PERIOD-5

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ FACTOR THEOREM

PREVIOUS KNOWLEDGE TEST

a) Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by x-1

LEARNING OUTCOME:~

Students will learn

a) factor theorem.

Question 2.

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution:

We have, $p(x) = x^3 - ax^2 + 6x - a$ and zero of $x - a$ is a.

$\therefore p(a) = (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Thus, the required remainder is 5a.

Question 3.

Check whether 7 + 3x is a factor of 3x3+7x.

Solution:

We have, p(x) = 3x3+7x. and zero of 7 + 3x is −73.

$$\therefore\ p\left(\frac{-7}{3}\right)=3\left(\frac{-7}{3}\right)^3+7\left(\frac{-7}{3}\right)$$

$$=3\left(\frac{-343}{27}\right)+\left(\frac{-49}{3}\right)=-\frac{343}{9}-\frac{49}{3}=-\frac{490}{9}$$

Since,(−4909) ≠ 0

i.e. the remainder is not 0.

∴ 3x3 + 7x is not divisib1e by 7 + 3x.

Thus, 7 + 3x is not a factor of 3x3 + 7x.

https://www.youtube.com/watch?v=4YXE7HDyInM

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

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Factorization of Polynomials

Factor Theorem- If $p(x)$ is a polynomial of degree n where n is greater than or equal to one and a is any real number then-

1) $x - a$ is a factor of $p(x)$ if $p(a) = 0$
2) $p(a) = 0$ if $x - a$ is a factor of $p(x)$

Proof-

Using Remainder theorem, we can write-

$p(x) = [(x - a) X q(x)] + p(a)$ Here, $r(x) = r = p(a)$

(1) Suppose, $p(a) = 0$. Then $p(x) = (x - a) X q(x)$
Clearly, $x - a$ is a factor of $p(x)$

(2)Suppose $x - a$ is a factor of $p(x)$. Then we can write, $p(x) = (x - a) X g(x)$ for some polynomial $g(x)$. So, $p(a) = 0$

Applications of Factor Theorem

Example 1. Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$

Solution: By Factor theorem, $p(1) = 0$

$$p(1) = 4(1) + 3(1) - 4(1) + k = 0$$

$$K = -3$$

Factorization of Quadratic polynomials using Splitting the middle term method

Consider the quadratic polynomial $a x^2 + b x + c$

Suppose $(p x + q)$ and $(r x + s)$ be its factors.

So we can write, $a x^2 + b x + c = (p x + q) \times (r x + s)$

On comparing the coefficients of the like terms in L.H.S and R.H.S we get,

$a = p r$, $b = p s + q r$, $c = q s$

And clearly, $(p s) \times (q r) = (p r) \times (q s) = (a c)$.

So, to factorize the quadratic polynomial, we need to write b as the sum of two numbers whose product is (a c)

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For example- Factorize $6x^2 + 17x + 5$ by-
1)Splitting the middle term method
2)Factor theorem

Solution-
1) We can write $6x^2 + 17x + 5 = 6x^2 + (15 + 2)x + 5$

$$= 6x^2 + 15x + 2x + 5$$
$$= 3x(2x + 5) + 1(2x + 5)$$
$$= (3x + 1)(2x + 5)$$

2) We can write $6x^2 + 17x + 5 = 6(x^2 + (17/6)x + (5/6))$
Taking, $p(x) = x^2 + (17/6)x + (5/6)$
So, $6x^2 + 17x + 5 = 6\,p(x)$
Suppose a and b are zeroes of p(x). Then $x – a$ and $x – b$ are factors of p(x) and we have $p(x) = (x – a)(x – b)$
So, $6x^2 + 17x + 5 = 6(x – a)(x – b)$ ⟶ **(*)**
On comparing the coefficients of like terms, we get, $ab = 5/6$
Possibilities for the values of a and b satisfying $ab = 5/6$ are- $\frac{\pm1}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm1$

Also, these values of a and b must satisfy $p(a) = 0$ and $p(b) = 0$
So, by checking all the values found above, we get that $a = -1/3$ and $b = -5/2$ satisfies the above conditions. So,**(*)**, becomes- $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

To factorize a cubic polynomial p(x) , we first find a zero of the polynomial by hit and trial method (by putting different values of x in the polynomial equation p(x) = 0)

Then the obtained root of the polynomial equation say x = a (or simply, the zero of the polynomial p(x)) implies that x – a is a factor of p(x).

So we can write, $p(x) = (x - a)\, g(x) \longrightarrow$ **(**)**

where g(x) is a polynomial such that the degree of g(x) < degree of p(x)

Here, as degree of p(x) = 3 So, degree of g(x) = 2 i.e. g(x) is a quadratic polynomial and using factor theorem or splitting the middle term method we can factorize g(x) and thus we get the desired factorization by putting the resultant g(x) in **(**)**

Evaluation:-

1. Find the value of 'k' if x-1is a factor of $4x^3 + 3x^2 - 4x - \text{k}$.
2. Factorize $6x^2 + 17x + 5$ by splitting the middle term and by using factor theorem.

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HOMEWORK:-EXERCISE - 2.4

QUESTION NUMBER-1,2,3

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AHA:~

1. For what value of 'a' is $2x^3 + ax^2 - 11x + a + 3$ is exactly divisible by $(2x - 1)$?

2. Find the value of 'a' and 'b' so that the polynomial $x^3 + 10x^2 + ax + a$ is exactly divisible by $x - 1$ and $x - 2$.

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THANKING YOU

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