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WAVES

INTRODUCTION OF WAVES

What is wave motion?

* When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (one way of transport energy from one place to another place)
* There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

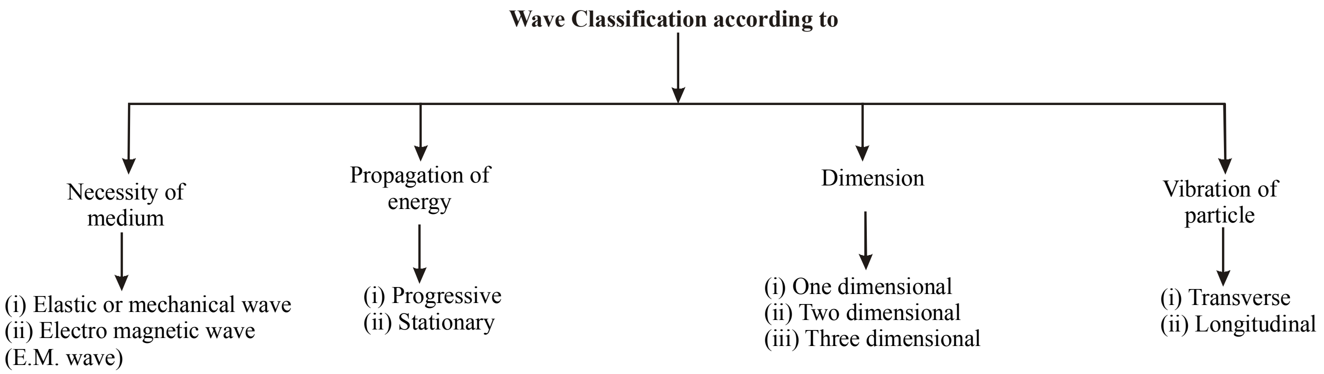
When you say "Namaste" to your friend no material particle is ejected from your lips to falls on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transfered to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

Note : In the above example air itself does not move.

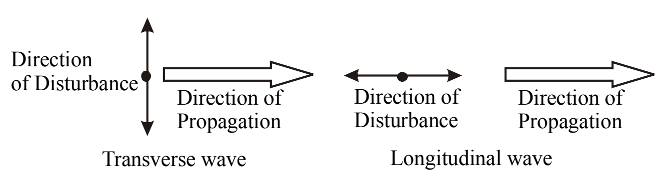
A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

Few examples of waves : The ripples on a pond(water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES



1. **Based on medium necessity :** A wave mayor may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as elastic waves.
   Example : Sound waves in water, seismic waves in earth's crust.
2. **Based on energy propagation:** Waves can be divided into two parts on the basis of energy propagation
   (i) Progressive wave (ii) Stationary waves.
   The progressive wave propagates with fixed velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.
3. **Based on direction of propagation :** Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.
4. **Based on the motion of particles of medium :** Waves are of two types on the basis of motion of particles of the medium.
   (i) Longitudinal waves (ii) Transverse waves
   In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave.



In the longitudinal wave the direction of disturbance is along the direction of propagation.

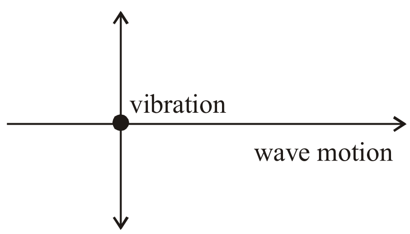
TRANSVERSE WAVE MOTION

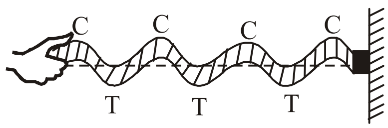
Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave.

Note : Shearing is the property of a body by which it changes its shape on application of force.

Mechanical transverse waves are generated only in solids & surface of liquid.

In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.



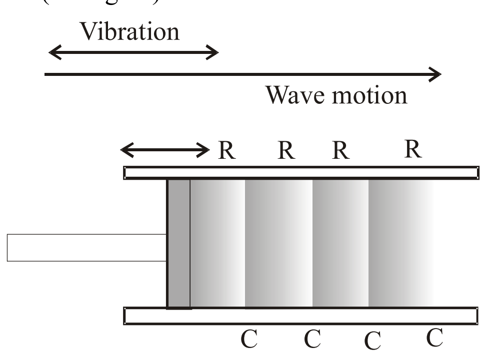


A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarafaction (low pressure) which propagated in space with time (see figure)



Note: The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

The propagation of sound waves in air is visualised as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

MECHANICAL WAVES IN DIFFERENT MEDIA

A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation. In strings mechanical waves are always transverse that too when string is under a tension. In gases and liquids mechanical waves are always longitudinal, .e.g., sound waves in air or water. This is because fluids cannot sustain shear. In solids mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle the waves in the rod will be transverse while if the rod is struck at the side or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal. Further more in case of seismic waves produced by earth-quakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds ($V_S \approx 5$ km/s while $V_p \approx 9$ km/s] S-waves are transverse while P-waves longitudinal. Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called ripple and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.

CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows:

(i) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(ii) The energy is transferred from place to another without any actual transfer of the particles of the medium.

(iii) Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.

(iv) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.

(v) The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.

(vi) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

Wavelength (λ) [length of one wave] : Distance travelled by the wave during the time, anyone particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.

Frequency (n) : Number of vibrations (Number of complete wavelengths) complete by a particle in one second.
Time period (T) : Time taken by wave to travel a distance equal to one wavelength.
Amplitude (A) : maximum displacement of vibrating particle from its equilibrium position.

Angular frequency (ω) : It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$

Phase : Phase is a quantity which contains all information related to any vibrating particle in a wave.
For equation $y = y = A \sin(\omega t - kx)$, $(\omega t - kx)$ = phase.

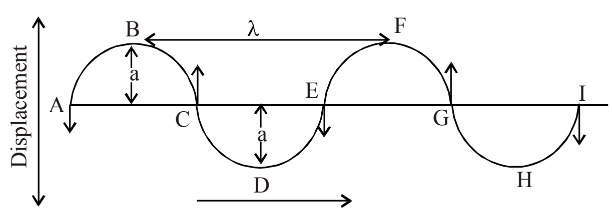
Angular wave number (k) : It is defined as $k = \frac{2\pi}{\lambda}$

Wave number (κ) : It is defined as

$\kappa = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.

Particle velocity, wave velocity and particle's acceleration
In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae in SHM apply to the particles here also. For example, maximum particle velocity is $\pm(A\omega)$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position.
However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium.

Particle velocity in wave motion : The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



Direction of propagation of wave

Wave velocity : The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.

Relation between particle velocity and wave velocity :
Wave equation : $y = A\sin(\omega t - kx)$

Particle velocity $= \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx) = v$

Wave velocity $= v_p = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{K}$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k\cos(\omega t - kx)$$

$$= -\frac{1}{v_p}\frac{\partial y}{\partial t} \Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_p}\frac{\partial y}{\partial t}$$

Note: $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.
Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.
Differential equation of harmonic progressive waves:

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx);\quad \frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_y^2}\frac{\partial^2 y}{\partial t^2}$$

Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave: The acceleration of the particle is the second partial derivative of y(x, t) with respect to t,

$$\therefore \frac{\partial^2 y(x,t)}{\partial t^2} = \omega^2 A\sin(kx - \omega t) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus,
$a_p = -\omega^2$ (displacement)

Relation between Phase difference, Path difference & Time difference :

Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wavelength (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3\lambda}{2}$
Time period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}$$

$\Rightarrow$ Path difference $= \left(\frac{\lambda}{2\pi}\right)$ Phase difference

Example 1 :

A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Sol. We known that for a wave $v = f\lambda$

So $\lambda = \dfrac{v}{f} = \dfrac{360}{500} = 0.72\text{ m}$

Now as in a wave path difference is related to phase difference by the relation

Phase difference $\Delta\phi = 60° = (\pi/180)\times 60 = (\pi/3)\text{ rad}$

Path difference $\Delta x = \dfrac{\lambda}{2\pi}(\Delta\phi) = \dfrac{0.72}{2\pi}\times\dfrac{\pi}{3} = 0.12\text{ m}$

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) Displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) Pressure difference (dP) or density difference (dp) in case of sound wave or

(iii) Electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature.

Consider one-dimensional wave travelling along x-axis.

In this case y is a function of x and t. i.e. $y = f(x, t)$

But only those function of x & t, represent a wave motion which satisfy the differential equation.

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2} \qquad .....(i)$$

The general solution of this equation is of the form

$$y(x,t) = f(ax \pm bt) \qquad .....(ii)$$

Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times, Further, if these conditions are satisfied, then speed of wave (v) is given by,

$$v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$$

Example 2 :

Which of the following functions represent a travelling wave

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$

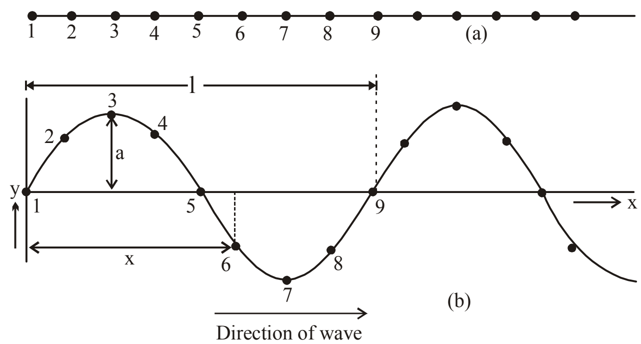
(c) $e^{-(x-vt)^2}$ (d) $\dfrac{1}{x + vt}$

Sol. (c). Although all the four functions are written in the form $f(ax \pm bt)$, only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

EQUATION OF A PLANE PROGRESSIVE WAVE

If on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a simple harmonic progressive wave.

Suppose, a simple harmonic progressive wave is advancing in a medium along the positive direction of the x-axis (from left to right). In Fig. (a) are shown the equilibrium positions of the particles 1, 2, 3.



When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave.

Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then

$$y = a\sin\omega t \qquad .....(1)$$

where a is the amplitude of oscillation and $\omega = 2\pi n$, where n is the frequency.

As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will , start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t - (x/v). The displacement of particle 1 at time t - (x/v) can be the particle 6, distant x from the origin (particle 1), at time t is

given by $y = a\sin\omega\left(t - \dfrac{x}{v}\right)$ but $\omega = 2\pi n$

$$y = a\sin(\omega t - kx) \quad \left(k = \frac{\omega}{v}\right) \qquad ....(2)$$

$$y = a\sin\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right] \qquad ....(3)$$

$$y = a\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right] \qquad ......(4)$$

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign.

For example, equation (4) will be of the following form :

$$y = a\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be $y = a\sin\left\{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\pm\phi\right\}$

Example 3 :

The equation of a wave is,

$$y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x-40t)-\frac{\pi}{4}\right]\text{m}$$

Find: (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x = 0.5 m and t = 0.05 s

Sol. (a) The equation may be rewritten as,

$$y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)\text{m}$$

Comparing this with equation of plane progressive harmonic wave, $u(x,t) = A\sin(kx-\omega t+\phi)$

wave number $k = \frac{2\pi}{\lambda} = 5\pi$ rad/m $\therefore\ \lambda = 0.4$ m

The angular frequency is, $\omega = 2\pi f = 20\pi$ rad/s $\therefore f = 10$ Hz

The wave velocity is,

$$v = f\lambda = \frac{\omega}{k} = 4\ \text{ms}^{-1}\ \text{in} + x\ \text{direction}$$

(b) The particle velocity and acceleration are,

$$v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right) = 2.22\ \text{m/s}$$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right) = 140\ \text{m/s}^2$$

INTENSITY OF WAVE

The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2.

$$I = 2\pi^2 f^2 A^2 \rho v \quad \text{i.e. } I \propto f^2 \text{ and } I \propto A^2$$

If P is the power of an isotropic point source, intensity at a distance r is given by,

$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2} \quad \text{(for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \quad \text{or} \quad I \propto \frac{1}{r} \quad \text{(for a line source)}$$

As, $I \propto A^2$

Therefore, $A \propto \frac{1}{r}$ (for a point source)

and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

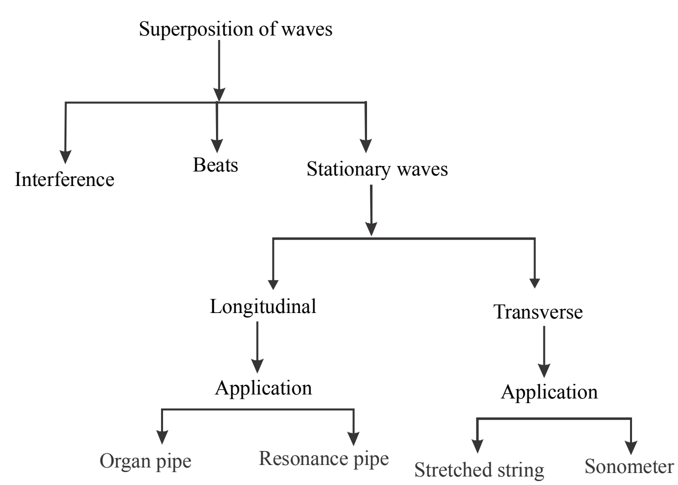
SUPERPOSITION PRINCIPLE

Two or more waves can traverse the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle

The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves. If $\vec{y}_1, \vec{y}_2, \vec{y}_3......$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude. Due to superposition of waves the following phenomenon can be seen

1. **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
2. **Beats :** Superposition of two waves of nearly equal frequency in same direction.
3. **Stationary waves :** Superposition of equal wave from opposite direction.
4. **Lissajous figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is

different at different points. At some points intensity is large, whereas at other points it is nearly zero.
Consider two waves

$y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition

$y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

where $A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$,

and $\tan\delta = \dfrac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Constructive interference (maximum intensity) :
Phase difference, $\phi = 2n\pi$ or path difference $= n\lambda$
where $n = 0, 1, 2, 3, ...$

$\Rightarrow A_{max} = A_1 + A_2$ and $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

Destructive interference (minimum intensity) :
Phase difference $= \phi = (2n+1)\pi$,

or path difference $= (2n-1)\dfrac{\lambda}{2}$; where $n = 0, 1, 2, 3, ...$

$\Rightarrow A_{max} = A_1 - A_2$ and $I_{max} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

Results :

(1) Maximum and minimum intensities in any interference wave form. $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{a_1+a_2}{a_1-a_2}\right)^2$

(2) Average intensity of interference wave form :

$< I >$ or $I_{av} = \dfrac{I_{max} + I_{min}}{2}$

Put the value of I_{max} & I_{min}
or $I_{av} = I_1 + I_2$
If $a = a_1 = a_2$ and $I_1 = I_2 = I$
then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$

(3) Condition of maximum contrast in interference wave form

$a_1 = a_2$ and $I_1 = I_2$

then $I_{max} = 4I$; $I_{min} = 0$
For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE

Velocity of transverse Wave in any wire $v = \sqrt{\dfrac{T}{m}}$

where m = mass per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$.

where d = Density of matter

$V = \sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}} \quad \because \pi r^2 = A$

(1) If m is constant then, $V \propto \sqrt{T}$ it is called tension law.

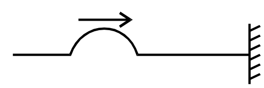
(2) If tension is T then $v \propto \sqrt{\dfrac{1}{m}}$ ← it is called law of mass.

(3) If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ ← it is called law of radius

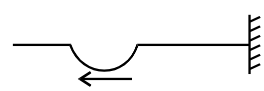
(4) If T is constant & take wire of same radius for different material. Then $V \propto \sqrt{\dfrac{1}{d}}$ ← law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall. The element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down. The resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.
Equation of wave propagating in +ve x-axis
Incident wave : $y_1 = a \sin(\omega t - kx)$



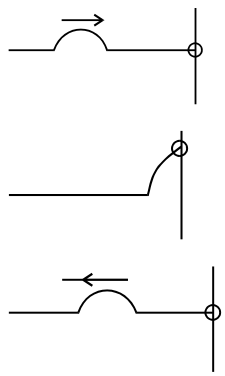
reflected wave : $y_2 = a \sin(\omega t + kx + \pi)$
or $y_2 = -a \sin(\omega t + kx)$



REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one.
The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$



Reflected wave $y_2 = a \sin(\omega t + kx)$

Example 4 :

A uniform rope of length 12 meter and mass 6kg, is swinging vertically from rigid base. From its free end, one 2kg mass is attached. At its bottom end one transverse wave is produced of wavelength 0.06 meter. At upper end of rope, wavelength will be -

(1) 1.2 m (2) 0.12 m (3) 0.12 cm (4) 1.12 cm

Sol. **(2).** Tension at bottom end of rope $= T_1 = 2 \times 9.8$ N

$\because$ weight of rope acts on gravity center

Therefore, tension at upper end of rope,

$T_2 = (6+2) \times 9.8 = 8 \times 9.8$ N. Thus, $T_2 = 4T_1$

if v_1 and v_2 are respective velocity at bottom and upper end, then $\sqrt{\frac{T_1}{m}}$ and $v_2 = \sqrt{\frac{T_2}{m}}$ $\therefore v_2 = 2v_1 (\because T_2 = 4T_1)$

Frequency n does not depend on medium,

therefore $v \propto \lambda$

If λ_1 and λ_2 are respective wavelength at bottom and upper end of rope $\lambda_2 = 2\lambda_1 = 2 \times 0.06 = 0.12$ m

STATIONARY WAVES

The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave.

Formation of stationary wave is possible only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

(i) From rigid end : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$

After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi)$

$y_2 = -a \sin(\omega t + kx)$

By principle of super position.

$y = y_1 + y_2$

$y = a \sin(\omega t - kx) - a \sin(\omega t + kx)$

$y = -2a \sin kx \cos \omega t$

This is equation of stationary wave reflected from rigid end

Amplitude = 2a sin kx ...(1)

Velocity of particle

$$V_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t \quad ....(2)$$

Strain, $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ ....(3)

Elasticity $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$

Change in pressure , $dp = E\frac{dy}{dx}$ .....(4)

(1) Node : $x = 0, \frac{\lambda}{2}, \lambda$ .... ; $A = 0$; $V_{pa} = 0$

strain $\to$ max ; change in pressure $\to$ max

This point is known as node.

Position of nodes $\to 0, \frac{\lambda}{2}, \lambda$ ....

(2) Antinode $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... ; $A \to$ max ; $V_{pa} \to$ max

Strain = 0 ; Change in pressure = 0

This point is known as antinode.

Position of Antinodes $\to \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ......

(ii) From free end : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$

After reflection from free end $y_2 = a \sin(\omega t + kx)$

By Principle of superposition.

$y = y_1 + y_2$

$y = a \sin(\omega t - kx) + a \sin(\omega t + kx)$

$y = 2a \sin \omega t \cos kx$

Amplitude = 2a cos kx

Velocity of particle $= \frac{dy}{dt} = 2a\omega \cos \omega t \cos kx$

Strain $= \frac{dy}{dt} = -2ak \sin \omega t \sin kx$

Change in pressure $dp = E\frac{dy}{dx}$

(1) Antinode : $0, \frac{\lambda}{2}, \lambda$.... ; $A \to$ Max ; $V_{pa} \to \frac{dy}{dt} \to$ Max

strain = 0 ; dp = 0

This point is known as antinode.

Position of antinode $\to 0, \frac{\lambda}{2} \lambda$ ....

(2) Node :

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ..... ; $A = 0$; $\frac{dy}{dt} = 0$

strain $\to$ Max ; dp $\to$ Max

This point is known as node.

Position of node $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ....

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed. Important characteristics of stationary waves are :

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is $\frac{\lambda}{2}$.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°.

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(ix) In a standing wave the medium is splited into segments and each segment is vibrating up and down as a whole.

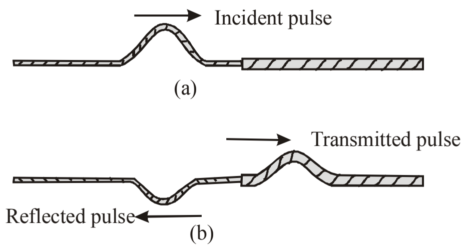
(x) In longitudinal stationary waves, condensation (compression) and rarefactions don't travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium.

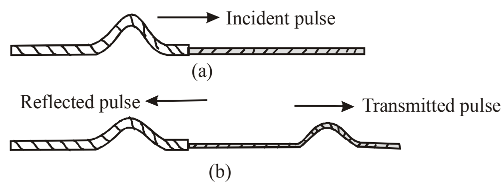
Transmission of energy is not possible in a stationary wave

TRANSMISSION OF WAVES

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, part of it is reflected and inverted and part of it is transmitted to the heavier string. As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behavior of a pulse meeting a rigid boundary, when it is totally reflected.



Where a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative .densities of the two string.



The speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B &$v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

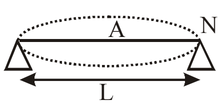
STATIONARY WAVE

Stationary wave are of two types –

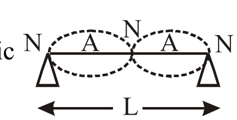
(I) Transverse st. wave (stretched string)

(II) Longitudinal st. wave (organ pipes)

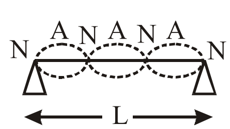
Transverse stationary wave :

First Harmonic  $\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$

(a)

Second Harmonic  $\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell$

(b)

Third Harmonic  $\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3}$

(c)

p^{th} harmonic

$$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p}$$

(i) Law of length: For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e,

$$n \propto \frac{1}{\ell} \text{ (T and m are constant)}$$

(ii) Law of tension : The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant.

$$n \propto \sqrt{T} \text{ (}\ell\text{ and m are constant)}$$

(iii) Law of mass : The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e.,

$$n \propto \frac{1}{\sqrt{m}} \text{ (}\ell\text{ and T are constant)}$$

(1) Melde's experiment:

In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be suitable loaded.,

Transverse arrangement :

Case 1 : In a vibrating string of fixed length, the product of number of loops in a vibrating string and square root of tension is a constant or $p\sqrt{T} =$ constant.

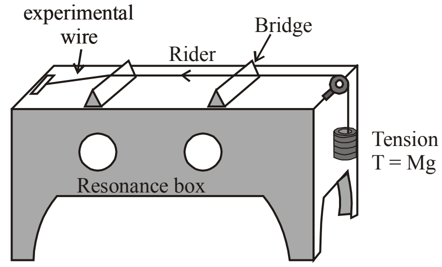
Case 2 : When the tuning fork is set vibrating such that the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3 : If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread sags only when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement : The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork. Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is, $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire. There are three laws of vibration of a wire.

SOUND WAVES

Displacement and pressure waves

A sound wave (i.e. longitudinal mechanical wave) can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement-wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure-wave).

Consider a sound wave travelling in the x-direction in a medium. Suppose at time t, the particle at the undisturbed position x suffers a displacement y in the x-direction. The displacement wave then will be described by

$$y = A\sin(\omega t - kx) \qquad .....(1)$$

Now consider the element of medium which is confined with in x and x + Δx in the undisturbed state. If S is the cross-section, the volume element in undisturbed state will be $V = S\Delta x$. As the wave passes the ends at x and x + Δx are displaced by amount y and y + Δy so that increase in volume of the element will be $\Delta V = S\,\Delta y$. This in turn implies that volume strain for the element under consideration

$$\frac{\Delta V}{V} = \frac{S\Delta y}{S\Delta x} = \frac{\Delta y}{\Delta x} \qquad ....(2)$$

So, corresponding stress, i.e. excess pressure

$$p = B\left[\frac{-\Delta V}{V}\right] \left[\text{as } B = -V\frac{\Delta P}{\Delta V} = -V\frac{P}{\Delta V}\right]$$

$$\text{or} \quad p = -B\frac{\Delta y}{\Delta x} \text{ [from eqn. 2]} \qquad ....(3)$$

Note: For a harmonic progressive waveform

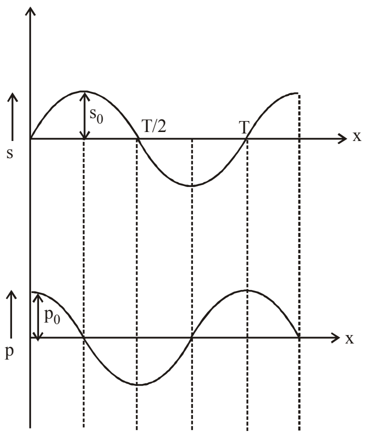
$$\frac{dy}{dx} = \frac{v_{pa}}{v},\ p = -B\frac{dy}{dx} = B\left(\frac{v_{Pa}}{v}\right)$$

i.e. pressure in a sound wave is equal to the product of elasticity of gas with the ratio of particle speed to wave speed. $\frac{\Delta y}{\Delta x} = -Ak\cos(\omega t - kx)$

so $p = AkB\cos(\omega t - kx)$ i.e. $p = p_0\cos(\omega t - kx)$

with $p_0 = ABk$ ....(4)

From eqn. (1) and (4) is clear that



(1) A sound wave may be considered as either a displacement wave $y = A\sin(\omega t - kx)$ or a pressure wave $p = p_0\cos(\omega t - kx)$

(2) The pressure wave is 90° out of phase with respect to displacement wave, i.e, displacement will be maximum when pressure is minimum and vice-versa. This is shown in fig.

(3) The amplitude of pressure wave

$p_0 = ABk = Ak\rho v^2$ [as $v = \sqrt{B/\rho}$]

$p_0 = \rho vA\omega$ [as $v = \omega/k$] ....(5)

(4) As sound-sensors (e.g., ear or mic) detects pressure changes, description of sound as pressure-wave is preferred over displacement wave.

Ultrasonic, Infrasonic and Audible Sound

Sound waves can be classified in three groups according to their range of frequencies.

Infrasonic Waves : Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.

Audible Waves : Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.

Ultrasonic Waves : Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.

Applications of Ultrasonic Waves : Ultrasonic waves have a large range of application. Some of them are :

(i) The fine internal cracks in metal can be detected by ultrasonic waves.

(ii) Ultrasonic waves can be used for determining the depth of the sea, lakes etc.

(iii) Ultrasonic waves can be used to detect submarines, icebergs etc.

(iv) Ultrasonic waves can be used to clean clothes, fine machinery parts etc.

(v) Ultrasonic waves can be used to kill smaller animals like rates, fish and frogs etc.

Shock Waves : If the speed of the body in air is greater than the speed of the sound, then it is called supersonic speed. Such a body leaves behind a conical region of disturbance which spreads continuously such a disturbance is called a Shock Waves.

This wave carries huge energy. If it strikes a building, then the building may be damage.

Sound intensity in decibels :

The physiological sensation of loudness is closely related to the intensity of wave producing the sound. At a frequency of 1 kHz people are able to detect sounds with intensities as low as 10^{-12} W/m^2. On the other hand an intensity of 1 W/m^2 can cause pain, and prolonged exposure to sound at this level will damage a person's ears. Because the range in intensities over which people hear is so large, it is convenient to use a logarithmic scale to specify intensities.

If the intensity of sound in watts per square meter is I, then β in decibels (dB) is given by,

$$\beta = 10\log\frac{I}{I_0}$$

where the base of the logarithm is 10, and $I_0 = 10^{-12}$W/m^2 (roughly the minimum intensity that can be heard).

On the decibel scale, the pain threshold of 1 W/m^2 is then

$$\beta = 10\log\frac{1}{10^{-12}} = 120\text{dB}$$

Example 5 :

Calculate the change in intensity level when the intensity of sound increases by 10^6 times its original intensity.

Sol. Here, $\frac{I}{I_0} = \frac{\text{Final intensity}}{\text{Initial intensity}} = 10^6$

Increase in intensity level, $L = 10\log_{10}(I/I_0)$

$= 10\log_{10}(10^6) = 10 \times \log_{10}10 = 10 \times 6 \times 1$

$\Rightarrow$ L = 60 decibal

CHARACTERISTICS OF SOUND

Sound is characterised by the following three parameters :

Loudness: It is the sensation received by the ear due to intensity of sound. Experimentally Weber-Fechner established that $L \propto K\log I$

i.e., greater the amplitude of vibration, greater will be the intensity I ($\propto A^2$) and so louder will be the sound as in about and lesser then intensity, feeble will be the sound as in whispering.

The unit of loudness is phon which is equal to the intensity level in dB of equally loud sound of a kHz [for which the ear is mot sensitive].

Pitch : It is the sensation received by the ear due to frequency and is the characteristic which distinguishes a shrill (or sharp) sound from a grave (or flat) sound. As pitch depends on frequency, higher the frequency higher will be the pitch and shriller will be the sound. Regarding pitch it is worth noting that:

(1) The buzzing of a bee or humming of a mosquito has high pitch but low loudness while the roar of a lion has large loudness but low pitch.

(2) Due to more harmonics usually the pitch of female voice is higher than male.

Quality (or timbre) : It is the sensation received by the ear due to 'waveform'. Two sounds of same intensity and frequency will produce different sensation on the ear if their waveforms are different. Now as waveform depends on overtones present, quality of sound depends on number of overtones, i,e., harmonics present and their relative intensities.

Musical Scale : The arrangement of notes having definite ratio in respective fundamental frequencies is called a musical scale. Musical scales are of two types.

(a) Diatonic scale : It is known as 'Sargam' in Indian system. It contains eight notes with definite ratio in their frequencies. The note of lowest frequency is called key note and the highest (which is double of first) is called an octave. Harmonium, piano etc. are based on this scale.

(b) Tempered Scale : It contains 13 notes. the ratio of frequencies of successive notes is $2^{1/12}$.

Musical Interval : The ratio between the frequencies of two notes is called the musical interval. Following are the names of some musical intervals.

(i) Unison $\frac{n_2}{n_1} = 1$ (ii) Octave $\frac{n_2}{n_1} = 2$

(iii) Major tone $\frac{n_2}{n_1} = \frac{9}{8}$ (iv) Minor tone $\frac{n_2}{n_1} = \frac{10}{9}$

(v) Semi tone $\frac{n_2}{n_1} = \frac{16}{15}$ (vi) Fifth tone $\frac{n_2}{n_1} = \frac{3}{2}$

Reverberation : If a loud sound wave is produced in a ordinary room with good reflecting walls, the wave undergoes a large number of reflections at the walls. The repeated reflections produce persistence of sound, this phenomenon is called reverberation. In an auditorium or class room excessive reverberation is not desirable. However, some reverberation is necessary in a concert hall.

Time of reverberation : It is the time required by the energy density to fall to the minimum audibility value (E) from an initial steady value 10^6E (i.e. million times minimum audibility) when the source of the sound wave is removed. The optimum time of reverberation is about 0.5s for a medium sized room, 0.8s to 1.5s for an auditorium, 1s to 2s for a music room and greater than 2s for a temple.

ECHO

Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then, 2d = vt

Hence, v = speed of sound and t, the time of echo.

$$\therefore \quad d = \frac{vt}{2}$$

Since, the effect of ordinary sound remains on our ear for 1/10 second, therefore, if the sound returns to the starting point before 1/10 second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is,

$$d_{min} = \frac{v \times t}{2} = \left(\frac{330}{2}\right)\left(\frac{1}{10}\right) = 16.5\text{m}$$

SPEED OF LONGITUDINAL (SOUND) WAVES

Newton Formula : Use for every medium

$V_{medium} = \sqrt{\frac{E}{\rho}}$, Where, E = Elasticity coefficient of medium , ρ = Density of medium

(a) For solid medium :

$V_{solid} = \sqrt{\frac{y}{\rho}}$ E = y = Young's modulus

for soft iron $V_{soft\ iron} = 5150$ m/s

(b) For liquid Medium : $V_{liquid} = \sqrt{\frac{B}{\rho}}$

E = B, where B = volume elasticity coefficient of liquid

for water $V_{Water} = 1450$ m/s

(c) For gas medium :

The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air temperature remains constt. (i.e. the process is isothermal) So, Isothermal Elasticity = P

$$\therefore \quad v_{air} = \sqrt{(P/\rho)}$$

At NTP for air, $P = 1.01 \times 10^5$ N/m^2 and $\rho = 1.3$ kg/m^3

$$\text{So, } V_{air} = \sqrt{\frac{1.01 \times 10^5}{1.3}} = 279 \text{ m/s}$$

However, the experimental value of sound in air is 332 m/s which is much higher than given by Newton's formula

Laplace Correction : In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

Adiabatic Elasticity = γ P

so that $v = \sqrt{(\gamma P/\rho)}$ .....(3)

i.e., $v = \sqrt{1.41} \times 279 = 331.3$ m/s [as $\gamma_{air} = 1.41$]

Which is in good agreement with the experimental value (332 m/s). This in turn establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light & radio-waves (3×10^8m/s)

In case of gases

$$v_s = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{E_\phi}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

i.e. $v_s = \sqrt{\frac{\gamma PV}{mass}}$ $\left[as\ \rho = \frac{mass}{volume} = \frac{M}{V}\right]$

or $v = \sqrt{\frac{\gamma \mu RT}{M}}$ [as $PV = \mu RT$]

or $v_s = \sqrt{\frac{\gamma RT}{M_w}}$ as $\mu = \frac{mass}{M_W} = \frac{M}{M_W}$ ...(4)

M_w = Molecular weight

And from kinetic-theory of gases

$$V_{rms} = \sqrt{(3RT/M_w)}$$

So, $\frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}}$. i.e. $v_s = \left[\frac{\gamma}{3}\right]^{1/2} v_{rms}$

i.e., velocity of sound in a gas is of the order of rms speed of gas molecules ($v \sim v_{rms}$)

As velocity of sound in a according to Eqn. (4) is -

$$\sqrt{\frac{\gamma RT}{M_w}}$$

Velocity of sound in case of gases at constant temperature depends on the nature of gas i.e., its atomicity (γ) and molecular weight.

Effect of Various Quantities

(1) Effect of temp.

For a gas γ & M_W is constant

$$v \propto \sqrt{T} \quad ; \quad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \quad ; \quad \frac{v_t}{v_0} = \sqrt{\frac{t+273}{273}}$$

$$v_t = v_0 \left[1 + \frac{t}{273}\right]^{1/2}$$

By applying Binomial theorem.

(i) For any gas medium $v_t = v_0 \left[1 + \frac{t}{546}\right]$

(ii) For air $v_0 = 332$ m/sec. ; $v_t = v_0 + 0.61t$ m/sec

(2) Effect of Relative Humidity :

With increase in humidity, density decreases so in the light of $v = \sqrt{\gamma P/\rho}$

We conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature.

(3) Effect of Pressure :

As velocity of sound

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

So pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why in going up in the atmosphere, through both pressure and density decreases, velocity of sound remains constant as long as temperature remains constant. Further more it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has practically no effect on velocity of sound.

(4) Effect of Motion of Air :

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be $(v + w)$, and in the opposite direction it will be $(v - w)$.

(5) Effect of Frequency :

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

Example 6 :

Determine the change in volume of 6 litres of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, Density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/ cc and g = 9.81 m/s^2]

Sol. For propagation of sound in liquid

$$v = \sqrt{(B/\rho)}, \text{ i.e. } B = v^2\rho$$

But by definition $B = -V\frac{\Delta P}{\Delta V}$

So, $-V\frac{\Delta P}{\Delta V} = v^2\rho$ i.e. $\Delta V = \frac{V(-\Delta P)}{\rho v^2}$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981$

$= -1.667 \times 10^6$ dynes/cm^2

$$\text{So, } \Delta V = \frac{(6 \times 10^3)(1.667 \times 10^6)}{0.81 \times (1.280 \times 10^5)^2} = 0.75\text{cc.}$$

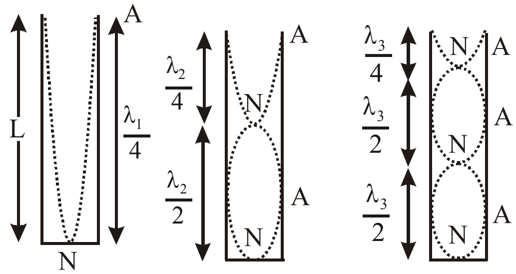
VIBRATION IN ORGAN PIPES

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition, standing waves are produced.

These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

1. Vibration of air column in closed organ pipe :

The tube which is closed at one end and open at the other end is called organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the waves reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.



(a) (b) (c)

Hence on blowing air at the open end, the column vibrates forming antinode at free end and node at closed end. If ℓ is length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then

Case (a) $\ell = \frac{\lambda}{4} \Rightarrow \lambda = 4\ell \Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{4\ell}$

Fundamental frequency.

Case (b) $\ell = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4\ell}{3} \Rightarrow n_2 = \frac{v}{\lambda} = \frac{3v}{4\ell}$

First overtone frequency.

Case (c) $\ell = \frac{5\lambda}{5} \Rightarrow \lambda = \frac{4\ell}{5} \Rightarrow n_3 = \frac{v}{\lambda} = \frac{5v}{4\ell}$

Second overtone frequency.

When closed organ pipe vibrate in mth overtone then

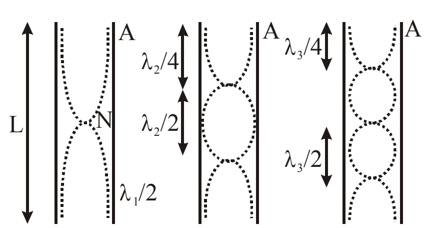
$$\ell = (2m+1)\frac{\lambda}{4} \quad \text{So} \quad \lambda = \frac{4\ell}{(2m+1)} \Rightarrow n = (2m+1)\frac{v}{4\ell}$$

Hence frequency of overtones is given by

$n_1 : n_2 : n_3 .... = 1 : 3 : 5....$

2. Vibration of air columns in open organ pipe :

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travel towards the other end from waves travel in opposite direction to superpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and λ be the wavelength and v is velocity of sound in organ pipe.



(a) (b) (c)

Case (a)

$$\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell \Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{2\ell}$$

Fundamental frequency.

Case (b)

$$\ell = \frac{2\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{2} \Rightarrow n_2 = \frac{v}{\lambda} = \frac{2v}{2\ell}$$

First overtone frequency.

Case (c)

$$\ell = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{3} \Rightarrow n_3 = \frac{v}{\lambda} = \frac{3v}{2\ell}$$

Second overtone frequency

Hence frequency of overtones are given by the relation

$n_1 : n_2 : n_3 .... = 1 : 2 : 3.........$

When open organ pipe vibrate in mth overtone then

$$\ell = (m+1)\frac{\lambda}{4} \quad \text{so,} \quad \lambda = \frac{4\ell}{m+1} \Rightarrow n = (m+1)\frac{v}{2\ell}$$

End correction :

Due to finite momentum of air molecular in organ pipes reflection takes place not exactly at open end but some what it so in an organ pipe antinode is not formed exactly at free-end but above it at a distance e = 0.6r (called end correction or Rayleigh correction) with r being the radius of pipe. So for closed organ pipe $L \to L + 0.6r$ while for open

$L \to L + 2 \times 0.6r$ (as both ends are open) so that

$$f_C = \frac{v}{4(L+0.6r)} \text{ while } f_0 = \frac{v}{2(L+1.2r)}$$

This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound.

Example 7 :

For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed for sound in air to be 340 m/s (a) Explain whether the pipe is closed at one end or open at both ends (b) determine the fundamental frequency and length of the pipe.

Sol. (a) The given frequencies are in the ratio

425 : 595 : 765 i.e., 5 : 7 : 9

And as clearly these are odd integers so the given pipe is closed end.

(b) From part (b) it is clear that the frequency of 5th harmonic (which is third overtone) is 425 Hz

so $425 = 5f_C$ or $f_C = 85$ Hz

Further as $f_C = \frac{v}{4L}$, $L = \frac{v}{4f_C} = \frac{340}{4 \times 85} = 1\text{m}$

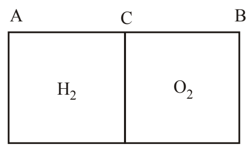
Example 8 :

AB is a cylinder of length 1 m fitted with a thin flexible diaphragm C at middle and two other thin flexible diaphragm A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of the same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node? Under the condition of the experiment the velocity of sound in hydrogen is 1100 m/s and oxygen 300 m/s.

Sol. As diaphragm C is a node, A and B will be antinode (as in a organ pipe either both ends are antinode or one end node and the other antinode), i.e., each part will behave as closed end organ pipe so that

$$f_H = \frac{V_H}{4L_H} = \frac{1100}{4 \times 0.5} = 550\text{ Hz}$$

$$f_0 = \frac{v_0}{4L_0} = \frac{300}{4 \times 0.5} = 150\text{Hz}$$



and $\frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$

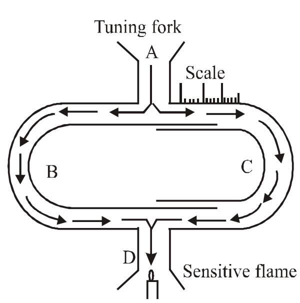
As the two fundamental frequencies are different, the system will vibrate with a common frequency f such that

$$f = n_H f_H = n_0 f_0 \quad \text{i.e.,} \quad \frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$$

i.e., the third harmonic of hydrogen and 6th harmonic of oxygen or 6th harmonic of hydrogen and 22nd harmonic of oxygen will have same frequency. So the minimum common frequency $f = 3 \times 550 \text{ or } 11 \times 150 = 1650\text{ Hz}$

APPARATUS FOR DETERMINING SPEED OF SOUND

1. **Quinck's Tube :** It consists of two U shaped metal tubes. Sound waves with the help of tuning fork are produced at A which travel through B & C and T) comes out at D where a sensitive flame is present. Now the two waves coming through different path interfere and flame flares up. But if they are not in phase destructive interference occurs and flame remains undisturbed.

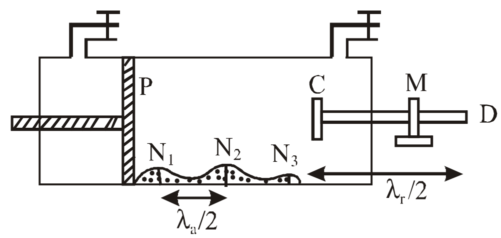


Suppose destructive interference occurs at D for some position of C. If now the tube C is moved so that interference condition is disturbed and again by moving a distance x, destructive interference occurs so that 2x = λ. Similarly if the distance moved between successive constructive and destructive interference is x then $2x = \frac{\lambda}{2}$

Now by having value of x, speed of sound is given by

$v = n\lambda$

2. **Kundt's tube :** It is the used to determine speed of sound in different gases. It consists of a glass tube in which a small quantity of lycopodium powder is spread. The tube is rotated so that powder starts slipping. Now rod is rubbed at end so that stationary waves form. The disc vibrates so that air column also vibrates with the frequency of the rod. The piston is adjusted so that frequency of air column become same as that of rod. So resonance occurs and column is thrown into stationary waves. The powder sets into oscillations at antinodes while heaps of powder are formed at nodes.



Let n is the frequency of vibration of the rod then, this is also the frequency of sound wave in the air column in the tube.

For rod: $\frac{\lambda_{rod}}{2} = \ell_{rod}$; For air: $\frac{\lambda_{air}}{2} = \ell_{air}$

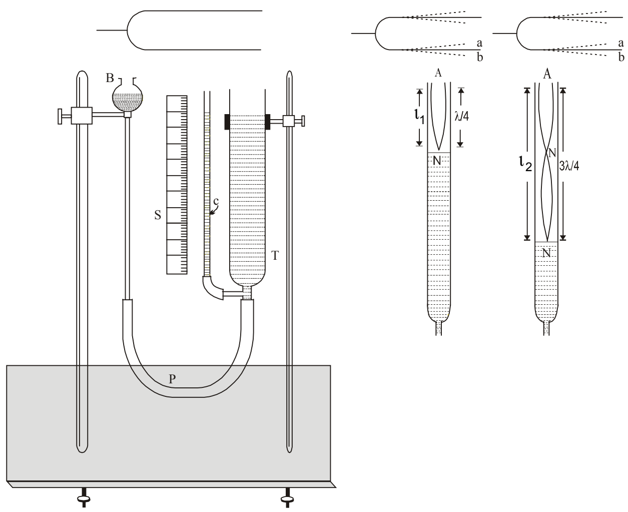
Where ℓ_{air} is the distance between two heaps of powder in the tube (i.e. distance between two nodes). if v_{air} and v_{rod} are velocity of sound waves in the air and rod respectively,

then $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}} = \frac{\ell_{air}}{\ell_{rod}}$

Thus knowledge of v_{rod} determines v_{air}

3. **Resonance tube**

Construction : The resonance tube is a tube T (Fig.) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air-column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the speed of sound in air by resonance tube : First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch-cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube . The pinch-cock P is opened slowly so that the water level in T falls and the length of the air-column increases. At a particular length of air-column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

(i) For first resonance $\ell_1 = \lambda/4$ .....(1)

(ii) For second resonance $\ell_2 = 3\lambda/4$ .....(2)

Substract Eq. (2) from Eq. (1)

$\ell_2 - \ell_1 = \lambda/2$; $\lambda = 2(\ell_2 - \ell_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by $v_t = n\lambda = 2n(\ell_2 - \ell_1)$

The speed of sound wave at 0°C ; $v_0 = (v_t - 0.61\ t)$ m/s.

End Correction : In the resonance tube, the antinode is not formed exactly at the open but slightly outside at a distance x. Hence the length of the air -column in the first and second states of resonance are $(\ell_1 + x)$ and $(\ell_2 + x)$ then

(i) For first resonance

$\ell_1 + x = \lambda/4$ .... (1)

(ii) For second resonance

$\ell_2 + x = 3\lambda/4$ .... (2)

Substract Eq. (2) from Eq. (1)

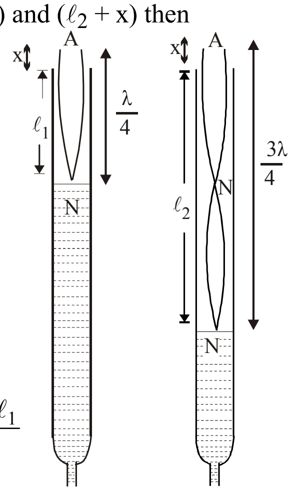
$\ell_2 - \ell_1 = \lambda/2$

$\lambda = 2(\ell_2 - \ell_1)$

Put the value of λ in eq. (1)

$$\ell_1 + x = \frac{2(\ell_2 - \ell_1)}{4}$$

$$\Rightarrow \ell_1 + x = \frac{\ell_2 - \ell_1}{2} ; x = \frac{\ell_2 - 3\ell_1}{2}$$

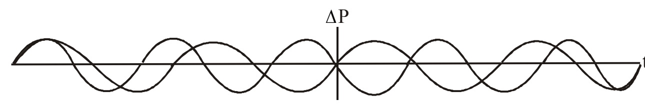


BEATS

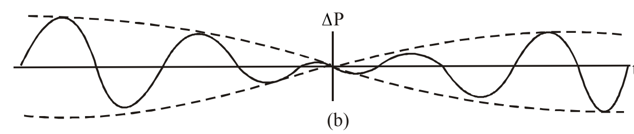
When two sound waves of same amplitude travelling in same direction with different frequency superimpose, then intensity varies periodically with time. This effect is called Beats. Suppose two waves of frequencies f_1 and f_2 ($< f_1$) are meeting at some point in space. The corresponding periods are T_1 and T_2 ($> T_1$). If the two waves are in phase at t = 0, they will again be in phase when the first wave has gone through exactly one more cycle than the second. This will happen at a time t = T, the period of the beat. Let n be the number of cycles of the first wave in time T, then the number of cycles of the second wave in the same time is (n – 1). Hence, $T = nT_1 = (n-1)T_2$

Eliminating n we have

$$T = \frac{T_1 T_2}{T_2 - T_1} = \frac{1}{\frac{1}{T_1} - \frac{1}{T_2}} = \frac{1}{f_1 - f_2}$$



(a)



(b)

The reciprocal of the beat period is the beat frequency

$$f = \frac{1}{T} = f_1 - f_2$$

Waves Interference on the bases of beats:

Conditions: Two equal frequency wave travel in same direction. Mathematical analysis

If displacement of first wave

$y_1 = a \sin \omega_1 t \rightarrow (N_1, a)$ $\qquad I \propto N^2 a^2$

Displacement of second wave

$y_2 = a \sin \omega_2 t \rightarrow (N_2, a)$

By superposition

$y = y_1 + y_2$

Eq. of resulting wave

$y = a(\sin 2\pi N_1 t + \sin 2\pi N_2 t)$

$$y = a\left\{2\sin 2\pi t\frac{(N_1 + N_2)}{2}\cos 2\pi t\frac{(N_1 - N_2)}{2}\right\}$$

$$\text{or} \quad y = \left\{2a\cos 2\pi t\frac{(N_1 - N_2)}{2}\right\}\sin 2\pi t\frac{(N_1 + N_2)}{2}$$

or $y = A \sin 2\pi N' t$

Amplitude $\quad A = 2a\cos 2\pi t\left(\frac{N_1 - N_2}{2}\right)$

$A = 2a\cos 2\pi t\,(N_1 - N_2)$

Frequency $\quad N' = \frac{N_1 + N_2}{2}$

(1) For max Intensity: $A = \pm 2a$

If $\cos \pi (N_1 - N_2)t = \pm 1$

$\cos \pi (N_1 - N_2) = \cos n\pi$; $n = 0, 1, 2$...........

$$\pi (N_1 - N_2)t = n\pi ; t = \frac{n}{N_1 - N_2} ; t = 0, \frac{1}{\Delta N}, \frac{2}{\Delta N}, \frac{3}{\Delta N} ....$$

(2) For Minimum Intensity: $A = 0$

$\cos\pi (N_1 - N_2) t = 0$

$$\cos\pi (N_1 - N_2)t = \cos (2n + 1) \frac{\pi}{2} \quad n = 0, 1, 2...........$$

$$\pi (N_1 - N_2)t = (2n + 1) \frac{\pi}{2} ; \ t = \frac{2n+1}{2(N_1 - N_2)}$$

$$t = \frac{1}{2\Delta N}, \frac{3}{2\Delta N}, \frac{5}{2\Delta N} ....$$

TUNING FORK

When tuning fork is sounded by striking its one end on rubber pad, then:

(a) The ends of prongs vibrate in and out while the stem Vibrates up and down or vibrations of the prongs are transverse and that of the stem is longitudinal. Generally tuning fork produces fundamental note.

(b) At the free end of a fork antinodes are formed. At the place where stem is fixed antinode is formed. In between these antinodes, nodes are formed.

(c) Frequency of tuning fork $n \propto \left(\frac{t}{\ell^2}\right)\sqrt{\frac{E}{d}}$

t = thickness of tuning fork

ℓ = length of arm of fork

E = coefficient of elasticity for the material of fork

d = density of the material of a fork.

(d) Frequency of tuning fork decrease with increase in temperature.

(e) increasing the weight, the frequency of a tuning fork decreases while on filling the prongs near stem the frequency decreases.

PRACTICAL APPLICATIONS OF BEATS

Determination of Frequency : If we know the frequency n_1 of a tuning fork, then we can determine the exact frequency of another fork of nearly equal frequency by the phenomenon of beats. For this, both the tuning forks are sounded together and the beats are heard. Suppose, x beats are heard in 1 second. Then the frequency of the second fork will be either $(n_1 + x)$ or $(n_1 - x)$. Now one prong of this fork is loaded with a small wax so that its frequency is slightly lowered. Again, the two forks are sounded together and beats are heard. If the number of beats per second decreases then it means that the new (lowered) frequency of the second tuning forks is more nearer to the frequency of the first tuning fork. This would happen if the frequency of the second tuning fork is higher than the frequency of the first fork. Hence the frequency of the second fork is $(n_1 + x)$. On the other hand, if on loading with wax, the number of beats per second increases, then the frequency of the second fork is $(n_1 - x)$.

Example 9 :

A tuning fork having n = 300 Hz produces 5 beats/s with another tuning fork. If impurity (wax) is added on the arm of known tuning fork, the number of beats decreases then calculate the frequency of unknown tuning fork.

Sol. The frequency of unknown tuning fork should be

$300 \pm 5 = 295$ Hz or 305 Hz.

When wax is added, if it would be 305 Hz, beats would have increases but with 295 Hz beats decreases so frequency of unknown tuning fork is 295 Hz.

Example 10 :

A tuning fork having n = 158 Hz, produce 3 beats/s with another. As we file the arm of unknown, beats become 7 then calculate the frequency of unknown. .

Sol. The frequency of unknown tuning fork should be

$158 \pm 3 = 155$ Hz or 161 Hz.

After filling the number of beats = 7 so frequency of unknown tuning fork should be $158 \pm 7 = 165$ Hz or 151 Hz.

As both above frequency can be obtain by filing so frequency of unknown = 155/161 Hz.

TRY IT YOURSELF-1

Q.1 Water waves produced by a motor boat sailing in water are

(A) neither longitudinal nor transverse.

(B) both longitudinal and transverse.

(C) only longitudinal.

(D) only transverse.

Q.2 Speed of sound wave in air

(A) is independent of temperature.

(B) increases with pressure.

(C) increases with increase in humidity.

(D) decreases with increase in humidity.

Q.3 Change in temperature of the medium changes

(A) frequency of sound waves.

(B) amplitude of sound waves.

(C) wavelength of sound waves.

(D) loudness of sound waves.

Q.4 String of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in

(A) one second (B) 0.5 second

(C) 2 seconds (D) data given is insufficient.

Q.5 Speed of sound waves in a fluid depends upon

(A) directly on density of the medium.

(B) square of Bulk modulus of the medium.

(C) inversly on the square root of density.

(D) directly on the square root of bulk modulus of the medium.

Q.6 During propagation of a plane progressive mechanical wave
(A) all the particles are vibrating in the same phase.
(B) amplitude of all the particles is equal.
(C) particles of the medium executes S.H.M.
(D) wave velocity depends upon the nature of the medium.

Q.7 Which of the following statements are true for a stationary wave?
(A) Every particle has a fixed amplitude which is different from the amplitude of its nearest particle.
(B) All the particles cross their mean position at the same time.
(C) All the particles are oscillating with same amplitude.
(D) There is no net transfer of energy across any plane.

Q.8 A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 16 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is
(A) 0.05 s (B) 0.10 s
(C) 0.20 s (D) 0.40 s

Q.9 $Y(x, t) = \dfrac{0.8}{[(4x+5t)^2+5]}$ represents a moving pulse where x and y are in metres and t is in second. Then
(A) pulse is moving in positive x-direction
(B) in 2 s it will travel a distance of 2.5 m
(C) its maximum displacement is 0.16 m
(D) it is a symmetric pulse

Q.10 Two vibrating strings of the same material but of lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency v_1 and the other with frequency v_2. The ratio v_1/v_2 is
(A) 2 (B) 4 (C) 8 (D) 1

Q.11 In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.
(A) 0.012 m (B) 0.025 m
(C) 0.05 m (D) 0.024 m

Q.12 A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is –
(A) 344 (B) 336
(C) 117.3 (D) 109.3

ANSWERS

(1) (B)	**(2)** (C)	**(3)** (C)
(4) (B)	**(5)** (CD)	**(6)** (BCD)
(7) (ABD)	**(8)** (B)	**(9)** (BCD)
(10) (D)	**(11)** (B)	**(12)** (A)

DOPPLER EFFECT

Acoustic Doppler effect (Doppler effect for sound waves) :

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called Doppler effect.

While deriving these expressions, we make the following assumptions:

(i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

(ii) The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

Notations :

$n \rightarrow$ actual frequency
$n' \rightarrow$ observed frequency (apparent frequency)
$\lambda \rightarrow$ actual wave length
$\lambda' \rightarrow$ observed (apparent) wave length
$v \rightarrow$ velocity of sound, $v_s \rightarrow$ velocity of source
$v_0 \rightarrow$ velocity of observer, $v_w \rightarrow$ wind velocity

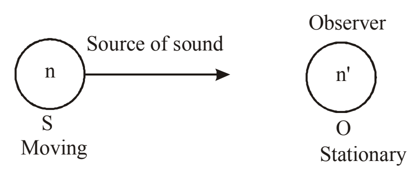
Case I : Source in motion, observer at rest, medium at rest

Suppose the source S and observer O are separated by distance v. Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v. So, wave length;

$$\lambda = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v}{n}$$

(1) Source moving towards stationary observer:

Let the source start moving towards the observer with velocity v_s After one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wavelength λ' and frequency n'



Now apparent wavelength

$$\lambda' = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v - v_s}{n}$$

and changed frequency,

$$n' = \frac{v}{\lambda} = \frac{v}{\left(\frac{v - v_s}{n}\right)} = n\left(\frac{v}{v - v_s}\right)$$

As the source of sound approaches the observer the apparent frequency n' becomes greater than the true frequency n.

(2) When source move away from stationary observer :

For this situation n waves will be crowded in distance $v+v_S$.

So, apparent wavelength , $\lambda' = \frac{v+v_s}{n}$

and apparent frequency,

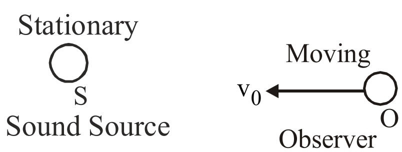
$$n' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v+v_s}{n}\right)} = n\left(\frac{v_s}{v+v_s}\right)$$

So, n' becomes less than n. (n' < n)

Case II : Observer in motion, source at rest, medium at rest: Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source S would be crossing observer O in one second and fill the space OA (=v)

(1) Observer move towards stationary source :

When observer O moves towards S with velocity v_0, it will cover v_0 distance in one second. So the observer has received not only the n waves occupying OA but also received additional number of Δn waves occupying the distance OO' (= v_0).



So, total waves received by observer in one second i.e., apparent frequency

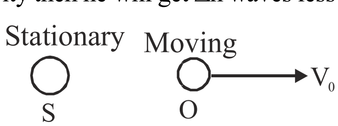
(n') = Actual waves (n) + Additional waves (Δn)

$$n' = \frac{v}{\lambda} + \frac{v_0}{\lambda} = \frac{v+v_0}{(v/n)} = n\left(\frac{v+v_0}{v}\right) \qquad \left(\because \lambda = \frac{v}{n}\right)$$

(so, n' > n)

(2) Observer move away from stationary source :

For this situation n waves will be crowded in distance v - v_0. When observer move away from source with v_0 velocity then he will get Δn waves less than real number of waves.



So, total number of waves received by observer i.e., Apparent frequency = (n') = Actual waves (n) – reduction in number of waves (Δn)

$$n' = \frac{v}{\lambda} - \frac{v_0}{\lambda} = \frac{v-v_0}{\lambda} = \frac{v-v_0}{(v/n)} = \left(\frac{v-v_0}{v}\right)n \qquad \left(\because \lambda = \frac{v}{n}\right)$$

(so, n' < n)

Case III : Effect of motion of medium :

General formula for doppler effect

$$n' = n\left[\frac{v \pm v_0}{v \mp v_s}\right] \qquad \text{..........(i)}$$

If medium (air) is also moving with v_m velocity in direction of source and observer. Then velocity of sound relative to observer will be v ± v_m (–ve sign, if v_m is opposite to sound velocity). So, $n' = n\left(\frac{v \pm v_m \pm v_0}{v \pm v_m \mp v_s}\right)$

[On replacing v by v ± v_m in equation (i)]

Note : When both S and O are in rest (i.e. $v_s = v_0 = 0$) then there is no effect of frequency due to motion of air

Case-IV : Both source and observer are moving away from each other. Medium at rest.

$n' = \left(\frac{v-v_0}{v+v_s}\right)n$: Clearly, n' > n

If moving towards each other $n' = \left(\frac{v+v_0}{v-v_s}\right)n$

If source moving towards observer which is moving away from source $n' = \left(\frac{v-v_0}{v-v_s}\right)n$

If observer moving towards source which is moving away from observer $n' = \left(\frac{v+v_0}{v+v_s}\right)n$

DOPPLER'S EFFECT IN REFLECTION OF SOUND (ECHO)

When the sound is reflected from the reflector the observer receives two notes one directly from the source and other from the reflector. If the two frequencies are different then superposition of these waves result in beats and by the beat frequency we can calculate speed of the source.

If the source is at rest and reflector is moving towards the source with speed u, then apparent frequency heard by reflector $n_1 = \left(\frac{v+u}{v}\right)n$

Now this frequency n_1 acts as a source so that apparent frequency received by observer is

$$n_2 = \left(\frac{v}{v-u}\right)n_1 = \left(\frac{v}{v-u}\right) \times \left(\frac{v+u}{v}\right)n = \left(\frac{v+u}{v-u}\right)n$$

If u << v then,

$$n_2 = n\left(1+\frac{u}{v}\right)\left(1-\frac{u}{v}\right)^{-1} = n\left(1+\frac{u}{v}\right)^2 = n\left(1+\frac{2u}{v}\right)$$

Beat frequency $\Delta n = n_2 - n = \left(\frac{2u}{v}\right) n$

So speed of the source $u = \frac{v}{2}\left(\frac{\Delta n}{n}\right)$

CONDITIONS WHEN DOPPLER'S EFFECT IS NOT OBSERVED FOR SOUND-WAVES

1. When the source of sound and observer both are at rest then doppler effect is not observed.
2. When the source and observer both are moving with same velocity in same direction.
3. When the source and observer are moving mutually in perpendicular directions.
4. When the medium only is moving.
5. When the distance between the source and observer is constant.

DOPPER EFFECT IN LIGHT

Dopper effect holds also for em waves. As speed of light is independent of relative motion between source and observer, the formulae are different from that of sound. Here when either source or observer (detector) or both are in motion, only two cases are possible (approach or recession). In case of approach

$$n' = \left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}} \quad \text{and } \lambda' = \lambda\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{\frac{1}{2}}$$

In case of recession

$$n' = n\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{\frac{1}{2}} \quad \text{and } \lambda' = \lambda\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}}$$

If $v << c$ then in case of approach $n' \approx n\left(1+\frac{v}{c}\right)$

in case of recession $n' \approx n\left(1-\frac{v}{c}\right)$

So at low speeds dopper effect in light and sound is governed by the same formulae,

DOPPLER'S SHIFT

When radiation coming from distance stars are analysed by radio telescopes and compared with their natural radiation wavelength focussed on mean wavelength on a visible spectrum, it is observed that coming radiation has a shift towards red or violet end.

Red shift $\Delta\lambda = \lambda' - \lambda = \left(\frac{v}{c}\right)\lambda$

Violet shift (or blue shift) $\Delta\lambda = \lambda - \lambda' = \left(\frac{v}{c}\right)\lambda$

In case of approach frequency increases while wavelength decreases i.e. shift $\Delta\lambda$ is towards blue end of the spectrum while in case of recession frequency decreases and wavelength increases i.e, shift is towards red end.

Example 11 :

How fast one must move to see a red light signal as a green one? Given $n_R = 4.8 \times 10^{14}$ Hz & $n_G = 5.6 \times 10^{14}$ Hz

Sol. $\because n' = n\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}} \quad \therefore 5.6 \times 10^{14} = 4.8 \times 10^{14}\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}}$

$$\Rightarrow \frac{7}{6} = \left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}} \Rightarrow \frac{49}{36} = \left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$$

$$\Rightarrow 49 - 49\left(\frac{v}{c}\right) = 36 + 36\left(\frac{v}{c}\right) \Rightarrow 85\left(\frac{v}{c}\right) = 13$$

$$\Rightarrow v = \frac{13}{85} \times 3 \times 10^8 = \frac{39}{85} \times 10^8 = 4.59 \times 10^7 \text{ ms}^{-1}$$

TRY IT YOURSELF-2

Q.1 A train, standing in a station yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10m/s.Given that the speed of sound in still air is 340m/s,

(A) the frequency of sound as heard by an observer standing on the platform is 400Hz.

(B) the speed of sound for the observer standing on the platform is 350m/s.

(c) the frequency of sound as heard by the observer standing on the platform will increase.

(d) the frequency of sound as heard by the observer standing on the platform will decrease.

Q.2 A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s then the ratio f_1/f_2 is

(A) 18/19 (B) 1/2

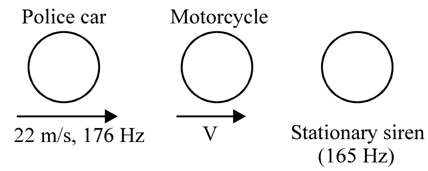
(C) 2 (D) 19/18

Q.3 A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz, while the train approaches the siren. During his return journey in a different train B he records a frequency of 6 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is

(A) 242/252 (B) 2

(C) 5/6 (D) 11/6

Q.4 A police car moving at 22 m/s chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle. If it is given that the motorcyclist does not observe any beats.



(A) 33 m/s (B) 22 m/s
(C) zero (D) 11 m/s

Q.5 A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer who is standing in air is –
(A) 200 Hz (B) 3000 Hz
(C) 120 Hz (D) 600 Hz

Q.6 An observer standing on a railway crossing receives frequency of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. [The speed of the sound in air is 300 m/s]

Q.7 A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320m/s. Frequency of the siren heard by the car driver
(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHz

Q.8 Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are) –
(A) If the wind blows from the observer to the source, $f_2 > f_1$.
(B) If the wind blows from the source to the observer, $f_2 > f_1$.
(C) If the wind blows from observer to the source, $f_2 < f_1$.
(D) If the wind blows from the source to the observer, $f_2 < f_1$.

ANSWERS

(1) (AB)	**(2)** (D)	**(3)** (B)
(4) (B)	**(5)** (D)	**(6)** 30 m/s
(7) (A)	**(8)** (AB)	

ADDITIONAL EXAMPLES

Example 1 :
The length of a wire between the two ends of a sonometer is 105cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15.
(1) 75 cm, 25 cm (2) 25 cm, 75 cm
(3) 75 cm, 100 cm (4) none

Sol. **(3).** From the law of length of stretched string,
we have $n_1\ell_1 = n_2\ell_2 = n_3\ell_3$
Here $n_1 : n_2 : n_3 = 1 : 3 : 15$

$$\frac{\ell_1}{\ell_2} = \frac{n_2}{n_1} = \frac{3}{1} \text{ and } \frac{\ell_1}{\ell_3} = \frac{n_3}{n_1} = \frac{15}{1}\,;\ \ell_2 = \frac{\ell_1}{3} \text{ and } \ell_3 = \frac{\ell_1}{15}$$

The total length of the wire is 105 cm
Therefore, $\ell_1 + \ell_2 + \ell_3 = 105$

$$\text{or } \ell_1 + \frac{\ell_1}{3} + \frac{\ell_1}{15} = 105 \text{ or } \frac{21\ell_1}{15} = 105$$

$$\ell_1 = \frac{105 \times 15}{15} = 75 \text{ cm} \quad \therefore \quad \ell_2 = \frac{\ell_1}{3} = \frac{75}{3} = 25 \text{ cm}$$

$$\ell_3 = \frac{\ell_1}{15} = \frac{75}{15} = 5 \text{ cm}$$

Hence the bridge should be placed at 75 cm and (75 + 25) = 100 cm from one end.

Example 2 :
(a) Compute the fundamental frequency of a sonometer wire of the length 20.0 cm, T = 20 N, m = 5.2×10^{-3}kg/m.
(b) A resonance air column resonates with a turning fork of the frequency 512 Hz at the length 17.4cm Neglecting the end correction, deduce the speed of the sound in air is your answer unique for the given data.

Sol. (a) The fundamental frequency of a sonometer wire is given

by $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

Here, $\ell = 20$ cm $= 0.2$m, T = 20N, m = $5\,.\,2 \times 10^{-3}$kg/m

$$\text{Hence, } n = \frac{1}{2 \times 0.2}\sqrt{\frac{20}{5.2 \times 10^{-3}}} = 155 \text{ Hz}$$

(b) Speed of sound in air is given by,
$v = n\,4\,\ell = 512 \times 4 \times 0.174 = 356.35 \text{ ms}^{-1}$
This answer is unique for the fundamental mode.

Example 3 :
A piezo electric quartz plate of thickness 0.005m is vibrating in resonant conditions. Calculate its fundamental frequency if for quartz Y = 8×10^{10}N/m^2 & $\rho = 2.65 \times 10^3$ kg/m^3

Sol. We know that for longitudinal waves in solids

$$v = \sqrt{\frac{Y}{\rho}}, \text{ so } v = \sqrt{\frac{8 \text{x} 10^{10}}{2.65 \text{x} 10^{3}}} = 5.5 \times 10^3 \text{ m/s}$$

Further more for fundamental mode of plate
$(\lambda/2) = L$ so $\lambda = 2 \times 5 \times 10^{-3} = 10^{-2}$m
$v = f\lambda$, i.e, $f = (v/\lambda)$
So, $f = [5.5 \times 10^3/10^{-2}] = 5.5 \times 10^5$ Hz = 550 kHz

Example 4 :
A 5 watt source sends out waves in air at frequency 1000s^{-1}. Deduce the intensity at a 100 meter distance, assuming spherical distribution. If v = 350 ms^{-1} and ρ = 1.3 kg/m^3, deduce the displacement amplitude.

Sol. We know that intensity is given by,

$$I = \frac{\text{Power}}{\text{Area}} = \frac{5\text{Watt}}{4\pi(100)^2\text{m}^2}$$

Also, $I = 2\rho v\pi^2 n^2 a^2 \quad \therefore \quad a^2 = \frac{I}{2\rho\pi n^2 v}$

Hence, displacement amplitude $a = \frac{1}{\pi n}\sqrt{\frac{I}{2\rho v}}$

Given that, $n = 1000\ s^{-1}$, $I = 4 \times 10^{-5} Wm^{-2}$ $\rho = 1.3$ kg m^{-3} and $v = 350\ ms^{-1}$

$\therefore \quad a = \frac{7}{22 \times 1000}\sqrt{\frac{4 \times 10^{-5}}{2 \times 1.3 \times 350}} = 6.67 \times 10^{-8} m$

Example 5 :

The equation of plane progressive wave motion is $y = a \sin 2\pi/\lambda\ (vt - x)$. Velocity of particle is -

(1) $y\frac{dv}{dx}$ (2) $v\frac{dy}{dx}$

(3) $-y\frac{dv}{dx}$ (4) $-v\frac{dy}{dx}$

Sol. (4). Velocity of particle

$$\frac{dy}{dt} = a\cos\left\{\frac{2\pi}{\lambda}(vt - x)\right\}\frac{2\pi v}{\lambda} \quad ....(1)$$

Slope of curve,

$$\frac{dy}{dx} = a\cos\left\{\frac{2\pi}{\lambda}(vt - x)\right\}\left\{\frac{-2\pi}{\lambda}\right\} \quad ....(2)$$

By equation (1) and (2)

$$\frac{dy/dt}{dy/dx} = -v \quad \therefore \quad \frac{dy}{dt} = -v\frac{dy}{dx}$$

Example 6 :

If equation of transverse wave is $y = x_0 \cos 2\pi\left(nt - \frac{x}{\lambda}\right)$.

Maximum velocity of particle is twice of wave velocity, if λ is-

(1) $\pi/2x_0$ (2) $2\pi x_0$

(3) πx (4) πx_0

Sol. (4). $y = x_0 \cos 2\pi\left(nt - \frac{x}{\lambda}\right)$; $y = x_0 \cos\frac{2\pi}{\lambda}(vt - x)$

$[\because v = n\lambda]$

$$\left(\frac{dy}{dt}\right)_{max} = x_0 \times \frac{2\pi}{\lambda} v = 2v \text{ (given)} \qquad \therefore \lambda = \pi x_0$$

Example 7 :

For the travelling harmonic wave $y = 2.0\cos(10t - 0.0080x + 0.35)$. Where x and y are in cm and t in sec. What is the phase difference between oscillatory motion at two points separated by a distance of (i) 4m, (ii) 0.5m (iii) A/2 (iv) $3\lambda/4$

Sol. The given equation of harmonic wave is

$$y = 2.0\cos(10t - 0.0080x + 0.35) \quad ....(1)$$

The standard equation of harmonic wave is

$$y = a\cos\left[2\pi\left(\frac{1}{T} - \frac{x}{\lambda}\right) + \phi\right] \quad ....(2)$$

Comparing equations (1) and (2),

$$\frac{2\pi}{\lambda} = 0.0080 \text{ or } \lambda = \frac{2\pi}{0.0080}\text{ cm} = \frac{2\pi}{0.0080 \times 100}$$

(ii) Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{2\pi} \times .00080 \times 100 \times 4 = 3.2$ rad

(ii) Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{2\pi} \times .00080 \times 100 \times 0.5 = 0.40$ rad

(iii) Phase difference $= \frac{2\pi}{\lambda} \times$ path diff. $= \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad

(iv) Phase difference $= \frac{2\pi}{\lambda} \times$ path diff. $= \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad

Example 8 :

Wavelength of two notes in air are (80/195) m and (80/195) m. Each note produces five beats per second with a note of a fixed frequency. Calculate the velocity of sound in air.

Sol. Given that $\lambda_1 = \frac{v}{\lambda_1} = \frac{195v}{80}$ and $n_2 = \frac{v}{\lambda_2} = \frac{193}{80}$

This show that, $n_1 > n_2$

Let the frequency of third note be n, then

$n_1 - n = 5$ and $n - n_2 = 5$

$\therefore \quad n_1 - n_2 = 10$

$\frac{195v}{80} - \frac{193v}{80} = 10$

$2v = 80 \times 10 = 800$; $v = 400$ m/sec

Example 9 :

Tuning fork A has frequency 1% greater than that of standard fork B while tuning fork C has frequency 2% smaller than that of B. When A and C are sounded together, the number of beats heard per second is 5. What is the frequency of each fork.

Sol. Let the frequencies of forks be n_1, n_2 & n_3 respectively.

Then, $n_1 = n_2(1 + 0.01) = 1.01\ n_2$

and $n_3 = n_2(1 - 0.02) = 0.98\ n_2$

Further, $n_1 - n_3 = 5$

Substituting the values, we get $(1.01n_2 - 0.98n_2) = 5$

$n_2 = 166.7$ Hz. Now, $n_1 = 1.01 \times 166.7 = 168.3$ Hz

and $n_3 = 0.98 \times 166.7 = 163.3$ Hz

Example 10 :

A metal rod 1.5m length is clamped at the center. When it is set with longitudinal vibrations it emits a note of 1KHz. If the density of the material is 8 × 10^3, then determine the Young's modulus.

Sol. For longitudinal waves in a rod the velocity of sound is

$v = \sqrt{\frac{Y}{\rho}}$, where Y is Young's modulus and ρ density.

Also for a clamped rod in the middle, the frequency of fundamental note is $n = \frac{v}{2\ell}$

Comparing we get $2n\ell = \sqrt{\frac{Y}{\rho}}$ or $Y = 4n^2\ell^2\rho$

Substituting the data from question

$$Y = 4 \times (10^3)^2 \times (1.5)^2 \times 8 \times 10^3 = 7.2 \times 10^{10}\ \text{N/m}^2$$

Example 11 :

The ratio in the densities of oxygen and nitrogen is 16 : 14. At what temperature the speed of sound will be the same which is in nitrogen at 15ºC.

Sol. If M be the molecular weight of the gas and T be the absolute temperature, then speed of sound in a gas.

$$v = \sqrt{\left(\frac{\gamma RT}{M}\right)} \qquad \left(\because \frac{P}{d} = \frac{RT}{M}\right)$$

Where R is universal gas constant. Velocity of sound in oxygen at tºC $= \sqrt{\left[\frac{\gamma R(273+t)}{M_0}\right]}$

Velocity of sound in nitrogen at 15°C $= \sqrt{\left[\frac{\gamma R(273+15)}{M_N}\right]}$

According to the given problem

$$\sqrt{\left[\frac{\gamma R(273+t)}{M_0}\right]} = \sqrt{\left[\frac{\gamma R(273+15)}{M_N}\right]}$$

$$\therefore \frac{M_0}{M_n} = \frac{273+t}{288}\ ;\ \frac{16}{14} = \frac{273+t}{288} \qquad \left(\because \frac{M_0}{M_N} = \frac{16}{14}\right)$$

Solving we get, $t = 56.1^\circ C$

Example 12 :

What is the intensity level of sound in dB for (i) threshold of hearing and (ii) threshold of pain.

Sol. (i) For $I = I_0$, $\beta = 10\log\left(\frac{I_0}{I_0}\right)$ or $\beta = 10\log(1) = 10 \times 0 = 0$ dB

(ii) $I = 1 W/m^2$, with $I_0 = 10^{-2} W/m^2$

$\beta = 10\log(10^{12})$ or $\beta = 120$ dB

Thus a pain begins to occur in ears at sound levels of 120 dB.

Example 13 :

A trumpet player plays a note of frequency 400Hz with an amplitude of 8 × 10^{-3} mm. If the density of air is taken as 1.3kg/m^3, and the speed of sound 330 m/s, find the intensity of the sound wave.

Sol. The intensity of sound wave is given by,

$$I = 2\pi^2\rho v n^2 a^2$$
$$= .2 \times (3.14)^2 \times 1.3 \times 330 \times (400)^2 \times (8 \times 10^{-6})^2$$
$$= 2 \times 9.86 \times 1.3 \times 330 \times 16 \times 64 \times 10^{-8}$$
$$= 8.66 \times 10^{-2} = 0.087\ \text{W/m}^2$$

Example 14 :

At what temperature will the speed of sound in hydrogen be the same as in oxygen at 100ºC. Densities of oxygen and hydrogen are in the ratio 16 : 1 -

(1) – 250ºC (2) 249.7ºC
(3) 250º (4) – 249.7ºC

Sol. **(4).** Velocity $v = \sqrt{\frac{\gamma RT}{M}}$

For oxygen and hydrogen γ = 1.4 and R is constant

$$\because \sqrt{\frac{T}{M_H}} = \sqrt{\frac{T_{100}}{M_0}} \Rightarrow \frac{T}{T_{100}} = \frac{M_H}{M_0}$$

$$\Rightarrow \frac{273+t}{273+100} = \frac{273+t}{273+100} = \frac{1}{16}\ ;\ t = -249.7^\circ C$$

Example 15 :

The length of an organ pipe open of both ends is 0.5 meter. Calculate the fundamental frequency of the pipe, if the velocity of sound in air be 350 m/sec. If one end of the pipe is closed, then the fundamental frequency will be -

(1) 350, 700 (2) 700, 350
(3) 175, 350 (4) 350, 175

Sol. **(4).** Speed of sound v = 350 m/sec

length of pipe ℓ = 0.5 m

The frequency of the fundamental tone of a pipe open at both end is given by

$$n = \frac{v}{2\ell} = \frac{350}{2 \times 0.5} = 350\ \text{sec}^{-1}$$

The frequency of the fundamental tone of a pipe open at one end is given by

$$n = \frac{v}{4\ell} = \frac{350}{4 \times 0.5} = 175\ \text{sec}^{-1}.$$

Example 16 :

The length of a pipe open at both ends is 48cm and its fundamental frequency is 320 Hz. If the speed of sound be 320 m/sec, then determine the diameter of the pipe. If one end of the pipe be closed, then what will be the fundamental frequency ?

(1) 3 cm, 160 Hz (2) 3.3 cm, 160.3 Hz
(3) 3.33 cm, 163 Hz (4) 3.33 cm, 163.3 Hz

Sol. **(4).** Fundamental frequency of the pipe of diameter D, open at both ends, is

$$n = \frac{v}{2(\ell + 2e)} = \frac{v}{2(\ell + 2x0.3D)}$$

$\Rightarrow\ 320 = \dfrac{32000}{2(48+2x0.3D)}\ \Rightarrow D = 3.33\text{ cm}$

For a pipe closed at one end,

$$n = \frac{v}{4(\ell+e)} = \frac{v}{4(1+0.3D)}\ ;\ n = \frac{32000}{4(48+0.3\times 33.3)}$$

$n = 163.3\text{Hz}$

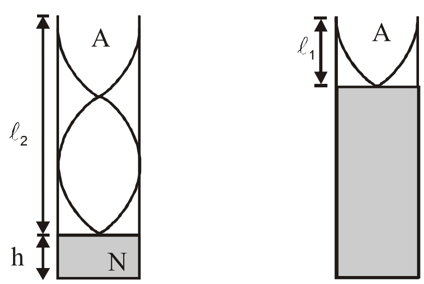
Example 17 :

A tuning fork of frequency 340 Hz is allowed to vibrate just above a 120cm high tube. Water is being filled slowly in the tube. What minimum height of water will be necessary for resonance. (speed of sound in air = 340 m/s)

(1) 45 cm (2) 25 cm

(3) 75 cm (4) 95 cm

Sol. (1). From the data n = 340 Hz and v = 340 m/s, we find the $\lambda = \dfrac{v}{n} = \dfrac{340}{340} = 1\text{m}$. For a closed pipe, the possible lengths at which air – column can resonate with a given frequency (wavelength) are



(i) $\ell_1 = \dfrac{\lambda}{4} = \dfrac{1}{4} = 0.25\text{ m}$ (ii) $\ell_2 = \dfrac{3\lambda}{4} = \dfrac{3}{4} = 0.75\text{m}$

(iii) $\ell_3 = \dfrac{5\lambda}{4} = \dfrac{5}{4} = 1.25\text{ m}$ and so on

Thus for the tube of length 1.20m only two possibilities occur. When water is filled slowly in the tube then the available length of air column starts decreasing from value 1.20m, and when it reaches a value 0.75m, resonance occurs. The minimum height of water should be 120 – 0.75 = 0.45m

Example 18 :

What should be the length of a closed organ pipe to produce a fundamental frequency of 512Hz at 0ºC.

Sol. $n_1 = \dfrac{v}{4\ell}$ or $\ell = \dfrac{v}{4n_1}$

$$\ell = \frac{332}{4\times 512}\text{m} = \frac{33200}{4\times 512}\text{cm} = 16.2\text{ cm}$$

Example 19 :

A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15ºC. When the temperature falls to 10ºC, the two produce 3 beats per second. Find the frequency of the fork

(1) 210 Hz (2) 113 Hz

(3) 112 Hz (4) 110 Hz

Sol. (4). Let the frequency of the tuning fork be n Hz

Then frequency of air column at 15ºC = n + 4

Frequency of air column at 10ºC = n + 3

According to v = nλ, we have

$v_{15} = (n+4)\lambda$ and $v_{10} = (n+3)\lambda$

$$\therefore\ \frac{v_{15}}{v_{10}} = \frac{n+4}{n+3}$$

The speed of sound is directly proportional to the square-root of the absolute temperature.

$$\frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}}\ \therefore\ \frac{n+4}{n+3} = \sqrt{\frac{288}{283}} = \left(1+\frac{5}{283}\right)^{1/2}$$

$$\Rightarrow 1+\frac{1}{n+3} = 1+1/2\times\frac{5}{283} = 1+\frac{5}{566}$$

$$\Rightarrow \frac{1}{n+3} = \frac{5}{566} \Rightarrow n+3 = 113 \Rightarrow n = 110\text{ Hz}$$

Example 20 :

5 beat per second are produced by simultaneously blowing two closed organ pipes of different lengths. If the shorter organ pipe is 25 cm in length and the speed of sound is 320 m/sec., determine the length of the other organ pipe.

Sol. Given that $n_1 - n_2 = 5$

$$\therefore\ \frac{320}{4\times 0.25} - \frac{320}{4\times\ell} = 5\ ;\ \text{Solving we get, } \ell = 25.4\text{ cm}$$

Example 21 :

The frequency of whistle of an engine appears to be $(4/5)^{th}$ of initial frequency when it crosses a stationary observer. If the velocity of sound is 330 m/s, then the speed of engine will be

(1) 30 m/s (2) 36.6 m/s

(3) 40 m/s (4) 330 m/s

Sol. (2). $n' = \dfrac{nv}{v-v_s}$ ....(1) ; $n'' = \dfrac{nv}{v+v_s}$ ....(2)

From (1) and (2) $\dfrac{n'}{n''} = \dfrac{v+v_s}{v-v_s}$ ....(3)

According to question $\frac{n'}{n''} = \frac{5}{4}$; $v_s = ?$ v = 330 m/s ....(4)

From eq. (3) and (4)

$$\frac{5}{4} = \left[\frac{330 + v_s}{330 - v_s}\right] \; ; \; 9v_s = 330 \quad \therefore \quad v_s = 36.6 \text{ m/s}$$

Example 22 :

The wavelength of light received from a milky way is 0.4% higher than that from the same source on earth. The velocity of milky way with respect to earth will be -

(1) 5×10^6 m/s (2) 1.2×10^6 m/s

(3) 0.2×10^6 m/s (4) 2×10^6 m/s

Sol. (2). $\Delta\lambda = \frac{v_s}{c}\lambda$; $\frac{\Delta\lambda}{\lambda} = \frac{v_s}{c}$

$$\frac{\Delta\lambda}{\lambda} \times 100 = \frac{v_s}{c} \times 100$$

$$v_s = \frac{0.4}{100} \times 3 \times 10^8 = 1.2 \times 10^6 \text{ m/s}$$

Example 23 :

A train approaching a hill at a speed of 40km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1km from a hill. A wind with a speed of 40Km/hr is blowing in the direction of motion of the train. Find the frequency of the whistle as heard by an observer on the hill.

(velocity of sound in air = 1200 km/hr)

(1) 580 Hz (2) 620 Hz

(3) 600 Hz (4) 720 Hz

Sol. (3). According to Doppler's effect, the apparent frequency when both source and observer move along the same direction is

$$n' = \frac{(v + w) - v_0}{(v + w) - v_s}$$

Velocity of observer $v_0 = 0$

$$\therefore \quad n' = \frac{(v + w)}{v + w - v_s} n$$

Given v = 1200 km/hr, w = 40 km/hr, v_s = 40 km/hr.

and n = 580 Hz

$$\therefore \quad n' = \frac{1200 + 40}{(1200 + 40) - 40} \times 580 = 599.33 \text{ Hz} = 600 \text{ Hz}$$

Example 24 :

A source of sound of frequency 256Hz is moving rapidly towards a wall with a velocity of 5m/s. How many beats per second will be heard if sound travels at a speed of 330 m/s by an observer behind the source.

Sol. When the source S is between the wall (W) and the observer (O) For direct sound the source is moving away from the observer, therefore the apparent frequency

$$n'' = \frac{v}{v + v_s} n = \frac{330}{330 + 5} \times 256$$

and frequency of reflected sound

$$n' = \frac{v}{v - v_s} n = \frac{330}{330 - 3} \times 256 = 259.9$$

Number of beats/sec = n′ – n″ = 259 – 252.2 = 7.7

Example 25 :

A siren is fitted on a car going towards a vertical wall at a speed of 36 km/hr A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave (a) coming directly from the siren to the person and (b) coming after reflection. Take the speed of sound to be 340 m/s.

(1) 515 Hz, 486 Hz (2) 486 Hz, 515 Hz

(3) 510 Hz, 490 Hz (4) 490 Hz, 510 Hz

Sol. (2). Here the observer is at rest with respect to the medium and the source is going away from the observer. The apparent frequency heard by the observer is, therefore,

$$\nu' = \frac{v}{v + v_s} \nu = \frac{340}{340 + 10} \times 500 \text{ Hz}$$

$$\nu' = 486 \text{ Hz}$$

(b) The frequency received by the wall is

$$\nu'' = \frac{v}{v - v_s} \nu = \frac{340}{340 - 10} \times 500 = 515 \text{ Hz}$$

The wall reflects this sound without changing the frequency. Thus, the frequency of the reflected wave as heard by the ground observer is 515 Hz.

QUESTION BANK — CHAPTER 13 : WAVES

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : WAVE MOTION

Q.1 The type of waves that can be propagated through solid is
(A) Transverse (B) Longitudinal
(C) Both (A) and (B) (D) None of these

Q.2 Water waves produced by a motor boat sailing in water are
(A) neither longitudinal nor transverse.
(B) both longitudinal and transverse.
(C) only longitudinal.
(D) only transverse.

Q.3 The equation of a transverse wave is given by $y = 10\sin\pi(0.01x - 2t)$, where x and y are in cm and t is in second. Its frequency is
(A) 10 sec^{-1} (B) 2 sec^{-1}
(C) 1 sec^{-1} (D) 0.01 sec^{-1}

Q.4 Change in temperature of the medium changes–
(A) frequency of sound waves.
(B) amplitude of sound waves.
(C) wavelength of sound waves.
(D) loudness of sound waves.

Q.5 With propagation of longitudinal waves through a medium, the quantity transmitted is –
(A) matter (B) energy
(C) energy and matter (D) energy, matter & momentum

Q.6 Energy is not carried by which of the following waves –
(A) Stationary (B) Progressive
(C) Transverse (D) Electromagnetic

Q.7 What will be the wave velocity, if the radar gives 54 waves per min and wavelength of the given wave is 10 m
(A) 4 m/sec (B) 6 m/sec
(C) 9 m/sec (D) 5 m/sec

Q.8 A wave of frequency 500 Hz has velocity 360 m/sec. The distance between two nearest points 60° out of phase, is
(A) 0.6 cm (B) 12 cm
(C) 60 cm (D) 120 cm

Q.9 A travelling wave passes a point of observation. At this point, the time interval between successive crests is 0.2 seconds and -
(A) The wavelength is 5 m
(B) The frequency is 5 Hz
(C) The velocity of propagation is 5 m/s
(D) The wavelength is 0.2 m

Q.10 A plane wave is described by the equation $y = 3\cos\left(\frac{x}{4} - 10t - \frac{\pi}{2}\right)$. The maximum velocity of the particles of the medium due to this wave is
(A) 30 (B) $3\pi/2$
(C) 3/4 (D) 40

Q.11 The path difference between the two waves $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a_2\cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is –
(A) $\frac{\lambda}{2\pi}\phi$ (B) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$
(C) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (D) $\frac{2\pi}{\lambda}\phi$

Q.12 Wave equations of two particles are given by $y_1 = a\sin(\omega t - kx)$, $y_2 = a\sin(kx + \omega t)$, then
(A) They are moving in opposite direction
(B) Phase between them is 90°
(C) Phase between them is 180°
(D) Phase between them is 0°

Q.13 The intensity of sound from a radio at a distance of 2 metres from its speaker is $1 \times 10^{-2}\ \mu\ W/m^2$ The intensity at a distance of 10 meters would be
(A) $0.2 \times 10^{-2}\ \mu\ W/m^2$ (B) $1 \times 10^{-2}\ \mu\ W/m^2$
(C) $4 \times 10^{-4}\ \mu\ W/m^2$ (D) $5 \times 10^{-2}\ \mu\ W/m^2$

Q.14 The disturbance of wave propagating in positive x-direction at t = 0 is $y = \frac{1}{(1+x^2)}$ and at t = 2s it becomes $y = \frac{1}{1+(x-1)^2}$ then the phase velocity of the wave will be
(A) 1/2 m/s (B) 1/4 m/s
(C) 1/6 m/s (D) 1/8 m/s

PART - 2 : SPEED OF SOUND WAVE

Q.15 Velocity of sound in air is –
(A) Faster in dry air than in moist air
(B) Directly proportional to pressure
(C) Directly proportional to temperature
(D) Independent of pressure of air

Q.16 Speed of sound wave in air
(A) is independent of temperature.
(B) increases with pressure.
(C) increases with increase in humidity.
(D) decreases with increase in humidity.

Q.17 A man sets his watch by the sound of a siren placed at a distance 1 km away. If the velocity of sound is 330 m/s –
(A) His watch is set 3 sec. faster
(B) His watch is set 3 sec. slower
(C) His watch is set correctly
(D) None of the above

Q.18 Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$

(C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

Q.19 Velocity of sound in air
I. Increases with temperature
II. Decreases with temperature
III. Increase with pressure
IV. Is independent of pressure
V. Is independent of temperature
Choose the correct answer –
(A) Only I and II are true (B) Only I and III are true
(C) Only II and III are true (D) Only I and IV are true

PART - 3 : SPEED OF TRANSVERSE WAVE

Q.20 A string of 7 m length has a mass of 0.035 kg. If tension in the string is 60.5 N, then speed of a wave on the string is
(A) 77 m/s (B) 102 m/s
(C) 110 m/s (D) 165 m/s

Q.21 A string is producing transverse vibration whose equation is $y = 0.0021 \sin(x + 30t)$, where x and y are in meters and t is in seconds. If the linear density of the string is 1.3×10^{-4} kg/m, then the tension in the string in N will be
(A) 10 (B) 0.5
(C) 1 (D) 0.117

Q.22 The speed of longitudinal wave in a wire is 100 times the speed of transverse wave. If Young's modulus of the wire material is 1×10^{11} N/m^2 then the stress in the wire is –
(A) 1×10^7 N/m^2 (B) 1.5×10^7 N/m^2
(C) 1×10^{11} N/m^2 (D) 1.5×10^{11} N/m^2

PART - 4 : INTERFERENCE

Q.23 When two sound waves with a phase difference of $\pi/2$, and each having amplitude A and frequency ω, are superimposed on each other, then the maximum amplitude and frequency of resultant wave is –

(A) $\frac{A}{\sqrt{2}} : \frac{\omega}{2}$ (B) $\frac{A}{\sqrt{2}} : \omega$ (C) $\sqrt{2}A : \frac{\omega}{2}$ (D) $\sqrt{2}A : \omega$

Q.24 If the phase difference between the two wave is 2π during superposition, then the resultant amplitude is –
(A) Maximum (B) Minimum
(C) Maximum or minimum (D) None of these

Q.25 The superposition takes place between two waves of frequency f and amplitude a. The total intensity is directly proportional to –
(A) a (B) 2a
(C) $2a^2$ (D) $4a^2$

Q.26 Consider interference between waves from two sources of intensities I and 4I. Find the intensities at points where the phase difference is (i) $\pi/2$ (ii) π.
(A) 5I, I (B) 4I, I/2
(C) 2I, I (D) 8I, I

Q.27 The displacement of the interfering light waves are

$y_1 = 4 \sin \omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{2}\right)$. What is the amplitude of the resultant wave
(A) 5 (B) 7
(C) 1 (D) 0

PART - 5 : BEATS

Q.28 Two tuning forks have frequencies 450 Hz and 454 Hz respectively. On sounding these forks together, the time interval between successive maximum intensities will be
(A) 1/4 sec (B) 1/2 sec
(C) 1 sec (D) 2 sec

Q.29 Beats are produced by two waves
$y_1 = a\sin 1000\pi t,\ y_2 = a\sin 998\pi t$. The number of beats heard/sec is –
(A) 0 (B) 2
(C) 1 (D) 4

Q.30 Maximum number of beats frequency heard by a human being is –
(A) 10 (B) 4
(C) 20 (D) 6

Q.31 Two waves of lengths 50 cm and 51 cm produced 12 beats per second. The velocity of sound is
(A) 306 m/s (B) 331 m/s
(C) 340 m/s (D) 360 m/s

Q.32 When a tuning fork of frequency 341 is sounded with another tuning fork, six beats per second are heard. When the second tuning fork is loaded with wax and sounded with the first tuning fork, the number of beats is two per second. The natural frequency of the second tuning fork is –
(A) 334 (B) 339
(C) 343 (D) 347

Q.33 The wavelengths of two waves are 50 and 51 cm respectively. If the temperature of the room is 20^{o}C, then what will be the no. of beats produced per second by these waves, when the speed of sound at 0^{o}C is 332m/sec
(A) 14 (B) 10
(C) 24 (D) None of these

Q.34 A source of sound gives five beats per second when sounded with another source of frequency 100s^{-1}. The second harmonic of the source together with a source of frequency 205s^{-1} gives five beats per second. What is the frequency of the source
(A) 105 s^{-1} (B) 205 s^{-1}
(C) 95 s^{-1} (D) 100 s^{-1}

Q.35 The frequency of tuning forks A and B are respectively 3% more and 2% less than the frequency of tuning fork C. When A and B are simultaneously excited, 5 beats per second are produced. Then the frequency of the tuning fork 'A' (in Hz) is
(A) 98 (B) 100
(C) 103 (D) 105

PART - 6 : STANDING WAVES IN STRINGS

Q.36 At a certain instant a stationary transverse wave is found to have maximum kinetic energy. The appearance of string at that instant is –
(A) Sinusoidal shape with amplitude A/3
(B) Sinusoidal shape with amplitude A/2
(C) Sinusoidal shape with amplitude A
(D) Straight line

Q.37 The distance between the nearest node and antinode in a stationary wave is
(A) λ (B) $\lambda/2$
(C) $\lambda/4$ (D) 2λ

Q.38 In stationary wave –
(A) Strain is maximum at nodes
(B) Strain is maximum at antinodes
(C) Strain is minimum at nodes
(D) Amplitude is zero at all the points

Q.39 The phase difference between the two particles situated on both the sides of a node is –
(A) 0° (B) 90°
(C) 180° (D) 360°

Q.40 Frequency of a sonometer wire is n. Now its tension is increased 4 times and its length is doubled then new frequency will be –
(A) n/2 (B) 4n
(C) 2n (D) n

Q.41 A string on a musical instrument is 50 cm long and its fundamental frequency is 270 Hz. If the desired frequency of 1000 Hz is to be produced, the required length of the string is
(A) 13.5 cm (B) 2.7 cm
(C) 5.4 cm (D) 10.3 cm

Q.42 The first overtone of a stretched wire of given length is 320Hz. The first harmonic is –
(A) 320 Hz (B) 160 Hz
(C) 480 Hz (D) 640 Hz

Q.43 In a stationary wave all the particles –
(A) On either side of a node vibrate in same phase
(B) In the region between two nodes vibrate in same phase
(C) In the region between two antinodes vibrate in same phase
(D) Of the medium vibrate in same phase

Q.44 The stationary wave produced on a string is represented by the equation $y = 5\cos(\pi x/3)\sin 40\pi t$. Where x and y are in cm and t is in seconds. The distance between consecutive nodes is
(A) 5 cm (B) π cm
(C) 3 cm (D) 40 cm

Q.45 A tuning fork of frequency 392 Hz, resonates with 50 cm length of a string under tension (T). If length of the string is decreased by 2%, keeping the tension constant, the number of beats heard when the string and the tuning fork made to vibrate simultaneously is –
(A) 4 (B) 6
(C) 8 (D) 12

Q.46 A tuning fork and a sonometer wire were sounded together and produce 4 beats per second. When the length of sonometer wire is 95 cm or 100 cm, the frequency of the tuning fork is
(A) 156 Hz (B) 152 Hz
(C) 148 Hz (D) 160 Hz

Q.47 A piece of wire is cut into two pieces A and B, and stretched to the same tension and mounted between two rigid walls. Segment A is longer than segment B. Which of the following quantities will always be larger for waves on A than for waves on B.
(A) amplitude of the wave
(B) frequency of the fundamental mode
(C) wave velocity
(D) wavelength of the fundamental mode

Q.48 The equation $y = 0.15\sin 5x\cos 300t$, describes a stationary wave. The wavelength of the stationary wave
(A) Zero (B) 1.256 metres
(C) 2.512 metres (D) 0.628 metre

Q.49 Equation of a stationary wave is $y = 10\sin\frac{\pi x}{4}\cos 20\pi t$. Distance between two consecutive nodes is
(A) 4 (B) 2
(C) 1 (D) 8

PART - 7 : STANDING WAVES IN ORGAN PIPES

Q.50 If the velocity of sound in air is 350 m/s. Then the fundamental frequency of an open organ pipe of length 50cm, will be –
(A) 350 Hz (B) 175 Hz
(C) 900 Hz (D) 750 Hz

Q.51 An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz, then the fundamental frequency of open pipe is
(A) 480 Hz (B) 300 Hz
(C) 240 Hz (D) 200 Hz

Q.52 If fundamental frequency of closed pipe is 50Hz then frequency of 2^{nd} overtone is –
(A) 100 Hz (B) 50 Hz
(C) 250 Hz (D) 150 Hz

Q.53 What should be the length of a closed organ pipe to produce a fundamental frequency of 512Hz at 0°C.
(A) 16.2 cm. (B) 15.2 cm.
(C) 18.2 cm. (D) 19.2 cm.

Q.54 The first overtone of an open pipe has the same frequency as the first overtone of a closed pipe 3m long. What is the length of the open pipe.
(A) 8m (B) 2m
(C) 6m (D) 4m

Q.55 An organ pipe emits fundamental tone of frequency 320Hz at 47ºC. What would be the fundamental tone emitted by pipe at 27ºC.
(A) 310 Hz (B) 720 Hz
(C) 512 Hz (D) 440 Hz

Q.56 Two closed pipes, one filled with O_2 and the other with H_2, have the same fundamental frequency. Find the ratio of their lengths.
(A) 1 : 2 (B) 1 : 4
(C) 1 : 6 (D) 2 : 6

Q.57 An air column in a pipe, which is closed at one end, will be in resonance with a vibrating body of frequency 166Hz, if the length of the air column is –
(A) 2.00 m (B) 1.50 m
(C) 1.00 m (D) 0.50 m

Q.58 If the length of a closed organ pipe is 1m and velocity of sound is 330 m/s, then the frequency for the second note is
(A) $4\times\frac{330}{4}$Hz (B) $3\times\frac{330}{4}$Hz
(C) $2\times\frac{330}{4}$Hz (D) $2\times\frac{4}{330}$Hz

Q.59 The frequency of fundamental tone in an open organ pipe of length 0.48 m is 320 Hz. Speed of sound is 320 m/sec. Frequency of fundamental tone in closed organ pipe will be –
(A) 153.8 Hz (B) 160.0 Hz
(C) 320.0 Hz (D) 143.2 Hz

Q.60 A resonance air column of length 20 cm resonates with a tuning fork of frequency 250 Hz. The speed of sound in air is
(A) 300 m/s (B) 200 m/s
(C) 150 m/s (D) 75 m/s

Q.61 Two closed pipe produce 10 beats per second when emitting their fundamental nodes. If their length are in ratio of 25 : 26. Then their fundamental frequency in Hz,
(A) 270, 280 (B) 260, 270
(C) 260, 250 (D) 260, 280

PART - 8 : DOPPLER EFFECT

Q.62 The frequency of a whistle of an engine is 600 cycles/sec is moving with the speed of 30 m/sec towards an observer. The apparent frequency will be –
(Velocity of sound = 330m/s)
(A) 600 cps (B) 660 cps
(C) 990 cps (D) 330 cps

Q.63 An observer is moving away from source of sound of frequency 100 Hz. His speed is 33 m/s. If speed of sound is 330 m/s, then the observed frequency is
(A) 90 Hz (B) 100 Hz
(C) 91 Hz (D) 110 Hz

Q.64 With what velocity an observer should move relative to a stationary source so that he hears a sound of double the frequency of source –
(A) Velocity of sound towards the source.
(B) Velocity of sound away from the source.
(C) Half the velocity of sound towards the source.
(D) Double the velocity of sound towards the source.

Q.65 A source of sound emitting a note of frequency 200 Hz moves towards an observer with a velocity v equal to the velocity of sound. If the observer also moves away from the source with the same velocity v, the apparent frequency heard by the observer is –
(A) 50 Hz (B) 100 Hz
(C) 150 Hz (D) 200 Hz

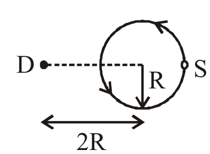
Q.66 A source of sound is travelling towards a stationary observer. The frequency of sound heard by the observer is of three times the original frequency. The velocity of sound is v m/sec. The speed of source will be
(A) (2/3) v (B) v
(C) (3/2) v (D) 3v

Q.67 The speed of sound in air at a given temperature is 350m/s. An engine blows whistle at a frequency of 1200cps. It is approaching the observer with velocity 50 m/s. The apparent frequency in cps heard by the observer will be
(A) 600 (B) 1050
(C) 1400 (D) 2400

Q.68 A source of frequency 150 Hz is moving in the direction of a person with a velocity of 110 m/s. The frequency heard by the person will be
(Speed of sound in medium = 330 m/s)
(A) 225 Hz (B) 200 Hz
(C) 150 Hz (D) 100 Hz

Q.69 A source of sound of frequency 500 Hz is moving towards an observer with velocity 30 m/s. The speed of sound is 330 m/s. the frequency heard by the observer will be
(A) 550 Hz (B) 458.3 Hz
(C) 530 Hz (D) 545.5 Hz

Q.70 A whistle S of frequency f revolves in a circle of radius R at a constant speed v. What is the ratio of largest and smallest frequency detected by a detector D at rest at a distance 2R from the centre of circle as shown in figure (take c as speed of sound)



(A) $\left(\frac{c+v}{c-v}\right)$ (B) $\sqrt{2}\left(\frac{c+v}{c-v}\right)$
(C) $\sqrt{2}$ (D) $\frac{(c+v)}{c\sqrt{2}}$

PART - 9 : MISCELLANEOUS

Q.71 A tube of length L_1 is open at both ends. A second tube of length L_2 is closed at one end and open at the other end. Both tubes have the same fundamental frequency of vibration of air in it. What is the value of L_2 ?

(A) $4L_1$ (B) $2L_1$
(C) $L_1/2$ (D) $L_1/4$

Q.72 In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x. The fraction of energy that is reflected is –

(A) $\left[\frac{x-1}{x}\right]^2$ (B) $\left[\frac{x}{x+1}\right]^2$

(C) $\left[\frac{x-1}{x+1}\right]^2$ (D) $\left[\frac{1}{x}\right]^2$

Q.73 A sound wave of frequency 440 Hz is passing through air. An O_2 molecule (mass $= 5.3 \times 10^{-26}$ kg) is set in oscillation with an amplitude of 10^{-6} m. Its speed at the centre of its oscillation is –

(A) 1.70×10^{-5} m/s (B) 17.0×10^{-5} m/s
(C) 2.76×10^{-3} m/s (D) 2.77×10^{-5} m/s

Q.74 A siren placed at a railway platform is emitting sound of frequency 5kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B be records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is :

(A) 242/252 (B) 2
(C) 5/6 (D) 11/6

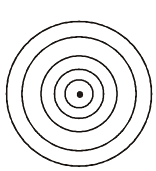
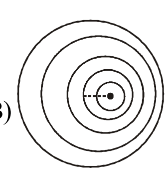
Q.75 A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 18cm. and 16cm respectively. The minimum possible length of the string is –

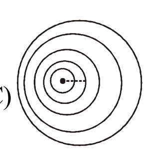
(A) 144 cm. (B) 152 cm.
(C) 176 cm. (D) 200 cm.

Q.76 The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to 1.5x, the speed of sound will be

(A) 1.22 v (B) 0.61 v
(C) 1.50 v (D) 0.75 v

Q.77 If the source is moving towards right, wave front of sound waves get modified to –

(A)  (B) 

(C)  (D) None of these

Q.78 Equation of a stationary and travelling waves are as follows $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin (\omega t - kx)$. The phase difference between two points

$x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ is ϕ_1 in the standing wave (y_1)

and is ϕ_2 in travelling wave (y_2) then ratio $\frac{\phi_1}{\phi_2}$ is–

(A) 1 (B) 5/6
(C) 3/4 (D) 6/7

Q.79 The beat frequency produced by two tuning forks when sounded together is observed to be 4 Hz. One of the forks makes 384 vibrations per second. When the other fork is loaded with a small piece of wax, the beats disappear 1st. The frequency of the second tuning fork is

(A) 388 Hz (B) 380 Hz
(C) more than 388 Hz (D) less than 380 Hz.

Q.80 In the resonance tube experiment, the first resonance is heard when length of air column is ℓ_1 and second resonance is heard when length of air column is ℓ_2. What should be the minimum length of the tube so that third resonance can also be heard –

(A) $2\ell_2 - \ell_1$ (B) $2\ell_1$
(C) $5\ell_1$ (D) $7\ell_1$

Q.81 A stationary observer receives sonic oscillations from two tuning forks, one of which approaches and the other recedes with same speed. As this takes place the observer hears the beat frequency of 2 Hz. Find the speed of each tuning fork, if their oscillation frequency is 680 Hz and the velocity of sound in air is 340 m/s –

(A) 1 m/s (B) 2 m/s
(C) 0.5 m/s (D) 1.5 m/s

Q.82 The average density of earth's crust 10 km beneath the surface is 2.7 gm/cm³. The speed of longitudinal seismic waves at that depth is 5.4 km/s. The bulk modulus of earth's crust considering its behaviour as fluid at that depth is–

(A) 7.9×10^{10} Pa (B) 5.6×10^{10} Pa
(C) 7.9×10^{7} Pa (D) 1.46×10^{7} Pa

Q.83 A note has a frequency of 128 Hz. The frequency of a note which is two octave higher than this is

(A) 256 Hz (B) 320 Hz
(C) 400 Hz (D) none of these

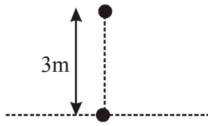
Q.84 Two coherent sources of different intensities send waves which interfere. The ratio of the maximum intensity to the minimum intensity is 25. The intensities are in the ratio –

(A) 25 : 1 (B) 5 : 1
(C) 9 : 4 (D) 625 : 1

EXERCISE - 2 [LEVEL-2]

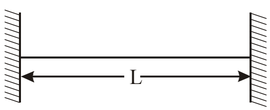
ONLY ONE OPTION IS CORRECT

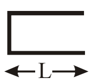
Q.1 S_1 and S_2 are two coherent sources of sound having no initial phase difference. The velocity of sound is 330m/s. No minima will be formed on the line passing through S_2 and perpendicular to the line joining S_1 and S_2, if the frequency of both the sources is

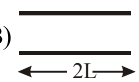


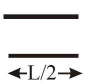
(A) 50 Hz (B) 60 Hz
(C) 70 Hz (D) 80 Hz

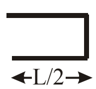
Q.2 Figure shows a stretched string of length L and pipes of length L, 2L, L/2 and L/2 in options (A), (B), (C) and (D) respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance.



(A) 

(B) 

(C) 

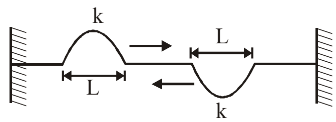
(D) 

Q.3 The speed of sound wave in a mixture of 1 mole of helium and 2 moles of oxygen at 27° C is
(A) 400 m/s (B) 600 m/s
(C) 800 m/s (D) 1200 m/s.

Q.4 In resonance tube experiment, if 400 Hz tuning fork is used, the first resonance occurs when length of air columns is 19 cm. If the 400 Hz tuning fork is replaced by 1600 Hz tuning fork then to get resonance, the water level in the tube should be further lowered by (take end correct = 1 cm.)
(A) 5 cm. (B) 10 cm.
(C) 15 cm. (D) 20 cm.

Q.5 Three coherent sonic sources emitting sound of single wavelength 'λ' are placed on the x-axis at points $\rightarrow (-\lambda\sqrt{11}/6\ ,\ 0)\ ,(0\ ,0)\ ,\ (\lambda\sqrt{11}/6\ ,\ 0)$. The intensity reaching a point $(0\ ,\ 5\lambda/6)$ from each source has the same value I_0. Then the resultant intensity at this point due to the interference of the three waves will be :
(A) $6\,I_0$ (B) $7\,I_0$
(C) $4\,I_0$ (D) $5\,I_0$

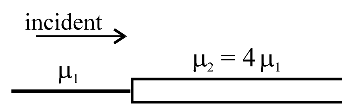
Q.6 Two identical pulses move in opposite directions with same uniform speeds on a stretched string. The width and kinetic energy of each pulse is L and k respectively. At the instant they completely overlap, the kinetic energy of the width L of the string where they overlap is –



(A) k (B) 2k
(C) 4k (D) 8k

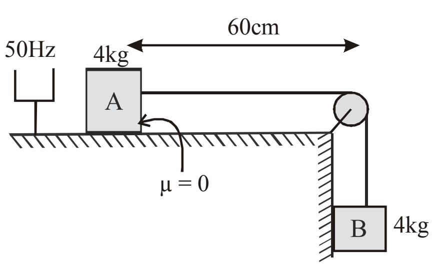
Q.7 Two vibrating strings of the same material but length L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is given by :
(A) 2 (B) 4
(C) 8 (D) 1

Q.8 String # 1 is connected with string # 2. The mass per unit length in string # 1 is μ_1 and the mass per unit length in string # 2 is 4 μ_1. The tension in the strings is T. A travelling wave is coming from the left. What fraction of the energy in the incident wave goes into string # 2 ?



(A) 1/8 (B) 4/9
(C) 2/3 (D) 8/9

Q.9 In the system shown, the wire connecting two masses has linear mass density of 1/20 kg/m. A tuning fork of 50Hz is found to be in resonance with the horizontal part of wire between pulley and block A. (Assuming nodes at block A and pulley). Now at t = 0 system is released from rest. The ratio of time gap between successive resonance with the same tuning fork starting from t = 0 (Take g = 10 m/s^2)



(A) 2 : 1 (B) 1 : 2
(C) $1:(\sqrt{2}-1)$ (D) $1:\sqrt{2}$

Q.10 The displacement y of a particle executing periodic motion is given by y = 4 cos^2 (t) sin (1000t). This expression may be considered to be a result of the superposition of waves :
(A) two (B) three
(C) four (D) five

Q.11 When beats are produced by two progressive waves of nearly the same frequency, which one of the following is correct –

(A) The particles vibrate simple harmonically, with the frequency equal to the difference in the component frequencies.

(B) The amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves.

(C) The frequency of beats depends upon the position, where the observer is.

(D) The frequency of beats changes as the time progresses.

Q.12 Sound waves of frequency 16 kHz are emitted by two coherent point sources of sound placed 2m apart at the centre of a circular train track of large radius. A person riding the train observes 2 maxima per second when the train is running at a speed of 36 km/h. Calculate the radius of the track. [Velocity of sound in air 320 m/s]

(A) $\frac{1000}{\pi}$ m (B) $\frac{500}{\pi}$ m

(C) $\frac{250}{\pi}$ m (D) $\frac{700}{\pi}$ m

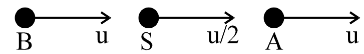
Q.13 A composite string is made up by joining two strings of different masses per unit length $\rightarrow \mu$ and 4μ. The composite string is under the same tension. A transverse wave pulse $Y = (6 \text{ mm}) \sin (5t + 40x)$, where 't' is in seconds and 'x' is in metres, is sent along the lighter string towards the joint. The joint is at $x = 0$. The equation of the wave pulse reflected from the joint is :

(A) $(2 \text{ mm}) \sin (5t - 40x)$ (B) $(4 \text{ mm}) \sin (40x - 5t)$
(C) $-(2 \text{ mm}) \sin (5t - 40x)$ (D) $(2 \text{ mm}) \sin (5t - 10x)$

Q.14 A pipe open at the top end is held vertically with some of its lower portion dipped in water. At a certain depth of immersion, the air column of length (3/8) m in the pipe resonates with a tuning fork of frequency 680 Hz. The speed of sound in air is 340 m/s. The pipe is now raised up by a distance 'a' until it resonates in the "next overtone" with the same tuning fork. The value of 'x' is:

(A) 20 cm (B) 40 cm
(C) 50 cm (D) 25 cm

Q.15 A source of sound is moving with velocity u/2 and two observers A and B are moving with velocity 'u' as shown. Find ratio of wavelength received by A and B. Given that velocity of sound is 10 u .



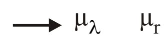
(A) 19/21 (B) 17/21
(C) 21/23 (D) 17/23

Q.16 A closed organ pipe has length ℓ. The air it is vibrating in 3^{rd} overtone with maximum amplitude 'a'. The amplitude at a distance of $\ell/7$ from closed end of the pipe is equal to –

(A) a (B) a/2

(C) $\frac{a\sqrt{3}}{2}$ (D) zero

Q.17 A string consists of two parts attached at $x = 0$. The right part of the string $(x > 0)$ has mass μ_r per unit length and the left part of the string $(x < 0)$ has mass μ_ℓ per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string.



(A) 1 (B) $\frac{2}{1+\sqrt{\mu_\ell/\mu_r}}$

(C) $\frac{2\sqrt{\mu_\ell/\mu_r}}{1+\sqrt{\mu_\ell/\mu_r}}$ (D) $\frac{\sqrt{\mu_\ell/\mu_r}-1}{\sqrt{\mu_\ell/\mu_r}+1}$

Q.18 A parachutist jumps from the top of a very high tower with a siren of frequency 800 Hz on his back. Assume his initial velocity to be zero. After falling freely for 12s, he observes that the frequency of sound heard by him reflected from level ground below him is differing by 700Hz w.r.t. the original frequency. What was the height of tower. Velocity of sound in air is 330 m/s, and

$g = 10 \text{ m/s}^2$.

(A) 511.5m. (B) 1057.5m.
(C) 757.5m. (D) 1215.5m.

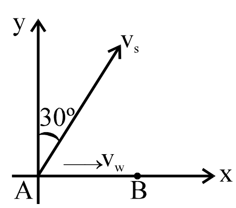
EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 A glass tube of 1.0 meter length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500Hz is brought at the upper end of the tube and the velocity of sound is 330m/s then the total number of resonances obtained will be –

Q.2 In the figure shown a source of sound of frequency 510 Hz moves with constant velocity v_s = 20 m/s in the direction shown. The wind is blowing at a constant velocity v_w = 20 m/s towards an observer who is at rest at point B. Find the frequency (in Hz) detected by the observer corresponding to the sound emitted by the source at initial position A.

[Speed of sound relative to air = 330 m/s]



Q.3 A straight line source of sound of length L = 10m, emits a pulse of sound that travels radially outward from the source. What sound energy (in mW) is intercepted by an acoustic cylindrical detector of surface area 2.4 cm², located at a perpendicular distance 7m from the source. The waves reach perpendicularly at the surface of the detector. The total power (in mW) emitted by the source in the form of sound is 2.2×10^4 W (Use $\pi = 22/7$)

Q.4 A wire of length ℓ having tension T and radius r vibrates with fundamental frequency f. Another wire of the same metal with length 2ℓ having tension 2T and radius 2r will vibrate with fundamental frequency $f / a\sqrt{2}$, then find the value of a.

Q.5 Two vibrating strings of same length, same cross section area and stretched to same tension is made of materials with densities ρ & 2ρ. Each string is fixed at both ends. If v_1 represents the fundamental mode of vibration of the one made with density ρ and v_2 for another, then v_1/v_2 is $\sqrt{x}$ then find the value of x.

Q.6 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, it gets partially reflected back and partially refracted (transmitted) in water. Difference of wavelength transmitted to wavelength reflected is 117/x (in m). Find the value of x. (speed of sound in air = 330 m/sec, Bulk modulus of water = 2.25×10^9, ρ_{water} =1000 kg/m^3).

Q.7 A 40 cm long wire having a mass 3.2 gm and area of cross section 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode. It vibrate with a frequency 1000/64 Hz then Young's modulus the wire = 10^a N/m^2. Find the value of a.

Q.8 A wall is moving with velocity u and a source of sound moves with velocity u/2 in the same direction as shown in the figure. Assuming that the sound travels with velocity 10u. The ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall, is equal to A/11. Find the value of A.

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A wire of length 40 cm is taut at both ends. The maximum wavelength of standing waves generated in the wire will be – **[AIEEE-2002]**
(A) 40 cm (B) 80 cm
(C) 20 cm (D) None

Q.2 A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at x = 0. Then the equation of the unknown wave is – **[AIEEE-2002]**
(A) $y = a \sin(\omega t + kx)$ (B) $y = -a \sin(\omega t + kx)$
(C) $y = a \sin(\omega t - kx)$ (D) $y = -a \sin(\omega t - kx)$

Q.3 A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. Frequency n of the alternating source is –
(A) 100 Hz (B) 200 Hz **[AIEEE-2003]**
(C) 25 Hz (D) 50 Hz

Q.4 A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was – **[AIEEE-2003]**
(A) 256 – 2 Hz (B) 256 – 5 Hz
(C) 256 + 5 Hz (D) 256 + 2 Hz

Q.5 When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per seconds are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? **[AIEEE-2005]**
(A) 200 Hz (B) 202 Hz
(C) 196 Hz (D) 204 Hz

Q.6 An observer moves towards a stationary source of sound, with a velocity one fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(A) zero (B) 0.5% **[AIEEE-2005]**
(C) 5% (D) 20%

Q.7 A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v upto which he can hear the whistle is – **[AIEEE-2006]**
(A) 30 ms^{-1} (B) $15\sqrt{2}$ ms^{-1}
(C) $15/\sqrt{2}$ ms^{-1} (D) 15 ms^{-1}

Q.8 A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420Hz and 315Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is – **[AIEEE 2006]**
(A) 1050 Hz (B) 10.5 Hz
(C) 105 Hz (D) 1.05 Hz

Q.9 A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of – **[AIEEE 2007]**
(A) 1000 (B) 10000
(C) 10 (D) 100

Q.10 A wave travelling along the x- axis is described by the equation $y(x,t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[AIEEE 2008]**
(A) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (B) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$
(C) $\alpha = 12.50\,\pi, \beta = \frac{\pi}{2.0}$ (D) $\alpha = 25.00\pi, \beta = \pi$

Q.11 Three sound waves of equal amplitudes have frequencies $(v - 1)$, v, $(v + 1)$. They superpose to give beats. The number of beats produced per second will be – **[AIEEE 2009]**
(A) 4 (B) 3
(C) 2 (D) 1

Q.12 A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest ? (Speed of sound = 330 ms^{-1}) **[AIEEE-2009]**
(A) 49 m (B) 98 m
(C) 147 m (D) 196 m

Q.13 The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02\,(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

Then tension in the string is – **[AIEEE 2010]**
(A) 4.0 N (B) 12.5 N
(C) 0.5 N (D) 6.25 N

Q.14 The transverse displacement y (x, t) of a wave on a string is given by $y(x, t) = e^{(-ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$
This represents a – **[AIEEE 2011]**
(A) wave moving in +x.direction with speed $(a/b)^{1/2}$
(B) wave moving in –x-direction with speed $(b/a)^{1/2}$
(C) standing wave of frequency $(b)^{1/2}$
(D) standing wave of frequency $1/\sqrt{b}$

Q.15 A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now – **[AIEEE 2012]**
(A) f (B) f/2
(C) 3f/4 (D) 2f

Q.16 A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? **[JEE MAIN 2013]**
(A) 188.5 Hz (B) 178.2 Hz
(C) 200.5 Hz (D) 770 Hz

Q.17 A pipe of length 85cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE MAIN 2014]**
(A) 6 (B) 4
(C) 12 (D) 8

Q.18 A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (Speed of sound = 320 ms^{-1}) close to **[JEE MAIN 2015]**
(A) 12% (B) 18%
(C) 24% (D) 6%

Q.19 A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (Take g=10 ms^{-2}) **[JEE MAIN 2015]**
(A) 2s (B) $2\sqrt{2}$ s
(C) $\sqrt{2}$ s (D) $2\pi\sqrt{2}$ s

Q.20 A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now : **[JEE MAIN 2016]**
(A) 3f/4 (B) 2f
(C) f (D) f/2

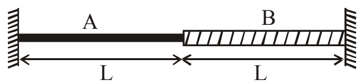
Q.21 An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (Speed of light = 3×10^8 m/s) **[JEE MAIN 2017]**
(A) 12.1 GHz (B) 17.3 GHz
(C) 15.3 GHz (D) 10.1 GHz

Q.22 A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg / m^3 and its Young's modulus is 9.27×10^{10} Pa . What will be the fundamental frequency of the longitudinal vibrations? **[JEE MAIN 2018]**
(A) 10 kHz (B) 7.5 kHz
(C) 5 kHz (D) 2.5 kHz

Q.23 Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: **[JEE MAIN 2019]**
(A) 4 : 1 (B) 25 : 9
(C) 16 : 9 (D) 5 : 3

Q.24 A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to : **[JEE MAIN 2019 (JAN)]**
(A) g / 5 (B) g / 20
(C) g / 10 (D) g / 30

Q.25 A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is : **[JEE MAIN 2019 (APRIL)]**



(A) 4 : 9 (B) 3 : 5
(C) 1 : 4 (D) 1 : 2

Q.26 In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be **[JEE MAIN 2019 (APRIL)]**
(A) 4 (B) 2
(C) 9 (D) 18

Q.27 A string of length 60 cm, mass 6gm and area of cross section 1mm^2 and velocity of wave 90m/s. Given Young's modulus is Y = 16×10^{11} N/m^2. Find extension in string **[JEE MAIN 2020 (JAN)]**
(A) 0.03 mm (B) 0.02 mm
(C) 0.01 mm (D) 0.04 mm

Q.28 A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is ν_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to : **[JEE MAIN 2020 (JAN)]**
(A) (1/4) m/s (B) 4 m/s
(C) 2 m/s (D) (1/2) m/s

Q.29 A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ________ Hz. **[JEE MAIN 2020 (JAN)]**

Q.30 Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, π/4 and –π/4 respectively. When they are superimposed the intensity of the resultant wave is close to : **[JEE MAIN 2020 (JAN)]**
(A) 5.8 I_0 (B) 0.2 I_0
(C) I_0 (D) 3 I_0

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

Q.1 Atransverse wave is represented by $y = A\sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity? **[AIPMT (PRE) 2010]**

(A) $\pi A/2$ (B) πA
(C) $2\pi A$ (D) A

Q.2 A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per sec when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[AIPMT (PRE) 2010]**

(A) 510 Hz (B) 514 Hz
(C) 516 Hz (D) 508 Hz

Q.3 Two waves are represented by the equations
$y_1 = a\sin(\omega t + kx + 0.57)$ m and
$y_2 = a\cos(\omega t + kx)$ m, where x is in meter and t in s. The phase difference between them is **[AIPMT (PRE) 2011]**

(A) 0.57 radian (B) 1.0 radian
(C) 1.25 radian (D) 1.57 radian

Q.4 Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air **[AIPMT (PRE) 2011]**

(A) Decreases by a factor 20 (B) Decreases by a factor 10
(C) Increases by a factor 20 (D) Increases by a factor 10

Q.5 Two identical piano wires kept under the same tension T have a fundamental frequency of 600Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be – **[AIPMT (MAINS) 2011]**

(A) 0.02 (B) 0.03
(C) 0.04 (D) 0.01

Q.6 When a string is divided into three segments of length ℓ_1, ℓ_2 and ℓ_3 the fundamental frequencies of these three segments are ν_1, ν_2 and ν_3 respectively. The original fundamental frequency (ν) of the string is **[AIPMT (PRE) 2012]**

(A) $\sqrt{\nu} = \sqrt{\nu_1} + \sqrt{\nu_2} + \sqrt{\nu_3}$ (B) $\nu = \nu_1 + \nu_2 + \nu_3$

(C) $\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$ (D) $\frac{1}{\sqrt{\nu}} = \frac{1}{\sqrt{\nu_1}} + \frac{1}{\sqrt{\nu_2}} + \frac{1}{\sqrt{\nu_3}}$

Q.7 Two sources of sound placed close to each other are emitting progressive waves given by
$y_1 = 4\sin 600\pi t$ and $y_2 = 5\sin 608\,\pi t$.
An observer located near these two sources of sound will hear : **[AIPMT (PRE) 2012]**

(A) 4 beats per second with intensity ratio 25 : 16 between waxing and waning.
(B) 8 beats per second with intensity ratio 25 : 16 between waxing and waning
(C) 8 beats per second with intensity ratio 81 : 1 between waxing and waning
(D) 4 beats per second with intensity ratio 81 : 1 between waxing and waning

Q.8 A train moving at a speed of $220\,\text{ms}^{-1}$ towards a stationary object, emits a sound of frequency 1000 Hz. Some of the sound reaching the object gets reflected back to the train as echo. The frequency of the echo as detected by the driver of the train is (Speed of sound in air is 330 m/s) **[AIPMT (MAINS) 2012]**

(A) 3500 Hz (B) 4000 Hz
(C) 5000 Hz (D) 3000 Hz

Q.9 A wave travelling in the +ve x-direction having displacement along y-direction as 1m, wavelength 2π m and frequency of $1/\pi$ Hz is represented by –**[NEET 2013]**

(A) $y = \sin(2\pi x + 2\pi t)$
(B) $y = \sin(x - 2t)$
(C) $y = \sin(2\pi x - 2\pi t)$
(D) $y = \sin(10\pi x - 20\pi t)$

Q.10 A source of unknown frequency gives 4beats/s, when sounded with a source of known frequency 250 Hz, The second harmonic of the source of unknown frequency gives five beats per second, when sounded with a source of frequency 513 Hz, The unknown frequency is – **[NEET 2013]**

(A) 260 Hz (B) 254 Hz
(C) 246 Hz (D) 240 Hz

Q.11 If we study the vibration of a pipe open at both ends, then the following statement is not true –**[NEET 2013]**

(A) Pressure change will be maximum at both ends.
(B) Open end will be antinode.
(C) Odd harmonics of the fundamental frequency will be generated.
(D) All harmonics of the fundamental frequency will be generated.

Q.12 The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (Velocity of sound = 340 ms^{-1}) **[AIPMT 2014]**

(A) 4 (B) 5
(C) 7 (D) 6

Q.13 If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by – **[AIPMT 2014]**

(A) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$ (B) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

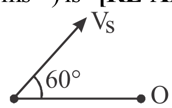
(C) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$ (D) $n = n_1 + n_2 + n_3$

Q.14 A speeding motorcyclist sees traffic jam ahead of him. He slows down to 36 km/hour. He finds that traffic has eased and a car moving ahead of him at 18 km/hour is honking at a frequency of 1392 Hz. If the speed of sound is 343 m/s, the frequency of the honk as heard by him will be **[AIPMT 2014]**

(A) 1332 Hz (B) 1372 Hz
(C) 1412 Hz (D) 1454 Hz

Q.15 The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is – **[AIPMT 2015]**
(A) 100 cm (B) 120 cm
(C) 140 cm (D) 80 cm

Q.16 A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer
(Velocity of sound in air 330 ms^{-1}) is – **[RE-AIPMT 2015]**
(A) 97 Hz (B) 100 Hz
(C) 103 Hz (D) 106 Hz



Q.17 A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequencies for this string is **[RE-AIPMT 2015]**
(A) 105 Hz (B) 155 Hz
(C) 205 Hz (D) 10.5 Hz

Q.18 A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of 15 m/s. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (Take velocity of sound in air = 330 m/s) **[NEET 2016 PHASE 1]**
(A) 765 Hz (B) 800 Hz
(C) 838 Hz (D) 885 Hz

Q.19 A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2. The ratio λ_2/λ_1 is **[NEET 2016 PHASE 1]**
(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_1+m_2}{m_2}}$
(C) $\sqrt{\frac{m_2}{m_1}}$ (D) $\sqrt{\frac{m_1+m_2}{m_1}}$

Q.20 An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is – **[NEET 2016 PHASE 1]**
(A) 66.7 cm (B) 100 cm
(C) 150 cm (D) 200 cm

Q.21 The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be **[NEET 2016 PHASE 2]**
(A) L (B) 2L
(C) L/2 (D) 4L

Q.22 Three sound waves of equal amplitudes have frequencies (n – 1), n, (n + 1). They superimpose to give beats. The number of beats produced per second will be **[NEET 2016 PHASE 2]**
(A) 1 (B) 4
(C) 3 (D) 2

Q.23 The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? **[NEET 2017]**
(A) 20 Hz (B) 30 Hz
(C) 40 Hz (D) 10 Hz

Q.24 Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a horn having a a frequency 400 Hz. The frequency heard by the driver of the second car is [Velocity of sound 340 m/s] **[NEET 2017]**
(A) 361 Hz (B) 411 Hz
(C) 448 Hz (D) 350 Hz

Q.25 The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20cm, the length of the open organ pipe is **[NEET 2018]**
(A) 12.5 cm (B) 8 cm (C) 13.2 cm (D) 16 cm

Q.26 A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of 27ºC two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320Hz, the velocity of sound in air at 27ºC is **[NEET 2018]**
(A) 350 m/s (B) 339 m/s
(C) 330 m/s (D) 300 m/s

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	B	C	C	B	A	C	B	B	A	B	A	C	A	D	C	B	B	D	C	D	A	D	A	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	A	A	A	C	A	A	D	A	A	C	D	C	A	C	D	A	B	B	C	C	A	D	B	A	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	C	A	D	A	B	D	B	B	B	C	B	A	A	D	A	C	A	A	A	C	C	C	B	A
Q	76	77	78	79	80	81	82	83	84																
A	A	B	D	A	A	C	A	D	C																

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	A	B	A	C	B	C	D	D	C	B	B	A	C	D	A	A	C	B

EXERCISE - 3

Q	1	2	3	4	5	6	7	8
A	3	525	12	2	2	100	9	9

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A	B	B	D	B	C	D	D	C	D	D	C	B	D	B	A	B	A	A	B	C	B	C	B	A
Q	25	26	27	28	29	30																		
A	D	A	A	A	106	A																		

EXERCISE - 5

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	C	D	B	D	A	C	D	C	B	B	A	D	A	C	B	C	A	C	B	C	B	D	A	C	C	B

WAVES

TRY IT YOURSELF-1

(1) (B)

(2) (C)

(3) (C)

(4) (B)

(5) (CD)

(6) (BCD)

(7) (ABD)

(8) **(B).** Mass per unit length of the string,

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{ kg / m}$$

$\therefore$ Velocity of wave in the string,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = 8 \text{ m/s}$$

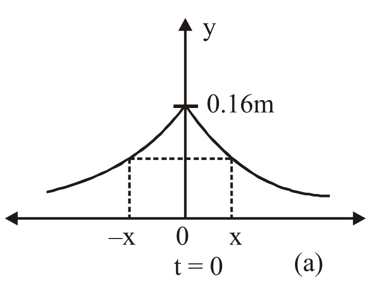
For constructive interference between successive pulses: $\Delta t_{min} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10\text{s}$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

(9) **(BCD).**

The shape of pulse at x = 0 and t = 0 would be as shown in figure (a).

$$y(0, 0) = \frac{0.8}{5} = 0.16\text{m}$$



(a)

From the figure it is clear that $y_{max} = 0.16$m.

Pulse will be symmetric (Symmetry is checked about y_{max}) if at t = 0.

$$y(x) = y(-x)$$

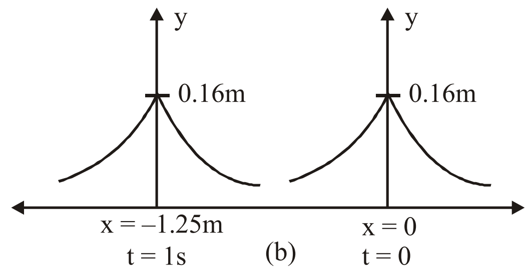
From the given equation,

$$y(x) = \frac{0.8}{16x^2 + 5} \text{ and } y(-x) = \frac{0.8}{16x^2 + 5} \text{ at } t = 0$$

or $\quad y(x) = y(-x)$

Therefore, pulse is symmetric.

Speed of pulse : At t = 1s and x = –1.25m



(b)

Value of y is again 0.16m, i.e., pulse has travelled a distance of 1.25m in 1s in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5m in 2s. The above statement can be better understood from figure (b).

Alternate : If equation of a wave pulse is $y = f(ax \pm bt)$

The speed of wave is b/a in negative x direction for $y = f(ax + bt)$ and positive x direction for $y = f(ax - bt)$.

Comparing this from given equation we can find that speed of wave is 5/4 = 1.25 m/s and it is travelling in negative x-direction.

(10) **(D).** Fundamental frequency is given by

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \text{ (with both the ends fixed)}$$

$\therefore$ Fundamental frequency

$$\nu \propto \frac{1}{\ell\sqrt{\mu}} \qquad \text{[for same tension in both strings]}$$

where μ = mass per unit length of wire

$= \rho.A$ (ρ = density) $= \rho(\pi r^2)$

or $\sqrt{\mu} \propto r \quad \therefore \nu \propto \frac{1}{r\ell}$

$$\therefore \frac{\nu_1}{\nu_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

(11) **(B).** Let Δl be the end correction.

Given that fundamental tone for a length 0.1m = first overtone for the length 0.35m.

$$\frac{v}{4(0.1 + \Delta l)} = \frac{3v}{4(0.35 + \Delta l)}$$

Solving this equation, we get $\Delta l = 0.025$m.

(12) **(A).** $n_s = \frac{3}{4}\left(\frac{340}{0.75}\right) = n - 4 \quad \therefore n = 344\,\text{Hz}$

TRY IT YOURSELF-2

(1) (AB)

(2) **(D).** $f_1 = f\left(\frac{v}{v - v_s}\right)$; $f_1 = f\left(\frac{340}{340-34}\right) = f\left(\frac{340}{306}\right)$

and $f_2 = f\left(\frac{340}{340-17}\right) = f\left(\frac{340}{323}\right) \quad \therefore \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$

(3) **(B).** Using the formula, $f' = f\left(\frac{v + v_0}{v}\right)$

We get, $5.5 = 5\left(\frac{v + v_A}{v}\right)$ .... (1)

and $6.0 = 5\left(\frac{v + v_B}{v}\right)$ .... (2)

Here, v = speed of sound,

v_A = speed of train A

v_B = speed of train B

Solving eqs. (1) and (2), we get $\frac{v_B}{v_A} = 2$

(4) **(B).** The motorcyclists observes no beats.

So, the apparent frequency observed by him from the two sources must be equal.

$f_1 = f_2$

$\therefore \quad 176 = \left(\frac{330 - v}{330 - 22}\right) = 165\left(\frac{330 + v}{330}\right)$

Solving this equation, we get

$v = 22\text{m/s}$

(5) **(D).** The frequency is a characteristic of source.

It is independent of the medium.

(6) While approaching,

$f' = f_0\left(\frac{v}{v - v_s}\right)$; $2200 = f_0\left(\frac{300}{300 - v_s}\right)$

While reducing ,

$f'' = f_0\left(\frac{v}{v + v_s}\right)$; $1800 = f_0\left(\frac{300}{300 + v_s}\right)$

On solving velocity of source (train) $v_s = 30$ m/s

(7) **(A).** $f = f_0\left(\frac{v}{v-u}\right)\left(\frac{v+u}{v}\right) = 8\left(\frac{320+10}{320-10}\right) = 8.5\,\text{kHz}$

(8) **(AB).** If wind blows from source to observer

$f_2 = f_1\left(\frac{V + w + u}{V + w - u}\right)$

When wind blows from observer towards source

$f_2 = f_1\left(\frac{V - w + u}{V - w - u}\right)$

In both cases, $f_2 > f_1$.

CHAPTER-13 : WAVES
EXERCISE-1

(1) **(C).** Since solid has both the properties (rigidity and elasticity).

(2) **(B).** Water waves produced by a motor boat sailing in water are both, longitudinal and transverse.

(3) **(C).** Comparing with the standard equation,

$$y = A\sin\frac{2\pi}{\lambda}(vt - x),$$

We have $v = 200$cm/sec, $\lambda = 200$cm $\therefore\ n = \frac{v}{\lambda} = 1\ \sec^{-1}$

(4) **(C).** Change in temperature of the medium changes the velocity of sound waves and hence the wavelength of sound waves. This is because frequency ($\nu = v/\lambda$) is fixed.

(5) **(B).** During the propagation of longitudinal wave in a medium, energy, not the matter is transmitted through the medium.

(6) **(A).** Energy is not carried by stationary waves

(7) **(C).** $n = \frac{54}{60}\text{Hz}, \lambda = 10\,\text{m} \Rightarrow \quad v = n\lambda = 9\,\text{m/s}$.

(8) **(B).** The distance between two points i.e. path difference

between them $\Delta = \frac{\lambda}{2\pi}\times\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{3} = \frac{\lambda}{6} = \frac{v}{6n}$

$$\Rightarrow \Delta = \frac{360}{6\times 500} = 0.12\,\text{m} = 12\,\text{cm}$$

(9) **(B).** Phase difference between two successive crest is 2π.

Also, phase difference $(\Delta\phi) = \frac{2\pi}{T}$ time interval (Δt)

$$\Rightarrow 2\pi = \frac{2\pi}{T}\times 0.2 \Rightarrow \frac{1}{T} = 5\sec^{-1} \Rightarrow n = 5\,\text{Hz}$$

(10) **(A).** $v_{max} = a\omega = 3\times 10 = 30$

(11) **(B).** $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ & $y_2 = a_2\cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$

$$= a_2\sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$$

So phase difference $= \phi + \frac{\pi}{2}$ and $\Delta = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

(12) **(A).** Both waves are moving opposite to each other

(13) **(C).** $I \propto \frac{1}{r^2} \Rightarrow \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \Rightarrow \frac{I_2}{1\times 10^{-2}} = \frac{2^2}{10^2} = \frac{4}{100}$

$$\Rightarrow I_2 = \frac{4\times 10^{-2}}{100} = 4\times 10^{-4}\ \mu\ \text{W/m}^2$$

(14) **(A).** Displacement of wave in x direction

$\Delta x = x - (x - 1) = 1$ m

Time intervel $\Delta t = 2 - 0 = 2$ sec.

$\therefore$ Phase velocity $= \frac{\Delta x}{\Delta t} = \frac{1}{2}$ m/s

(15) **(D).** $v = \sqrt{\frac{\gamma P}{\rho}}$; as P changes, ρ also changes. Hence P/ρ remains constant so speed remains constant.

(16) **(C).** Speed of sound wave in air increases with increase in humidity. This is because presence of moisture decreases the density of air.

(17) **(B).** $\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{1000}{330} = 3.03$ second

Sound will be heard after 3.03 sec. So his watch is set 3sec, slower.

(18) **(B).** Speed of sound in gases is given by

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

(19) **(D).** Speed of sound $v \propto \sqrt{T}$ and it is independent of pressure.

(20) **(C).** $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{60.5}{(0.035/7)}} = 110\,\text{m/s}$

(21) **(D).** $y = 0.0021\sin(x + 30t) \Rightarrow v = \frac{\omega}{k} = \frac{30}{1} = 30\,\text{m/s}$.

Using, $v = \sqrt{\frac{T}{m}} \Rightarrow 30 = \sqrt{\frac{T}{1.3\times 10^{-4}}} \Rightarrow T = 0.117\text{N}$

(22) **(A).** $v_{long.} = 100 v_{trans.}$; $\sqrt{\frac{Y}{d}} = 100\sqrt{\frac{\text{stress}}{d}}$

$$\sqrt{1\times 10^{11}} = 100\sqrt{\text{stress}}\ ;\quad \text{Stress} = \frac{10^{11}}{10^4} = 10^7$$

(23) **(D).** $A_{max} = \sqrt{A^2 + A^2} = A\sqrt{2},$

frequency will remain same i.e. ω.

(24) **(A).** Phase difference is 2π means constructive interference so resultant amplitude will be maximum.

(25) **(D).** Resultant amplitude

$$A = \sqrt{a^2 + a^2 + 2aa\cos\phi} = \sqrt{4a^2\cos^2\left(\frac{\phi}{2}\right)}$$

$\because I \propto A^2 \Rightarrow I \propto 4a^2$

(26) **(A).** Resultant intensity

$I_R = A^2 = {a_1}^2 + {a^2}_2 + 2a_1a_2\cos\phi = I + 4I + 4I\cos\phi$

$\therefore\ I_R = 5I + 4I\cos(\pi/2) = 5I$; $I_R = 5I + 4I\cos\pi = I$

(i) $\phi = \pi/2$, $I_R = 5I$ (ii) $\phi = \pi$, $I_R = I$

(27) **(A).** Since $\phi = \frac{\pi}{2} \Rightarrow A = \sqrt{a_1^2 + a_2^2} = \sqrt{(4)^2 + (3)^2} = 5$

(28) **(A).** The time interval between successive maximum intensities will be $\frac{1}{n_1 - n_2} = \frac{1}{454 - 450} = \frac{1}{4}$ sec.

(29) **(C).** $n_1 = \frac{1000\pi}{2\pi} = 500$Hz and $n_2 = \frac{998\pi}{2\pi} = 499$ Hz

Hence beat frequency $= n_1 - n_2 = 1$

(30) **(A).** Persistence of hearing is 10 sec^{-1}.

(31) **(A).** $n_1 = \frac{v}{\lambda_1} = \frac{v}{0.50}$ and $n_2 = \frac{v}{\lambda_2} = \frac{v}{0.51}$

$$\Delta n = n_1 - n_2 = v\left[\frac{1}{0.50} - \frac{1}{0.51}\right] = 12$$

$$\Rightarrow \quad v = \frac{12 \times 0.51 \times 0.50}{0.01} = 306 m/s$$

(32) **(D).** n_A = Known frequency = 341 Hz, n_B = ?
x = 6 bps, which is decreasing (i.e. x↓) after loading (from 6 to 1 bps)
Unknown tuning fork is loaded so $n_B\downarrow$
Hence $n_A - n_B\downarrow = x\downarrow$ ... (i) Wrong
$n_B\downarrow - n_A = x\downarrow$ ... (ii) Correct
$\Rightarrow n_B = n_A + x = 341 + 6 = 347$ Hz.

(33) **(A).** v_0 = 332m/s. Velocity sound at t°C is $v_t = (v_0 + 0.61t)$

$$\Rightarrow v_{20} = v_0 + 0.61 \times 20 = 344.2 \text{ m/s}$$

$$\Rightarrow \Delta n = v_{20}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = 344.2\left(\frac{100}{50} - \frac{100}{51}\right) = 14$$

(34) **(A).** Frequency of the source = 100 ± 5 = 105 Hz or 95 Hz.
Second harmonic of the source = 210 Hz or 190 Hz.
As the second harmonic gives 5 beats/sec with sound of frequency 205 Hz, the second harmonic should be 210Hz.
⇒ Frequency of the source = 105 Hz.

(35) **(C).** Let n be the frequency of fork C then

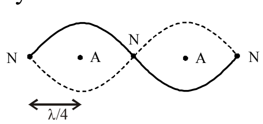
$$n_A = n + \frac{3n}{100} = \frac{103n}{100} \text{ and } n_B = n - \frac{2n}{100} = \frac{98}{100}$$

$$\text{but } n_A - n_B = 5 \Rightarrow \frac{5n}{100} = 5 \Rightarrow n = 100 \text{ Hz}$$

$$\therefore n_A = \frac{(103)(100)}{100} = 103 \text{Hz}$$

(36) **(D).** Particles have kinetic energy maximum at mean position.

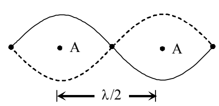
(37) **(C).** The distance between the nearest node and antinode in a stationary wave is λ/4



(38) **(A).** At nodes strain is maximum.

(39) **(C).** Both the sides of a node, two antinodes are present with separation λ/2. So phase difference between then

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$$



(40) **(D).** $n \propto \frac{1}{\ell}\sqrt{T} \Rightarrow \frac{n'}{n} = \sqrt{\frac{T'}{T}} \times \frac{\ell}{\ell'} = \sqrt{4} \times \frac{1}{2} = 1 \Rightarrow n' = n$

(41) **(A).** $n \propto \frac{1}{\ell} \Rightarrow \frac{\ell_2}{\ell_1} = \frac{n_1}{n_2} \Rightarrow \ell_2 = \ell_1\left(\frac{n_1}{n_2}\right) = 50 \times \frac{270}{1000} = 13.5\text{cm}$

(42) **(B).** Frequency of first overtone or second harmonic (n_2) = 320 Hz. So, frequency of first harmonic

$$n_1 = \frac{n_2}{2} = \frac{320}{2} = 160\text{Hz}$$

(43) **(B).** In stationary wave all the particles in one particular segment (i.e., between two nodes) vibrates in the same phase.

(44) **(C).** On comparing the given equation with standard equation $\Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{3} \Rightarrow \lambda = 6\,\text{cm}$.
Distance between two consecutive nodes ⇒ λ = 3cm

(45) **(C).** $n \propto \frac{1}{\ell} \Rightarrow \frac{\Delta n}{n} = -\frac{\Delta \ell}{\ell}$
If length is decreased by 2% then frequency increases by 2% i.e., $\frac{n_2 - n_1}{n_1} = \frac{2}{100}$

$$\Rightarrow n_2 - n_1 = \frac{2}{100} \times n_1 = \frac{2}{100} \times 392 = 7.8 \approx 8.$$

(46) **(A).** Probable frequencies of tuning fork be n + 4 or n – 4
Frequency of sonometer wire $n \propto \frac{1}{\ell}$

$$\therefore \frac{n+4}{n-4} = \frac{100}{95} \text{ or } 95\ (n+4) = 100\ (n-4)$$

or $95n + 380 = 100n - 400$ or $5n = 780$ or $n = 156$

(47) **(D).** For fundamental mode = $\frac{\lambda}{2} = L, \lambda = 2L$
High L assure high λ.

(48) **(B).** On comparing the given equation with standard equation $\frac{2\pi}{\lambda} = 5 \Rightarrow \lambda = \frac{6.28}{5} = 1.256\text{m}$

(49) **(A).** On comparing the given equation with standard equation $\frac{2\pi}{\lambda} = \frac{\pi}{4} \Rightarrow \lambda = 8$
Hence distance between two consecutive nodes $\frac{\lambda}{2} = 4$

(50) **(A).** Fundamental frequency of open pipe

$$n_1 = \frac{v}{2\ell} = \frac{350}{2 \times 0.5} = 350 \text{ Hz}$$

(51) **(D).** Fundamental frequency of open organ pipe = $v/2\ell$
Frequency of third harmonic of closed pipe = $3v/4\ell$

$$\therefore \frac{3v}{4\ell} = 100 + \frac{v}{2\ell};$$

$$\frac{3v}{4\ell} - \frac{2v}{4\ell} = \frac{v}{4\ell} = 100 \Rightarrow \frac{v}{2\ell} = 200\,\text{Hz}$$

(52) **(C).** Frequency of 2nd overtone $n_3 = 5n_1 = 5 \times 50 = 250$ Hz

(53) **(A).** $n_1 = \frac{v}{4\ell}$ or $\ell = \frac{v}{4n_1}$

$\therefore\ \ell = \frac{332}{4\times 512}\text{m} = \frac{33200}{4\times 512}\text{cm} = 16.2\text{cm}$

(54) **(D).** $\frac{2v_0}{2\ell_0} = \frac{3v}{4\ell_c} = \frac{3v}{4\times 3}$; $\ell_0 = 4$ meter

(55) **(A).** $n \propto \sqrt{T}$ (assuming no change in length)

or $\frac{n_1}{n} = \sqrt{\left(\frac{T_1}{T}\right)} = \sqrt{\left(\frac{273+27}{273+47}\right)}$

$\frac{n_1}{320} = \sqrt{\left(\frac{300}{320}\right)}$; $n_1 = 320\sqrt{\left(\frac{15}{16}\right)} \approx 310\text{Hz}.$

(56) **(B).** $n = \frac{v_1}{4\ell_1} = \frac{v_2}{4\ell_2}$

$\therefore\ \frac{\ell_1}{\ell_2} = \frac{v_1}{v_2} = \sqrt{\left(\frac{d_2}{d_1}\right)} = \sqrt{\left(\frac{1}{16}\right)} = \frac{1}{4}\ \therefore\ \ell_1 : \ell_2 = 1:4$

(57) **(D).** For closed pipe

$n_1 = \frac{v}{4\ell} \Rightarrow \ell = \frac{v}{4n} = \frac{332}{4\times 166} = 0.5\text{m}$

(58) **(B).** For closed pipe $n_1 = \frac{v}{4\ell} = \frac{330}{4}$ Hz

Second note $= 3n_1 = \frac{3\times 300}{4}$ Hz.

(59) **(B).** $n_{\text{Closed}} = \frac{1}{2}(n_{\text{Open}}) = \frac{1}{2}\times 320 = 160\,\text{Hz}$

(60) **(B).** For closed pipe

$n_1 = \frac{v}{4l} \Rightarrow 250 = \frac{v}{4\times 0.2} \Rightarrow v = 200\,\text{m/s}$

(61) **(C).** $n_1 - n_2 = 10$ .....(i)

Using $n_1 = \frac{v}{4\ell_1}$ and $n_2 = \frac{v}{4\ell_2} \Rightarrow \frac{n_1}{n_2} = \frac{\ell_2}{\ell_1} = \frac{26}{25}$ .....(ii)

After solving these equation $n_1 = 260$ Hz, $n_2 = 250$ Hz

(62) **(B).** $n' = n\left(\frac{v}{v-v_S}\right) = 600\left(\frac{330}{300}\right) = 660$ cps

(63) **(A).** $n' = n\left(\frac{v-v_0}{v}\right) = \left(\frac{330-33}{330}\right)\times 100 = 90\text{Hz}$

(64) **(A).** $2n = n\left(\frac{v-v_0}{v-0}\right) \Rightarrow v_0 = -v = -$ (Speed of sound)

Negative sign indicates that observer is moving opposite to the direction of velocity of sound.

(65) **(D).** Since there is no relative motion between observer and source, therefore there is no apparent change in frequency.

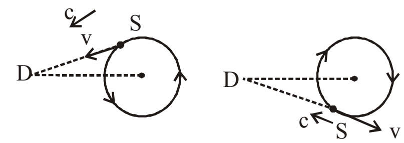
(66) **(A).** $n' = n\left(\frac{v}{v-v_S}\right) \Rightarrow \frac{n'}{n} = \frac{v}{v-v_S} \Rightarrow \frac{v}{v-v_S} = 3 \Rightarrow v_s = \frac{2v}{3}$

(67) **(C).** $n' = n\left(\frac{v}{v-v_S}\right) = 1200\times\left(\frac{350}{350-50}\right) = 1400$ cps

(68) **(A).** $n' = \frac{v}{v-v_s}\times n = \left(\frac{330}{330-110}\right)\times 150 = 225\,\text{Hz}$

(69) **(A).** $n' = n\left(\frac{v}{v-v_S}\right) \Rightarrow n' = 500\left(\frac{330}{330-30}\right) = 550\text{Hz}$

(70) **(A).**

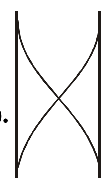


Largest frequency (f_1) Lowest frequency (f_2)

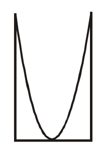
Largest frequency will be detected when the source approaches detector along the line joining and the smallest frequency will be detected when the source recedes the detectors along the line joining them

$$\frac{f_1}{f_2} = \frac{\left(\frac{c}{c-v}\right)f}{\left(\frac{c}{c+v}\right)f} = \frac{c+v}{c-v}$$

(71) **(C).**



$\frac{\lambda}{2} = L_1$



$\frac{\lambda}{4} = L_2$

$2L_1 = 4L_2$ $L_1 = 2L_2$

(72) **(C).** $\frac{A_1 + A_2}{A_1 - A_2} = x$; $\frac{A_2}{A_1} = \frac{x-1}{x+1}$; Energy $\propto A^2$

$\Rightarrow \left(\frac{x-1}{x+1}\right)^2$

(73) **(C).** $v_{max} = \omega_n A = (2\pi f)\,A = (2\pi)\,(440)\,(10^{-6})$
$= 2.76\times 10^{-3}$ m/sec

(74) **(B).** $f_A - \frac{v+v_A}{v}f$, or $v_A = \frac{v}{f}(f_A - f)$

$f_B = \frac{v+v_B}{v}f$, or $v_B = \frac{v}{f}(f_B - f)$

$\therefore\ \frac{v_B}{v_A} = \frac{f_B - f}{f_A - f} = \frac{6.0-5}{5.5-5} = \frac{1}{0.5} = 2$

(75) **(A).** $L = \frac{m\lambda_1}{2}$ and $L(m+1)\frac{\lambda_2}{2}$,

where m is no. of harmonic

$m.36 = (m+1)\,32 \Rightarrow m = 8$; $L = 8\times 18 = 144$ cm.

(76) **(A).** Tension $T = kx,\ T' = 1.5x$

Since $v \propto \sqrt{T}$ $\therefore \frac{v'}{v} = \sqrt{\frac{T'}{T}} = \sqrt{1.5} = 1.22$ or $v' = 1.22v$

(77) **(B).** Towards right wavelength gets compressed, towards left, wavelength gets expanded.

(78) **(D).** x_1 and x_2 are in successive loops of std. waves.

so, $\phi_1 = \pi$ and

$$\phi_2 = K(\Delta x) = K\left(\frac{3\pi}{2K} - \frac{\pi}{3K}\right) = \frac{7\pi}{6} = \frac{\phi_1}{\phi_2} = \frac{6}{7}$$

(79) **(A).** Since the frequency of beats in 4 Hz, the frequency of the second tuning fork will be either 384 + 4 = 388Hz, or 384 – 4 = 380 Hz. On loading, the frequency decreases. If 388 Hz be the true frequency, the beats after loading may disappear since frequency may decrease from 388Hz to 384 Hz. The frequency 380Hz is not permissible, since if will decrease further and cannot increase to 384Hz. Hence 388 Hz is the true frequency.

(80) **(A).** $\ell_1 + \varepsilon = \frac{v}{4f_0} \Rightarrow \ell_2 + \varepsilon = \frac{3v}{4f_0} \Rightarrow \ell_3 + \varepsilon = \frac{5v}{4f_0}$

Solving we get, $\ell_3 = 2\ell_2 - \ell_1$

(81) **(C).** $\left[\left(\frac{v}{v - v_s}\right) - \left(\frac{v}{v + v_s}\right)\right] f_0 = 2\text{Hz}$; $v_s = 0.5$ m/s

(82) **(A).** $V = \sqrt{\frac{B}{\rho}} \Rightarrow B = V^2\rho$

$= (5.40 \times 10^3\ \text{m/s})^2 (2.7 \times 10^3) = 7.9 \times 10^{10}$ Pa

(83) **(D).** One octave higher means the note whose frequency is 2 times the given frequency.
Similarly 2 octave higher means 3 times the given frequency, which is 3 × 128 Hz = 384 Hz.

(84) **(C).** $\frac{a_1 + a_2}{a_1 - a_2} = 5 \Rightarrow a_1 + a_2 = 5(a_1 - a_2)$

$\frac{a_1}{a_2} = \frac{3}{2}$; $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{9}{4}$

EXERCISE-2

(1) **(A).** For minimum, $\Delta x = (2n - 1)\frac{\lambda}{2}$

The maximum possible path difference = distance between the sources = 3m.

For no minimum $\frac{\lambda}{2} > 3 \Rightarrow \lambda > 6$ $\therefore f = \frac{V}{\lambda} < \frac{330}{6} = 55$

$\therefore$ If f < 55 Hz, no, minimum will occur.

(2) **(B).** Fundamental frequency of wire $(f_{wire}) = \frac{v}{2\ell}$

(A)  $f = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}$ cannot match with f_{wire}

(B)  $f = \frac{v}{2(2\ell)}, \frac{2v}{2(2\ell)}, \frac{3v}{2(2\ell)}$

its second harmonic $\frac{2v}{2(2\ell)}$ matches with f_{wire}.

(C)  $f = \frac{v}{2(\ell/2)}, \frac{2v}{2(\ell/2)}$

cannot match with f_{wire}.

(D)  $f = \frac{v}{4(\ell/2)}, \frac{3v}{4(\ell/2)}$.......

cannot match with f_{wire}.

(3) **(A).** The speed of sound wave is

$$v = \sqrt{\frac{\gamma RT}{M}},\ \text{where } \gamma = \left(\frac{c_p}{c_v}\right)_{mixture}$$

The molecular weight of the mixture

$$M = \frac{n_1M_1 + n_2M_2}{n_1 + n_2} = \frac{1 \times 4 + 2 \times 32}{1 + 2}$$

$$= \frac{68}{3}\text{g/mol} = \frac{68}{3} \times 10^{-3}\text{kg/mol}$$

$$(c_v)_{mixture} = \frac{(n_1c_v)_{He} + (n_1c_v)_{O_2}}{n_1 + n_2} = \frac{1 \times \frac{3}{2}R + 2 \times \frac{5}{2}R}{1 + 2} = \frac{13}{6}R$$

$$(c_p)_{mixture} = (c_v)_{mixture} + R = \frac{19}{6}R \quad \therefore$$

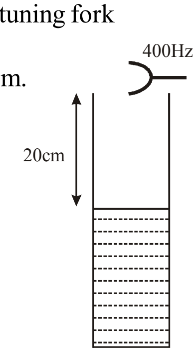
$$(\gamma)_{mixture} = \frac{(c_p)_{mixture}}{(c_v)_{mixture}} = \frac{19}{13}$$

Substituting these values, we get

$$v = \sqrt{\frac{19}{13} \times 8.31 \times (273 + 27) / \frac{68}{3} \times 10^{-3}} = 400.93\ \text{m/s}$$

(4) **(C).** For first resonance with 400 Hz tuning fork

$$\ell_{eq} = \frac{V}{4f_0} = \frac{V}{4(400)} = (19 + 1) = 20\text{cm}.$$



If we use 1600 Hz tuning fork

$$\frac{V}{4f_0} = \frac{V}{4 \times (1600)} = \frac{20}{4} = 5\ \text{cm}.$$

For resonance

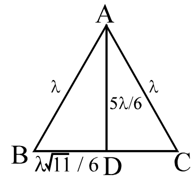
$$\ell_{eq} = \frac{V}{4f_0}, \frac{3V}{4f_0}, \frac{5V}{4f_0}, \frac{7V}{4f_0}, ...$$

1cm + ℓ = 5cm , 15 cm, 25 cm, 35 cm, 45 cm. ....
ℓ = 4cm , 14 cm, 24 cm, 34 cm, 44 cm, ...
water level should be further lowered by,
24 – 19 = 5 cm
34 – 19 = 15 cm

(5) $AB = \sqrt{AD^2 + BD^2} = \lambda = AC$

So sound reaching from B and C will be in same phase

Now $AD = 5\lambda/6$



$$\Delta\varphi = \frac{2\pi}{\lambda} \times \frac{5\lambda}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}$$

$$A = \sqrt{(2A)^2 + (A)^2 + 2 \times 2A \times A \times \cos(\pi/3)}$$

$$= \sqrt{5A^2 + 2A^2} = A\sqrt{7} \qquad \therefore\ I = 7I_0$$

(6) **(C).**

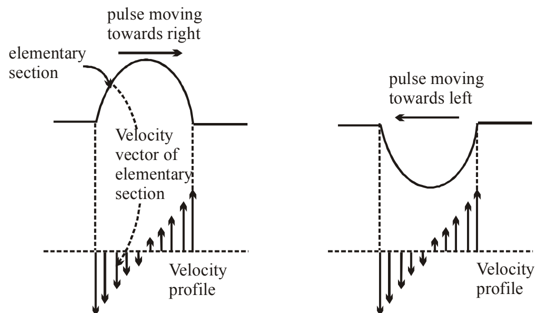


Figure-1 Figure-2

The velocity of profile of each elementary section of the pulse is shown in figure 1 and figure 2.

When both the pulses completely overlaps, the velocity profiles of both the pulses in overlap region are identical. By superposition, velocity of each elementary section doubles. Therefore, KE of each section becomes four times. Hence the K.E. in the complete width of overlap becomes four times.

(7) **(D).** Fundamental frequency $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$$\therefore\ \frac{f_1}{f_2} = \frac{\ell_1}{\ell_2}\sqrt{\frac{\mu_2}{\mu_1}} \quad \text{(since tension is same)}$$

$$= \frac{2L}{L}\sqrt{\frac{\pi r^2 \rho}{\pi \cdot 4r^2 \rho}}, \text{ (since the wires are of same material)} = 1$$

(8) **(D).** $\frac{E_r}{E_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{V_2 - V_1}{V_1 + V_2}\right)^2 = 1/9; \quad \frac{E_t}{E_i} = \frac{8}{9}$

(9) **(C).** $v = \frac{n}{2\ell}\sqrt{\frac{T}{m}} \Rightarrow n = 3$ initially.

Hence two more resonance will occur at 40cm. and 20cm. length of the horizontal part of wire. As acceleration of system is constant therefore ratio of time gap must be $1 : (\sqrt{2} - 1)$

(10) **(B).** Given :

$$y = 4\cos^2(t)\sin(1000t) = 2[1 + \cos 2t]\sin(1000t)$$
$$2\sin 1000t + 2\sin 1000t \cdot \cos 2t$$
$$= 2\sin 1000t + \sin 1002t + \sin 998t$$

Thus the periodic motion consists of three components.

(11) **(B).** As $y = A_b \sin(2\pi n_{av} t)$; where

$A_b = 2A\cos(2\pi n_A t)$, where $n_A = \frac{n_1 - n_2}{2}$

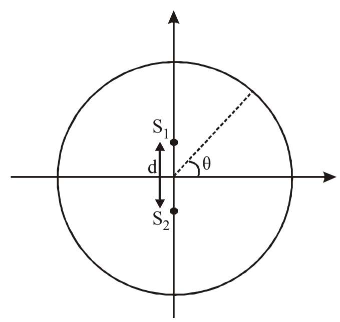
(12) **(A).** The wavelength of the sound wave in air is

$$\lambda = \frac{320}{16 \times 10^3} = 2 \times 10^{-2}\,\text{m}.$$

The positions of maxima on the circumference of the circular track will be given by

$$d\sin\theta = n\lambda$$

When d is the separation between the sources and θ is the angular position of n^{th} maximum as shown in the figure



$$2\sin\theta = n(2 \times 10^{-2}) \Rightarrow \sin\theta = \frac{n}{100}$$

Since sin θ lies between 0 and 1 there are 400 maxima on the entire circle.

These 400 maxima will be heard by the person in the time

$$t = \frac{400}{2} = 200\text{s}$$

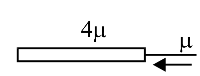
Speed of the train = 36 km/h = $36 \times \frac{5}{18} = 10\,\text{m/s}$

From the obtained values so far we get length of the track

$\ell = (10\,\text{m/s})(200\text{s}) = 2000\text{m}$

So radius of the track = $\frac{2000}{2\pi} = \frac{1000}{\pi}\,\text{m}$

(13) **(C).** $V_1 = \sqrt{\frac{T}{\mu}}$; $V_2 = \sqrt{\frac{T}{4\mu}}$



$V_2 < V_1 \Rightarrow$ 2nd is denser $\Rightarrow$ phase change of π wave reflected from denser medium

$$\Rightarrow A_r = \frac{V_2 - V_1}{V_2 + V_1} \times 6 = \frac{\frac{V_1}{2} - V_1}{\frac{V_2}{2} + V_1} \times 6 = -2\text{mm}$$

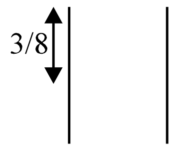
$\Rightarrow eq^n \Rightarrow -(2\text{mm})\sin(5t - 40x)$

(14) **(D).** $\frac{3}{8} = (2n+1)\frac{\lambda}{4}$;

$\lambda = \frac{340}{680} = \frac{1}{2}m \Rightarrow n = 1$

next overtone $\Rightarrow$ n = 2

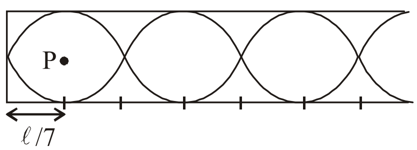
$x = \frac{\lambda}{2} = \frac{1}{4}m = 25$ cm



(15) **(A).** $\lambda_A = \left(10u - \frac{u}{2}\right)\frac{1}{f} = \frac{9.5u}{f}$

$\Rightarrow \lambda_B = \left(10u + \frac{u}{2}\right)\frac{1}{f} = \frac{10.5u}{f} \Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{19}{21}$

(16) **(A).** The figure shows variation of displacement of particles in a closed organ pipe for 3^{rd} overtone.



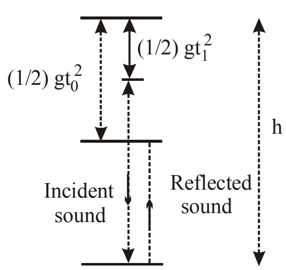
For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$

Hence the amplitude at P at a distance $\ell/7$ from closed end is 'a' because there is an antinode at that point.

(17) **(C).** $A_t = \frac{2\sqrt{\mu_\ell}}{\sqrt{\mu_\ell} + \sqrt{\mu_r}} A_i$

(18) **(B).** Let the sound observed by the parachutist at $t_0 = 12$s be produced at t_1s. Velocity of source at the instant of sound = gt_1 and velocity of observer at the instant of observing same sound = gt_0. Hence the relation between apparent frequency f ' and original frequency f will be

$f' = f\left(\frac{v + gt_0}{v - gt_1}\right)$.



Here f = 800 Hz, g = 10 m/s^2, v = 330 m/s, $t_0 = 12$s and f' = 800 + 700 = 1500 Hz

Putting these, we get, $t_1 = 9$s

Now the distance traveled by sound in $(t_0 - t_1)$ sec is

$v(t_0 - t_1) = \left(h - \frac{1}{2}gt_0^2\right) + \left(h - \frac{1}{2}gt_1^2\right)$

Putting the values, we get, h = 1057.5m.

EXERCISE-3

(1) **3.** $\lambda = \frac{v}{f} = \frac{330}{500} = 0.66m = \frac{4\ell}{2n-1} \Rightarrow n = 3$

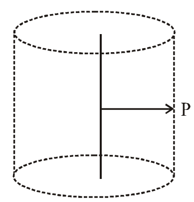
(2) **525.** $f = \frac{c + v_w}{c + v_w - v_s \sin 30^o}$

$f_0 = \frac{300 + 20}{330 + 20 - 10} = \frac{350}{340} \times 510 = 525$ Hz

(3) **12.** Imagine a cyclinder of radius 7m and length 10m. Intensity of sound at the surface of cylinder is same everywhere. Therefore, $I = \frac{P}{2\pi rL}$

(As sound is propagating radially out only, sound energy

$\therefore$ I = 50 W/m^2



Energy intercepted by the detector = I × A = 12 mW

(4) **2.** $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

If radius is doubled and length is doubled, mass per unit length will become four times.

Hence, $f' = \frac{1}{2 \times 2\ell}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$

(5) **2.** Velocity of sound is inversely proportional to the square root of density of the medium.

i.e. V.ρ = constant $\Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{2\rho}{\rho}} = \sqrt{2}$

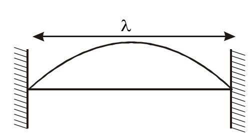
(6) **100.** $\lambda_{air} = \frac{V_{air}}{f} = \frac{330}{1000} = 0.33m$

$\lambda_{water} = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.25 \times 10^9}{1000}} = 1.5 \times 10^3 = 1500$

$\lambda_{water} = \frac{1500}{1000} = 1.5m$; $\lambda_{water} - \lambda_{air} = 1.5 - 0.33 = 1.17m$

(7) **9.** For fundamental frequency,

$\mu = \frac{3.2gm}{40cm} = \frac{3.2 \times 10^{-3}}{40 \times 10^{-2}} = \frac{3.2}{40} = \frac{32}{4000} kg/m$



$\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell$ .... (1) $f = \frac{v}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$$\Rightarrow \frac{1000}{64} = \frac{1}{2\times40\times10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\Rightarrow \left[\frac{1000}{64}\times2\times40\times10^{-2}\right]^2 \frac{32}{4000} = T$$

$$\frac{1000}{64}\times\frac{32}{4000} = T \Rightarrow T = \frac{10}{8}N$$

$$\text{Now, } Y = \frac{\frac{10/8}{10^{-6}}}{\frac{0.05\times10^{-2}}{40\times10^{-2}}} = 10^9 N/m^2$$

(8) **9.** λ_1 = wavelength of the incident sound

$$= \frac{10u-(u/2)}{f} = \frac{19u}{2f}$$

f_1 = frequency of the incident sound

$$= \frac{10u-u}{10u-(u/2)}f = \frac{18}{19}f = f_r$$

= frequency of the reflected sound

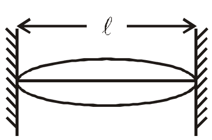
λ_r = wavelength of the reflected sound

$$= \frac{10u+u}{f_r} = \frac{11u}{18f}\times19 = \frac{11\times19}{18}\cdot\frac{u}{f}$$

$$\frac{\lambda_i}{\lambda_r} = \frac{19u}{2f}\times\frac{18f}{11\times19u} = \frac{9}{11}$$

EXERCISE-4

(1) **(B).**



Wire vibrate with minimum frequency or fundamental mode. $\lambda_{max} = 2\ell = 2\times40 = 80cm$

(2) **(B).** It is possible when wave reflected from rigid end. So phase difference found between incident and reflected wave is 180° and direction changed.

Incident wave equation $\Rightarrow y = a\sin(\omega t - kx)$

Reflected wave equation $\Rightarrow y = a\sin(\omega t + kx + 180^o)$

$y = -a\sin(\omega t + kx)$

(3) **(D).** $m = 9.8$ g/m $= 9\times10^{-3}$ kg/m

$T = 10$ kg-wt $= 10$ gN ; $L = 1m$

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2\times1}\sqrt{\frac{10g}{9\times10^{-3}}} = 50\text{ Hz}$$

(4) **(B).** $n = 256$Hz

Piano wire frequency may be 256 +5 or 256 –5

On increasing tension frequency decreases 2 beats per sec.

$T\uparrow \Rightarrow n\uparrow \Rightarrow$ If
- 256 +5 $\Rightarrow$ no. of beats $\uparrow$
- 256 –5 $\Rightarrow$ no. of beats $\downarrow$

So answer is 256 – 5.

(5) **(C).** $|f_1 - f_2| = 4$

Since mass of second tuning fork increases so f_2 decrease and beats increase so $f_1 > f_2$.

$\Rightarrow f_2 = f_1 - 4 = 196$

(6) **(D).** $f = \frac{v+v/5}{v}f = \frac{6f}{5}$; % increase in frequency = 20%

(7) **(D).** $f_{app} = \frac{f\,(300)}{300-v} \Rightarrow v = 15$ m/s

(8) **(C).** Ratio of vibrations frequency in stretched string

1 : 2 : 3 : 4 : ....

Even and odd both harmonics are present.

Given frequency ratio 315 : 420

3 : 4

315 Hz is 3rd harmonic so fundamental frequency is one third of 3rd harmonic

$$n_{fundamental} = \frac{315}{3} = 105\text{Hz}$$

(9) **(D).** $B_1 = 10\log\left(\frac{I}{I_0}\right)$, $B_2 = \log\left(\frac{I'}{I_0}\right)$

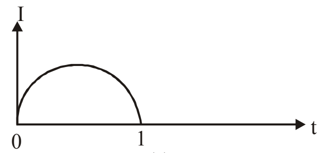
Given $B_2 - B_1 = 20$

$$20 = 10\log\left(\frac{I'}{I}\right) \;;\; I' = 100I$$

(10) **(D).** $\lambda = 0.08$ m, $T = 280$

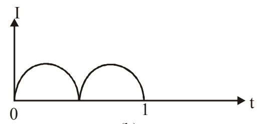
$$\alpha = K = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08}\times100 = 25\pi \;;\; \beta = \omega = \frac{2\pi}{T} = \pi$$

(11) (C). Variation of intensity due to superposition of waves having frequencies v –1 & v and v & v + 1 is as shown



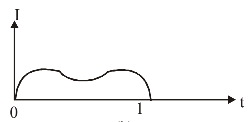
(a)

Variation of intensity due to superposition of waves having frequencies v – 1 and v + 1 is as shown



(b)

Resultant of (a) and (b) be as shown



(b)

So number of beats/s = 2

(12) (B). $\frac{94}{100}V = \frac{V - V_0}{V}V$; $0.94V = V - V_0$

$V_0 = 0.06V = 0.06 \times 330 = 19.8$ m/s

$V_0^2 = u^2 + 2as$; $(19.8)^2 = 0^2 + (2)(2)s$; $s = 98$m

(13) (D). $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} = 6.25$ N

(14) (B). $y(x,t) = e^{-[\sqrt{ax}+\sqrt{bt}]^2}$

It is transverse type $y(x,t) = e^{-(ax+bt)^2}$

Speed $v = \frac{\sqrt{b}}{\sqrt{a}}$

and wave is moving along x direction.

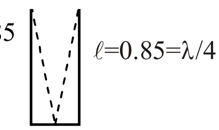
(15) (A). $f = v/2\ell$ Now, it will act like one end opened and other closed. So, $f_0 = \frac{v}{4\ell} = \frac{v}{4(\ell/2)} = \frac{v}{2\ell} = f$

(16) (B). $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{Ad}}$

Also, $Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{Y\Delta\ell}{\ell d}}$

$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}$ Hz ; $f \approx 178.2$ Hz

(17) (A). In fundamental mode, $\frac{\lambda}{4} = 0.85$



$\lambda = 4 \times 0.85$

$f = \frac{v}{\lambda} = \frac{340}{4 \times 0.85} = 100$ Hz

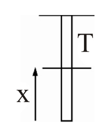
$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 hz, 700Hz, 900 Hz 1100 Hz below 1250 Hz.

(18) (A). $f_1 = f\left[\frac{v}{v - v_s}\right] = f\left[\frac{320}{320-20}\right] = f \times \frac{320}{300}$Hz

$f_2 = f\left[\frac{v}{v+v_s}\right] = f \times \frac{320}{340}$Hz

$100 \times \left(\frac{f_2}{f_1} - 1\right) = \left(\frac{f_2 - f_1}{f_1}\right) \times 100 = 100\left[\frac{300}{340} - 1\right] = 12\%$

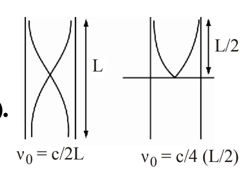
(19) (B). $T = \frac{Mgx}{L}$; $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{Mgx}{L}}{\frac{M}{L}}} = \sqrt{gx}$



$\frac{dx}{dt} = \sqrt{gx}$; $\int_0^L \frac{dx}{\sqrt{x}} = \sqrt{g}t$

$[2\sqrt{x}]_0^{20} = \sqrt{10}t$; $2\sqrt{20} = \sqrt{10}t$; $t = 2\sqrt{2}$s

(20) (C).



Fundamental frequency remains same.

(21) (B). This question involves the use of relativistic Doppler's effect. The usual non-relativistic Doppler formula will NOT applicable here as the velocity of observer is not small as compared to light.

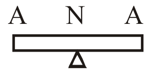
The relativistic Doppler's formula is

ν (observed) = ν (actual) $\sqrt{\frac{1+\beta}{1-\beta}}$, where $\beta = \frac{V}{C}$

V is relative velocity of observer w.r.t. the source and is taken to be positive if observer and source are moving towards each other.

So, here ν (observed)=(10 GHz) $\sqrt{\frac{1+1/2}{1-1/2}} = 17.3$ GHz.

(22) **(C).** $f = \frac{c}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}}$



$$= \frac{1}{2\times 0.6}\sqrt{\frac{9.27\times 10^{10}}{2.7\times 10^{3}}} = \frac{1}{1.2}\sqrt{\frac{9.27\times 10^{7}}{2.7}}$$

$= 4.88 \times 10^{3}$ Hz $\approx$ 5 kZ

(23) **(B).** $\frac{I_{max}}{I_{min}} = 16 \Rightarrow \frac{A_{max}}{A_{min}} = 4 \Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$

Using componendo & dividendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

(24) **(A).** $60 = \sqrt{\frac{Mg}{\mu}}$; $60.5 = \sqrt{\frac{M\,(g^2 + a^2)^{1/2}}{\mu}}$

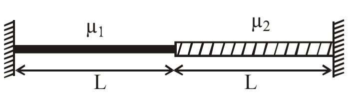
$$\Rightarrow \frac{60.5}{60} = \sqrt{\sqrt{\frac{g^2 + a^2}{g^2}}}$$

$$\left(1 + \frac{0.5}{60}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{60}$$

$$g^2 + a^2 = g^2 + g^2 \times \frac{2}{60}$$

$$a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47} \approx \frac{g}{5}$$

(25) **(D).**



Let mass per unit length of wires are μ_1 and μ_2 respectively.

Materials are same, so density ρ is same.

$$\therefore \quad \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu \text{ and } \mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension in both are same = T.

Let speed of wave in wires are V_1 and V_2

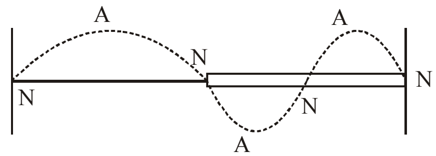
$$V_1 = \sqrt{\frac{T}{\mu}} = V \ ; \ V_2 = \sqrt{\frac{T}{4\mu}} = \frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_{01} = \frac{V_1}{2L} = \frac{V}{2L} \ \& \ f_{02} = \frac{V_2}{2L} = \frac{V}{4L}$$

Frequency at which both resonate is L.C.M of both frequencies i.e. V / 2L.

Hence no. of loops in wires are 1 and 2 respectively.



So, ratio of no. of antinodes is 1 : 2.

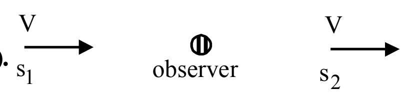
(26) **(A).** Given $\frac{a_1}{a_2} = \frac{1}{3}$; Ratio of intensities, $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{9}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{1+3}{1-3}\right)^2 = 4$$

(27) **(A).** $v = \sqrt{\frac{T}{\mu}}$; $T = \mu v^2$; $\frac{\mu v^2}{A} = Y\frac{\Delta\ell}{\ell}$; $\Delta\ell = \frac{\mu v^2 \ell}{AY}$

After substituting value of μ, v, ℓ, A and Y we get, $\Delta\ell = 0.03$ mm

(28) **(A).**



$$v_0\left(\frac{C}{C-V}\right) - v_0\left(\frac{C}{C+V}\right) = 2 \ ; \ V = \frac{1}{4}\text{ m/s}$$

(29) **106.00**

$$v_s = \sqrt{\frac{\gamma P}{\rho}} ; \frac{v_{gas}}{v_{air}} = \sqrt{\frac{\rho_{air}}{\rho_{gas}}} \Rightarrow \frac{v_{gas}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v_{gas} = \frac{300}{\sqrt{2}} \ \therefore v_{gas} = 150\sqrt{2}$$

$$n_2 - n_1 = \frac{v_{gas}}{2\ell} = \frac{150\sqrt{2}}{2\,(1)} = 75\sqrt{2} \Rightarrow \quad \Delta n = 106.06 \text{ Hz}$$

(30) **(A).** Let amplitude of each wave is A.

Resultant wave equation

$$= A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$= A\sin\omega t + \sqrt{2}\ A\sin\omega t \ = (\sqrt{2} + 1)\,A\sin\omega t$$

Resultant wave amplitude $= (\sqrt{2} + 1)\,A$

as $I \propto A^2$ so $\frac{I}{I_0} = (\sqrt{2}+1)^2$; $I = 5.8\, I_0$

$\frac{I_{max}}{I_{min}} = \frac{(A_1+A_2)^2}{(A_1-A_2)^2} = \frac{(5+4)^2}{(5-4)^2} = \frac{81}{1}$

EXERCISE-5

(1) **(C).** Maximum particle velocity, $(v_p)_{max} = A\omega$

$v = (v_p)_{max}$; $\frac{\omega}{k} = A\omega$

$\frac{1}{k} = A$ or $\frac{\lambda}{2\pi} = A$ $\left(\because k = \frac{2\pi}{\lambda}\right)$

$\lambda = 2\pi A$

(2) **(D).** Let the frequencies of tuning fork and piano string be ν_1 and ν_2 respectively.

$\therefore$ $\nu_2 = \nu_1 \pm 4 = 512 \pm 4 = 516$ Hz or 508Hz

Increase in the tension of a piano string increases its frequecy.

If ν_2 = 516 Hz, further increase in ν_2, resulted in an increase in the beat frequency. But this is not given in the question.

If ν_2 = 508 Hz, further increase in ν_2 resulted in decrease in the beat frequency. This is given in the question. When the beat frequency decreases to 2 beats per second.

Therefore, the frequency of the piano string before increasing the tension was 508 Hz.

(3) **(B).** $y_2 = a\sin(\omega t + kx + \pi/2)$

$y_1 = a\sin(\omega t + kx + 0.57)$

Phase difference $= \frac{\pi}{2} - 0.57 = 1$ radian.

(4) **(D).** $\therefore$ Frequency is same in both the medium

$\therefore$ $\lambda \propto$ speed

(5) **(A).** $\frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = f$ (for fundamental mode)

Taking ln on both side & differentiating

$\frac{dT}{2T} = \frac{df}{f} \Rightarrow \frac{dT}{T} = \frac{2\times df}{f} = 2\times\frac{6}{600} = 0.02$

(6) **(C).** Fundamental frequency is given by

$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow \nu \propto \frac{1}{\ell}, \ell = \ell_1 + \ell_2 + \ell_3$; $\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$

(7) **(D).** $2\pi f_1 = 600\pi$; $f_1 = 300$ ....(1)

$2\pi f_2 = 608\pi$; $f_2 = 304$ ......(2)

$|f_1 - f_2| = 4$ beats

(8) **(C).** Fequency of the echo detected by the driver of the train is

$f' = \left(\frac{v+u}{v-u}\right) f = \left(\frac{330+220}{330-220}\right) \times 1000 = 5000$ Hz

(9) **(B).** $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi} = 1$ and $\omega = 2\pi f = (2\pi)(1/\pi) = 2$

So, equation of wave

$y = \sin(kx - \omega t) = \sin(x - 2t)$

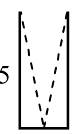
(10) **(B).** Frequency of unknown source = 246 Hz or 254Hz. Second harmonic of this source = 492Hz or 508 Hz, which gives 5 beats per second, when sounded with a source of frequency 513 Hz.

Therefore unknown frequency = 254 Hz

(11) **(A).** Pressure change will be minimum at both open ends.

(12) **(D).** In fundamental mode,

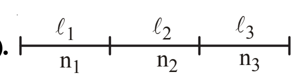
$\frac{\lambda}{4} = 0.85$; $\lambda = 4 \times 0.85$


ℓ=0.85=λ/4

$f = \frac{v}{\lambda} = \frac{340}{4\times 0.85} = 100$ Hz

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700Hz, 900 Hz & 1100 Hz below 1250Hz.

(13) **(A).**



$n \propto \frac{1}{\ell}$; $\ell = \ell_1 + \ell_2 + \ell_3 \Rightarrow \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

(14) **(C).** $v_0 = 36$ km/h = 10 m/s

$v_s = 18$ km/h = 5 m/s

$f' = f\left[\frac{v+v_0}{v+v_s}\right] = 1392 \times \left(\frac{343+10}{343+5}\right)$ Hz

$= 1392 \times \frac{353}{348}$ Hz = 1412 Hz

(15) **(B).** $\frac{V}{4\,(20cm)} = \frac{3V}{2\ell_{open}} \Rightarrow \ell_{open} = 120$cm.

(16) **(C).** $f_0 = f_s\left(\frac{v}{v - v_s\cos 60°}\right) = 100\left(\frac{330}{330 - \frac{19.4}{2}}\right) \approx 103$ Hz

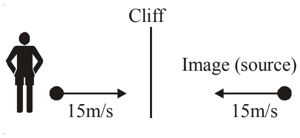
(17) **(A).** The two consecutive resonant frequencies for a string

$$\frac{nv}{2\ell} \text{ and } \frac{(n+1)\,v}{2\ell}$$

$$\frac{(n+1)\,v}{2\ell} - \frac{nv}{2\ell} = 420 - 315 \;;\; \frac{v}{2\ell} = 105 \text{ Hz}$$

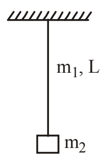
Which is the minimum resonant frequency.

(18) **(C).**



$$f' = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{330 - 15}\right) \times 800 = 838 \text{ Hz}$$

(19) **(B).** $\lambda = \frac{v}{f}$ $\left(v = \sqrt{\frac{T}{\mu}}\right)$



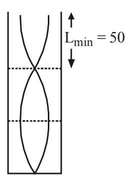
$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{m_1 + m_2}{m_2}}$$

(20) **(C).** $L_{min} = 50$ cm.

So other lengths for resonance are

$3L_{min}, 5L_{min}, 7L_{min}$, etc.

$\Rightarrow$ 150 cm, 250 cm, 350 cm, etc.



(21) **(B).** $\frac{3V}{2L_1} = \frac{3V}{4L} \Rightarrow L_1 = 2L$

(22) **(D).** Net beat frequency

$= $ LCM of individual beat frequencies

$= $ LCM of $[(n, n-1), (n, n+1), (n-1, n+1)]$

$= $ LCM of $(1, 1, 2) = 2$ Hz

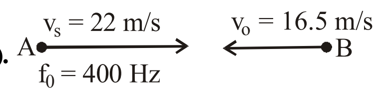
So, number of beats per second = 2

(23) **(A).** Difference between any two consecutive frequencies

of COP $= \frac{2v}{4\ell} = 260 - 220 = 40$ Hz $\Rightarrow \frac{v}{4\ell} = 20$ Hz .

So fundamental frequency = 20 Hz.

(24) **(C).**



As we know for given condition

$$f_{app} = f_0\left(\frac{v + v_{observer}}{v - v_{source}}\right) = 400\left(\frac{340 + 16.5}{340 - 22}\right)$$

$f_{app} = 448$ Hz

(25) **(C).** For closed organ pipe, third harmonic $= 3v/4\ell$

For open organ pipe, fundamental frequency $= v/2\ell'$

$$\frac{3v}{4\ell} = \frac{v}{2\ell'} \Rightarrow \ell' = \frac{4\ell}{3 \times 2} = \frac{2\ell}{3} = \frac{2 \times 20}{3} = 13.33 \text{ cm.}$$

(26) **(B).** $v = 2\,(\nu)\,[L_2 - L_1] = 2 \times 320\,[73 - 20] \times 10^{-2}$

$= 339.2 \text{ ms}^{-1} = 339$ m/s