PERIOD-8

MATHEMATICS

CHAPTER NUMBER :~ 2
CHAPTER NAME :~ POLYNOMIALS
SUB TOPIC :~ APPLICATIONS OF ALGEBRAIC IDENTITIES

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PREVIOUS KNOWLEDGE TEST

Algebraic Identities

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

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LEARNING OUTCOME:~

Students will learn

a) Applications of Algebraic identities.

EXERCISE-2.5

Question 1.

Use suitable identities to find the following products

(i) $(x + 4)(x + 10)$

(ii) $(x+8)(x - 10)$

(iii) $(3x + 4)(3x – 5)$

Solution:

(i) We have, $(x+ 4)(x + 10)$

Using identity,

$(x+ a)(x+ b) = x^2 + (a + b)x + ab$.

We have, $(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10)$

$= x^2 + 14x + 40$

(ii) We have, $(x+8)(x-10)$

Using identity,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

We have, $(x + 8)(x - 10) = x^2 + [8 + (-10)]x + (8)(-10)$

$= x^2 - 2x - 80$

(iii) We have, $(3x + 4)(3x - 5)$

Using identity,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

We have, $(3x + 4)(3x - 5) = (3x)^2 + (4 - 5)x + (4)(-5)$

$= 9x^2 - x - 20$

Question 2.
Evaluate the following products without multiplying directly

(i) 103 x 107
(ii) 95 x 96
(iii) 104 x 96

Solution:

(i) We have, 103 x 107 = (100 + 3) (100 + 7)
$= (100)^2 + (3 + 7)(100) + (3 \times 7)$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 10000 + (10) \times 100 + 21$
$= 10000 + 1000 + 21 = 11021$

(ii) We have, 95 x 96 = (100 – 5) (100 – 4)

$= (100)^2 + [(-5) + (-4)]\ 100 + (-5 \times -4)$

[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 10000 + (-9) + 20 = 9120$

$= 10000 + (-900) + 20 = 9120$

(iii) We have 104 x 96 = (100 + 4) (100 – 4)

$= (100)^2 - 4^2$

[Using $(a + b)(a - b) = a^2 - b^2$]

$= 10000 - 16 = 9984$

Question 3.

Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - y2100$

Solution:

(i) We have, $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2$

[Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= (3x + y)(3x + y)$

(ii) We have, $4y^2 - 4y + 1^2$

$= (2y)^2 + 2(2y)(1) + (1)^2$

$= (2y - 1)^2$

[Using $a^2 - 2ab + b^2 = (a - b)^2$]

$= (2y - 1)(2y - 1)$

Question 4.

Expand each of the following, using suitable identity

(i) $(x+2y+ 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(- 2x + 3y + 2z)^2$

(iv) $(3a -7b - c)^z$

(v) $(- 2x + 5y - 3z)^2$

Solution:

We know that

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2$

$= x^2 + (2y)^2 + (4z)^2 + 2 (x) (2y) + 2 (2y) (4z) + 2(4z) (x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8 zx$

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

Question 5.

Factorise

(i) $4\ x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solution:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2 = (2x + 3y + 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2y + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

Question 6.

Write the following cubes in expanded form

Solution:

We have, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ …(1)

and $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ …(2)

(i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$ [By (1)]

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a – 3b)^3 = (2a)^3 – (3b)^3 – 3(2a)(3b)(2a – 3b)$ [By (2)]

$= 8a^3 – 27b^3 – 18ab(2a – 3b)$

$= 8a^3 – 27b^3 – 36a^2b + 54ab^2$

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Question 7.

Evaluate the following using suitable identities

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solution:

(i) We have, $99 = (100 - 1)$

$\therefore 99^3 = (100 - 1)^3$

$= (100)^3 - 1^3 - 3(100)(1)(100 - 1)$

[Using $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$]

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 1000300 - 30001 = 970299$

(ii) We have, 102 =100 + 2

$\therefore 102^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

[Using $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$]

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200 = 1061208$

(iii) We have, 998 = 1000 – 2

$\therefore (998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

[Using $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$]

$= 1000000000 - 8 - 6000(1000 - 2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

https://www.youtube.com/watch?v=C46fQQeHgQE

"As great a genius as Archimedes could not invent analytical geometry, for the algebraic knowledge necessary for such as achievement was not available in his time…"

~ *Nathan A. Court…*

Application of Algebraic Identities

- To find- 99 X 101 without actual multiplication
  Solution: We can write, 99 X 101 = (100 – 1) (100 + 1)
  $= (100)^2 - (1)^2$
  = 10000 – 1
  = 9999

- Evaluate: $(999)^3$
  Solution: We can write, $(999)^3 = (1000 - 1)^3$
  $= (1000)^3 - (1)^3 - 3\ (1000)\ (1)\ (1000 - 1)$
  = 997002999

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Evaluation:~

1. Factorize: $8x^3+27y^3+36x^2y+54xy^2$.

HOMEWORK:-

EXERCISE - 2.5

QUESTION NUMBER-8 TO 16

AHA:-

1. $a+b+c=6$, $ab+bc+ca=11$, Find $a^3+b^3+c^3-3abc$.

2. $x+1/x=3$, Find x^2+1/x^2.

THANKING YOU

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