

# **MATHEMATICS**

**CHAPTER NUMBER :~ 1**

**CHAPTER NAME :~ NUMBER SYSTEMS**

**SUB TOPIC :~ RATIONALIZATION**

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**CHANGING YOUR TOMORROW**

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## PREVIOUS KNOWLEDGE TEST

1. Visualise  $4.262626\dots$  on the number line, up to 4 decimal places.
2. Visualise  $3.765$  on the number line, using successive magnification.

## LEARNING OUTCOME:~

Students will learn

a) More on Rationalization

b) Laws of Exponents for Real Numbers

## Question 1.

Classify the following numbers as rational or irrational.

Solution:

(i) Since, it is a difference of a rational and an irrational number.

$\therefore 2 - \sqrt{5}$  is an irrational number.

(v)  $\because 2\pi = 2 \times \pi =$  Product of a rational and an irrational number is an irrational number.

$\therefore 2\pi$  is an irrational number.

## Question 2.

Simplify each of the following expressions

Solution:

$$\begin{aligned} \text{(i)} \quad & (3 + \sqrt{3})(2 + \sqrt{2}) \\ &= 2(3 + \sqrt{3}) + \sqrt{2}(3 + \sqrt{3}) \\ &= 6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6} \end{aligned}$$

$$\text{Thus, } (3 + \sqrt{3})(2 + \sqrt{2}) = 6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6}$$

$$\begin{aligned} \text{(ii)} \quad & (3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 \\ &= 9 - 3 = 6 \end{aligned}$$

$$\text{Thus, } (3 + \sqrt{3})(3 - \sqrt{3}) = 6$$

$$\begin{aligned} \text{(iii)} \quad & (\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2}) \\ &= 5 + 2 + 2\sqrt{10} = 7 + 2\sqrt{10} \end{aligned}$$

$$\text{Thus, } (\sqrt{5} + \sqrt{2})^2 = 7 + 2\sqrt{10}$$

$$\text{(iv)} \quad (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$$

$$\text{Thus, } (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = 3$$

### Question 3.

Recall,  $\pi$  is defined as the ratio of the circumference (say  $c$ ) of a circle to its diameter (say  $d$ ). That is  $\pi = \frac{c}{d}$ . This seems to contradict the fact that  $\pi$  is irrational. How will you resolve this contradiction?

Solution:

When we measure the length of a line with a scale or with any other device, we only get an approximate rational value, i.e.  $c$  and  $d$  both are rational.

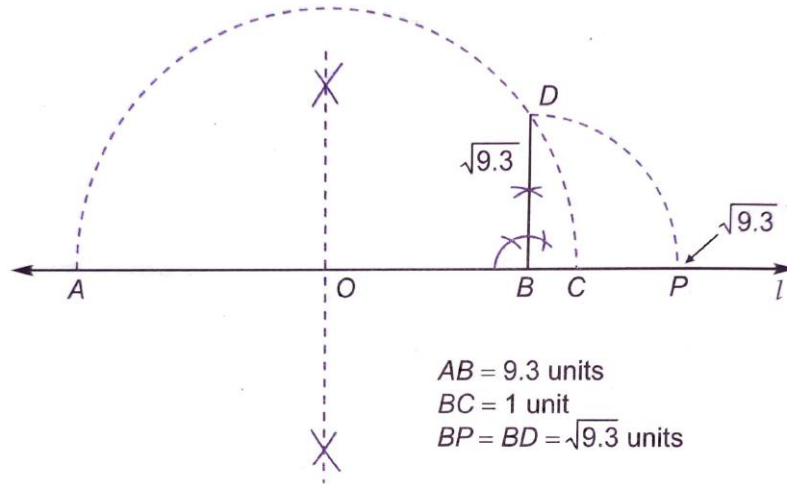
$\therefore \frac{c}{d}$  is rational and hence  $\pi$  is irrational.

Thus, there is no contradiction in saying that  $\pi$  is irrational.

Represent  $\sqrt{9.3}$  on the number line.

**Solution.** Steps of construction :

1. On the number line  $l$ , draw  $AB = 9.3$  units.
2. Extend  $AB$  to point  $C$  such that  $BC = 1$  unit.
3. Draw the perpendicular bisector of  $AC$  and mark the midpoint of  $AC$  as  $O$ .
4. With  $O$  as centre, draw a semicircle of radius  $OA = OC$ .
5. From  $B$ , draw  $BD \perp AC$ , intersecting the semicircle at  $D$ .
6. With  $B$  as centre, draw an arc of radius  $BD$  intersecting the number line at  $P$ .



<https://www.youtube.com/watch?v=JMnm6xeHC58>

*“Numbers Are Intellectual Witness That Belong Only To Mankind...”*



## RATIONALIZATION:-

The word rationalize literally means making something more efficient. Rationalization is the process of eliminating a radical or imaginary number from the denominator or numerator of an algebraic fraction.

## Laws of Exponents for Real Numbers

If  $a > 0$  and  $b > 0$  be real numbers and  $p$  and  $q$  be rational numbers, then

$$(i) a^p \cdot a^q = a^{p+q} \quad (ii) (a^p)^q = a^{pq} \quad (iii) \frac{a^p}{a^q} = a^{p-q} \quad (iv) a^p b^p = (ab)^p$$

$$(v) \frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p \quad (vi) a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

**REMARKS** An exponent is a number or symbol that indicates the power to which another number or expression is raised. For example,  $(x + y)^n$  indicates that the expression  $(x + y)$  is raised to the  $n$ th power ;  $n$  is the exponent.

### Commit To Memory (C.T.M.)

1.  $a^m \cdot a^n = a^{m+n}$

When numbers with equal bases are multiplied, their powers get added.

2.  $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

When numbers with equal bases are divided, their powers get subtracted.

3.  $(a^m)^n = a^{mn}$

When number with some power is raised to another power, powers get multiplied.

4.  $a^0 = 1, a \neq 0$

Any non-zero number raised to power zero is equal to 1.

5.  $a^m = a^n \Rightarrow m = n$

When bases are equal, powers are equal.

6.  $a^m \cdot b^m = (ab)^m$

When numbers with different bases but equal powers are multiplied, their bases get multiplied retaining the same power.

7.  $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m, b \neq 0$

When numbers with different bases but equal powers are divided, their bases get divided retaining the same power.

8.  $a^{-m} = \frac{1}{a^m}, a \neq 0$

A number with negative power is equal to the reciprocal of that number raised to equal positive power.

Evaluation:

Question: Rationalize the denominator

a.  $\frac{1}{5+\sqrt{2}}$

b.  $\frac{2}{3\sqrt{3}}$

## Homework : Exercise 1.6

AHA:~

1. If  $x = 3 + \sqrt{8}$ , find  $x^2 + \frac{1}{x^2}$ .

2. If  $x = 2 + \sqrt{3}$ , find  $x^1 + \frac{1}{x^1}$ .

3. If  $a^x = b$ ,  $b^y = c$ ,  $c^z = a$  then prove that:  $xyz = 1$ .

**THANKING YOU**  
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