1

SETS, RELATIONS AND FUNCTIONS
(CLASS-XI)

SECTION - A : SETS

INTRODUCTION

The concept of set serves as a fundamental part of the present day mathematics. Today this concept is being used in almost every branch of mathematics. Sets are used to define the concepts of relations and functions. The study of geometry, sequences, probability, etc. requires the knowledge of sets.

SETS AND THEIR REPRESENTATIONS

A set is a well-defined collection of objects.

The following points may be noted :

(i) Objects, elements and members of a set are synonymous terms.

(ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.

(iii) The elements of a set are represented by small letters a, b, c, x, y, z, etc.

If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write $a \in A$. If 'b' is not an element of a set A, we write $b \notin A$ and read "b does not belong to A".

Thus, in the set V of vowels in the English alphabet, $a \in V$ but $b \notin V$. In the set P of prime factors of 30, 3 $\in$ P but 15 $\notin$ P.

There are two methods of representing a set : (i) Roster or tabular form (ii) Set-builder form.

(a) In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

(b) In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write

V = {x : x is a vowel in English alphabet}

Example 1 :

Write the set {x : x is a positive integer and $x^2 < 40$} in the roster form.

Sol. The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is {1, 2, 3, 4, 5, 6}.

Example 2 :

Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form.

Sol. We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the set-builder form the given set is

$$\left\{x : x = \frac{n}{n+1}, \text{ where n is a natura number and } 1 \le n \le 6\right\}$$

TYPES OF SETS

1. The Empty set :

A set which does not contain any element is called the empty set or the null set or the void set.

According to this definition, B is an empty set while A is not an empty set. The empty set is denoted by the symbol ϕ or { }.

We give below a few examples of empty sets.

(i) Let A = {x : 1 < x < 2, x is a natural number}. Then A is the empty set, because there is no natural number between 1 and 2.

(ii) B = {x : $x^2 - 2 = 0$ and x is rational number}. Then B is the empty set because the equation $x^2 - 2 = 0$ is not satisfied by any rational value of x.

(iii) C = {x : x is an even prime number greater than 2}. Then C is the empty set, because 2 is the only even prime number.

(iv) D = { x : $x^2 = 4$, x is odd }. Then D is the empty set, because the equation $x^2 = 4$ is not satisfied by any odd value of x.

2. Finite and Infinite Sets :

A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

Consider some examples :

(i) Let W be the set of the days of the week. Then W is finite.

(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.

(iii) Let G be the set of points on a line. Then G is infinite. When we represent a set in the roster form, we write all the elements of the set within braces { }. It is not possible to write all the elements of an infinite set within braces { } because the numbers of elements of such a set is not finite.

So, we represent some infinite set in the roster form by writing a few elements which clearly indicate the structure of the set followed (or preceded) by three dots. For example, {1, 2, 3 . . .} is the set of natural numbers, {1, 3, 5, 7, . . .} is the set of odd natural numbers, {. . .,–3, –2, –1, 0,1, 2 ,3, . . .} is the set of integers. All these sets are infinite.

3. Equal sets :

Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write $A \neq B$.

(i) Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

(ii) Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

4. Singleton set :

A set, consisting of a single element is called a singleton set. The sets {0}, {5}, {–7} are singleton sets.

$\{x : x + 6 = 0, x \in Z\}$ is a singleton set, because this set contains only integer namely, – 6.

Example 3 :

Which of the following pairs of sets are equal? Justify your answer.

(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".

(ii) $A = \{n : n \in Z \text{ and } n^2 \leq 4\}$ and $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$.

Sol. (i) We have, X = {A, L, L, O, Y}, B = {L, O, Y, A, L}. Then X and B are equal sets as repetition of elements in a set do not change a set. Thus, X = {A, L, O, Y} = B

(ii) A = {–2, –1, 0, 1, 2}, B = {1, 2}. Since $0 \in A$ and $0 \notin B$, A and B are not equal sets.

Example 4 :

Are the following pair of sets equal ? Give reason.

(i) $A = \{2, 3\}, B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

(ii) A = {x : x is a letter in the word FOLLOW}

B = {y : y is a letter in the word WOLF}.

Sol. We have,

(i) $A = \{2, 3\}, B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

Now, $x^2 + 5x + 6 = 0 \Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x + 3) + 2(x + 3) = 0$

$\Rightarrow (x + 3)(x + 2) = 0 \Rightarrow x = -2, -3$

Therefore, B = {–2, –3}

Here, we observe that the elements of set A are not exactly the same to that of set B, hence A & B are not equal sets.

(ii) We have, A = {x : x is a letter in the word FOLLOW}

$\Rightarrow$ A = {F, O, L, W}

And B = {x : x is a letter in the word WOLF}

$\Rightarrow$ B = {W, O, L, F}

Here, we observe that the elements of both sets are exactly same, hence the sets are equal.

SUBSETS

A set A is said to be a subset of a set B if every element of A is also an element of B. In other words, $A \subset B$ if whenever $a \in A$, then $a \in B$. It is often convenient to use the symbol "$\Rightarrow$" which means implies. Using this symbol, we can write the definition of subset as follows: $A \subset B$ if $a \in A \Rightarrow a \in B$

We read the above statement as "A is a subset of B if a is an element of A implies that a is also an element of B". If A is not a subset of B, we write $A \not\subset B$. For example :

(i) The set Q of rational numbers is a subset of the set R of real numbers, and we write $Q \subset R$.

(ii) If A is the set of all divisors of 56 and B the set of all prime divisors of 56, then B is a subset of A and we write $B \subset A$.

(iii) Let A = {1, 3, 5} and B = {x : x is an odd natural number less than 6}. Then $A \subset B$ and $B \subset A$ and hence A = B.

(iv) Let A = { a, e, i, o, u} and B = { a, b, c, d}. Then A is not a subset of B, also B is not a subset of A.

Proper subset : If $A \subset B$ and $A \neq B$, then A is called a proper subset of B, written as $A \subset B$.

For example : Let A = {x : x is an even natural number} and B = {x : x is a natural number}. Then,

A = {2, 4, 6, 8, ...} and B = {1, 2, 3, 4, 5, ...} $\Rightarrow A \subset B$.

Theorems on subsets :

Theorem 1 : Every set is a subset of itself.

Proof : Let A be any set. Then, each element of A is clearly in A. Hence $A \subseteq A$.

Theorem 2 : The empty set is a subset of every set.

Proof : Let A be any set and ϕ be the empty set. In order to show that $\phi \subseteq A$, we must show that every element of ϕ is an element of A also. But, ϕ contains no element, So, every element of ϕ is in A. Hence $\phi \subset A$.

Theorem 3 : The total number of subsets of a finite set containing n element is 2^n.

Proof : Let A be a set of n elements.

The null set is a subset of A containing no element.

$\therefore$ Number of subsets of A containing no element $= 1 = {}^nC_0$.

Number of subsets of A containing 1 element = Number of groups of n elements taking 1 at a time $= {}^nC_1$.

Number of subsets of A containing 2 elements = Number of groups of n elements taking 2 at a time $= {}^nC_2$.

....

Number of subsets of A containing n elements i.e.

$A = 1 = {}^nC_n$

Total no. of subsets of $A = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

[Using binomial theorem]

Some properties of subsets :

(a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Let $x \in A \Rightarrow x \in B$ ($\because A \subseteq B$) and $x \in B \Rightarrow x \in C$ ($\because B \subseteq C$) $\Rightarrow A \subseteq C$

(b) If A = B and only if $A \subseteq B$ and $B \subseteq A$.

Let $A \subseteq B$ and $B \subseteq A$ $\therefore$ $x \in A \Rightarrow x \in B$ ($\because A \subseteq B$) and $x \in B \Rightarrow x \in A$ ($\because B \subseteq A$) $\therefore A = B$

Conversely, Let $A \subseteq B$

$\therefore\ x \in A \Rightarrow x \in B\ (\because A = B)\ \therefore\ A \subseteq B$

Similarly, $x \in B \Rightarrow x \in A\ (\because A = B)$

$\therefore\ B \subseteq A$

Subsets of set of real numbers :

There are many important subsets of R. We give below the names of some of these subsets.

The set of natural numbers N = {1, 2, 3, 4, 5, . . .}

The set of integers Z = {. . ., –3, –2, –1, 0, 1, 2, 3, . . .}

The set of rational numbers $Q = \{ x : x = \frac{p}{q}, p, q \in Z \ \&\ q \neq 0\}$

which is read " Q is the set of all numbers x such that x equals the quotient $\frac{p}{q}$, where p and q are integers and q is not zero". Members of Q include –5 (which can be expressed as $-\frac{5}{1}$), $\frac{5}{7}$, $3\frac{1}{2}$ (which can be expresses as $\frac{7}{2}$) and $-\frac{11}{3}$.

The set of irrational numbers, denoted by T, is composed of all other real numbers. Thus $T = \{ x : x \in R \text{ and } x \notin Q\}$, i.e., all real numbers that are not rational. Members of T include $\sqrt{2}, \sqrt{5}$ and π.

Some of the obvious relations among these subsets are:

$N \subset Z \subset Q,\ Q \subset R,\ T \subset R,\ N \not\subset T.$

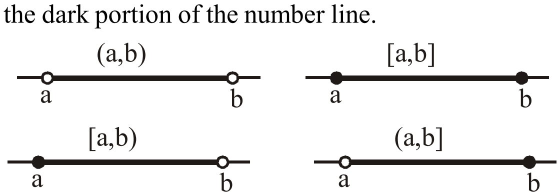
Intervals as subsets of R : Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{ y : a < y < b\}$ is called an open interval and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval.

The interval which contains the end points also is called closed interval and is denoted by [a, b]. Thus

$[a, b] = \{x : a \le x \le b\}$

We can also have intervals closed at one end and open at the other, i.e., $[a, b) = \{x : a \le x < b\}$ is an open interval from a to b, including a but excluding b.

$(a, b] = \{ x : a < x \le b \}$ is an open interval from a to b including b but excluding a. These sets can be shown by the dark portion of the number line.



The number (b – a) is called the length of any of the intervals (a, b), [a, b), (a, b], [a, b]

POWER SET

The collection of all subsets of a set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

If A has n elements then its power set has 2^n elements.

UNIVERSAL SET

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set, denoted by U or ξ.

For example, (i) In the context of human population studies, the universal set consists of all the people in the world.

(ii) Let A = {1, 2, 3, 4}, B = {2, 5, 6}, C = {1, 3, 7, 8, 9}, then U = {1, 2, 3, 4, 5, 6, 7, 8, 9} is the universal set.

Example 5 :

Show that : n {P {P (P (ϕ))}} = 4

Sol. We have, $P(\phi) = \{\phi\}$

$\therefore\ P(P(\phi)) = \{\phi, \{\phi\}\}$

$\Rightarrow\ P\{P(P(\phi))\} = \{\phi, \{\phi\}\}, \{\{\phi\}\}\ \{\phi, \{\phi\}\}$

Hence, n n {P {P (P (ϕ))}} = 4

Example 6 :

Prove that $A \subset \phi \Rightarrow A = \phi$

Sol. We know that,

Two sets A and B are equal if an only if $A \subset B$ and $B \subset A$

Also, we know that, $\phi \subset A$ and $A \subset \phi$

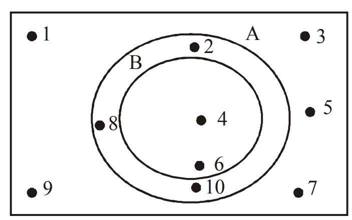
$\therefore\ A = \phi$

VENN DIAGRAMS

In order to illustrate in a clear and simple way, the ideas involving universal sets, subsets there of, and certain operations on sets, we make use of geometric figures. These figures are called Venn-Diagrams.

In Venn diagrams, the elements of the sets are written in their respective circles.

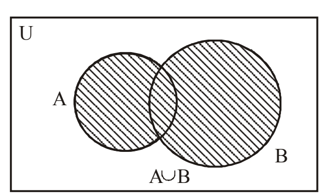
In Fig 1.3, U = {1, 2, 3, ..., 10} is the universal set of which A = {2,4,6,8,10} and B = {4, 6} are subsets, and also $B \subset A$.



OPERATION ON SETS

Union of sets : The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

The union of two sets can be represented by a Venn diagram as shown in Fig. The shaded portion in Fig represents $A \cup B$.



Some Properties of the Operation of Union

(i) $A \cup B = B \cup A$ (Commutative law)

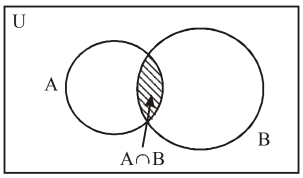
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

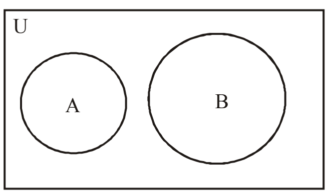
(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (Law of U)

Intersection of sets : The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$ The shaded portion in Fig. indicates the intersection of A and B.



If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets. For example, let $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$. Then A and B are disjoint sets, because there are no elements which are common to A and B. The disjoint sets can be represented by means of Venn diagram as shown in the Fig.



In the above diagram, A and B are disjoint sets.

Some properties of operation of intersection

(i) $A \cap B = B \cap A$ (Commutative law).

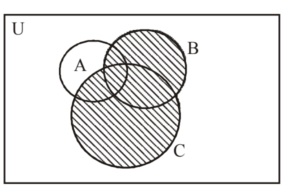
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) ϕ n $A = \phi$, $U \cap A = A$ (Law of f and U).

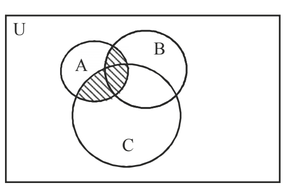
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)
i.e., $\cap$ distributes over $\cup$

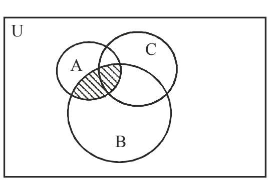
This can be seen easily from the following Venn diagrams [Figs (i) to (v)].



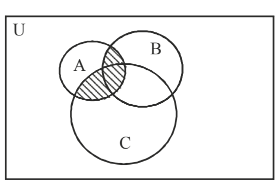
(i) (B∪C)



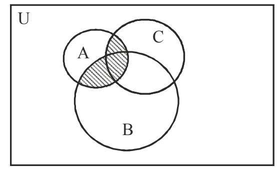
(ii) A∩ (B∪C)



(iii) (A∩B)



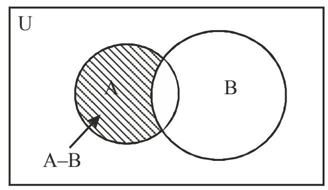
(iv) (A∩C)



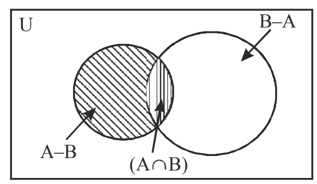
(v) (A ∩ B) ∪(A∩ B)

Difference of sets : The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write A – B and read as "A minus B". Using the setbuilder notation, we can rewrite the definition of difference as $A - B = \{x : x \in A \text{ and } x \notin B\}$

The difference of two sets A and B can be represented by Venn diagram as shown in Fig.



The shaded portion represents the difference of the two sets A and B. The sets A – B, A n B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set as shown in Fig.

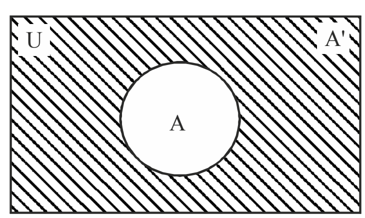


Complement of set : Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U. Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$.

Obviously $A' = U - A$

We note that the complement of a set A can be looked upon, alternatively, as the difference between a universal set U and the set A. The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. These are called De Morgan's laws. These are named after the mathematician De Morgan.

The complement A' of a set A can be represented by a Venn diagram as shown in Fig. The shaded portion represents the complement of the set A.



Some Properties of Complement Sets :

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = f$
2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
3. Law of double complementation : $(A')' = A$
4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$.

These laws can be verified by using Venn diagrams.

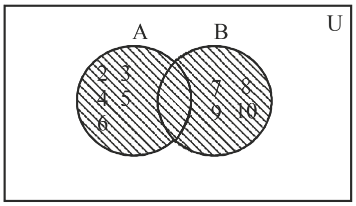
Example 7 :

Find the union of each of the following pairs of sets :

(i) $A = \{x : x$ is a natural number and $1 < x \le 6\}$

$B = \{ x : x$ is a natural number and $6 < x \le 10\}$

(ii) $A = \{1, 2, 3\}$, $B = \phi$



Sol. (i) $A = \{ x : x$ is a natural number and $1 < x \le 6\}$

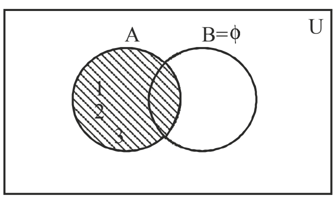
$\Rightarrow$ $A = \{2, 3, 4, 5, 6\}$

$B = \{ x : x$ is a natural number and $6 < x \le 10\}$

$\Rightarrow$ $B = \{7, 8, 9, 10\}$

$\therefore$ $A \cup B = \{2, 3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$\Rightarrow$ $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$



(ii) We have, $A = \{1, 2, 3\}$, $B = \phi$

$\Rightarrow$ $A \cup B = \{1, 2, 3\} \cup \phi$

$\Rightarrow$ $A \cup B = \{1, 2, 3\}$

Example 8 :

If $A = \{x : x = 3n, n \in Z\}$ and $B = \{x : x = 4n, n \in Z\}$, then find $(A \cap B)$.

Sol. Let $x \in (A \cap B) \Leftrightarrow x \in A$ and $x \in B$

$\Leftrightarrow$ x is a multiple of 3 and x is a multiple of 4.

$\Leftrightarrow$ x is a multiple of 3 and 4 both

$\Leftrightarrow$ x is a multiple of 12

$\Leftrightarrow$ $x = 12n$, $n \in Z$

Hence $A \cap B = \{x : x = 12n, n \in Z\}$

Example 9 :

If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

Sol. Given that, $n(X \cup Y) = 50$, $n(X) = 28$, $n(Y) = 32$, $n(X \cap Y) = ?$

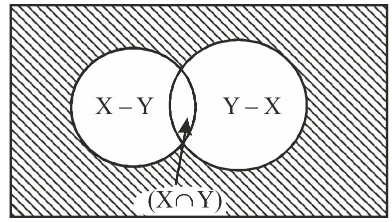
By using the formula,

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$,

we find that $n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$

$= 28 + 32 - 50 = 10$

Alternatively, suppose $n(X \cap Y) = k$, then



$n(X - Y) = 28 - k$, $n(Y - X) = 32 - k$

(by Venn diagram in Fig.)

This gives $50 = n(X \cup Y) = n(X - Y) + n(X \cap Y) + n(Y - X)$

$= (28 - k) + k + (32 - k)$

Hence $k = 10$.

Example 10 :

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football ?

Sol. Let X be the set of students who like to play cricket and Y be the set of students who like to play football. Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games.

Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y = ?$

Using the formula $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$,

we get $35 = 24 + 16 - n(X \cap Y)$

Thus, $n(X \cap Y) = 5$ i.e., 5 students like to play both games.

TRY IT YOURSELF-1

Q.1 Let B be a subset of a set A and let

$P(A : B) = \{X \in P(A) : B \subset X\}$

(i) Show that $P(A : \phi) = P(A)$

(ii) If $A = \{a, b, c, d\}$ and $B = \{a, b\}$.

List all the members of the set $P(A : B)$.

Q.2 A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products ?

Q.3 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket ? How many like tennis.

Q.4 In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both Hindi and English.

Q.5 Let $A = \{ a, e, i, o, u \}$ and $B = \{ a, i, u \}$. Show that $A \cup B = A$

ANSWERS

(1) (ii) {a, b, c}, {a, b, d}, {a, b, c, d}

(2) 170 **(3)** 35, 25 **(4)** 50

SECTION - B :
RELATIONS AND FUNCTIONS

INTRODUCTION

Functions are an extremely useful way of describing many real-world situations in which the value of one quantity varies with, depends on, or determines the value of another. In this chapter you will be introduced to functions, learn how to use functional notation, develop skill in constructing and interpreting the graphs of functions, and, finally, learn to apply this knowledge in a variety of situations.

CARTESIAN PRODUCTS OF SETS

Given two non-empty sets P and Q. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, i.e., $P \times Q = \{ (p,q) : p \in P, q \in Q \}$

If either P or Q is the null set, then $P \times Q$ will also be empty set, i.e., $P \times Q = \phi$

(i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

(ii) If there are p elements in A and q elements in B, then there will be pq elements in $A \times B$, i.e., if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(iii) If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$.

(iv) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

Example 11 :

If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B.

Sol. A = set of first elements = {p, m}

B = set of second elements = {q, r}.

Example 12 :

If $P = \{1, 2\}$, form the set $P \times P \times P$.

Sol. We have, $P \times P \times P = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$.

RELATIONS

A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$. The second element is called the image of the first element. The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R. The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range $\subseteq$ codomain.

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

Example 13 :

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Find the number of relations from A to B.

Sol. We have, $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$.

Since n (A×B) = 4, the number of subsets of A×B is 24. Therefore, the number of relations from A into B will be 2^4.

FUNCTION

INTRODUCTION

* A function is like a machine which gives unique output for each input that is fed into it. But every machine is designed for certain defined inputs for eg. a juicer is designed for fruits & not for wood. Similarly functions are defined for certain inputs which are called as its "domain and corresponding outputs are called "Range".

General Definition :

Definition-1 :

* Let A and B be two sets and let there exist a rule or manner or correspondence ' f ' which associates to each element of A to a unique element in B, then f is called a function or mapping from A to B. It is denoted by the symbol

$$f : A \to B \text{ or } A \xrightarrow{f} B$$

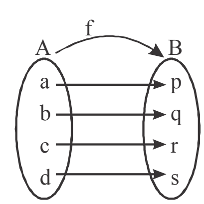
which reads ' f ' is a function from A to B' or 'f maps A to B'.

* If an element $a \in A$ is associated with an element $b \in B$ then b is called 'the f image of a' or 'image of a under f' or 'the value of the function f at a'. Also a is called the pre-image of b or argument of b under the function f. We write it as $b = f(a)$ or $f : a \to b$ or $f : (a, b)$

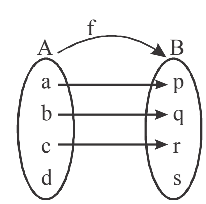
Function as a set of ordered pairs :

* A function $f : A \to B$ can be expressed as a set of ordered pairs in which each ordered pair is such that its first element belongs to A and second element is the corresponding element of B.

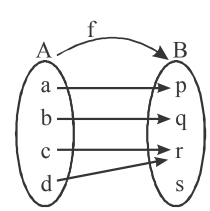
* As such a function $f : A \to B$ can be considered as a set of ordered pairs $(a, f(a))$ where $a \in A$ and $f(a) \in B$ which is the f image of a. Hence f is a subset of $A \times B$.



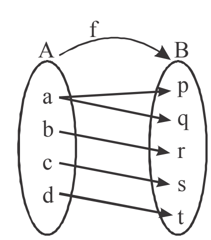
Function



Not a function



Function



Not a function

As a particular type of relation, we can define a function as follows :

Definition-2 :

* A relation R from a set A to a set B is called a function if
(i) each element of A is associated with some element of B.
(ii) each element of A has unique image in B.

Thus a function ' f ' from a set A to a set B is a subset of $A \times B$ in which each 'a' belonging to A appears in one and only one ordered pair belonging to f. Hence a function f is a relation from A to B satisfying the following properties :

* Every function from $A \to B$ satisfies the following conditions.
(i) $f \subset A \times B$
(ii) $\forall\ a \in A \Rightarrow (a, f(a)) \in f$ and
(iii) $(a, b) \in f\ \&\ (a, c) \in f \Rightarrow b = c$.

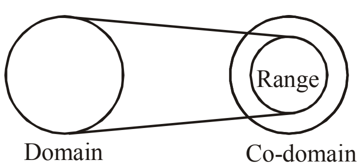
* Thus the ordered pairs of f must satisfy the property that each element of A appears in some ordered pair and no two ordered pairs have same first element.

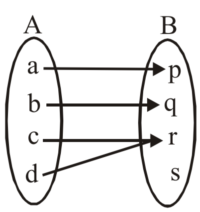
Note : Every function is a relation but every relation is not necessarily a function.

Domain, Co-domain & Range of A Function :

* Let $f : A \to B$, then the set A is known as the domain of f & the set B is known as co–domain of f. The set of all f images of elements of A is known as the range of f.
Domain of $f = \{a \mid a \in A, (a, f(a)) \in f\}$
Range of $f = \{f(a) \mid a \in A, f(a) \in B, (a, f(a)) \in f\}$

* It should be noted that range is a subset of co–domain . If only the rule of function is given then the domain of the function is the set of those real numbers, where function is defined.





Domain = {a, b, c, d} = A

Co-domain = {p, q, r, s} = B

Range = = {p, q, r}

* For a continuous function, the interval from minimum to maximum value of a function gives the range.

Let f and g be function with domain D_1 and D_2 then the functions

Note :

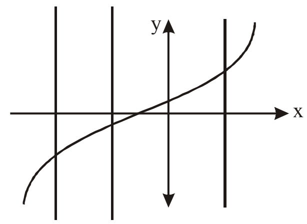
$f+g,\ f-g,\ fg,\ f/g$ are defined as

$(f+g)(x) = f(x) + g(x)$; Domain $D_1 \cap D_2$
$(f-g)(x) = f(x) - g(x)$; Domain $D_1 \cap D_2$
$(fg)(x) = f(x) \cdot g(x)$; Domain $D_1 \cap D_2$

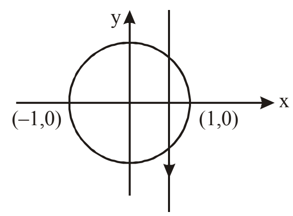
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$; Domain $= \{x \in D_1 \cap D_2 \mid g(x) \neq 0\}$

Graphical Representation of function :

* Let f be a mapping with domain D such that y = f(x) should assume angle value for each x,(i.e. the straight line drawn parallel to y-axis in its domain should cut at only one point.
Eg. $y = x^3$ Here all the straight lines parallel to y-axis cut $y = x^3$ only at one print.



Eg. $x^2 + y^2 = 1^2$. Here line parallel to y-axis is intersecting the circle at two points hence it is not a function.



Domain :

* **Rule for finding Domain :**
(i) Expression under even root (i.e. square root, fourth root etc) ≥ 0.
(ii) Denominator $\neq 0$
(iii) If domain of y = f(x) & y = g(x) are D_1 & D_2 respectively then the domain of $f(x) \pm g(x)$ or $f(x) \cdot g(x)$ is $D_1 \cap D_2$
(iv) Domain of $\frac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{g(x) = 0\}$.

Example 14 :

Find the domain of $f(x) = \frac{2x-1}{x-2}$

Sol. It is meaningful for all real value of x except at x = 2
so domain = R – {2}

Classification of Functions:

(i) **Polynomial Function:** If a function f is defined by $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + a_n$ where n is a non negative integer and $a_0, a_1, a_2, ..., a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n. A polynomial function is always continuous.

NOTE

* A polynomial of degree one with no constant term is called an odd linear function i.e. $f(x) = ax$, $a \neq 0$
* There are two polynomial functions , satisfying the relation ;$f(x).f(1/x) = f(x) + f(1/x)$. They are :
(a) $f(x) = x^n + 1$ **(b)** $f(x) = 1 - x^n$, where n is a positive integer.
* A polynomial of degree odd has its range $(-\infty, \infty)$ put a polynomial of degree even has a range which is always subset of R.

(ii) Algebraic Function : A function f is called an algebraic function if it can be constructed using algebraic operations such as addition, substraction, multiplication, division and taking roots, started with polynomials.

e.g. $f(x)=\sqrt{x^2+1}$; $g(x)=\dfrac{x^4-16x^2}{x+\sqrt{x}}+(x-2)\times\sqrt[3]{x+1}$

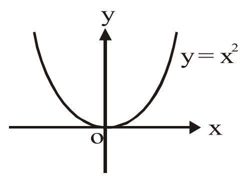
Note that all polynomial are algebraic but converse is not true. Functions which are not algebraic, are known as Transcidential function.

Basic algebraic function :

(a) $y=x^2$

Domain : R

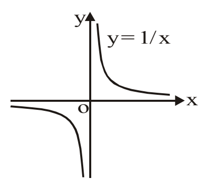
Range : $R^+ \cup \{0\}$ or $[0, \infty)$



(b) $y=\dfrac{1}{x}$

Domain : $R-\{0\}$ or R_0

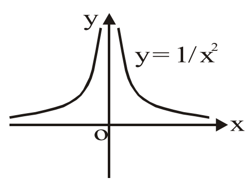
Range : $R-\{0\}$



(c) $y=\dfrac{1}{x^2}$

Domain : R_0

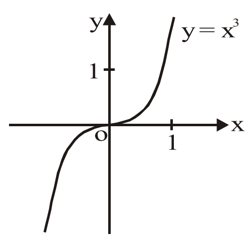
Range : R^+ or $(0, \infty)$



(d) $y=x^3$

Domain : R

Range : R

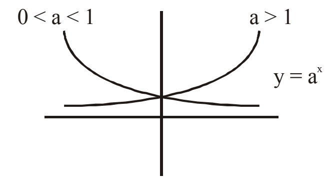


(iii) Fractional Rational Function: A rational function is a function of the form. $y=f(x)=\dfrac{g(x)}{h(x)}$, where g (x) & h (x) are polynomials & $h(x)\neq 0$. The domain of f (x) is set of real x such that $h(x)\neq 0$.

e.g. $f(x)=\dfrac{2x^4-x^2+1}{x^2-4}$; $D=\{x \mid x \neq \pm 2\}$

(iv) Exponential Function: A function $f(x)=a^x=e^{x\ln a}$ $(a>0,\ a\neq 1,\ x\in R)$ is called an exponential function. $f(x)=a^x$ is called an exponential function because the variable x is the exponent. It should not be confused with power function. $g(x)=x^2$ in which variable x is the base.

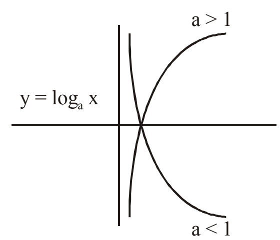
For $f(x)=e^x$ domain is R and range is R^+.



For $f(x)=e^{1/x}$ domain is $R-\{0\}$ and range is $R^+-\{1\}$. i.e. $(0,1)\cup(1,\infty)$

$f(x)=\dfrac{1}{\ln x}$ with domain $R^+-\{1\}$, range is $R-\{0\}$

(v) Logarithmic function : A function of the form $y=\log_a x$, $x>0$, $a>0$, $a\neq 1$, is called Logarithmic function.



(vi) Absolute Value Function: A function $y=f(x)=|x|$ is called the absolute value function or Modulus function.

It is defined as: $y=|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$

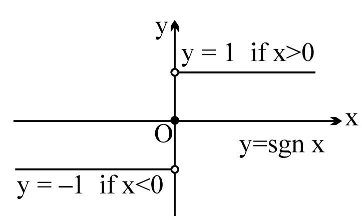
For $f(x)=|x|$, domain is R and range is $R^+\cup\{0\}$.

For $f(x)=\dfrac{1}{|x|}$ or $\dfrac{|x|}{x^2}$, domain is $R-\{0\}$ and range is R^+.

(vii) Signum Function:

A function y= f (x) = Sgn (x) is defined as follows :

$$y=f(x)=\begin{cases} 1 & \text{for } x>0 \\ 0 & \text{for } x=0 \\ -1 & \text{for } x<0 \end{cases}$$



It is also written as Sgn x = |x| / x or $\dfrac{x}{|x|}$

$x\neq 0$; $f(0)=0$

Note that Sgn (Sgn x) = Sgn x;

$$y=\mathrm{Sgn}(x^2-1)=\begin{cases} 1, & |x|>1 \\ 0, & |x|=1 \\ -1, & |x|<1 \end{cases}$$

(viii) Greatest Integer Or Step Up Function : The function $y = f(x) = [x]$ is called the greatest integer function where $[x]$ denotes the greatest integer less than or equal to x. Note that for :

$-1 \le x < 0 : [x] = -1 \qquad 0 \le x < 1 : [x] = 0$

$1 \le x < 2 : \quad [x] = 1 \qquad 2 \le x < 3 : [x] = 2$

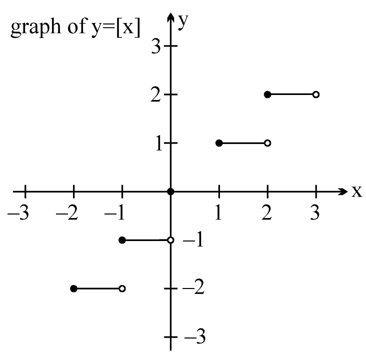
For $f(x) = [x]$, domain is R and range is I.

For $f(x) = \dfrac{1}{[x]}$ domain is $R - [0, 1)$ and range is

$\left\{ \dfrac{1}{n} \,\middle|\, n \in I - \{0\} \right\}$

Properties of greatest integer function :

(a) $[x] \le x < [x] + 1$ and
$x - 1 < [x] \le x$, $0 \le x - [x] < 1$



(b) $[x + m] = [x] + m$, if m is an integer.

(c) $[x] + [y] \le [x + y] \le [x] + [y] + 1$

(d) $[x] + [-x] = \begin{cases} 0 & \text{if } x \text{ is an integer} \\ -1 & \text{otherwise.} \end{cases}$

(e) $[x] \ge n \Rightarrow x \in [n, \infty) \ \forall \ n \in I$

(f) $[x] > n \Rightarrow x \in [n + 1, \infty) \ \forall \ n \in I$

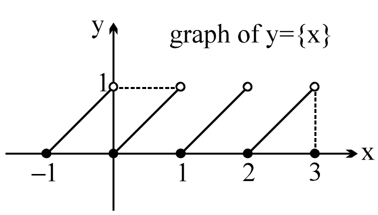
(g) $[x] \le n \Rightarrow x \in (-\infty, n + 1) \ \forall \ n \in I$

(h) $[x] < n \Rightarrow x \in (-\infty, n) \ \forall \ n \in I$

(ix) Fractional Part Function : It is defined as :

$g(x) = \{x\} = x - [x]$.

e.g. the fractional part of the number 2.1 is $2.1 - 2 = 0.1$ and the fractional part of -3.7 is 0.3. The period of this function is 1 and graph of this function is as shown.



For $f(x) = \{x\}$, domain is R and range is $[0, 1)$

For $f(x) = \dfrac{1}{\{x\}}$, domain is $R - I$, range is $(1, \infty)$

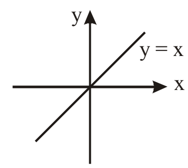
Properties of fractional part :

(a) $0 \le \{x\} < 1$

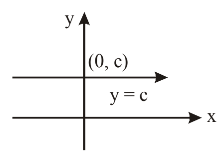
(b) $\{x + n\} = \{x\}$, $n \in I$

(c) $\{x\} + \{-x\} = \begin{cases} 0, & x \in I \\ 1, & x \notin I \end{cases}$

(x) Identity function : The function $f : A \to A$ defined by $f(x) = x \ \forall \ x \in A$ is called the identity of A and is denoted by I_A. $\quad f : R \to R, \quad f(x) = x$



(xi) Constant function : A function $f : A \to B$ is said to be a constant function if every element of A has the same f image in B. Thus $f : A \to B$; $f(x) = c$, $\forall \ x \in A$, $c \in B$ is a constant function. Note that the range of a constant function is a singleton.



e.g. $f(x) = [\{x\}]$; $g(x) = \sin^2 x + \cos^2 x$;
$h(x) = \text{sgn}(x^2 - 3x + 4)$ etc, all are constant functions.

Example 15 :

Find the domain of $f(x) = \sin x + \cos x + e^x \tan x$

Sol. Domain $\sin x = R$; Domain $\cos x = R$

Domain $\tan x = R - \left\{ \dfrac{2n+1}{2} \pi \right\}$; Domain $e^x = R$

$\therefore$ Domain f = Domain $\sin x \cap$ Domain $\cos x \cap$ Domain $e^x \tan x$

$= R \cap R \cap R - \left\{ \dfrac{2n+1}{2} \pi \right\} = R - \left\{ \dfrac{2n+1}{2} \pi \right\}$

Example 16 :

Find the domain of following function

(i) $f(x) = \sqrt{x^2 - 5x + 6}$

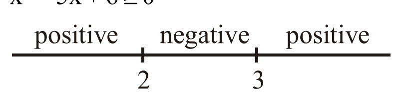
(ii) $f(x) = \dfrac{2}{x^2 - 4} + \log_{10}(x^3 - x)$

(iii) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

(iv) $f(x) = \log_4 \log_2 \log_{1/2}(x)$

Sol. (i) $f(x) = \sqrt{x^2 - 5x + 6}$

$\Rightarrow x^2 - 5x + 6 \ge 0$



$\Rightarrow (x - 2)(x - 3) \ge 0 \Rightarrow x \in (-\infty, 2] \cup [3, \infty)$

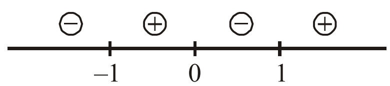
(ii) $f(x) = \frac{2}{x^2-4} + \log_{10}(x^3 - x)$

Following conditions should be followed

$x^2 - 4 \neq 0$ & $x^3 - x > 0$

$x \neq \pm 2$ $x(x^2 - 1) > 0$

$x \in R - \{-2, 2\}$ ...(i)



$\Rightarrow$ $x(x-1)(x+1) > 0$

$x \in (-1, 0) \cup (1, \infty)$ ...(ii)

Taking union of (i) & (ii)

$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

(iii) $f(x) = \frac{1}{\sqrt{|x| - x}}$

$|x| - x > 0$

$|x| > x$

This is possible only when x is negative i.e. $x < 0$, hence $x \in (-\infty, 0)$

(iv) $f(x) = \log_4 \log_2 \log_{1/2}(x)$

$\Rightarrow \log_2 \log_{1/2}(x) > 0 \Rightarrow \log_{1/2}(x) > (2^0)$

$\Rightarrow \log_{1/2} x > 1 \Rightarrow x < \left(\frac{1}{2}\right)^1$

Also $0 < x \Rightarrow x \in (0, 1/2)$

Example 17 :

If $y = 2[x] + 3$ & $y = 3[x-2] + 5$ then find $[x + y]$ where [.] denotes greatest integer function.

Sol. $y = 3[x-2] + 5 = 3[x] - 1$

so $3[x] - 1 = 2[x] + 3$

$[x] = 4 \Rightarrow 4 \leq x < 5$ then $y = 11$

so $x + y$ will lie in the interval [15, 16)

so $[x + y] = 15$

RANGE :

* Range of y = f(x) is the collection of all outputs corresponding to each real number of the domain.

To find the range of function

(i) First of all find the domain of y = f(x).

(ii) If domain is a set having only finite number of points, then range is the set of corresponding f(x) values.

(iii) If domain of y = f(x) is R or R – {Some finite points}, then express x in terms of y. From this find y for x to be defined or real or form an equation in terms of x & apply the condition for real roots.

Example 18 :

Find the range of the following function

(i) $f(x) = a \sin x + b$. $a > 0$, $b \in R$

(ii) $f(x) = 4 \tan x \cos x$

(iii) $y = \frac{x^2}{1+x^2}$

Sol. (i) . . . . $\in R$

$f(x) = a \sin x + b$

$\because$ $-1 \leq \sin x \leq 1$

$\therefore$ $-a + b \leq f(x) \leq a + b$

Range $\in [b - a, b + a]$

(ii) $f(x) = 4 \tan x \cos x$

$f(x) = 4 \sin x$ for $\cos x \neq 0$

$-1 \leq \sin x \leq 1$

but at $\sin x = \pm 1$, $\cos x = 0$

hence points with $\sin x = \pm 1$ will not be included in range. Range $\in (-4, 4)$.

(iii) $y = \frac{x^2}{1+x^2}$

y is defined $\forall x \in R$, domain is R

from $y = \frac{x^2}{1-x^2} \Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \sqrt{\frac{y}{1-y}} \geq 0 \Rightarrow \frac{y}{1-y} \geq 0 \Rightarrow 0 \leq y < 1$

Range [0, 1)

EQUAL OR IDENTICAL FUNCTION

Two functions f & g are said to be equal if

(i) The domain of f = the domain of g.

(ii) The range of f = the range of g

(iii) $f(x) = g(x)$, for every x belonging to their common domain.

e.g. $f(x) = \frac{1}{x}$ & $g(x) = \frac{x}{x^2}$ are identical functions.

Note : Functions are also equal if their graphs are same.

Example 19 :

Find the domain of x for which the function $f(x) = \ln x^2$ and $g(x) = 2\ln x$ are identical.

Sol. $f(x) = \ln x^2 = 2\ln|x|$

$g(x) = 2\ln x$

if $f(x) = g(x)$; $2\ln|x| = 2\ln x$

Function are equal only if $x \in (0, \infty)$

Some important points to remember :

If x, y are independent variables, then :

(i) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k \ln x$ or $f(x) = 0$.

(ii) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n$, $n \in R$

(iii) $f(x+y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx} \Rightarrow f(x) = A^x, A > 0$.

(iv) $f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx$, where k is a constant.

(v) If P(x) is a polynomial function of degree n and

$P(x) \cdot P\left(\frac{1}{x}\right) = P(x) + P\left(\frac{1}{x}\right)$ for $x \neq 0$. Then

$P(x) = 1 + x^n$ or $1 - x^n$.

FUNCTIONAL EQUATIONS

Example 20 :

If $f(0)=1, f(1)=2$ & $f(x)=\frac{1}{2}[f(x+1)+f(x+2)]$, find the value of $f(5)$.

Sol. $f(x+2)=2f(x)-f(x+1)$

thus $f(0+2)=f(2)=2f(0)-f(1)=2(1)-2=0$

$f(3)=2f(1)-f(2)=2(2)-0=4$

$f(4)=2f(2)-f(3)=0-4=-4$

$f(5)=2f(3)-f(4)=2(4)-(-4)=12$

Example 21 :

If $f(x)+2f(1-x)=x^2+2\ \forall\ x \in R$, find $f(x)$.

Sol. $f(x)+2f(1-x)=x^2$ ...(i)

Replacing x by $1-x$

$f(1-x)+2f(x)=(1-x)^2$ ...(ii)

Solving (i) & (ii), we get, $3f(x)=2x^2-(1-x)^2$

$$f(x)=\frac{x^2+2x-1}{3}$$

Example 22 :

Let f (x) & g (x) be functions which take integers as arguments let $f(x+y)=f(x)+g(y)+8$ for all integer x & y. Let $f(x)=x$ for all negative integers x let $g(8)=17$, find $f(0)$.

Sol. $f(x)=x$ for integers less than zero

$\therefore\ f(-8)=-8$

$f(x+y)=f(x)+g(y)+8$

$f(-8+8)=f(-8)+g(8)+8$

$f(0)=-8+g(8)+8$

$f(0)=17$

COMPOSITE FUNCTION

Let $f: A \to B$ & $g: B \to C$ be two functions. Then the function $gof: A \to C$ defined by

$(gof)(x)=g(f(x))\ \forall\ x \in A$

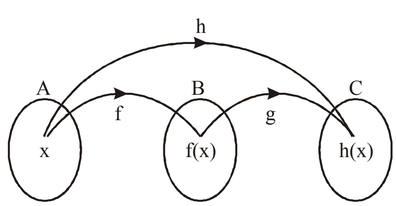
is called the composite of the two functions f & g. Diagramatically

$$\xrightarrow{x} \boxed{f} \xrightarrow{f(x)} \boxed{g} \longrightarrow g(f(x)).$$

Thus the image of every $x \in A$ under the function gof is the g–image of the f–image of x.

Note that gof is defined only if $\forall\ x \in A$, f(x) is an element of the domain of g so that we can take its g–image. Hence for gof of two functions f & g, the range of f must be a subset of the domain of g.

Note that gof in general not equal to fog.



$g\{f(x)\}=h(x)$

Properties of Composite Functions :

(i) The composite of functions is not commutative i.e. $gof \neq fog$.

(ii) The composite of functions is associative i.e. if f, g, h are three functions such that fo (goh) & (fog) oh are defined, then fo (goh) = (fog) oh.

Associativety : $f: (N) \to I_0\ \ f(x)=2x$

$g: I_0 \to Q\quad g(x)=\frac{1}{x}$

$h: Q \to R\quad h(x)=e^{\frac{1}{x}}$

(hog) of = ho (gof) $= e^{2x}$

Example 23 :

If $f: R \to R$, $f(x)=2x+3$ and $g: R \to R$, $g(x)=x^2+1$, then find the value of $gof(x)$.

Sol. $gof(x)=g\{f(x)\}$

$f(x)=2x+3$

$g\{f(x)\}=g(2x+3)=(2x+3)^2+1=4x^2+9+12x+1$

$gof(x)=4x^2+12x+10$

Example 24 :

Let $f(x)=\sqrt{x}$; $g(x)=\sqrt{2-x}$ find the domain of

(i) (fog) (x) (ii) (gof) (x)

(iii) (fof) (x) (iv) (gog) (x)

Sol. (i) $(fog)(x)=f(g(x))=f(\sqrt{2-x})=\sqrt{\sqrt{2-x}}$

Domain $2-x \geq 0$; $x \leq 2 \Rightarrow x \in (-\infty, 2]$

(ii) $(gof)(x)=g(f(x))=g(\sqrt{x})=\sqrt{2-\sqrt{x}}$

$\therefore\ 2-\sqrt{x} \geq 0 \Rightarrow 0 \leq \sqrt{x} \leq 2 \Rightarrow\ x \in [0, 4]$

(iii) $(fof)(x)=f\left(f(x)\right)=f(\sqrt{x})=\sqrt{\sqrt{x}}$

Domain $\Rightarrow\ x \geq 0$; $x \in [0, \infty)$

(iv) $(gog)(x)=g\left(g(x)\right)=g(\sqrt{2-x})=\sqrt{2-\sqrt{2-x}}$

$\therefore\ 0 \leq \sqrt{2-x} \leq 2\ \Rightarrow\ 0 \leq 2-x \leq 4 \Rightarrow -2 \leq x \leq 4$

$x \in [-2, 2]$

Example 25 :

Let $f(x)=x^x$ & $g(x)=x^{2x}$, then find $f(g(x))$.

Sol. $f(g(x))=f(x^{2x})=(x^{2x})^{x^{2x}}=x^{2x.x^{2x}}=x^{2x^{2x+1}}$

HOMOGENEOUS FUNCTIONS

* A function is said to be homogeneous with respect to any set of variables when each of its terms is of the same degree with respect to those variables.

* For example $5x^2+3y^2-xy$ is homogeneous in x & y.

f (x, y) is a homogeneous function iff

$f(tx, ty)=t^n\ f(x, y)$

or $f(x, y)=x^n g\left(\frac{y}{x}\right)=y^n h\left(\frac{x}{y}\right)$, where n is the degree of homogenity

$f(x, y) = \dfrac{x - y\cos x}{y\sin x + x}$ is not a homogeneous function and

$f(x, y) = \dfrac{x}{y}\ln\dfrac{y}{x} + \dfrac{y}{x}\ln\dfrac{x}{y}$; $\sqrt{x^2 - y^2} + x$; $x + y\cos\dfrac{y}{x}$ are homogeneous functions of degree one.

BOUNDED FUNCTION

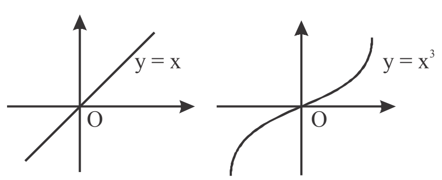
A function is said to be bounded if $|f(x)| \le M$, where M is a finite quantity. e.g. $f(x) = \sin x$ is bounded in [–1, 1]

IMPLICIT & EXPLICIT FUNCTION

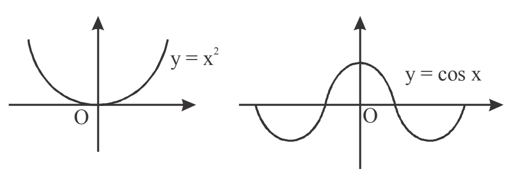
A function defined by an equation not solved for the dependent variable is called an **Implicit Function**.
For eg. the equation $x^3 + y^3 = 1$ defines y as an implicit function. If y has been expressed in terms of x alone then it is called an **Explicit Function** .

ODD & EVEN FUNCTIONS

* A function f (x) defined on the symmetric interval (–a, a)
* If $f(-x) = f(x)$ for all x in the domain of 'f' then f is said to be an even function.
  e.g. $f(x) = \cos x$; $g(x) = x^2 + 3$.
* If $f(-x) = -f(x)$ for all x in the domain of 'f' then f is said to be an odd function.
  e.g. $f(x) = \sin x$; $g(x) = x^3 + x$.



Odd functions (Symmetric about origin)



Even functions (Symmetric about y-axis)

NOTE :

(a) $f(x) - f(-x) = 0 \Rightarrow f(x)$ is even & $f(x) + f(-x) = 0 \Rightarrow f(x)$ is odd .

(b) A function may neither be odd nor even .

(c) Every even function is symmetric about the y–axis & every odd function is symmetric about the origin.

(d) Every function can be expressed as the sum of an even & an odd function.

e.g. $f(x) = \dfrac{f(x) + f(-x)}{2} + \dfrac{f(x) - f(-x)}{2}$

EVEN ODD

$2^x = \dfrac{2^x + 2^{-x}}{2} + \dfrac{2^x - 2^{-x}}{2}$

EVEN ODD

(e) The only function which is defined on the entire number line & is even and odd at the same time is $f(x) = 0$. Any non zero constant is even.

(f) If f and g both are even or both are odd then the function f.g will be even but if any one of them is odd then f.g will be odd.

Example 26 :

Identity the functions, as even, odd or neither even nor odd.

(i) $f(x) = (\ln x + \sqrt{1 + x^2})$ (ii) $f(x) = x \cdot \left(\dfrac{2^x + 1}{2^x - 1}\right)$

(iii) $f(x) = 2x^3 - x + 1$ (iv) $f(x) = 3$

(v) $f(x) = x^2 - |x|$

Sol. (i) $f(x) = (\ln x + \sqrt{1 + x^2})$

$f(-x) = \ln(-x + \sqrt{1 + x^2}) = \ln(\sqrt{1 + x^2} - x)$

$= \ln\left(\dfrac{(\sqrt{1 + x^2} - x)(\sqrt{1 + x^2} + x)}{(\sqrt{1 + x^2} + x)}\right)$

$= \ln(\sqrt{1 + x^2}) = -f(x)$. Hence odd function.

(ii) $f(x) = x \cdot \left(\dfrac{2^x + 1}{2^x - 1}\right)$; $f(-x) = (-x)\left(\dfrac{2^{-x} + 1}{2^{-x} - 1}\right)$

$= (-x)\left(\dfrac{1 + 2^x}{1 - 2^x}\right) = x\left(\dfrac{2^x + 1}{2^x - 1}\right) = f(x)$

Hence even function

(iii) $f(x) = 2x^3 - x + 1$

$f(-x) = -2x^3 + x + 1 \ne f(x)$ or $-f(x)$

Hence neither even nor odd function

(iv) $f(x) = 3$

$f(-x) = 3 = f(x)$. Hence even function

(v) $f(x) = x^2 - |x|$

$f(-x) = x^2 - |-x| = f(x)$; even function

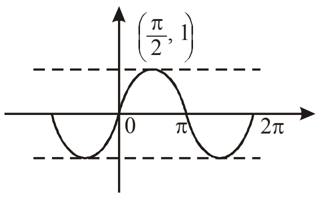
PERIODIC FUNCTION

A function f (x) is called periodic if there exists a positive number T(T > 0) called the period of the function such that f(x + T) = f(x), for all values of x within the domain of x.

e.g. The function sin x & cos x both are periodic over 2π & tan x is periodic over π.

Graphically :

If the graph repeats at fixed interval then function is said to be periodic and its period is the width of that interval. For example graph of sin x repeats itself at an interval of 2π



f (x) = sin x

Properties of periodic function :

(i) $f(T) = f(0) = f(-T)$, where 'T' is the period.

(ii) Every constant function is always periodic, with no fundamental period .

(iii) If $f(x)$ has a period p, then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has a period p.

(iv) if f(x) has a period T then f(ax + b) has a period $\frac{T}{|a|}$.

(v) If $f(x)$ has a period T & $g(x)$ also has a period T then it does not mean that $f(x) + g(x)$ must have a period T. e.g. $f(x) = |\sin x| + |\cos x|$; $\sin^4 x + \cos^4 x$ has fundamental period equal to $\pi/2$.

(vi) If f (x) and g (x) are periodic then f (x) + g (x) need not be periodic. e.g. $f(x) = \cos x$ and $g(x) = \{x\}$

Example 27 :

Find the period of the following functions.

(i) $f(x) = \cos\frac{2x}{3} - \sin\frac{4x}{5}$

(ii) $f(x) = \cos(\sin x)$

(iii) $f(x) = \sin(\cos x)$

(iv) $f(x) = [x] + [2x] + [3x] + \ldots\ldots + [nx] - \frac{n(n+1)}{2}x$

where $n \in N$ & [ ] denotes greatest integer function

Sol. (i) $f(x) = \cos\left(\frac{2x}{3}\right) - \sin\left(\frac{4x}{5}\right)$

Period of $\cos\left(\frac{2x}{3}\right) = \frac{2\pi(3)}{2} = 3\pi$

Period of $\sin\left(\frac{4x}{5}\right) = \frac{2\pi}{4} \times 5 = \frac{5}{2}\pi$

L.C.M. of 3π & $\frac{5}{2}\pi = 15\pi$

(ii) $f(x) = \cos(\sin x)$

Since cos is even functions

$f(\pi + x) = \cos(\sin(\pi + x)) = \cos(-\sin x)$

$= \cos(\sin x) = f(x)$

Hence π is period.

(iii) $f(x) = \sin(\cos x)$

Period is 2π

(iv) $f(x) = [x] + [2x] + [3x] + \ldots\ldots + [nx] - \frac{n(n+1)}{2}x$

$= -\{x\} - \{2x\} - \ldots\ldots - \{nx\}$

Period of $\{x\} = 1$

period of $\{2x\} = 1/2$

period of $\{3x\} = 1/3$

……………………

……………………

L.C.M. of $\left(1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}\right) = 1$

Example 28 :

If $f(x) = \frac{\sin nx}{\sin\frac{x}{n}}$ has its period as 4π, then find the integral values of n.

Sol. Period of $\sin nx = \frac{2\pi}{n}$; Period of $\sin\frac{x}{n} = 2n\pi$

L.C.M. of $\left(\frac{2\pi}{n}, 2n\pi\right) = 2n\pi$; $2n\pi = 4\pi$; $n = 2, -2$

Example 29 :

Find the period of $f(x) = |\sin x| + |\cos x|$.

Sol. $|\sin x|$ has period π ; $|\cos x|$ has period π

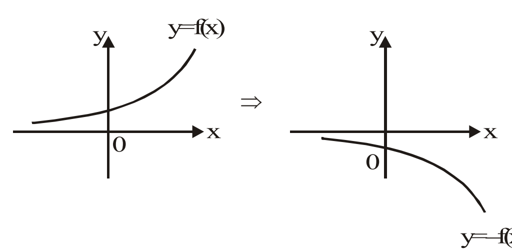
f (x) is an even function & sin x, cos x are complementary

then period of $f(x) = \frac{1}{2}\{\text{LCM of } \pi \;\&\; \pi\} = \frac{\pi}{2}$

BASIC TRANSFORMATIONS ON GRAPHS :

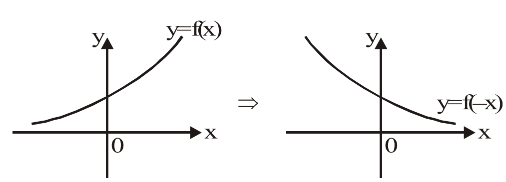
(i) Drawing the graph of y = –f (x) from the known graph of y = f (x)

To draw y = –f (x), take the image of the curve y = f (x) in the x-axis as plane mirror.



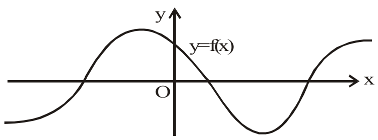
(ii) Drawing the graph of y = f(–x) from the known graph of y = f (x)

To draw y = f (–x), take the image of the curve y = f (x) in the y-axis as plane mirror.



(iii) Drawing the graph of y = |f (x)| from the known graph of y = f (x)

$|f(x)| = f(x)$ if $f(x) \geq 0$ and $|f(x)| = -f(x)$ if $f(x) < 0$. It means that the graph of $f(x)$ and $|f(x)|$ would coincide if $f(x) \geq 0$ and for the portions where $f(x) < 0$ graph of $|f(x)|$ would be image of $y = f(x)$ in x-axis.



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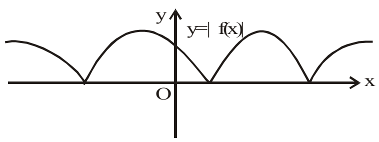
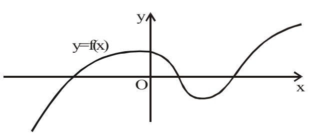


(iv) Drawing the graph of y = f (|x|) from the known graph of y = f (x)

It is clear that, $f(|x|) = \begin{cases} f(x), & x \geq 0 \\ f(-x), & x < 0 \end{cases}$. Thus $f(|x|)$ would be a even function, graph of f(|x|) and f(x) would be identical in the first and the fourth quadrants (ax $x \geq 0$) and as such the graph of f(|x|) would be symmetric about the y-axis (as (|x|) is even).



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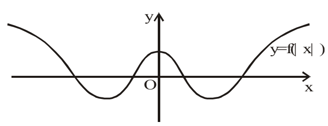
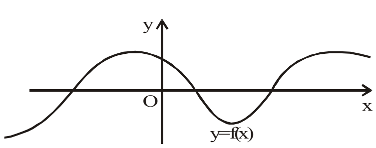


(v) Drawing the graph of |y| = f (x) from the known graph of y = f (x)

Clearly $|y| \geq 0$. If $f(x) < 0$, graph of $|y| = f(x)$ would not exist. And if $f(x) \geq 0$, $|y| = f(x)$ would give $y = \pm f(x)$. Hence graph of $|y| = f(x)$ would exist only in the regions where $f(x)$ is non-negative and will be reflected about the x-axis only in those regions.



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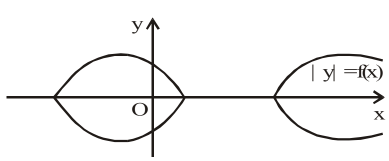
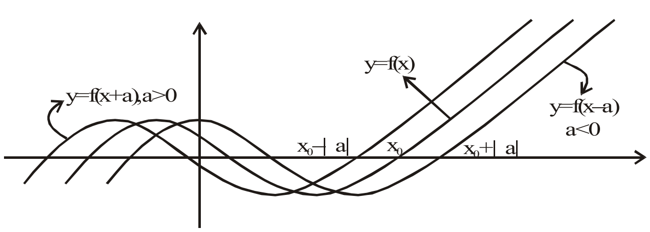


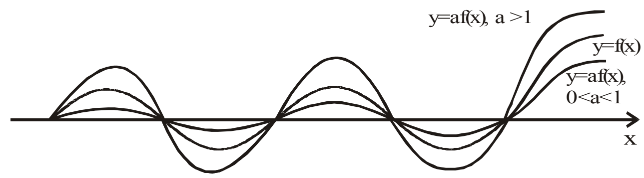
(vi) Drawing the graph of y = f (x + a), a ∈ R from the known graph of y = f (x)



(a) If $a > 0$, shift the graph of f(x) through 'a' units towards left of f(x).

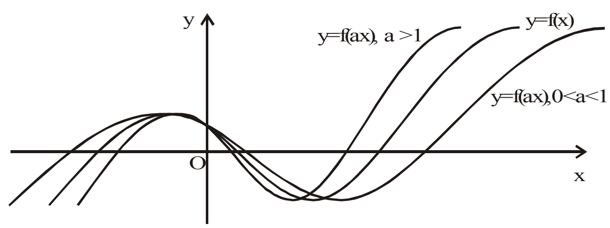
(b) If $a < 0$, shift the graph of f(x) through 'a' units towards right of f(x).

(vii) Drawing the graph of y = af (x) from the known graph of y = f (x)



It is clear that the corresponding points (points with same x co-ordinates) would have their ordinates in the ratio of 1 : a.

(viii) Drawing the graph of y = f (ax) from the known graph of y = f (x).



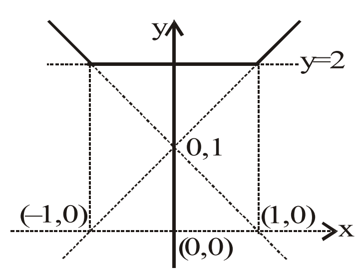
Let us take any point $x_0 \in$ domain of $f(x)$.

Let $ax = x_0$ or $x = \dfrac{x_0}{a}$.

Clearly if $0 < a < 1$, then $x > x_0$ and $f(x)$ will stretch by $\dfrac{1}{a}$ units along the y-axis and if $a > 1$, $x < x_0$, then f(x) will compress by 'a' units along the y-axis.

Example 30 :

Find $f(x) = \max\{1 + x, 1 - x, 2\}$.



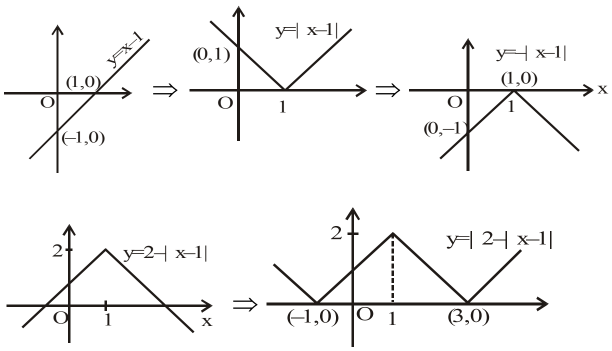
Sol. From the graph it is clear that

$$f(x) = \begin{cases} 1 - x & ; \quad x < -1 \\ 2 & ; \quad -1 \leq x \leq 1 \\ 1 + x & ; \quad x > 1 \end{cases}$$

Example 31 :

Draw the graph of $y = |2 - |x - 1||$.

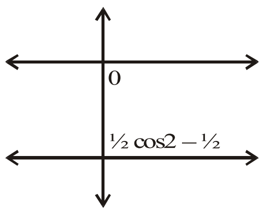
Sol.



Example 32 :

Draw the graph of $f(x) = \cos x \cos(x + 2) - \cos^2(x + 1)$.

Sol. $f(x) = \cos x \cos(x + 2) - \cos^2(x + 1)$



$$= \frac{1}{2}[\cos(2x + 2) + \cos 2] - \frac{1}{2}[\cos(2x + 2) + 1]$$

$$= \frac{1}{2}\cos 2 - \frac{1}{2} < 0$$

TRY IT YOURSELF-2

Q.1 Which of the following relation is a function -

(A) {(a, b) (b, e) (c, e) (b, x)}

(B) {(a, d) (a, m) (b, e) (a, b)}

(C) {(a, d) (b, e) (c, d) (e, x)}

(D) {(a, d) (b, m) (b, y) (d, x)}

Q.2 If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A×B).

Q.3 Find the domain of the function $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$

Q.4 Find the domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 7[x] + 10}}$, where [.] denotes the greatest integer function.

Q.5 If $y = f(x) = \frac{1 - x}{1 + x}$, prove that $x = f(y)$.

Q.6 If $f(x) = x^2$ find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$

Q.7 Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Q.8 Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

Q.9 Let $f(x) = \left[\left(x, \frac{x^2}{1 + x^2}\right) : x \in R\right]$ be a function from R into R. Determine the range of f.

Q.10 Let $f, g : R \to R$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and f/g.

ANSWERS

(1) (C) **(2)** 9 **(3)** $R - \{1, 4\}$

(4) $x \in (-\infty, 2) \cup [6, \infty)$ **(6)** 2.1

(7) $R - \{2, 6\}$ **(8)** Domain = R, Range = $[0, \infty)$

(9) $[0, 1)$

(10) $3x - 2\,;\, -x + 4\,;\, \frac{x + 1}{2x - 3},\ x \neq \frac{3}{2}$

USEFUL TIPS

1. In roster form, the order in which the elements are listed is immaterial.
2. All infinite sets cannot be described in the roster form. For example, the set of real numbers cannot be described in this form, because the elements of this set do not follow any particular pattern.
3. A set does not change if one or more elements of the set are repeated. For example, the sets A = {1, 2, 3} and B = {2, 2, 1, 3, 3} are equal, since each element of A is in B and vice-versa. That is why we generally do not repeat any element in describing a set.
4. If A is a subset of the universal set U, then its complement A' is also a subset of U.
5. Union rule for counting : $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
6. $f(T) = f(0) = f(-T)$, where T is the period.
7. Every constant function is always periodic, with no fundamental period.
8. If f (x) has a period T and g (x) also has a period T then it does not mean that f (x) + g (x) must have a period T. e.g., $f(x) = |\sin x| + |\cos x|$.

DOMAINS AND RANGES OF COMMON FUNCTIONS

Function	Domain	Range
($y = f(x)$)	(i.e. values taken by x)	(i.e. values taken by $f(x)$)
(i) x^n , ($n \in N$)	R = (set of real numbers)	R , if n is odd $R^+ \cup \{0\}$, if n is even
(ii) $\frac{1}{x^n}$, ($n \in N$)	$R - \{0\}$	$R - \{0\}$, if n is odd R^+, if n is even
(iii) $x^{1/n}$, ($n \in N$)	R, if n is odd $R^+ \cup \{0\}$, if n is even	R, if n is odd $R^+ \cup \{0\}$, if n is even
(iv) $\frac{1}{x^{1/n}}$, ($n \in N$)	$R - \{0\}$, if n is odd R^+, if n is even	$R - \{0\}$, if n is odd R^+, if n is even
(v) sin x	R	$[-1, +1]$
(vi) cos x	R	$[-1, +1]$
(vii) tan x	$R - (2k+1)\frac{\pi}{2}, k \in I$	R
(viii) sec x	$R - (2k+1)\frac{\pi}{2}, k \in I$	$(-\infty, -1] \cup [1, \infty)$
(ix) cosec x	$R - k\pi,\ k \in I$	$(-\infty, -1] \cup [1, \infty)$
(x) cot x	$R - k\pi,\ k \in I$	R
(xi) $\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
(xii) $\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
(xiii) $\tan^{-1}x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
(xiv) $\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
(xv) $y = \sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
(xvi) $y = \text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$
(xvii) e^x	R	R^+
(xviii) $e^{1/x}$	$R - \{0\}$	$R^+ - \{1\}$
(xix) a^x, $a > 0$	R	R^+
(xx) $a^{1/x}$, $a > 0$	$R - \{0\}$	$R^+ - \{1\}$
(xxi) $\log_a x$, $(a > 0)(a \neq 1)$	R^+	R
(xxii) $\log_x a = \frac{1}{\log_a x}$ $(a > 0)$ $(a \neq 1)$	$R^+ - \{1\}$	$R - \{0\}$
(xxiii) [x]	R	I
(xxiv) $\frac{1}{[x]}$	$R - [0, 1)$	$\left\{\frac{1}{n}, n \in I - \{0\}\right\}$
(xxv) {x}	R	[0, 1)
(xxvi) 1/{x}	R – I	$(1, \infty)$

(xxvii) $\|x\|$	R	$R^+ \cup \{0\}$
(xxviii) $1/\|x\|$	$R - \{0\}$	R^+
(xxix) $\text{sgn}(x) = \frac{\|x\|}{x}, x \neq 0$ $= 0, x = 0$	R	$\{-1, 0, 1\}$
(xxx) $f(x) = c$	R	$\{c\}$

ADDITIONAL EXAMPLES

Example 1 :

Let R and S be the sets defined as follows:

$R = \{x \in Z \mid x \text{ is divisible by } 2\}$

$S = \{y \in Z \mid y \text{ is divisible by } 3\}$

then prove that $R \cap S \neq \phi$

Sol. Since 6 is divisible by both 3 and 2. Thus $R \cap S \neq \phi$

Example 2 :

From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible no. that could have passed all three examination?

Sol. Let M be the set of students passing in Mathematics.

P be the set of students passing in Physics.

C be the set of students passing in Chemistry.

Now, $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$, $n(C) = 43$

$n(M \cap P) \leq 19$, $n(M \cap C) \leq 29$, $n(P \cap C) \leq 20$ (Given)

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$
$- n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \leq 50$

$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$

$\Rightarrow n(M \cap P \cap C) \leq 50 - 36 \Rightarrow n(M \cap P \cap C) \leq 14$

Thus, the largest possible number that could have passed all the three examinations is 14.

Example 3 :

Find the domain of the function :

$$f(x) = \frac{x}{x^2 + 3x + 2}$$

Sol. f is a rational function of the form $\frac{g(x)}{h(x)}$,

where $g(x) = x$ and $h(x) = x^2 + 3x + 2$.

Now $h(x) \neq 0 \Rightarrow x^2 + 3x + 2 \neq 0 \Rightarrow (x+1)(x+2) \neq 0$ and hence domain of the given function is $R - \{-1, -2\}$.

Example 4 :

Find the range of the function given by $\frac{|x-4|}{x-4}$.

Sol.
$$f(x) = \frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1, & x > 4 \\ \frac{-(x-4)}{x-4} = -1, & x > 4 \end{cases}$$

Thus, the range of $\frac{|x-4|}{x-4} = \{1, -1\}$

Example 5 :

(i) Find the domain of the function $f(x) = [x] + x$.

(ii) Find the range of the function $\sqrt{16 - x^2}$.

Sol. (i) $f(x) = [x] + x$, i.e., $f(x) = h(x) + g(x)$

where $h(x) = [x]$ and $g(x) = x$

The domain of $h = R$ and the domain of $g = R$.

Therefore, Domain of $f = R$

(ii) The domain of f, where $f(x) = \sqrt{16 - x^2}$ is given by $[-4, 4]$.

For the range, let $y = \sqrt{16 - x^2}$ then $y^2 = 16 - x^2$ or $x^2 = 16 - y^2$. Since $x \in [-4, 4]$. Range of $f = [0, 4]$.

Example 6 :

(i) If $f(x) = x^3 - \frac{1}{x^3}$, then find $f(x) + f(1/x)$.

(ii) Let f and g be two functions given by

$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$

$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}$ then find the domain of $f + g$.

Sol. (i) Since, $f(x) = x^3 - \frac{1}{x^3}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

Hence, $f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$

(ii) Since Domain of $f = D_f = \{2, 5, 8, 10\}$ and Domain of $g = D_g = \{2, 7, 8, 10, 11\}$,

therefore the domain of $f + g = \{x \mid x \in D_f \cap D_g\}$
$= \{2, 8, 10\}$

Example 7:

Find the value of $\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{1000}\right]+......\left[\frac{1}{2}+\frac{2946}{1000}\right]$

where [.] denotes greatest integer function ?

Sol. $\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{1000}\right]+......\left[\frac{1}{2}+\frac{499}{1000}\right]+\left[\frac{1}{2}+\frac{500}{1000}\right]$

$+......\left[\frac{1}{2}+\frac{1499}{1000}\right]+\left[\frac{1}{2}+\frac{1500}{1000}\right]+......$

$+\left[\frac{1}{2}+\frac{2499}{1000}\right]+\left[\frac{1}{2}+\frac{2500}{1000}\right]+......\left[\frac{1}{2}+\frac{2946}{1000}\right]$

$=0+1\times1000+2\times1000+3\times447=3000+1341=4341$

Example 8 :

Find the domain $f(x)=\dfrac{1}{\sqrt{\|[|x|-5]\|-11}}$ where [.] denotes greatest integer function.

Sol. $\|[|x|-5]\|>11$

so $[|x|-5]>11$ or $[|x|-5]<-11$

$[|x|]>16$ $\qquad [|x|]<-6$

$|x|\geq17$ or $|x|<-6$ (Not Possible)

$\Rightarrow x\leq-17$ or $x\geq17$

so $x\in(-\infty,-17]\cup[17,\infty)$

Example 9 :

Find the range of $f(x)=\dfrac{x-[x]}{1+x-[x]}$, where [.] denotes greatest integer function.

Sol. $y=\dfrac{x-[x]}{1+x-[x]}=\dfrac{\{x\}}{1+\{x\}}$

$\therefore\ \dfrac{1}{y}=\dfrac{1}{\{x\}}+1\Rightarrow\dfrac{1}{\{x\}}=\dfrac{1-y}{y}\Rightarrow\{x\}=\dfrac{y}{1-y}$

$0\leq\{x\}<1\Rightarrow 0\leq\dfrac{y}{1-y}<1$

Range = [0, 1/2)

Example 10 :

If $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1\ (x\neq0)$ then find $f(x^2)$.

Sol. $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$ ...(i)

Replace x by $\frac{1}{x}$

$2f\left(\frac{1}{x^2}\right)+3f(x^2)=\frac{1}{x^2}-1$ ...(ii)

Solving (i) & (ii) we get

$9f(x)^2-4f(x^2)=3\left(\frac{1}{x^2}-1\right)-2(x^2-1)$

$5f(x^2)=\frac{3}{x^2}-2x^2-1$

$f(x^2)=-\left(\frac{2x^4+x^2-3}{5x^2}\right)$

Example 11 :

Find the range of the following function

(i) $y=\log_e(3x^2-4x+5)$

(ii) $y=\dfrac{x^2-x}{x^2+2x}=\dfrac{x(x-1)}{x(x+2)}$

(iii) $y=3-2^x$

Sol. (i) $y=\log_e(3x^2-4x+5)$

y is defined if $3x^2-4x+5>0$

$D<0$ and coefficeint of $x^2>0$

hence domain is R and log is increaisng function.

Minimum value of $3x^2-4x+5$ is $-\dfrac{D}{4a}$

$=\dfrac{-(-44)}{4(3)}=\dfrac{11}{3}\quad\Rightarrow y\geq\log_e\left(\frac{11}{3}\right)$

Range $\in\left[\log_e\left(\frac{11}{3}\right),\infty\right)$

(ii) $y=\dfrac{x^2-x}{x^2+2x}=\dfrac{x(x-1)}{x(x+2)}$

Domain is $x\in R-\{-2,0\}$

$y=\dfrac{x(x-1)}{x(x+2)}$

when $x\neq0$, $y=\dfrac{x-1}{x+2}\Rightarrow x=\dfrac{1+2y}{1-y}$

$\therefore$ If x is real $y-1\neq0\Rightarrow y\neq1$

Also for $x=\dfrac{1+2y}{1-y}$; $x\neq0$

hence $y\neq-\frac{1}{2}$. Hence range $y\in R-\left\{\frac{-1}{2},1\right\}$

(iii) $y=3-2^x$

Domain is $x\in R$

$0\leq2^x<\infty$

Range $\in(-\infty,3)$

QUESTION BANK

CHAPTER 1 : SETS, RELATIONS AND FUNCTIONS (CLASS XI)

EXERCISE - 1 [LEVEL-1]

PART - 1 : SETS, SUBSETS, OPERATION ON SETS

Q.1 If a set $A = \{a, b, c\}$ then find the number of subsets of the set A –
(A) 4 (B) 6
(C) 8 (D) 9

Q.2 $n\{P[P(\phi)]\}$ is equal to –
(A) 4 (B) 6
(C) 8 (D) 9

Q.3 If $A = \{x : x = 2n + 1, n \in Z$ and $B = \{x : x = 2n, n \in Z\}$, then find $A \cup B$ –
(A) Z (B) W
(C) I (D) None

Q.4 If $A = \{x : x = 3n, n \in Z\}$ and $B = \{x : x = 4n, n \in Z\}$ then find $A \cap B$.
(A) $\{x : x = 8n, n \in Z\}$ (B) $\{x : x = 12n, n \in Z\}$
(C) $\{x : x = 7n, n \in Z\}$ (D) None of these

Q.5 If A and B be two sets containing 3 & 6 elements respectively, what can be the minimum no. of elements in $A \cup B$?
(A) 4 (B) 6
(C) 8 (D) 9

Q.6 In above question, find the maximum number of elements in $A \cup B$.
(A) 4 (B) 6
(C) 8 (D) 9

Q.7 If $A = \{2, 3, 4, 5, 6, 7\}$ and $B = \{3, 5, 7, 9, 11, 13\}$ then find $A - B$
(A) $\{2, 4, 6\}$ (B) $\{9, 11, 13\}$
(C) $\{5, 7, 9\}$ (D) $\{7, 11, 13\}$

Q.8 In the above question, find B – A.
(A) $\{2, 4, 6\}$ (B) $\{9, 11, 13\}$
(C) $\{5, 7, 9\}$ (D) $\{7, 11, 13\}$

Q.9 Let A and B be two non-empty sets having elements in common, then no. of common elements in $A \times B$ & $B \times A$ are
(A) n^2 (B) n
(C) 2n (D) n^3

Q.10 Choose the correct relation –
(A) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
(B) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) - n(A \cap B \cap C)$
(C) $n(A \cup B \cup C) = n(A) + n(B) + n(C) + n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
(D) None of these

Q.11 Choose the correct relation –
(A) $n(A' \cup B') = n(A \cap B)' = n(U) + n(A \cap B)$
(B) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cup B)$
(C) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
(D) None of these

Q.12 Choose the correct relation –
(A) $n(A' \cap B') = n(A \cup B)' = n(U) + n(A \cup B)$
(B) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
(C) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cap B)$
(D) None of these

Q.13 For any set A, choose the correct options –
(A) $A \cup A = A$ (B) $A \cap A = A$
(C) $A \cup \phi = A$ (D) All of these

Q.14 For any set A and B, we have –
(A) $A \cup B = B \cup A$ (B) $A \cap B = B \cap A$
(C) $A \cup B \neq B \cup A$ (D) Both (A) and (B)

Q.15 If A, B and C are any three sets then –
(A) $(A \cup B) \cup C = A \cup (B \cup C)$
(B) $(A \cap B) \cap C = A \cap (B \cap C)$
(C) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(D) All of these

Q.16 If A and B are any two sets, then
(A) $(A \cup B)' = A' \cap B'$ (B) $(A \cup B)' = A' \cup B'$
(C) $(A \cap B)' = A' \cup B'$ (D) Both (A) and (C)

Q.17 In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only ?
(A) 600 (B) 250
(C) 150 (D) 400

Q.18 In the above question how many can speak Bangali ?
(A) 600 (B) 250
(C) 150 (D) 400

Q.19 In the above question many can speak both Hindi and Bengali ?
(A) 600 (B) 250
(C) 150 (D) 400

Q.20 The set $A = [x : x \in R, x^2 = 16$ and $2x = 6]$ equal –
(A) ϕ (B) [14, 3, 4]
(B) [3] (D) [4]

Q.21 Let $A = [x : x \in R, |x| < 1]$, $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is –
(A) $[x : 1 < x \leq 2]$ (B) $[x : 1 \leq x < 2]$
(C) $[x : 1 \leq x \leq 2]$ (D) None of these

Q.22 The set $(A \cup B \cup C) \cap (A \cap B^C \cap C^C)^C \cap C^C$ is equal to
(A) $B \cap C$ (B) $A \cap C$
(C) $B \cap C^C$ (D) None

Q.23 In a class of 45 students. 22 can speak Hindi only and 12 can speak English only. The no. of students, who can speak both Hindi and English is –
(A) 9 (B) 11
(C) 23 (D) 17

Q.24 Let $y = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, $B = \{3, 4, 5\}$; Then $(y \times A) \cap (y \times B) =$
(A) y (B) A
(C) B (D) ϕ

Q.25 In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is
(A) 45 (B) 0
(C) 25 (D) 35

Q.26 The set $A = \{x : |2x + 3| < 7\}$ is equal to
(A) $D = \{x : 0 < x + 5 < 7\}$ (B) $B = \{x : -3 < x < 7\}$
(C) $E = \{x : -7 < x < 7\}$ (D) $C = \{x : -13 < 2x < 4\}$

Q.27 Write the set builder from $A = \{-1, 1\}$
(A) $A = \{x : x$ is a real number$\}$
(B) $A = \{x : x$ is an integer$\}$
(C) $A = \{x : x$ is a root of the equation $x^2 = 1\}$
(D) $A = \{x : x$ is a root of the equation $x^2 + 1 = 0\}$

PART - 2 : RELATIONS, DOMAIN AND RANGE OF FUNCTION

Q.28 Function $f(x) = x^{-2} + x^{-3}$ is-
(A) a rational function (B) an irrational function
(C) an inverse function (D) None of these

Q.29 The period of $|\sin 2x|$ is-
(A) $\pi/4$ (B) $\pi/2$
(C) π (D) 2π

Q.30 If $f(x) = 2|x - 2| - 3|x - 3|$, then the value of f(x) when $2 < x < 3$ is
(A) $5 - x$ (B) $x - 5$
(C) $5x - 13$ (D) None

Q.31 The domain of function $f(x) = \sqrt{2^x - 3^x}$ is -
(A) $(-\infty, 0]$ (B) R
(C) $[0,\infty)$ (D) No value of x

Q.32 The range of function $f(x) = \dfrac{x^2}{1 + x^2}$ is –
(A) $R - \{1\}$ (B) $R^+ \cup \{0\}$
(C) $[0,1]$ (D) None

Q.33 Let $A = \{1, 2, 3\}$, the total number of distinct relations that can be defined over A is –
(A) 2^9 (B) 6 (C) 8 (D) None

Q.34 Find the range of $f(x) = x - [x]$
(A) [0, 1) (B) [0, 1]
(C) (0, 1) (D) [–1, 0)

Q.35 Find the range of $f(x) = 3 + x - [x+2]$
(A) [0, 2) (B) [0, 1]
(C) [1, 2) (D) [–1, 0)

Q.36 The domain of the function $f(x) = \sqrt{\cos x}$ is –
(A) $[3\pi/2, 2\pi]$ (B) $[0, \pi/2]$
(C) $[-\pi/2, \pi/2]$ (D) $[0, \pi/2] \cup [3\pi/2, 2\pi]$

Q.37 If $f(x) = 2x^2$, find $\dfrac{f(3.8) - f(4)}{3.8 - 4}$
(A) 1.56 (B) 156
(C) 15.6 (D) 0.156

Q.38 $f(x) = \dfrac{1}{2} - \tan\left(\dfrac{\pi x}{2}\right) -1 < x < 1$ and
$g(x) = \sqrt{3 + 4x - 4x^2}$. Find domain of $(f + g)$.
(A) [–1/2, 1) (B) (–1/2, 1]
(C) [–1/2, 3/2] (D) (–1, 1)

PART - 3 : COMBINATIONS OF FUNCTIONS

Q.39 If $f(x) = \dfrac{x - 3}{x + 1}$, then $f[f\{f(x)\}]$ equals-
(A) x (B) 1/x
(C) –x (D) –1/x

Q.40 If $f(x) = \cos(\log x)$, then $f(x) f(y) - \dfrac{1}{2}[f(x/y) + f(xy)] =$
(A) –1 (B) 1/2
(C) –2 (D) 0

Q.41 If $f(x) = \dfrac{2^x + 2^{-x}}{2}$, then $f(x + y) . f(x - y)$ is equal to
(A) $\dfrac{1}{2}[f(x+y) + f(x-y)]$ (B) $\dfrac{1}{2}[f(2x) + f(2y)]$
(C) $\dfrac{1}{2}[f(x+y) . f(x-y)]$ (D) None of these

Q.42 If $f : R \to R$. $f(x) = 2x + |x|$, then $f(3x) - f(-x) - 4x$ equals-
(A) f(x) (B) –f(x)
(C) f(–x) (D) 2f(x)

Q.43 If $f : R \to R$ is defined by $f(x) = \dfrac{x}{x^2 + 1}$, find $f(f(2))$
(A) 1/29 (B) 10/29
(C) 29/10 (D) 29

PART - 4 : MISCELLANEOUS

Q.44 Let $f(x) = \tan x$, $g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where f(x) and g(x) are real valued functions. For all possible value of x, $f(g(x)) =$
(A) $\tan\left(\dfrac{x - 1}{x + 1}\right)$ (B) $\tan(x - 1) - \tan(x + 1)$
(C) $\dfrac{f(x) + 1}{f(x) - 1}$ (D) $\dfrac{x - \pi/4}{x + \pi/4}$

Q.45 The period of the function
$4\sin^4\left(\dfrac{4x - 3\pi}{6\pi^2}\right) + 2\cos\left(\dfrac{4x - 3\pi}{3\pi^2}\right)$ is –
(A) $3\pi^2/4$ (B) $3\pi^3/4$ (C) $3\pi^3/2$ (D) $4\pi^3/2$

Q.46 $f(x) = |x-1|$, $f: R^+ \to R$ and $g(x) = e^x$, $g: [-1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are –

(A) $(0, \infty)$ and $[0, \infty)$
(B) $[-1, \infty)$ and $[0, \infty)$
(C) $[-1, \infty)$ and $\left[1-\frac{1}{e}, \infty\right)$
(D) $[-1, \infty)$ and $\left[\frac{1}{e}-1, \infty\right)$

Q.47 $h(x) = |kx+5|$, domain of $f(x)$ is $[-5, 7]$, domain of $f(h(x))$ is $[-6, 1]$ and range of $h(x)$ is the same as the domain of $f(x)$, then value of k is –

(A) 2
(B) 4
(C) 3
(D) 1

Q.48 If $f(x)$ is a polynomial function that $f(x) \cdot f(-x) = f(2x)$, then–

(A) No such function exists
(B) $f(x)$ is linear
(C) Number of such functions are exactly one
(D) Number of such functions are exactly two

Q.49 If domain of $f(x) = \dfrac{\sin^{-1}(\sin x)}{\sqrt{-\log_{\left(\frac{x+4}{2}\right)} \log_2\left(\frac{2x-1}{3+x}\right)}}$ is $(a, b) \cup (c, \infty)$, then find the value of $a + b + 3c$.

(A) 5
(B) 4
(C) 3
(D) 2

Q.50 If $f: (0, \infty) \to (0, \infty)$ satisfy $f(x\,f(y)) = x^2 y^a$ $(a \in R)$, then find the value of a.

(A) 4
(B) 1
(C) 3
(D) 2

EXERCISE - 2 [LEVEL-2]

Q.1 Domain of definition of the function

$$f(x) = \log\left(\sqrt{10 \cdot 3^{x-2} - 9^{x-1} - 1}\right) + \sqrt{\cos^{-1}(1-x)} \text{ is}$$

(A) $[0, 1]$
(B) $[1, 2]$
(C) $(0, 2)$
(D) $(0, 1)$

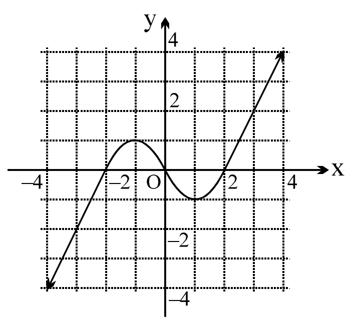
Q.2 The period of the function $f(x) = \dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is

(A) $\pi/2$
(B) $\pi/4$
(C) π
(D) 2π

Q.3 Let $f(x) = \sin^2 x + \cos^4 x + 2$ and $g(x) = \cos(\cos x) + \cos(\sin x)$. Also let period of $f(x)$ and $g(x)$ be T_1 and T_2 respectively then

(A) $T_1 = 2T_2$
(B) $2T_1 = T_2$
(C) $T_1 = T_2$
(D) $T_1 = 4T_2$

Q.4 The graph of the function $y = g(x)$ is shown.



The number of solutions of the equation $\left||g(x)| - 1\right| = \frac{1}{2}$, is

(A) 4
(B) 5
(C) 6
(D) 8

Q.5 Consider the function g (x) defined as

$$g(x) \cdot \left(x^{(2^{2008}-1)} - 1\right) = (x+1)(x^2+1)(x^4+1) \ldots\ldots (x^{2^{2007}}+1) - 1.$$

the value of g (2) equals

(A) 1
(B) 2
(C) 2^{2008}
(D) $2^{2008} - 1$

Q.6 If $f(x) = x^2 + bx + c$ and $f(2+t) = f(2-t)$ for all real numbers t, then which of the following is true?

(A) $f(1) < f(2) < f(4)$
(B) $f(2) < f(1) < f(4)$
(C) $f(2) < f(4) < f(1)$
(D) $f(4) < f(2) < f(1)$

Q.7 Let
$f(x) = \max.\{\sin t : 0 \le t \le x\}$
$g(x) = \min.\{\sin t : 0 \le t \le x\}$
and $h(x) = [f(x) - g(x)]$

where [ ] denotes greatest integer function, then the range of h (x) is

(A) $\{0, 1\}$
(B) $\{1, 2\}$
(C) $\{0, 1, 2\}$
(D) $\{-3, -2, -1, 0, 1, 2, 3\}$

Q.8 Range of the function $f(x) = \dfrac{\{x\}}{1+\{x\}}$

where $\{x\}$ denotes the fractional part function is

(A) $[0, 1)$
(B) $[0, 1/2]$
(C) $[0, 1/2)$
(D) $(0, 1/2)$

Q.9 Let $f(x) = (3x+2)^2 - 1$, $-\infty < x \le -2/3$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to line $y = x$, then $g(x)$ equals

(A) $\frac{1}{3}(-2-\sqrt{x+1})$, $x \ge -1$
(B) $\frac{1}{3}(-2+\sqrt{x+1})$, $x \ge -1$
(C) $\frac{1}{3}(-1-\sqrt{x+2})$, $x \ge -2$
(D) $\frac{1}{3}(-1+\sqrt{x+2})$, $x \ge -2$

Q.10 If $g(x) = \left(4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7\right)^{\frac{1}{7}}$,

then the value of g (g (100)) is equal to

(A) – 1 (B) 0
(C) 1 (D) 100

Find the domain of the functions : (Q.11-Q.17)

Q.11 $f(x) = \sqrt{\frac{1-5^x}{7^{-x}-7}}$

(A) $(-\infty, -1) \cup [0, \infty)$
(B) $(3-2\pi < x < 3-\pi) \cup (3 < x \le 4)$
(C) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$
(D) $(-1 < x < -1/2) \cup (x > 1)$

Q.12 $y = \log_{10} \sin(x-3) + \sqrt{16-x^2}$

(A) $(-\infty, -1) \cup [0, \infty)$
(B) $(3-2\pi < x < 3-\pi) \cup (3 < x \le 4)$
(C) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$
(D) $(-1 < x < -1/2) \cup (x > 1)$

Q.13 $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

(A) $(-\infty, -1) \cup [0, \infty)$
(B) $(3-2\pi < x < 3-\pi) \cup (3 < x \le 4)$
(C) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$
(D) $(-1 < x < -1/2) \cup (x > 1)$

Q.14 $f(x) = \frac{1}{\sqrt{4x^2-1}} + \ln x\,(x^2-1)$

(A) $(-\infty, -1) \cup [0, \infty)$
(B) $(3-2\pi < x < 3-\pi) \cup (3 < x \le 4)$
(C) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$
(D) $(-1 < x < -1/2) \cup (x > 1)$

Q.15 $f(x) = \sqrt{\log_{\frac{1}{2}} \frac{x}{x^2-1}}$

(A) $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$
(B) $(-3, -1] \cup \{0\} \cup [1, 3)$
(C) $\{4\} \cup [5, \infty)$
(D) $(-4, -1/2) \cup (2, \infty)$

Q.16 $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9-x^2}}$

(A) $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$
(B) $(-3, -1] \cup \{0\} \cup [1, 3)$
(C) $\{4\} \cup [5, \infty)$
(D) $(-4, -1/2) \cup (2, \infty)$

Q.17 $f(x) = \sqrt{(x^2-3x-10).\ln^2(x-3)}$

(A) $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$
(B) $(-3, -1] \cup \{0\} \cup [1, 3)$
(C) $\{4\} \cup [5, \infty)$
(D) $(-4, -1/2) \cup (2, \infty)$

Q.18 The function $f(x) = \frac{3x-2}{x+4}$ has an inverse that can be written in the form $f^{-1}(x) = \frac{x+b}{cx+d}$. Find the value of $(b+c+d)$.

(A) 1 (B) 2
(C) 3 (D) 4

Q.19 Suppose that f (x) is a function of the form

$f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$ $(x \ne 0)$. If $f(5) = 2$ then find the value of $f(-5)$.

(A) 11 (B) 28
(C) 33 (D) 48

Q.20 The function f is not defined for $x = 0$, but for all non zero real numbers x, $f(x) + 2f(1/x) = 3x$. Find the number of real numbers satisfying equation $f(x) = f(-x)$.

(A) 1 (B) 2
(C) 3 (D) 4

Q.21 Suppose that f is an even, periodic function with period 2, and that $f(x) = x$ for all x in the interval [0, 1]. Find the value of f (3.14).

(A) 0.86 (B) 0.76
(C) 0.56 (D) 0.46

Q.22 Find the domain of the function $\log|x^2 - 9|$.

(A) R– [–1,1] (B) R– [–2,2]
(C) R– [–3,3] (D) R– [–4,4]

Q.23 Find the domain of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$.

(A) [–1,2] (B) [1,6]
(C) [1,4] (D) [–1,1]

Q.24 Find the domain of the function $f(x) = \sqrt{(2-2x-x^2)}$.

(A) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$ (B) $-1-\sqrt{2} \le x \le -1+\sqrt{5}$
(C) $-1-\sqrt{3} \le x \le -1+\sqrt{5}$ (D) $1-\sqrt{3} \le x \le -1+\sqrt{7}$

Q.25 If the range of rational algebraic function

$f(x)=\dfrac{x^2+x+k}{x^2+2x+k}$ is $\left[\dfrac{5}{6},\dfrac{3}{2}\right]$, then find value of k.

(A) 1 (B) 2
(C) 3 (D) 4

Q.26 Find fundamental period of function

$$f(x)=\operatorname{cosec}\left(\frac{x}{2}\right)+\sec\left(\frac{x}{3}\right)+\cot\left(\frac{x}{4}\right)+\tan\left(\frac{x}{5}\right)+\cos\left(\frac{x}{6}\right)+\sin\left(\frac{x}{7}\right)$$

(A) 120π (B) 220π
(C) 320π (D) 420π

Q.27 Find fundamental period of function

$$f(x)=4\cos^4\left(\frac{x-\pi}{4\pi^2}\right)-2\cos\left(\frac{x-\pi}{2\pi^2}\right)$$

(A) $2\pi^3$ (B) $2\pi^2$
(C) $2\pi^4$ (D) π^5

Q.28 Let $f(x)=x^2+4x-1$ and $g(x)=|x|$
If $h(x)=f(g(x))+10$. Find range of h (x).

(A) $[9,\infty)$ (B) $[4,\infty)$
(C) $[5,\infty)$ (D) $[8,\infty)$

Q.29 Let $f: R-\{-4/3\}\to R-\{4/3\}$ be a function defined as

$f(x)=\dfrac{4x}{3x+4}$. If inverse of map f is map g given by

$g: R-\{4/3\}\to R-\{-4/3\}$. Find g (1)

(A) 1 (B) 2
(C) 3 (D) 4

Q.30 The function f(x) is defined as

$$f(x)=\begin{cases}\dfrac{1}{2^x} & \text{if } x\ge 4\\ f(x+1) & \text{if } x<4\end{cases}$$

then find value of $f\left(2+\log_2\left(\dfrac{3}{2}\right)\right)$.

(A) 1/6 (B) 1/8
(C) 1/12 (D) 1/24

Q.31 f (x) and g (x) are linear function such that for all x, f (g (x)) and g (f (x)) are Identity functions. If f (0) = 4 and g (5) = 17, compute f (136).

(A) 11 (B) 12
(C) 13 (D) 14

Q.32 Let $f(x)=(x+1)(x+2)(x+3)(x+4)+5$ where $x\in[-6,6]$. If the range of the function is [a, b] where $a, b\in N$ then find the value of (a + b).

(A) 5049 (B) 2049
(C) 1049 (D) 3049

Q.33 Let a and b be real numbers and let $f(x)=a\sin x+b\sqrt[3]{x}+4, \forall x\in R$. If $f(\log_{10}(\log_3 10))=5$ then find the value of $f(\log_{10}(\log_{10}3))$.

(A) 1 (B) 2
(C) 3 (D) 4

Q.34 Let $f(x)=-x^{100}$. If f(x) is divided by x^2+x, then the remainder is r(x). Find the value of r (10).

(A) 10 (B) 12
(C) 13 (D) 14

Q.35 Let $P(x)=x^4+ax^3+bx^2+cx+d$, where $a, b, c, d\in R$. Suppose $P(0)=6$, $P(1)=7$, $P(2)=8$ and $P(3)=9$, then find the value of P (4).

(A) 11 (B) 18
(C) 34 (D) 44

Q.36 Match the column

Column I

(a) $f(x)=\sqrt{\cos 2x}+\sqrt{16-x^2}$

(b) $f(x)=\log_7\log_5\log_3\log_2(2x^3+5x^2-14x)$

(c) $f(x)=\ln(\sqrt{x^2-5x-24}-x-2)$

Column II (Domains)

(p) $\left[-\dfrac{5\pi}{4},\dfrac{-3\pi}{4}\right]\cup\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]\cup\left[\dfrac{3\pi}{4},\dfrac{5\pi}{4}\right]$

(q) $(-\infty,-3]$

(r) $\left(-4,-\dfrac{1}{2}\right)\cup(2,\infty)$

(A) a – p, b – q, c – r (B) a – p, b – r, c – q
(C) a – q, b – p, c – r (D) a – r, b – p, c – q

Passage (Q.37-Q.38)

If the function f (x) defined on the interval [–2, 2] as

$$f(x)=\begin{cases}1; & -2\le x\le -1\\ x+2; & -1<x<1\\ 4-x; & 1\le x\le 2\end{cases}$$

Q.37 Find the range of f (x).

(A) [1, 3] (B) [1, 2]
(C) [2, 4] (D) [3, 5]

Q.38 Find the number of solution of the equation $\{f(x)\}=1/2$. (where {x} denotes fractional part of x)

(A) 1 (B) 2
(C) 3 (D) 4

Passage (Q.39-Q.40)

Consider the function $f(x)=\dfrac{3x+a}{x^2+3}$ which has the greatest value 3/2.

Q.39 Find the value of constant number 'a'.

(A) 1 (B) 2
(C) 3 (D) 4

Q.40 Find the minimum value of f (x).

(A) –1/2 (B) –1/3
(C) –1/4 (D) –1

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 If X and Y are two sets such that n (X) = 17, n (Y) = 23 and n (X ∪ Y) = 38, find n (X ∩ Y).

Q.2 If X and Y are two sets such that X ∪ Y has 18 elements, X has 8 elements and Y has 15 elements ; how many elements does X ∩ Y have?

Q.3 In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Q.4 In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Q.5 In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Q.6 In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

Q.7 Let f : {1, 3, 4} → {1, 2, 5} and g : {l, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4,1)} and g = {(1,3), (2,3), (5,1)}. If gof is {(1, X), (3, Y), (4, 3)}. Find the value of X + Y.

Q.8 Let f : R – (–4/3) → R be a function defined as

$\dfrac{4x}{3x+4}$. The inverse of f is the map g : Range

$f \to R - \left\{-\dfrac{4}{3}\right\}$ given by : $g(y) = \dfrac{Py}{4-3y}$. Find the value of P.

Q.9 Let $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that P (1) = 1, P (2) = 2, P (3) = 3, P (4) = 4, P (5)=5 and P (6) = 6 then the value of P (7) is

Q.10 If $\sum_{i=1}^{7} i^2 x_i = 1$, $\sum_{i=1}^{7} (i+1)^2 x_i = 12$ & $\sum_{i=1}^{7} (i+2)^2 x_i = 123$, then find the value of $\sum_{i=1}^{7} (i+3)^2 x_i$.

Q.11 h (x) = | kx + 5 |, domain of f (x) is [–5, 7], domain of f (h (x)) is [–6, 1] and range of h (x) is the same as the domain of f (x), then value of k is –

Q.12 Find the period of function $f(x) = \tan\dfrac{\pi}{2}[x]$, where [.] denotes greatest integer function.

Q.13 The set of real values of 'x' satisfying the equality

$\left[\dfrac{3}{x}\right] + \left[\dfrac{4}{x}\right] = 5$ (where [ ] denotes the greatest integer function) belongs to the interval (a, b/c] where a,b,c ∈N and b/c is in its lowest form. Find the value of a + b + c + abc.

Q.14 Let l_1 be the line 4x + 3y = 3 and l_2 be the line y = 8x. L_1 is the line formed by reflecting l_1 across the line y = x and L_2 is the line formed by reflecting l_2 across the x-axis. If θ is the acute angle between L_1 and L_2 such that tan θ = a/b, where a and b are coprime then find (a + b).

Q.15 Let E = {1, 2, 3, 4} & F = {1, 2}. Then the number of onto functions from E to F is

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 Which of the following is not a periodic function - **[AIEEE 2002]**
(A) $\sin 2x + \cos x$ (B) $\cos \sqrt{x}$
(C) $\tan 4x$ (D) $\log \cos 2x$

Q.2 The period of $\sin^2 x$ is- **[AIEEE 2002]**
(A) $\pi/2$ (B) π
(C) $3\pi/2$ (D) 2π

Q.3 The range of the function $f(x) = \frac{2+x}{2-x}$, $x \neq 2$ is - **[AIEEE-2002]**
(A) R (B) $R-\{-1\}$
(C) $R-\{1\}$ (D) $R-\{2\}$

Q.4 Domain of definition of the function
$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is- **[AIEEE 2003]**
(A) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (B) $(1, 2)$
(C) $(-1, 0) \cup (1, 2)$ (D) $(1, 2) \cup (2, \infty)$

Q.5 If $f : R \to R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is- **[AIEEE 2003]**
(A) $\frac{7n(n+1)}{2}$ (B) $\frac{7n}{2}$
(C) $\frac{7(n+1)}{2}$ (D) $7n(n+1)$

Q.6 The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then- **[AIEEE 2004]**
(A) $f(x+2) = f(x-2)$ (B) $f(2+x) = f(2-x)$
(C) $f(x) = f(-x)$ (D) $f(x) = -f(-x)$

Q.7 A real valued function f(x) satisfies the functional equation $f(x - y) = f(x) f(y) - f(a - x) f(a + y)$ where a is a given constant and $f(0) = 1$, then $f(2a - x)$ is equal to - **[AIEEE-2005]**
(A) $-f(x)$ (B) $f(x)$
(C) $f(a) + f(a - x)$ (D) $f(-x)$

Q.8 If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then - **[AIEEE 2009]**
(A) A = B (B) A = C
(C) B = C (D) $A \cap B = \phi$

Q.9 Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE 2012]**
(A) 5^2 (B) 3^5
(C) 2^5 (D) 5^3

Q.10 If $f(x) + 2f(1/x) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$, then S **[JEE MAIN 2016]**
(A) contains exactly one element
(B) contains exactly two elements
(C) contains more than two elements
(D) is an empty set

Q.11 Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall\, x, y \in R$ then $\sum_{n=1}^{10} f(n)$ is equal to : **[JEE MAIN 2017]**
(A) 190 (B) 255
(C) 330 (D) 165

Q.12 Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S: **[JEE MAIN 2018]**
(A) contains exactly two elements.
(B) contains exactly four elements.
(C) is an empty set.
(D) contains exactly one element.

Q.13 Two sets A and B are as under:
$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$
$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then: **[JEE MAIN 2018]**
(A) $A \cap B = \phi$ (an empty set)
(B) neither $A \subset B$ nor $B \subset A$
(C) $B \subset A$
(D) $A \subset B$

Q.14 For $x \in R-\{0, 1\}$, let $f_1(x) = 1/x$, $f_2(x) = 1 - x$ & $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to :- **[JEE MAIN 2019 (JAN)]**
(A) $f_3(x)$ (B) $f_1(x)$
(C) $f_2(x)$ (D) $(1/x) f_3(x)$

Q.15 Let $A = \{x \in R : x$ is not a positive integer$\}$
Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$ then f is **[JEE MAIN 2019 (JAN)]**
(A) injective but not surjective.
(B) not injective.
(C) surjective but not injective.
(D) neither injective nor surjective

Q.16 If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to : **[JEE MAIN 2019 (APRIL)]**
(A) $2f(x)$ (B) $2f(x^2)$
(C) $(f(x))^2$ (D) $-2f(x)$

Q.17 Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals **[JEE MAIN 2019 (APRIL)]**
(A) $2f_1(x) f_1(y)$ (B) $2f_1(x) f_2(y)$
(C) $2f_1(x + y) f_2(x - y)$ (D) $2f_1(x + y) f_1(x - y)$

Q.18 If the function $f : R - \{1, -1\} \to A$ defined by

$f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to

[JEE MAIN 2019 (APRIL)]

(A) $R - [-1, 0)$ (B) $R - (-1, 0)$

(C) $R - \{-1\}$ (D) $[0, \infty)$

Q.19 Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :

[JEE MAIN 2019 (APRIL)]

(A) 12.8 (B) 13.5

(C) 13.9 (D) 13

Q.20 The domain of the definition of the function

$f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is

[JEE MAIN 2019 (APRIL)]

(A) $(1, 2) \cup (2, \infty)$

(B) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(C) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(D) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

Q.21 Let $f(x) = x^2, x \in R$. For any $A \subseteq R$, define

$g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

[JEE MAIN 2019 (APRIL)]

(A) $f(g(S)) \neq f(S)$ (B) $f(g(S)) = S$

(C) $g(f(S)) = g(S)$ (D) $g(f(S)) \neq S$

Q.22 For $x \in (3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$.

If $\phi(x) = ((hof)og)(x)$, then $\phi = \pi/3$ is equal to :

[JEE MAIN 2019 (APRIL)]

(A) $\tan(\pi/12)$ (B) $\tan(7\pi/12)$

(C) $\tan(11\pi/12)$ (D) $\tan(5\pi/12)$

Q.23 Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

[JEE MAIN 2019 (APRIL)]

(A) If $(A - C) \subseteq B$, then $A \subseteq B$ (B) $(C \cup A) \cap (C \cup B) = C$

(C) If $(A - B) \subseteq C$, then $A \subseteq C$ (D) $B \cap C \neq \phi$

Q.24 If $g(x) = x^2 + x - 1$ and $gof(x) = 4x^2 - 10x + 5$, then find $f(5/4)$. **[JEE MAIN 2020 (JAN)]**

(A) 1/2 (B) –1/2

(C) –1/3 (D) 1/3

Q.25 The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$, is

[JEE MAIN 2020 (JAN)]

(A) $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$ (B) $\frac{1}{2}\log_8\left(\frac{1-x}{1+x}\right)$

(C) $\frac{1}{4}\log_8\left(\frac{1-x}{1+x}\right)$ (D) $\frac{1}{2}\log_8\left(\frac{1+x}{1-x}\right)$

Q.26 Let $f(x) = \frac{x[x]}{x^2+1} : (1,3) \to R$ then range of $f(x)$ is

(where [.] denotes greatest integer function)

[JEE MAIN 2020 (JAN)]

(A) $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$ (B) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(C) $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$ (D) $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

Q.27 If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x - 2| \geq 3\}$; then :

[JEE MAIN 2020 (JAN)]

(A) $A \cup B = R - (2, 5)$ (B) $A \cap B = (-2, -1)$

(C) $B - A = R - (-2, 5)$ (D) $A - B = [-1, 2)$

Q.28 Let $X = \{n \in N : 1 \leq n \leq 50\}$. If

$A = \{n \in X : n \text{ is a multiple of } 2\}$ and

$B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is_____ **[JEE MAIN 2020 (JAN)]**

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	C	A	A	B	B	D	A	B	A	A	C	B	D	D	D	D	A	B	C	A	B	C	B	D	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	A	C	A	B	C	A	D	A	A	C	D	C	A	A	D	B	D	B	A	B	B	A	D	A	A

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	C	C	D	B	B	C	C	A	D	A	B	C	D	A	B	C	A	B	B
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	A	C	B	A	D	D	A	A	D	D	B	A	C	A	C	B	A	C	C	A

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	2	5	60	325	125	11	4	4	727	334	2	2	20	57	14

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	B	B	B	A	A	B	A	C	B	B	C	A	D	A
Q	15	16	17	18	19	20	21	22	23	24	25	26	27	28
A	A	A	A	A	C	C	C	C	A	B	A	B	C	29

CHAPTER-1 : SOLUTIONS TO TRY IT YOURSELF

TRY IT YOURSELF-1

(1) (i) We have, $P(A:B) = \{X \in P(A) : B \subset X\}$

= Set of all those subsets of A which contain B.

$\therefore P(A:\phi)$ = Set of all those subsets of A which contain ϕ

= Set of all subsets of set A = P (A)

(ii) If $A = \{a, b, c, d\}$ and $B = \{a, b\}$. Then,

$P(A:B)$ = Set of all those subsets of set A which contain B = {a, b, c}, {a, b, d}, {a, b, c, d}

(2) Let U be the set of consumers questioned, S be the set of consumers who liked the product A and T be the set of consumers who like the product B.

Given that $n(U) = 1000, n(S) = 720, n(T) = 450$

So $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 720 + 450 - n(S \cap T) = 1170 - n(S \cap T)$

Therefore, $n(S \cup T)$ is maximum when $n(S \cap T)$ is least.

But $S \cup T \subset U$ implies $n(S \cup T) \le n(U) = 1000$.

So, maximum values of $n(S \cup T)$ is 1000.

Thus, the least value of $n(S \cap T)$ is 170.

Hence, the least number of consumers who liked both products is 170.

(3) Let A be the set of people who like cricket and B be the set of people who like tennis. Then

$(A \cup B) = 65,\ n(A) = 40,\ n(A \cap B) = 10$

We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow\ 65 = 40 + n(B) - 10 \Rightarrow n(B) = 35$

No. of people who like tennis only and not cricket = $n(B - A)$

Also, $n(B) = n(B - A) + n(A \cap B)$

$35 = n(B - A) + 10\ ;\ n(B - A) = 35 - 10 = 25$

(4) Let H denote the set of peoples who speak Hindi and E denote the set of peoples who speak English. We are given that, $n(H) = 250,\ n(E) = 200,\ n(H \cup E) = 400$

We know that, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow\ 400 = 250 + 200 - n(H \cap E) \Rightarrow n(H \cap E) = 450 - 400 = 50$

Hence, 50 people can speak Hindi as well as English.

(5) We have, $A \cup B = \{a, e, i, o, u\} = A$.

This example illustrates that union of sets A and its subset B is the set A itself, i.e., if $B \subset A$, then $A \cup B = A$.

TRY IT YOURSELF-2

(1) **(C).** Since in (C) each element is associated with unique element. while in (1) element b is associated with two elements, in (2) element a is associated with three elements and in (D) element b is associated with two elements so. (C) is function.

(2) Number of elements in $A \times B = 3 \times 3 = 9$

(3) Since $x^2 - 5x + 4 = (x - 4)(x - 1)$, the function f (x) is defined for all real numbers except at x = 4 and x = 1.

Hence the domain of f is $R - \{1, 4\}$.

(4) Domain $[x]^2 - 7[x] + 10 > 0$

$([x] - 5)([x] - 2) > 0 \Rightarrow [x] < 2$ and $[x] > 5$

$[x] < 2 \Rightarrow x < 2$ and $[x] > 5 \Rightarrow x \ge 6$

$\Rightarrow\ x \in (-\infty, 2) \cup [6, \infty)$

(5) Given, $y = f(x) = \dfrac{1-x}{1+x}$ ...(i)

Now, $f(y) = \dfrac{1-y}{1+y} = \dfrac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = \dfrac{1+x-1+x}{1+x+1-x} = \dfrac{2x}{2} = x$

(6) At $x = 1.1 : f(1.1) = (1.1)^2 = 1.21\ ;\ f(1) = (1)^2 = 1$

$\therefore\ \dfrac{f(1.1) - f(1)}{(1.1 - 1)} = \dfrac{1.21 - 1}{0.1} = \dfrac{0.21}{0.1} = 2.1$

(7) f (x) is a rational function of x.

f (x) assumes real values of all x except for those values of x for which $x^2 - 8x + 12 = 0 \Rightarrow (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6$ $\therefore$ Domain of function = $R - \{2, 6\}$

(8) Here, $f(x) = |x - 1|$

The function f (x) is defined for all values of x.

$\therefore$ Domain of f (x) = R

when $x > 1,\ |x - 1| = x - 1 > 0$

when $x = 1,\ |x - 1| = 0$

when $x < 1,\ |x - 1| = -x + 1 > 0$

$\therefore$ Range of f (x) = all real number $\ge 0 = [0, \infty)$

(9) Put $y = \dfrac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow x^2(1 - y) = y$

$\Rightarrow\ x^2 = \dfrac{y}{1-y} \Rightarrow x = \pm\sqrt{\dfrac{y}{1-y}}$

For x to be defined y and y – 1 must either be both positive or both negative.

$y \ge 0$ and $1 - y > 0 \Rightarrow y \ge 0$ and $y - 1 < 0$

$\Rightarrow\ y \ge 0$ and $y < 1 \Rightarrow 0 \le y < 1$

Also when $y \le 0$ and $1 - y < 0 \Rightarrow y \le 0$ and $y - 1 > 0$

$\Rightarrow\ y \le 0$ and $y > 1$ which is not possible.

$\therefore$ Range of f (x) = [0, 1)

Any positive real number x such that $0 \le x < 1$.

(10) Now, $(f + g)(x) = f(x) + g(x) = x + 1 + 2x - 3 = 3x - 2$

$(f - g)(x) = f(x) - g(x) = x + 1 - (2x - 3)$

$= x + 1 - 2x + 3 = -x + 4$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+1}{2x-3},\ x \ne \dfrac{3}{2}$

CHAPTER-1 : SETS, RELATIONS AND FUNCTIONS
EXERCISE-1

(1) **(C).** Since $n(A) = 3$
$\therefore$ number of subsets of A is $2^3 = 8$
and set of all those subsets is P(A) named as power set
$P(A) : \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

(2) **(A).** We have $P(\phi) = \{\phi\}$
$\therefore\ P(P(\phi)) = \{\phi, \{\phi\}\}$
$\Rightarrow P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$.
Hence, $n\{P[P(\phi)]\} = 4$

(3) **(A).** $A \cup B = \{x : x$ is an odd integer$\} \cup \{x : x$ is an even integer$\} = \{x : x$ is an integer$\} = Z$

(4) **(B).** $x \in A \cap B \Leftrightarrow x = 3n, n \in Z$ and $x = 4n, n \in Z$
$\Leftrightarrow$ x is a multiple of 3 and x is a multiple of 4
$\Leftrightarrow$ x is a multiple of 3 and 4 both
$\Leftrightarrow$ x is a multiple of 12 $\Leftrightarrow x = 12n, n \in Z$
Hence $A \cap B = \{x : x = 12n, n \in Z\}$

(5) **(B).** We have, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively. When $n(A \cap B)$ is maximum.
This is possible only when $A \subseteq B$.
In this case, $n(A \cap B) = 3$
$\therefore\ n(A \cup B) = n(A) + n(B) - n(A \cap B) = (3 + 6 - 3) = 6$
So, minimum number of elements in $A \cup B$ is 6.

(6) **(D).** When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$
This is possible only when $A \cap B = \phi$.
$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$.
So, maximum number of elements in $A \cup B$ is 9.

(7) **(A).** $A - B = \{2, 4, 6\}$

(8) **(B).** $B - A = \{9, 11, 13\}$

(9) **(A).** We have $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
On replacing C by B and D by A, we get
$\Rightarrow\ (A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$
It is given that AB has n elements so $(A \cap B) \times (B \cap A)$ has n^2 elements
But $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$
$\therefore\ (A \times B) \cap (B \times A)$ has n^2 elements
Hence $A \times B$ and $B \times A$ have n^2 elements in common.

(10) **(A).** $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(11) **(C).** $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(12) **(B).** $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

(13) **(D).** (i) $A \cup A = \{x : x \in A$ or $x \in A\} = \{x : x \in A\} = A$
(ii) $A \cap A = \{x : x \in A\ \&\ x \in A\} = \{x : x \in A\} = A$
(iii) $A \cup \phi = \{x : x \in A$ or $x \in \phi\} = \{x : x \in A\} = A$

(14) **(D).** For any set A and B, we have
(i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$
i.e. union and intersection are commutative.

(15) **(D).** If A, B and C are any three sets then
(i) $(A \cup B) \cup C = A \cup (B \cup C)$
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$
i.e. union and intersection are associative.
(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
i.e. union and intersection are distributive over intersection and union respectively.

(16) **(D).** Let x be an arbitrary element of $(A \cup B)'$.
Then $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B \Rightarrow x \in A' \cap B'$
Again let y be an arbitrary element of $A' \cap B'$.
$y \in A' \cap B' \Rightarrow y \in A'$ and $y \in B' \Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B) \Rightarrow y \in (A \cup B)'$
$\therefore A' \cap B' \subseteq (A \cup B)'$. Hence $(A \cup B)' = A' \cap B'$

(17) **(A), (18) (B), (19) (C).**
Let A and B be the sets of persons who can speak Hindi and Bengali respectively.
then $n(A \cap B) = 1000$, $n(A) = 750$, $n(B) = 400$
Number of persons whose can speak both hindi and bengali $= n(A \cup B) = n(A) + n(B) - n(A \cup B)$
$= 750 + 400 - 1000 = 150$
Number of persons who can speak Hindi only
$= n(A - B) = n(A) - n(A \cap B) = 750 - 150 = 600$
Number of persons whose can speak Bengali only
$= n(B - A) = n(B) - n(A \cap B) = 400 - 150 = 250$

(20) **(A).** $x^2 = 16 \Rightarrow x = \pm 4$; $2x = 6 \Rightarrow x = 3$
There is no value of x which satisfies both the above equations. Thus $A = \phi$.

(21) **(B).** $A = [x : x \in R, -1 < x < 1]$
$B = [x : x \in R, x - 1 \le -1$ or $x - 1 \ge 1]$
$= [x : x \in R, x \le 0$ or $x \ge 2]$
$\therefore\ A \cup B = R - D$ where $D = [x : x \in R, 1 \le x < 2]$

(22) **(C).** $(A \cup B \cup C) \cap (A \cap B^C \cap C^C)^C \cap C^C$
$[(A \cap A^C)(B \cup C)] \cap C^C = (B \cup C) \cap C^C$
$(B \cup C^C) \cup (C \cap C^C) = B \cap C^C$

(23) **(B).** $\because n(H \cup E) = n(H \cap E') + n(H \cap E) + n(H' \cap E)$
$45 = 22 + n(H \cap E) + 12 \qquad \therefore\ n(H \cap E) = 11$

(24) **(D).** Clearly, $A \cap B = \phi \qquad \therefore (Y \times A)\ \&\ (Y \times B)$
Can't have any commom element

(25) **(C).** $n(C) = 25$, $n(T) = 20$
$n(C \cap T) = 10$ then $n(C \cap T)' = ?$
$n(C \cup T) = n(C) + n(T) - n(C \cap T) = 25 + 20 - 10 = 35$
$\therefore\ n(C \cap T)' = n(n) - n(C \cup T) = 60 - 35 = 25$

(26) **(A).** $|2x + 3| < 7 \rightarrow -7 < 2x + 3 < 7$
$-10 < 2x < 4$; $-5 < x < 2$; $0 < x + 5 < 7$

(27) **(C).** $x^2 = 1 \Rightarrow x = \pm 1$

(28) **(A).** $\because\ f(x) = \dfrac{1}{x^2} + \dfrac{1}{x^3} = \dfrac{x+1}{x^3}$ = ratio of two polynomials
$\therefore$ f(x) is a rational function.

(29) **(B).** Here $|\sin 2x| = \sqrt{\sin^2 2x} = \sqrt{\dfrac{(1 - \cos 4x)}{2}}$
Period of $\cos 4x$ is $\pi/2$; Period of $|\sin 2x|$ will be $\pi/2$

(30) **(C).** $2 < x < 3 \Rightarrow |x - 2| = x - 2$; $|x - 3| = 3 - x$
$\therefore\ f(x) = 2(x - 2) - 3(3 - x) = 5x - 13$.

(31) **(A).** Domain $= \{x ; 2^x - 3^x \ge 0\} = \{x ; (2/3)^x \ge 1\}$
$= x \in (-\infty, 0]$

(32) **(D).** Range is containing those real numbers y for which $f(x) = y$ where x is real number.

Now $f(x) = y \Rightarrow \frac{x^2}{1+x^2} = y \Rightarrow x = \sqrt{\frac{y}{(1-y)}}$ ....(1)

by (1) clearly $y \neq 1$, and for x to be real

$\frac{y}{1-y} \geq 0 \Rightarrow y \geq 0$ and $y < 1$. $\therefore\ 0 \leq y < 1$

$\therefore$ Range of function $= (0 \leq y < 1) = [0,1)$

(33) **(A).** $n(A \times A) = n(A) . n(A) = 32 = 9$.
So the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times$ a is a relation over the set A.

(34) **(A).** $\because$ We know $x - [x] = \{x\}$:
{where {x} is fractional part function}
and $0 \leq \{x\} < 1$ $\therefore$ Range of f(x) is [0, 1)

(35) **(C).** $f(x) = 3 + x - [x+2] = 1 + 2 + x - [x+2]$
{where {.} is fractional part function}
$= 1 + \{2 + x\}$ $\because 0 \leq \{2+x\} < 1$
$\therefore 0 + 1 \leq \{2+x\} + 1 < 1 + 1$ $\therefore 1 \leq f(x) < 2$
$\therefore$ Range of f(x) is [1, 2)

(36) **(D).** $f(x) = \sqrt{\cos x} \Rightarrow \cos x \geq 0$
$0 \leq \cos x \leq 1$, x is in I quad or IV quad
i.e., x varies from 0 to $\pi/2$ (in I quadrant)
Also from to $3\pi/2$ to 2π, cos x is ≥ 0
$\therefore$ In $= [0, \pi/2] \cup [3\pi/2, 2\pi]$, $\cos x > 0$
However, $[-\pi/2, \pi/2]$ also is the domain of the function. Infact $[3\pi/2, 2\pi]$ and $[0, \pi/2]$ are also domains since cos x > 0 when x belongs to either of these two intervals.

(37) **(C).** $\frac{f(a) - f(b)}{a - b} = \frac{2(a^2 - b^2)}{a - b}$
$= 2(a+b) = 2(3.8 + 4) = 2(7.8) = 15.6$

(38) **(A).** Domain of (f + g) is Dom f $\cap$ Dom g
As domain of f is (–1, 1), the option (C) can't be the answer
$3 + 4x - 4x^2 = -[4x^2 - 4x - 3] = -[(2x-1)^2 - 4]$
$= 4 - (2x-1)^2$

$\therefore\ -2 \leq 2x - 1 \leq 2 \Rightarrow -1 \leq 2x \leq 3\ \therefore\ -\frac{1}{2} \leq x \leq \frac{3}{2}$

$\therefore$ Domain of (f + g) is (is $(-1, 1) \cap \left[-\frac{1}{2}, \frac{3}{2}\right] = \left[-\frac{1}{2}, 1\right)$

(39) **(A).** Here $f\{f(x)\} = f\left(\frac{x-3}{x+1}\right) = \frac{\left(\frac{x-3}{x+1}\right) - 3}{\left(\frac{x-3}{x+1}\right) + 1} = \frac{x+3}{1-x}$

$\therefore f[f\{f(x)\}] = \frac{\frac{x+3}{1-x} - 3}{\frac{x+3}{1-x} + 1} = \frac{4x}{4} = x$

(40) **(D).** cos (log x) cos (log y)

$-\frac{1}{2}[\cos(\log x/y) + \cos(\log xy)]$

$= \frac{1}{2}[\cos(\log x + \log y) + \cos(\log x - \log y)]$

$-\frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)] = 0$

(41) **(B).** $f(x+y).\ f(x-y) = \frac{2^{x+y} + 2^{-x-y}}{2} . \frac{2^{x+y} + 2^{-x-y}}{2}$

$= \frac{2^{2x} + 2^{2y} + 2^{-2x} + 2^{-2y}}{4}$

$= \frac{1}{2}\left[\frac{2^{2x} + 2^{-2x}}{2} + \frac{2^{2y} + 2^{-2y}}{2}\right] = \frac{1}{2}[f(2x) + f(2y)]$

(42) **(D).** $f(3x) - f(-x) - 4x = 6x + |3x| - \{-2x + |-x|\} - 4x$
$= 6x + 3|x| + 2x - |x| - 4x = 4x + 2|x| = 2f(x)$.

(43) **(B).** $f(2) = \frac{2}{4+1} = \frac{2}{5}$

$\therefore\ f(f(2)) = f\left(\frac{2}{5}\right) = \frac{2/5}{\frac{4}{25} + 1} = \frac{2 \times 5}{4 + 25} = \frac{10}{29}$

(44) **(A).** $g(f(x)) = \tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$

$\Rightarrow g(x) = \frac{x-1}{x+1}$; $f(g(x)) = \tan\left(\frac{x-1}{x+1}\right)$

(45) **(B).** $f(x) = \left[2\sin^2\left(\frac{4x - 3\pi}{6\pi^2}\right)\right]^2 + 2\cos\left(\frac{4x - 3\pi}{3\pi^2}\right)$

$= \frac{3}{2} + \frac{1}{2}\cos\left(\frac{8x - 6\pi}{3\pi^2}\right)\ \therefore\ T = \frac{3\pi^3}{4}$

(46) **(B).** $f(x) = |x - 1| = \begin{cases} 1 - x & 0 < x < 1 \\ x - 1 & x \geq 1 \end{cases}$

$g(x) = e^x\quad x \geq -1$

$(fog)(x) = \begin{cases} 1 - g(x) & 0 < g(x) < 1 \text{ i.e. } -1 \leq x < 0 \\ g(x) - 1 & g(x) \geq 1 \text{ i.e. } 0 \leq x \end{cases}$

$= \begin{cases} 1 - e^x & -1 \leq x < 0 \\ e^x - 1 & x \geq 0 \end{cases}$

$\therefore$ domain $= [-1, \infty]$
fog is decreasing in [–1, 0] and increasing in $(0, \infty)$

$fog(-1) = 1 - \frac{1}{e}$ and $fog(0) = 0$

$x \to \infty\ fog(x) = \infty$ $\therefore$ range $[0, \infty)$

(47) **(A).** $-5 \leq |kx + 5| \leq 7 \Rightarrow -12 \leq kx \leq 2$
where $-6 \leq x \leq 1$

$-6 \le \frac{k}{2} x \le 1$ where $-6 \le x \le 1$

$\therefore\ k = 2$ {$\because$ range of h (x) = domain of f (x)}

(48) **(D).** Let degree of f (x) is n

Equating the degree of LHS and RHS, we get

$n + n = n \Rightarrow n = 0$

$\Rightarrow f(x) = c \Rightarrow c^2 = c$

$\Rightarrow c = 0, 1 \Rightarrow f(x) = 0, 1$

(49) **(A).** Domain of $\sin^{-1}(\sin x)$ is whole of R

$$-\log_{\left(\frac{x+4}{2}\right)} \log_2\left(\frac{2x-1}{3+x}\right) > 0$$

i.e. $$\log_{\left(\frac{x+4}{2}\right)}\left(\log_2\left(\frac{2x-1}{3+x}\right)\right) < 0$$

Case I : $0 < \frac{x+4}{2} < 1$ i.e. $-4 < x < -2$

then $\log_2 \frac{2x-1}{3+x} > 1$ i.e. $\frac{2x-1}{3+x} > 2$

i.e. $\frac{2x-1-6-2x}{3+x} > 0$

i.e. $x + 3 < 0$ i.e. $x < -3$

$\therefore\ -4 < x < -3$. .... (1)

Case II : If $\frac{x+4}{2} > 1$ i.e. $x > -2$

then $0 < \log_2 \frac{2x-1}{3+x} < 1$ i.e. $1 < \frac{2x-1}{3+x} < 2$

i.e. $\frac{2x-1-3-x}{x+3} > 0$ and $\frac{2x-1-6-2x}{x+3} < 0$

i.e. $\frac{x-4}{x+3} > 0$ and $\frac{-7}{x+3} < 0$

i.e. $\{x < -3 \text{ or } x > 4\}$ and $x > -3$

i.e. $x > 4$ .... (2)

From eq. (1) and (2), $x \in (-4, -3) \cup (4, \infty)$

$\therefore\ a = -4, b = -3, c = 4$ and so, $a + b + 3c = 5$

(50) **(A).** $f : (0, \infty) \to (0, \infty)$

$f(x\, f(y)) = x^2 y^a\ (a \in R)$

Put x = 1, we get $f(f(y)) = y^a$

Put $f(y) = \frac{1}{x}$, we get $f(1) = \frac{1}{(f(y))^2} \cdot y^a$

put y = 1 we get $(f(1))^3 = 1$

$\therefore\ f(1) = 1$

for y = 1, we have $f(x(f(1))) = x^2$

$\therefore\ f(x) = x^2 ; f(x\, f(x)) = x^2 x^a ; f(x^3) = x^2 x^a$

$f(x^3) = x^6 \quad \therefore\ x^6 = x^2 x^a$ thus a = 4

EXERCISE-2

(1) **(C).** We have for $\cos^{-1}(1 - x) \ge 0 \Rightarrow -1 \le (1 - x) \le 1$

$\Rightarrow -2 \le -x \le 0 \qquad \Rightarrow 0 \le x \le 2$ ....(1)

also $10 \cdot 3^{x-2} - 9^{x-1} - 1 > 0$

$10 \cdot 3^x - 9^x - 9 > 0$

$10 \cdot 3^x - 3^{2x} - 9 > 0$

$3^{2x} - 10 \cdot 3^x + 9 < 0$

$(3^x - 1)(3^x - 9) < 0$

$1 < 3^x < 9 \Rightarrow 0 < x < 2$ .....(2)

from (1) and (2)

$0 < x < 2$

(2) **(C).** When $p = \pi/2$ then $D^r \to \cos x + \sin x \Rightarrow \pi/2$ can not be the period

(3) **(C).** $f(x) = \sin^2 x + (1 - \sin^2 x)^2 + 2$

$= 3 - \sin^2 x + \sin^4 x \quad = 3 - \sin^2 x \cos^2 x$

$= 3 - \frac{\sin^2 2x}{4} \quad \Rightarrow \quad T_1 = \frac{\pi}{2}$, and $T_2 = \frac{\pi}{2}$

(4) **(D).** If $g(x) \ge 0$, $|g(x) - 1| = \frac{1}{2}$

$g(x) - 1 = \frac{1}{2}$ or $\frac{-1}{2} \Rightarrow \quad g(x) = \frac{3}{2}$ or $\frac{1}{2}$

At $g(x) = \frac{3}{2}$ we get only 1 solution ......(1)

At $g(x) = \frac{1}{2}$ we get 3 solution ......(2)

Again if $g(x) < 0$

$1 + g(x) = \frac{1}{2}$ or $-\frac{-1}{2}$; $g(x) = \frac{-1}{2}$ or $\frac{-3}{2}$

At $g(x) = \frac{-3}{2}$ we get only 1 solution ......(3)

At $g(x) = \frac{-1}{2}$ we get 3 solution ......(4)

From (1), (2), (3), (4) we get total 8 solutions

(5) **(B).** $\text{LHS} = g(x)\left(\frac{x^{2^{2008}}}{x} - 1\right) = \frac{g(x)\left(x^{2^{2008}} - x\right)}{x}$

$$\text{RHS} = \frac{(x-1)\left[(x+1)(x^2+1)(x^4+1)........(x^{2^{2007}}+1) - 1\right]}{(x-1)}$$

$$= \frac{(x^{2^{2008}} - 1) - (x - 1)}{x - 1} = \frac{(x^{2^{2008}} - x)}{x - 1} = \frac{x(x^{(2^{2008}-1)} - 1)}{x - 1}$$

From question

$$\therefore\ g(x)\left[x^{(2^{2008}-1)} - 1\right] = \frac{x\left(x^{(2^{2008}-1)} - 1\right)}{x - 1}$$

$\therefore\ g(x) = \frac{x}{x-1} \quad \Rightarrow \quad g(2) = 2$

(6) **(B).** Since $f(2+t) = f(2-t)$

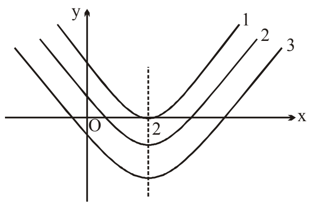
$\Rightarrow$ function is symmetric about the line $x = 2$

Also $x^2 + bx + c = 0$ is symmetric about $x = -b/2$

$\therefore\ -b/2 = 2 \Rightarrow b = -4$

$\therefore\ f(x) = x^2 - 4x + c$

Now 3 graphs are possible. In (1) and (2) 'c' is positive and in (3) 'c' is negative. $f(0) = c$



Let c is positive

Now $f(1) = c - 3$

$f(2) = c - 4$; $f(4) = c$, say $c = 3$

then $f(1) = 0$; $f(2) = -1$;

$f(4) = 3 \Rightarrow f(2) < f(1) < f(4)$

again c is negative Let $c = -3$

$f(1) = -6$; $f(2) = -7$; $f(4) = -3$

$\therefore\ f(2) < f(1) < f(4) \quad \Rightarrow \quad$ (B)

Also if $c = 0$ the statement 'B' is true.

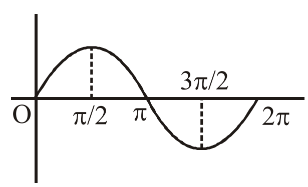
[**Note :** If must have minimum value at $x = 2$ as 1 is closed to 2 or compared to $4 \Rightarrow$ B]

(7) **(C).** $f(x) = \sin x$; $0 \le x < \frac{\pi}{2}$

$= 1$; $\frac{\pi}{2} \le x \le 2\pi$

$g(x) = 0$; $0 \le x \le \pi$

$= \sin x$; $\pi < x < \frac{3\pi}{2}$



$= -1$; $\frac{3\pi}{2} \le x \le 2\pi$

$h(x) = 0$; $0 \le x < \frac{\pi}{2}$

$= 1$; $\frac{\pi}{2} \le x < \frac{3\pi}{2}$

$= 2$; $x \ge \frac{3\pi}{2}$

Hence the range of $h(x)$ is $\{0, 1, 2\}$

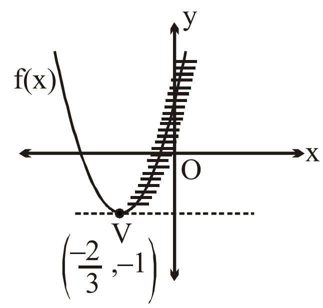
(8) **(C).** $y = \frac{\{x\}}{1+\{x\}} \Rightarrow \{x\} = \frac{y}{1-y}$.

Hence $0 \le \frac{y}{1-y} < 1 \quad \Rightarrow$ (C)

(9) **(A).** We have $f(x) = y = (3x+2)^2 - 1$

$\Rightarrow (3x+2)^2 = y + 1 \Rightarrow 3x + 2 = -\sqrt{y+1}$ (As $x \le \frac{-2}{3}$)

$\Rightarrow\ x = \left(\frac{-2-\sqrt{y+1}}{3}\right) = f^{-1}(y) = g(y)$



Hence $g(x) = \left(\frac{-2-\sqrt{x+1}}{3}\right)$

(10) **(D).** We have $4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7$

$= 4\cos^4 x - 2(2\cos^2 x - 1) - \frac{1}{2}(2\cos^2 2x - 1) - x^7$

$= 4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2 + \frac{1}{2} - x^7$

$= \left(\frac{3}{2} - x^7\right)$

We get $g(x) = \left(\frac{3}{2} - x^7\right)^{\frac{1}{7}}$

$g(g(x)) = \left(\frac{3}{2} - (g(x))^7\right)^{\frac{1}{7}} = \left(\frac{3}{2} - \left(\frac{3}{2} - x^7\right)\right)^{\frac{1}{7}} = x$

Hence $g(g(100)) = 100$

(11) **(A).** $f(x) = \sqrt{\frac{1-5^x}{7^{-x}-7}}$

$(N^r \ge 0 \text{ and } D^r > 0) \cup (N^r \le 0 \text{ and } D^r < 0)$

$f(x)\ \sqrt{\frac{1-5^x}{7^{-x}-7}} = \sqrt{\frac{7^x(1-5^x)}{1-7^{x+1}}}$

$7^x > 0 \Rightarrow \left(\dfrac{1-5^x}{1-7^{x+1}}\right) \geq 0$

Case I: $1-5^x \geq 0$ & $1-7^{x+1} > 0$

$5^x \leq 1$; $7^{x+1} < 1$

$x \leq 0$ ... (1) ; $x+1 < 0$

$x < -1$ ... (1)

$(1) \cap (2)$

$x < -1$

Case II : $1-5^x \leq 0$ & $1-7^{x+1} < 0$

$5^x \geq 1$; $7^{x+1} \geq 1$

$x \geq 0$ ... (3) & $x+1 > 0 \Rightarrow x > -1$ ... (4)

$(3) \cap (4)$

$x \geq 0$

$\therefore$ Case I $\cup$ Case II

$x \in (-\infty, -1) \cup [0, \infty)$

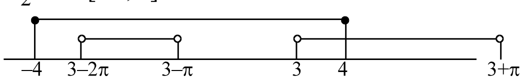
(12) **(B).** $f(x) = \log_{10} \sin(x-3) + \sqrt{16-x^2}$

D_1: $\sin(x-3) > 0 \Rightarrow 0 < x-3 < \pi$

$\Rightarrow 2k\pi < x-3 < (2k+1)\pi$

$\Rightarrow 3+2k\pi < x < 3+(2k+1)\pi$

$D_2 : x \in [-4, 4]$



$(3-2\pi, 3-\pi) \cup (3, 4]$

(13) **(C).** $f(x) = \log_{100x}\left(\dfrac{2\log_{10}^x + 1}{-x}\right)$

$100x \neq 1$ & $100x > 0 \Rightarrow x > 0$

$x \neq 1/100$

$\dfrac{2\log_{10}^x + 1}{-x} > 0 \Rightarrow 2\log_{10}^x + 1 < 0$

$\Rightarrow \log_{10}^x < -\dfrac{1}{2} \Rightarrow x < \dfrac{1}{\sqrt{10}}$

$\therefore x \in \left(0, \dfrac{1}{100}\right) \cup \left(\dfrac{1}{100}, \dfrac{1}{\sqrt{10}}\right)$

(14) **(D).** $f(x) = \dfrac{1}{\sqrt{4x^2-1}} + \ln x(x^2-1)$

$4x^2-1 > 0$ and $x(x^2-1) > 0$

$\Rightarrow (-1, -1/2) \cup (1, \infty)$

(15) **(A).** $f(x) = \sqrt{\log_{\frac{1}{2}} \dfrac{x}{x^2-1}} = \sqrt{-\log_2 \dfrac{x}{x^2-1}}$

$0 < \dfrac{x}{x^2-1} \leq 1$

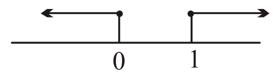
$\Rightarrow \left[\dfrac{1-\sqrt{5}}{2}, 0\right) \cup \left[\dfrac{1+\sqrt{5}}{2}, \infty\right)$

(16) **(B).** $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9-x^2}}$

Let $|x| = t$

$f(x) = \sqrt{t^2 - t} + \dfrac{1}{\sqrt{9-t^2}}$

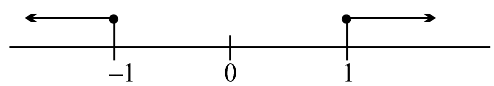
$t(t-1) \geq 0$ and $9 - t^2 > 0$

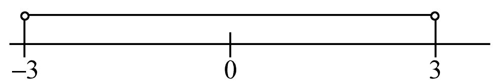


and $t \in (-3, 3)$

$|x| \geq 1$; $x \geq 1$ or $x \leq -1$

or $|x| \leq 0 \Rightarrow x = 0$



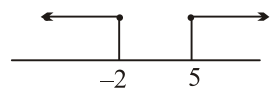


$(-3, -1] \cup [1, 3) \cup \{0\}$

(17) **(C).** $f(x) = \sqrt{(x^2 - 3x - 10)\ln^2(x-3)}$

$f(x) = \sqrt{(x-5)(x+2)\ln^2(x-3)}$

$(x-5)(x+2)\ln^2(x-3) \geq 0$ and $x > 3$



at $x = 4 \Rightarrow \ln(x-3) = 0$

$[5, \infty) \cup \ln(x-3) = 0 \Rightarrow [5, \infty) \cup \{4\}$

(18) **(A).** $y = \dfrac{3x-2}{x+4}$

$\Rightarrow xy + 4y = 3x - 2 \Rightarrow x(y-3) = -4y - 2$

$\Rightarrow x = \dfrac{4y+2}{3-y} \Rightarrow f^{-1}(y) = \dfrac{4y+2}{3-y}$

$\Rightarrow f^{-1}(x) = \dfrac{4x+2}{3-x} = \dfrac{x + \frac{1}{2}}{\frac{-x}{4} + \frac{3}{4}}$

$b = 1/2, c = -1/4, d = 3/4;\ b + c + d = 1.$

(19) **(B).** $x f(x) = ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1$ ....(1)

$x \to -x, -x f(-x) = ax^8 + bx^6 + cx^4 + dx^2 - 15x + 1$

$x f(-x) = -ax^8 - bx^6 - cx^4 - dx^2 + 15x - 1$ ....(2)

Add (1) and (2)

$x(f(x) + f(-x)) = 30x \Rightarrow (f(x) + f(-x)) = 30$

$\Rightarrow (f(5) + f(-5)) = 30 \Rightarrow f(-5) = 28$

(20) **(B).** $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x$ ... (1)

Replacing x by $\dfrac{1}{x}$

$f\left(\frac{1}{x}\right) + 2f(x) = 3\left(\frac{1}{x}\right)$; $2f\left(\frac{1}{x}\right) + 4f(x) = 6\left(\frac{1}{x}\right)$ ... (2)

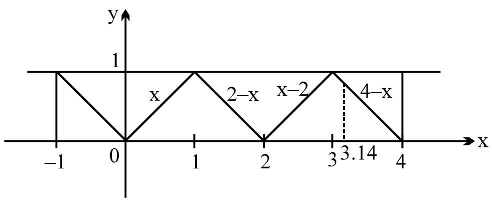
(2) – (1) we get, $3f(x) = \frac{6}{x} - 3x$

Now if $f(x) = f(-x)$

$\frac{6}{x} - 3x = \frac{6}{x} + 3x$; $\frac{12}{x} = 6x$; $x^2 = 2$; $x = \pm\sqrt{2}$

(21) f is periodic with period 2 and $f(x) = x \ \forall x \in [0, 1]$ also $f(x)$ is even $\Rightarrow$ symmetry about y-axis

$\therefore$ graph of $f(x)$ is as shown



$f(3.14) = 4 - 3.14 = 0.86$

(22) **(C).** Domain of $\log|x^2 - 9|$

$|x^2 - 9| > 0 \Rightarrow x^2 - 9 \neq 0$

$x^2 \neq \pm 3$

Domain $= R - \{-3, 3\}$

(23) **(B).** $f(x) = \sqrt{x-1} + \sqrt{6-x}$

$x - 1 \geq 0$ and $6 - x \geq 0$

$x \geq 1$ and $x \leq 6 \Rightarrow x \in [1, 6]$

(24) **(A).** $f(x) = \sqrt{(2 - 2x - x^2)}$

$2 - 2x - x^2 \geq 0$

$\Rightarrow 3 - (x+1)^2 \geq 0 \Rightarrow (x+1)^2 \leq 3$

$\Rightarrow -\sqrt{3} \leq x + 1 \leq \sqrt{3}$

$\Rightarrow -\sqrt{3} - 1 \leq x \leq \sqrt{3} - 1$

Domain, x $[-\sqrt{3} - 1, \sqrt{3} - 1]$

(25) **(D).** If maximum value of $\frac{x^2 + x + k}{x^2 + 2x + k} = \frac{3}{2}$

then solve and put $D = 0 \Rightarrow k = 4$

(26) **(D).** Period of

$f(x) = \text{L.C.M. of } \left\{\frac{2\pi}{1/2}, \frac{2\pi}{1/3}, \frac{\pi}{1/4}, \frac{\pi}{1/5}, \frac{\pi}{1/6}, \frac{2\pi}{1/7}\right\}$

L.C.M. of $\{4\pi, 6\pi, 4\pi, 5\pi, 12\pi, 14\pi\} = 420\pi$.

(27) **(A).** $f(x) = 4\cos^4\left(\frac{x-\pi}{4\pi^2}\right) - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$

$= \left[2\cos^2\left(\frac{x-\pi}{4\pi^2}\right)\right]^2 - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$

$= \left[1 + \cos\left(\frac{x-\pi}{2\pi^2}\right)\right]^2 - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$

$= 1 + \cos^2\left(\frac{x-\pi}{2\pi^2}\right)$

$\Rightarrow$ Period of $f(x) = \left(\frac{\pi}{1/2\pi^2}\right) = 2\pi^3$

(28) **(A).** $h(x) = f(g(x)) + 10 = |x|^2 + 4|x| - 1 + 10$

$h(x) = |x|^2 + 4|x| + 9 = (|x| + 2)^2 + 5$

Hence range of $h(x)$ is $[9, \infty)$.

(29) **(D).** $y = \frac{4x}{3x+4}$; $(4 - 3y)x = 4y$

$x = \frac{4y}{4-3y} = g(y) \Rightarrow g(1) = \frac{4}{4-3} = 4$

(30) **(D).** $f\left(2 + \log_2\left(\frac{3}{2}\right)\right) = f\left(\log_2 4 + \log_2\left(\frac{3}{2}\right)\right)$

$= f(\log_2 6) = f(\log_2 6 + 1) = f(\log_2 12)$

$= f(\log_2 12 + 1)$; Hence $f\left(2 + \log_2\left(\frac{3}{2}\right)\right)$

$= f(\log_2 24) = \frac{1}{2^{\log_2 24}} = \frac{1}{24}$.

(31) **(B).** Let $f(x) = ax + b$ and $g(x) = cx + d$

as $f(g(x)) = x$ hold for all x, we have

$f(g(x)) = a(g(x)) + b$

$= a(cx + d) + b = acx + ad + b$

$\therefore$ $acx + (ad + b) = x$ for all x,

on comparing coefficients

$\therefore$ $ac = 1$ and $ad + b = 0$

$c = 1/a$ and $d = -b/a$

$\therefore$ $g(x) = \frac{x}{a} - \frac{b}{a} = \frac{x-b}{a}$

also $f(x) = ax + b$

Now, $f(0) = 4 \Rightarrow b = 4$ and

$g(5) = 17 \Rightarrow \frac{5-4}{a} = 17 \Rightarrow a = \frac{1}{17}$

$\therefore f(x) = \frac{x}{17} + 4 \Rightarrow f(136) = \frac{136}{17} + 4 = 12$.

(32) **(A).** $f(x) = (x^2 + 5x + 4)(x^2 + 5x + 6) + 5$

$= [(x^2 + 5x + 5) - 1][(x^2 + 5x + 5) + 1] + 5$

$= (x^2 + 5x + 5)^2 - 1 + 5$

$f(x) = (x^2 + 5x + 5)^2 + 4$

Hence $f(x)$ has a minimum value 4 when

$x^2 + 5x + 5 = 0$

i.e. $x = \frac{-5 \pm \sqrt{5}}{2}$; $x = \frac{-(5 + \sqrt{5})}{2} \in [-6, 6]$

Also maximum occurs at $x = 6$

$f(x)|_{max} = (36 + 30 + 5)^2 + 4 = (71)^2 + 4$

$= 5041 + 4 = 5045$

Range is $[4, 5045]$

$\therefore \quad a = 4;\ b = 5045 \Rightarrow \qquad a + b = 5049$

Alternatively: $f(x) - 5 = g(x)]$

(33) **(C).** $\log(\log_3 10) = \log\left(\dfrac{1}{\log_{10} 3}\right) = -\log_{10}(\log_{10} 3)$

Given $\quad f(-\log_{10}(\log_{10} 3)) = 5 \qquad ....(1)$

now $\quad f(x) = a\sin x + b x^{1/3} + 4$

$f(-x) = -a\sin x - b x^{1/3} + 4$

$f(x) + f(-x) = 8$

$f\left(\log_{10}(\log_{10} 3)\right) + f\left(-\log_{10}(\log_{10} 3)\right) = 8$

$f\left(\log_{10}(\log_{10} 3)\right) + 5 = 8$

$f\left(\log_{10}(\log_{10} 3)\right) = 3$ Ans.

(34) **(A).** We have

$f(x) = x(x+1)\underbrace{Q(x)}_{\text{Quotient}} + \underbrace{(ax+b)}_{r(x)\text{(Remainder)}} \qquad ....(1)$

Put $x = 0$ and $x = -1$ in equation (1),

we get $b = 0 \qquad ....(2)$

and $-1 = -a + b \qquad ....(3)$

$\left(\begin{array}{l}\text{As } f(0) = 0 \\ \text{and } f(-1) = -1\end{array}\right)$

$\therefore \quad a = 1, b = 0.$ So, $r(x) = ax + b = x$

Hence, $r(10) = 10.$

(35) **(C).** Let $P(x) = 6 + x$, for $x = 0, 1, 2, 3$

Let $f(x) = P(x) - x - 6$

$\Rightarrow f(x)$ is also a polynomial of degree 4.

$\Rightarrow f(x) = x(x-1)(x-2)(x-3)$

As $f(x)$ vanishes at $x = 0, 1, 2, 3$, so

$f(x) = P(x) - x - 6 \equiv (x-0)(x-1)(x-2)(x-3)$

$\Rightarrow \quad P(x) = x + 6 + x(x-1)(x-2)(x-3)$

Hence $P(4) = 10 + (4 \cdot 3 \cdot 2 \cdot 1) = 10 + 24 = 34$

(36) **(B).**

(a) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

$D_1 : \cos 2x \geq 0 \Rightarrow -\dfrac{\pi}{2} \leq 2x \leq \dfrac{\pi}{2}$

$\Rightarrow \quad 2k\pi - \dfrac{\pi}{2} \leq 2x \leq \dfrac{\pi}{2} + 2k\pi$

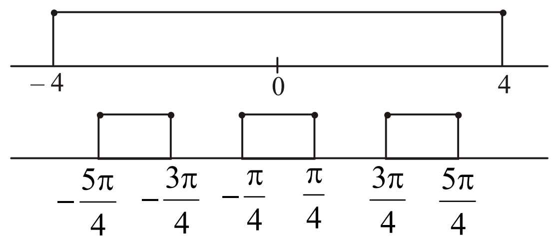
$\Rightarrow \quad k\pi - \dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{4} + k\pi \quad k \in I$

$D_2 : 16 - x^2 > 0$

$x \in [-4, 4]$

$\therefore \quad D_1 \cap D_2$

$\Rightarrow \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right] \cup \left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right] \cup \left[-\dfrac{5\pi}{4}, -\dfrac{3\pi}{4}\right]$



(b) $f(x) = \log_7 \log_5 \log_3 \log_2(2x^3 + 5x^2 - 14x)$

$\log_7 \underbrace{\log_5 \overbrace{\log_3 \overbrace{\log_2(2x^3 + 5x^2 - 14x)}^{w}}^{z}}_{y}$

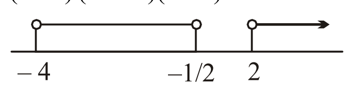
$\Rightarrow \quad \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$\Rightarrow \quad \log_2(2x^3 + 5x^2 - 14x) > 3$

$\Rightarrow \quad 2x^3 + 5x^2 - 14x > 8$

$\Rightarrow \quad 2x^3 + 5x^2 - 14x - 8 > 0$

$\Rightarrow \quad (x-2)(2x+1)(x+4) > 0$



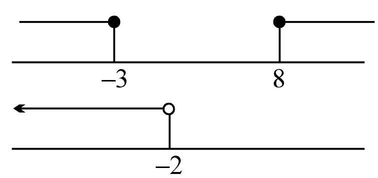
$\therefore \quad x \in \left(-4, -\dfrac{1}{2}\right) \cup (2, \infty)$

(c) $f(x) = \ln\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

$\sqrt{x^2 - 5x - 24} > x + 2$

If $x^2 - 5x - 24 \geq 0 \qquad$ and $\qquad x + 2 < 0$

(1) $(x-8)(x+3) \geq 0$ and $x < -2$



$\Rightarrow (-\infty, -3]$

(2) $x^2 - 5x - 24 \geq 0 \qquad$ and $\qquad x + 2 \geq 0$

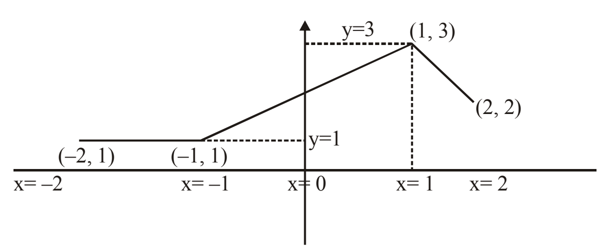
and $x^2 - 5x - 24 > (x+2)^2$

$x^2 - 5x - 24 > x^2 + 4x + 4 \Rightarrow 9x + 28 < 0$

$x < -\dfrac{28}{9}$; no answer ϕ

$\therefore \quad (1) \cup (2) \Rightarrow x \in (-\infty, -3]$

(37) **(A).** Drawing graph of $y = f(x)$



From above graph range is [1, 3]

(38) **(C).** When $-2 \le x \le -1$; $\{f(x)\} = 0$
$\Rightarrow$ (As $f(x) = 1, x \in [-2, 1]$)
when $-1 \le x \le 0$; $\{f(x)\} = 1/2$ is possible for one value of x, which is 1/2 when $1 \le x \le 2$, $\{f(x)\} = 1/2$ is possible for one value of x which is 3/2. Hence total number of solutions for $\{f((x)\} = 1/2$ is three given as $\left\{\frac{-1}{2}, \frac{1}{2}, \frac{3}{2}\right\}$.

(39) **(C).** We have $f(x) = \frac{3x+a}{x^2+3}$

Now, $\frac{3}{2} = \frac{3x+a}{x^2+3} \Rightarrow 3x^2 - 6x + 9 - 2a = 0$

This must be a perfect square, so
$D = 0 \Rightarrow 36 = 12(9 - 2a) \Rightarrow a = 3$

(40) **(A).** $F = F(x) = \frac{3x+3}{x^2+3}$

Solving by usual method

Range $\left[\frac{-1}{2}, \frac{3}{2}\right]$

$\Rightarrow$ Minimum value $= -1/2$

EXERCISE-3

(1) **2.** We know that
$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$38 = 17 + 23 - n(X \cap Y)$
$\Rightarrow n(X \cap Y) = 17 + 23 - 38 = 2$

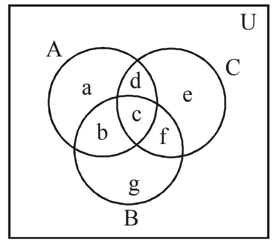
(2) **5.** Here, $n(X \cup Y) = 18$, $n(X) = 8$ and $n(Y) = 15$
We know that $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$\therefore\ 18 = 8 + 15 - n(X \cap Y)$
$\therefore\ n(X \cap Y) = 23 - 18 = 5$

(3) **60.** Let F be the set of people who speak French and 'S' be the set of people who speak Spanish.
Here, $n(F) = 50$, $n(S) = 20$ and $n(F \cap S) = 10$
We know that $n(F \cup S) = n(F) + n(S) - n(F \cap S)$
$\therefore\ n(F \cup S) = 50 + 20 - 10 = 60$
$\therefore$ Number of people who speak at least one of the these two languages = 60.

(4) **325.** Let T be the set of students who like tea and C be the set of students who like coffee.
Here, $n(T) = 150$, $n(C) = 225$ and $n(C \cap T) = 100$
We know that, $n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$= 150 + 225 - 100 = 275$
$\therefore$ Number of students taking either tea or coffee = 275
$\therefore$ Number of students taking neither tea nor coffee
$= 600 - 275 = 325$

(5) **125.** Let H be the set of students who know Hindi and E be the set of students who know English.
Here, $n(H) = 100$, $n(E) = 50$ and $n(H \cap E) = 25$
We know that, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$
$= 100 + 50 - 25 = 125.$

(6) **11.** Here, $n(A) = a + b + c + d = 21$ .... (1)
$n(B) = b + c + f + g = 26$ .... (2)
$n(C) = c + d + e + f = 29$ .... (3)
$n(A \cap B) = b + c = 14$ .... (4)
$n(C \cap A) = c + d = 12$ .... (5)
$n(B \cap C) = c + f = 14$ .... (6)
$n(A \cap B \cap C) = c = 8$ .... (7)



Putting value of c in (4), (5) and (6),
$b + 8 = 14 \Rightarrow b = 6$
$8 + d = 12 \Rightarrow d = 4$
$8 + f = 14 \Rightarrow f = 6$
Putting value of c, d, f in (3),
$8 + 4 + e + 6 = 29$
$\Rightarrow e = 29 - 18 = 11$
Number of people who like product C only = 11.

(7) **4.** $f: \{1, 3, 4\} \to \{1, 2, 5\}$; $f = \{(1, 2), (3, 5), (4, 1)\}$ and
$g: \{1, 2, 5\} \to \{1, 3\}$; $g = \{(1, 3), (2, 3), (5, 1)\}$
Now gof exists because range of f is equal to domain of g.
Then gof : $\{1, 3, 4\} \to [1, 3]$
$gof(1) = g[f(1)] = g(2) = 3$, $gof(3) = g[f(3)] = g(5) = 1$, $gof(4) = g[f(4)] = g(1) = 3$
Thus, gof $= \{(1, 3), (3, 1), (4, 3)\}$

(8) **4.** $f(x) = \frac{4x}{3x+4}$. Let $y = \frac{4x}{3x+4}$

$\Rightarrow 3xy + 4y = 4x \Rightarrow 3xy - 4x = -4y$

$\Rightarrow x(3y - 4) = -4y \Rightarrow x = \frac{4y}{4-3y}$

Now, $f^{-1}(y) = g(y) = \frac{4y}{4-3y}$

(9) **727.** Let $g(x) = P(x) - x$.
Hence g (x) vanishes at x = 1, 2, 3, 4, 5 and 6.
Since it is a polynomial of degree six
so $g(x) = P(x) - x = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$
$\therefore\ P(7) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 7 = 727$

(10) **334.** Let, $f(x) = x^2 x_1 + (x+1)^2 x_2 + \ldots\ldots + (x+6)^2 x_7$
[if x = 1, we get 1st relation, and so on]
note that degree of f (x) is 2
hence $f(x) = ax^2 + bx + c$ where $f(1) = 1$, $f(2) = 12$ and $f(3) = 123$ to find $f(4) = ?$
hence $a + b + c = 1$
$4a + 2b + c = 12$
$9a + 3b + c = 123$
solving $a = 50$, $b = -139$, $c = 90$
$\therefore\ f(4) = 16a + 4b + c = 800 - 556 + 90 = 334$

(11) **2.** $-5 \le |kx + 5| \le 7 \Rightarrow -12 \le kx \le 2$ where $-6 \le x \le 1$

$-6 \le \frac{k}{2} x \le 1$ where $-6 \le x \le 1$

$\therefore$ k = 2 {$\because$ range of h(x) = domain of f(x)}

(12) **2.** $f(x) = \tan\frac{\pi}{2}[x]$

$f(x+1) = \tan\frac{\pi}{2}[x+1] = -\cot\frac{\pi}{2}[x]$

$f(x+2) = \tan\frac{\pi}{2}[x+2] = \tan\frac{\pi}{2}(2+[x])$

$= \tan\left(\pi + \frac{\pi}{2}[x]\right) = \tan\frac{\pi}{2}[x] = f(x)$

$\therefore$ 2 is a period of f(x)

1 is not a period of f(x)

$\therefore$ 2 is the fundamental period

(13) **20.** Case-I : If $x < 0$ then $\left[\frac{3}{x}\right]$ and $\left[\frac{4}{x}\right]$ is – ve

hence $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right]$ can never be equal to 5

Case-II : If $x > 0$

we have $\frac{3}{x} < \frac{4}{x} \quad \therefore \quad \left[\frac{3}{x}\right] \le \left[\frac{4}{x}\right]$

Since each of $\left[\frac{3}{x}\right]$ and $\left[\frac{4}{x}\right]$ is an integer

$\therefore$ 3 possibilities are there

(1) $\left[\frac{3}{x}\right] = 0$ and $\left[\frac{4}{x}\right] = 5$

(2) $\left[\frac{3}{x}\right] = 1$ and $\left[\frac{4}{x}\right] = 4$

As $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$

(3) $\left[\frac{3}{x}\right] = 2$ and $\left[\frac{4}{x}\right] = 3$

now, If $\left[\frac{3}{x}\right] = 0 \Rightarrow 0 \le \frac{3}{x} < 1$

$\Rightarrow 0 \le 3 < x \Rightarrow x > 3$

and $\left[\frac{4}{x}\right] = 5 \Rightarrow 5 \le \frac{4}{x} < 6$

$\Rightarrow \frac{1}{6} < \frac{x}{4} \le \frac{1}{5} \Rightarrow \frac{2}{3} < x \le \frac{4}{5}$

these two equations are not possible. Hence no solutions in these cases.

now, If $\left[\frac{3}{x}\right] = 1 \Rightarrow 1 \le \frac{3}{x} < 2$

$\Rightarrow \frac{1}{2} < \frac{x}{3} \le 1 \Rightarrow \frac{3}{2} < x \le 3$

and $\left[\frac{4}{x}\right] = 4 \Rightarrow 4 \le \frac{4}{x} < 5$

$\Rightarrow \frac{1}{5} < \frac{x}{4} \le \frac{1}{4} \Rightarrow \frac{4}{5} < x \le 1$

not possible simultaneously $\Rightarrow$ no solution

again If $\left[\frac{3}{x}\right] = 2 \Rightarrow 2 \le \frac{3}{x} < 3$

$\Rightarrow \frac{1}{3} < \frac{x}{3} \le \frac{1}{2} \Rightarrow 1 < x \le \frac{3}{2}$

and $\left[\frac{4}{x}\right] = 3 \Rightarrow 3 \le \frac{4}{x} < 4$

$\Rightarrow \frac{1}{4} < \frac{x}{4} \le \frac{1}{3} \Rightarrow 1 < x \le \frac{4}{3}$

common solution $1 < x \le 4/3$. Hence $x \in (1, 4/3]$

$\therefore$ a = 1, b = 4, c = 3;

$\therefore$ a + b + c + abc = 1 + 4 + 3 + 12 = 20

(14) **57.** $l_1 : 4x + 3y = 3$

$f(x) = y = \frac{3-4x}{3}$ ....(1)

since f(x) and $f^{-1}(x)$ are the mirror images of each other in the line y = x hence we find $f^{-1}(x)$.

now $y = f(x) \Rightarrow f^{-1}(y) = x$

from (1) $x = \frac{3(1-y)}{4}$; $f^{-1}(y) = \frac{3(1-y)}{4}$

$\therefore f^{-1}(x) = \frac{3(1-x)}{4}$

$4y = 3 - 3x$; $L_1 = 3x + 4y - 3 = 0$; $m_1 = -3/4$

|||ly $L_2 = y = -8x$ with $m_2 = -8$

If θ is the acute angle between the lines

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{-8 + \frac{3}{4}}{1 + (-8)\left(-\frac{3}{4}\right)}\right| = \left|\frac{-29}{28}\right|$$

$\Rightarrow \frac{29}{28} \Rightarrow a = 29$ and $b = 28$

$\therefore$ a + b = 29 + 28 = 57

(15) **14.** From E to F we can define, in all, 2 × 2 × 2 × 2 = 16 functions (2 options for each element of E) out of which 2 are into, when all the elements of E either map to 1 or to 2.

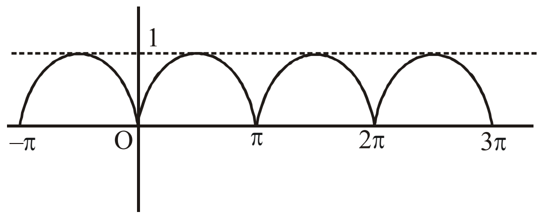
$\therefore$ No. of onto functions = 16 – 2 = 14

EXERCISE-4

(1) **(B).** $\cos\sqrt{x}$ is composite function of periodic and non periodic function

$\therefore$ $\cos\sqrt{x}$ is not periodic function.

(2) **(B).** Period of $\sin^2 x$ is π.



From graph of sin2x
Period of $\sin^2 x$ is π.

(3) **(B).** $y = f(x) = \dfrac{2+x}{2-x}$; $x \neq 2$

$\therefore$ Domain of $f(x) = R - \{2\}$

Now, $y = \dfrac{2+x}{2-x} \Rightarrow 2y - yx = 2 + x \Rightarrow 2y - 2 = x(1+y)$

$\Rightarrow$ $x = \dfrac{2y-2}{1+y}$

For defining x, $1 + y \neq 0 \Rightarrow y \neq -1$

$\therefore$ $y \in R - \{-1\}$

(4) **(A).** $f(x) = \dfrac{3}{4-x^2} + \log_{10}(x^3 - x)$

Let $f_1(x) = \dfrac{3}{4-x^2}$

for domain f_1

$4 - x^2 \neq 0 \Rightarrow x^2 \neq 4 \Rightarrow x \neq \pm 2$

$\therefore$ $D_1 = R - \{\pm 2\}$

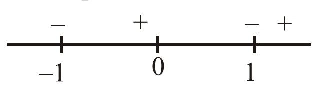
$f_2(x) = \log_{10}(x^3 - x)$

for domain of f_2

$x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0$

$\Rightarrow$ $x(x+1)(x-1) > 0$

critical points are 0, 1, –1



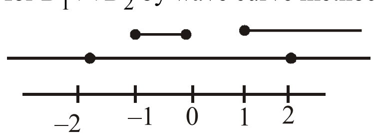
$\therefore$ $x \in (-1, 0) \cup (1, \infty)$

$\therefore$ $D_2 = (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of $f(x)$, $D = D_1 \cap D_2$

$= (-1, 0) \cup (1, 2) \cup (2, \infty)$

for $D_1 \cap D_2$ by wave curve method



$\therefore$ Common region $= (-1, 0) \cup (1, 2) \cup (2, \infty)$

(5) **(A).** $f : R \to R$ $f(x+y) = f(x) + f(y)$

and $f(1) = 7$ .... (1)

$\therefore$ $f(x+y) = f(x) + f(y)$

$\Rightarrow$ $f(1+1) = f(1) + f(1) = 7 + 7$

$f(2) = 14 = 7.2$ .... (2)

$f(2+1) = f(2) + f(1)$

$f(3) = 14 + 7 = 21 = 7.3$ .... (3)

$f(3+1) = f(3) + f(1)$

$f(4) = 21 + 7 = 28 = 7.4$ .... (4)

Similarly, $f(n) = 7 . n$

$\therefore$ $\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$

$= 7.1 + 7.2 + 7.3 + ... 7.n$

$= 7(1 + 2 + 3 + ... + n) = \dfrac{7n(n+1)}{2}$

(6) **(B).** We know that if curve is symmetric about x = 0 then $f(x) = f(-x) \Rightarrow f(0+x) = f(0-x)$

$\therefore$ given curve is symmetric about x = 2

$\therefore$ $f(2+x) = f(2-x)$

(7) **(A).** $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ .... (1)

Put x = 0, y = 0

$f(0) = f(0) . f(0) - f(a)f(a)$ $\{\because f(0) = 1 \text{ given}\}$

$1 = 1 - [f(a)]^2 \Rightarrow f(a) = 0$ .... (2)

Now if we put in (1) x = a and y = x – a we get

$f(2a - x) = f(a)f(x-a) - f(0)f(x) = 0 \times f(x-a) - f(x)$

$= -f(x)$ $\left\{\begin{array}{l}\because f(a) = 0 \text{ from (2)} \\ \text{and } f(0) = 1 \text{ given}\end{array}\right.$

(8) **(C).** $A \cap B = A \cap C$ and $A \cup B = A \cup C \Rightarrow B = C$

(9) **(B).** Every element has 3 options. Either set Y or set Z or none.So number of ordered pairs $= 3^5$

(10) **(B).** $f\left(\dfrac{1}{x}\right) + 2f(x) = \dfrac{3}{x}$; $3f(x) = \dfrac{6}{x} - 3x \Rightarrow f(x) = \dfrac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \dfrac{2}{x} - x = \dfrac{-2}{x} + x$

$\dfrac{4}{x} = 2x \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

(11) **(C).** $f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3$

Now $f(x+y) = f(x) + f(y) + xy$

Put y = 1

$f(x+1) = f(x) + f(1) + x$

$f(x+1) = f(x) + x + 3$

Now, $f(2) = 7$; $f(3) = 12$

$S_n = 3 + 7 + 12 + ... t_n$ ...(1)

$S_n = \quad 3 + 7 + ...t_{n-1} + t_n$ ...(2)

On subtracting (2) from (1)

$t_n = 3 + 4 + 5 + ...$ upto n terms

$t_n = \dfrac{n^2 + 5n}{2}$; $S_n = \Sigma t_n = \sum \dfrac{(n^2 + 5n)}{2}$

$S_n = \dfrac{1}{2}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{5n(n+1)}{2}\right]$

$S_{10} = 330$

(12) **(A).** $2|\sqrt{x}-3|+\sqrt{x}\,(\sqrt{x}-6)+6=0$

Case-I : $\sqrt{x}\ge 3$

$2(\sqrt{x}-3)+x-6\sqrt{x}+6=0$

$x-4\sqrt{x}=0 \Rightarrow x=0,16$

As $x\ge 9 \Rightarrow x=16$

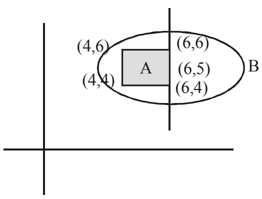
Case-II: $\sqrt{x}<3 \Rightarrow -2\sqrt{x}+6+x-6\sqrt{x}+6=0$

$\Rightarrow x-8\sqrt{x}+12=0 \Rightarrow (\sqrt{x}-6)(\sqrt{x}-2)=0$

$x=36, 4.$ As $\sqrt{x}<3 \Rightarrow x=4$

$\therefore$ There are exactly two elements in the given set.

(13) **(D).**



Since Set A is, $|a-5|<1$; $4<a<6$
and $|b-5|<1$; $4<b<6$

Now B is, $\frac{(a-6)^2}{9}+\frac{(b-5)^2}{4}\le 1$

It can be seen that all vertices of rectangle lie inside the ellipse, therefore $A\subset B$

(14) **(A).** Given $f_1(x)=1/x$, $f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$

$(f_2 o J o f_1)(x)=f_3(x)$
$f_2 o (J(f_1(x)))=f_3(x)$

$f_2 o (J(1/x))=\frac{1}{1-x}$; $1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$

$J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}=\frac{x}{x-1}$

Now $x\to\frac{1}{x}$, $J(x)=\frac{\frac{1}{x}}{\frac{1}{x}-1}=\frac{1}{1-x}=f_3(x)$

(15) **(A).** $f(x)=2\left(1+\frac{1}{x-1}\right)$

$f'(x)=-\frac{2}{(x-1)^2}<0$

$f(x)$ is strictly decreasing.
$\Rightarrow$ f is one-one but not onto.

(16) **(A).** $f(x)=\log_e\left(\frac{1-x}{1+x}\right)$, $|x|<1$

$$f\left(\frac{2x}{1+x^2}\right)=\ln\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$=\ln\left(\frac{(x-1)^2}{(x+1)^2}\right)=2\ln\left|\frac{1-x}{1+x}\right|=2f(x)$$

(17) **(A).** $f(x)=a^x, a>0$

$$f(x)=\frac{a^x+a^{-x}+a^x-a^{-x}}{2}$$

$\Rightarrow f_1(x)=\frac{a^x+a^{-x}}{2}$; $f_2(x)=\frac{a^x-a^{-x}}{2}$

$\Rightarrow f_1(x+y)+f_1(x-y)$

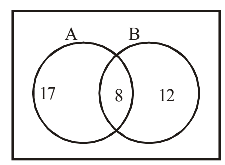
$$=\frac{a^{x+y}+a^{-x-y}}{2}+\frac{a^{x-y}+a^{-x+y}}{2}$$

$$=\frac{(a^x+a^{-x})}{2}(a^y+a^{-y})=f_1(x)\times 2f_1(y)=2f_1(x)f_1(y)$$

(18) **(A).** $y=\frac{x^2}{1-x^2}$; Range of y : $R-[-1,0)$

For surjective funciton, A must be same as above range.

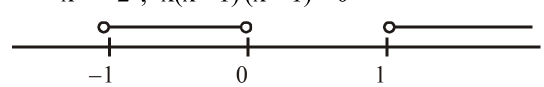
(19) **(C).** Let population = 100
$n(A)=25$
$n(B)=20$
$n(A\cap B)=8$
$n(A\cap\bar{B})=17$
$n(\bar{A}\cap B)=12$



$\frac{30}{100}\times 17+\frac{40}{100}\times 12+\frac{50}{100}\times 8$

$5.1+4.8+4=13.9$

(20) **(C).** $4-x^2\ne 0$; $x^3-x>0$
$x=\pm 2$; $x(x-1)(x+1)>0$



$\therefore$ $D_f\in(-1,0)\cup(1,2)\cup(2,\infty)$

(21) **(C).** $g(S)=[-2,2]$
So, $f(g(S))=[0,4]=S$
And $f(S)=[0,16]\Rightarrow f(g(S))\ne f(S)$
Also, $g(f(S))=[-4,4]\ne g(S)$
So, $g(f(S)\ne S$

(22) **(C).** $f(x)=\sqrt{x}$, $g(x)=\tan x$,

$h(x)=\frac{1-x^2}{1+x^2}$, $fog(x)=\sqrt{\tan x}$

$$hofog(x)=h\sqrt{\tan x}=\frac{1-\tan x}{1+\tan x}=-\tan\left(\frac{\pi}{4}-x\right)$$

$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

(23) **(A).** For $A = C, A - C = \phi \Rightarrow \phi \subseteq B$. But $A \not\subseteq B$

$\Rightarrow$ Option A is NOT true

Let $x \in (Cx \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C$ or $x \in A)$ and $(x \in C$ or $x \in B)$

$\Rightarrow x \in C$ or $x \in (A \cap B)$

$\Rightarrow x \in C$ or $x \in C$ (as $(A \cup B) \subseteq C) \Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$ ..... (1)

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

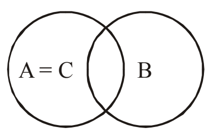
$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$C \subseteq (C \cup A) \cap (C \cup B)$ ...... (2)

From (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

Option 2 is true



Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$x \in (A \cap B) \Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence, $x \in A \Rightarrow x \in C \Rightarrow A \subseteq C$

Option 3 is true.

As $C \supseteq (A \cap B)$

$B \cap C \supseteq (A \cap B)$

$A \cap B \neq \phi$

$A \cap B \neq \phi$.

Option D is true.

(24) **(B).** $g(f(x)) = f^2(x) + f(x) - 1$

$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10 \cdot \frac{5}{4} + 5 = -\frac{5}{4}$

$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$; $f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$

$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$; $f\left(\frac{5}{4}\right) = -\frac{1}{2}$

(25) **(A).** $y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$; $\frac{1+y}{1-y} = \frac{8^{2x}}{8^{-2x}}$

$8^{4x} = \frac{1+y}{1-y}$; $4x = \log_8\left(\frac{1+y}{1-y}\right)$; $x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

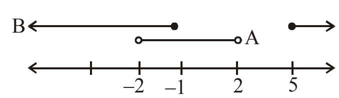
(26) **(B).** $f(x) = \begin{cases} \frac{x}{x^2+1}; & x \in (1, 2) \\ \frac{2x}{x^2+1}; & x \in [2, 3) \end{cases}$

$\therefore$ f (x) is a decreasing function

$\therefore$ $y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right] \Rightarrow y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(27) **(C).** $A : x \in (-2, 2); B : x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



(28) **29.** $n(A) = 25, n(B) = 7, n(A \cap B) = 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3 = 29$