least one 7 in m throws) = $1 - q^m = 1 - \left(\frac{5}{6}\right)^m$.

According to the question $1-\left(\frac{5}{6}\right)^m > 0.95$

$$\Rightarrow \left(\frac{5}{6}\right)^m > 0.05$$

$$\Rightarrow m\{\log_{10} 5 - \log_{10} 6\} < \log_{10} 1 - \log_{10} 20$$

$\therefore m > 16.44$

Hence, the least number of trails = 17.

(40) **(A).** Total number of sample points in the sample space $= 6^4 = 1296$

Number of sample points in favour of the event

= Coefficient of x^{10} in the expansion of $(1 + x + x^2 + \ldots + x^5)^2 (x + x^2 + \ldots + x^6)^2$

= Coefficient of x^{10} in the expansion of $x^2(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansion of $(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansion of $\left(\frac{1-x^6}{1-x}\right)^4$

= Coefficient of x^8 in the expansion of $(1 - x^6)^4 (1 - x)^{-4}$

= Coefficient of x^8 in the expansion of

$$(1-4x^6)\left(1+4x+\frac{4\times 5}{2!}x^2+\frac{4\times 5\times 6}{3!}x^3+\ldots\right)$$

$$= 1 \times {}^{11}C_8 - 4 \times {}^{5}C_2 = 125.$$

$\therefore$ Required probability = $\frac{125}{1296}$

(41) **(C).** Let the number of marble be 2n (where n is large)

$$\text{Required probability} = \lim_{n\to\infty} \frac{n \times {}^nC_4}{{}^{2n}C_5} \times \frac{{}^nC_3 \times {}^nC_2}{{}^{2n}C_5}$$

$$= \lim_{n\to\infty} \frac{n\times n(n-1)(n-2)(n-3)}{4!} \times \frac{n(n-1)(n-2)}{3!} \times \frac{n(n-1)}{2!} \times \frac{(5)^2((2n-5)!)^2}{(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{n^4(n-1)^3(n-2)^2(n-3)((2n-5)!)^2 \times 5\times 5\times 4\times 3!}{3!2!(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{50.n^4(n-1)^3(n-2)^2(n-3)}{(2n(2n-1)(2n-2)(2n-3)(2n-4))^2}$$

$$= \frac{50}{1024} = \frac{25}{512}$$

(42) **(B).** The number of heads that they can get is either 0 or 1 or 2 or 3.

$\therefore$ the required probability

$$= {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 . {}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 + {}^4C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 . {}^3C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 + {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 . {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) . {}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^7}(1 + 12 + 18 + 4) = \frac{35}{128}.$$

(43) **(B).** n < A,B ; (n – 2) others

r books from the remaining (n – 2) books can be selected $^{n-2}C_r$ ways and arranged between A and B in r! ways, also A and B can be interchanged in 2! ways.

Hence n (E) = $\boxed{^{n-2}C_r \cdot r!\ 2!}\ (n-r-1)!$;

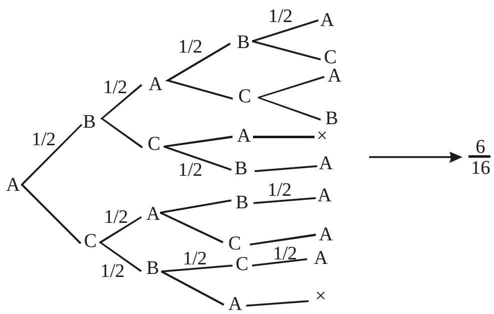
A	B_1 B_1 ...... B_r	B	(n – r – 2) other books

$$n(E) = \frac{(n-2)!\ 2!\ (n-r-1)!\ r!}{r!\,(n-r-2)!} = 2! \cdot (n-2)! \cdot (n-r-1)$$

also n (S) = n!

$$P(E) = \frac{2\,(n-2)!\,(n-r-1)}{n!} = \frac{2\,(n-r-1)}{n\,(n-1)}$$

(44) **(B).**



(45) **(A).** A : exactly one child, B : exactly two children, C : exactly 3 children

$$P(A) = \frac{1}{4},\ P(B) = \frac{1}{2},\ P(C) = \frac{1}{4}$$

E : couple has exactly 4 grandchildren

P(E) = P(A) · P(E/A) + P(B) · P(E/B) + P(C) · P(E/C)

$$= \frac{1}{4}.0 + \frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^2}_{2/2} + \underbrace{\frac{1}{4}.\frac{1}{4}.2}_{(1,3)}\right] + \frac{1}{4}\left[3\underbrace{\left(\frac{1}{4}.\frac{1}{4}.\frac{1}{2}\right)}_{1\quad 1\quad 2}\right]$$

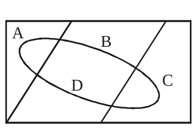
$$= \frac{1}{8} + \frac{1}{16} + \frac{3}{128} = \frac{27}{128}$$

Similarly 2 / 2 denotes each child having two children

$2.\frac{1}{4}.\frac{1}{4}$ denotes each child having 1 and 3 or 3 and 1

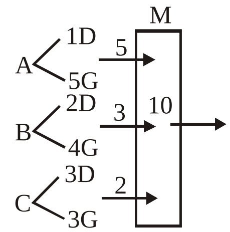
$$\text{children} = \frac{16}{128} + \frac{8}{128} + \frac{3}{128} = \frac{27}{128}$$

(46) **(A).** $P(A) = \frac{5}{10},\ P(B) = \frac{3}{10},\ P(C) = \frac{2}{10}$



P (D) = P (A) . P (D/A) + P (B) . P (D/B) + P (C) . P (D/C)

$$= \frac{5}{10}.\frac{1}{6} + \frac{3}{10}.\frac{2}{6} + \frac{2}{10}.\frac{3}{6} = \frac{5+6+6}{60} = \frac{17}{60}$$



(47) **(A).** $P\left(A \cup \bar{B}\right) = 1 - \left(\overline{A \cup \bar{B}}\right)$

$$= 1 - \left(\bar{A} \cap B\right) = 1 - P\left(\bar{A}\right)P\left(B\right)$$

0.8 = 1 – 0.7 × P(B) ⇒ P(B) = 2/7

(48) **(D).** P(A∪B) = P(A) + P(B) – P(A∩B)

P(A∪B) = P(A) + P(B) – P(A) × P(B)

0.8 = 0.3 + P(B) – 0.3 × P(B)

P(B) = 5/7

(49) **(A).** $\because\ P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$\geq P(A) + P(B) - 1$$

$$\therefore\ P(A \cap B) \geq \frac{3}{5} + \frac{2}{3} - 1 \ \Rightarrow\ P(A \cap B) \geq \frac{4}{15} \quad ..(i)$$

$$\because\ P(A \cap B) \leq P(A)\ \Rightarrow P(A \cap B) \leq \frac{3}{5} \quad ...(ii)$$

from (i) and (ii), $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$ ...(iii)

from (iii), $\frac{4}{15P(B)} \leq \frac{P(A \cap B)}{P(B)} \leq \frac{3}{5P(B)}$

$$\Rightarrow\ \frac{2}{5} \leq P\left(\frac{A}{B}\right) \leq \frac{9}{10}$$

(50) **(C).** Statement – I is true as there are six equally likely possibilities of which only two are favourable (4 and 6).

Hence P(obtained number is composite) = $\frac{2}{6} = \frac{1}{3}$.

Statement – II is not true, as the three possibilities are not equally likely.

Hence (c) is the correct answer.

(51) **(A).** ∴ Reqd. probability = 35/55.

(52) **(A).** P(E) = P(14, 41, 32, 23) = 1/9

P(F) = P(2, 4, 6) = 1/2

P(E ∩ F) = P (41, 23) = 1/18 = P(E) . P(F)

Hence E and F are independent

(53) **(A).** The statement-1 A is true and follows from statement-2

indeed $P(A/B) = \frac{P(A \cap B)}{P(B)} \leq P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} \le P(B) \Rightarrow P(B/A) \le P(B)$

(54) **(A).** $P\{A \cap (B \cap C)\} = P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
$\therefore P[A \cup (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$
$= P[(A \cap B) + (A \cap C) - P[(A \cap B) \cap (A \cap C)]$
$= p(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$
$= P(A)\,P(B) + P(A)\,P(C) - P(A)\,P(B)\,P(C)$
$= P(A)\,[P(B) + P(C) - P(B)\,P(C)] = P(A).P(B \cup C)$
$\therefore$ A & $B \cup C$ are independent events

(55) **(C).** Required probability is
$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$
$= 0.39$

(56) **(D).** Statement-1 is false. Since if the colour white is first to exhaust then last ball must be black.
$\Rightarrow$ favourable sample points $((a + b - 1)!)b$
req. probability $= \frac{b(a+b-1)!}{a+b!} = \frac{b}{a+b}$

(57) **(D).** For statement I, $n(S) = {}^6C_3 = 20$
only two triangle formed are equilateral, they are $\Delta A_1A_3A_5$ and $\Delta A_2A_4A_6$. $\therefore$ n(E) = 2
$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{2}{20} = \frac{1}{10}$.
For statement – II n(S) = 216
No. of favorable ways $= \sum_{i=1}^{6} (i-1)(6-i) = 20$
$\therefore$ Required probability $= \frac{20}{216} = \frac{5}{64}$.

(58) **(D).** In case of probability whether dice are identical or distinct probability remains same.

(59) **(D).** 2n+1 = 5, n = 2
$P(E) = \frac{3n}{4n^2 - 1} = \frac{6}{15} = \frac{2}{5}$
For a, b, c are in A.P. a + c = 2b $\Rightarrow$ a + c is even
$\therefore$ a and c are both even or both odd.
So, number of ways of choosing a and c is
${}^nC_2 + {}^{n+1}C_2 = n^2$ ways. $P(E) = \frac{n^2}{{}^{2n+1}C_3} = \frac{3n}{4n^2 - 1}$

(60) **(B).** Both A and R are correct but R is not the correct explanation of A.

(61) **(A).** $\because P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
using all the given values we get that
$P(B \cap C) \in (0.23, 0.48)$.

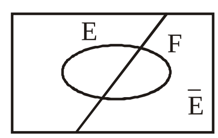
(62) **(A).** Statement –II is true as this is the definition of the independent events.
Statement – I is also true, as if events are independent, then P (A/B) = P(A)
$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \Rightarrow P(A \cap B) = P(A).\,P(B)$.
Obviously statement – II is a correct reasoning of statement – I

(63) **(B).** Area of the board $= 5 \times 5 = 25\ m^2$
Area of inner circle $= \pi \times 1^2 = \pi\, m^2$
Area of outer circle $= \pi \times 2^2 = 4\pi\, m^2$
Area between the two circle $= 4\pi - \pi = 3\pi$
Probability (Hitting the inner circle)= Probability (Hitting the dart board)×Probability of hitting the are enclosed by the inner circle
$= 0.75 \times \frac{\text{Area of inner circle}}{\text{Area of dart board}}$
$= 0.75 \times \frac{\pi}{25} = \frac{3\pi}{100} = \frac{3 \times 22}{100 \times 7} = \frac{33}{350}$

(64) **(D).** Probability of hitting the space between the two circles $= 0.75 \times \frac{\text{Area of between circle}}{\text{Area of board}}$
$= \frac{3}{4} \times \frac{3\pi}{25} = \frac{9}{100} \times \frac{22}{7} = \frac{99}{350}$

(65) **(C).** Probability of hitting outside both circle
$= \frac{3}{4} \times \frac{\text{Area of outside the circle}}{\text{Area of board}} = \frac{3}{4} \times \frac{(25 - 4\pi)}{25}$
$= \frac{3}{4} \times \frac{(175 - 88)}{25 \times 7} = \frac{3 \times 87}{100 \times 7} = \frac{261}{700}$

(66) (D), (67) (A), (68) (B).
P(E) = p
$P(F) = P(E \cap F) + P(\bar{E} \cap F)$
$P(F) = P(E)\,P(F/E) + P(\bar{E})\,P(F/\bar{E}) = p.1 + (1 - p).\frac{1}{5}$
$= \frac{4p}{5} + \frac{1}{5}$



(i) If p = 0.75
$P(F) = \frac{1}{5}(4p + 1) = \frac{1}{5}(4) = 0.8$
$\therefore P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0.75}{0.80} = \frac{15}{16}$
(ii) Now $P(E/F) = \frac{5p}{(4p + 1)} \ge p$
Equality holds for p = 0 or p = 1
for all others value of $p \in (0, 1)$, L.H.S. > R.H.S.
(iii) If each question has n alternatives then
$P(F) = p + (1 - p)\frac{1}{n} = P\left(1 - \frac{1}{n}\right) + \frac{1}{n} = \frac{(n-1)\,p + 1}{n}$

$\therefore\ P(E/F) = \dfrac{np}{(n-1)p+1}$ which increases as n increases for a fixed p.

(69) (B), (70) (A), (71) (C).

Urn-I $\begin{cases} 5R \\ 1B \end{cases}$ Urn-II $\begin{cases} 2R \\ 4B \end{cases}$

A : first two draws resulted in a blue ball.

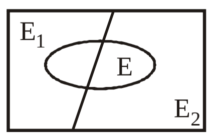
B_1 : urn-I is used $P(B_1) = \dfrac{1}{2}$

B_2 : urn-II is used $P(B_2) = \dfrac{1}{2}$

$$P(A/B_1) = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36},\quad P(A/B_2) = \frac{4}{6}\cdot\frac{4}{6} = \frac{16}{36} = \frac{4}{9}$$

$$\underbrace{P(B_1/A)}_{E_1} = \frac{\frac{1}{2}\cdot\frac{1}{36}}{\frac{1}{2}\cdot\frac{1}{36}+\frac{1}{2}\cdot\frac{16}{36}} = \frac{1}{17}$$

$$\underbrace{P(B_2/A)}_{E_2} = \frac{\frac{1}{2}\cdot\frac{16}{36}}{\frac{1}{2}\cdot\frac{16}{36}+\frac{1}{2}\cdot\frac{1}{36}} = \frac{16}{17}$$



E : third ball drawn is red $P(E) = P(E \cap E_1) + P(E \cap E_2)$

$$= \frac{1}{17}\cdot\frac{5}{6} + \frac{16}{17}\cdot\frac{2}{6} = \frac{5}{102} + \frac{32}{102} = \frac{37}{102}$$

(72) (B).

$2n \begin{cases} n \text{ white} \\ n \text{ black} \end{cases}$ $\underbrace{P_1\ P_2\ P_3 \ldots\ldots P_n}_{n \text{ persons}}$

$$n(S) = \frac{(2n)!\,n!}{(2!)^n . n!}$$

[n equal groups each of 2, distributed in n persons]

$$= \frac{(2n)!}{2^n}$$

$n(E) = ({}^nC_1)^2 . ({}^{n-1}C_1)^2 . ({}^{n-2}C_1)^2 \ldots\ldots\ldots ({}^1C_1)^2 = (n!)^2$

where

E: denotes the event that each of the n persons draw balls of different colour.

$$P(E) = \frac{(n!)^2.2^n}{(2n)!} = \frac{8}{35} = \frac{(n!)^2.2^n}{2^n.n![1.3.5....(2n-1)]} = \frac{8}{35}$$

$$= \frac{n!}{[1.3.5....(2n-1)]} = \frac{8}{35} \text{ which hold true for } n = 4.$$

(73) (C). If n = 4 we have $8 \begin{cases} W\ W\ W\ W \\ B\ B\ B\ B \end{cases}$; $P_1\ P_2\ P_3\ P_4$

$$n(S) = \frac{8!.4!}{(2!)^4.4!} = \frac{7!}{2}$$

Now 4 white can be grouped, 2 in each in $\dfrac{4!}{2!2!2!2!}$ ways

and similarly 4 black can also be grouped in $\dfrac{4!}{2!2!2!2!}$ ways. Hence number of ways in which we have 4 groups each of two balls of same colour, is $\dfrac{4!}{8!}.\dfrac{4!}{8!}$.

These groups can be distributed in 4 person in 4! ways.

Hence $n(A) = \dfrac{4!}{8!}.\dfrac{4!}{8!}.4! = 6.6.6$

$$\therefore\ P(A) = \frac{6.36.2}{7!} = \frac{72.6}{7.720} = \frac{3}{35}$$

(74) **(A).** When n = 7 i.e. n is odd the number of favourable cases (and consequently also the required probability) equals zero. When the total number white balls is odd, atleast one of the people must draw one white ball and one black ball. In this case p = 0

(75) (C), (76) (D), (77) (A).

Probability of success in each trial is p_1

Probability of failure in each trial is $q_1 = 1 - p_1$

$P_r(X = r) = {}^nC_r \cdot p_1^r . q_1^{n-r}$

Given P_r (at least one success) = p

$P_r(1) + P_r(2) + \ldots\ldots + P_r(n) = p$

or $1 - P_r(0) = p$

$P_r(0) = 1 - p$

$(P_r(F))^n = 1 - p$

$(1 - p_1)^n = 1 - p$

$\therefore\ 1 - p_1 = (1-p)^{1/n}$

$p_1 = 1 - (1-p)^{1/n}$

If $p = \dfrac{65}{81}$ and n = 4

$$p_1 = 1 - \left(\frac{16}{81}\right)^{1/4} = 1 - \frac{2}{3} = \frac{1}{3}$$

P_r (A occurs at most once)

$= P_r(1) + P_r(0) = P_r(1) + 1 - p = 1 - p + P_r(1)$

Now $P_r(1) = {}^nC_1 - p_1 q_1^{\,n-1}$

$$= n[1-(1-p)^{1/n}](1-p)^{\frac{n-1}{n}} = n\left[(1-p)^{\frac{n-1}{n}} - (1-p)\right]$$

$$P(\text{at most once}) = (1-p) + n\left[(1-p)^{\frac{n-1}{n}} - (1-p)\right]$$

$$= n(1-p)^{\frac{n-1}{n}} - (n-1)(1-p)$$

(78) (C), (79) (A), (80) (B).

A : 3 balls drawn found to be one each of different colours.

B_1 : 1(W) + 1(G) + 4(R) are drawn; $P(B_1) = \dfrac{1}{10}$

$B_2 : 1(W) + 4(G) + 1(R)$ are drawn; $P(B_2) = \frac{1}{10}$

$B_3 : 4(W) + 1(G) + 1(R)$ are drawn; $P(B_3) = \frac{1}{10}$

B4 : They are drawn in groups of 1, 2, 3 (WGR)

– (6 cases); $P(B_4) = \frac{6}{10}$

$B_5 : 2(W) + 2(G) + 2(R); P(B_5) = 1/10$

$P(A/B_1) = \frac{4}{20}$ W G R R R R

$P(A/B_2) = \frac{4}{20}$ W G G G G R

$P(A/B_3) = \frac{4}{20}$ W W W W G R

$P(A/B_4) = 6.\frac{{}^1C_1.\ {}^2C_1.\ {}^3C_1}{{}^6C_3} = \frac{36}{20}$ W G G R R R

$P(A/B_5) = \frac{{}^2C_1.\ {}^2C_1.\ {}^2C_1}{{}^6C_3} = \frac{8}{20}$ W W G G R R

$\sum_{R=1}^{5} P(B_i).P(A/B_i)$

$= \frac{1}{10}\cdot\frac{4}{20} + \frac{1}{10}\cdot\frac{4}{20} + \frac{1}{10}\cdot\frac{4}{20} + \frac{1}{10}\cdot\frac{36}{20} + \frac{1}{10}\cdot\frac{8}{20} = \frac{56}{200}$

$P(B_1/A) = \frac{\frac{1}{10}\cdot\frac{4}{20}}{\frac{56}{200}} = \frac{4}{56} = \frac{1}{14}$

$P(B_5/A) = \frac{\frac{1}{10}\cdot\frac{8}{20}}{\frac{56}{200}} = \frac{8}{56} = \frac{2}{14}$

Hence P (bag had equals number of W and G balls/A)

$= P(B_1/A) + P(B_5/A) = \frac{1}{14} + \frac{2}{14} = \frac{3}{14}$

(81) (C), (82) (B), (83) (A).

(i) $P(E_1) = \frac{1}{10}\times 1 + \frac{2}{10}\times\frac{1}{2} + \frac{3}{10}\times\frac{1}{3} + \frac{4}{10}\times\frac{1}{4} = \frac{2}{5}$

(ii) $P(A_3/E_2) = \frac{\frac{3}{10}\times\frac{1}{3}}{\frac{2}{10}\times\frac{1}{2} + \frac{3}{10}\times\frac{1}{3} + \frac{4}{10}\times\frac{1}{4}} = \frac{1}{3}$

(iii) Expectation $= \frac{4}{10}\times 1 + \frac{3}{10}\times 2 + \frac{2}{10}\times 3 + \frac{1}{10}\times 4 = 2$

(84) **7.** Total number of ways = 7!

Favourable number of ways = 7! – 2.(6!)

$\therefore$ Probability $= \frac{7! - 2(6!)}{7!} = 1 - \frac{2}{7} = \frac{5}{7}$

(85) **3.** Out of 16 possible pairs of (x, y) only three are favourable which are (1, 1), (1, 2), (1, 3).

(86) **1.** Total numbers of thirteen digits = 9×10^{12}

Favourable = 9×10^6

(87) **91.**

Total number of ways of selecting x and y = 11 × 11 = 121

Favourable ways are as follows.

x	y
0	0 to 5
1	0 to 6
2	0 to 7
3	0 to 8
4	0 to 9
5	0 to 10
6	1 to 10
7	2 to 10
8	3 to 10
9	4 to 10
10	5 to 10

So, number of ways = 2 (6 + 7 + 8 + 9 + 10) + 11

= 2 × 40 + 11 = 91

$\therefore\ P = \frac{91}{121}\ \therefore\ 121P = 91$

(88) **0.** $P(A' \cap B') = (P \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= P(B') - P(A) + P(A \cap B)$

$\Rightarrow P(A \cap B) = 0$

(89) **229.** The total number of squares is

$1^2 + 2^2 + 3^2 + 4^2 + ...+ 14^2$

$+ [1 \times 13^2 + 2 \times 12^2 + 3 \times 11^2 .... 13 \times 1^2] = 4200$

The total number of squares with length of side as integer is

$(1^2 + 2^2 + 3^2 + ...+ 14^2) + (2 \times 8 \times 8) + (2 \times 1 \times 1) = 1145$

Hence probability $= \frac{1145}{4200} = \frac{229}{840}$

(90) **1.** We have $P(A) = P(7) = \frac{6}{36}$, $P(B) = P(4) = \frac{3}{36}$

Since equal throws are disregarded,

Hence in each throw A is twice as likely to win as B.

Let P(B) = p, P(A) = 2p

$\therefore\ 3p = 1 \Rightarrow p = 1/3$

(91) **17.** We have P(E) = P(T T T T H + T T T T T)

$= \frac{1}{32} + \frac{1}{32} = \frac{1}{16} = \frac{p}{q} \Rightarrow p + q = 17$

(92) **6.** $52 \xrightarrow{\text{face card removed}} 40 \xrightarrow{\text{20 drawn randomly}}$

Let E_0 : 20 cards randomly removed has no aces.

E_1 : 20 cards randomly removed has exactly one ace.

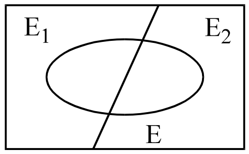
E_2 : 20 cards randomly removed has exactly 2 aces.

E : event that 2 drawn from the remaining 20 cards has both the aces.

$P(E) = P(E \cap E_0) + P(E \cap E_1) + P(E \cap E_2)$

$= P(E_0)\cdot P(E/E_0) + P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2)$

$= 40 \left\langle \begin{matrix} 4 \text{ aces} \\ 36 \text{ other} \end{matrix} \right.$



$$= \frac{{}^4C_0 \cdot {}^{36}C_{20}}{{}^{40}C_{20}} \cdot \frac{{}^4C_2}{{}^{20}C_2} + \frac{{}^4C_1 \cdot {}^{36}C_{19}}{{}^{40}C_{20}} \cdot \frac{{}^3C_2}{{}^{20}C_2}$$

$$+ \frac{{}^4C_2 \cdot {}^{36}C_{18}}{{}^{40}C_{20}} \cdot \frac{{}^2C_2}{{}^{20}C_2}$$

$$= \frac{{}^{36}C_{20} \cdot {}^4C_2 + {}^4C_1 \cdot {}^{36}C_{19} \cdot {}^3C_2 + {}^4C_2 \cdot {}^{36}C_{18} \cdot {}^2C_2}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6 \cdot {}^{36}C_{20} + 12 \cdot {}^{36}C_{19} + 6 \cdot {}^{36}C_{18}}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6[{}^{36}C_{20} + {}^{36}C_{19} + {}^{36}C_{19} + {}^{36}C_{18}]}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6({}^{37}C_{20} + {}^{37}C_{19})}{{}^{40}C_{20} \cdot {}^{20}C_2} = \frac{6({}^{38}C_{20})}{{}^{40}C_{20} \cdot {}^{20}C_2} \Rightarrow p = 6$$

(93) **6.** Let $P(E_1) = x$, $P(E_2) = y$ and $P(E_3) = z$

then $(1-x)(1-y)(1-z) = p$

$x(1-y)(1-z) = \alpha$

$(1-x)y(1-z) = \beta$

$(1-x)(1-y)(1-z) = \gamma$

so $\frac{1-x}{x} = \frac{p}{\alpha}$; $x = \frac{\alpha}{\alpha + p}$. Similarly, $z = \frac{\gamma}{\gamma + p}$

$$\text{So, } \frac{P(E_1)}{P(E_3)} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$$

$$\text{Also given, } \frac{\alpha\beta}{\alpha - 2\beta} = p = \frac{2\beta\gamma}{\beta - 3\gamma} \Rightarrow \beta = \frac{5\alpha\gamma}{\alpha + 4\gamma}$$

$$\text{Substituting back } \left(\alpha - 2\left(\frac{5\alpha\gamma}{\alpha + 4\gamma}\right)\right) p = \frac{\alpha \cdot 5\alpha\gamma}{\alpha + 4\gamma}$$

$\Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$

$$\Rightarrow \left(\frac{p}{\gamma} + 1\right) = 6\left(\frac{p}{\alpha} + 1\right) = \frac{\frac{p}{\gamma} + 1}{\frac{p}{\alpha} + 1} = 6$$

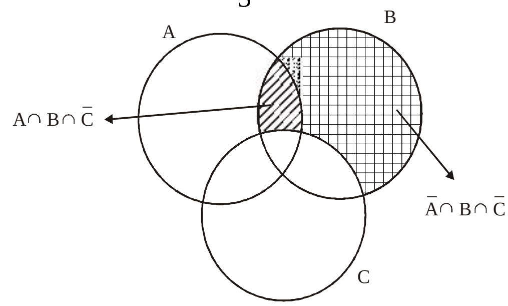
(94) **5.** The minimum of two numbers will be less than 4 or at least one of the numbers is less than 4.

P (at least one numbers < 4) = 1 – P(both the numbers ≥ 4)

$$= 1 - \frac{3}{6} \times \frac{2}{5} = 1 - \frac{6}{30} = 1 - \frac{1}{4} = \frac{4}{5}$$

(95) **12.** Given that P (B) = 3/4, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$

$$P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}.$$



From Venn's diagram, we have

$P(B \cap C) = P(B) - P(A \cap B \cap \bar{C}) - P(\bar{A} \cap B \cap \bar{C})$

$$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{9-4-4}{12} = \frac{1}{12}$$

(96) **4.** If a number is to be divisible by both 2 and 3, it should be divisible by their L.C.M.

L.C.M. of 2 and 3 is 6.

The number are 6, 12, 18, ..., 96.

The total number is 16. Hence, the probability is

$$\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$$

(97) **5.** In single throw of a dice, probability of getting 1 is 1/6 and probability of not getting 1 is 5/6.

Then, getting 1 in even number of chances is getting 1 in 2nd chance or in 6th chance and so on.

Therefore, the required probability is

$$\frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + ...\infty$$

$$= \frac{5}{36}\left[\frac{1}{1 - \frac{25}{36}}\right] = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

EXERCISE-3

(1) **(C).** Let the probability of solving a problem by first student is P (A) by second is P (B) and by third is P (C).

$P(A) = 1/2 \Rightarrow P(\bar{A}) = 1/2$, $P(B) = 2/3 \Rightarrow P(\bar{B}) = 1/3$

$P(C) = 1/4 \Rightarrow P(\bar{C}) = 3/4$

prorbability that problem is not solved by any of them is

$$P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{8}$$

$\therefore$ Probability that the problem will be solved $= 1 - \frac{1}{8} = \frac{7}{8}$

(2) **(C).** $P(A \cup B) = \frac{3}{4}$

$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$ .... (1)

$P(\bar{A} \cap B) = P(B) - (A \cap B)$ .... (2)

$\because P(A \cup B) = \frac{3}{4} \Rightarrow P(A) + P(B) - P(A \cap B) = 3/4$

$\Rightarrow$ From eq. (1) & (2), $\frac{1}{3} + P(\bar{A} \cap B) = \frac{3}{4}$

$\Rightarrow P(\bar{A} \cap B) = \frac{3}{4} - \frac{1}{3} = \frac{9-4}{12} = \frac{5}{12}$

(3) **(C).** Let A is the event that 5 appears on at least one of the dice = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)} = 11

$\therefore P(A) = \frac{11}{36}$ {incase of two dice total no. of case are 36}

B | A is the event getting sum is 10 or greater when 5 appears on at least one of the dice.

$A \cap B$ is 5 appears on at least one of the dice and sum is 10 or greater then 10 = {(5, 5), (6, 5), (5, 6)} = 3 case

$\therefore P(A \cap B) = 3/36$

Now $P(A \cap B) = P(A) \times P(B/A)$

$$\Rightarrow P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{3/36}{11/36} = \frac{3}{11}$$

Alternate :

Total no. of case = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)} = 11 case

P. case = {(5, 5), (6, 5), (5, 6)} = 3 case

$P = \frac{3}{11}$

(4) **(B).** $\because$ Probability of any event is always lies in [0, 1]

Now $P(A) = \frac{3x+1}{3} \Rightarrow 0 \le \frac{3x+1}{3} \le 1$

$\Rightarrow 0 \le 3x + 1 \le 3 \Rightarrow \frac{-1}{3} \le x \le \frac{2}{3}$ .... (1)

$P(B) = \frac{1-x}{4} \Rightarrow 0 \le \frac{1-x}{4} \le 1$

$\Rightarrow 0 \le 1 - x \le 4 \Rightarrow -3 \le x \le 1$ .... (2)

$P(C) = \frac{1-2x}{2} \Rightarrow 0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow 0 \le 1 - 2x \le 2 \Rightarrow \frac{-1}{2} \le x \le \frac{1}{2}$ .... (3)

and $0 \le P(A) + P(B) + P(C) \le 1$

$\Rightarrow 0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$\Rightarrow 0 \le \frac{12x + 4 + 3 - 3x + 6 - 12x}{12} \le 1$

$\Rightarrow 0 \le 13 - 3x \le 12 \Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$ .... (4)

From (1), (2), (3) & (4), $x \in [1/3, 1/2]$

(5) **(A).** Total number of ways of selecting two horses is $^5C_2 = 10$ ways

If Mr. A selects the wining horse then he is to choose one more horse from the remaining four horses which he can do in $^4C_1 = 4$ way.

$\therefore$ Required probability $= \frac{4}{10} = \frac{2}{5}$

(6) **(C).** Let the probability of A speaks truth is P (A) = 4/5

$\Rightarrow P(\bar{A}) = \frac{1}{5}$

and probability of B speaks truth is P (B) = 3/4

$\Rightarrow P(\bar{B}) = \frac{1}{4}$

$\therefore$ Required probability $= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$$

(7) **(B).**

X :	1	2	3	4	5
		6	7	8	
P(X) :	0.15	0.23	0.12	0.10	0.20
		0.08	0.07	0.05	

E = {X is prime number} = X {X = 2, 3, 5, 7}

F = {X < 4} = {X = 1, 2, 3}

E U F = {X = 1, 2, 3, 5, 7}

$\therefore$ P (E U F) = p (1) + p (2) + p (3) + p (5) + p (7)

= 0.15 + 0.23 + 0.12 + 0.20 + 0.07 = 0.77

(8) **(D).** Mean = np = 4 .... (1)

Variance = npq = 2 .... (2)

From eq. (1) and eq. (2), q = 1/2

but $p + q = 1 \Rightarrow p = 1/2$

put value of p in (1) we get, n = 8

Now probability of 2 success is $^8C_2\,(1/2)^2\,(1/2)^6$

{Probability of exactly r success is $^nC_r\,p^r q^{n-r}$}

$$= \frac{8}{2} \times \frac{7}{1} \times \frac{1}{2^8} = \frac{7}{2^6} = \frac{7}{64} = \frac{7 \times 4}{64 \times 4} = \frac{28}{256}$$

(9) **(B).** Person 1 has three option to apply similarly person 2 has three option to apply and person 3 has three option to apply. Total cases 3^3

Now favourable cases = 3 (either all the three apply for 1 or 2 or for 3)

$\therefore$ Probability = $\frac{3}{3^3} = \frac{1}{9}$

(10) **(C).** Given, $P(\overline{A \cup B}) = \frac{1}{6}$

$P(A \cap B) = \frac{1}{4}$; $P(\bar{A}) = \frac{1}{4}$

$\because \; P(\bar{A}) = \frac{1}{4} \Rightarrow P(A) = \frac{3}{4}$ .... (1)

Now, $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6} \Rightarrow 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{6}$

$\Rightarrow \; 1 - \frac{3}{4} - P(B) + \frac{1}{4} = \frac{1}{6} \; \Rightarrow \; P(B) = \frac{1}{3}$ .... (2)

Clearly from (1) & (2), $P(A) \times P(B)$

$= \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

$\therefore$ Event are independent but no equally likely.

(11) $P(X = r) = \frac{e^{-m} m^r}{r!}$

$P(X \le 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = 6/e^5$.

(12) **(D).** In throw of two dice the total no. of case = 36

No. of cases in which sum is 9 are

{(3, 6), (4, 5), (5, 4), (6, 3)} = 4 cases

$\therefore$ Probability of getting sum 9 is $p = \frac{4}{36} = \frac{1}{9}$

$\Rightarrow$ Probability of not getting sum 9 is $q = 1 - \frac{1}{9} = \frac{8}{9}$

dice are thrown three time $\therefore$ n = 3

Now probability of getting score of exactly 9 twice is

$= {}^3C_2 \left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right)^{3-2}$ {from binonial distribution probability of exactly succes is ${}^nC_r\, p^r\, q^{n-r}$}

$= \frac{3}{2} \times \frac{2}{1} \times \frac{1}{81} \times \frac{8}{9} = \frac{8}{243}$

(13) **(B).** Let probability of 1st plane hit correctly is

P (A) = 0.3 (given)

$\therefore \; P(\bar{A}) = 0.7$

and probability of IInd plane hit target correctly is

P (B) = 0.2 (given) $\therefore \; P(\bar{B}) = 0.8$

$\therefore$ required probability $P(\bar{A}) \times P(B) = (0.7 \times 0.2) = 0.14$

(14) **(B).** $A = \{4, 5, 6\} \Rightarrow P(A) = 3/6$

$B = \{1, 2, 3, 4\} \Rightarrow P(B) = 4/6$

$(A \cap B) = \{4\} \Rightarrow P(A \cap B) = 1/6$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = \frac{6}{6} = 1$

(15) **(A).** P (A) = 1/4

P (A | B) = 1/2

P (B | A) = 2/3

$\because \; P(A) \times P(B|A)$

$\Rightarrow \; P(B) = \frac{P(A) \times P(B|A)}{P(A|B)} = \frac{1/4 \times 2/3}{1/2} = \frac{1}{3}$

(16) **(D).** P (at least 1 success) = 1 – P (all failures)

Given that $1 - \left(\frac{3}{4}\right)^n \ge \frac{9}{10}$

[P (at least 1 success) = 1 – $(3/4)^n$]

$\Rightarrow \frac{1}{10} \ge \left(\frac{3}{4}\right)^n \; \Rightarrow \; -1 \ge n\,(\log_{10}^3 - \log_{10}^4)$

$\Rightarrow \; n \ge \frac{1}{\log_{10}^4 - \log_{10}^3}$

(17) **(D).** 00 10 20 30 40

01 11 21 31 40

: : : : :

09 19 29 39 49

n (S) = 14, n (A) = 1

$\Rightarrow P(A) = \frac{1}{14}$

(18) **(B).** $N(S) = {}^{20}C_4$

Statement-1: common difference is 1

Total number of cases = 17

Common difference is 2; total number of cases = 14

Common difference is 3; total number of cases = 11

Common difference is 4; total number of cases = 8

Common difference is 5; total number of cases = 5

Common difference is 6; total number of cases = 2

Prob. = $\frac{17 + 14 + 11 + 8 + 5 + 2}{{}^{20}C_4} = \frac{1}{85}$

(19) **(A).** $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

Probability = $\frac{3 \times 4 \times 2}{{}^9C_3} = \frac{24 \times 3!}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$

(20) **(C).** $1 - P^5 \ge \frac{31}{32}$; $P^5 \le \frac{1}{32}$; $P \le \frac{1}{2}$; $P \in \left[0, \frac{1}{2}\right]$

(21) **(B).** $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$

$\frac{1}{P(D)} \ge 1$; $\frac{P(C)}{P(D)} \ge P(C)$; $P(C) \le P\left(\frac{C}{D}\right)$

(22) **(B).** Let Event (Given : {1, 2, 3,.........8})

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

(23) **(C).** p = 1/3, q = 2/3

$$^5C_4\left(\frac{1}{3}\right)^4.\frac{2}{3}+{}^5C_5\left(\frac{1}{3}\right)^5=5.\frac{2}{3^5}+\frac{1}{3^5}=\frac{11}{3^5}$$

(24) **(C).** $P(\overline{A\cup B})=\frac{1}{6}$; $P(A\cup B)=\frac{5}{6}$, $P(A)=\frac{3}{4}$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{5}{6}$$

$$P(B)=\frac{5}{6}-\frac{3}{4}+\frac{1}{4}=\frac{1}{3}$$

$$P(A\cap B)=P(A)\cdot P(B)\ ;\ \frac{1}{4}=\frac{3}{4}\times\frac{1}{3}$$

(25) **(D).** Question is wrong but the best suitable option is (D). Required probability $={}^{12}C_3\frac{2^9}{3^{12}}=\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

(26) **(C).** $P(E_1)=1/6, P(E_2)=1/6$

$$P(E_3)=\frac{2+4+6+4+2}{36}=\frac{1}{2}$$

$$P(E_2\cap E_3)=\frac{1}{6}\times\frac{1}{2}=P(E_2)\times P(E_3)$$

$$P(E_1\cap E_3)=\frac{1}{6}\times\frac{1}{2}=P(E_1)\times P(E_3)$$

$$P(E_1\cap E_2\cap E_3)=0\neq P(E_1)\,P(E_2)\,P(E_3)$$

(27) **(C).** n = 10, p = 15/25, q = 10/25

$$\text{var}(X)=npq=10\times\frac{15}{25}\times\frac{10}{25}=10\times\frac{3}{5}\times\frac{2}{5}=\frac{12}{5}$$

(28) **(C).** Let $A\equiv\{0,1,2,3,4 ......, 10\}$

$n(s)={}^{11}C_2$ (where 'S' denotes sample space)

Let E be the given event

$\therefore\ E\equiv\{(0,4),(0,8),(2,6),(2,10),(4,8),(6,10)\}$

$n(E)=6$ $\therefore\ P(E)=6/55$

(29) **(D).** $P(A)+P(B)-2P(A\cap B)=1/4$

$P(B)+P(C)-2P(B\cap C)=1/4$

$P(C)+P(A)-2P(C\cap A)=1/4$

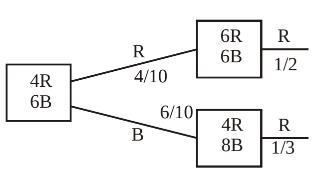
$P(A\cap B\cap C)=1/16$

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=3/8$

$$P(A\cup B\cup C)=\Sigma P(A)-\Sigma P(A\cap B)+P(A\cap B\cap C)$$

$$=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$$

(30) **(D).**



(31) **(B).** Two cards are drawn successively with replacement

4 Aces 48 Non-Aces

$$P(x=1)=\frac{^4C_1}{^{52}C_1}\times\frac{48C_1}{52C_1}+\frac{48C_1}{52C_1}\times\frac{4C_1}{52C_1}=\frac{24}{169}$$

$$P(x=2)=\frac{^4C_1}{^{52}C_1}\times\frac{^4C_1}{^{52}C_1}=\frac{1}{169}$$

$$P(x=1)+P(x=2)=\frac{25}{169}$$

(32) **(D).** $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}$

(as $A\subset B\Rightarrow P(A\cap B)=P(A)$)

$\Rightarrow\ P(A|B)\geq P(A)$

(33) **(D).** Probability of observing at least one head out of n tosses $=1-\left(\frac{1}{2}\right)^n\geq 0.9\Rightarrow\left(\frac{1}{2}\right)^n\leq 0.1\ \Rightarrow n\geq 4$

$\Rightarrow$ Minimum number of tosses = 4

(34) **(C).** Let persons be A, B, C, D

P (Hit) = 1 – P (none of them hits)

$$=1-P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D})$$

$$=1-P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C})\cdot P(\bar{D})=1-\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{7}{8}=\frac{25}{32}$$

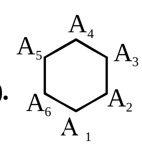
(35) **(A).** P(B) = P(G) = 1/2

$$\text{Required Proballity}=\frac{\text{all 4 girls}}{\text{(all 4 girls)+(exactly 3 girls+1 boy)+(exactly 2 girls+2 boys)}}$$

$$=\frac{(1/2)^4}{(1/2)^4+{}^4C_3(1/2)^4+{}^4C_2\,(1/2)^4}=\frac{1}{11}$$

(36) **(C).** $1-\left(\frac{1}{2}\right)^n>\frac{99}{100}\Rightarrow\left(\frac{1}{2}\right)^n<\frac{1}{100}\ \Rightarrow\ n=7$

(37) **(B).**



Only two equilateral tringles are possible

$A_1\ A_3\ A_5$ and $A_2A_5A_6$

$$\frac{2}{^6C_3}=\frac{2}{20}=\frac{1}{10}$$

(38) **(D).** np = 8, npq = 4

$q=1/2\ \Rightarrow p=1/2$; $n=16$

$$p(x=r)={}^{16}C_r\left(\frac{1}{2}\right)^{16}$$

$$P(x\leq 2)=\frac{^{16}C_0+{}^{16}C_1+{}^{16}C_2}{2^{16}}=\frac{137}{2^{16}}$$

(39) **(B).** Let X be random varibale which denotes number of problems that candidate is unbale to solve p = 1/5, X < 2

$\Rightarrow$ P (X < 2) = P (X = 0) + P (X = 1)

$$=\left(\frac{4}{5}\right)^{50}+{}^{50}C_1\cdot\left(\frac{1}{5}\right)\cdot\left(\frac{4}{5}\right)^{49}$$

(40) **(B).** Win Rs.15 $\rightarrow$ number of cases = 6

Win Rs.12 $\rightarrow$ number of cases = 4

Loss Rs.6 $\rightarrow$ number of cases = 26

p (expected gain/loss)

$$=15\times\frac{6}{36}+12\times\frac{4}{36}-6\times\frac{26}{36}=-\frac{1}{2}$$

(41) **(A).**

k	0	1	2	3	4	5
P (k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = no. of times head occur consecutively

Now expectation = $\sum x\,P(k)$

$$=(-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}$$

$$+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$$

(42) **(C).** Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

$$={}^5C_0\left(\frac{3}{4}\right)^5+{}^5C_1\frac{1}{4}\left(\frac{3}{4}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$=\frac{243}{1024}+\frac{405}{1024}+\frac{270}{1024}=\frac{918}{1024}=\frac{459}{512}=\frac{27\times17}{64\times8}$$

$$=\left(\frac{3}{4}\right)^3\times k=\left(\frac{3}{4}\right)^3\times\frac{17}{8}\quad\therefore\ k=\frac{17}{8}$$

(43) **(B).** A & B are independent events so P (A/B') = 1/3

(44) **(A).** P(exactly one) = 2/5

$P(A)+P(B)-2P(A\cap B)=2/5$

$(A\cup B)=1/2$

$P(A)+P(B)-P(A\cap B)=1/2$

$$\therefore\ P(A\cap B)=\frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}$$

(45) **(A).** A : Event when card A is drawn

B : Event when card B is drawn.

P(A) = P(B) = 1/2

Required probability = P(AA or (AB) A

or (BA) A or (ABB) A or (BAB)A or (BBA)A)

$$=\frac{1}{2}\times\frac{1}{2}+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times2+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times3$$

$$=\frac{1}{4}+\frac{1}{4}+\frac{3}{16}=\frac{11}{16}$$

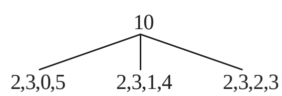
(46) **(B).** $\sum P(X)=1\Rightarrow K^2+2K+K+2K+5K^2=1$

$\Rightarrow 6K^2+5K-1=0\Rightarrow(6K-1)(K+1)=0$

$\Rightarrow K=-1$ (rejected) $\Rightarrow K=1/6$

$$P(X>2)=K+2K+5K^2=\frac{23}{36}$$

(47) **(D).** 10 different balls in 4 different boxes.



$$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times3!\times0!\times5!}+4!\times\frac{10!}{2!\times3!\times1!\times4!}+4!\times\frac{10!}{(2!)^2\times2!\times(3!)^2\times2!}\right)$$

$$=\frac{17\times945}{2^{15}}$$