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PERIOD-6

# MATHEMATICS

**CHAPTER NUMBER :~ 2**
**CHAPTER NAME :~ POLYNOMIALS**
**SUB TOPIC :~ FACTOR THEOREM AND REMAINDER THEOREM**

# PREVIOUS KNOWLEDGE TEST

Factorize $6x^2 + 17x + 5$ by splitting the middle term and by using factor theorem.

# LEARNING OUTCOME:~

Students will learn

a) factor theorem and remainder theorem.

# EXERCISE-2.4

Question 1.
Determine which of the following polynomials has (x +1) a factor.

(i) $x^3+x^2+x+1$
(ii) $x^4 + x^3 + x^2 + x + 1$
(iii) $x^4 + 3x^3 + 3x^2 + x + 1$
(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

The zero of x + 1 is -1.
(i) Let $p(x) = x^3 + x^2 + x + 1$
$\therefore p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$ .
$= -1 + 1 - 1 + 1$
$\Rightarrow p(-1) = 0$
So, (x+ 1) is a factor of $x^3 + x^2 + x + 1$.

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(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

$\therefore P(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1)+1$

$= 1 - 1 + 1 - 1 + 1$

$\Rightarrow P(-1) \neq 1$

So, $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$ .

$\therefore p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

$\Rightarrow p(-1) \neq 0$

So, $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$\therefore p(-1) = (-1)3 - (-1)2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2}$

$\Rightarrow p(-1) \neq 0$

So, $(x + 1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Question 2.
Use the Factor Theorem to determine whether g (x) is a factor of p (x) in each of the following cases
(i) p (x)= $2x^3 + x^2 - 2x - 1$, g (x) = x + 1
(ii) p(x)= $x^3 + 3x^2 + 3x + 1$, g (x) = x + 2
(iii) p (x) = $x^3 - 4x^2 + x + 6$, g (x) = x – 3

Solution:

(i) We have, p (x)= $2x^3 + x^2 - 2x - 1$ and g (x) = x + 1
$\therefore p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$= 2(-1) + 1 + 2 - 1$
$= -2 + 1 + 2 - 1 = 0$
$\Rightarrow p(-1) = 0$, so g(x) is a factor of p(x).

(ii) We have, p(x) $x^3 + 3x^2 + 3x + 1$ and g(x) = x + 2

$\therefore p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -14 + 13$

$= -1$

$\Rightarrow p(-2) \neq 0$, so g(x) is not a factor of p(x).

(iii) We have, $= x^3 - 4x^2 + x + 6$ and g (x) = x – 3

$\therefore p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 4(9) + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

$\Rightarrow p(3) = 0$, so g(x) is a factor of p(x).

Question 3.
Find the value of k, if $x - 1$ is a factor of p (x) in each of the following cases

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

For $(x - 1)$ to be a factor of p(x), p(1) should be equal to 0.
(i) Here, $p(x) = x^2 + x + k$
Since, $p(1) = (1)^2 + 1 + k$
$\Rightarrow p(1) = k + 2 = 0$
$\Rightarrow k = -2$.

(ii) Here, $p(x) = 2x^2 + kx + \sqrt{2}$

Since, $p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$= 2 + k + \sqrt{2} = 0$

$k = -2 - \sqrt{2} = -(2 + \sqrt{2})$

(iii) Here, $p(x) = kx^2 - \sqrt{2}\, x + 1$

Since, $p(1) = k(1)^2 - (1) + 1$

$= k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

(iv) Here, $p(x) = kx^2 - 3x + k$

$p(1) = k(1)^2 - 3(1) + k$

$= k - 3 + k$

$= 2k - 3 = 0$

$\Rightarrow k = 34$

https://www.youtube.com/watch?v=M0YifLWM-4M

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

## zation of Polynomials

eorem- If p(x) is a polynomial of degree n where n is greater than or
ne and a is any real number then-
 a factor of p(x) if p(a) = 0
) if x – a is a factor of p(x)

Jsing Remainder theorem, we can write-

p(x) = [(x – a) X q(x)] + p(a)      Here, r(x) = r = p(a)

se, p(a) = 0 . Then p(x) = (x – a) X q(x)
 x – a is a factor of p(x)
e x – a is a factor of p(x). Then we can write, p(x) = (x – a) X g(x) for
lynomial g(x). So, p(a) = 0

# Applications of Factor Theorem

**Example 1**. Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$

Solution: By Factor theorem, $p(1) = 0$

$$p(1) = 4(1) + 3(1) - 4(1) + k = 0$$
$$K = -3$$

**Factorization of Quadratic polynomials using Splitting the middle term method**

Consider the quadratic polynomial $a x^2 + b x + c$
Suppose $(p x + q)$ and $(r x + s)$ be its factors.
So we can write, $a x^2 + b x + c = (p x + q) \times (r x + s)$
On comparing the coefficients of the like terms in L.H.S and R.H.S we get,
$a = p r$, $b = p s + q r$, $c = q s$
And clearly, $(p s) \times (q r) = (p r) \times (q s) = (a c)$ .

So, to factorize the quadratic polynomial, we need to write b as the sum of two numbers whose product is $(a c)$

**For example**- Factorize $6x^2 + 17x + 5$ by-
1)Splitting the middle term method
2)Factor theorem

Solution-
1) We can write $6x^2 + 17x + 5 = 6x^2 + (15 + 2)x + 5$
$= 6x^2 + 15x + 2x + 5$
$= 3x(2x + 5) + 1(2x + 5)$
$= (3x + 1)(2x + 5)$

2) We can write $6x^2 + 17x + 5 = 6(x^2 + (17/6)x + (5/6))$
Taking, $p(x) = x^2 + (17/6)x + (5/6)$
So, $6x^2 + 17x + 5 = 6\,p(x)$
Suppose a and b are zeroes of p(x). Then x – a and x – b are factors of p(x) and we have
$p(x) = (x - a)(x - b)$
So, $6x^2 + 17x + 5 = 6(x - a)(x - b)$ ⟶ **(*)**
On comparing the coefficients of like terms, we get, $ab = 5/6$
Possibilities for the values of a and b satisfying $ab = 5/6$ are- $\frac{\pm 1}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm 1$

Also, these values of a and b must satisfy $p(a) = 0$ and $p(b) = 0$
So, by checking all the values found above, we get that $a = -1/3$ and $b = -5/2$ satisfies the above conditions. So,**(*)**, becomes- $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

**To factorize a cubic polynomial** $p(x)$ , we first find a zero of the polynomial by hit and trial method (by putting different values of x in the polynomial equation $p(x) = 0$ )

Then the obtained root of the polynomial equation say $x = a$ (or simply, the zero of the polynomial $p(x)$ ) implies that $x - a$ is a factor of $p(x)$.

So we can write, $p(x) = (x - a)\, g(x) \longrightarrow$ **(**)**

where $g(x)$ is a polynomial such that the degree of $g(x) <$ degree of $p(x)$

Here, as degree of $p(x) = 3$ So, degree of $g(x) = 2$ i.e. $g(x)$ is a quadratic polynomial and using factor theorem or splitting the middle term method we can factorize $g(x)$ and thus we get the desired factorization by putting the resultant $g(x)$ in **(**)**

## Evaluation:~

1. Factorize: $-y^2 - 5y + 6$.
2. Factorize:~ $2x^2 + 7x + 3$.

HOMEWORK:-

EXERCISE - 2.4

QUESTION NUMBER-4,5

# AHA:~

1. If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + \mathrm{d}x + \mathrm{c}$

Show that a+c+e = b+d =0.

2. Without actual division , prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is actually divisible by $x^2 - 3\mathrm{x} + 2$.

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