**12**

HEAT & THERMODYNAMICS

THERMAL EXPANSION

INTRODUCTION :

Solid are made of atoms and molecules. At a given temperature, the atoms and molecules are placed at some equilibrium distance. When heat is supplied to solid, the interatomic separation increases by which there is an expansion of solids. This expansion can be in terms of length/area/volume. The thermal expansion of solid is classified as

Thermal expansion of solid

Linear expansion   Superficial expansion   Cubical expansion
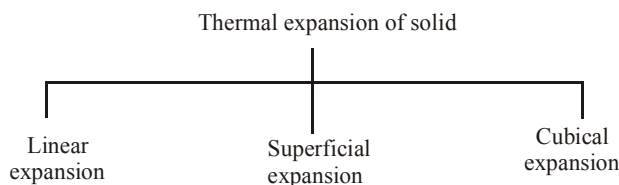
LINEAR EXPANSION OF SOLID

Almost all solids expand on heating. On increasing the temperature of a solid, its length increases. This change in length of a solid on heating is called linear expansion.

Coefficient of linear expansion is defined as fractional increase in length per °C rise in temperature. If ℓ is the length of the rod at T K and as the temperature is changed to T + ΔT its length becomes $\ell + \Delta\ell$, so coefficient of linear expansion is given by $\alpha = \dfrac{\left(\dfrac{\Delta\ell}{\ell}\right)}{\Delta T} = \dfrac{d\ell}{\ell.dT}$

It is positive for metals except carbon. The value of α is negative for plastic because in plastic when the temperature increases, length decreases. The numerical value of α is same in both the units i.e. in Per Kelvin or per °C

If α is coefficient of linear expansion at t_1°C.

ℓ_1 = length of the rod at t_1°C

ℓ_2 = length of the rod at t_2°C

$\quad \ell_2 = \ell_1[1 + \alpha (t_2 - t_1)]$

t may be in any unit °C or K because in the formula there is a difference of temperature which remains same for °C or K.

Bimetallic strip : If two strips of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in the form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is :

$$R = \frac{d}{(\alpha_2 - \alpha_1)(t_2 - t_1)} \quad \text{or} \quad R = \frac{d}{(\alpha_2 - \alpha_1)t}$$
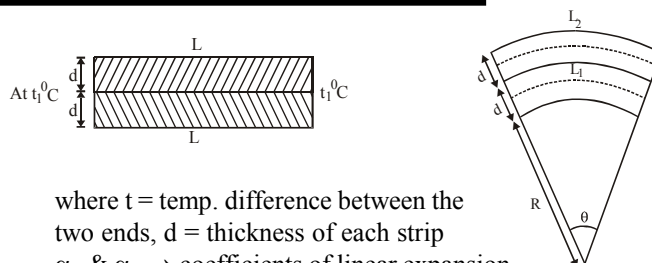
where t = temp. difference between the two ends, d = thickness of each strip

α_1 & $\alpha_2 \rightarrow$ coefficients of linear expansion

Example 1 :

Find the length of the steel rod which would have the same difference in length with a copper rod of length 24 cm at all temperatures. ($\alpha_{copper} = 18 \times 10^{-6} k^{-1}$, $\alpha_{steel} = 12 \times 10^{-6} k^{-1}$).

Sol. By linear expansion of solids, we have

$\quad \Delta\ell = \ell_0 \alpha.\Delta T$

so $\quad \ell_{steel} \cdot \alpha_{steel}.\Delta T = \ell_{copper} \cdot \alpha_{copper}.\Delta T$

$\quad \ell_{steel} = \dfrac{\ell_{copper}.\alpha_{copper}}{\alpha_{steel}} = \dfrac{24 \times 18 \times 10^{-6}}{12 \times 10^{-6}} = 36$ cm

**Example 2 :**

A metal sheet with a circular hole is heated. The hole

(1) Gets larger   (2) Gets smaller

(2) Remains of the same size   (4) Gets deformed

Sol. (1). When a body is heated, the distance between any two points on it increases. So due to thermal expansion of solids, the hole gets larger.

Thermal stress : When a rod is heated or cooled, it expands or contracts. It is turned as free expansion of the rod
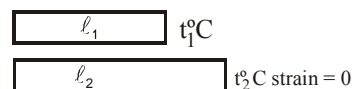
Actually no strain is being developed because on increasing the temperature the length of the rod increase so at t_2 °C, ℓ_2 because natural length of rod.

Now if the ends of the rods are rigidly fixed so as to prevent it from expansion or contraction than stress is produced in the rod. By virtue of this thermal stress the rod exerts a large force on the supports.
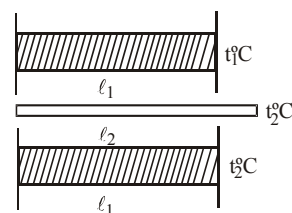
The first figure indicates the rod kept at a temperature t_1°C having ℓ_1. The rod is between two rigid supports. If the supports were not there then on increasing the temperature the free expansion occurs by which length becomes ℓ_2 at higher temperature t_2°C. The third figure indicates that on increasing the temperature to t_2°C, the length remains same but a compressive strain is induced in the rod.

If ℓ_1 is the length of the rod at t_1°C and on increasing temperature to t_2°C its length becomes ℓ_2 then

$$\ell_2 = \ell_1 [1 + \alpha (t_2 - t_1)] \;\; ; \;\; \ell_2 = \ell_1 + \ell_1 \alpha (t_2 - t_1)$$

$$\frac{\ell_2 - \ell_1}{\ell_1} = \alpha (t_2 - t_1)$$

Above relationship indicates the thermal strain developed

in the rod. i.e. Thermal strain $= \dfrac{\ell_2 - \ell_1}{\ell_2} = \alpha (t_2 - t_1)$

In elasticity, Young's modulus $= \dfrac{\text{stress}}{\text{strain}}$

Stress $= Y \alpha (t_2 - t_1)$

As force $= \dfrac{\text{stress}}{\text{Area}}$; Force $= \dfrac{Y \alpha (t_2 - t_1)}{A}$

If the rod is in its natural length at t_1°C while at t_2°C it is in compressed state, then

t_1°C

ℓ

Strain $= \dfrac{\text{Change in length}}{\text{Original length}}$

t_2°C

ℓ

In this case first figure represents the rod of length ℓ at t_1°C when the temperature is lowered to t_2°C the length of the rod remains same but a tensile strain is developed in the

$$\text{rod} = \frac{(\ell + \ell \alpha \Delta t) - \ell}{\ell + \ell \alpha \Delta t} = \frac{\ell \alpha \Delta t}{\ell + \ell \alpha \Delta t} = \frac{\ell \Delta t}{\ell + \ell \Delta t} \simeq \alpha \Delta t$$

Stress $= \alpha \, \Delta t$

When the temperature of the rod is increased, the compressive stress is developed while on decreasing the temperature of the rod the tensile stress is developed.

F | Heated | F | F | Cooled | F
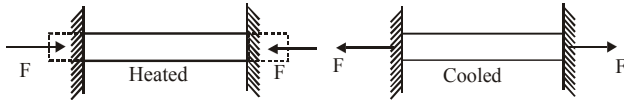
Effect of temperature on Pendulum clock

A pendulum clock consists of a metal rod or wire with the bob at one end.

Let ℓ_1 be the length of the simple pendulum at θ_1°C than

time period T_1 is given by $\qquad T_1 = 2\pi \sqrt{\dfrac{\ell_1}{g}}$ ....(1)

Now when the temperature increases to θ_2°C the effective

length becomes ℓ_2 so that $T_2 = 2\pi \sqrt{\dfrac{\ell_2}{g}}$ ....(2)

Dividing eq^n (2) by (1), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{\ell_2}{\ell_1}} \quad \text{but } \ell_2 = \ell_1 [1 + \alpha (\theta_2 - \theta_1)]$$

so $\quad \dfrac{T_2}{T_1} = \sqrt{\dfrac{\ell_1 [1 + \alpha(\theta_2 - \theta_1)]}{\ell_1}}$

$\dfrac{T_2}{T_1} = [1 + \alpha (\theta_2 - \theta_1)]^{1/2}$ or $\dfrac{T_2}{T_1} = 1 + \dfrac{1}{2} \alpha (\theta_2 - \theta_1)$

or $\quad \dfrac{T_2 - T_1}{T_1} = \alpha (\theta_2 - \theta_1)$

Change in time period $\Delta T = T_2 - T_2 = \dfrac{1}{2} \alpha . \Delta \theta . T_1$

The above expression represents the time lost per oscillation. Thus a pendulum clock loses time in summer and gains time in winter.

Note : If a pendulum clock is giving correct time when time period is T then

If T increases, clock becomes slow.

If T decreases, clock becomes fast.

Example 3 :

A steel rod of length 1m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed ?

Sol. In this case rod rests on a horizontal base which is the free expansion on heating. Hence no strain is developed in the rod i.e. Strain = 0

Example 4 :

A steel rod of length 50 cm has a cross-sectional area of 0.4 cm^2. What force would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C. ($\alpha = 10^{-5}$ k^{-1} and $Y = 2 \times 10^{11}$ N/m^2)

Sol. Force $= YA \alpha . \Delta \theta = 2 \times 10^{11} \times 0.4 \times 10^{-4} \times 10^{-5} \times 10$
$= 0.8 \times 10^3 = 800$ N

Example 5 :

A pendulum clock with a pendulum made of invar ($\alpha = 0.7 \times 10^{-6}$°C^{-1}) has a period of 0.5 s and is accurate at 25°C. If the clock is used in a country where the temperature averages 35°C, what correction in necessary at the end of a month (30 days) to the time given by the clock -

Sol. In time interval t, the clock will become slow by

$$\Delta t = \frac{1}{2} \alpha . t \Delta \theta = \frac{1}{2} \times 7 \times 10^{-7} \times 30 \times 86400 \times (35 - 25) = 9.1 \text{s}$$

SUPERFICIAL EXPANSION OF SOLID

On increasing the temperature of solid, its area increases. This change in area is referred as superficial expansion of solids. If the area of solid at temperature t_1°C is A_1 and on heating the rod, the area becomes A_2 at t_2°C than

$$A_2 = A_1 [1 + \beta (t_2 - t_1)]$$

where β is coefficient of superficial expansion at t_1°C.
Coefficient of superficial expansion is defined as fractional increase in area per °C rise in temperature.

$$\frac{\Delta A / A}{\Delta T} = \frac{dA}{A.dT}$$

VOLUME EXPANSION OF SOLID

On increasing the temperature of rod, its volume changes. If V_1 is the volume of solid at t_1°C and on increasing the temperature to t_2°C the volume becomes V_2 then,

$$V_2 = V_1 [1 + \gamma(t_2 - t_1)]$$

where γ is coefficient of volume expansion at t_1°C.
Coefficient of volume expansion is defined as the fractional increase in volume per °C rise in temperature.

$$\gamma = \frac{\Delta V / V}{\Delta T} = \frac{dV}{V.dT}$$

Relation between coefficient of linear expansion (α), coefficient of superficial expansion (β) & coefficient of cubical expansion (γ) $\beta = 2\alpha$; $\gamma = 3\alpha$; $\beta = (2/3)\gamma$

Example 6 :
The coefficient of linear expansion of a crystal in one direction is α_1 and that in every direction perpendicular to it is α_2. Find the coefficient of cubical expansion.

Sol. $V = V_0(1 + \gamma T)$
or $L^3 = L_0(1 + \alpha_1 T) L_0^2(1 + \alpha_2 T)^2$
or $= L_0^3 (1 + \alpha_1 T) (1 + \alpha_2 T)^2$
or $V = V_0(1 + \alpha_1 T)(1 + \alpha_2 T)^2$
Hence $1 + \gamma T = (1 + \alpha_1 T)(1 + \alpha_2 T)^2$
or $1 + \gamma T = (1 + \alpha_1 T)(1 + 2\alpha_2 T) = 1 + (\alpha_1 + 2\alpha_2)T$
Hence $\gamma = \alpha_1 + 2\alpha_2$

CHANGE IN DENSITY OF SOLID WITH TEMPERATURE

Suppose m is the mass of a solid which at a given temperature occupies a volume V so that density at 0°C is

$d_0 = \dfrac{m}{V}$. Now if the temperature is increased by t°C, mass

will remain unchanged but due to thermal expansion volume increases so that, $V' = V(1 + \gamma t)$

Now density, $d_t = \dfrac{m}{V'} = \dfrac{m}{V(1 + \gamma t)}$; $d_t = \dfrac{d_0}{(1 + \gamma t)}$

Here γ is coefficient of cubical expansion at 0°C.

TRY IT YOURSELF-1

Q.1 Two identical rectangular strips one of copper and other of steel are riveted together to form a bimetallic strip $(\alpha_{copper} > \alpha_{steel})$. On heating this strip will
(A) Remains straight
(B) Bend with copper on convex side
(C) Bend with steel on convex side
(D) Get twisted

Q.2 Consider the following statements -
(a) Coefficient of linear expansion has dimension K^{-1}
(b) Coefficient of volume expansion has dimension K^{-1}

(A) Both a and b are correct
(B) a is correct but b is wrong
(C) b is correct but a is wrong
(D) a and b are both wrong

Q.3 A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly
(A) its speed of rotation increases.
(B) its speed of rotation decreases.
(C) its speed of rotation remains same.
(D) its speed increases because its moment of inertia increases.

Q.4 An aluminium sphere is dipped into water. Which of the following is true?
(A) Buoyancy will be less in water at 0°C than that in water at 4°C.
(B) Buoyancy will be more in water at 0°C than that in water at 4°C.
(C) Buoyancy in water at 0°C will be same as that in water at 4°C.
(D) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.

Q.5 As the temperature is increased, the time period of a pendulum
(A) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
(B) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.
(C) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.
(D) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.

Q.6 The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α . The sphere is heated a little by a temperature ΔT so that its new temperature is T + ΔT . The increase in the volume of the sphere is approximately
(A) $2\pi R \alpha \Delta T$ (B) $\pi R^2 \alpha \Delta T$
(C) $4\pi R^3 \alpha \Delta T /3$ (D) $4\pi R^3 \alpha \Delta T$

Q.7 Calculate the temperature which has same numeral value on celsius and Fahrenheit scale.

ANSWERS

(1) (B)   **(2)** (A)   **(3)** (B)
(4) (A)   **(5)** (A)   **(6)** (D)
(7) –40°C = –40°F

CALORIMETRY

HEAT

Heat is a form of energy. It is measured in joule. It is also measured in the unit calorie
Heat is felt by its effects. Some of the effects of heat are :
(a) Change in the degree of hotness
(b) Expansion in length, surface area and volume
(c) Change in state

(d) Change in the resistance of a conductor

(e) Thermo e.m.f. effect

When a hot body is put in contact with a cold one, the former gets colder and the latter warmer. From this observation it is natural to conclude that a certain quantity of heat has passed from the hot body to the cold one.

Definition of 1 Calorie : Amount of heat required to raise temperature of one gram of water from 14.5 °C to 15.5 °C is known as one calorie.

Specific heat : The amount of energy needed to raise the temperature of unit mass of that substance by 1ºC (or 1K). It is denoted by s or c.

If the temperature of a substance of mass m changes from T to T + dT when it exchanges an amount of heat dQ with its surroundings then its specific heat is $c = \dfrac{1}{m} \cdot \dfrac{dQ}{dT}$

Unit :

(a) CGS unit of specific heat is cal/g-°C

(b) MKS unit of specific heat is kcal/kg-K

(c) SI unit of specific heat is joule/kg-K

For example, the specific heat of water is :

$C_{water} = 1$cal/g-°C = 1cal/g-K=1kcal/kg-K=4200 Joule/kg-K

When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), then the amount of heat required to raise the temperature of mass m of the substance by an amount $\Delta\theta$ is

$\Delta Q = ms\Delta\theta$

The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made a constant pressure as functions of temperature, because constant pressure is quite easy to produce experimentally.

The temperature dependence of the specific heat of water at 1 atmospheric pressure is shown in figure. Its variation is less than 1% over the interval from 0 to 100ºC. Such a small variation is typical for most solids and liquids, so their specific heats can generally be taken to be constant over fairly large temp. ranges.
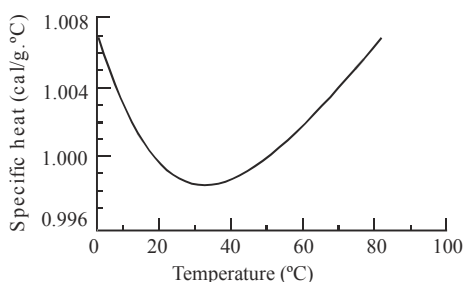
The temperature dependence of the specific heat of water at 1 atm.

Gram Specific Heat "c" : The amount of heat required to raise the temperature of the 1 gram substance by 1°C is known as "gram specific heat".

Heat required to raise the temperature of m gram substance by $\Delta\theta$°C, $Q = m\,c\,\Delta\theta$.

Unit : $\dfrac{cal}{gm°C}$ or $\dfrac{cal}{gm - kelvin}$

Molar Specific Heat "C" : "Molar heat capacity C is the heat required to raise the temperature of 1 mole of a gas by 1°C (or 1 K)." Heat required to raise the temperature of µ gram-mole substance by $\Delta\theta$°C, $Q = \mu M_w\,c\,\Delta\theta$

If $\mu = 1$ and $\Delta\theta = 1$°C then $Q = C$, so $C = M_w\,c$

For a gas the value of C depends on the process through which its temperature is raised.

For example, in an isothermal process $\Delta T = 0$ or $C_{iso} = \infty$.

In an adiabatic process $\Delta Q = 0$. Hence, $C_{adi} = 0$.

Thus, molar heat capacity of a gas varies from 0 to ∞ depending on the process.

MOLAR HEAT CAPACITY

The amount of energy needed to raise the temperature of one mole of a substance by 1ºC (or 1K).

The molar heat capacity is the product of molecular weight and specific heat i.e., Molar heat capacity

C = (Molecular weight M) × (Specific heat c)

$$C = \frac{1}{\mu} \cdot \frac{dQ}{dT}$$

when µ is the number of moles of the substance. If M is the molecular mass of the substance, then $\mu = \dfrac{m}{M}$ where m is the mass of the substance and, $C = \dfrac{M}{m} \cdot \dfrac{dQ}{dT}$

Unit : SI - J/mol-K.

THERMAL CAPACITY

The quantity of heat required to raise the temperature of the whole of that substance through 1°C. The thermal capacity of mass m of the whole of substance of specific heat s is equal to (ms) kilocalorie i.e.,

Thermal capacity = mass × specific heat

Water equivalent of a body : The thermal capacity of a body ms represents also its water equivalent, since the specific heat of water is unity.

The water equivalent of a body is the amount of water that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through 1°C.

Mass of water having the same thermal capacity as the body is called the water equivalent of the body

W = mass of body × specific heat

Latent heat or Hidden heat : When state of a body changes, change of state takes place at constant temperature [m.pt. or b.pt.] and heat released or absorbed is given by,

Q=mL

where L is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is released if liquid converts into solid or vapours converts into liquid.

Latent heat of fusion : The quantity of heat (in kilocalories) required to change its 1 kg mass from solid to liquid state at its melting point

For ice latent heat of fusion = 80 kilocal/kg.

Latent heat of vaporization : The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point.

For water latent heat of vaporisation = 536 kilocal/kg.

Evaporation : Conversion of liquid into gaseous state at all the temperatures is called evaporation.

It is a phenomenon that occurs at the surface of liquids. The rate of evaporation increases with rise in temperature. Heat required to change unit mass of liquid into vapour at a given temperature is called heat of evaporation at that temperature.

Sublimation : Sublimation is the conversion of a solid directly into vapours. Sublimation takes place when boiling point is less than the melting point. A block of ice sublimates into vapours on the surface of moon because of very-very low pressure on its surface. Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.

Heating curve : If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time, the graph is as shown in fig. and is called heating curve. From this curve it is clear that :
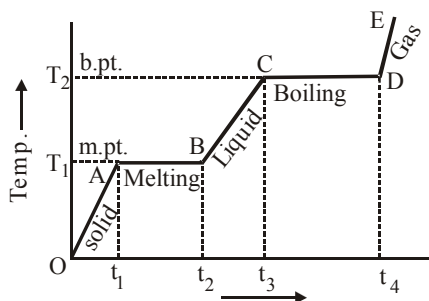
(a) In the region OA temperature of solid is changing with time so, $Q = mc_S \Delta T$

or $P \Delta t = mc_S \Delta T$ [as $Q = P \Delta t$]

But as $\dfrac{\Delta T}{\Delta t}$ is the slope of temp/time curve

$c_S \propto$ (1/slope of line OA)

i.e., specific heat (or thermal capacity) is inversely proportional to the slope of temp/time curve.

(b) In the region AB temperature is constant, so it represents change of state, i.e., melting of solid with melting point T_1. At A melting starts and a B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion

$$Q = mL_F \text{ or } L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)]$$

or $L_F \propto$ length of line AB

i.e., Latent heat of fusion is proportional to the length of line of zero slope.

[In this region specific heat $\propto (1/\tan 0) = \infty$]

(c) In the region BC temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, i.e., $c_L \propto$ (1/slope of line BC)

(d) In the region CD temperature is constant, so it represents change of state, i.e., boiling with boiling point T_2. At C all substance is in liquid state while at D is vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation, i.e.,

$L_V \propto$ Length of line CD

[In this region specific heat $\propto (1/\tan 0) = \infty$]

(e) The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

LAW OF MIXTURES

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature released heat while body at lower temperature absorbs it, so that :

Heat lost = Heat gained,

principle of calorimetry represents the law of conservation of heat energy. Temperature of mixture (T) is always $\geq$ lower temperature (T_L) and $\leq$ higher temperature (T_H),

$$T_L \leq T \leq T_H$$

The temp. of mixture can never be lesser than lower temp. (as a body cannot be cooled below the temp. of cooling body) and greater than higher temp. (as a body cannot be heated above the temp. of heating body). Furthermore usually rise in temp. of one body is not equal to the fall in temp. of the other body though heat gained by one body is equal to the heat lost by the other.

Example 7 :

Steam at 100ºC is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15ºC till the temp. of the calorimeter and its contents rises to 80ºC. What is the mass of steam condensed? Latent heat of steam = 536 cal/g.

Sol. Heat required by (calorimeter + water)

$Q = (m_1 c_1 + m_2 c_2) \Delta\theta = (0.02 + 1.1 \times 1)(80 - 15)$

   $= 1.12 \times 65 = 72.8$

If m is mass of steam condensed, then heat given by steam

$Q = mL + mc \Delta\theta = m \times 536 + m \times 1 \times (100 - 80) = 556 \, m$

$\therefore 556 \, m = 72.8$

$\therefore$ mass of steam condensed $m = \dfrac{72.8}{556} = 0.130$ kg

Example 8 :

5g ice at 0ºC is mixed with 5g of steam at 100ºC. What is the final temperature?

Sol. Heat required by ice to raise its temperature to 100ºC,

$Q_1 = m_1 L_1 + m_1 c_1 \Delta\theta_1$

   $= 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 = 900$ cal

Heat given by steam when condensed,

$Q_2 = m_2 L_2 = 5 \times 536 = 2680$ cal

As $Q_2 > Q_1$. This means that whole steam is not even condensed. Hence temperature of mixture will remain at 100ºC.

Example 9 :

Find the quantity of heat required to convert 40 gm of ice at –20°C into water at 20°C.Given $L_{ice} = 0.336 \times 10^6$ J/kg. Sp heat of ice = 2100 J/kg-K sp heat of water = 4200 J/kg-K

Sol. Mass of ice = $\dfrac{40}{1000} = 0.04$ Kg

Heat required to raise the temperature of ice from –20°C to 0°C = $0.04 \times 2100 \times 20 = 1680$ J

Heat required to convert the ice into water at 0°C = mL

$\qquad = 0.04 \times 0.336 \times 10^6$

$\qquad = 13440$ J

Heat required to heat water from 0°C to 20°C

$\qquad = 0.04 \times 4200 \times 20 = 3360$ J

Total heat required = $1680 + 13440 + 3360 = 18480$ J

TRY IT YOURSELF-2

Q.1 A calorimeter of heat capacity 100 J/k is at room temperature of 30°C. 100 gm of water at 40°C of specific heat 4200 J/kg-K is poured into the calorimeter. What is the temperature of water in calorimeter?

Q.2 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter of water equivalent 0.025 kg containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.

Q.3 A geyser heats water flowing at the rate of 3.0 litres per minute from 27 °C to 77 °C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g ?

Q.4 Which one of following would raise the temperature of 20 g of water at 30°C most when mixed with (Specific heat of water is 1 cal/g-°C)
(A) 20 g of water at 40°C   (B) 40 g of water at 35°C
(C) 10 g of water at 50°C   (D) 4 g of water at 80°C

Q.5 2 kg of ice at – 20 °C is mixed with 5 kg of water at 20 °C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg/ºC and 0.5 kcal/kg/ºC , while the latent heat of fusion of ice is 80 kcal/kg.
(A) 7 kg   (B) 6 kg
(C) 4 kg   (D) 2 kg

Q.6 Water of volume 2 litre in a container is heated with a coil of 1 kW at 27ºC. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27 ºC to 77 ºC ?
(A) 8 min 20s   (B) 6 min 2s
(C) 7 min   (D) 14 min

ANSWERS

(1) 38.07°C   **(2)** 0.103 cal g^{-1}°C^{-1}

(3) 15.675 g/min.   **(4)** (D)   **(5)** (B)   **(6)** (A)

KINETIC THEORY OF GASES

BASIC DEFINITIONS

Avogadro Number N_A : The number of molecules present in 1 gm. mol of a gas is defined as Avogadro's Number.
$N_A = 6.02 \times 10^{23}$ per gm. mole

1 gram mol weight M_w : Mass of one mole gas is known as molecular weight of gas.

Number of moles $\mu = \dfrac{\text{Mass of gas}}{\text{Mol wt. of gas}}$ or $\mu = \dfrac{M}{M_w}$

If $\mu = 1$ mol then $M = M_w$.

$M_w = N_A . m$; $m = \dfrac{M_w}{N_A}$

Gram mol. : The quantity of matter in which the number of molecules is equal to the Avogadro's number, is defined as gram mol.

Free path : The distance covered by the molecules between two successive collisions is called the free path.

Mean free path : The average distance covered by the molecule between two successive collisions is called the

mean free path. i.e. $\lambda = \dfrac{1}{\sqrt{2} . \pi n d^2}$

Mean free path depends on nature of molecule (d) and with increase in n it decreases.

At N.T.P. λ for air molecules is 0.01 μm.

Average velocity : Since gas molecules moves randomly in all possible directions so average velocity of gas molecules is zero.

Mean speed : Mean speed of average speed is the arithmetic mean of the speeds of molecules in a gas at a given temperature.

$$\overline{v} = \dfrac{v_1 + v_2 + ... + v_N}{N}$$

Mean square velocity : Mean square velocity is defined as the average of the square of the velocities of all molecules. Let v_1, v_2, ..., v_N be the velocities of N molecules of a gas, then mean square velocity is,

$$\overline{v^2} = \dfrac{v_1^2 + v_2^2 + ... + v_N^2}{N}$$

Root mean square velocity (R.M.S. velocity) : The square root of the average of the squares of the velocities of gas molecules is called the R.M.S. velocity.

$$v_{rms} = \sqrt{\dfrac{v_1^2 + v_2^2 + ... + v_N^2}{N}} = \sqrt{\overline{v^2}}$$

Example 10 :

Velocities of four gas molecules are 2, 3, –4, –1 m/sec respectively. Find out mean velocity mean speed, mean square velocity, root mean square velocity.

Sol. Mean velocity = $\dfrac{2+3-4-1}{4} = 0$

Mean speed = $\dfrac{2+3+4+1}{4} = 2.5$ m/sec

Mean square velocity

$\overline{v^2} = \dfrac{2^2 + 3^2 + (-4)^2 + (-1)^2}{4}$

$= \dfrac{4+9+16+1}{4} = 7.5$ m^2/sec^2

Root mean square velocity

$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{7.5} = 2.7386$ m/sec.

IDEAL GAS EQUATION

For μ mols $\quad PV = \mu RT$ and
For 1 gm. mol $\quad\quad PV = RT$
$PV = N_A kT \quad\quad (N_A . k = R)$

For one gram of gas $PV = \dfrac{RT}{M_w}$

Values of R, its unit and dimensions :
(a) R = 2 Calorie/mol/°C, (b) R = 8.31 Joule/mol/°C
(c) R = 8.31 × 10^7 erg/mol/°C
Unit S.I. : Joule/mol/K.
Dimensions : $ML^2T^{-2}\theta^{-1}$ mole^{-1}

POSTULATES OF KINETIC THEORY

The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.
Assumptions of the kinetic theory of gases
The kinetic theory of gases is based on the following assumptions.

(1) All the molecules of a gas are identical as regards their shape and mass. The molecules of different gases are different.
(2) The molecules are rigid and perfectly elastic spheres of very small diameters.
(3) Gas molecules occupy very small space. The actual volume occupied by the molecule is very small compared to the total volume of the gas. Therefore volume of the gas is equal to volume of the vessel.
(4) The molecules are in a state of random motion, i.e., they are constantly moving with all possible velocities in all possible directions.
(5) Normally no force acts between the molecules. Hence they move in straight lines with constant speeds.
(6) The molecules can have all possible velocities lying between zero and infinity.
(7) The molecules collide with one another and also with the walls of the container and change there direction and speed due to collision. These collisions are perfectly elastic i.e., there is no loss of kinetic energy in these collisions.
(8) The molecules do not exert any force of attraction or repulsion on each other except during collision. So, the molecules do not posses any potential energy. Their energy is wholly kinetic.
(9) The collisions are instantaneous i.e., the time spent by a molecule in a collision is very small as compared to the time elapsed between two consecutive collisions.
(10) Though the molecules are constantly moving from one place to another, the average number of molecules per unit volume of the gas remains constant.
(11) The molecules inside the vessel keep on moving continuously in all possible directions, the distribution of molecules in the whole vessel remains uniform.
(12) The mass of a molecule is negligibly small and the speed is very large, these is no effect of gravity on the motion of the molecules. If this effect were there, the density of the gas would have been greater at the bottom of the vessel.

EXPRESSION FOR THE PRESSURE EXERTED BY A GAS

A gas exerts pressure on the walls of the containing vessel due to continuous collisions of the molecules against the wall. Consider a gas enclosed in a cube having each side length ℓ. The area of each face of the cube A = ℓ^2.

The volume of the cube is V = ℓ^3. Let the total number of molecules of the gas inside the cube = N
The mass of each molecule = m
Suppose that the three intersecting edges of the cube are along the rectangular co-ordinate axes X, Y, and Z with the origin O at one corner of the cube.
Consider a molecule which has a velocity

$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

If this molecule collides the face EFGH, it will rebound with its x component of velocity ($-v_x$). There will be no effect on v_y or v_z and after collision molecule will move with constant velocity along straight path along –ive X-axis.
∴ Change in momentum of the molecule = final momentum - initial momentum = $-mv_x - (mv_x) = -2mv_x$
Since the total momentum is conserved, the momentum imparted by the molecule to the wall in this impact to the face EFGH = $2mv_x$.
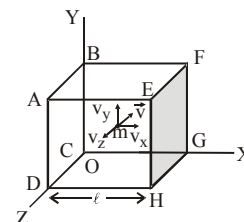∴ Change in momentum of the surface EFGH = $+2mv_x$
Some time later, the molecule strikes the opposite wall ABCD and eventually returns to the wall EFGH.
Between two successive collisions with the same face EFGH, the molecule covers a distance 2ℓ.
Therefore, the time between two successive collisions at

the same face EFGH = $\dfrac{2\ell}{v_x}$.

Hence in one second, the number of collisions of the molecule with face EFGH = $\dfrac{v_x}{2\ell}$.

Change in momentum of surfaces EFGH per second due to this molecule = [Change in momentum in one collision]
$$\times \text{[Number of collisions per second]}$$

$$= 2mv_x \times \dfrac{v_x}{2\ell} = \dfrac{mv_x^2}{\ell}$$

To find the total change in momentum per second at the wall, we add the contributions of all these molecules.
Change in momentum per second

$$= \dfrac{m}{\ell}\,(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots\ldots + v_{xN}^2)$$

By Newton's second law of motion the change in momentum per second is equal to the force.

$$\therefore \quad F_x = \dfrac{m}{\ell}(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots.. + v_{xN}^2)$$

So pressure on this wall

$$P_x = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F_x}{\ell^2} = \dfrac{m}{\ell^3}\,(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots\ldots + v_{xN}^2)$$

Similarly, on the walls normal to Y and Z axes

$$P_y = \dfrac{m}{\ell^3} = (v_{y1}^2 + v_{y2}^2 + \ldots\ldots + v_{yN}^2)$$

and $P_z = \dfrac{m}{\ell^3} = (v_{z1}^2 + v_{z2}^2 + \ldots\ldots + v_{zN}^2)$

Since molecule are in random motion so they exert same pressure on all the phases of cubical vessel. As the choice of the axes has been arbitrary, so $P_x = P_y = P_z = P$

Therefore, $P = \dfrac{P_x + P_y + P_z}{3}$

$$= \dfrac{m}{3\ell^3}\left[\begin{array}{c}(v_{x1}^2 + v_{y1}^2 + v_{z1}^2) + (v_{x2}^2 + v_{y2}^2 + v_{z2}^2) \\ + \ldots.(v_{xN}^2 + v_{yN}^2 + v_{zN}^2)\end{array}\right]$$

$$P = \dfrac{mN}{3\ell^3}\left[\dfrac{v_1^2 + v_2^2 + \ldots.. + v_N^2}{N}\right] = \dfrac{Nm}{3V}v_{rms}^2$$

$PV = \dfrac{1}{3}Nm.v_{rms}^2$ or $P = \dfrac{1}{3}\dfrac{M}{V}\overline{v^2}$ $(v_{rms}^2 = \overline{v^2})$

$$P = \dfrac{1}{3}\rho v_{rms}^2 \qquad (\dfrac{Nm}{V} = \rho \text{ density of gas})$$

Relation between Pressure and Kinetic Energy :
From kinetic theory of gases,

$$P = \dfrac{1}{3}\dfrac{Nm}{V}v_{rms}^2 \quad \text{or} \quad PV = \dfrac{2}{3}N\left(\dfrac{1}{2}mv_{rms}^2\right)$$

$$\therefore \quad N\left(\dfrac{1}{2}mv_{rms}^2\right) = \dfrac{3}{2}PV$$

But, $\dfrac{1}{2}mv_{rms}^2 = $ average K.E. of a gas molecule.

$\therefore$ Total K.E. of a gas $E = N\left(\dfrac{1}{2}mv_{rms}^2\right) = \dfrac{3}{2}PV$

$\therefore$ K.E. per unit volume of the gas $E = \dfrac{3}{2}P$

The pressure exerted by a gas is numerically equal to $\dfrac{2}{3}$rd of the kinetic energy of the molecules present per unit volume of the gas.

Root mean square velocity :

$\because$ $P = \dfrac{1}{3}\dfrac{M}{V}\overline{v^2}$ or $3PV = M\overline{v^2}$

$$\overline{v^2} = \dfrac{3PV}{M} \qquad M = \text{mass of gas}$$

$$\sqrt{\overline{v^2}} = \dfrac{3P}{\rho} \qquad \left(\rho = \dfrac{M}{V}\right)$$

$$v_{rms} = \sqrt{\dfrac{3PV}{M}} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M_w}} = \sqrt{\dfrac{3kT}{m}}$$

$(\because R = N_A k \text{ and } M_w = m . N_A)$

(a) For the molecules of a given gas the root mean square velocity is proportional to the square root of its temperature in Kelvin scale. $v_{rms} \propto \sqrt{T}$; $v_{rms} = \sqrt{\dfrac{T}{M_w}}$

For two different temperatures of a gas $\dfrac{v_{rms_2}}{v_{rms_1}} = \sqrt{\dfrac{T_2}{T_1}}$

(b) For two different gases $\dfrac{v_{rms_2}}{v_{rms_1}} = \sqrt{\dfrac{T_2}{T_1} \times \dfrac{M_{w1}}{M_{w2}}}$

(c) For given temp. lighter the gas, larger the root mean square velocity of the molecules.

$$v_{rms} \propto \sqrt{\dfrac{1}{m}}$$

(d) If v_{rms} is equal or greater than escape velocity v_e than gas will escape from earth or any other planet and so a planet or satellite will have atmosphere only and only if

$$v_{rms} < v_e \; (= \sqrt{2gR})$$

In earth's atmosphere hydrogen molecules acquires this velocity due to elastic collisions with other molecules and escape, so free hydrogen is not present in atmosphere.

Kinetic Interpretation of Temperature :

It is clear that the mean kinetic energy of a molecule is directly proportional to the kelvin (or absolute) temperature of a gas. When the temperature of the gas is increased, the mean kinetic energy of the molecule increases. On the other hand, when heat is withdrawn from a gas, the mean kinetic energy of the molecules decreases. Thus, the temperature of a gas is a measure of the mean translational kinetic energy per molecule of the gas.

Effect of temperature on pressure of the gas :

The pressure of a gas is the rate of change of momentum of molecules striking the walls of container per unit area. When the temp. increases, the root mean square speed of gas molecules also increases. This causes an increase in change in momentum of the molecules as well as the frequency of collisions. Consequently the increase in temp. results in an increase of rate of change of momentum of striking molecules per unit area and hence an increase in pressure.

Example 11 :

Calculate the temperature at which rms velocity of SO_2 molecules is the same as that of O_2 molecules at 27°C. Molecular weights of oxygen and sulphur dioxide are 32 and 64 respectively.

Sol. For oxygen, $T = (27 + 273) \, K = 300 \, K$

Molecular weight of oxygen, $M_w = 32$

rms velocity of O_2 molecules at 27°C,

$$v_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3 \times R \times 300}{32}} \text{ . Since } v_{rms} = \sqrt{\frac{3RT}{M_w}}$$

$\therefore$ for same v_{rms}, $\dfrac{T}{M_w} = $ constant $\therefore \dfrac{T_{SO_2}}{T_{O_2}} = \dfrac{M_{SO_2}}{M_{O_2}}$

or $T_{SO_2} = T_{O_2} \times \dfrac{M_{SO_2}}{M_{O_2}} = 300 \times \dfrac{64}{32} = 600 \, K$

$t_{SO_2} = T_{SO_2} - 273 = (600 - 273)°C = 327°C$

Example 12 :

At what temperature will helium molecules have the same R.M.S. velocity as hydrogen molecules at N.T.P.? Take molecular weight of Hydrogen & Helium as 2 & 4 respectively.

Sol. Given : $v_{rmsHe} = v_{rmsH}$, $M_{wH} = 2$, $M_{wHe} = 4$

But $v_{rmsH} = v_{rmsHe}$

$\therefore \dfrac{3RT_H}{M_H} = \dfrac{3RT_{He}}{M_{He}}$ $\therefore \dfrac{273}{2} = \dfrac{T_{He}}{4}$

$\therefore T_{He} = 2 \times 273 = 546 \, K$ or 273 °C

EXPLANATION OF GAS LAWS FROM KINETIC THEORY

Boyle's Law : According to this law, the product of the pressure and the volume of a given mass of gas at constant temperature is constant. From the kinetic theory of gases, the pressure of a given mass of an ideal gas is given by

$P = \dfrac{1}{3} \dfrac{mN}{V} v_{rms}^2$, where, mN is the mass of the gas which is

constant. If the temp. remains constant, the mean-square-velocity of the molecules, v_{rms}^2, also remains constant. Thus, from the above equation, we have PV = constant. This is Boyle's law.

$V \propto \dfrac{1}{P}$ (T = constant) or $P_i V_i = P_f V_f$

Thus, P-V graph in an isothermal process is a rectangular hyperbola, or PV versus P or V graph is a straight line parallel to P or V axis.
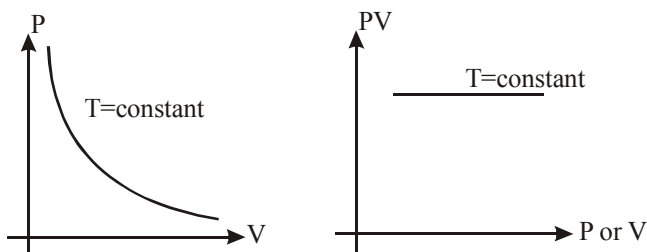
Charle's Law : According to this law, the volume of a given mass of gas at constant pressure is directly proportional to the absolute temperature of the gas. From kinetic theory, we have

$V = \dfrac{2}{3} \dfrac{N}{P} \left(\dfrac{1}{2} mv_{rms}^2 \right)$; $V = \dfrac{2}{3} \dfrac{N}{P} \left(\dfrac{3}{2} kT \right) = \dfrac{N}{P} kT$
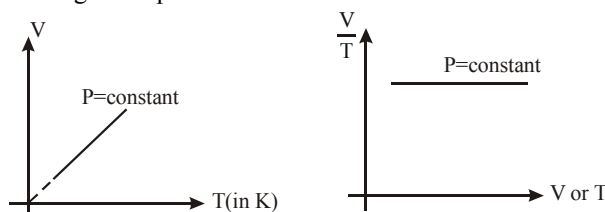
$\left[\because \dfrac{1}{2} mv_{rms}^2 = \dfrac{3}{2} kT \right]$

If the pressure P is constant, then for a given mass of the gas, we have $V \propto T$. This is Charle's Law.

so $\dfrac{V}{T} = $ constant or $\dfrac{V_i}{T_i} = \dfrac{V_f}{T_f}$

Thus, V-T graph in an isobaric process is a straight line passing through origin, or $\dfrac{V}{T}$ versus V or T graph is a straight line parallel to V or T axis.

Gay Lussac's law of Pressure law

For a given mass of a gas the pressure of a gas at constant volume (called isochoric process) is directly proportional to its absolute temperature, i.e. $P \propto T$ (V = constant)

or $\dfrac{P}{T} = $ constant or $\dfrac{P_i}{T_i} = \dfrac{P_f}{V_f}$

P-T graph in an isochoric process is a straight line passing through origin or $\dfrac{P}{T}$ versus P or T graph is a straight line parallel to P or T axis.
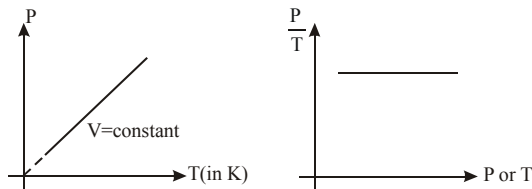
Avogadro's Law : Equal volumes of 'all' gases under the same conditions of temperature and pressure contain equal number of molecules.

At same pressure equal volumes V of different gases contain . . . . $_1$ and N_2 molecules of masses m_1 and m_2.

$$PV = \frac{1}{3}m_1 N_1 v_{rms_1}^2 \quad \text{and} \quad PV = \frac{1}{3}m_2 N_2 v_{rms_2}^2$$

$$\Rightarrow \quad m_1 N_1 v_{rms_1}^2 = m_2 N_2 v_{rms_2}^2$$

Now, if the gases are at the same temperature, their average kinetic energies of translation per molecule are equal. That

is $\dfrac{1}{2}m_1 v_{rms_1}^2 = \dfrac{1}{2}m_2 v_{rms_2}^2$

Applying this result in the last expression, we get $N_1 = N_2$.

Dalton's Law of Partial Pressures : The total pressure exerted by a mixture of non-reacting gases occupying a vessel is equal to the sum of the individual pressures which each gas would exert if it alone occupied the whole vessel. Let as consider a mixture of gases occupying a volume V. Suppose the first gas contains N_1 molecules, each of mass m_1 having mean-square-speed $v_{rms_1}^2$, the second gas contains N_2 molecules each of mass m_2 and mean-square-speed $v_{rms_2}^2$, and so on. Let P_1, P_2,........ respectively the partial pressures of the gases. Each gas fills the whole volume V. According to kinetic theory, we have

$$P_1 V = \frac{1}{3}m_1 N_1 v_{rms_1}^2 , \quad P_2 V = \frac{1}{3}m_2 N_2 v_{rms_2}^2 , \text{ and so on.}$$

Adding, we get

$$(P_1 + P_2 +.......) V = \frac{1}{3}(m_1 N_1 v_{rms_1}^2 + m_2 N_2 v_{rms_2}^2 +......) \quad ...(i)$$

Now, the whole mixture is at the same temperature.

$$\therefore \quad \frac{1}{2}m_1 v_{rms_1}^2 = \frac{1}{2}m_2 v_{rms_2}^2 ...... = \frac{1}{2}mv_{rms}^2 \text{ (say)}.$$

Substituting this result in eqn. (i) we have

$$(P_1 + P_2 +......) V = \frac{1}{3}(N_1 + N_2 +......) mv_{rms}^2$$

Mixture has a total number of molecules $(N_1 + N_2 + ...)$.

Hence the pressure P exerted by the mixture is given by

$$PV = \frac{1}{3}(N_1 + N_2 +......) mv_{rms}^2$$

That $P = P_1 + P_2 + ...$
This is Dalton's law of partial pressures.

DIFFERENT TYPES OF SPEEDS OF GAS MOLECULES

Most probable speed v_{mp} :
It is the speed which maximum number of molecules in a gas have at constant temperature and is given by

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \left(\sqrt{\frac{2}{3}}\right) v_{rms} = 0.816 \; v_{rms}$$

Average speed v_{av} :
It is the arithmetic mean of the speeds of molecules in a gas at a given temperature, i.e,

$$v_{av} = \frac{v_1 + v_2 + v_3 +....+ v_N}{N}$$

and according to kinetic theory of gases,

$$v_{av} = \sqrt{\frac{8RT}{\pi M_w}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{rms} = 0.92 \; v_{rms}$$

$$v_{rms} > v_{av} > v_{mp}$$

Example 13 :
Nitrogen is in equilibrium state at T = 421 K. Find the value of most probable speed, v_{mp}.

Sol. $v_{mp} = \sqrt{\dfrac{2}{3}} \; v_{rms} = \sqrt{\dfrac{2}{3}} \times \sqrt{\dfrac{3RT}{M_w}}$

$$= \sqrt{\frac{2 \times 8.31 \times (421)}{28 \times 10^{-3}}} = \sqrt{\frac{2 \times 8.31 \times 421}{28 \times 10^{-3}}} \cong 500 \; \text{m/s}.$$

MAXWELLIAN DISTRIBUTION OF MOLECULAR SPEEDS

At a given temperature, the root-mean-square speed of the molecules of a gas is constant. However, the speeds of individual molecules vary over a wide range.
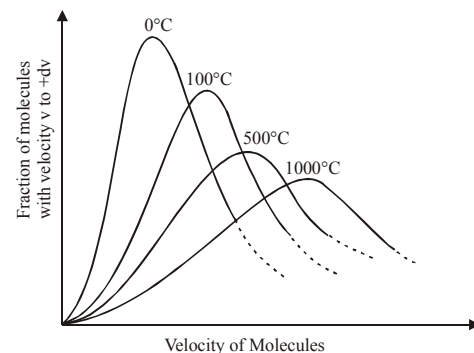
Maxwell derived mathematically a relation for the most probable distribution of speeds among the molecules of a gas (in the steady state).

This relation can be called mathematical statement of a law known as Maxwell's law of distribution of molecular speeds.

(1) At a given temperature the velocities of gas molecules may have any value between zero and infinity.

(2) The number of molecules having speeds zero and infinity is very small. The number of molecules with most probable velocity is the maximum.

(3) The most probable velocity of gas molecules increases with increasing temperature $v_{mp} \propto \sqrt{T}$.

(4) The mean molecular energy goes on increasing with increasing temperature.

(5) At a given temperature the area enclosed between the curve and velocity axis represents the total number of molecules.

(6) At a given temperature the velocities of different molecules are different.

(7) The number of molecules with higher velocity increases with increasing temperature.

This law agrees remarkably well with observations under ordinary conditions. But it fails at high densities.

DEGREE OR FREEDOM (f)

The term degree of freedom refers to the number of possible independent ways in which a system can have energy.

Degree of freedom of gas molecules

A gas molecule can have following types of energies :

(i) Translational K.E.   (ii) Rotational K.E.

(iii) Vibrational energy (potential + kinetic)

Vibrational energy : The forces between different atoms of a gas molecule may be visualized by imagining every atom as being connected to its neighbours by springs. Each atom can vibrate along the line joining the atoms. Energy associated with this is called vibrational energy.


Degree of freedom of monoatomic gas

A monoatomic gas molecule (like He) consists of a single atom. It can have translational motion in any direction in space. Thus, it has 3 translational degrees of freedom.

f = 3   (all translational)

It can also rotate but due to its small moment of inertia, rotational kinetic energy is neglected.

Degree of freedom of a diatomic and linear polyatomic gas

The molecules of a diatomic and linear polyatomic gas (like O_2, CO_2 and H_2) cannot only move bodily but also rotate about any one of the three co-ordinate axes as shown in fig. However, its moment of inertia about the axis joining the two atoms (x-axis) is negligible. Hence, it can have only two rotational degrees of freedom.
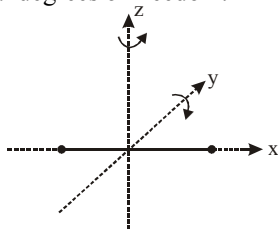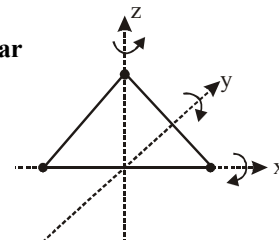

Thus, a diatomic molecule has 5 degrees of freedom : 3 translational and 2 rotational. At sufficiently high temperatures it has vibrational energy as well providing it two more degrees of freedom (one vibrational kinetic energy and another vibrational potential energy). Thus, at high temperatures a diatomic molecule has 7 degrees of freedom, 3 translational, 2 rotational and 2 vibrational.

f = 5 (3 translational + 2 rotational) at room temperatures and f = 7 (3 translational + 2 rotational + 2 vibrational) at high temperatures.

Degree of freedom of nonlinear polyatomic gas :

A nonlinear polyatomic molecule (such as NH_3) can rotate about any of three co-ordinate axes.

Hence, it has 6 degrees of freedom 3 translational and 3 rotational. At room temperatures a polyatomic gas molecule has vibrational energy greater than that of a diatomic gas. But at high enough temp. it is also significant. So it has 8 degrees of freedom 3 rotational, 3 translational and 2 vibrational. Thus,

f = 6 (3 translational + 3 rotational) at room temperatures and f = 8 (3 translational + 3 rotational + 2 vibrational) at high temperatures.

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

For a dynamical system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2}kT$,

Translational kinetic energy of a molecule $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

we have, $\frac{1}{2}mv_{rmsx}^2 + \frac{1}{2}mv_{rmsy}^2 + \frac{1}{2}mv_{rmsz}^2 = \frac{3}{2}kT$

The molecular motion is a random one and as no direction of motion is a preferred one, the average kinetic energy corresponding to each degree of freedom is the same, i.e.

$\frac{1}{2}mv_{rmsx}^2 = \frac{1}{2}mv_{rmsy}^2 = \frac{1}{2}mv_{rmsz}^2$

$\frac{1}{2}mv_{rmsx}^2 = \frac{1}{2}mv_{rmsy}^2 = \frac{1}{2}mv_{rmsz}^2 = \frac{1}{2}kT$

Mean kinetic energy per molecule per degree of freedom

$= \frac{1}{2}kT$

In each degree of freedom energy of one mole of an ideal

gas $= \frac{1}{2}RT$. If f be the number of degrees of freedom, the

internal energy of 1 mole of the gas will be $\frac{f}{2}RT$ or internal

energy of μ moles of the gas will be $\frac{\mu}{2}fRT.$; $U = \frac{\mu}{2}f\ RT$

TWO SPECIFIC HEATS FOR A GAS

When gases are heated, small change in temp. is accompanied with considerable change in both, volume and pressure. Hence it is necessary to define two specific heats of gases.

Specific heat of a gas at constant volume "c_v" :

It is the quantity of heat required to raise the temperature of unit mass of a gas through 1°C or 1K at constant volume.

Molar specific heat constant volume "C_v" :

The amount of heat required to raise the temperature of 1 mole of gas through 1 K (or 1°C) at constant volume. It is denoted by C_v.

If M_w is the molecular weight of the gas in gram, then
$$C_v = M_w \cdot c_v$$

At constant volume no external work is done by gas and total heat given to it is utilized for increase in internal energy so $dU = \mu\, C_v\, dT$.

Specific heat of a gas at constant pressure "c_P" :

It is the quantity of heat required to raise the temperature of 1 gram of a gas through 1°C or 1 K at constant pressure.

Molar specific heat constant pressure :

The amount of heat required to raise the temperature of 1 gram-mole of gas by 1 K (or 1°C) at constant pressure. It is denoted by C_p. $\qquad C_p = M_w \cdot c_{p_1}$

Unit : Both C_p and C_v : $J\,K^{-1}\,mol^{-1}$.

C_p is greater than C_v :

If a gas is heated at constant volume, the gas does no work against external pressure. In this case, the whole of the heat energy supplied to the gas is spent in raising the temp. of the gas.

If a gas is heated at constant pressure, its volume increases. In this case, heat energy is required for the following two purposes :

(i) To increase the volume of the gas against external pressure.

(ii) To increase the temperature of 1 mole of gas through 1 K. Thus, more heat energy is required to raise the temperature of 1 mole of a gas through 1 K when it is heated at constant pressure than when it is heated at constant volume.
$$\therefore\quad C_p > C_v$$
According to Mayor's formula $C_P - C_V = R$

Example 14 :

The difference between the two principal specific heats of oxygen is 0.062 cal/gm°C. Find the value of universal gas constant R. Molecular weight of oxygen is 32 & J= 4.2 J/cal.

Sol. $c_p - c_v = 0.062$ cal/g°C, J = 4.2 J/cal M = 32

$$c_p - c_v = \frac{R}{MJ}$$
or $R = (c_p - c_v)\,M\,J = 0.062 \times 32 \times 4.2\ = 8.332$ J/mol °C

Example 15 :

One mole of a gas occupies 22.4 lit at N.T.P. Calculate the difference between two molar specific heats of the gas. J = 4200 J/kcal.

Sol. V = 22.4 litre = 22.4 × 10^{-3} m^3, J = 4200 J/kcal
by ideal gas equation for one mole of a gas,

$$R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$

$$C_p - C_v = \frac{R}{J} = \frac{1.013 \times 10^5 \times 22.4}{273 \times 4200} = 1.979\ \text{kcal/kmol K}$$

RELATION BETWEEN C_v, C_p AND γ FOR IDEAL GAS :

Internal energy for μ moles, $\quad U = \mu\dfrac{f}{2}RT$

For 1 gm. mole gas $\quad U = \dfrac{f}{2}\,RT \quad \therefore\ \dfrac{dU}{dT} = \dfrac{f}{2}\,R$

$dU = \mu\,C_v\,dT \qquad$ If $\mu = 1$ mole $\qquad$ then $C_v = \dfrac{dU}{dT}$

So $\quad C_v = \dfrac{1}{2}fR$ and hence $C_p = \dfrac{1}{2}Rf + R$

Now $\gamma = \dfrac{C_p}{C_v} = \dfrac{\frac{1}{2}fR + R}{\frac{1}{2}fR}$, so $\gamma = 1 + \dfrac{2}{f}$ and $f = \dfrac{2}{\gamma - 1}$

$$C_V = \frac{R}{\gamma - 1}\ ; \quad C_P = \frac{\gamma R}{\gamma - 1}$$

(1) For a monoatomic gas,
$$C_v = \frac{3R}{2}, C_p = \frac{5R}{2}\ \text{and}\ \gamma = \frac{5}{3} = 1.67$$

(2) For a diatomic gas,
$$C_v = \frac{5R}{2}, C_p = \frac{7R}{2}\ \text{and}\ \gamma = \frac{7}{5} = 1.4$$

(3) For a triatomic or polyatomic gas,
$$C_v = 3R, C_p = 4R\ \text{and}\ \gamma = \frac{4}{3} = 1.33$$

Example 16 :

Find the number of degrees of freedom of molecules in a gas whose molar heat capacity at constant pressure is 29 J/mole.K. R = 8.31 J/mole.K

Sol. $C_p = 29$ J/mole.K, $C_v = C_p - R = 29 - 8.31 = 20.69$

$$\gamma = \frac{C_p}{C_v} = \frac{29}{20.69} = 1.40$$

But, $\gamma = \dfrac{2}{f} + 1$; $\dfrac{2}{f} + 1 = 1.4$ or $f = \dfrac{2}{0.4} = 5$

Example 17 :

What is the total kinetic energy of 2gm of nitrogen at 300 K? Given : molecular weight of nitrogen = 28.

Sol. $M_w = 28$, $T = 300$ K, $R = 8.31 \times 10^7$ erg mol^{-1} K^{-1}

Kinetic energy of 2gm of nitrogen

$$= 2 \times \frac{3}{2} \frac{RT}{M_w} = \frac{3RT}{M_w} = \frac{3 \times 8.31 \times 10^7 \times 300}{28} \text{ erg}$$

$$= 267.1 \times 10^7 \text{ erg}$$

THERMODYNAMIC PARAMETERS FOR A MIXTURE OF GASES

Equivalent molar mass : When μ_1 moles of a gas with molar mass M_1 are mixed with μ_2 moles of a gas with molar mass M_2, the equivalent molar mass of the mixture is given

by $M = \dfrac{\mu_1 M_1 + \mu_2 M_2}{\mu_1 + \mu_2}$

Internal energy of the mixture : The total energy of the mixture $U = U_1 + U_2$

C_V of the mixture :

$dU = dU_1 + dU_2$

or $\mu C_V dT = \mu_1 C_{V1} dT + \mu_2 C_{V2} dT$

or $(\mu_1 + \mu_2) C_V = \mu_1 C_{V1} + \mu_2 C_{V2}$ (as $\mu = \mu_1 + \mu_2$)

$\therefore \quad C_V = \dfrac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2}$

C_P of the mixture : $C_P = C_V + R$

or $C_P = \dfrac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} + R = \dfrac{\mu_1(C_{V_1} + R) + \mu_2(C_{V_2} + R)}{\mu_1 + \mu_2}$

$C_P = \dfrac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 + \mu_2}$

γ of the mixture : $\quad (\mu_1 + \mu_2) C_V = \mu_1 C_{V1} + \mu_2 C_{V2}$

$\dfrac{(\mu_1 + \mu_2) R}{\gamma_{mix} - 1} = \dfrac{\mu_1 R}{\gamma_1 - 1} + \dfrac{\mu_2 R}{\gamma_2 - 1}$

or $\dfrac{(\mu_1 + \mu_2)}{\gamma_{mix} - 1} = \dfrac{\mu_1}{\gamma_1 - 1} + \dfrac{\mu_2}{\gamma_2 - 1}$

Similarly we can solve for more than two gases.

Example 18 :

Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively, find the temperature of the mixture.

Sol. Total energy of molecules of first gas $= \dfrac{3}{2} n_1 k T_1$

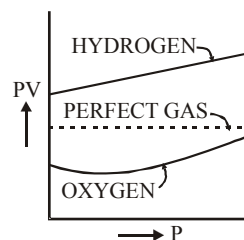Total energy of molecules of second gas $= \dfrac{3}{2} n_2 k T_2$

Total energy of molecules of mixture $= \dfrac{3}{2} k (n_1 T_1 + n_2 T_2)$

$\therefore \dfrac{3}{2}(n_1 + n_2) kT = \dfrac{3}{2} k (n_1 T_1 + n_2 T_2)$

$\Rightarrow T = \dfrac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}$

EQUATION OF STATE FOR REAL GASES OR VANDER WAAL'S GAS EQUATION

The gas equation for 1 mole of an ideal gas is $PV = RT$.

If the temperature T remains constant, then PV = constant. This is Boyle's law. In practice, no real gas strictly follows this law. At low temperatures and high pressures, gases show marked deviation from Boyle's Law. in fig. graphs are drawn between PV and P at 0°C temperature for hydrogen and oxygen. If these gases were strictly obedient to Boyle's law, then a graph between PV and P would have been a straight line parallel to the P-axis.

But it is evident from the graphs that for hydrogen, PV increases on increasing the pressure P (instead of remaining constant). For oxygen, as pressure increases, the value of PV first decreases and then increases. Gases like carbon-dioxide show still more deviation from Boyle's law.

Vander Waal's Equation : In order to explain the behaviour of real gases, Vander Waals made the following two corrections in the ideal gas model :

(i) **Finite Size of Molecules :** It was assumed in the ideal-gas model that gas molecules are infinitesimally small so that the volume occupied by them is negligible in comparison to the total volume of the gas. But the diameter of the molecule of a real gas is of the order of 10^{-8} cm while the mean distance between two molecules of a gas is of the order of 10^{-7} cm.

From this it is evident that of the total volume V of a gas, a part is occupied by the molecules themselves. Hence the volume actually available for the molecules to move about is slightly less than V. The effect cannot be ignored at high pressure when the volume of the gas is decreased. Thus, if the observed volume of a gas is V, then its effective volume will be $(V - b)$.

Where b is a small constant. Value of v is nearly four times the volume of gas-molecules, because around each molecule there is a region within which no other molecules can enter.

Hence, in the ideal-gas equation $PV = RT$, we shall put $(V - b)$ instead of V.

(ii) Inter-molecular Force : It was also assumed in the ideal-gas model that the molecules of a gas do not exert force on one another.
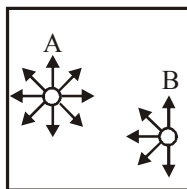
This assumption is not true for real gases. Infact, each molecule of a real gas exerts a force on other molecules which is called the 'inter-molecular force'. At ordinary pressures the gas-molecules are far apart, and so the inter-molecular force between them is nearly zero. But on increasing the pressure the molecules come closer and begin to attract one another. The molecules like A (fig.) which are completely in the interior of the gas, are equally attracted in all directions by the surrounding molecules. Hence the net inter molecular force on them is zero. But molecules like B (fig.) which are near the wall of the vessel, experience an inward pull due to a net force acting on them. On account of this pull, the molecule collides against the wall with a smaller momentum. The effect is no more negligible at high pressures and low temperatures (when the motion of the molecules is comparatively slow).

Then the molecule does not exert so much force on the wall as it would have exerted in the absence of the inter-molecular force. Hence the observed pressure of the gas is less than its ideal pressure. If this reduction is β, then we shall put P + β instead of P in the ideal-gas eq^n. (here P is observed value of pressure). The value of β depends upon the net inward force exerted on the molecule colliding against unit area of the wall per second.

$\beta \propto$ net force × number of molecules colliding per second. Each of these factors is proportional to the density of the gas. $\qquad \because \beta \propto$ density × density $\propto$ density^2.

For a given mass of the gas; density $\propto \dfrac{1}{\text{volume}}$.

$\therefore \ \beta \propto \dfrac{1}{V^2}$ or $\beta = \dfrac{a}{V^2}$, where a is constant for 1 gram-

molecule of the gas. So we must substitute $P + \dfrac{a}{V^2}$ for P in

the ideal-gas equation PV = RT.

On applying the corrections for the finite size of the molecules and the inter-molecular force, the ideal-gas equation PV = RT is converted into the following form :

For 1 gram mole $\left(P + \dfrac{a}{V^2} \right) (V - b) = RT$

For μ gram mole $\left(P + \mu^2 \dfrac{q}{V^2} \right) (V - \mu b) = \mu RT$

This equation is called Vandar Waal's equation. Real gases obey this equation to a sufficient extent at high pressures and low temperatures.

BOYLE TEMPERATURE

The temperature, at which the value of PV remains constant with respect to P, is defined as the Boyle temperature.
The temperature at which the virial coefficient (B) becomes zero, is defined as the Boyle temperature.
Boyle temperature : The specific temperature at which the real gases obey Boyle's law, is defined as Boyle temperature.

$T_B = a/Rb$

Critical temperature (T_c) : The limiting temperature at which a gas can be liquefied only by increasing the pressure and above which it can not the be liquefied, how large the pressure may be increased, is defined as critical temperature.

$T_c = \dfrac{8a}{27Rb} \ ; \quad T_c = \dfrac{8}{27} T_B$

For CO_2, $\quad T_c = 304.3$ K
For O_2, $\qquad T_c = -118°C$
For N_2, $\qquad T_c = -147.1°C$
For steam, $\quad T_c = 365°C$

Critical Pressure (P_c) : The minimum pressure necessary to liquefied a gas at critical temperature is defined as critical

pressure (P_c). $\qquad P_c = \dfrac{a}{27b^2}$

For CO_2, $P_c = 73.87$ bar, For O_2, 49.7 atmosphere.
Critical Volume (V_c) : The volume of 1 mol of gas at critical pressure and critical temperature is defined as critical volume (Vc). $V_c = 3b$
For CO_2, $V_c = 95 \times 10^{-6}$ m^3.

Relation between : T_c, V_c and P_c $\qquad \dfrac{P_c V_c}{T_c} = \dfrac{3}{8} R$

Values of a and b in terms of T_c, V_c and P_c.

$a = \dfrac{27R^2 \ T_c^2}{64 \ P_c} \quad ; \quad b = \dfrac{RT_c}{8P_c}$

TRY IT YOURSELF-3

Q.1 Boyle's law is applicable for an
(A) adiabatic process. (B) isothermal process.
(C) isobaric process. (D) isochoric process.

Q.2 In a diatomic molecule, the rotational energy at a given temperature
(A) obeys Maxwell's distribution.
(B) have the same value for all molecules.
(C) equals the translational KE for each molecule.
(D) is (2/3)rd the translational KE for each molecule.

Q.3 The volume of a given mass of a gas at 27°C, 1 atm is 100cc. What will be its volume at 327°C?

Q.4 Two molecules of a gas have speeds of 9×10^6 ms^{-1} and 1×10^6 ms^{-1}, respectively. What is the root mean square speed of these molecules.

Q.5 In kinetic theory of gases, it is assumed that molecules-
(A) Have same mass but can have different volume
(B) Have same volume but mass can be different
(C) Have both mass and volume different
(D) Have same mass but negligible volume

Q.6 A sample of gas is at 0°C. to what temperature it must be raised in order to double the r.m.s. speed of the molecule-
(A) 270°C   (B) 819°C
(C) 1090°C   (D) 100°C

Q.7 O_2 is 16 times heavier than H_2. If at some temperature the O_2 molecules have average kinetic energy E than at the same temperature. The average kinetic energy of H_2 molecules will be-
(A) E/4   (B) 4E
(C) E   (D) E/16

Q.8 C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then
(A) $C_p – C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(C) C_p/C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas
(D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Q.9 A real gas behaves like an ideal gas if its –
(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high

Q.10 A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds

$\left(\dfrac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})} \right)$ is

(A) 0.32   (B) 0.45
(C) 2.24   (D) 3.16

ANSWERS

(1) (B)   **(2)** (AD)   **(3)** 200cc

(4) $\sqrt{41} \times 10^6$ m/s **(5)** (D)   **(6)** (B)

(7) (C)   **(8)** (BD)   **(9)** (D)

(10) (D)

THERMODYNAMICS

Thermodynamics is a branch of physics which deals with the inter-conversion between heat energy and any other form of energy. It is that branch of Physics which deals with the processes involving heat, work and internal energy. The thermodynamics is the branch of science in which the conversion of heat into mechanical work and vice versa is studied.

System : A specified portion of matter consisting of one or more substances on which the effects of variables such as temperature, volume and pressure are to be studied, is called a system. e.g. A gas enclosed in a cylinder fitted with a piston in a system.

Surroundings : Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surroundings.

Universe : The system and its surroundings are together known as the universe.

Homogeneous System : A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

Heterogeneous System : A system which is not uniform throughout is said to be heterogeneous.
e.g. A system consisting of two or more immiscible liquids.

Isolated System : A system in which there can be no exchange of matter and energy with the surroundings is said to be an isolated system.

Thermodynamic state :

The state of a system can be described completely by (i) composition (ii) temperature (iii) volume and (iv) pressure. These parameters are called the thermodynamic parameters or variables of the system. If a system is homogeneous and has definite mass and composition, then the state of the system can be described by the remaining three variables namely temperature, pressure and volume. These three variables are interrelated by equation $PV = \mu RT$.

The thermodynamic state of the system is its condition as identified by two independent thermodynamic co-ordinates.

TRIPLE POINT OF WATER

The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the co-existence of all the three phases of water ice water and water vapour in equilibrium. The pressure corresponding to triple point of water is 6.03×10^{-3} atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16 K.

Significance of triple point of water : Triple point of water represents a unique condition and it is used to define the absolute temperature. While making Kelvin's absolute scale upper fixed point is 273.16 K and lower fixed point is 0 K.

One kelvin of temperature is fraction $\dfrac{1}{273.16}$ of the temperature of triple point of water.

MECHANICAL EQUIVALENT OF HEAT

According to Joule, work may be converted into heat and vice-versa. The ratio of work done to heat produced is always constant. i.e. $\dfrac{W}{H}$ = constant, J   or $W = J H$

where J is called mechanical equivalent of heat.

Its value is 4.2 Joule/cal or 4.2×10^3 Joule per kilo-cal. It is not a physical quantity but simply a conversion factor. It converts unit of work into that of heat and vice-versa.

In eq^n. W = JH, W must be in joule, irrespective of nature of energy or work and H must be in calorie or kilo-calorie.

Example 19 :
Water falls through a height of 250 m. Assuming that whole of the energy due to fall is converted into heat, calculate the rise in temperature of water.
(g = 9.8 m/s², J = 4.18 × 10³ Joule/Kilocal)

Sol. Here energy converted to heat
$$W = mgh = m \times 9.8 \times 250 \text{ Joule.} \qquad ....(1)$$
If ΔT is the rise in temperature, then heat absorbed by water $= ms\Delta T = m \times 1 \times \Delta T \qquad ....(2)$
From relation W = JH we have
$$m \times 9.8 \times 250 = 4.18 \times 10^3 \times m \times 1 \times \Delta T.$$
This gives $\quad \Delta T = \dfrac{9.8 \times 250}{4.18 \times 10^3} = 0.59°C$

Example 20 :
A heavy box having a mass of 300 kg is pulled along a level road for a distance of 10 metre. How many kilocalorie of heat are produced ? Given : Coefficient of sliding friction=0.2.

Sol. Mass of box, m = 300 kg, distance, S = 10 m; Coefficient of sliding friction, $\mu = 0.2$
If f be the frictional force, then
Work done, $W = f \times S = \mu\, RS = \mu\, mg\, S$, where R is the normal reaction.
$$W = 0.2 \times 300 \times 9.8 \times 10 \text{ J}$$
Heat produced,
$$Q = \frac{W}{J} = \frac{0.2 \times 300 \times 9.8 \times 10}{4.2 \times 10^3} \text{k cal} = 1.4 \text{ kilocalorie}$$

ZEROTH LAW OF THERMODYNAMICS

If objects A and B are separately in thermal equilibrium with a third object C (the thermometer), then objects A and B are in thermal equilibrium with each other.

Zeroth law of thermodynamics introduces thermodynamic quantity called temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are the same. In measuring the temperature of a body, it is important that the thermometer be in the thermal equilibrium with the body whose temperature is to be measured.

Different types of temperature scales

The Kelvin temperature scale is also known as thermodynamic scale. The SI unit of temperature is the kelvin and is defined as (1/273.16) of the temperature of the triple point of water. The triple point of water is that point on a P-T diagram where the three phase of water, the solid, the liquid and the gas, can coexist in equilibrium.

In addition to Kelvin temperature scale, there are other temperature scales also like Celsius, Fahrenheit, Reaumer, Rankine, etc. Temperature on one scale can be converted into other scale by using the following identity :

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point (LFP)}}$$
$$= \text{constant for all scales}$$

Hence, $\dfrac{t°C - 0°}{100° - 0°} = \dfrac{t°F - 32°}{212° - 32°} = \dfrac{tK - 273.15}{373.15 - 273.15}$

INTERNAL ENERGY

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p).
$$dU = dU_k + dU_p$$
If there no intermolecular forces, then $dU_p = 0$.
$$\therefore \quad dU = dU_k = mC_v\, dT$$
where m is mass of system, c_v specific heat at constant volume and dT is change in temperature. If C_v is molar heat capacity C_v ($= Mc_v$, M being molecular weight), then for μ-moles of ideal gas

$$dU = \mu C_v dT = \frac{m}{M} C_v dT$$

It is obvious that internal energy in the absence of inter-molecular forces is simply the function of temperature and state only, it is independent of path followed.
$$dU = U_f - U_i$$
where U_i and U_f are internal energies in initial and final states.

THERMODYNAMIC PROCESSES

Thermodynamic process is said to take place if some change occurs in the state of a thermodynamic system, i.e. the thermodynamic variables of the system pressure, volume, temperature and entropy change with time.

Sign Conventions for the study of thermodynamic processes:
The following sign conventions are adopted in the study of thermodynamical process :
(i) Heat gained by a system is taken as positive while that lost by a system is taken as negative.
(ii) The work done by a system is taken as positive while that done on the system is taken as negative.
(iii) Increase in the internal energy of a system is taken as positive. Decrease in the internal energy of system is taken as negative.

Indicator diagram or P-V diagram :
The equation of state of a gas is $PV = \mu RT$. Out of the three variables P, V and T, if any two are known, the third can be calculated. So, two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.
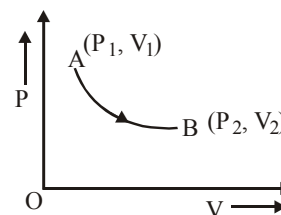
P-V diagram is a graph between the volume V of a system and the pressure P of the system. The volume is plotted against X-axis while the pressure is plotted against Y-axis. Fig. shows an indicator or P-V diagram. The point A represents the initial stage of the system. P_1 and V_1 are the

initial pressure and initial volume respectively of the system. The point B represents the final state of the system. P_2 and V_2 are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system.

The indicator diagram helps us to calculate the amount of work done by the gas or on the gas during expansion or compression.

Work done by thermodynamic system

A gas in a cylinder with a movable piston is a simple example of a thermodynamic system.

A = area of piston
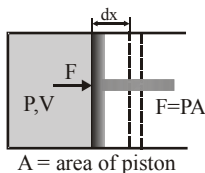
Figure shows a gas confined to a cylinder that has a movable piston at one end. If the gas expands against the piston, it exerts a force on the piston and displace it through a distance and does work on the piston. If the piston compresses the gas as it is moved inward, work is also done–in this case on the gas. The work associated with such volume changes can be determined as follows.

Let pressure of gas on the piston = P.

Then the force on the piston due to gas is F = PA, where A is the area of the piston. When the piston is pushed outward an infinitesimal distance dx, the work done by the gas is

Shaded area = work done
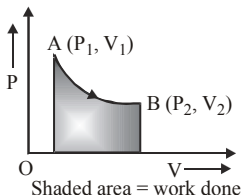
$$dW = F \cdot dx = PA\,dx$$

since the change in volume of the gas is dV = Adx,

$$\therefore \quad dW = PdV$$

For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the net work
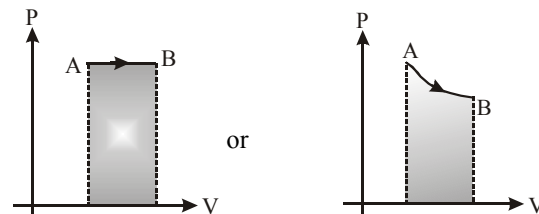
$$W = \int dW = \int_{V_i}^{V_f} PdV$$

Work done by a gas is also equal to the area under P-V graph. Following different cases are possible.

Case 1. When volume is constant
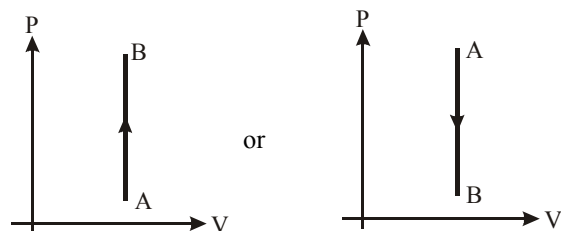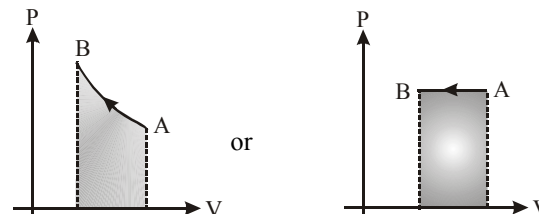

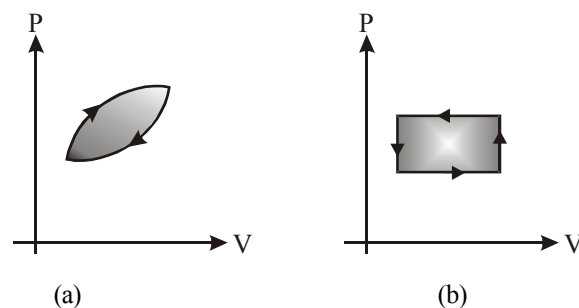V = constant
$$W_{AB} = 0$$

Case 2. When volume is increasing

V is increasing
$$W_{AB} > 0 \qquad W_{AB} = \text{Shaded area}$$
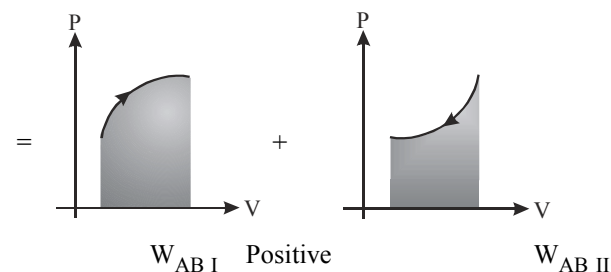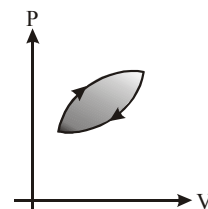
Case 3. When volume is decreasing

V is decreasing $\qquad W_{AB} < 0 \qquad W_{AB} = -$ Shaded area

Case 4. Cyclic process

(a)   (b)

$$W_{\text{clockwise cycle}} = + \text{ Shaded area}$$
$$W_{\text{anticlockwise cycle}} = - \text{ Shaded area}$$

Work Done in Clockwise Cycle

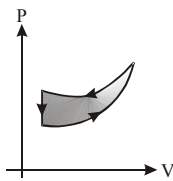
$W_{AB\,I}$ Positive $\qquad\qquad\qquad W_{AB\,II}$

Negative

$W_{\text{cyclic}} = W_{AB\,I}$ Positive $+ W_{AB\,II}$ Negative

$= $ area of close path

Example 21 :

For one complete cycle as shown .... ΔE_{int} for the gas and (b) the net heat transfer Q is positive, negative, or zero.

Sol. (a) Zero (b) Negative W_{net} = -ive

For the case of closed cycle, $\Delta E_{int.} = 0$
and Q = W = area enclosed by the close path.

FIRST LAW OF THERMODYNAMICS

A gas in a cylinder with a movable piston is a simple example of a thermodynamic system.

Let this system changes from an initial equilibrium state **i** to a final equilibrium state **f** in a definite way, the heat absorbed by the system being δQ_1 and the work done by the system being δW_1 and system change its state through path 1, again for path 2 heat absorbed by the system is δQ_2 and work done by the system is δW_2. Then the $\delta Q_1 - \delta W_1 = \delta Q_2 - \delta W_2$. Difference of heat absorbed by the system δQ and work done by the system δW i.e. $\delta Q - \delta W$ does not depends on path in between two definite states of the system. Both δQ and δW depend on the thermodynamic path taken between two equilibrium states, their difference $\delta Q - \delta W$ does not. So, there is a function (internal energy) of the thermodynamic coordinates (P, V and T) whose final value (U_f) minus its initial value (U_i) equals the change $\delta Q - \delta W$ in the process.
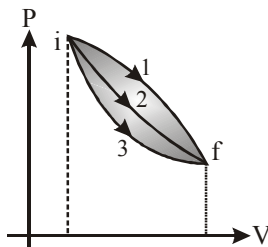
Hence, $dU = \delta Q - \delta W$

This equu^n. is known as the first law of thermodynamics.

Change in internal energy $dU = U_f - U_i$ does not depends on path it depends only on initial and final positions of the system. So, it is denoted by dU, but heat supplied to the system and work done by the system are path dependent so they are denoted by δQ and δW respectively.

First Law of Thermodynamics :

If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system. $\delta Q = dU + \delta W$

The first law of thermodynamics is essentially a restatement of the law of conservation of energy, i.e., energy can neither be created nor be destroyed but may be converted from one form to another.

Note :

(i) In applying the first law of thermodynamics, all the three quantities, i.e., δQ, dU and δW must be expressed in the same units, i.e., either in units of work or in units of heat.

(ii) This law is applicable to every process in nature.

(iii) This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.

(iv) dU may be any type of internal energy-translational kinetic energy, rotational kinetic energy, binding energy etc. It is a characteristic of the state of a system.

Limitations of First Law of Thermodynamics :

(a) It does not explain the direction of heat flow.

(b) It does not explain how much amount of heat given will be converted into work.

Example 22 :

When a system is taken from state a to state b, in fig. along the path $a \rightarrow c \rightarrow b$, 60 J of heat flow into the system, and 30 J of work are done : (i) How much heat flows into the system along the path $a \rightarrow d \rightarrow b$ if the work is 10 J.
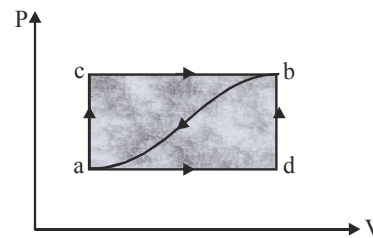
(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process $a \rightarrow d$ and $d \rightarrow b$.

Sol. For the path a, c, b,

$dU = dQ - dW = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a, d, b, $dQ = dU + dW = 30 + 10 = 40$ J

(ii) Along the curved path b, a,

$dQ = (U_a - U_b) + W = (-30) + (-20) = -50$ J,

heat flows out the system

(iii) $Q_{ad} = 32$ J; $Q_{db} = 8$ J

Application of first law of thermodynamics

(a) Melting Process : When a substance melts, the change in volume (dV) is very small and can, therefore, be neglected. The temperature of a substance remains unchanged during melting process. Let us consider the melting of a mass m of the solid. Let L be the latent heat of fusion i.e., the heat required to change a unit mass of a solid to liquid phase at constant temperature. Heat absorbed during melting process, $\delta Q = mL$

According to the first law of thermodynamics,

$\delta Q = dU + \delta W$

or $mL = dU$ $[\because \delta W = P\delta V = P \times 0 = 0]$

So, the internal energy increases by mL during the melting process.

(b) Boiling Process : When a liquid is heated, it changes into vapour at constant temperature (called boiling point) and pressure. When water is heated at normal atmospheric pressure, it boils at 100°C. The temperature remains unchanged during the boiling process.

Let us consider the vaporisation of liquid of mass m. Let V_ℓ and V_v be the volume of the liquid and vapours respectively. Then, the work done in expanding at constant temperature and pressure P, $\delta W = P\delta V = P(V_v - V_\ell)$

Let L be the latent heat of vaporisation, i.e., the heat required to change a unit mass of a liquid to vapour phase at constant temperature and pressure.

∴ Heat absorbed during boiling process, $\delta Q = mL$

Let U_ℓ and U_v be the internal energies of the liquid and vapours respectively.

Change in internal energy, $dU = U_v - U_\ell$

According to the first law of thermodynamics,

$\delta Q = dU + \delta W$

∴ $mL = (U_v - U_\ell) + P(V_v - V_\ell)$

or $U_v - U_\ell = mL - P(V_v - V_\ell)$

Knowing all the quantities on right hand side, we can calculate the gain in the internal energy.

DIFFERENT THERMODYNAMIC PROCESSES
ISOCHORIC OR ISOMETRIC PROCESS

Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.

Equation of state : $P = \text{const.} \times T$
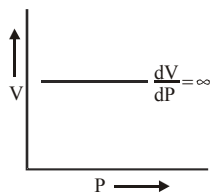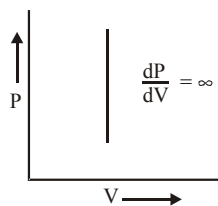
(since V is constant, P and T are variable)

Work Done : This is the process in which volume remains constant i.e. $\Delta V = 0$

Work done, $\Delta W = \int_{V_i}^{V_f} PdV = 0$

Form of First Law : $\delta Q = dU$

Whole of heat supplied is utilized for change in internal energy of the system. $\delta Q = dU = \mu C_v dT$

Slope of the P-V curve : $\dfrac{dP}{dV} = \infty$

Specific heat at constant volume C_v : The specific heat at constant volume is the quantity of heat required to raise the temperature of 1 gram mole gas through 1 °C at constant volume.

Ex. (1) A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.

(2) When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.

ISOBARIC PROCESS

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

Equation of state : $V = \text{const.} \times T$

Work Done : This is the process in which pressure remains constant, i.e. $\Delta P = 0$
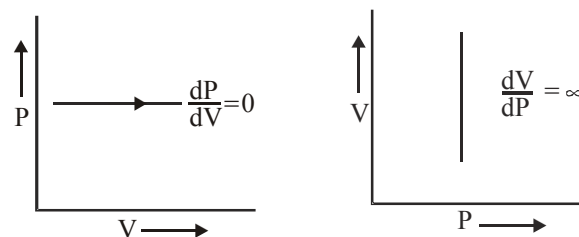
∴ Work done $W = \int_{V_i}^{V_f} PdV = P(V_f - V_i)$

Form of First Law : $\delta Q = dU + P(V_f - V_i)$

or $\mu C_p dT = \mu C_v dT + P(V_f - V_i)$

Heat supplied to the system is utilized for :

(a) Increasing internal energy and

(b) Work done against the surrounding atmosphere.

Slope of the PV curve : $\left(\dfrac{dP}{dV}\right)_{isobaric} = 0$

Specific heat at constant pressure C_p : The specific heat at constant pressure is the quantity of heat required to raise the temperature of 1 gram mole gas through 1 °C at constant pressure.

Ex. Heating of water at atmospheric pressure.

ISOTHERMAL PROCESS

It is a thermodynamic process in which the pressure and volume of system change but temperature remains constant.

In an isothermal process, the exchange of heat between the system and the surroundings is allowed. In other words, as isothermal process is carried out by either supplying heat to the substance or by extracting heat from it.

A process has to be extremely slow to be isothermal.

Equation of state : $P.V. = \text{constant} \ \mu RT$ (since T is constant)

Work Done : Consider μ moles of an ideal gas, enclosed in a cylinder, at absolute temperature T, fitted with a frictionless piston.

Suppose that gas undergoes an isothermal expansion from the initial state (P_1, V_1) to the final state (P_2, V_2).

∴ Work done $W = \int_{V_1}^{V_2} \dfrac{\mu RT}{V} dV$

or $W = 2.303 \mu RT \log_{10}\left[\dfrac{V_2}{V_1}\right]$

or $W = 2.303 \mu RT \log_{10}\left[\dfrac{P_1}{P_2}\right]$ [∵ $P_1 V_1 = P_2 V_2$.]

Form of First Law : Since there is no change in temperature and internal energy of the system depends on temperature only so $dU = 0$. So, $\delta Q = 2.303 \ \mu RT \log_{10}\left[\dfrac{V_2}{V_1}\right]$

Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

Slope of the Isothermal curve : For an isothermal process, PV = constant

Isothermal expansion of an ideal gas
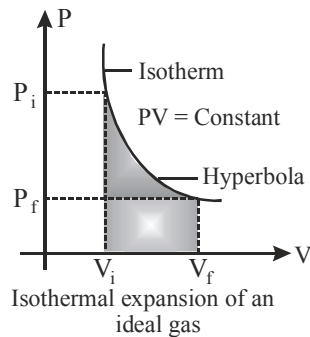
Differentiating, $PdV + VdP = 0$

or $VdP = -PdV$ or $\dfrac{dP}{dV} = -\dfrac{P}{V}$

$\therefore$ Slope of isothermal, $\left(\dfrac{dP}{dV}\right)_{iso} = -\dfrac{P}{V}$

Example :

(1) The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.

(2) When a substance boils, its temperature remains constant. So, boiling is an isothermal process.

(3) An ideal gas enclosed in a conducting cylinder fitted with a conducting piston.
Let the gas be allowed to expand very-very slowly. This shall cause a very slow cooling of the gas. But heat will be conducted into the cylinder from the surrounding. Thus, the temperature of the gas remains constant.
If the gas is compressed very-very slowly, heat will be produced. But this heat will be conducted to the surroundings. So, the temperature of the gas shall remain constant.

(4) If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

Note :

For a given system : The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process. In other words, Boyle's law is obeyed in an isothermal process.

At const. P
Hence $T_2 > T_1$
$V \propto T$
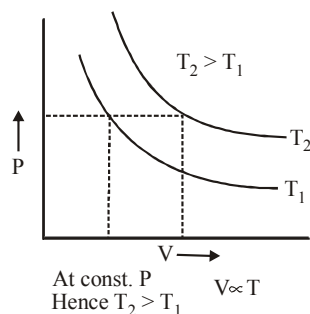
A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal of the gas. In fig., two isotherms for a given gas at two different temperatures T_1 and T_2 are shown.

Example 23 :

An ideal gas expands isothermally along AB and does 700 J of work.
(a) How much heat does the gas exchange along AB.
(b) The gas then expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhausts 100 J of heat to its surroundings. How much work is done on the gas along this path.
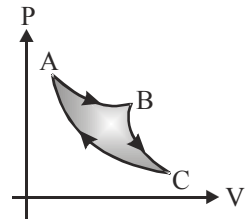
Sol. (a) AB is an isothermal process. Hence, $\Delta U_{AB} = 0$
and $\quad Q_{AB} = W_{AB} = 700$ J

(b) BC is an adiabatic process. Hence, $Q_{BC} = 0$ and
$W_{BC} = 400$ J $\therefore$ $\Delta U_{BC} = -W_{BC} = -400$ J
ABCA is a cyclic process and internal energy is a state function $(\Delta U)_{whole\ cycle} = 0$

$\therefore$ $\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$
from first law of thermodynamics,
$Q_{AB} + Q_{BC} + Q_{CA} = W_{AB} + W_{BC} + W_{CA}$
or $700 + 0 - 100 = 700 + 400 + \Delta W_{CA}$

$\therefore$ $\Delta W_{CA} = -500$ J
Negative sign implies that work is done on the gas.

ADIABATIC PROCESS

It is that thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and the surroundings. A process has to be sudden and quick to be adiabatic.

Equation of state : $P.V = \mu RT$

Equation for adiabatic process $PV^\gamma = $ constant

Work Done : Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write $P_1 V_1^\gamma = P_2 V_2^\gamma = K$ (here K is const.)

Work done $W = \int_{V_1}^{V_2} PdV$ or

$$W = \dfrac{1}{(\gamma - 1)}[P_1 V_1 - P_2 V_2]$$

$$W = \dfrac{\mu R}{(\gamma - 1)}(T_1 - T_2) \qquad (\because PV = \mu RT)$$

Form of First Law : $dU = -\delta W$

The work done by an ideal gas during adiabatic expansion (or compression) is proportional to the fall (or rise) in the temperature of the gas.
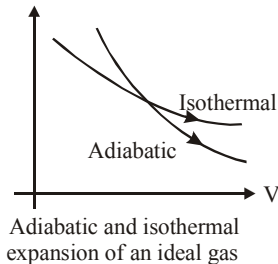
If the gas expands adiabatically, work is done by the gas. So, W_{adia} is positive.

$\therefore$ $T_1 > T_2$. So, the gas cools during adiabatic expansion.
If the gas is compressed adiabatically, work is done on the gas. So, W_{adia} is negative.

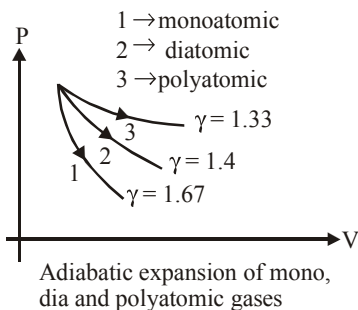$T_1 < T_2$. So, the gas heats up during adiabatic compression.

Slope of the adiabatic curve

Adiabatic and isothermal
expansion of an ideal gas

For an adiabatic process, $PV^\gamma = $ constant

Differentiating, $P\gamma V^{\gamma-1} dV + V^\gamma dP = 0$

or $V^\gamma dP = -\gamma PV^{\gamma-1} dV$

or $\dfrac{dP}{dV} = -\dfrac{\gamma PV^{\gamma-1}}{V^\gamma} = -\dfrac{\gamma P}{V}$

Slope of adiabatic curve, $\left(\dfrac{dP}{dV}\right)_{adia} = -\dfrac{\gamma P}{V}$

$1 \rightarrow$ monoatomic
$2 \rightarrow$ diatomic
$3 \rightarrow$ polyatomic
$\gamma = 1.33$
$\gamma = 1.4$
$\gamma = 1.67$

Adiabatic expansion of mono,
dia and polyatomic gases

$\left(\dfrac{dP}{dV}\right)_{adia} = \gamma\left(-\dfrac{P}{V}\right) = \gamma\left(\dfrac{dP}{dV}\right)_{iso}$

$\dfrac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$

But γ is always greater than one.

So, slope of adiabatic is greater than the slope of isotherm. Thus, we conclude that an adiabatic is steeper than an isotherm.

Example :

(1) A gas enclosed in a thermally insulated cylinder fitted with a non-conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape the cylinder. Consequently, there will be an increase in the temperature of the gas.

(2) If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.

(3) Bursting of a cycle tube.

(4) Propagation of sound waves in a gas.

(5) Expansion of hot gases in internal combustion engine. In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture.

(6) Expansion of steam in the cylinder of a steam engine.

Adiabatic relation between P and V for ideal gas

Let one mole of a gas contained in a perfectly non-conducting cylinder fitted with a non-conducting piston. If P, V and T be the pressure, volume and temperature respectively of the gas. Now the gas be compressed adiabatically by moving piston for a infinitely small distance dx, so for this small displacement, we can consider pressure P of the gas unchanged.
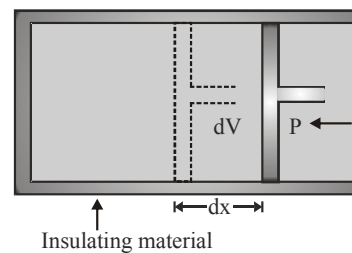
Decreased volume of the gas = dV

Work done in this small displacement dW = P dV

The heat generated due to compression causes a rise of temperature dT. Change in the internal energy of the gas $dU = C_V dT$ First law of thermodynamics, $dU + dW = 0$.

(In an adiabatic process, $dQ = 0$)

$\therefore C_v dT + PdV = 0$ ....(1)

Insulating material

For one gram mole ideal gas, $PV = RT$

Differentiating, $PdV + VdP = RdT$

or $dT = \dfrac{PdV + VdP}{R}$ ....(2)

$C_v\left[\dfrac{PdV + VdP}{R}\right] + PdV = 0$ [From eq^n. (1) and eq^n. (2)]

or $C_v PdV + C_v VdP + RPdV = 0$

or $(C_v + R) PdV + C_v VdP = 0$

But $C_p - C_v = R$ or $C_p = C_v + R$ $\therefore C_p PdV + C_v VdP = 0$

Dividing both sides by $C_v P V$, we get

$\dfrac{C_p PdV}{C_v PV} + \dfrac{C_v VdP}{C_v PV} = 0$ or $\gamma\dfrac{dV}{V} + \dfrac{dP}{P} = 0$ $\left[\because \dfrac{C_p}{C_v} = \gamma\right]$

Integrating, $\int\gamma\dfrac{dV}{V} + \int\dfrac{dP}{P} = 0$

or $\gamma\int\dfrac{1}{V}dV + \int\dfrac{1}{P}dP = 0$

or $\gamma \log V = \log P = $ constant $\left[\because \int\dfrac{1}{x}dx = \log_e x\right]$

or $\log V^\gamma + \log P = $ constant

or $\log PV^\gamma = $ constant

or $PV^\gamma = $ antilog (constant) = another constant K

$\therefore PV^\gamma = K$

If P_1, V_1 be the initial and P_2, V_2 the final pressures and volumes respectively of the gas for an adiabatic change.

Then $P_1 V_1{}^\gamma = P_2 V_2{}^\gamma$

(ii) Adiabatic relation between volume and temperature

$\therefore TV^{\gamma-1} = $ constant

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If V_1, T_1 be the initial and V_2, T_2 be the final volumes and temperatures respectively of the gas for an adiabatic change, then $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

(iii) Adiabatic relation between pressure and temperature

$\therefore\ T^{\gamma} P^{1-\gamma} = $ constant

If P_1, T_1 be the original and P_2, T_2 the final pressures and temperatures respectively of gas for an adiabatic change, then $T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma}$
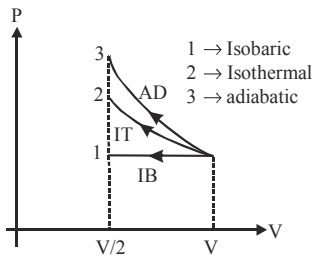
Note : When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.
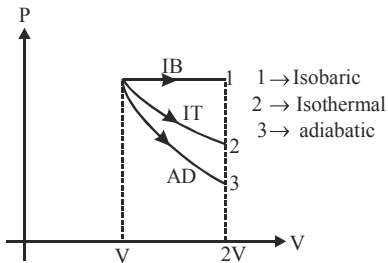
$$W_{IB} > W_{IT} > W_{AD}$$
$$P_f \quad P_{IB} > P_{IT} > P_{AD}$$
$$\Delta P \quad \Delta P_{IB} > \Delta P_{IT} > \Delta P_{AD}$$

$$W_{AD} > W_{IT} > W_{IB}$$
$$P_f \quad P_{AD} > P_{IT} > P_{IB}$$
$$\Delta P \quad \Delta P_{AD} > \Delta P_{IT} > \Delta P_{IB}$$

Example 24 :

Two samples of a gas initially at same temperature and pressure are compressed from a volume V to V/2. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Sol. Let initial volume, $V_1 = V$ and final volume, $V_2 = V/2$
Initial pressure, $P_1 = P$; final pressure, $P_2 = ?$
For isothermal compression
    Adiabatic compression

$$P_2 V_2 = P_1 V_1 \text{ or } P_2 = \frac{P_1 V_1}{V_2} = \frac{PV}{V/2} = 2P$$

$$P_2' V_2^{\gamma} = P_1 V_1^{\gamma} \text{ or } P_2' = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} \quad P\left(\frac{V}{V/2}\right)^{\gamma} = 2^{\gamma} P$$

or $\quad P_2{}' = 2^{\gamma} P$
Since $\gamma > 1\ \ \therefore 2^{\gamma} > 2\ \ \therefore P_2' > P_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Example 25:

A motor tyre pumped to a pressure of 3 atm. suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27ºC. Given $\gamma = 1.4$.

Sol. Initial pressure, $P_1 = 3$ atm ; Final pressure, $P_2 = 1$ atm
Initial temperature, $T_1 = (27 + 273)\,K = 300\,K$
Final temperature, $= T_2$
Fall in temperature, $T_1 - T_2 = ?$
We know that $T_2^{\gamma} P_2^{1-\gamma} = T_1^{\gamma} P_1^{1-\gamma}$

or $\left(\dfrac{T_2}{T_1}\right)^{\gamma} = \left(\dfrac{P_1}{P_2}\right)^{1-\gamma}$ or $\left(\dfrac{T_2}{300}\right)^{1.4} = \left(\dfrac{3}{1}\right)^{1-1.4}$

or $\left(\dfrac{T_2}{300}\right)^{1.4} = \left(\dfrac{1}{3}\right)^{0.4}$ or $T_2 = 219.2\,K$

$$T_1 - T_2 = (300 - 219.2)\,K = 80.8\,K$$

Example 26 :

Calculate the work done when 1 mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litre respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is 3R/2. $[(3)^{5/3} = 6.19]$

Sol. For an adiabatic change $PV^{\gamma} = $ constant $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$
As molar specific heat of gas at constant volume

$$C_v = \frac{3}{2}R$$

$$C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R\ ;$$

$$\gamma = \frac{C_P}{C_V} = \frac{(5/2)\,R}{(3/2)\,R} = \frac{5}{3}$$

$\therefore$ From eq^n. (1)

$$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} P_1 = \left(\frac{6}{2}\right)^{5/3} \times 10^5\,N/m^2$$

$$= (3)^{5/3} \times 10^5 = 6.19 \times 10^5\,N/m^2$$

Work done

$$= \frac{1}{1-(5/3)}[6.19 \times 10^5 \times 2 \times 10^{-3} - 10^{-5} \times 6 \times 10^{-3}]$$

$$= -\left[\frac{2 \times 10^2 \times 3}{2}(6.19 - 3)\right] = -3 \times 10^2 \times 3.19 = -957\ \text{joules}$$

[–ve sign shows external work done on the gas]

Example 27 :

Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then, it undergoes and adiabatic change until the temperature returns to the initial value.

(i) Sketch the process on a P–V diagram.

(ii) What are the final volume and pressure of the gas?

(iii) What is the work done by the gas?

Sol. For a perfect gas, $PV = \mu RT$

$$P_1 = \frac{\mu RT}{V} = \frac{2 \times 8.31 \times (273 + 27)}{20 \times 10^{-3}}$$

(i) Fig. shows the sketch of the process
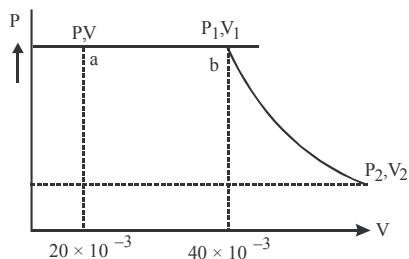
or $P_1 = 2.5 \times 10^5 \text{ N/m}^2$

(ii) At constant pressure, $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

$$\therefore \quad T_2 = \left(\frac{V_2}{V_1}\right) T_1 = 2 \times 300 = 600 \text{ K}$$

(iii) The gas now undergoes an adiabatic change.

$T_1 = 600$ K, $T_2 = 300$ K, $V_1 = 40$ lit., $V_2 = ?$

$\gamma - 1 = 5/3 - 1 = 2/3$

$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$600 (40)^{2/3} = 300 (v_2)^{2/3}$

$(2)^{3/2} \times 40 = V_2 \quad$ or $V_2 = 112.4$ lit.

But, $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$P_2 = \left(\frac{40}{112.4}\right)^{5/3} \times 2.5 \times 10^5 = 0.45 \times 10^5 \text{ N/m}^2$$

FREE EXPANSION

Consider a gas which initially occupies one compartment of a two-chambered container as illustrated in fig. A membrance separates the two chambers, and the one on the right is evacuated. The entire assembly is insulated from the exterior. Suppose now that the membrance separating the two chambers spontaneously breaks and the gas expands freely to fill the entire container. This process is called a free expansion.
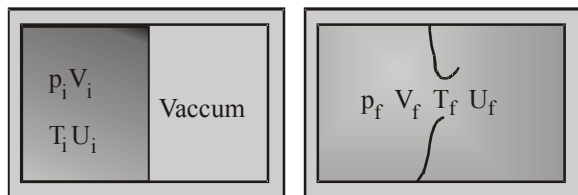
The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free-expansion process.

The process is adiabatic because of the insulation, so Q=0. No part of the surroundings moves (we consider the rupturing membrane to be an inert part of the system) so the system does no work on its surroundings.

For ideal gas :

$(\delta W)_{ext.}$ = work done against external atmosphere
$= P\,dV = 0$ (because $P = 0$)

$(\delta W)_{int.}$ = work done against internal molecular forces = 0
$0 = dU + 0 \qquad (\delta Q = dU + \delta W)$

Therefore, the internal energy does not change. $dU = 0$

$U \to$ const. $T \to$ const.

The initial and final states of this gas have the same internal energy. Which implies that the internal energy of an ideal gas does not depend on the volume at all. The free-expansion process has led us to the following conclusion : The internal energy U(T) of an ideal gas depends only on the temperature.

For real gas :

$\delta Q = 0$, $(\delta W)_{ext.} = 0$ $(\because P = 0)$
$\qquad (\delta W)_{int.} \neq 0$

(intermolecular forces are present in real gases)

$\quad 0 = dU + (\delta W)_{int.}$

$\quad dU = -(\delta W)_{int.} \quad U\downarrow \; T\downarrow$

Free expansion real gases, measurements show that the temperature changes slightly in a free expansion. Which implies that the internal energy of a real gas depends on the volume also.

CARNOT CYCLE

Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession : isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression.
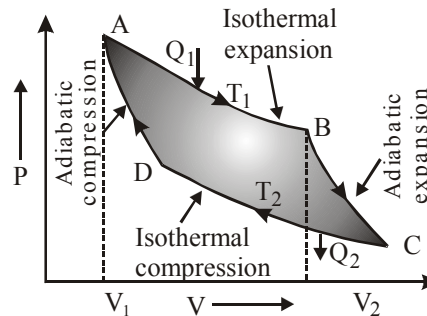
There are four thermodynamical processes isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression.

Main parts of Carnot's engine are :

Source of Heat : A hot body of high temperature T_1 from which the heat can be drawn. It is a hot body of very large heat capacity kept at a constant high temperature T_1 K. Its upper surface is perfectly conducting so that working substance can take heat from heat.

Mechanical Arrangements and working substance :
A cylinder whose walls are perfectly non-conducting but base perfectly conducting fitted with non-conducting piston which can move without any frictional losses. Ideal gas enclosed in these system as a working substance.

Heat sink : A cold body at low temperature T_2 K to which the heat can be rejected. It is a body of large heat capacity. Its upper surface is highly conducting so that working substance can reject heat to it.

Stand : It is made by perfectly insulating material so that when cylinder placed on it working substance can expended or compressed adiabatically .

Working : To get maximum work from this type of ideal engine there is a set of reversible processes through then working substance is taken back to Initial condition. Complete Carnot cycle is divided in four steps.

Processes of Carnot's cycle can be denoted by an indicator diagram a shown here in these diagram variation of pressure (P) and volume (V) of working substance (ideal gas), are plotted.

(a) **First process : Isothermal expansion A → B -** Initially the cylinder is taken to be in thermal equilibrium with the high temperature T_1 represented by point A (P_1, V_1, T_1)
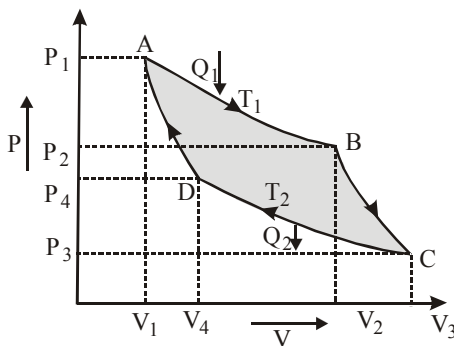
This is initial state of working substance (ideal gas)
Then the piston is allowed to move outward slowly. With the movement of the piston, the temperature of the gas tends to fall. The process is very slow so that it is isothermal. Heat from reservoir flows into the gas and temperature of the gas remains T_1. In this expansion gas receive heat Q_1 from source and gets state B (P_2, V_2, T_1)
The total heat input Q_1 to the gas occurs over the path from A to B and comes from a large reservoir of heat (source) at temperature T_1 and utilized for doing work W_1.
Over the path from A to B the heat input to the gas equals the work done against the external pressure.

$$Q_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{\mu RT_1}{V} \, dV = \mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right)$$

$$W_1 = \mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right) \qquad (W_1 = Q_1)$$

(b) **Second process : adiabatic expansion B → C**
Now the contact of cylinder with source is removed and

cylinder is put in contact with a non-conducting stand. The piston is allowed to move outward the gas now expands adiabatically because no heat can enter in or leave out. The temperature falls to T_2 and gas describes the adiabatic BC to point C (P_3, V_3, T_2) during which more work is done (W_2) at the expense of the internal energy.
Work done over this adiabatic path BC is

$$W_2 = \frac{\gamma R}{\gamma - 1}(T_1 - T_2)$$

(c) **Third process : Isothermal compression C → D**
The gas cylinder is placed in contact with sink at temperature T_2. The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed to point D. (P_4, V_4, T_2)
The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over the path from C to D.
The amount of work done on the gas W_3 is equal to the amount of heat rejected to the sink
$W_3 = Q_2$ so that

$$W_3 = \mu RT_2 \ \text{In} \left(\frac{V_4}{V_3} \right) \ \text{or} \ Q_2 = \mu RT_2 \ \text{In} \left(\frac{V_4}{V_3} \right)$$

(d) **Fourth process : Adiabatic compression D → A**
The cylinder is removed from the sink and is put in contact with insulating stand and piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Then work done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).
No heat exchanges occur over the adiabatic path.

Work done on the system is $W_4 = \frac{\gamma R}{\gamma - 1}(T_1 - T_2)$

This cycle of operations is called a Carnot cycle. In first two steps work is done by engine (W_1 and W_2 are positive) and in last two steps work is done on gas (W_3 and W_4 are negative). The work done in complete cycle W is equal to the area of the closed part of the P–V cycle.
$$W = W_1 + W_2 + W_3 + W_4$$

$$\text{or} \ \ W = \mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right) + \frac{\gamma R}{\gamma - 1}(T_1 - T_2)$$

$$+ \mu RT_2 \ \text{In} \left(\frac{V_4}{V_3} \right) + \frac{\gamma R}{\gamma - 1}(T_2 - T_1)$$

$$= \mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right) + \mu RT_2 \ \text{In} \left(\frac{V_4}{V_3} \right)$$

Efficiency of Carnot Engine,

$$\eta = \frac{W}{Q_1} = \frac{\mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right) + \mu RT_2 \ \text{In} \left(\frac{V_4}{V_3} \right)}{\mu RT_1 \ \text{In} \left(\frac{V_2}{V_1} \right)}$$

The points B and C are connected by an adiabatic path as are the points D and A. Hence, using eq^n. and the adiabatic gas eq^n.

$T_1V_2^{(\gamma-1)} = T_2 V_3^{(\gamma-1)}$ and $T_1V_1^{(\gamma-1)} = T_2 V_4^{(\gamma-1)}$.

Combination of the above eq^n. gives $\dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}$, and, substituting into the ratio of eq^n., we find

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \; ; \; \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}.$$

$$\eta = \frac{T_1 - T_2}{T_1} \times 100\% \; ; \; \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

It can be shown that the efficiency for the Carnot engine is the best that can be obtained for any heat engine and eq^n. gives an upper limit to the efficiency of any heat engine operating between temperatures T_1 and T_2.

The efficiency depends upon the temperatures T_1 and T_2 which approaches unity only when the temperature of the cold reservoir approaches absolute zero. A steam engine using steam at 373 K and with the cold reservoir at 273 K .... $\eta \sim 1 - (273/373) \sim 27\%$.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0$ which is impossible.
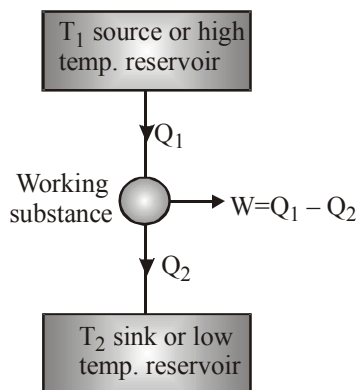
In a Carnot cycle $\dfrac{Q_2}{Q_1} = \dfrac{T_2}{T_1}$ or $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$.

Carnot Theorem : No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working between same hot and cold reservoirs i.e.,

$$\eta_R > \eta_I \; \text{or} \; 1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}$$

HEAT ENGINE

Heat engine is a device which converts heat into work. A heat engine, in general, consists of three parts :

(1) A source of high temperature reservoir at temperature T_1.
(2) A sink or low temperature reservoir at temperature T_2.
(3) A working substance.

In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to sink.

Efficiency of heat engine,

$$\eta = \frac{\text{Work done (W)}}{\text{Heat taken from soruce (Q}_1)}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

The useful work is done in third stroke called work stroke or power stroke.

The efficiency of internal combustion engine is 40 to 60%.

Example 28 :

A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Sol. W = 800 J, $T_1 = 600$ K $T_2 = 300$ K

$$\therefore \; \eta = 1 - \frac{T_1}{T_1} = \frac{W}{Q_1}$$

$$1 - \frac{300}{600} = \frac{800}{Q_1} \qquad \text{or } 0.5 = \frac{800}{Q_1}$$

Heat energy supplied by source

$$Q = \frac{800}{0.5} = 1600 \text{ Joule per cycle}$$

SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

(1) A device called engine with a working substance is essential.

(2) The engine must work in a reversible cyclic process.

(3) The engine must operate between two temperatures. It will absorb heat from a hot body (called source), convert a part of into useful work and reject the rest to a cold body (called sink). There are two conventional statements of second law dependent on common experience.

1. **Kelvin-Planck Statement :** It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

2. **Clausius Statement :** It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

ENTROPY

It is a measure of molecular disorder of a system. Greater is disorder, greater is entropy.

The change in entropy,

$$\delta S = \frac{\text{heat absorbed}}{\text{absolute temperature}} \quad \text{i.e., } \delta S = \frac{\delta Q}{T}$$

$$\delta Q = T\delta S$$

This is mathematical form of second law of thermodynamics.

For solids and liquids : The change in entropy during a change of state of mass m of substance $\Delta S = \pm \dfrac{mL}{T}$

where (+) sign refers to heat absorption and (–) sign for heat evolution.

$$\Delta S = mc \log_e \frac{T_f}{T_i} = 2.3mc \log_{10} \frac{T_f}{T_i}$$

For a perfect gas : The change in entropy for µ moles of a gas

$$\Delta S = n\left[C_v \log_e \frac{T_f}{T_i} + R \log_e \frac{V_f}{V_i} \right]$$

$$= n\left[C_p \log_e \frac{T_f}{T_i} - R \log_e \frac{P_f}{P_i} \right]$$

$$= n\left[C_v \log_e \frac{P_f}{P_i} + C_p \log_e \frac{V_f}{V_i} \right]$$

Example 29 :

Calculate change in entropy to convert 1 gm water at 100°C to steam.

Sol. $\Delta s = \dfrac{mL}{T} = \dfrac{1 \times 540}{373} = 1.4 \text{ cal/°C}$

Example 30 :

Determine the change in entropy when 0.1 kg of ice at 0°C is converted completely into water 0°C. Latent heat of fusion $= 3.36 \times 10^5 \text{ J kg}^{-1}$.

**Sol.** During a change in state, the change in entropy is given by

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL}{T} = \frac{0.1 \times 3.36 \times 10^5}{273} = 123 \text{ J K}^{-1}.$$

TRY IT YOURSELF-4

Q.1 Which of the processes described below are irreversible?
(A) The increase in temprature of an iron rod by hammering it.
(B) A gas in a small cantainer at a temprature T_1 is brought in contact with a big reservoir at a higher temprature T_2 which increases the temprature of the gas.
(C) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston.

(D) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight W is added to the piston, resulting in compression of gas.

Q.2 If a refrigerator's door is kept open, will the room become cool or hot? Explain.

Q.3 Is it possible to increase the temperature of a gas without adding heat to it? Explain.

Q.4 Air pressure in a car tyre increases during driving.
True/False

Q.5 The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,
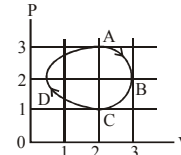
(A) the process during the path A → B is isothermal
(B) heat flows out of the gas during the path B → C → D
(C) work done during the path A → B → C is zero
(D) positive work is done by the gas in the cycle ABCDA

Q.6 One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following
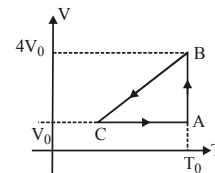
(A) Internal energies at A and B are the same
(B) Work done by the gas in process AB is $P_0V_0 \ln 4$
(C) Pressure at C is $P_0/4$
(D) Temperature at C is $T_0/4$

Q.7 A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

Q.8 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is :

(A) $\dfrac{9}{8}RT_1$　　(B) $\dfrac{3}{2}RT_1$　　(C) $\dfrac{15}{8}RT_1$ (D) $\dfrac{9}{2}RT_1$

Q.9 Starting with the same initial conditions , an ideal gas expands from volume V_1 to V_2 in three different ways , the work done by the gas W_1 if the process is purely isothermal W_2 if purely isobaric & W_3 if purely adiabatic, then :
(A) $W_2 > W_1 > W_3$　　　　(B) $W_2 > W_3 > W_1$
(C) $W_1 > W_2 > W_3$　　　　(D) $W_1 > W_3 > W_2$

ANSWERS

(1) (ABD)　　　**(2)** hotter　　　**(3)** Yes

(4) True　　　**(5)** (BD)　　　**(6)** (AB)

(7) (4)　　　**(8)** (A)　　　**(9)** (A)

HEAT TRANSFER

CONDUCTION

Conduction is that mode of transmission of heat in which heat is transferred from a region of higher temperature to a region of lower temperature by the aid of particles of the body without their actual migration.

In conduction atoms or molecules near the hot end absorb energy and hence their amplitude of vibration about their mean position increases due to which they collide with other neighbouring atoms or molecules and thus transfer a part of energy, to them and so on. In solids, heat transfer is mainly by conduction.

In metals the conduction of heat takes place by means of free electrons present in the metal.

Steady State :

When one end of a rod is heated, then initially the temperature of various points of the rod changes continuously and the rod is said to exist in a variable state. After some time, a state is reached, when the temperature of each cross-section becomes steady. This state is known as steady state. In this state, any heat received by any cross-section is partly conducted to the next section and partly radiated, i.e., no heat is absorbed by the cross-section.

By steady state it does not, imply that the temperature of the whole rod is same. In steady state, the temperature of the different parts of the rod are different but temperature of the each part remains constant. The temperature decreases, as we move away from the hot end.

In this state rate of heat flow depends only on the heat conductivity of the material of the rod so it remains constant. If rod is in contact with atmosphere or surroundings and it is heated at one end then distance versus temperature curve is as shown in fig. (a).

fig. (a)
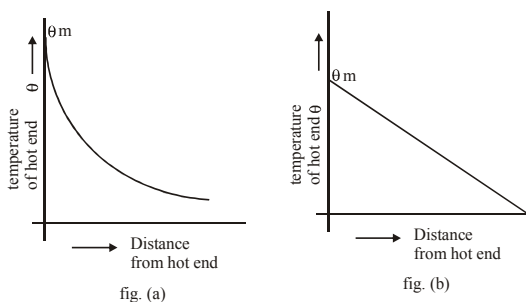
fig. (b)

And if rod is insulated with surroundings then graph is straight line as shown in fig. (b)

Isothermal surface :

In steady state if heat loss due to radiation or convection in negligible then the temperature of every transverse section of the rod is same on the whole area of section (Because the cross-section is perpendicular to the direction of flow of heat hence there will be no heat flow from one point to other on the cross-section) . Thus every transverse section of the rod behaves as on Isothermal surface.
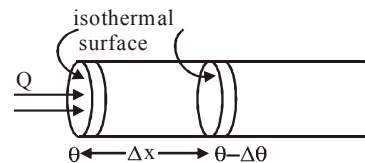
Temperature gradient :

The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient. If the temperature of the isothermal surfaces be θ and $\theta - \Delta\theta$ and perpendicular distance between then in Δx Temperature gradient between them

$$= \frac{\text{change in temperature}}{\text{distance}} = \frac{(\theta - \Delta\theta) - \theta}{\Delta x} = -\frac{\Delta\theta}{\Delta x}$$

The negative sign show that temperature θ decreases as the distance x increases in the direction of heat flow.

Unit: ºC/meter

Thermal Conductivity

If a steady temperature difference $(\theta_1 - \theta_2)$ is to be maintained between the ends of a rod, heat must be supplied at a steady rate at one end and the same must be taken out at the other end. Suppose an amount of heat Q flows through the rod in time t then rate of heat flow or Heat current $H = \dfrac{Q}{t}$

It has been found experimentally that in steady state, Heat current $H = \dfrac{Q}{t}$ will be

(a) proportional to the area A of the cross-section of the rod. The larger the cross-sectional area, the greater will be the number of molecules which conduct heat via collisions.

(b) proportional to $(\theta_1 - \theta_2)$. The greater the temperature difference to be maintained, the greater must be the rate of heat flow.

(c) inversely proportional to ℓ, the distance between ends of the rod. The smaller the distance ℓ, the greater must be the rate of heat flow to maintain a given temperature difference. Thus we find that

$$\frac{Q}{t} \propto \frac{A(\theta_1 - \theta_2)}{\ell} \quad \text{or} \quad H = \frac{K A (\theta_1 - \theta_2)}{\ell}$$
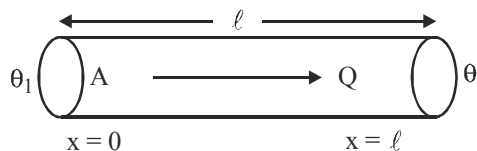
where K is a constant of proportionality called the thermal conductivity of the substance. It is a measure of how quickly heat energy can conduct (flow) through the substance.

Coefficient of thermal conductivity (K) :

The coefficient of thermal conductivity, K, of a material is defined as the amount of heat that flowing per second through a rod of that material of having unit length and

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unit area of cross-section in the steady state, when the difference of temperature between two ends of the rod is 1°C and flow of heat is perpendicular to the end faces of the rod.

Unit of K :   CGS = cal/cm-sec-°C
   MKS = kcal/m-sec-K   SI = watt/m-K

Dimensions : $[M\ L\ T^{-3}\ \theta^{-1}]$

For a perfect conductor of heat K is infinite and for a perfect insulator K is zero.

In general, solids are better conductors than liquids and liquids are better conductors than gases. (Heat transfer through process of conduction is possible in liquids and gases also, if they are heated from the top.) Metals are much better conductors than non-metals.

A good conductor of heat is also a good conductor of electricity. The conduction of both heat and electricity is due to the movement of free electrons.

Thermal Resistance : The thermal resistance of a body is a measure of its opposition to the flow of heat through it. It is defined as the ratio of temperature difference to the heat current (= rate of flow of heat)

Now, temperature difference $= (\theta_1 - \theta_2)$, $\ell = $ Length of rod

heat current, $H = \dfrac{Q}{t}$

A = Area of cross section of rod

$$\therefore \quad R_{Th} = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{(Q/t)} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/\ell} = \frac{\ell}{KA}$$

or $\quad R_{Th} = \dfrac{\Delta\theta}{H} = \dfrac{\ell}{KA}$

Unit : °C × sec/cal or K × sec/k-calorie.

Dimensions : $[M^{-1}\ L^{-2}\ T^3\ \theta]$

|	**Electricity**	**Heat**
Potential difference	$V_2 - V_1$	$\theta_2 - \theta_1$
Current	$I = q/t$ rate of flow of charge	$H = Q/t$ rate of flow of heat
Resistance	$R = \dfrac{V_2 - V_1}{I}$	$R_{Th} = \dfrac{\theta_2 - \theta_1}{H}$
Conductivity	$\sigma = \dfrac{\ell}{AR}$	$K = \dfrac{\ell}{A\left(\frac{\theta_2 - \theta_1}{H}\right)}$

SERIES COMBINATION OF CONDUCTORS

Suppose two conductors of areas of cross-section A_1 and A_2, lengths ℓ_1 and ℓ_2, coefficients of thermal conductivities K_1 and K_2 are connected in series.

Let heat is allowed to flow through this combination after the steady state reached. Let the temperatures of the two outer faces be θ_1 and θ_2 and the temperature of the junction be θ. The heat current in them will be
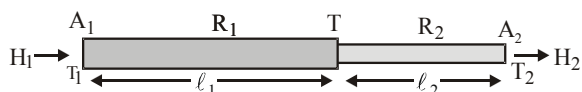
$$H_1 = \frac{(\theta_1 - \theta)}{R_1} \quad \text{and} \quad H_2 = \frac{(\theta - \theta_2)}{R_2}$$

But here, i.e., in series $H_1 = H_2 = H$

So $\theta_1 - \theta = HR_1$ and $\theta - \theta_2 = HR_2$

or $\theta_1 - \theta_2 = H(R_1 + R_2)$, i.e., $H = (\theta_1 - \theta_2)/(R_1 + R_2)$

or $H = \dfrac{(\theta_1 - \theta_2)}{R}$ with $R = R_1 + R_2$

i.e., in series the total thermal resistance is equal to the sum of individual resistances.

This case is similar to two resistances in series. The potential difference is replaced by the temperature difference, the electrical resistance is replaced by thermal resistance and electrical current (rate of flow of charge) is replaced by heat current (the rate of flow of heat).

$$R = R_1 + R_2$$

$$R_1 = \frac{\ell_1}{K_1 A} \text{ and } R_2 = \frac{\ell_2}{K_2 A}$$

and $\quad \dfrac{\ell_{eq.}}{K_{eq.}\ A_{eq.}} = \dfrac{\ell_1}{K_1 A_1} + \dfrac{\ell_2}{K_2 A_2}$

If $\quad A_1 = A_2 = A$

$$\therefore \quad \frac{\ell_1 + \ell_2}{K_{eq.}} = \frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} \quad \text{or} \quad K_{eq.} = \frac{\ell_1 + \ell_2}{\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2}}$$

If $\ell_1 = \ell_2 = \ell$; $K_{eq.} = \dfrac{2K_1\ K_2}{K_1 + K_2}$

If we have more than two slabs in the compound slab. The total thermal resistance of the slabs connected in series.
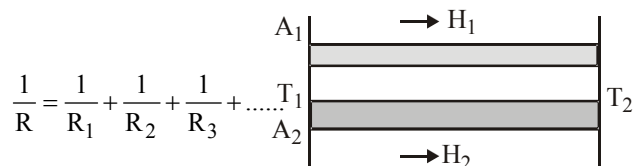
$R_{eq.} = R_1 + R_2 + R_3 + ...$

PARALLEL COMBINATION OF CONDUCTORS

If the two slabs are connected in parallel (i.e. one placed on top of the other) as shown in fig. the thermal resistance R

of the parallel combination is given by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

For more than two slabs, we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

Suppose two conductors each of thickness ℓ but with areas of cross-section A_1 and A_2 are connected in parallel. Let the temperatures of the first and second faces be θ_1 and θ_2. The total heat current H_p is the sum of heat currents H_1 and H_2 in the first and second conductors respectively, i.e., $H_p = H_1 + H_2$.

Now, $H_1 = \dfrac{K_1 A_1 (\theta_1 - \theta_2)}{\ell}$ and $H_2 = \dfrac{K_2 A_2 (\theta_1 - \theta_2)}{\ell}$

and $H_p = \dfrac{K_p (A_1 + A_2)(\theta_1 - \theta_2)}{\ell}$

As $H_p = H_1 + H_2$, hence,

$\dfrac{K_p (A_1 + A_2)(\theta_1 - \theta_2)}{\ell} = \dfrac{K_1 A_1 (\theta_1 - \theta_2)}{\ell} + \dfrac{K_2 A_2 (\theta_1 - \theta_2)}{\ell}$

or $K_p (A_1 + A_2) = K_1 A_1 + K_2 A_2$ or $K_p = \dfrac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$

If $A_1 = A_2$, then $K_p = \dfrac{K_1 + K_2}{2}$

Example 31 :

Two rods of equal length and area of cross section are kept parallel and logged between the temperature θ_1 and θ_2. The ratio of their thermal conductivity is 2 : 3. Find the ratio of effective thermal conductivity (i) to that of first rod and (ii) to that of second rod.

**Sol.** $K = \dfrac{K_1 + K_2}{2} = \dfrac{2K_1 + 3K_1}{2} = \dfrac{5K_1}{2}$

(i) ratio of effective coefficient of that of first rod

$= K = 2K_1 = \dfrac{5K_1}{2} : 2K_1 = 5 : 4$

(ii) ratio of effective coefficient to that of second rod

$= K : 3K_1 = \dfrac{5}{2} K_1 : 3K_1 = 5 : 6$

Example 32 :

Two rods of equal area of cross-section and length in ratio 1 : 2 and coefficient of thermal conductivity in ratio 2 : 1 are joined end to end and logged between steam and melting ice. Find the

(i) ratio of the temperature difference across the two rods.
(ii) temperature of common point in the rods.
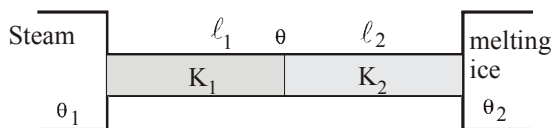

Sol. In the present case

(i) $\dfrac{\theta_1 - \theta}{\ell_1 / K_1} = \dfrac{\theta - \theta_2}{\ell_2 / K_2}$ or $\dfrac{\theta_1 - \theta}{\theta - \theta_2} = \dfrac{\ell_1}{K_1} \times \dfrac{K_1}{\ell_2} = \dfrac{1}{2K_1} \times \dfrac{K_1}{2\ell} = \dfrac{1}{4}$

hence ratio of temperature difference = 1 : 4

(ii) As $\dfrac{\theta_1 - \theta}{\theta - \theta_2} = \dfrac{1}{4}$ or $4(100 - \theta) = \theta - 0$ ∴ $\theta = 80°C$

GROWTH OF ICE ON PONDS

When atmospheric temperature falls below 0°C the water in the lake will start freezing. Let at any time t, the thickness of ice in the lake be y and atmospheric temperature is $-\theta°C$. The temperature of water in contact with the lower surface of ice will be 0ºC.
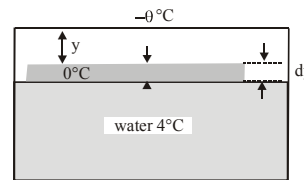
the area of the lake = A
heat escaping through ice in time dt is

$dQ_1 = KA \dfrac{[0 - (-\theta)]}{y} dt = \dfrac{K A \theta}{y} dt$

Now due to escaping of this heat if dy thickness of water in contact with lower surface of ice freezes,

$dQ_2 = mL = \rho (dy\,A)L$ [as m = ρV = ρA dy]
But as $dQ_1 = dQ_2$, the rate of growth of ice will be

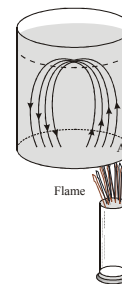$\dfrac{dy}{dt} = \dfrac{K\theta}{\rho L} \times \dfrac{1}{y}$

and so time taken by ice to grow a thickness y,

$t = \dfrac{\rho L}{K\theta} \int_0^y y\,dy = \dfrac{1}{2} \dfrac{\rho L}{K\theta} y^2$

Time taken to double and triple the thickness will be in the ratio $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$,

CONVECTION

Convection requires a medium and is the process in which heat is transferred from one place to other by actual movement of heated substance (usually fluid).

The type of convection which results from difference in densities is called natural convection (for example, a fluid in a container heated through its bottom). However, if a heated fluid is forced to move by a blower, fan or pump, the process is called forced convection. The rate of heat convection from an object is proportional to the temperature difference ($\Delta\theta$) between the object and convective fluid

and the area of contact A, i.e., $\left[\dfrac{dQ}{dt}\right]_{convection} = hA\,\Delta\theta$

where, h represents a constant of proportionality called convection coefficient and depends on the properties of fluid such as density, viscosity, specific heat and thermal conductivity, etc.
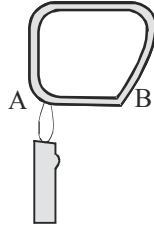
Note :

1. Natural convection takes place from bottom to top while forced convection in any direction.

2. In case of natural convection, convection currents move warm air upwards and cool air downwards. This is why heating is done from base, while cooling from the top.

3. Natural convection is not possible in a gravity free region such as a freely falling lift or an orbiting satellite.

4. Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.

5. The forced convection of blood in our body by a pump (heart) helps in keeping the temperature of body constant.

Example 33 :

Water in a closed tube is heated with one are vertically placed above the lamp. In what direction water will begin the circulate along the tube ?

Sol. On heating the liquid at A will become lighter and will rise up. This

will push the liquid in the tube upwards and so the liquid in the tube will move clockwise i.e. from B to A.
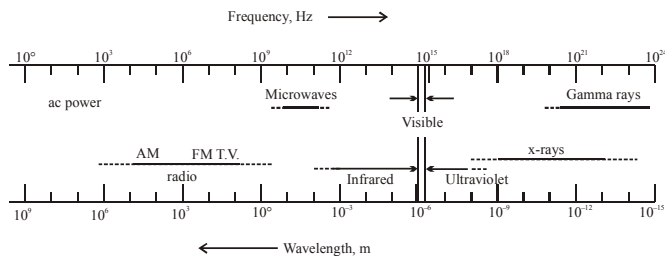

RADIATION
THERMAL RADIATION

When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat can not go out from the body by the process of conduction or convection since both of these process require the presence of a material medium between source and surrounding objects. The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium. It is by radiation that the heat from the sun reaches the earth. Radiation has the following properties :

(a) Radiant energy travels in straight lines and when some object is placed in the path, it's shadow is formed at the detector.

(b) It is reflected and refracted or can be made to interfere. The reflection or refraction are exactly as in case of light.

(c) It can travel through vacuum.

(d) Intensity of radiation follows the law of inverse square.

(e) Thermal radiation can be polarised in the same way as light by transmission through a nicol.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of heat radiation is larger than that of visible light.

Radiations have a wavelength greater than that of red colour and they form the infrared region of the electromagnetic spectrum. The wavelength ranges from 10^{-6} m to 10^{-3} m.

The electromagnetic spectrum :

Radiations	Wavelength range
Cosmic Rays	$< 10^{-13}$ m.
γ-Rays	10^{-10} m–10^{-14} m.
X-Rays	From 10^{-9} m to 6×10^{-12} m.

Ultraviolet Rays	3.8×10^{-7} m to 6×10^{-10} m
Visible waves	7.8×10^{-7} to 3.8×10^{-7} m
Infra Red waves	10^{-3} m to 7.8×10^{-7} m
Microwaves	0.3 m to 10^{-3} m
Radio waves	Few Kms to 0.3 m
Television	0.1 m – 1 m waves

The wavelength of the visible spectrum :
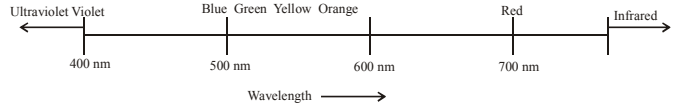
When the radiant energy is incident on the surface of any body, it is used up in three different ways.

(I) Part of this energy is absorbed by the body.

When radiant energy is incident on matter, it is partly absorbed and converted into heat.

(II) Some part is reflected and

(III) Remaining part is transmitted by the body.

Let Q be the quantity of radiant heat incident on the surface of the body and Q_a, Q_r and Q_t be the the quantities of heat absorbed, reflected & transmitted by the body respectively.

Absorption coefficient "a"

$$a = \frac{Q_a}{Q} = \frac{\text{Quantity of radiant energy absorbed by the body}}{\text{Quantity of radiant energy incident on the surface of the body}}$$

$$a = \frac{Q_a}{Q}$$

Reflection Coefficient :

$$r = \frac{\text{Quantity of radiant energy reflected}}{\text{Quantity of radiant energy incident on the surface of the body}} \quad ; \quad r = \frac{Q_r}{Q}$$

Transmission Coefficient :

$$t = \frac{\text{Quantity of radiant energy transmitted by the body}}{\text{Quantity of radiant energy incident on the surface of the body}}$$

$$t = \frac{Q_t}{Q} . \text{ Then } Q_a + Q_r + Q_t = Q$$

$$\therefore \quad \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1 \quad \therefore \quad a + r + t = 1$$

Diathermanous substances : The substances which can transmit the radiant heat incident upon their surfaces are called diathermanous substances. ($t \neq 0$). The best diather-manous solids are rock salt, sylvine, quartz and certain other crystals. Pure air is highly diathermanous. e.g. glass, quartz, gases etc.

Athermanous substances : The substances which cannot transmit the radiant heat incident upon their surfaces are called athermanous substances. (t = 0). e.g. wood, iron, copper etc. Water vapour & carbon dioxide are athermanous.

Emissive Power (e) : For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface.

Spectral Emissive Power (e_λ) : If we consider emissive power of a surface for a particular wavelength instead of all wavelengths, it is called spectral emissive power and is represented by e_λ

Total emissive power $e = \int_0^\infty e_\lambda \, d\lambda$.

Emissive power of a surface depends on its nature and temperature.

It is maximum for a perfectly black body and minimum for a smooth shining body. **Unit :** W/m^2

Absorptive Power (a) : Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it is the same time. If Q calories fall on the surface and Q′ calories out of it are absorbed, the absorptive power of the surface

is given by $a = \dfrac{Q'}{Q}$

For a perfectly black body, absorptive power is maximum and unity. It has no units and dimensions.

Spectral absorptive power "a_λ" : Spectral absorptive power (a_λ) is defined as the ratio of the radiant energy of a given wavelength absorbed by a given surface in a given time to the total radiant energy of that wavelength incident in the same time on the same surface within a unit wavelength

range. Total absorptive power $a = \int_0^\infty a_\lambda \, d\lambda$.

ABSOLUTE EMISSIVITY (e)

The emissivity e of a surface is defined as the quantity of heat radiated per second by one unit area of its surface when its temperature is 1°C above the surroundings.

Relative Emissivity (e) : It is defined as the ratio of radiant energy emitted per unit time per unit surface area of a body at a given temperature to the radiant energy emitted per unit time per unit surface area of a perfectly black body at

the same temperature i.e.   ∴ $e = \dfrac{E}{E_b}$.

A perfectly black body : A black body is defined as one that will completely absorb all the radiation of whatever wavelength which falls on it. For perfectly black body, $a_\lambda = 1$.

A body which can absorb all the quantity of radiant heat incident on it is called a perfectly black body. A body which is a good absorber of a radiation of a certain wavelength is also a good emitter of the radiation of that wavelength. Consequently, a black body, which absorbs every wavelength, must also emit every wavelength.

Properties of perfectly black body :

(a) A perfectly black body absorbs all the radiant heat incident upon it. (i.e. a = 1)

(b) A perfectly black body does not reflect or transmit the radiant heat incident upon it. (i.e. t = 0, r = 0)

(c) The coefficient of emission of a perfectly black body is 1. It is very good emitter of heat.

(d) A perfectly black body does not exist in nature. For practical purposes lamp black is treated as a perfectly black body as its coefficient of absorption is 0.98.

Ferry's black body :

This is shown in fig. it consists of a hollow copper sphere which is blackened inside with a small fine hole in the surface. The conical projection opposite the hole is to avoid direct reflection from surface opposite the hole.
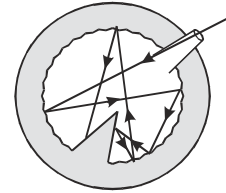
If the whole of the sphere (described above) is raised to a uniform very high temperature, the radiation emerging out from the hole would contain every possible wavelength. The hole (inside the sphere) would then appear to be white-hot, if the hole were viewed. These radiations are known as black body radiations.

"The glowing heart of a furnace is an ideal (black body) radiator, for it is practically a small hole surrounded by glowing bodies all at one temperature". A hot copper sphere, or the glowing filament of an electric lamp, are not the black body radiators, as certain wavelengths are missing from their spectra.

PREVOST'S THEORY OF HEAT EXCHANGES

All bodies at all temperatures above the absolute zero are continually radiating, the amount increasing with temperature, unaffected by the presence of surrounding bodies. The rise or fall of temperature which is observed in a body is due to its exchange of radiant energy with the surrounding bodies. This is known as Prevost's theory of exchange.

(a) Every body radiates heat at all temperatures except at absolute zero of temperature. At the same time, every body absorbs heat from the surroundings at all temperatures.

(b) The radiant energy emitted per unit time by the body depends on its absolute temperature and does not depend upon the temperature of the surroundings.

Explanation :

All the observed facts of heat exchanges can be explained on the basic of prevosts's theory of heat exchanges.

(a) When a hot body is placed in cooler surroundings, the body radiates more energy and absorb less energy from the surroundings. Therefore temperature of body decreases.

(b) When a cold body is placed in the hot surrounding, the body radiates less energy and absorbs more energy from the surrounding. Therefore the temperature of body increases.

(c) When the temperature of a body is equal to the temperature of the surroundings, the energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body therefore its temperature remains constant.

KIRCHHOFF'S LAW

At any given temperature and for radiations of the same wavelength the ratio of the emissive power (e_λ) to the absorptive power (a_λ) is the same for all substances and is equal to the emissive power of a perfectly black body (E_λ).

$$\frac{e_\lambda}{a_\lambda} = \text{const.} = E_\lambda$$

Proof : Let us consider a body placed inside a uniformly heated enclosure maintained at temperature T. The radiation within the enclosure will be independent of the nature of the walls of the enclosure and of the presence of the body. Let dQ be the quantity of radiant energy of wavelength laying between λ and $\lambda + d\lambda$, falling on the body per unit area of its surface per second.

Let a_λ be the absorptive power of the body, a_λ is the ratio of the radiant energy absorbed by the body to the total incident energy at the temperature T and for the wavelength λ. Then the amount of radiant energy absorbed by the body per unit area per second = a_λ dQ. The remainder energy is reflected or transmitted = $(1 - a_\lambda)$ dQ.

If e_λ be the emissive power of the body, the amount of energy radiated per unit area per second under the same conditions of temperature and wavelength = e_λ dλ.

Hence the total energy given out by the body per unit area per second = $(1 - a_\lambda)$ dQ + e_λ dλ. Since the body is in temperature equilibrium with the enclosure, the radiant energy given out must be equal to that received.

$\therefore$ dQ = $(1 - a_\lambda)$ dQ + e_λ dλ

or a_λ dQ = e_λ dλ ....(1)

For a perfectly black body, since $a_\lambda = 1$ and e_λ has a maximum value, which we may denote by E_λ,

dQ = E_λ dλ ....(2)

Substituting this value of dQ in (1), we have

$$a_\lambda \, E_\lambda \, d\lambda = e_\lambda \, d\lambda \quad \text{or} \quad \frac{e_\lambda}{a_\lambda} = E_\lambda \qquad \text{.......(3)}$$

This relation (3) shows that, at any given temperature T and for radiations of the same wavelengths λ, the ratio of the emissive power to the absorptive power of a substance is constant and equal to the emissive power of a perfectly black body. This is Kirchhoff's law which is here proved for bodies inside a uniform temperature enclosure. Since, however, the emissive and absorptive powers depend only on the nature of the body and not on its surroundings, it follows that the law is general, holding good under all conditions for pure temperature radiation. In the above relation, since E_λ is a constant, if e_λ is larger a_λ also is large and vice versa. The shows that good emitters are also good absorbers.

Applications of kirchhoff's law :

If a body emits strongly the radiation of a particular wavelength, it must also absorb the same radiation strongly.

(1) Let a piece of china clay plate with some dark painting on it be first heated to nearly 1300 K and then examined in dark room. It will be observed that the dark paintings appear much brighter than the white portion. This is because the paintings being better absorbers emit also much greater light.

(2) The silvered surface of a thermos flask does not absorb much heat from outside. This stops ice from melting quickly. Also, the silvered surface does not radiate much heat from inside. This prevents hot liquids from becoming cold quickly.

(3) A red glass appears red at room temperature. This is because it absorbs green light strongly. However, if it is heated in a furnace, it glows with green light. This is because it emits green light strongly at a higher temperature.

(4) When white light is passed through sodium vapours and the spectrum of transmitted light is seen, we find two dark lines in the yellow region. These dark lines are due to absorption of radiation by sodium vapours which it emits when heated.

Fraunhofer lines are dark lines in the spectrum of the sun. When white light emitted from the central core of the sun (photosphere) passes through its atmosphere (chromosphere) radiations of those wavelengths will be absorbed by the gases present there which they usually emit (as a good emitter is a good absorber) resulting in dark lines in the spectrum of sun.

At the time of solar eclipse direct light rays emitted from photosphere cannot reach on the earth and only rays from chromosphere are able to reach on the earth surface. At that time we observe bright fraunhofer lines.
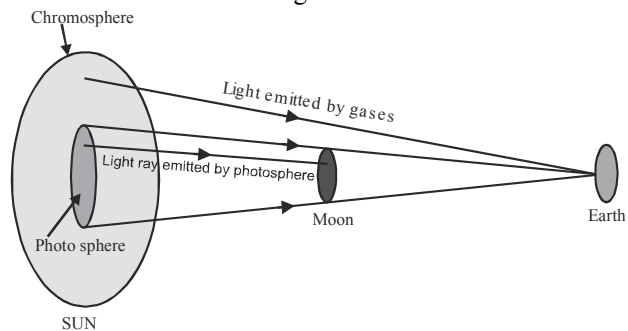
STEFAN BOLTZMANN LAW OF RADIATION

A perfectly black body is the best possible emitter of radiation at a given temperature. Stefan's Law gives the total amount of radiation emitted by a perfectly black body. According to Stefan's law, the quantity of energy radiated in one second per unit area of a perfectly black body is directly proportional to the fourth power of its absolute temperature. $E_b \propto T^4$ or $E_b = \sigma T^4$

where σ is a universal constant known as Stefan-Boltzmann constant and T is its temperature on absolute scale. The measured value of σ is -

$\sigma = 5.67 \times 10^{-8}$ W/m² –K⁴ or $\sigma = 5.67 \times 10^{-5}$ erg/cm²sK⁴.

eq^n. $E_b = \sigma T^4$ is called the Stefan-Boltzmann law.

Stefan has suggested this law from experimental data available on radiation and Boltzmann derived it from thermodynamical considerations. The law is also quoted as Stefan's law and the constant σ as Stefan constant.

Now, $\dfrac{Q_b}{A.t} \quad \therefore \dfrac{Q_b}{A.t} = \sigma T^4$

Q_b = total energy radiated by the surface of a black body in time t. or $Q_b = \sigma \, A \, t \, T^4$

From eq^n., the emissive power E of a non-black body is given by : $E = e\,E_b$ or $\therefore E = e\,\sigma T^4$

$\therefore$ Total energy radiated by the surface of a body in time t

$Q = e\,\sigma\,A\,t\,T^4$

APPLICATION OF PREVOST CONCEPT

Cooling of a body by radiation

Consider a perfectly black body of absolute temperature T is surrounded by another black body at absolute temp. T_0.
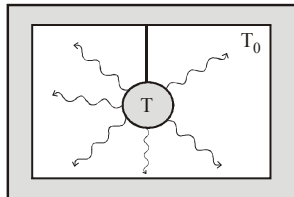
Energy radiated per unit time per unit surface area by the perfectly black body $= \sigma\,T^4$.

Energy absorbed per unit time per unit surface area by the perfectly black body from the surroundings $= \sigma\,T_0{}^4$.

The net loss of heat by the perfectly black body per unit time per unit surface area $= \sigma\,(T^4 - T_0{}^4)$

Let, the surface area of the perfectly black body $= A$

$\therefore$ The rate of loss of heat of the perfectly black body

$$\frac{dQ}{dt} = \varepsilon\sigma A(T^4 - T_0^4)$$

Let, the emissivity of an ordinary body $= e$

$\therefore$ The rate of loss of heat of the ordinary body

$$\frac{dQ}{dt} = A\,e\,\sigma\,(T^4 - T_0{}^4) \text{ J/sec.}$$

Rate of fall of temp. $\dfrac{d\theta}{dt} = \dfrac{A\,e\,\sigma\left[T^4 - T_0^4\right]}{msJ}$ cal/sec.

NEWTON'S LAW OF COOLING

Newton was the first to study experimentally in systematic manner and derive the relation between heat radiated by a body in a given enclosure and its temperature based on experimental facts as given below.

The rate of cooling of a body (rate of loss of heat) is directly proportional to the excess of temp. of the body over the surroundings, provided that this excess is small and loss of heat by radiation only.

If θ and θ_0 are the temperatures of the body and its surroundings respectively,

Rate of loss of heat $R = (-dQ/dt)$

The negative sign indicates that the amount of heat is decreasing with time.

$$-\frac{dQ}{dt} \propto (\theta - \theta_0) \quad \text{or} \quad -\frac{dQ}{dt} = k\,(\theta - \theta_0) \quad \text{.........(1)}$$

where k is the constant of proportionality.

Rate of fall of temperature : $\dfrac{d\theta}{dt}$

The rate of decrease of temperature with respect to time is known as rate of fall of temperature.

If m is mass and s is specific heat of the body, then,

$dQ = m\,s\,J\,d\theta$

$$\therefore \quad \frac{dQ}{dt} = m\,s\,J\,\frac{d\theta}{dt} \quad \text{.........(2)}$$

$$\therefore \quad m\,s\,J\,\frac{d\theta}{dt} = k\,(\theta - \theta_0) \quad \text{[From eq}^n\text{. (1) and (2)]}$$

$$\therefore \quad \text{Rate of fall of temp.} -\frac{d\theta}{dt} = \frac{k}{m\cdot s}(\theta - \theta_0)$$

i.e. $-\dfrac{d\theta}{dt} \propto (\theta - \theta_0)$

(m and s are constant.)

Thus, Newton's law of cooling can also be stated as,

The rate of fall of temperature of a body is directly proportional to excess of temperature of the body over the surroundings, provided that excess is small.

Excess of temperature : If the temperature of body decreases from θ_1 to θ_2 and temperature of surroundings is
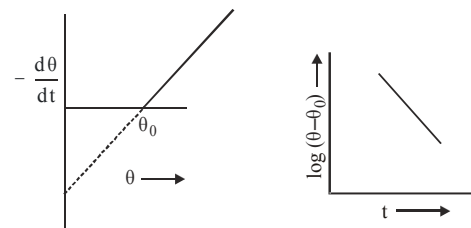
θ_0 then average excess of temperature $= \left[\dfrac{\theta_1 + \theta_2}{2} - \theta_0\right]$

The graph drawn between the temperature of a body and time is known as cooling curve. The slope of the tangent to the curve at any point gives the rate of fall of temperature.
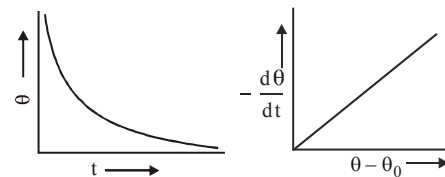
If rate of fall of temperature is plotted against excess temperature, we get a straight line passing through the origin.

Curve between the rate of cooling and body temperature.

$R = k\,(\theta - \theta_0)$ or $R = k\theta - k\theta_0$.

$$\frac{d\theta}{dt} = -\frac{k}{msJ}(\theta - \theta_0)$$

$$\int \frac{1}{(\theta - \theta_0)}\,d\theta = -\frac{k}{msJ}\int dt \quad \text{or} \quad \log_e(\theta - \theta_0) = -\frac{k}{msJ}\,t + B$$

or $\log(\theta - \theta_0) = -At + B$

This is equ^n. of a straight line, hence the curve between log $(\theta - \theta_0)$ and t will be a straight line.

Example 34 :

A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. What will be the temperature after 15 minutes and what is the temperature of surroundings?

Sol. Let θ_0 be the temperature of surroundings. According to Newton's law of cooling,

The rate of cooling $\propto$ excess temperature

Case I : Average rate of cooling $= \dfrac{80 - 64}{5}$ °C / min

Mean temperature $= \left[\dfrac{80 + 64}{2}\right]$ °C

$\therefore$ Excess temperature $= \left[\dfrac{80 + 64}{2}\right] - \theta_0$

$\therefore \dfrac{80 - 64}{5} = K\left[\dfrac{80 + 64}{2} - \theta_0\right]$ .... (1)

where K is a constant of proportionality.

Case II : Average rate of cooling $= \dfrac{64 - 52}{5}$

Mean temperature $= \dfrac{64 + 52}{2}$

$\therefore \dfrac{64 - 52}{5} = K\left[\dfrac{64 + 32}{2} - \theta_0\right]$ ....(2)

Dividing (1) by (2), we get $\dfrac{16}{12} = \dfrac{72 - \theta_0}{58 - \theta_0}$

Solving $\theta_0 = 16$°C

Case III : Let θ be the temp. at the end of next 5 minutes (or after 15 minutes from start)

$\therefore$ Average rate of cooling $= \dfrac{52 - \theta}{5}$ °C / min

Mean temperature $= \dfrac{52 + \theta}{2}$

$\therefore \dfrac{52 - \theta}{5} = K\left[\dfrac{52 + \theta}{2} - \theta_0\right]$ .....(3)

Dividing (3) by (1) and putting $\theta_0 = 16$°C

$\dfrac{52 - \theta}{16} = \dfrac{\dfrac{52 + \theta}{2} - 16}{72 - 16}$; Solving, we get $\theta = 43$°C

Derivation Newton's law of cooling from Stefan's law of radiation : Consider a perfectly black body of surface area A whose absolute temperature is T. Let T_0 be the absolute temperature of the surroundings.

By Stefan's law of radiation,

(Rate of loss of heat) $= \sigma A (T^4 - T_0^4)$

$\therefore \dfrac{dQ}{dt} = \sigma A (T^4 - T_0^4)$

Let $T - T_0 = \Delta T$ $\therefore T = T_0 + \Delta T$

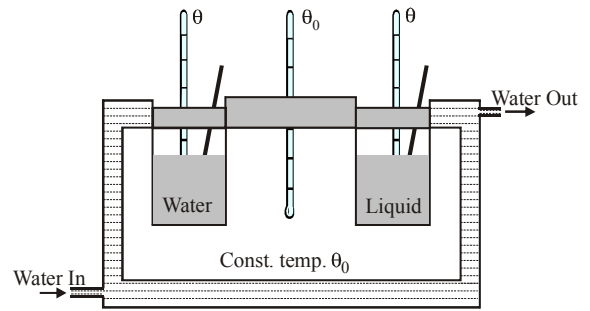$\therefore \dfrac{dQ}{dt} = \sigma A [(T_0 + \Delta T)^4 - T_0^4]$

$\dfrac{dQ}{dt} = 4\sigma A T_0^3 \Delta T$ (Let $4\sigma A T_0^3 = $ const. k)

$\therefore \dfrac{dQ}{dt} \propto \Delta T$ $\therefore \dfrac{dQ}{dt} \propto (T - T_0)$

But, $T - T_0 = (\theta + 273) - (\theta_0 + 273) = (\theta - \theta_0)$

$\therefore \dfrac{dQ}{dt} \propto (\theta - \theta_0)$ This is Newton's law of cooling.

Comparision of specific heat of two liquids using Newton's law of cooling :

If equal volume of two liquids of densities and specific heats ρ_1, s_1 and ρ_2, s_2 respectively are filled in calorimeters having same surface area and finish cool from same initial temperature θ_1 to same final temperature θ_2 with same temperature of surroundings, i.e., θ_0, in time intervals t_1 and t_2 respectively and water equivalent of calorimeter is w. According to Newton's cooling law

$\left(\dfrac{dQ}{dt}\right)_{liq.} = \left(\dfrac{dQ}{dt}\right)_{water}$

$\dfrac{(w + m_1 s_1)(\theta_1 - \theta_2)}{t_1} = \dfrac{(w + m_2 s_2)(\theta_1 - \theta_2)}{t_2}$

or $\dfrac{w + m_1 s_1}{t_1} = \dfrac{w + m_2 s_2}{t_2}$

If water equivalent of calorimeter w is negligible then

$\dfrac{m_1 s_1}{t_1} = \dfrac{m_2 s_2}{t_2}$

So, $\dfrac{m_1 s_1}{m_2 s_2} = \dfrac{t_1}{t_2}$ or $\dfrac{\rho_1 s_1}{\rho_2 s_2} = \dfrac{t_1}{t_2}$ ($v_1 = v_2$, volume are equal)

with the help of this eq^n. we can find specific heat of liquid.

Example 35 :

A calorimeter of water equivalent 5×10^{-3} kg contains

25×10^{-3} kg of water. It takes 3 minutes to cool from 28°C to 21°C. When the same calorimeter is filled with 30×10^{-3} kg of turpentine oil then it takes 2 minutes to cool from 28°C to 21°C. Find out the specific heat of turpentine oil.

Sol. $R_{water} = R_{turpentine} \quad \dfrac{(m_1 + w)}{t_1} = \dfrac{(m_2 s_2 + w)}{t_2}$

or $\dfrac{(25 \times 10^{-3} + 5 \times 10^{-3})}{3} = \dfrac{30 \times 10^{-3} s_2 + 5 \times 10^{-3}}{2}$

$10 = \dfrac{30\ s_2 + 5}{2}, \qquad 20 = 30\ s_2 + 5$

∴ specific heat of turpentine $s_2 = 1/2 = 0.5$ kcal/kg/°C

Example 36 :

A man, the surface area of whose skin is 2m², is sitting in a room where the air temperature is 20°C. If his skin temperature is 28°C, find the rate at which his body loses heat. The emissivity of his skin = 0.97.

Sol. Absolute room temperature $(T_0) = 20 + 273 = 293$ K
Absolute skin temperature $(T) = 28 + 273 = 301$ K
Rate of heat loss $= \sigma e A (T^4 - T_0^4)$
$= 5.67 \times 10^{-8} \times 0.97 \times 2 \times \{(301)^4 - (293)^4\} = 92.2$ W

Example 37 :

Compare the rate of loss of heat from a metal sphere of the temperature 827°C, with the rate of loss of heat from the same sphere at 427 °C, if the temperature of surroundings is 27°C.

Sol. Given : $T_1 = 827\,°C = 1100$ K, $\quad T_2 = 427\,°C = 700$ K
and $T_0 = 27\,°C = 300$ K
According to Steafan's law of radiation,

$\dfrac{dQ}{dt} = \sigma\, Ae\, (T^4 - T_0{}^4)$

∴ $\dfrac{\left(\dfrac{dQ}{dt}\right)_1}{\left(\dfrac{dQ}{dt}\right)_2} = \dfrac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)} = \dfrac{[(1100)^4 - (300)^4]}{[(700)^4 - (300)^4]} = 6.276$

or $\left(\dfrac{dQ}{dt}\right)_1 : \left(\dfrac{dQ}{dt}\right)_2 = 6.276 : 1$

Example 38 :

The filament of an evacuated light bulb has a length 10 cm, diameter 0.2 mm and emissivity 0.2, calculate the power it radiates at 2000 K. $(\sigma = 5.67 \times 10^{-8}$ W/m² K⁴$)$

Sol. $\ell = 10$ cm $= 0.1$ m, $\qquad d = 0.2$ mm,

$r = 0.1$ mm $= 1 \times 10^{-4}$ m,
$e = 0.2$, $T = 2000$ K, $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴
According to stefan's law of radiation, rate of emission of heat for an ordinary body,
$E = \sigma A e T^4 = \sigma (2\,\pi\, r\, \ell)\, e T^4$

$= 5.67 \times 10^{-8} \times 2 \times 3.14 \times 1 \times 10^{-4} \times 0.1 \times 0.2 \times (2000)^4$
$= 11.4$ W
∴ Power radiated by the filament $= 11.4$ W $\quad [A = 2\pi r\ell]$

Example 39 :

The energy radiated from a black body at a temperature of 727°C is E. By what factor the radiated energy shall increase if the temperature is raised to 2227°C?

Sol. $\dfrac{E_2}{E_1} = \left[\dfrac{T_2}{T_1}\right]^4 = \left[\dfrac{2227 + 273}{727 + 273}\right]^4 = \left[\dfrac{2500}{1000}\right]^4 = 39$

DISTRIBUTION OF ENERGY IN A BLACK BODY RADIATION

Fig. shows how the energy of a black body radiation varies with temperature and wavelength.
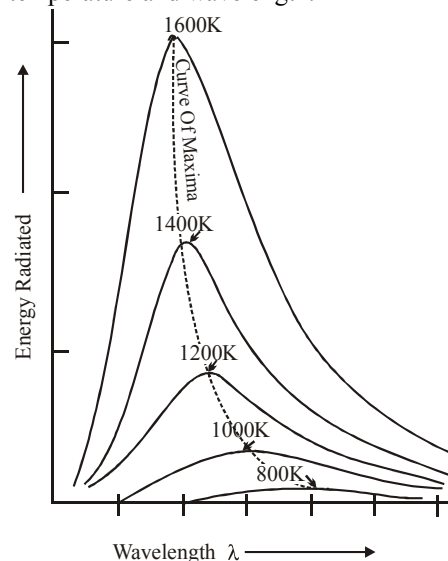
Each curve represents the variation of monochromatic emittance E_λ with wavelength λ for a certain temperature. The following conclusions can be drawn from these curves

(a) At each temperature, the black body emits continuous heat radiation spectrum.

(b) The energy associated with the radiation of a particular wavelength increases with increases in temperature of the black body.

(c) At a given temperature of the black body, the amount of energy associated with the radiation initially increases with wavelength and after becoming maximum corresponding to a wavelength λ_m, it decreases. The wavelength λ_m corresponding to which energy associated is maximum is called the wavelength of the maximum emission.

(d) The wavelengths of maximum emission at various temperatures of the black body decreases as temp. increases.
Therefore, the wavelength of maximum emission shifts towards smaller wavelength as the temperature of the black body increases. It is known as Wien's displacement law.
$$\lambda_m \times T = b$$
const. b is known as Wien's constant

(e) The area under each curve represents the total energy emitted by the perfect black body per second per unit area

over the complete wavelength range at that temperature.
Area under the curve $\propto T^4$
Thus, if E is the total energy emitted per second per unit area at temperature T, then
E $\propto$ area under the curve at temperature T.
Total energy emitted per second per unit area by a perfect black body is directly proportional to the fourth power of its absolute temperature. $E \propto T^4$
It is known as Stefan's law of heat radiation.
(f) The energy (E_{max}) emitted corresponding to the wavelengths of maximum emission (λ_m) increases with the fifth power of the absolute temperature of the black body i.e. $E_{max} \propto T^5$
It is known as Wien's fifth power law.

Example 40 :
Draw typical curves showing the energy distribution with wavelength of black body radiation at two different temperatures T and 2T.
Sol. For plotting the energy distribution curve or E_λ versus λ at two different temperatures, we keep following points in mind.
(i) The total area under the curves represent the total radiation per unit volume in the isothermal enclosure at the given temperature which is proportional to T^4. Therefore as temp. rises, the total area under the curve also increases. The ratio of area at temperature 2T and T is $2^4 : 1$ i.e., $16 : 1$.
(ii) As temperature rises, the λ_m (the wavelength at which E_λ is maximum) shifts to lower wavelength side because $\lambda_m T$ is constant.

$$\frac{\lambda_m \text{ at } 2T}{\lambda_m \text{ at } T} = \frac{1}{2}$$
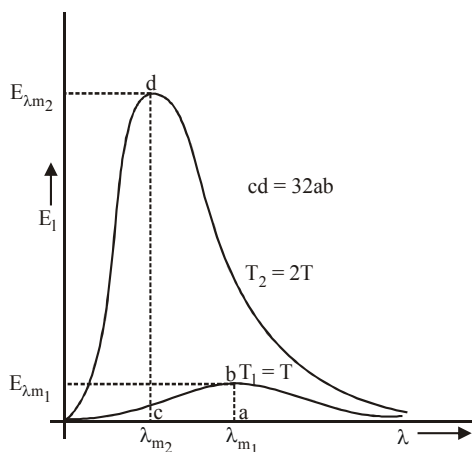
(iii) For any $d\lambda$ range, the energy increases with temperature much faster at small λ values than at higher ones.
(iv) The maximum value of E_λ is proportional to T^5.

$$\therefore \quad \frac{E_{\lambda max} \text{ at } 2T}{E_{\lambda max} \text{ at } T} = \left(\frac{2T}{T}\right)^5 = 32 : 1$$

The plot for the energy distribution for E_λ versus λ at temperatures T and 2T is as shown in fig.

Example 41 :
Experimental investigations show that the intensity of the solar radiation is maximum for wavelength $\lambda_m = 4753$ Å in the visible region. Estimate the surface temperature of the sun. Assume the sun to be a black body.
Wien's constant (b) = 2.892×10^{-3} m K.
Sol. Wien's law states that
$\lambda_m T = $ constant b = 2.892×10^{-3} m K
where T (in kelvin) is the temperature of the radiating body.
$\lambda_m = 4753$ A° = 4753×10^{-10} m

Hence, $T = \dfrac{b}{\lambda_m} = \dfrac{2.892 \times 10^{-3} \text{ mK}}{4753 \times 10^{-10} \text{ m}} = 6060$ K

This is only a rough estimate of the temperature of the surface of the sun as the sun does not actually radiate like a black body.

SOLAR CONSTANT
The sun emits radiant energy continuously in space of which an insignificant part reaches the earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the sun's rays and placed at the mean distance of the earth (in the absence of atmosphere) is called Solar Constant. The solar constant S is taken to be 1400 watts/m^2.
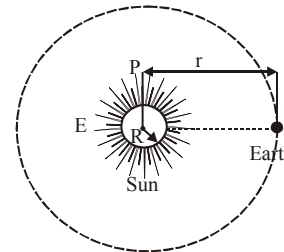
Temperature of the Sun : Let R be the radius of the Sun and r be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. Then, the total energy emitted by the Sun in one second = $4\pi R^2 \times E$. This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area $4\pi r^2$.

The energy falling per unit area = $\dfrac{4\pi R^2 \times E}{4\pi r^2} = \dfrac{E \times R^2}{r^2}$

By definition, this is the solar constant S

i.e., $S = \dfrac{ER^2}{r^2}$ But $E = \sigma T^4$ According to Stefan's law,

$S = \dfrac{\sigma T^4 R^2}{r^2}$ or $T^4 = \dfrac{Sr^2}{\sigma R^2}$ or $T = \left[\dfrac{S \times r^2}{\sigma \times R^2}\right]^{1/4}$

Now, S = 1400 W m^{-2}, R = 6.96×10^8 m, r = 1.496×10^{11} m, $\sigma = 5.68 \times 10^{-8}$ SI units

On substituting these values above, we get T, the surface temperature of the Sun. It comes out to be equal to 5791 K. In this way, the surface temp. of Sun has been estimated.

TRY IT YOURSELF-5

Q.1 The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that :
(A) $T_1 > T_2 > T_3$
(B) $T_1 > T_3 > T_2$

(C) $T_2 > T_3 > T_1$
(D) $T_3 > T_2 > T_1$
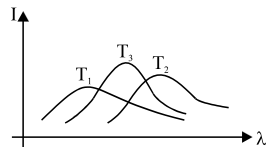
Q.2 Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0 °C and 90 °C respectively. The temperature of junction of the three rods will be :
(A) 45ºC
(B) 60ºC
(C) 30ºC
(D) 20ºC
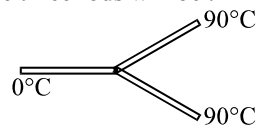
Q.3 An ideal black-body at room temperature is thrown into a furnace. It is observed that :
(A) initially it is the darkest body and at later times the brightest
(B) it is the darkest body at all times
(C) it cannot be distinguished at all times
(D) initially it is the darkest body and at later times it cannot be distinguished

Q.4 Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

Q.5 A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a const. K) and sizes (given in terms of length, L) as shown in the fig. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then in steady state
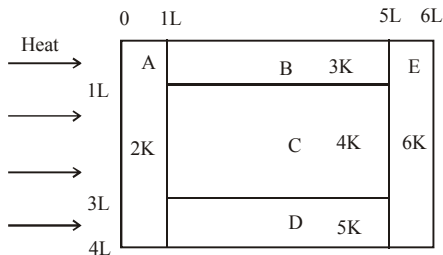
(A) heat flow through A and E slabs are same
(B) heat flow through slab E is maximum
(C) temperature difference across slab E is smallest
(D) heat flow through C = heat flow through B + heat flow through D.

Q.6 Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity κ and the other 2κ. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is –
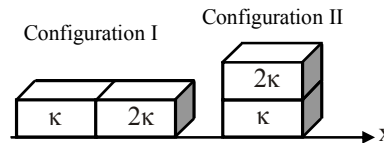
(A) 2.0 s   (B) 3.0 s
(C) 4.5 s   (D) 6.0 s

Q.7 Parallel rays of light of intensity I = 912 Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ Wm^{-2}K^{-4} and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to –
(A) 330 K   (B) 660 K
(C) 990 K   (D) 1550 K

Q.8 Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio (λ_A/λ_B) for their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is :

ANSWERS

(1) (B)   **(2)** (B)   **(3)** (A)
(4) (9)   **(5)** (ACD)   **(6)** (A)
(7) (A)   **(8)** (2)

ADDITIONAL EXAMPLES

Example 1 :
A clock which keeps correct time at 25ºC has a pendulum made of brass whose coefficient of linear expansion is 0.000019. How many seconds a day will it gain if the temperature fall to 0ºC.

Sol. Let L_0 and L_{25} be the length of pendulum at 0ºC and 25ºC respectively. We know that
$L_{25} = L_0(1 + \alpha T) = T_0(1 + 0.000019 \times 25) = 1.000475\, L_0$
If T_{25} & T_0 be the time periods at 25ºC & 0ºC respectively, then

$$T_{25} = 2\pi \sqrt{\left(\frac{L_{25}}{g}\right)} \text{ and } T_0 = 2\pi \sqrt{\left(\frac{L_0}{g}\right)}$$

$$\therefore \frac{T_{25}}{T_0} = \sqrt{\left(\frac{L_{25}}{L_0}\right)} = \sqrt{\left(\frac{1.000475 L_0}{L_0}\right)}$$

$$= \sqrt{(1.000475)} = 1.000237$$

Now $\dfrac{T_{25} - T_0}{T_0} = 0.000237$

$\therefore$ Gain in time for one vibration $= 2 \times 0.000237$ sec.

Number of vibration in one day $= \dfrac{24 \times 60 \times 60}{2}$ (T = 2 sec.)

Hence the gain in time in one day

$$= 2 \times 0.000237 \times \dfrac{24 \times 60 \times 60}{2} = 20.52 \text{sec.}$$

Example 2 :
A circular hole in an aluminium plate is 2.54 cm in diameter at 0ºC. What is the diameter when the temperature of the plate is raised to 100ºC ? Given $\alpha_A = 2.3 \times 10^{-5}$ (ºC)$^{-1}$

Sol. Let D_0 & D_t be diameters of hole at 0ºC and tºC respectively.
Circumference of hole at 0ºC, $\qquad \ell_0 = 2\pi r_0 = \pi D_0$
Circumference of hole at t = 100ºC, $\quad \ell_t = 2\pi r_t = \pi D_t$
From relation $\ell_t = \ell_0 (1 + \alpha.t)$, we get
$\pi D_t = \pi D_0 (1 + 2.3 \times 10^{-5} \times 100)$
$D_t = 2.54 (1 + 0.0023) = 2.5458$ cm

Example 3 :
If the volume of a block of a metal changes by 0.12% when it is heated through 20ºC, what is the coefficient of linear expansion of metal ?

Sol. Coefficient of cubical expansion of metal is given by

$\gamma = \dfrac{\Delta V}{V_t}$. Here $\gamma = \dfrac{\Delta V}{V_t} = \dfrac{0.12}{100}$, t = 20ºC

$\therefore \gamma = \dfrac{0.12}{100 \times 20} = 6.0 \times 10^{-5}$ per ºC

Coefficient of linear expansion

$\alpha = \dfrac{\gamma}{3} = \dfrac{6.0 \times 10^{-5}}{3} = 2.0 \times 10^{-5}$ Per ºC

Example 4 :
1 g of steam at 100ºC can melt how much ice at 0ºC ? Latent heat of ice = 80 cal/g and latent heat of steam = 540 cal/g.

Sol. Heat required by ice for melting of m g of ice = mL = m×80 cal
Heat available with steam for being condensed and then brought to 0ºC $= 1 \times 540 \times 100 = 640$ cal
$m \times 80 = 640$ or $m = 640/80 = 8$ grams

Example 5 :
A tap supplies water at 10ºC and another tap at 100ºC. How much hot water must be taken so that we get 20 kg of water at 35ºC ?

Sol. Let mass of hot water = m kg ?
mass of cold water $= (20 - m) \times 1 \times (35 - 10)$
Heat given by hot water $= m \times 1 \times (100 - 35)$
Law of mixture gives
Heat given by hot water = Heat taken by cold water
$m \times 1 \times (100 - 35) = (20 - m) \times (35 - 10)$
$\qquad 65m = (20 - m) \times 25$
$\qquad 65\,m = 500 - 25\,m$ or $\qquad 90\,m = 500$
$\qquad m = 500/90 = 5.56$ kg

Example 6 :
5g ice of 0ºC is mixed with 5g of steam at 100ºC. What is the final temperature ?

Sol. Heat required by ice to raise its temperature to 100ºC,
$Q_1 = m_1 L_1 + m_1 c_1 \Delta\theta_1$
$\qquad = 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 = 900$ cal
Heat given by steam when condensed,
$\qquad Q_2 = m_2 L_2 = 5 \times 536 = 2680$ cal
As $Q_2 > Q_1$. This means that whole steam is not even condensed. Temperature of mixture will remain at 100ºC

Example 7 :
Certain perfect gas is found obey $PV^{3/2}$ = const. during adiabatic process. If such a gas at initial temperature T is adiabatically compressed to half the initial volume, find its final temperature will be

Sol. $\therefore PV^{3/2} = $ constant (given)

Put $P = \dfrac{nRT}{V}$ $\quad \therefore \left(\dfrac{nRT}{V}\right) (V^{3/2}) = $ constant

When V change to V/2 the temperature becomes $\sqrt{2}T$.

Example 8 :
An air bubble of volume v_0 is released by a fish at a depth h in a lake. The bubble rises to the surface. Assume constant temperature and standard atmospheric pressure above the lake. Find the volume of the bubble just before touching the surface [(density) of water is ρ]

Sol. As the bubble rises the pressure gets reduced for constant temperature, if P is the standard atmospheric pressure, then

$$(p + \rho gh) v_0 = PV \quad \text{or} \quad V = V_0 \left(1 + \dfrac{\rho gh}{P}\right)$$

Example 9 :
Two gases occupy two containers A and B the gas in A, of volume 0.10m^3, exerts a pressure of 1.40 MPa and that in B of volume 0.15m^3 exerts a pressure 0.7 MPa. The two containers are united by a tube of negligible volume and the gases are allowed to intermingle. Then if the temperature remains constant, find the final pressure in the container. (in MPa)

Sol. We know that
$\qquad P_A V_A = nRT, P_B V_B = n_B RT$
and $P_f (V_A + V_B) = (n_A + n_B) RT$
$\qquad P_f (V_A + V_B) = P_A V_A + P_B V_B$

$$P_f = \left(\dfrac{P_A V_A + P_B V_B}{V_A + V_B}\right) = \dfrac{1.4 \times 0.1 + 0.7 \times 0.15}{0.1 + 0.15} \text{ MPa} = 9.8 \text{MPa}$$

Example 10 :
What is the degree of freedom of gas ? If at STP the velocity of sound in it is 330 m/sec gas density = 1.3 mg/cm^3.

Sol. $v = \sqrt{\dfrac{\gamma P}{\rho}}$, $P = 1.013 \times 10^5$ N/m^2, $\rho = 1.3$ kg/m^3, $v = 330$ m/s

$$\gamma = \dfrac{v^2 \rho}{p} = 1.4$$

Let f be the number of degree of freedom then

$C_v = fR/2$ and $C_p = fR/2 + R = R(1 + f/2)$

$$\therefore \ \gamma = \frac{C_p}{C_v} = \frac{2+f}{f} = 1.4 \Rightarrow \ (f=5)$$

Example 11 :

A body of mass 2Kg is dragged on a horizontal surface with constant speed of 2m/s. if the coefficient of friction between the body and the surface is 0.2, then find the heat generated in 5 sec.

Sol. The work done against the force of friction

$= \mu R \times$ distance $= 0.2 \times 2 \times 9.8 \times 2$ (in one second)

$= (0.2 \times 2 \times 9.8 \times 2) \times 5 = 39.2$ J (in 5 second)

Heat generated $= \dfrac{39.2}{4.2} = 9.33$ cal.

Example 12 :

A bullet, moving with velocity v, is stopped by the target and then completely melts if the mass of bullet is m. Its specific heat is 's', initially temperature is 25ºC, melting point is 475ºC and latent heat is L, then find the velocity V is given by the relation.

Sol. $W = JQ$ and $W = \dfrac{mv^2}{2}$ and $Q = mL + ms\,(\theta_2 - \theta_1)$

$\therefore \dfrac{mv^2}{2} = [mL + ms\,(\theta_2 - \theta_1)] \ \therefore \ \dfrac{mv^2}{2} = mL + ms\,(475 - 25)$

Example 13 :

One mole of an ideal monatomic gas is caused to go through the cycle shown in figure. Then find the change in the internal energy in expanding the gas from a to c along path abc.
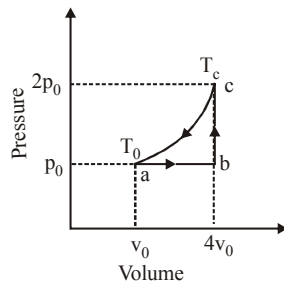
Sol. $\because \dfrac{PV}{T} = \mu R = $ constant
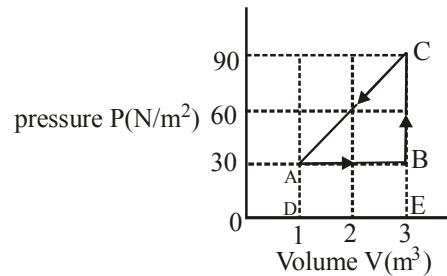
For any state of an ideal gas. Therefore

$\dfrac{P_a V_a}{T_a} = \dfrac{P_c V_c}{T_C}$ or $\dfrac{P_0 V_0}{T_0} = \dfrac{2P_0 4V_0}{T_c}$; $T_c = 8T_0$

Thus change in internal energy

$\Delta U = \mu\, C_v \Delta T = 1 \times \dfrac{3}{2} \times R \times 7T_0 = \dfrac{21}{2}\, RT_0 = 10.5\, RT_0$

Example 14 :

The figure shows the change in a thermodynamic system is going from an initial state A to the state B and C and returning to the state A. If $U_A = 0$, $U_B = 30$J an the heat given to the system in the process B → C, 50J, then determine :

(i) internal energy in the state C

(ii) heat given to the system in the process A → B

Sol. (i) Work done in the process B → C, W = 0 [∵ volume is constant] and heat given to the system

Q = 50J (given)

Hence, by the first law of thermodynamics, the change in the internal energy is

$\Delta U = (U_C - U_B) = Q - W = 50$J

$\Rightarrow \quad U_C = U_B + \Delta U = 30 + 50 = 80$J

(ii) For the process A → B, $\Delta U = U_B - U_A = 30$Joule and

W = area ABCD = DE × DA = 2 × 30 = 60J

$\therefore \quad Q = \Delta U + W = 30 + 60 = 90$J

Example 15 :

An ideal gas expands from state (P_1, V_1) to state (P_2, V_2) where $P_2 = 2P$ and $V_2 = 2V_1$. The path of the gas is expressed by the following relation $P = P_1 \left[1 + \left(\dfrac{V - V_1}{V_1}\right)^2\right]$ find work done

Sol. $W = \displaystyle\int_{v_1}^{2v_1} P dv = \int_{v_1}^{2v_1} P_1 \left[1 + \left(\dfrac{V - V_1}{V_1}\right)^2\right] dv$

$\Rightarrow W = P_1 \displaystyle\int_{v_1}^{2v_1} \left(1 + \dfrac{v^2 + v_1^2 - 2vv_1}{v_1^2}\right) dv$

$\Rightarrow W = P_1 \left[2V + \dfrac{V^3}{3V_1^2} - \dfrac{2V^2}{2V_1}\right]_{V_1}^{2V_1} \Rightarrow W = 4/3\, P_1 V_1$

Example 16 :

Calculate the root mean square velocity of the molecules of hydrogen at a temperature of 0°C and a pressure of 76 cm of mercury column when the density of hydrogen at NTP is 0.00009 g cm⁻³.

Sol. Pressure,

P = 76 cm of mercury column = 76 × 13.6 × 981 dyne/cm²

Density, $\rho = 0.00009$ gm/cm³ $\left[\because P = \dfrac{1}{3}\rho\right] v_{rms}^2$

rms velocity, $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3 \times 76 \times 13.6 \times 981}{0.0009}}$ cm/s

$= 1.838 \times 10^5$ cm/s.

Example 17 :

Calculate the molecular kinetic energy of a unit mass of helium at NTP.

Sol. Molecular weight, $M_w = 4$; Normal temperature, $T = 273$ K

Molar gas constant, $R = 8.31 \times 10^7$ erg mol^{-1} K^{-1}.

Molecular kinetic energy of a unit mass

$$= \frac{3}{2} \frac{RT}{M_w} = \frac{3}{2} \times \frac{8.31 \times 10^7 \times 273}{4} \text{ erg} = 8.507 \times 10^9 \text{ erg}$$

Example 18 :

From what height must a block of ice be dropped into a well of water so that 5% of it may melt. Given : both ice and water are at 0°C, $L = 80$ cal g^{-1}, $J = 4.2$ J cal^{-1} & g=980cm/s^2.

Sol. Let m be the mass of ice. Let h be the height from which block of ice is dropped. Work done, W=mgh=m×980× h erg

Mass of ice to be melted $= \frac{5}{100} \times mg$

Heat required, $Q = \frac{5}{100} \times m \times 80$ cal or $Q = 4$ m cal

$$[\because L = 80 \text{ cal g}^{-1}] [\because W = JQ]$$

Now,m × 980 × h = J × 4 m

$$h = \frac{4.2 \times 10^7 \times 4m}{m \times 980} \text{ cm} = 1714.3 \text{ m}$$

$$[\because J = 4.2 \times 10^7 \text{ erg cal}^{-1}]$$

Example 19 :

If hot air rises, why is it cooler at the top of a mountain than near sea level?

Sol. Pressure decreases with height. Therefore if hot air rises, it suffers adiabatic expansion. Therefore form first law of thermodynamics $\Delta Q = \Delta U + \Delta W$

$$\Delta U = -\Delta W \text{ (as } \Delta Q = 0)$$

This causes a decrease in internal energy and hence a fall of temperature. That is why it is cooler at the top of a mountain than at sea level.

Example 20 :

The temperatures T_1 and T_2 of the two heat reservoirs in an ideal Carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?

Sol. The efficiency of a Carnot's engine is given by $\eta = 1 - \dfrac{T_2}{T_1}$

Given $T_1 = 1500°C = 1500 + 273 = 1773$ K

and $T_2 = 500°C = 500 + 273 = 773$ K.

When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is $T'_1 = 1500 + 100 = 1600°C = 1873$ K. The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T'_1} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is $T'_2 = 500 - 100 = 400°C = 673$ K. The efficiency

now becomes $\eta'' = 1 - \dfrac{T'_2}{T_1} = 1 - \dfrac{673}{1773} = 0.62$

Since η'' is greater than η', decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperature of the source by 100°C.

Example 21 :

Calculate the change in entropy to convert 1 gm ice at −10°C to steam at 100°C.

Sol. −10°C ice → 0°C ice → 0°C water → 100°C water → 100°C steam

$$\Delta s = ms \log \frac{273}{263} + \frac{mL}{273} + ms \log \frac{373}{273} + \frac{mL}{373}$$

$$= 1 \times 0.5 \ \log \frac{273}{263} + \frac{1 \times 80}{273} + 1 \times 1 \ \log \frac{373}{273} + \frac{1 \times 540}{373}$$

Example 22 :

A copper sphere cools from 62°C to 50°C in 10 minutes and to 42°C in the next 10 minutes. Calculate the temperature of the surroundings.

Sol. By Newton's law of cooling,

$$\frac{\theta_1 - \theta_2}{t} = -k \left[\frac{\theta_1 + \theta_2}{2} - \theta_0 \right] \quad \text{....(1)}$$

A sphere cools from 62°C to 50°C in 10 min.

$$\frac{62 - 50}{10} = -k \left[\frac{62 + 50}{2} - \theta_0 \right] \quad \text{....(2)}$$

Now, sphere cools from 50°C to 42°C in next 10 min.

$$\frac{50 - 42}{10} = -k \left[\frac{50 + 42}{2} - \theta_0 \right] \quad \text{....(3)}$$

Dividing eqⁿ. (2) by (3) we get,

$$\frac{56 - \theta_0}{46 - \theta_0} = \frac{1.2}{0.8} \quad \text{or} \quad 0.4\theta_0 = 10.4$$

Hence $\theta_0 = 26°C$

Example 23 :

Two spheres of the same material have radii 6 cm and 9 cm respectively. They are heated to the same temperature and allowed to cool in the same enclosure. Compare their initial rates of emission of heat and initial rates of fall of temperature.

Sol. Given : $r_1 = 6$ cm, $r_2 = 9$ cm, $\quad \therefore \dfrac{r_1}{r_2} = \dfrac{2}{3}$

According to Stefan's law of radiation, the rate of emission of heat by an ordinary body,

$$R_h = \left(\frac{dQ}{dt}\right) = \sigma A e T^4 \text{ or } R_h \propto r^2 \qquad (A = 4\pi r^2)$$

$$\therefore \quad \frac{R_{h1}}{R_{h2}} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9} ; \quad \frac{R_{FT1}}{R_{FT2}} = \frac{r_2}{r_1} = \frac{3}{2}$$

[Rate of fall of temp. R_{FT},

$$\frac{d\theta}{dt} = \frac{\sigma A e (T^4 - T_0^4)}{msJ} \text{ or } R_{FT} \propto \frac{A}{m} \text{ or } R_{FT} \propto \frac{1}{r}$$

$\therefore$ Initial rates of emission of heat are in the ratio 4 : 9 and initial rates of fall of temperature are in the ratio 3 : 2.

Example 24 :

An aluminium rod and a copper rod of equal length 1m and cross-sectional area 1cm^2 are welded together as shown in the figure. One end is kept at a temperature of 20ºC and other at 60ºC. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminum is 200 W/mºC and of copper is 390 W/mºC.
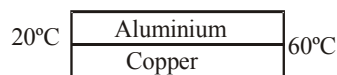
Sol. Heat current through the aluminium rod

$$= \frac{200 \times (1 \times 10^{-4})}{1} . (60 - 20)$$

Heat current through the copper rod

$$= \frac{390 \times (1 \times 10^{-4})}{1} . (60 - 20)$$

Total heat $= 200 \times 10^{-4} \times 40 + 390 \times 10^{-4} \times 40$

$= 590 \times 40 \times 10^{-4} = 2.36$ Joule

Example 25 :

The three rods shown in the figure (1) have identical geometrical dimensions. Heat flows from the hot end at the rate of 40W in arrangement (1). Find the rate of heat flow when the rods are joined in arrangement (2). Thermal conductivity of aluminum and copper are 200 W/mºC and 400 W/mºC respectively.

(1)

(2)
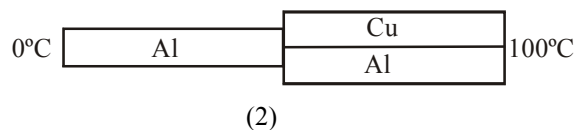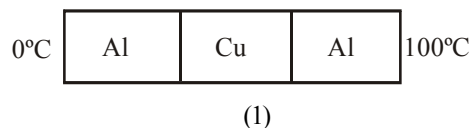
Sol. (a) In the arrangement (1), the three rods are joined in series. The rate of flow of heat,

$$\frac{d\theta}{dt} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{\theta_1 - \theta_2}{R}$$

But, $R = R_1 + R_2 + R_3$ [In series]

$$\therefore \quad 40 = \frac{100 - 0}{R_1 + R_2 + R_1} ; \quad 40 = \frac{100}{\dfrac{\ell}{K_1 A} + \dfrac{\ell}{K_2 A} + \dfrac{\ell}{KA}}$$

$$40 = \frac{100}{\dfrac{\ell}{A}\left[\dfrac{2}{K_1} + \dfrac{1}{K_2}\right]} ; \quad \frac{\ell}{A}\left[\frac{2}{200} + \frac{1}{400}\right] = \frac{100}{40}$$

$$\frac{\ell}{A} = 200 \text{ per m}$$

(b) In figure (2) two rods all in parallel and resultant of both is in series the first rod

$$\therefore \quad \frac{dQ}{dt} = \frac{\theta_1 - \theta_2}{R} \quad \text{But} \quad R = R_1 + \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

$$\frac{dQ}{dt} = \frac{100 - 0}{\dfrac{\ell}{K_1 A} + \dfrac{1}{\dfrac{K_1 A}{\ell} + \dfrac{K_2 A}{\ell}}}$$

$$= \frac{100 - 0}{\dfrac{\ell}{A}\left[\dfrac{1}{K_1} + \dfrac{1}{K_1 + K_2}\right]} = \frac{600 \times 100}{200 \times 4} = 75 W$$

Example 26 :

An ice box made of 1.5 cm thick styrofoam has dimensions 60 cm $\times$ 30cm. It contains ice at 0ºC and is kept in a room at 40ºC. Find the rate at which the ice is melting. Latent heat of fusion of ice $= 3.36 \times 10^5$ J/kg. and thermal conductivity of stryrofoam $= 0.4$ W/m-ºC.

Sol. The total surface area of the walls

$= 2(60 \text{ cm} \times 60 \text{ cm} + 60 \text{ cm} \times 30 \text{ cm} + 60 \text{ cm} \times 30 \text{ cm}) = 1.44 \text{ m}^2$

The thickness of the wall $= 1.5$ cm $= 0.015$m

The rate of heat flow into the box is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{x} = \frac{(0.04 \text{ W/m} - ºC)(1.44 \text{ m}^2)(40º C)}{0.015 \text{ m}}$$

$$= 154 \text{ W}.$$

Rate at which the ice melts is $= \dfrac{154 W}{3.36 \times 10^5 \text{ J/kg}} = 0.46$ g/s

Example 27 :

A black body emits 10 watts per cm^2 at 327ºC. The sun radiates 105 watt per cm^2 then what is the temperature of the sun ?

Sol. $\because \dfrac{E_{sun}}{E_{body}} = \left(\dfrac{E_{sun}}{E_{body}}\right)^4$ $\therefore \dfrac{T_{sun}}{T_{body}} = \left(\dfrac{10^5}{10}\right)^{1/4}$; T_{sun}=6000 K

Example 28 :

A bulb made of tungsten filament of surface are a 0.5 cm^2 is heated to a temperature 3000K when operated at 220V. The emissivity of the filament is $\in = 0.35$ and take $\sigma = 5.7 \times 10^{-8}$ mks units. Calculate the wattage of the bulb.

Sol. The emissive power watt/m^2 is $E = \in \sigma T^4$

Therefore the power of the bulb is $P = \in$ x area (Watts)

$\therefore P = \in A\sigma T^4$

$\therefore P = 0.35 \times 0.5 \times 10^{-4} \times 5, 7 \times 10^{-8} \times (3000)^4 = 80.8$ W

Example 29 :

In the above example, if the temperature of the filament falls to 2000K due to a drop of mains voltage, then what will be the wattage of the bulb ?

Sol. Now the power of the bulb will be such that

$\dfrac{P_2}{P_1} = \left(\dfrac{T_2}{T_1}\right)^4$. Thus $P_2 = P_1 \times \left(\dfrac{2}{3}\right)^4$

$\therefore P_2 = 80.8 \times \dfrac{16}{81} \Rightarrow P_2 = 15.96$

Example 30 :

Two liquids of same volume are cooled under same conditions from 65ºC to 50ºC. Time taken are 200sec and 480 sec. If ratio of their specific heats is 2 : 3 then find the ratio of their densities. (neglect the water equivalent of calorimeter)

Sol. From Newton's law of cooling

$\left(\dfrac{m_1 s_1 + w_1}{t_1}\right) (\theta_1 - \theta_2) = \left(\dfrac{m_2 s_2 + w_2}{t_2}\right) (\theta_1 - \theta_2)$

here $w_1 = w_2 = 0$

$\dfrac{m_1 s_1}{t_1} = \dfrac{m_2 s_2}{t_2} \Rightarrow \dfrac{V d_1 s_1}{t_1} = \dfrac{V d_2 s_2}{t_2}$

$\Rightarrow \dfrac{d_1}{d_2} = \dfrac{t_1 s_2}{t_2 s_1} = \dfrac{200}{480} \times \dfrac{3}{2} = \dfrac{5}{8}$

Example 31 :

A liquid takes 30 seconds to cool from 95ºC to 90ºC and 70 seconds to cool from 55 to 50ºC. Find the room temperature and the time it will take to cool from 50ºC to 45ºC

Sol. From the first data

$\dfrac{95 - 90}{30} = k\left(\dfrac{95 \times 90}{2} - \theta_0\right)$ ....(1)

From the second data

$\dfrac{55 - 50}{70} = k\left(\dfrac{55 - 50}{0} - \theta_0\right)$ ....(2)

Dividing (1) and (2) we get, $\dfrac{7}{3} = \dfrac{92.5 - \theta_0}{52.5 - \theta_0}$

Solution : ve to ge $\theta_0 = 22.5º$ ....(3)

Let the time taken in cooling from 50ºC to 45ºC is t, then

$\dfrac{50 - 45}{t} = k\left(\dfrac{50 - 45}{2} - \theta_0\right)$ ....(4)

Using $\theta_0 = 22.5ºC$, and dividing (1) by (2) we get

$\dfrac{t}{30} = \left(\dfrac{92.5 - 22.5}{47.5 - 22.5}\right) \Rightarrow t = 84$ sec

Example 32 :

At 1600 K maximum radiation is emitted at a wavelength of 2µm. Then find the corresponding wavelength at 2000 K.

Sol. Using $\lambda_{m_1} T = \lambda_{m_2} T_2$

$\therefore \lambda_{m_2} = \dfrac{\lambda_{m_2} T_1}{T_2}$ $\therefore \lambda_{m_2} = \dfrac{2 \times 10^{-6} \times 1600}{2000} = 1.6$ µm

Example 33 :

If the temperature of a body is increased by 50%, then find the increase in the amount of radiation emitted by it.

Sol. Percentage increase in the amount of radiations emitted

$\therefore \dfrac{E_2 - E_1}{E_1} \times 100 = \dfrac{(1.5T_1)^4 - T_1^4}{T_1^4} \times 100$

$\dfrac{E_2 - E_1}{E_1} \times 100 = [(1.5)^4 - 1] \times 10$; $\dfrac{E_2 - E_1}{E_1} \times 100 = 400\%$

Example 34 :

A blackened platinum wire of length 5cm and perimeter 0.02cm is maintained at a temperature of 300K. Then at what rate the wire is losing its energy ?

(Take s = 57 × 10⁻⁸ units)

Sol. The rate of radiation heat loss is $\dfrac{dQ}{dt} = \in A\sigma T^4$ (watts)

for blackened surface $\in = 1$

and $A = (2\pi r)\ell = P_{erimeter} \times$ length

$\therefore A = 0.02 \times 5 \times 10^{-2}$ Thus

$\therefore \dfrac{dQ}{dt} = 0.02 \times 5 \times 10^{-4} \times 5.7 \times 10^{-8} \times (3000)^4 = 46.2W$

QUESTION BANK   CHAPTER 12 : HEAT & THERMODYNAMICS

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : THERMAL EXPANSION

Q.1 Coefficient of volumetric expansion of solids and liquids is
(A) high with respect to gases.
(B) low with respect to gases.
(C) equals to that of gases
(D) high at low temperature and low at high temperature.

Q.2 There is a hole in the middle of a copper plate. When heating the plate, diameter of hole would
(A) increases   (B) decreases
(C) remains the same   (D) none of these

Q.3 With rise in temperature, density of a given body changes according to one of the following relations
(A) $\rho = \rho_0 [1 + \gamma \Delta T]$ (B) $\rho = \rho_0 [1 - \gamma \Delta T]$
(C) $\rho = \rho_0 \gamma \Delta T$ (D) $\rho = \rho_0 / \gamma \Delta T$

Q.4 The temperature at which centigrade and fahrenheit scales give the same reading, is –
(A) $-40°$ (B) $40°$
(C) $-30°$ (D) $30°$

Q.5 When temperature of water is raised from 0°C to 4°C, it –
(A) expands
(B) contracts
(C) expands upto 2°C and then contracts upto 4°C.
(D) contracts upto 2°C and then expands upto 4°C.

Q.6 The density of water at 4°C is 1000 kg m^{-3} and at 100°C it is 958.4 kg m^{-3}. The coefficient of volume expansion of water is –
(A) 4.5×10^{-3} °C^{-1} (B) 5.4×10^{-5} °C^{-1}
(C) 4.5×10^{-4} °C^{-1} (D) 5.4×10^{-6} °C^{-1}

PART - 2 : CALORIMETRY

Q.7 The latent heat of vaporization of a substance is always
(A) Greater than its latent heat of fusion
(B) Greater than its latent heat of sublimation
(C) Equal to its latent heat of sublimation
(D) Less than its latent heat of fusion

Q.8 540 g of ice at 0°C is mixed with 540 g of water at 80°C. The final temperature of the mixture is
(A) 0°C   (B) 40°C
(C) 80°C   (D) Less than 0°C

Q.9 100 gm of ice at 0°C is mixed with 100 g of water at 100°C. What will be the final temperature of the mixture
(A) 10°C   (B) 20°C
(C) 30°C   (D) 40°C

Q.10 Place the following in increasing order of the amount of heat required:
(a) vaporizing 1.0 kg of water
(b) melting 1.0 kg of ice
(c) heating 1.0 kg of water from 0°C to 100°C.
(A) a, b, c   (B) a, c, b
(C) b, a, c   (D) b, c, a

Q.11 Steam burns are more serious as
(A) steam at 100°C carries same heat as that of water 100°C but pressure of steam is more.
(B) steam is more reactive.
(C) steam has less surface tension and so it burns surface more rapidly.
(D) steam, at 100°C carries more heat than water at 100°C.

Q.12 Two liquids A and B are at 32°C and 24°C. When mixed in equal masses the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of –
(A) 3 : 2   (B) 2 : 3
(C) 1 : 1   (D) 4 : 3

PART - 3 : KINETIC THEORY OF AN IDEAL GASES

Q.13 When the temperature of a gas is raised from 27°C to 90°C, the percentage increase in the r.m.s. velocity of the molecules will be
(A) 10%   (B) 15%
(C) 20%   (D) 17.5%

Q.14 Choose the correct statement –
(A) Mean free path is proportional to temperature if pressure is kept constant.
(B) Mean free path is inversely proportional to pressure if temperature is kept constant.
(C) Both (A) and (B)
(D) None of these

Q.15 At what temperature, the mean kinetic energy of O_2 will be the same for H_2 molecules at $-73°C$
(A) 127°C   (B) 527°C
(C) $-73°C$ (D) $-173°C$

Q.16 Which one of the following is not an assumption of kinetic theory of gases?
(A) The volume occupied by the molecules of the gas is negligible.
(B) The force of attraction between the molecules is negligible.
(C) The collision between the molecules are elastic.
(D) All molecules have same speed.

Q.17 The kinetic theory of gases gives the formula
$PV = \frac{1}{3} Nm\bar{v}^2$ for the pressure P exerted by a gas enclosed in a volume V. The term Nm represents –
(A) the mass of a mole of the gas.
(B) the mass of the gas present in the volume V.
(C) the average mass of one molecule of the gas.
(D) the total number of molecules present in volume V.

Q.18 The energy of a gas/litre is 300 joules, then its pressure will
(A) 3×10^5 N/m^2 (B) 6×10^5 N/m^2
(C) 10^5 N/m^2 (D) 2×10^5 N/m^2

Q.19 Two molecules of a gas have speeds of 9×10^6 m/s and 1×10^6 m/s, respectively. What is the root mean square speed of these molecules.

    (A) 21×10^6 m/s   (B) $\sqrt{21} \times 10^6$ m/s

    (C) $\sqrt{41} \times 10^6$ m/s   (D) 40×10^6 m/s

Q.20 A jar has a mixture of hydrogen and oxygen gas in the ratio of 1 : 5. The ratio of mean kinetic energies of hydrogen and oxygen molecules is –

    (A) 1 : 16   (B) 1 : 4

    (C) 1 : 5   (D) 1 : 1

PART - 4 : SPECIFIC HEAT CAPACITY

Q.21 One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is –

    (A) 8R   (B) (3/2) R

    (C) 2R   (D) 2.5 R

Q.22 The heat capacity per mole of water is

(R is universal gas constant)

    (A) 9R   (B) (9/2) R

    (C) 6R   (D) 5R

Q.23 If specific heat of a substance is infinite, it means

    (A) Heat is given out

    (B) Heat is taken in

    (C) No change in temperature takes place whether heat is taken in or given out

    (D) All of the above

Q.24 The ratio of two specific heats (C_p/C_v) for CO is –

    (A) 1.33   (B) 1.40

    (C) 1.29   (D) 1.66

Q.25 If C_p and C_v denoted the specific heats of unit mass of nitrogen at constant pressure and volume respectively, then

    (A) $C_P - C_V = R/28$ (B) $C_P - C_V = R/7$

    (C) $C_P - C_V = R/14$ (D) $C_P - C_V = R$

Q.26 A mixture of ideal gases N_2 and He are taken in the mass ratio of 14 : 1 respectively. Molar heat capacity of the mixture at constant pressure is –

    (A) 6R/19   (B) 13R/6

    (C) 6R/13   (D) 19R/6

Q.27 A mixture of n_1 moles of monatomic gas and n_2 moles of diatomic gas has $\dfrac{C_p}{C_v} = \gamma = 1.5$.

    (A) $n_1 = n_2$ (B) $2n_1 = n_2$

    (C) $n_1 = 2n_2$ (D) $2n_1 = 3n_2$

PART - 5 : HEAT, WORK AND INTERNAL ENERGY

Q.28 In a thermodynamic system working substance is ideal gas, its internal energy is in the form of –

    (A) Kinetic energy only

    (B) Kinetic and potential energy

    (C) Potential energy

    (D) None of these

Q.29 Heat is not being exchanged in a body. If its internal energy is increased, then –

    (A) Its temperature will increase

    (B) Its temperature will decrease

    (C) Its temperature will remain constant

    (D) None of these

Q.30 Choose the correct option –

    (A) U is a state variable.

    (B) ΔU is path independent

    (C) ΔQ and ΔW are path dependent

    (D) All of the above

Q.31 If a system undergoes contraction of volume then the work done by the system will be

    (A) Zero   (B) Negligible

    (C) Negative   (D) Positive

Q.32 Work done by air when it expands from 50 litres to 150 litres at a constant pressure of 2 atm is

    (A) 2×10^4 J   (B) 2×100 J

    (C) $2 \times 10^5 \times 100$ J   (D) $2 \times 10^{-5} \times 100$ J

PART - 6 : FIRST LAW OF THERMODYNAMICS

Q.33 In thermodynamic process, 200 Joules of heat is given to a gas and 100 Joules of work is also done on it. The change in internal energy of the gas is

    (A) 100 J   (B) 300 J

    (C) 419 J   (D) 24 J

Q.34 In a given process for an ideal gas, dW = 0 and dQ < 0. Then for the gas –

    (A) The temperature will decrease

    (B) The volume will increase

    (C) The pressure will remain constant

    (D) The temperature will increase

Q.35 Choose the correct statement –

    (a) If Q > 0, heat is added to the system

    (b) If Q < 0, heat is removed to the system

    (c) If W > 0, Work is done by the system

    (d) If W < 0, Work is done on the system

    (A) a, b, c   (B) b, c, d

   (C) a, c, d   (D) all of these

PART - 7 : THERMODYNAMICS PROCESSES

Q.36 From the following V-T diagram we can conclude

    (A) $P_1 = P_2$ (B) $P_1 > P_2$

    (C) $P_1 < P_2$ (D) None
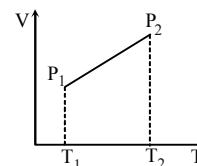
Q.37 A perfect gas contained in a cylinder is kept in vacuum. If the cylinder suddenly bursts, then the temperature of the gas –

    (A) Remains constant   (B) Becomes zero

    (C) Increases   (D) Decreases

Q.38 When an ideal gas is compressed adiabatically, its temperature rises; the molecules on the average have more kinetic energy than before. The kinetic energy increases –
(A) because of collisions with moving parts of the wall only.
(B) because of collisions with the entire wall.
(C) because the molecules get accelerated in their motion inside the volume.
(D) because of redistribution of energy amongst the molecules.

Q.39 In an isothermal expansion
(A) Internal energy of the gas increases
(B) Internal energy of the gas decreases
(C) Internal energy remains unchanged
(D) Average kinetic energy of gas molecule decreases

Q.40 Work done per mol in an isothermal change is

(A) $RT \log_{10} \dfrac{V_2}{V_1}$ 　　　　(B) $RT \log_{10} \dfrac{V_1}{V_2}$

(C) $RT \log_e \dfrac{V_2}{V_1}$ 　　　　(D) $RT \log_e \dfrac{V_1}{V_2}$

Q.41 The isothermal Bulk modulus of an ideal gas at pressure P is –
(A) P 　　　　(B) γP
(C) P / 2 　　　　(D) P/γ

Q.42 A container that suits the occurrence of an isothermal process should be made of
(A) Copper 　　　　(B) Glass
(C) Wood 　　　　(D) Cloth

Q.43 In an isothermal process the volume of an ideal gas is halved. One can say that
(A) Internal energy of the system decreases
(B) Work done by the gas is positive
(C) Work done by the gas is negative
(D) Internal energy of the system increases

Q.44 A monoatomic gas ($\gamma = 5/3$) is suddenly compressed to 1/8 of its original volume adiabatically, then the pressure of the gas will change to
(A) 24/5 　　　　(B) 8
(C) 40/3 　　　　(D) 32 times its initial pressure

Q.45 The pressure and density of a diatomic gas ($\gamma = 7/5$) change adiabatically from (P, d) to (P', d'). If d'/d = 32, then P'/P should be
(A) 1/128 　　　　(B) 32
(C) 128 　　　　(D) None of these

Q.46 Which is incorrect –
(A) In an isobaric process, $\Delta P = 0$
(B) In an isochoric process, $\Delta W = 0$
(C) In an isothermal process, $\Delta T = 0$
(D) In an isothermal process, $\Delta Q = 0$

Q.47 When heat in given to a gas in an isobaric process, then
(A) The work is done by the gas
(B) Internal energy of the gas increases
(C) Both (A) and (B)
(D) None of these

Q.48 The P-V diagram for adiabatic expansion of 0.2 mole of an ideal diatomic gas is shown in the figure. Work done by the gas during the process is approximately –
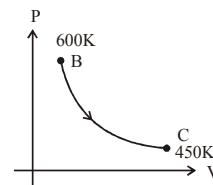

(A) zero 　　　　(B) – 623 J
(C) 623 J 　　　　(D) 301 J

PART - 8 : HEAT ENGINES AND REFRIGERATORS

Q.49 Choose the correct option.
(A) Heat engine cannot have 100% efficiency.
(B) Refrigerator cannot work without external work.
(C) Both (A) and (B) are correct
(D) None of these

Q.50 A Carnot engine operates between 227°C and 27°C. Efficiency of the engine is –
(A) 1/3 　　　　(B) 2/5
(C) 3/4 　　　　(D) 3/5

Q.51 The coefficient of performance (α) of a refrigerator is (when heat extractred is Q_2)

(A) $\dfrac{Q_2}{Q_2 - Q_1}$ 　　　　(B) $\dfrac{Q_2}{Q_1}$

(C) $\dfrac{Q_2 - Q_1}{W}$ 　　　　(D) $\dfrac{Q_2}{Q_1 - Q_2}$

Q.52 A heat engine has an efficiency η. Temperatures of source and sink are each decreased by 100K. The efficiency of the engine
(A) increases 　　　　(B) decreases
(C) remains constant 　　　　(D) becomes 1

Q.53 The temperature of sink of Carnot engine is 27°C. Efficiency of engine is 25%. Then temperature of source is
(A) 227°C 　　　　(B) 327°C
(C) 127°C 　　　　(D) 27°C

PART - 9 : CONDUCTION

Q.54 Mud houses are cooler in summer and warmer in winter because –
(A) Mud is superconductor of heat
(B) Mud is good conductor of heat
(C) Mud is bad conductor of heat
(D) None of these

Q.55 A slab consists of two parallel layers of copper and brass of the same thickness and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100°C and that of copper at 0°C, the temperature of interface is
(A) 80°C 　　　　(B) 20°C
(C) 60°C 　　　　(D) 40°C

Q.56 A metal rod of length 2m has cross sectional areas 2A and A as shown in figure. The ends are maintained at temperatures 100°C and 70°C. The temperature at middle point C is
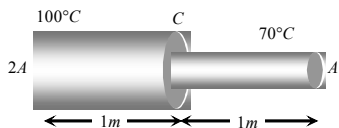
(A) 80°C   (B) 85°C
(C) 90°C   (D) 95°C

PART - 10 : CONVECTION

Q.57 It is hotter for the same distance over the top of a fire than it is in the side of it, mainly because –
(A) Air conducts heat upwards
(B) Heat is radiated upwards
(C) Convection takes more heat upwards
(D) Convection, conduction and radiation all contribute significantly transferring heat upwards.

Q.58 Which of the following statement(s) is/are correct?
I.   Convection is a mode of heat transfer by actual motion of matter.
II. Convection is possible only in gases.
III. Convection can be natural or forced.
(A) Only I   (B) Both I and III
(C) Only II   (D) All of these

Q.59 In which of the following process convection does not take place primarily?
(A) Sea and land breeze
(B) Trade wind
(C) Boiling of water
(D) Warming of glass of bulb due to filament

Q.60 The common example of forced convection systems are
I.   human blood circulatory system.
II. cooling system of an automobile engine.
III. human-liver system.
IV. water cycle.
(A) Only I   (B) Both I and II
(C) I. II and IV   (D) Both II and III

PART - 11 : RADIATION

Q.61 A black body at 2000K emits radiation with $\lambda_m = 1250$ nm. If for the radiation coming from the star SIRUS λ_m is 71 nm, then find the temperature of this star.
(A) 35211 K   (B) 32211 K
(C) 25211 K   (D) 15211 K

Q.62 Heat travels through vacuum by
(A) Conduction   (B) Convection
(C) Radiation   (D) Both (A) and (B)

Q.63 If between wavelength λ and $\lambda + d\lambda$, e_λ and a_λ be the emissive and absorptive powers of a body and E_λ be the emissive power of a perfectly black body, then according to Kirchoff's law, which is true
(A) $e_\lambda = a_\lambda = E_\lambda$ (B) $e_\lambda E_\lambda = a_\lambda$
(C) $e_\lambda = a_\lambda E_\lambda$ (D) $e_\lambda a_\lambda E_\lambda$ = constant

Q.64 An ideal black body at room temperature is thrown into a furnace. It is observed that
(A) Initially it is the darkest body and at later times the brightest.
(B) It is the darkest body at all times.
(C) It cannot be distinguished at all times.
(D) Initially it is the darkest body and at later times it cannot be distinguished.

Q.65 If black wire of platinum is heated, then its colour first appear red, then yellow and finally white. It can be understood on the basis of
(A) Wien's displacement law
(B) Prevost theory of heat exchange
(C) Newton's law of cooling
(D) None of the above

Q.66 If a body cools down from 80°C to 60°C in 10min when the temperature of the surrounding is 30°C, then the temperature of the body after next 10 min will be –
(A) 50°C   (B) 48°C
(C) 30°C   (D) None of these

Q.67 There is a black spot on a body. If the body is heated and carried in dark room then it glows more. This can be explained on the basis of –
(A) Newton's law of cooling (B) Wien's law
(C) Kirchoff's law   (D) Stefan's

Q.68 A students performs cooling experiment with a solid sphere and hollow sphere of same material and size which are heated to the same temperature. If the temperature difference between each sphere and surroundings is around 30°C, then –
(A) The hollow sphere will cool at a faster rate
(B) The solid sphere will cool at a faster rate
(C) Both spheres will cool at the same rate.
(D) Both spheres will cool at the same rate if temperature difference more than 30°C.

PART - 12 : MISCELLANEOUS

Q.69 A pendulum clock (fitted with a small heavy bob that is connected with a metal rod) is 5 seconds fast each day at a temperature of 15°C and 10 seconds slow at a temperature of 30°C. The temperature at which it is designed to give correct time, is –
(A) 18°C   (B) 20°C   (C) 24°C   (D) 25°C

Q.70 If H_C, H_K and H_F are heat required to raise the temperature of one gram of water by one degree in Celcius, Kelven and Fahrenheit temperature scales respectively then –
(A) $H_K > H_C > H_F$ (B) $H_F > H_C > H_K$
(C) $H_K = H_C > H_F$ (D) $H_K = H_C = H_F$

Q.71 One end of a uniform rod of length 1 m is placed in boiling water while its other end is placed in melting ice. A point P on the rod is maintained at a constant temperature of 800°C. The mass of steam produced per second is equal to the mass of ice melted per second. If specific latent heat of steam is 7 times the specific latent heat of ice, the distance of P from the steam chamber must be
(A) (1/7) m   (B) (1/8) m   (C) (1/9) m   (D) (1/10) m

Q.72 On an X temperature scale, water freezes at $-125.0°$ X and boils at $375.0°$X. On a Y temperature scale, water freezes at $-70.0°$ Y and boils at $-30.0°$ Y. The value of temperature on X-scale equal to the temperature of $50.0°$Y on Y-scale is –

(A) $455.0°$ X   (B) $-125.0°$ X
(C) $1375.0°$ X   (D) $1500.0°$ X

Q.73 A steel rod of length 1m is heated from 25°C to 75°C keeping its length constant. The longitudinal strain developed in the rod is (Given : Coefficient of linear expansion of steel $= 12 \times 10^{-6}$/°C)

(A) 6×10^{-6} (B) -6×10^{-5}
(C) -6×10^{-4} (D) zero

Q.74 A power radiated by a black body is P_0 and the wavelength corresponding to the maximum energy is around λ_0. On changing the temperature of the black body, it was observed that the power radiated is increased to $(256/81) P_0$. The shift in the wavelength corresponding to the maximum energy will be –

(A) $+ \lambda_0/4$ (B) $+\lambda_0/2$
(C) $- \lambda_0/4$ (D) $-\lambda_0/2$

Q.75 Two elastic rods are joined between fixed supports as shown in figure. Condition for no change in the lengths of individual rods with the increase of temperature.

(α_1, α_2 = linear expansion coefficient, A_1, A_2 = area of rods, y_1, y_2 = Young modulus)
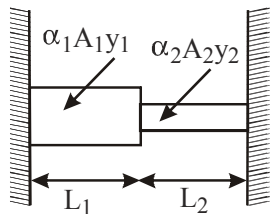
(A) $\dfrac{A_1}{A_2} = \dfrac{\alpha_1}{\alpha_2} \dfrac{y_1}{y_2}$ (B) $\dfrac{A_1}{A_2} = \dfrac{L_1\alpha_1 y_1}{L_2\alpha_2 y_2}$

(C) $\dfrac{A_1}{A_2} = \dfrac{L_2\alpha_2 y_2}{L_1\alpha_1 y_1}$ (D) $\dfrac{A_1}{A_2} = \dfrac{\alpha_2 y_2}{\alpha_1 y_1}$

Q.76 There are four objects A, B, C and D. It is observed that A and B are in thermal equilibrium and C and D are also in thermal equilibrium. However, A and C are not in thermal equilibrium. We can conclude that –

(A) B and D are in thermal equilibrium
(B) B and D could be in thermal equilibrium but might not be
(C) B and D cannot be in thermal equilibrium
(D) The zeroth law of thermodynamics does not apply here because there are more than three objects.

Q.77 In an adiabatic expansion the product of pressure and volume –

(A) decreases   (B) increases
(C) remains constant   (D) first increase then decreases

Q.78 Some of the thermodynamic parameters are state variables while some are process variables. Some grouping of the parameters are given. Choose the correct one –

(A) State variables : Temperature, No. of moles ; Process variables : Internal energy, work done by the gas
(B) State variables : Volume, Temperature ; Process variables : Internal energy, work done by the gas
(C) State variables : work done by the gas, heat rejected by the gas. ; Process variables : Temperature, volume
(D) State variables : Internal energy, volume ; Process variables : Work done by the gas, heat absorbed by the gas

Q.79 A cube, a pyramid (with all four faces identical) and a sphere (all of them hollow) are made from the same material and have equal mass and bound equal volume. They are heated to the same temperature and then left to cool. After some time,
1. sphere will have the highest temperature
2. pryamid will have the highest temperature.
3. cube will have the lowest temperature.
4. sphere will have the lowest temperature.
5. pyramid will have the lowest temperature
Correct option will be :

(A) 2 and 3   (B) 3 and 1
(C) 1 and 5   (D) 2 and 4

Q.80 A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric process, process 2 is isothermal and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas is these three processes.Then –
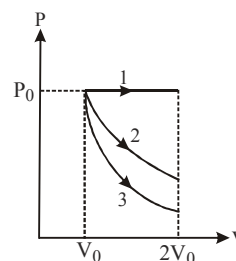
(A) $\Delta U_1 > \Delta U_2 > \Delta U_3$ (B) $\Delta U_1 < \Delta U_2 < \Delta U_3$
(C) $\Delta U_2 < \Delta U_1 < \Delta U_3$ (D) $\Delta U_2 < \Delta U_3 < \Delta U_1$

Q.81 Which of the following will have maximum total kinetic energy at temperature 300 K –

(A) 1 kg, H_2 (B) $\dfrac{1}{2}$ kg $H_2 + \dfrac{1}{2}$ kg He

(C) $\dfrac{1}{2}$ kg $H_2 + \dfrac{3}{4}$ kg He   (D) 1 kg, He

Q.82 A body cools from 50°C to 40°C in 5 minutes. The surrounding temperature is 20°C. In what further time (in minutes) will it cool to 30°C?

(A) 5   (B) 15/2
(C) 25/3   (D) 10

Q.83 Four particles have velocities 1, 0, 2, 3 m/s. The root mean square of the particles is (in m/s)

(A) 3.5　　　　　　　　(B) $\sqrt{3.5}$

(C) 1.5　　　　　　　　(D) $\sqrt{14/3}$

Q.84 The specific heat of Ar at constant volume is $0.075\ \text{kg}^{-1}\ \text{K}^{-1}$. Calculate the atomic weight ($R = 2\ \text{cal mol}^{-1}\ \text{K}^{-1}$)

(A) 40　　　　　　　　(B) 40.4

(C) 40.2　　　　　　　(D) 40.80

Q.85 In a process the density of gas remain constant. If the temperature is doubled, then the change in the pressure will be

(A) 100% increase　　　　(B) 200% increase

(C) 50% decrease　　　　(D) 25% decrease

Q.86 What is the average velocities of particles is a gas –

(A) $\sqrt{\dfrac{3RT}{M}}$　　　　　(B) zero

(C) $\sqrt{\dfrac{2RT}{M}}$　　　　　(D) $\sqrt{\dfrac{8RT}{\pi M}}$

Q.87 Consider two identical iron spheres , one which lie on a thermally insulating plate, while the other hangs from an insulatory thread. Equal amount of heat is supplied to the two spheres–
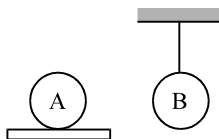
(A) Temperature of A will be greater than B.

(B) Temperature of B will be greater than A.

(C) Their temperature will be equal.

(D) Can't be predicted.

Q.88 Let V denote the root mean square speed of the molecules in an ideal diatomic gas at absolute temperature T. The mass of a molecule is 'm'. Neglecting vibrational energy terms, which is false?

(A) A molecule can have a speed greater than $\sqrt{2}$ v.

(B) V is proportional to $\sqrt{T}$.

(C) The average rotational kinetic energy of a molecule is $mv^2/4$.

(D) The average kinetic energy of a molecule is $5mv^2/6$.

**Q.89** Two holes of unequal diameters d_1 and d_2 ($d_1 > d_2$) are cut in a metal sheet. If the sheet is heated
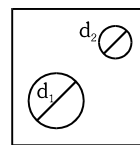
(A) both d_1 and d_2 will decrease.

(B) both d_1 and d_2 will increase.

(C) d_1 will increase, d_2 will decrease.

(D) d_1 will decrease, d_2 will increase

Q.90 A horizontal tube, open at both ends, contains a column of liquid. The length of this liquid column does not change with temperature. Let γ = coefficient of volume expansion of the liquid and a = coefficient of linear expansion of the material of the tube.

(A) $\gamma = \alpha$　　　　　　(B) $\gamma = 2\alpha$

(C) $\gamma = 3\alpha$　　　　　　(D) $\gamma = 0$

Q.91 In a vertical U-tube containing a liquid, the two arms are maintained at different temperatures, t_1 and t_2. The liquid columns in the two arms have heights ℓ_1 and ℓ_2 respectively. The coefficient of volume expansion of the liquid is equal to

(A) $\dfrac{\ell_1 - \ell_2}{\ell_2 t_1 - \ell_1 t_2}$　　　　(B) $\dfrac{\ell_1 - \ell_2}{\ell_1 t_1 - \ell_2 t_2}$
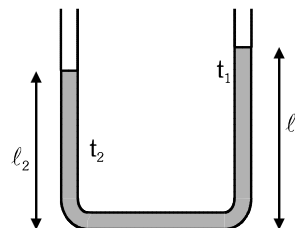
(C) $\dfrac{\ell_1 + \ell_2}{\ell_2 t_1 + \ell_1 t_2}$　　　　(D) $\dfrac{\ell_1 + \ell_2}{\ell_1 t_1 + \ell_2 t_2}$

Q.92 Maxwell's velocity distribution curve is given for two different temperature. For the given curves.
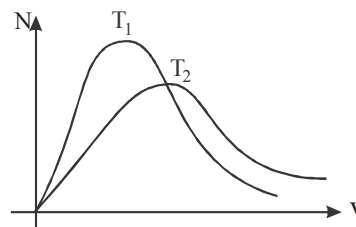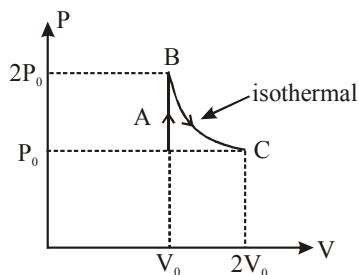

(A) $T_1 > T_2$　　　　　　(B) $T_1 < T_2$

(C) $T_1 \le T_2$　　　　　　(D) $T_1 = T_2$

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

Q.1 A diatomic ideal gas undergoes a thermodynamic change according to the P-V diagram shown in the figure. The total heat given to the gas is nearly (use ln 2 = 0.7)


(A) $2.5\,P_0V_0$ (B) $1.4\,P_0V_0$
(C) $1.1\,P_0V_0$ (D) $3.9\,P_0V_0$

Q.2 An ideal gas with adiabatic exponent ($\gamma = 1.5$) undergoes a process in which work done by the gas is same as increase in internal energy of the gas. The molar heat capacity of gas for the process is –
(A) $C = 4R$ (B) $C = 0$
(C) $C = 2R$ (D) $C = R$

Q.3 For an ideal gas four processes are marked as 1, 2, 3 and 4 on P-V diagram as shown in figure. The amount of heat supplied to the gas in the process 1, 2, 3 and 4 are Q_1, Q_2, Q_3 and Q_4 respectively, then correct order of heat supplied to the gas is – [AB is process-1, AC is process-2, AD is adiabatic process-3 &AE is process-4]
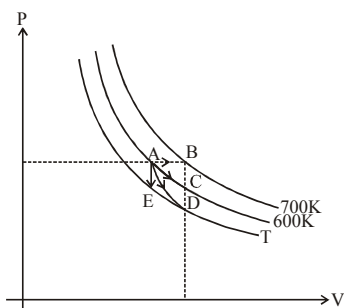
(A) $Q_1 > Q_2 > Q_3 > Q_4$ (B) $Q_1 > Q_2 > Q_4 > Q_3$
(C) $Q_1 > Q_4 > Q_2 > Q_3$ (D) $Q_1 < Q_2 < Q_3 < Q_4$

Q.4 In a process the pressure of a gas is inversely proportional to the square of the volume. If temperature of the gas is increases, then work done by the gas –
(A) is positive   (B) is negative
(C) is zero   (D) may be positive

**Q.5** One mole of an ideal gas is taken from state A to state B by three different processes, (A) ACB (B) ADB (C) AEB as shown in the P-V diagram. The heat absorbed by the gas is–
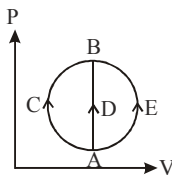
(A) greater in process (B) than in (A).
(B) the least in process (B).
(C) the same in (A) and (C).
(D) less in (C) then in (B).

Q.6 All the rods are made of same material and have equal cross-sectional area. Length of rods are such that $L_1 = L_2 = L_4 = L_5 \neq L_3$.
Temperature $\theta_A = \theta_B > \theta_C > \theta_D > \theta_E = \theta_F$. Consider the following statements for steady state.
(i) As $\alpha_1 \neq \alpha_2$, power through AC $\neq$ power through DF.
(ii) Power through CD = 2 (power through DF) which of the following is correct –
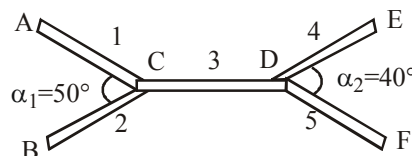
(A) statement (i) is correct but (ii) is incorrect
(B) statement (i) is incorrect but (ii) is correct
(C) both are incorrect
(D) both are correct

Q.7 A black body emits radiation at the rate P when its temperature is T. At this temperature the wavelength at which the radiation has maximum intensity is λ_0. If at another temperature T' the power radiated is P' and wavelength at maximum intensity is $\lambda_0/2$ then –
(A) $P'T' = 32\,PT$ (B) $P'T' = 16\,PT$
(C) $P'T' = 8\,PT$ (D) $P'T' = 4\,PT$

Q.8 Suppose 0.5 mole of an ideal gas undergoes an isothermal expansion as energy is added to its heat Q. Graph shows the final volume V_f versus Q.
The temperature of the gas is
(use ln 9 = 2 and R = 25/3 J/mol-K)
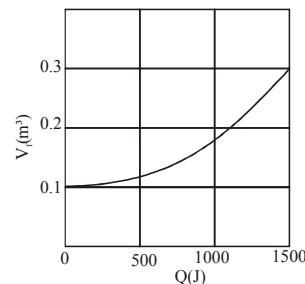
(A) 360 K   (B) 293 K
(C) 386 K   (D) 412 K

Q.9 The emissive power of a black body at T = 300 K is 100 Watt/m². Consider a body B of area A = 10m² coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5. Its temperature is 300 K. Then which of the following is incorrect –
(A) The emissive power of B is 20 W/m²
(B) The emissive power of B is 200 W/m²
(C) The power emitted by is 200 watts
(D) The emissivity of B is 0.2

Q.10 When a sample of a gas is taken from stable i to state f along the path 'iaf', heat supplied to the gas is 50 cal and work done by the gas is 20 cal. If it is taken by path 'ibf', then heat supplied is 36 cal.
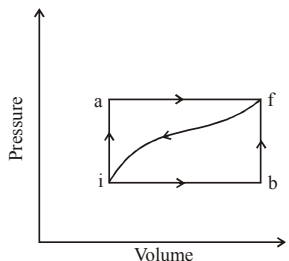
(a) Work done by the gas along path ibf is 6 cal

(b) If work done upon the gas is 13 cal for the return path 'fi', then heat rejected by the gas along path 'fi' is 43 cal.

(c) If internal energy of the gas at state i is 10 cal, then internal energy at state 'f' is 40 cal.

(d) If internal energy at stable 'b' is 22 cal and at 'i' is 10cal then heat supplied to the gas along path 'ib' is 18 cal.

(A) ab   (B) ac
(C) abc   (D) abcd

Q.11 5 moles of nitrogen gas are enclosed in an adiabatic cylindrical vessel. The piston itself is a rigid light cylindrical container containing 3 moles of Helium gas. There is a heater which gives out a power 100 cal to the nitrogen gas. A power of 30 cal is transferred to Helium through the bottom surface of the piston. The rate of increment of temperature of the nitrogen gas assuming that the piston moves slowly :
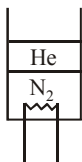
(A) 2K/sec   (B) 4K/sec
(C) 6K/sec   (D) 8K/sec

Q.12 A parallel plate capacitor of capacitance 200 μF is charged by a battery of emf 100 V. The battery is now disconnected and temperature of the plates is equal to atmospheric temperature. The plates are now connected by a thin wire of negligible heat capacity. Assume 50 % of their stored energy increases their temperature till the capacitor gets completely discharged and energy equally distributes over the plates. If thermal capacity of each plate is 0.5 J K^{-1} and co-efficient of linear expansion is 2×10^{-5} °C^{-1} , percentage increase in the volume of the plates is :

(A) 0.001 %   (B) 0.002 %
(C) 0.003 %   (D) 0.004 %

Q.13 Two thin walled spheres of different materials, one with double the radius and one-fourth wall thickness of the other, are filled with ice. If the time taken for complete melting of ice in the sphere of larger radius is 25 minutes

and that for smaller one is 16 minutes, the ratio of thermal conductivities of the materials of larger sphere to the smaller sphere is :

(A) 4 : 5   (B) 25 : 1
(C) 1 : 25   (D) 8 : 25

Q.14 In two experiments with a continuous flow calorimeter to determine the specific heat capacity of a liquid , an input power of 60 W produced a rise of 10 K in the liquid. When the power was doubled, the same temperature rise was achieved by making the rate of flow of liquid three times faster. The power lost to the surrounding in each case was

(A) 20 W   (B) 30 W
(C) 40 W   (D) 120 W

Q.15 0.5 mole of an ideal gas at constant temperature 27°C kept inside a cylinder of length L and cross-section area A closed by a massless piston. The cylinder is attached with a conducting rod of length L , cross-section area (1/9) m^2 and thermal conductivity k, whose other end is maintained at 0°C. If piston is moved such that rate of heat flow through the conducing rod is constant then velocity of piston when it is at height L/2 from the bottom of cylinder is : [neglect any kind of heat loss from system]
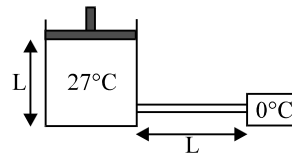
(A) $\left(\dfrac{k}{R}\right)$ m / sec   (B) $\left(\dfrac{k}{10R}\right)$ m / sec

(C) $\left(\dfrac{k}{100R}\right)$ m / sec   (D) $\left(\dfrac{k}{1000R}\right)$ m / sec

Q.16 In an H$_2$ gas process , PV^2 = constant. The ratio of work done by gas to change in internal energy is :

(A) 2/3   (B) 0.4
(C) -0.4 (D) $-2/3$

Q.17 The density of steel at 0 °C is 8.0 g/cc. At what temperature is density lesser by 0.1 %? Co-efficient of linear expansion of steel is 10^{-5} / °C .

(A) 37.2 °C   (B) 33.3 °C
(C) 55.4 °C   (D) 40.2 °C

Q.18 A process $1 \rightarrow 2$ using diatomic gas is shown on the P-V diagram below. $P_2 = 2 P_1 = 10^6$ N/m^2, $V_2 = 4 V_1 = 0.4$ m^3 . The molar heat capacity of the gas in this process will be :
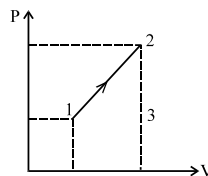
(A) 35R/12   (B) 25R/13
(C) 35R/11   (D) 22R/7

Q.19 A thin steel ring of inner diameter 40 cm and cross-sectional area 1 mm^2, is heated until it easily slides on a rigid cylinder of diameter 40.05 cm .When the ring cools down, the tension in the ring will be :

[For steel, $\alpha = 10^{-5}$ /°C , Y = 200 GPa]

(A) 1000 N   (B) 500 N
(C) 250 N   (D) 100 N

Q.20 The power radiated by a black body is P, and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around a wavelength $3\lambda_0/4$, the power radiated by it will increase by a factor of

(A) 4/3   (B) 16/9
(C) 64/27   (D) 256/81

Q.21 For two thermodynamics process temperature and volume diagram are drawn. In first process, it is a straight line having initial and final coordinates as (V_0, T_0) and $(2V_0, 2T_0)$, where as in second process it is a rectangular hyperbola having initial and final coordinates (V_0, T_0) and $(2V_0, T_0/2)$. Then ratio of work done in the two processes must be –
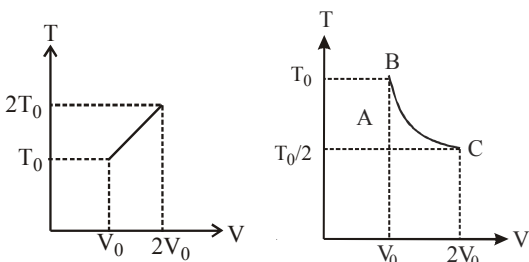
(A) 1 : 2   (B) 2 : 1
(C) 1 : 1   (D) None of these

Q.22 The molar heat capacity C for an ideal gas going through a given process is given by C = a/T , where 'a' is a constant. If $\gamma = C_P/C_V$, the work done by one mole of gas during heating from T_0 to ηT_0 through the given process will be:

(A) $\dfrac{1}{a} \ln \eta$ (B) $a \ln \eta - \left(\dfrac{\eta - 1}{\gamma - 1}\right) RT_0$

(C) $a \ln \eta - (\gamma - (A) RT_0$ (D) none of these

Q.23 The average degree of freedom per molecule for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. The heat absorbed by the gas is

(A) 75 J   (B) 100 J
(C) 150 J   (D) 125 J

Q.24 In a 10-metre-deep lake, the bottom is at a constant temperature of 4°C. The air temperature is constant at -4°C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be

(A) 7.5 m   (B) 6 m
(C) 5 m   (D) 2.5 m

Q.25 A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

(A) 100 J/°C   (B) 300 J/°C
(C) 750 J/°C   (D) 1500 J/°C

Q.26 Equal masses of three liquids A, B and C have temperatures 10°C, 25°C and 40°C respectively. If A and B are mixed, the mixture has a temperature of 15°C. If B and C are mixed, the mixture has a temperature of 30°C. If A and C are mixed, the mixture will have a temperature of
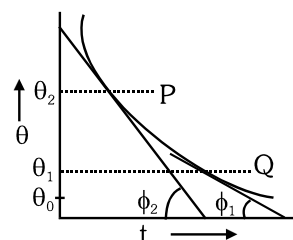
(A) 16°C   (B) 20°C
(C) 25°C   (D) 29°C

Q.27 If water at 0°C, kept in a container with an open top, is placed in a large evacuated chamber,

(A) all the water will vaporize

(B) all the water will freeze

(C) part of the water will vaporize and the rest will freeze

(D) ice, water and water vapour will be formed and reach equilibrium at the triple point

Q.28 A and B are two points on a uniform metal ring whose centre is C. The angle ACB = θ. A and B are maintained at two different constant temperatures. When θ = 180°, the rate of total heat flow from A to B is 1.2 W. When θ = 90°, this rate will be

(A) 0.6 W   (B) 0.9 W   (C) 1.6 W   (D) 1.8 W

Q.29 A body cools in a surrounding which is at a constant temperature of θ_0. Assume that it obeys Newton's law of cooling. Its temperature θ is plotted against time t. Tangents are drawn to the curve at the points P ($\theta = \theta_1$) and Q($\theta - \theta_2$). These tangents meet the time axis at angle of ϕ_2 and ϕ_1, as shown.

(A) $\dfrac{\tan \phi_2}{\tan \phi_1} = \dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (B) $\dfrac{\tan \phi_2}{\tan \phi_1} = \dfrac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

(C) $\dfrac{\tan \phi_1}{\tan \phi_2} = \dfrac{\theta_1}{\theta_2}$ (D) $\dfrac{\tan \phi_1}{\tan \phi_2} = \dfrac{\theta_2}{\theta_1}$

Q.30 The temperature of an air bubble while rising from bottom to surface of a lake remains constant but its diameter is doubled if the pressure on the surface is equal to h meter of mercury column and relative density of mercury is ρ then the depth of lake is

(A) 2ρhm   (B) 4 ρhm   (C) 8ρhm   (D) 7 ρhm

Q.31 Two identical containers A and B have frictionless pistons. They contain the same volume of a ideal gas at the same temperature. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. The changes in the pressure in A and B are found to be Δp and $1.5\ \Delta p$ respectively.

(A) $4m_A = 9m_B$ (B) $2m_A = 3m_B$

(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

Q.32 Three moles of an ideal monoatomic gas perform a cycle shown in figure. The gas temperatures in different states are $T_1 = 200K$, $T_2 = 400K$, $T_3 = 1600\ K$, and $T_4 = 800K$. The work done by the gas during the cycle is

(Take $R = 25/3$ J/mol-K)
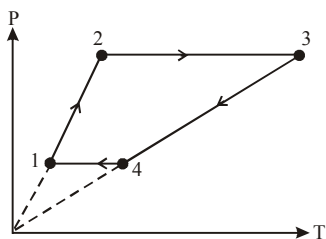
(A) 5 kJ   (B) 25 kJ

(C) 15 kJ   (D) 20 kJ

Q.33 The state of an ideal gas is changed through an isothermal process at temperature T_0 as shown in figure. The work done by gas in going from state B to C is double the work done by gas in going from state A to B. If the pressure in the state B is $P_0/2$ then the pressure of the gas in state C is–
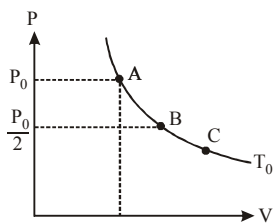

(A) $P_0/2$ (B) $P_0/4$

(C) $P_0/6$ (D) $P_0/8$

Q.34 A copper sphere is suspended in an evacuated chamber maintained at 300K. The sphere is maintained at a constant temperature of 500K by heating it electrically. A total of 300W of electric power is needed to do it. When half of the surface of the copper sphere is completely blackened, 600W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper –

(A) e = 1/3   (B) e = 2/3

(C) e = 1/2   (D) e = 1/6

Q.35 In the P-V diagram show, the gas does 5J of work in isothermal process ab and 4J in adiabatic process bc. What will be the change in internal energy of the gas in straight path c to a ?
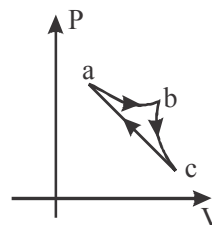
(A) 9J   (B) 1J

(C) 4J   (D) 5J

Q.36 Figure shows three different arrangements of material 1, 2 and 3 to form a wall. Thermal conductivities are $k_1 > k_2 > k_3$. The left side of the wall is 20°C higher than the right side. Temperature difference ΔT across the material 1 has following relation in three cases :
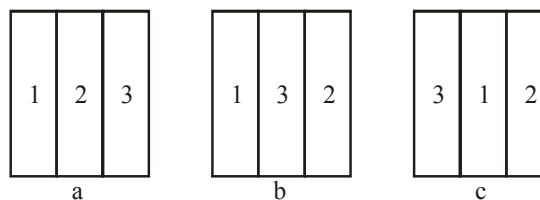

(A) $\Delta T_a > \Delta T_b > \Delta T_c$ (B) $\Delta T_a = \Delta T_b = \Delta T_c$

(C) $\Delta T_a = \Delta T_b > \Delta T_c$ (D) $\Delta T_a = \Delta T_b < \Delta T_c$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

**NOTE : The answer to each question is a NUMERICAL VALUE.**

Q.1 A diatomic gas is heated at constant volume until the pressure is doubled and again heated at constant pressure until volume is doubled. The average molar heat capacity for whole process is 19R/A. Find the value of A

Q.2 There are two thin spheres A and B of the same material and same thickness. They emit like black bodies. Radius of A is double that of B. A and B have same temperature T. When A and B are kept in a room of temperature T_0 (< T), the ratio of their rates of cooling (rate of fall of temperature) is 1 : a. Find the value of a.
[assume negligible heat exchange between A and B]

Q.3 A gas is suddenly expanded such that its final volume becomes 3 times its initial volume. If the specific heat at constant volume of the gas is 2R, then find approximately the ratio of initial to final pressure.

Q.4 In a process the density of a gas remains constant. If the temperature is doubled, then new pressure becomes x time initial pressure. Find the value of x.

Q.5 The narrow tube with one of its ends scaled as shown in the figure, is in a vertical plane. In the 3L long horizontal part of the tube a mercury column of length L blocks some oxygen gas of length L. The outside air-pressure of p_0 equals with the pressure of a mercury column of height L. Increasing the temperature of the surrounding, the volume of the blocked gas doubles while the gas absorbs Q = 7J of heat from its surroundings. How much work (in joule) is done by the expanding gas ?

Q.6 If hydrogen were monoatomic, what temperature would the average translational kinetic energy be equal to the energy required to raise a hydrogen atom from the ground state to the n = 2 excited state ? (Rhc = 14 eV, k = 1×10^{-23} J/K) k is Boltzmann's constant. The answer is $x \times 10^3$ K. Find x.

Q.7 Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 litres of gas at 300 K and 2×10^5 N/m² pressure while vessel B has 4 litres of gas at 350 K and 4×10^5 N/m² pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. Calculate new temperature (in K) upto 3 significant figures.

Q.8 A resistance coil, wired to an external battery is placed inside a thermally insulated cylinder fitted with a frictionless piston and containing an ideal gas. A current I = 240 mA flows through the coil, which has resistance R = 490Ω. At what constant speed v (in cm/s) must the piston of mass m = 12kg move upward in order that the temperature of the gas remains unchanged ? [Assume that 50% energy is utilized to do work by the gas, and neglect the work done by atmospheric pressure.]
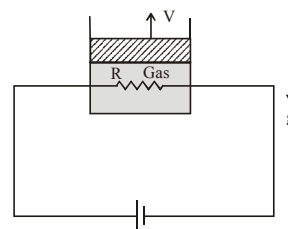
Q.9 0.5 mole of an ideal gas is kept inside an adiabatic cylinder of length L and cross-sectional area A closed by massless adiabatic piston. The cylinder is attached with a conducting rod of length L, cross-section area $\frac{1}{900}$ m² and thermal conductivity 415.5 W/m-K, whose other end is maintained at 0°C. The piston is moved such that the temperature of the gas remains constant at 27°C. Find the velocity (in mm/sec) of piston when it is at height L/2 from the bottom of cylinder. Rod is well lagged and has negligible heat capacity. R = 8.31 J/mol-K.
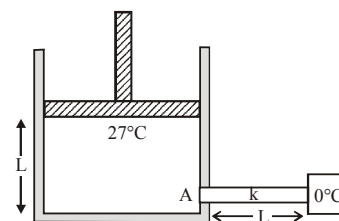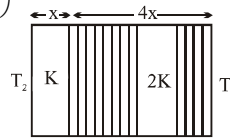

Q.10 A heat engine uses an ideal gas (γ = 1.40) that undergoes the reversible cycle shown in figure. Obtain the thermodynamic percentage efficiency of the engine.
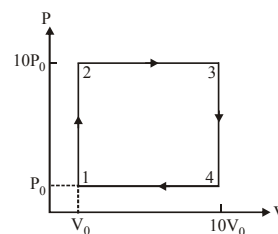
Q.11 A flask of volume 2 liter, containing a piece of iron, volume 1 dm³, is filled to the brim with water. How much water will overflow in mm³ if the temperature is raised from 10°C to 60°C ? (The coefficient of linear expansion of iron is 12×10^{-6}/°C, that of glass 8×10^{-6}/°C and the coefficient of volume expansion of water is 1.8×10^{-4}/°C)

Q.12 We want to cool down 15 litre of 80°C water to 20°C by putting 0°C ice into it. What minimum volume (in litres) should the vessel have to prevent the water from overflowing at the rim ?
(Latent heat of fusion of ice is 80 cal/g)

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 Heat given to a body which raises its temperature by 1°C is– **[AIEEE-2002]**
(A) Water equivalent
(B) Thermal capacity
(C) Specific heat
(D) Temperature gradient

Q.2 If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should **[AIEEE-2002]**
(A) Increase
(B) Remain unchanged
(C) Decrease
(D) First increase then decrease

Q.3 Which of the following is more close to a black body ? **[AIEEE-2002]**
(A) Black board paint
(B) Green leaves
(C) Black holes
(D) Red roses

Q.4 Infrared radiation is detected by– **[AIEEE-2002]**
(A) Spectrometer
(B) Pyrometer
(C) Nanometer
(D) Photometer

Q.5 Two spheres of the same material have radii 1m and 4m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is **[AIEEE-2002]**
(A) 1 : 1
(B) 16 : 1
(C) 4 : 1
(D) 1 : 9

Q.6 Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will **[AIEEE-2002]**
(A) increase
(B) decrease
(C) remain same
(D) decrease for some, while increase for others

Q.7 At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47° C?
(A) 80 K
(B) – 73 K **[AIEEE-2002]**
(C) 3 K
(D) 20 K

Q.8 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of γ for the resulting mixture is– **[AIEEE-2002]**
(A) 7/5
(B) 2/5
(C) 24/16
(D) 12/7

Q.9 Even Carnot engine cannot give 100% efficiency because we cannot – **[AIEEE-2002]**
(A) prevent radiation
(B) find ideal sources
(C) reach absolute zero temperature
(D) eliminate friction

Q.10 Which statement is incorrect ? **[AIEEE-2002]**
(A) all reversible cycles have same efficiency.
(B) reversible cycle has more efficiency than an irreversible one.
(C) Carnot cycle is a reversible one.
(D) Carnot cycle has the maximum efficiency in all cycles.

Q.11 "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of **[AIEEE-2003]**
(A) second law of thermodynamics
(B) conservation of momentum
(C) conservation of mass
(D) first law of thermodynamics

Q.12 Which of the following parameters does not characterize the thermodynamic state of matter? **[AIEEE-2003]**
(A) temperature
(B) Pressure
(C) Work
(D) Volume

Q.13 A Carnot engine takes 3×10^6 cal. of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is – **[AIEEE-2003]**
(A) 4.2×10^6 J
(B) 8.4×10^6 J
(C) 16.8×10^6 J
(D) Zero

Q.14 Which of the following statements is correct for any thermodynamic system? **[AIEEE-2003]**
(A) The internal energy changes in all processes.
(B) Internal energy and entropy are state functions.
(C) The change in entropy can never be zero.
(D) The work done in an adiabatic process is always zero.

Q.15 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is **[AIEEE-2003]**
(A) 4/3
(B) 2
(C) 5/3
(D) 3/2

Q.16 The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by – **[AIEEE-2003]**
(A) Planck's law of radiation
(B) Stefan's law of radiation
(C) Wien's law
(D) Rayleigh Jeans law

Q.17 According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to – **[AIEEE-2003]**
(A) Three
(B) Four
(C) One
(D) Two

Q.18 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is

$\left(\dfrac{A(T_2 - T_1)K}{x} \right)$ f, with f equal to – **[AIEEE-2004]**

(A) 1
(B) 1/2
(C) 2/3
(D) 1/3

Q.19 If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously will be– **[AIEEE-2004]**
(A) 4
(B) 16
(C) 32
(D) 64

Q.20 A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is – **[AIEEE-2004]**
(A) E/c
(B) 2E/c
(C) Ec
(D) E/c^2

Q.21 One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume. **[AIEEE-2004]**
(A) 3/2
(B) 23/15
(C) 35/23
(D) 4/3

Q.22 Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) & pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be – **[AIEEE-2004]**
(A) $T_1 + T_2$
(B) $(T_1 + T_2)/2$
(C) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$
(D) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T_2}$

**Q.23** Which of the following is incorrect regarding the first law of thermodynamics? **[AIEEE-2005]**
(A) It is not applicable to any cyclic process.
(B) It is a restatement of the principle of conservation of energy.
(C) It introduces the concept of the internal energy.
(D) It introduces the concept of the entropy.

Q.24 The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is **[AIEEE-2005]**
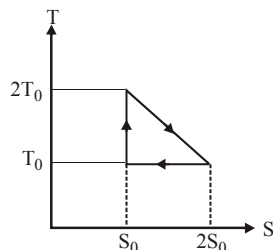
(A) 1/2
(B) 1/4
(C) 1/3
(D) 2/3

Q.25 A system goes from A to B via two processes I and II as shown in the figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, the **[AIEEE-2005]**
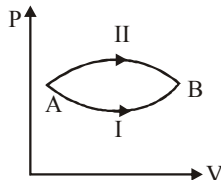

(A) $\Delta U_1 = \Delta U_2$
(B) relation between ΔU_1 and ΔU_2 can not be determined
(C) $\Delta U_2 > \Delta U_1$
(D) $\Delta U_2 < \Delta U_1$

Q.26 A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_p/C_v of the mixture is –**[AIEEE-2005]**
(A) 1.59
(B) 1.62
(C) 1.4
(D) 1.54

Q.27 The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to **[AIEEE-2005]**
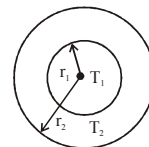
(A) $(r_2 - r_1)/(r_1 r_2)$
(B) $\ln (r_2/r_1)$
(C) $\dfrac{r_1 r_2}{(r_2 - r_1)}$
(D) $(r_2 - r_1)$

Q.28 Assuming the Sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on Earth, at a distance r from the Sun – **[AIEEE-2006]**
(A) $r_0^2 R^2 \sigma T^4/4\pi r^2$
(B) $R^2 \sigma T^4/r^2$
(C) $4\pi r_0^2 R^2 \sigma T^4/r^2$
(D) $\pi r_0^2 R^2 \sigma T^4/r^2$
where r is the radius of the Earth and σ is Stefan's constant.

Q.29 Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature $(7/3) T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is **[AIEEE-2006]**
(A) $T_f = \dfrac{5}{2} T_0$
(B) $T_f = \dfrac{3}{7} T_0$
(C) $T_f = \dfrac{7}{3} T_0$
(D) $T_f = \dfrac{3}{2} T_0$

Q.30 The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is ($R = 8.3$ J mol^{-1} K^{-1}) **[AIEEE-2006]**
(A) monoatomic   (B) diatomic
(C) triatomic   (D) a mixture of monoatomic and diatomic

Q.31 When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf Q = 36 cal. W along the path ibf is –
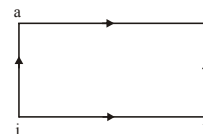
(A) 16 cal.
(B) 66 cal. **[AIEEE-2007]**
(C) 14 cal.
(D) 6 cal.

Q.32 A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10J, the amount of energy absorbed from the reservoir at lower temperature is **[AIEEE-2007]**
(A) 90J
(B) 1J
(C) 100J
(D) 99J

Q.33 If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[AIEEE-2007]**

(A) $C_p - C_v = R/28$ (B) $C_p - C_v = R/14$
(C) $C_p - C_v = R$ (D) $C_p - C_v = 28\,R$

Q.34 One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities k_1 and k_2 respectively. The temperature at the interface of the two sections is **[AIEEE-2007]**
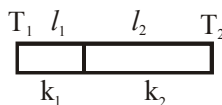
(A) $(k_2 l_2 T_1 + k_1 l_1 T_2) / (k_1 l_1 + k_2 l_2)$
(B) $(k_2 l_1 T_1 + k_1 l_2 T_2) / (k_2 l_1 + k_1 l_2)$
(C) $(k_1 l_2 T_1 + k_2 l_1 T_2) / (k_1 l_2 + k_2 l_1)$
(D) $(k_1 l_1 T_1 + k_2 l_2 T_2) / (k_1 l_1 + k_2 l_2)$

Q.35 An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be– **[AIEEE-2008]**

(A) $\dfrac{P_1 V_1 T_1 + P_2 V_2 T_2}{P_1 V_1 + P_2 V_2}$ (B) $\dfrac{P_1 V_1 T_2 + P_2 V_2 T_1}{P_1 V_1 + P_2 V_2}$

(C) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$ (D) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T_2}$

Q.36 One kg of a diatomic gas is at a pressure of $8 \times 10^4\,\text{N/m}^2$. The density of the gas is $4\,\text{kg/m}^3$. What is the energy of the gas due to its thermal motion? **[AIEEE-2009]**
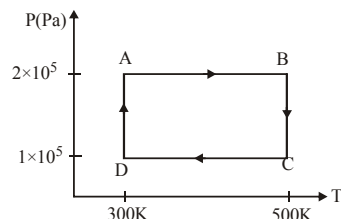
(A) $7 \times 10^4\,\text{J}$ (B) $3 \times 10^4\,\text{J}$
(C) $5 \times 10^4\,\text{J}$ (D) $6 \times 10^4\,\text{J}$

Q.37 A long metallic bar is carrying heat from one of its ends of the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures? **[AIEEE 2009]**
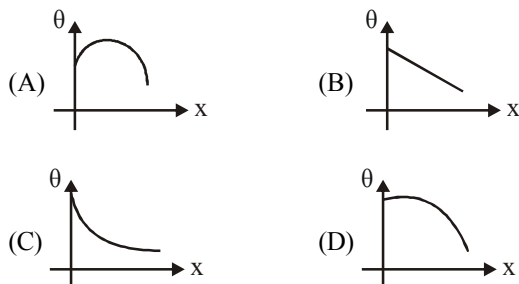
For Q.38-Q.40

Two moles of helium gas are taken over the cycle ABCDA, as shown in the P-T diagram. **[AIEEE 2009]**

Q.38 Assuming the gas to be ideal the work done on the gas in taking it from A to B is –

(A) $500\,R$ (B) $200\,R$
(C) $300\,R$ (D) $400\,R$

Q.39 The work done on the gas in taking it from D to A is –

(A) $+690\,R$ (B) $-414\,R$
(C) $+414\,R$ (4) $-690\,R$

Q.40 The net work done on the gas in the cycle ABCDA is –

(A) $1904\,R$ (B) Zero
(C) $276\,R$ (D) $1076\,R$

Q.41 A diatomic ideal gas is used in a Car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32V the efficiency of the engine is – **[AIEEE 2010]**

(A) 0.5 (B) 0.75
(C) 0.99 (D) 0.25

Q.42 100g of water is heated from 30°C to 50°C ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K) : **[AIEEE 2011]**

(A) $4.2\,\text{kJ}$ (B) $8.4\,\text{kJ}$
(C) $84\,\text{kJ}$ (D) $2.1\,\text{kJ}$

Q.43 A Carnot engine operating between temperatures T_1 and T_2 has effeiciency 1/6. When T_2 is lowered by 62 K, its efficiency increases to 1/3. Then T_1 & T_2 are, respectively **[AIEEE 2011]**

(A) 372 K and 310 K   (B) 372 K and 330 K
(C) 330 K and 268 K   (D) 310 K and 248 K

Q.44 Three perfect gases at absolute temperature T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : **[AIEEE 2011]**

(A) $\dfrac{(T_1 + T_2 + T_3)}{3}$ (B) $\dfrac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

(C) $\dfrac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$ (D) $\dfrac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$

Q.45 A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by – **[AIEEE 2011]**

(A) $\dfrac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

(B) $\dfrac{(\gamma-1)}{2\gamma R}Mv^2K$

(C) $\dfrac{\gamma\,Mv^2}{2R}K$

(D) $\dfrac{(\gamma-1)\,Mv^2}{2R}K$

Q.46 Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) **[AIEEE 2012]**
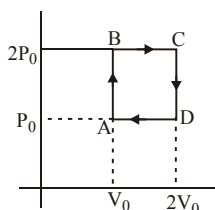

(A) 15.4% (B) 9.1%
(C) 10.5% (D) 12.5%

Q.47 A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is **[AIEEE 2012]**
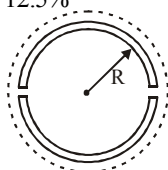
(A) $2\pi SY\alpha\Delta T$ (B) $SY\alpha\Delta T$
(C) $\pi SY\alpha\Delta T$ (D) $2SY\alpha\Delta T$

Q.48 A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta-\theta_0)$ and t is – **[AIEEE 2012]**
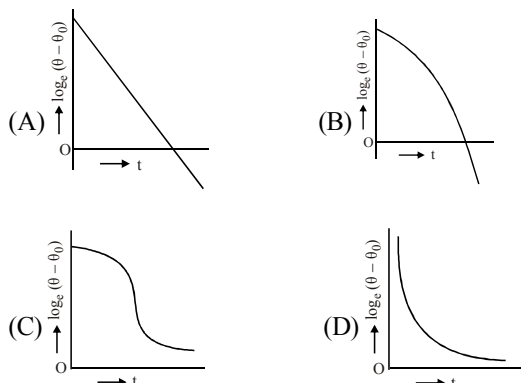

Q.49 A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500K. It is

desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be : **[AIEEE 2012]**

(A) efficiency of carnot engine cannot be made larger than 50% (B) 1200 K
(C) 750 K (D) 600 K

Q.50 The p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is – **[JEE MAIN 2013]**
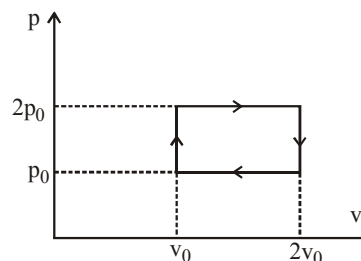
(A) p_0v_0 (B) $(13/2)\,p_0v_0$
(C) $(11/2)\,p_0v_0$ (D) $4\,p_0v_0$

Q.51 If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to – **[JEE MAIN 2013]**
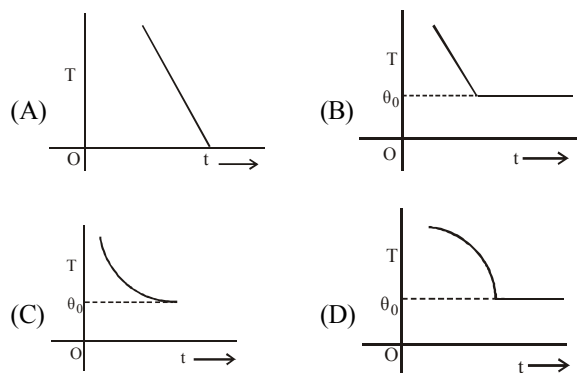
Q.52 One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800K and 600 K respectively. Choose the correct statement: **[JEE MAIN 2014]**
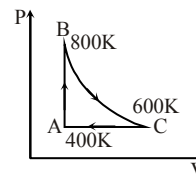
(A) The change in internal energy in the process AB is –350R.
(B) The change in internal energy in the process BC is –500 R.
(C) The change in internal energy in whole cyclic process is 250 R.
(D) The change in internal energy in the process CA is 700 R.

Q.53 Three rods of Copper, Brass and Steel are welded together to from a Y-shaped structure. Area of cross-section of $\ldots\ldots\ldots^2$. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is : **[JEE MAIN 2014]**
(A) 4.8 cal/s   (B) 6.0 cal/s
(C) 1.2 cal/s   (D) 2.4 cal/s

Q.54 Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \dfrac{U}{V} \propto T^4$ and pressure $P = \dfrac{1}{3}\left(\dfrac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is – **[JEE MAIN 2015]**
(A) $T \propto e^{-3R}$ (B) $T \propto 1/R$
(C) $T \propto 1/R^3$ (D) $T \propto e^{-R}$

Q.55 Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is ($\gamma = C_p/C_v$) **[JEE MAIN 2015]**
(A) $\dfrac{3\gamma - 5}{6}$ (B) $\dfrac{\gamma + 1}{2}$
(C) $\dfrac{\gamma - 1}{2}$ (D) $\dfrac{3\gamma + 5}{6}$

Q.56 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways **[JEE MAIN 2015]**
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.
In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is
(A) ln 2, ln 2   (B) ln 2, 2ln 2
(C) 2ln 2, 8ln 2   (D) ln 2, 4ln 2

Q.57 'n' mole of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE MAIN 2016]**
(A) $\dfrac{3P_0V_0}{2nR}$ (B) $\dfrac{9P_0V_0}{2nR}$
(C) $\dfrac{9P_0V_0}{nR}$ (D) $\dfrac{9P_0V_0}{4nR}$
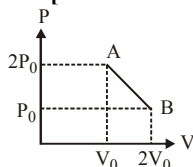
Q.58 A pendulum clock loses 12s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:
(A) 60°C; $\alpha = 1.85 \times 10^{-4}$ /°C   **[JEE MAIN 2016]**
(B) 30°C; $\alpha = 1.85 \times 10^{-3}$ /°C
(C) 55°C; $\alpha = 1.85 \times 10^{-2}$ /°C
(D) 25°C; $\alpha = 1.85 \times 10^{-5}$ /°C

Q.59 An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively): **[JEE MAIN 2016]**
(A) $n = \dfrac{C - C_p}{C - C_V}$ (B) $n = \dfrac{C_p - C}{C - C_V}$
(C) $n = \dfrac{C - C_V}{C - C_p}$ (D) $n = \dfrac{C_P}{C_V}$

Q.60 C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is: **[JEE MAIN 2017]**
(A) $a = b$ (B) $a = 14\,b$
(C) $a = 28\,b$ (D) $a = (1/14)\,b$

Q.61 A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given : Room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C) **[JEE MAIN 2017]**
(A) 885°C   (B) 1250°C
(C) 825°C   (D) 800°C

Q.62 An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by: **[JEE MAIN 2017]**
(A) $\dfrac{P}{\alpha K}$ (B) $\dfrac{3\alpha}{PK}$
(C) $3PK\alpha$ (D) $\dfrac{P}{3\alpha K}$

**Q.63** The temperature of an open room of volume 30m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa . If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be **[JEE MAIN 2017]**
(A) 1.38×10^{23} (B) 2.5×10^{25}
(C) -2.5×10^{25} (D) -1.61×10^{23}

Q.64 The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly **[JEE MAIN 2018]**
(A) 2.35×10^2 N/m^2 (B) 4.70×10^2 N/m^2
(C) 2.35×10^3 N/m^2 (D) 4.70×10^3 N/m^2

Q.65 Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy. **[JEE MAIN 2018]**
(A) (a) 189 K (b) – 2.7 kJ   (B) (a) 195 K (b) 2.7 kJ
(C) (a) 189 K (b) 2.7 kJ   (D) (a) 195 K (b) –2.7 kJ

Q.66 A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10J. The heat flow into the system in path ADB is
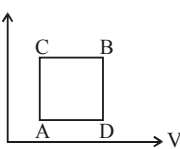
[JEE MAIN 2019 (JAN)]
(A) 80 J   (B) 20 J
(C) 100 J   (D) 40 J

Q.67 A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds

$\left[\dfrac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})} \right]$, is close to **[JEE MAIN 2019 (JAN)]**

(A) 2.24   (B) 0.45
(C) 0.32   (D) 3.16

Q.68 Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length 3L/2 , is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to

[JEE MAIN 2019 (JAN)]
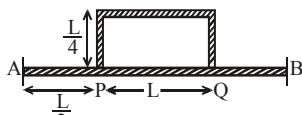
(A) 60ºC   (B) 75ºC
(C) 35ºC   (D) 45ºC

Q.69 Two identical breakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (Assume the emissivity of both the beakers to be the same)

[JEE MAIN 2019 (APRIL)]
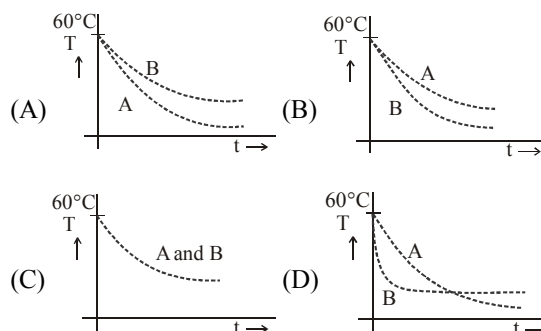
Q.70 A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to

(Latent heat of vaporization of water $= 2.10 \times 10^6$ J kg^{-1} and Latent heat of Fusion of water $= 3.36 \times 10^5$ J kg^{-1})

[JEE MAIN 2019 (APRIL)]
(A) 130 g   (B) 35 g
(C) 20 g   (D) 150 g

Q.71 If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of :

[JEE MAIN 2019 (APRIL)]
(A) 8 N/m^2 (B) 4 N/m^2
(C) 2 N/m^2 (D) 16 N/m^2

Q.72 The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by : **[JEE MAIN 2019 (APRIL)]**
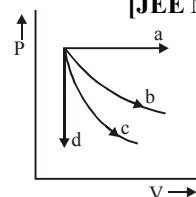
(A) d a c b   (B) a d c b
(C) a d b c   (D) d a b c

Q.73 Three moles of ideal gas A with $\dfrac{C_p}{C_v} = \dfrac{4}{3}$ is mixed with two moles of another ideal gas B with $\dfrac{C_p}{C_v} = \dfrac{5}{3}$. The $\dfrac{C_p}{C_v}$ of mixture is (Assuming temperature is constant)

[JEE MAIN 2020 (JAN)]
(A) 1.5   (B) 1.42
(C) 1.7   (D) 1.3

Q.74 1 litre of a gas at STP is expanded adiabatically to 3 litre. Find work done by the gas. Given $\gamma = 1.40$ and $3^{1.4} = 4.65$
[Take air to be an ideal gas]   **[JEE MAIN 2020 (JAN)]**
(A) 100.8 J   (B) 90.5 J
(C) 45 J   (D) 18 J

Q.75 A Carnot's engine operates between two reservoirs of temperature 900K and 300K. The engine performs 1200 J of work per cycle. The heat energy delivered by the engine to the low temperature reservoir in a cycle is:

[JEE MAIN 2020 (JAN)]

Q.76 A non-isotropic solid metal cube has coefficients of linear expansion as 5×10^{-5} /°C along the x-axis and 5×10^{-6} /°C along y-axis and z-axis. If coefficient of volume expansion of the solid is $C \times 10^{-6}$ /°C then the value of C is **[JEE MAIN 2020 (JAN)]**

Q.77 Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2 . If $\dfrac{C_p}{C_v} = \gamma$ for this gas then a good estimate for τ_2 / τ_1 is given by : **[JEE MAIN 2020 (JAN)]**

(A) 1/2 (B) 2

(C) $\left(\dfrac{1}{2}\right)^{\gamma}$ (D) $(2)^{\frac{\gamma+1}{2}}$

Q.78 Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work **[JEE MAIN 2020 (JAN)]**

(A) $\dfrac{2T_1T_2}{T_1+T_2}$ (B) $\dfrac{T_1+T_2}{2}$

(C) 0 (D) $\sqrt{T_1T_2}$

Q.79 When m gram of steam at 100°C is mixed with 200 gm of ice at 0°C. it results in water at 40°C. Find the value of m in gram. (Given : Latent heat of fusion (L_f) = 80 cal/gm, Latent heat of vaporisation (L_v) = 540 cal/gm., specific heat of water (C_w) = 1 cal/gm/°C)

[JEE MAIN 2020 (JAN)]

Q.80 The plot that depicts the behavior of the mean free time t (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale) **[JEE MAIN 2020 (JAN)]**
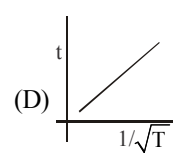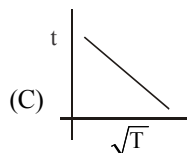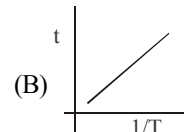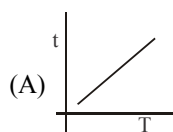
Q.81 A thermodynamic cycle xyzx is shown on a V-T diagram.
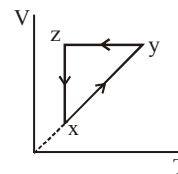
The P-V diagram that best describes this cycle is : (Diagrams are schematic and not to scale)

[JEE MAIN 2020 (JAN)]
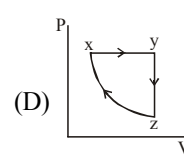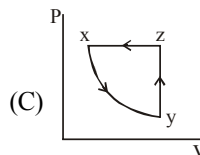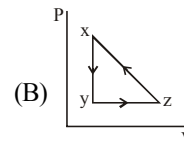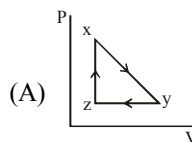
Q.82 A carnot engine having an efficiency of (1/10) is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is : **[JEE MAIN 2020 (JAN)]**

(A) 99 J (B) 100 J

(C) 90 J (D) 1 J

Q.83 Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p / C_v value will be : **[JEE MAIN 2020 (JAN)]**

(A) 67/45 (B) 19/13

(C) 23/15 (D) 40/27

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

PART- A (THERMAL EXPANSION AND CALORIMETRY)

Q.1 Which of the following circular rods (given radius r and length ℓ), each made of the same material and whose ends are maintained at the same temperature will conduct most heat – **[AIPMT 2005]**
(A) $r = r_0, \ell = \ell_0$ (B) $r = 2r_0, \ell = \ell_0$
(C) $r = r_0, \ell = 2\ell_0$ (D) $r = 2r_0, \ell = 2\ell_0$

Q.2 On a new new scale of temperature (which is linear) and called the W scale, the freezing and boiling points of water are 39°W and 239°W respectively. What will be the temperature on the new scale, corresponding to a temperature of 39°C on the Celsius scale **[AIPMT 2008]**
(A) 139°W   (B) 78°W
(C) 117°W   (D) 200°W

Q.3 Steam at 100°C is passed into 20 g of water at 10°C. When water acquires a temperature of 80°C, the mass of water present will be –[Take specific heat of water = 1 calg^{-1}°C^{-1} and latent heat of steam = 540 cal g^{-1}]
(A) 24 g   (B) 31.5 g **[AIPMT 2014]**
(C) 42.5 g   (D) 22.5 g

Q.4 The value of coefficient of volume expansion of glycerin is 5×10^{-4} K^{-1}. The fractional change in the density of glycerin for a rise of 40°C in its temperature, is –
[RE-AIPMT 2015]
(A) 0.010   (B) 0.015
(C) 0.020   (D) 0.025

Q.5 Coefficient of linear expansion of brass and steel rods are α_1 and α_2. Lengths of brass and steel rods are ℓ_1 and ℓ_2 respectively. If $(\ell_2 - \ell_1)$ is maintained same at all temperatures, which one of the following relations holds good? **[NEET 2016 PHASE 1]**
(A) $\alpha_1\ell_2 = \alpha_2\ell_1$ (B) $\alpha_1\ell_2{}^2 = \alpha_2\ell_1{}^2$
(C) $\alpha_1{}^2\ell_2 = \alpha_2{}^2\ell_1$ (D) $\alpha_1\ell_1 = \alpha_2\ell_2$

Q.6 A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of h is [Latent heat of ice is 3.4×10^5 J/kg and g = 10 N/kg]
[NEET 2016 PHASE 1]
(A) 34 km   (B) 544 km
(C) 136 km   (D) 68 km

Q.7 A copper rod of 88 cm and an aluminium rod of unknown length have their increase in length independent of increase in temperature. The length of aluminium rod is ($\alpha_{Cu} = 1.7 \times 10^{-5}$ K^{-1} and $\alpha_{Al} = 2.2 \times 10^{-5}$K^{-1})
(A) 6.8 cm   (B) 113.9 cm **[NEET 2019]**
(C) 88 cm   (D) 68 cm

PART- B (KINETIC THEORY OF GASES)

Q.1 In the given (V – T) diagram, what is the relation between pressure P_1 and P_2 ? **[NEET 2013]**
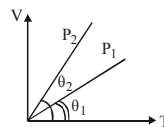
(A) Cannot be predicted   (B) $P_2 = P_1$
(C) $P_2 > P_1$ (D) $P_2 < P_1$

Q.2 The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_P and C_V, respectively. If $\gamma = \dfrac{C_P}{C_V}$ and R is the universal gas constant, then C_V is equal to – **[NEET 2013]**
(A) γR (B) $\dfrac{1+\gamma}{1-\gamma}$ (C) $\dfrac{R}{(\gamma-1)}$ (D) $\dfrac{(\gamma-1)}{R}$

Q.3 The mean free path of molecules of a gas, (radius r) is inversely proportional to – **[AIPMT 2014]**
(A) r^3 (B) r^2 (C) r   (D) $\sqrt{r}$

Q.4 The ratio of the specific heats $\dfrac{C_p}{C_v} = \gamma$ in terms of degrees of freedom (f) is given by: **[AIPMT 2015]**
(A) $\left(1+\dfrac{f}{3}\right)$ (B) $\left(1+\dfrac{2}{f}\right)$ (C) $\left(1+\dfrac{f}{2}\right)$ (D) $\left(1+\dfrac{1}{f}\right)$

Q.5 Two vessels separately contain two ideal gases A and B at the same temperature, the pressure of A being twice that of B. Under such conditions, the density of A is found to be 1.5 times the density of B. The ratio of molecular weight of A and B is : **[RE-AIPMT 2015]**
(A) 1/2   (B) 2/3
(C) 3/4   (D) 2

Q.6 4.0 g of a gas occupies 22.4 litres at NTP. The specific heat capacity of the gas at constant volume is 5.0 JK^{-1} mol^{-1} . If the speed of sound in this gas at NTP is 952m/ s, then the heat capacity at constant pressure is (Take gas constant R = 8.3 JK^{-1} mol^{-1})
[RE-AIPMT 2015]
(A) 8.5 JK^{-1} mol^{-1} (B) 8.0 JK^{-1} mol^{-1}
(C) 7.5 JK^{-1} mol^{-1} (D) 7.0 JK^{-1} mol^{-1}

Q.7 The molecules of a given mass of a gas have r.m.s. velocity of 200 ms^{-1} at 27°C and 1.0×10^5 Nm^{-2} pressure. When the temperature and pressure of the gas are respectively, 127°C and 0.05×10^5 Nm^{-2}, the r.m.s. velocity of its molecules in ms^{-1} is
[NEET 2016 PHASE 1]
(A) $100\sqrt{2}$ (B) $400/\sqrt{3}$ (C) $100\sqrt{2}/3$ (D) 100/3

Q.8 A given sample of an ideal gas occupies a volume V at a pressure P and absolute temperature T. The mass of each molecule of the gas is m. Which of the following gives the density of the gas? **[NEET 2016 PHASE 2]**
(A) P/kT   (B) Pm/kT
(C) P/kTV   (D) mkT

Q.9 A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is **[NEET 2017]**
(A) 15 RT   (B) 9 RT
(C) 11 RT   (D) 4 RT

Q.10 At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere? (Given :Mass of oxygen molecule $(m) = 2.76 \times 10^{-26}$ kg. Boltzmann's constant $k_B = 1.38 \times 10^{-23} JK^{-1}$) **[NEET 2018]**
(A) $5.016 \times 10^4 K$ (B) $8.360 \times 10^4 K$
(C) $2.508 \times 10^4 K$ (D) $1.254 \times 10^4 K$

Q.11 Increase in temperature of a gas filled in a container would lead to : **[NEET 2019]**
(A) Increase in its mass.
(B) Increase in its kinetic energy.
(C) Decrease in its pressure.
(D) Decrease in intermolecular distance.

PART- C (THERMODYNAMICS)

Q.1 When 1 kg of ice at 0°C melts to water at 0° C, the resulting change in its entropy, taking latent heat of ice to be 80 cal/°C is – **[AIPMT (PRE) 2011]**
(A) 293 cal/K   (B) 273 cal/K
(C) 8×10^4 cal/K   (D) 80 cal/K

Q.2 Liquid oxygen at 50K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which one of the following graphs represents the variation of temperature with time **[AIPMT (PRE) 2012]**
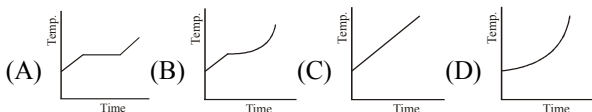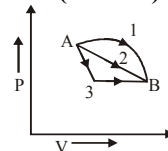

(A)   (B)   (C)   (D)

Q.3 A thermodynamic system is taken through the cycle ABCD as shown in figure. Heat rejected by the gas during the cycle is

(A) 2PV   (B) 4PV **[AIPMT(PRE) 2012]**
(C) (1/2) PV   (D) 4PV
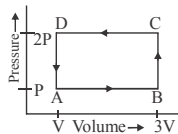
Q.4 One mole of an ideal gas goes from an initial state A to final state B via two processes : It first undergoes isothermal expansion from volume V to 3V and then its volume is reduced from 3V to V at constant pressure. The correct P-V diagram representing the two processes is : **[AIPMT (PRE) 2012]**

(A)   (B)   (C)   (D)
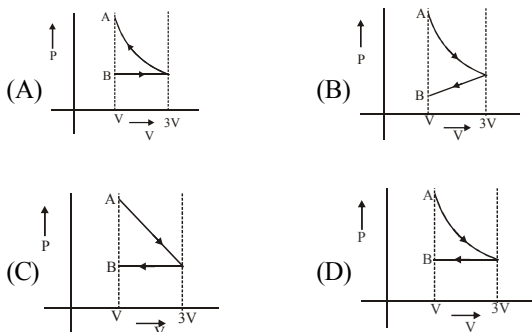
Q.5 An ideal gas goes from state A to state B via three different processes as indicated in the P-V diagram. If Q_1, Q_2, Q_3 indicate the heat a absorbed by the gas along the three processes and ΔU_1, ΔU_2, ΔU_3 indicate the change in internal energy along the three processes respectively , then **[AIPMT (MAINS) 2012]**
(A) $Q_1 > Q_2 > Q_3$ and $\Delta U_1 = \Delta U_2 = \Delta U_3$
(B) $Q_3 > Q_2 > Q_1$ and $\Delta U_1 = \Delta U_2 = \Delta U_3$
(C) $Q_1 = Q_2 = Q_3$ and $\Delta U_1 > \Delta U_2 > \Delta U_3$
(D) $Q_3 > Q_2 > Q_1$ and $\Delta U_1 > \Delta U_2 > \Delta U_3$

Q.6 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of C_p/C_v for the gas is – **[NEET 2013]**
(A) 3/2   (B) 4/3
(C) 2   (D) 5/3

Q.7 The amount of heat energy required to raise the temperature of 1 g of Helium at NTP, from T_1 K to T_2 K is **[NEET 2013]**
(A) $\frac{3}{4} N_a k_B \left(\frac{T_2}{T_1} \right)$ (B) $\frac{3}{8} N_a k_B (T_2 - T_1)$
(C) $\frac{3}{2} N_a k_B (T_2 - T_1)$ (D) $\frac{3}{4} N_a k_B (T_2 - T_1)$

Q.8 A gas is taken through the cycle $A \to B \to C \to A$, as shown, What is the net work done by the gas ?
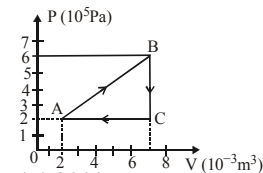
(A) – 2000 J   (B) 2000 J
(C) 1000 J   (D) Zero **[NEET 2013]**

Q.9 A monoatomic gas at a pressure P, having a volume V expands isothermally to a volume 2 V and then adiabatically to a volume 16 V. The final pressure of the gas is (Take $\gamma = 5/3$) **[AIPMT 2014]**
(A) 64 P   (B) 32 P
(C) P/64   (D) 16 P

Q.10 A thermodynamic system undergoes cyclic process ABCDA as shown in figure. The work done by the system in the cycle is **[AIPMT 2014]**
(A) $P_0 V_0$ (B) $2P_0 V_0$
(C) $P_0 V_0/2$ (D) Zero
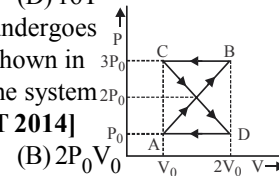
Q.11 One mole of an ideal diatomic gas undergoes a transition from A to B along a path AB as shown in the figure. The change in internal energy of the gas during the transition is
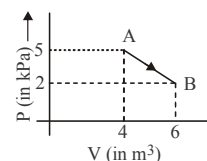
[AIPMT 2015]
(A) – 20 kJ   (B) 20 J
(C) –12 kJ   (D) 20 kJ

Q.12 A Carnot engine, having efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10J, the amount of energy absorbed from the reservoir at lower temperature is : **[AIPMT 2015]**
(A) 99 J 　　　　　　　 (B) 90 J
(C) 1 J 　　　　　　　　(D) 100 J

Q.13 Figure shows two paths that may be taken by a gas to go from a state A to a state C. In process AB, 400J of heat is added to the system and in process BC, 100 J of heat is added to the system. The heat absorbed by the system in the process AC will be: **[AIPMT 2015]**
(A) 500 J 　(B) 460 J 　(C) 300 J 　(D) 380 J

Q.14 The coefficient of performance of a refrigerator is 5. If the temperature inside freezer is –20°C, the temperature of the surroundings to which it rejects heat is :
[RE-AIPMT 2015]
(A) 21°C 　　　　　　　(B) 31°C
(C) 41°C 　　　　　　　(D) 11°C

Q.15 An ideal gas is compressed to half its initial volume by means of several processes. Which of the process results in the maximum work done on the gas ?
[RE-AIPMT 2015]
(A) Isothermal 　　　　(B) Adiabatic
(C) Isobaric 　　　　　(D) Isochoric

Q.16 A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second in order to keep the temperature of the refrigerated space constant. The power required is [Take 1 cal = 4.2 joules]
[NEET 2016 PHASE 1]
(A) 2.365 W 　　　　　(B) 23.65 W
(C) 236.5 W 　　　　　(D) 2365 W

Q.17 A gas is compressed isothermally to half its initial volume. The same gas is compressed separately through an adiabatic process until its volume is again reduced to half. Then **[NEET 2016 PHASE 1]**
(A) Compressing the gas isothermally will require more work to be done.
(B) Compressing the gas through adiabatic process will require more work to be done.
(C) Compressing the gas isothermally or adiabatically will require the same amount of work.
(D) Which of the case (whether compression through isothermal or through adiabatic process) requires more work will depend upon the atomicity of the gas.
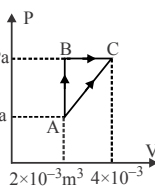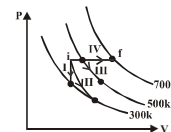
Q.18 One mole of an ideal monatomic gas undergoes a process described by the equation PV^3 = constant. The heat capacity of the gas during this process is
[NEET 2016 PHASE 2]
(A) (3/2) R 　　　　　　(B) (5/2) R
(C) 2R 　　　　　　　　(D) R

Q.19 The temperature inside a refrigerator is t_2°C and the room temperature is t_1°C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be **[NEET 2016 PHASE 2]**

(A) $\dfrac{t_1}{t_1 - t_2}$ 　(B) $\dfrac{t_1 + 273}{t_1 - t_2}$ 　(C) $\dfrac{t_2 + 273}{t_1 - t_2}$ 　(D) $\dfrac{t_1 + t_2}{t_1 + 273}$

Q.20 Thermodynamic processes are indicated in the following diagram :
Match the following

Column-1	Column-2	**[NEET 2017]**
P. Process I	a. Adiabatic	
Q. Process II	b. Isobaric	
R. Process III	c. Isochoric	
S. Process	d. Isothermal	

(A) P → c, Q → a, R → d, S → b
(B) P → c, Q → d, R → b, S → a
(C) P → d, Q → b, R → a, S → c
(D) P → a, Q → c, R → d, S → b

Q.21 A carnot engine having an efficiency of (1/10) as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is – **[NEET 2017]**
(A) 90 J 　　　　　　　(B) 99 J
(C) 100 J 　　　　　　(D) 1 J

Q.22 The volume (V) of a monatomic gas varies with its temperature (T), as shown in the graph. The ratio of work done by the gas, to the heat absorbed by it, when it undergoes a change from state A to state B, is **[NEET 2018]**
(A) 1/3 　　　　　　　　(B) 2/3
(C) 2/5 　　　　　　　　(D) 2/7
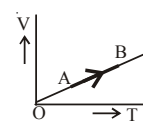
Q.23 The efficiency of an ideal heat engine working between the freezing point and boiling point of water, is
(A) 6.25% 　　　　　(B) 20% 　**[NEET 2018]**
(C) 26.8% 　　　　　(D) 12.5%

Q.24 A sample of 0.1 g of water at 100°C and normal pressure $(1.013 \times 10^5 \ Nm^{-2})$ requires 54 cal of heat energy to convert to steam at 100°C. If the volume of the steam produced is 167.1 cc, the change in internal energy of the sample, is **[NEET 2018]**
(A) 42.2 J 　　　　　　(B) 208.7 J
(C) 104.3 J 　　　　　(D) 84.5 J

Q.25 In which of the following processes, heat is neither absorbed nor released by a system? **[NEET 2019]**
(A) Isothermal 　　　　(B) Adiabatic
(C) Isobaric 　　　　　(D) Isochoric

PART - D (HEAT TRANSFER)

Q.1 A black body at 1227°C emits radiations with maximum intensity at a wavelength of 5000 Å. If the temperature of the body is increased by 1000°C, the maximum intensity will be observed at **[AIPMT 2006]**
(A) 5000 Å 　　　　　　(B) 6000 Å
(C) 3000 Å 　　　　　　(D) 4000 Å

Q.2 Assuming the sun to have a spherical outer surface of radius r, radiating like a black body at temperature t°C, the power received by a unit surface, (normal to the incident rays) at a distance R from the centre of the sun is – **[AIPMT 2007]**

(A) $\dfrac{r^2\sigma(t+273)^4}{4\pi R^2}$ (B) $\dfrac{16\pi^2 r^2\sigma t^4}{R^2}$

(C) $\dfrac{r^2\sigma\,(t+273)^4}{R^2}$ (D) $\dfrac{4\pi^2\sigma t^4}{R^2}$

where σ is the Stefan's constant.

Q.3 A black body is at 727°C. It emits energy at a rate which is proportional to – **[AIPMT 2007]**
(A) $(1000)^4$ (B) $(1000)^2$
(C) $(727)^4$ (D) $(727)^2$

Q.4 The two ends of a rod of length L and a uniform cross-sectional area A are kept at two temperatures T_1 and T_2 ($T_1 > T_2$). The rate of heat transfer, dQ/dt through the rod in a steady state is given by – **[AIPMT 2009]**

(A) $\dfrac{dQ}{dt}=\dfrac{k\,(T_1-T_2)}{LA}$ (B) $\dfrac{dQ}{dt}=k\,LA\,(T_1-T_2)$

(C) $\dfrac{dQ}{dt}=\dfrac{k\,A(T_1-T_2)}{L}$ (D) $\dfrac{dQ}{dt}=\dfrac{k\,L\,(T_1-T_2)}{A}$

Q.5 A black body at 227°C radiates heat at the rate of 7 cals/ cm²s. At a temperature of 727°C, the rate of heat radiated in the same units will be: **[AIPMT 2009]**
(A) 50   (B) 112
(C) 80   (D) 60

Q.6 A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in thermal contact with the two reservoirs in time t? **[AIPMT (PRE) 2010]**
(A) Q/4   (B) Q/16
(C) 2Q   (D) Q/2

Q.7 The total radiant energy per unit area, normal to the direction of incidence, received at a distance R from the centre of a star of radius r, whose outer surface radiates as a black body at a temperature T K is given by **[AIPMT (PRE) 2010]**

(A) $\dfrac{\sigma r^2 T^4}{R^2}$ (B) $\dfrac{\sigma r^2 T^4}{4\pi r^2}$ (C) $\dfrac{\sigma r^4 T^4}{r^4}$ (D) $\dfrac{4\pi\sigma r^2 T^4}{R^2}$

Q.8 A slab of stone of area 0.36 m² and thickness 0.1 m is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on the upper surface of the slab. In one hour 4.8 kg of ice is melted. The thermal conductivity of slab is **[AIPMT (MAINS) 2010]**
(Given latent heat of fusion of ice $= 3.36 \times 10^5$ J kg^{-1}) :
(A) 1.24 J/m/s/°C   (B) 1.29 J/m/s/°C
(C) 2.05 J/m/s/°C   (D) 1.02 J/m/s/°C

Q.9 If the radius of a star is R and it acts as a black body, what would be the temperature of the star, in which the rate of energy production is Q ? **[AIPMT (PRE) 2012]**
(A) $Q/4\pi R^2\sigma$ (B) $(Q/4\pi R^2\sigma)^{-1/2}$
(C) $(4\pi R^2 Q/\sigma)^{1/4}$ (D) $(Q/4\pi R^2\sigma)^{1/4}$
(σ stands for Stefan's constant)

Q.10 A piece of iron is heated in a flame. It first becomes dull red then becomes reddish yellow and finally turns to white hot. The correct explanation for the above observation is possible by using – **[NEET 2013]**
(A) Newton's Law of cooling (B) Stefan's Law
(C) Wien's displacement Law (D) Kirchoff's Law

Q.11 Certain quantity of water cools from 70°C to 60°C in the first 5 minutes and to 54°C in the next 5 minutes. The temperature of the surroundings is – **[AIPMT 2014]**
(A) 45°C   (B) 20°C
(C) 42°C   (D) 10°C

Q.12 On observing light from three different stars P, Q and R, it was found that intensity of violet colour is maximum in the spectrum of P, the intensity of green colour is maximum in the spectrum of R and the intensity of red colour is maximum in the spectrum of Q. If T_P, T_Q and T_R are the respective absolute temperature of P, Q and R, then it can be concluded from the above observations that : **[AIPMT 2015]**
(A) $T_P > T_R > T_Q$ (B) $T_P < T_R < T_Q$
(C) $T_P < T_Q < T_R$ (D) $T_P > T_Q > T_R$

Q.13 The two ends of a metal rod are maintained at temperatures 100ºC and 110ºC. The rate of heat flow in the rod is found to be 4.0 J/s. If the ends are maintained at temperatures 200ºC and 210ºC, the rate of heat flow will be: **[AIPMT 2015]**
(A) 16.8 J/s   (B) 8.0 J/s
(C) 4.0 J/s   (D) 44.0 J/s

Q.14 Two metal wires of identical dimensions are connected in series. If σ_1 and σ_2 are the conductivities of the metal wires respectively, the effective conductivity of the combination is **[RE-AIPMT 2015]**

(A) $\dfrac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (B) $\dfrac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (C) $\dfrac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$ (D) $\dfrac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

Q.15 A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500nm is U_2 and that at 1000 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nmK. Which of the following is correct? **[NEET 2016 PHASE 1]**
(A) $U_1 = 0$ (B) $U_3 = 0$
(C) $U_1 > U_2$ (D) $U_2 > U_1$

Q.16 Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100°C, while the other one is at 0°C. If the two bodies are brought into contact, then assuming no heat loss, the final common temperature is
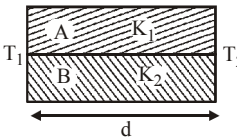(A) 50°C   **[NEET 2016 PHASE 2]**
(B) More than 50°C
(C) Less than 50°C but greater than 0°C
(D) 0°C

Q.17 A body cools from a temperature 3T to 2T in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be **[NEET 2016 PHASE 2]**
(A) (7/4) T   (B) (3/2) T
(C) (4/3) T   (D) T

Q.18 Two rods A and B of different materials are welded together as shown in figure. Their thermal conductivities are K_1 and K_2. The thermal conductivity of the composite rod will be – **[NEET 2017]**

(A) $\dfrac{3\,(K_1+K_2)}{2}$ (B) K_1+K_2

(C) $2\,(K_1+K_2)$ (D) $\dfrac{K_1+K_2}{2}$

Q.19 A spherical black body with a radius of 12 cm radiates 450 watt power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be **[NEET 2017]**

(A) 450 (B) 1000
(C) 1800 (D) 225

Q.20 The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0. If the temperature of the black body is now changed so that it radiates maximum energy at wavelength $(3/4)\,\lambda_0$, the power radiated by it becomes nP. The value of n is **[NEET 2018]**

(A) 256 / 81 (B) 4 / 3
(C) 3 / 4 (D) 81 / 256

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	B	A	B	C	A	A	A	D	D	C	A	C	C	D	B	D	C	D	C	A	C	B	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	A	A	A	D	C	A	B	A	D	B	A	A	C	C	A	A	C	D	C	D	C	C	C	B
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	A	C	C	A	C	C	B	D	B	A	C	C	A	A	B	C	A	B	C	C	C	C	C	D
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92								
A	C	A	D	C	A	A	C	B	A	A	B	B	C	B	B	A	B								

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	D	A	B	B	D	B	A	A	A	D	A	C	D	B	C	C	B	D	C	D	B	B	B	A	D
Q	26	27	28	29	30	31	32	33	34	35	36														
A	A	C	C	B	D	C	C	D	A	C	B														

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12
A	6	1	5	2	2	112	339	12	5	24	8400	24

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	C	C	B	A	C	D	C	C	A	A	C	B	B	D	A	C	D	D	B
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	A	C	A	C	A	B	C	D	D	B	D	A	A	C	C	C	B	D	B	C
Q	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	B	B	A	B	D	A	D	A	C	B	C	B	A	B	B	A	D	D	A	B
Q	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
A	A	D	C	C	A	D	D	D	A	C	C	D	B	B	600	60	D	B	40	D
Q	81	82	83																	
A	D	C	B																	

EXERCISE - 5 (PART-A)

Q	1	2	3	4	5	6	7
A	B	C	D	C	D	C	D

EXERCISE - 5 (PART-B)

Q	1	2	3	4	5	6	7	8	9	10	11
A	D	C	B	B	C	B	B	B	C	B	B

EXERCISE - 5 (PART-C)

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	A	A	D	A	A	B	C	C	D	A	B	B	B	B	C	B	D	B	A	A	C	C	B	B

EXERCISE - 5 (PART-D)

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	A	C	B	B	A	A	D	C	A	A	C	B	D	B	B	D	C	A

HEAT & THERMODYNAMICS

TRY IT YOURSELF-1

(1) **(B).** When a bimetallic strip of different metals is heated, the strip bends due to unequal linear expansion of the two metals. The strip will bends metal of greater α on the convex side i.e. outer side.

(2) **(A).** Coefficient of linear expansion is given by $\alpha = \dfrac{d\ell}{\ell.dt}$

and coefficient of volume expansion is by $\gamma = \dfrac{dV}{V.dt}$.

So from above formula it is clear that both have units of per Kelvin.

(3) (B)

(4) (A)

(5) (A)

(6) (D)

(7) $-40°C = -40°F$

TRY IT YOURSELF-2

(1) Let the temperature of water in calorimeter is t.
Heat lost by water = heat gained by calorimeter

or $\dfrac{100}{1000} \times 4200 \times (40 - t) = 100\,(t - 30)$

or $420\,(40 - t) - 100 = (t - 30)$

or $42 \times 40 - 42t = 10t - 300$

$t = 38.07°C$

(2) Here mass of metal block, $m = 0.20\,kg = 200g$
Fall in temperature of metal $\Delta t = 150 - 40 = 110°C$
Value of water $M_w = 150\,cc = 150g$
Water equivalent of calorimeter $w = 0.025\,kg = 25g$
Rise in the temperature of water and calorimeter $\Delta t' = 40 - 27 = 13°C$
According to principle of calorimetry
Heat given = Heat taken

$$m s \Delta t = (m_w + w)\, s_w \Delta t'$$

$\therefore$ $s = \dfrac{(150 + 25) \times 1 \times 13}{200 \times 110} = \dfrac{175 \times 13}{200 \times 110} = 0.103$ cal $g^{-1}°C^{-1}$

(3) Here, m = mass of water flowing per unit time = 3000g /min.
$\Delta t = 77 - 27 = 50°C$
Heat of combustion $= 4 \times 10^4$ J/g
Heat required to heat the water ,

$Q = ms\,\Delta t = \dfrac{3000g}{mm} \times \dfrac{4.2\,J}{°Cg} \times 50°C = 63 \times 10^4$ J/min.

$\therefore$ Rate of consumption of the fuel

$= \dfrac{\text{fuel required per min}}{\text{Heat of combustion}} = \dfrac{63 \times 10^4}{4 \times 10^4} = 15.675$ g / min.

(4) (D)

(5) **(B).** Heat released by 5 kg of water when its temperature falls from 20°C to 0°C is

$Q_1 = mC\Delta\theta = (5)\,(10)^3\,(20 - 0) = 10^5$ cal

When 2 kg ice at –20°C comes to temperature of 0°C, it takes an energy

$Q_2 = mC\Delta\theta = (2)\,(500)\,(20) = 0.2 \times 10^5$ cal

The remaining heat

$Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt a mass m of

the ice, where, $m = \dfrac{Q}{L} = \dfrac{0.8 \times 10^5}{80 \times 10^3} = 1$ kg

So, the temperature of the mixture will be 0°C, mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 – 1 = 1 kg.

(6) **(A).** Rate of heat gain = 1000 – 160 = 840 J/s

$\therefore$ Required time $= \dfrac{2 \times 4.2 \times 10^3 \times (77 - 27)}{840}$

$= 500$ sec $= 8$ min 20 sec

TRY IT YOURSELF-3

(1) (B)

(2) (AD)

(3) Keeping P constant, we have

$V_2 = \dfrac{V_1 T_2}{T_1} = \dfrac{100 \times 600 3}{300} = 200cc$

(4) $v_{rms} = \sqrt{\dfrac{v_1 + v_2}{2}} = \sqrt{\dfrac{(9 \times 10^6)^2 + (1 \times 10^8)^2}{2}}$

$= \sqrt{\dfrac{(81 + 1)^2 \times 10^{12}}{2}} = \sqrt{41} \times 10^6$ m / s

(5) (D)   **(6)** (B)   **(7)** (C)

(8) **(BD).** For monoatomic gas,

$$C_p = \dfrac{5}{2}R, C_v = \dfrac{3}{2}R , C_p - C_v = R$$

For diatomic gas

$$C_p = \dfrac{7}{2}R, C_v = \dfrac{5}{2}R , C_p - C_v = R$$

$C_p - C_v$ is same for both $C_p + C_v = 6R$ (for diatomic)
$C_p + C_v = 4R$ (for mono)   so $(C_p + C_v)_{dia} > (C_p + C_v)_{mono}$

$\dfrac{C_p}{C_v} = \dfrac{7}{5} = 1.4$ (for diatomic)

$\dfrac{C_p}{C_v} = \dfrac{5}{3} = 1.66$ (for monoatomic)

$(C_p)\,(C_v) = \dfrac{35}{4}R^2 = 8.75\,R^2$ (for diatomic)

$(C_p)\,(C_v) = \dfrac{15}{4}R^2$ (for monoatomic)

so $(C_p . C_v)_{diatomic} > (C_p . C_v)_{monoatomic}$

(9) **(D).** At low pressure and high temperature inter molecular forces become ineffective. So a real gas behaves like an ideal gas.

(10) **(D).** $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$\Rightarrow \dfrac{(v_{rms})_{He}}{(v_{rms})_{Ar}} = \sqrt{\dfrac{M_{Ar}}{M_{He}}} = \sqrt{\dfrac{40}{4}} = \sqrt{10} = 3.16$

TRY IT YOURSELF-4

(1) (ABD)

(2) Here heat removed is less than the heat supplied and hence the room, including the refrigerator (which is not insulated from the room) becomes hotter.

(3) Yes. When the gas undergoes adiabatic compression, its temperature increases.

$dQ = dU + dW$

As $dQ = 0$ (adiabatic process), so $dU = -dW$

In compression, work is done on the system So, $dW = $ -ve

$\Rightarrow dU = +$ ve

So internal energy of the gas increases, i.e. its temperature increases.

(4) True. During driving, temperature of the gas increases while its volume remains constant.

So according to Charle's law, at constant V, $P \propto T$.

Therefore, pressure of gas increases.

(5) **(BD).**

(A) Process is not isothermal (For isothermal P-V curve should be rectangular hyperbola)

(B) Volume decreases and temperature decreases

$\Delta U = $ negative, $\Delta W = $ negative so $\Delta Q = $ negative

(C) Work done in process $A \to B \to C$ is positive

(D) Cycle is clockwise, so work done by the gas is positive.

(6) **(AB).** $\because U = \dfrac{f}{2}nRT$, where f,n,R are constants.

Also temperature T is same at A & B.

$\therefore U_A = U_B$

Also, $\Delta W_{AB} = nRT_0 \ln\left(\dfrac{V_f}{V_i}\right) = nRT_0 \ln \dfrac{4V_0}{V_0}$

$= nRT_0 \ln 4 = P_0 V_0 \ln 4$

So, answers are (A) & (B).

If process BC is consider as isobaric then answer will be ABCD.

(7) **4.** $TV^{\gamma-1} = $ constant ; $TV^{7/5-1} = aT\left(\dfrac{v}{32}\right)^{7/5-1}$
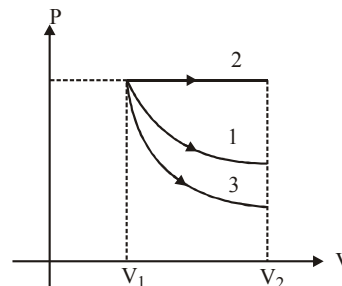
$\therefore a = 4$

(8) **(A).** Number of moles of He $= 1/4$

Now, $T_1 (5.6)^{\gamma-1} = T_2 (0.7)^{\gamma-1}$

$T_1 = T_2 \left(\dfrac{1}{8}\right)^{2/3}$; $4T_1 = T_2$

Work done $= -\dfrac{nR[T_2 - T_1]}{\gamma - 1} = -\dfrac{\dfrac{1}{4}R[3T_1]}{2/3} = -\dfrac{9}{8}RT_1$

(9) **(A).** The corresponding P-V graphs (also called indicator diagram) in three different processes will be as shown:

Area under the graph gives the work done by the gas.

$(Area)_2 > (Area)_1 > (Area)_3$

$\therefore W_2 > W_1 > W_3$

TRY IT YOURSELF-5

(1) **(B).** Wien's displacement law for a perfectly black body is

$\lambda_m T = $ constant $= $ Wien's constant b

Here, λ_m is the minimum wavelength corresponding to maximum intensity I.

or $\lambda_m \propto \dfrac{1}{T}$

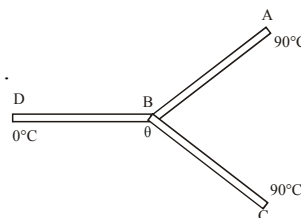From the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

Therefore, $T_1 > T_3 > T_2$

(2) **(B).** Let θ be the temperature of the junction (say B).

Thermal resistance of all the three rods is equal. Rate of heat flow through AB + Rate of heat flow through CB = Rate of heat flow through BD

$\therefore \dfrac{90° - \theta}{R} + \dfrac{90° - \theta}{R} = \dfrac{\theta - 0}{R}$

Here, R = Thermal resistance

$\therefore 3\theta = 180°$

or $\theta = 60°C$

(3) **(A).** Black body radiates maximum number of wavelength and maximum energy if all other conditions (e.g., temperature area etc.) are some.

So, when the temperature of the black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially it will absorb all the radiant energy incident on it, so it is the darkest one.

(4)　**9.**　$(\lambda_m)_B = 3(\lambda_m)_A \Rightarrow T_A = 3T_B$

$E_1 = 4\pi\,(6)^2\,\sigma T_A^4 = 4\pi\,(6)^2\,(3T_B)^4$

$E_2 = 4\pi\,(18)^2\,\sigma T_B^4$; $E_1/E_2 = 9$

(5)　**(ACD).**

A : At steady state, heat flow through A and E are same.

C : $\Delta T = i \times R$

'i' is same for A and E but R is smallest for E.

D : $i_B = \dfrac{\Delta T}{R_B}, i_C = \dfrac{\Delta T}{R_C}, i_D = \dfrac{\Delta T}{R_D}$

If $i_C = i_B + i_D$

Hence, $\dfrac{1}{R_C} = \dfrac{1}{R_B} + \dfrac{1}{R_D} \Rightarrow \dfrac{8KA}{\ell} = \dfrac{3KA}{\ell} + \dfrac{5KA}{\ell}$

(6)　**(A).** $R_1 = \dfrac{L}{\kappa A} + \dfrac{L}{2\kappa A} = \dfrac{3L}{2\kappa A}$

$\dfrac{1}{R_2} = \dfrac{1}{\left(\dfrac{L}{\kappa A}\right)} + \dfrac{1}{\left(\dfrac{L}{2\kappa A}\right)} = \dfrac{3\kappa A}{L}$; $R_2 = \dfrac{L}{3\kappa A}$

$\Delta Q_1 = \Delta Q_2$

$\dfrac{\Delta T}{R_1}t_1 = \dfrac{\Delta T}{R_2}t_2 \Rightarrow t_2 = \dfrac{R_2}{R_1}t_1 = 2\ \text{sec}$

(7)　**(A).** $\sigma \times 4\pi R^2\,(T^4 - T_0^{\ 4}) = 912\,\pi R^2$

$T^4 - T_0^4 = \dfrac{912}{4 \times \sigma} = \dfrac{912}{4 \times 5.7 \times 10^{-8}} = 40 \times 10^8$

$T^4 = 40 \times 10^8 + (300)^4 = (40 + 81) \times 10^8$

$T = 330\,\text{K}$

(8)　**2.**　According to Wien's displacement law

$\lambda_{m_A} T_A = \lambda_{m_B} T_B$

Ratio of energy radiated per unit time

$\dfrac{E_A}{E_B} = \dfrac{\sigma T_A^4 A_A}{\sigma T_B^4 A_B}$; $\dfrac{10^4\,E}{E} = \dfrac{(\sigma)\,(4\pi)\,(400r)^2\,T_A^4}{(\sigma)\,(4\pi)\,(r)^2\,T_B^4}C$

$\left\{\dfrac{\lambda_B}{\lambda_A}\right\}^4 \cdot (400)^2 = 10^4 \Rightarrow \left\{\dfrac{\lambda_B}{\lambda_A}\right\}^4 = 2^4 \Rightarrow \dfrac{\lambda_A}{\lambda_B} = 2$

CHAPTER-12 : HEAT & THERMODYNAMICS
EXERCISE-1

(1) **(B).** Gases show a greater expansion, than solids and liquids at ordinary temperature.

(2) **(A).** A metal expands on heating. Therefore, diameter of the hole also increases.

(3) **(B).** Since, with increase in temperature, volume of given body increases, while mass remains constant so that density will decrease.

i.e. $\dfrac{\rho}{\rho_0} = \dfrac{m/V}{m/V_0} = \dfrac{V_0}{V} = \dfrac{V_0}{V_0(1+\gamma\Delta T)} = (1-\gamma\Delta T)$

$\therefore \quad \rho = \rho_0(1-\gamma\Delta T)$

(4) **(A).** Using relation, $\dfrac{t_F - 32}{180} = \dfrac{t_C}{100}$

If $t_F = t_C = x$, then

$\dfrac{x-32}{180} = \dfrac{x}{100} \Rightarrow \dfrac{x}{5} = \dfrac{x-32}{9}$

$\Rightarrow 9x = 5x - 160 \Rightarrow 160 = -4x \Rightarrow x = -40°$

(5) **(B).** Water exhibits an anomalous behaviour, it contracts on heating between 0°C and 4°C.

(6) **(C).** $\rho_{T_2} = \dfrac{\rho T_1}{1+\gamma\Delta T} = \dfrac{\rho T_1}{1+\gamma\,(T_2-T_1)}$

$T_1 = 4°C, T_2 = 100°C$

$\rho_{T_1} = 1000\text{ kg m}^{-3}$, $\rho_{T_2} = 958.4\text{ kg m}^{-3}$

$958.4 = \dfrac{1000}{1+\gamma\,(100-4)}$; $\gamma = \dfrac{41.6}{958.4 \times 96}$

$= 4.5 \times 10^{-4}\,°C^{-1}$

(7) **(A).** The latent heat of vaporization is always greater than latent heat of fusion because in liquid to vapour phase change there is a large increase in volume. Hence more heat is required as compared to solid to liquid phase change.

(8) **(A).** Heat taken by ice to melt at 0°C is

$Q_1 = mL = 540 \times 80 = 43200\text{ cal}$

Heat given by water to cool upto 0°C is

$Q_2 = ms\Delta\theta = 540 \times 1 \times (80-0) = 43200\text{ cal}$

Hence heat given by water is just sufficient to melt the whole ice and final temperature of mixture is 0°C.

(9) **(A).** Heat lost = Heat gain

$100 \times 1 \times (100-T) = 100 \times 80 + 100 \times T$; $T = 10°C$

(10) **(D).** Vapourisation of 1.0 kg requires 2.3×10^6J and melting requires 3.3×10^5 J. Heating from 0° C to 100°C will require approximately 4.2×10^5 J. Thus, in increasing order, they are melting, heating, and vapourisation.

(11) **(D).** Steam at 100°C carries 22.6×10^5 J kg^{-1} more heat than water at 100°C. This is why burns from steam are usually more serious than those from boiling water.

(12) **(C).** Heat lost = Heat gain

$mS_A(32-38) = mS_B(28-24)$

$S_A = S_B$

(13) **(A).** $v_{rms} = \sqrt{\dfrac{3RT}{M}}$; $\dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{(273+90)}{(273+30)}} = 1.1$

% increase $= \left(\dfrac{v_2}{v_1} - 1\right) \times 100 = 0.1 \times 100 = 10\%$

(14) **(C).** $\lambda = \dfrac{k_B T}{4\pi\sqrt{2}r^2 P}$

(15) **(C).** Mean kinetic energy of molecule depends upon temperature only. For O_2 it is same as that of H_2 at the same temperature of $-73°C$.

(16) **(D).** Molecules of an ideal gas move randomly with different speeds.

(17) **(B).** The kinetic theory of gases gives the formula

$PV = \dfrac{1}{3}Nm\overline{v}^2$ for the pressure P exerted by a gas

enclosed in a volume V. The term Nm represents the average mass of one molecule of the gas

(18) **(D).** Energy $= 300\text{ J / litre} = 300 \times 10^3\text{ J / m}^3$

$P = \dfrac{2}{3}E = \dfrac{2 \times 300 \times 10^3}{3} = 2 \times 10^5\text{ N / m}^2$

(19) **(C).** By definition, $v_{rms} = \sqrt{\dfrac{v_1^2 + v_2^2}{2}}$,

$\therefore \quad v_{rms} = \sqrt{\dfrac{(9\times10^6)^2 + (1\times10^6)^2}{2}} = \sqrt{\dfrac{81\times10^{12} + 1\times10^{12}}{2}}$

$= \sqrt{\dfrac{(81+1)\times10^{12}}{2}} = \sqrt{\dfrac{82\times10^{12}}{2}} = \sqrt{41}\times10^6\text{ m / s}$

(20) **(D).** In mixture gases will acquire thermal equilibrium (i.e., same temperature) so their kinetic energies will also be same.

(21) **(C).** $(C_V)_{mix} = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \dfrac{1\times\dfrac{3}{2}R + 1\times\dfrac{5}{2}R}{1+1} = 2R$

$(C_V)_{mono} = \dfrac{3}{2}R,\ (C_V)_{di} = \dfrac{5}{2}R$

(22) **(A).** We treat water like a solid. For each atom average energy is $3k_B T$. Water molecule has three atoms, two hydrogen and one oxygen. The total energy of one mole of water is $U = 3 \times 3k_B T \times N_A = 9RT$

$[\because k_B = R/N_A]$

Heat capacity per mole of water is

$C = \dfrac{\Delta Q}{\Delta T} = \dfrac{\Delta U}{\Delta T} = 9R$

(23) **(C).** $Q = m.s.\Delta T \Rightarrow s = \dfrac{Q}{m.\Delta T}$

When $\Delta T = 0 \Rightarrow s = \infty$

(24) (B). CO is diatomic gas, for diatomic gas

$$C_P = \frac{7}{2}R \text{ and } C_V = \frac{5}{2}R \Rightarrow \gamma_{di} = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = 1.4$$

(25) (A). Using Mayers equation, for 1 mole of gas,

$$C_P - C_V = R$$

$$\therefore \text{ For unit mass, } C_P - C_V = \frac{R}{M} = \frac{R}{28}$$

$$(\because M \text{ for nitrogen} = 28)$$

(26) (D). $\dfrac{M_{N_2}}{M_{He}} = \dfrac{14}{1} M_{N_2} = 14m \text{ and } M_{He} = m$

$$C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2} = \frac{\dfrac{14m}{28} \times \dfrac{7}{2}R + \dfrac{m}{4} \times \dfrac{5}{2}R}{\dfrac{14m}{28} + \dfrac{m}{4}}$$

$$= \frac{2 \times \dfrac{7}{2}R + \dfrac{5}{2}R}{2 + 1} = \frac{7R + \dfrac{5}{2}R}{3} = \frac{19R}{6}$$

(27) (A). Average number of degrees of freedom per molecule,

$$f = \frac{\text{total number of degree of freedom}}{\text{total number of molecules}}$$

$$= \frac{n_1 N_A f_1 + n_2 N_A f_2}{n_1 N_A + n_2 N_A} \text{ ; where } N_A = \text{Avogadro constant.}$$

Here, $f_1 = 3, f_2 = 5$ $\therefore$ $f = \dfrac{n_1 f_1 + n_2 f_2}{n_1 + n_1} = \dfrac{3n_1 + 5n_2}{n_1 + n_2}$

Also, $\gamma = 1 + \dfrac{2}{f} = 1.5$ or $f = 4$ $\therefore$ $\dfrac{3n_1 + 5n_2}{n_1 + n_2} = 4$

(28) (A). Ideal gas possess only kinetic energy.
(29) (A). With rise in temperature, internal energy also increases.
(30) (D). Internal energy is a state variable and ΔU depends the initial and final states only, it is path independent. ΔQ and ΔW are path dependent.
(31) (C). $\Delta W = P\Delta V$.
Here ΔV is negative so ΔW will be negative

(32) (A). $W = P \times \Delta V = 2 \times 10^5 (150 - 50) \times 10^{-3} = 2 \times 10^4 J$

(33) (B). $\Delta Q = \Delta U + \Delta W$

$$\Delta Q = 200J \text{ and } \Delta W = -100J$$

$$\Rightarrow \Delta U = \Delta Q - \Delta W = 200 - (-100) = 300J$$

(34) (A). From first law of thermodynamics

$$\Rightarrow dU = dQ - dW \Rightarrow dU = dQ(<0)$$

$$\Rightarrow dU < 0. \text{ So temperature will decrease.}$$

(35) (D). (a) If $Q > 0$, heat is added to the system
(b) If $Q < 0$, heat is removed to the system
(c) If $W > 0$, Work is done by the system
(d) If $W < 0$, Work is done on the system

(36) (B). In case of given graph (straight line), V and T are related as $V = aT - b$, where a and b are constants. From ideal gas equation, $PV = \mu RT$

We find $P = \dfrac{\mu RT}{aT - b} = \dfrac{\mu R}{a - b/T}$

Since $T_2 > T_1$, therefore $P_2 < P_1$.

(37) (A). During free expansion of a perfect gas no work is done and also no heat is supplied from outside. Therefore, no change in internal energy. Hence, temperature remain constant.

(38) (A). Conceptually, elastic collisions with a moving object leads to change in its energy.

(39) (C). In isothermal expansion temperature remains constant, hence no change in internal energy.

(40) (C). For isothermal process

$$PV = RT \Rightarrow P = \frac{RT}{V}$$

$$W = PdV = \int_{V_1}^{V_2} \frac{RT}{V} dV = RT \log_e \frac{V_2}{V_1}$$

(41) (A). Bulk modulus $= -\dfrac{dP}{(dV/V)}$

For isothermal process, $PV = $ constant

$$P + \frac{VdP}{dV} = 0 \text{ ; } \frac{dP}{dV} = -\frac{P}{V}$$

(42) (A). An isothermal process takes place at constant temperature, must be carried out in a vessel with conducting wall so that heat generated should go out at once.

(43) (C). For isothermal process, $dU = 0$
and work done $= dW = P(V_2 - V_1)$

$$\because V_2 = \frac{V_1}{2} = \frac{V}{2} \therefore dW = -\frac{PV}{2}$$

(44) (D). $PV^\gamma = $ constant

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \Rightarrow P_2 = (8)^{5/3} P_1 = 32 P_1$$

(45) (C). Volume of the gas $V = m/d$
and using $PV^\gamma = $ constant

$$\frac{P'}{P} = \left(\frac{V}{V'}\right)^\gamma = \left(\frac{d'}{d}\right)^\gamma = (32)^{7/5} = 128$$

(46) (D). In isothermal process $\Delta Q \neq 0$.
(47) (C). When heat is supplied at constant pressure, a part of it goes in the expansion of gas and remaining part is used to increase the temperature of the gas which in turn increases the internal energy.

(48) (C). $Q = \Delta U + W$; $0 = \Delta U + W$

$$\therefore \Delta W = -\Delta U = -nC_V \Delta T$$

$$= \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{0.2 \times 8.314 (150)}{1.4 - 1} \approx 623 J$$

(49) (C). Heat engine cannot have 100% efficiency and refrigerator cannot work without external work. This is because of second law of thermodynamics.

(50) (B). $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300}{500} = \dfrac{2}{5}$

(51) (D). Coefficient of performance (α) of refrigerator is defined as heat extracted per unit work done. It may be more that one. It cannot be infinite.

$\alpha = \dfrac{Q_2}{W} = \dfrac{\text{Heat extracted}}{\text{Work done}}$ (for refrigerator)

Here, $Q_1 = W + Q_2$ = heat released to hot reservoir

$\alpha = \dfrac{Q_2}{Q_1 - Q_2}$

(52) (A). $\eta = 1 - \dfrac{T_2}{T_1} = \dfrac{T_1 - T_2}{T_1}$

When T_1 and T_2 both are decreased by 100 K each, $(T_1 - T_2)$ stays constant.
T_1 decreases. $\therefore$ η increases.

(53) (C). $\eta = 1 - \dfrac{T_2}{T_1} \Rightarrow \dfrac{25}{100} = 1 - \dfrac{300}{T_1}$

$\Rightarrow \dfrac{1}{4} = 1 - \dfrac{300}{T_1}$; $T_1 = 400\,K = 127°C$

(54) (C). Mud is bad conductor of heat. So it prevents the flow of heat between surroundings and inside.

(55) (A). Temperature of interface $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

$(\because \dfrac{K_1}{K_2} = \dfrac{1}{4} \Rightarrow$ If $K_1 = K$ then $K_2 = 4K)$

$\Rightarrow \theta = \dfrac{K \times 0 + 4K \times 100}{5K} = 80°C$

(56) (C). Let θ be temperature middle point C and in series rate of heat flow is same
$\Rightarrow K(2A)(100 - \theta) = KA(\theta - 70)$
$\Rightarrow 200 - 2\theta = \theta - 70 \Rightarrow 3\theta = 270 \Rightarrow \theta = 90°C$

(57) (C). Convection significantly transferring heat upwards (Gravity effect).

(58) (B). Convection is a mode of heat transfer by actual motion of matter. It is possible only in fluids. Convection can be natural or forced. In natural convection, gravity plays an important part.

(59) (D). In convection process heat is transferred by the motion of the heated particles. Warming of glass bulb is due to radiation.

(60) (B). The common examples of forced convection systems are forced-air heating systems in home, the human circulatory system and the cooling system of an automobile engine. In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection and maintaining it at a uniform temperature.

(61) (A). Using wien's displacement law

$\dfrac{T_2}{T_1} = \dfrac{(\lambda_m)_1}{(\lambda_m)_2}$ $\therefore$ $T_2 = \dfrac{2000 \times 1250}{71} = 35211\,K$

(62) (C). Heat radiations are electromagnetic waves of high wavelength.

(63) (C). According to Kirchoff's law, the ratio of emissive power to absorptive power is same for all bodies is equal to the emissive power of a perfectly black body

i.e., $\left(\dfrac{e}{a}\right)_{\text{body}} = E_{\text{Black body}}$

For a particular wavelength

$\left(\dfrac{e_\lambda}{a_\lambda}\right)_{\text{body}} = (E_\lambda)_{\text{Black body}} \Rightarrow e_\lambda = a_\lambda E_\lambda$

(64) (A). Initially black body absorbs all the radiant energy incident on it, So it is the darkest one. Black body radiates maximum energy if all other condition are same. So when the temperature of the black body becomes equal to the temperature of furnace it will be brightest of all.

(65) (A). According to Wein's law $\lambda_m T$ = constant, on heating up to ordinary temperatures, only long wavelength (red) radiation is emitted. As the temperature rises, shorter wavelengths are also emitted in more and more quantity. Hence the colour of radiation emitted by the hot wire shifts from red to yellow, then to blue and finally to white.

(66) (B). Apply Newton's law of cooling,

$\dfrac{\Delta T}{t} = -k(T_{av} - T_\theta)$; $\dfrac{20}{10} = -k(70 - 30)$ ..... (1)

$\dfrac{60 - T}{10} = -k\left(\dfrac{60 + T}{2} - 30\right)$ ..... (2)

Divide eq. (2) $\div$ eq. (1)

$\dfrac{60 - T}{20} = \dfrac{T/2}{40} \Rightarrow 120 - 2T = \dfrac{T}{2} \Rightarrow T = 48°C$

(67) (C). According to Kirchoff's law, a good emitter is also a good absorber.

(68) (A). Rate of cooling $\dfrac{d\theta}{dt} \propto \dfrac{dQ}{ms}$

dQ is same so $\dfrac{d\theta}{dt} \propto \dfrac{1}{m}$; $m_{\text{solid}} > m_{\text{hollow}}$
Hence, hollow sphere will cool fast.

(69) (B). Fractional loss of time per second $= \dfrac{1}{2}\alpha.\Delta T$

Therefore, $\dfrac{1}{2}\alpha(T_0 - 15) \times (24\text{ hrs.}) = 5$

and $\dfrac{1}{2}\alpha(30 - T_0) \times (24\text{ hrs.}) = 10$

On solving, $T_0 = 20°C$

(70) **(C).** $\dfrac{C}{5} = \dfrac{F-3^2}{9} \Rightarrow \dfrac{\Delta C}{5} = \dfrac{\Delta F}{9}$; $\Delta C = \dfrac{5}{9}\Delta F \Rightarrow \Delta C = \Delta K$

(71) **(C).** Let Q_1 and Q_2 amounts of heat flow from P in any time t. Let m be the masses of steam formed and ice melted in time t. Let k and A be the thermal conductivity and the area of cross-section respectively of the rod.

$Q_1 = kA\left(\dfrac{800-100}{x}\right)t = mL_{steam}$;

$Q_2 = kA\left(\dfrac{800-0}{1-x}\right)t = mL_{ice}$

Dividing, $\left(\dfrac{700}{x}\right)\left(\dfrac{1-x}{800}\right) = 7$ or $1-x = 8x$ or $x = \dfrac{1}{9}m$

(72) **(C).** $\dfrac{X-(-25)}{500} = \dfrac{Y-(-70)}{40}$ For $Y = 50, X = 1375.0°X$

(73) **(C).** Strain developed

$\varepsilon = \alpha\Delta T = (12 \times 10^{-6})(50) = 6 \times 10^{-4}$

Strain will be negative, as the rod is in a compressed state.

(74) **(C).** Power radiated $P \propto T^4$ also $\lambda T = $ constant.

(75) **(D).** Since tension is the two rods will be same, hence

$A_1 Y_1 \alpha_1 \Delta\theta = A_2 Y_2 \alpha_2 \Delta\theta \Rightarrow A_1 Y_1 \alpha_1 = A_2 Y_2 \alpha_2$

(76) **(C).** Thermal equilibrium means same temperature.

(77) **(A).** In an adiabatic expansion, internal energy decreases and hence temperature decreases.
from equation of state of ideal gas, $PV = nRT$
$\Rightarrow$ The product of P and V decreases.

(78) **(D).** Internal energy and volume depend upon states.

(79) **(C).** More the surface area, more will be the rate of cooling and hence lesser will be the temperature. Greater the number of surrounding surfaces of the objects having equal area and equal volume, lesser will be the surface area. Among all the given objects, sphere has least and pyramid has largest surface area. Hence the temperature of sphere is highest while that of the pyramid is lowest.

(80) **(A).** Since volume is same in all three process therefore temperature will be least having least pressure.

(81) **(A).** In case of H_2 degree of freedom is greatest and number of moles n is highest.
So this is the case of maximum kinetic energy.

(82) **(C).** $\dfrac{50-40}{5} = k\left(\dfrac{50+40}{2}-20\right)$ .... (1)

$\dfrac{40-30}{t} = k\left(\dfrac{40+30}{2}-20\right)$ .... (2)

Divide eq.1 by eq. 2 ; t = 25/3 min.

(83) **(B).** $V_{rms} = \sqrt{\dfrac{1^2+0^2+2^2+3^2}{4}} = \sqrt{3.5}$

(84) **(A).** $M = \dfrac{C_V}{S_V} = \dfrac{\frac{3\times2}{2}}{0.075} = 40$ $\therefore C_V = \dfrac{3}{2}R$

(85) **(A).** We have, $\rho = \dfrac{PM}{RT} \Rightarrow \dfrac{P_1 M}{RT_1} = \dfrac{P_2 M}{RT_2} \Rightarrow \dfrac{P_1}{T_1} = \dfrac{P_2}{2T_1}$

$P_2 = 2P_1$

(86) **(B).** Since molecules of the gas do not favour any particular direction.

(87) **(B).** Temperature of B will be higher because, due to expansion centre of mass B will come down same heat is supplied but in B, Potential energy is decreased therefore internal energy gain will be more.

(88) **(C).** Avg.K.E. $= \dfrac{5}{2}KT = \dfrac{5}{6}mv^2 \left[\therefore \dfrac{1}{2}mv^2 = \dfrac{3}{2}KT\right]$

(89) **(B).** When a body is heated, the distance between any two points on it increases. The increase is in the same ratio for any set of two point.

(90)   Let A_0 and At be the areas of cross-section of the tube at temperature 0 and t respectively,
ℓ = length of the liquid column (constant)
V_0 and V_t be the volumes of the liquid at temperature 0 and t respectively,
$V_0 = \ell A_0$ $V_t = \ell A_t$
$V_t = V_0(1 + \gamma t)$ $A_t = A_0(1+2\alpha t)$
$\therefore V_t = \ell A_0(1+2\alpha t) = V_0(1+\gamma t) = \ell A_0(1+\gamma t)$ or $\gamma = 2\alpha$.

(91) **(A).** Let ρ_0, ρ_1 and ρ_2 be the densities of the liquid at temperature 0, t_1 and t_2 respectively
To balance pressure, $\rho_1 \ell_1 g = \rho_2 \ell_2 g$

or $\left(\dfrac{\rho_0}{1+\gamma t_1}\right)\ell_1 = \left(\dfrac{\rho_0}{1+\gamma t_2}\right)\ell_2$

(92) **(B).** Higher is the temperature greater is the most probable velocity.

EXERCISE-2

(1) **(D).** $Q_{AB} = \Delta U_{AB} + W_{AB}$; $W_{AB} = 0$

$\Delta U_{AB} = \dfrac{f}{2}nR\Delta T \Rightarrow \dfrac{f}{2}(\Delta PV)$

$\Delta U_{AB} = \dfrac{5}{2}(\Delta PV) \Rightarrow Q_{AB} = 2.5\, P_0 V_0$

Process BC :
$Q_{BC} = \Delta U_{BC} + W_{BC} = 0 + 2P_0 V_0 \ln 2 = 1.4\, P_0 V_0$
$Q_{net} = Q_{AB} + Q_{BC} = 3.9\, P_0 V_0$

(2) **(A).** $\int PdV = \int nC_v dt$

$\Rightarrow$ $dQ = 2\, dU \Rightarrow nCdT = 2nC_v dt \Rightarrow C = 2C_v$

$\Rightarrow$ $C = \dfrac{2R}{1.5-1} = 4R$

(3) **(B).** In process-1 heat supplied
= area under AB curve + $n \times c_v \times 100$ (isobaric process)
In process-2 heat supplied = area under AC curve (isothermal process)
In process-3 heat supplied = 0 (adiabatic process)
In process-4 heat supplied = $n \times c_v (T - 600)$ (isobaric process)

(4) **(B).** $P \propto \dfrac{1}{V^2} \Rightarrow P = \dfrac{k}{V^2} \Rightarrow PV^2 = k$

$PV.V = k \Rightarrow nRTV = k \Rightarrow TV = k_1$
Since temperature increases therefore volume decreases.

(5) **(D).** Heat absorbed by gas in three processes us given

by $Q_{ACB} = \Delta U + W_{ACB}$; $Q_{ADB} = \Delta U$

$Q_{AEB} = \Delta U + W_{AEB}$
The change in internal energy in all the three cases is same and W_{ACB} is +ve, W_{AEB} is –ve.
Hence $Q_{ACB} > Q_{ADB} > Q_{AEB}$

(6) **(B).** Since thermal resistance
$R_{AC} = R_{BC} = R_{DE} = R_{DF}$
$\therefore$ Heat current $H_{AC} = H_{BC} = H_{DE} = H_{DF}$
Also $2H_{AC} = H_{CD}$ $\therefore$ $P_{AC} = P_{DF}$
Also, $P_{CD} = 2P_{DF}$

(7) **(A).** For a block body, wavelength for maximum intensity

$$\lambda \propto \frac{1}{T} \text{ and } P \propto T^4 \Rightarrow P \propto \frac{1}{\lambda^4} \Rightarrow P' = 16P$$

$\therefore$ $P'T' = 32 PT$

(8) **(A).** $Q = W = nRT \ln \dfrac{V_f}{V_i}$

$$T = \frac{Q}{nR \ln(V_f / V_i)} = \frac{1500}{0.5 \times 25/3 \times \ln 3}$$

$$\Rightarrow T = \frac{1500}{0.5 \times 25/3 \times 1} = 360 \text{ K}$$

(9) **(A).** Since, $e = a = 0.2$ (since $a = (1 - r - t) = 0.2$
for the body B)
$E = (100)(0.2) = 20 \text{ W/m}^2$.

(10) **(D).** $U_f - U_i = dU = Q - W = 50 - 20 = 30$ cal.

$Q_{ibf} = 36$ cal, $dU = 30$ cal.

Therefore, $W_{ibf} = Q_{ibf} - dU = 60$ cal.
For path, 'fi' $dU = -30$ cal
$W = -13$ cal , $Q = -30 - 13 = -43$ cal
Negative Q means heat has been rejected by the gas.
Again as $U_f - U_i = 30$ cal and $U_i = 10$ cal
Then $U_f = 40$ cal.
Along path 'ibf' $W = 6$ cal
again as $W_{bf} = 0$, therefore $W_{ib} = 6$ cal

$U_b - U_i = 22 - 10 = 12$ cal

Hence, $Q = W_{ib} + (U_b - U_i) = 18$ cal]

(11) **(A).** Net power given to N_2 gas = $100 - 30 = 70$ cal/sec
The nitrogen gas expands isobarically.

$\therefore$ $Q = nC_P \dfrac{dT}{dt}$ or $70 = 5 \times \dfrac{7}{2} R \dfrac{dT}{dt}$ $\therefore$ $\dfrac{dT}{dt} = 2k / \text{sec}$

(12) **(C).** $2 \times S \times \Delta T = \dfrac{1}{2} \left(\dfrac{1}{2} C V^2 \right)$
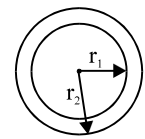
where $S = 0.5 \text{ J K}^{-1}$ (for each plate)

$\therefore$ $\Delta T = 0.5 °C$ Now, $\dfrac{\Delta V}{V} = \gamma (\Delta T) = 3 \alpha (\Delta T)$

$\therefore$ Percentage change = $3 \alpha (\Delta T) \times 100\% = 0.003\%$

(13) **(D).** $\int dR = \int \dfrac{dr}{4\pi r^2 K} = \dfrac{1}{4\pi K} \left[\dfrac{r_2 - r_1}{r_2 r_1} \right]$
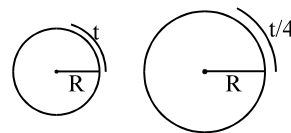
[R = thermal resistance]

$$\dot{Q} = \frac{4\pi K \Delta \theta}{\left(\dfrac{r_2 - r_1}{r_1 r_2} \right)} = \frac{4\pi K \Delta \theta}{\left(\dfrac{t}{r^2} \right)}$$

$$\frac{mL}{\text{time}} = \frac{4\pi K \Delta \theta}{(t / r^2)} \qquad \left[m = \rho \times \frac{4}{3} \pi r^3 \right]$$

$$\frac{\rho L}{\text{time}} = \left(\frac{K}{tr} \right) \times \text{constant} \quad ; \quad \text{time} \propto \frac{tr}{K}$$

$$\frac{25}{16} = \frac{\dfrac{t}{4} 2rK_s}{trK_L} = \frac{1}{2} \frac{K_S}{K_L} \quad ; \quad \frac{K_L}{K_S} = \frac{8}{25}$$

(14) **(B).** Let power lost to surrounding is Q

$\therefore$ $60 - Q = \left(\dfrac{dm}{dt} \right) S(10)$ and $120 - Q = 3\left[\left(\dfrac{dm}{dt} \right) S(10) \right]$

$\therefore$ $\dfrac{120 - Q}{60 - Q} = 3$ $\therefore$ $Q = 30 \text{ W}$

(15) **(C).** $\dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta \omega}{\Delta t}$ = work done per unit time = $\dfrac{k a \theta}{L}$

$\dfrac{d\omega}{dt} = P \dfrac{dv}{dt} = k \dfrac{a\theta}{L}$,

$P = \dfrac{nRT}{V} \Rightarrow \dfrac{0.5 R(300)}{V} A . \dfrac{d\ell}{dt} = \dfrac{ka\theta}{L}$

$\Rightarrow \dfrac{0.5 R(300)}{A . \dfrac{L}{2}} A.v = \dfrac{ka\theta}{L} \Rightarrow v = \dfrac{ka}{R} \left(\dfrac{27}{300} \right) = \dfrac{k}{100R}$

(16) (C). $W = \dfrac{P_1V_1 - P_2V_2}{2-1} = \dfrac{P_1V_1 - P_2V_2}{1}$

$\Delta U = \dfrac{nR}{(\gamma-1)}\Delta T = \dfrac{nR\Delta T}{(\gamma-1)} = \dfrac{P_2V_2 - P_1V_1}{\gamma-1}$; $\dfrac{W}{\Delta U} = -0.4$

(17) (B). $\rho = \dfrac{\rho_0}{1+\gamma\Delta\theta} \cong \rho_0(1-\gamma\Delta\theta)$; $\dfrac{\Delta\rho}{\rho} = -\gamma\Delta\theta$

$\dfrac{1}{100} = 3\times10^{-5}\Delta\theta$; $\Delta\theta = \dfrac{100}{3} = 33.3°C$

(18) (D). $P_2 = 2P_1, V_2 = 4V_1, n = 1$

$C = C_V + \dfrac{PdV}{dT}$;

$dW = PdV = Area = \dfrac{1}{2}[(P_1 + P_2)(V_2 - V_1)]$

$= \dfrac{1}{2}(3P_1 \times 3V_1) = \dfrac{9}{2}P_1V_1$

$dT = T_2 - T_1 = \dfrac{P_2V_2}{R} - \dfrac{P_1V_1}{R} = \dfrac{2P_1 \times 4V_1}{R} - \dfrac{P_1V_1}{R} = \dfrac{7P_1V_1}{R}$

$C = \dfrac{5}{2}R + \dfrac{9}{2}\dfrac{P_1V_1R}{7P_1V_1} = \dfrac{22R}{7}$

(19) (C). $\dfrac{\Delta\ell}{\ell} = \dfrac{\pi \times 0.05}{\pi \times 40} = \dfrac{1}{800}$

$T = Y\dfrac{\Delta\ell}{\ell} \times A = 200\times10^9 \times \dfrac{1}{800} \times 1\times10^{-6} = 250 \text{ N}$

(20) (D). Let T_0 = initial temperature of the black body.
∴ $\lambda_0 T_0 = b$ (constant)
Power radiated $= P_0 = C.T_0^4.$ (c = constant)
Let T = new temperature of black body.

∴ $\dfrac{3\lambda_0}{4}T = b = \lambda_0 T_0$ or $T = \dfrac{4T_0}{3}$

Power radiated $= c.T^4 = (cT_0^4)\left(\dfrac{4}{3}\right)^4 = P_0\left(\dfrac{256}{81}\right)$
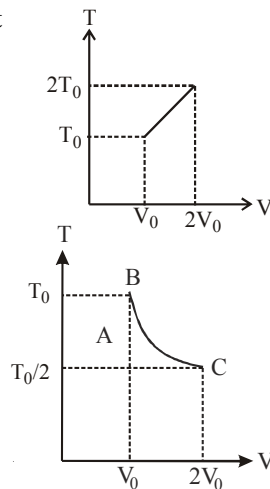
(21) (B). First process is constant
pressure process
$_1 = nR(2T_0 - T_0) = nRT_0$
Equation of second

process is $T = \dfrac{c}{V}$

$P = \dfrac{nRT}{V} = \dfrac{nRc}{V^2}$

$w = \int_{V_0}^{2V_0} P\,dV = \dfrac{nRT_0}{2}$

∴ $\dfrac{w_1}{w_2} = 2:1$

(22) (B). $Q = \int_{T_0}^{\eta T_0} CdT = a\ln\dfrac{\eta T_0}{T_0} = a\ln\eta$

$\Delta U = C_V\Delta T = \dfrac{R}{r-1}(\eta-1)T_0$

$W = Q - \Delta U = a\ln\eta - \left[\dfrac{\eta-1}{r-1}\right]RT_0$

(23) (B). $\gamma + 1 + \dfrac{2}{\gamma} = 1 + \dfrac{2}{6} = \dfrac{4}{3}$

$\dfrac{\Delta W}{\Delta Q} = \dfrac{\Delta Q - \Delta U}{\Delta Q} = 1 - \dfrac{nC_V\Delta T}{nC_p\Delta T} = 1 - \dfrac{1}{\gamma} = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

∴ $\Delta Q = 4\Delta W = 100J$

(24) (A). The rate of heat flow is the same through water and ice in the steady state.

$I = kA\dfrac{4-0}{10-x} = 3kA\dfrac{0-(-4)}{x}$

or $x = (10-x).3.$ or $x = 7.5$ m

(25) (D). Rate of loss of heat ∝ difference in temperature with the surroundings.

At 50°C, $\dfrac{dQ}{dt} = k(50-20) = 10$, where k = constant

∴ k = 1/3
At an angle temperature of 35°C,

$\dfrac{dQ}{dt} = \dfrac{1}{3}(35-20)J/s = 5J/s$

Heat lost in 1 minutes $= \dfrac{dQ}{dt} \times 60J = 5 \times 60J = 300J = Q$

Fall in temperature $= 0.2°C = \Delta\theta$. $Q = c\Delta\theta$.

Heat capacity $= c = \dfrac{Q}{d\theta} = \dfrac{300J}{0.2°C} = 1500J/°C$.

(26) (A). When systems of masses m_1, m_2, ... specific heat capacities s_1, s_2 .... and initial temperatures θ_1, θ_2 ... are mixed, the temperature of the mixture is

$\theta = \dfrac{\sum ms\theta}{\sum ms} = \dfrac{m_1s_1\theta_1 + m_2s_2\theta_2 + ...}{m_1s_1 + m_2s_2 + ...}$

For systems of equal mass, $\theta = \dfrac{\sum s\theta}{\sum s}$

Let s_1, s_2 and s_3 be the specific heat capacities of A, B and C respectively.

For A + B, $15 = \dfrac{10s_1 + 25s_2}{s_1 + s_2}$ or $5s_1 = 10s_2$ or $s_1 = 2s_2$.

Similarly, calculate θ for A + C systems.

(27) (C). Let L_S = specific latent heat of vaporization,
L_I = specific latent heat of freezing.

Given, $L_s = nL_1$. Let m = initial mass of water, f = fraction of water frozen.

Mass of vapour formed = $m(1-f)$,

Mass of ice formed = mf.

$\therefore$ $m(1-f)L_s = mf L_I$

or $\dfrac{1-f}{f} = \dfrac{L_I}{L_s} = \dfrac{1}{\eta}$ or $\eta - \eta f = f$ or $f = \dfrac{\eta}{\eta+1}$

(28) (C). Let R = total thermal resistance of the ring.

ΔT = difference in temperature between A and B.

For q = 180°, two sections of resistance R/2 each are in parallel. Equivalent resistance = R/4

Rate of total heat flow = $I_1 = 1.2 = \dfrac{\Delta T}{R/4}$ or $0.3 = \dfrac{\Delta T}{R}$

For q = 90°, two sections of resistance R/4 and 3R/4 are in

parallel. Equivalent resistance = $\dfrac{(R/4)(3R/4)}{R/4+3R/4} = \dfrac{3R}{16}$

Rate of total heat flow

$I_2 = \dfrac{\Delta T}{3R/16} W = \dfrac{16}{3}\left(\dfrac{\Delta T}{R}\right) W = \dfrac{16}{3} \times 0.3 W = 1.6 W$

(29) (B). For q-t plot, rate of cooling = $\dfrac{dQ}{dt}$ = slope of the curve.

AT P, $\dfrac{d\theta}{dt} = |\tan(180-\phi_2)| = \tan\phi_2 = k(\theta_2 - \theta_1)$

where k = constant.

At Q, $\dfrac{d\theta}{dt} = |\tan(180-\phi_1)| = \tan\phi_1 = k(\theta_1 - \theta_0)$

$\therefore \dfrac{\tan\phi_2}{\tan\phi_1} = \dfrac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

(30) (D). From Boyle's law (T = constant)

$P_1 V_1 = P_2 V_2$

$\therefore (Hd_{waer} + h\, d_{mercury}) g (\dfrac{4}{3}\pi r^3) = hd_{mercury} g (\dfrac{4}{3}\pi(2r)^3)$

$\Rightarrow Hd_{water} = 8hd_{mercury} - hd_{mercury}$

$\Rightarrow H = 7h \dfrac{d_{mercury}}{d_{water}}$ $\therefore H = 7h\rho$

(31) (C). Let pA, pB be the initial pressures in A and B respectively. When the gases double their volumes at constant temperature, their pressures fall to $\dfrac{p_A}{2}$ and $\dfrac{p_B}{2}$

$\therefore$ for A, $p_A - \dfrac{p_A}{2} = \Delta p$ or $p_A = 2\Delta p$

for B, $p_B - \dfrac{p_B}{2} = 1.5\Delta p$ or $p_B = 3\Delta p$ $\therefore \dfrac{p_A}{p_B} = \dfrac{2}{3}$

Also, $p_A V = \dfrac{m_A}{M} RT$ and $p_B V = \dfrac{m_B}{M} RT$

$\therefore \dfrac{p_A}{p_B} = \dfrac{m_A}{m_B}$ $\therefore \dfrac{m_A}{m_B} = \dfrac{2}{3}$ or $3m_A = 2m_B$.

(32) (C). Process$_{1\to2}$ and Process$_{3\to4}$ are isochoric process.

$W_{12} = 0$, $W_{34} = 0$, $W_{23} = nR(T_3 - T_2)$
$= 3R(1600-400) = 3600\,R$

$W_{41} = nR(T_1 - T_4) = 3R(200-800) = -1800\,R$

$W = (3600-1800)R = 1800R = 15\,kJ$

(33) (D). Work done by gas in going isothermally from state A

to B is $\Delta W_{AB} = nRT \ln \dfrac{P_A}{P_B} = nRT \ln 2$ ....(1)
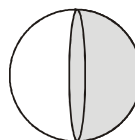
Work done by gas in going isothermally from state B to C

is $\Delta W_{BC} = nRT \ln \dfrac{P_B}{P_C} = nRT \dfrac{P_0}{2P_C}$ ....(2)

It is given that $\Delta W_{BC} = 2\Delta W_{AB}$

$\ln \dfrac{P_0}{2P_C} = \ln(2)^2$ $\therefore P_C = \dfrac{P_0}{8}$

(34) (A). Initially T = 300 T_s = 500

$300 = \sigma e A[500^4 - 300^4]$ ....(1)

afterwards half of the surface of sphere is completely blackened

$600 = \dfrac{\sigma e A}{2}[500^4 - 300^4] + \dfrac{\sigma A}{2}[500^4 - 300^4]$ ....(2)
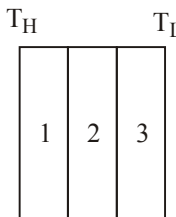
Dividing (2) by (1)

$2 = \dfrac{\left(\dfrac{e}{2}+\dfrac{1}{2}\right)}{e} \Rightarrow 2e = \dfrac{e}{2} + \dfrac{1}{2} \Rightarrow e = \dfrac{1}{3}$

(35) (C). $\Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$

$\Delta U_{ab} = 0$ (isothermal); $\Delta U_{bc} = -\Delta W_{bc}$ (adiabatic)

Hence, $\Delta U_{ca} = 4\,J$

(36) (B). Heat current $H = \dfrac{d\theta}{dt} = \dfrac{T_H - T_L}{(R_{th})_{eq}}$

$T_H \to$ heat temperature,

$T_L \to$ low temperature

Equivalent thermal resistance $(R_{th})_{eq}$

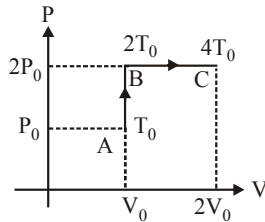$= \dfrac{L_1}{k_1 A} + \dfrac{L_2}{k_2 A} + \dfrac{L_3}{k_3 A}$

In each case : $H = \dfrac{T_H - T_L}{(R_{th})_{eq}} = \dfrac{\Delta T_1}{(R_{th})_1}$

$(R_{th})_1 = \dfrac{L_1}{k_1 A}$. Hence, $(\Delta T_1)_a = (\Delta T_1)_b = (\Delta T_1)_c$

EXERCISE-3

(1) **6.** Let initial pressure, volume, temperature be P_0, V_0, T_0 indicated by state A in P-V diagram.

The gas is then isochorically taken to state B ($2P_0$, V_0, $2T_0$) and then taken from state B to state C ($2P_0$, $2V_0$, $4T_0$) isobarically.

Total heat absorbed by 1 mole of gas

$\Delta Q = C_v (2T_0 - T_0) + C_p (4T_0 - 2T_0)$

$= \frac{5}{2} RT_0 + \frac{7}{2} R \times 2T_0 = \frac{19}{2} RT_0$

Total change in temperature from state A to C is $\Delta Q = 3T_0$

$\therefore$ Molar heat capacity $= \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6} R$

(2) **1.** The rate of heat loss by a thin hollow sphere of thickness Δx, mean radius r and made of density ρ is given by

$ms \frac{dT}{dt} = - \in \sigma A (T^4 - T_0^4)$

$(\rho.4\pi r^2 \Delta x) S \frac{dT}{dt} = - \in \sigma 4\pi r^2 (T^4 - T_0^4)$

$\Rightarrow \frac{dT}{dt} = \frac{- \in \sigma (T^4 - T_0^4)}{S\Delta x}$ is independent of radius.

Hence, rate of cooling is same for both spheres.

$1 : a = 1 : 1 \Rightarrow a = 1$

(3) **5.** A gas is suddenly expanded so it is an adiabatic expansion i.e. $PV^\gamma = $ constant

or $P_1 V_1^\gamma = P_2 V_2^\gamma$

Given : $V_2 = 3V_1$, $C_v = 2R$ $\therefore$ $C_P = 2R + R = 3R$

$\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = 1.5$ $\therefore$ $\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^\gamma = (3)^{1.5} = 5.1 \approx 5$
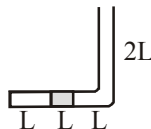
(4) **2.** We have, $\rho = \frac{PM}{RT}$; $\frac{P_1M}{RT_1} = \frac{P_2M}{RT_2}$; $\frac{P_1}{T_1} = \frac{P_2}{2T_1}$

$P_2 = 2P_1$

(5) **2.** $Q = 7J$

$\Delta Q = \Delta U + W$

$7 = nC_v\Delta T + PdV = n\frac{5}{2}R\Delta T + nR\Delta T$

$7 = \frac{7}{2}(nR\Delta T) \Rightarrow nR\Delta T = 2J$

(6) **112.** $\frac{3}{2} kT = 14\left(1 - \frac{1}{4}\right)$; $T = 112 \times 10^3$ K. So x is 112

(7) **339.** For ideal gas, $U = \frac{3}{2} nRT = \frac{3}{2} PV$

$\frac{3}{2} P_1 V_1 + \frac{3}{2} P_2 V_2 = \frac{3}{2} PV$ $\therefore$ $P = \frac{P_1V_1 + P_2V_2}{V}$

On putting the values we have

$P = \frac{2 \times 2 \times 10^5 + 4 \times 4 \times 10^5}{2 + 4} = 3.3 \times 10^5$ N/m^2

$n = n_1 + n_2$ and therefore, $\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{PV}{RT}$

$\frac{2 \times 10^5 \times 2}{300} + \frac{4 \times 10^5 \times 4}{350} = \frac{10 \times 10^5 \times 6}{3T}$

$\frac{4}{300} + \frac{16}{350} = \frac{20}{T}$; $T = 339$ K

(8) **12.** $mgh = 0.5 \ I^2Rt$

$\Rightarrow h = \frac{0.5I^2Rt}{mg} \Rightarrow v = \frac{dh}{dt} = \frac{0.5I^2R}{mg} = 12$ cm/s

(9) **5.** Temperature is constant

$\Delta E = 0$, $dW = nRT \frac{dv}{v} = nRT \frac{Adx}{AL/2}$

$Q = \Delta E + W \Rightarrow \frac{dW}{dt} = \frac{nRT}{L/2} \frac{dx}{dt}$

$\frac{dQ}{dt} = \frac{dW}{dt} \Rightarrow k\frac{1}{900} \frac{\Delta T}{L} = \frac{2nRT}{L} \left(\frac{dx}{dt}\right)$

$\Rightarrow \frac{dx}{dt} = \frac{k \times 27}{900 \times 2nRT} = \frac{415.5 \times 27}{900 \times 2 \times 0.5 \times 8.31 \times 300} = \frac{1}{200}$ m/s

$= 5$mm/s

(10) **24.** Heat supplied $(1 \rightarrow 2 \ \& \ 2 \rightarrow 3)$

$= nC_v 9T_0 + nC_p 90T_0$

Heat rejected $(3 \rightarrow 4 \ \& \ 4 \rightarrow 1)$

$= nC_v 90T_0 + nC_p 9T_0$

$\therefore \eta = \left[1 - \frac{10C_v + C_p}{C_v + 10C_p}\right] = 1 - \frac{10 + \gamma}{1 + 10\gamma} = 1 - \frac{11.4}{15} = \frac{3.6}{15} = \frac{12}{50}$

$\therefore$ %efficiency $= 24$%

(11) **8400.** New volume of flask $= 2 (1 + 3\alpha_{Glass} \Delta T)$ litres

$(1$dm$^3 = $ Litre$)$

New volume of water $= 1 (1 + Y_{water} \Delta T)$ litres

New volume of iron $= 1(1 + 3\alpha_{iron} \Delta T)$ dm^3

Volume of water overflow $= V_{water} + V_{iron} - V_{flask}$

$= (Y_{water} + 3\alpha_{iron} - 6\alpha_{Glass})\Delta T$

$= [(180 + 3 \times 12 - 6 \times 8) \times 10^{-6}] \times 50$

$= 168 \times 50 \times 10^{-6} = 8400 \times 10^{-6}$ litres $= 8400$ mm^3

(12) **24.** Heat released by water = Heat absorbed by ice

$(15 \times 10^3 (\text{gm}))(\text{cal/gm}) \times (80 - 20)°C = ms(20°C - 0°C) + mL$

$= (100 \, \text{cal/g}) \, m \, (\text{gram})$

$\Rightarrow m = \dfrac{15 \times 60}{100} \times 10^3 \, \text{gm} = 9\text{kg}$

Total water = $15 + 9 = 24$ kg

∴ minimum volume of vessel = 24 litres.

EXERCISE-4

(1) **(B).** Thermal capacity (defination)

(2) **(C).** Water rejected heat and convert in ice.
If mass-energy equivalence consider then energy decreases "mass".

(3) **(C).** Black holes.

(4) **(B).** Pyrometer

(5) **(A).** According to Stefan law, $P = \dfrac{Q}{t} \propto T^4 R^2$

$\dfrac{P_1}{P_2} = \left(\dfrac{T_1}{T_2}\right)^2 \left(\dfrac{R_1}{R_2}\right)^2 = \left(\dfrac{4000}{2000}\right)^4 \left(\dfrac{1}{4}\right)^2 = \dfrac{2^4}{2^4} = 1:1$

(6) **(C).** Remain same

(7) **(D).** $(V_T)_{H_2} = (V_{47C})_{O_2}$

$\sqrt{\dfrac{3RT_{H_2}}{2}} = \sqrt{\dfrac{3R(320)}{32}}$; $T_{H_2} = \dfrac{320}{16} = 20K$

(8) **(C).** $n_1 = 1$ mole, $\gamma_1 = \dfrac{7}{5}$ (diatomic gas)

$n_2 = 1$ mole, $\gamma_2 = \dfrac{5}{3}$ (Monoatomic gas)

$C_{v_{mix}} = \dfrac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \dfrac{1 \times \dfrac{5}{2}R + 1 \times \dfrac{3}{2}R}{1 + 1}$

$Cv_{mix} = 2R$; $Cp_{mix} = 3R$

$\gamma_{mix} = \dfrac{C_{p_{mix}}}{C_{v_{mix}}} = \dfrac{3}{2} = \dfrac{24}{16}$

(9) **(C).** For $\eta = 100\%$ $\begin{cases} T_1 = \infty \text{ (source temperature)} \\ T_2 = 0 \text{ (sin k temperature)} \end{cases}$

(10) **(A).** All reversible cycles have same efficiency.

(11) **(A).** Second law of thermodynamics.

(12) **(C).** Work

(13) **(B).** $Q = 3 \times 10^6$ cal
$T_1 = 627°C = 900K, T_2 = 27°C = 300 \, K, W = ?$

$\eta = \dfrac{T_1 - T_2}{T_1} = \dfrac{W}{Q}$

$W = Q\left(\dfrac{T_1 - T_2}{T_1}\right) = 3 \times 10^6 \times \dfrac{600}{900} \text{cal}$

$= 2 \times 10^6 \times 4.2 \, J = 8.4 \times 10^6 \, J$

(14) **(B).** Internal energy and entropy are state functions both are depends on initial and final position.

(15) **(D).** $P \propto T^3$
In adiabatic process $P \propto T^{\gamma/\gamma - 1}$

$\dfrac{\gamma}{\gamma - 1} = 3$; $\left[\gamma = \dfrac{3}{2}\right]$

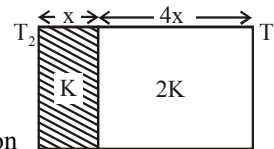**(16)** **(A).** Spectrum explain by Planck.

(17) **(C).** Rate of cooling $R_c = -C(\theta - \theta_0) = -C(\Delta\theta)$
So n = 1

(18) **(D).** $(T_2 > T_1)$

$\dfrac{dQ}{dt} = \dfrac{K_{eq}A}{5x}(T_2 - T_1)$

K_{eq} in series combination

$K_{eq} = \dfrac{5}{3}K$; $\dfrac{dQ}{dt} = \dfrac{KA}{3x}(T_2 - T_1)$

Compare with given expression f = 1/3

(19) **(D).** $P \propto T^4 R^2$
$T_2 = 2T_1$
$R_2 = 2R_1$; $P_2 = 64P_1$

(20) **(B).** Momentum transferred to the surface $p_i = \dfrac{E}{C}$

Strike normally and collide elastically
So momentum of reflected radiations.

$P_R = -\dfrac{E}{C}$

Change in momentum, $\Delta p = p_R - p_i = -2E/C$

(21) **(A).** $\gamma_{mix} = \dfrac{C_{P_{min}}}{C_{V_{mix}}} = \dfrac{n_1 C_{P_1} + n_2 C_{P_1}}{n_1 C_{V_1} + n_2 C_{V_2}}$

$= \dfrac{1 \times \dfrac{7}{2}R + 1 \times \dfrac{5}{2}R}{1 \times \dfrac{3}{2}R + 1 \times \dfrac{5}{2}R} = \dfrac{12}{8} = \dfrac{3}{2}$

(22) **(C).**

(23) **(A).** It is not applicable to any cyclic process.

(24) **(C).** $\eta = \dfrac{Q_1 - Q_2}{Q_1}$; $Q_1 = \dfrac{1}{2}(T_0 S_0) + T_0 S_0 = \dfrac{3}{2}T_0 S_0$

$Q_2 = T_0 S_0$; $\eta = \dfrac{\dfrac{3}{2}T_0 S_0 - T_0 S_0}{\dfrac{3}{2}T_0 S_0} = \dfrac{1}{3}$

(25) **(A).** Internal energy path independent $\Delta U_I = \Delta U_{II}$
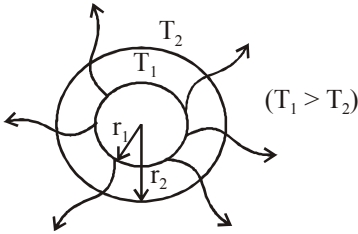
(26) **(B).** $m_{He} = 16\text{gm} \Rightarrow n_{He} = \dfrac{m_{He}}{m_{W_{He}}} = \dfrac{16}{4} = 4$

$m_{O_2} = 16\text{gm} \Rightarrow n_{O_2} = \dfrac{m_{O_2}}{M_{w_{O_2}}} = \dfrac{16}{32} = \dfrac{1}{2}$

$$\gamma_{mix} = \frac{n_{He}(C_P)_{He} + n_{O_2}(C_P)_{O_2}}{n_{He}(C_V)_{He} + n_{O_2}(C_V)_{O_2}}$$

$$= \frac{4 \times \frac{5}{2}R + \frac{1}{2} \times \frac{7}{2}R}{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R} = \gamma_{mix} = 1.62$$

(27) **(C).** In variable cross section case heat flow rate

$$(T_1 > T_2)$$

$$\frac{dQ}{dt} = KA\left(\frac{-dT}{dx}\right)$$
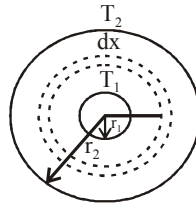
In steady state condition $\frac{dQ}{dt} = H = \text{constant}$

Take "dx" thickness portion at distance "x" from centre

$$H = \frac{dQ}{dt} = -\frac{K(4\pi x^2)dT}{dx}$$

$$\int_{r_1}^{r_2}\frac{dx}{x^2} = -\frac{4\pi K}{H}\int_{T_1}^{T_2}dT$$

$$\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{4\pi K}{H}(T_2 - T_1)$$

$$H = \frac{4\pi K(T_1 - T_2)}{\left(\frac{r_2 - r_1}{r_1 r_2}\right)}$$

$$H = 4\pi K(T_1 - T_2)\left(\frac{r_1 r_2}{r_2 - r_1}\right) \; ; \; H \propto \left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

(28) **(D).** $P = (\pi r_0^2)\,\sigma T^4\left(\frac{R}{r}\right)^2$

(29) **(D).** $T_{mix} = \frac{n_1 f_1 T_1 + n_2 f_2 T_2}{n_1 f_1 + n_2 f_2}$ (from energy conservation)

$$= \frac{1 \times 5 \times T_0 + 1 \times 3 \times \frac{7}{3}T_0}{1 \times 5 + 1 \times 3} = \frac{5T_0 + 7T_0}{8} = \frac{12}{8}T_0 = \frac{3}{2}T_0$$

(30) **(B).** $W_{AD} = \frac{nR(T_1 - T_2)}{\gamma - 1} \; ; \; \gamma = 1 + \frac{nR(T_1 - T_2)}{W_{AD}}$

$$n = 1K \text{ mole} = 10^3 \text{ mole}$$

$$T_1 - T_2 = 7°C$$
$$W_{AD} = 146\,KJ = 146 \times 10^3\,J$$
$$\gamma = 1.66$$

(31) **(D).** $dU_{iaf} = \delta Q_{iaf} - \delta W_{iaf} = 50 - 20$

$dU_{iaf} = 30\,cal$

Internal energy path independent

$dU_{ibf} = dU_{iaf} = 30$ cal

$\delta W_{ibf} = \delta Q_{ibf} - dU_{ibf} = 36 - 30 = 6$ cal

(32) **(A).** $\eta = \frac{1}{10} = \frac{W}{Q_1} \; ; \; W = 10J$

$$Q_1 = 10W = 100J$$
$$Q_2 = Q_1 - W = 100 - 10 = 90J$$

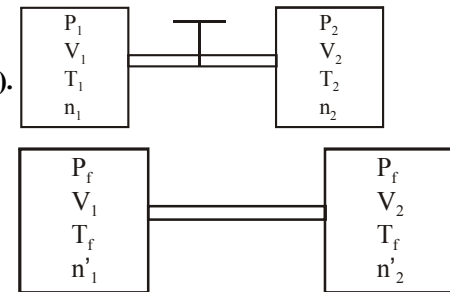**(33)** **(A).** Molar specific heat $C_P - C_V = R$

gm specific heat $C_P - C_V = \frac{R}{M_w}$

For $N_2 \Rightarrow C_P - C_V = \frac{R}{28}$

(34) **(C).** $K_1\left(\frac{T_1 - T_C}{\ell_1}\right) = K_2\left(\frac{T_C - T_2}{\ell_2}\right)$

$$T_C = \frac{K_1 \ell_2 T_1 + K_2 \ell_1 T_2}{K_1 \ell_2 + K_2 \ell_1}$$

(35) **(C).**

$$n_1 + n_2 = n'_1 + n'_2$$

$$\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{P_f V_1}{RT_f} + \frac{P_f V_2}{RT_f}$$

$$T_f = \frac{P_f(V_1 + V_2)}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$$

$$P_f(V_1 + V_2) = P_1 V_1 + P_2 V_2$$

As total energy remain conserved

$$\frac{n_1 f R T_1}{2} + \frac{n_2 f R T_2}{2} = \frac{n'_1 f R T}{2} + \frac{n'_2 f R T}{2}$$

$$P_1 V_1 + P_2 V_2 = P_f V_1 + P_f V_2$$

$$T_f = \frac{T_1 T_2(P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

(36) **(C).** $K = \frac{5}{2}pV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4\,J$
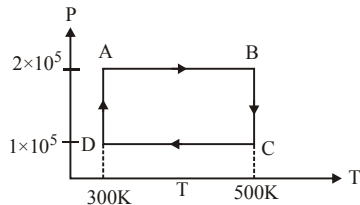
(37) (B). $r = -KA \dfrac{d\theta}{dx}$

In steady state, r = constant

$\dfrac{d\theta}{dx} = -\dfrac{KA}{r}$; $d\theta = -\dfrac{KA}{r}dx$; $\theta = C - \dfrac{KA}{r}x$

(38) (D), (39) (B), (40) (C).

(i) $W_{AB} = P\Delta V = nR\Delta T = 2 \times R \times (500 - 300) = 400R$

(ii) $W_{DA} = 2.3 \, nRT \log \dfrac{P_1}{P_2} = (2.302)(2)(R)(300) \log$

$\left(\dfrac{1 \times 10^5}{2 \times 10^5}\right) = -(2.3)(600R((0.3) = -414\,R$

(iii) $W_{BC} = (2.3)(2)(R)(500) \log 2 = 690\,R$
$W_{CD} = -400\,R$
$W_{total} = (690 - 414)R = 276\,R$

(41) (B). The efficiency of cycle is $\eta = 1 - \dfrac{T_2}{T_1}$

For adiabatic process, $TV^{\gamma-1}$ = constant
For diatomic gas $\gamma = 7/5$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$; $T_1 = T_2 \left(\dfrac{V_2}{V_1}\right)^{\gamma-1}$

$T_1 = T_2 (32)^{\frac{7}{5}-1} = T_2 (2^5)^{2/5} = T_2 \times 4$

$T_1 = 4T_2$

$\eta = \left(1 - \dfrac{1}{4}\right) = \dfrac{3}{4} = 0.75$

(42) (B). $\Delta Q = MS\Delta T = 100 \times 10^{-3} \times 4.184 \times 20 = 8.4 \times 10^3$
$\Delta Q = 84\,kJ, \Delta W = 0$
$\Delta Q = \Delta V + \Delta W$
$\therefore \Delta V = 8.4\,kJ$

(43) (A). $\eta = 1 - \dfrac{T_2}{T_1} = \dfrac{1}{6} \Rightarrow \dfrac{T_2}{T_1} = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

$\dfrac{1}{3} = 1 - \dfrac{(T_2 - 62)}{T_1} \Rightarrow \dfrac{T_2 - 62}{T_1} = \dfrac{2}{3}$; $\dfrac{5(T_2 - 62)}{T_2} = \dfrac{2}{3}$

$5T_2 - 310 = 4T_2$

$T_2 = 310$ and $T_1 = \dfrac{6 \times 310}{5} = 372\,K$

(44) (B). $T = \dfrac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

(45) (D). $\dfrac{1}{2}Mv^2 = C_v \Delta T$

$\dfrac{1}{2}Mv^2 = \dfrac{R}{\gamma - 1}\Delta T$ $\Delta T = \dfrac{Mv^2 (\gamma - 1)}{2R} = \dfrac{(\gamma - 1)Mv^2}{2R}$

(46) (A).

$\eta = \dfrac{p_0 v_0}{\dfrac{f}{2}(p_0 v_0) + \dfrac{f}{2}(2p_0)v_0 + 2p_0 v_0} = \dfrac{1}{\dfrac{3}{2} + 3 + 2} = \dfrac{200}{13}$

$= 15.4\%$

(47) (D). $\dfrac{F}{A} = \dfrac{\Delta L}{L}Y$; $F = \alpha\,\Delta T\,YS$. So, $T = 2F$; $T = 2\alpha\Delta TYS$
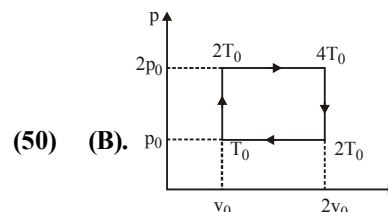
(48) (A). $\dfrac{d\theta}{dt} = -k(\theta - \theta_0)$; $\displaystyle\int_{\theta_0}^{\theta} \dfrac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$

$\ln(\theta - \theta_0) = -kt + C$. So graph is straight line.

(49) (C). For 1st case, efficiency $= \eta = \left(1 - \dfrac{T_1}{T_2}\right) \times 100$

$\left(1 - \dfrac{T_1}{500}\right) \times 100 = 40$; $T_1 = 300\,K$

For 2nd case, $\eta = \left(1 - \dfrac{300}{T_2}\right) \times 100 = 60$

(50) (B).

Heat supplied $= nC_v(2T_0 - T_0) + nC_p(4T_0 - 2T_0)$

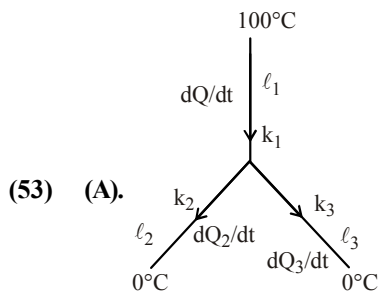$= \dfrac{n3RT_0}{2} + \dfrac{n5R}{2}(2T_0) = \dfrac{13}{2}nRT_0 = \dfrac{13}{2}p_0 v_0$

(51) (C). According to Newtons cooling law option (C) is correct

(52) (B). $\Delta U_{AB} = nC_v(T_B - T_A) = 1 \times \dfrac{5R}{2}(800 - 400) = 1000\,R$

$\Delta U_{BC} = nC_v(T_C - T_B) = 1 \times \dfrac{5R}{2}(600 - 800) = -500\,R$

$\Delta U_{total} = 0$

$\Delta U_{CA} = nC_v(T_A - T_C) = 1 \times \dfrac{5R}{2}(400 - 600) = -500\,R$

(53) (A).

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$$

$$\Rightarrow \quad \frac{0.92\,(100-T)}{46} = \frac{0.26\,(T-0)}{13} + \frac{0.12\,(T-0)}{12}$$

$$\Rightarrow \quad T = 40°$$

$$\frac{dQ_1}{dt} = \frac{0.92 \times 4\,(100-40)}{40} = 4.8\ \text{cal}\,/\,\text{s}$$

(54) (B). $P = \dfrac{1}{3}\left(\dfrac{U}{V}\right) = \dfrac{1}{3} kT^4$ .... (1) ; $P = \mu\,RT$ .... (2)

$$\frac{\mu RT}{V} = \frac{1}{3} kT^4 \propto V \propto T^{-3}\ ;\ R \propto 1/T$$

(55) (B). $\tau = \dfrac{\lambda}{v_{rms}} = \dfrac{1}{\sqrt{2}\pi d^2 \left(\dfrac{N}{V}\right)\sqrt{\dfrac{3RT}{M}}}$ .... (1)

$$\tau = \frac{V}{\sqrt{T}}\ ......(2)\ ;\ TV^{\gamma-1} = k\ ......(3) \Rightarrow \tau \propto V^{\frac{\gamma+1}{2}}$$

(56) (A). Since entropy is a state function, therefore change in entropy in both the processes should be same.

(57) (D). $P = \dfrac{-P_0}{V_0} V + 3P_0$

$$PV = nRT\ ;\ T = \frac{1}{nR}\left(\frac{-P_0}{V_0} V^2 + 3P_0 V\right)$$

For T_{max}, $\dfrac{dT}{dV} = 0$; $\dfrac{-P_0}{V_0} 2V + 3P_0 = 0 \Rightarrow V = \dfrac{3}{2} V_0$

$$T_{max} = \frac{1}{nR}\left[\frac{-P_0}{V_0}\frac{9}{4}V_0^2 + 3P_0\frac{3}{2}V_0\right] = \frac{9}{4}\frac{P_0 V_0}{nR}$$

Alternate : Directly by using isotherms we can see that highest temperature will occur at the midpoint.

(58) (D). Time loss per day $= \dfrac{1}{2}\alpha\,\Delta\theta\,(86400)$

$$\frac{1}{2}\alpha\,(40-\theta)\times 86400 = 12$$

$$\frac{1}{2}\alpha\,(\theta-20)\times 86400 = 4\ ;\ \frac{40-\theta}{\theta-20} = 3$$

$$\theta = 25°C;\ \alpha = 1.85 \times 10^{-5}\,/°C$$

(59) (A). $C = C_V + \dfrac{R}{1-n}$

$$C - C_V = \frac{C_P - C_V}{1-n}\ ;\quad 1-n = \frac{C_P - C_V}{C - C_V}$$

$$n = 1 - \left(\frac{C_P - C_V}{C - C_V}\right) = \frac{C - C_V - C_P + C_V}{C - C_V} = \frac{C - C_P}{C - C_V}$$

(60) (B). If C_p & C_v are specific heat capacity per gram
$a = C_p - C_v = R/2$ (for hydrogen)
(As R is per mole of H_2 i.e. 2 gm of H_2)
$b = C_p - C_v = R/28$ (for Nitrogen)
(As R is per mole of N_2 i.e. 28 gm of N_2)
$\Rightarrow a = 14b$

(61) (A). Heat gained by (water + calorimeter)
$\qquad\qquad\qquad\qquad$ = Heat lost by copper ball
$\Rightarrow 100 \times 0.1 \times (75-30) + 170 \times 1 \times (75-30)$
$\quad = 100 \times 0.1 \times (T-75)$
$\Rightarrow 450 + 7650 = 10\,(T-75) \Rightarrow T - 75 = 810 \Rightarrow T = 885°C$

(62) (D). $K = \dfrac{P}{(-\Delta V / V)} \Rightarrow \dfrac{\Delta V}{V} = \dfrac{P}{K} \Rightarrow -\Delta V = \dfrac{PV}{K}$

$$\frac{PV}{K} = V\,(3\alpha)\,\Delta T \Rightarrow \Delta T = \frac{P}{3\alpha K}$$

(63) (C). $n_f - n_i = \dfrac{PV}{RT_f} \times N_0 - \dfrac{PV}{RT_i} N_0$

$$\qquad\qquad (N_0 = \text{Avogadro's number})$$

$$N_0 = \frac{10^5 \times 30}{8.314} \times 6.02 \times 10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5 \times 10^{25}$$

(64) (C). Change in momentum of a single molecule.

$$\Delta P_0 = m_0 \frac{u}{\sqrt{2}} \times 2$$

Total change in momentum per second

$$\Delta P = n\Delta P_0 = n\,m_0 u\sqrt{2}$$

$$\text{Pressure} = \frac{F}{A} = \frac{nm_0 u\sqrt{2}}{A}$$

Substituting values: $P = 2.35 \times 10^3\ \text{N}\,/\text{m}^2$.

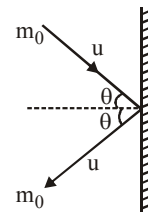**(65) (A).** For mono atomic gas $\gamma = 5/3$
Using $TV^{\gamma-1} = \text{constant}$

$$(300)V^{2/3} = T\,(2V)^{2/3} \Rightarrow T = \frac{300}{(2)^{2/3}} \approx 189\ K$$

$$\Delta U = n\frac{3}{2}R\Delta T = 2\left(\frac{3}{2} \times 8.314\right)(189-300)$$

$$= -2768 \approx -2.7\ kJ$$

(66) (D). $\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$
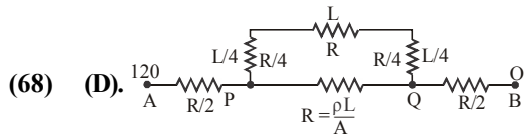$\Rightarrow 60\,J = 30\,J + \Delta U_{ACB} \Rightarrow \Delta U_{ACB} = 30\,J$

$\Rightarrow \Delta U_{ADB} = \Delta U_{ACB} = 30$ J

$\Delta Q_{ADB} = \Delta U_{ACB} + \Delta W_{ADB} = 30$ J $+ 10$ J $= 40$ J

(67) (D). $\dfrac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\dfrac{M_{Ar}}{M_{He}}} = \sqrt{\dfrac{40}{4}} = 3.16$

(68) (D).

$\dfrac{\Delta T}{R_{eq}} = I = \dfrac{120 \times 5}{8R}$; $\Delta T_{PQ} = \dfrac{120 \times 5}{8R} \times \dfrac{3}{5} R = \dfrac{360}{8}$

$= 45^\circ C$

(69) (A). $-ms\dfrac{dT}{dt} = e\sigma A \, (T^4 - T_0^4)$

$-\dfrac{dT}{dt} = \dfrac{e\sigma A}{ms}(T^4 - T_0^4)$; $-\dfrac{dT}{dt} = \dfrac{4e\sigma A T_0^3}{ms}(\Delta T)$

$T = T_0 + (T_i - T_0) \, e^{-kt}$

where $k = \dfrac{4e\sigma A T_0^3}{ms}$; $k = \dfrac{4e\sigma A T_0^3}{\rho vs}$

$\left| \dfrac{dT}{dt} \right| \propto k$ $\therefore$ $\left| \dfrac{dT}{dt} \right| \propto \dfrac{1}{\rho s}$

$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A s_A < \rho_B s_B$

$\left| \dfrac{dT}{dt} \right|_A > \left| \dfrac{dT}{dt} \right|_B$

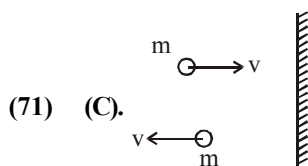**(70) (C).** Suppose 'm' gram of water evaporates then,heat required $\Delta Q_{req} = mL_v$

Mass that converts into ice $= (150 - m)$

So, heat released in this process

$\Delta Q_{rel} = (150 - m) \, L_f$

Now, $\Delta Q_{rel} = \Delta Q_{req}$; $(150 - m) \, L_f = mL_V$

$m(L_f + L_v) = 150 \, L_f$; $m = \dfrac{150 L_f}{L_f + L_v} = 20g$

(71) (C).

Magnitude of change in momentum per collision $= 2mv$

Pressure $= \dfrac{Force}{Area} = \dfrac{N(2mv)}{1}$

$= \dfrac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1} = 2$ N/m^2

(72) (D). Isochoric $\rightarrow$ Process d

Isobaric $\rightarrow$ Process a

Adiabatic slope will be more than isothermal

Isothermal $\rightarrow$ Process b

Adiabatic $\rightarrow$ Process c

Order $\rightarrow$ d a b c

(73) (B). $\gamma_{mixture} = \dfrac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{v_1} + n_2 C_{v_2}} = \dfrac{n_1 \dfrac{\gamma_1 R}{\gamma_1 - 1} + n_2 \dfrac{\gamma_2 R}{\gamma_2 - 1}}{\dfrac{n_1 R}{\gamma_1 - 1} + \dfrac{n_2 R}{\gamma_2 - 1}}$

On rearranging we get

$\dfrac{n_1 + n_2}{\gamma_{mix} - 1} = \dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}$; $\dfrac{5}{\gamma_{mix} - 1} = \dfrac{3}{1/3} + \dfrac{2}{2/3}$

$\dfrac{5}{\gamma_{mix} - 1} = 9 + 3 = 12$; $\gamma_{mixture} = \dfrac{17}{12} = 1 + \dfrac{5}{12}$

$\gamma_{mix} = 1.42$

(74) (B). $P_1 = 1$ atm, $T_1 = 273$ K

$P_1 V_1^\gamma = P_2 V_2^\gamma$; $P_2 = P_1 \left[\dfrac{V_1}{V_2} \right]^\gamma = 1 \, atm \left(\dfrac{1}{3} \right)^{1.4}$

Now work done $= \dfrac{P_1 V_1 - P_2 V_2}{\gamma - 1} = 88.7$ J

Closest ans is 90.5 J

(75) 600. $\eta = \dfrac{W}{Q_h} = 1 - \dfrac{300}{900} = \dfrac{2}{3}$; $Q_h = \dfrac{3}{2} W = 1800$ J

$Q_L = Q_h - W = 600$ J

(76) 60. $V = 2\alpha_2 + \alpha_1 = 10 \times 10^{-6} + 5 \times 10^{-5}$

$= 60 \times 10^{-6} /^\circ C$

(77) (D).

Relaxation time $(\tau) \propto \dfrac{V}{\sqrt{T}}$; $TV^{\gamma - 1} = $ constant $\therefore$ $t \propto V^{\frac{\gamma + 1}{2}}$

(78) (B). Let, Q_H : Heat input to Ist engine

Q_L : Heat rejected from Ist engine

$Q_{L'}$: Heat rejected from IInd engine

Work done by Ist engine $=$ work done by IInd engine

$Q_H - Q_L = Q_L - Q_{L'}$

$2Q_L = Q_H + Q_{L'}$

$2 = \dfrac{T_1}{T} + \dfrac{T_2}{T}$; $T = \dfrac{T_1 + T_2}{2}$

(79) 40. $M_{ice} L_f + m_{ice} (40 - 0) C_w$

$= m_{steam} L_v + m_{steam} (100 - 40) C_w$

$\Rightarrow 200 [80 + 40 (1)] = m [540 + 60 (1)]$

$\Rightarrow 200 (120) = m (600) \Rightarrow m = 40$ gm

(80) (D). Mean free time $= \dfrac{\text{Mean free path}}{\text{Average speed}} = \dfrac{\dfrac{1}{\sqrt{2}\pi D^2 n}}{\sqrt{\dfrac{8RT}{\pi M_w}}}$; $t \propto \dfrac{1}{\sqrt{T}}$

(81) (D). $x \Rightarrow y \Rightarrow$ Isobaric ; $y \Rightarrow z \Rightarrow$ Isochoric

$z \Rightarrow x \Rightarrow$ Isothermal

(82) (C). Refrigerator cycle is : $\eta = \dfrac{W}{Q_+} = \dfrac{W}{W + Q_-}$

$\dfrac{1}{10} = \dfrac{10}{10 + Q_-} \Rightarrow Q_- = 90J$

Heat absorbed from the reservoir at lower temperature is 90 J.

(83) (B). $\dfrac{C_p}{C_v} mix = \dfrac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$

$\dfrac{C_p}{C_v} mix = \dfrac{n \times \left(\dfrac{5R}{2}\right) + 2n \left(\dfrac{7R}{2}\right)}{n \times \left(\dfrac{3R}{2}\right) + 2n \left(\dfrac{5R}{2}\right)}$; $\dfrac{C_p}{C_v} = \dfrac{19}{13}$

EXERCISE-5
PART- A (THERMAL EXPANSION AND CALORIMETRY)

(1) (B).

(i) Rate of heat flow is directly proportional to area.

(ii) Inversely proportional to length.

∴ Heat flow will be maximum when r is maximum and ℓ is minimum.

(2) (C). $\left(\dfrac{t - LFP}{UFP - LFP}\right)_{true} = \left(\dfrac{t - LFP}{UFP - LFP}\right)_{faulty}$

$\dfrac{39°C - 0°C}{100°C - 0°} = \dfrac{t - 39°W}{239°W - 39°W}$; $t = 117°W$

(3) (D). Heat gain by water = Heat lost by steam

$20 \times 1 \times (80 - 10) = m \times 540 + m \times 1 \times (100 - 80)$

$\Rightarrow 1400 = 560 \, m \Rightarrow m = 2.5$ g

Total mass of water $= 20 + 2.5 = 22.5$ g

(4) (C). $d_f = \dfrac{d_i}{1 + \gamma \Delta T}$; Fractional change $= \dfrac{d_i - d_f}{d_i} = 1 - \dfrac{d_i}{d_f}$

$= 1 - (1 + \gamma \Delta T)^{-1} = 1 - (1 - \gamma \Delta T)$

∵ $(1 + x)^n \approx 1 + nx = \gamma \Delta T = 5 \times 10^{-4} \times 40 = 0.020$

(5) (D). $\ell'_2 - \ell'_1 = \ell_2 - \ell_1$

$\Rightarrow \ell_2 (1 + \alpha_2 \Delta t) - \ell_1 (1 + \alpha_1 \Delta t) = \ell_2 - \ell_1$

$\ell_2 \alpha_2 = \ell_1 \alpha_1$

(6) (C). $\dfrac{mgh}{4} = mL_f \Rightarrow h = \dfrac{4L_f}{g} = \dfrac{4 \times 3.4 \times 10^5}{10} = 136$ km

(7) (D). $\alpha_{Cu} L_{Cu} = \alpha_{Al} L_{Al}$

$1.7 \times 10^{-5} \times 88$ cm $= 2.2 \times 10^{-5} \times L_{Al}$

$L_{Al} = \dfrac{1.7 \times 88}{2.2} = 68$ cm

PART- B (KINETIC THEORY OF GASES)

(1) (D). $PV = nRT \Rightarrow V = \left(\dfrac{nR}{P}\right) T \Rightarrow$ Slope $= \dfrac{nR}{P}$

As $\theta_2 > \theta_1$, so, $\dfrac{1}{P_2} > \dfrac{1}{P_1} \Rightarrow P_1 > P_2$

(2) (C). $C_P - C_V = R$ and $\gamma = \dfrac{C_P}{C_V} \Rightarrow C_V = \dfrac{R}{\gamma - 1}$

(3) (B). $\lambda = \dfrac{1}{\lambda d^2 n \sqrt{2}} = \dfrac{1}{4\pi r^2 n \sqrt{2}}$; $\lambda \propto \dfrac{1}{r^2}$

(4) (B). $\gamma = \dfrac{C_p}{C_v} = \dfrac{\left(\dfrac{f}{2} + 1\right) R}{\left(\dfrac{f}{2}\right) R} = \left(1 + \dfrac{2}{f}\right)$

(5) (C). $P = \dfrac{\rho RT}{M} \Rightarrow M = \dfrac{\rho RT}{P}$

$\dfrac{M_A}{M_B} = \dfrac{\rho_A}{\rho_B} \cdot \dfrac{T_A}{T_B} \cdot \dfrac{P_B}{P_A} = (1.5) (1) \left(\dfrac{1}{2}\right) = \dfrac{3}{4}$

(6) (B). Molecular mass M = 4.0g

$v_{sound} = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow \gamma = \dfrac{Mv^2}{RT} = 1.6$

So, $C_p = \gamma \, C_v = 1.6 \times 5.0 = 8.0 \, JK^{-1} mol^{-1}$

(7) (B). $v_{rms} = \sqrt{\dfrac{3RT}{M}} \Rightarrow \dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}}$

$\Rightarrow v_2 = \sqrt{\dfrac{400}{300}} \times 200 \, ms^{-1} = \dfrac{400}{\sqrt{3}} \, ms^{-1}$

(8) (B). $\dfrac{P}{\rho} = \dfrac{kT}{m} \Rightarrow \rho = \dfrac{Pm}{kT}$

(9) (C). $U = \dfrac{f}{2} nRT$; $U_{total} = \dfrac{5}{2}(2) RT + \dfrac{3}{2}(4) RT = 11 RT$

(10) **(B).** $V_{escape} = 11200$ m/s

Say at temperature T it attains V_{escape}

So, $\sqrt{\dfrac{3k_B T}{m_{O_2}}} = 11200$ m/s

On solving, $T = 8.360 \times 10^4$ K

(11) **(B).** Increase in temperature would lead to the increase in kinetic energy of gas (assuming far as to be ideal) as

$U = \dfrac{f}{2} nRT$

PART- C (THERMODYNAMICS)

(1) **(A).** $\Delta S = \dfrac{\Delta Q}{T} = \dfrac{80 \times 100}{273} \approx 293$ cal / K

(2) **(A).** Initially liquid oxygen will gain the temperature up to its boiling temperature then it change its state to gas. After this again its temperature will increase.

(3) **(A).** In cyclic process, $\Delta U = 0$.

So, heat absorbed

$\Delta Q = W =$ Area under the curve $= -(2V)(P) = -2PV$

So, heat rejected $= 2PV$

(4) **(D).** 1st process is isothermal expansion.2nd process is isobaric compression.

(5) **(A).** Initial and final condition is same for all process

$\Delta U_1 = \Delta U_2 = \Delta U_3$; $\Delta Q = \Delta U + \Delta W$

Work done $\Delta W_1 > \Delta W_2 > \Delta W_3$ (Area of P-V graph)

So $\Delta Q_1 > \Delta Q_2 > \Delta Q_3$

(6) **(A).** $P \propto T^3$; $PV = nRT$; $PV^{3/2} = $ constant $\Rightarrow \gamma = \dfrac{C_p}{C_v} = \dfrac{3}{2}$

(7) **(B).** Number of moles in 1g He $= 1/4$

Amount of heat energy required to raise its temperature from T_1K to T_2K

$= nC_v \, \Delta T = \left(\dfrac{1}{4}\right)\left(\dfrac{3}{2}R\right)(T_2 - T_1) = \dfrac{3}{8} k_B N_A (T_2 - T_1)$
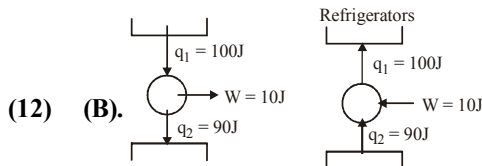
(8) **(C).** Net work done = Area of triangle ABC

$= \dfrac{1}{2} \times [(7-2) \times 10^{-3}] [(6-2) \times 10^5] = 1000$ J

(9) **(C).** **Step-1 :** Isothermal expansion, $PV = P_2 2V \Rightarrow P_2 = P/2$

Step - 2 : Adiabatic expansion

$P_2 V_2^\gamma = P_3 V_3^\gamma$; $\dfrac{P}{2}(2V)^{5/3} = P_3 (16V)^{5/3}$

$P_3 = \dfrac{P}{2}\left(\dfrac{2V}{16V}\right)^{5/3} = \dfrac{P}{2} \times \left(\dfrac{1}{8}\right)^{5/3} = \dfrac{P}{64}$

(10) **(D).** W = Area bound by closed curve

$= \dfrac{1}{2}(P_0 + 3P_0) \, V_0 - 3P_0 V_0 + \dfrac{1}{2}(P_0 + 3P_0) \, V_0 - P_0 V_0 = 0$

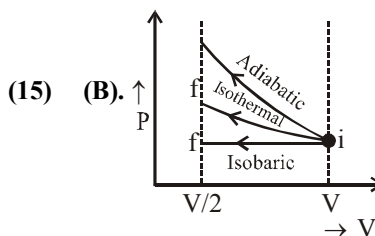**(11)** **(A).** $\Delta U = \dfrac{f}{2} nR (T_f - T_i) = \dfrac{5}{2}(P_f V_f - P_i V_i) = -20$ KJ

(12) **(B).**

90 J heat is absorbed at lower temperature.

(13) **(B).** For a complete cycle, $Q_{cycle} = W_{cycle}$
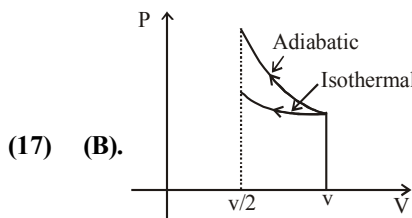
$400 + 100 + Q_{C \to A} = \dfrac{1}{2}(2 \times 10^{-3})(4 \times 10^4)$

$\Rightarrow Q_{C \to A} = -460$ J $\Rightarrow Q_{A \to C} = +460$ J

(14) **(B).** $COP = \dfrac{T_L}{T_H - T_L}$; $5 = \dfrac{T_L}{T_H - T_L}$

$T_H = \dfrac{6}{5} T_L = \dfrac{6}{5}(253) \approx 303.6$ K $\approx 31°C$

(15) **(B).**

Work done on the gas, $W_{isochoric} = 0$

$W_{adiabatic} > W_{Isothermal} > W_{Isobaric}$

(16) **(C).** $T_2 = 4°C = 277$ K ; $T_1 = 303$ K, $Q_2 = 600$ cal

$\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} \Rightarrow \dfrac{Q_2 + W}{Q_2} = \dfrac{T_1}{T_2}$; $W = 236.5$ J/s

(17) **(B).**

(18) **(D).** $PV^3 = $ constant polytropic process with

$n = 3$; $C = C_v + \dfrac{R}{1-n} = \dfrac{R}{\gamma - 1} + \dfrac{R}{1-n}$

$= \dfrac{R}{(5/3)-1} + \dfrac{R}{1-3} = R$

(19) (B). $K = \dfrac{Q_2}{W} = \dfrac{t_2}{t_1 - t_2}$; $Q_2 = \dfrac{t_2 W}{t_1 - t_2}$

$Q_1 = Q_2 + W = \dfrac{t_2 W}{t_1 - t_2} + W = \dfrac{t_1 W}{t_1 - t_2}$; $\dfrac{Q_1}{W} = \dfrac{t_1 + 273}{t_1 - t_2}$

(20) (A). Process (I) $\rightarrow$ volume constant $\rightarrow$ Isochroic

Process (II) $\rightarrow$ adiabatic

Process (III) $\rightarrow$ Temperature constant $\rightarrow$ Isothermal

Process (IV) $\rightarrow$ Pressure constant $\rightarrow$ Isobaric

(21) (A). $\beta = \dfrac{Q_2}{W} = \dfrac{1 - \eta}{\eta} \Rightarrow \dfrac{Q_2}{9} = \dfrac{1 - 0.1}{0.1}$

$\Rightarrow Q_2 = 9 \times 10 = 90\,J$

(22) (C). Given process is isobaric

$dQ = n\,C_p\,dT = n\left(\dfrac{5}{2}R\right)dT$

$dW = P\,dV = n\,RdT$

Required ratio $= \dfrac{dW}{dQ} = \dfrac{nRdT}{n\left(\dfrac{5}{2}R\right)dT} = \dfrac{2}{5}$

(23) (C). Efficiency of ideal heat engine,

$\eta = \left(1 - \dfrac{T_2}{T_1}\right)$; T_2 : Sink temperature

T_1 : Source temperature

$\%\eta = \left(1 - \dfrac{T_2}{T_1}\right) \times 100 = \left(1 - \dfrac{273}{373}\right) \times 100$

$= \left(\dfrac{100}{373}\right) \times 100 = 26.8\%$

(24) (B). $\Delta Q = \Delta U + \Delta W$

$54 \times 4.18 = \Delta U + 1.013 \times 10^5 (167.1 \times 10^{-6} - 0)$

$\Delta U = 208.7\,J$

(25) (B). In adiabatic process, there is no exchange of heat.

PART - D (HEAT TRANSFER)

(1) (C). Applying wein's displacement law,

$\lambda_m T = $ constant

$5000\,\text{Å} \times (1227 + 273)$

$= (2227 + 273) \times \lambda_m$

$\lambda_m = \dfrac{5000 \times 1500}{2500} = 3000\text{Å}$

(2) (C). Power radiated by the sun at t°C

$= \sigma\,(t + 273)^4 \times 4\pi r^2$

Power received by a unit surface

$= \dfrac{\sigma\,(t + 273)^4\,4\pi r^2}{4\pi R^2} = \dfrac{r^2 \sigma (t + 273)^4}{R^2}$

(3) (A). According to Stefan's law

$E \propto T^4 \propto (727 + 273)^4\,K \propto (1000)^4\,K$

(4) (C). For a rod of length L and area of cross-section A whose faces are maintained at temperatures T_1 and T_2 respectively. Then in steady state the rate of heat flowing from one face to the other face in time t is given by $\dfrac{dQ}{dt} = \dfrac{k\,A(T_1 - T_2)}{L}$

(5) (B). $E = \sigma T^4$, $\dfrac{E_2}{E_1} = \left(\dfrac{T_2}{T_1}\right)^4 = \left(\dfrac{1000}{500}\right)^4 = 16$

$T_1 = 500\,K,\ T_2 = 1000\,K$

$\therefore\ E_2 = 16 \times 7 = 112\,\text{cal/cm}^2\text{s}$

(6) (B). $Q = \dfrac{KA\,(T_1 - T_2)}{L}$ ...... (i)

Area of cross-section of new rod

$A' = \pi\left(\dfrac{R}{2}\right)^2 = \dfrac{\pi R^2}{4} = \dfrac{A}{4}$ ...... (ii)

Volume $= AL = A'L'$, where L' is the length the new rod

or $L' = L\dfrac{A}{A'} = 4L$ ....(iii)

$Q' = \dfrac{KA'\,(T_1 - T_2)\,t}{L'}$ .....(iv)

We get $Q' = \dfrac{K\,(A/4)\,(T_1 - T_2)\,t}{4L}$

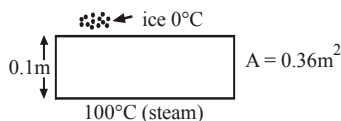$= \dfrac{1}{16}\dfrac{KA\,(T_1 - T_2)\,t}{L} = \dfrac{1}{16}Q$

(7) (A). Radiant power per unit area received at a distance R from the centre of a star is

$S = \dfrac{P}{4\pi R^2} = \dfrac{4\pi r^2 \sigma T^4}{4\pi R^2} = \dfrac{\sigma r^2 T^4}{R^2}$

(8) (A). Rate of heat given by steam

= Rate of heat taken by ice

$\dfrac{dQ}{dt} = \dfrac{KA\,(100 - 0)}{\ell} = m\dfrac{dL}{dt}$

$$\frac{K \times 100 \times 0.36}{0.1} = \frac{4.8 \times 3.36 \times 10^5}{60 \times 60}$$

$$K = 1.24 \text{ J/m/s/°C}$$

(9) **(D).** $Q = e\sigma AT^4$; $e = 1$; $A = 4\pi R^2$

(10) **(C).** $\lambda_m T = b$, Wien's displacement law.

(11) **(A).** Newtons law of cooling

$$\frac{\theta_1 - \theta_2}{\Delta t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

$$\text{First} \Rightarrow \frac{70 - 60}{5} = K\left[65 - \theta_0\right]$$

$$\Rightarrow \quad 2 = K\left[65 - \theta_0\right] \qquad \text{.... (1)}$$

$$\text{Next} \Rightarrow \frac{60 - 54}{5} = K\left[57 - \theta_0\right] \qquad \text{.... (2)}$$

Dividing eq. (1) and (2)

$$\frac{5}{3} = \frac{65 - \theta_0}{57 - \theta_0} \; ; \; 285 - 5\theta_0 = 195 - 3\theta_0$$

$$\Rightarrow \quad 2\theta_0 = 90 \Rightarrow \theta_0 = 45°$$

(12) **(A).** P – maximum intensity is at violet

$\Rightarrow \lambda_m$ is minimum $\Rightarrow$ temperature maximum
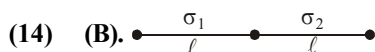
R – maximum intensity is at green

$\Rightarrow \lambda_m$ is moderate

$\Rightarrow$ temperature moderate

Q – maximum intensity is at red

$\Rightarrow \lambda_m$ is maximum $\Rightarrow$ temperature minimum

$$T_P > T_R > T_Q$$

(13) **(C).** As the temperature difference as well as the thermal resistance is same for both the cases, so thermal current will also be same for both the cases.

(14) **(B).**

$$R_{eq} = R_1 + R_2$$

$$\frac{2\ell}{\sigma_{eq} A} = \frac{\ell}{\sigma_1 A} + \frac{\ell}{\sigma_2 A} \Rightarrow \sigma_{eq} = \frac{2\sigma_1 \sigma_2}{\sigma_1 + \sigma_2}$$

(15) **(D).** $T_1 = 5760 \text{ K}$, $\lambda_m T = 2.88 \times 10^6 \text{ nmK}$

$$\lambda_m = \frac{2.88 \times 10^6 \text{ nmK}}{5760 \text{ K}} = 500 \text{ nm}$$

λ_m = Wavelength corresponding to maximum energy
$U_2 > U_1$.

(16) **(B).** Loss of heat by hot body=Gain of heat by cold body

$$T_{c_1} \Delta\theta_1 = T_{c_2} \Delta\theta_2$$

$$T_{c_1} > T_{c_2} \Rightarrow \Delta\theta_1 < \Delta\theta_2$$

(17) **(B).** $\Delta T = \Delta T_0 e^{-\lambda t}$; $T = 2Te^{-\lambda \,(10min)}$

$$\Delta T' = 2Te^{-\lambda \,(20min)} = 2T\left(\frac{1}{2}\right)^2 = \frac{T}{2}$$

$$\text{So, } T_f = T + \frac{T}{2} = \frac{3T}{2}$$

(18) **(D).** In parallel $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$\frac{K_{eq}(2A)}{\ell} = \frac{K_1 A}{\ell} + \frac{K_2 A}{\ell}$$

$$K_{eq} = \frac{K_1 + K_2}{2}$$

(19) **(C).** $P \propto r^2 T^4 \Rightarrow \dfrac{P_1}{P_2} = \left(\dfrac{r_1}{r_2}\right)^2 \left(\dfrac{T_1}{T_2}\right)^4$

$$P_2 = 1800 \text{ watt}$$

(20) **(A).** We know, $\lambda_{max} T = $ constant (Wien's law)

So, $\lambda_{max_1} T_1 = \lambda_{max_2} T_2$

$$\lambda_0 T = \frac{3\lambda_0}{4} T' \Rightarrow T' = \frac{4}{3} T$$

$$\text{So, } \frac{P_2}{P_1} = \left(\frac{T'}{T}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$