12

MATHEMATICAL REASONING

STATEMENT

A statement which is either true or false but cannot be both is called a statement. A sentence which is exclamatory or a wish or imperative or interrogative cannot be a statement. If a statement is true then its truth value is T and if it is false then its truth value is F.

For Ex.

(i) New Delhi is the capital of India, a true statement

(ii) $3 + 2 = 6$, a false Statement

(iii) where are you going? not a statement, because it can not be defined as true or false

Note : A statement cannot be both true and false at a time

SIMPLE STATEMENT

Any statement whose truth value does not depend on other statement is said to be a simple statement. For Ex.

(i) $\sqrt{2}$ is an irrational number.

(ii) The set of real numbers is an infinite set.

COMPOUND STATEMENT

A statement which is combination of two or more simple statements is said to be a compound statement.

The simple statement which form a compound statement are known as its sub-statements.

For Ex.

(i) If x is divisible by 2 then x is even number.

(ii) Δ ABC is equilateral if and only if its three sides are equal.

LOGICAL CONNECTIVES

The words or phrases which combined simple statements to form compound statement are called logical connectives. In the following table some possible connectives, their symbols and the nature of the compound statement formed by them.

S.N.	Connective	Symbol	Operations	Use
1.	and	$\wedge$	conjunction	$p \wedge q$
2.	or	$\vee$	disjunction	$p \vee q$
3.	not	$\sim$ or $'$	negation	$\sim p$ or p'
4.	If .... then	$\Rightarrow$ or $\rightarrow$	Implication or conditional	$p \Rightarrow q$ or $p \rightarrow q$
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	Equivalence or Bi-conditional	$p \Leftrightarrow q$ or $p \leftrightarrow q$

Explanation :

$p \wedge q \equiv$ statement p and q

($p \wedge q$ is true if and only if p and q both are true)

$p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true)

$\sim p \equiv$ statement p not

($\sim p$ is true if p is false and $\sim p$ is false if p is true)

$p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false if and only if p is true and q is false otherwise it is true all other cases)

$p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true if and only if statements p and q both are true or false)

TRUTH TABLE

A table that shows the relationship between the truth value of compound statement. S (p, q, r, ...) and the truth values of its sub-statements p, q, r, ... etc., is called the truth table of statement S.

If p and q are two simple statements then truth table for basic logical connectives are :

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$\sim p$
T	F
F	T

Conditional

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Leftrightarrow q$ or $(p \Rightarrow q) \wedge (q \Leftrightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement is made up of n sub-statements then its truth table will contain 2^n rows.

LOGICAL EQUIVALENCE

Two compound statements S_1 (p, q, r, ...) and S_2 (p, q, r, ...) are said to be logically equivalent if they have the same truth values for all logically possibilities.

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of the truth tables are same.

If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For Ex. The truth tables for $(p \to q)$ and $(\sim p \vee q)$ are as given below :

p	q	$(\sim p)$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that the last columns of the truth tables are identical, hence $p \to q \equiv \sim p \vee q$.

TAUTOLOGY AND CONTRADICTION

Tautology : A statement is said to be a tautology if it is true for all logical possibilities.

For ex. The statement $p \vee \sim (p \wedge q)$ is a tautology.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Clearly, the truth value of $p \vee \sim (p \wedge q)$ is T for all values of p and q, so it is a Tautology.

Contradiction : A statement is a contradiction if it is false for all logical possibilities i.e. its truth value always F.

For ex. The statements $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim p \wedge \sim q$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Clearly the truth table of $(p \vee q) \wedge (\sim p \wedge \sim q)$ is F for all values of p and q. So it is a contradiction

Note : The negation of a tautology is a contradiction and vice-versa.

DUALITY

Two compound statements S_1 and S_2 are said to the duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

1. The connectives $\wedge$ & $\vee$ are also called dual of each other

2. If $S^*(p,q)$ is the dual of the compound statement S (p, q) then (a) $S^*(\sim p, \sim q) \equiv \sim S(p, q)$ (b) $\sim S^*(p, q) \equiv S(\sim p, \sim q)$

For ex. The duals of the following statements.

(i) $(p \wedge q) \vee (r \vee s)$

(ii) $(p \vee t) \wedge (p \vee c)$

(iii) $\sim (p \wedge q) \vee [p \wedge \sim (q \vee \sim s)]$

are as given below :

(i) $(p \vee q) \wedge (r \wedge s)$

(ii) $(p \wedge c) \vee (p \wedge t)$

(iii) $\sim (p \vee q) \wedge [p \vee \sim (q \wedge \sim s)]$

CONVERSE, INVERSE AND CONTRAPOSITIVE OF THE CONDITIONAL STATEMENT $(p \to q)$

Converse: The converse of the conditional statement $p \to q$ is defined as $q \to p$.

Inverse : The inverse of the conditional statement $p \to q$ is defined as $\sim p \to \sim q$

Contrapositive : The contrapositive of the conditional statement $p \to q$ is defined as $\sim q \to \sim p$

NEGATION OF COMPOUND STATEMENTS

(i) Negation of conjunction : If p and q are two statements then $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Truth table of $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim(p \vee q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table of $\sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Clearly, truth tables of $\sim (p \wedge q)$ and $(\sim p \vee \sim q)$ are identical. Hence $\sim (p \wedge q) \equiv \sim p \vee \sim q$.

(ii) Negation of disconjunction : If p and q are two statements then $\sim (p \vee q) \equiv \sim p \wedge \sim q$

Truth table of $\sim (p \vee q)$

p	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table of $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(\sim p) \wedge (\sim q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Clearly, truth tables of $\sim (p \vee q)$ and $(\sim p \wedge \sim q)$ are identical. Hence $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

(iii) Negation of implication : If p and q are two statements, then $\sim (p \Rightarrow q) \equiv p \wedge \sim q$

Truth table of $\sim (p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\sim (p \Rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Truth table of $p \wedge \sim q$

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Clearly, truth tables of $\sim (p \Rightarrow q)$ and $(p \wedge \sim q)$ are identical hence, $\sim (p \Rightarrow q) \equiv p \wedge \sim q$.

(iv) Negation of Biconditional : If p and q are two statements, then $\sim (p \Rightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

We know that,

$$p \Rightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$$

$$\therefore \ \sim (p \Leftrightarrow q) = \sim \{(p \Rightarrow q) \wedge (q \Rightarrow p)\}$$

$$= \{\sim (p \Rightarrow q)\} \wedge \{-(q \Rightarrow p)\}$$

$$= (p \wedge \sim q) \vee (q \wedge \sim p)$$

Note : The above result can also be proved by preparing truth tables of $\sim (p \Leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

ALGEBRA OF STATEMENTS

If p, q, r are any three statements then the some law of algebra of statements are as follow :

(i) Idempotent laws :

(a) $p \vee p \equiv p$ (b) $p \wedge p \equiv p$

p	$p \vee p$	$p \wedge p$
T	T	T
F	F	F

i.e. $p \vee p \equiv p$ and $p \wedge p \equiv p$

(ii) Commutative laws : (a) $p \vee q \equiv q \vee p$ (b) $p \wedge p \equiv q \wedge p$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

(iii) Associative law :

(a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ (b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Similarly, (b) can be proved.

(iv) Distributive laws : (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Similarly, (b) can be proved.

(v) De 'Morgan's law :

(a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	~p	~q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Similarly, (b) can be proved.

(vi) Contrapositive laws : For any statement p, we have
$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$

p	q	~p	~q	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

(vii) Involution laws (for Double negation laws) :
$\sim(\sim p) \equiv p$

p	~p	~(~p)
T	F	T
F	T	F

QUANTIFIERS

Quantifiers are phrases like, "There exists" and "For all". Another phrase which appears in mathematical statements is "there exists". For example, consider the statement. p: There exists a rectangle whose all sides are equal. This means that there is atleast one rectangle whose all sides are equal. A word closely connected with "there exists" is "for every" (or for all). Consider a statement.

p: For every prime number p, $\sqrt{p}$ is an irrational number.

This means that if S denotes the set of all prime numbers, then for all the members p of the set S, $\sqrt{p}$ is an irrational number. In general, a mathematical statement that says "for every" can be interpreted as saying that all the members of the given set S where the property applies must satisfy that property.

TRY IT YOURSELF

Q.1 The negation of the statement "A circle is an ellipse" is
(A) An ellipse is a circle. (B) An ellipse is not a circle.
(C) A circle is not an ellipse. (D) A circle is an ellipse.

Q.2 The negation of the statement "101 is not a multiple of 3" is
(A) 101 is a multiple of 3. (B) 101 is a multiple of 2.
(C) 101 is an odd number. (D) 101 is an even number.

Q.3 The contrapositive of the statement
"If 7 is greater than 5, then 8 is greater than 6" is
(A) If 8 is greater than 6, then 7 is greater than 5.
(B) If 8 is not greater than 6, then 7 is greater than 5.
(C) If 8 is not greater than 6, then 7 is not greater than 5.
(D) If 8 is greater than 6, then 7 is not greater than 5.

Q.4 The contrapositive of the statement "If p, then q", is
(A) If q, then p. (B) If p, then ~ q.
(C) If ~ q, then ~ p. (D) If ~ p, then ~ q.

Q.5 Which of the following is the conditional $p \to q$?
(A) q is sufficient for p. (B) p is necessary for q.
(C) p only if q. (D) if q, then p.

Q.6 Which of the following statement is a conjunction ?
(A) Ram and Shyam are friends.
(B) Both Ram and Shyam are tall.
(C) Both Ram and Shyam are enemies.
(D) None of the above.

ANSWERS

(1) (C) **(2)** (A) **(3)** (C)
(4) (C) **(5)** (C) **(6)** (D)

ADDITIONAL EXAMPLES

Example 1 :
Which of the following is true for the statements p and q ?
(1) $p \wedge q$ is true when at least one of p and q is true
(2) $p \to q$ is true when p is true and q is false
(3) $p \leftrightarrow q$ is true only when both p and q are true
(4) $\sim(p \vee q)$ is true only when both p and q are false

Sol. **(4).** We know that $p \wedge q$ is true when both p and q are true.
So, option (1) is not true.
We know that $p \to q$ is false when p is true and q is false.
So, option (2) is not true.
We know that $p \leftrightarrow q$ is true when either both p and q are true or both are false. So, option (3) is not true.
If p and q both are false, then
$p \vee q$ is false $\Rightarrow \sim(p \vee q)$ is true. Hence, option (4) is true.

Example 2 :
$\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to –
(1) p (2) ~ p
(3) q (4) ~q

Sol. **(2).** We have
$\cong \sim(p \vee q) \vee (\sim p \wedge q) \cong (\sim p \wedge \sim q) \vee (\sim p \wedge q)$
$\cong \sim p \wedge (\sim q \vee q) \cong \sim p \wedge t \cong \sim p$

Example 3 :

$(p \wedge \sim q) \wedge (\sim p \vee q)$ is –

(1) a tautology

(2) a contradiction

(3) both a tautology and a contradiction

(4) neither a tautology nor a contradiction

Sol. (2). The truth table of $(p \wedge \sim q) \wedge (\sim p \vee q)$ is as given below

p	q	~p	~q	$p \vee \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

The last column of the above truth table contains F only. So, then given statement is a contradiction.

Example 4 :

If $x = 5$ and $y = -2$, then $x - 2y = 9$. Then contrapositive of this proposition is

(1) If $x - 2y \neq 9$, then $x \neq 5$ or $y \neq -2$.

(2) If $x - 2y = 9$ then $x \neq 5$ and $y \neq -2$

(3) $x - 2y = 9$ if and only if $x = 5$ and $y = -2$

(4) None of these

Sol. (1). Let p, q and r be three propositions given by

$p : x = 5$, $q : y = -2$ and $r : x - 2y = 9$

Then, the given statement is $(p \wedge q) \rightarrow r$

Its contrapositive is

$$\sim r \rightarrow \sim (p \wedge q)$$

i.e., $\sim r \rightarrow \sim p \vee \sim q$

i.e., If $x - 2y \neq 9$, then $x \neq 5$ or $y \neq -2$

QUESTION BANK

CHAPTER 12 : MATHEMATICAL REASONING

EXERCISE - 1

Q.1 The inverse of the statement $(p \wedge \sim q) \rightarrow r$ is –
(A) $\sim (p \vee \sim q) \rightarrow \sim r$ (B) $(\sim p \wedge q) \rightarrow \sim r$
(C) $(\sim p \vee q) \rightarrow \sim r$ (D) None of these

Q.2 If the compound statement $p \rightarrow (\sim p \vee q)$ is false then the truth value of p and q are respectively –
(A) T, T (B) T, F
(C) F, T (D) F, F

Q.3 The statement $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ is –
(A) a tautology
(B) a contradiction
(C) neither a tautology nor a contradiction
(D) None of these

Q.4 Negation of the statement $(p \wedge r) \rightarrow (r \vee q)$ is –
(A) $\sim (p \wedge r) \rightarrow \sim (r \vee q)$ (B) $(\sim p \vee \sim r) \vee (r \vee q)$
(C) $(p \wedge r) \wedge (r \wedge q)$ (D) $(p \wedge r) \wedge (\sim r \wedge \sim q)$

Q.5 Which of the following is always true –
(A) $(\sim p \vee \sim q) \equiv (p \wedge q)$
(B) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
(C) $\sim (p \rightarrow \sim q) \equiv (p \wedge \sim q)$
(D) $\sim (p \leftrightarrow q) \equiv (p \rightarrow q) \rightarrow (q \rightarrow p)$

Q.6 The contrapositive of $p \rightarrow (\sim q \rightarrow \sim r)$ is –
(A) $(\sim q \wedge r) \rightarrow \sim p$ (B) $(q \rightarrow r) \rightarrow \sim p$
(C) $(q \vee \sim r) \rightarrow \sim p$ (D) None of these

Q.7 If p and q are two statement then $(p \leftrightarrow \sim q)$ is true when
(A) p and q both are true (B) p and q both are false
(C) p is false and q is true (D) None of these

Q.8 Which of the following statement is a contradiction –
(A) $(\sim p \vee \sim q) \vee (p \vee \sim q)$ (B) $(p \rightarrow q) \vee (p \wedge \sim q)$
(C) $(\sim p \wedge q) \wedge (\sim q)$ (D) $(\sim p \wedge q) \vee (\sim q)$

Q.9 If p, q, r are simple statement with truth values T, F, T respectively then the truth value of $((\sim p \vee q) \wedge \sim r) \rightarrow p$ is
(A) True (B) False
(C) True if r is false (D) False if q is true

Q.10 Which of the following is wrong –
(A) $p \vee \sim p$ is a tautology
(B) $\sim (\sim p) \leftrightarrow p$ is a tautology
(C) $p \wedge \sim p$ is a contradiction
(D) $((p \wedge q) \rightarrow q) \rightarrow p$ is a tautology

Q.11 The statement "If $2^2 = 5$ then I get first class" is logically equivalent to –
(A) $2^2 = 5$ and I do not get first class
(B) $2^2 = 5$ or I do not get first class
(C) $2^2 \neq 5$ or I get first class
(D) None of these

Q.12 Which of the following is logically equivalent to $(p \wedge q)$?
(A) $p \rightarrow \sim q$ (B) $\sim p \vee \sim q$
(C) $\sim (p \rightarrow \sim q)$ (D) $\sim (\sim p \wedge \sim q)$

Q.13 If $p \rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively,
(A) T, F, F (B) F, F, F
(C) F, T, T (D) T, T, F

Q.14 Negation of the statement $p \rightarrow (q \wedge r)$ is
(A) $\sim p \rightarrow \sim (q \vee r)$ (B) $\sim p \rightarrow \sim (q \wedge r)$
(C) $(q \wedge r) \rightarrow p$ (D) $p \wedge (\sim q \vee \sim r)$

Q.15 The negation of the proposition "If a quadrilateral is a square, then it is a rhombus" is –
(A) if a quadrilateral is not a square, then it is a rhombus
(B) if a quadrilateral is a square, then it is not a rhombus
(C) a quadrilateral is a square and it is not a rhombus
(D) a quadrilateral is not a square and it is a rhombus

Q.16 Which of the following is wrong –
(A) $p \rightarrow q$ is logically equivalent to $\sim p \vee q$
(B) If the truth values of p, q, r are T, F, T respectively, then the truth value of $(p \vee q) \wedge (q \vee r)$ is T
(C) $\sim (p \vee q \vee r) \cong \sim p \wedge \sim q \wedge \sim r$
(D) The truth value of $p \wedge \sim (p \vee q)$ is always T.

Q.17 The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
(A) $p \rightarrow (p \vee q)$ (B) $p \rightarrow (p \wedge q)$
(C) $p \rightarrow (p \leftrightarrow q)$ (D) $p \rightarrow (p \rightarrow q)$

Q.18 Let p be the statement 'x is an irrational number", q be the statement 'y is a transcendental number", and r be the statement:
"x is an irrational number iff y is a transcendental number.".
Statement-1 : r is equivalent to either q or p.
Statement -2 : r is equivalent ot $\sim (p \leftrightarrow \sim q)$
(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement -2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.19 **Statement-1** : $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement -2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.
(A) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1.
(B) Statement-1 is true, Statement -2 is false.
(C) Statement-1 is false, Statement -2 is true.
(D) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Q.20 Let S be a non-empty subset of R.
Consider the following statement:
P: There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(A) There is no rational number $x \in S$ such that $x \leq 0$
(B) Every rational number $x \in S$ satisfies $x \leq 0$
(C) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational
(D) There is a rational number $x \in S$ such that $x \leq 0$

Q.21 Which of the following is true for the statements p and q
(A) $p \wedge q$ is true when at least one of p and q is true
(B) $p \rightarrow q$ is true when p is true and q is false
(C) $p \leftrightarrow q$ is true only when both p and q are true
(D) $\sim (p \vee q)$ is true only when both p and q are false

Q.22 $\sim (p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to –
(A) p (B) ~ p
(C) q (D) ~q

Q.23 $(p \wedge \sim q) \wedge (\sim p \vee q)$ is –
(A) a tautology
(B) a contradiction
(C) both a tautology and a contradiction
(D) neither a tautology nor a contradiction

Q.24 If x = 5 and y = – 2, then x – 2y = 9. Then contrapositive of this proposition is
(A) If $x - 2y \neq 9$, then $x \neq 5$ or $y \neq -2$.
(B) If $x - 2y = 9$ then $x \neq 5$ and $y \neq -2$
(C) $x - 2y = 9$ if and only if $x = 5$ and $y = -2$
(D) None of these

EXERCISE - 2 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 The statement $p \to (q \to p)$ is equivalent to **[AIEEE-2008]**
(A) $p \to (p \vee q)$ (B) $p \to (p \wedge q)$
(C) $p \to (p \leftrightarrow q)$ (D) $p \to (p \to q)$

Q.2 Let p be the statement 'x is an irrational number", q be the statement 'y is a transcendental number", and r be the statement.
"x is an irrational number iff y is a transcendental number.".
Statement-1 : r is equivalent to either q or p.
Statement -2 : r is equivalent ot $\sim (p \leftrightarrow \sim q)$ **[AIEEE-2008]**
(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement -2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.3 **Statement-1** : $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement -2 : $\sim (p \leftrightarrow \sim q)$ is a tautology. **[AIEEE-2009]**
(A) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1.
(B) Statement-1 is true, Statement -2 is false.
(C) Statement-1 is false, Statement -2 is true.
(D) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Q.4 Let S be a non-empty subset of R. Consider the following statement: **[AIEEE 2010]**
P: There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(A) There is no rational number $x \in S$ such that $x \leq 0$
(B) Every rational number $x \in S$ satisfies $x \leq 0$
(C) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational
(D) There is a rational number $x \in S$ such that $x \leq 0$

Q.5 Consider the following statements **[AIEEE 2011]**
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest.
The negation of the statement . Suman is brilliant and dishonest if and only if Suman is rich. can be expressed as :
(A) $\sim P \wedge (Q \leftrightarrow \sim R)$ (B) $\sim (Q \leftrightarrow (P \wedge \sim R))$
(C) $\sim Q \leftrightarrow \sim P \wedge R$ (D) $\sim (P \wedge \sim R) \leftrightarrow Q$

Q.6 The negation of the statement **[AIEEE 2012]**
"If I become a teacher, then I will open a school", is :
(A) I will become a teacher and I will not open a school.
(B) Either I will not become a teacher or I will not open a school.
(C) Neither I will become a teacher nor I will open a school
(D) I will not become a teacher or I will open a school.

Q.7 Consider **[JEE MAIN 2013]**
Statement-I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement-II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.
(A) Statement-I is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is true; Statement-II is true; Statement-II is not a correct explanation for Statement-I.
(C) Statement-I is true; Statement-II is false.
(D) Statement-I is false; Statement-II is true.

Q.8 The statement $\sim (p \leftrightarrow \sim q)$ is – **[JEE MAIN 2014]**
(A) equivalent to $p \leftrightarrow q$ (B) equivalent to $\sim p \leftrightarrow q$
(C) a tautology (D) a fallacy

Q.9 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to **[JEE MAIN 2015]**
(A) $s \wedge (r \wedge \sim s)$ (B) $s \vee (r \vee \sim s)$
(C) $s \wedge r$ (D) $s \wedge \sim r$

Q.10 The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to : **[JEE MAIN 2016]**
(A) $p \wedge q$ (B) $p \vee q$
(C) $p \vee \sim q$ (D) $\sim p \wedge q$

Q.11 The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is **[JEE MAIN 2017]**
(A) equivalent to $p \to \sim q$ (B) a fallacy
(C) a tautology (D) equivalent to $\sim p \to q$

Q.12 The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to: **[JEE MAIN 2018]**
(A) q (B) ~q
(C) ~p (D) p

Q.13 If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is: **[JEE MAIN 2019 (JAN)]**
(A) $(\wedge, \vee)$ (B) $(\vee, \vee)$
(C) $(\wedge, \wedge)$ (D) $(\vee, \wedge)$

Q.14 The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : **[JEE MAIN 2019 (APRIL)]**
(A) If you are born in India, then you are not a citizen of India.
(B) If you are not a citizen of India, then you are not born in India.
(C) If you are a citizen of India, then you are born in India.
(D) If you are not born in India, then you are not a citizen of India.

Q.15 For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is **[JEE MAIN 2019 (APRIL)]**
(A) $p \wedge q$ (B) $p \leftrightarrow q$
(C) $\sim p \vee \sim q$ (D) $\sim p \wedge \sim q$

Q.16 If $P \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively : **[JEE MAIN 2019 (APRIL)]**
(A) F, T, T (B) T, F, F
(C) T, T, F (D) F, F, F

Q.17 Which one of the following Boolean expressions is a tautology ? **[JEE MAIN 2019 (APRIL)]**
(A) $(p \vee q) \wedge (\sim p \vee \sim q)$ (B) $(p \wedge q) \vee (p \wedge \sim q)$
(C) $(p \vee q) \wedge (p \vee \sim q)$ (D) $(p \vee q) \vee (p \vee \sim q)$

Q.18 The negation of the boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[JEE MAIN 2019 (APRIL)]**
(A) r (B) $s \wedge r$
(C) $s \vee r$ (D) $\sim s \wedge \sim r$

Q.19 If the truth value of the statement $P \to (\sim p \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively **[JEE MAIN 2019 (APRIL)]**
(A) F, T, T (B) T, F, F
(C) T, T, F (D) T, F, T

Q.20 The Boolean expression $\sim (p \Rightarrow (\sim q))$ is equivalent to : **[JEE MAIN 2019 (APRIL)]**
(A) $(\sim p) \Rightarrow q$ (B) $p \vee q$
(C) $q \Rightarrow \sim p$ (D) $p \wedge q$

Q.21 $(p \to q) \wedge (q \to \sim p)$ is equivalent to **[JEE MAIN 2020 (JAN)]**
(A) $\sim p$ (B) p
(C) $p \wedge q$ (D) $p \vee q$

Q.22 Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$ " is : **[JEE MAIN 2020 (JAN)]**
(A) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(B) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
(C) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
(D) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

Q.23 Which of the following is tautology **[JEE MAIN 2020 (JAN)]**
(A) $(p \wedge (p \to q)) \to q$ (B) $q \to p \wedge (p \to q)$
(C) $p \vee (p \wedge q)$ (D) $(p \wedge (p \vee q)$

Q.24 Which of the following is tautology **[JEE MAIN 2020 (JAN)]**
(A) $\sim (p \vee \sim q) \to (p \vee q)$ (B) $(\sim p \vee q) \to (p \vee q)$
(C) $\sim(p \wedge \sim q) \to (p \vee q)$ (D) $\sim(p \vee \sim q) \to (p \wedge q)$

Q.25 Negation of the statement : $\sqrt{5}$ is an integer or 5 is irrational is : **[JEE MAIN 2020 (JAN)]**
(A) $\sqrt{5}$ is irrational or 5 is an integer.
(B) $\sqrt{5}$ is not an integer and 5 is not irrational.
(C) $\sqrt{5}$ is an integer and 5 is irrational.
(D) $\sqrt{5}$ is not an integer or 5 is not irrational.

Q.26 If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively : **[JEE MAIN 2020 (JAN)]**
(A) F, T (B) T, T
(C) F, F (D) T, F

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A	C	B	B	D	B	A	C	C	D	D	C	C	A	D	C	D	A	D	C	B	D	B	B	A

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13
A	A	D	C	B	B	A	B	A	C	B	C	C	A
Q	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	D	B	D	B	C	D	A	B	A	A	B	B

CHAPTER- 12 :
MATHEMATICAL REASONING
TRY IT YOURSELF

(1) (C) **(2)** (A) **(3)** (C) **(4)** (C) **(5)** (C) **(6)** (D)

CHAPTER- 12 : MATHEMATICAL REASONING

EXERCISE-1

(1) **(C).** The inverse of the proposition $(p \wedge \sim q) \rightarrow r$ is
$\sim (p \wedge \sim q) \rightarrow \sim r \equiv \sim p \vee \sim(\sim q) \rightarrow \sim r \equiv \sim p \vee q \rightarrow \sim r$

(2) **(B).** We know that $p \rightarrow q$ is false only when p is true and q is false. So $p \rightarrow (\sim p \vee q)$ is false only when p is true and $(\sim p \vee q)$ is false.
But $(\sim p \vee q)$ is false if q is false because ~p is false.
Hence $p \rightarrow (\sim p \vee q)$ is false when truth value of p and q are T and F respectively.

(3) **(B).** The truth table of $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ as

p	~p	p→~p	~p→p	(p→~p) ∧ (~p→p)
T	F	F	T	F
F	T	T	F	F

Clearly last column of the above truth table contains only F. Hence $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ is a contradiction.

(4) **(D).** We know that $\sim (p \rightarrow q) \equiv p \wedge \sim q$
$\therefore \sim((p \wedge r) \rightarrow (r \vee q)) \equiv (p \wedge r) \wedge [\sim(r \vee q)]$
$\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$

(5) **(B).** Since $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ and $\sim (p \wedge q) \equiv (\sim p \vee q)$
So option (B) and (D) are not true.
$(p \rightarrow q) \equiv p \wedge \sim q)$, so option (C) is not true.
Now $p \rightarrow q \sim p \vee q$
$\sim q \rightarrow \sim p \equiv [\sim (\sim q) \vee \sim p] \equiv q \vee \sim p \equiv \sim p \vee q$
$p \rightarrow q \equiv \sim q \rightarrow \sim p$

(6) **(A).** We know that the contropositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$. So contra positive of $p \rightarrow (\sim q \rightarrow \sim r)$ is
$\sim (\sim q \rightarrow \sim r) \rightarrow \sim p \equiv \sim q \wedge [\sim (\sim r)]\ \sim p$
$\because \sim (p \rightarrow q) \equiv p \wedge \sim q \equiv \sim q \wedge r \rightarrow \sim p$

(7) **(C).** We know that $p \leftrightarrow q$ is true if p and q both are true or false. So $p \leftrightarrow \sim q$ is true when if p and ~q is true.
i.e., p is true and q is false.
or p and ~q is false, i.e. p is false and q is true.
Hence, option (C) is correct

(8) **(C).** We consider following truth table.

p	q	~p	~q	p ∧ q	p ∨ q	(~(p ∨ q)	(p ∧ q) ∧ (~(p ∨ q)
T	T	F	F	T	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	F	F	T	F

Clearly last column of the above truth table contains only F. Hence $(p \wedge q) \wedge (\sim(p \vee q)$ is a contradiction

(9) **(D).** Since p, q, r have truth values T, F, T respectively $(p \rightarrow q) \wedge r$ is true only when $(p \rightarrow q)$ and r both are true but $p \rightarrow q$ is false only when p is true and q is false.
So option (A) are not true.
Here truth value of $p \rightarrow q$ is F. So truth value of $(p \rightarrow q) \wedge \sim r$ is F.
So option (B) is not true.
Again $(p \wedge q) \wedge (p \vee r)$ is true only when $(p \wedge q)$ and $(p \vee r)$ both are true but here truth value of $(p \wedge q)$ is F. So option (C) is not true since truth value of $(p \wedge r)$ is T and $q \rightarrow (p \wedge r)$ is false only when q is true and $(p \wedge r)$ is false.

(10) **(D).** The truth value of $\sim(\sim p) \leftrightarrow p$ as follow

p	~p	~(~p)	~(~p)→p	p→~(~p)	~(~p)↔p
T	F	T	T	T	T
F	T	F	T	T	T

Since last column of above truth table contains only T.
Hence $\sim (\sim p) \rightarrow p$ is a tautology.

(11) **(C).** Let p and q be two proposition given by p : $2^2 = 5$,
q : 1 get first class
Here give statement is $p \rightarrow q$
So contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
i.e. if I do not get first class then $2^2 \neq 5$.

(12) **(C).** We know that $p \rightarrow q \cong \sim p \vee q$
$\therefore\ p \rightarrow \sim q \cong \sim p \vee \sim q \cong \sim (p \wedge q)$
So, option (A) is not correct.
We have, $\sim p \vee \sim q \cong \sim (p \wedge q)$
So, option (B) is not correct.
We have, $\sim (p \rightarrow \sim q) \equiv \sim (p \wedge q) \Rightarrow \sim (p \rightarrow \sim q) \equiv p \wedge q$
So, option (C) is correct.
We have, $\sim (\sim p \wedge \sim q) = \sim (\sim (p \vee q)) = p \vee q$
So, option (D) is not correct

(13) **(A).** We know that $p \rightarrow q \vee r$ is false only when p is true and $q \vee r$ is false.
But, $q \vee r$ is false when both q and r are false.
Hence, truth values of p, q , r are respectively, T, F, F

(14) **(D).** We know that
$\sim (p \rightarrow q) \cong p \wedge \sim q$
$\therefore\ \sim (p \rightarrow \sim (q \wedge r)) \cong p \wedge (\sim (q \wedge r))$
$\cong p \wedge (\sim q \vee \sim r))$ [By De'Morgan's laws]

(15) **(C).** Let p and q be the propositions as given below :
p : A quadrilateral is a square
q : A quadrilateral is a rhombus
The given proposition is $p \rightarrow q$
Now, $\sim (p \rightarrow q) \cong p \wedge \sim q$
Therefore, the negation of the given proposition is :
A quadrilateral is a square and it is not a rhombus

(16) **(D).** The truth tables of $p \rightarrow q$ and $\sim p \vee q$ are given below:

p	q	~p	p → q	~ (p ∨ q)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Clearly, truth tables of $p \rightarrow q$ and $\sim p \vee q$ are same.
So, $p \rightarrow q$ is logically equivalent to $\sim p \vee q$.
Hence, option (A) is correct.
If the truth value of p, q, r are T, F, T respectively, then the truth values of $p \vee q$ and $q \vee r$ are each equal to T.
Therefore, the truth value of $(p \vee q) \wedge (q \vee r)$ is T.
Hence, option (B) is correct.
We have, $\sim (p \vee q \vee r) \cong (\sim p \wedge \sim q \wedge \sim r)$

So, option, (C) Is correct.
If p is true and q is false, then $p \vee q$ is true. Consequently, $\sim(p \vee q)$ is false and hence $p \wedge \sim(p \vee q)$ is false.
Hence, option (D) is wrong.

(17) **(A).** Truth table of $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
F	T	F	T
T	F	T	T
F	F	T	T

Truth table of option (i)
$p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Option (ii) $p \to (p \wedge q)$

p	q	$p \wedge q$	$p \to (p \wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

$\therefore$ $p \to (q \to p)$ equivalent to option (1)

(18) **(D).** p : x is an irrational no.
q : y is an transscedental number
r : $p \Leftrightarrow q$.   Truth table for r : $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\because$ r is not equivalent to either p or q
Truth table of $\sim(p \Leftrightarrow \sim q)$

p	q	$\sim q$	$p \Leftrightarrow \sim q$	$\sim(p \Leftrightarrow \sim q)$
T	T	F	F	T
F	T	F	T	F
T	F	T	T	F
F	F	T	F	T

$\therefore$ r is equivalent of $\sim(p \Leftrightarrow \sim q)$ $\therefore$ Statement (2) is true.

(19) **(C).** Statement 1 : $\sim(p \leftrightarrow \sim q)$ means $\sim p \leftrightarrow q$
Let us break statement 1 in two parts : (A) $\sim p \leftrightarrow q$ and (B) $p \leftrightarrow q$
(A) $\sim p \leftrightarrow q$ gives us two statements
(1) $\sim p$ – True $\Rightarrow$ q – True   (2) q – True $\Rightarrow \sim p$ – True
(B) $p \leftrightarrow q$ gives us following two statements
(i) p – True $\Rightarrow$ q – True   (ii) q – True $\Rightarrow$ p – True
Observing the above statements we find statement (ii) of (A) and (B) contradict each other. So, statement-1 is false.
Again, in statement-2 we have
(1) $\sim p$ – True $\Rightarrow$ q – True and q – True $\Rightarrow \sim p$ – True
(2) q – True $\Rightarrow \sim p$ – True and $\sim p$ – True $\Rightarrow$ q – True
The above two statements clearly show tautology.

(20) **(B).** P: there is a rational number $x \in S$ such that $x > 0$
~P: Every rational number $x \in S$ satisfies $x \le 0$

(21) **(D).** We know that $p \wedge q$ is true when both p and q are true. We know that $p \to q$ is false when p is true and q is false.
We know that $p \leftrightarrow q$ is true when either both p and q are true or both are false. If p and q both are false, then $p \vee q$ is false $\Rightarrow \sim(p \vee q)$ is true. Hence, option (D) is true.

(22) **(B).** We have
$\cong \sim(p \vee q) \vee (\sim p \wedge q) \cong (\sim p \wedge \sim q) \vee (\sim p \wedge q)$
$\cong \sim p \wedge (\sim q \vee q) \cong \sim p \wedge t \cong \sim p$

(23) **(B).** The truth table of $(p \wedge \sim q) \wedge (\sim p \vee q)$ is as :

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

The last column of the above truth table contains F only. So, then given statement is a contradiction.

(24) **(A).** Let p, q and r be three propositions given by
p : x = 5, q : y = –2 and r : x – 2y = 9
Then, the given statement is $(p \wedge q) \to r$
Its contrapositive is $\sim r \to \sim(p \wedge q)$ i.e., $\sim r \to \sim p \vee \sim q$
i.e., If $x - 2y \ne 9$, then $x \ne 5$ or $y \ne -2$

EXERCISE-2

(1) **(A).** Truth table of $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
F	T	F	T
T	F	T	T
F	F	T	T

Truth table of option (i)
$p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Option (ii) $p \to (p \wedge q)$

p	q	$p\wedge q$	$p\to(p\wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

$\therefore$ $p \to (q \to p)$ equivalent to option (1)

(2) **(D).** p : x is an irrational no.

q : y is an transscedental number

r : $p \Leftrightarrow q$

Truth table for r : $p \Leftrightarrow q$

p	q	$p\Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\because$ r is not equivalent to either p or q

Truth table of $\sim(p \Leftrightarrow \sim q)$

p	q	$\sim q$	$p\Leftrightarrow\sim q$	$\sim(p\Leftrightarrow\sim q)$
T	T	F	F	T
F	T	F	T	F
T	F	T	T	F
F	F	T	F	T

$\therefore$ r is equivalent of $\sim(p \Leftrightarrow \sim q)$ $\therefore$ Statement (2) is true.

(3) **(C).** Statement 1 : $\sim(p \leftrightarrow \sim q)$ means $\sim p \leftrightarrow q$

Let us break statement 1 in two parts :

(A) $\sim p \leftrightarrow q$ and (B) $p \leftrightarrow q$

(A) $\sim p \leftrightarrow q$ gives us two statements

(1) $\sim p$ – True $\Rightarrow$ q – True

(2) q – True $\Rightarrow$ $\sim p$ – True

(B) $p \leftrightarrow q$ gives us following two statements

(i) p – True $\Rightarrow$ q – True (ii) q – True $\Rightarrow$ p – True

Observing the above statements we find statement (ii) of (A) and (B) contradict each other. So, statement-1 is false. Again, in statement-2 we have

(1) $\sim p$ – True $\Rightarrow$ q – True and q – True $\Rightarrow$ $\sim p$ – True

(2) q – True $\Rightarrow$ $\sim p$ – True and $\sim p$ – True $\Rightarrow$ q – True

The above two statements clearly show tautology.

(4) **(B).** P: there is a rational number $x \in S$ such that $x > 0$

~P: Every rational number $x \in S$ satisfies $x \le 0$

(5) **(B).** Negation of $(P \wedge \sim R) \leftrightarrow Q$ is $\sim((P \wedge - R) \leftrightarrow Q)$

It may also be written as $\sim(Q \leftrightarrow (P \wedge \sim R))$

(6) **(A).** Let p : I become a teacher

q : I will open a school

Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

(7) **(B).** Statement-II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true, so statement -II is true

Statement-I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$= p \wedge \sim q \wedge \sim p \wedge q = p \wedge \sim p \wedge \sim q \wedge q = f \wedge f = f$

so statement -I is true

Alternate : Statement-II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\sim q \to \sim p$ is contrapositive of $p \to q$ hence

$(p \to q) \leftrightarrow (p \to q)$ will be a tautology

Statement -I $(p \wedge \sim q) \wedge (\sim p \wedge q)$

p	q	$p\wedge\sim q$	$-p\wedge q$	$(p\wedge\sim q)\wedge(\sim p\wedge q)$
T	T	F	F	F
T	F	T	F	F
F	T	F	T	F
F	F	F	F	F

$\therefore$ It is a fallacy.

(8) **(A).**

P	q	$\sim q$	$p\leftrightarrow\sim q$	$\sim(p\leftrightarrow\sim q)$	$p\leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

(9) **(C).** $\sim(\sim s \vee (\sim r \wedge s)) = s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s) = s \wedge r$

(10) **(B).** $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= [(p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q) = [(p \vee q) \wedge t] \vee (\sim p \wedge q)$

$= (p \vee q) \vee (\sim p \wedge q) = [(p \vee q) \vee \sim p) \wedge (p \vee q \vee q)]$

$= (t \vee q) \wedge (p \vee q) = t \wedge (p \vee q) = p \vee q$

(11) **(C).** $(p \to q) \to [(\sim p \to q) \to q]$

p	q	$(p\to q)$	$(\sim p\to q)$	$(\sim p\to q)\to q$	$(p\to q)\to[(\sim p\to q)\to q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

(12) **(C).** $\sim(p \vee q) \vee (\sim p \wedge q)$

p	q	$\sim(p\vee q)$	$\sim p\wedge q$	$\sim p$
T	F	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	F	T

(13) **(A).** $(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$ (Given)

p	q	$\sim p$	$p\wedge q$	$p\vee q$	$\sim p\vee q$	$\sim p\wedge q$	$(p\wedge q)\wedge(\sim p\vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

From truth table $(\oplus, \odot) = (\wedge, \vee)$

(14) **(B).** The contrapositive of statement $p \to q$ is $\sim q \to \sim p$
Here, p : you are born in India.
q : you are citizen of India.
So, contrapositive of above statement is
"If you are not a citizen of India, then you are not born in India".

(15) **(D).** $\sim (p \vee (\sim p \wedge q))$
$= \sim p \wedge \sim (\sim p \wedge q) = \sim p \wedge (p \vee \sim q)$
$= (\sim p \wedge p) \vee (\sim p \wedge \sim q) = c \vee (\sim p \wedge \sim q) = (\sim p \wedge \sim q)$

(16) **(B).** $P \Rightarrow (q \vee r) : F$
$P : T \; q \vee r : F$
$P : T : q : F : r : F$

(17) **(D).**
(A) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim (p \wedge q)$
$\to$ Not tautology (Take both p and q as T)
(B) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$
(C) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$
(D) $(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$

(18) **(B).** $\sim (\sim s \vee (\sim r \wedge s))$
$s \wedge (r \vee \sim s)$
$(s \wedge r) \vee (s \wedge \sim s)$
$(s \wedge r) \vee (c)$
$(s \wedge r)$

(19) **(C).** $P \to (\sim p \vee r)$
$\sim p \vee (\sim q \vee r)$

$$\left.\begin{matrix} \sim p \to F \\ \sim q \to F \\ r \to F \end{matrix}\right\} \Rightarrow \left.\begin{matrix} p \to T \\ q \to T \\ r \to F \end{matrix}\right\}$$

(20) **(D).** $\sim (p \to (\sim q)) = \sim (\sim p \vee \sim q) = p \wedge q$

(21) **(A).**

p	q	$p \to q$	$\sim p$	$q \to \sim p$	$(p \to q) \wedge (p \to \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \to q) \wedge (q \to \sim p)$ is equivalent to $\sim p$

(22) **(B).** Contrapositive of $p \to q$ is $\sim q \to \sim p$
$(A \subseteq B) \wedge (B \subseteq D) \to (A \subseteq C)$
Contrapositive is $\sim (A \subseteq C) \to \sim (A \subseteq B) \vee \sim (B \subseteq D)$
$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

(23) **(A).**

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$	$q \to p \wedge (p \to q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

(24) **(A).** $(\sim p \wedge q) \to (p \vee q)$
$\sim \{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

(25) **(B).** $p = \sqrt{5}$ is an integer.
q : 5 is irrational
$\sim (p \vee q) \equiv \sim p \wedge \sim q$
$= \sqrt{5}$ is not an integer and 5 is not irrational.

(26) **(B).** $p \to (p \wedge \sim q)$ is F
$\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T
$\therefore$ p is T, q is T.