PERIOD-7

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ ALGEBRAIC IDENTITIES

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PREVIOUS KNOWLEDGE TEST

Without actual division , prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is actually divisible by $x^2 - 3x + 2$.

LEARNING OUTCOME:~

Students will learn

a) algebraic identities.

Question 4.

Factorise

(i) $12x^2 – 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x – 6$

(iv) $3x^2 – x – 4$

Solution:

(i) We have,

$12x^2 – 7x + 1 = 12x^2 – 4x - 3x + 1$

$= 4x (3x – 1) - 1 (3x – 1)$

$= (3x - 1) (4x - 1)$

Thus, $12x^2 - 7x + 3 = (2x – 1) (x + 3)$

(ii) We have, $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$

$= x(2x + 1) + 3(2x + 1)$

$= (2x + 1)(x + 3)$

Thus, $2\times2 + 7x + 3 = (2x + 1)(x + 3)$

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(iii) We have, $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

Thus, $6x^2 + 5x - 6 = (2x + 3)(3x - 2)$

(iv) We have, $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$

Thus, $3x^2 - x - 4 = (3x - 4)(x + 1)$

Question 5.

Factorise

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Solution:

(i) We have, $x^3 - 2x^2 - x + 2$

Rearranging the terms, we have $x^3 - x - 2x^2 + 2$

$= x(x^2 - 1) - 2(x^2 - 1) = (x^2 - 1)(x - 2)$

$= [(x)^2 - (1)^2](x - 2)$

$= (x - 1)(x + 1)(x - 2)$

$[\because (a^2 - b^2) = (a + b)(a - b)]$

Thus, $x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$

(ii) We have, $x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$,

$= x^2 (x + 1) - 4x(x + 1) - 5(x + 1)$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

Thus, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

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(iii) We have, $x^3 + 13x^2 + 32x + 20$

$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$

$= x^2(x + 1) + 12x(x + 1) + 20(x + 1)$

$= (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 2x + 10x + 20)$

$= (x + 1)[x(x + 2) + 10(x + 2)]$

$= (x + 1)(x + 2)(x + 10)$

Thus, $x^3 + 13x^2 + 32x + 20$

$= (x + 1)(x + 2)(x + 10)$

(iv) We have, $2y^3 + y^2 - 2y - 1$

$= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$

$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$

Thus, $2y^3 + y^2 - 2y - 1$

$= (y - 1)(y + 1)(2y + 1)$

https://www.youtube.com/watch?v=_IUCfKBHAl0

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

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Algebraic Identities

What is a mathematical identity?

An identity is an equality relation A = B where A and B can be variables.
Here, A and B can be differently defined functions but the equality between the two still holds.
For example: $\cos^2 x + \sin^2 x = 1$ is a trigonometric identity where x is a variable and for any value of x the above result holds true.

So, algebraic identities are algebraic equations that holds true for all values of the variables occurring in it.

Relevance:

Algebraic identities are very important in mathematics. They are helpful in computing the values without actually performing lengthy calculations and for factorizing the polynomials.

Algebraic Identities

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Evaluation:~

1. Write in expanded form $(3a+4b)^3$.
2. Evaluate:~ $(999)^3$.

HOMEWORK:-

EXERCISE - 2.5

QUESTION NUMBER-1 TO 7

AHA:~

1.x+y=12, xy=27 find $x^3 + y^3$.

2. $x^4 + \frac{1}{x^4} = 47$, find $x^3 + \frac{1}{x^3}$.

THANKING YOU

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