1

MATHEMATICS IN PHYSICS

SECTION - A (VECTOR)

SCALAR AND VECTOR QUANTITIES

The physical quantities are of two types : scalar and vector.

Scalar Quantities : The quantities which require only magnitude to express them, are called 'scalar quantities', e.g. mass, distance, time, speed, volume, density, pressure, work, energy, power, charge, electric current, temperature, potential, specific heat, frequency, etc.

Vector Quantities : Certain quantities have both magnitude and direction, e.g. position, displacement, velocity, acceleration, force, weight, momentum, impulse, electric field, magnetic field, current density, etc. Such quantities are called vector quantities.

Note :

If a physical quantity in addition to magnitude :

(a) Has a specified direction.

(b) Obeys the law of parallelogram of addition, i.e., $R = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$

(c) And its addition is commutative, i.e., $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

then and then only it is said to be a vector. If any of the above conditions is not satisfied the physical quantity cannot be a vector.

Vector $\Rightarrow$ has a definite direction

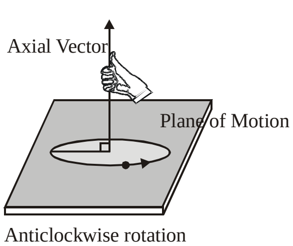
Converse may or may not be true, i.e., if a physical quantity has a direction, it may or may not be a vector, e.g., time, pressure, surface tension or current, etc., have direction but are not vectors.

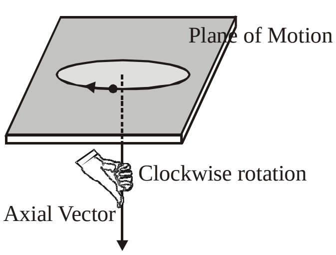
Vectors are divided into two types :

(a) Polar Vectors : It have starting point (like displacement) or a point of application (like force).

(b) Axial Vectors : Rotational effects are represented by axial vectors. They are along axis of rotation, direction denoted by right hand thumb rule or right hand screw rule.

Ex. Angular displacement, Angular velocity, torque, angular momentum.



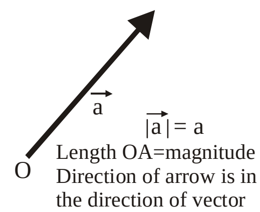


REPRESENTATION OF A VECTOR

(i) A vector is represented by a line with an arrow head.

(ii) The point O from where the arrow starts is called the tail or initial point or origin of the vector.

The point A where the arrow ends is called the tip or head or terminal point or terminus of the vector.



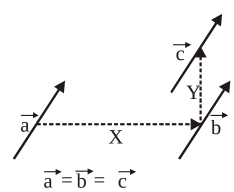
Vector have their own algebra which is different from ordinary algebra.

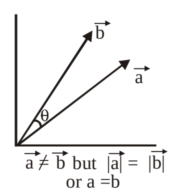
In ordinary algebra 5 + 9 = 14

and $A \times B = B \times A$

But in vector algebra $\vec{5}+\vec{9}$ = can have value in between $\vec{4}$ to $\vec{14}$ or $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

* A vector displaced parallel to itself remains unchanged (fig.)





* If a vector is rotated through an angle other then 360° it changes.

* A vector can be replaced by other when its direction & magnitude is same.

MULTIPLICATION OF A VECTOR WITH SCALAR

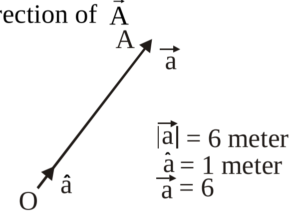
If a vector multiplied by a scalar quantity then we get a new vector in the same direction but having new magnitude =scalar quantity × magnitude of the vector quantity.

UNIT VECTOR

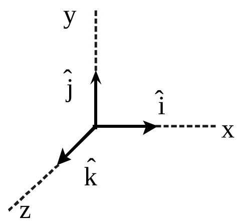
A vector having unit magnitude. It is used to denote the direction of a given vector. $\vec{A} = \hat{a}\cdot A$

$\hat{a}$ is unit vector along the direction of $\vec{A}$

Ex. As shown in fig. if OA is displacement of vector in any arbitrary direction then $\hat{a}$ is unit vector in the same direction.



Orthogonal Unit vectors : The unit vectors along the X-axis, Y-axis and Z-axis of the right-handed Cartesian coordinate system are written as $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.



These are called orthogonal unit vectors.

TYPES OF VECTORS

(a) **Negative of a vector :** It has direction just opposite to given vector and have same magnitude.

(b) **Zero vector or null vector :** A vector with zero magnitude having no specific direction is called zero vector .

(i) Multiplying a vector by zero. i.e. $0\,(\vec{A}) = \vec{0}$

(ii) By adding a negative vector to the given vector.

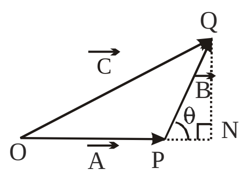
(c) **Equal vector :** Two vectors are called equal (or equivalent) vectors if they have equal magnitude, and same direction.

(d) **Collinear vectors :** Two vectors acting along same straight line or along parallel straight lines in same direction or in opposite direction.

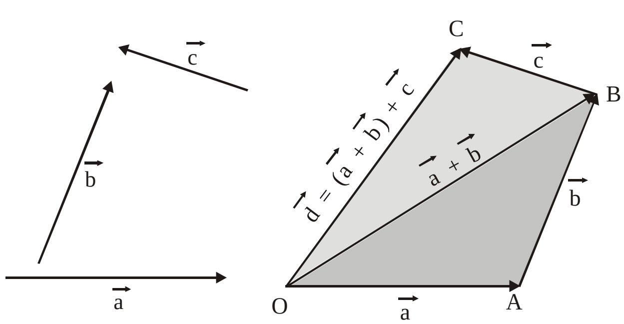
(e) **Coplanar vectors :** Three (or more) vectors are called coplanar vectors if they lie in the same plane. Two (free) vectors are always coplanar.

VECTOR ADDITION

(i) **Law of Triangle :** If two sides of triangle are shown by two continuous vectors $(\vec{A}$ and $\vec{B})$ then third side of triangle in opposite direction shown resultant of two vectors $(\vec{C})$. $\vec{C} = \vec{A} + \vec{B}$



(ii) **Polygon Method :** We use this method for more than two vector. Suppose $\vec{a}, \vec{b}, \vec{c}$ are three vectors to be added, a diagram is drawn in which the tail of $\vec{b}$ coincides with the head of $\vec{a}$. And tail of $\vec{c}$ coincides with head of $\vec{b}$. The vector joining the tail of $\vec{a}$ and head of $\vec{c}$ is called the resultant vector and this is the vector sum of three given vectors $\vec{d} = (\vec{a} + \vec{b} + \vec{c})$.



From Δ OAB, $\vec{OB} = \vec{OA} + \vec{AB}$

$\Rightarrow \vec{OB} = \vec{a} + \vec{b}$ (resultant of $\vec{a}$ and $\vec{b}$)

From Δ OBC

$$\vec{OC} = \vec{OB} + \vec{BC} \Rightarrow \vec{OC} = (\vec{a} + \vec{b}) + \vec{c}$$

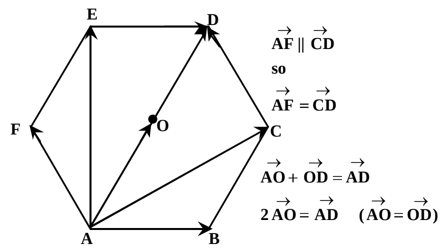
$$\text{or} \quad \vec{d} = \vec{a} + \vec{b} + \vec{c}$$

(resultant of $\vec{a}$, $\vec{b}$ and $\vec{c}$)

Example 1 :

ABCDEF is A regular hexagon. O is its centre then prove that $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 6\vec{AO}$.

Sol.



$$\vec{AC} + \vec{CD} = \vec{AD} \qquad (\vec{CD} = \vec{AF})$$

$$\vec{AE} + \vec{ED} = \vec{AD} \qquad (\vec{ED} = \vec{AB})$$

$$\vec{AD} = 2\vec{AO}$$

$$\text{LHS} = (\vec{AE} + \vec{AB}) + \vec{AD} + (\vec{AC} + \vec{AF})$$

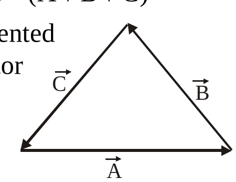
$$= \vec{AD} + \vec{AD} + \vec{AD} = 3\vec{AD} = 6\vec{AO}$$

NOTE

* Vectors of the same nature alone can be added .
Ex. A force vector cannot be added to velocity vector but can be added to force vector only.

* Vectors addition is commutative.
Sum of the vectors remains the same in whatever order they may be added. $(\vec{A} + \vec{B}) + \vec{C} = (\vec{A} + \vec{B} + \vec{C})$

* If all sides of polygon are represented by continuous vectors then vector sum of all sides is zero.



$\vec{A} + \vec{B} + \vec{C} = 0$

* If a vector $\vec{A}$ is multiplied by zero, we get a vector whose magnitude is zero called null vector or zero vector.

* The unit of n $\vec{A}$ is same as that of $\vec{A}$, if n is a pure real number .

* The unit of vector does not change on being multiplied by a dimensionless scalar

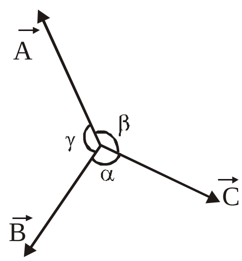
* The unit of n$\vec{A}$ is different from that of $\vec{A}$, if n is a dimensional scalar.

The multiplication of velocity vector by time gives us displacement.

* Sum of three non-coplanar forces can never be zero.
* Minimum number of equal forces required for a zero resultant is two.
* Minimum number of unequal forces required for a zero resultant is three.

LAMI'S THEOREM

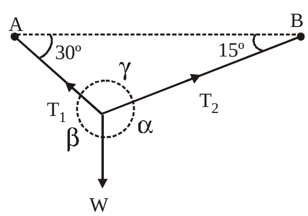
If three forces acting at a point are in equilibrium then each force is proportional to sine of the angle between the other two.



or $\dfrac{A}{\sin\alpha} = \dfrac{B}{\sin\beta} = \dfrac{C}{\sin\gamma}$

Example 2 :

A rope is stretched between two poles. A 50 N boy hangs from it, as shown in Fig. Find the tensions in the two parts of the rope.



Sol. In Fig. $\alpha = 90^o + 15^o = 105^o$, $\beta = 90^o + 30^o = 120^o$
and $\gamma = 180^o - (30^o + 15^o) = 135^o$

Using Lami's Theorem, we have, $\dfrac{T_1}{\sin\alpha} = \dfrac{T_2}{\sin\beta} = \dfrac{W}{\sin\gamma}$

$$\therefore\ T_1 = W \times \frac{\sin\alpha}{\sin\gamma} = 50 \times \frac{\sin 105^o}{\sin 135^o}$$

$$= 50 \times \frac{\sin 75^o}{\sin 45^o} = \frac{50 \times 0.9659}{0.7071} = 68.3\ \text{N}$$

$$T_2 = \frac{W\sin\beta}{\sin\gamma} = \frac{50\sin 120^o}{\sin 135^o} = \frac{50 \times \sin 60^o}{\sin 45^o}$$

$$= \frac{50 \times 0.8660}{0.7071} = 61.24\ \text{N}$$

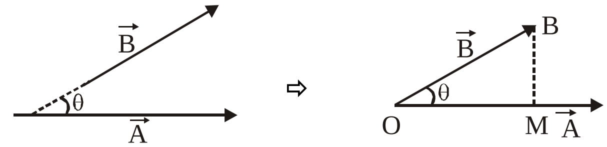
COMPONENT OF A VECTOR ALONG ANOTHER VECTOR

Two vector $\vec{A}$ and $\vec{B}$ there as shown below to get component of $\vec{B}$ along $\vec{A}$

$\Rightarrow$ shift $\vec{B}$ || to it self and made it co-initial with $\vec{A}$ draw a perpendicular from B to OA let it meets $\vec{A}$ at M

θ is angle in between $\vec{A}$ and $\vec{B}$.

Component of $\vec{B}$, along $\vec{A}$ = OM = OB cos θ



Note : (i) θ = 0, OM maximum (ii) θ = 90°, OM = zero

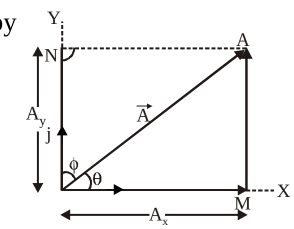
RESOLUTION OF A VECTOR

It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is splitted are called component vectors. Infact, the resolution of a vector is just opposite to composition of vectors.

(i) Rectangular Components of a Vector in a Plane

A given vector represented by $\vec{A}$, makes $\angle\theta$ with x-axis and $\angle\phi$ with y-axis.

$\vec{ON} = ON \cdot \hat{j} = A_y\hat{j}$



$\vec{OM} = OM \cdot \hat{i} = A_x\hat{i}$

$OM = A_x$, $ON = A_y$

$A_x = A\cos\theta$, $A_y = A\sin\theta = A\cos\phi$

$\vec{OA} = \vec{OM} + \vec{MA}$; $\vec{A} = A_x\hat{i} + A_y\hat{j}$

We can write

$\vec{A} = (A\cos\theta)\hat{i} + (A\sin\theta)\hat{j}$ or $\vec{A} = (A\cos\theta)\hat{i} + (A\cos\phi)\hat{j}$

From Δ OMA

(i) $OA^2 = OM^2 + AM^2 = (A\cos\theta)^2 + (A\sin\theta)^2$

or $A^2 = A_x^2 + A_y^2$ or $A = \sqrt{A_x^2 + A_y^2}$; $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

(ii) $\tan\theta = \dfrac{AM}{OM}$; $\tan\theta = \dfrac{A_y}{A_x}$; $\theta = \tan^{-1}\dfrac{A_y}{A_x}$

Example 3 :

$\vec{A} = 5\hat{i} + 10\hat{j}$ find out magnitude and direction of $\vec{A}$.

Sol. $A_x = 5$, $A_y = 10$

$|\vec{A}| = \sqrt{(5)^2 + (10)^2} = \sqrt{125} = 5\sqrt{5}$.

If $\vec{A}$ makes angle θ with x-axis, then

$$\tan\theta = \frac{10}{5} = 2\ ;\ \ \theta = \tan^{-1}(2)$$

Example 4 :

Given $\vec{B} = 2\hat{i} + 2\hat{j}$ find out magnitude and direction of $\vec{B}$.

Sol. $B_x = 2$, $B_y = 2$; $B = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}$

If $\vec{B}$ makes $\angle\phi$ with x-axis, then

$$\tan\phi = \frac{B_y}{B_x} = \frac{2}{2} = 1 \quad \therefore \angle\phi = 45°$$

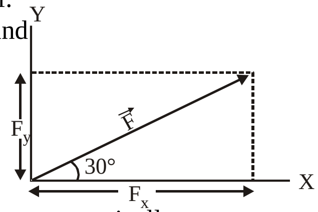
Example 5 :

A man pulls a rope attached to a box with a force of 80 N. The rope makes an angle of 30° to the ground.
(a) Calculate the effective value of the pull tending to move the stone along the ground.
(b) Calculate the force tending to lift the stone vertically.
(c) Write down force vector.

Sol. (a) The pull along the ground

$= F_x = F\cos 30°$

$= 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3}$ N



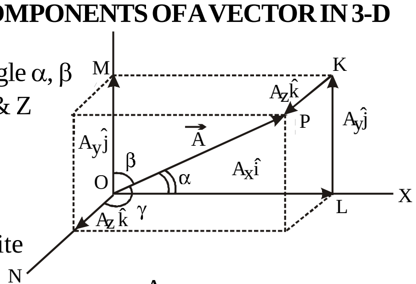
(b) Force tending to lift the stone vertically

$= F_y = F\sin 30° = 80 \times \frac{1}{2} = 40$ N

(c) $\vec{F} = F_x\hat{i} + F_y\hat{j} \Rightarrow \vec{F} = (40\sqrt{3})\,\hat{i} + (40)\,\hat{j}$

RECTANGULAR COMPONENTS OF A VECTOR IN 3-D

Let $\vec{A}$ makes angle α, β and γ with X, Y & Z axis respectively.



$\vec{A} = \vec{OP}$

Then we can write

$A_x = A\cos\alpha \Rightarrow \cos\alpha = \frac{A_x}{A} = \ell$

$A_y = A\cos\beta \Rightarrow \cos\beta = \frac{A_y}{A} = m$

$A_z = A\cos\gamma \Rightarrow \cos\gamma = \frac{A_z}{A} = n$

$\vec{OP} = \vec{OL} + \vec{LK} + \vec{KP}$ or $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

With the help of plane geometry we can prove

$A^2 = A_x^{\,2} + A_y^{\,2} + A_z^{\,2}$

or $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ (Magnitude of $\vec{A}$)

$\cos\alpha$, $\cos\beta$, $\cos\gamma$, gives direction of $\vec{A}$ in space, so these are known as Direction Cosines (D.C.) of $\vec{A}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{A_x}{A}\right)^2 + \left(\frac{A_y}{A}\right)^2 + \left(\frac{A_z}{A}\right)^2$$

$$= \frac{A_x^2 + A_y^2 + A_z^2}{A^2} = \frac{A^2}{A^2} = 1$$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$; $\ell^2 + m^2 + n^2 = 1$
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Example 6 :

Write D.C. of $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$.

Sol. Let $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

so, $a_x = 2$, $a_y = -3$, $a_z = -1$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(2)^2 + (-3)^2 + (-1)^2} = \sqrt{14}$$

$$\therefore \quad \cos\alpha = \frac{a_x}{a} = \frac{2}{\sqrt{14}}, \quad \cos\beta = \frac{a_y}{a} = \frac{-3}{\sqrt{14}},$$

$$\cos\gamma = \frac{a_z}{a} = \frac{-1}{\sqrt{14}}$$

Example 7 :

Find out unit vector in the direction of $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$.

Sol. Unit vector along $\vec{A}$ is $= \hat{a}$ Given by $\hat{a} = \frac{\vec{A}}{A}$

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A_x = 3,\ A_y = -2,\ A_z = 4$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(3)^2 + (-2)^2 + (4)^2} = \sqrt{29}$$

$$\therefore \quad \hat{a} = \frac{\vec{A}}{A} = \frac{3\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{29}}$$

SUBTRACTION OF VECTORS

Vector which is want to subtracted just change direction of that vector and then add. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Example 8 :

Given $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$.

Determine (i) $\vec{a} + \vec{b}$ (ii) $\vec{a} - \vec{b}$

Sol. $\vec{a} + \vec{b} = (3\hat{i} + 2\hat{j} - \hat{k}) + (\hat{i} + \hat{j} + 3\hat{k})$

$\vec{a} - \vec{b} = (3\hat{i} + 2\hat{j} - \hat{k}) - (\hat{i} + \hat{j} + 3\hat{k})$

$= 3\hat{i} + 2\hat{j} - \hat{k} + \hat{i} + \hat{j} + 3\hat{k}$

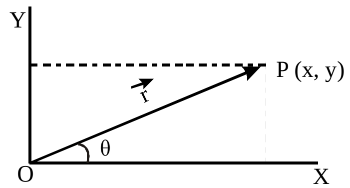
$= 3\hat{i} + 2\hat{j} - \hat{k} - \hat{i} - \hat{j} - 3\hat{k}$

$= 4\hat{i} + 3\hat{j} + 2\hat{k} = 2\hat{i} + \hat{j} - 4\hat{k}$

POSITION VECTOR

A vector which gives the position of a point with reference to the origin of the co-ordinate system is called position vector. $\vec{OP}$ is the position vector $\Rightarrow$ gives the position of the particle with reference to origin O.
Magnitude of position vector
$\Rightarrow$ distance of the point P from origin O

Position vector of point P in two dimension.



Let co-ordinates of point P are (x, y)

$$\vec{OP} = \vec{r} \; ; \; \vec{r} = x\hat{i} + y\hat{j}$$

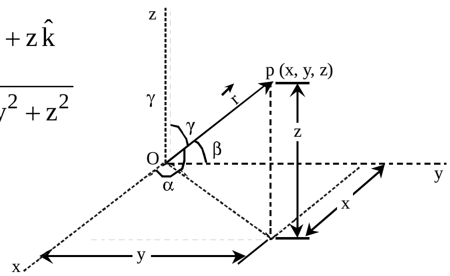
$x = r\cos\theta, \;\; y = r\sin\theta \;;\;\; |\vec{r}| = \sqrt{x^2 + y^2}$

Position vector of point P in three dimension.
Co-ordinates of point P are (x, y, z)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

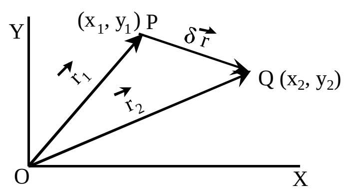
$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$x = r\cos\alpha$
$y = r\cos\beta$
$z = r\cos\gamma$



DISPLACEMENT VECTOR

Suppose at time t a particle is at P,



Position vector of point P, $\vec{OP} = \vec{r_1} \;;\;\; \vec{r_1} = x_1\hat{i} + y_1\hat{j}$

Point P move to Q

position vectors of Q, $\vec{OQ} = \vec{r_2} \;;\;\; \vec{r_2} = x_2\hat{i} + y_2\hat{j}$

displacement vector = difference of two positions

$$\vec{\delta r} = \vec{r_2} - \vec{r_1} \;;\; |\vec{\delta r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Generalising this result for three dimensions,

$$\vec{\delta r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\vec{\delta r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Let P moves to Q in time Δt. The vector $\vec{PQ} = (\Delta\vec{r})$ with tail P and tip Q is the corresponding to the motion from t to t´. Then velocity of particle $\vec{v} = \lim_{\delta t \to 0} \dfrac{\vec{\delta r}}{\delta t}$.

Example 9 :

Find the unit vector of $4\hat{i} - 3\hat{j} + 5\hat{k}$.

Sol. Let $\vec{A} = 4\hat{i} - 3\hat{j} + 5\hat{k}$

Comparing with $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A_x = 4, \; A_y = 3, \; A_z = 5$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(4)^2 + (-3)^2 + (5)^2}$$

or $\quad A = \sqrt{16 + 9 + 25} = 5\sqrt{2}$

$$\hat{A} = \frac{\vec{A}}{A} = \frac{4\hat{i} - 3\hat{j} + 5\hat{k}}{5\sqrt{2}} = \frac{4}{5\sqrt{2}}\hat{i} - \frac{3}{5\sqrt{2}}\hat{j} + \frac{5}{5\sqrt{2}}\hat{k}$$

Example 10 :

If $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$ obtain the scalar magnitude and directions from x axis of $\vec{A}, \vec{B}, \; \vec{A} + \vec{B}$; $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$.

Sol. Magnitude of $\vec{A} = \sqrt{(4)^2 + (-3)^2} = 5$

$$\tan\theta = -\frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(-\frac{3}{4}\right)$$

Magnitude of $\vec{B} = \sqrt{6^2 + 8^2} = 10$

$$\tan\theta = \frac{8}{6} = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$$

$$\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$$

$$\tan\theta = \frac{5}{10} = \frac{1}{2} = 0.5 \;;\;\; |\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 11.2$$

$$\theta = \tan^{-1}(0.5) = 26.5° \text{ approx.}$$

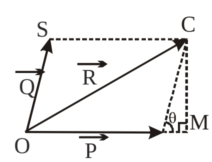
$$\vec{A} - \vec{B} = 4\hat{i} - 3\hat{j} - (6\hat{i} + 8\hat{j}) = 4\hat{i} - 6\hat{j} - 3\hat{j} - 8\hat{j}$$

$$|\vec{A} - \vec{B}| = \sqrt{(-2)^2 + (-11)^2} = -2\hat{i} - 11\hat{j} = \sqrt{4 + 121} = 5\sqrt{5}$$

$$\tan\theta = \frac{(-11)}{(-2)} = \frac{11}{2} \;;\;\; \theta = \tan^{-1}\left(\frac{11}{2}\right)$$

PARALLELOGRAM LAW OF VECTOR

To determine magnitude & direction of resultant vector, when two vectors act at an angle θ.



According to this law if two vectors $\vec{P}$ and $\vec{Q}$ are represented by two adjacent sides of a parallelogram both pointing outwards. The diagonal drawn through the intersection of the two vectors represents the resultant $\vec{R}$. $\vec{R} = \vec{P} + \vec{Q}$

From triangle OCM

$$OC^2 = OM^2 + CM^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

(Magnitude of resultant vector)

$$\tan\phi = \frac{CM}{OM} = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{or} \quad \phi = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

(Angle of resultant vector with P)

IMPORTANT RESULTS

[a] $\theta = 0^\circ \Rightarrow \vec{P} \parallel \vec{Q}$ then $R_{max} = P + Q$

[b] $\theta = 180^\circ \Rightarrow$ anti $\parallel$ then $R_{min} = P - Q$

[c] $\theta = 90^\circ \Rightarrow \vec{P} \perp \vec{Q}$

$$R = \sqrt{a^2 + b^2} \quad \text{here } \tan\phi = \left(\frac{Q}{P}\right)$$

[d] $|\vec{P}| = |\vec{Q}| = a$

$$|\vec{R}| = 2P\cos\frac{\theta}{2} \text{ and } \tan\phi = \tan\frac{\theta}{2} \qquad \therefore \phi = \frac{\theta}{2}$$

[e] $|\vec{P}| = |\vec{Q}| = a$ and

(i) $\theta = 60^\circ$; $R = \sqrt{3}P$

(ii) $\theta = 90^\circ$; $R = \sqrt{2}P$

(iii) $\theta = 120^\circ$; $R = P$

[f] If three vectors of equal magnitudes makes an angle of 120° with each other then the resultant vector will be zero. $R = 0$

[g] If n vectors of equal magnitude makes the angle of equal measure with each other then the resultant vector will be zero.

SCALAR PRODUCT OR DOT PRODUCT OF TWO VECTORS

If θ is the angle between $\vec{A}$ and $\vec{B}$.

Then $A(B\cos\theta) = \vec{A} \cdot \vec{B}$,   A and B are the magnitude of $\vec{A}$ and $\vec{B}$.

The quantity $AB\cos\theta$ is a scalar quantity.

$B\cos\theta$ is the component of vector $\vec{B}$ in the direction of $\vec{A}$. Hence, the scalar product of two vectors is equal to the product of the magnitude of one vector and the component of the second vector in the direction of the first vector.

Ex. : [1] $W = \vec{F} \cdot \vec{S}$ [2] $P = \vec{F} \cdot \vec{V}$ [3] $\phi = \vec{B} \cdot \vec{A}$

PROPERTIES OF SCALAR PRODUCT

(a) The scalar product is commutative.

i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

$$\vec{A} \cdot \vec{B} = A(B\cos\theta) = (A\cos\theta)B = \vec{B} \cdot \vec{A}$$

(b) The scalar product is distributive.

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

(c) The scalar product of two mutually perpendicular vectors is zero.

$$\vec{A} \cdot \vec{B} = AB\cos 90^\circ = 0$$

$$\vec{A} \cdot \vec{B} = 0 \text{ for } \vec{A} \perp \vec{B}$$

$$\hat{i} \cdot \hat{j} = 0,\ \hat{j} \cdot \hat{k} = 0,\ \hat{k} \cdot \hat{i} = 0$$

(d) The scalar product of two parallel vectors is equal to the product of their magnitudes.

$$\vec{A} \cdot \vec{B} = AB\cos 0^\circ = AB \qquad ; \cos 0^\circ = 1$$

(e) The scalar product of a vector with itself is equal to the square of the magnitude of the vector.

$$\vec{A} \cdot \vec{A} = AA\cos 0^\circ = A^2$$

$$\hat{i} \cdot \hat{i} = 1,\ \hat{j} \cdot \hat{j} = 1,\ \hat{k} \cdot \hat{k} = 1$$

(f) The scalar product of two vectors is equal to the sum of the products of their corresponding x, y, z components.

Let $\vec{A}$ and $\vec{B}$ be two vectors.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\ ;\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Their scalar product is given by

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z.$$

Now, the scalar magnitudes of $\vec{A}$ and $\vec{B}$ are

$$\sqrt{A_x^2 + A_y^2 + A_z^2} \text{ and } \sqrt{B_x^2 + B_y^2 + B_z^2}.$$

Therefore, the angle between $\vec{A}$ and $\vec{B}$ is given by

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}\ ;\ \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

Example 11 :

Two vectors $\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} + 6\hat{k}$.

Find (i) Dot product (ii) Angle between them,

Sol. (i) Dot product

$$A = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$B = \sqrt{(1)^2 + (3)^2 + (6)^2} = \sqrt{46}$$

$$\vec{A} \bullet \vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k}) \bullet (\hat{i} + 3\hat{j} + 6\hat{k})$$

$$= 1 \times 1 + 2 \times 3 + 2 \times 6 = 19$$

(ii) Angle between them

$$\vec{A} \bullet \vec{B} = AB\cos\theta \Rightarrow \cos\theta = \frac{\vec{A} \bullet \vec{B}}{AB}$$

$$\cos\theta = \frac{19}{3\sqrt{46}} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{19}{3\sqrt{46}}\right)$$

Example 12 :

Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k},\ \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$. Evaluate

(i) $|\vec{a}|, |\vec{b}|$ (ii) $\vec{a} \cdot \vec{b}$

(iii) the angle between the vectors $\vec{a}$ and $\vec{b}$

(iv) the component of $\vec{a}$ along $\vec{b}$

(v) the component of $\vec{b}$ along $\vec{a}$

Sol. Given $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k},\ \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

(i) $|a| = \sqrt{2^2 + 3^3 + (-1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$$|b| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26}$$

(ii) $\vec{a} \cdot \vec{b} = 2(-1) + 3 \text{ x } 3 + (-1)(4) = 3$

(iii) The angle θ between the $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|} = \frac{3}{\sqrt{14}\sqrt{26}} = \frac{3}{2\sqrt{91}}$$

or $\theta = \cos^{-1}\left(\frac{3}{2\sqrt{91}}\right)$

(iv) The component of $\vec{a}$ on $\vec{b}$ = a $\cos\theta$

$$= \sqrt{14} \times \frac{3}{\sqrt{14}\sqrt{26}} = \frac{3}{\sqrt{26}}$$

(v) The component of $\vec{b}$ on $\vec{a}$ = b $\cos\theta$

$$= \sqrt{26} \times \frac{3}{\sqrt{14}\sqrt{26}} = \frac{3}{\sqrt{14}}$$

Example 13 :

Find the value of λ so that the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ & $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are (i) parallel (ii) perpendicular to each other

Sol. Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

(i) $\vec{a}$ and $\vec{b}$ are parallel to each other

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \quad \text{i.e. if} \quad \frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \text{ or } \lambda = 2$$

(ii) $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} \cdot \vec{b} = 0$

i.e. if $2(-4) + 3(-6) + (-1)(\lambda) = 0$

or $\lambda = -8 - 18$ or $\lambda = -26$

Example 14 :

Under what condition the sum and difference of two vectors will be equal in magnitude.

Sol. $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

Squaring both the side

$$A^2 + B^2 + 2\vec{A} \cdot \vec{B} = A^2 - 2\vec{A} \cdot \vec{B} + B^2$$

$$4\vec{A} \cdot \vec{B} = 0 \qquad \text{or} \quad \vec{A} \cdot \vec{B} = 0 \qquad \text{i.e.} \quad \vec{A} \perp \vec{B}$$

When the two vectors are equal in magnitude and perpendicular to each other

Example 15 :

There are two displacement vectors, one of magnitude 3m and other of 4m. How should the two vectors be added so that the resultant vector be : (a) 7m (b) 1m (c) 5m

Sol. (a) For 7m both the vector should be parallel i.e. angle between them should be zero.

(b) For 1m both the vectors should be antiparallel i.e. angle between them should be 180°.

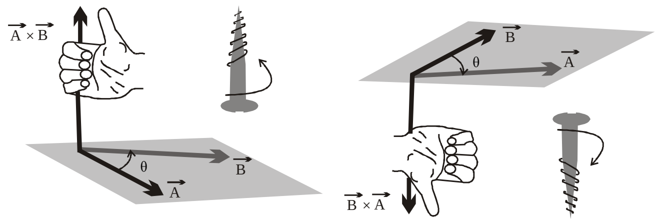
(c) For 5m both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

CROSS PRODUCT OF TWO VECTORS

Cross product of $\vec{A}$ and $\vec{B}$ inclined to each other at an angle θ is defined as : $A\ B\sin\theta\ \hat{n} = \vec{A} \times \vec{B}$

$\hat{n} \perp$ to plane of $\vec{A}$ and $\vec{B}$.

Direction of $\hat{n}$ is given by right hand thumb rule. Curl the fingers of your right hand from $\vec{A}$ to $\vec{B}$. Then the direction of the erect thumb will point in the direction $\vec{A} \times \vec{B}$.



The vector product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between them, and having the direction perpendicular to the plane containing the two vectors. Thus, If $\vec{A}$ and $\vec{B}$ be two vectors, then their vector product, written as $\vec{A}\times\vec{B}$, is a vector $\vec{C}$ defined by $\vec{C}=\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$, where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$; and θ is the angle between them and $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$.

The direction of $\vec{C}$ (or of $\hat{n}$) is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and its sense is decided by right-hand screw rule. If a right-handed screw whose axis is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ is rotated from the first vector $\vec{A}$ to the second vector $\vec{B}$ through the smaller angle between them, then the direction of advance of the screw gives the direction of $\vec{C}$ (or of $\hat{n}$).

Examples : (i) $\vec{\tau}=\vec{r}\times\vec{F}$ (ii) $\vec{J}=\vec{r}\times\vec{p}$ (iii) $\vec{v}=\vec{\omega}\times\vec{r}$ (iv) $\vec{a}=\vec{\alpha}\times\vec{r}$

PROPERTIES OF VECTOR PRODUCT

(a) The vector product is 'not' commutative i.e.
$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$; $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

(b) The vector product is distributive i.e.
$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

(c) The magnitude of the vector product of two vectors mutually at right angles is equal to the product of the magnitudes of the vectors.

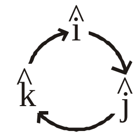
$\vec{A}\times\vec{B}=AB\sin 90^\circ\,\hat{n}=AB\,\hat{n}$,

(d) The vector product of two parallel vectors is a null vector (or zero vector).
$\vec{A}\times\vec{B}=AB(\sin 0)\hat{n}=\vec{0}$ or 0

(e) The vector product of a vector by itself is a null vector (zero vector). $\vec{A}\times\vec{A}=AA\sin(0)\hat{n}=\vec{0}$ or 0

(f) The vector product of unit orthogonal vectors $\hat{i},\hat{j},\hat{k}$ have the following relations in the right-handed coordinate system.

(a) $\hat{i}\times\hat{j}=\hat{k}$ $\quad\hat{j}\times\hat{i}=-\hat{k}$
$\hat{j}\times\hat{k}=\hat{i}$ $\quad\hat{k}\times\hat{j}=-\hat{i}$
$\hat{k}\times\hat{i}=\hat{j}$ $\quad\hat{i}\times\hat{k}=-\hat{j}$



(b) $\hat{i}\times\hat{i}=0$ $\quad\hat{j}\times\hat{j}=0$ $\quad\hat{k}\times\hat{k}=0$

The magnitude of each of the vectors $\hat{i},\hat{j}$ and $\hat{k}$ is 1 and the angle between any two of them is 90°. Therefore, we write $\hat{i}\times\hat{j}=(1)(1)\sin 90^\circ\,\hat{n}=\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the plane of $\hat{i}$ and $\hat{j}$ i.e. it is just the third unit vector $\hat{k}$.

(g) The vector product of two vectors in terms of their x, y & z components can be expressed as a determinant.

Let $\vec{A}$ and $\vec{B}$ be two vectors. Let us write their rectangular components :

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\ ;\ \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B}=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

$$=(A_yB_z-B_yA_z)\hat{i}-(A_xB_z-B_xA_z)\hat{j}+(A_xB_y-B_xA_y)\hat{k}$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}$$

Examples of some physical quantities which can be expressed as cross product of two vectors :

(a) The instantaneous velocity $(\vec{v})$ of a particle is equal to the cross product of its angular velocity $(\vec{\omega})$ and the position vector $(\vec{r})$ i.e. $\vec{v}=\vec{\omega}\times\vec{r}$

(b) The tangential acceleration $(\vec{a}_t)$ of a particle is equal to cross product of its angular acceleration $(\vec{\alpha})$ and the position vector $(\vec{r})$ i.e. $\vec{a}_t=\vec{\alpha}\times\vec{r}$

(c) The centripetal acceleration $(\vec{a}_c)$ of a particle is equal to the cross product of its angular velocity and the linear velocity $(\vec{v})$ i.e. $\vec{a}_c=\vec{\omega}\times\vec{v}$

(d) The force $\vec{F}$ on a charge q moving in side magnetic field is equal to charge times the cross product of its velocity $(\vec{v})$ and magnetic induction $(\vec{B})$ i.e. $\vec{F}=q(\vec{v}\times\vec{B})$

(e) The torque of a force $\vec{\tau}$ is equal to cross product of the position vectors $(\vec{r})$ and the force $(\vec{F})$ applied i.e. $\vec{\tau}=\vec{r}\times\vec{F}$

(f) The angular momentum $(\vec{L})$ is equal to cross product of position vector $(\vec{r})$ and linear momentum $(\vec{p})$ of the particle i.e., $\vec{L} = \vec{r} \times \vec{p}$

Example 16 :

Find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ if $\vec{a} = 3\hat{k} + 4\hat{j},\ \vec{b} = \hat{i} + \hat{j} - \hat{k}$

Sol. $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix} = -7\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 0 & 4 & 3 \end{vmatrix} = 7\hat{i} - 3\hat{j} + 4\hat{k}$$

TRY IT YOURSELF-1

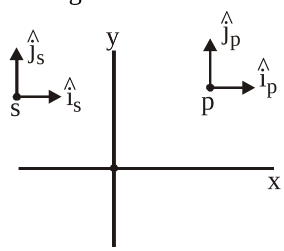
Q.1 Consider the pair of units vectors $(\hat{i}_p, \hat{j}_p)$ located at the point P, and the pair of units vectors $(\hat{i}_s, \hat{j}_s)$ located at the point S . Which of the following statements is true?

(A) $\hat{i}_p \neq \hat{i}_s$

(B) $\hat{j}_p \neq \hat{j}_s$

(C) $\hat{i}_p = \hat{i}_s$

(D) $\hat{j}_p = \hat{j}_s$

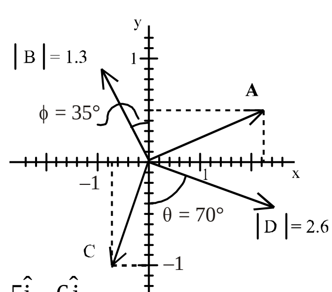


Q.2 Consider a vector $\vec{A}$ with $|\vec{A}| > 1$. The unit vector pointing in the same direction as the vector $\vec{A}$ is given by

(A) $\dfrac{|\vec{A}|}{\vec{A}}$ (B) $\dfrac{\vec{A}}{|\vec{A}|}$ (C) $|\vec{A}|\,\vec{A}$ (D) $\dfrac{1}{|\vec{A}|\,\vec{A}}$

Q.3 The polar coordinates of a point are r = 5.50 m & θ = 240°. What are the cartesian coordinates of this point?

Q.4 From the graph in Figure, characterize vectors $\vec{A}$ and $\vec{C}$ in unit vector notation and vectors $\vec{B}$ and $\vec{D}$ in polar notation



Q.5 Let $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 5\hat{i} - 6\hat{j}$.

(i) Find $\vec{A} + \vec{B},\ \vec{A} - \vec{B},\ 2\vec{A} + 3\vec{B}$, and $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = 0.$

(ii) Find A , the length of $\vec{A}$ and the angle it makes with the x-axis.

Q.6 Consider two vectors $\vec{A} = 2\hat{i} + 3\hat{k}$ and $\vec{B} = -6\hat{i} + 4\hat{k}$. The two vectors are –

(A) parallel

(B) perpendicular.

(C) neither parallel or perpendicular.

Q.7 Using the dot product show that if $\vec{A} + \vec{B}$ is perpendicular to $\vec{A} - \vec{B}$, then A = B.

Q.8 Two vectors are given by $\vec{A} = -3\hat{i} + 4\hat{j}$ and $\vec{B} = 2\hat{i} + 3\hat{j}$. Find (a) $\vec{A} \times \vec{B}$ and (b) the angle between $\vec{A}$ and $\vec{B}$.

Q.9 A student moves in a straight line at constant speed from the point $\vec{r}_1$ to $\vec{r}_2$ in a time of 5s.

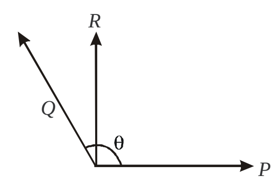
$\vec{r}_1$ = (1.0 m; 1.0 m; 0.0m) ; $\vec{r}_2$ = (5.0 m;.–2.0 m; 0.0m)

What is the unit vector, $\hat{v}$, in the direction of the student's motion?

Q.10 The resultant of two vectors of magnitudes 2A and $\sqrt{2}A$ acting at an angle θ is $\sqrt{10}A$. Find the value of θ.

Q.11 If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{13}Q$, find P/Q.

Q.12 The resultant of $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. What is the angle between $\vec{P}$ and $\vec{Q}$.



Q.13 One of the two rectangular components of a force is 25N and it makes an angle of 45° with the force. Find the magnitude of the other component.

Q.14 One of the rectangular components of a velocity of 60 kmh^{-1} is 30 kmh^{-1}. Find other rectangular component ?

Q.15 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then find the angle between $\vec{a}$ and $\vec{b}$.

ANSWERS

(1) (CD) **(2)** (B) **(3)** x = –2.75 m, y = –4.76 m

(4) $\vec{A} = 2.25\hat{i} + 0.5\hat{j},\ \vec{C} = -0.72\hat{i} - \hat{j}$

$\vec{B} = 1.3\angle 125°,\ \vec{D} = 2.6\angle -20°.$

(5) (i) $\vec{A} + \vec{B} = 8\hat{i} - 2\hat{j}$; $\vec{A} - \vec{B} = -2\hat{i} + 10\hat{j}$

$2\vec{A} + 3\vec{B} = 21\hat{i} - 10\hat{j},\ \vec{C} = -8\hat{i} + 2\hat{j}$ (ii) 5

(6) (B) **(8)** $-17\hat{k}$, 70.6° **(9)** $\dfrac{4}{5}\hat{x} - \dfrac{3}{5}\hat{y}$

(10) 45° **(11)** 3 **(12)** $\cos^{-1}\left(\dfrac{-P}{Q}\right)$

(13) 25 N **(14)** $30\sqrt{3}$ kmh^{-1} **(15)** 2π/3

SECTION - B
CALCULUS IN PHYSICS

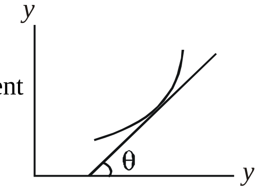
DIFFERENTIATION

Meaning 1 : Literally it means to divide into very small part.

Meaning 2 : The derivative of variable y with respect to variable x is defined as the instantaneous rate of change of y w.r.t. x. It is denoted by $\frac{dy}{dx}$.

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$, $\frac{dy}{dx}$ means change in y w.r.t. change in x when change in x is very small.

Meaning 3 : Geometrically, $\frac{dy}{dx}$ is equal to the slope of the tangent to the curve representing y = f (x) at that point



i.e. Slope = $\frac{dy}{dx}$ = tan θ

(1) Slope of distance (s) – time (t) graphs shows speed

$v = \frac{ds}{dt}$

(2) Slope of v – t graph shows acceleration

$$a = \frac{dv}{dt} = \frac{d\left(\frac{ds}{dt}\right)}{dt} = \frac{d^2s}{dt^2} \rightarrow \text{(double diff. of s w.r.t. t)}$$

(3) Slope of work – time graph shows power. Power (P) = $\frac{dw}{dt}$

(4) Slope of momentum (P) - time graph shows force $F = \frac{dp}{dt}$

(5) Slope of angular momentum (L) - time graph shows torque

$\tau = \frac{dL}{dt}$

Fundamental Formulae of Differentiation :

(1) If c is some constant, then $\frac{d}{dx}(c) = 0$

(2) If y= cx, where c is a constant, then $\frac{dy}{dx} = \frac{d}{dx}(cx) = c\frac{dx}{dx} = c$.

(3) If y = x^n, where n is a real number, then $\frac{dy}{dx} = nx^{n-1}$

$$\frac{d}{dx}(x^7) = 7x^6,\ \frac{d}{dx}(x^{\sqrt{2}}) = \sqrt{2}x^{\sqrt{2}-1},\ \frac{d}{dx}(x^{-5}) = -5x^{-6}.$$

(4) If y = cu, where c is a constant and u is a function of x, then

$$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}.$$

Example 17 :

(i) if $y = 8x^7$, $\frac{dy}{dx} = \frac{d}{dx}(8x^7) = 8 \times 7x^6 = 56x^6$

(ii) if $y = -5x^{-5}$, $\frac{dy}{dx} = \frac{d}{dx}(-5x^{-5}) = -5 \times (-5)x^{-6} = 25x^{-6}$

(5) If y = u^n, where n is real number and u is a function of x, then $\frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$.

(6) If y = u + v, where u and v are the functions of x,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}.$$

Product Rule :

(7) If y = u v, where u and v are the functions of x, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Example : If y = $(3x^3 + 7)(6x^2 + 3)$, then

$$\frac{dy}{dx} = (3x^3+7)(12x) + (6x^2+3)(9x^2) = 90x^4 + 27x^2 + 84x$$

(8) If y = $\frac{u}{v}$, where u and v are the functions of x, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{\text{denominator} \times \text{derivative of numerator} - \text{numerator} \times \text{derivative of denominator}}{(\text{denominator})^2}$$

Example 18 :

If $y = \frac{x^2+1}{x-2}$, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x-2}\right) = \frac{(x-2)(2x+0) - (x^2+1)(1-0)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2 - 1}{(x-2)^2} = \frac{x^2 - 4x - 1}{(x-2)^2}$$

Chain rule :

(9) If y = f (u) and u = f (x), then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Example 19 :

y = $(ax^2 + b)^3$. Find $\frac{dy}{dx}$

Sol. Put u = $ax^2 + b$ Then y = u^3

From these equations, we have

$$\frac{du}{dx} = 2ax \quad \text{and} \quad \frac{dy}{du} = 3u^2 = 3\,(ax^2 + b)^2$$

$$\therefore \quad \frac{dy}{du} = \frac{dy}{du} \times \frac{du}{dx} = 3\,(ax^2 + b)^2\,(2ax)$$

(10) If $y = (ax + b)^n$, then $\frac{dy}{dx} = n\,(ax + b)^{n-1} \times \frac{d}{dx}(ax + b)$

Derivatives of Trigonometrical Functions :

(1) If $y = \sin x$, then $\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$.

(2) If $y = \cos x$, then $\frac{dy}{dx} = \frac{d}{dx}(\cos x) = -\sin x$.

(3) If $y = \tan x$, then $\frac{dy}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$.

Example 20 :

Find out the differential coefficient of sec x

Sol. Let cos x = t, then

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}(\cos x)^{-1} = \frac{d}{dt}(t)^{-1}\left(\frac{dt}{dx}\right)$$

$$= -t^{-2}\frac{d}{dx}(\cos x) = -\sec^2 x\,(-\sin x) = \sec x.\tan x$$

Example 21 :

Determine $\frac{dy}{dx}$ when $y = \cos(2x)$

Sol. $\frac{dy}{dx} = [-\sin(2x)] \times [2] = -2\sin 2x$

Derivatives of Logarithmic And Exponential Functions :

(1) If $y = \log_e x$, then $\frac{dy}{dx} = \frac{1}{x}\log_e e = \frac{1}{x}$.

(2) If $y = e^x$, then $\frac{dy}{dx} = e^x \log_e e = e^x$.

(3) $\frac{d\,(a^x)}{dx} = a^x \log_e a$

Example 22 :

If y = log (sin x), then

$$\frac{dy}{dx} = \frac{1}{\sin x}\frac{d}{dx}(\sin x) = \frac{1}{\sin x}(\cos x) = \cot x$$

Similarly, if y = log (ax + b), then

$$\frac{dy}{dx} = \frac{d}{dx}[\log(ax+b)] = \frac{1}{(ax+b)}\frac{d}{dx}(ax+b) = \frac{1}{(ax+b)}(a+0) = \frac{a}{ax+b}$$

Example 23 :

If $y = xe^x$, then $\frac{dy}{dx} = e^x + xe^x = e^x(x+1)$

Similarly, if $y = e^{\tan x}$, then

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\tan x}) = e^{\tan x}\frac{d}{dx}(\tan x) = e^{\tan x}\sec^2 x$$

Note : While differentiating in Physics, remember, with respect to which parameter, you have to differentiate.

USE OF DIFFERENTIATION IN PHYSICS

Differentiation is a mathematical tool and can be use in any section of physics for developing the concepts we are dealing only in (A) Motion analysis

(B) Error analysis

(C) Maxima & minima based problem

(A) **In Motion Analysis :** Motion Analysis deals with position, velocity, acceleration to find acc^n from velocity. Velocity from position function you can use differentiation

$$x \to v \to a \quad v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt}\ ;\ v = \sqrt{v_x^2 + v_y^2}$$

$$a = \frac{dv}{dt} = \frac{d\,(dx/dt)}{dt} = \frac{d^2x}{dt^2}$$

(read as double differentiation of x wrt t)

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

Example 24 :

The displacement x of a particle at a time t is given by $t = \sqrt{x} + 3$. Find the displacement of the particle at the instant velocity is zero.

Sol. As $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

Squaring both sides $x = (t-3)^2$ ....(1)

Differentiating at x w.r.t. time we get

$$v = \frac{dx}{dt} = 2\,(t-3)$$

Velocity is zero at $2\,(t-3) = 0$ or $t = 3$

$\therefore$ From eq. (1) at t = 3 the value of x is $x = (3-3)^2$ or $x = 0$

Example 25 :

If the motion of a particle is given by $x = at^2$ and $y = bt^2$, then find the velocity of the particle.

Sol. As, $x = at^2$ ....(1) and $y = bt^2$ ....(2)

Differentiating eq. (1) and (2) w.r.t. time we get

$\frac{dx}{dt} = 2at$ & $\frac{dy}{dt} = 2bt$; Let $v_1 = 2at$ & $v_2 = 2bt$

Resultant velocity $v = \sqrt{v_1^2 + v_2^2}$ or $v = \sqrt{4a^2t^2 + 4b^2t^2}$

or $v = 2t\sqrt{a^2 + b^2}$

Example 26 :

The relation between displacement 'x' of a particle with time 't' is given by $t = \alpha x^2 + \beta x$, where α and β are constants. Find acceleration of the particle.

Sol. To get acceleration, we should have relation between v & t

To obtain v $\left(\text{i.e. } \frac{dx}{dt}\right)$ differentiate given equation w.r.t. 't'

Differentiating we get

$$1 = 2\alpha x \frac{dx}{dt} + \beta \frac{dx}{dt} \quad \text{or} \quad 1 = \frac{dx}{dt}(2\alpha x + \beta)$$

$$\Rightarrow v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$$

$$\Rightarrow a = \frac{dv}{dt} = \frac{(2\alpha x+\beta)\frac{d(1)}{dt} - \frac{(1)d}{dt}(2\alpha x+\beta)}{(2\alpha x+\beta)^2}$$

$$a = -\frac{2\alpha \frac{dx}{dt}}{(2\alpha x+\beta)^2} = -\frac{2\alpha}{(2\alpha x+\beta)^3} \quad \left[\because v = \frac{dx}{dt} = \frac{1}{(2\alpha x+\beta)}\right]$$

or $a = -2\alpha v^3$

Example 27 :

The displacement x of a particle along a straight line at a time t is $x = a_0 + a_1 t + a_2 t^2$. What is the acceleration of the particle.

Sol. As $x = a_0 + a_1 t + a_2 t^2$

Differentiating above w.r.t. time t, we get $v = \frac{dx}{dt} = a_1 + 2a_2 t$

Again Differentiating w.r.t. time t,we get $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 2a_2$

(B) Use of Differentiation in Calculation of Error :

In this section we will consider only application of differentiation in error, for details on error please refer chapter units and dimensions.

Let $Z = x^a \times y^b$, Taking log and differentiating, we get,

$$\frac{dZ}{Z} = a\frac{dx}{x} + b\frac{dy}{y} \quad \text{or} \quad \frac{dZ}{Z}\% = a\frac{dx}{x}\% + b\frac{dy}{y}\%$$

Example 28 :

If an error of 2% occurs in measuring the radius of a sphere then find the percentage (%) error in surface area of the sphere.

Sol. Surface area of sphere, $s = 4\pi r^2$

or $\frac{ds}{s} = 2\left(\frac{dr}{r}\right)$ [4π is constant] or $\frac{ds}{s} = 2 \times 2\%$

or $\frac{ds}{s}\% = 4\%$

Example 29 :

If an error of 1% occurs in the momentum of a particle then find the % error in the kinetic energy of the particle.

Sol. As kinetic Energy $K = \frac{1}{2}mv^2$ or $K = \frac{1}{2}mv^2 \times \frac{m}{m}$

or $K = \frac{1}{2m}(mv)^2$ or $K = \frac{1}{2m}(P)^2$

or $\frac{dK}{K} = 2\frac{dP}{P}$ [$\therefore$ m is constant] or $\frac{dK}{K}\% = 2 \times 1\% = 2\%$

This formula will be applied only when the given % error is less than 5% (because differentiation generated formulae are applicable for small changes only).

Example 30 :

If the momentum of a particle increases by 50% then, what will be the % increase in kinetic energy (K)?

Sol. Here given % change (50%) is more than 5% therefore do not use differentiation based formula.

As, Initial momentum = P

Initial KE, $K = \frac{P^2}{2m} \quad \left[\because K = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{P^2}{2m}\right]$

Now, Final momentum $= P + \frac{50}{100}P = 1.5\,P$

Final kinetic energy $= \frac{(1.5P)^2}{2m} = \frac{2.25\,P^2}{2m}$

$$\therefore \ \% \text{ error} = \frac{(K)_F - (K)_I}{(K)_I} \times 100 = \frac{\frac{2.25P^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}} \times 100$$

$$= \frac{2.25 - 1}{1} \times 100 = 1.25 \times 100 = 125\%$$

Example 31 :

If the % change in the length of a simple pendulum is 1% and in g is 2% then find the % change in the time period of the simple pendulum.

Sol. As the time period of simple pendulum is

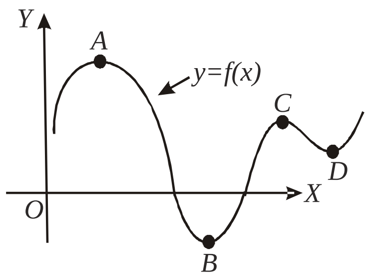
$$2\pi\sqrt{\frac{\ell}{g}} \quad \text{or} \quad T = 2\pi\frac{\ell^{1/2}}{g^{1/2}} \quad \text{or} \quad T = 2\pi\,\ell^{1/2} g^{-1/2}$$

$$\therefore \ \frac{dT}{T}\% = \frac{1}{2}\frac{d\ell}{\ell}\% + \frac{1}{2}\frac{dg}{g}\%$$

$$= \frac{1}{2} \times 1\% + \frac{1}{2} 2\% = \frac{1}{2}\% + 1\% = \frac{3}{2}\%$$

Note : Always add the error to get maximum error.

(C) Use of differentiation in Maxima and Minima based problems : Consider a function y = f (x) which is continuous in a given domain. The graph of the function is shown in fig. The graph, evidently shows, certain points such as A, B, C and D. The points A and C are such that, the function values
there, are more than the points in the immediate neighbourhood.



Such points are termed as maxima. Similarly, the points B and D reveal that, the function values, there are smaller than that of the points in the immediate neighbourhood. Such points are termed as minima.

It is interesting to note that, as one passes from lower to higher values of x through a maxima, the slope of the curve changes from positive to negative and is zero at the point of maxima. Similarly, as one passes from lower to higher values of x through a minima, the slope of the curve changes from negative to positive, and is zero exactly at the point of minima. Therefore it is evident that, the condition for a point to be maxima or minima is that dy/dx = 0 at that point.

1. A function may or may not have one or more maxima and minima.
2. Points when dy/dx = 0 are known as stationary points.

For a maxima, the rate of change of slope is negative w.r.t. x i.e., $\frac{d}{dx}\left(\frac{d}{dx}\right) = -\text{ve}$, since the slope changes from positive value to negative value.

$$\because \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \quad \therefore \text{ For maxima } \frac{d^2y}{dx^2} = -\text{ve}$$

Similarly, for minima, it can be shown that $\frac{d^2y}{dx^2} = +ve$

To find out maximum and minimum value of varying physical quantity : Important steps to be used in physics :

1. First of all find that the given question is based on maxima or minima.
2. Then identify that quantity, of which we have to find maximum or minimum value.
3. Now find variable quantity.
4. Now establish a relation between both quantities.
5. Now differentiate that quantity which is to be maximised or minimised w.r.t. the variable quantity and equate it to zero.

Example 32 :

A particle moves along a straight line according to the law $s = 2t - 3t^2$ where s is the displacement of the particle and t the time. What will be its maximum positive displacement

Sol. Given $s = 2t - 3t^2$

Differentiating w.r.t. time t, we get $\frac{ds}{dt} = 2 - 6t$

Now, for the maximum value of s,

$$\frac{ds}{dt} = 0 \ \therefore \ 2 - 6t = 0 \Rightarrow t = \frac{1}{3}$$

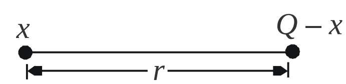
Again $\frac{d^2s}{dt^2} = -6$ (–ve) which shows that $t = \frac{1}{3}$ corresponds to maximum value of s.

$$\therefore s_{max} = 2\left(\frac{1}{3}\right) - 3\left(\frac{1}{3}\right)^2 \Rightarrow \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{units}$$

Example 33 :

Divide charge Q in two charges such that when they are placed r distance apart then a maximum force acts between them.

Sol.



(i) We have to find maximum value i.e. maxima (ii) Force F is to be maximised. (iii) x is the variable quantity.

(iv) The relation between F and x is ⇒ As $F = K\frac{q_1q_2}{r^2}$

Let $q_1 = x$ and $q_2 = Q - x$ $\quad \therefore F = K$

$$\frac{x(Q-x)}{r^2} = \frac{K}{r^2}(Qx - x^2)$$

Differentiating F w.r.t. x we get $\frac{dF}{dx} = \frac{K}{r^2}(Q - 2x)$

Now putting $\frac{dF}{dx} = 0$ we get,

$$\frac{K}{r^2}(Q - 2x) = 0 \text{ or } Q - 2x = 0 \text{ or } x = Q/2$$

Example 34 :

The height 'h' reached by a particle thrown, as a function of time t, is given by $h = ut - ½ gt^2$, where u and g are constants (u = initial speed, g = acceleration due to gravity). Find the time t when h is maximum?

Sol. For h to be maximum, dh/dt = 0

$$\text{Now, } \frac{dh}{dt} = \frac{d}{dt}(ut) - \frac{d}{dt}\left(\frac{1}{2}gt^2\right) = u\frac{dt}{dt} - \frac{1}{2}g\frac{d(t^2)}{dt}$$

$= u - \frac{1}{2}g.\,2t = u - gt$; At max. height $\frac{dh}{dt} = 0$

or $u - gt = 0$ or $t = u/g$

INTEGRATION

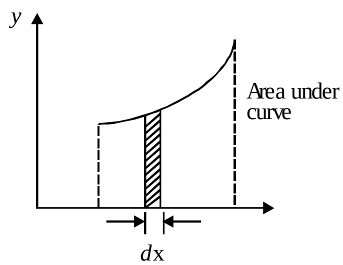
Meaning of Integration is to 'join' or 'combine'. The process of Integration is just the reverse of differentiation.

Important Points

(i) Meaning of Integration is scalar sum.

(ii) Integration gives the area under the x – y curve.

(iii) Area represents that physical quantity obtained by the product of x & y.



$$A = \int y . dx$$

Remember :

Area of Velocity – time curve $\rightarrow$ Displacement

Area of Acceleration – time curve $\rightarrow$ change in Velocity

Area of Force – time curve $\rightarrow$ Impulse (change in linear momentum)

Area of Force – displacement curve $\rightarrow$ Work

Area of Power – time curve $\rightarrow$ Work

Area of Pressure – volume curve $\rightarrow$ Work

Area of Torque – time curve $\rightarrow$ change in Angular momentum

Points to be remembered while using Integration in Physics :

(i) Before doing integration ensure that the given physical quantity is scalar or vector.

(ii) It the given physical quantity is scalar, then integrate directly and if it is vector quantity then first see the direction. If direction remains constant, then integrate directly and if direction varies then first find the components and then integrate.

(iii) When doing integration there should not be more than two variables, each separated on different sides.

(iv) If there are more than two variables then first convert them into two variables.

(v) Be careful while deciding limits and use relative limits.
"Always put limits of that variable only with respect to which integration is being done."

$$\int_{x_1}^{x_2} f(x)\, .dx$$

Fundamental Formulae of Integration :

(i) $\int c\, dx = cx + k$ where c is a constant and k is constant of integration

(ii) $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + k$, except when n = –1

This is the most used formula and you must learn it by heart. You can remember it easily by "Increase the power by one and divided by the increased power."

(iii) $\int (ax+b)^n\, dx = \dfrac{(ax+b)^{n+1}}{a\,(n+1)} + k$

(iv) $\int x^{-1}\, dx = \int \dfrac{1}{x}\, dx = \log_e x + k$

(v) $\int e^x\, dx = e^x + k$

(vi) $\int \sin x\, dx = -\cos x + k$

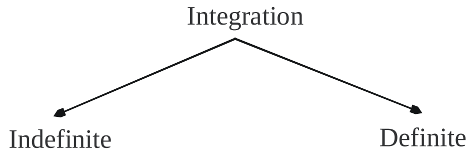
(vii) $\int \cos x\, dx = \sin x + k$

(viii) $\int \sec^2 x\, dx = \tan x + k$

(ix) $\int \sin nx\, dx = \dfrac{-\cos nx}{n} + k$

(x) $\int \cos nx\, dx = \dfrac{\sin nx}{n} + k$

Integration is of two types :



Indefinite Integration : $\int f(x)\, .dx = F(x) + c$

"In physics c is found by the condition given in the question."

Reason for 'c'

We know that , $\dfrac{d}{dx}(x) = 1$ .... (1)

$\dfrac{d}{dx}(x+1) = 1$ .... (2)

$\dfrac{d}{dx}(x+2) = 1$ .... (3)

$\dfrac{d}{dx}(x+3) = 1$ .... (4)

Since integration is the inverse of differentiation,

$\therefore \int 1\, dx = x$ [from (1)] ;

$\int 1\, dx = x+1$ [from (2)] ;

$\int 1\, dx = x+2$ [from (3)] ;

$\int 1\, dx = x+3$ [from (4)]

In general, we may write : $\int 1\, dx = x + c$;

where c is a constant of integration. In all indefinite integrals, constant of integration is supposed to be present even if it is not specifically mentioned.

Definite Integration

Definite Integration is given as

$$\int_{x_1}^{x_2} f(x)\,.\,dx = \left[F(x)\right]_{x_1}^{x_2} = F(x_2) - F(x_1)$$

Where x_1 and x_2 are the limits. $x_2 \rightarrow$ upper limit and $x_1 \rightarrow$ lower limit.

Example 35 :

Evaluate the definite integral $\int_0^{\pi/2} \cos x\, dx$.

Sol. $\because \int \cos x\, dx = \sin x$

$$\therefore \quad \int_0^{\pi/2} \cos x\, dx = \left[\sin x\right]_0^{\pi/2} = \sin(\pi/2) - 0 = 1 - 0 = 1$$

Example 36 :

Evaluate $\int_0^{\pi/4} \sin 2x\, dx$

Sol.

$$\int_0^{\pi/4} \sin 2x\, dx = \left[-\frac{\cos 2x}{2}\right]_0^{\pi/4} = -\frac{1}{2}[\cos(\pi/2) - \cos 0)] = \frac{1}{2}$$

Example 37 :

$$\int_0^{\infty} \frac{GMm}{x^2} dx$$

Sol. $\int_0^{\infty} \frac{GMm}{x^2} dx = \left[-\frac{GMm}{x}\right]_R^{\infty} = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{GMm}{R}$

Example 38 :

Integrate $x^2 - \cos x + \frac{1}{x}$ w.r.t. x

Sol. $\int\left(x^2 - \cos x + \frac{1}{x}\right) dx = \int x^2 dx - \int \cos x\, dx + \int \frac{1}{x} dx$

$$= \frac{x^3}{3} - \sin x + \log_e x + c$$

Example 39:

Evaluate $\int_1^{\infty} \frac{1}{x^4} dx$

Sol. $\int_1^{\infty} x^{-4} dx = \left[\frac{x^{-4+1}}{-4+1}\right]_1^{\infty} = -\frac{1}{3}\left[\frac{1}{\infty} - \frac{1}{1}\right] = -\frac{1}{3}(0-1) = \frac{1}{3}$

Integration by substitution

It is not always possible to find the integral of a complicated function only by observation, so we need some methods of integration and integration by substitution is one of them.

Example 40 :

Evaluate : $\int (x+2)^5\, dx$

Sol. We can put the integral in the form $\int u^n du$

by substituting $u = x + 2,\ du/dx = 1$

Then $\int (x+2)^5\, dx = \int u^5 du = \frac{u^6}{6} + C = \frac{(x+2)^6}{6} + C$

Example 41 :

Evaluate : $\int \sin^4 t \cos t\, dt$

Sol. Let $u = \sin t$, $du = \cos t\, dt$

$$I = \int u^4\, du = \frac{u^5}{5} + C = \frac{\sin^5 t}{5} + C$$

USE OF INTEGRATION IN PHYSICS

In physics, there are numerous situations in which we have to calculate the integral of a given function.
For example,

(i) Velocity function $\vec{v}$ of a particle is the integral of its acceleration function $\vec{a}$. That is, $\vec{v}(t) = \int \vec{a}\, dt$

(ii) Displacement function $\vec{x}$ of a particle is the integral function of its velocity function $\vec{v}$. That is $\vec{x} = \int \vec{v}\, dt$

(iii) Impulse I is defined as the integral of force with respect to time. That is $\vec{I} = \int \vec{F}\, dt$

(iv) Work W is defined as the integral of force with respect to displacement. $W = \int \vec{F}\,.\,d\vec{s}$

Mathematically, definite integral is a number unlike the indefinite integral which is a function. In physics, definite integral is not merely a number but a physical concept or physical quantity with certain units. For example,

(v) The integral of acceleration function between two time limits gives the change in velocity between the time interval. $\Delta v_{21} = v_2 - v_1 = \int_1^2 a\, dt$

(vi) The integral of velocity function between two time limits gives the change in position (i.e. displacement) between the given time interval $\Delta x_{21} = x_2 - x_1 = \int_1^2 v\, dt$

Integration can be used for calculating 'v' from 'a' & position from 'v'. $a \rightarrow v \rightarrow x$.

(vii) The integral of force within a given time interval is equal to the change in momentum of the particle during that time interval. $\Delta p_{21} = p_2 - p_1 = \int_1^2 F\,dt$

(viii) The integral of force within the specific space is equal to the change in kinetic energy of the particle.

$$\Delta K_{21} = K_2 - K_1 = \int_1^2 F\,dx$$

Example 42 :

A particle moves with a constant acceleration $a = 2\ \text{ms}^{-2}$. along a straight line. If it starts with an initial velocity of $5\ \text{ms}^{-1}$., then obtain an expression for its instantaneous velocity.

Sol. Using the relation between velocity and acceleration, we get $\quad v = \int a\,dt = 2\int dt = 2t + c$

where c is the constant of integration. Its value can be obtained by using the initial condition. That is at

$t = 0;\ v = 5\ \text{ms}^{-1}$ Thus, $5 = 2(0) + c \Rightarrow c = 5\ \text{ms}^{-1}$

Therefore, v = 2t + 5 is the required expression for instantaneous velocity.

Example 43 :

In the above example if the particle occupies a position x = 7m at t = 1s, then obtain an expression for the instantaneous displacement of the particle.

Sol. Using the relation between displacement and velocity, we get $\int v\,dt = \int (2t+5)\,dt$ or $x = t^2 + 5t + c$

where c is the constant of integration. Its value can be determined by using the given conditions.

That is, at t = 1 s ; x = 7m $\therefore\ 7 = (1)^2 + 5(1) + c \Rightarrow c = 1\text{m}$

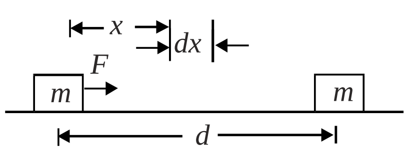
Thus, $x = t^2 + 5t + 1$

Example 44 :

If a force F is being applied on a block of mass m, which is proportional to displacement x, then find the work done in displacing the block by distance d.

Sol. **Method I :** $F \propto x$ or $F = kx$

Now small work done in displacing the body by a small distance dx is $\quad dw = F.dx$



So total work done in displacing the body by a distance d

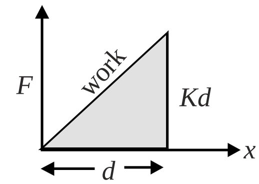
is $\int dw = \int_0^d Kx.dx \therefore w = K\int_0^d xdx$ or $w = K\left[\frac{x^2}{2}\right]_0^d$

$$= K\left[\frac{d^2}{2} - 0\right] \text{ or } w = K\frac{d^2}{2}$$

Method II : Work done is equal to the area under the force-displacement graph.

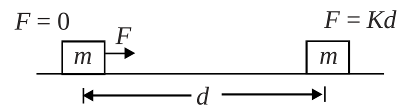
w = Area under F – x graph

$$= \frac{1}{2} \times d \times Kd = \frac{Kd^2}{2}$$



Method III : Work done is given by the product of average force and total displacement.

$$w = F_{av} \times d = \left(\frac{0 + Kd}{2}\right) d = \frac{Kd^2}{2}$$



This method of average is used when relation is linear.

TRY IT YOURSELF-2

Q.1 If y = sin log x, find dy/dx

Q.2 Find the slope of the curve $y = x^2 + 3x + 4$ at (–1, 2)

Q.3 A particle starts from rest with a uniform acceleration. Its displacement after t seconds is given in metres by the relation $x = 5 + 6t + 7t^2$. Calculate the magnitude of its (i) initial velocity (ii) velocity at t = 3 s (iii) uniform acceleration and (iv) displacement at t = 5s.

Q.4 The mass of a body is 2.5 kg. It is in motion and its velocity v after time t is $v = \frac{t^3}{3} + \frac{t^2}{2} + 1$

Calculate the force acting on the body at the time t = 3s.

Q.5 A particle in uniform acceleration in a straight line has a speed $v = (180 - 16x)^{1/2}\ \text{ms}^{-1}$. What is its acceleration ?

Q.6 The position of a body is given by $x = 2t^3 - 6t^2 + 12t + 6$. Find the value of t when acceleration is zero.

Q.7 Where is the minimum of the potential energy occur in $U(x) = 100 - 50x + 1000x^2$ joule ?

Q.8 Evaluate: $\int (\cos x - \sin x)(3 + 4\sin 2x)dx$.

Q.9 Evaluate : $\int \frac{2z\,dz}{\sqrt[3]{z^2+1}}$

Q.10 If a particle is moving with velocity u and a resistive force acts on it which is proportional to v^2, then find the velocity of the particle after travelling distance x.

Q.11 A force F = a + bx acts on a particle in the x-direction where a and b are constants. Find the work done by this force during a displacement from x_1 to x_2.

ANSWERS

(1) $\frac{\cos\log x}{x}$ **(2)** 45°

(3) (i) 6 m/s (ii) 48 m/s, (iii) 14 m/s^2 (iv) 210m.

(4) 30 N **(5)** – 8 ms^{-2} **(6)** 1 sec.

(7) 0.025 **(8)** $\left(\frac{\sin x + \cos x}{3}\right)(1 + 4\sin 2x) + C$

(9) $\frac{3}{2}(z^2 + 1)^{2/3} + C$ **(10)** $v = ue^{-\frac{Kx}{m}}$

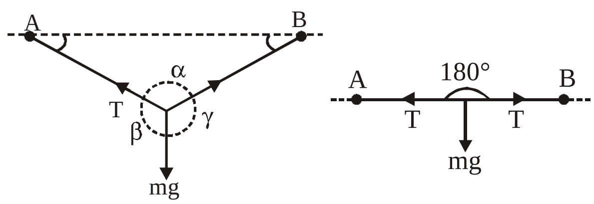
(11) $\frac{(x_2 - x_1)}{2}[2a + b\,(x_1 + x_2)]$

ADDITIONAL EXAMPLES

Example 1 :

A weight mg is suspended from the middle of a rope whose ends are at the same level. The rope is no longer horizontal. Find the minimum tension required to completely straighten the rope.

Sol. According to Lami's theorem, $\frac{T}{\sin\beta} = \frac{T}{\sin\gamma} = \frac{mg}{\sin\alpha}$



For Straighten, $\alpha = 180^o, \beta = 90^o, \gamma = 90^o$

$$\frac{T}{\sin 90^o} = \frac{mg}{\sin 180^o} \;;\; T = \frac{mg}{0} = \infty$$

So the minimum tension required to completely straighten will be infinity.

Example 2 :

A man rows a boat with a speed of 18 km h^{-1} in the north-west direction. The shoreline makes an angle of 15° south of west. Obtain the components of the velocity of the boat along the shoreline and perpendicular to the shoreline.

Sol. The north-west direction of the boat makes an angle of 60° with the shoreline.

Component of the velocity of boat along the shoreline

$= 18 \cos 60°$ km h^{-1} = 9 km h^{-1}

Component of the boat velocity along a line normal to the

shoreline = 18 sin 60° km h^{-1} = $18 \times \frac{\sqrt{3}}{2}$ km h^{-1}

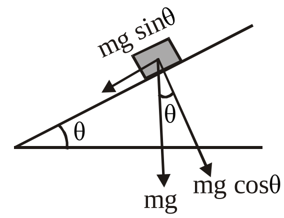
= 15.59 km h^{-1}.

Example 3 :

A mass of 2 kg lies on a plane making an angle 30° to the horizontal. Resolve its weight along and perpendicular to the plane. Assume g = 10 ms^{-2}.

Sol. In fig., the component of weight along the plane

$= mg \sin\theta = 2 \times 10 \times \sin 30 = 10$ N.

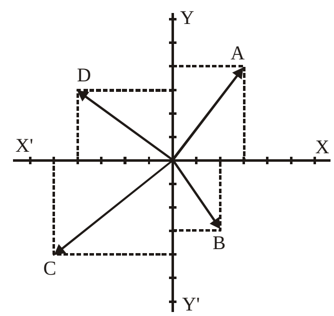


The component of weight perpendicular to plane

$= mg \cos 30° = 2 \times 10 \times \sqrt{3}/2 = 17.3$ N.

Example 4 :

Write vector shown Graphically.



Sol. (a) $\vec{A} = 3\hat{i} + 4\hat{j}$ (b) $\vec{B} = 2\hat{i} - 3\hat{j}$

(c) $\vec{C} = -5\hat{i} - 4\hat{j}$ (d) $\vec{D} = -4\hat{i} + 3\hat{j}$

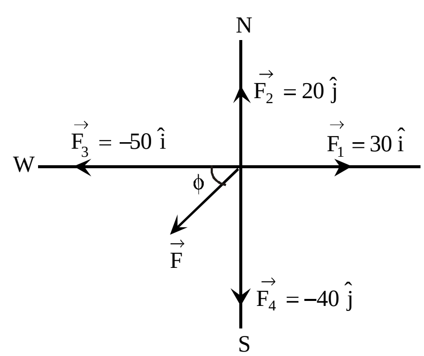
Example 5 :

Find the resultant force of the following forces which act upon a particle.

(a) 30 N due East (b) 20 N due North

(c) 50 N due West (d) 40 N due South.

Sol. Resultant Force



$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$= 30\,\hat{i} + 20\,\hat{j} - 50\,\hat{i} - 40\,\hat{j} = -20\,\hat{i} - 20\,\hat{j}$$

$$|\vec{F}| = \sqrt{(-20)^2 + (-20)^2} = 20\sqrt{2}$$

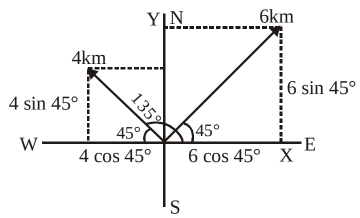
$$\tan\phi = \frac{20}{20} = 1 \text{ or } \phi = 45°$$

Example 6 :

A car travels 6 km towards north at an angle 45° to the east and then travels distance of 4 km towards north at an angle of 135° to the east. How far is its final position due east and due north? How far is the point from the starting point? What angle does the straight line joining its initial and final position makes with the east? What is the total distance travelled by the car?

Sol. Net movement along X-direction

$$= (6-4)\cos 45° = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \text{ km.}$$



Net movement along Y -direction

$$= (6+4)\sin 45° = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

Net movement form starting point = 6 + 4 = 10 km

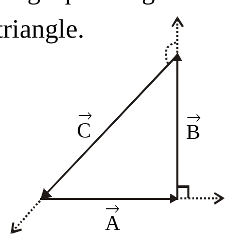
Angle which makes with the east direction

$$\tan\theta = \frac{\text{Y} - \text{component}}{\text{X} - \text{component}} = \frac{5\sqrt{2}}{\sqrt{2}} \;;\; \theta = \tan^{-1}(5) \;;\; \theta = 79°$$

Example 7 :

Given that $\vec{A} + \vec{B} + \vec{C} = 0$ But of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then find the angles between vectors .

Sol. From polygon law, three vectors having summation zero should form a closed polygon, (triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. triangle should be right angled triangle.



Angle between $\vec{A}$ and $\vec{B}$ is 90°

Angle between $\vec{B}$ and $\vec{C}$ is 135°

Angle between $\vec{A}$ and $\vec{C}$ is 135°

Example 8 :

What is the angle in between two vectors

$-2\hat{i} + 3\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$.

Sol. Let $\vec{a} = -2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{a} \cdot \vec{b} = (-2\hat{i} + 3\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} - 4\hat{k})$$

$$= -2 \times 1 + 3 \times 2 + 1 \times (-4) = -2 + 6 - 4 = 0$$

or $\vec{a} \cdot \vec{b} = 0$ hence, $\theta = 90°$

Example 9 :

Given $\vec{r} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{p} = 4\hat{j} - 3\hat{k}$

Calculate $|\vec{L}|$, Here $\vec{L} = \vec{r} \times \vec{p}$.

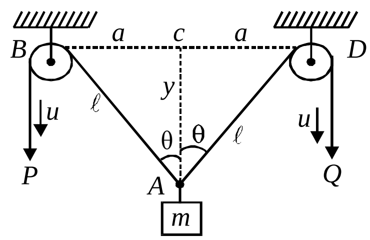
L = Angular momentum, p = Linear momentum

Sol. $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$

$$= \hat{i}(6-8) + \hat{j}(0+3) + \hat{k}(4+0) = -2\hat{i} + 3\hat{j} + 4\hat{k}$$

Example 10:

In the arrangement shown in figure, the ends P and Q of an unstretchable string move downwards with speed u. Pulleys are fixed. Find the speed with which mass M moves upwards.



Sol. Let AB = ℓ , BC = a and AC = y

from Δ ABC, by pythagoras theorem,

$$\ell^2 = a^2 + y^2 \quad \text{or} \quad y^2 = \ell^2 - a^2$$

Differentiating both sides w.r.t. time we get,

$$2y\frac{dy}{dt} = 2\ell\frac{d\ell}{dt} + 0 \quad \text{or} \quad \frac{dy}{dt} = \frac{\ell}{y}\frac{d\ell}{dt}$$

$$\text{But } \frac{\ell}{y} = \frac{1}{\cos\theta} \quad \text{also} \quad \frac{d\ell}{dt} = u \;\therefore$$

$$\frac{dy}{dt} = \frac{1}{\cos\theta} \times u \quad \text{or} \quad \frac{dy}{dt} = \frac{u}{\cos\theta}$$

Example 11 :

The acceleration of a particle is given by $a = t^3 - 3t^2 + 5$, where a is in m/s^2 and t in sec. At t = 1 sec., the displacement and velocity are 8.30 m and 6.25 m/s respectively. Calculate the displacement and velocity at t = 2 sec.

Sol. Given : $a = t^3 - 3t^2 + 5$ i.e., $\frac{dv}{dt} = t^3 - 3t^2 + 5$

$$\Rightarrow dv = (t^3 - 3t^2 + 5)\,dt$$

Integrating both sides we get $v = \frac{t^4}{4} - t^3 + 5t + C_1$

At t = 1 sec, v = 6.25

$\therefore \quad 6.25 = \frac{1}{4} - 1 + 5 + C_1 \Rightarrow C_1 = +2$

Again, $\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2 \quad \left(v = \frac{ds}{dt}\right)$

Integrating we get, $\quad s = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$

At t = 1 sec, s = 8.30 m

$\therefore\ 8.30 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + C_2 \ \Rightarrow C_2 = 4$

The expressions for displacement & velocity are as follows :

$\therefore$ At t = 2 sec, $s = \frac{2^5}{20} - \frac{2^4}{4} + \frac{5}{2}(2)^2 - 2\,(2) + 4 = 7.6$ m

and $v = \frac{t^4}{4} - t^3 + 5t + 2 \quad \therefore$ at t = 2 sec ; v = 8 m/s

Example 12 :

A train starting from rest is accelerated and the instantaneous acceleration is given by $\frac{10}{v+1}$ m/s where v is its velocity in m/s. Find the distance in which the train attains a velocity of 54 km/hr, and the corresponding time.

Sol. Given : $a = \frac{10}{v+1} \Rightarrow \frac{dv}{dt} = \frac{10}{v+1} \Rightarrow (v+1)\,dv = 10\,dt$

Integrating $\frac{v^2}{2} + v = 10t + C$ [$\because$ At t = 0, v = 0 $\therefore$ C = 0]

Eq. is $\frac{v^2}{2} + v = 10t \qquad [v = 54 \text{ km/hr} = 54 \times \frac{15}{18} = 15 \text{ m/s}]$

$\therefore \ \frac{15^2}{2} + 15 = 10t \Rightarrow t = 12.75$ sec

Again , $v.\frac{dv}{ds} = \frac{10}{v+1} \Rightarrow (v^2 + v)\,dv = 10\,ds$

Integrating, $\frac{v^3}{3} + \frac{v^2}{2} = 10s + C'$

Using v = 0 at s = 0 ; C' = 0 $\therefore \ \frac{v^3}{3} + \frac{v^2}{2} = 10s$

Putting v = 15 ; s = 123.75m

QUESTION BANK

CHAPTER 1 : MATHEMATICS IN PHYSICS

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : VECTOR AND ITS APPLICATIONS

Q.1 The vector that must be added to the vector $\hat{i}-3\hat{j}+2\hat{k}$ and $3\hat{i}+6\hat{j}-7\hat{k}$ so that the resultant vector is a unit vector along the y-axis is –

(A) $4\hat{i}+2\hat{j}+5\hat{k}$ (B) $-4\hat{i}-2\hat{j}+5\hat{k}$

(C) $3\hat{i}+4\hat{j}+5\hat{k}$ (D) Null vector

Q.2 How many minimum number of coplanar vectors having different magnitudes can be added to give zero resultant

(A) 2 (B) 3

(C) 4 (D) 5

Q.3 The unit vector along $\hat{i}+\hat{j}$ is

(A) $\hat{k}$ (B) $\hat{i}+\hat{j}$

(C) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (D) $\frac{\hat{i}+\hat{j}}{2}$

Q.4 If a unit vector is represented by $0.5\hat{i}+0.8\hat{j}+c\hat{k}$, then the value of 'c' is

(A) 1 (B) $\sqrt{0.11}$

(C) $\sqrt{0.01}$ (D) $\sqrt{0.39}$

Q.5 The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ & $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is –

(A) $\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$ (B) $\frac{1}{7}(3\hat{i}+6\hat{j}+2\hat{k})$

(C) $\frac{1}{49}(3\hat{i}+6\hat{j}-2\hat{k})$ (D) $\frac{1}{49}(3\hat{i}-6\hat{j}+2\hat{k})$

Q.6 The angle between the two vectors $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$ will be

(A) 90° (B) 0°

(C) 60° (D) 45°

Q.7 If the sum of two unit vectors is a unit vector, then magnitude of difference is

(A) $\sqrt{2}$ (B) $\sqrt{3}$

(C) $1/\sqrt{2}$ (D) $\sqrt{5}$

Q.8 The magnitude of vector $\vec{A},\vec{B}$ and $\vec{C}$ are respectively 12, 5 and 13 units and $\vec{A}+\vec{B}=\vec{C}$ then the angle between $\vec{A}$ and $\vec{B}$ is –

(A) 0 (B) π

(C) $\pi/2$ (D) $\pi/4$

Q.9 The resultant of two vectors A and B is perpendicular to the vector A and its magnitude is equal to half the magnitude of vector B. The angle between A and B is

(A) 120° (B) 150°

(C) 135° (D) None of these

Q.10 A body moves due East with velocity 20km/hour and then due North with velocity 15 km/hour. The resultant velocity

(A) 5 km/hour (B) 15 km/hour

(C) 20 km/hour (D) 25 km/hour

Q.11 A person goes 10 km north and 20 km east. What will be displacement from initial point

(A) 22.36 km (B) 2 km

(C) 5 km (D) 20 km

Q.12 If $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is

(A) 90° (B) 120°

(C) 0° (D) 60°

Q.13 The vector sum of two forces is perpendicular to their vector differences. In that case, the forces–

(A) Are equal to each other in magnitude

(B) Are not equal to each other in magnitude

(C) Cannot be predicted

(D) Are equal to each other

Q.14 The vectors $5i+8j$ and $2i+7j$ are added. The magnitude of the sum of these vector is

(A) $\sqrt{274}$ (B) 38

(C) 238 (D) 560

Q.15 A particle moves from position $3\hat{i}+2\hat{j}-6\hat{k}$ to $14\hat{i}+13\hat{j}+9\hat{k}$ due to a uniform force of $(4\hat{i}+\hat{j}+3\hat{k})$ N. If the displacement in meters then work done will be

(A) 100 J (B) 200 J

(C) 300 J (D) 250 J

PART - 2 : DIFFERENTIATION & ITS APPLICATIONS

Q.16 Equation of displacement for any particle is; $s = 3t^3 + 7t^2 + 14t + 8$m. Its acceleration at time t = 1 sec is.

(A) 10 m/s^2 (B) 16 m/s^2

(C) 25 m/s^2 (D) 32 m/s^2

Q.17 The motion of a particle along a straight line is given by the equation: $x = 6 + 4t^2 - t^4$ (where x is in metres and t is in sec). The acceleration a of the particle at t equal to 2 sec, is.

(A) – 30 m/s^2 (B) – 40 m/s^2

(C) – 50 m/s^2 (D) – 60 m/s^2

Q.18 A particle covers a distance x in time t in a straight line according to the equation $x = A + Bt + Ct^2$. The acceleration of the particle is –
(A) A (B) B
(C) C (D) 2C

Q.19 The displacement S of a particle varies with time (t) as $S = at^2 - bt^3$. At what time acceleration becomes zero –
(A) a/b (B) a/3b
(C) 3b/a (D) 2a/3b

Q.20 A particle starts from the origin at t = 0 and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity in the x-direction
(A) $\frac{\alpha}{2\beta}$ (B) $\frac{2\alpha}{\beta}$
(C) $\sqrt{\frac{\alpha}{2\beta}}$ (D) $\sqrt{\frac{2\alpha}{\beta}}$

Q.21 A car moves along a straight line whose motion is given by: $s = 12t + 3t^2 - 2t^3$, where (s) is in metres and (t) is in seconds. The velocity of the car at start will be.
(A) 7 m/s (B) 9 m/s
(C) 12 m/s (D) 16 m/s

Q.22 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in kinetic energy obtained by measuring mass and speed, will be.
(A) 12% (B) 10%
(C) 8% (D) 2%

Q.23 The velocity of a particle moving along x-axis is given as $v = x^2 - 5x + 4$ (in m/s) where x denotes the x-coordinate of the particle in metres. Find the magnitude of acceleration of the particle when the velocity of particle is zero
(A) 0 m/s^2 (B) 2 m/s^2
(C) 3 m/s^2 (D) none of these

Q.24 A particle of mass m moves along a curve $y = x^2$. When particle has x -coordinate as 1/2 and x-component of velocity as 4m/s then –
(A) the position coordinate of particle are (1/2, 1/4)
(B) the velocity of particle will be along the line $4x - 4y - 1 = 0$
(C) the magnitude of velocity at that instant is $4\sqrt{2}$ m/s
(D) All of these

Q.25 The displacement of a particle is given by : $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively.
(A) b, –4d (B) –b, 2c
(C) b, 2c (D) 2c, – 4d

Q.26 If an error of 2% occurs in measuring the radius of a sphere then find the percentage (%) error in surface area of the sphere.
(A) 1 (B) 2
(C) 3 (D) 4

Q.27 If an error of 1% occurs in the momentum of a particle then find the % error in the kinetic energy of the particle.
(A) 1 (B) 2
(C) 3 (D) 4

Q.28 Divide charge Q in two charges such that when they are placed r distance apart then a maximum force acts between them. If one charge is X then find X.
(A) Q/2 (B) Q/3
(C) Q/4 (D) Q/5

PART - 3 : INTEGRATION AND ITS APPLICATIONS

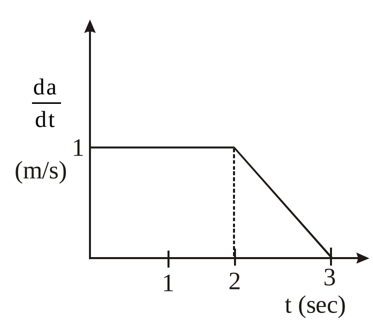
Q.29 Evaluate $\int_0^\infty \frac{GMm}{x^2}dx$
(A) $\frac{GMm}{R}$ (B) $\frac{GMm}{2R}$
(C) $\frac{GMm}{3R}$ (D) $\frac{GMm}{4R}$

Q.30 A particle moves with a constant accelerationa=2m/s^2. along a straight line. If it starts with an initial velocity of 5 ms^{-1}., then obtain an expression for its instantaneous velocity.
(A) $v = 2t^2 + 5$ (B) $v = 2t^3 + 5$
(C) $v = 2t + 5$ (D) $v = 2t^2 + 5t$

Q.31 In the above question if the particle occupies a position x = 7m at t = 1s, then obtain an expression for the instantaneous displacement of the particle.
(A) $x = t^3 + 5t + 1$ (B) $x = t^2 + 5t + 1$
(C) $x = 5t^2 + 5t + 1$ (D) $x = 2t^2 + 5t - 7$

Q.32 If a force F is being applied on a block of mass m, which is proportional to displacement x, then find the work done in displacing the block by distance d.
(A) Kd^2 (B) Kd
(C) $Kd^2/3$ (D) $Kd^2/2$

Q.33 Figure given below shows the variation of "rate of change in acceleration" with time, for a particle moving along a straight line. (Assume the body starts from rest) Find the acceleration at instant t = 2 sec.



(A) 2 m/s^2 (B) 1 m/s^2
(C) 3 m/s^2 (D) 4 m/s^2

PART - 4 : MISCELLANEOUS

Q.34 If $\vec{A}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4\hat{k}$; $|\vec{A}\times\vec{B}|$ =

(A) $8\sqrt{2}$ (B) $8\sqrt{3}$

(C) $8\sqrt{5}$ (D) $5\sqrt{8}$

Q.35 Consider two vectors $\vec{F}_1=2\hat{i}+5\hat{k}$ and $\vec{F}_2=3\hat{j}+4\hat{k}$. The magnitude of the scalar product of these vectors is

(A) 20 (B) 23

(C) $5\sqrt{33}$ (D) 26

Q.36 A particle moves with a velocity $6\hat{i}-4\hat{j}+3\hat{k}\ m/s$ under the influence of a const. force $\vec{F}=20\hat{i}+15\hat{j}-5\hat{k}$ N. The instantaneous power applied to the particle is –

(A) 35 J/s (B) 45 J/s

(C) 25 J/s (D) 195 J/s

Q.37 A particle moves in the x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $(\vec{P})$ at anytime t is $P_x=2\cos t, p_y=2\sin t$. The angle θ between $\vec{F}$ and $\vec{p}$ at a given time t will be

(A) $\theta=0°$ (B) $\theta=30°$

(C) $\theta=90°$ (D) $\theta=180°$

Q.38 The resultant of the two vectors having magnitude 2 and 3 is 1. What is their cross product

(A) 6 (B) 3

(C) 1 (D) 0

Q.39 The linear velocity of a rotating body is given by $\vec{v}=\vec{\omega}\times\vec{r}$, where $\vec{\omega}$ is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body is $\vec{\omega}=\hat{i}-2\hat{j}+2\hat{k}$ & the radius vector $\vec{r}=4\hat{j}-3\hat{k}$, $|\vec{v}|$ =

(A) $\sqrt{29}$ units (B) $\sqrt{31}$ units

(C) $\sqrt{37}$ units (D) $\sqrt{41}$ units

Q.40 The position of a particle is given by $\vec{r}=(\vec{i}+2\vec{j}-\vec{k})$ momentum $\vec{P}=(3\vec{i}+4\vec{j}-2\vec{k})$. The angular momentum is perpendicular to

(A) x-axis

(B) y-axis

(C) z-axis

(D) Line at equal angles to all the three axes

Q.41 A force vector applied on a mass is represented as $\vec{F}=6\hat{i}-8\hat{j}+10\hat{k}$ and accelerates with 1 m/s^2. What will be the mass of the body in kg.

(A) $10\sqrt{2}$ (B) 20

(C) $2\sqrt{10}$ (D) 10

Q.42 When $\vec{A}.\vec{B}=-|A||B|$, then

(A) $\vec{A}$ and $\vec{B}$ are perpendicular to each other

(B) $\vec{A}$ and $\vec{B}$ act in the same direction

(C) $\vec{A}$ and $\vec{B}$ act in the opposite direction

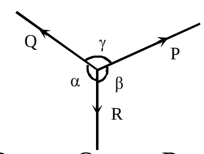
(D) $\vec{A}$ and $\vec{B}$ can act in any direction

Q.43 If a vector $\vec{A}$ is parallel to another vector $\vec{B}$ then the resultant of the vector $\vec{A}\times\vec{B}$ will be equal to

(A) A (B) $\vec{A}$

(C) Zero vector (D) Zero

Q.44 A body is in equilibrium under the action of three coplanar forces P, Q and R as shown in the figure. Select the correct statement



(A) $\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$ (B) $\frac{P}{\cos\alpha}=\frac{Q}{\cos\beta}=\frac{R}{\cos\gamma}$

(C) $\frac{P}{\tan\alpha}=\frac{Q}{\tan\beta}=\frac{R}{\tan\gamma}$ (D) $\frac{P}{\sin\beta}=\frac{Q}{\sin\gamma}=\frac{R}{\sin\alpha}$

Q.45 The vectors from origin to the points A and B are $\vec{A}=3\hat{i}-6\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}+\hat{j}-2\hat{k}$ respectively. The area of the triangle OAB be

(A) $\frac{5}{2}\sqrt{17}$ sq.unit (B) $\frac{2}{5}\sqrt{17}$ sq.unit

(C) $\frac{3}{5}\sqrt{17}$ sq.unit (D) $\frac{5}{3}\sqrt{17}$ sq.unit

Q.46 The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to minimum force, then the forces are –

(1) 6 N and 10 N (2) 8 N and 8 N

(3) 4 N and 12 N (4) 2 N and 14 N

Q.47 If $|\vec{A}\times\vec{B}|=\sqrt{3}\,\vec{A}\,.\,\vec{B}$ then the value of $|\vec{A}+\vec{B}|$ is –

(1) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$ (2) $A+B$

(3) $(A^2+B^2+\sqrt{3}AB)^{1/2}$ (4) $(A^2+B^2+AB)^{1/2}$

Q.48 The relation $3t=\sqrt{3x}+6$ describes the displacement of a particle in one direction where x is in meters and t in sec. The displacement, when velocity is zero, is –

(A) 24 m (B) 12 m

(C) 5 m (D) Zero

Q.49 The displacement 'x' of a particle moving along a straight line at time t is given by $x=a_0+a_1t+a_2t^2$. What is the acceleration of the particle –

(A) a_1 (B) a_2

(C) $2a_2$ (D) $3a_2$

Q.50 The displacement x of a particle moving in one dimension is related to time by the equation $t = \sqrt{x} + 3$ where x is in meters and t in seconds. The displacement when velocity is zero is –
(A) 0m (B) 1m
(C) 9m (D) 4m

Q.51 The distances covered by a particle thrown in a vertical plane, in horizontal and vertical directions at any instant of time t are $x = \sqrt{21}\,t$ and $y = 2t - 4t^2$ respectively. The initial velocity of the particle will be –
(A) 2 m/s (B) 3 m/s
(C) 4 m/s (D) 5 m/s

Q.52 The coordinates of a moving particle at any time are given by $x = at^2$ and $y = bt^2$. The speed of the particle at any moment is.
(A) 2t (a + b) (B) $2t\sqrt{(a^2 - b^2)}$
(C) $t\sqrt{a^2 - b^2}$ (D) $2t\sqrt{(a^2 + b^2)}$

Q.53 The x and y coordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in metre and t in seconds. The acceleration of particle at t = 5s is.
(A) Zero (B) 8 m/s^2
(C) 20 m/s^2 (D) 40 m/s^2

Q.54 The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to.
(A) a/b (B) 2a/3b
(C) a/3b (D) Zero

Q.55 The displacement of a particle starting from rest (at t = 0) is given by $s = 6t^2 - t^3$. The time in, seconds at which the particle will attain zero velocity again, is –
(A) 2 (B) 4
(C) 6 (D) 8

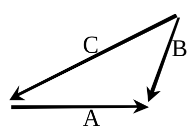
Q.56 A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dx} = c = \frac{dy}{dt}$. Where a, b and c are constants. The acceleration of the particle is –
(A) $(6ac^2x + 2bc^2)\,\hat{k}$ (B) $(2ax^2 + 6by^2)\,\hat{k}$
(C) $(4bc^2x + 3ac^2)\,\hat{k}$ (D) $(bc^2x + 2by)\,\hat{k}$

Q.57 A particle moves along a straight line such that its displacement at any time t is given by
$S = t^3 - 6t^2 + 3t + 4$ metres. The velocity when the acceleration is zero is.
(A) 3 ms^{-1} (B) – 12 ms^{-1}
(C) 42 ms^{-1} (D) – 9 ms^{-1}

EXERCISE - 2 [LEVEL-2]

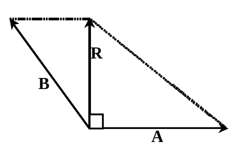
Q.1 If A = B + C and magnitudes of A, B and C are 5, 4, and 3 units respectively, the angle between A and C is :-
(A) $\sin^{-1}(3/4)$ (B) $\cos^{-1}(4/5)$
(C) $\cos^{-1}(3/5)$ (D) $\pi/2$

Q.2 For the figure :



(A) $\vec{A} + \vec{B} = \vec{C}$ (B) $\vec{B} + \vec{C} = \vec{A}$
(C) $\vec{C} + \vec{A} = \vec{B}$ (D) $\vec{A} + \vec{B} + \vec{C} = 0$

Q.3 The resultant of two vectors A and B is perpendicular to the vector A and its magnitude is equal to half the magnitude of vector B. The angle between A and B is :-



(A) 120º (B) 150º
(C) 135º (D) None of these

Q.4 If $\vec{A}$ and $\vec{B}$ denote the sides of a parallelogram and its area is AB/2, the angle between $\vec{A}$ and $\vec{B}$ is :-
(A) $\pi/2$ (B) π
(C) $\pi/6$ (D) $\pi/3$

Q.5 What displacement must be added to the displacement $25\hat{i} - 6\hat{j}\,m$ to give a displacement of 7.0 m pointing in the x-direction ?
(A) $18\hat{i} - 6\hat{j}$ (B) $32\hat{i} - 13\hat{j}$
(C) $-18\hat{i} + 6\hat{j}$ (D) $-25\hat{i} + 13\hat{j}$

Q.6 The value of a unit vector in the direction of vector A = $5\hat{i} - 12\hat{j}$ is :
(A) $\hat{i}$ (B) $\hat{j}$
(C) $(\hat{i} + \hat{j})\,/\,13$ (D) $(5\hat{i} - 12\hat{j})/13$

Q.7 Two vectors A and B lie in X-Y plane. The vector B is perpendicular to vector A. If A $\hat{i} + \hat{j}$, then B may be :-
(A) $\hat{i} - \hat{j}$ (B) $-\hat{i} + \hat{j}$
(C) $-2\hat{i} + 2\hat{j}$ (D) Any of the above

Q.8 The two vectors A = $2\hat{i} + \hat{j} + 3\hat{k}$ and B = $7\hat{i} - 5\hat{j} - 3\hat{k}$ are
(A) parallel (B) perpendicular
(C) anti-parallel (D) none of these

Q.9 $0.4\hat{i} + 0.8\hat{j} + c\hat{k}$ represents a unit vector, when c is :-
(A) 0.2 (B) $\sqrt{0.2}$
(C) $\sqrt{0.8}$ (D) 0

Q.10 The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and ω with $\vec{B}$, then :-
(A) $\alpha < \beta$ (B) $\alpha < \beta$ if A < B
(C) $\alpha < \beta$ if A > B (D) $\alpha < \beta$ if A = B

Q.11 I started walking down a road to day-break facing the sun. After walking for some-time, I turned to my left, then I turned to the right once again. In which direction was I going then ?

(A) East (B) North-west
(C) North-east (D) South

Q.12 If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$, the angle between $\vec{A}$ and $\vec{B}$ is :-

(A) 60º (B) 0º
(C) 120º (D) 90º

Q.13 $\vec{a}=3\hat{i}+5\hat{j}$, $\vec{b}=2\hat{i}+7\hat{j}$ and $\vec{c}=\hat{i}+9\hat{j}$ are three vectors which of the following combinations is in the same direction as $\vec{a}+\vec{b}-\vec{c}$:

(A) $2\vec{a}+\vec{b}$ (B) $2\vec{a}-\vec{b}$
(C) $\vec{a}-2\vec{b}$ (D) None of these

Q.14 If vectors A = $\hat{i}+2\hat{j}+4\hat{k}$ and B = $5\hat{i}$ represent the two sides of a triangle, then the third side of the triangle has length equal to :-

(A) $\sqrt{56}$ (B) $\sqrt{21}$
(C) 5 (D) 6

Q.15 If $\vec{a}$ be a unit vector, then :-

(A) direction of $\vec{a}$ is constant
(B) magnitude of $\vec{a}$ is constant
(C) both (A) and (B)
(D) any one of direction or magnitude is constant.

Q.16 If two numerical equal forces P and P acting at a point produce a resultant force of magnitude P itself, then the angle between the two original forces is :-

(A) 0º (B) 60º
(C) 90º (D) 120º

Q.17 Given that $\vec{A}+\vec{B}=\vec{R}$ and $\vec{A}+2\vec{B}$ is perpendicular to $\vec{A}$. Then :-

(A) 2B = R (B) B = 2R
(C) B = R (D) $B^2 = 2R^2$

Q.18 The value of λ for two perpendicular vectors $\vec{A}=2\vec{i}+\lambda\vec{j}+\vec{k}$ and $\vec{B}=4\vec{i}-2\vec{j}-2\vec{k}$ is :-

(A) 4 (B) 3
(C) 1 (D) 6

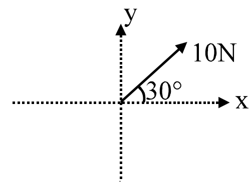
Q.19 The resultant of two vectors of magnitude 3 units and 4 units is 1 unit. What is the value of their dot product.?

(A) –12 units (B) –7 units
(C) –1 unit (D) zero

Q.20 If $\vec{i}$ and $\vec{j}$ are unit vectors along x-axis and y-axis respectively, the magnitude of vector $\vec{i}+\vec{j}$ will be :-

(A) 1 (B) $\sqrt{2}$
(C) $\sqrt{3}$ (D) 2

Q.21 Write a force of 10 N in x–y plane in terms of unit vectors $\hat{i}$ and $\hat{j}$ if it makes an angle 30° with x–axis as shown in figure :-



(A) $5\sqrt{3}\hat{i}+5\hat{j}$ (B) $5\hat{i}+5\sqrt{3}\hat{j}$
(C) $5\hat{i}-5\sqrt{3}\hat{j}$ (D) $5\sqrt{3}\hat{i}-5\hat{j}$

Q.22 Moment about point whose coordinate is (1, 2, 3) of a force represented by $\vec{i}+\vec{j}+\vec{k}$ acting at the point (–2, 3,1) is :-

(A) $6\vec{i}+2\vec{j}-8\vec{k}$ (B) $\vec{i}$
(C) $-3\vec{i}-\vec{j}+4\vec{k}$ (D) $3\vec{i}+\vec{j}-4\vec{k}$

Q.23 A truck travelling due north with 20 m/s turns towards west and travels at the same speed. Then the change in velocity is :-

(A) 40 m/s north-west
(B) 20 $\sqrt{2}$ m/s north-west
(C) 40 m/s south-west
(D) 20 $\sqrt{2}$ m/s south-west

Q.24 A vector perpendicular to $(\hat{i}+\hat{j}-\hat{k})$ and $(\hat{i}-\hat{j}-\hat{k})$ is :-

(A) $\hat{i}+\hat{j}+\hat{k}$ (B) $\hat{i}+\hat{k}$
(C) $-\hat{i}+\hat{j}+\hat{k}$ (D) $\hat{j}+\hat{k}-2\hat{i}$

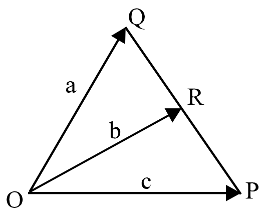
Q.25 Which of the following is **not true** ? If $\vec{A}=3\hat{i}+4\hat{j}$ and $\vec{B}=6\hat{i}+8\hat{j}$ where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$?

(A) $\vec{A}\times\vec{B}=0$ (B) $\frac{A}{B}=\frac{1}{2}$
(C) $\vec{A}.\vec{B}=48$ (D) A = 5

Q.26 Two constant forces $\vec{F}_1$ and $\vec{F}_2$ acts on a body of mass 8 kg. These forces displaces the body from point P(1, –2, 3) to Q(2, 3, 7) in 2s starting from rest. Force $\vec{F}_1$ is of magnitude 9 N and is acting along vector $(2\hat{i}-2\hat{j}+\hat{k})$. Work done by the force $\vec{F}_2$ is :-

(A) 80 J (B) –80 J
(C) –180 J (D) 180 J

Q.27 Figure shows three vectors a, b and c. If RQ = 2PR, which of the following relations is correct ?



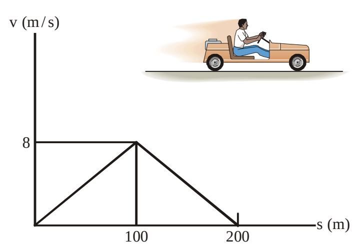
(A) 2a + c = 3b (B) a + 3c = 2b
(C) 3a + c = 2b (D) a + 2c = 3b

Q.28 Component of $-10\hat{j}$ in the direction of $3\hat{i}-4\hat{j}$ is :-
(A) 8 (B) 2
(C) –4 (D) 1

Q.29 If $3\hat{i}-2\hat{j}+8\hat{k}$ and $2\hat{i}+x\hat{j}+\hat{k}$ are at right angles then x=
(A) 7 (B) – 7
(C) 5 (D) – 4

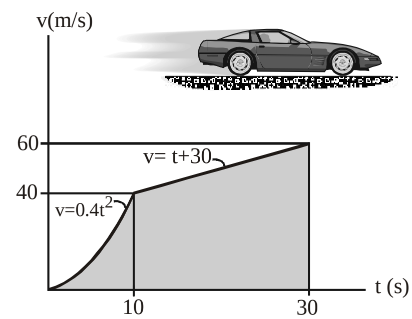
Q.30 If $\vec{a}=4\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{c}=-\hat{i}-2\hat{j}+\hat{k}$, then the value of $|\vec{a}-\vec{b}-\vec{c}|$ is :-
(A) $\sqrt{52}$ (B) $\sqrt{45}$
(C) $\sqrt{54}$ (D) $\sqrt{50}$

Q.31 The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at s = 50 m.



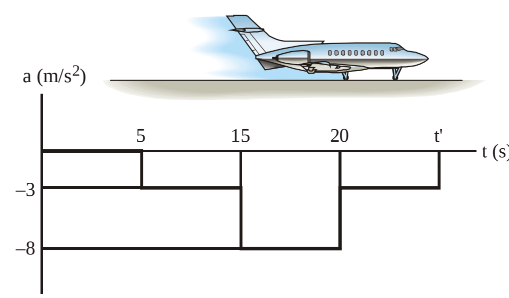
(A) 0.16 m/s^2 (B) 0.32 m/s^2
(C) 0.48 m/s^2 (D) 0.62 m/s^2

Q.32 The v–t graph for the motion of a car as it moves along a straight road is shown. Determine the maximum acceleration during the 30-s time interval. The car starts from rest at s = 0.



(A) 4 m/s^2 (B) 5 m/s^2
(C) 6 m/s^2 (D) 8 m/s^2

Q.33 An airplane lands on the straight runway, originally traveling at 110 m/s when s = 0. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane.



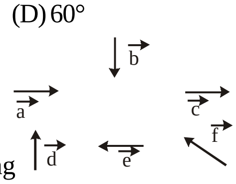
(A) 13.3 s (B) 23.3 s
(C) 33.3 s (D) 43.3 s

EXERCISE - 3

PREVIOUS YEARS AIPMT / NEET QUESTIONS

Q.1 $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A}\times\vec{B}|=\sqrt{3}\,(\vec{A}.\vec{B})$, the value of θ is – **[AIPMT 2007]**
(A) 45° (B) 30°
(C) 90° (D) 60°

Q.2 Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure.Which of the following statements is true? **[AIPMT (PRE) 2010]**



(A) $\vec{b}+\vec{c}=\vec{f}$ (B) $\vec{d}+\vec{c}=\vec{f}$
(C) $\vec{d}+\vec{e}=\vec{f}$ (D) $\vec{b}+\vec{e}=\vec{f}$

Q.3 If vectors $\vec{A}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ and $\vec{B}=\cos\frac{\omega t}{2}\hat{i}+\sin\frac{\omega t}{2}\hat{j}$ are are fcunctions of time, then the value of t at which they are orthogonal to each other is : **[RE-AIPMT 2015]**
(A) t = 0 (B) t = π/4ω
(C) t = π/2ω (D) t = π/ω

Q.4 Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At their initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$. respectively. The condition for particle A and B for their collision is **[RE-AIPMT 2015]**

(A) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (B) $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

(C) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$ (D) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

Q.5 A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true? **[NEET 2016 PHASE 1]**

(A) Velocity and acceleration both are perpendicular to $\vec{r}$.

(B) Velocity and acceleration both are parallel to $\vec{r}$.

(C) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

(D) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

Q.6 If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is **[NEET 2016 PHASE 1]**

(A) 0° (B) 90°

(C) 45° (D) 180°

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	B	C	B	A	A	B	C	B	D	A	C	A	A	A	D	B	D	B	C	C	C	A	D	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	D	B	A	A	C	B	D	A	B	D	B	C	D	A	A	A	C	C	A	A	A	D	D	A	A
Q	51	52	53	54	55	56	57																		
A	D	D	B	C	B	A	D																		

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	C	B	C	C	D	D	B	B	C	A	C	B	A	C	D	C	B	A	B	A	D	D	B	C
Q	26	27	28	29	30	31	32	33																	
A	D	D	A	A	B	B	D	C																	

EXERCISE-3

Q	1	2	3	4	5	6
A	D	C	D	B	C	B

SOLUTIONS

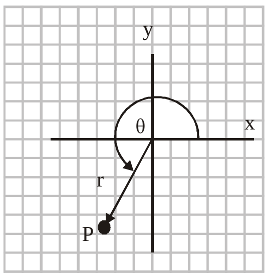
MATHEMATICS IN PHYSICS

TRY IT YOURSELF-1

(1) **(CD).** Vectors are equal if they have the same magnitude and point in the same direction. It doesn't matter where they are drawn.

(2) **(C).** If you divide a vector by its length then the resulting vector has length one hence is a unit vector pointing in the same direction as the original vector.

(3) When the polar coordinates (r, θ) of a point P are known, the cartesian coordinates can be found:



$x = r\cos\theta\,;\; y = r\sin\theta$
$x = (5.50\text{ m})\cos 240° = -2.75\text{ m}$
$y = (5.50\text{ m})\sin 240° = -4.76\text{ m}$

(4) Using the graph given in problem-Figure

In unit vector notation: $\vec{A} = 2.25\hat{i} + 0.5\hat{j}$

$$\vec{C} = -0.72\hat{i} - \hat{j}$$

In polar notation: $\vec{B} = 1.3\angle 125°$.

$$\vec{D} = 2.6\angle -20°.$$

Note: All angles must be measured either clockwise or counterclockwise from the +x-axis.

(5) (i) $\vec{A}+\vec{B} = 8\hat{i} - 2\hat{j}\ ;\ \vec{A}-\vec{B} = -2\hat{i} + 10\hat{j}$

$2\vec{A}+3\vec{B} = 6\hat{i}+8\hat{j}+15\hat{i}-18\hat{j} = 21\hat{i}-10\hat{j}$

$\vec{C} = -\vec{A}-\vec{B} = -8\hat{i}+2\hat{j}$

(ii) $A = \sqrt{3^2+4^2} = 5$

(6) **(B).** We can calculate the scalar product between two vectors in a Cartesian coordinates system as follows:

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = (2)(-6) + (3)(4) = 0$$

When the scalar product of two vectors is zero then are perpendicular.

(7) $(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = \vec{A}\cdot\vec{A} - \vec{B}\cdot\vec{B} + \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} = A^2 - B^2$

Since, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

So, in order for the dot product to be 0, A^2 must equal B^2 and $A = B$.

(8) (a) $\vec{A}\times\vec{B} = (-3\hat{i}+4\hat{j})\times(2\hat{i}+3\hat{j})$

$$\vec{A}\times\vec{B} = (-6\hat{i}\times\hat{i}) - (9\hat{i}\times\hat{j}) + (8\hat{j}\times\hat{i}) + (12\hat{j}\times\hat{j})$$

$$= 0 - 9\hat{k} + 8(-\hat{k}) + 0 = -17\hat{k}$$

(b) Since $|A\times B| = |AB\sin\theta|$

$$\theta = \sin^{-1}\frac{|\vec{A}\times\vec{B}|}{AB} = \sin^{-1}\left(\frac{17}{\sqrt{3^2+4^2}\,\sqrt{2^2+3^2}}\right) = 70.6°$$

(9) The vector describing the displacement is

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1\ ;\ \vec{r}_{12} = (4\text{m})\,\hat{x} + (-3\text{m})\,\hat{y}.$$

The length of this vector is the magnitude of the vector, or 5 m. From this, our unit vector is

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{4}{5}\hat{x} - \frac{3}{5}\hat{y}$$

(10) $\sqrt{10}A = \sqrt{(2A)^2 + (\sqrt{2}A)^2 + 2\times 2A\times\sqrt{2}A\cos\theta}$

Square both sides $\cos\theta = 1/\sqrt{2} \Rightarrow \theta = 45°$

(11) $\sqrt{13}Q = \sqrt{P^2+Q^2+2PQ\cos 60°}$,

$$\therefore\ \sqrt{13} = \sqrt{\left(\frac{P}{Q}\right)^2 + 1 + 2 + \left(\frac{P}{Q}\right)\frac{1}{2}}$$

$$\therefore\ \left(\frac{P}{Q}\right)^2 + \left(\frac{P}{Q}\right) - 12 = 0\ ;\ \frac{P}{Q} = -4 \text{ or } 3$$

Neglecting –4 we have, (P/Q) = 3

(12) $\tan 90° = \dfrac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow P + Q\cos\theta = 0$

$$\cos\theta = \frac{-P}{Q} \qquad \therefore \theta = \cos^{-1}\left(\frac{-P}{Q}\right)$$

(13) Let the force be F. Then, $F\cos 45° = 25$

or $F(1/\sqrt{2}) = 25$ or $F = 25\sqrt{2}N$

The magnitude of the other component is ,

$$F\sin 45° = 25\sqrt{2}\times\frac{1}{\sqrt{2}} = 25\ N$$

(14) Let other comp. to be Q.

$$60 = \sqrt{30^2+Q^2} \Rightarrow 60^2 - 30^2 = Q^2$$

$$\Rightarrow Q = \sqrt{60^2-30^2} = \sqrt{2700} = 10\sqrt{27} = 30\sqrt{3}\text{ kmh}^{-1}$$

(15) Given : $\vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow \vec{c} = \vec{a}+\vec{b}$

Also, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Let angle between $\vec{a}$ and $\vec{b} = \theta$

$$\therefore\ 1 = \sqrt{1^2+1^2+2\times1\times1\times\cos\theta}$$

$$\therefore\ \cos\theta = -1/2 \Rightarrow \theta = 120° = 2\pi/3$$

TRY IT YOURSELF-2

(1) Let log x = t .... (1)

$\therefore$ y = sin t

$\therefore \frac{dy}{dt} = \cos t$ .... (2)

Eq. (1) gives $\frac{dt}{dx} = \frac{1}{x}$ .... (3)

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times \frac{1}{x} = \cos\log x \times \frac{1}{x} = \frac{\cos\log x}{x}$

(2) Given curve is $y = x^2 + 3x + 4$

Differentiating both sides wrt x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) + \frac{d}{dx}(4) \Rightarrow \frac{dy}{dx} = 2x + 3$$

$$\left[\because \frac{d}{dx}(x) = 1 \,\&\, \frac{d}{dx}(4) = 0\right]$$

$$\therefore \left[\frac{dy}{dx}\right]_{at\ (-1,2)} = 2(-1) + 3 = 1$$

i.e. slope of the curve at (–1, 2) is 1.

$\therefore$ $\tan\phi = 1$ i.e. $\phi = 45°$

(3) The velocity v at any instant is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}(5 + 6t + 7t^2) = 0 + 6 \times 1 + 7 \times 2t = 6 + 14t$$

(i) For initial velocity, t = 0

$\therefore$ Initial velocity = 6 + 14 × 0 = 6 m/s

(ii) When t = 3, velocity, v = 6 + 14 × 3 = 6 + 42 = 48 m/s

(iii) Acceleration, a at any time t is

$$a = \frac{dv}{dt} = \frac{d}{dt}(6 + 14t) = 0 + 14 \times 1 = 14 \text{ m/s}^2$$

(iv) At t = 5s, displacement,

$x = 5 + 6 \times 5 + 7 \times 5^2 = 5 + 30 + 175 = 210$m.

(4) Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{t^3}{3} + \frac{t^2}{2} + 1\right) = \frac{d}{dt}$

$$= \left(\frac{t^3}{3}\right) + \frac{d}{dt}\left(\frac{t^2}{2}\right) + \frac{d}{dt}(1)$$

$$a = \frac{1}{3} \times 3t^{3-1} + \frac{1}{2}2t^{2-1} + 0 = t^2 + t$$

When t = 3 s, $a = 3^2 + 3 = 12$ m/s^2

As, Force = mass × acceleration = 2.5 × 12 = 30 N

(5) Here $v = (180 - 16x)^{1/2}$ or $\frac{dx}{dt} = (180 - 16x)^{1/2}$

Differentiate w.r.t. 't'

$$\frac{dv}{dt} = a = \frac{1}{2}(180 - 16x)^{-1/2}.(-16)\frac{dx}{dt}$$

$$= \frac{1}{2}(180 - 16x)^{-1/2}(-16)(180 - 16x)^{1/2}$$

$$= -8(180 - 16x)^{-1/2}(180 - 16x)^{1/2} = -8 \text{ ms}^{-2}.$$

(6) Here $x = 2t^3 - 6t^2 + 12t + 6$

Differentiate w.r.to t. $v = \frac{dx}{dt} = 6t^2 - 12t + 12$

Again differentiate w.r.t t. $a = \frac{dv}{dt} = 12t - 12$

Now a = 0 $\therefore$= 12t – 12 or t = 1 sec.

(7) At minimum, $\frac{dU}{dx} = 0$

Now $\frac{dU}{dx} = \frac{d}{dx}(100) - \frac{d}{dx}(50x) + \frac{d}{dx}(1000x^2) =$ 0 – 50 + 2000 x

At minimum $\frac{dU}{dx} = 0$ or $0 = -50 + 2000x$ or $x = \frac{1}{40} = 0.025$

The minimum occurs at x = 0.025

(8) $I = \int(\cos x - \sin x)(3 + 4\sin 2x)dx$

Here integration of cos x – sin x = sin x + cos x and 3 + 4 sin 2x = 3 + 4 ((sin x + cos x)^2 – 1)

Put sin x + cos x = t $\Rightarrow$ (cos x – sin x)dx = dt

So $I = \int(3 + 4(t^2 - 1)dt = \frac{t}{3}[4t^2 - 3] + c$

$$= \left(\frac{\sin x + \cos x}{3}\right)[4(\sin x + \cos x)^2 - 3]$$

$$= \left(\frac{\sin x + \cos x}{3}\right)(1 + 4\sin 2x) + C$$

(9) Substitute u = z^2 + 1, du = 2z dz

$$I = \int \frac{2z\,dz}{\sqrt[3]{z^2+1}} = \int \frac{du}{u^{1/3}}$$

$$\frac{u^{2/3}}{2/3} + C = \frac{3}{2}u^{2/3} + C = \frac{3}{2}(z^2+1)^{2/3} + C$$

(10) As given in the question, $F \propto v^2$ or $F = -K v^2$

or $m.a = -Kv^2$ or $m.v.\frac{dv}{dx} = -Kv^2$

or $\int_u^v \frac{1}{v}\,.\,dv = -\frac{K}{m}\int_0^x dx$ or $[\log_e v]_u^v = -\frac{K}{m}[x]_0^x$

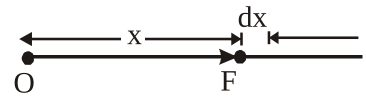
or $\log_e v - \log_e u = -\frac{K}{m}(x-0)$ or $\log_e \frac{v}{u} = -\frac{K}{m}(x)$

or $\frac{v}{u} = e^{-\frac{Kx}{m}}$ or $v = ue^{-\frac{Kx}{m}}$

(11) Let the particle at any instant be at a position x. Let, under the action of force F = a + bx, it describe a small displacement dx. (fig.)

Work done during the displacement dx will be

$$dW = F\,dx = (a+bx)\,dx$$



Total work done can be obtained by "summing up" the work done in individual element displacements (i.e., by integrating)

$$W = \int dW = \int_{x_1}^{x_2} (a+bx)\,dx \qquad [\because \text{ x varies from } x_1 \text{ to } x_2]$$

$$\Rightarrow W = \left[a + \frac{bx^2}{2}\right]_{x_1}^{x_2} = a(x_2 - x_1) + \frac{b}{2}(x_2^2 - x_1^2)$$

$$= \frac{(x_2 - x_1)}{2}[2a + b(x_1 + x_2)]$$

CHAPTER-1 : MATHEMATICS IN PHYSICS
EXERCISE-1

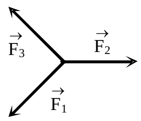
(1) **(B).** Unit vector along y axis $= \hat{j}$

So the required vector

$= \hat{j} - [(\hat{i} - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} - 7\hat{k})]$

$= -4\hat{i} - 2\hat{j} + 5\hat{k}$

(2) **(B).** $\vec{F}_3 = \vec{F}_1 + \vec{F}_2$



There should be minimum three coplaner vectors having different magnitude which should be added to give zero resultant.

(3) **(C).** $\hat{R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{\hat{i} + \hat{j}}{\sqrt{1^2 + 1^2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

(4) **(B).** Magnitude of unit vector = 1

$\hat{G}$ $\quad \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

By solving we get $c = \sqrt{0.11}$

(5) **(A).** Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

(6) **(A).** $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{(3\hat{i} + 4\hat{j} + 5\hat{k})(3\hat{i} + 4\hat{j} - 5\hat{k})}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}}$

$= \dfrac{9 + 16 - 25}{50} = 0 \Rightarrow \cos\theta = 0$, $\therefore \theta = 90°$

(7) **(B).** Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $\vec{n}_s = \hat{n}_1 + \hat{n}_2$

or $\quad n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta \; = 1 + 1 + 2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta$

$\Rightarrow \quad \cos\theta = -\dfrac{1}{2} \; \therefore \theta = 120°$

Now the difference vector is $\hat{n}_d = \hat{n}_1 - \hat{n}_2$

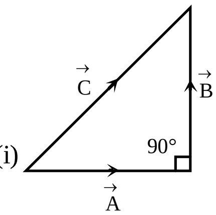
or $\quad n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta \; = 1 + 1 - 2\cos(120°)$

$\therefore \quad n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3$

$\Rightarrow n_d = \sqrt{3}$

(8) **(C).** $C = \sqrt{A^2 + B^2}$

The angle between A and B is $\pi/2$



(9) **(B).** $\dfrac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ...(i)

$\therefore \quad \tan 90° = \dfrac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$\therefore \quad \cos\theta = -\dfrac{A}{B}$. Hence, from (i)

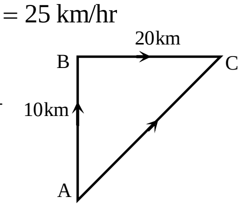
$\dfrac{B^2}{4} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3}\dfrac{B}{2}$

$\Rightarrow \quad \cos\theta = -\dfrac{A}{B} = -\dfrac{\sqrt{3}}{2} \; \therefore \; \theta = 150°$

(10) **(D).** Resultant velocity $= \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225} = \sqrt{625} = 25$ km/hr

(11) **(A).** $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$



$AC = \sqrt{(AB)^2 + (BC)^2}$

$= \sqrt{(10)^2 + (20)^2}$

$= \sqrt{100 + 400} = \sqrt{500} = 22.36$ km

(12) **(C).** Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by

$\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $\theta = 0°$ then $|\vec{R}| = A + B \; = |\vec{A}| + |\vec{B}|$

(13) **(A).** If two vectors are perpendicular then their dot product must be equal to zero.

According to problem $(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

$\Rightarrow \quad \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$

$\Rightarrow \quad A^2 - B^2 = 0 \; \Rightarrow A^2 = B^2$

$\therefore \quad$ A = B i.e. two vectors are equal to each other in magnitude.

(14) **(A).** Sum of the vectors

$\vec{R} = 5\hat{i} + 8\hat{j} + 2\hat{i} + 7\hat{j} = 7\hat{i} + 15\hat{j}$

Magnitude of $\vec{R} = |\vec{R}| = \sqrt{49 + 225} \; = \sqrt{274}$

(15) **(A).** $S = \vec{r}_2 - \vec{r}_1$

$W = \vec{F}.\vec{S} \; = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4 \times 11 + 1 \times 11 + 3 \times 15) = 100$ J.

(16) **(D).** $v = \dfrac{ds}{dt} = 9t^2 + 14t + 14$

$a = \dfrac{dv}{dt} = 18t + 14$; at t = 1, a = 32 m/s^2

(17) **(B).** $v = \frac{dx}{dt} = 8t - 4t^3$; $a = \frac{dv}{dt} = 8 - 12t^2$

$t = 2, a = -40\ m/s^2$

(18) **(D).** Use $v = \frac{dx}{dt}$; $a = \frac{dv}{dt}$

(19) **(B).** $v = \frac{dS}{dt} = 2at - 3bt^2$

Acceleration $= \frac{dv}{dt} = 2a - 6bt$

Acceleration $= 0 \Rightarrow 2a = 6bt \Rightarrow t = a/3b$

(20) **(C).** $v_y = \frac{dy}{dt} = \beta 2x\frac{dx}{dt} = 2\beta x v_x$

$a_y = \frac{dv_y}{dt} = 2\beta v_x \frac{dx}{dt}$; $\alpha = 2\beta v_x^2$; $v_x = \sqrt{\alpha / 2\beta}$

(21) **(C).** $v = \frac{ds}{dt} = 12 + 6t - 6t^2$ at $t = 0, v = 12$ m/s

(22) **(C).** $KE = \frac{1}{2}mv^2$; $\frac{dKE}{KE} = \frac{dm}{m} + 2\frac{dv}{v} = 2 + 2 \times 3 = 8\%$

(23) **(A).** $a = \frac{dv}{dt} = v\frac{dx}{dt}$; $\frac{dv}{dx} = 2x - 5$

at $x = 1, v = 0, a = 0$

(24) **(D).** $v_y = \frac{dy}{dt} = 2x\frac{dx}{dt}$; at $x = \frac{1}{2}, v_y = 4$

$v = \sqrt{v_x^2 + v_y^2} = 4\sqrt{2} m/s$

(25) **(C).** $\frac{dy}{dt} = b + 2ct - 4dt^3$; at $t = 0$ $\frac{dy}{dt} = b$

Acc. $= 2c - 12dt^2$; at $t = 0$, acc. $= 2c$

(26) **(D).** Surface area of sphere, $s = 4\pi r^2$

or $\frac{ds}{s} = 2\left(\frac{dr}{r}\right)$ [4π is constant]

or $\frac{ds}{s} = 2 \times 2\%$ or $\frac{ds}{s}\% = 4\%$

(27) **(B).** As kinetic Energy $K = \frac{1}{2}mv^2$

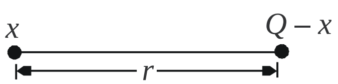
or $K = \frac{1}{2}mv^2 \times \frac{m}{m}$ or $K = \frac{1}{2m}(mv)^2$ or $K = \frac{1}{2m}(P)^2$

or $\frac{dK}{K} = 2\frac{dP}{P}$ [$\therefore$ m is constant]

or $\frac{dK}{K}\% = 2 \times 1\% = 2\%$

This formula will be applied only when the given % error is less than 5% (because differentiation generated formulae are applicable for small changes only).

(28) **(A).**



(i) We have to find maximum value i.e. maxima
(ii) Force F is to be maximised.
(iii) x is the variable quantity.
(iv) The relation between F and x is

$\Rightarrow$ As $F = K\frac{q_1 q_2}{r^2}$

Let $q_1 = x$ and $q_2 = Q - x$ $\therefore$ $F = K$

$\frac{x(Q-x)}{r^2} = \frac{K}{r^2}(Qx - x^2)$

Differentiating F w.r.t. x we get

$\frac{dF}{dx} = \frac{K}{r^2}(Q - 2x)$

Now putting $\frac{dF}{dx} = 0$ we get,

$\frac{K}{r^2}(Q - 2x) = 0$ or $Q - 2x = 0$ or $x = Q/2$

(29) **(A).** $\int_0^{\infty}\frac{GMm}{x^2}dx = \left[-\frac{GMm}{x}\right]_R^{\infty}$

$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{GMm}{R}$

(30) **(C).** Using the relation between velocity and acceleration, we get

$v = \int a\,dt = 2\int dt = 2t + c$

where c is the constant of integration.

Its value can be obtained by using the initial condition. That is at $t = 0; v = 5\ ms^{-1}$

Thus, $5 = 2(0) + c \Rightarrow c = 5\ ms^{-1}$

Therefore, $v = 2t + 5$ is the required expression for instantaneous velocity.

(31) **(B).** Using the relation between displacement and velocity, we get $\int v\,dt = \int (2t+5)\,dt$

or $x = t^2 + 5t + c$

where c is the constant of integration. Its value can be determined by using the given conditions. That is, at $t = 1$ s ; $x = 7m$

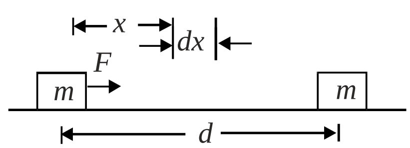
$\therefore$ $7 = (1)^2 + 5(1) + c \Rightarrow c = 1m$

Thus, $x = t^2 + 5t + 1$

(32) **(D).** **Method I :** $F \propto x$ or $F = kx$

Now small work done in displacing the body by a small distance dx is

$dw = F.dx$



So total work done in displacing the body by a distance d is

$\int dw = \int_0^d Kx.dx \quad \therefore\ w = K\int_0^d xdx$

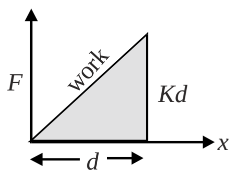
or $\quad w = K\left[\frac{x^2}{2}\right]_0^d = K\left[\frac{d^2}{2} - 0\right]$

or $\quad w = K\frac{d^2}{2}$

Method II : Work done is equal to the area under the force-displacement graph.

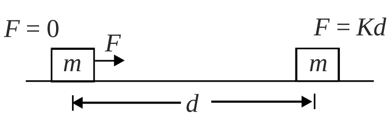
w = Area under F – x graph

$= \frac{1}{2} \times d \times Kd = \frac{Kd^2}{2}$



Method III : Work done is given by the product of average force and total displacement.

$w = F_{av} \times d = \left(\frac{0 + Kd}{2}\right) d = \frac{Kd^2}{2}$



This method of average is used when relation is linear.

(33) **(A).** Between t = 0 and t = 2 sec, it is evident that $\frac{da}{dt} = 1 \Rightarrow da = dt$

Integrating both sides, we have

$\int_0^a da = \int_0^t dt \Rightarrow a = t$ [$\because$ Initial acceleration is 0]

$\therefore$ At t = 2 sec, acceleration = 2 m/s²

Alternative : Change in acceleration = Area below the graph, $\quad a - 0 = 2 \times 1 = 2\ m/s^2$

(34) **(B).** $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= (1\times 4 - 2\times -2)\hat{i} + (2\times 2 - 4\times 3)\hat{j} + (3\times -2 - 1\times 2)\hat{k}$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\therefore$ Magnitude of

$\vec{A}\times\vec{B} = |\vec{A}\times\vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2} \quad = 8\sqrt{3}$

(35) **(D).** $\vec{F}_1.\vec{F}_2 = (2\hat{j} + 5\hat{k})(3\hat{j} + 4\hat{k}) \ = 6 + 20 = 20\ + 6 = 26$

(36) **(B).** $P = \vec{F}.\vec{v} = 20\times 6 + 15\times(-4) + (-5)\times 3$

$= 120 - 60 - 15 = 120 - 75 = 45\ J/s$

(37) **(C).** $P_x = 2\cos t\ ,\ P_y = 2\sin t \quad \therefore \vec{P} = 2\cos t\ \hat{i} + 2\sin t\ \hat{j}$

$\vec{F} = \frac{d\vec{P}}{dt} = -2\sin t\ \hat{i} + 2\cos t\ \hat{j}\ ;\ \vec{F}.\vec{P} = 0 \quad \therefore\ \theta = 90°$

(38) **(D).** $\sqrt{2^2 + 3^2 + 2\times 2\times 3\times\cos\theta} = 1$

By solving we get θ = 180° $\therefore\ \vec{A}\times\vec{B} = 0$

(39) **(A).** $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix} = \hat{i}(6-8) - \hat{j}(-3) + 4\hat{k}$

$-2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{(-2)^2 + (3)^2 + 4^2} \ = \sqrt{29}$ unit

(40) **(A).** $\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$

i.e. the angular momentum is perpendicular to x-axis.

(41) **(A).** Mass $= \frac{\text{Force}}{\text{Acceleration}} = \frac{|\vec{F}|}{a}$

$= \frac{\sqrt{36+64+100}}{1} = 10\sqrt{2}$ kg

(42) **(C).** $\vec{A}.\vec{B} = AB\cos\theta$

In the problem $\vec{A}.\vec{B} = -AB$ i.e. cos θ = –1 $\therefore\ \theta = 180°$

i.e. $\vec{A}$ and $\vec{B}$ acts in the opposite direction.

(43) **(C).** $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

for parallel vectors θ = 0° or 180°, sin θ = 0

$\therefore\ \vec{A}\times\vec{B} = \hat{0}$

(44) **(A).** According to Lami's theorem, $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$

(45) **(A).** $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\overrightarrow{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\therefore\ (\vec{a}\times\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$

$= (12-2)\hat{i} + (4+6)\hat{j} + (3+12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k} \ \Rightarrow |\vec{a}\times\vec{b}| = \sqrt{10^2 + 10^2 + 15^2}$

$= \sqrt{425} = 5\sqrt{17}$

(46) $F_1 + F_2 = 16$; $F_1^2 + (8)^2 = 2$
$F_2^2 = (16 - F_1^2)^2$ $\therefore$ $F_1 = 6$ N, $F_2 = 10$ N

(47) **(D).** $|\vec{A} \times \vec{B}| = \sqrt{3}\vec{A}.\vec{B} \Rightarrow AB\sin\theta = \sqrt{3}\ AB\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

Therefore, $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 + 2AB\cos 60°} = \sqrt{A^2 + B^2 + AB}$

(48) **(D).** $x = \dfrac{(3t-6)^2}{3}$; $\dfrac{dx}{dt} = 2\,(3t-6) = 0$
$t = 2, x = 0$

(49) **(A).** Use $v = \dfrac{dx}{dt}$; $a = \dfrac{dv}{dt}$

(50) **(A).** $x = (t-3)^2$; $\dfrac{dx}{dt} = 2\,(t-3)$ at $t = 3, v = 0, x = 0$

(51) **(D).** $v_x = \dfrac{dx}{dt} = \sqrt{21}$; $v_y = \dfrac{dy}{dt} = 2 - 8t$

at $t = 0$, $v_y = 2$; $v = \sqrt{v_x^2 + v_y^2} = 5$

(52) **(D).** $\dfrac{dx}{dt} = 2at$; $\dfrac{dy}{dt} = 2bt$; $v = 2t\sqrt{a^2 + b^2}$

(53) **(B).** $\dfrac{dx}{dt} = 7 + 8t$; $\dfrac{dy}{dt} = 5$; $\dfrac{d^2x}{dt^2} = 8$; $\dfrac{d^2y}{dt^2} = 0$

(54) **(C).** $\dfrac{dx}{dt} = 2at - 3bt^2$; Acc. $= 2a - 6bt = 0$; $t = a/3b$

(55) **(B).** $v = \dfrac{ds}{dt} = 12t - 3t^2 = 0$; $t = 4$

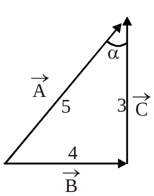
(56) **(A).** $\dfrac{dz}{dt} = 3ax^2\dfrac{dx}{dt} + 2by\dfrac{dy}{dt} = 3acx^2 + 2byc$

$\dfrac{d^2z}{dt^2} = 3ac\ 2x\dfrac{dx}{dt} + 2bc\dfrac{dy}{dt} = 6ac^2x + 2bc^2$

(57) **(D).** $v = \dfrac{ds}{dt} = 3t^2 - 12t + 3$; acc. $= \dfrac{dv}{dt} = 6t - 12$
acc. = 0, at t = 2
at t = 2, v = – 9m/s

EXERCISE-2

(1) **(C).**

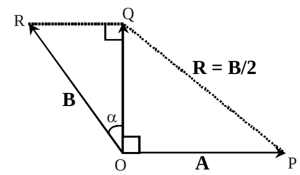


$\vec{A} = \vec{B} + \vec{C}$

$\cos\alpha = 3/5 \Rightarrow \alpha = 53°$; $\alpha = \cos^{-1}(3/5)$

(2) **(C).** From triangle law of vector addition.
$\vec{C} + \vec{A} = \vec{B}$

(3) **(B).**



$\Delta OQR : \cos\alpha = \dfrac{B/2}{B} = \dfrac{1}{2} \Rightarrow \alpha = 60°$

Hence, Angle = 90 + 60 = 150°

(4) **(C).** $|\vec{A} \times \vec{B}| = |AB\sin\theta|$

$\dfrac{AB}{2} = AB\sin\theta \Rightarrow \sin\theta = 1/2 \Rightarrow \theta = 60°$

(5) **(C).** Let $\vec{X}$ the requred displacement

We have $25\hat{i} - 6\hat{j} + \vec{X} = 7\hat{i}$

$\Rightarrow \vec{X} = -18\hat{i} + 6\hat{j}$

(6) **(D).** $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{5\hat{i} - 12\hat{j}}{13}$

(7) **(D).** Option A $\rightarrow$ $\vec{A}\cdot\vec{B} = 1 - 1 = 0$
Option B $\rightarrow$ $\vec{A}\cdot\vec{B} = -1 + 1 = 0$
Option C $\rightarrow$ $\vec{A}\cdot\vec{B} = -2 + 2 = 0$

(8) **(B).** $\vec{A}\cdot\vec{B} = 14 - 5 - 9 = 0$

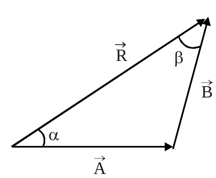
(9) **(B).** Let $\vec{A} = 0.4\hat{i} + 0.8\hat{j} + C\hat{k}$ is a unit vector

then $|\vec{A}| = 1$

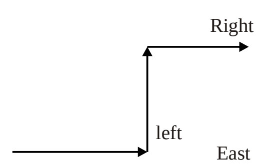
$\Rightarrow \sqrt{0.16 + 0.64 + C^2} = 1 \Rightarrow 0.80 + c^2 = 1^2$

$\Rightarrow C^2 = 0.2 \Rightarrow C = \sqrt{0.2}$

(10) **(C).**



(11) **(A).**



Hence 'East'

(12) **(C).** $R^2 = A^2 + B^2 + 2AB\cos\theta$
We have R = A = B
So, $A^2 = A^2 + 2A^2\cos\theta$
$\Rightarrow \cos\theta = -1/2 \Rightarrow \theta = 120°$

(13) **(B).** $\vec{a} + \vec{b} - \vec{c} = 4\hat{i} + 3\hat{j}$; $2\vec{a} - \vec{b} = 4\hat{i} + 3\hat{j}$

(14) **(A).** $\vec{A}+\vec{B}+\vec{C}=0$. So, $\hat{i}+2\hat{j}+4\hat{k}+5\hat{i}+\vec{C}=0$

$\vec{C}=-6\hat{i}-2\hat{j}-4\hat{k}$

$|\vec{C}|=\sqrt{36+4+16}=\sqrt{56}$

(15) **(C).**

(16) **(D).** We have $P^2=P^2+P^2+2P^2\cos\theta$

$\Rightarrow \cos\theta=\dfrac{-1}{2}$; $\theta=120^o$

(17) **(C).** $\vec{A}+2\vec{B}=(\vec{R}-\vec{B})+2\vec{B}=\vec{R}+\vec{B}$

$\vec{A}=(\vec{R}-\vec{B})$

$(\vec{A}+2\vec{B})\cdot\vec{A}=(\vec{R}+\vec{B})\cdot(\vec{R}-\vec{B})=R^2-B^2=0$

$R=B$

(18) **(B).** $\vec{B}\cdot\vec{A}=0$ (since $\vec{A}\perp\vec{B}$)

$\Rightarrow 8-2\lambda-2=0\Rightarrow\lambda=3$

(19) **(A).** We have $1^2=3^2+4^2+2\times3\times4\cos\theta$

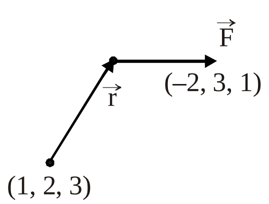
$\Rightarrow \cos\theta=\dfrac{-24}{24}=-1\Rightarrow\theta=\pi$

$\vec{A}\cdot\vec{B}=3\times4\cos\theta=-12$ units

(20) **(B).** Let $\vec{A}=\hat{i}+\hat{j}$; $|\vec{A}|=\sqrt{1^2+1^2}=\sqrt{2}$

(21) **(A).** $10\left(\cos30^o\,\hat{i}+\sin30^o\,\hat{j}\right)=\left(5\sqrt{3}\,\hat{i}+5\hat{j}\right)$

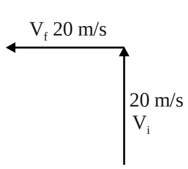
(22) **(D).**



$\vec{\tau}=\vec{r}\times\vec{F}=(\vec{r}_i-\vec{r}_0)\times\vec{F}=\left(-3\hat{i}+\hat{j}-2\hat{k}\right)\times\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-3&1&-2\\1&1&1\end{vmatrix}=3\vec{i}+\vec{j}-4\vec{k}$

(23) **(D).**



$\vec{V}_i=20\hat{j}$; $V_f=-20\hat{j}$; $\Delta\vec{V}=\vec{V}_f-\vec{V}_i=-20\,\hat{j}-20\,\hat{i}$

Hence change in velcoity is $20\sqrt{2}$ m/s SW

(24) **(B).** Let the vector be $(x\hat{i}+y\hat{j}+z\hat{k})$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})=0$

$x+y-z=0$ .... (1)

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})=0$

$x-y-z=0$ .... (2)

Subtracting (1) & (2) $\Rightarrow$ $y=0$ & $x=z$

Hence possible vector is Ans. B

(25) **(C).** Checking options : $\vec{A}=3\hat{i}+4\hat{j}$; $\vec{B}=6\hat{i}+8\hat{j}$

(A) $\vec{A}\times\vec{B}=24\hat{k}-24\hat{k}=0$

(B) $\dfrac{|\vec{A}|}{|\vec{B}|}=\dfrac{5}{10}=\dfrac{1}{2}$

(C) $\vec{A}\cdot\vec{B}=18+32=50$; $\vec{A}\cdot\vec{B}\neq48$

(D) $A=5$

(26) **(D).** $\vec{F}_1=6\hat{i}-6\hat{j}+3\hat{k}$; $\vec{S}=\vec{u}\,t-\dfrac{1}{2}\vec{a}\,t^2$

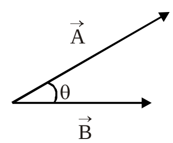
$(1\hat{i}+5\hat{j}+4\hat{k})=0+\dfrac{1}{2}\vec{a}(2)^2$

$\vec{a}=\dfrac{1\hat{i}+5\hat{j}+4\hat{k}}{2}=\dfrac{\vec{F}_1+\vec{F}_2}{8}$; $\vec{F}_2=-2\hat{i}+26\hat{j}+13\hat{k}$

$w=\vec{F}_2\cdot\vec{S}=(-2\hat{i}+26\hat{j}+13\hat{k})\cdot(1\hat{i}+5\hat{j}+4\hat{k})=180$ J

(27) **(D).** $\dfrac{a+2c}{3}=b$

(28) **(A).**



$\vec{A}\cdot\vec{B}=AB\cos\theta$; $\dfrac{\vec{A}\cdot\vec{B}}{B}=A\cos\theta$

$(A\cos\theta)=\dfrac{(-10\hat{j})\cdot(3\hat{i}-4\hat{j})}{\sqrt{3^2+4^2}}=\dfrac{+40}{5}=8$

(29) **(A).** $\left(3\hat{i}-2\hat{j}+8\hat{k}\right)\cdot\left(2\hat{i}+x\hat{j}+\hat{k}\right)=0$

$\Rightarrow 6-2x+8=0\Rightarrow x=7$

(30) **(B).** $|\vec{a}-\vec{b}-\vec{c}|=|4\hat{i}+\hat{j}-\hat{k}-3\hat{i}+2\hat{j}-2k+\hat{i}+\hat{i}+2\hat{j}-\hat{k}|$

$=|2\hat{i}+5\hat{j}-4\hat{k}|=\sqrt{4+25+16}=\sqrt{45}$

(31) **(B).** For $0 \le s < 100$

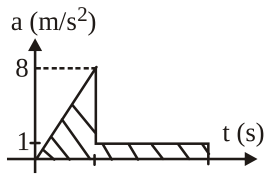
$v = 0.08s,\ dv = 0.08\ ds$

$a = v\dfrac{dv}{ds}$; $a = 6.4\ (10^{-3})\ s$

At $s = 50$ m, $a = 0.32\ m/s^2$

(32) **(D).** For $t < 10s$:

$v = 0.4t$; $a = \dfrac{dv}{dt} = 0.8\ t$



At $t = 10s$: $a = 8\ m/s^2$

For $10 < t \le 30$ s :

$v = t + 30$

$a = \dfrac{dv}{dt} = 1$

$a_{max} = 8\ m/s^2$

(33) **(C).** $v_0 = 110$ m/s

$\Delta v = \int a\ dt$

$0 - 110 = -3\ (15 - 5) - 8\ (20 - 15)$

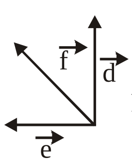
$-3\ (t' - 20)$; $t' = 33.3$ s

EXERCISE-3

(1) **(D).** $|\vec{A} \times \vec{B}| = \sqrt{3}\ (\vec{A}.\vec{B})$

$\Rightarrow AB \sin\theta = \sqrt{3}\ AB\cos\theta \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

(2) **(C).**



From figure, $\vec{d} + \vec{e} = \vec{f}$

(3) $\because\ \vec{A}\cdot\vec{B} = 0$

$\cos\omega t \cos\dfrac{\omega t}{2} + \sin\omega t \sin\dfrac{\omega t}{2} = 0$

$\cos\left(\omega t - \dfrac{\omega t}{2}\right) = 0 \Rightarrow \cos\dfrac{\omega t}{2} = 0 \Rightarrow \dfrac{\omega t}{2} = \dfrac{\pi}{2} \Rightarrow t = \dfrac{\pi}{\omega}$

(4) **(B).** For two particles to collide, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle

i.e. $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} \rightarrow$ direction of relative position of 1 w.r.t.2.

$\dfrac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|} \rightarrow$ direction of velocity of 2 w.r.t. 1

so for collision of A & B : $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \dfrac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

(5) **(C).** $\vec{r} = \cos\omega\ t\ \hat{x} + \sin\omega t\ \hat{y}$,

$\vec{v} = \dfrac{d\vec{r}}{dt} = -\omega\sin\omega t\ \hat{x} + \omega\cos\omega t\ \hat{y}$

$\vec{v}\cdot\vec{r} = 0$

$\vec{a} = -\omega^2\cos\omega t\ \hat{x} - \omega^2\sin\omega t\ \hat{y} = -\omega^2\vec{r}$

(6) **(B).** $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\cos\theta = 0 \Rightarrow \theta = 90°$