

MATHEMATICS

CHAPTER NUMBER :~ 1

CHAPTER NAME :~ NUMBER SYSTEMS

SUB TOPIC :~ OPERATIONS ON DECIMAL AND NUMBER LINE

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

1. Show how $\sqrt{5}$ can be represented on the number line.
2. Represent 4.236 on the number line by successive magnification.

LEARNING OUTCOME:~

Students will learn

a) Operations on Real Numbers

b) Representation of square root of decimals on number line c) Rationalization of expressions.

EXERCISE-1.3

Question 1.

Write the following in decimal form and say what kind of decimal expansion each has

$$(i) \frac{36}{100} \quad (ii) \frac{1}{11} \quad (iii) 4\frac{1}{8} \quad (iv) \frac{3}{13} \quad (v) \frac{2}{11} \quad (vi) \frac{329}{400}$$

Solution:

(i) We have, $\frac{36}{100} = 0.36$

Thus, the decimal expansion of $\frac{36}{100}$ is terminating.

(ii) Dividing 1 by 11, we have

$$\begin{array}{r}
 11 \overline{) 1.00000(0.090909 \dots\dots} \\
 \underline{-0} \\
 10 \\
 \underline{-00} \\
 100 \\
 \underline{-99} \\
 10 \\
 \underline{-00} \\
 100 \\
 \underline{-99} \\
 10 \\
 \underline{-00} \\
 100 \\
 \underline{-99} \\
 1
 \end{array}$$

Thus, the decimal expansion of $\frac{1}{11}$ is non-terminating repeating.

$$\therefore \frac{1}{11} = 0.090909\dots = 0.\overline{09}$$

(iii) We have, $418 = 338$
Dividing 33 by 8, we get

$$\begin{array}{r} 8 \overline{) 33.000} \quad (4.125 \\ \underline{-32} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

$\therefore 418 = 4.125$. Thus, the decimal expansion of 418 is terminating.

(iv) Dividing 3 by 13, we get

$$\begin{array}{r} 13 \overline{) 3.00000000} \quad (0.23076923\dots \\ \underline{-26} \\ 40 \\ \underline{-39} \\ 10 \\ \underline{-00} \\ 100 \\ \underline{-91} \\ 90 \\ \underline{-78} \\ 120 \\ \underline{-117} \\ 30 \\ \underline{-26} \\ 40 \\ \underline{-39} \\ 1 \end{array}$$

Here, the repeating block of digits is 230769

$$\therefore 313 = 0.23076923\dots = 0.230769\overline{}$$

Thus, the decimal expansion of $\frac{3}{13}$ is non-terminating repeating.

(v) Dividing 2 by 11, we get

$$\begin{array}{r} 11 \overline{) 2.0000} (0.1818..... \\ \underline{-11} \\ 90 \\ \underline{-88} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ 2 \end{array}$$

Here, the repeating block of digits is 18.

$$\therefore 211 = 0.1818... = 0.18\bar{18}$$

Thus, the decimal expansion of $\frac{2}{11}$ is non-terminating repeating.

Question 2.

You know that $17 = 0.142857\bar{}$. Can you predict what the decimal expansions of 27 , 137 , 47 , 57 , 67 are , without actually doing the long division? If so, how?

Solution:

We are given that $17 = 0.142857\bar{}$.

$$\therefore 27 = 2 \times 17 = 2 \times (0.142857\bar{}) = 0.285714\bar{}$$

$$37 = 3 \times 17 = 3 \times (0.142857\bar{}) = 0.428571\bar{}$$

$$47 = 4 \times 17 = 4 \times (0.142857\bar{}) = 0.571428\bar{}$$

$$57 = 5 \times 17 = 5 \times (0.142857\bar{}) = 0.714285\bar{}$$

$$67 = 6 \times 17 = 6 \times (0.142857\bar{}) = 0.857142\bar{}$$

Thus, without actually doing the long division we can predict the decimal expansions of the given rational numbers.

Question 3.

Express the following in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $0.6\bar{6}$

(ii) $0.47\bar{6}$

(iii) $0.001\bar{6}$

Solution:

(i) Let $x = 0.6\bar{6} = 0.6666\dots$ (1)

As there is only one repeating digit, multiplying (1) by 10 on both sides, we get

$$10x = 6.6666\dots \quad (2)$$

Subtracting (1) from (2), we get

$$10x - x = 6.6666\dots - 0.6666\dots$$

$$\Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$$

Thus, $0.6\bar{6} = \frac{2}{3}$

Question 4.

Express $0.99999\dots$ in the form $\frac{p}{q}$ Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:

Let $x = 0.99999\dots$ (i)

As there is only one repeating digit, multiplying (i) by 10 on both sides, we get

$10x = 9.9999\dots$ (ii)

Subtracting (i) from (ii), we get

$10x - x = (9.9999\dots) - (0.9999\dots)$

$\Rightarrow 9x = 9 \Rightarrow x = \frac{9}{9} = 1$

Thus, $0.9999\dots = 1$

As $0.9999\dots$ goes on forever, there is no such a big difference between 1 and $0.9999\dots$

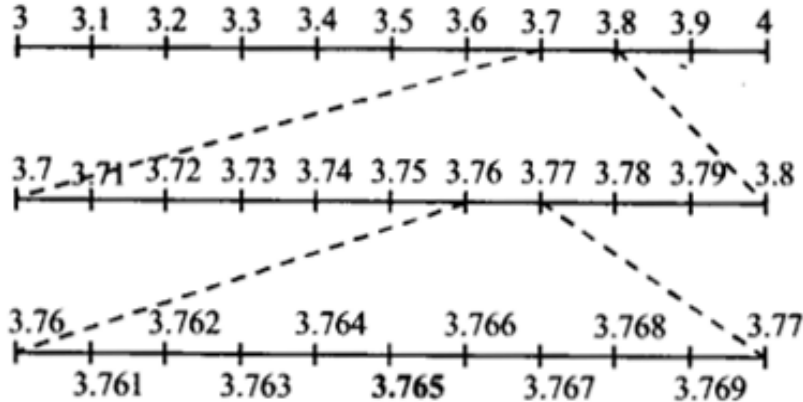
Hence, both are equal.

EXERCISE~2.4

Question 1.

Visualise 3.765 on the number line, using successive magnification.

Solution:



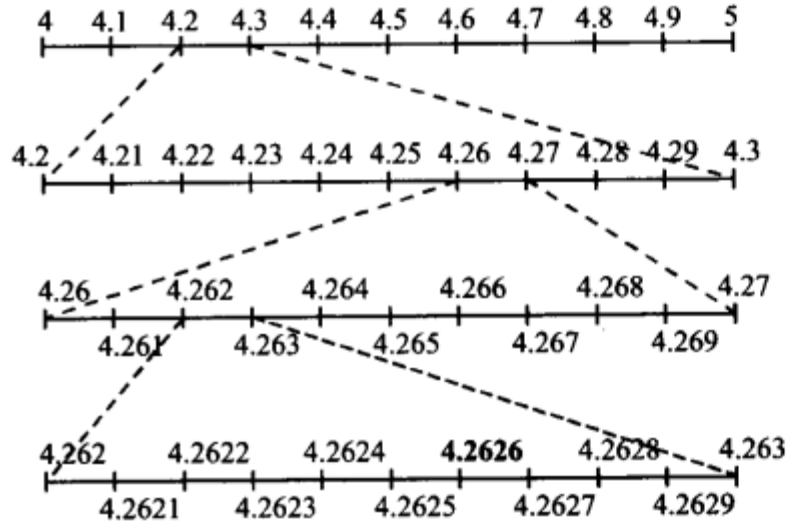
(i) 3.7 lies between 3 and 4

(ii) 3.76 lies between 3.7 and 3.8

(iii) 3.765 lies between 3.76 and 3.77

Question 2.

Visualise $4.26\bar{7}$ on the number line, upto 4 decimal places.



(i) 4.2 lies between 4 and 5

(ii) 4.26 lies between 4.2 and 4.3

(iii) 4.262 lies between 4.26 and 4.27

(iv) 4.2626 lies between 4.262 and 4.263

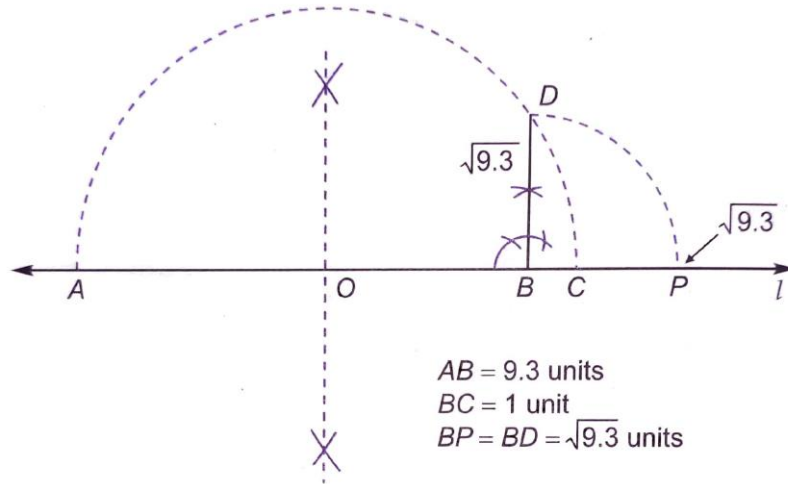
https://www.youtube.com/watch?v=8Br5b_RCwcs

“Numbers Are Intellectual Witness That Belong Only To Mankind...”

Represent $\sqrt{9.3}$ on the number line.

Solution. Steps of construction :

1. On the number line l , draw $AB = 9.3$ units.
2. Extend AB to point C such that $BC = 1$ unit.
3. Draw the perpendicular bisector of AC and mark the midpoint of AC as O .
4. With O as centre, draw a semicircle of radius $OA = OC$.
5. From B , draw $BD \perp AC$, intersecting the semicircle at D .
6. With B as centre, draw an arc of radius BD intersecting the number line at P .



Add $(3\sqrt{2} + 4\sqrt{3})$ and $(2\sqrt{2} - \sqrt{3})$.

Solution. $(3\sqrt{2} + 4\sqrt{3}) + (2\sqrt{2} - \sqrt{3}) = (3\sqrt{2} + 2\sqrt{2}) + (4\sqrt{3} - \sqrt{3}) = (3+2)\sqrt{2} + (4-1)\sqrt{3} = 5\sqrt{2} + 3\sqrt{3}$.

Multiply $4\sqrt{5}$ by $3\sqrt{5}$.

Solution. $4\sqrt{5} \times 3\sqrt{5} = 4 \times 3 \times \sqrt{5} \times \sqrt{5} = 12 \times 5 = 60$.

Divide $12\sqrt{15}$ by $4\sqrt{3}$.

Solution. $12\sqrt{15} \div 4\sqrt{3} = \frac{12\sqrt{15}}{4\sqrt{3}} = \frac{3 \times \sqrt{5} \times \sqrt{3}}{\sqrt{3}} = 3\sqrt{5}$.

Simplify each of the following expressions :

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

(iii) $(\sqrt{5} + \sqrt{2})^2$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Solution.

(i) $(3 + \sqrt{3})(2 + \sqrt{2}) = 3(2 + \sqrt{2}) + \sqrt{3}(2 + \sqrt{2}) = 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$.

(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$.

$$[(a+b)(a-b) = a^2 - b^2]$$

(iii) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \cdot \sqrt{5}\sqrt{2}$
 $= 5 + 2 + 2\sqrt{10} = 7 + 2\sqrt{10}$.

$$[(a+b)^2 = a^2 + b^2 + 2ab]$$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$.

$$[(a+b)(a-b) = a^2 - b^2]$$

Evaluation:~

1. Represent $\sqrt{5.2}$ on the number line.
2. $(\sqrt{5} - \sqrt{2}) + (\sqrt{5} + \sqrt{2})$.
3. $(\sqrt{3} + 2)(\sqrt{3} - 2)$.

Homework :

Exercise 1.5

AHA:~

1. Find a and b : $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$.
2. $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$ evaluate.

THANKING YOU
ODM EDUCATIONAL GROUP