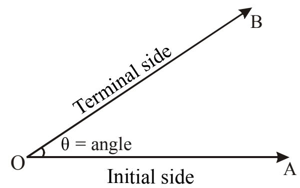
2

TRIGONOMETRY

TRIGONOMETRIC RATIOS

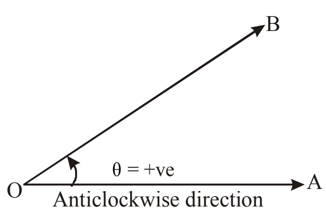
Trigonometry is the branch of science in which we study about the angles and sides of a triangle.

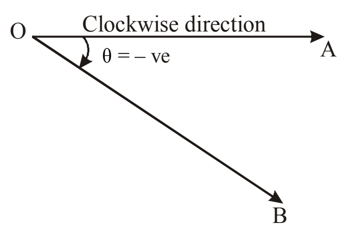
Angle: Consider a ray $\overrightarrow{OA}$. If this ray rotates about its end point O and takes the position OB, then the angle $\angle$ AOB has been generated.



An angle is considered as the figure obtained by rotating a given ray about its end-point. The initial position OA is called the initial side and the final position OB is called terminal side of the angle. The end point O about which the ray rotates is called the vertex of the angle.

Sense of an angle : The sense of an angle is said to be positive or negative according as the initial side rotates in anticlockwise or clockwise direction to get to the terminal side.

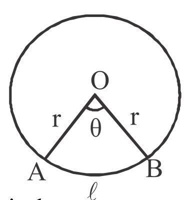




Measurement of angle :

English system : One right angle = 90° (degree)
$1° = 60'$ (minutes)
$1' = 60''$ (seconds)

Circular system : If length of arc of a circle equal's to radius then angle impose by that arc on centre of circle is called one radian.



Length of arc, $\ell = r\,\theta$; r = Radius of circle

θ = Angle in radian

Some Important Conversion :

π Radian = 180° One Radian = $\left(\dfrac{180}{\pi}\right)^{\circ}$

$\dfrac{\pi}{6}$ Radian = 30° $\dfrac{\pi}{4}$ Radian = 45°

$\dfrac{\pi}{3}$ Radian = 60° $\dfrac{\pi}{2}$ Radian = 90°

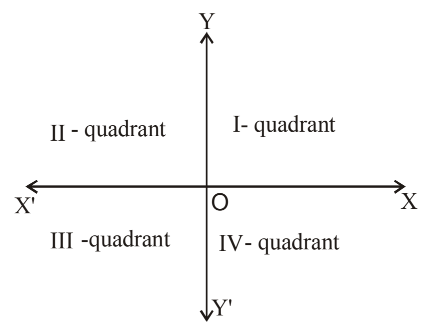
$\dfrac{2\pi}{3}$ Radian = 120° $\dfrac{3\pi}{4}$ Radian = 135°

$\dfrac{5\pi}{6}$ Radian = 150° $\dfrac{7\pi}{6}$ Radian = 210°

$\dfrac{5\pi}{4}$ Radian = 225° $\dfrac{5\pi}{3}$ Radian = 300°

SOME USEFUL TERMS

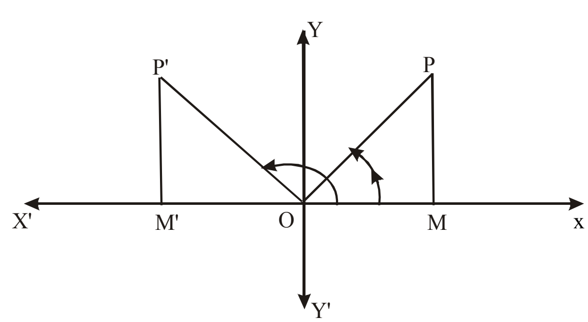
Quadrants : Let X′OX and YOY′ be two lines at right angles in the plane of the paper. These lines divide the plane of the paper into four equal parts which are known as quadrants.



The lines X′OX and YOY′ are known as x-axis and y-axis respectively. These two lines taken together are known as the coordinate axes. The regions XOY, YOX′, X′OY′ and Y′OX are known as the first, the second, the third and the fourth quadrant respectively.

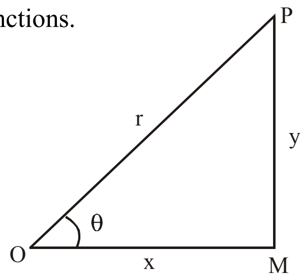
Angle in Standard Position : An angle is said to be in standard position if its vertex coincides with the origin O and the initial side coincides with OX i.e. the positive direction of x-axis.

Co-terminal Angles : Two angles with different measures but having the same initial sides and the same terminal sides are known as co-terminal angles.



TRIGONOMETRICAL RATIOS OR FUNCTIONS

In the right angled triangle OMP, we have base (OM) = x, perpendicular (PM) = y and hypotenuse (OP) = r, then we define the following trigonometric ratios which are also known as trigonometric functions.



$\sin\theta = \frac{P}{H} = \frac{y}{r}$,

$\cos\theta = \frac{B}{H} = \frac{x}{r}$,

$\tan\theta = \frac{P}{B} = \frac{y}{x}$.

$\cot\theta = \frac{B}{p} = \frac{x}{y}$, $\sec\theta = \frac{H}{B} = \frac{r}{x}$, $\text{cosec}\theta = \frac{H}{p} = \frac{r}{y}$

Fundamental trigonometric Identities :

(i) $\sin\theta = \frac{1}{\text{cosec}\theta}$ (ii) $\cos\theta = \frac{1}{\sec\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ (iv) $\sin^2\theta + \cos^2\theta = 1$

(v) $1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta) = \frac{1}{\sec\theta + \tan\theta}$

(vi) $1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow (\text{cosec}\theta - \cot\theta) = \frac{1}{\text{cosec}\theta + \cot\theta}$

Example 1 :

If cosec A + cot A = 11/2, then find the value tanA.

Sol. cosec A + cot A = 11/2 .....(1)

$\Rightarrow \frac{1}{\text{cosec}A + \cot A} = \frac{2}{11}$

$\Rightarrow \text{cosec}\,A - \cot A = \frac{2}{11}$ ....(2)

$(1) - (2) \Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$

Example 2 :

Find $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

Sol. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$

Example 3 :

Find the value of $\tan^2\theta\sec^2\theta(\cot^2\theta - \cos^2\theta)$.

Sol. $\tan^2\theta\sec^2\theta(\cot^2\theta - \cos^2\theta)$

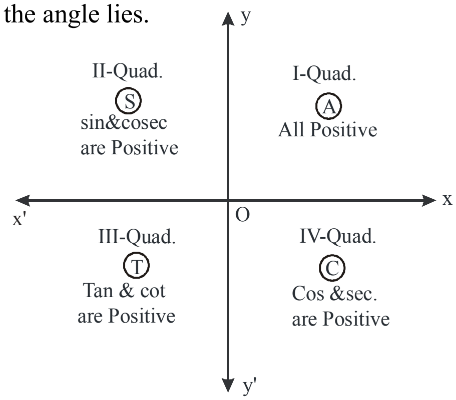
$= \sec^2\theta(\tan^2\theta\cot^2\theta - \tan^2\theta\cos^2\theta)$

$= \sec^2\theta\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right) = \sec^2\theta(1-\sin^2\theta)$

$= \sec^2\theta.\cos^2\theta = 1$

Sign of the trigonometrical Ratios or functions :

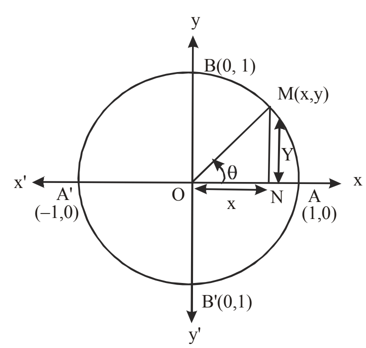
Their signs depends on the quadrant in which the terminal side of the angle lies.



Variations in values of trigonometric functions in different quadrants :

Let X'OX and YOY' be the coordinate axes. Draw a circle with centre at origin O and radius unity.

Let M (x, y) be a point on the circle such that $\angle AOP = \theta$ then $x = \cos\theta$ and $y = \sin\theta$



$-1 \le \cos\theta \le 1$ and $-1 \le \sin\theta \le 1$ for all values of θ

II - Quadrant (S)		I - Quadrant (A)	
sinθ →	Decrease from 1 to 0	sinθ →	Increase from 0 to 1
cosθ →	Decrease from 0 to -1	cosθ →	Decrease from 1 to 0
tanθ →	Increase from - ∞ to 0	tanθ →	Increase from 0 to ∞
cotθ →	Decrease from 0 to -∞	cotθ →	Decrease from ∞ to 0
secθ →	Increase from - ∞ to -1	secθ →	Increase from 1 to ∞
cosecθ →	Increase from 1 to ∞	cosecθ →	Decrease from ∞ to 1

III - Quadrant (T)		IV - Quadrant (C)	
sinθ →	Decrease from 0 to -1	sinθ →	Increase from -1 to 0
cosθ →	Increase from -1 to 0	cosθ →	Increase from 0 to 1
tanθ →	Increase from 0 to ∞	tanθ →	Increase from - ∞ to 0
cotθ →	Decrease from ∞ to 0	cotθ →	Decrease from 0 to -∞
secθ →	Decrease from -1 to -∞	secθ →	Decrease from ∞ to 1
cosecθ →	Increase from - ∞ to -1	cosecθ →	Decrease from -1 to -∞

Note : + ∞ and – ∞ are two symbols. These are not real number. When we say that tanθ increases from 0 to ∞ for as θ varies from 0 to π/2 it means that tanθ increases in the interval (0, π/2) and it attains large positive values as θ tends to π/2 . Similarly for other trigonometric functions.

Table : Summarizes the trigonometric functional values of some special angles.

θ	In radians	sin θ	cos θ	tan θ
0°	0	0	1	0
30°	π/6	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	π/4	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	π/3	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	π/2	1	0	Undefined
180°	π	0	–1	0
270°	3π/2	–1	0	Undefined

Example 4 :

Find the value of sinθ and tanθ if cosθ = – (12/13) and θ lies in the third quadrant.

Sol. We have $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

In the third quadrant sinθ is negative, therefore

$$\sin\theta = -\sqrt{1-\cos^2\theta} \Rightarrow \sin\theta = -\sqrt{1-\left(-\frac{12}{13}\right)^2} = -\frac{5}{13}$$

$$\text{then, } \tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = -\frac{5}{13}\times\frac{13}{-12} = \frac{5}{12}$$

Example 5 :

$$\text{If } \frac{\pi}{2} < \theta < \pi, \text{ then find } \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}.$$

Sol. $$\text{Exp.} = \frac{(1-\sin\theta)+(1+\sin\theta)}{\pm\sqrt{1-\sin^2\theta}} = -\frac{2}{\cos\theta} = -2\sec\theta$$

TRIGONOMETRICAL RATIO OF ALLIED ANGLES

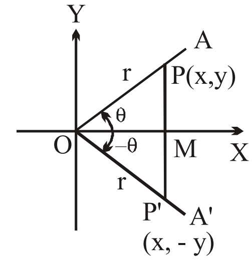
Two angles are said to be allied when their sum or difference is either zero or a multiple of 90°.

1. **Trigonometric Ratios of (–θ) :** Let a revolving ray starting from its initial position OX, trace out an angle ∠ XOA = θ. Let P (x, y) be a point on OA such that OP = r. Draw PM ⊥ from P on x-axis. Angle ∠ XOA′ = –θ in the clockwise sense. Let P' be a point on OA' such that OP' = OP. Clearly M and M' coincide and ΔOMP is congruent to ΔOMP', then P' are (x, – y)

$$\sin(-\theta) = \frac{-y}{r} \Rightarrow \frac{-y}{r} = -\sin\theta$$

$$\cos(-\theta) = \frac{x}{r} = \cos\theta$$

$$\tan(-\theta) = \frac{-y}{x} = -\tan\theta$$



Taking the reciprocal of these trigonometric ratios,

cosec (–θ) = – cosec θ

sec (–θ) = sec θ and cot (–θ) = – cot θ

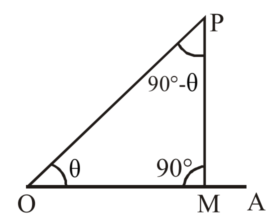
Note : A function f (x) is said to be even function if f (–x) = f (x) for all x in its domain.

A function f (x) is an odd function if f (–x) = – f (x) for all x in its domain.

sinθ, tanθ, cotθ, cosecθ all odd functions and cosθ, secθ are even functions.

2. **Trigonometric function of (90 – θ) :**

Let the revolving line, starting from OA, trace out any acute angle AOP, equal to θ. From any point P, draw PM ⊥ to OA.



Three angles of a triangle are together equal to two right angles, and since OMP is a right angle, the sum of the two angles MOP and OPM is right angle. ∠OPM = 90º – θ

[When the angle OPM is consider, the line PM is the 'base' and MO is the 'perpendicular']

$$\sin(90° - \theta) = \sin\ MPO = \frac{MO}{PO} = \cos\ AOP = \cos\theta$$

$$\cos(90° - \theta) = \cos\ MPO = \frac{PM}{PO} = \sin\ AOP = \sin\theta$$

$$\tan(90° - \theta) = \tan\ MPO = \frac{MO}{PM} = \cot\ AOP = \cot\theta$$

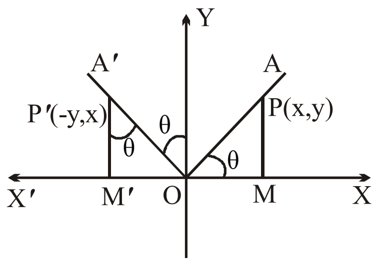
$$\cot(90° - \theta) = \cot\ MPO = \frac{PM}{MO} = \tan\ AOP = \tan\theta$$

$$\operatorname{cosec}(90° - \theta) = \operatorname{cosec}\ MPO = \frac{PO}{MO} = \sec\ AOP = \sec\theta$$

$$\text{and } \sec(90° - \theta) = \sec\ MPO = \frac{PO}{PM} = \operatorname{cosec}\ AOP = \operatorname{cosec}\theta$$

3. **Trigonometric Function of (90 + θ) :**

Let a revolving ray OA starting from its initial position OX, trace out an angle ∠ XOA = θ and Let another revolving ray OA′ starting from the same initial position OX, First trace out an angle θ so as to coincide with OA and then it revolves through an angle of 90° in anticlockwise direction to form an angle $\angle XOA' = 90^\circ + \theta$



Let P and P' be points on OA and OA' respectively such that OP = OP' = r.
Draw perpendicular PM and PM' from P and P' respectively on OX. Let the coordinates of P be (x, y).
Then OM = x and PM = y clearly
OM' = PM = y and P' M' = OM = x
so the coordinates of P' are $(-y_1, x)$

$$\sin(90+\theta) = \frac{M'P'}{OP'} = \frac{x}{y} = \cos\theta$$

$$\cos(90+\theta) = \frac{OM'}{OP'} = \frac{-y}{r} = -\sin\theta$$

$$\tan(90+\theta) = \frac{M'P'}{OM'} = \frac{x}{-y} = \frac{-x}{y} = -\cot\theta$$

Similarly, $\cot(90+\theta) = -\tan\theta$
$\sec(90+\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90+\theta) = \sec\theta$

$\sin(120°) = \sin(90° + 30°) = \cos 30° = \sqrt{3}/2$
$\tan(135°) = \tan(90° + 45°) = -\cot 45° = -1$
$\cos(150°) = \cos(90° + 60°) = -\sin 60° = -\sqrt{3}/2$

4. Reduction (180 – θ) :
$\sin(180-\theta) = \sin\theta$; $\cos(180-\theta) = -\cos\theta$;
$\tan(180-\theta) = -\tan\theta$; $\cot(180-\theta) = -\cot\theta$;
$\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$; $\sec(180-\theta) = -\sec\theta$;
$\sin(120°) = \sin(180° - 60°) = \sin 60° = \sqrt{3}/2$
$\cos(150°) = \cos(180° - 30°) = -\cos 30° = -\sqrt{3}/2$

5. Reduction (180 + θ) :
$\sin(180+\theta) = -\sin\theta$; $\cos(180+\theta) = -\cos\theta$;
$\tan(180+\theta) = \tan\theta$; $\cot(180+\theta) = \cot\theta$;
$\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$; $\sec(180+\theta) = -\sec\theta$;
$\sin(210°) = \sin(180° + 30°) = -\sin 30° = -1/2$
$\cos(240°) = \cos(180° + 60°) = -\cos 60° = -1/2$
$\tan(225°) = \tan(180° + 45°) = \tan 45° = 1$

6. Reduction (360 – θ) :
$\sin(2\pi-\theta) = \sin(-\theta)$; $\cos(2\pi-\theta) = \cos(-\theta)$
$\tan(2\pi-\theta) = \tan(-\theta)$
$\cos(315°) = \cos(360° - 45°) = \cos(-45°) = \cos(45°) = 1/\sqrt{2}$
$\tan(330°) = \tan(360° - 30°) = \tan(-30°) = -\tan 30°$
$= -1/\sqrt{3}$

Allied angles / Trigo.Ratio	(–θ)	(90 – θ) or $\left(\frac{\pi}{2}-\theta\right)$	(90 + θ) or $\left(\frac{\pi}{2}+\theta\right)$	(180 – θ) or (π – θ)	(180 + θ) or (π + θ)	(270 – θ) or $\left(\frac{3\pi}{2}-\theta\right)$	(270 + θ) or $\left(\frac{3\pi}{2}+\theta\right)$	(360 – θ) or (2π – θ)
sinθ	–sinθ	cosθ	cosθ	sin θ	–sin θ	–cos θ	– cosθ	– sin θ
cosθ	cos θ	sinθ	–sinθ	–cosθ	–cosθ	–sinθ	sinθ	cosθ
tanθ	–tanθ	cotθ	–cotθ	–tanθ	tanθ	cotθ	– cotθ	– tanθ

Example 6 :

Find the value of

$$\frac{\cos(90° + \theta)\sec(-\theta)\tan(180° - \theta)}{\sin(360° + \theta)\sec(180° + \theta)\cot(90° - \theta)}$$

Sol. Given expression, $\dfrac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sin\theta)(-\sec\theta)\tan\theta} = -1$

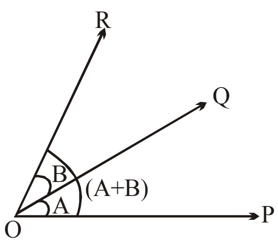
Example 7 :
Find the value of cot 5° cot 10° ..... cot 85°.
Sol. cot5° cot10° .... cot 85°
= cot5° cot10°....cot(90° – 10°) cot (90°– 5°)
= cot5° cot 10°...... tan10° tan5°
= (tan5° cot5°) (tan 10° cot 10°)........
= (1) (1) (1)....... = 1

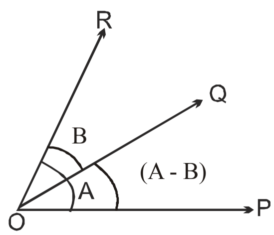
Example 8 :
Find the value of sin 10° + sin 20° + sin 30° +......+sin360°.
Sol. ∵ sin 190° = sin (180° + 10°) = – sin 10°
sin 200° = – sin 20°
sin 210° = – sin 30°
....
sin 360° = sin 180° = 0
∴ Exp. = 0

SUM AND DIFFERENCE OF ANGLE (COMPOUND ANGLE FORMULAE)

The algebraic sums of two or more angles are generally called compound angles and the angles are known as the constituent angles.

For example : If A, B, C are three angles then $A \pm B, A+B+C, A-B+C$ etc. are compound angles.





(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$
(ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$
(iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$
(iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(vii) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(viii) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Some more results :

(i) $\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
(ii) $\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
(iii) $\sin(A+B+C) = \sin A \cos B \cos C + \cos A \sin B \sin C + \cos A \cos B \sin C - \sin A \sin B \sin C$
(iv) $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$
(v) $\tan(A+B+C) =$

$$\frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

Example 9 :

Find the value of $\tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40°$

Sol. $\sqrt{3} = \tan 60° = \tan(40° + 20°)$

$$= \frac{\tan 40° + \tan 20°}{1 - \tan 40° \tan 20°}$$

$\therefore \sqrt{3} - \sqrt{3}\tan 40° \tan 20° = \tan 40° + \tan 20°$

Hence $\tan 40° + \tan 20° + \sqrt{3}\tan 40° \tan 20° = \sqrt{3}$

Example 10 :

If $\tan A = \dfrac{1}{2}$ and $\tan B = \dfrac{1}{3}$, then find the value of A + B i.e. $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3}$

Sol. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = \dfrac{5/6}{5/6} = 1$

$\therefore A + B = 45° = \pi/4$

Example 11 :

If $\sin A = \dfrac{3}{5}, 0 < A < \dfrac{\pi}{2}$ and $\cos B = \dfrac{-12}{13}, \pi < B < \dfrac{3\pi}{2}$ then find the value of $\sin(A-B)$.

Sol. We have : $\sin A = \dfrac{3}{5}$, where $0 < A < \dfrac{\pi}{2}$

$\therefore \cos A = \pm\sqrt{1 - \sin^2 A}$

$\Rightarrow \cos A = +\sqrt{1-\sin^2 A} = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$

[$\because$ cos is positive in first quadrant]

It is given that : $\cos B = \dfrac{-12}{13}$ and $\pi < B < \dfrac{3\pi}{2}$

$\therefore \sin B = \pm\sqrt{1-\cos^2 B}$

$\Rightarrow \sin B = -\sqrt{1-\cos^2 B}$

[$\because$ Sine is negative in the third quadrant]

$\Rightarrow \sin B = -\sqrt{1 - \left(\dfrac{-12}{13}\right)^2} = -\dfrac{5}{13}$

Now, $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \dfrac{3}{5} \times \dfrac{-12}{13} - \dfrac{4}{5} \times \dfrac{-5}{13} = -\dfrac{16}{65}$

TRANSFORMATION FORMULAE (PRODUCT INTO SUM OR DIFFERENCE)

We know that,

$\sin A \cos B + \cos A \sin B = \sin(A+B)$ ......(i)
$\sin A \cos B - \cos A \sin B = \sin(A-B)$ ......(ii)
$\cos A \cos B - \sin A \sin B = \cos(A+B)$ ......(iii)
$\cos A \cos B + \sin A \sin B = \cos(A-B)$ ......(iv)

Adding (i) and (ii),

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$ ......(v)

Subtracting (ii) and (i),

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$ ......(vi)

Adding (iii) and (iv),

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$ ......(vii)

Substracting (iii) and (iv).

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$ ......(viii)

Example 12 :

Find the value of $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$.

Sol. $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\frac{9\pi}{13}+\frac{\pi}{13}\right)+\cos\left(\frac{9\pi}{13}-\frac{\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

[$\because \cos(\pi-\theta)=-\cos\theta$]

Example 13 :

Find the value of $2\sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$.

Sol. $2\sin A\sin B=\cos(A-B)-\cos(A+B)$

$\therefore\ 2\sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)=\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$=\cos\frac{\pi}{3}-\cos\frac{\pi}{2}=\frac{1}{2}-0=\frac{1}{2}$

TRANSFORMATION FORMULAE
(SUM OR DIFFERENCE INTO PRODUCT)

We know that,

$\sin(A+B)+\sin(A-B)=2\sin A\cos B$ .....(i)

Let $A+B=C$ and $A-B=D$

then $A=\frac{C+D}{2}$ and $B=\frac{C-D}{2}$

Substituting in (i),

(a) $\sin C+\sin D=$ $2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

Similarly other formulae are,

(b) $\sin C-\sin D=$ $2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

(c) $\cos C+\cos D=$ $2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(d) $\cos C-\cos D=$ $2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

Finding values of 15° & 75°

(i) $\sin 15°=\sin\frac{\pi}{12}=\sin(45°-30°)$

$=\sin 45°.\cos 30°-\cos 45°.\sin 30°$

$=\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}=\cos 75°=\cos\frac{5\pi}{12}$

(ii) $\sin 75°=\sin\frac{5\pi}{12}=\sin(45°+30°)$

$=\sin 45°.\cos 30°+\cos 45°.\sin 30°$

$=\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}=\cos 15°=\cos\frac{\pi}{12}$

(iii) $\tan 15°=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}=\cot 75°$

(iv) $\tan 75°=\frac{\sqrt{3}+1}{\sqrt{3}-1}=2+\sqrt{3}=\cot 15°$

NOTE

(i) $\sin A\sin(60°-A)\sin(60°+A)=\frac{1}{4}\sin 3A$

(ii) $\cos A\cos(60°-A)\cos(60°+A)=\frac{1}{4}\cos 3A$

(iii) $\tan A\tan(60°-A)\tan(60°+A)=\tan 3A$

Example 14 :

Find the value of $\cos 52°+\cos 68°+\cos 172°$

Sol. $\cos 52°+\cos 68°+\cos 172°$

$=\cos 52°+\cos 68°+\cos(180°-8°)$

$=\cos 52°+\cos 68°-\cos 8°$

$=2\cos\frac{52°+68°}{2}\cos\frac{68°-52°}{2}-\cos 8°$

$=2\cos 60°\cos 8°-\cos 8°=\cos 8°-\cos 8°=0$

Example 15 :

If $\sin 2\theta+\sin 2\phi=1/2$, $\cos 2\theta+\cos 2\phi=3/2$ then find the value of $\cos^2(\theta-\phi)$.

Sol. Using cosine formula

$2\sin(\theta+\phi)\cos(\theta-\phi)=1/2$ .....(1)

$2\cos(\theta+\phi)\cos(\theta-\phi)=3/2$ .....(2)

Squaring (1) and (2) and then adding

$4\cos^2(\theta-\phi)=\frac{1}{4}+\frac{9}{4}=\frac{5}{2}\Rightarrow\cos^2(\theta-\phi)=\frac{5}{8}$

Example 16 :

Find : $\sin 47°+\sin 61°-\sin 11°-\sin 25°$.

Sol. Given value

$=(\sin 47°+\sin 61°)-(\sin 11°+\sin 25°)$

$=2\sin 54°\cos 7°-2\sin 18°\cos 7°$

$=2\cos 7°(\sin 54°-\sin 18°)$

$= 2 \cos 7° \, 2 \cos 36° \sin 18°$

$= 2\cos 7° \dfrac{2\sin 18° \cos 18°}{\cos 18°} \times \cos 36°$

$= \cos 7° \dfrac{2\sin 36° \cos 36°}{\cos 18°}$

$= \cos 7° \dfrac{\sin 72°}{\cos 18°} = \cos 7°$ $\quad [\because \sin 72° = \cos 18°]$

TRIGONOMETRICAL RATIOS OF MULTIPLE ANGLES

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(iv) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(v) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(vi) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(vii) $\sin\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}}$

(viii) $\cos\dfrac{\theta}{2} = \sqrt{\dfrac{1+\cos\theta}{2}}$

(ix) $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

Example 17 :

If $\sin A = \dfrac{1}{2}$, then find $4\cos^3 A - 3\cos A$ $(0° < A < 90°)$

Sol. $4\cos^3 A - 3\cos A = \cos 3A = \cos 90° = 0$

$[\because \sin A = \dfrac{1}{2} \Rightarrow A = 30°]$

Example 18 :

If α and β be between 0 and $\dfrac{\pi}{2}$ and if $\cos(\alpha+\beta) = \dfrac{12}{13}$ and

$\sin(\alpha-\beta) = \dfrac{3}{5}$, then find the value of $\sin 2\alpha$.

Sol. $\sin(2\alpha) = \sin(\alpha+\beta+\alpha-\beta)$

$= \sin(\alpha+\beta)\cos(\alpha-\beta) + \cos(\alpha+\beta)\sin(\alpha-\beta)$

$= \dfrac{5}{13}.\dfrac{4}{5} + \dfrac{12}{13}.\dfrac{3}{5} = \dfrac{56}{65}$

TRIGONOMETRIC FUNCTIONS OF AN ANGLE OF 18°

Let θ stands for 18°, so that 2θ is 36° and 3θ is 54°.

Hence, $2\theta = 90° - 3\theta$, and therefore,

$\sin 2\theta = \sin(90° - 3\theta) = \cos 3\theta$

$\therefore$ $2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

Hence, either $\cos\theta = 0$, which gives $\theta = 90°$, or

$2\sin\theta = 4\cos^2\theta - 3 = 1 - 4\sin^2\theta$

$\therefore$ $4\sin^2\theta + 2\sin\theta = 1$

By solving quadratic equation, we have

$\sin\theta = \dfrac{\sqrt{5}-1}{4} = \sin 18°$

$\cos 18° = \sqrt{1-\sin^2 18°} = \sqrt{1-\dfrac{6-2\sqrt{5}}{16}} = \sqrt{\dfrac{10+2\sqrt{5}}{16}}$

Table : sine, cosine and tangent of some angle less than 90º

	15°	**18°**	**22½º**	**36°**
sin	$\dfrac{\sqrt{3}-1}{2\sqrt{2}}$	$\dfrac{\sqrt{5}-1}{4}$	$\dfrac{1}{2}\sqrt{2-\sqrt{2}}$	$\dfrac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\dfrac{\sqrt{3}+1}{2\sqrt{2}}$	$\dfrac{\sqrt{10+2\sqrt{5}}}{4}$	$\dfrac{1}{2}\sqrt{2+\sqrt{2}}$	$\dfrac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\dfrac{1}{\sqrt{5+2\sqrt{5}}}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

CONDITIONAL TRIGONOMETRICAL IDENTITIES

We have certain trigonometry identities like $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ etc.

Such identities are identities in the sense that they hold for all value of the angles which satisfy the given condition among them and they are called conditional identities.

If A, B, C denote the angle of a triangle ABC, then the relation $A + B + C = \pi$ enables us to establish many important identities involving trigonometric ratios of these angles.

(i) If $A + B + C = \pi$, then $A + B = \pi - C$, $B + C = \pi - A$ and $C + A = \pi - B$

(ii) If $A + B + C = \pi$, then $\sin(A+B) = \sin(\pi - C) = \sin C$
similarly, $\sin(B+C) = \sin(\pi - A) = \sin A$
and $\sin(C+A) = \sin(\pi - B) = \sin B$

(iii) If $A + B + C = \pi$, then $\cos(A+B) = \cos(\pi - C) = -\cos C$
similarly, $\cos(B+C) = \cos(\pi - A) = -\cos A$
and $\cos(C+A) = \cos(\pi - B) = -\cos B$

(iv) If $A + B + C = \pi$, then $\tan(A+B) = \tan(\pi - C) = -\tan C$
similarly, $\tan(B+C) = \tan(\pi - A) = -\tan A$
and $\tan(C+A) = \tan(\pi - B) = -\tan B$

(v) If $A + B + C = \pi$, then $\tan\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$ and

$$\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2} \text{ and } \frac{C+A}{2} = \frac{\pi}{2} - \frac{B}{2}$$

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\left(\frac{C}{2}\right)$$

$$\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\left(\frac{C}{2}\right)$$

$$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right)$$

All problems on conditional identities are broadly divided into the following four types

(i) Identities involving sines and cosines of the multiple or sub-multiples of the angles involved.

(ii) Identities involving squares of sines and cosines of the multiple or sub-multiples of the angles involved.

(iii) Identities involving tangents and cotangents of the multiples or sub-multiples of the angles involved.

(iv) Identities involving cubes and higher powers of sines and cosines and some mixed identities.

Type I :

Identities involving sines and cosines of the multiple or sub-multiple of the angles involved.

Working Method : Step - 1- Use C & D formulae.

Step - 2- Use the given relation $(A + B + C = \pi)$ in the expression obtained in step -1 such that a factor can be taken common after using multiple angles formulae in the remaining term.

Step - 3- Take the common factor outside.

Step - 4- Again use the given relation $(A + B + C = \pi)$ within the bracket in such a manner so that we can apply C & D formulae.

Step -5- Find the result according to the given options.

Type II :

Identities involving squares of sines and cosines of multiple or sub-multiples of the angles involved

Working Method :

Step -1 Arrange the terms of the identity such that either

$$\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B)$$

or $\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$ can be used.

Step - 2 Take the common factor outside.

Step - 3 Use the given relation $(A + B + C = \pi)$ within the bracket in such a manner so that we can apply C & D formulae.

Step - 4 Find the result according to the given options.

Example 19 :

If $A + B + C = \pi$, then find $\cos^2 A + \cos^2 B + \cos^2 C$.

Sol. L.H.S.$= \cos^2 A + \cos^2 B + \cos^2 C$

$= \cos^2 A + (1 - \sin^2 B) + \cos^2 C$

$= (\cos^2 A - \sin^2 B) + \cos^2 C + 1$

$= \cos(A+B)\cos(A-B) + \cos^2 C + 1$

$[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)]$

$= \cos(\pi - C)\cos(A-B) + \cos^2 C + 1$

$= -\cos C\cos(A-B) + \cos^2 C + 1$

$= -\cos C[\cos(A-B) - \cos C] + 1$

$= -\cos C[\cos(A-B) - \cos\{\pi - (A+B)\}] + 1$

$= -\cos C[\cos(A-B) + \cos(A+B)] + 1$

$= -\cos C[(\cos A\cos B + \sin A\sin B)$
$+ (\cos A\cos B - \sin A\sin B)] + 1$

$= -\cos C(2\cos A\cos B) + 1 = 1 - 2\cos A\cos B\cos C$

Type III : Identities for tan and cot of the angles:

Working Method :

Step-1 - Express the sum of the two angles in terms of third angle by using the given relation $(A + B + C = \pi)$.

Step-2 - Taking tangent or cotangent of the angles of both the sides.

Step-3-Use sum and difference formulae in the left hand side.

Step-4 - Use cross multiplication in the expression obtained in the step 3.

Step-5 - Arrange the terms as per the result required.

THE GREATEST AND LEAST VALUE OF THE EXPRESSION [a sinθ + b cosθ] :

Let $a = r\cos\alpha$ ......(i) and $b = r\sin\alpha$ ......(ii)

Squaring and adding (i) and (ii)

then $a^2 + b^2 = r^2$ or, $r = \sqrt{a^2 + b^2}$

$\therefore a\sin\theta + b\cos\theta = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = r\sin(\theta + \alpha)$

But $-1 \le \sin\theta \le 1$ so $-1 \le \sin(\theta + \alpha) \le 1$

then $-r \le r\sin(\theta + \alpha) \le r$

Hence, $-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$

then the greatest and least values of $a\sin\theta + b\cos\theta$ are respectively $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$.

Example 20 :

The value of the expression $\sin\theta + \cos\theta$ lies between

(1) –2 and 2 both inclusive

(2) 0 and $\sqrt{2}$ both inclusive

(3) $-\sqrt{2}$ and $\sqrt{2}$ both inclusive

(4) 0 and 2 both inclusive

Sol. (3). Since $\sin\theta + \cos\theta = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right]$

$$= \sqrt{2}\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right] = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

which lies between $-\sqrt{2}$ and $\sqrt{2}$

$[\because \sin\left(\theta + \frac{\pi}{4}\right)$ lies between – 1 and 1]

Example 21 :

y = cos 2x + 3 sin x. Find range of y.

Sol. $y = 1 - 2\sin^2 x + 3\sin x$

$$= 1 - 2\left[\sin^2 x - \frac{3}{2}\sin x + \frac{9}{16} - \frac{9}{16}\right] = 1 - 2\left[\sin x - \frac{3}{4}\right]^2 + \frac{9}{8}$$

$$y = \frac{17}{8} - 2\left[\sin x - \frac{3}{4}\right]^2 \;;\; y_{max} = \frac{17}{8} \text{ at } \sin x = \frac{3}{4}$$

$y_{min} = -4$ at $\sin x = -1$; $y \in [-4, 17/8]$

SOME USEFUL SERIES

(i) $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots\ldots$ to n terms

$$= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)} \;;\; \beta \neq 2n\pi$$

(ii) $\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots\ldots$ to n terms

$$= \frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)} \;;\; \beta \neq 2n\pi$$

An Increasing product series:

$p = \cos\alpha \cdot \cos 2\alpha \cdot \cos 2^2\alpha \ldots\ldots \cos(2^{n-1}\alpha)$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}, & \text{if } \alpha \neq n\pi \\ 1, & \text{if } \alpha = 2k\pi \\ -1, & \text{if } \alpha = (2k+1)\pi \end{cases}$$

Example 22 :

Find the sum of series

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$

Sol. Here the angles are in A.P. of 5 terms.

$$n = 5,\; \alpha = \frac{\pi}{11},\; \beta = \frac{2\pi}{11} \quad \therefore \frac{\beta}{2} = \frac{\pi}{11}$$

$$S = \frac{\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} \cdot \cos\frac{1}{2}\left[\frac{\pi}{11} + \frac{9\pi}{11}\right]$$

$$S = \frac{2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{\sin\left(\pi - \frac{\pi}{11}\right)}{2\sin\frac{\pi}{11}} = \frac{1}{2}$$

Example 23 :

Find the value of $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$

Sol. $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$

$$= \left(\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\right) \times \cos\frac{3\pi}{9}$$

$$= \frac{\sin(2^3, .\pi/9)}{2^3.\sin\pi/9} \times \cos\frac{\pi}{3} = \frac{\sin 8\pi/9}{8\sin\pi/9} \times \frac{1}{2} = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

TRY IT YOURSELF-1

Q.1 Prove that sin 240° sin 510° – sin 330° cos 390° = 0

Q.2 To prove that
cot 16°. cot 44° + cot 44° . cot 76° – cot 76° . cot 16° = 3
or (cot 16°) (cot 44°) – 1 + (cot 44°.cot 76° – 1)
– (cot 76° . cot 16° + 1) = 0

Q.3 Prove that cos (36° – A) cos (36° + A) + cos (54° + A) cos (54° – A) = cos 2A

Q.4 Prove that $\sin 20° \sin 40° \sin 80° = \frac{\sqrt{3}}{8}$

Q.5 Prove that $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14} = \frac{1}{8}$

Q.6 If A, B, C are the angles of triangle ABC and tan A, tan B, tan C are the roots of the equation $x^4 - 3x^3 + 3x^2 + 2x + 5 = 0$, then find the fourth root of the equation.

Q.7 Find minimum and maximum values of
$y = 1 + \cos x + \cos^2 x + \cos^3 x + \cos^4 x$.

Q.8 $y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$. Find range of y.

Q.9 Compute the square of the value of the expression
$\frac{4 + \sec 20°}{\operatorname{cosec} 20°}$

Q.10 Find the value of tan15°. tan45° tan75°.

ANSWERS

(6) 5/3 **(7)** $y_{max} = 5$; $y_{min} = 1$

(8) $y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ **(9)** 3 **(10)** 1

TRIGONOMETRIC EQUATIONS

An equation involving one or more trigonometrical ratios of unknown angle is known as trigonometrical equation.

Ex. $\cos\theta = \frac{1}{2}$, $\tan\theta = \frac{1}{\sqrt{3}}$ and $\sin\theta = \frac{1}{2}$ etc. are trigonometrical equations.

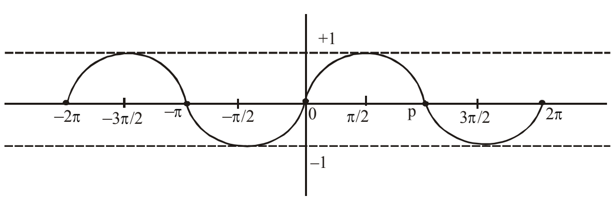
PERIODIC FUNCTION

A function f (x) is said to be periodic if there exists T > 0 such that f (x + T) = f (x) for all x in the domain of definitions of f (x). If T is the smallest positive real numbers such that f (x + T) = f (x), then it is called the period of f (x).
The period of sinx, cosx, secx, cosecx is 2π and period of tan x and cotx is π.

GENERAL SOLUTION OF TRIGONOMETRICAL EQUATIONS

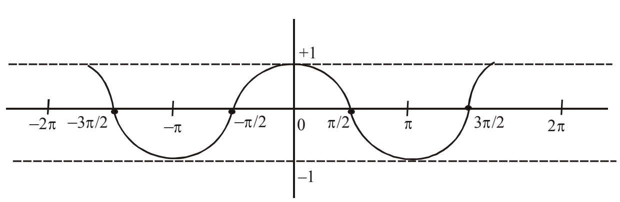
Since Trigonometrical functions are periodic functions, therefore, solutions of Trigonometrical equations can be generalised with the help of periodicity of Trigonometrical functions. The solution consisting of all possible solutions of a Trigonometrical equation is called its general solution.

General Solution of the equation $\sin\theta = 0$:
By Graphical approach,



The above graph of $\sin\theta$ clearly shows that $\sin\theta = 0$ at
$\theta = 0, \pm\pi, \pm 2\pi, \pm 3\pi$.............,
it follows that when $\sin\theta = 0$
$\theta = n\pi : n \in I$ i.e. $n = 0, \pm 1, \pm 2$...........

General solution of the equation $\cos\theta = 0$:
By graphical approach,



The above graph of $\cos\theta$ clearly shows that $\cos\theta = 0$ at
$\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$,..................
It follows that when $\cos\theta = 0$
$\theta = (2n + 1)\pi/2, n \in I$. i.e. $n = 0, \pm 1, +2$..........

Example 24 :

Find the general solution of $\cos\left(\frac{3}{2}\theta\right) = 0$.

Sol. We know that $\cos\theta = 0$, then $\theta = (2n+1)\frac{\pi}{2}; n \in I$

$\therefore \cos\left(\frac{3}{2}\theta\right) = 0 \Rightarrow \frac{3}{2}\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{3}; n \in I$

General solution of the equation $\tan\theta = 0$:
Proof : If $\tan\theta = 0$

or, $\frac{\sin\theta}{\cos\theta} = 0 \Rightarrow \sin\theta = 0$ it follows that general solution of $\tan\theta = 0$ it same as of $\sin\theta = 0$ so that, general solution of $\tan\theta = 0$ is $\theta = n\pi; n \in I$

Note: General solution of $\sec\theta = 0$ and $\text{cosec}\theta = 0$ does not exist because $\sec\theta$ and $\text{cosec}\theta$ can never be equal to 0.

Example 25 :
Find the general solution of $\tan(\theta/2) = 0$.

Sol. We know that $\tan\theta = 0$

$\theta = n\pi; n \in I$ then $\tan(\theta/2) = 0$, so, $\frac{\theta}{2} = n\pi$; $\theta = 2n\pi$

General solution of the equation $\sin\theta = \sin\alpha$:
Proof : If $\sin\theta = \sin\alpha$
or, $\sin\theta - \sin\alpha = 0$

or, $2\sin\left(\frac{\theta-\alpha}{2}\right)\cos\left(\frac{\theta+\alpha}{2}\right) = 0$

or, $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$ or $\cos\left(\frac{\theta+\alpha}{2}\right) = 0$

or, $\frac{\theta-\alpha}{2} = m\pi; m \in I$ and $\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2}; m \in I$

$\theta = 2m\pi + \alpha; m \in I$ and $\theta = (2m + 1)\pi - \alpha; m \in I$
θ = (any even multiple of π) + α and θ
= (any odd multiple of π) − α
$\theta = n\pi + (-1)^n\alpha; n \in I$

General solution of the equation $\sin\theta = k$, where $-1 \le k \le 1$.
Let α be the numerically least angle such that $k = \sin\alpha$
then, $\sin\theta = \sin\alpha$
$\therefore \theta = n\pi + (-1)^n\alpha$, where $n \in I$ and $\alpha = \sin^{-1}k$
Note: The equation $\text{cosec}\theta = \text{cosec}\alpha$ is equivalent to $\sin\theta = \sin\alpha$ so these two equation having the same general solution.

General solution of the equation $\cos\theta = \cos\alpha$:
Proof : If $\cos\theta = \cos\alpha$
or $\cos\theta - \cos\alpha = 0$

or $-2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right)=0$

or, $\sin\left(\frac{\theta+\alpha}{2}\right)=0$ and $\sin\left(\frac{\theta-\alpha}{2}\right)=0$

$\frac{\theta+\alpha}{2}=n\pi;\ n\in I$ and $\frac{\theta-\alpha}{2}=n\pi;\ n\in I$

$\theta=2n\pi-\alpha;\ n\in I$ and $\theta=2n\pi+\alpha;\ n\in I$

For the general solution of $\cos\theta=\cos\alpha$, combine these two result which gives

$\theta=2n\pi\pm\alpha\ n\in I$

General solution of the equation $\cos\theta=k$, where $-1\le k\le 1$.

Let α be the numerically least angle such that $k=\cos\alpha$, then $\cos\theta=\cos\alpha$

$\therefore\ \theta=2n\pi\pm\alpha$, where $n\in I$ and $\alpha=\cos^{-1}k$

Note: The equation $\sec\theta=\sec\alpha$ is equivalent to $\cos\theta=\cos\alpha$, so the general solution of these two equations are same.

Example 26 :

If $\cos\theta=-1/2$ and $0<\theta<360°$, then find the solutions.

Sol. $\cos\theta=-1/2=\cos 120°$ or $\cos 240°$ $[0<\theta<360°]$

$\therefore\ \theta=120°, 240°$

General solution of the equation $\tan\theta=\tan\alpha$:

Proof : If $\tan\theta=\tan\alpha$

or, $\frac{\sin\theta}{\cos\theta}=\frac{\sin\alpha}{\cos\alpha}$ or, $\sin\theta.\cos\alpha-\cos\theta\sin\alpha=0$

or, $\sin(\theta-\alpha)=0$

or, $\theta-\alpha=n\pi;\ n\in I$

$\theta=n\pi+\alpha;\ n\in I$

General solution of the equation $\tan\theta=k$,

Let α be the numerically least angle such that $k=\tan\alpha$, then $\tan\theta=\tan\alpha$

$\therefore\ \theta=n\pi+\alpha$, where $n\in I$ and $\alpha=\tan^{-1}k$

Note : The equation $\cot\theta=\cot\alpha$ is equivalent to $\tan\theta=\tan\alpha$ so these two equations having the same general solution.

Example 27 :

If $\tan\theta=-\frac{1}{\sqrt{3}}$, then find the general solution of the equation,

Sol. $\tan\theta=-\frac{1}{\sqrt{3}}=\tan\left(-\frac{\pi}{6}\right)$ $\therefore\ \theta=n\pi-\frac{\pi}{6}$

GENERAL SOLUTION OF SQUARE OF THE TRIGONOMETRICAL EQUATIONS

General solution of $\sin^2\theta=\sin^2\alpha$

Proof : If $\sin^2\theta=\sin^2\alpha$

or $2\sin^2\theta=2\sin^2\alpha$ (Both the sides multiple by 2)

or $1-\cos2\theta=1-\cos2\alpha$

or $\cos2\theta=\cos2\alpha$

$2\theta=2n\pi\pm2\alpha;\ n\in I$

$\theta=n\pi\pm\alpha;\ n\in I$

General solution of $\cos^2\theta=\cos^2\alpha$

Proof : If $\cos^2\theta=\cos^2\alpha$

or, $2\cos^2\theta=2\cos^2\alpha$ (multiply both the side by 2)

or, $1+\cos2\theta=1+\cos2\alpha$ or, $\cos2\theta=\cos2\alpha$

or, $2\theta=2n\pi\pm2\alpha$

$\theta=n\pi\pm\alpha;\ n\in I$

General solution of $\tan^2\theta=\tan^2\alpha$

Proof : If $\tan^2\theta=\tan^2\alpha$

or $\frac{\tan^2\theta}{1}=\frac{\tan^2\alpha}{1}$ (using compo. and divid. rule)

$\frac{\tan^2\theta+1}{\tan^2\theta-1}=\frac{\tan^2\alpha+1}{\tan^2\alpha-1}$; $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\tan^2\alpha}{1-\tan^2\alpha}$

or $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ or $\cos2\theta=\cos2\alpha$

$\theta=n\pi\pm\alpha;\ n\in I$

Example 28 :

If $2\tan^2\theta=\sec^2\theta$, then find the general value of θ.

Sol. $2\tan^2\theta=\sec^2\theta=1+\tan^2\theta$

$\tan^2\theta=1=(1)^2=\tan^2\frac{\pi}{4}$

$\theta=n\pi\pm\frac{\pi}{4}, n\in I$

EQUATION REDUCIBLE TO THE STANDARD FORM

If equation is not in the standard form then with the help of any trigonometric formula we convert the equation in standard form then we find the general solution.

Example 29 :

Find the solution of $\tan 2\theta\tan\theta=1$.

Sol. $\tan2\theta\tan\theta=1\Rightarrow\frac{2\tan\theta}{1-\tan^2\theta}.\tan\theta=1$

$\Rightarrow 2\tan^2\theta=1-\tan^2\theta\Rightarrow 3\tan^2\theta=1$

$\Rightarrow \tan\theta=\pm\frac{1}{\sqrt{3}}=\tan\left(\pm\frac{\pi}{6}\right)$

$\Rightarrow \theta=n\pi\pm\frac{\pi}{6}\ (m\in Z)=(6n\pm1)\frac{\pi}{6}$

or $\tan2\theta=\cot\theta=\tan\left(\frac{\pi}{2}-\theta\right)$

$\Rightarrow\ 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow 3\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow\ \theta = \frac{n\pi}{3} + \frac{\pi}{6} = (2n+1)\frac{\pi}{6}$

GENERAL SOLUTION OF TRIGONOMETRICAL EQUATION – a cosθ + bsinθ = C :

Consider a trigonometrical equation a $\cos\theta$ + b$\sin\theta$ = c, where a, b, c $\in$ R and $|c| \le \sqrt{a^2+b^2}$. To solve this type of equation, first we reduce them in the form $\cos\theta = \cos\alpha$ or $\sin\theta = \sin\alpha$. Algorithm to solve equation of the form a $\cos\theta$ + b$\sin\theta$ = c.

Step I Obtain the equation a $\cos\theta$ + b$\sin\theta$ = c

Step II Put a = r $\cos\alpha$ and b = r$\sin\alpha$,

where $r = \sqrt{a^2+b^2}$ and $\tan\alpha = \frac{b}{a}$ i.e $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

Step III using the substitution in step–II, the equation reduces r cos $(\theta - \alpha)$ = c

$\Rightarrow \cos(\theta - \alpha) = \frac{c}{r}$

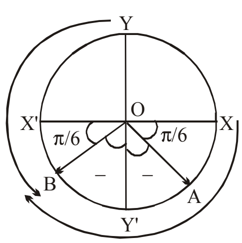
$\Rightarrow \cos(\theta - \alpha) = \cos\beta$ (say)

Step IV : Solve the equation obtained in step III by using the formula.

METHOD FOR FINDING PRINCIPAL VALUE

Suppose we have to find the principal value of θ satisfying the equation $\sin\theta = -1/2$

Since sinθ is negative, θ will be in 3rd or 4th quadrant. We can approach 3rd or 4th quadrant from two directions. If we take anticlockwise direction the numerical value of the angle will be greater than π. If we approach it in clockwise direction the angle will be numerically less than π. For principal value, we have to take numerically smallest angle.



For principal value :

1. If the angle is in 1st or 2nd quadrant we must select anticlockwise direction and if the angle is in 3rd or 4th quadrant, we must select clock wise direction.
2. Principal value is never numerically greater than π.
3. Principal value always lies in the first circle (i.e. in first rotation) On the above criteria, θ will be $-\frac{\pi}{6}$ or $-\frac{5\pi}{6}$.

Among these two $-\frac{\pi}{6}$ has the least numerical value. Hence $-\frac{\pi}{6}$ is the principal value of θ satisfying the equation $\sin\theta = -1/2$.

From the above discussion, the method for finding principal value can be summed up as follows:

1. First draw a Trigonometrical circle and mark the quadrant, in which the angle may lie.
2. Select anticlockwise direction for 1st and 2nd quadrants and select clockwise direction for 3rd and 4th quadrants.
3. Find the angle in the first rotation.
4. Select the numerically least angle among these two values. The angle thus found will be the principal value.
5. In case, two angles one with positive sign and the other with negative sign qualify for the numerically least angle, then it is the convention to select the angle with positive sign as principal value.

SOLUTIONS IN THE CASE OF TWO EQUATIONS ARE GIVEN

Two equation are given and we have to find the values of variable θ which may satisfy both the given equations, like

$\cos\theta = \cos\alpha$ and $\sin\theta = \sin\alpha$

so the common solution is

$\theta = 2n\pi + \alpha,\ n \in I$

Similarly $\sin\theta = \sin\alpha$ and $\tan\theta = \tan\alpha$

so the common solution is, $\theta = 2n\pi + \alpha,\ n \in I$

Rule: Find the common values of θ between 0 and 2π and then add 2πn to this common value.

IMPORTANT POINTS

(a) Solving equations by introducing an Auxiliary argument. To solve equation, we convert the equation to the form $\cos\theta = \cos\alpha$ or $\sin\theta = \sin\alpha$, etc.
Consider the equation:
Example : $\sin x + \cos x = \sqrt{2}$ and $\sqrt{3}\cos x + \sin x = 2$

(b) Solving equations by transforming a sum of Trigonometrical functions into a product. Consider the example :
$\cos 3x + \sin 2x - \sin 4x = 0$

(c) Solving equations by transforming a product of Trigonometrical functions into a sum. Consider the equation:
Example % $\sin 5x.\cos 3x = \sin 6x.\cos 2x$.

(d) Solving equations by a change of variable.

(i) Equations of the form P (sinx ± cosx.sinx.cosx) = 0. Where P(y, z) is a polynomial can be solved by the change.
$\cos x \pm \sin x = t \Rightarrow 1 \pm 2\sin x.\cos x = t^2$
Consider the equation: $\sin x + \cos x = 1 + \sin x.\cos x$

(ii) Many equations can be solved by introducing a new variable. eg. the equation
$\sin^4 2x + \cos^4 2x = \sin 2x.\cos 2x$ changes to
$2(y+1)(y-1/2) = 0$ by substituting $\sin 2x.\cos 2x = y$

(e) Many Trigonometrical equations can be solved by different methods. The form of solution obtained in different methods may be different. From these different forms of solutions, the students should not think that the answer obtained by one method are wrong and those obtained by another method are correct. The solutions obtained by different methods may be shown to be equivalent by some supplementary transformations.
To test the equivalence of two solutions obtained from two methods, the simplest way is to put values of
n = ...–2, –1, – 0, 1, 2, 3..........etc and then to find the angles in [0, 2π]. If all the angles in both solutions are same, the solutions are equivalent.

(f) While manipulating the Trigonometrical equation, avoid the danger of losing roots. Generally, some roots are lost by cancelling a common factor from the two sides of an equation. For example, suppose we have the equation tanx = 2sinx. Here by dividing both sides by sinx, we get cosx = 1/2. This is not equivalent to the original equation. Here the roots obtained by sinx = 0, are lost. Thus in place of dividing an equation by a common factor, the students are advised to take this factor out as a common factor from all terms of the equation.

(g) While equating one of the factors to zero, take care of the other factor that it should not become infinite. For example, if we have the equation sinx = 0, which can be written as cosx tanx = 0. Here we cannot put cosx = 0, since for cosx = 0, tanx = sinx/cosx is infinite.

(h) **Avoid squaring:** When we square both sides of an equation, some extraneous roots appear. Hence it is necessary to check all the solutions found by substituting them in the given equation and omit the solutions not satisfying the given equation.
For example: Consider the equation, $\sin\theta + \cos\theta = 1$.....(1)
Squaring we get $1 + \sin 2\theta = 1$ or $\sin 2\theta = 0$ .....(2)
i.e. $2\theta = n\pi$ or $\theta = n\pi/2$
This gives $\theta = 0, \pi/2, \pi, 3\pi/2$..........
Verification shows that π and 3π/2 do not satisfy the equation as $\sin\pi + \cos\pi = -1, \neq 1$
and $\sin 3\pi/2 + \cos 3\pi/2 = -1, \neq 1$.
The reason for this is simple.
The equation (2) is not equivalent to (1) and (2) contains two equations: $\sin\theta + \cos\theta = 1$
and $\sin\theta + \cos\theta = -1$. Therefore we get extra solutions.
Thus, if squaring is must, verify each of the solution.

(i) **Some necessary Restrictions**
If the equation involves tanx, secx, take cosx ≠ 0. If cotx or cosecx appear, take sinx ≠ 0.
If log appear in the equation, i.e. log [f(θ)] appear in the equation, use f(θ) > 0 and base of log > 0, ≠1
Also not that $\sqrt{[f(\theta)]}$ is always positive, for example
$\sqrt{\sin^2\theta} = |\sin\theta|$, not $\pm \sin\theta$.

(j) **Verification:** Student are advice to check whether all the roots obtained by them satisfy the equation and lie in the domain of the variable of the given equation.

Example 30 :

If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = 1/\sqrt{3}$ then find θ

(1) 2nπ + π/6 (2) 2nπ +11π/6
(3) 2nπ +7π/6 (4) 2nπ + π/4

Sol. **(3).** We shall first consider values of θ between 0 and 2π

$$\sin\theta = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right)$$

or $\sin(2\pi - \pi/6)$ ∴ $\theta = 7\pi/6\ ;\ 11\pi/6$

$$\tan\theta = 1/\sqrt{3} = \tan(\pi/6) = \tan(\pi + \pi/6)$$

∴ θ = π/6 , 7π/6
Hence the value of θ between 0 and 2π which satisfies both the equations is 7π/6
Hence the general value of θ is 2nπ + 7π/6 where n ∈ I

Example 31 :
If sin 5x + sin 3x + sinx = 0 and 0 ≤ x ≤ π/2, then find the value of x.

Sol. $\sin 5x + \sin x = -\sin 3x \Rightarrow 2\sin 3x\cos 2x + \sin 3x = 0$
$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$
$\Rightarrow \sin 3x = 0, \cos 2x = -1/2$
$\Rightarrow x = n\pi, x = n\pi \pm (\pi/3)$ So $x = \pi/3$

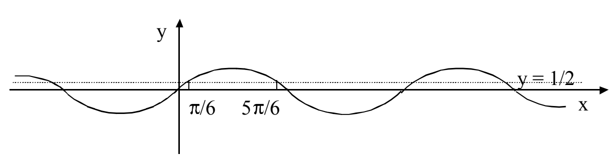
TRIGONOMETRIC INEQUALITIES

Evaluating trigonometric ratios is a direct process in which we make use of known values, trigonometric identities and transformations or even pre-defined trigonometric tables. The evaluation of trigonometric inequalities is somewhat inverse of this process.
At solving of trigonometric inequalities we use the properties of inequalities, known from algebra and also the trigonometric transformations and formulas.
To solve trigonometric inequation of the type f (x) ≤ a, or f (x) ≥ a where f(x) is some trigonometric ratio we take following steps.

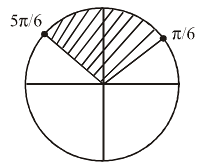
(i) Draw the graph of f(x) in a interval length equal to fundamental period of f (x).
(ii) Draw the line y = a.
(iii) Take the portion of the graph for which inequation is satisfied.
(iv) To generalise add pn (n ∈ I) and take union over set of integers , where p is fundamental period of f(x).

Example 32 :
Find the solution set of the inequation sin x > 1/2.

Sol. **Method I :** When sin x = 1/2, the two values of x between 0 and 2π are π/6 and 5π/6.



From, the graph of y = sin x, it is obvious that, between 0 & 2π



$\sin x > 1/2$ for $\frac{\pi}{6} < x < \frac{5\pi}{6}$. Hence $\sin x > 1/2$

$\Rightarrow 2n\pi + \pi/6 < x < 2n\pi + 5\pi/6$

The required solution set = $\bigcup\limits_{n \in I}\left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$

Method II : $\frac{\pi}{6} + 2\pi n < x < \frac{5\pi}{6} + 2\pi n$

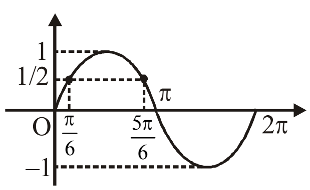
Example 33 :

Solve : $2\sin^2\theta - \sin\theta \geq 0$, where $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$

Sol. $\sin\theta\,(2\sin\theta - 1) \geq 0$ which is possible only where

$\sin\theta \geq \frac{1}{2}$ or $\sin\theta \leq 0$

$\sin\theta \geq \frac{1}{2} \Rightarrow \frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}$



$\sin\theta \leq 0 \Rightarrow \pi \leq \theta \leq \frac{3\pi}{2}$; $\theta \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

SEVEN WONDERS OF THE WORLD

The following list of inequalities comprise the seven wonders of the world based on major theme.

$$\frac{f(x_1) + f(x_2) + f(x_3)}{3} \leq f\left(\frac{x_1 + x_2 + x_3}{3}\right)$$

If $\alpha + \beta + \gamma = 180°$

W1 : $\sin\alpha + \sin\beta + \sin\gamma \leq \frac{3\sqrt{3}}{2}$

W2 : $\operatorname{cosec}\alpha + \operatorname{cosec}\beta + \operatorname{cosec}\gamma \geq 2\sqrt{3}$

W3 : $1 < \cos\alpha + \cos\beta + \cos\gamma \leq \frac{3}{2}$

W4 : $\cot\alpha\cot\beta\cot\gamma \leq \frac{\sqrt{3}}{9}$

W5 : $\cot\alpha + \cot\beta + \cot\gamma \geq \sqrt{3}$

W6 : $\sin^2\alpha + \sin^2\beta + \sin^2\gamma \leq \frac{9}{4}$

W 7 : $\cot^2\alpha + \cot^2\beta + \cot^2\gamma \geq 1$

Example 34 :

Find the least value of sec A + sec B + sec C in acute angle triangle.

Sol. In an acute angle triangle sec A, sec B, and sec C are positive.

Now, A.M. ≥ H.M.

$\Rightarrow \frac{\sec A + \sec B + \sec C}{3} \geq \frac{3}{\cos A + \cos B + \cos C}$

But in Δ ABC, $\cos A + \cos B + \cos C \leq 3/2$

$\Rightarrow \frac{\sec A + \sec B + \sec C}{3} \geq 2 \Rightarrow \sec A + \sec B + \sec C \geq 6$

TRY IT YOURSELF-2

Q.1 Solve : $\sqrt{3}\sec 2\theta = 2$

Q.2 Solve : $7\cos^2\theta + 3\sin^2\theta = 4$

Q.3 Find the general solutions of equation $1 - \cos x \cot x = \cot x - \cos x$.

Q.4 Find the general solutions of equation $2\sin^2x + 6\sin^2x = 5$

Q.5 Find the general solutions of equation $\sqrt{3}\cos x + \sin x = 2$.

Q.6 General solution of sin x + sin 5x = sin 2x + sin 4x is –

(A) $n\pi/3$ (B) $2n\pi/3$
(C) $2n\pi$ (D) $n\pi$

Q.7 Find the number of solution of the equation in $[0, 2\pi]$, $\tan(5\pi\cos\alpha) = \cot(5\pi\sin\alpha)$

Q.8 Find general value of x satisfying the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$.

Q.9 Solve : $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

Q.10 If $3\tan^2\theta - 2\sin\theta = 0$, then θ is equal to –

(A) $\frac{n\pi}{4}$ (B) $n\pi + (-1)^n\frac{\pi}{6}$
(C) $n\pi + (-1)^n\frac{\pi}{3}$ (D) $n\pi + \frac{\pi}{3}$

Q.11 If $\sin x + \cos x = \sqrt{\left(y + \frac{1}{y}\right)}$, $x \in [0, \pi]$ then

(A) $x = \pi/4$. $y = 1$ (B) $y = 0$
(C) $y = 2$ (D) $x = 3\pi/4$

Q.12 If $\tan\left(\frac{p\pi}{4}\right) = \cot\left(\frac{q\pi}{4}\right)$ and $n \in I$, then –

(A) $p + q = 0$ (B) $p + q = 2n + 1$
(C) $p + q = 2n$ (D) $p + q = 2(2n + 1)$

Q.13 Find the general solution of the equation $\sin^{100}x - \cos^{100}x = 1$.

Q.14 Find the set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$.

ANSWERS

(1) $\theta = n\pi \pm \frac{\pi}{12},\ n \in I$ **(2)** $\theta = n\pi \pm \frac{\pi}{3},\ n \in I$

(3) $(2n+1)\pi,\ \ n\pi + \frac{\pi}{4},\ n \in I$ **(4)** $n\pi \pm \frac{\pi}{4}, n \in I$

(5) $x = 2n\pi + \frac{\pi}{6},\ n \in I$ **(6)** (A)

(7) 28 **(8)** $x = (4n+1)\frac{\pi}{8}$

(9) $\theta = \frac{n\pi}{3} + \frac{\pi}{9}$ **(10)** (B) **(11)** (A)

(12) (D) **(13)** $x = n\pi \pm \frac{\pi}{2}$

(14) $x \in \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$

PROPERTIES & SOLUTIONS OF TRIANGLE

A triangle has three sides and three angles. In this section we shall find the relation between the sides and trigonometrical ratios of angles of a triangle.

We shall denote the angle BAC, CBA and ACB by A, B, C and the corresponding sides opposite to them by a, b and c respectively. These six elements of a triangle are connected by the following relations:

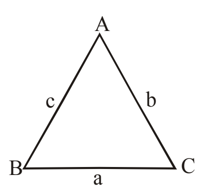
(i) $A + B + C = 180^\circ$ or π (ii) $a + b > c,\ b + c > a,\ c + a > b$

(iii) $a > 0,\ b > 0,\ c > 0$

SINE RULE

The sides of a triangle (any type of triangle) are proportional to the sines of the angle opposite to them. In triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Note : (i) The above rule may also be expressed as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(ii) The sine rule is very useful tool to express sides of a triangle in terms of sines of angle and vice-versa in the following manner.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\ (\text{Let})$$

$\Rightarrow a = k\sin A,\ b = k\sin B,\ c = k\sin C$

Similarly, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \lambda\ (\text{Let})$

$\Rightarrow \sin A = \lambda a,\ \sin B = \lambda b,\ \sin C = \lambda c$

Example 35 :

In a triangle ABC, if $a = 3$, $b = 4$ and $\sin A = \frac{3}{4}$, then find $\angle B$

Sol. We have, $\frac{\sin A}{a} = \frac{\sin B}{b}$ or, $\sin B = \frac{b}{a}\sin A$

Since, $a = 3,\ b = 4,\ \sin A = \frac{3}{4}$, we get, $\sin B = \frac{4}{3} \times \frac{3}{4} = 1$

$\therefore \angle B = 90^\circ$

Example 36 :

If $\sin^2 A + \sin^2 B = \sin^2 C$, the Δ is

(1) acute angled (2) right angled

(3) obtuse angled (4) None of these

Sol. **(2).** $\sin^2 A + \sin^2 B = \sin^2 C$ (given)

$$\Rightarrow \frac{a^2}{K^2} + \frac{b^2}{K^2} = \frac{c^2}{K^2}$$

$$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \Rightarrow \sin A = \frac{a}{k} \text{etc.}\right]$$

$\Rightarrow a^2 + b^2 = c^2$

$\Rightarrow \Delta$ is right angled at C.

COSINE RULE

In any triangle ABC

(i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (ii) $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

(iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Example 37 :

In a ΔABC, $a = 2$ cm, $b = 3$ cm, $c = 4$ cm then find the value of cos A.

Sol. By the cosine rule,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}\ ;\ \cos A = \frac{3^2 + 4^2 - 2^2}{2\,(3)\,(4)}$$

$$\cos A = \frac{21}{24}\ ;\ \cos A = \frac{7}{8}$$

Example 38 :

Find the smallest angle of the triangle whose sides are $6+\sqrt{12}, \sqrt{48}, \sqrt{24}$.

Sol. Let $a = 6 + \sqrt{12}$, $b = \sqrt{48}$, $c = \sqrt{24}$

Here c is the smallest side.

$\angle C$ is the smallest angle of the triangle.

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(48+24\sqrt{3})+48-24}{4(3+\sqrt{3}).\ 4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

so, $\angle C = \pi/6$

PROJECTION FORMULAE

In any ΔABC ;

(i) $a = b \cos C + c \cos B$

(ii) $b = c \cos A + a \cos C$

(iii) $c = a \cos B + b \cos A$

i.e. any side of a triangle is equal to the sum of the projection of other two sides on it.

Example 39 :

In any Δ ABC find $2\left[a\sin^2\left(\frac{C}{2}\right)+c\sin^2\left(\frac{A}{2}\right)\right]$

Sol. $2\left[a\sin^2\left(\frac{C}{2}\right)+c\sin^2\left(\frac{A}{2}\right)\right]$

$\Rightarrow$ [a (1 – cos C) + c (1 – cos A)]

$\Rightarrow$ a + c – (a cos C + c cos A)

$\Rightarrow$ a + c – b [By projection formulae]

Example 40 :

In any triangle ABC,

(b + c) cos A + (c + a) cos B + (a + b) cos C equals

(1) 0 (2) a + b + c

(3) a + b – c (4) a – b – c

Sol. **(2).** (b cos A + c cos A) + (c cos B + a cos B) + (a cos C + b cos C)

= (b cos A + a cos B) + (c cos A + a cos C) + (c cos B + b cos C)

= c + b + a = a + b + c

NAPIER'S ANALOGY (TANGENT RULE)

In any Δ ABC

(i) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\tan\frac{B+C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

(ii) $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$

(iii) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

Example 41 :

In a Δ ABC, $b = \sqrt{3}+1$, $c = \sqrt{3}-1$, $\angle A = 60°$ then find the value of $\tan\left(\frac{B-C}{2}\right)$

Sol. $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\left(\frac{A}{2}\right)$

Putting the value of b, c and $\angle A$

$$\tan\left(\frac{B-C}{2}\right) = \frac{(\sqrt{3}+1)-(\sqrt{3}-1)}{(\sqrt{3}+1)+(\sqrt{3}-1)}\cot(30°) = 1$$

Example 42 :

If $\tan\left(\frac{B-C}{2}\right) = x\cot\left(\frac{A}{2}\right)$, find the value of x.

Sol. By the formulae

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$$

$$\therefore\quad x = \frac{b-c}{b+c}$$

HALF ANGLE FORMULAE FOR TRIGONOMETRIC RATIOS

If the perimeter of a triangle ABC is denoted by 2s then 2s = a + b + c and area denoted by Δ then.

Formulae for $\sin\left(\frac{A}{2}\right)$, $\sin\left(\frac{B}{2}\right)$, $\sin\left(\frac{C}{2}\right)$

In any Δ ABC

(i) $\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(ii) $\sin\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{ac}}$

(iii) $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{ab}}$

Example 43 :

In a Δ ABC, if a = 13, b = 14 and c = 15, then find the value of sin (A/2).

Sol. we know that, 2s = a + b + c

2s = 42

s = 21

s – a = 8, s – b = 7, and s – c = 6

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{7\times 6}{14\times 15}} = \frac{1}{\sqrt{5}}$$

Formulae for $\cos\left(\frac{A}{2}\right)$, $\cos\left(\frac{B}{2}\right)$, $\cos\left(\frac{C}{2}\right)$

In any Δ ABC

(i) $\cos\left(\frac{A}{2}\right) = \sqrt{\frac{s(s-a)}{bc}}$ (ii) $\cos\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{ac}}$

(iii) $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{ab}}$

Example 44 :

In a triangle ABC, if $\cos\frac{A}{2} = \sqrt{\frac{b+c}{2c}}$, then

(1) $a^2 + b^2 = c^2$ (2) $b^2 + c^2 = a^2$

(3) $c^2 + a^2 = b^2$ (4) $b - c = c - a$

Sol. (1). $\cos\frac{A}{2} = \sqrt{\frac{b+c}{2c}} \Rightarrow \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{b+c}{2c}}$

$\Rightarrow 2s(s-a) = b^2 + bc$

$\Rightarrow (a+b+c)(b+c-a) = 2b^2 + 2bc$

$\Rightarrow a^2 + b^2 = c^2$

Formulae for $\tan\left(\frac{A}{2}\right)$, $\tan\left(\frac{B}{2}\right)$, $\tan\left(\frac{C}{2}\right)$

In any Δ ABC

(i) $\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (ii) $\tan\left(\frac{B}{2}\right) = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

(iii) $\tan\left(\frac{C}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Example 45 :

In a Δ ABC, the sides a, b and c are in A. P.

Then find $\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right) : \cot\frac{B}{2}$.

Sol. $\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right) : \cot\frac{B}{2}$

$= \left[\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}\right] : \sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$

$= \frac{(s-c)+(s-c)}{\sqrt{s}} : \sqrt{s} = 2s - (a+c) : s \Rightarrow b : \frac{a+b+c}{2}$

$\Rightarrow 2b : a+b+c = 2b : 3b$ [$\because$ a, b, c are in A.P. $\therefore 2b = a + c$]

$= 2 : 3$

AREA OF TRIANGLE

In a triangle ABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{2\Delta}{abc}$

The area of Δ ABC is given by

(i) $\Delta = \frac{1}{2}bc\sin A$ (ii) $\Delta = \frac{1}{2}ca\sin B$ (iii) $\Delta = \frac{1}{2}ab\sin C$

Hero's Formula :

In any ΔABC

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where

$s = \frac{a+b+c}{2}$ = semi perimeter of Δ

Example 46 :

Find the area of a triangle ABC in which $\angle A = 60°$, b = 4 cm and $c = \sqrt{3}$ cm.

Sol. The area of triangle ABC is given by

$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2} \times 4\sqrt{3} \times \sin 60° = 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 3$ sq. cm.

Example 47 :

In any triangle ABC, if $a = \sqrt{2}$ cm, $b = \sqrt{3}$ cm and $c = \sqrt{5}$ cm, show that its area is $\frac{1}{2}\sqrt{6}$ sq. cm.

Sol. We know that, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = 0$

then, $\angle C = \frac{\pi}{2}$. So, $\Delta = \frac{1}{2}ab\sin C$ [$\because \sin C = 1$]

$\Delta = \frac{1}{2} \times (\sqrt{2})(\sqrt{3})(1)$; $\Delta = \frac{\sqrt{6}}{2}$ sq. cm

SOLUTION OF TRIANGLES

In a triangle, there are six elements three sides and three angles. In plane geometry if three of the elements are given at least one of which must be side, then the other three elements can be uniquely determined. The procedure of determining unknown elements from the known elements is called solving a triangle.

Solution of a right angled triangle :

Case - I : When two sides are given :

Let the triangle be right angled at C. Then we can determine the remaining elements as given in the table.

Given	Required
(i) a, b	$\tan A = \frac{a}{b}$, $B = 90° - A$, $c = \frac{a}{\sin A}$
(ii) a, c	$\sin A = \frac{a}{c}$, $b = c\cos A$, $B = 90° - A$

Case - II : When a side and an acute angle are given :

In this case, we can determine the remaining elements as given in the table.

Given	Required
(i) a, A	$B = 90° - A,\ b = a\cot A,\ c = \dfrac{a}{\sin A}$
(ii) c, A	$B = 90° - A,\ a = c\sin A,\ b = c\cos A$

Example 48 :

Solve the triangle, where $\angle C = 90°$, $\angle A = 30°$ and $c = 10$.

Sol. Since $\angle A + \angle B + \angle C = 180°$

$\therefore\ \angle B = 180° - (90° + 30°) = 60°$

$$\text{Now } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore\ \frac{a}{\sin 30°} = \frac{10}{\sin 90°} = \frac{b}{\sin 60°}$$

$$\therefore\ a = \frac{10\times 1}{2} = 5 \text{ and } b = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$$

Thus $\angle B = 60°$, $b = 5\sqrt{3}$ and $a = 5$

Solution of oblique Triangles :

The triangle which are not right angled are known as the oblique triangles. The problems on solving an oblique triangle are divided in the following categories.

Case - I : When three sides a, b, c are given :

In this case, the remaining elements are determined by using the following formulae.

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $2s = a + b + c$

$$\sin A = \frac{2\Delta}{bc},\ \sin B = \frac{2\Delta}{ac},\ \sin C = \frac{2\Delta}{ab}$$

$$\text{and } \tan\left(\frac{A}{2}\right) = \frac{\Delta}{s(s-a)},\ \tan\left(\frac{B}{2}\right) = \frac{\Delta}{s(s-b)},$$

$$\tan\left(\frac{C}{2}\right) = \frac{\Delta}{s(s-c)}$$

Example 49 :

Solve the triangle ABC, in which $a = 2$, $b = \sqrt{6}$ & $c = \sqrt{3} - 1$.

Sol. By cosine formula,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(\sqrt{6})^2 + (\sqrt{3}-1)^2 - 2^2}{2\sqrt{6}\,(\sqrt{3}-1)}$$

$$= \frac{6 + 3 + 1 - 2\sqrt{3} - 4}{2\sqrt{6}\,(\sqrt{3}-1)}$$

$$= \frac{6 - 2\sqrt{3}}{2\sqrt{6}\,(\sqrt{3}-1)} = \frac{2\sqrt{3}(\sqrt{3}-1)}{2\sqrt{6}\,(\sqrt{3}-1)} = \frac{1}{\sqrt{2}} \quad \therefore A = 45°$$

$$\cos B = \frac{b^2 + a^2 - c^2}{2ca} = \frac{(\sqrt{3}-1)^2 + 2^2 - (\sqrt{6})^2}{2.2(\sqrt{3}-1)}$$

$$= \frac{3 + 1 - 2\sqrt{3} + 4 - 6}{4(\sqrt{3}-1)} = \frac{2 - 2\sqrt{3}}{4(\sqrt{3}-1)}$$

$$= \frac{-2(\sqrt{3}-1)}{4(\sqrt{3}-1)} = -\frac{1}{2} \qquad \therefore B = 120°$$

Since $A + B + C = 180°$

$\therefore\ 45° + 120° + C = 180° \quad \therefore\ C = 180° - 165° = 15°$

Case - II :

When two sides a, b and the included angle C are given :

In this case, we use the following formulae.

$$\Delta = \frac{1}{2}ab\sin C,\ \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$\frac{A+B}{2} = 90° - \frac{C}{2} \text{ and } c = \frac{a\sin C}{\sin A}$$

Example 50 :

If $a = \sqrt{3} + 1$, $b = \sqrt{3} - 1$ and $\angle C = 60°$, find the other side and angles.

Sol. Here $a = \sqrt{3} + 1$, $b = \sqrt{3} - 1$ and $a > b$.

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$= \frac{(\sqrt{3}+1) - (\sqrt{3}-1)}{(\sqrt{3}+1) + (\sqrt{3}-1)}\cot 30° = \frac{2}{2\sqrt{3}}\cdot\sqrt{3} = 1$$

$$\therefore\ \frac{A-B}{2} = 45° \Rightarrow A - B = 90°$$

Also $A + B = 180° - C = 180° - 60° = 120°$

$\therefore\ 2A = 210° \Rightarrow A = 105°$

$\therefore\ B = 120° - 105° = 15°$

Also, $c^2 = a^2 + b^2 - 2ab\cos C$

$= (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1)\cos 60°$

$= 3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3} - 2(3-1) - \dfrac{1}{2}$

$= 8 - 2 = 6 \qquad \therefore c = \sqrt{6}$

Case - III : When one side a and two angles A & B are given

In this case, we use the following formulae to determine the remaining elements.

$A + B + C = 180°$ or $C = 180° - (A + B)$

$$b = \frac{a\sin B}{\sin A} \quad \text{and} \quad c = \frac{a\sin C}{\sin A}$$

$$\Delta = \frac{1}{2}ca\sin B$$

Example 51 :

In a triangle ABC, if a = 2, B = 60° and C = 75°, then find the value of b.

Sol. A = 180° – 60° – 75° = 180° – 135° = 45°

$$\text{Now, } \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin 45°} = \frac{b}{\sin 60°}$$

$$\Rightarrow b = \frac{2.\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \sqrt{6}$$

Case - IV : When all the three angles are given :

In this case unique solution of triangle is not possible. In this case only the ratio of the sides can be determined. For this the formula.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{can be used}$$

Example 52 :

The angles of a triangle are in ratio 2 : 3 : 7, find the ratio of its sides

Sol. Let A = 2k, B = 3k, C = 7k

Then $\because$ A + B + C = 180°

$\therefore$ 12 k = 180° $\therefore$ k = 15°

$\therefore$ A = 30°, B = 45° and C = 105°

$$\text{Now, } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \frac{a}{\sin 30°} = \frac{b}{\sin 45°} = \frac{c}{105°}$$

$$\text{or, } \frac{a}{\frac{1}{2}} = \frac{b}{\frac{1}{\sqrt{2}}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \quad \text{or, } \frac{a}{\frac{2\sqrt{2}}{2}} = \frac{b}{\frac{2\sqrt{2}}{\sqrt{2}}} = \frac{c}{(\sqrt{3}+1)}$$

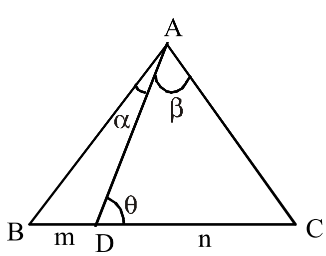
$\therefore$ a : b : c = $\sqrt{2}$: 2 : $\sqrt{3}$ + 1

m-n THEOREM

In any triangle ABC, if BD : DC = m : n and ∠ BAD = α, ∠ CAD = β and ∠ ADC = θ, then

(i) (m + n) cot θ = m cot α – n cot β

(ii) (m + n) cot θ = n cot B – m cot C



Example 53 :

In a triangle ABC, ∠ ABC = 45°. Point D is on BC so that 2BD = CD and ∠ DAB = 15°. ∠ ACB in degree equals.

(A) 30°   (B) 60°

(C) 75°   (D) 90°

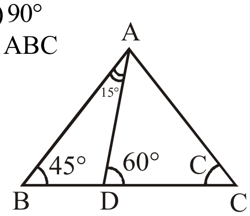
(11) **(C).** Applying m-n theorem, in Δ ABC

(BD + DC) cot 60°

= CD cot 45° – BD cot C

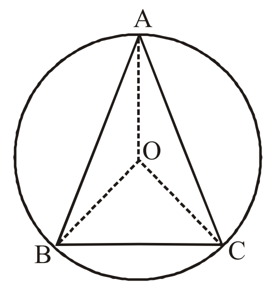
$\Rightarrow$ 3 cot 60° = 2 cot 45° – cot C

$\Rightarrow$ cot C = 2 – $\sqrt{3}$ $\Rightarrow$ C = 75°



CIRCUMCIRCLE OF A TRIANGLE AND ITS RADIUS

The circle which passes through the angular points of a triangle is called its circumcircle. In a triangle the point of interaction of perpendicular bisector of the sides and is called the circumcentre. Its radius is always denoted by R.



The circumcentre may lie within, outside or upon one of the sides of the triangle. In a right angled triangle the circumcentre is the midpoint of the hypotenuse.

In a triangle ABC circumradius is given by

(i) $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

(ii) $R = \frac{abc}{4\Delta}$

(iii) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$

Example 54 :

If any Δ ABC , a cosA + b cos B + c cos C equals

(1) 4 R cos A sin B sin C   (2) 4 R sin A cos B sin C

(3) 4 R sin A sin B sin C   (4) None of these

Sol. **(3).** a cos A + b cos B + c cos C

= 2R sin A cos A + 2R sin B cos B + 2R sin C cosC

[$\because$ a = 2R sin A, b = 2R sin B, c = 2 R sin C]

= R (sin 2A + sin 2B + sin 2C)

= R [2 sin (A + B) cos (A – B) + 2 sin C cos C]

= R [2 sin C cos (A – B) + 2 sin C cos C]

= 2 R sin C [cos (A – B) + cos C]

= 2R sin C [cos (A – B) + cos (π – (A + B))]

= 2R sin C [cos (A – B) – cos (A + B)]

= 2 R sin C [2 sin A sin B] = 4R sin A sin B sin C

Example 55 :

Find the diameter of the circumcircle of a triangle with sides 5 cm, 6 cm and 7 cm.

Sol. Radius of circumcircle is given by $R = \frac{abc}{4\Delta}$ and

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}$$

Here a = 5 cm, b = 6 cm, and c = 7 cm

$$\therefore \quad s = \frac{5+6+7}{2} = 9$$

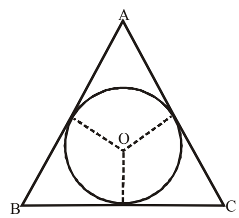
$$\Delta = \sqrt{9(9-5)(9-6)(9-7)} = \sqrt{216} = 6\sqrt{6}$$

$$\Rightarrow \quad R = \frac{5.6.7}{4.6.\sqrt{6}} = \frac{35}{4\sqrt{6}} \; ; \; \text{Diameter} = 2R = \frac{35}{2\sqrt{6}}$$

INCIRCLE OF A TRIANGLE AND ITS RADIUS

Incircle or Inscribed circle:

The circle which can be inscribed with in a triangle and touch each of the sides is called its inscribed circle or incircle. The centre of this circle is the point of intersection of the bisector of the angle of the triangle.



The radius of this circle is always denoted by r and is equal to the length of the perpendicular from its centre to any one of the sides of triangle.

In-Radius: The radius r of the inscribed circle of a triangle ABC is given by–

(i) $r = \frac{\Delta}{s}$

(ii) $r = (s-a)\tan\left(\frac{A}{2}\right)$, $r = (s-b)\tan\left(\frac{B}{2}\right)$

and $r = (s-c)\tan\left(\frac{C}{2}\right)$

(iii) $r = \dfrac{a\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)}{\cos\left(\frac{A}{2}\right)}$, $r = \dfrac{b\sin\left(\frac{A}{2}\right).\sin\left(\frac{C}{2}\right)}{\cos\left(\frac{B}{2}\right)}$

and $r = \dfrac{c\sin\left(\frac{B}{2}\right).\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{C}{2}\right)}$

(iv) $r = 4R\sin\left(\frac{A}{2}\right).\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)$

Example 56 :

Find the ratio of the circumradius and inradius of an equilateral triangle.

Sol. $\frac{r}{R} = \frac{a\cos A + b\cos B + c\cos C}{a+b+c}$

In equilateral triangle A = B = C = 60°

$$= \frac{(a+b+c)\cos 60°}{a+b+c} = \frac{1}{2}$$

Example 57 :

A Δ ABC is right angled at B. Then find the diameter of the incircle of the triangle.

Sol. $r = \frac{\Delta}{s} = \frac{\left(\frac{1}{2}\right)ac}{\left(\frac{1}{2}\right)(a+b+c)} = \frac{ac}{(a+b+c)}$

$$= \frac{ac(c+a-b)}{(c+a)^2 - b^2} = \frac{ac(c+a-b)}{c^2 + 2ca + a^2 - b^2}$$

$$= \frac{ac(c+a-b)}{2ca + b^2 - b^2} = \frac{c+a-b}{2} \qquad (\because a^2 + c^2 = b^2)$$

Example 58 :

In a Δ ABC, if a = 4 cm, b = 6 cm and c = 8 cm then find the value of r.

Sol. If a = 4 cm, b = 6 cm and c = 8 cm

then, 2s = a + b + c

s = 9 cm

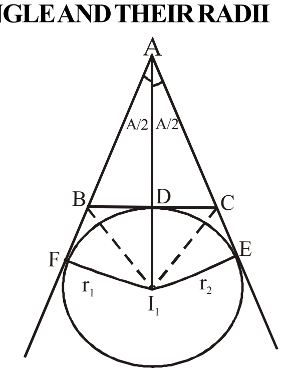
But, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ [by Hero's formula]

$$= \sqrt{9 \times 5 \times 3 \times 1}$$

so, $r = \frac{\Delta}{s}$ or, $r = \frac{3\sqrt{15}}{9}$ or, $r = \frac{\sqrt{15}}{3}$ cm or, $r = \sqrt{\frac{5}{3}}$ cm

ESCRIBED CIRCLES OF A TRIANGLE AND THEIR RADII

The circle which touches the side BC and two sides AB and AC produced of a triangle ABC is called the Escribed circle opposite to the angle A. Its radius is denoted by r_1. Similarly, r_2 and r_3 denote the radii of the escribed circle opposite to the angle B & C respectively.



The centres of the escribed circle are called the Ex-centres. The centre of the escribed circles opposite to the angle A is the point of intersection of the external bisector of angle B and C. The internal bisector of angle A also passes through the same point. The centre is generally denoted by I_1.

Radii of Ex-circles: In any ΔABC,

(i) $r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$

(ii) $r_1 = s \tan\frac{A}{2}, r_2 = s \tan\frac{B}{2}, r_3 = s \tan\frac{C}{2}$

(iii) $r_1 = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}, r_2 = \frac{b\cos\frac{C}{2}\cos\frac{A}{2}}{\cos\frac{B}{2}},$

$r_3 = \frac{c\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$

(iv) $r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}, r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2},$

$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Important results regarding r_1, r_2 and r_3 :

Given r_1, r_2 and r_3

(i) Semiperimeter, $s = \sqrt{(r_1r_2 + r_2r_3 + r_3r_1)} = \sqrt{\Sigma r_1r_2}$

(ii) $\Delta = \frac{r_1r_2r_3}{\sqrt{\Sigma r_1r_2}}$ **(iii)** $r = \frac{r_1r_2r_3}{\Sigma r_1r_2}$

(iv) $R = \frac{(r_1+r_2)(r_2+r_3)(r_3+r_1)}{4\Sigma r_1r_2}$

(v) $a = \frac{r_1(r_2+r_3)}{\sqrt{\Sigma r_1r_2}}, b = \frac{r_2(r_3+r_1)}{\sqrt{\Sigma r_1r_2}}, c = \frac{r_3(r_1+r_2)}{\sqrt{\Sigma r_1r_2}}$

(vi) $\sin A = \frac{2r_1\sqrt{\Sigma r_2r_3}}{(r_1+r_2)(r_1+r_3)}$

Example 59 :

In a Δ ABC, if a = 18 cm and b = 24 cm and c = 30 cm then find the value of r_1, r_2 and r_3.

Sol. a = 18 cm, b = 24 cm, c = 30 cm

$\therefore$ 2s = a + b + c = 72 cm

s = 36 cm

But, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$; $\Delta = 216$ sq. units

then, $r_1 = \frac{\Delta}{s-a} = \frac{216}{18} = 12$ cm or, $r_2 = \frac{\Delta}{s-b} = \frac{216}{12} = 18$ cm

or, $r_3 = \frac{\Delta}{s-c} = \frac{216}{6} = 36$ cm

so, r_1, r_2, r_3 are 12 cm, 18 cm, and 36 cm

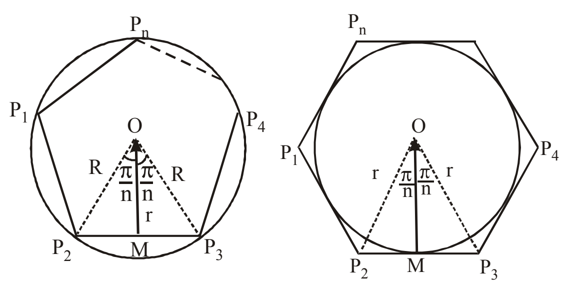
REGULAR POLYGON

If a polygon has all its sides equal in length and also all its angles equal then the polygon is called a regular polygon. The circle inscribed in the regular polygon and touching all the sides of the regular polygon is called the Inscribed circle.

The circle which passes through all the vertices of the regular polygon is called its Circumscribed circle.

Important Formulae to remember : Consider a regular polygon $P_1 P_2 .....P_n$ of n sides each of length a.

(i) (a) Radius R of circumscribed circle $R = \frac{1}{2}$ a cosec (π/n)



(a) **(b)**

(b) The radius r of inscribed circle is given by

$r = \frac{1}{2}$ a cot (π/n)

(ii) The area S of the regular polygon is given by

(a) $S = \frac{1}{4} na^2 \cot(\pi/n)$

(b) $S = \frac{1}{2} nR^2 \sin(2\pi/n)$, where R is the radius of circle circumscribing the polygon

(c) $S = nr^2 \tan(\pi/n)$

Where r is the radius of the circle inscribed in the polygon

Note : Formula (b) gives the area of a polygon inscribed in a circle of radius R. Formula (c) gives the area of polygon circumscribed the circle of radius r.

Example 60 :

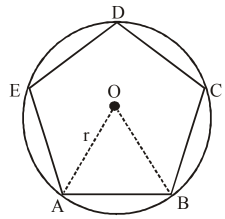
The area of a circle is A_1 and the area of a regular pentagon inscribed in the circle is A_2. Find the ratio of $A_1 : A_2$.

Sol. **(2).** If in the ΔOAB, OA = OB = r and $\angle AOB = \frac{360°}{5} = 72°$

$\therefore$ ar (Δ AOB) $= \frac{1}{2}$. r. r sin 72° $= \frac{1}{2} r^2 \cos 18°$

$\therefore$ ar (pentagon) $= \frac{5}{2} r^2 \cos 18°$

$\therefore$ $A_1 : A_2 = \frac{2\pi r^2}{5r^2 \cos 18°} = \frac{2\pi}{5} \sec \frac{\pi}{10}$



TRY IT YOURSELF-3

Q.1 Prove that $a \cos \frac{B-C}{2} = (b+c)\sin\left(\frac{A}{2}\right)$.

Q.2 If A = 75°, B = 45°, then prove that $b + c\sqrt{2} = 2a$.

Q.3 If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side a = 2, then find the area of the triangle ?

Q.4 If $a = \sqrt{3}, b = \frac{1}{2}(\sqrt{6}+\sqrt{2}), c = \sqrt{2}$, then find $\angle A$?

Q.5 If the sides of a triangle are $(x^2 + x + 1), (x^2 - 1)$ & $(2x + 1)$, then find the largest angle ?

Q.6 In any triangle $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$, then find the relation between the sides of the triangle?

Q.7 In any triangle ABC, if $b = \sqrt{3}, c = 1$ and A = 30°. Find the value of a, c, A & C ?

Q.8 If the sides of a triangle are 17, 25, 28, then find the greatest length of the altitude.

Q.9 If the sides of a triangle be in arithmetic progression, prove that the contangents of half the angles are also in arithmetic progression.

Q.10 If in a triangle ABC, if $\Delta = a^2 - (b - c)^2$, then find the value of tan A?

Q.11 If the median of a triangle ABC through A is perpendicular is AB then $\frac{\tan A}{\tan B}$ has the value equal to –

(A) 1/2 (B) 2
(C) –2 (D) –1/2

Q.12 If the area of the circle is A_1 and the area of the regular pentagon inscribed in the circle is A_2, then find the ratio $\frac{A_1}{A_2}$?

Q.13 In a triangle ABC, 3 sin A + 4 cos B = 6 and 3 cos A + 4 sin B = 1, then $\angle$ C can be

(A) 30° (B) 60°
(C) 90° (D) 150°

Q.14 If in a triangle ABC, $\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$, then the Δ ABC is

(A) right angled (B) obtuse angled
(C) isoscles (D) equilateral

ANSWERS

(3) $\sqrt{3}$ **(4)** $\pi/3$ **(5)** $2\pi/3$

(6) a, b, c are in A.P. **(7)** a = c = 1, C = 30°

(8) 420/17 **(10)** 8/15 **(11)** (C)

(12) $\frac{A_1}{A_2} = \frac{2\pi}{5}\sec\left(\frac{\pi}{10}\right)$ **(13)** (A) **(14)** (B)

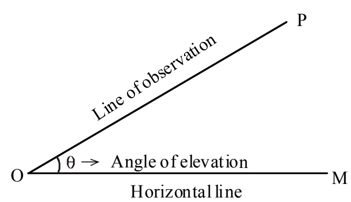
HEIGHT AND DISTANCE

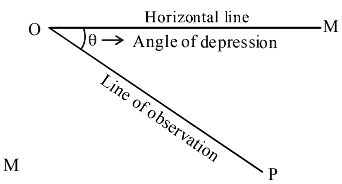
One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

DEFINITIONS

Angle of elevation : Let O and P be two points where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight. Then $\angle$ POM = θ is called the angle of elevation of P as observed from O.

Angle of depression : In the figure, if P be at a lower level than O, then $\angle$ MOP = θ is called the angle of depression.



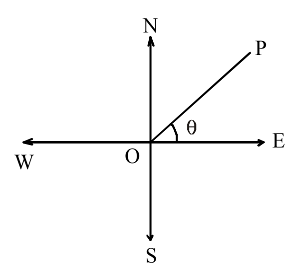


Note :

(i) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.

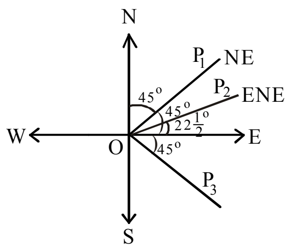
(ii) The angle of elevation or depression is always measured from horizontal line through the point of observation.

Bearing : In the figure, if the observer and the object i.e., O and P be on the same level then bearing is defined. To measure the 'Bearing', the four standard directions East, West, North and South are taken as the cardinal directions. Angle between the line of observation i.e. OP and any one standard direction-east, west , north or south is measured. Thus, $\angle$ POE = θ is called the bearing of point P with respect to O measured from east to north.



In other words the bearing of P as seen from O is the direction in which P is seen from O.

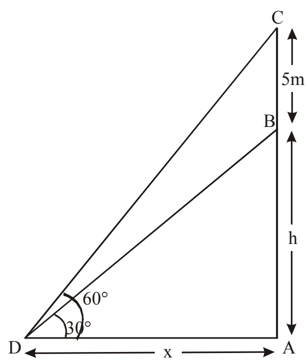
North-east : North-east means equally inclined to north and east, south-east means equally inclined to south and east. ENE means equally inclined to east and north-east.



Example 61 :

A vertical to stands on a horizontal plane and is surmounted by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.

Sol. Let AB be the tower of height h meter and BC be the height of flag staff surmounted on the tower. Let the point of the plane be D at a distance m meter from the foot of the tower.



In Δ ABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \qquad \text{........ (1)}$$

In Δ ADC,

$$\tan 60° = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{5+h}{x} \Rightarrow x = \frac{5+h}{\sqrt{3}} \qquad \text{......... (2)}$$

From (1) and (2), $\sqrt{3}h = \frac{5+h}{\sqrt{3}}$

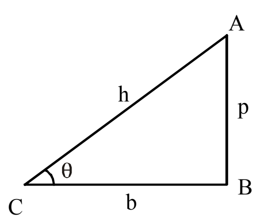
$\Rightarrow 3h = 5 + h \Rightarrow 2h = 5 \Rightarrow h = 5/2 = 2.5$ m

So, the height of tower = 2.5 m

SOME USEFUL RESULTS

1. In a triangle ABC,

$$\sin\theta = \frac{p}{h}, \quad \cos\theta = \frac{b}{h}$$

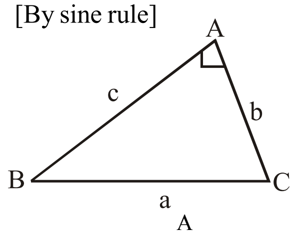


2. In any triangle ABC,

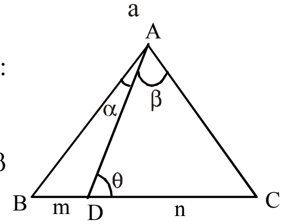
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{[By sine rule]}$$

or cosine formula

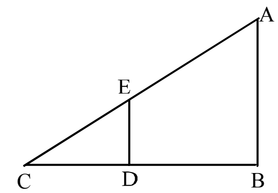
i.e., $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



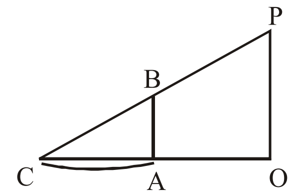
3. In any triangle ABC, if BD : DC = m : n and ∠BAD = α, ∠CAD = β and ∠ADC = θ, $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$



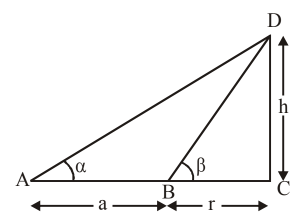
4. In a triangle ABC, if DE || AB then, $\frac{AB}{DE} = \frac{BC}{DC}$



5. To find the shadow of line object AB with respect to the light source P, we first join the upper points P and B. Let O be the projection of P on the plane on which object AB is situated, join OA. The section AC obtained by the intersection of the lines PB and OA, extended represents the shadow of AB with respect to light source P.

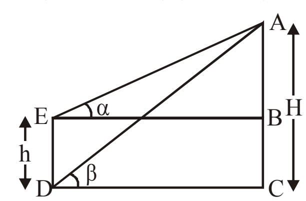


6. $a = h(\cot\alpha - \cot\beta)$

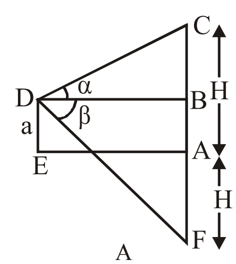


7. $h = \frac{H\sin(\alpha-\beta)}{\cos\alpha\sin\beta}$

and $H = \frac{h\cot\alpha}{\cot\alpha - \cot\beta}$

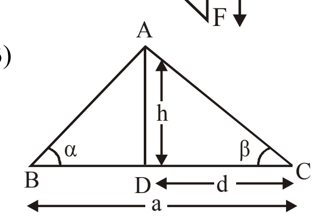


8. $H = \frac{\alpha\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$



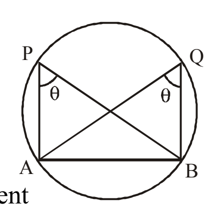
9. $a = h(\cot\alpha + \cot\beta)$

$h = a\sin\alpha\sin\beta\,\text{cosec}(\alpha+\beta)$

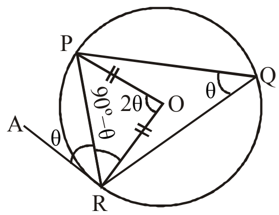


10. Angles on the same segment of a circle are equal.

In other words, we can say that if the angles APB and AQB subtended on the segment AB are equal, a circle will pass through the points A, B, Q, and P, i.e. the point A,B, Q and P will be concyclic.



11. If AR be the tangent to the circle through the points P, Q, R then ∠PRA = ∠PQR = θ.



[Angle between any chord and the tangent to the circle is equal to the angle subtended by the chord at the circumference.]

TRY IT YOURSELF-4

Q.1 The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° & 60° respectively. Find the height of the tower.

Q.2 If a flagtaff 6m high placed on the top of a tower throws a shadow of $2\sqrt{3}$ m on the ground, then the angle that the sun makes with the ground is –
(A) 15° (B) 30°
(C) 45° (D) 60°

Q.3 An observer on the top of a tree finds the angle of depressioin of a car moving towards the tree to be 30°. After 3 min, this angle becomes 60°. After how much more time, will the car reach the tree ?
(A) 4 min (B) 4.5 min.
(C) 1.5 min. (D) 2 min.

Q.4 From the top of a cliff of height a, the angle of depression of the foot of a certain tower is found to be double the angle of elevation of the top of the tower of height h. If θ is the angle of elevation, then its value is –

(A) $\cos^{-1}\sqrt{\frac{2h}{a}}$ (B) $\sin^{-1}\sqrt{\frac{2h}{a}}$

(C) $\sin^{-1}\sqrt{\frac{a}{2-h}}$ (D) $\tan^{-1}\sqrt{3-\frac{2h}{a}}$

Q.5 The angles of depression of the top and the foot of a chimney as seen from the top of a second chimney, which is 150m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when tan θ = 4/3 and tan ϕ = 5/2 is –

(A) $\frac{150}{\sqrt{3}}$ m (B) $100\sqrt{3}$ m
(C) 150m (D) 100 m

Q.6 A flagstaff 5m high is placed on a building 25m high. If the flag and building both subtend equal angles on the observer at a height 30m, the distance between the observer and the top of the flag is –

(A) $\frac{5\sqrt{3}}{2}$ (B) $5\sqrt{\frac{3}{2}}$

(C) $5\sqrt{\frac{2}{3}}$ (D) $\frac{5\sqrt{2}}{3}$

Q.7 A tower of x metres high has a flagstaff at its top. The tower and the flagstaff subtend equal angles at a point distant y metres from the foot of the tower. Then the length of the flagstaff (in metres) is –

(A) $\frac{y(x^2-y^2)}{(x^2+y^2)}$ (B) $\frac{x(y^2+x^2)}{(y^2-x^2)}$

(C) $\frac{x(x^2+y^2)}{(x^2-y^2)}$ (D) $\frac{x(x^2-y^2)}{(x^2+y^2)}$

Q.8 When the length of the shadow of the pole is equal to the height of the pole, then the elevation of the source of light
(A) 75° (B) 60°
(C) 45° (D) 30°

Q.9 A house of height 100m subtends a right angle at the window of an opposite house. If the height of the window is 64m, then the distance between the two houses is –
(A) 48m (B) 36m
(C) 54m (D) 72m

ANSWERS

(1) 15 m	**(2)** (D)	**(3)** (C)
(4) (D)	**(5)** (D)	**(6)** (B)
(7) (B)	**(8)** (C)	**(9)** (A)

ADDITIONAL EXAMPLES

Example 1 :

If $y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$ then find $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

Sol. $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$$= \frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$= \frac{(1+\sin^2\alpha+2\sin\alpha)-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$= \frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)\,(1+\cos\alpha+\sin\alpha)} = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$$

Example 2 :

If $A = \cos^2\theta + \sin^4\theta$, then for all real values of θ

(1) $1 \le A \le 2$ (2) $\frac{3}{4} \le A \le 1$

(3) $\frac{13}{10} \le A \le 1$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

Sol. (2). $A = \cos^2\theta + \sin^4\theta = \cos^2\theta + (1-\cos^2\theta)^2$
$= \cos^2\theta + 1 - 2\cos^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$
$\therefore\ 1 - A = \cos^2\theta - \cos^4\theta = \cos^2\theta\,[1-\cos^2\theta] = \cos^2\theta\sin^2\theta$

$$= \left(\frac{2\sin\theta\cos\theta}{2}\right)^2 = \frac{1}{4}(\sin 2\theta)^2$$

$\Rightarrow 4(1-A) = \sin^2 2\theta$

$0 \le 4(1-A) \le 1 \quad [\because 0 \le \sin^2 2\theta \le 1]$

$\Rightarrow \; 0 \le 1 - A \le \frac{1}{4} \quad \Rightarrow \; 1 - A \ge 0$ i.e. $A \le 1$ and

$1 - A \le \frac{1}{4}$ i.e. $\frac{3}{4} \le A \quad \therefore \; \frac{3}{4} \le A \le 1$

Example 3 :

If for $n \in N$, $f_n(\theta) = \tan\theta/2\,(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)$ ....$(1 + \sec 2^n\theta)$ then correct statement is

(1) $f_2(\pi/16) = 1$ (2) $f_3(\pi/32) = 1$

(3) $f_4(\pi/64) = 1$ (4) all above

Sol. (4). $\because \tan\frac{\theta}{2}(1+\sec\theta) = \tan\frac{\theta}{2}\left(\frac{1+\cos\theta}{\cos\theta}\right)$

$= \frac{\sin\theta/2}{\cos\theta/2}.\frac{2\cos^2\theta/2}{\cos\theta}$ ----- $= \frac{\sin\theta}{\cos\theta} = \tan\theta$ ....(1)

$\therefore f_1(\theta) = \tan\theta/2\,(1+\sec\theta)(1+\sec 2\theta)$

$= [\tan\theta/2\,(1+\sec\theta)](1+\sec 2\theta)$

$= (\tan\theta)(1+\sec 2\theta)$ [from(1)]

$= \tan 2\theta$ [replacing θ by 2θ as above]

$\Rightarrow f_1(\theta) = \tan 2^1\theta$ ....(2)

Similarly $f_2(\theta) = \tan 2^2\theta$, $f_3(\theta) = \tan 2^3\theta$, $f_4(\theta) = \tan 2^4\theta$ etc.

$\Rightarrow \quad f_2\left(\frac{\pi}{16}\right) = \tan\left(2^2\frac{\pi}{16}\right) = \tan\frac{\pi}{4} = 1$

$f_3\left(\frac{\pi}{32}\right) = \tan\left(2^3\frac{\pi}{32}\right) = \tan\frac{\pi}{4} = 1$

$f_4\left(\frac{\pi}{64}\right) = \tan\left(2^4\frac{\pi}{64}\right) = \tan\frac{\pi}{4} = 1$

Example 4 :

Find the value of $\sqrt{3}$ cosec $20° - \sec 20°$

Sol. The given expression

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20°\cos 20°}$

$= \frac{2(\sin 60°\cos 20° - \cos 60°\sin 20°)}{\sin 20°\cos 20°}$

$= \frac{2\sin(60°-20°)}{\sin 20°\cos 20°} = \frac{2\sin 40°}{\sin 20°\cos 20°}$

$= \frac{4\sin 40°}{2\sin 20°\cos 20°} = \frac{4\sin 40°}{\sin 40°}$

Example 5 :

Find the value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$.

Sol. $\tan 9° + \tan 81° - (\tan 27° + \tan 63°)\,(\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \left(\frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°}\right) - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$

$= \frac{1}{\sin 9°\cos 9°} - \frac{1}{\cos 27°\sin 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = \frac{2}{\sin 18°} - \frac{2}{\cos 36°}$

$= \frac{2\times 4}{\sqrt{5}-1} - \frac{2\times 4}{\sqrt{5}+1} = 8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}\right] = \frac{16}{4} = 4$

Example 6 :

If ABCD is a cyclic quadrilateral such that $12\tan A - 5 = 0$ and $5\cos B + 3 = 0$ then find the quadratic equation whose roots are $\cos C$, $\tan D$

Sol. In a quadrilateral no angle is greater then 180°

Here $\tan A = \frac{5}{12}$ so, $0 < A < \frac{\pi}{2}$

and $\frac{\pi}{2} < C < \pi \qquad (\because A + C = 180°)$

$\therefore \tan(\pi - C) = \frac{5}{12}$, i.e. ,$\tan C = -\frac{5}{12} \quad \therefore \cos C = -\frac{12}{13}$

Also $\cos B = -\frac{3}{5}$, so, $\frac{\pi}{2} < B < \pi$

and $0 < D < \frac{\pi}{2} \qquad (\because B + D = 180°)$

$\therefore \cos(\pi - D) = -\frac{3}{5}$, i.e., $\cos D = \frac{3}{5}$. $\quad \therefore \tan D = \frac{4}{3}$

$\therefore$ The required equation is

$x^2 - \left(-\frac{12}{13} + \frac{4}{3}\right)x + \left(-\frac{12}{13}\right).\frac{4}{3} = 0 \Rightarrow 39x^2 - 16x - 48 = 0$

Example 7 :

If $\tan(\cot x) = \cot(\tan x)$, then

(1) $\sin 2x = \frac{2}{(2n+1)\pi}$ (2) $\sin x = \frac{4}{(2n+1)\pi}$

(3) $\sin 2x = \frac{4}{(2n+1)\pi}$ (4) none of these

Sol. (3). $\tan(\cot x) = \cot(\tan x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \quad \cot x = n\pi + \frac{\pi}{2} - \tan x \quad [\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha]$

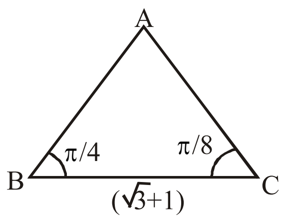
$\Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2} \Rightarrow \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \frac{1}{\sin x \cos x} = (2n+1)\frac{\pi}{2} \Rightarrow \frac{1}{\sin 2x} = \frac{(2n+1)\pi}{4}$

$\Rightarrow \sin 2x = \frac{4}{(2n+1)\pi}$

Example 8 :

Two angles of a triangle are $\frac{\pi}{6}$ and $\frac{\pi}{4}$ and the length of the included side is $(\sqrt{3}+1)$ cm. Find the area of the triangle.



Sol. $A = \pi - \frac{\pi}{4} - \frac{\pi}{6} = 180° - 45° - 30° = 105°$

$\frac{\sqrt{3}+1}{\sin 105°} = \frac{C}{\sin\frac{\pi}{6}}$ i.e. $\frac{C}{\sin 30°}$ $\therefore C = \frac{1}{2} \cdot \frac{\sqrt{3}+1}{\sin(60°+45°)}$

$= \frac{\sqrt{3}+1}{2\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right)} = \frac{(\sqrt{3}+1)}{2.(\sqrt{3}+1)} . 2\sqrt{2} = \sqrt{2}$

$\Rightarrow$ Reqd. area $= \frac{1}{2}$ ca sin B $= \frac{1}{2}\sqrt{2}(\sqrt{3}+1)\sin\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}(\sqrt{3}+1) . \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2}$ cm^2

Example 9 :

If $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$, then the two curves y = cos x and y = sin 3x intersect at

(1) $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ & $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)$ (2) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$ & $\left(\frac{-\pi}{8}, \cos\frac{\pi}{8}\right)$

(3) $\left(\frac{\pi}{4}, \frac{-1}{\sqrt{2}}\right)$ & $\left(\frac{\pi}{8}, -\cos\frac{\pi}{8}\right)$ (4) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$

Sol. (1). At the intersection point of y = cos x and y = sin 3x,
We have cos x = sin 3x

$\Rightarrow \cos x = \cos\left(\frac{\pi}{2} - 3x\right) \Rightarrow x = 2n\pi \pm \left(\frac{\pi}{2} - 3x\right)$

$\Rightarrow x = \frac{\pi}{4}, \frac{\pi}{8}$ $\quad [\because -\pi/2 \le x \le \pi/2]$

So, $y = \cos\frac{\pi}{4}$ at $x = \frac{\pi}{4}$ and $y = \cos\frac{\pi}{8}$, at $x = \frac{\pi}{8}$

Thus, the points are $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)$

Example 10 :

The solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

(1) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (2) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(3) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (4) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

Sol. (4). We have $\sin^2\theta - \sin\theta - 2 = 0$

$\Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

$\therefore \sin\theta = -1 = \sin\frac{3\pi}{2}$ $\therefore \theta = \frac{3\pi}{2} = \frac{6\pi}{4} \in \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

Example 11 :

If $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0, -\pi < \theta < \pi$, then find θ.

Sol. Change in terms of cos θ

$5(2\cos^2\theta - 1) + (1 + \cos\theta) + 1 = 0$

or $10\cos^2\theta + \cos\theta - 3 = 0$

or $(5\cos\theta + 3)(2\cos\theta - 1) = 0$.

$\therefore \theta = \pi/3, -\pi/3,$

$\cos^{-1}\left(-\frac{3}{5}\right)$ i.e. $\pi - \cos^{-1}\frac{3}{5}$ $\quad \because -\pi < \theta < \pi$

Example 12 :

Let the angles A, B, C of Δ ABC be in A.P. and let $b : c = \sqrt{3} : \sqrt{2}$. Then find angle A.

Sol. Since A, B, C are in A. P.

$\therefore A + C = 2B \Rightarrow A + B + C = 3B$

$\Rightarrow 180° = 3B \qquad \therefore B = 60°$

$\Rightarrow A + C = 120°$

Now $\sin B : \sin C = \sqrt{3} : \sqrt{2}$ $\quad [\because b : c = \sqrt{3} : \sqrt{2}]$

$\therefore \sin C = \frac{\sqrt{2}}{\sqrt{3}}\sin 60° = \frac{\sqrt{2}}{\sqrt{3}}.\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$

$\therefore C = 45° \quad \therefore A = 120° - 45° = 75°$

Example 13 :

Find the sides of a triangle are a, b, $\sqrt{a^2 + b^2 + ab}$ the greatest angle.

Sol. As the greater side of a triangle has greater angle opposite to it.

$\therefore$ The angle (say C) opposite to $\sqrt{a^2 + b^2 + ab} = c$ (say) is the greatest in this case

Now, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$= \dfrac{a^2+b^2-(a^2+b^2+ab)}{2ab} = \dfrac{-ab}{2ab} = \dfrac{-1}{2}$; $C = 120°$

$[\because c^2 = a^2+b^2+ab]$

Example 14 :

The positive integer value of $n > 3$ satisfying the equation $\dfrac{1}{\sin\left(\frac{\pi}{n}\right)} = \dfrac{1}{\sin\left(\frac{2\pi}{n}\right)} + \dfrac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Sol. **7.** $\dfrac{1}{\sin\left(\frac{\pi}{n}\right)} - \dfrac{1}{\sin\left(\frac{3\pi}{n}\right)} = \dfrac{1}{\sin\left(\frac{2\pi}{n}\right)}$;

$$\frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$\sin\dfrac{4\pi}{n} = \sin\dfrac{3\pi}{n}$; $\dfrac{4\pi}{n} = (-1)^k\dfrac{3\pi}{n} + k\pi,\ k \in I$

If $k = 2m \Rightarrow \dfrac{\pi}{n} = 2m\pi$; $\dfrac{1}{n} = 2m$, not possible

If $k = 2m+1 \Rightarrow \dfrac{7\pi}{n} = (2m+1)\pi \Rightarrow n = 7, m = 0$

Example 15 :

If in a Δ ABC, $a = 6$, $b = 3$ and $\cos(A-B) = 4/5$, then find its area.

Sol. Since $\cos(A-B) = 4/5 \Rightarrow \dfrac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)} = \dfrac{4}{5}$

$\Rightarrow 5 - 5\tan^2\dfrac{A-B}{2} = 4 + 4\tan^2\dfrac{A-B}{2}$

$\Rightarrow 9\tan^2\dfrac{A-B}{2} = 1 \Rightarrow \tan\dfrac{A-B}{2} = \dfrac{1}{3}$

$= \tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$

$\therefore \dfrac{1}{3} = \dfrac{6-3}{6+3}\cot\dfrac{C}{2} = \dfrac{1}{3}\cot\dfrac{C}{2}$

$\therefore \cot\dfrac{C}{2} = 1 \Rightarrow \dfrac{C}{2} = 45° \Rightarrow C = 90°$

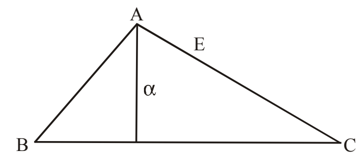
Hence area of Δ ABC $= \dfrac{1}{2} \times 6 \times 3 = 9$ sq.units

Example 16 :

In a Δ ABC, the sides are in the ratio 4 : 5 : 6. Find the ratio of the circumradius and the inradius.

Sol. Here $a = 4k$, $b = 5k$, $c = 6k$

$\therefore s = \dfrac{15k}{2}$



$\therefore \Delta = \sqrt{\dfrac{15k}{2}\left(\dfrac{15k}{2}-4k\right)\left(\dfrac{15k}{2}-5k\right)\left(\dfrac{15k}{2}-6k\right)} = \dfrac{15\sqrt{7}}{4}k^2$

But $R = \dfrac{abc}{4\Delta} = \dfrac{4k.5k.6k}{15\sqrt{7}\,k^2} = \dfrac{8}{\sqrt{7}}k$

and $r = \dfrac{\Delta}{s} = \dfrac{15\sqrt{7}}{4}k^2.\dfrac{2}{15k} = \dfrac{\sqrt{7}}{2}k \therefore \dfrac{R}{r} = \dfrac{\frac{8k}{\sqrt{7}}}{\frac{\sqrt{7}k}{2}} = \dfrac{16}{7}$

Example 17 :

In an equilateral triangle, the in radius, circum- radius and one of the ex-radii are in the ratio

(1) 2 : 3 : 5 (2) 1 : 2 : 3

(3) 3 : 7 : 9 (4) 3 : 7 : 9

Sol. **(2).** We have $\Delta = \dfrac{\sqrt{3}}{4}a^2$, $s = \dfrac{3a}{2}$

$\therefore r = \dfrac{\Delta}{s} = \dfrac{a}{2\sqrt{3}}$, $R = \dfrac{abc}{4\Delta} = \dfrac{a^3}{\sqrt{3}a^2} = \dfrac{a}{\sqrt{3}}$

and $r_1 = \dfrac{\Delta}{s-a} = \dfrac{\sqrt{3}/4a^2}{a/2} = \dfrac{\sqrt{3}}{2}a$

Hence, $r : R : r_1 = \dfrac{a}{2\sqrt{3}} : \dfrac{a}{\sqrt{3}} : \dfrac{\sqrt{3}}{2}a = 1 : 2 : 3$

Example 18 :

If R and r are the radii of the circumcircle and incircle of a regular polygon of n sides, each side being of length a, then find 2 (R + r).

Sol. $R = \dfrac{1}{2}a\,\text{cosec}\left(\dfrac{\pi}{n}\right)$ and $r = \dfrac{1}{2}a\cot\left(\dfrac{\pi}{n}\right)$, so that

$$2(R+r) = a\left(\frac{1}{\sin\left(\frac{\pi}{n}\right)} + \frac{\cos\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}\right) = a\frac{1+\cos\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}$$

$$= a\frac{2\cos^2\left(\frac{\pi}{2n}\right)}{2\sin\left(\frac{\pi}{2n}\right)\cos\left(\frac{\pi}{2n}\right)} = a\cot\frac{\pi}{2n}$$

QUESTION BANK

CHAPTER 2 : TRIGONOMETRY

EXERCISE - 1 [LEVEL-1]

PART 1 : TRIGONOMETRIC RATIOS

Q.1 If $\cos 3x = -1$, where $0^\circ \le x \le 360^\circ$, then x =
(A) 60º, 180º, 300º (B) 180º
(C) 60º, 180º (D) 180º, 300º

Q.2 $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ =$
(A) 2 (B) – 1
(C) 0 (D) $1/\sqrt{2}$

Q.3 $\dfrac{\operatorname{cosec}(2\pi+\theta).\cos(2\pi+\theta)\tan(\pi/2+\theta)}{\sec(\pi/2+\theta).\cos\theta.\cot(\pi+\theta)} =$
(A) 2 (B) – 1
(C) 4 (D) 1

Q.4 $\dfrac{\sin 2\theta}{1+\cos 2\theta} =$
(A) $\cot\theta$ (B) $\tan\theta$
(C) $\sin\theta$ (D) $\operatorname{cosec}\theta$

Q.5 If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ then $(m^2n)^{2/3} + (n^2m)^{2/3}$ equals to –
(A) 0 (B) 1
(C) –1 (D) 2

Q.6 The value of
$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is
(A) $\frac{1}{2}$ (B) $\cos\frac{\pi}{8}$
(C) $\frac{1}{8}$ (D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

Q.7 The value of $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$ is -
(A) 3/8 (B) 1/8
(C) 3/16 (D) None of these

Q.8 $\cos\left(\frac{\pi}{14}\right) + \cos\left(\frac{3\pi}{14}\right) + \cos\left(\frac{5\pi}{14}\right) =$
(A) $\frac{1}{2}\tan\left(\frac{\pi}{14}\right)$ (B) $\frac{1}{2}\cos\left(\frac{\pi}{14}\right)$
(C) $\frac{1}{2}\cot\left(\frac{\pi}{14}\right)$ (D) None of these

Q.9 The value of the sum
$\frac{1}{\sin 1^\circ \sin 2^\circ} + \frac{1}{\sin 2^\circ \sin 3^\circ} + ... + \frac{1}{\sin 89^\circ \sin 90^\circ}$
(A) $\frac{2\sin 1^\circ}{1+\cos 2^\circ}$ (B) $\frac{\sin 1^\circ}{\cos^2 1^\circ}$
(C) $\frac{\cos 1^\circ}{\sin^2 1^\circ}$ (D) None of these

Q.10 Exact value of $\dfrac{\cos 21^\circ - \sin 21^\circ}{\sin^2 57^\circ - \sin^2 33^\circ}$ is equal to –
(A) 2 (B) $-\sqrt{2}$
(C) 1 (D) $\sqrt{2}$

Q.11 $\cot 12^\circ \cot 102^\circ + \cot 102^\circ \cot 66^\circ + \cot 66^\circ \cot 12^\circ =$
(A) –2 (B) 1
(C) –1 (D) 2

Q.12 The value of $\sin 10^\circ . \sin 30^\circ . \sin 70^\circ$ is
(A) 1/16 (B) $\sqrt{3}/16$
(C) 3/16 (D) 1/8

Q.13 $\dfrac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} =$
(A) 1 (B) $1/\sqrt{3}$
(C) $\sqrt{3}$ (D) 1/2

Q.14 $\log(\sin 1^\circ) . \log(\sin 2^\circ) . \log(\sin 3^\circ) .... \log(\sin 179^\circ)$
(A) is positive (B) is negative
(C) lies between 1 and 180 (D) is zero

Q.15 The value of $\tan(1^\circ) + \tan(89^\circ)$ is –
(A) $\frac{1}{\sin(1^\circ)}$ (B) $\frac{2}{\sin(2^\circ)}$
(C) $\frac{2}{\sin(1^\circ)}$ (D) $\frac{1}{\sin(2^\circ)}$

PART 2 : TRIGONOMETRIC IDENTITIES

Q.16 If $A + B + C = \pi$, then $\sin 2A + \sin 2B + \sin 2C =$
(A) 4sin A sin B cos C (B) 4sin A sin B sin C.
(C) 4cos A sin B sin C (D) None of these

Q.17 If $A + B + C = \pi$, then $\tan A + \tan B + \tan C =$
(A) cotA.tanB.tanC (B) tanA.cotB.tanC
(C) tanA.tanB.tanC (D) None of these

Q.18 $\cos A + \cos B + \cos C = 1 + x\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. Find the value of x. [A, B, C are the angles of triangle]
(A) 4 (B) 3
(C) 2 (D) 1

Q.19 $\cos 2A + \cos 2B + \cos 2C + 4\cos A\cos B\cos C = ?$
[A, B, C are the angles of triangle]
(A) –1 (B) 1
(C) 2 (D) –2

Q.20 $\cos 2A + \cos 2B - \cos 2C = 1 - x\sin A\sin B\sin C$
Find the value of x. [A, B, C are the angles of triangle]
(A) 4 (B) 3
(C) 2 (D) 1

Q.21 $\sin A + \sin B - \sin C - 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = ?$

[A, B, C are the angles of triangle]

(A) 4 (B) 3
(C) 0 (D) 1

Q.22 $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = ?$

[A, B, C are the angles of triangle]

(A) 4 (B) 3
(C) 1 (D) 0

Q.23 $\cot B \cot C + \cot C \cot A + \cot A \cot B = ?$

[A, B, C are the angles of triangle]

(A) 4 (B) 1
(C) 2 (D) 3

Q.24 $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = x \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Find the value of x. [A, B, C are the angles of triangle]

(A) 8 (B) 3
(C) 2 (D) 1

Q.25 Find the sum of the series :

$\cos^3\alpha + \cos^3 3\alpha + \cos^3 5\alpha + ... + \cos^3(2n-1)\alpha$

(A) $\frac{\sin n\alpha \cos n\alpha}{4\sin\alpha} + \frac{\sin 3n\alpha \cos 3n\alpha}{4\sin 3\alpha}$

(B) $\frac{3\sin n\alpha \cos n\alpha}{4\sin\alpha} + \frac{\sin 3n\alpha \cos 3n\alpha}{4\sin 3\alpha}$

(C) $\frac{3\sin n\alpha \cos n\alpha}{4\sin\alpha} - \frac{\sin 3n\alpha \cos 3n\alpha}{4\sin 3\alpha}$

(D) None of these

PART 3 : TRIGONOMETRIC EQUATIONS

Q.26 If $\sin 3\theta = \sin\theta$, then the general value of θ is -

(A) $2n\pi,\ (2n+1)\frac{\pi}{3}$ (B) $n\pi,\ (2n+1)\frac{\pi}{4}$

(C) $n\pi,\ (2n+1)\frac{\pi}{3}$ (D) None of these

Q.27 The number of solutions of equation,

$\sin 5x \cos 3x = \sin 6x \cos 2x$, in the interval $[0, \pi]$ are -

(A) 3 (B) 4
(C) 5 (D) 6

Q.28 The number of solutions of the equation

$5\sec\theta - 13 = 12\tan\theta$ in $[0, 2\pi]$ is

(A) 2 (B) 1
(C) 4 (D) 0

Q.29 The general solution of $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$ is

(A) $\theta = 2r\pi + \frac{\pi}{2}, r \in Z$ (B) $\theta = 2r\pi, r \in Z$

(C) $\theta = 2r\pi + \frac{\pi}{2}$ and $\theta = 2r\pi, r \in Z$ (D) None of these

Q.30 The general solution of the equation $\sec 4\theta - \sec 2\theta = 2$ is–

(A) $(2n+1)\frac{\pi}{2}, n\pi + \frac{\pi}{10}$ (B) $(2n+1)\frac{\pi}{2}, (2n+1)\frac{\pi}{16}$

(C) $(2n-1)\frac{\pi}{2}, (2n+1)\frac{\pi}{4}$ (D) $(2n+1)\frac{\pi}{2}, (2n+1)\frac{\pi}{10}$

Q.31 The general solution of the equation

$\sin^4 x + \cos^4 x = \sin x \cos x$ is –

(A) $\left(\frac{2n+1}{4}\right)\pi\ ; n \in I$ (B) $\left(\frac{4n+1}{4}\right)\pi\ ; n \in I$

(C) $2n\pi + \frac{\pi}{4}; n \in I$ (D) $n\pi - \frac{\pi}{4}\ ; n \in I$

Q.32 The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}\ (0 \le x \le 2\pi)$ is -

(A) 0 (B) 1
(C) 2 (D) 3

Q.33 Let n be positive integer such that

$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then -

(A) $6 \le n \le 8$ (B) $4 < n \le 8$
(C) $6 \le n \le 8$ (D) $4 < n < 8$

Q.34 The general solution of $\sin 2\theta = 0$ is –

(A) $n\pi; n \in I$ (B) $\frac{n\pi}{2}; n \in I$

(C) $(2n+1)\frac{\pi}{2}; n \in I$ (D) $2n\pi; n \in I$

Q.35 The general solution of $\cos\theta = 1/2$ is –

(A) $2n\pi \pm \frac{\pi}{6}; n \in I$ (B) $n\pi \pm \frac{\pi}{6}; n \in I$

(C) $2n\pi \pm \frac{\pi}{3}; n \in I$ (D) $n\pi \pm \frac{\pi}{3}; n \in I$

Q.36 If $\sin\theta + \sin 3\theta + \sin 5\theta = 0$, then the general value of θ is

(A) $\frac{n\pi}{6}, \frac{m\pi}{12}; m, n \in I$ (B) $\frac{n\pi}{3}, m\pi \pm \frac{\pi}{3}; m, n \in I$

(C) $\frac{n\pi}{3}, m\pi \pm \frac{\pi}{6}; m, n \in I$ (D) None of these

Q.37 If $2\cos^2\theta + 3\sin\theta = 0$, then general value of θ is –

(A) $n\pi + (-1)^n\frac{\pi}{6}\ ; n \in I$ (B) $2n\pi \pm \frac{\pi}{6}; n \in I$

(C) $n\pi + (-1)^{n+1}\frac{\pi}{6}; n \in I$ (D) None of these

Q.38 The general solution of $\cos^2\theta = 1/2$ is –

(A) $n\pi \pm \frac{\pi}{3}$; $n \in I$ (B) $2n\pi \pm \frac{\pi}{4}$; $n \in I$

(C) $n\pi \pm \frac{\pi}{4}$; $n \in I$ (D) None of these

Q.39 General solution of equation $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is –

(A) $n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$; $n \in I$ (B) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$; $n \in I$

(C) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}$; $n \in I$ (D) None of these

Q.40 Principal value of $\tan\theta = -1$ is -

(A) $-\pi/4$ (B) $\pi/4$

(C) $3\pi/4$ (D) $-3\pi/4$

Q.41 The most general value of θ satisfying the equations $\cos\theta = 1/\sqrt{2}$ and $\tan\theta = -1$ is –

(A) $n\pi + \frac{7\pi}{4}$; $n \in I$ (B) $n\pi + (-1)^n \frac{7\pi}{4}$; $n \in I$

(C) $2n\pi + \frac{7\pi}{4}$; $n \in I$ (D) None of these

Q.42 Solve $\sin x + \cos x = \sqrt{2}$, if $0 \le x < 2\pi$

(A) $\pi/4$ (B) $7\pi/4$

(C) $3\pi/4$ (D) π

Q.43 Find the solution of the equation $\sin^4x + \cos^4x = \sin x \cos x$

(A) $\frac{(4n+1)\pi}{4} (n \in I)$ (B) $\frac{(4n+1)\pi}{2} (n \in I)$

(C) $\frac{(2n+1)\pi}{4} (n \in I)$ (D) None of these

Q.44 Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equal area. Then the angle θ between the lines is given by –

(A) $3\theta + 3\sin\theta = \pi$ (B) $6\theta + 3\sin\theta = \pi$

(C) $2\theta + \sin\theta = \pi$ (D) $\theta + \sin\theta = \pi/2$

Q.45 The number of values of $x \in [0, n\pi]$, $n \in z$ that satisfied $\log_{|\sin x|}(1+\cos x) = 2$ is –

(A) 0 (B) n

(C) 2n (D) None

Q.46 If $2\sec(2\alpha) = \tan\beta + \cot\beta$, then one of the value of $\alpha + \beta$ is –

(A) π (B) $n\pi - \pi/4, n \in z$

(C) $\pi/4$ (D) $\pi/2$

Q.47 The smallest positive angle θ satisfying the equation $\sin 2\theta = \cos 3\theta$, is equal to –

(A) $\pi/5$ (B) $\pi/10$

(C) $\pi/12$ (D) $\pi/6$

PART 4 : TRIGONOMETRIC INEQUALITIES

Q.48 The maximum value of $3\sin\theta + 4\cos\theta$ is-

(A) 2 (B) 3

(C) 4 (D) 5

Q.49 The set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x is –

(A) ϕ (B) $(-2, 2)$

(C) R (D) None

Q.50 For any real θ, the max. value of $\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is

(A) 1 (B) $1 + \sin^2 1$

(C) $1 + \cos^2 1$ (D) None

Q.51 In Δ ABC, $\tan C < 0$ then

(A) $\tan A \cdot \tan B > 1$ (B) $\tan A + \tan B + \tan C < 0$

(C) $\tan A + \tan B + \tan C > 0$ (D) None

Q.52 If $\log_{\cos x}\sin x \ge 2$, and $0 \le x \le 3\pi$ then sinx lies in the interval

(A) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ (B) $\left[0, \frac{\sqrt{5}-1}{2}\right]$

(C) $\left[0, \frac{1}{2}\right]$ (D) None of these

Q.53 The number of solutions of the inequality

$$\frac{1}{2^{\sin^2 x_2}} \cdot \frac{1}{3^{\sin^2 x_3}} \cdot \frac{1}{4^{\sin^2 x_4}} \cdots\cdots \frac{1}{n^{\sin^2 x_n}} \le n!$$

where $x_i \in (0, 2\pi)$ for $i = 2, \ldots\ldots n$ is –

(A) 1 (B) 2^{n-1}

(C) n^n (D) Infinite

Q.54 $\operatorname{cosec}^2\theta\,(\cos^2\theta - 3\cos\theta + 2) \ge 1$ if θ lies in –

(A) $\left(0, \frac{\pi}{3}\right)$ (B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (D) $\left(0, \frac{\pi}{4}\right)$

PART 5 : PROPERTIES AND SOLUTIONS OF TRIANGLE

Q.55 The sides of a triangle are $6+\sqrt{12}, 4\sqrt{3}, \sqrt{24}$. The tangent of the smallest angle of the triangle is

(A) $1/\sqrt{3}$ (B) $\sqrt{2}-1$

(C) $\sqrt{3}$ (D) 1

Q.56 In Δ ABC, if $a = 2$, $b = \tan^{-1}(1/2)$ and $c = \tan^{-1}(1/3)$, then $(a, b) =$

(A) $\left(\frac{3\pi}{4}, \frac{2}{\sqrt{5}}\right)$ (B) $\left(\frac{\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$

(C) $\left(\frac{3\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$ (D) $\left(\frac{\pi}{4}, \frac{2}{\sqrt{5}}\right)$

Q.57 In any triangle ABC, the simplified form of

$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$ is –

(A) $a^2 + b^2$ (B) $\frac{1}{a^2} - \frac{1}{b^2}$

(C) $\frac{1}{a^2 - b^2}$ (D) $a^2 - b^2$

Q.58 In a triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and a = 2, then its area is –

(A) $2\sqrt{3}$ (B) $\sqrt{3}$

(C) $\sqrt{3}/2$ (D) $\sqrt{3}/4$

Q.59 In a triangle ABC, a [b cos C – c cos B] =

(A) 0 (B) a^2

(C) $b^2 - c^2$ (D) b^2

PART 6 : HEIGHT AND DISTANCE

Q.60 The angle of elevation of the top of a tower from a point on the ground is 30° and it is 60° when it is viewed from a point located 40m away from the initial point towards the tower. The height of the tower is –

(A) $-20\sqrt{3}$m (B) $\sqrt{3}/20$ m

(C) $-\sqrt{3}/20$ m (D) $20\sqrt{3}$ m

Q.61 On one bank of a river, there is a tree. On another bank, an observer makes an angle of elevation of 60° at the top of the tree. The angle of elevation of the top of the tree at a distance 20m away from the bank is 30°. The width of the river is –

(A) 20m (B) 10m

(C) 5m (D) 1m

Q.62 A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is 60°. If the distance between the two houses is 6m, then the height of the first house is –

(A) $8\sqrt{3}$ m (B) $6\sqrt{3}$ m

(C) $4\sqrt{3}$ m (D) None of these

Q.63 A and B are two points 30m apart in a line on the horizontal plane through the foot of a tower lying on opposite sides of the tower if the distance of the top of the tower from A and B are 20m and 15m respectively. The angle of elevation of the top of the tower at A is –

(A) $\cos^{-1}\frac{43}{48}$ (B) $\sin^{-1}\frac{43}{48}$

(C) $\sin^{-1}\frac{29}{36}$ (D) $\cos^{-1}\frac{29}{36}$

Q.64 Two flag staffs on a horizontal plane. A and B are two points on the line joining their feet and are between them. The angle of elevation of tops of the flag staffs as seen from A are 30° and 60° and as seen from B are 60° and 45°. If AB is 30m. The distance between flag staffs in meters is–

(A) $30+15\sqrt{3}$ (B) $45+15\sqrt{3}$

(C) $60-15\sqrt{3}$ (D) $60+15\sqrt{3}$

Q.65 Angles of elevation of the top of a tower from three points (collinear) A, B and C on a road leading to the foot of the tower are 30°, 45° and 60° respectively. The ratio of AB to BC is –

(A) $2 : \sqrt{3}$ (B) 1:2

(C) $\sqrt{3} : 2$ (D) $\sqrt{3} : 1$

PART 7 : MISCELLANEOUS

Q.66 The value of (1 + tan 1°) (1 + tan 2°) (1 + tan 3°) ...... (1 + tan 44°) (1 + tan 45°) is –

(A) 2^{21} (B) 2^{24}

(C) 2^{23} (D) 2^{22}

Q.67 If $\sin x + \sin^2 x + \sin^3 x = 1$, then $\cos^6 x - 4\cos^4 x + 8\cos^2 x =$

(A) 2 (B) 1

(C) 3 (D) 4

Q.68 The sum $\cos\frac{\pi}{9} + \cos\frac{2\pi}{9} + \cos\frac{3\pi}{9} + ... + \cos\frac{17\pi}{9} =$

(A) 1/2 (B) –1/2

(C) 1 (D) –1

Q.69 If tan A and tan B are the roots of the quadratic equation $x^2 - ax + b = 0$, then the value of $\cos^2(A + B)$ is –

(A) $\frac{a^2}{a^2+(1-b)^2}$ (B) $\frac{(1-b)^2}{a^2+(1-b)^2}$

(C) $\frac{(1-a)^2}{a^2+(1-b)^2}$ (D) $\frac{b^2}{a^2+(1-b)^2}$

Q.70 The value of

cos 20° cos 40° cos 80° cos 160° cos 320° cos 640° is equal to –

(A) 1/32 (B) 1/64

(C) 1/128 (D) –1/64

Q.71 If (1 – cot 1°) . (1 – cot 2°) . (1– cot 3°) .... (1 – cot 44°) = 2^n, then n is equal to –

(A) 21 (B) 22

(C) 23 (D) 24

Q.72 If $\tan B = \frac{n \sin A \cos A}{1 - n\cos^2 A}$ then $\tan(A + B)$ equals

(A) $\frac{\sin A}{(1-n)\cos A}$ (B) $\frac{(n-1)\cos A}{\sin A}$

(C) $\dfrac{\sin A}{(n-1)\cos A}$ (D) $\dfrac{\sin A}{(n+1)\cos A}$

Q.73 Given $a^2 + 2a + \mathrm{cosec}^2\left(\dfrac{\pi}{2}(a+x)\right) = 0$ then, which of the following holds good?

(A) $a = 1$; $\dfrac{x}{2} \in I$ (B) $a = -1$; $\dfrac{x}{2} \in I$

(C) $a \in R$; $x \in \phi$

(D) a , x are finite but not possible to find

Q.74 The set of angles between 0 & 2π satisfying the equation $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$ is

(A) $\left\{\dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}\right\}$ (B) $\left\{\dfrac{\pi}{12}, \dfrac{7\pi}{12}, \dfrac{17\pi}{12}, \dfrac{23\pi}{12}\right\}$

(C) $\left\{\dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{19\pi}{12}\right\}$ (D) $\left\{\dfrac{\pi}{12}, \dfrac{7\pi}{12}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}\right\}$

Q.75 In a triangle ABC, angle B < angle C and the values of B & C satisfy the equation $2\tan x - k(1+\tan^2 x) = 0$ where $(0 < k < 1)$. Then the measure of angle A is :

(A) $\pi/3$ (B) $2\pi/3$

(C) $\pi/2$ (D) $3\pi/4$

Q.76 If $x\sin\theta = y\sin\left(\theta + \dfrac{2\pi}{3}\right) = z\sin\left(\theta + \dfrac{4\pi}{3}\right)$ then :

(A) $x+y+z=0$ (B) $xy+yz+zx=0$

(C) $xyz+x+y+z=1$ (D) none

Q.77 The value of $\cos\dfrac{\pi}{10}\cos\dfrac{2\pi}{10}\cos\dfrac{4\pi}{10}\cos\dfrac{8\pi}{10}\cos\dfrac{16\pi}{10}$ is :

(A) 1/32 (B) 1/16

(C) $\dfrac{\cos(\pi/10)}{16}$ (D) $-\dfrac{\sqrt{10+2\sqrt{5}}}{64}$

Q.78 If $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ and $a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$. Then $(m+n)^{2/3} + (m-n)^{2/3}$ is equal to :

(A) $2a^2$ (B) $2a^{1/3}$

(C) $2a^{2/3}$ (D) $2a^3$

Q.79 The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$ is equal to:

(A) $\cot 3x$ (B) $\tan 3x$

(C) $3\tan 3x$ (D) $\dfrac{3 - 9\tan^2 x}{3\tan x - \tan^3 x}$

Q.80 The value of $\dfrac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ}$ is :

(A) $\cot 44^\circ$ (B) $\tan 44^\circ$

(C) $\tan 2^\circ$ (D) $\cot 46^\circ$

Q.81 The number of solutions of $\tan(5\pi\cos\theta) = \cot(5\pi\sin\theta)$ for θ in $(0, 2\pi)$ is :

(A) 28 (B) 14

(C) 4 (D) 2

Q.82 If $A = 340^\circ$ then $2\sin\dfrac{A}{2}$ is identical to

(A) $\sqrt{1+\sin A} + \sqrt{1-\sin A}$

(B) $-\sqrt{1+\sin A} - \sqrt{1-\sin A}$

(C) $\sqrt{1+\sin A} - \sqrt{1-\sin A}$

(D) $-\sqrt{1+\sin A} + \sqrt{1-\sin A}$

Q.83 If the value of the expression $\sin 25^\circ . \sin 35^\circ . \sin 85^\circ$ can be expressed as $\dfrac{\sqrt{a}+\sqrt{b}}{c}$ where $a, b, c \in N$ and are in their lowest form, find the value of $(a + b + c)$.

(A) 22 (B) 18

(C) 24 (D) 12

Q.84 The value of expression $\left(1+\cos\dfrac{3\pi}{10}\right)\left(1-\cos\dfrac{\pi}{10}\right)\left(1+\cos\dfrac{7\pi}{10}\right)\left(1-\cos\dfrac{9\pi}{10}\right)$ is

(A) 1/8 (B) 1/16

(C) 1/4 (D) 0

Q.85 If $m\tan(q - 30^\circ) = n\tan(q + 120^\circ)$, then $\dfrac{m+n}{m-n} =$

(A) $2\cos 2\theta$ (B) $\cos 2\theta$

(C) $2\sin 2\theta$ (D) $\sin 2\theta$

Q.86 The set of values of 'a' for which the equation, $\cos 2x + a\sin x = 2a - 7$ possess a solution is :

(A) $(-\infty, 2)$ (B) $[2, 6]$

(C) $(6, \infty)$ (D) $(-\infty, \infty)$

Q.87 In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(A) $\dfrac{\pi}{3}$ & $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{8}$ & $\dfrac{3\pi}{8}$

(C) $\dfrac{\pi}{4}$ & $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{5}$ & $\dfrac{3\pi}{10}$

Q.88 If $\tan \pi/8$ is a root of the equation $x^2 + bx + c = 0$, where $b, c \in Q$, then the ordered pair (b, c) is –

(A) (–2, 1) (B) (–2, –1)

(C) (2, –1) (D) (2, 1)

Q.89 Find the values of $\tan 15^\circ$

(A) $2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $1-\sqrt{3}$ (D) $\sqrt{3}$

Q.90 Find the value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$.

(A) 3 (B) 8

(C) 2 (D) 4

Q.91 $\sqrt{23}\sin 2x + \sqrt{2}\cos 2x = a^2 + 4a - 9$ has infinitely many solution for x if –

(A) $a = 1$ (B) $a < 1$

(C) $a = 2$ (D) $a > 2$

Q.92 The value of tan 6° tan 54° tan 66° is equal to –
(A) tan 36° (B) tan 18°
(C) cot 18° (D) tan 54°

Q.93 A circular wire of radius 7 cm is cut and bend again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is –
(A) 50° (B) 210°
(C) 100° (D) 60°

Q.94 The value of tan 100° + 4 sin 100° is equal to –
(A) $-\sqrt{3}$ (B) $\sqrt{3}$
(C) 2 (D) $\sqrt{3}\tan 10°$

Q.95 If $\operatorname{cosec}\theta = x + \frac{1}{4x}$ then the value of $\operatorname{cosec}\theta + \cot\theta$ is–
(A) 2x (B) –2x
(C) 1/2x (D) –1/2x

Q.96 The general solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is
(A) $2n\pi$ (B) $n\pi$
(C) $n\pi/3$ (D) $2n\pi/3$
[where $n \in I$]

Q.97 If $\tan\theta = \sqrt{n}$ for some non-square natural number n, then sec 2θ is
(A) a rational number (B) an irrational number
(C) a positive number (D) none of these

Q.98 If $\tan\theta = a \neq 0$, $\tan 2\theta = b \neq 0$ and $\tan\theta + \tan 2\theta = \tan 3\theta$ then
(A) a = b (B) ab = 1
(C) a + b = 0 (D) b = 2a

Q.99 Out of the following four relations :
(I) $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A} = \frac{2}{\sin A}$
(II) $\left(\frac{1+\cos A}{\sin A}\right)^2 = \frac{1+\cos A}{1-\cos A}$
(III) $\frac{\sin 10°}{\cos 80°} = 1$
(IV) $\sin^4 A - \cos^4 A = 1 + \sin^2 A$
the wrong one is –
(A) I (B) II
(C) III D) IV

Q.100 If $\tan q + \sin q = m$ and $\tan q - \sin q = n$, the value of $m^2 - n^2$ is equal to –
(A) 4 mn (B) $2\sqrt{mn}$
(C) $4\sqrt{mn}$ (D) $2\sqrt{m/n}$

EXERCISE - 2 [LEVEL-2]

Q.1 In a triangle ABC, D is on AC such that AD = BC, BD = DC, $\angle DBC = 2x$, $\angle BAD = 3x$, all angles in degrees, then the value of x, is –
(A) 5° (B) 10°
(C) 12° (D) 15°

Q.2 If $x \in (0, \pi)$ and $\cos x + \sin x = \frac{1}{2}$ then tan x is equal to –
(A) $\frac{4-\sqrt{7}}{3}$ (B) $\frac{4+\sqrt{7}}{3}$
(C) $\frac{-(4+\sqrt{7})}{3}$ (D) 6

Q.3 If $x + y = 3 - \cos 4\theta$ and $x - y = 4\sin 2\theta$ then
(A) $x^4 + y^4 = 9$ (B) $\sqrt{x} + \sqrt{y} = 16$
(C) $x^3 + y^3 = 2(x^2 + y^2)$ (D) $\sqrt{x} + \sqrt{y} = 2$

Q.4 $\frac{\tan\left(x - \frac{\pi}{2}\right) . \cos\left(\frac{3\pi}{2} + x\right) - \sin^3\left(\frac{7\pi}{2} - x\right)}{\cos\left(x - \frac{\pi}{2}\right) . \tan\left(\frac{3\pi}{2} + x\right)}$ when simplified reduces to :
(A) $\sin x \cos x$ (B) $-\sin^2 x$
(C) $-\sin x \cos x$ (D) $\sin^2 x$

Q.5 $\sin 3\theta = 4 \sin\theta \sin 2\theta \sin 4\theta$ in $0 \le \theta \le \pi$ has :
(A) 2 real solutions (B) 4 real solutions
(C) 6 real solutions (D) 8 real solutions.

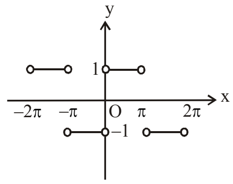
Q.6 tan 20° tan 40° tan 60° tan 80° =
(A) 1 (B) 2
(C) 3 (D) $\sqrt{3}/2$

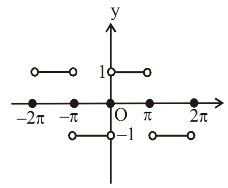
Q.7 Find the value of
$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$
(A) 1/2 (B) 1
(C) 3/4 (D) $\sqrt{3}/2$

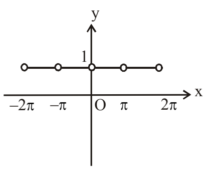
Q.8 $\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}.\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14}$ is equal to
(A) 1/64 (B) 1/32
(C) 1/16 (D) 1/8

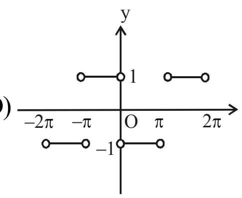
Q.9 The number of points inside or on the circle $x^2 + y^2 = 4$ satisfying $\tan^4 x + \cot^4 x + 1 = 3\sin^2 y$ is
(A) one (B) two
(C) four (D) infinite

Q.10 Graph of $y = \frac{|\sin x|}{\sin x}$ is –

(A)


(B)


(C)


(D)


Q.11 Let **R** denotes the value of $\sin\frac{\pi}{16}\cos\frac{\pi}{16}\left(\cos^4\frac{\pi}{16} - \sin^4\frac{\pi}{16}\right)$ and **S** denotes the value of $\tan^2 x$ if $\frac{\sin^2 x - 2\cos^2 x + 1}{\sin^2 + 2\cos^2 x - 1} = 4$ and the expression $\cos 2x - \frac{\sin x \cos 3x}{\sin 2x}$ has the value **T**, wherever defined. Find the value of RST.
(A) $\frac{5}{16\sqrt{2}}$ (B) $\frac{5}{4\sqrt{2}}$
(C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{3}{2\sqrt{2}}$

Q.12 Number of roots of the equation $\cos^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$ which lie in the interval $[-\pi, \pi]$ is
(A) 2 (B) 4
(C) 6 (D) 8

Q.13 Suppose that $\sin^3 x \sin 3x = \sum_{m=0}^{n} c_m \cos mx$ is an identity in x, where $c_0, c_1, c_2, ..., c_n$ are constants and $c_n \ne 0$, find the value of n.
(A) 2 (B) 6
(C) 3 (D) 4

Q.14 If $\frac{\sin(\theta+\alpha)}{\cos(\theta-\alpha)} = \frac{1-m}{1+m}$, find $\tan\left(\frac{\pi}{4} - \theta\right).\tan\left(\frac{\pi}{4} - \alpha\right)$
(A) m (B) 1/m
(C) –m (D) 2m

Q.15 For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi,\ y = \sum_{n=0}^{\infty} \sin^{2n}\phi,\ z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$$

then –
(A) $xyz = xz + y$ (B) $xyz = xy + z$
(C) $xy^2 = y^2 + x$ (D) none of these

Q.16 A circular wire of radius 7 cm is cut and bend again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is –
(A) 50° (B) 210°
(C) 100° (D) 60°

Q.17 If $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$ and $\tan\phi = \frac{y\sin\theta}{1 - y\cos\theta}$, then $x/y =$

(A) $\frac{\sin\phi}{\sin\theta}$ (B) $\frac{\sin\theta}{\sin\phi}$ (C) $\frac{\sin\theta}{1-\cos\theta}$ (D) $\frac{\sin\theta}{1-\cos\phi}$

Q.18 If angle θ be divided into two parts such that the tangent of one part is k times the tangent of the other and ϕ is their difference, then $\sin\theta =$

(A) $\frac{k+1}{k-1}\sin\phi$ (B) $\frac{k-1}{k+1}\sin\phi$

(C) $\frac{2k-1}{2k+1}\sin\phi$ (D) None of these

Q.19 The equation, $\cos 2x + a\sin x = 2a - 7$ possesses a solution if :
(A) $a < 2$ (B) $2 \le a \le 6$
(C) $a > 6$ (D) a is any integer

Q.20 The maximum value of $(7\cos\theta + 24\sin\theta) \times (7\sin\theta - 24\cos\theta)$ for every $\theta \in \mathbb{R}$.
(A) 25 (B) 625

(C) $\frac{625}{2}$ (D) $\frac{625}{4}$

Q.21 4 sin 5° sin55° sin65° has the values equal to

(A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (D) $\frac{3(\sqrt{3}-1)}{2\sqrt{2}}$

Q.22 If $x = \frac{n\pi}{2}$, satisfies the equation $\sin\frac{x}{2} - \cos\frac{x}{2} = 1 - \sin x$ & the inequality $\left|\frac{x}{2} - \frac{\pi}{2}\right| \le \frac{3\pi}{4}$, then:

(A) $n = -1, 0, 3, 5$ (B) $n = 1, 2, 4, 5$
(C) $n = 0, 2, 4$ (D) $n = -1, 1, 3, 5$

Q.23 The value of

$$\left(1+\cos\frac{\pi}{9}\right)\left(1+\cos\frac{3\pi}{9}\right)\left(1+\cos\frac{5\pi}{9}\right)\left(1+\cos\frac{7\pi}{9}\right) \text{ is}$$

(A) 9/16 (B) 10/16
(C) 12/16 (D) 5/16

Q.24 If θ is eliminated from the equations $x = a\cos(\theta - \alpha)$ and $y = b\cos(\theta - \beta)$ then $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos(\alpha - \beta)$ is equal to
(A) $\cos^2(\alpha - \beta)$ (B) $\sin^2(\alpha - \beta)$
(C) $\sec^2(\alpha - \beta)$ (D) $\text{cosec}^2(\alpha - \beta)$

Q.25 If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{p}{q}$ then $\tan\alpha \cdot \cot\beta$ has the value equal to

(A) $\frac{p+q}{p-q}$ (B) $\frac{p-q}{p+q}$ (C) $\frac{p+q}{q}$ (D) $\frac{p-q}{q}$

Q.26 Given ΔABC is inscribed in the semicircle with diameter AB. The area of ΔABC equals 2/9 of the area of the semicircle. If the measure of the smallest angle in ΔABC is x then sin 2x is equal to
(A) π/9 (B) 2π/9
(C) π/18 (D) π/8

Q.27 The average of the numbers $n\sin n°$ for $n = 2, 4, 6, ... 180$
(A) 1 (B) cot 1°
(C) tan 1° (D) 1/2

Q.28 An equilateral triangle, with sides of 10 inches, is inscribed in a square ABCD in such a way that one vertex is at A, another vertex on BC and one on CD. The area of the square is

(A) $25(2-\sqrt{3})$ (B) $25(2+\sqrt{3})$

(C) 25 (D) $\frac{100}{2+\sqrt{3}}$

Q.29 The value of
cos 5° + cos 77° + cos 149° + cos 221° + cos 293°
is equal to
(A) 0 (B) 1
(C) – 1 (D) 1/2

Q.30 If $\begin{cases} x\sec\theta + y\tan\theta = 2\cos\theta \\ x\tan\theta + y\sec\theta = \cot\theta \end{cases}$ then y equals

(A) $\frac{\cos 2\theta}{\sin\theta}$ (B) $\sin\theta$
(C) $\cos\theta$ (D) $\sin 2\theta$

Q.31 If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$

(C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

Q.32 If the graphs of $y = \cos x$ and $y = \tan x$ intersect at some value say θ in the first quadrant. Then the value of $\sin\theta$ is

(A) $\frac{-1+\sqrt{2}}{2}$ (B) $\frac{-1+\sqrt{3}}{2}$

(C) $\frac{-1+\sqrt{5}}{2}$ (D) $\frac{-1\pm\sqrt{5}}{2}$

Q.33 The point A $(\sin\theta, \cos\theta)$ is 3 units away from the point B $(2\cos 75°, 2\sin 75°)$. If $0° \le \theta < 360°$. Then θ is

(A) 15° (B) 165°
(C) 195° (D) 255°

Q.34 The most general solutions of the equation $x^{3\sin 2x + 2} = \sqrt{x}$ is

(A) $x = n\pi + (-1)^n \frac{\pi}{12}$ (B) $x = \frac{n\pi}{2} - (-1)^n \frac{\pi}{12}$

(C) $x = 0$ (D) $x = n\pi - (-1)^n \frac{\pi}{12}$

where $n \in I$

Q.35 The sum (in radians) of all values of x with $0 \le x \le 2\pi$ which satisfy $\sqrt{2}(\cos 2x - \sin x - 1) = 1 + 2\sin x$, is

(A) 2π (B) 3π
(C) 4π (D) 6π

Q.36 Find the solutions of the equation, $\log_{\sqrt{2}\sin x}(1+\cos x) = 2$ in the interval $x \in [0, 2\pi]$.

(A) $\pi/3$ (B) $\pi/2$
(C) $\pi/4$ (D) π

Q.37 Find the general solution of the equation, $2 + \tan x \cdot \cot\frac{x}{2} + \cot x \cdot \tan\frac{x}{2} = 0$

(A) $x = 2n\pi \pm \frac{2\pi}{3}$ (B) $x = 2n\pi \pm \frac{\pi}{3}$

(C) $x = 2n\pi \pm \frac{\pi}{4}$ (D) $x = 2n\pi \pm \frac{\pi}{2}$

Q.38 Evaluate $\sum_{n=1}^{89} \frac{1}{1+(\tan n°)^2}$

(A) 89 (B) 44.5
(C) 98 (D) 108

Q.39 Let $\tan\alpha \cdot \tan\beta = \frac{1}{\sqrt{2005}}$. Find the value of $(1003 - 1002\cos 2\alpha)(1003 - 1002\cos 2\beta)$

(A) 2005 (B) 2006
(C) 2008 (D) 2010

ASSERTION AND REASON QUESTIONS

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 5 choices (A), (B), (C), (D) and (E) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.40 If $P = 2\sin 2° + 4\sin 4° + \ldots\ldots\ldots 180\sin 180°$

Statement 1 : $\frac{\sqrt{1+\left(\frac{P}{90}\right)^2}}{P}$ is irrational.

Statement 2 : $\tan 1°$ is irrational.

Q.41 **Statement 1 :** $[\sin x] = 0$, if $x \in [0, \pi/2]$ (where [.] represents greatest integer function.

Statement 2 : $0 \le \sin x \le 1,\ x \in [0, \pi/2]$.

Q.42 **Statement 1 :** The number of integral values of λ, for which the equation $7\cos x + 5\sin x = 2\lambda + 1$ has a solution, is 8.

Statement 2 : $a\cos\theta + b\sin\theta = c$ has atleast one solution if $|c| > \sqrt{a^2+b^2}$.

Q.43 **Statement 1 :** $\sin 2 > \sin 3$

Statement 2 : If $x, y \in \left(\frac{\pi}{2}, \pi\right)$, $x < y$, then $\sin x > \sin y$.

Q.44 **Statement 1 :** $\cos 36° > \tan 36°$

Because

Statement 2 : $\cos 36° > \sin 36°$

MATCH THE COLUMN TYPE QUESTIONS

Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

Q.45 Match the column –

Column I	Column II
(a) Value of $3 + \left[\frac{1+\cos^2 x}{4}\right] + \left[\frac{1}{3}\sin^2 x\right]$ is (where [.] represents greatest integer function)	(p) 0
(b) If $N = \sin^2\alpha + \cos\left(\frac{\pi}{3}-\alpha\right).\cos\left(\frac{\pi}{3}+\alpha\right)$, then $5 + \log_2 N$ is equal to	(q) 2
(c) The number of integral values of λ, for which equation $6\cos x + 8\sin x = 2\lambda + 1$ has a solution, is 6.	(r) 3

(d) Least value of $3\sin^2\theta + 4\cos^2\theta$ is (s) 10

Code :

(A) a-r, b-r, c-s, d-r (B) a-p, b-q, c-r, d-s
(C) a-r, b-q, c-p, d-p (D) a-s, b-s, c-r, d-p

Q.46 Match the column –

Column I	Column II
(a) The number of real roots of the equation $\cos^7x + \sin^4x = 1$ in $(-\pi, \pi)$ is	(p) 1
(b) The value of $\sqrt{3}\operatorname{cosec} 20° - \sec 20°$ is	(q) 4
(c) $4\cos 36° - 4\cos 72° + 4\sin 18°\cos 36°$ equals	(r) 0
(d) The number of values of $x \in [-2\pi, 2\pi]$, which satisfy cosec x = 1 + cot x	(s) 3
	(t) 2

Code :

(A) a-r, b-r, c-s, d-t (B) a-s, b-q, c-s, d-t
(C) a-r, b-q, c-p, d-p (D) a-s, b-s, c-r, d-t

Q.47 Match the column –

Column I	Column II
(a) The number of solutions of the equation $\lvert\cot x\rvert = \cot x + \dfrac{1}{\sin x}\ (0 < x < \pi)$ is	(p) No solution
(b) If $\sin\theta + \sin\phi = \dfrac{1}{2}$ and $\cos\theta + \cos\phi = 2$, then value of $\cot\left(\dfrac{\theta+\phi}{2}\right)$ is	(q) 1/3
(c) The value of $\sin^2\alpha + \sin\left(\dfrac{\pi}{3}-\alpha\right)\sin\left(\dfrac{\pi}{3}-\alpha\right)$ is	(r) 1
(d) If $\tan\theta = 3\tan\phi$, then maximum value of $\tan^2(\theta - \phi)$ is	(s) 2
	(t) 3/4

Code :

(A) a-r, b-q, c-s, d-t (B) a-s, b-q, c-s, d-t
(C) a-r, b-q, c-t, d-p (D) a-r, b-p, c-t, d-q

PASSAGE BASED QUESTIONS

Passage 1- (Q.48-Q.50)

α is a root of the equation $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$; β is a root of the equation $3\cos^2x - 10\cos x + 3 = 0$ and γ is a root of the equation $1 - \sin 2x = \cos - \sin x$, $0 \le \alpha, \beta, \gamma \le \pi/2$

Q.48 $\cos\alpha + \cos\beta + \cos\gamma$ can be equal to –

(A) $\dfrac{3\sqrt{6}+2\sqrt{2}+6}{6\sqrt{2}}$ (B) $\dfrac{3\sqrt{3}-4}{6}$

(C) $\dfrac{3\sqrt{3}+2}{6}$ (D) None of these

Q.49 $\sin\alpha + \sin\beta + \sin\gamma$ can be equal to –

(A) $\dfrac{7-3\sqrt{2}}{6\sqrt{2}}$ (B) 5/6

(C) $\dfrac{3+4\sqrt{2}}{6}$ (D) $\dfrac{1+\sqrt{2}}{2}$

Q.50 $\sin(\alpha - \beta)$ is equal to –

(A) 1 (B) 0

(C) $\dfrac{1-2\sqrt{6}}{6}$ (D) $\dfrac{\sqrt{3}-2\sqrt{2}}{6}$

Passage 2- (Q.51-Q.53)

Let p be the product of the sines of the angles of a triangle ABC and q is the product of the cosines of the angles. Then

Q.51 tan A + tan B + tan C + tan A tan B tan C is equal to –

(A) 2p/q (B) p/q
(C) 2 (D) 2q/p

Q.52 tan A + tan B + tan C + tan A tan B + tan B tan C + tan C tanA is equal to –

(A) $\dfrac{1+p}{p}$ (B) $\dfrac{1+p+q}{q}$

(C) $\dfrac{1+p+q}{p}$ (D) $\dfrac{1+q}{p}$

Q.53 The value of $\tan^2A + \tan^2B + \tan^2C$ is equal to –

(A) $\dfrac{q^2-2p^2-2p}{p^2}$ (B) $\dfrac{q^2-2q^2+2q}{q^2}$

(C) $\dfrac{p^2-2q^2-2q}{q^2}$ (D) $\dfrac{p^2+2q^2-2q}{q^2}$

Passage 3- (Q.54-Q.56)

If θ is an angle one measured in radian and $\theta \in [0, 2\pi]$, then $r\theta$ is length of arc AB, of circle of radius r, subtending angle θ at the centre O, of the circle. Area of sector OAB is $\dfrac{1}{2}r^2\theta$.

Q.54 The angle between minute hand and hour hand of a clock at "half past 4" equals –

(A) 42° (B) 43°
(C) 44° (D) None of these

Q.55 The wheel of a train is 1 meter in diameter and it makes 5 revolutions per second. Then the speed of the train is approximately equal to –

(A) 57 km/hr (B) 66 km/hr
(C) 68 km/hr (D) 42.6 km/hr

Q.56 Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equal area. Then the angle θ between the lines is given by –

(A) $3\theta + 3\sin\theta = \pi$ (B) $6\theta + 3\sin\theta = \pi$
(C) $2\theta + \sin\theta = \pi$ (D) $\theta + \sin\theta = \pi/2$

Passage 4- (Q.57-Q.59)

Consider a triangle ABC, where x , y, z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a, b, c respectively let the letters R, r, S, Δ denote the circumradius, inradius semiperimeter and area of the triangle respectively.

Q.57 If $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}=\frac{a^2+b^2+c^2}{k}$, then the value of k is –

(A) R (B) S
(C) 2R (D) (3/2) R

Q.58 If $\cot A+\cot B+\cot C = k\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$, then the value of k is –

(A) R^2 (B) rR
(C) Δ (D) $a^2+b^2+c^2$

Q.59 The value of

$$\frac{c\sin B+b\sin C}{x}+\frac{a\sin C+c\sin A}{y}+\frac{b\sin A+a\sin B}{z}=$$

(A) R/r (B) S/R
(C) 2 (D) 6

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Find the value of tan 9° – tan 27° – cot 27° + cot 9°.

Q.2 If $x\in\left(\pi,\frac{3\pi}{2}\right)$, then find the value of

$$4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sqrt{4\sin^4 x+\sin^2 2x}.$$

Q.3 How many solutions are there for the equation $\cos^2x-\sin^2 2x=0$ on $[0, 2\pi]$?

Q.4 If $\sin^{16}a=\frac{1}{5}$, then the value of

$$\frac{1}{\cos^2 a}+\frac{1}{1+\sin^2 a}+\frac{2}{1+\sin^4 a}+\frac{4}{1+\sin^8 a}, \text{ is}$$

Q.5 If the sum of the solutions of the equation $\sin^2x-\sin x=\cos^2 x$ on the interval $[0, 2\pi]$ is expressed as $a\pi/b$, where a and b are positive integers, a/b in lowest terms, then (a + b) is

Q.6 Suppose x and y are real numbers such that tan x + tan y = 42 and cot x + cot y = 49. Find the value of tan (x + y).

Q.7 If $(\sin x+\cos x)^2+k\sin x\cos x=1$ holds $\forall\, x\in R$ then find the value of | k |.

Q.8 If the expression

$$\cos\left(x-\frac{3\pi}{2}\right)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(32\pi+x)$$
$$-18\cos(19\pi-x)+\cos(56\pi+x)-9\sin(x+17\pi)$$

is expressed in the form of a sin x + b cos x find the value of a + b.

Q.9 If m and n are positive integers satisfying

$$1+\cos 2\theta+\cos 4\theta+\cos 6\theta+\cos 8\theta+\cos 10\theta=\frac{\cos m\theta\cdot\sin n\theta}{\sin\theta}$$

then m + n is equal to

Q.10 If $a\sin^2x+b$ lies in the interval [–2, 8] for every $x\in R$ then find the value of (a – b).

Q.11 Find the exact value of

$$\tan^2\frac{\pi}{16}+\tan^2\frac{3\pi}{16}+\tan^2\frac{5\pi}{16}+\tan^2\frac{7\pi}{16}.$$

Q.12 If $\frac{1-\cos a-\tan^2\left(\frac{a}{2}\right)}{\sin^2\left(\frac{a}{2}\right)}=\frac{k\cos a}{w+p\cos a}$ where k, w and p have no common factor other than 1, then the value of $k^2+w^2+p^2$ is equal to

Q.13 The value of $\sqrt{3}$ cosec 20° – sec 20° is :

Q.14 If cos A, cos B and cos C are the roots of the cubic $x^3+ax^2+bx+c=0$ where A, B, C are the angles of a triangle then find the value of $a^2-2b-2c$.

Q.15 A triangle with sides a = 15, b = 28 and c = 41. The length of the altitude from the vertex B on the side AC is

Q.16 If the orthocentre and circumcentre of a triangle ABC be at equal distances from the side BC and lie on the same side of BC then tanB tanC has the value equal to :

Q.17 In a Δ ABC if b + c = 3a then $\cot\frac{B}{2}\cdot\cot\frac{C}{2}$ has the value equal to :

Q.18 In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and $l(CD)=6$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ has the value equal to 1/x. Find the value of x.

Q.19 If the medians AD and BE of a Δ ABC intersect at right angles then (a^2+b^2) is equal to xc^2. Find the value of x.

Q.20 If A, B, C are angles of a triangle then the minimum value of $\dfrac{\cos\frac{B-C}{2}}{\cos\frac{B+C}{2}}+\dfrac{\cos\frac{C-A}{2}}{\cos\frac{C+A}{2}}+\dfrac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}$

Q.21 Line ℓ is a tangent to a unit circle S at a point P. Point A and the circle S are on the same side of ℓ, and the distance from A to ℓ is 3. Two tangents from point A intersect line ℓ at the point B and C respectively. Find the value of (PB) (PC).

Q.22 The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations $(y + z) \cos 3\theta = (xyz) \sin 3\theta$;

$x \sin 3\theta = \dfrac{2\cos 3\theta}{y}+\dfrac{2\sin 3\theta}{z}$;

$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0z_0 \neq 0$, is

Q.23 The number of values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

Q.24 The maximum value of the expression

$\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Q.25 The positive integer value of $n > 3$ satisfying the equation $\dfrac{1}{\sin\left(\frac{\pi}{n}\right)}=\dfrac{1}{\sin\left(\frac{2\pi}{n}\right)}+\dfrac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 If cos x + cos y + cos α = 0 and sin x + sin y + sin α = 0, then $\cot\left(\frac{x+y}{2}\right) =$ **[AIEEE-2002]**

(A) sin α (B) cos α
(C) cot α (D) 2 sin α

Q.2 cos 1°. cos 2°. cos 3°.... cos 179° = **[AIEEE-2002]**

(A) 0 (B) 1
(C) 2 (D) 3

Q.3 Find the no. of roots of the equation tan x + sec x = 2 cos x in the interval [0, 2π]- **[AIEEE-2002]**

(A) 1 (B) 2
(C) 3 (D) 4

Q.4 General solution of tan 5θ = cot 2θ is- **[AIEEE-2002]**

(A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$

(C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (D) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}, n \in Z$

Q.5 The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is -**[AIEEE 2003]**

(A) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (B) $a\cot\left(\frac{\pi}{n}\right)$

(C) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (D) $a\cot\left(\frac{\pi}{2n}\right)$

Q.6 In a triangle ABC, medians AD and BE are drawn. In AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is- **[AIEEE 2003]**

(A) 64/3 (B) 8/3
(C) 16/3 (D) None of these

Q.7 If in a triangle ABC $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c **[AIEEE 2003]**

(A) satisfy a + b = c (B) are in A.P.
(C) are in G.P. (D) are in H.P.

Q.8 The sides of a triangle are sin α, cos α and $\sqrt{1+\sin\alpha\cos\alpha}$ for some 0 < α < π/2. Then the greatest angle of the triangle is- **[AIEEE 2004]**

(A) 60° (B) 90°
(C) 120° (D) 150°

Q.9 Let α, β be such that π < α – β < 3π. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is- **[AIEEE-2004]**

(A) $-\frac{3}{\sqrt{130}}$ (B) $\frac{3}{\sqrt{130}}$

(C) $\frac{6}{65}$ (D) $\frac{-6}{65}$

Q.10 A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meter away from the tree the angle of elevation becomes 30° . The breadth of the river is – **[AIEEE-2004]**

(A) 20 m (B) 30 m
(C) 40 m (D) 60 m

Q.11 In a triangle ABC, let ∠C = π/2. If r is the in-radius and R is the circumradius of the triangle ABC, then 2(r + R) equals - **[AIEEE-2005]**

(A) b + c (B) a + b
(C) a + b + c (D) c + a

Q.12 The number of values of x in the interval [0, 3π] satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is – **[AIEEE-2006]**

(A) 6 (B) 1
(C) 2 (D) 4

Q.13 If 0 < x < π, and cos x + sin x = 1/2, then tan x is – **[AIEEE-2006]**

(A) $(4 - \sqrt{7})/3$ (B) $-(4 + \sqrt{7})/3$

(C) $(1 + \sqrt{7})/4$ (D) $(1 - \sqrt{7})/4$

Q.14 A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is – **[AIEEE-2007]**

(A) $2a/\sqrt{3}$ (B) $2a\sqrt{3}$

(C) $a/\sqrt{3}$ (D) $a\sqrt{3}$

Q.15 AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7m. From D the angle of elevation of the point A is 45°. Then the height of the pole is – **[AIEEE-2008]**

(A) $\frac{7\sqrt{3}}{2}.\frac{1}{\sqrt{3}-1}$ m (B) $\frac{7\sqrt{3}}{2}.(\sqrt{3}+1)$ m

(C) $\frac{7\sqrt{3}}{2}.(\sqrt{3}-1)$ m (D) $\frac{7\sqrt{3}}{2}.\frac{1}{\sqrt{3}+1}$ m

Q.16 Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$, then : **[AIEEE-2009]**

(A) A is false and B is true (B) both A and B are true
(C) both A and B are false (D) A is true and B is false

Q.17 Let $\cos(\alpha + \beta) = 4/5$ and let $\sin(\alpha - \beta) = 5/13$, where $0 \le \alpha, \beta \le \pi/4$ then $\tan 2\alpha =$

(A) 56/33 (B) 19/12
(C) 20/7 (D) 25/16

Q.18 For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is – **[AIEEE 2010]**

(A) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(B) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(C) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

(D) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

Q.19 If $A = \sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011]**

(A) $\frac{3}{4} \le A \le 1$ (B) $\frac{13}{16} \le A \le 1$

(C) $1 \le A \le 2$ (D) $\frac{3}{4} \le A \le \frac{13}{16}$

Q.20 In a Δ PQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to : **[AIEEE 2012]**

(A) $5\pi/6$ (B) $\pi/6$
(C) $\pi/4$ (D) $3\pi/4$

Q.21 ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to – **[JEE MAIN 2013]**

(A) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (B) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(C) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (D) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

Q.22 The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : **[JEE MAIN 2013]**

(A) sin A cos A + 1 (B) sec A cosec A + 1
(C) tan A + cot A (D) sec A + cosec A

Q.23 A bird is sitting on the top of a vertical pole 20m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is – **[JEE MAIN 2014]**

(A) $40(\sqrt{2}-1)$ (B) $40(\sqrt{3}-\sqrt{2})$

(C) $20\sqrt{2}$ (D) $20(\sqrt{3}-1)$

Q.24 Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals – **[JEE MAIN 2014]**

(A) 1/6 (B) 1/3
(C) 1/4 (D) 1/12

Q.25 If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC, is **[JEE MAIN 2015]**

(A) $\sqrt{3} : \sqrt{2}$ (B) $1 : \sqrt{3}$

(C) 2 : 3 (D) $\sqrt{3} : 1$

Q.26 If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is **[JEE MAIN 2016]**

(A) 5 (B) 7
(C) 9 (D) 3

Q.27 A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30° . After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is : **[JEE MAIN 2016]**

(A) 10 (B) 20
(C) 5 (D) 6

Q.28 Let a vertical tower AB have its end A on the level ground. Let C be the mid point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to : **[JEE MAIN 2017]**

(A) 2/9 (B) 4/9
(C) 6/7 (D) 1/4

Q.29 If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is : **[JEE MAIN 2017]**

(A) 2/9 (B) –7/9
(C) –3/5 (D) 1/3

Q.30 If sum of all the solutions of the equation

$$8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$$

in $[0, \pi]$ is $k\pi$, then k is equal to: **[JEE MAIN 2018]**

(A) 8/9 (B) 20/9
(C) 2/3 (D) 13/9

Q.31 PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : **[JEE MAIN 2018]**

(A) $100\sqrt{3}$ (B) $50\sqrt{2}$
(C) 100 (D) 50

Q.32 For any $\theta \in (\pi/4, \pi/2)$ the expression
$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :
[JEE MAIN 2019 (JAN)]

(A) $13 - 4\cos^6\theta$
(B) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(C) $13 - 4\cos^2\theta + 6\cos^4\theta$
(D) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

Q.33 If $\cos(\alpha + \beta) = 3/5$, $\sin(\alpha + \beta) = 5/13$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : **[JEE MAIN 2019 (APRIL)]**

(A) 21/16 (B) 63/52
(C) 33/52 (D) 63/16

Q.34 If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :
[JEE MAIN 2019 (APRIL)]

(A) 5 : 9 : 13 (B) 5 : 6 : 7
(C) 4 : 5 : 6 (D) 3 : 4 : 5

Q.35 Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :
[JEE MAIN 2019 (APRIL)]

(A) 12 (B) 15
(C) 16 (D) 18

Q.36 Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is **[JEE MAIN 2019 (APRIL)]**

(A) $13\pi/6$ (B) π
(C) 2π (D) $5\pi/3$

Q.37 The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is
[JEE MAIN 2019 (APRIL)]

(A) $\frac{3}{2}(1 \cos 20°)$ (B) 3/4

(C) $\frac{3}{4} + \cos 20°$ (D) 3/2

Q.38 The value of sin 10º sin 30º sin 50º sin 70º is :
[JEE MAIN 2019 (APRIL)]

(A) 1/36 (B) 1/32
(C) 1/18 (D) 1/16

Q.39 Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15º with ground. Then the distance (in m) between the poles, is : **[JEE MAIN 2019 (APRIL)]**

(A) $\frac{5}{2}(2+\sqrt{3})$ (B) $5(\sqrt{3}+1)$

(C) $5(2+\sqrt{3})$ (D) $10(\sqrt{3}-1)$

Q.40 All the pairs (x, y) that satisfy the inequality

$2\sqrt{\sin^2 x - 2\sin x + 5}\cdot\frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation.

[JEE MAIN 2019 (APRIL)]

(A) $\sin x = |\sin y|$ (B) $\sin x = 2 \sin y$
(C) $2|\sin x| = 3 \sin y$ (D) $2 \sin x = \sin y$

Q.41 ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :
[JEE MAIN 2019 (APRIL)]

(A) $10\sqrt{5}$ (B) $\frac{100}{3\sqrt{3}}$

(C) 20 (D) 25

Q.42 The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:
[JEE MAIN 2019 (APRIL)]

(A) $15(3-\sqrt{3})$ (B) $15(3+\sqrt{3})$

(C) $15(1+\sqrt{3})$ (D) $15(5-\sqrt{3})$

Q.43 Let $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$

where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Then $\tan(\alpha + 2\beta)$ is equal to

[JEE MAIN 2020 (JAN)]

Q.44 The value of $\cos^3\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)$ is –

[JEE MAIN 2020 (JAN)]

(A) 1/4 (B) $1/\sqrt{2}$

(C) $1/2\sqrt{2}$ (D) 1/2

Q.45 The number of distinct solutions of the equation $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is ______. **[JEE MAIN 2020 (JAN)]**

Q.46 If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then – **[JEE MAIN 2020 (JAN)]**

(A) $y(1+x) = 1$
(B) $x(1+y) = 1$
(C) $y(1-x) = 1$
(D) $x(1-y) = 1$

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	B	D	B	B	C	C	C	C	D	B	A	C	D	B	B	C	A	A	A	C	C	B	A	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	C	A	B	D	B	C	D	B	C	B	C	C	B	A	C	A	A	A	A	C	B	D	A	B
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	B	B	B	C	A	C	B	B	C	D	B	A	A	D	D	C	D	D	B	D	B	A	B	B	C
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
A	B	D	C	D	A	A	D	C	B	A	B	B	C	B	D	C	B	B	A	A	C	A	C	D	C

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	C	D	D	D	C	A	A	C	A	A	B	B	A	B	B	B	A	B	C
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	B	B	A	B	A	A	B	B	A	A	A	C	C	B	D	A	A	B	A	B
Q	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	
A	D	C	A	B	A	B	D	A	C	C	A	B	C	D	A	A	C	C	D	

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	4	2	6	10	9	294	2	27	11	12	28	6	4	1	9	3	2	9	5	6	3	3	3	2	7

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	A	B	A	C	D	B	C	A	A	B	D	B	C	B	B	A	B	A	B
Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	A	B	D	D	D	A	C	A	B	D	C	A	D	C	C	C	B	D	C	A
Q	41	42	43	44	45	46														
A	C	B	1	C	8	C														

CHAPTER- 2:

TRY IT YOURSELF-1

(1) $\sin(270°-30°)\sin(540°-30°)\cos(360°+30°)$
$= -\cos 30°\sin 30° + \sin 30°\cos 30° = 0$

(2) $\cot A\cos B \mp 1 = \dfrac{\cos(A\pm B)}{\sin A\sin B}$

$44°+16°=60°$, $44°+76°=120°$
$76°-16°=60°$

$$\therefore \text{LHS} = \frac{\cos 60°\sin 76° + \cos 120°\sin 16° - \cos 60°\sin 44°}{\sin 16°\sin 44°\sin 76°}$$

$$= \frac{1}{2}\left[\frac{\sin 76° + \sin 16° - \sin 44°}{\sin 16°\sin 44°\sin 76°}\right]$$

$$= \frac{1}{2}\frac{2\sin 30°\cos 46° - \sin 44°}{\sin 16°\sin 44°\sin 76°}$$

$$= \frac{1}{2\sin 16°\sin 44°\sin 76°}(\cos 46° - \cos 46°) = 0$$

(3) $\cos(36°-A)\cos(36°+A) + \cos[90°-(54°+A)]$
$\sin[90°-(54°-A)] = \cos 2A$
$= \cos(36°-A)\cos(36°+A) + \sin(36°-A)\sin(36°+A)$
$= \cos[(36°+A)-(36°-A)] = \cos 2A$

(4) $\dfrac{1}{2}(2\sin 20°\sin 40°)\sin 80° = \dfrac{1}{2}(\cos 20° - \cos 60°)\sin 80°$

$$= \frac{1}{2}(\cos 20°\sin 80° - \frac{1}{2}\sin 80°)$$

$$= \frac{1}{4}(2\cos 20°\sin 80° - \sin 80°)$$

$$= \frac{1}{4}(\sin 100° + \sin 60° - \sin 80°)$$

$$= \frac{1}{4}(2\cos 90°\sin 10° + \sin 60°) = \frac{1}{4}\sin 60° = \frac{\sqrt{3}}{8}$$

(5) $\sin\dfrac{\pi}{14} = \sin\left(\dfrac{\pi}{2}-\dfrac{6\pi}{14}\right) = \cos\dfrac{6\pi}{14} = \cos\left(\pi - \dfrac{8\pi}{14}\right) = -\cos\dfrac{8\pi}{14}$

$$\sin\frac{3\pi}{14} = \sin\left(\frac{\pi}{2}-\frac{4\pi}{14}\right) = \cos\frac{4\pi}{14}$$

$$\sin\frac{5\pi}{14} = \sin\left(\frac{\pi}{2}-\frac{2\pi}{14}\right) = \cos\frac{2\pi}{14}$$

$$\therefore \text{LHS} = -\cos\frac{2\pi}{14}\cos\frac{4\pi}{14}\cos\frac{8\pi}{14}$$

$$= -\frac{1}{2^3\sin A}\sin(2^3 A),\ A = \frac{2\pi}{14}$$

$$= -\frac{1}{8\sin\frac{\pi}{7}}\sin\frac{8\pi}{7} = -\frac{1}{8\sin\frac{\pi}{7}}\sin\left(\pi+\frac{\pi}{7}\right)$$

$$= -\frac{1}{8}(1) = \frac{1}{8} \quad ;\ \sin(\pi+\theta) = -\sin\theta$$

(6) Let the fourth root of the equation be tan D
Now, $A+B+C=\pi$
Consider $\tan(A+B+C+D)$

$$= \frac{\Sigma\tan A - \Sigma\tan A\tan B\tan C}{1-\Sigma\tan\tan B + \Pi\tan A} = \frac{3-(-2)}{1-3+5} = \frac{5}{3}$$

$\tan(\pi+D) = \tan D = 5/3$

(7) For $x = 0°$, $\cos x = 1$
For $x = \pi$, $\cos x = -1$
For $x = \pi/2, \cos x = 0$
For $\cos x = 1$, $y_{max} = 5$
For $\cos x = -1$ or 0, $y_{min} = 1$

(8) $\sin\left(\dfrac{15\pi}{8} - 4x + \dfrac{17\pi}{8} - 4x\right)\sin\left(\dfrac{15\pi}{8} - \dfrac{17\pi}{8}\right)$

$$\Rightarrow \sin(4\pi - 8x)\sin\left(-\frac{\pi}{4}\right) \Rightarrow -\sin\frac{\pi}{4}\cdot\sin 8x$$

$$\Rightarrow -\frac{1}{\sqrt{2}}\cdot\sin 8x \Rightarrow -\frac{1}{\sqrt{2}} \le y \le \frac{1}{\sqrt{2}}$$

Ans. $y \in \left[-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right]$

(9) $\dfrac{4+\sec 20°}{\operatorname{cosec} 20°} = \left(\dfrac{4\cos 20° + 1}{\cos 20°}\right)\sin 20°$

$$= \frac{4\sin 20°\cos 20° + \sin 20°}{\cos 20°} = \frac{2\sin 40° + \sin 20°}{\cos 20°}$$

$$= \frac{\sin 40° + 2\sin 30°\cos 10°}{\cos 20°} = \frac{\sin 40° + \cos 10°}{\cos 20°}$$

$$= \frac{\sin 40° + \sin 80°}{\cos 20°} = \frac{2\sin 60°\cos 20°}{\cos 20°} = \sqrt{3}$$

Required value $= (\sqrt{3})^2 = 3$

(10) tan15°. tan45° tan75°
$= \tan 15°.\ \tan(60°-15°).\tan(60°+15°)$
$= \tan(3\times 15°) = \tan 45° = 1$

TRY IT YOURSELF-2

(1) $\cos 2\theta = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6} \Rightarrow 2\theta = 2n\pi \pm \dfrac{\pi}{6}$

$$\theta = n\pi \pm \frac{\pi}{12},\ n \in I$$

(2) $7(1-\sin^2\theta) + 3\sin^2\theta = 4$, $4\sin^2\theta = 3$

$$\sin^2\theta = \frac{3}{4} = \left(\frac{\sqrt{3}}{2}\right)^2 = \sin^2\frac{\pi}{3}$$

$\theta = n\pi \pm \dfrac{\pi}{3},\ n \in I$ $\quad 1 - \cos x\cot x = \cot x - \cos x.$

(3) $(1+\cos x)(1-\cot x)=0 \Rightarrow \cos x=-1$ or $\cot x=1$

$\Rightarrow (2n+1)\pi,\ \ n\pi+\frac{\pi}{4},\ \ n\in I$

(4) $8\sin^2 x\,.\,\cos^2 x+6\sin^2 x=5$

$8\sin^4 x-14\sin^2 x+5=0$

$\Rightarrow (2\sin^2 x-1)(4\sin^2 x-5)=0$

$\Rightarrow \sin^2 x=\frac{1}{2},\frac{5}{4}\Rightarrow \sin^2 x=\frac{1}{2}\Rightarrow n\pi\pm\frac{\pi}{4},\ n\in I$

(5) $\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1,\ \cos x\cos\frac{\pi}{6}+\sin x\sin\left(\frac{\pi}{6}\right)=1$

$\cos\left(x-\frac{\pi}{6}\right)=1$ so, $x-\frac{\pi}{6}=2n\pi,\ \ x=2n\pi+\frac{\pi}{6},\ \ n\in I$

(6) **(A).** $2\sin 3x\cos 2x=2\sin 3x\cos x$

$\Rightarrow \sin 3x=0$ or $\cos 2x=\cos x$

$\Rightarrow 3x=n\pi$ or $2x=2n\pi\pm x$

$\Rightarrow x=\frac{n\pi}{3},2n\pi,\frac{2n\pi}{3}\Rightarrow x=\frac{n\pi}{3}$

(7) $5\pi\cos\alpha=n\pi+\left(\frac{\pi}{2}-5\pi\sin\alpha\right)$

$\Rightarrow \sin\alpha+\cos\alpha=\frac{(2n+1)}{10}$ as $-\sqrt{2}\le\sin\alpha+\cos\alpha\le\sqrt{2}$

$n=0,\pm1,\pm2,\pm3,\pm4,\pm5,\pm6,-7$

For each value of n, we get two values of $\alpha\in[0,2\pi]$

$\therefore\ 2\times 14=28$ solutions.

(8) Let $\sin 2x\cos 2x=y$

$\Rightarrow \sin^4 2x+\cos^4 2x=\sin 2x\cos 2x.$

$\Rightarrow (\sin^2 2x+\cos^2 2x)-2\sin^2 2x\cos^2 2x=\sin 2x\cos 2x$

$\Rightarrow 1-2y^2=y\Rightarrow 2y^2+y-1=0$

$\Rightarrow y=-1,\ 1/2\Rightarrow \sin 2x\,.\,\cos 2x=y=-1,\ 1/2$

$\Rightarrow \sin 4x=-2$ (rejected), 1

$\Rightarrow \sin 4x=1\Rightarrow 4x=2n\pi+\frac{\pi}{2},\ \ n\in I\Rightarrow x=(4n+1)\frac{\pi}{8}$

(9) $\tan\theta+\tan 2\theta=\sqrt{3}\ (1-\tan\theta\tan 2\theta)\Rightarrow \tan 3\theta=\sqrt{3}$

$\therefore\ 3\theta=n\pi+\frac{\pi}{3}\ \ ;\ \ \theta=\frac{n\pi}{3}+\frac{\pi}{9}$

(10) **(B).** $\frac{3\sin^2\theta}{\cos^2\theta}-2\sin\theta=0,\ \cos\theta\ne 0$

$\Rightarrow 3\sin^2\theta-2\sin\theta(\cos^2\theta)=0,$

$3\sin^2\theta-2\sin\theta(1-\sin^2\theta)=0$

$\Rightarrow \sin\theta(2\sin^2\theta+3\sin\theta-2)=0$

$\Rightarrow \sin\theta(2\sin\theta-1)(\sin\theta+2)=0$

$\Rightarrow \sin\theta=0,\ 1/2,\ -2$ (rejected)

$\Rightarrow \theta=n\pi,\ n\pi+(-1)^n\frac{\pi}{6}$

(11) **(A).** $\sqrt{y+\frac{1}{y}}\ge\sqrt{2}$ assuming $y>0$

But $|\sin x+\cos x|\le\sqrt{2}$ so, $y=1$ & $x=\pi/4$

(12) **(D).** $\tan\left(\frac{p\pi}{4}\right)=\tan\left(\frac{\pi}{2}-\frac{q\pi}{4}\right)$

$\Rightarrow \frac{p\pi}{4}=n\pi+\frac{\pi}{2}-\frac{q\pi}{4}\Rightarrow\frac{(p+q)}{4}=n+\frac{1}{2}$

$\Rightarrow p+q=2(2n+1)$

(13) $\sin^{100}x=1+\cos^{100}x$

LHS ≤ 1, RHS ≥ 1. So, LHS = RHS = 1

$\cos^{100}x=0,\ \sin^{100}x=1\ ;\ x=n\pi\pm\frac{\pi}{2}$

(14) $-\sqrt{5}<4\sin x-1<\sqrt{5},\ \frac{-(\sqrt{5}-1)}{4}<\sin x<\frac{(\sqrt{5}-1)}{4}$

$\Rightarrow -\sin\left(\frac{\pi}{10}\right)<\sin x<\cos\left(\frac{2\pi}{10}\right)$

$\Rightarrow \sin\left(\frac{-\pi}{10}\right)<\sin x<\sin\left(\frac{3\pi}{10}\right)\ ;\ \ x\in\left(\frac{-\pi}{10},\frac{3\pi}{10}\right)$

TRY IT YOURSELF-3

(1) $\frac{b+c}{a}=\frac{\sin B+\sin C}{\sin A}=\frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$

$=\frac{\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}\cos\frac{A}{2}}=\frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$

So, $a\cos\left(\frac{B-C}{2}\right)=(b+c)\sin\frac{A}{2}$

(2) $A=75°, B=45°\Rightarrow C=60°$

$\frac{a}{\sin 75°}=\frac{b}{\sin 45°}=\frac{c}{\sin 60°}$

So, $b+\sqrt{2}c=\frac{\sin 45°}{\sin 75°}a+\sqrt{2}\frac{\sin 60°}{\sin 75°}a$

$b+c\sqrt{2}=\frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}a+\sqrt{2}\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}a=\frac{2}{\sqrt{3}+1}a+\frac{2\sqrt{3}a}{\sqrt{3}+1}=2a$

(3) $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c},\ \frac{\cos A}{\sin A}=\frac{\cos B}{\sin B}=\frac{\cos C}{\sin C}$

So, $\cot A=\cot B=\cot C\Rightarrow$ equilateral triangle.

Area $=\frac{\sqrt{3}}{4}a^2=\frac{\sqrt{3}}{4}(2)^2=\sqrt{3}$

(4) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{\frac{1}{4}(8 + 4\sqrt{3}) + 2 - 3}{\sqrt{12} + \sqrt{4}}$

$\dfrac{(1+\sqrt{3})}{2\,(1+\sqrt{3})} = \dfrac{1}{2}$; $A = \dfrac{\pi}{3}$

(5) Let $a = x^2 + x + 1,\ b = 2x + 1,\ c = x^2 - 1$

$a > 0 \Rightarrow x \in R$ $\qquad$ $b > 0 \Rightarrow x > -1/2$

$c > 0 \Rightarrow x < -1$ or $x > 1$ $\qquad$ So, $x \in (1, \infty)$

$a - b = x^2 - x > 0 \Rightarrow a > b,$ $\qquad$ $a - c = x + 2 > 0 \Rightarrow a > c$

So, angle $\angle A$ is largest angle.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2\,(2x+1)\,(x^2-1)}$$

$$= \frac{-(2x^3 + x^2 - 2x - 1)}{2\,(2x^3 + x^2 - 2x - 1)} = \frac{-1}{2} \quad \therefore \quad \angle A = \frac{2\pi}{3}$$

(6) $a\cos^2\dfrac{C}{2} + c\cos^2\dfrac{A}{2} = \dfrac{3b}{2}$

$\Rightarrow$ $a\,(1 + \cos C) + c\,(1 + \cos A) = 3b$

$a + c + a\cos C + c\cos A = 3b,$

$a + c + b = 3b,\ a + c = 2b$

So, a, b, c are in A.P.

(7) We have, $\tan\left(\dfrac{B-C}{2}\right) = \left(\dfrac{b-c}{b+c}\right)\cot\left(\dfrac{A}{2}\right)$

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\cdot\cot 15^\circ$$

$$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\cdot\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \text{ as } \tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

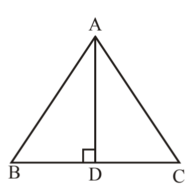
$\dfrac{B-C}{2} = 45^\circ, B - C = 90^\circ,\ A + B + C = 180^\circ,$

$B + C = 150^\circ, B = 120^\circ, C = 30^\circ$

Since A = C, we have a = c = 1

(8) The greatest altitude is perpendicular to the shortest side.

Let $a = 17, b = 25, c = 28$



Now, $\Delta = \dfrac{1}{2}AD\,(BC) \Rightarrow AD = \dfrac{2\Delta}{17}$

where $\Delta = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} = 210$

$AD = \dfrac{2 \times 210}{17} = \dfrac{420}{17}$

(9) $a + c = 2b$ (give)

We have to prove that $\cot\dfrac{A}{2} + \cot\dfrac{C}{2} = 2\cot\dfrac{B}{2}$

$$\sqrt{\frac{s\,(s-a)}{(s-b)\,(s-c)}} + \sqrt{\frac{s\,(s-c)}{(s-a)\,(s-c)}} = 2\sqrt{\frac{s\,(s-b)}{(s-c)\,(s-a)}}$$

Multiplying both side by $\sqrt{\dfrac{(s-a)\,(s-b)\,(s-c)}{s}}$

$(s-a) + (s-c) = 2\,(s-b)$

$a + c = 2b$ (which is the given relation)

(10) $\Delta = (a+b-c)\,(a-b+c) = (a+b+c-2c)\,(a+b+c-2b)$

$\Delta = (2s-2c)\,(2s-2b)$

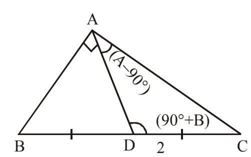
$\Rightarrow$ $\Delta^2 = [2\,(s-b)\,2\,(s-c)]^2$

$\Rightarrow$ $s\,(s-a)\,(s-b)\,(s-c) = 16\,(s-b)^2\,(s-c)^2$

$\Rightarrow$ $\dfrac{(s-b)\,(s-c)}{s\,(s-a)} = \dfrac{1}{16} \Rightarrow \tan^2\dfrac{A}{2} = \dfrac{1}{16} \Rightarrow \tan\dfrac{A}{2} = \dfrac{1}{4}$

$\Rightarrow$ $\tan A = \dfrac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}} = \dfrac{2 \times \frac{1}{4}}{1 - \frac{1}{16}} = \dfrac{8}{15}$

(11) **(C).** Applying (m – n) theorem, ΔABC



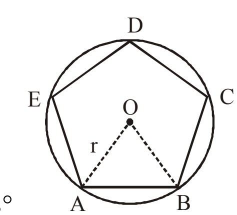
$(BD + CD)\cot(90^\circ + B) = BD\cot 90^\circ - CD\cot(A - 90^\circ)$

$\Rightarrow$ $-2\tan B = 0 + \tan A$

$\Rightarrow$ $\dfrac{\tan A}{\tan B} = -2$

(12) In ΔOAB, $OA = OB = r$

and $\angle AOB = \dfrac{360^\circ}{5} = 72^\circ$



$\therefore$ Area of ΔAOB

$= \dfrac{1}{2}(r)\,(r)\sin 72^\circ = \dfrac{r^2}{2}\cos 18^\circ$

A_2 = Area of pentagon $= \dfrac{5r^2}{2}\cos 18^\circ$

A_1 = Area of circle = πr^2 so, $\frac{A_1}{A_2} = \frac{2\pi}{5}\sec\left(\frac{\pi}{10}\right)$.

(13) **(A).** Given, $3\sin A + 4\cos B = 6$ ...... (1)

$3\cos A + 4\sin B = 1$ ...... (2)

Squaring and adding eqs. (1) and (2)

$(3\sin A + 4\cos B)^2 + (3\cos A + 4\sin B)^2 = 36 + 1$

$\Rightarrow 9 + 16 + 24(\sin A\cos B + \cos A\sin B) = 37$

$\Rightarrow \sin(A + B) = 1/2$

$\Rightarrow A + B = 30°$ or $150°$

When $A + B = 30°$ then

$(3\sin A + 4\cos B) < 3\sin 30° + 4\cos 30° < 6$

so, $A + B = 50°$

$\therefore \angle C = 30°$

(14) **(B).** $\sin A = \sin^2 B$ ...... (1)

$2\cos^2 A = 3\cos^2 B$ ...... (2)

$\Rightarrow 2(1 - \sin^2 A) = 3(1 - \sin A)$

$\Rightarrow \sin A = 1, 1/2$

But $\sin A \neq 1$ [From eq. (1)]

$\therefore \sin A = 1/2 \Rightarrow A = 30°$ or $150°$

$\sin^2 A = 1/2 \Rightarrow B = 45°$ or $135°$

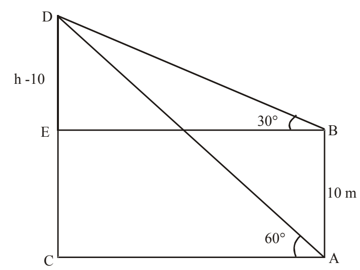
In each triangle is obtuse angled.

TRY IT YOURSELF-4

(1) Let AB and CD be the pole and tower respectively.

Let CD = h. Then $\angle DAC = 60°$ and $\angle DBE = 30°$

Now $\frac{CD}{CA} = \tan 60° = \sqrt{3}$ $\therefore$ $CD = \sqrt{3}\,CA \Rightarrow \frac{h}{\sqrt{3}} = CA$



Again $\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore (h - 10) = \frac{BE}{\sqrt{3}} = \frac{CA}{\sqrt{3}} = \frac{h/\sqrt{3}}{\sqrt{3}} = \frac{h}{3}$ [$\because$ BE = CA]

$\Rightarrow 3h - 30 = h \Rightarrow 2h = 30 \quad \Rightarrow h = 15$.

Hence, height of the tower = 15 m.

(2) (D) **(3)** (C) **(4)** (D) **(5)** (D)

(6) (B) **(7)** (B) **(8)** (C) **(9)** (A)

CHAPTER- 2 : TRIGONOMETRY

EXERCISE-1

(1) **(A).** If $\cos 3x = -1 = \cos(2n+1)\pi$
or, $3x = (2n+1)\pi$

$$x = (2n+1)\frac{\pi}{3} \text{ i.e., } x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

(2) **(B).** $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ$
$= \cos(360^\circ + 150^\circ)\cos(360^\circ - 30^\circ) + \sin(360^\circ + 30^\circ)\cos(90^\circ + 30^\circ)$
$= \cos 150^\circ \cos 30^\circ + \sin 30^\circ(-\sin 30^\circ)$

$$= \cos(180^\circ - 30^\circ)\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{4}$$

$$= -\cos 30^\circ\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{4} = -\frac{3}{4} - \frac{1}{4} = -1$$

(3) **(D).** $\dfrac{\operatorname{cosec}(2\pi+\theta).\cos(2\pi+\theta)\tan(\pi/2+\theta)}{\sec(\pi/2+\theta).\cos\theta.\cot(\pi+\theta)}$

$$= \frac{\operatorname{cosec}\theta\,.\cos\theta\,(-\cot\theta)}{(-\operatorname{cosec}\theta).\cos\theta.\cot\theta} = 1$$

(4) **(B).** $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta} = \tan\theta$

(5) **(B).** $\operatorname{cosec}\theta - \sin\theta = m$

$$m = \frac{1}{\sin\theta} - \sin\theta = \frac{\cos^2\theta}{\sin\theta} \quad ...(i)$$

$$n = \frac{1}{\cos\theta} - \cos\theta = \frac{\sin^2\theta}{\cos\theta} \quad ...(ii)$$

$$m \times n = \frac{\cos^2\theta}{\sin\theta}\,.\,\frac{\sin^2\theta}{\cos\theta} = \sin\theta\cos\theta$$

from (i) and (ii)
from (i) $\cos^2\theta = m\,.\sin\theta$
or $\cos^3\theta = m\sin\theta\cos\theta = m\,.(mn) = m^2n$
Similarly $\sin^3\theta = n^2m$
since $\sin^2\theta + \cos^2\theta = 1$
$(n^2m)^{2/3} + (m^2n)^{2/3} = 1$

(6) **(C).** $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)$

$$\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$= \frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right) = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$$

(7) **(C).** $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$$= \frac{\sqrt{3}}{2}\sin 20^\circ \sin(60^\circ - 20^\circ)\sin(60^\circ + 20^\circ)$$

$$= \frac{\sqrt{3}}{2}\sin 20^\circ(\sin^2 60^\circ - \sin^2 20^\circ)$$

$$= \frac{\sqrt{3}}{2}\sin 20^\circ\left(\frac{3}{4} - \sin^2 20^\circ\right) = \frac{\sqrt{3}}{8}(3\sin 20^\circ - 4\sin^3 20^\circ)$$

$$= \frac{\sqrt{3}}{8}\sin 60^\circ = \frac{\sqrt{3}}{8}\,.\,\frac{\sqrt{3}}{2} = \frac{3}{16}$$

(8) **(C).** Here $\alpha = \frac{\pi}{14}$, $\beta = \frac{2\pi}{14}$ and $n = 3$.

$$S = \frac{\cos\left[\frac{\pi}{14} + \left(\frac{3-1}{2}\right)\left(\frac{2\pi}{14}\right)\right]\sin\left(\frac{2\pi}{14}\times\frac{3}{2}\right)}{\sin\left(\frac{2\pi}{14}\times\frac{1}{2}\right)}$$

$$= \frac{2\cos\left(\frac{3\pi}{14}\right)\sin\left(\frac{3\pi}{14}\right)}{2\sin\left(\frac{\pi}{14}\right)}$$

$$S = \frac{\sin\left(\frac{6\pi}{14}\right)}{2\sin\left(\frac{\pi}{14}\right)} = \frac{\frac{1}{2}\sin\left(\frac{\pi}{2} - \frac{\pi}{14}\right)}{\sin\left(\frac{\pi}{14}\right)} = \frac{1}{2}\cot\left(\frac{\pi}{14}\right)$$

(9) **(C).** $\dfrac{1}{\sin 1^\circ}\left[\dfrac{\sin(2^\circ - 1^\circ)}{\sin 1^\circ \sin 2^\circ} + \dfrac{\sin(3^\circ - 2^\circ)}{\sin 2^\circ \sin 3^\circ} + ... + \dfrac{\sin(90^\circ - 89^\circ)}{\sin 89^\circ \sin 90^\circ}\right]$

$$\frac{1}{\sin 1^\circ}[\cot 1^\circ - \cot 2^\circ + \cot 2^\circ - \cot 3^\circ + ... + \cot 89^\circ - \cot 90^\circ]$$

$$\frac{1}{\sin 1^\circ}[\cot 1^\circ - \cot 90^\circ] = \frac{\cos 1^\circ}{\sin^2 1^\circ}$$

(10) **(D).** We have, $\dfrac{\sin 69^\circ - \sin 21^\circ}{\sin(57^\circ + 33^\circ)\sin(57^\circ - 33^\circ)}$

$$= \frac{2\cos 45^\circ \sin 24^\circ}{\sin 90^\circ \sin 24^\circ} = \sqrt{2}$$

(11) **(B).** $\cot(102°) = -\tan 12°$

$\therefore \cot 12° (-\tan 12°) + \cot 66° [-\tan 12° + \cot 12°]$

$= -1 + \cot 66° \left[1 - \frac{\tan^2 12°}{\tan 12°}\right]$

$= -1 + \cot 66° \times \cot 24° \times 2 = -1 + \cot 66° (\tan 66°) \times 2$

$= -1 + 2 = 1$

(12) **(A).** $\sin 10° \sin 30° \sin 50° \sin 70°$

$\sin 10° \left(\frac{1}{2}\right) \frac{1}{2} \{\cos 20° - \cos 120°\}$

$= \frac{1}{4} \sin 10° (\cos 20° + \frac{1}{2})$

$\frac{1}{4} \sin 10° \cos 20° - \frac{1}{8} \sin 10°$

$= \frac{1}{4} \times \frac{1}{2} (\sin 30° - \sin 10°) + \frac{1}{8} \sin 10° = \frac{1}{16}$

(13) **(C).** $\frac{\sin 70° + \cos 40°}{\cos 70° + \sin 40°} = \frac{\sin 70° + \sin 50°}{\sin 20° + \sin 40°}$

$= \frac{2\sin(60°)\cos(10°)}{2\sin 30° \cos 10°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

(14) **(D).** log sin 1. log sin 2….log sin 90…. log sin 179

= log sin 1. log sin 2….log 1……..log sin 179 = 0

(15) **(B).** $\tan 1° + \tan 89° = \tan 1° + \cot 1°$

$= 2 \text{ cosec } (2°) = \frac{2}{\sin(2°)}$

(16) **(B).** $\sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right) + \sin 2C$

$= 2\sin(A+B).\cos(A-B) + \sin 2C$

$= 2\sin(\pi - C).\cos(A-B) + \sin 2C$

$[\because A + B + C = \pi, A + B = \pi - C$

$\therefore \sin(A+B) = \sin(\pi - C) = \sin C]$

$= 2\sin C \cos(A-B) + 2\sin C \cos C$

$= 2\sin C [\cos(A-B) + \cos C]$

$= 2\sin C [\cos(A-B) - \cos(A+B)]$

$[\because \cos(A-B) - \cos(A+B) = 2\sin A.\sin B$, By C & D formula]

$= 2\sin C [2\sin A \sin B] = 4\sin A \sin B \sin C$

(17) **(C).** $A + B + C = \pi$

$A + B = \pi - C \Rightarrow \tan(A+B) = \tan(\pi - C)$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \tan A + \tan B = -\tan C + \tan A.\tan B.\tan C$

$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$

(18) **(A).** $2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + \cos C$

$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2}$

$= 1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right)$

$= 1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)$

$= 1 + 2\sin\frac{C}{2}\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right) = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(19) **(A).** $\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1$

$= 2\cos(180° - C)\cos(A-B) + 2\cos^2 C - 1$

$= 1 - 2\cos C \{\cos(A-B) - \cos C\}$

$= -1 - 2\cos C \{\cos(A-B) + \cos(A+B)$

$[\because \cos C = -\cos(A+B)]$

$= -1 - 2\cos C \{2\cos A \cos B\} = -1 - 4\cos A \cos B \cos C$

(20) **(A).** $\cos 2A + \cos 2B - \cos 2C$

$= 2\cos(A+B)\cos(A-B) - 2\cos^2 C + 1$

$= 1 + 2\cos(180° - C)\cos(A-B) - 2\cos^2 C$

$= 1 - 2\cos C \{\cos(A-B) + \cos C\}$

$= 1 - 2\cos C \{\cos(A-B) - \cos(A+B)\}$

$= 1 - 2\cos C (2\sin A \sin B)$

$= 1 - 4\sin A \sin B \cos C$

(21) **(C).** $\sin A + \sin B - \sin C$

$= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin\left(90° - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left\{\cos\frac{A-B}{2} - \sin\frac{C}{2}\right\}$

$= 2\cos\frac{C}{2}\left\{\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right\}$

$= 2\cos\frac{C}{2}\left\{2\sin\frac{A}{2}\sin\frac{B}{2}\right\} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(22) **(C).** Since, $\frac{B}{2} + \frac{C}{2} = \left(90° - \frac{A}{2}\right)$

$\therefore \tan\left(\frac{B}{2} + \frac{C}{2}\right) = \tan\left(90° - \frac{A}{2}\right)$

or $\tan\left(\frac{B}{2} + \frac{C}{2}\right) = \cot\frac{A}{2} = \frac{1}{\tan\frac{A}{2}}$

$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1 - \tan\frac{B}{2}\tan\frac{C}{2}$

or $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(23) **(B).** We know, $A + B = 180° - C$

$\therefore \cot(A+B) = -\cot C$

or $\dfrac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$

or $\cot A \cot B - 1 = -\cot A \cot C - \cot B \cot C$

or $\cot A \cot B + \cot A \cot C + \cot B \cot C = 1$

(24) **(A).** $\dfrac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C}$

$= \dfrac{2\sin(A+B)\cos(A-B) + 2\sin C\cos C}{\sin A + \sin B + \sin C}$

$= \dfrac{2\sin C\{\cos(A-B) + \cos C\}}{\sin A + \sin B + \sin C}$

$= \dfrac{2\sin C\{\cos(A-B) - \cos(A+B)\}}{\sin A + \sin B + \sin C} = \dfrac{2\sin C.2\sin A\sin B}{\sin A + \sin B + \sin C}$

$= \dfrac{32\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$\text{LHS} = 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(25) **(B).** $4S = (3\cos\alpha + \cos 3\alpha) + (3\cos 3\alpha + \cos 9\alpha)$

$+ (3\cos 5\alpha + \cos 15\alpha) + \ldots..$

$= 3(\cos\alpha + \cos 3\alpha + \cos 5\alpha + \ldots\ldots.)$

$(\cos 3\alpha + \cos 9\alpha + \cos 15\alpha + \ldots..)$

$= \dfrac{3\sin n\alpha}{\sin\alpha}\cos\left\{\dfrac{\alpha + (2n-1)\alpha}{2}\right\}$

$+ \dfrac{\sin 3n\alpha}{\sin 3\alpha}\cos\left\{\dfrac{3\alpha + (2n-1)3\alpha}{2}\right\}$

$\therefore\ S = \dfrac{3\sin n\alpha\cos n\alpha}{4\sin\alpha} + \dfrac{\sin 3n\alpha\cos 3n\alpha}{4\sin 3\alpha}$

(26) $\sin 3\theta = \sin\theta$ or, $3\theta = m\pi + (-1)^m\theta$

For (m) even i.e. $m = 2n$,

then $\theta = \dfrac{2n\pi}{2} = n\pi$ and for (m) odd i.e. $m = (2n+1)$

or, $\theta = (2n+1)\dfrac{\pi}{4}$

(27) **(C).** The given equation can be written as

$\dfrac{1}{2}(\sin 8x + \sin 2x) = \dfrac{1}{2}(\sin 8x + \sin 4x)$

or, $\sin 2x - \sin 4x$

$\Rightarrow\ -2\sin x\cos 3x = 0$. Hence $\sin x = 0$ or $\cos 3x = 0$.

That is, $x = n\pi\ (n \in I)$, or $3x = k\pi + \dfrac{\pi}{2}\ \ (k \in I)$.

Therefore, since $x \in [0, \pi]$, the given equation is satisfied

if $x = 0, \pi, \dfrac{\pi}{6}, \dfrac{\pi}{2}$ or $\dfrac{5\pi}{6}$

(28) **(A).** $5\sec\theta - 13 = 12\tan\theta$

or, $13\cos\theta + 12\sin\theta = 5$

or, $\dfrac{13}{\sqrt{13^2+12^2}}\cos\theta + \dfrac{12}{\sqrt{13^2+12^2}}\sin\theta = \dfrac{5}{\sqrt{13^2+12^2}}$

or, $\cos(\theta - \alpha) = \dfrac{5}{\sqrt{313}}$, where $\cos\alpha = \dfrac{13}{\sqrt{313}}$

$\therefore\ \theta = 2n\pi \pm \cos^{-1}\dfrac{5}{\sqrt{313}} + \alpha$

$= 2n\pi \pm \cos^{-1}\dfrac{5}{\sqrt{313}} + \cos^{-1}\dfrac{13}{\sqrt{313}}$

As $\cos^{-1}\dfrac{5}{\sqrt{313}} > \cos^{-1}\dfrac{13}{\sqrt{313}}$,

then $\theta \in [0, 2\pi]$, when $n = 0$ (One value, taking positive sign) and when $n = 1$ (One value, taking negative sign.)

(29) **(B).** We have, $\tan\left(\dfrac{\pi}{2}\sin\theta\right) = \cot\left(\dfrac{\pi}{2}\cos\theta\right)$

$\Rightarrow\ \tan\left(\dfrac{\pi}{2}\sin\theta\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\cos\theta\right)$

$\Rightarrow\ \dfrac{\pi}{2}\sin\theta = r\pi + \dfrac{\pi}{2} - \dfrac{\pi}{2}\cos\theta,\ r \in Z$

$\Rightarrow \sin\theta + \cos\theta = (2r+1),\ r \in Z$

$\Rightarrow\ \dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta = \dfrac{2r+1}{\sqrt{2}},\ r \in Z$

$\Rightarrow\ \cos\left(\theta - \dfrac{\pi}{4}\right) = \dfrac{2r+1}{\sqrt{2}},\ r \in Z$

$\Rightarrow\ \cos\left(\theta - \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$ or $-\dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta - \dfrac{\pi}{4} = 2r\pi \pm \dfrac{\pi}{4},\ r \in Z \Rightarrow\ \theta = 2r\pi \pm \dfrac{\pi}{4} + \dfrac{\pi}{4},\ r \in Z$

$\Rightarrow\ \theta = 2r\pi, 2r\pi + \dfrac{\pi}{2},\ r \in Z$ But $\theta = 2r\pi + \dfrac{\pi}{2},\ r \in Z$ gives extraneous roots as it does not satisfy the given equation.

Therefore $\theta = 2r\pi,\ r \in Z$

(30) **(D).** Given equation is, $\sec 4\theta - \sec 2\theta = 2$

or, $\dfrac{1}{\cos 4\theta} - \dfrac{1}{\cos 2\theta} = 2,\ \cos 4\theta \neq 0,\ \cos 2\theta \neq 0$

or, $\cos 2\theta - \cos 4\theta = 2\cos 4\theta\cos 2\theta$

or, $\cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta$

or, $\cos 6\theta + \cos 4\theta = 0$ or $2\cos 5\theta\cos\theta = 0$

$\therefore$ either $\cos 5\theta = 0$ or $\cos\theta = 0$

If $\cos 5\theta = 0$, then $5\theta = (2n+1)\pi/2$

or, $\theta = (2n+1)\pi/10$, where $n \in I$.
and if $\cos\theta = 0$, then $\theta = (2n+1)\pi/2$, where $n \in I$.
obviously for $\theta = (2n+1)\pi/2$ and $\theta = (2n+1)\pi/10$, $\cos 2\theta$ or $\cos 4\theta$ are not zero
Hence $\theta = (2n+1)\pi/2$, $(2n+1)\pi/10$ are the general solutions of the given equation.

(31) **(B).** The given equation can be written as
$4\sin^4 x + 4\cos^4 x = 4\sin x\cos x$
or, $(1-\cos 2x)^2 + (1+\cos 2x)^2 = 2\sin 2x$
or, $2(1+\cos^2 2x) = 2\sin 2x$
$\Rightarrow 1+\cos^2 2x = \sin 2x$ or, $1 + 1 - \sin^2 2x = \sin 2x$
$\Rightarrow \sin^2 2x + \sin 2x = 2$
This relation is possible if and only if $\sin 2x = 1$

or, $2x = 2n\pi + \dfrac{\pi}{2} \Rightarrow x = n\pi + \dfrac{\pi}{4} = \dfrac{(4n+1)\pi}{4}$ $(n \in I)$

(32) **(C).** If $\cot x > 0$ then $\dfrac{1}{\sin x} = 0$ (impossible)

Now if $\cot x < 0$ then $-\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{2\cos x + 1}{\sin x} = 0 \Rightarrow \cos x = -\dfrac{1}{2} \Rightarrow \cos x = \cos\left(\dfrac{2\pi}{3}\right)$

$x = 2n\pi \pm \dfrac{2\pi}{3}$; $n \in I$ and $0 \le x \le 2\pi$ then $x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

(33) **(D).** $\sin\dfrac{\pi}{2n} + \cos\dfrac{\pi}{2n} = \sqrt{2}\sin\left(\dfrac{\pi}{2n} + \dfrac{\pi}{4}\right)$

or, $\sin\left(\dfrac{\pi}{2n} + \dfrac{\pi}{4}\right) = \dfrac{\sqrt{n}}{2\sqrt{2}}$

since $\dfrac{\pi}{4} < \dfrac{\pi}{2n} + \dfrac{\pi}{4} < \dfrac{3\pi}{4}$ for $n > 1$

or, $\dfrac{1}{\sqrt{2}} < \dfrac{\sqrt{n}}{2\sqrt{2}} \le 1$ or, $2 < \sqrt{n} \le 2\sqrt{2}$ or, $4 < n \le 8$.
If $n = 1$, L.H.S. $= 1$, R.H.S. $= 1/2$

Similarly for $n = 8$, $\sin\left(\dfrac{\pi}{16} + \dfrac{\pi}{4}\right) \ne 1$ $\therefore 4 < n < 8$

(34) **(B).** We know that $\sin\theta = 0$, then $\theta = n\pi$
$\sin 2\theta = 0$
or, $2\theta = n\pi$; $n \in I$

$\theta = \dfrac{n\pi}{2}$; $n \in I$

(35) **(C).** If $\cos\theta = \dfrac{1}{2}$ or $\cos\theta = \cos\left(\dfrac{\pi}{3}\right)$; $\theta = 2n\pi \pm \dfrac{\pi}{3}$; $n \in I$

(36) **(B).** If $(\sin 5\theta + \sin\theta) + \sin 3\theta = 0$
or, $2\sin 3\theta\cos 2\theta + \sin 3\theta = 0$
or, $\sin 3\theta(2\cos 2\theta + 1) = 0$
Case I : $\sin 3\theta = 0 \Rightarrow 3\theta = n\pi$; $n \in I$

$\Rightarrow \theta = \dfrac{n\pi}{3}$; $n \in I$

Case II : $2\cos 2\theta + 1 = 0$

$\Rightarrow \cos 2\theta = -\dfrac{1}{2} \Rightarrow \cos 2\theta = \cos\dfrac{2\pi}{3}$

$\Rightarrow \theta = m\pi \pm \dfrac{\pi}{3}$; $m \in I$

So the general solution of the given equation is

$\theta = \dfrac{n\pi}{3}$ and $\theta = m\pi \pm \dfrac{\pi}{3}$, where $m, n \in I$

(37) **(C).** If $2\cos^2\theta + 3\sin\theta = 0$
$\Rightarrow 2(1-\sin^2\theta) + 3\sin\theta = 0$
$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$
$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$
$\Rightarrow 2\sin\theta(\sin\theta - 2) + (\sin\theta - 2) = 0$
$\Rightarrow (\sin\theta - 2)(2\sin\theta + 1) = 0$
Case I : If $\sin\theta - 2 = 0$
$\sin\theta = 2$
Which is not possible because $-1 \le \sin\theta \le 1$
Case II : If $2\sin\theta + 1 = 0$

$\Rightarrow \sin\theta = -\dfrac{1}{2}$ or, $\sin\theta = \sin\left(\dfrac{-\pi}{6}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\dfrac{-\pi}{6}\right)$; $n \in I$

$\Rightarrow \theta = n\pi + (-1)^{n+1}\left(\dfrac{\pi}{6}\right)$; $n \in I$

(38) **(C).** If $\cos^2\theta = \dfrac{1}{2}$ or, $\cos^2\theta = \left(\dfrac{1}{\sqrt{2}}\right)^2$

or, $\cos^2\theta = \cos^2\left(\dfrac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \dfrac{\pi}{4}$; $n \in I$

(39) **(B).** $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ ....(1)
this is the form of $a\cos\theta + b\sin\theta = c$
where $a = \sqrt{3}$, $b = 1$ and $c = \sqrt{2}$
Let $a = r\cos\alpha$, and $b = r\sin\alpha$
i.e., $\sqrt{3} = r\cos\alpha$ and $1 = r\sin\alpha$

then $r = 2$ and $\tan\alpha = \dfrac{1}{\sqrt{3}}$, so $\alpha = \dfrac{\pi}{6}$

Substituting $a = \sqrt{3} = r\cos\alpha$
and $b = 1 = r\sin\alpha$ in the equation (1)
so, $r[\cos\alpha\cos\theta + \sin\theta\sin\alpha] = \sqrt{2}$

or, $r\cos(\theta - \alpha) = \sqrt{2}$ or, $2\cos\left(\theta - \dfrac{\pi}{6}\right) = \sqrt{2}$

or, $\cos\left(\theta - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{2}}$

or, $\cos\left(\theta-\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{4}\right)$ or, $\theta-\frac{\pi}{6}=2n\pi\pm\frac{\pi}{4}$; $n\in I$

$\theta=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{6}$; $n\in I$

(40) **(A).** $\because$ $\tan\theta$ is negative

$\therefore$ θ will lie in 2nd or 4th quadrant.

For 2nd quadrant we will select anticlockwise and for 4th quadrant, we will select clockwise direction.

In the first circle two values $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$ are obtained.

Among these two, $-\frac{\pi}{4}$ is numerically least angle. Hence principal value is $-\frac{\pi}{4}$.

(41) **(C).** $\cos\theta=\frac{1}{\sqrt{2}}=\cos\left(\frac{\pi}{4}\right)$; $\theta=2n\pi\pm\frac{\pi}{4}$; $n\in I$

Put $n=1$, $\theta=\frac{9\pi}{4},\frac{7\pi}{4}$

$\tan\theta=-1=\tan\left(\frac{-\pi}{4}\right)$; $\theta=n\pi-\frac{\pi}{4}$; $n\in I$

put $n=1$, $\theta=\frac{3\pi}{4}$; put $n=2$, $\theta=\frac{7\pi}{4}$

The common value which satisfies both these equation is $\left(\frac{7\pi}{4}\right)$. Hence the general value is $2n\pi+\frac{7\pi}{4}$

(42) **(A).** $\sin x+\cos x=\sqrt{2}$

$\sin x=\sqrt{2}-\cos x$

$\sin^2 x=2-2\sqrt{2}\cos x+\cos^2 x$ [square both sides]

$1-\cos^2 x=2-2\sqrt{2}\cos x+\cos^2 x$

$0=2\cos^2 x-2\sqrt{2}\cos x+1$

$\cos x=\frac{2\sqrt{2}\pm\sqrt{8-8}}{4}=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$

If $\cos x=\frac{\sqrt{2}}{2}$, then $x=\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$\sin x+\cos x=\sqrt{2}$

$\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\overset{?}{=}\sqrt{2}$

$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\overset{?}{=}\sqrt{2}$

$\frac{2\sqrt{2}}{2}=2$

$\sin x+\cos x=\sqrt{2}$

$\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\overset{?}{=}\sqrt{2}$

$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\overset{?}{=}\sqrt{2}$

$\frac{2\sqrt{2}}{2}=2$

The only solution is $\pi/4$.

(43) **(A).** $4\sin^4 x+4\cos^4 x=4\sin x\cos x$

or, $(1-\cos 2x)^2+(1+\cos 2x)^2=2\sin 2x$

or $2(1+\cos^2 2x)=2\sin 2x$

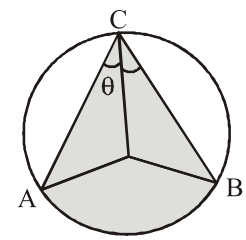
$\Rightarrow$ $1+\cos^2 2x=\sin 2x$ or $1+1-\sin^2 2x=\sin 2x$

$\Rightarrow$ $\sin^2 2x+\sin 2x=2$

$\sin 2x=1$ or $2x=2n\pi+\frac{\pi}{2}$

$\Rightarrow x=n\pi+\frac{\pi}{4}=\frac{(4n+1)\pi}{4}(n\in I)$

(44) **(A).** Area of region ABC $=\frac{\pi r^2}{3}$



Area of OAB $=\frac{1}{2}r^2.2\theta=r^2\theta$

Area of OAC $=\frac{1}{2}r^2\sin\theta$ = Area of OBC

$\therefore$ $\frac{1}{2}r^2\sin\theta+\frac{1}{2}r^2\sin\theta+r^2\theta=\frac{\pi r^2}{3}$

$\Rightarrow$ $3\sin\theta+3\theta=\pi$

(45) **(A).** $1+\cos x=|\sin x|^2=\sin^2 x$

$\cos x\,(1+\cos x)=0\Rightarrow 1+\cos x=0$ is not possible

then $\cos x=0\Rightarrow\sin x=1$ Not possible

(46) **(C).** $\frac{2}{\cos 2\alpha}=\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta}$

$\frac{2}{\cos 2\alpha}=\frac{2}{\sin 2\beta}\Rightarrow\cos 2\alpha=\sin 2\beta$

$\Rightarrow\cos 2\alpha=\cos\left(\frac{\pi}{2}-2\beta\right)$

$\Rightarrow 2\alpha=\frac{\pi}{2}-2\beta\Rightarrow\alpha+\beta=\frac{\pi}{4}$

(47) **(B).** Obviously, $\sin 2\theta=\sin\left(\frac{\pi}{2}-3\theta\right)$

$\Rightarrow$ $2\theta=\frac{\pi}{3}-3\theta\Rightarrow 5\theta=\frac{\pi}{2}$ $\therefore$ $\theta=18°=\frac{\pi}{10}$

(48) **(D).** $-\sqrt{25}\le 3\sin\theta+4\cos\theta\le\sqrt{25}$

[By the standard results]

or, $-5\le 3\sin\theta+4\cos\theta\le 5$

so the maximum value is 5.

(49) **(A).** $\sin^{-1}(\sin 4)=\pi-4$

$x^2-kx+\pi-4>0\ \forall\ x\in R$

$D<0$ i.e. $k^2-4(\pi-4)<0$

$k^2+4(4-\pi)<0$, which is not true

(50) **(B).** $\cos^2(\cos\theta)+\sin^2(\cos\theta)+\sin^2(\sin\theta)-\sin^2(\cos\theta)$

$1+\sin^2(\sin\theta)-\sin^2(\cos\theta)$

value will be maximum when, $\cos\theta=0$ then $\sin\theta=1$

$\therefore$ maximum value is $1+\sin^2 1$

(51) **(B).** $\tan C < 0, \Rightarrow C$ is obutse

A, B are acute and $A + B < \pi/2$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} > 0$$

$1 - \tan A \tan B > 0 \Rightarrow \tan A \tan B < 1$

In a Δ ABC

$\tan A + \tan B + \tan C = \tan A . \tan B. \tan C$

$= (+) \quad (+)\ (-) < 0$

(52) $\sin x \le \cos^2 x$; cos x will be +ve proper fraction

$\sin^2 x + \sin x - 1 \le 0$

$$\left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \le 0$$

From the defination of logrithm.

$\sin x > 0, \cos x > 0, \cos x \ne 1$

$$\sin x + \frac{1}{2} \le \frac{\sqrt{5}}{2} \Rightarrow 0 < \sin x \le \frac{\sqrt{5}-1}{2}$$

(53) **(B).** Only equality holds because $\sin^2 x_i \le 1$

$$\text{so } \frac{1}{\sin^2 x_i} \ge 1 \Rightarrow 2^{\frac{1}{\sin^2 x_2}} . 3^{\frac{1}{\sin^2 x_3}} .....n^{\frac{1}{\sin^2 x_n}} = n!$$

$$\Rightarrow \sin^2 x_i = 1 \Rightarrow x_i = \frac{\pi}{2}, \frac{3\pi}{2} \in (0, 2\pi) \text{ in given interval}$$

So, $(x_2, x_3, x_4, ... x_n)$

Every varible have two choice either $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

So Number of solution

$2 \times 2 \times ..... (n-1) \text{ times} = 2^{n-1}$

(54) **(C).** $\text{cosec}^2\theta\,(\cos 2\theta - 3\cos\theta + 2) \ge 1$

$\cos^2\theta - 3\cos\theta + 2 \ge \sin^2\theta$

$2\cos^2\theta - 3\cos\theta + 1 \ge 0$

$2\cos^2\theta - 2\cos\theta - \cos\theta + 1 \ge 0$

$2\cos\theta\,[\cos\theta - 1] - 1[\cos q - 1] \ge 0$

$(2\cos\theta - 1)(\cos\theta - 1) \ge 0$

$$\cos\theta \le \frac{1}{2}$$

(55) **(A).** Let $a = 6 + 2\sqrt{3}, b = 4\sqrt{3}, c = \sqrt{24}$, since c is smallest side, C is smallest angle

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{\sqrt{3}}{2} \Rightarrow C = 30°$$

Therefore $\tan C = 1/\sqrt{3}$.

(56) **(C).** $$A = \pi - \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

and $\tan B = \frac{1}{2} \Rightarrow \sin B = \frac{1}{\sqrt{5}}$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin\frac{3\pi}{4}} = \frac{b}{\frac{1}{\sqrt{5}}} \Rightarrow b = \frac{2\sqrt{2}}{\sqrt{5}}$$

(57) **(B).** $$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2}$$

$$= \frac{1}{a^2} - \frac{1}{2R^2} - \frac{1}{b^2} + \frac{1}{2R^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

(58) **(B).** We know $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .... (1)

Given, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ .... (2)

$\frac{\text{Eq. (1)}}{\text{Eq. (2)}}$; $\tan A = \tan B = \tan C$

$\Rightarrow \Delta ABC$ is equilateral

$$\therefore \text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}2^2 = \sqrt{3}$$

(59) **(C).** $a\,[b\cos C - c\cos B]$

$= (b\cos C + c\cos B)(b\cos C - c\cos B)$

$= b^2\cos^2 C - c^2\cos^2 B = b^2(1 - \sin^2 C) - c^2(1 - \sin^2 B)$

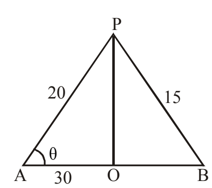
$$= b^2\left(1 - \frac{c^2}{4R^2}\right) - c^2\left(1 - \frac{b^2}{4R^2}\right)$$

$$= b^2 - \frac{b^2c^2}{4R^2} - c^2 + \frac{c^2b^2}{4R^2} = b^2 - c^2$$

(60) **(D).** **(61) (B).** **(62) (A).**

(63) **(A).** Let OP = tower

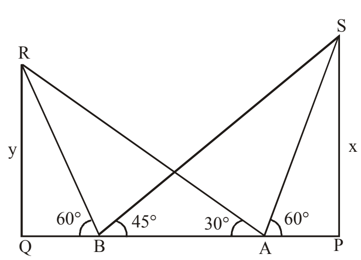
then AB = 30, AP = 20, BP = 15



If θ be angle of elevation at A

$$\cos\theta = \frac{(20)^2 + (30)^2 - (15)^2}{2 \times 20 \times 30} = \frac{43}{48} \;;\; \theta = \cos^{-1}\frac{43}{48}$$

(64) **(D).**



In $\Delta APS : AP = x\cot 60° = \dfrac{x}{\sqrt{3}}$

& $\Delta AQR : AQ = y\cot 30° = y\sqrt{3}$

$\Delta BPS : BP = x\cot 45° = x$

In $\Delta BRQ : BQ = y\cot 60° = y\sqrt{3}$

$\because\ BP - AP = AB$

$x - x\sqrt{3} = 30m\ ;\ x(1-\sqrt{3}) = 30m$

$x = 15(3+\sqrt{3})$

Similarly, $30 = y\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)\ ;\ y = 15\sqrt{3}$

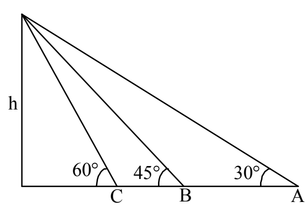
$PQ = BP + BQ = x + \dfrac{y}{\sqrt{3}}$

$= 15(3+\sqrt{3}) + 15 = (60 + 15\sqrt{3})\ m$

(65) **(D).** $AB = h(\cot 30° - \cot 45°) = h(\sqrt{3}-1)$

$BC = h(\cot 45° - \cot 60°)$

$= h\left(1 - \dfrac{1}{\sqrt{3}}\right)$



$\dfrac{AB}{BC} = \dfrac{\sqrt{3}-1}{1-\dfrac{1}{\sqrt{3}}} = \sqrt{3}$

$\sqrt{3} : 1$

(66) **(C).** If $A + B = 45$

$\tan(A+B) = 1$

$\tan A + \tan B = 1 - \tan A \tan B$

$(1+\tan A)(1+\tan B) = 2$

(67) **(D).** Given : $\sin x + \sin^2 x + \sin^3 x = 1$

$\sin x + \sin^3 x = 1 - \sin^2 x$

$\sin x[1 + \sin^2 x] = \cos^2 x$

$\sin x[2 - \cos^2 x] = +\cos^2 x$

Squaring we have,

$(1-\cos^2 x)(2-\cos^2 x)^2 = \cos^4 x$

or $4 + \cos^4 x - 4\cos^2 x - 4\cos^2 x - \cos^6 x + 4\cos^4 x = \cos^4 x$

or $\cos^6 x - 4\cos^4 x + 8\cos^2 x = 4$

(68) **(D).** $S = \dfrac{\sin(n\theta/2)}{\sin(\theta/2)}\cos\dfrac{(n+1)\theta}{2}$, $n = 17$, $\theta = \dfrac{\pi}{9}$

$= \dfrac{\sin(17\pi/18)}{\sin(\pi/18)}.\cos\pi = -1$

(69) **(B).** $\tan A + \tan B = a$, $\tan A \tan B = b$

$\therefore\ \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{a}{1-b}$

$\therefore\ \cos^2(A+B) = \dfrac{(1-b)^2}{a^2 + (1-b)^2}$

(70) **(D).** $\cos 20° \cos 40° \cos 80° \cos 160° \cos 320° \cos 640°$

$= \dfrac{\sin 1280°}{64\sin 20°} = \dfrac{\sin 200°}{64\sin 20°} = -\dfrac{1}{64}$

(71) **(B).** $(1-\cot 1°)(1-\cot 44°)$

$= 1 - \cot 1° - \cot 44° + \cot 1° \cot 44°$ .... (1)

Also, $1 = \cot(1° + 44°) = \dfrac{\cot 1° \cot 44° - 1}{\cot 1° + \cot 44°}$

$\cot 1° + \cot 44° = \cot 1° \cot 44° - 1$

$1 = \cot 1° \cot 44° - \cot 1° - \cot 44°$ .... (2)

From (1) and (2), we get

$\therefore\ (1-\cot 1°)(1-\cot 44°) = 2$

Similarly $(1-\cot 2°)(1-\cot 43°) = 2$

$\vdots$

$(1-\cot 22°)(1-\cot 23°) = 2$

$\therefore\ (1-\cot 1°)(1-\cot 2°)(1-\cot 3°)\ ......\ (1-\cot 44°) = 2^{22}$

$\therefore\ n = 22$

(72) **(A).** $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\tan A + \dfrac{n\sin A\cos A}{1 - n\cos^2 A}}{1 - \tan A\cdot\dfrac{n\sin A\cos A}{1-n\cos^2 A}}$

$= \dfrac{\sin A(1-n\cos^2 A) + n\sin A\cos^2 A}{\cos A(1-n\cos^2 A) - n\sin^2 A\cos A}$

$= \dfrac{\sin A - 0}{\cos A(1 - n\cos^2 A - n\sin^2 A)} = \dfrac{\sin A}{(1-n)\cos A}$

(73) **(B).** $(a+1)^2 + \text{cosec}^2\left(\dfrac{\pi a}{2} + \dfrac{\pi x}{2}\right) - 1 = 0$

or $(a+1)^2 + \cot^2\left(\dfrac{\pi a}{2} + \dfrac{\pi x}{2}\right) = 0$

from option [B] If $a = -1 \Rightarrow \tan^2 \pi x/2 = 0 \Rightarrow x/2 \in I$

(74) **(B).** $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = \dfrac{2\sqrt{2} \pm \sqrt{8+16}}{8} = \dfrac{\sqrt{2}\pm\sqrt{6}}{4}$

$\cos\theta = \dfrac{\sqrt{6}+\sqrt{2}}{4} \Rightarrow \theta = \dfrac{\pi}{12} ; 2\pi - \dfrac{\pi}{12} = \dfrac{23\pi}{12}$

$\cos\theta = -\dfrac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\theta = \cos(\pi - 5\pi/12)\ ;\ \cos(\pi + 5\pi/12)$

$\theta = 7\pi/12\ ;\ 17\pi/12$

(75) **(C).** $k = \dfrac{2\tan x}{1+\tan^2 x} = \sin 2x \Rightarrow \sin 2C = \sin 2B$

But $\angle C > \angle B$

$2C = \pi - 2B \Rightarrow B + C = \pi/2$

$\therefore \angle A = \pi/2$

(76) **(B).** $\frac{x}{y} = \frac{\sin 2\pi/3.\cos\theta + \cos 2\pi/3.\sin\theta}{\sin\theta}$

$= \frac{1}{2}\left[\frac{\sqrt{3}\cos\theta - \sin\theta}{\sin\theta}\right] = \frac{\sqrt{3}}{2}\cot\theta - \frac{1}{2}$ ....(1)

Similarly $\frac{x}{z} = \frac{\sin\theta.\cos 4\pi/3 + \cos\theta.\sin 4\pi/3}{\sin\theta}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}\cot\theta$ ....(2)

$\frac{x}{y} + \frac{x}{z} = -1 \Rightarrow xz + xy + yz = 0$

(77) **(D).** $\pi/10 = \theta$

$E = \frac{2\sin\theta(\cos\theta.\cos 2\theta.\cos 4\theta.\cos 8\theta.\cos 16\theta)}{2\sin\theta}$

$= \frac{\sin 32\theta}{32\sin\theta} = \frac{\sin(30\theta + 2\theta)}{32.\sin\theta} = -\frac{1}{16}\cos\frac{\pi}{10}$

$= -\frac{1}{16}\sqrt{1 - \sin^2\pi/10}$

(78) **(C).**

$m+n = a\{(\cos^3\alpha + \sin^3\alpha) + 3\cos\alpha\sin\alpha(\cos\alpha + \sin\alpha)\}$

$m+n = a\{\cos\alpha + \sin\alpha\}^3$

$|||^{ly}$ $m-n = a\{\cos\alpha - \sin\alpha\}^3$

$(m+n)^{2/3} = a^{2/3}(\cos\alpha + \sin\alpha)^2$

add. $(m-n)^{2/3} = a^{2/3}(\cos\alpha - \sin\alpha)^2$

$= a^{2/3}(2) \Rightarrow 2a^{2/3}$

(79) **(D).** $\cot x + \frac{\cos(60 + x)}{\sin(60 + x)} + \frac{\cos(x - 60)}{\sin(x - 60)}$

$= \frac{\cos x}{\sin x} + \frac{\sin(2x)}{\sin(x + 60)\sin(x - 60)}$

$= \frac{\cos x}{\sin x} + \frac{8\sin x\cos x}{4\sin^2 x - 3}$

$= \frac{4\sin^2 x\cos x - 3\cos x + 8\sin^2 x\cos x}{4\sin^3 x - 3\sin x}$

$= \frac{3[3\cos x - 4\cos^3 x]}{\sin^3 x} = 3\cot 3x$

$\Rightarrow \frac{3[1 - 3\tan^2 x]}{3\tan x - \tan^3 x}$

(80) **(A).** Using $\frac{3\sin 76°.\sin 16° + \cos 76°\cos 16°}{\cos 76°\sin 16° + \sin 76°\cos 16°}$

$= \frac{2\sin 76°\sin 16° + [\sin 76°\sin 16° + \cos 76°\cos 16°]}{\sin 92°}$

$= \frac{1 - \cos 92°}{\sin 92°} = \frac{2\sin^2 46°}{2\sin 46°\cos 46°} = \tan 46° = \cot 44°$

(81) **(A).** $\tan(5\pi\cos\theta) = \cot(5\pi\sin\theta)$

$\tan(5\pi\cos\theta) = \tan\left(\frac{\pi}{2} - 5\pi\sin\theta\right)$

$5\pi\cos\theta = n\pi + \pi/2 - 5\pi\sin\theta$

$(\cos\theta + \sin\theta) = \left(\frac{2n+1}{10}\right) \Rightarrow -1 < \frac{2n+1}{10\sqrt{2}} < 1$

$\Rightarrow -\frac{10\sqrt{2} - 1}{2} < n < \frac{10\sqrt{2} - 1}{2}$

n= 14 for each 'n' there are two values of θ

$\therefore$ no. of solutions = 28

(82) **(D).** $A/2 = 170^0$ hence $2\sin A/2 > 0$ now 340^0 lies in IV quadrant. Hence $\sin A < 0$.

So $1 + \sin A < 1 - \sin A$. Hence B & C are rejected because they give – values.

Now we will check A & D.

A : $|\sin A/2 + \cos A/2| + |\sin A/2 - \cos A/2|$

–ve +ve

$-\sin A/2 - \cos A/2 + \sin A/2 - \cos A/2 = -2\cos A/2$

(83) **(C).** $\frac{\sin 25°}{2}.2\sin 35°.\sin 85°$

$= \frac{\sin 25°}{2}[\cos 50° - \cos 120°]$

$= \frac{1}{4}2\sin 25°.\cos 50° + \frac{1}{4}\sin 25°$

$= \frac{1}{4}[\sin 75° - \sin 25°] + \frac{1}{4}\sin 25°$

$= \frac{1}{4}\sin 75° = \frac{1}{4}\frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{16}$

$\Rightarrow a + b + c = 6 + 2 + 16 \Rightarrow a + b + c = 24$

(84) **(B).** $\left(1 - \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 - \cos\frac{9\pi}{10}\right)$

$= \left(1 - \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 - \cos\frac{3\pi}{10}\right)$

$= \left(1 - \cos^2\frac{\pi}{10}\right)\left(1 - \cos^2\frac{3\pi}{10}\right)$

$= \sin^2\frac{\pi}{10}.\sin^2\frac{3\pi}{10} = \left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{1}{16}$

(85) **(A).** $\frac{m}{n}=\frac{\tan(120°+q)}{\tan(q-30°)}$

Þ $\frac{m+n}{m-n}=\frac{\tan(q+120°)+\tan(q-30°)}{\tan(q+120°)-\tan(q-30°)}$

[By componendo and dividendo]

$=\frac{\sin(q+120°)\cos(q-30°)+\cos(q+120°)\sin(q-30°)}{\sin(q+120°)\cos(q-30°)-\cos(q+120°)\sin(q-30°)}$

$=\frac{\sin(2q+90°)}{\sin(150°)}=\frac{\cos q}{1/2}=2\cos 2q$

(86) **(B).** $\cos 2x + a\sin x = 2a-7$

i.e. $2\sin^2 x - a\sin x + 2a - 8 = 0$

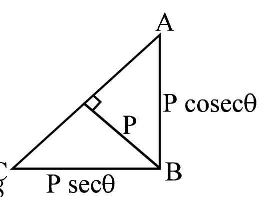
$\sin x=\frac{a\pm\sqrt{a^2-8(2a-8)}}{4}=\frac{a\pm(a-8)}{4}$

$\sin x=\frac{a-4}{2}$ or 2

Hence $-1\le(a-4)/2\le 1 \Rightarrow$ the range of a

(87) **(B).** $p^2\sec^2\theta + p^2\mathrm{cosec}^2\theta = \left(2\sqrt{2}\right)^2 p^2$

$\Rightarrow \frac{1}{\sin^2\theta\cos^2\theta}=8$



$\sin^2 2\theta = 1/2 = \left(\frac{1}{\sqrt{2}}\right)^2$

$2\theta = n\pi+\pi/4$; $\theta = n\pi/2+\pi/8$

for $n=0 \Rightarrow \theta=\pi/8$; for $n=1 \Rightarrow \theta = 3\pi/8$

(88) **(C).** $\tan\frac{\pi}{8}=\sqrt{2}-1 \therefore x=\sqrt{2}-1\Rightarrow x^2+2x-1=0$

$b=2, c=-1$

(89) **(B).** $\tan 15° = \tan(45°-30°)$

$=\frac{1-\tan 30°}{1+\tan 30°}=\frac{1-1\sqrt{3}}{1+1/\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$.

(90) **(D).** $\tan 9°+\tan 81°-(\tan 27°+\tan 63°)(\tan 9°+\cot 9°)$
$-(\tan 27°+\cot 27°)$

$=\left(\frac{\sin 9°}{\cos 9°}+\frac{\cos 9°}{\sin 9°}\right)-\left(\frac{\sin 27°}{\cos 27°}+\frac{\cos 27°}{\sin 27°}\right)$

$=\frac{1}{\sin 9°\cos 9°}-\frac{1}{\cos 27°\sin 27°}$

$=\frac{2}{\sin 18°}-\frac{2}{\sin 54°}=\frac{2}{\sin 18°}-\frac{2}{\cos 36°}$

$=\frac{2\times 4}{\sqrt{5}-1}-\frac{2\times 4}{\sqrt{5}+1}=8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}\right]=\frac{16}{4}=4$

(91) **(C).** If the equation

$\sqrt{23}\sin 2x+\sqrt{2}\cos 2x=-a^2+4a-9$ has infinite solution then

$-\sqrt{23+2}\le -a^2+4a-9\le\sqrt{23+2}$

$-5\le -a^2+4a-9\le 5$

$a^2-4a+4\le 0$ and $0\le a^2-4a+14$

$(a-2)^2\le 0$ and $0\le a^2-4a+14\Rightarrow a=2$

(92) **(B).** $\tan 6°\tan 54°\tan 66°$

$=\frac{\sin 6°}{\cos 6°}\cdot\frac{\sin(60°-6°)\sin(60°+6°)}{\cos(60°-6°)\cos(60°+6°)}$

$=\frac{\sin 6°}{\cos 6°}\cdot\frac{\sin^2 60°-\sin^2 6°}{\cos^2 6°-\sin^2 60°}$

$=\frac{\sin 6°}{\cos 6°}\left(\frac{3-4\sin^2 6}{4\cos^2 6°-3}\right)=\tan 18°$

(93) **(B).** Given that diameter of circular wire = 14 cm.

Therefore, length of circular wire = 14π cm

$\therefore$ Required angle

$=\frac{arc}{radius}=\frac{14p}{12}=\frac{7p}{6}=\frac{7}{6}p.\frac{180°}{p}=210°$

(94) **(A).** $\tan 100°+4\sin 100°$

$=\frac{\sin 100°+2\sin 200°}{\cos 100°}=\frac{\sin 100°+\sin 200°+\sin 200°}{\cos 100°}$

$=\frac{2\sin 150°\cos 50°-\sin 20°}{\cos 100°}$

$=\frac{\cos 50°-\cos 70°}{-\sin 10°}=\frac{2\sin 60°.\sin 10°}{-\sin 10°}=-\sqrt{3}$

(95) **(A).** Let, $\mathrm{cosec}\theta+\cot\theta=k\Rightarrow\mathrm{cosec}\theta-\cot\theta=\frac{1}{k}$

On adding, we get $2\,\mathrm{cosec}\theta=k+\frac{1}{k}$

or $2\left(x+\frac{1}{4x}\right)=k+\frac{1}{k}\Rightarrow 2x+\frac{1}{4x}=k+\frac{1}{k}\Rightarrow k=2x$

(96) **(C).** $\sin x+\sin 5x=\sin 2x+\sin 4x$

$2\sin 3x\cos 2x = 2\sin 3x\cos x$

$2\sin 3x[\cos 2x-\cos x]=0$

On solving we get $x=n\pi/3$

(97) **(A).** $\sec 2\theta = \dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1+n}{1-n}$,

where n is a non-square natural number so
$1-n \neq 0 \Rightarrow \sec 2\theta$ is a rational number.

(98) **(C).** As, $\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta$
$\therefore\ 0 = ab \tan 3\theta \Rightarrow \tan 3\theta = 0$
$\Rightarrow \tan\theta + \tan 2\theta = 0 \Rightarrow a + b = 0.$

(99) **(D).** $\sin^4 A - \cos^4 A = (\sin^2 A - \cos^2 A)$
$(\sin^2 A + \cos^2 A) = (\sin^2 A - \cos^2 A)$ which can never be expressed as $1 + \sin^2 A$

(100) **(C).** $m^2 - n^2 = (m+n)(m-n)$
$= 2\tan q \,´\, 2\sin q = 4\sin q \sin q$

$m \times n = \tan^2\theta - \sin^2\theta = \sin^2\theta . \sec^2\theta - \sin^2\theta$

$= \sin^2\theta(\sec^2\theta - 1) = \sin^2\theta \tan^2\theta$

$\therefore \sqrt{m \,´\, n} = \sin q . \tan q$

$\therefore\ m^2 - n^2 = 4\sqrt{m \times n}$

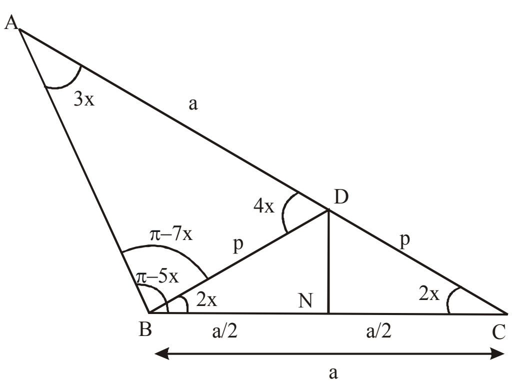
EXERCISE-2

(1) **(B).** In Δ ABC, $\dfrac{AC}{\sin 5x} = \dfrac{BC}{\sin 3x}$;

$\dfrac{a+p}{\sin 5x} = \dfrac{a}{\sin 3x}$ .... (1)

In Δ BDN, (note that Δ BDC is isosceles)

$\cos 2x = \dfrac{a}{2p}$; $a = 2p\cos 2x$



From (1), $\dfrac{2p\cos 2x + p}{\sin 5x} = \dfrac{2p\cos 2x}{\sin 3x}$

$2\sin 3x \cos 2x + \sin 3x = 2\sin 5x \cos 2x$
$\sin 5x + \sin x + \sin 3x = \sin 7x + \sin 3x$
$\sin 7x - \sin 5x = \sin x$
$2\cos 6x \sin x = \sin x$

$\cos 6x = \dfrac{1}{2} \Rightarrow x = 10°$

(2) **(C).** Given $\cos x + \sin x = \dfrac{1}{2} \Rightarrow 1 + \sin 2x = \dfrac{1}{4}$

$\sin 2x = -\dfrac{3}{4} \Rightarrow 2x \in (\pi, 2\pi)$

$\Rightarrow x \in (\pi/2, \pi) \Rightarrow \tan x < 0$

$\dfrac{2t}{1+t^2} = -\dfrac{3}{4} \Rightarrow 8t = -3 - 3t^2 \Rightarrow 3t^2 + 8t + 3 = 0$
where $t = \tan x$

$t = \dfrac{-8 \pm \sqrt{64-36}}{2 \times 3}$; $t = \dfrac{-8 \pm \sqrt{28}}{2 \times 3}$;

$t = \dfrac{-(4+\sqrt{7})}{3}$ or $\dfrac{-4+\sqrt{7}}{3}$ (rejected)

(3) **(D).** On adding and subtracting

$x = \dfrac{3 - \cos 4\theta + 4\sin 2\theta}{2}$; $y = \dfrac{3 - \cos 4\theta - 4\sin 2\theta}{2}$

$x = \dfrac{4(1+\sin 2\theta) - (1+\cos 4\theta)}{2}$;

$y = \dfrac{4(1-\sin 2\theta) - (1+\cos 4\theta)}{2}$

$x = 2(1+\sin 2\theta) - \cos^2 2\theta$; $y = 2(1 - \sin 2\theta) - \cos^2 2\theta$
$x = 1 + 2\sin 2\theta + \sin^2 2\theta$; $y = 1 - 2\sin 2\theta + \sin^2 2\theta$
$x = (1+\sin 2\theta)^2$; $y = (1-\sin 2\theta)^2$

$\Rightarrow \sqrt{x} + \sqrt{y} = 2$

(4) **(D).** $\dfrac{\tan\left(x - \frac{\pi}{2}\right) . \cos\left(\frac{3\pi}{2} + x\right) - \sin^3\left(\frac{7\pi}{2} - x\right)}{\cos\left(x - \frac{\pi}{2}\right) . \tan\left(\frac{3\pi}{2} + x\right)}$

$= \dfrac{-\cot x . \sin x + \cos^3 x}{-\sin x . \cot x} = \dfrac{\cos^3 x - \dfrac{\sin x . \cos x}{\sin x}}{-\sin x . \dfrac{\cos x}{\sin x}} = \sin^2 x$

(5) **(D).** Given equation can be written as
$3\sin\theta - 4\sin^3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$
hence either $\sin\theta = 0 \Rightarrow \theta = n\pi$
or $3 - 4\sin^2\theta = 4\sin 2\theta \sin 4\theta$
$3 - 2(1 - \cos 2\theta) = 2(\cos 2\theta - \cos 6\theta)$
or $1 = -2\cos 6\theta$

$\cos 6\theta = -\dfrac{1}{2} = \cos\dfrac{2\pi}{3}$; $6\theta = 2n\pi \pm \dfrac{2\pi}{3}$

if $0 \le \theta \le \pi$ then total solution are

$0, \dfrac{\pi}{9}, \dfrac{2\pi}{9}, \dfrac{4\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}, \dfrac{8\pi}{9}, \pi$ is 8 real solutions.

(6) **(C).** $\tan 20° \tan 40° \tan 60° \tan 80°$

$= \dfrac{\sin 20° \sin 40° \sin 80° \tan 60°}{\cos 20° \cos 40° \cos 80°}$

Here, $N^r = (\sin 20° \sin 40° \sin 80°)$

$= \frac{\sin 20°}{2}(2\sin 40° \sin 80°)$

$= \frac{\sin 20°}{2}(\cos 40° - \cos 120°)$

$= \frac{1}{2}\sin 20°\left(1 - 2\sin^2 20° + \frac{1}{2}\right)$

$= \frac{1}{2}\sin 20°\left(\frac{3}{2} - 2\sin^2 20°\right) = \frac{\sin 60°}{4} = \frac{\sqrt{3}}{8}$

Now, we take $D^r = \cos 20° \cos 40° \cos 80°$

$= \frac{\sin 2^3 20°}{2^3 \sin 20°} = \frac{\sin 160°}{8\sin 20°} = \frac{\sin 20°}{8\sin 20°} = \frac{1}{8}$

$\therefore$ Hence $\tan 20° \tan 40° \tan 80° = \frac{\sqrt{3}/8}{1/8}$

Therefore, $\tan 20° \tan 40° \tan 60° \tan 80° = \sqrt{3}.\sqrt{3} = 3$

(7) **(A).** $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

$= \cos\frac{\pi}{11} + \cos\left(\frac{\pi}{11} + \frac{2\pi}{11}\right) + \cos\left(\frac{\pi}{11} + \frac{2.2\pi}{11}\right)$
$+ \cos\left(\frac{\pi}{11} + \frac{3.2\pi}{11}\right) + \cos\left(\frac{\pi}{11} + \frac{4.2\pi}{11}\right)$

Use $\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots + \cos(\alpha + (n-1)\beta)$

$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\frac{2\alpha + (n-1)\beta}{2}$

Here $\alpha = \frac{\pi}{11}$, $\beta = \frac{2\pi}{11}$ and $n = 5$ then

$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

$= \frac{\sin\frac{5}{2} \times \frac{2\pi}{11}}{\sin\frac{2\pi}{2.11}} \cos\frac{2\frac{\pi}{11} + 4.\frac{2\pi}{11}}{2} = \frac{\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} \cos\frac{5\pi}{11}$

$= \frac{\frac{1}{2}\sin\frac{10\pi}{11}}{\sin\frac{\pi}{11}} = \frac{1}{2}\frac{\sin\left(\pi - \frac{\pi}{11}\right)}{\sin\frac{\pi}{11}} = \frac{1}{2}$

(8) **(A).** Exp. $= \sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.1$

$.\sin\left(\pi - \frac{5\pi}{14}\right)\sin\left(\pi - \frac{3\pi}{14}\right)\sin\left(\pi - \frac{\pi}{14}\right)$

$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2 .1$

$= \left\{\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right\}^2$

$= \left(\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right)^2 = \left(\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}\right)^2$

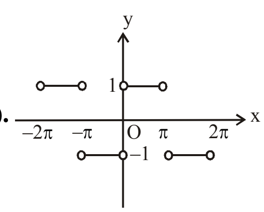
$= \left(-\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}\right)^2 = \left[-\frac{\sin 2^3\pi/7}{2^3 \sin\pi/7}\right]^2$

$\left[\because \cos\theta.\cos 2\theta.\cos 2^2\theta.....\cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}\right]$

$= \frac{1}{64}\left(\frac{\sin 8\pi/7}{\sin\pi/7}\right)^2 = \frac{1}{64}$

(9) **(C).** $\tan^4 x + \cot^4 x + 1 = (\tan^2 x - \cot^2 x)^2 + 3 \geq 3$
$3\sin^2 y \leq 3 \Rightarrow \tan^2 x = \cot^2 x, \sin^2 y = 1$
$\Rightarrow \tan x = \pm 1, \sin y = \pm 1 \Rightarrow \quad x = \pm\pi/4, \pm 3\pi/4, \ldots$
But $x^2 \leq 4 \Rightarrow -2 \leq x \leq 2 \Rightarrow x = \pm\pi/4$ only
$\sin y = \pm 1 \Rightarrow y = \pm\pi/2, \pm 3\pi/2, \ldots..$ But $y^2 \leq 4$
$\Rightarrow y = \pm\pi/2$ only. So four solutions are possible.

(10) **(A).**



(11) **(A).** $R = \frac{1}{2}\sin\frac{\pi}{8}\left(\cos^2\frac{\pi}{16} - \sin^2\frac{\pi}{16}\right)$
$= \frac{1}{2}\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{4\sqrt{2}}$

S $= \sin^2 x - 2\cos^2 x + 1 = 4\sin^2 x + 8\cos^2 x - 4$
$= 10\cos^2 x + 3\sin^2 x - 5 = 0$
$= 10 + 3\tan^2 x - 5(1 + \tan^2 x) = 0$ (multiplying by $\sec^2 x$)
$= 2\tan^2 x = 5$
$\therefore$ $S = \tan^2 x = 5/2$

$T = \cos 2x - \frac{2\sin x \cos 3x}{2\sin 2x} = \cos 2x - \frac{\sin 4x - \sin 2x}{2\sin 2x}$

$= \frac{\sin 4x - \sin 4x + \sin 2x}{2\sin 2x} = \frac{1}{2}$

$\therefore$ RST $= \frac{1}{4\sqrt{2}}.\frac{5}{2}.\frac{1}{2} = \frac{5}{16\sqrt{2}}$

(12) **(B).** $1 - \sin^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$

$\sin^2 x - \frac{\sqrt{3}+1}{2}\sin x + \frac{\sqrt{3}}{4} = 0$

$4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} = 0$

On solving we get

$\sin x = 1/2 \; ; \; \frac{\sqrt{3}}{2} = (\pi/6, 5\pi/6 \; ; \pi/3, 2\pi/3]$

(13) **(B).** Given that $\sin^3 x \sin 3x = \sum_{m=0}^{n} c_m \cos mx$

or $\left(\frac{3\sin x - \sin 3x}{4}\right).\sin x = \sum_{m=0}^{n} c_m \cos mx$

or $\frac{3}{8}.(2\sin 3x \sin x) - \frac{1}{8}.2\sin^2 3x = \sum_{m=0}^{n} c_m \cos mx$

or $\frac{3}{8}.[\cos 2x - \cos 4x] - \frac{1}{8}[1 - \cos 6x] = \sum_{m=0}^{n} c_m \cos mx$

or $-\frac{1}{8} + \frac{3}{8}\cos 2x - \frac{3}{8}\cos 4x + \frac{1}{8}\cos 6x = \sum_{m=0}^{n} c_m \cos mx$.

Comparing, we get $n = 6$.

(14) **(A).** Given $\frac{\sin(\theta+\alpha)}{\cos(\theta-\alpha)} = \frac{1-m}{1+m}$

or $\frac{\sin(\theta+\alpha)+\cos(\theta-\alpha)}{\sin(\theta+\alpha)-\cos(\theta-\alpha)} = \frac{1-m+1+m}{1-m-1-m}$

[by componendo and dividendo]

or $\frac{\sin(\theta+\alpha)+\sin\left[\frac{\pi}{2}-(\theta+\alpha)\right]}{\sin(\theta+\alpha)-\sin\left[\frac{\pi}{2}-(\theta=\alpha)\right]} = \frac{2}{-2m} = -\frac{1}{m}$

or $\frac{2\sin\frac{\theta+\alpha+\frac{\pi}{2}-\theta+\alpha}{2}\cos\frac{\theta+\alpha-\frac{\pi}{2}+\theta-\alpha}{2}}{2\cos\frac{\theta+\alpha+\frac{\pi}{2}-\theta+\alpha}{2}\sin\frac{\theta+\alpha-\frac{\pi}{2}+\theta-\alpha}{2}} = -\frac{1}{m}$

or $\frac{\sin\left(\frac{\pi}{4}+\alpha\right).\cos\left(\theta-\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}+\alpha\right).\sin\left(\theta-\frac{\pi}{4}\right)} = -\frac{1}{m}$

or $\tan\left(\frac{\pi}{4}+\alpha\right).\cot\left(\theta-\frac{\pi}{4}\right) = -\frac{1}{m}$

or $-\tan\left(\frac{\pi}{4}+\alpha\right).\cot\left(\frac{\pi}{4}-\theta\right) = -\frac{1}{m}$

[$\because \cot(-\theta) = -\cot\theta$]

or $m = \cot\left(\frac{\pi}{4}+\alpha\right).\tan\left(\frac{\pi}{4}-\theta\right)$

or $m = \tan\left[\frac{\pi}{2}-\left(\frac{\pi}{4}+\alpha\right)\right].\tan\left(\frac{\pi}{4}-\theta\right)$

$= \tan\left(\frac{\pi}{4}-\alpha\right)\tan\left(\frac{\pi}{4}-\theta\right)$

(15) **(B).** We have $x = \sum_{n=0}^{\infty}\cos^{2n}\phi = \frac{1}{1-\cos^2\phi} = \mathrm{cosec}^2\phi$

and $y = \sum_{n=0}^{\infty}\sin^{2n}\phi = \frac{1}{1-\sin^2\phi} = \sec^2\phi$

and $z = \sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$

$= \frac{1}{1-\cos^2\phi\sin^2\phi} = \frac{1}{1-1/xy} = \frac{xy}{xy-1}$

$\Rightarrow xyz - z = xy \Rightarrow xyz = xy + z$

(16) **(B).** Given that diameter of circular wire = 14 cm.
Therefore, length of circular wire = 14π cm
$\therefore$ Required angle

$= \frac{\text{arc}}{\text{radius}} = \frac{14p}{12} = \frac{7p}{6} = \frac{7}{6}p.\frac{180°}{p} = 210°$

(17) **(B).** We have $\tan\theta = \frac{x\sin\phi}{1-x\cos\phi}$

$\Rightarrow \frac{1}{x}\tan\theta - \tan\theta\cos\phi = \sin\phi \Rightarrow \frac{1}{x} = \frac{\sin\phi+\cos\phi\tan\theta}{\tan\theta}$

and $\tan\phi = \frac{y\sin\theta}{1-y\cos\theta}$

$\Rightarrow \tan\phi = \dfrac{\sin\theta}{\frac{1}{y}-\cos\theta} \Rightarrow \dfrac{1}{y}\tan\phi - \tan\phi\cos\theta = \sin\theta$

$\Rightarrow \dfrac{1}{y}\tan\phi = \sin\theta + \tan\phi\cos\theta \quad \therefore \dfrac{1}{y} = \dfrac{\sin\theta+\tan\phi\cos\theta}{\tan\phi}$

Now $\dfrac{x}{y} = \left[\dfrac{\tan\theta}{\sin\phi+\cos\phi\tan\theta}\right]\times\left[\dfrac{\sin\theta+\tan\phi\cos\theta}{\tan\phi}\right]$

$= \dfrac{\tan\theta}{\tan\phi} = \left[\dfrac{\sin\theta+\cos\theta\dfrac{\sin\phi}{\cos\phi}}{\sin\phi+\cos\phi\dfrac{\sin\theta}{\cos\theta}}\right] = \dfrac{\tan\theta\cos\theta}{\tan\phi\cos\phi} = \dfrac{\sin\theta}{\sin\phi}$

(18) **(A).** Let $A+B=\theta$ and $A-B=\phi$

Then $\tan A = k\tan B$ or $\dfrac{k}{1} = \dfrac{\tan A}{\tan B} = \dfrac{\sin A\cos B}{\cos A\sin B}$

Applying componendo and divdendo

$\Rightarrow \dfrac{k+1}{k-1} = \dfrac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B} = \dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{\sin\theta}{\sin\phi}$

$\Rightarrow \sin\theta = \dfrac{k+1}{k-1}\sin\phi$

(19) **(B).** $k = a$ (say)

$k\sin x + (1 - 2\sin^2 x) = 2k-7$

$\Rightarrow 2\sin^2 x - k\sin x + 2(k-4) = 0$

$\Rightarrow \sin x = \dfrac{k\pm\sqrt{k^2-16k+64}}{4} = \dfrac{k\pm(k-8)}{4}$

$= \dfrac{1}{2}(k-4), 2$

$\sin x \neq 2 \quad \therefore \sin x = \dfrac{k-4}{2}$; $-1 \le \dfrac{k-4}{2} \le 1 \Rightarrow 2\le k\le 6$

(20) **(C).** $y = (7\cos\theta + 24\sin\theta)\times(7\sin\theta - 24\cos\theta)$

$r\cos\phi = 7$; $r\sin\phi = 24$

$r^2 = 625$; $\tan\phi = \dfrac{24}{7}$

$y = r\cos(\theta-\phi) . r\sin(\theta-\phi)$

$= \dfrac{r^2}{2} . 2\sin(\theta-\phi)\cos(\theta-\phi) = \dfrac{r^2}{2} . (\sin 2(\theta-\phi))$

$y_{max} = \dfrac{25^2}{2} = \dfrac{625}{2}$

(21) **(B).** $2[2\sin5°\sin55°]\sin65° \Rightarrow 2[\cos50° - \cos60°]\sin65°$

$\Rightarrow 2\cos50°\sin65° - \sin65°$

$\Rightarrow \sin(115°) + \sin15° - \sin65° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(22) **(B).** $\left|\dfrac{x}{2}-\dfrac{\pi}{2}\right| \le \dfrac{3\pi}{4}$ possible x are

$-\dfrac{3\pi}{4} \le \dfrac{x}{2}-\dfrac{\pi}{2} \le \dfrac{3\pi}{4}$

$-\dfrac{\pi}{4} \le \dfrac{x}{2} \le \dfrac{5\pi}{4}$

$-\dfrac{\pi}{2} \le x \le \dfrac{5\pi}{4}$

$-\dfrac{\pi}{2}, 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi, \dfrac{5\pi}{2}$

$\sin\dfrac{x}{2} - \cos\dfrac{x}{2} = (\sin\dfrac{x}{2} - \cos\dfrac{x}{2})^2$

factors $\sin\dfrac{x}{2} - \cos\dfrac{x}{2} = 0$

or $\sin\dfrac{x}{2} - \cos\dfrac{x}{2} = 1$

only circled angle satisfy one of the above equation when n = 1, 2, 4, 5

(23) **(A).** $E = \dfrac{3}{2}(1+\cos20^0)(1+\cos100^0)(1+\cos140^0)$

$= \dfrac{3}{2}\, 2\cos^2 10^0 . 2\cos^2 50^0 . 2\cos^2 70^0$

$= 12(\cos10^0\cos50^0\cos70^0)^2 = 12\times\dfrac{3}{64} = \dfrac{9}{16}$

(24) **(B).** $(\alpha-\beta) = (\theta-\beta)-(\theta-\alpha)$

$\cos(\alpha-\beta) = \cos(\theta-\beta)\cos(\theta-\alpha) + \sin(\theta-\beta)\sin(\theta-\alpha)$

$\cos(\alpha-\beta) = \dfrac{y}{b}.\dfrac{x}{a} + \sqrt{1-\dfrac{x^2}{a^2}} . \sqrt{1-\dfrac{y^2}{b^2}}$

$\Rightarrow \left[\dfrac{xy}{ab} - \cos(\alpha-\beta)\right]^2 = \left(1-\dfrac{x^2}{a^2}\right)\left(1-\dfrac{y^2}{b^2}\right)$

$\Rightarrow \dfrac{x^2y^2}{a^2b^2} + \cos^2(\alpha-\beta) - \dfrac{2xy}{ab}\cos(\alpha-\beta)$

$= 1 - \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} + \dfrac{x^2y^2}{a^2b^2}$

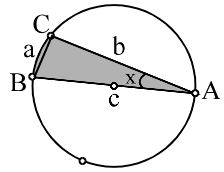
$\Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{2xy}{ab}\cos(\alpha-\beta) = \sin^2(\alpha-\beta)$

(25) **(A).** $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \dfrac{p+q}{p-q}$

$\dfrac{2\sin\alpha\cos\beta}{2\cos\alpha\sin\beta} = \dfrac{p+q}{p-q}$

$\tan\alpha\cdot\cot\beta = \dfrac{p+q}{p-q}$

(26) **(A).** $a^2 + b^2 = c^2 = 4r^2$ ....(1)



also $\frac{1}{2} a \cdot b = \frac{1 - t_1^2}{1 + t_1^2}$

$9ab = 2\pi r^2$ ....(2)

from (1) and (2)

$\frac{a^2 + b^2}{9ab} = \frac{2}{\pi}$ or $\frac{a}{b} + \frac{b}{a} = \frac{18}{\pi}$

Let $m\angle BAC = x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{18}{\pi} \Rightarrow \frac{\cos^2 x + \sin^2 x}{\sin x \cdot \cos x} = \frac{18}{\pi}$

$\Rightarrow \sin x \cdot \cos x = \frac{\pi}{18}$; $\sin 2x = \frac{\pi}{9}$

(27) **(B).**

$S = 2 \sin 2° + 4 \sin 4° + ... + 180 \sin 180°$

$S = 2[\sin 2° + 2 \sin 4° + 3 \sin 6° ...... + 89 \sin 178°]$

$S = 2[89 \sin 178° + 88 \sin 176° + ...+ 1 \cdot \sin 2°]$

$2S = 2[90(\sin 2° + \sin 4° + \sin 6° + ... + \sin 178°)]$

$S = 90 \cdot \frac{\sin\left(\frac{n\theta}{2}\right)}{\sin\frac{\theta}{2}} \sin\left(\frac{(n+1)\theta}{2}\right) = \frac{90 \sin(89)}{\sin 1} \cdot \sin 90°$

$S = 90 \cot 1°$

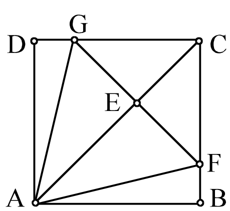
average value $= \frac{90 \cot 1}{90} = \cot 1°$

(28) **(B).** Draw diagonal AC.

Let x be the length of the side of the square

$x = 10 \cos 15°$

$x^2 = 100 \cos^2 15°$



$2 \cos^2 15° = \cos 30° + 1$

$x^2 = 100 \cos^2 15° = 50\left(\frac{\sqrt{3}}{2} + 1\right) = 25(2 + \sqrt{3})$

(29) **(A).**

$\cos 5° + (\cos 77° + \cos 293°) + (\cos 149° + \cos 221°)$

$\cos 5° + 2 \cos 185° \cos 108° + 2 \cos 185° \cos 36°$

$\cos 5° - 2 \cos 5° \cos 108° - 2 \cos 5° \cos 36°$

$\cos 5° [1 + 2(\sin 18° - \cos 36°)]$

$\cos 5° \left[1 + 2\left(\frac{(\sqrt{5} - 1) - (\sqrt{5} + 1)}{4}\right)\right]$

$= \cos 5°(1 - 1) = 0$

(30) **(A).**

$x \sec^2\theta + y \sec\theta \cdot \tan\theta = 2$

$x \tan^2\theta + y \sec\theta \cdot \tan\theta = 1$

– – –

$x = 1$

$\therefore$ $\sec\theta + y \tan\theta = 2 \cos\theta$

$y \tan\theta = 2 \cos\theta - \sec\theta$

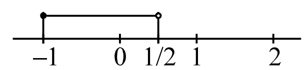
$y = \frac{2\cos^2\theta - 1}{\cos\theta \cdot \tan\theta} = \frac{\cos 2\theta}{\sin\theta}$

(31) **(A).**

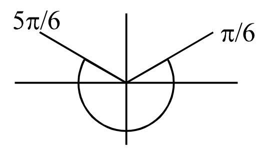
$2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow$ $2\sin^2\theta - 4 \sin\theta - \sin\theta + 2 > 0$

$(\sin\theta - 2)(2\sin\theta - 1) > 0$



$-1 \le \sin\theta \le 1/2$



$\Rightarrow$ (A)]

(32) **(C).** Since $\cos x = \tan x = \frac{\sin x}{\cos x}$, the angle x is in the first or second quadrant. Now $\cos^2 x = \sin x$. Thus $\sin x$ is non-negative and substituting $\cos^2 x = 1 - \sin^2 x$ in the second equation result in $\sin^2 x + \sin x - 1 = 0$. The only non-negative solution of this equation is $\sin x = \frac{-1 + \sqrt{5}}{2}$

(33) **(C).** $AB = 3$

$\sqrt{(2\cos 75° - \sin\theta)^2 + (2\sin 75° - \cos\theta)^2} = 3$

$\sqrt{4 + 1 - 4\sin(\theta + 75°)} = 3$

$5 - 4 \sin(\theta + 75°) = 9 \Rightarrow \sin(\theta + 75°) = -1$

$\therefore \theta + 75° = 270° \Rightarrow \theta = 195°$ Ans.]

(34) **(B).** $x^{3 \sin 2x + 2} = x^{1/2}$

$\Rightarrow \left[(3\sin 2x + 2) - \frac{1}{2}\right] \log x = 0$

$\Rightarrow 3 \sin 2x + 2 = -1/2 \Rightarrow \sin 2x = -1/2$

$2x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right) \Rightarrow x = \frac{n\pi}{2} - (-1)^n \frac{\pi}{12}$

(35) **(D).** $\sqrt{2} (\cos 2x - \sin x - 1) = 1 + 2 \sin x$

$\sqrt{2} (1 - 2 \sin^2 x - \sin x - 1) = 1 + 2 \sin x$

$-2\sqrt{2} \sin^2 x - \sqrt{2} \sin x = 1 + 2 \sin x$

$2\sqrt{2}\ \sin^2 x + (2+\sqrt{2}\ )\sin x + 1 = 0$

$\sqrt{2}\ \sin x\,[2\sin x + 1] + 2\sin x + 1 = 0$

$\Rightarrow (\sqrt{2}\ \sin x + 1)(1 + 2\sin x) = 0$

$\sin x = -1/2$ or $\sin x = -1/\sqrt{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}$; sum $= 6\pi$ Ans.

(36) **(A).** $2\sin^2 x = 1 + \cos x$; $2\cos^2 x + \cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2}$ or $-1 \Rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ but $x = \pi$ and $\frac{5\pi}{3}$

are rejected $\Rightarrow x = \frac{\pi}{3}$

(37) **(A).** $2 + \frac{2\tan(x/2)\cdot\cot(x/2)}{1-\tan^2(x/2)} = \frac{(1-\tan^2(x/2))\tan(x/2)}{2\tan(x/2)}$

$= 0$

$2 + \frac{1}{1-\tan^2(x/2)} + \frac{1-\tan^2(x/2)}{2} = 0$

$\left[\frac{\pi}{2} = \frac{\pi}{2} \text{ and } \pi\right]$ $x \neq \pi$ or 2π

$4\left(1-\tan^2\frac{x}{2}\right) + 4 + \left(1-\tan^2\frac{x}{2}\right) = 0$

$1 - \tan^2\frac{x}{2} = y$

$y^2 + 4y + 4 = 0$

$(y+2)^2 = 0 \Rightarrow y = -2$

$1 - \tan^2\frac{x}{2} = -1$; $\tan^2\frac{x}{2} = 3 = \tan^2\frac{\pi}{3}$; $\frac{x}{2} = n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}$, $n \in N$

(38) **(B).** $S = \frac{1}{1+(\tan 1°)^2} + \frac{1}{1+(\tan 2°)^2} + \frac{1}{1+(\tan 3°)^2} + \ldots\ldots + \frac{1}{1+(\tan 88°)^2} + \frac{1}{1+(\tan 89°)^2}$

reversing the sum

$S = \frac{1}{1+(\cot 1°)^2} + \frac{1}{1+(\cot 2°)^2} + \ldots\ldots\ldots + \frac{1}{1+(\cot 88°)^2} + \frac{1}{1+(\cot 89°)^2}$

$2S = \sum_{n=1}^{89} \frac{1}{1+(\tan n°)^2} + \frac{1}{1+(\cot n°)^2}$

$= \sum_{n=1}^{89} \frac{1}{1+(\tan n°)^2} + \frac{(\tan n°)^2}{1+(\tan n°)^2}$

$= 6\tan\frac{t}{2} + 3 - 3\tan^2\frac{t}{2} = 4 + 4\tan^2\frac{t}{2} = 1 + 1 + \ldots\ldots + 1 = 89$

$\therefore\ S = 44.5$

(39) **(A).** Using $\cos 2\alpha = \frac{1-t_1^2}{1+t_1^2}$

where $t_1 = \tan\alpha$ and $\cos 2\beta = \frac{1-t_2^2}{1+t_2^2}$ where $t_2 = \tan\beta$

we have

$\left[1003 - 1002\frac{(t-t_1^2)}{1+t_1^2}\right]\left[1003 - 1002\frac{(t-t_2^2)}{1+t_2^2}\right]$

$= \frac{1003(1+t_1^2) - 1002(1-t_1^2)}{(1+t_1^2)} \times \frac{1003(1+t_2^2) - 1002(1-t_2^2)}{(1+t_2^2)}$

$= \frac{(1+2005t_1^2)(1+2005t_2^2)}{1+t_1^2+t_2^2+t_1^2t_2^2}$

given $t_1t_2 = \frac{1}{\sqrt{2005}} \Rightarrow t_1^2t_2^2 = \frac{1}{2005}$

Hence $\frac{1+2005(t_1^2+t_2^2)+(2005)^2t_1^2t_2^2}{1+t_1^2+t_2^2+\frac{1}{2005}}$

$= \frac{1+2005(t_1^2+t_2^2)+2005}{1+t_1^2+t_2^2+\frac{1}{2005}}$

$= \frac{2005\left[\frac{1}{2005}+t_1^2+t_2^2+1\right]}{\left[1+t_1^2+t_2^2+\frac{1}{2005}\right]} = 2005$

(40) **(B).**

$P = 2\sin 2° + 4\sin 4° + 6\sin 6° + \ldots\ldots. + 178\sin 178°$

$P = 178\sin 2° + 176\sin 4° + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 2\sin 178°$

$2P = 180\,[\sin 2° + \sin 4° + \sin 6° + \ldots\ldots. + \sin 178°]$

$P = 90\left[\frac{\sin 89°}{\sin 1°}\sin 90°\right] = 90\cot 1° \Rightarrow \frac{P}{90} = \cot 1°$

S-1 : $\frac{\sqrt{1+\cot^2 1°}}{90\cot 1°} = \frac{\operatorname{cosec} 1°}{90\cot 1°} = \frac{\sec 1°}{90}$ which is irrational.

S-2 is not the correct reason as tan 60° is irrational but sec 60° is rational.

(41) **(D).** Statement-2 is true.
Statement-1 is false because [sin x] = 1, if x = π/2

(42) **(C).** $7\cos x + 5\sin x = 2\lambda + 1$

$|2\lambda+1| \le \sqrt{49+25} \Rightarrow |2\lambda+1| \le \sqrt{74}$

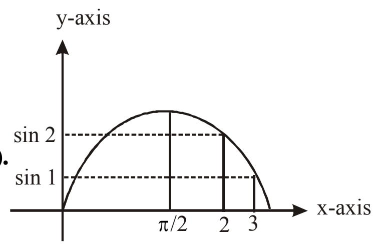
$-\sqrt{74} \le 2\lambda + 1 \le \sqrt{74}$;

$-8.6 \le 2\lambda \le 7.6, -4.8 \le \lambda \le 3.8$

$\lambda = -4, -3, -1, 0, 1, 2, 3$

$a\cos\theta + b\sin\theta = c$ has no solution if $|c| > \sqrt{a^2+b^2}$

(43) **(A).**



(44) **(B).** Statement 2 is true as $\cos\theta > \sin\theta$ for $0 \le \theta < \pi/4$
Statement 1 is true if $\cos^2 36° > \sin 36°$
or if $1 + \cos 72° > 2\sin 36° = 2\sin(30° + 6°)$
or if $1 + \sin 18° > 2(\sin 30°\cos 6° + \cos 30°\sin 6°)$
or $1 + 2\sin 9°\cos 9° > \cos 6° + 2\cos 30°\sin 6°$
which is true because
$1 > \cos 6°, \sin 9° > \sin 6°, \cos 9° > \cos 30°$
Statement 1 is also true but follows from
$\cos^2 36° > \sin 36°$ which does not follow from
$\cos 36° > \sin 36°$ as $0 < \cos 36° < 1$

(45) **(A).**

(a) $3 + \left[\dfrac{1+\cos^2 x}{4}\right] + \left[\dfrac{1}{3}\sin^2 x\right] = 3$

(b) $N = \sin^2\alpha + \cos\left(\dfrac{\pi}{3}-\alpha\right).\cos\left(\dfrac{\pi}{3}+\alpha\right)$

$= \sin^2\alpha + \cos^2\dfrac{\pi}{3} - \sin^2\alpha = \dfrac{1}{4}$

$\therefore\ 5 + \log_2 N = 5 + \log_2\dfrac{1}{4} = 3$

(c) $6\cos x + 8\sin x = 2\lambda + 1$ i.e. $\sin(x+\alpha) = \dfrac{2\lambda+1}{10}$

$\therefore\ -1 \le \dfrac{2\lambda+1}{10} \le 1 \Rightarrow -\dfrac{11}{2} \le \lambda \le \dfrac{9}{2}$

Number of integral values of λ is 10.
(d) Least value of $3\sin^2\theta + 4\cos^2\theta$ is 3.

(46) **(B).**
(a) $\cos^7 x + \sin^4 x = 1$
$\cos^7 x = (1 + \sin^2 x)\cos^2 x$
$\cos x = 0$ or $\cos^5 x = 1 + \sin^2 x$

$\cos x = 0 \Rightarrow x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$; $\cos^5 x = 1 + \sin^2 x \Rightarrow x = 0$

($\because$ LHS ≤ 1 and RHS ≥ 1)

$\therefore\ x = -\dfrac{\pi}{2}, 0, \dfrac{\pi}{2}$

(b) $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$= \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°} = \dfrac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$= \dfrac{2\left(\dfrac{\sqrt{3}}{2}\cos 20° - \dfrac{1}{2}\sin 20°\right)}{\sin 20°\cos 20°} = \dfrac{4\cos 50°}{\sin 40°} = 4$

(c) $4\cos 36° - 4\cos 72° + 4\sin 18°\cos 36°$

$= 4\left(\dfrac{\sqrt{5}+1}{4}\right) - 4\left(\dfrac{\sqrt{5}-1}{4}\right) + 4\left(\dfrac{\sqrt{5}-1}{4}\right)\left(\dfrac{\sqrt{5}+1}{4}\right)$

$= \sqrt{5} + 1 - \sqrt{5} + 1 + 1 = 3$

(d) $\text{cosec}\,x = 1 + \cot x$

$\dfrac{1}{\sin x} = \dfrac{\sin x + \cos x}{\sin x} \Rightarrow \sin x + \cos x = 1$ and $\sin x \ne 0$

$\cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} \Rightarrow x - \dfrac{\pi}{4} = -2\pi + \dfrac{\pi}{4}, \dfrac{\pi}{4}$

$\Rightarrow x = -\dfrac{3\pi}{2}, \dfrac{\pi}{2}$ $\left(\because x - \dfrac{\pi}{4} \in \left[-2\pi - \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}\right]\right)$

(47) **(D).**

(a) $|\cot x| = \cot x + \dfrac{1}{\sin x}$; If $0 < x < \dfrac{\pi}{2} \Rightarrow \cot x > 0$

So, $\cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \dfrac{1}{\sin x} = 0$ no solution

If $\dfrac{\pi}{2} < \cot x < \pi$, $-\cot x = \cot x + \dfrac{1}{\sin x}$

$\dfrac{2\cos x}{\sin x} + \dfrac{1}{\sin x} = 0$

$1 + 2\cos x = 0$ and $\sin x \ne 0 \Rightarrow x = \dfrac{2\pi}{3}$

(b) Since $\sin\phi + \sin\theta = 1/2$ and $\cos\theta + \cos\phi = 2$ has no solution.

(c) $\sin^2\alpha + \sin\left(\dfrac{\pi}{3}-\alpha\right)\sin\left(\dfrac{\pi}{3}+\alpha\right)$

$= \sin^2\alpha + \sin^2\dfrac{\pi}{3} - \sin^2\alpha = \dfrac{3}{4}$

(d) $\tan\theta = 3\tan\phi$

$$\tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi} = \frac{2\tan\phi}{1+3\tan^2\phi}$$

$$= \frac{2}{\cot\phi + 3\tan\phi}. \quad \text{Max if } \tan\phi > 0$$

$$\frac{\cot\phi + 3\tan\phi}{2} \geq \sqrt{3} \text{ (Using AM} \geq \text{GM)}$$

$$(\cot\phi + 3\tan\phi)^2 \geq 12\ ;\ \tan^2(\theta-\phi) \leq \frac{1}{3}$$

(48) (A) (49) (C) (50) (C)

$(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$\Rightarrow (1+\cos x)[2\sin x - \cos x - 1 + \cos x] = 0$

$\Rightarrow (1+\cos x)(2\sin x - 1) = 0$

$\Rightarrow \cos x = -1$ or $\sin x = 1/2$

so $\sin\alpha = 1/2$ [as $0 \leq \alpha \leq \pi/2$]

$\Rightarrow \cos\alpha = \sqrt{3}/2$ .... (1)

$3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow (3\cos x - 1)(\cos x - 3) = 0$

$\Rightarrow \cos x = 1/3$ as $\cos x \neq 3$

So, $\cos\beta = 1/3,\ \sin\beta = \frac{2\sqrt{2}}{3}$ .... (2)

and $1 - \sin 2x = \cos x - \sin x$

$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x\cos x = \cos x - \sin x$

$\Rightarrow (\cos x - \sin x)(\cos x - \sin x - 1) = 0$

$\Rightarrow$ Either $\sin x = \cos x \Rightarrow \sin\gamma = \cos\gamma = 1/\sqrt{2}$ .... (3)

or $\cos x - \sin x = 1 \Rightarrow \cos x = 1, \sin x = 0$

$\cos\gamma = 1, \sin\gamma = 0$ .... (4)

So that $\cos\alpha + \cos\beta + \cos\gamma$ can be equal to

$$\frac{\sqrt{3}}{2} + \frac{1}{3} + \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{3}}{2} + \frac{1}{3} + 1$$

i.e. $\frac{3\sqrt{6} + 2\sqrt{2} + 6}{6\sqrt{2}}$ or $\frac{3\sqrt{3}+8}{6}$

$\sin\alpha + \sin\beta + \sin\gamma$ can be equal to

$$\frac{1}{2} + \frac{2\sqrt{2}}{3} + \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{1}{2} + \frac{2\sqrt{2}}{3} + 0$$

i.e. $\frac{3\sqrt{2}+14}{6\sqrt{2}}$ or $\frac{3+4\sqrt{2}}{6}$

and $\sin(\alpha - \beta)$ is equal to

$$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{1}{2}\times\frac{1}{3} - \frac{\sqrt{3}}{2}\times\frac{2\sqrt{2}}{3} = \frac{1-2\sqrt{6}}{6}$$

(51) **(A).** $\tan A + \tan B + \tan C = \tan A\tan B\tan C = \frac{p}{q}$

$\therefore\ \tan A + \tan B + \tan C + \tan A\tan B\tan C = \frac{2p}{q}$

(52) **(B).** $\tan A\tan B + \tan B\tan C + \tan C\tan A$

$$= \frac{\sin A\sin B\cos C + \cos A\sin B\sin C + \cos B\sin C\sin A}{\cos A\cos B\cos C}$$

$$= \frac{\sin B.\sin(A+C) + \cos B\sin C\sin A}{q}$$

$$= \frac{1-\cos^2 B + \cos B\sin C\sin A}{q}$$

$$= \frac{1+\cos B(\sin C\sin A - \cos B)}{q}$$

$$= \frac{1+\cos B\cos C\cos A}{q} = \frac{1+q}{q}$$

$\therefore\ \tan A + \tan B + \tan C + \tan A\tan B$

$+ \tan B\tan C + \tan C\tan A = \frac{1+p+q}{q}$

(53) **(C).** $\tan^2 A + \tan^2 B + \tan^2 C = (\tan A + \tan B + \tan C)^2 - 2(\tan A\tan B + \tan B\tan C + \tan C\tan A)$

$$= \left(\frac{p}{q}\right)^2 - 2\left(\frac{1+q}{q}\right) = \frac{p^2 - 2(1+q)q}{q^2}$$

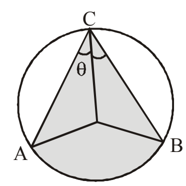
$$= \frac{p^2 - 2q}{q^2} - 2 = \frac{p^2 - 2q^2 - 2q}{q^2}$$

(54) **(D).** Angle subtended by two consecutive marks at centre = 30°. Hence at half past 4, is the angle is 45°.

(55) **(A).** Distance covered in 1 second $= 5\left(2\pi.\frac{1}{2}\right) = 5\pi$m

Distance covered in 1 hour $= \frac{5\pi}{1000}\times 60\times 60 = 56.52$

(56) **(A).** Area of region ABC $= \frac{\pi r^2}{3}$



Area of OAB $= \frac{1}{2}r^2.2\theta = r^2\theta$

Area of OAC $= \frac{1}{2}r^2\sin\theta$ = Area of OBC

$$\therefore\ \frac{1}{2}r^2\sin\theta + \frac{1}{2}r^2\sin\theta + r^2\theta = \frac{\pi r^2}{3} \Rightarrow 3\sin\theta + 3\theta = \pi$$

(57) **(C).** $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = b\sin B + c\sin C + a\sin A$

$$= \frac{b^2+c^2+a^2}{2R} \quad \therefore\ k = 2R$$

(58) **(C).** $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{a^2}{4\Delta^2} + \frac{b^2}{4\Delta^2} + \frac{c^2}{4\Delta^2} = \frac{a^2+b^2+c^2}{4\Delta^2}$

cost A + cos B + cot C

$= \frac{R}{abc}(b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2)$

$\frac{R}{abc}(b^2 + c^2 + a^2) = \frac{R}{abc}\left(\frac{4\Delta^2}{x^2} + \frac{4\Delta^2}{y^2} + \frac{4\Delta^2}{z^2}\right)$

$= \frac{4\Delta^2 R}{abc}\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$

$= \frac{4\Delta R}{abc}.\Delta\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = \Delta\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) \quad \therefore k = \Delta$

(59) **(D).** $\sum \frac{c\sin B + b\sin C}{x}, \sum \frac{x + x}{x} = 6$

EXERCISE-3

(1) **4.** $\tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°} = \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}} = \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1}$

$= \frac{8(\sqrt{5}+1) - 8(\sqrt{5}-1)}{4} = 4$

(2) **2.** $4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$

$= 2\left(1 + \cos\left(\frac{\pi}{2} - x\right)\right) + \sqrt{(1 - \cos 2x)^2 + \sin^2 2x}$

$= 2(1 + \sin x) + \sqrt{1 - 2\cos 2x + 1}$

$= 2(1 + \sin x) + \sqrt{2(1 - \cos 2x)}$

$= 2(1 + \sin x) + 2|\sin x| = 2 + 2\sin x - 2\sin x = 2$

(3) $\cos^2 x(1 - 4\sin^2 x) = 0$

$\cos^2 x = 0$ or $\sin x = \pm 1/2$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(4) **10.** $\frac{1}{1 - \sin^2 a} + \frac{1}{1 + \sin^2 a}$

$= \frac{2}{1 - \sin^4 a} + \frac{2}{1 + \sin^4 a} = \frac{4}{1 - \sin^8 a} + \frac{4}{1 + \sin^8 a}$

$= \frac{8}{1 - \sin^{16} a} = \frac{8}{1 - (1/5)} = \frac{8 \times 5}{4} = 10$

(5) **9.** $\sin x = 1$ or $\sin x = -1/2$

$\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad \therefore S = \frac{7\pi}{2}$

(6) **294.** $\tan x + \tan y = 42$ and $\cot x + \cot y = 49$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$, now, $\cot x + \cot y = 49$

$\Rightarrow \frac{1}{\tan x} + \frac{1}{\tan y} = 49 \Rightarrow \frac{\tan y + \tan x}{\tan x \cdot \tan y} = 49$

$\tan x \cdot \tan y = \frac{\tan x + \tan y}{49} = \frac{42}{49} = \frac{6}{7}$

$\tan(x + y) = \frac{42}{1 - (6/7)} = \frac{42}{1/7} = 294$

(7) **2.** $1 + \sin 2x + \frac{k}{2}\sin 2x = 1$

$\sin 2x\left[1 + \frac{k}{2}\right] = 0$ for this to be an identity

$1 + \frac{k}{2} = 0 \Rightarrow k = -2$

(8) **27.** $-\sin x - \cos x + \sin x + 18\cos x + \cos x + 9\sin x$

$18\cos x + 9\sin x = a\sin x + b\cos x$

$\therefore a = 9, \quad b = 18 \quad \therefore a + b = 27$

(9) **11.** Let $S = \cos 0° + \cos 2\theta + \cos 4\theta + ... + \cos 10\theta$

$2\sin\theta \cdot S = 2\sin\theta\,[\cos 0 + \cos 2\theta + ... + \cos 10\theta]$

$= \sin\theta + \sin\theta = \sin 3\theta - \sin\theta = \sin 5\theta - \sin 3\theta$

$= \sin 7\theta - \sin 5\theta = \sin 9\theta - \sin 7\theta = \sin 11\theta - \sin 9\theta$

$2\sin\theta \cdot S = \sin 11\theta + \sin\theta$

$2\sin\theta \cdot S = 2\sin 6\theta \cdot \sin 5\theta$

$S = \frac{2\sin 6\theta \cos 5\theta}{2\sin\theta} = \frac{\sin n\theta \cos m\theta}{\sin\theta}$

$\Rightarrow n = 6$ and $m = 5$

(10) **12.** $f(x) = a\sin^2 x + b$

f(x) has a maximum value of 8 which occurs when

$\sin^2 x = 1 \quad \therefore a + b = 8$ ....(1)

|||ly f(x) has a minimum value of – 2 which occurs where

$\sin x = 0 \quad \therefore b = -2$ ....(2)

from (1) and (2), $a = 10;\ b = -2 \Rightarrow \quad a - b = 12$

(11) **28.** Let $\pi/16 = \theta$

$\tan^2\theta + \tan^2 3\theta + \tan^2 5\theta + \tan^2 7\theta$

$= (\tan^2\theta + \cot^2\theta) + (\tan^2 3\theta + \cot^2 3\theta)$

[Note that $\tan 7\theta = \tan(8\theta - \theta) = \cot\theta$

and $\tan 5\theta = \tan(8\theta - 3\theta) = \cot 3\theta$]

$= (\cot\theta - \tan\theta)^2 + (\cot 3\theta - \tan 3\theta)^2 + 4$

$= 4[\cot^2 2\theta + \cot^2 6\theta] + 4 = 4[\cot^2 2\theta + \tan^2 2\theta] + 4$

$= 4[(\cot 2\theta - \tan 2\theta)^2 + 2] + 4$

$= 4(\cot 2\theta - \tan 2\theta)^2 + 12$

$= 4 \cdot 4\cot^2 4\theta + 12 = 16 \times 1^2 + 12 = 28$

(12) **6.** $\dfrac{1-\cos a-\dfrac{\sin^2(a/2)}{\cos^2(a/2)}}{\sin^2\left(\dfrac{a}{2}\right)}$

$$=\frac{\sin^2\left(\frac{a}{2}\right)\left[2\cos^2\left(\frac{a}{2}\right)-1\right]}{\sin^2\left(\frac{a}{2}\right)\cos^2\left(\frac{a}{2}\right)}=\frac{2\cos a}{1+\cos a}$$

$k=2;\ w=1;\ p=1 \quad \Rightarrow k^2+w^2+p^2=4+1+1=6$

(13) **4.** $\dfrac{\sqrt{3}}{\sin 20^0}-\dfrac{1}{\cos 20^0}=\dfrac{2\left(\dfrac{\sqrt{3}}{2}\right)\cos 20^0-\sin 20^0}{\dfrac{\sin 40^0}{2}}$

$$=\frac{\sin 80^0+\sin 40^0-\sin 20^0}{\frac{\sin 40^0}{2}}$$

$$=\frac{2\sin 30^0\cos 50^0+\sin 40^0}{\frac{\sin 40^0}{2}}=4$$

(14) **1.** $\sum\cos A=-a;\ \sum\cos A\cos B=b$

and $\prod\cos A=-c$

now $(\cos A+\cos B\cos C)^2$

$$=\left(\sum\cos^2 A\right)+2\left(\sum\cos A\cos B\right)$$

$\therefore\ \cos^2A+\cos^2B+\cos^2C=a^2-2b$

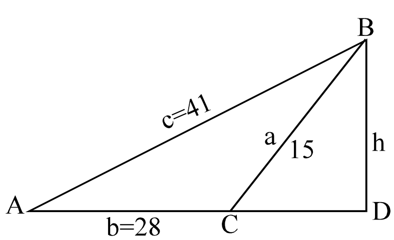
$1-2\cos A\cos B\cos C=a^2-2b^2$

$1-2c=a^2-2b^2 \ \Rightarrow a^2-2b-2c=1$

(15) **9.** Note that C must be largest

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{(15)^2+(28)^2-(41)^2}{2\cdot 15\cdot 28}=-\frac{4}{5}$$

Hence $\sin C=3/5$

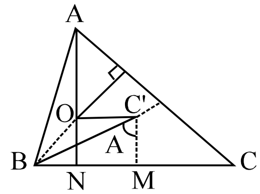


now $h=a\sin(180-C)=a\sin C=15\cdot\dfrac{3}{5}=9$

(16) **3.** $R\cos A=2R\cos B\cos C$

($C'M=ON=$ distance of orthocentre from the side)

$\therefore\ \dfrac{\cos(B+C)}{\cos B\cos C}=-2\ (ON=2R\cos B\cos C)$



$\dfrac{\cos B\cos C-\sin B\sin C}{\cos B\cos C}=-2 \qquad (C'B=R)$

$1-\tan B\tan C=-2 \quad \therefore \quad \tan B\tan C=3$

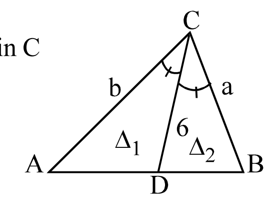
(17) **2.** $\cot\dfrac{B}{2}\cdot\cot\dfrac{C}{2}=\dfrac{s(s-b)}{\Delta}\cdot\dfrac{s(s-c)}{\Delta}\cdot\dfrac{(s-a)}{s-a}$

$$=\frac{s}{s-a}=\frac{2s}{2s-2a}$$

but given that $a+b+c=4a \ \Rightarrow\ 2s=4a$

Hence $\cot\dfrac{B}{2}\cdot\cot\dfrac{C}{2}=\dfrac{4a}{2a}=2$

(18) **9.** $\Delta=\Delta_1+\Delta_2=\dfrac{1}{2}ab\sin C$



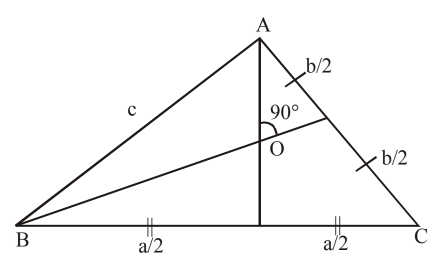
$=ab\sin\dfrac{C}{2}\cos\dfrac{C}{2}$

$=\dfrac{1}{2}6b\sin\dfrac{C}{2}+\dfrac{1}{2}6a\sin\dfrac{C}{2} \ \Rightarrow\ \dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{9}$

(19) **5.** $AD=\dfrac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$\Rightarrow AO=\dfrac{2}{3}AD$ (where O is centroid)

$\Rightarrow AO=\dfrac{1}{3}\sqrt{2b^2+2c^2-b^2}$



Similarly, $BO=\dfrac{1}{3}\sqrt{2c^2+2a^2-b^2}$

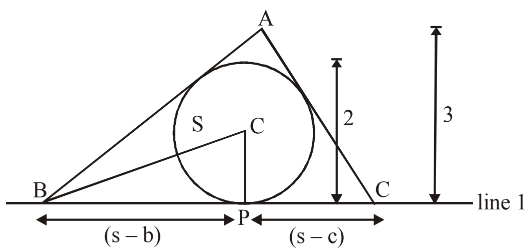
$\angle AOB=90°\Rightarrow AO^2+BO^2=AB^2\Rightarrow a^2+b^2=5c^2$

(20) **6.** $\dfrac{\cos\frac{B-C}{2}}{\cos\frac{B+C}{2}} = \dfrac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{\sin(B+C)} = \dfrac{\sin B+\sin C}{\sin A}$

Similarly, $\dfrac{\cos\frac{C-A}{2}}{\cos\frac{C+A}{2}} = \dfrac{\sin C+\sin A}{\sin B}$

and $\dfrac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}} = \dfrac{\sin A+\sin B}{\sin C}$

(21) **3.**



$(PB)(PC) = (s-b)(s-c) = \dfrac{s(s-a)(s-b)(s-c)}{s(s-a)}$

$= \dfrac{\Delta.\Delta}{s(s-a)} = r.\dfrac{\Delta}{s(s-a)}\ (r=1)$

$= \dfrac{\Delta}{s-a} = \dfrac{\Delta}{\Delta-a}\ \left(r = \dfrac{\Delta}{s} = 1 \Rightarrow s = \Delta\right)$

$= \dfrac{3a}{2\left(\frac{3a}{2}-a\right)} = \dfrac{3}{3-2} = 3$

(22) **3.** $(y+z)\cos 3\theta - (xyz)\sin 3\theta = 0$ .... (1)

$(xyz)\sin 3\theta = (2\cos 3\theta)z + (2\sin 3\theta)y$ .... (2)

$(y+z)\cos 3\theta = (2\cos 3\theta)z + (2\sin 3\theta)y$

$= (y+2z)\cos 3\theta + y\sin 3\theta$

$y(\cos 3\theta - 2\sin 3\theta) = z\cos 3\theta$ and

$y(\sin 3\theta - \cos 3\theta) = 0$

$\Rightarrow \sin 3\theta - \cos 3\theta = 0 \Rightarrow \sin 3\theta = \cos 3\theta$

$\therefore\ 3\theta = n\pi + \pi/4$

(23) **3.** $\tan\theta = \cot 5\theta \Rightarrow \cos 6\theta = 0$; $4\cos^3 2\theta - 3\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = 0$ or $\pm\dfrac{\sqrt{3}}{2}$; $\sin 2\theta = \cos 4\theta$

$\Rightarrow 2\sin^2 2\theta + 2\sin 2\theta - \sin 2\theta - 1 = 0$

$\sin 2\theta = -1$ or $\sin 2\theta = 1/2$; $\cos 2\theta = 0$ and $\sin 2\theta = -1$

$\Rightarrow 2\theta = -\dfrac{\pi}{2} \Rightarrow \theta = -\dfrac{\pi}{4}$; $\cos 2\theta = \pm\dfrac{\sqrt{3}}{2}$, $\sin 2\theta = \dfrac{1}{2}$

$\Rightarrow 2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6} \Rightarrow \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$; $\therefore\ \theta = \dfrac{\pi}{4}, \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

(24) **2.**

$\dfrac{1}{4\cos^2\theta + 1 + \frac{3}{2}\sin 2\theta} \Rightarrow \dfrac{1}{2[1+\cos 2\theta] + 1 + \frac{3}{2}\sin 2\theta}$

lies between $\dfrac{1}{2}$ to $\dfrac{11}{2}$ $\therefore$ Maximum value is 2.

Maximum value of $1 + 4\cos^2\theta + 3\sin\theta\cos\theta$

$1 + \dfrac{4(1+\cos 2\theta)}{2} + \dfrac{3}{2}\sin 2\theta = 1 + 2 + 2\cos 2\theta + \dfrac{3}{2}\sin 2\theta$

$= 3 + 2\cos 2\theta + \dfrac{3}{2}\sin 2\theta = 3 - \sqrt{4+\dfrac{9}{4}} = 3 - \dfrac{5}{2} = \dfrac{1}{2}$

So, maximum value of $\dfrac{1}{4\cos^2\theta + 1 + \frac{3}{2}\sin 2\theta}$ is 2.

(25) **7.** $\dfrac{1}{\sin\left(\frac{\pi}{n}\right)} - \dfrac{1}{\sin\left(\frac{3\pi}{n}\right)} = \dfrac{1}{\sin\left(\frac{2\pi}{n}\right)}$

$\dfrac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \dfrac{1}{\sin\frac{2\pi}{n}}$

$\sin\dfrac{4\pi}{n} = \sin\dfrac{3\pi}{n}$; $\dfrac{4\pi}{n} = (-1)^k\dfrac{3\pi}{n} + k\pi,\ k \in I$

If $k = 2m \Rightarrow \dfrac{\pi}{n} = 2m\pi$; $\dfrac{1}{n} = 2m$, not possible

If $k = 2m+1 \Rightarrow \dfrac{7\pi}{n} = (2m+1)\pi \Rightarrow n = 7, m = 0$

EXERCISE-4

(1) **(C).** $\cos x + \cos y + \cos\alpha = 0$ and $\sin x + \sin y + \sin\alpha = 0$

$\because\ \cos x + \cos y + \cos\alpha = 0$

$\Rightarrow 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2} + \cos\alpha = 0$

$\Rightarrow 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2} = -\cos\alpha$ .... (1)

and $\sin x + \sin y + \sin\alpha = 0$

$\Rightarrow 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2} + \sin\alpha = 0$

$\Rightarrow 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = -\sin\alpha$ .... (2)

Dividing (1) by (2) we get

$\cot\frac{x+y}{2} = \cot\alpha$

(2) **(A).** $\cos 1^\circ . \cos 2^\circ . \cos 3^\circ$ .... $\cos 179^\circ = 0$

$\{\because$ in between then $\cos 90^\circ$ occurs value of $\cos 90^\circ = 0\}$

(3) **(B).** $\tan x + \sec x = 2\cos x$ where $x \in [0, 2\pi]$

$\because$ In the equation tan x and sec x is given

$\therefore$ both are not define at $x = (2n-1)\frac{\pi}{2}$

$\therefore x \ne \pi/2, 3\pi/2 \; \{\because x \in [0, 2\pi]\}$ .... (1)

Now $\tan x + \sec x = 2\cos x$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x = 1$ or $\sin x = -1$

$\sin x = 1/2$ or $x = 3\pi/2$ {Not possible from (1)}

$x = \pi/6, 5\pi/6$

$\therefore$ total no. of solution in $[0, 2\pi]$ are 2.

(4) **(A).** $\tan 5\theta = \cot 2\theta$

$\Rightarrow \tan 5\theta = \tan(\pi/2 - 2\theta)$

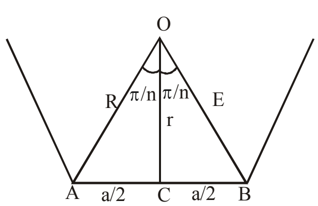
$\Rightarrow 5\theta = n\pi + (\pi/2 - 2\theta)$

$\Rightarrow 7\theta = n\pi + \pi/2$

$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$

$\{\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $n \in N\}$

(5) **(C).**



From Δ OAC $\operatorname{cosec}(\pi/n) = \frac{R}{a/2}$

$\Rightarrow R = \frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$ .... (1)

and from Δ OCB

$\cot\left(\frac{\pi}{n}\right) = \frac{r}{a/2} \Rightarrow r = \frac{a}{2}\cot\frac{\pi}{n}$ .... (2)

$\therefore$ From eq. (1) and eq. (2)

$R + r = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} + \frac{a}{2}\cot\frac{\pi}{n} = \frac{a}{2}\left[\frac{1+\cos(\pi/n)}{\sin(\pi/n)}\right]$

$= \frac{a}{2}\left[\frac{2\cos^2(\pi/2n)}{2\sin(\pi/2n)\cos(\pi/2n)}\right] = \frac{a}{2}\cot\frac{\pi}{2n}$

$\{1 + \cos\theta = 2\cos^2(\theta/2)$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)\}$

(6) **(D).** AD = 4

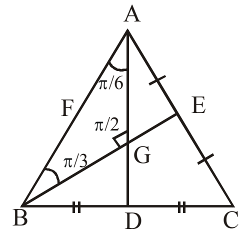
$\angle DAB = \pi/6$

$\angle ABE = \pi/3$

$\because$ G is centroid

$\therefore$ it divides AD in the ratio of 2 : 1 and AD = 4 (given)

$\therefore$ AG = 8/3 and GD = 4/3



Now $\tan\frac{\pi}{6} = \frac{BG}{AG} \Rightarrow BG = AG\tan\left(\frac{\pi}{6}\right)$

$= \frac{8}{3}\times\frac{1}{\sqrt{3}} = \frac{8}{3\sqrt{3}}$ ...... (1)

$\therefore$ Area of $\Delta ABD = \frac{1}{2}\times AD\times BG = \frac{1}{2}\times\frac{4\times 8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$ ...... (2)

$\therefore \Delta ABC = 2\Delta ABD = \frac{2\times 16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$

$\{\because$ a median divides Δ into two triangles of same area$\}$

(7) **(B).** $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$

$a\left(\sqrt{\frac{s(s-c)}{ab}}\right)^2 + c\left(\sqrt{\frac{s(s-a)}{bc}}\right)^2 = \frac{3b}{2}$

$\left\{\because \cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ba}}, \cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}\right.$

$\Rightarrow \frac{a.s(s-c)}{ab} + \frac{c.s(s-a)}{bc} = \frac{3b}{2}$

$\Rightarrow \frac{s}{b}[s-c+s-a] = \frac{3b}{2}$ $\{\because 2s = a+b+c\}$

$\Rightarrow \frac{s}{b}[2s-a-c] = \frac{3b}{2} \Rightarrow \frac{s}{b}[a+b+c-a-c] = \frac{3b}{2}$

$\Rightarrow \frac{s\times b}{b} = \frac{3b}{2} \Rightarrow s = \frac{3b}{2}$

$\Rightarrow \frac{a+b+c}{2} = \frac{3b}{2} \Rightarrow a+b+c = 3b \Rightarrow a+c = 2b$

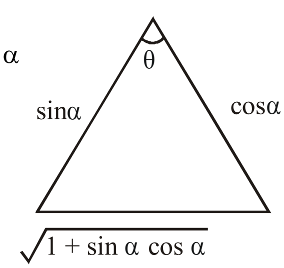
$\therefore$ a, b, c are in A.P.

(8) **(C).** sides of Δ are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ and

$0 < \alpha < \pi/2$

$\therefore 0 < \sin\alpha < 1$

$0 < \cos\alpha < 1$



but $\sqrt{1+\sin\alpha\cos\alpha} > 1$

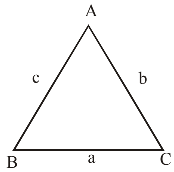
$\therefore$ largest side $\sqrt{1+\sin\alpha\cos\alpha}$

and we know angle opposite to largest side is greatest.

Let greatest angle is θ

$$\therefore \cos\theta = \frac{\cos^2\alpha + \sin^2\alpha - (\sqrt{1+\sin\alpha\cos\alpha})^2}{2\sin\alpha\cos\alpha}$$

$$= \frac{1-1-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$



$$\{\because \cos A = \frac{b^2+c^2-a^2}{2bc}\}$$

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$$

(9) **(A).** $\because \sin\alpha + \sin\beta = \frac{-21}{65}$

Squaring both side we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = \frac{441}{(65)^2} \quad .....(1)$$

and $\cos\alpha + \cos\beta = \frac{-27}{65}$

Square both side

$$\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \frac{729}{(65)^2} \quad .....(2)$$

Adding eq. (1) and eq. (2) we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha$

$$\cos\beta = \frac{441}{(65)^2} + \frac{729}{(65)^2}$$

$$\Rightarrow 2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{1170}{(65)^2}$$

$$\Rightarrow 2 + 2\cos(\alpha-\beta) = \frac{1170}{(65)^2} \Rightarrow 2[1+\cos(\alpha-\beta)] = \frac{1170}{(65)^2}$$

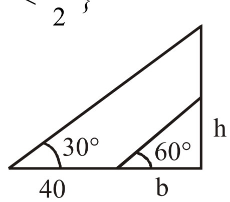
$$\Rightarrow 2.2\cos^2\frac{\alpha-\beta}{2} = \frac{1170}{(65)^2} \Rightarrow \cos^2\frac{\alpha-\beta}{2} = \frac{1170}{4.(65)^2}$$

$$\Rightarrow \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}}$$

$$\{\because \pi < \alpha-\beta < 3\pi \therefore \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}\}$$

(10) **(A).** $\tan 30° = \frac{h}{40+b}$

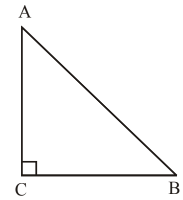
$\Rightarrow \sqrt{3}h = 40 + b$



$\tan 60° = \frac{h}{b} \Rightarrow h = \sqrt{3}b \quad \Rightarrow b = 20m$

(11) **(B).** $r \longrightarrow$ in radius

$R \longrightarrow$ circumradius



$$\because R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$

$$R = \frac{c}{2\sin C} \quad \text{but } \angle C = \frac{\pi}{2}$$

$$\therefore R = \frac{c}{2\sin(\pi/2)} = \frac{c}{2} \quad ....(1)$$

and $r = (s-c)\tan C/2 \qquad \because \angle C = \frac{\pi}{2}$

$= (s-c)\tan(\pi/4)\,; \qquad = s - c \quad ....(2)$

Adding eq. (1) and (2), $R + r = \frac{c}{2} + s - c$

$$R + r = \frac{c+2s-2c}{2} \qquad \{\because 2s = a+b+c\}$$

$\Rightarrow 2(R+r) = c + a + b + c - 2s = a + b$

(12) **(D).** $2\sin^2x + 5\sin x - 3 = 0 \qquad \because x \in [0, 3\pi]$

$2\sin^2x + 6\sin x - \sin x - 3 = 0$

$\Rightarrow 2\sin x[\sin x + 3] - 1[\sin x + 3] = 0$

$\Rightarrow [2\sin x - 1][\sin x + 3] = 0$

$\Rightarrow$ either $2\sin x - 1 = 0$ or $\sin x + 3 = 0$

$\Rightarrow \sin x = 1/2$ or $\sin x = -3$ (not possible)

$\Rightarrow x = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6$

= four value of x which satisfy $\because x \in [0, 3\pi]$

(13) **(B).** $\cos x + \sin x = 1/2$

$$\Rightarrow \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} + \frac{2\tan(x/2)}{1+\tan^2(x/2)} = \frac{1}{2}$$

$$\Rightarrow \frac{1-\tan^2(x/2)+2\tan(x/2)}{1+\tan^2(x/2)} = \frac{1}{2}$$

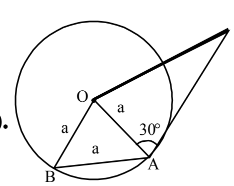
$\Rightarrow 3\tan^2(x/2) - 4\tan(x/2) - 1 = 0$

$$\tan(x/2) = \frac{4 \pm \sqrt{16-4.3.(-1)}}{2.3} = \frac{4 \pm 2\sqrt{7}}{2.3} = \left(\frac{2+\sqrt{7}}{3}\right)$$

$\{\because 0 < x < \pi \Rightarrow 0 < x/2 < \pi/2 \quad \therefore$ –ve sign neglected$\}$

$$\tan x = \frac{2\tan(x/2)}{1-\tan^2(x/2)} = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\frac{1}{3}(4+\sqrt{7})$$

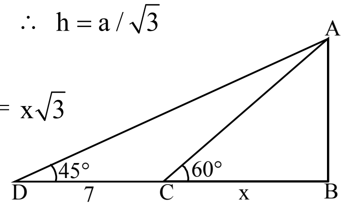
(14) **(C).**



Δ OAB is equilateral $\quad \therefore$ OA = OB = AB = a

Now, $\tan 30° = h/a \quad \therefore \quad h = a/\sqrt{3}$

(15) **(B).** $BD = AB = 7 + x$



Also, $AB = x \tan 60° = x\sqrt{3}$

$\therefore \; x\sqrt{3} = 7 + x$

$x = \dfrac{7}{\sqrt{3}-1}$; $AB = \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)$

(16) **(B).** $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -\dfrac{3}{2}$

$\Rightarrow 3 + 2(\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha)) = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

(17) **(A).** $\cos(\alpha+\beta) = \dfrac{4}{5} \quad \Rightarrow \quad \tan(\alpha+\beta) = \dfrac{3}{4}$

$\sin(\alpha-\beta) = \dfrac{5}{13} \quad \Rightarrow \quad \tan(\alpha-\beta) = \dfrac{5}{12}$

$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta) = \dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \dfrac{56}{33}$

(18) **(B).** $r = \dfrac{a}{2}\cot\dfrac{\pi}{n}$; a is side of polygon

$R = \dfrac{a}{2}\operatorname{cosec}\dfrac{\pi}{n}$; $\dfrac{r}{R} = \dfrac{\cot\frac{\pi}{n}}{\operatorname{cosec}\frac{\pi}{n}} = \cos\dfrac{\pi}{n}$

$\cos\dfrac{\pi}{n} \neq \dfrac{2}{3}$ for any $n \in N$

(19) **(A).** $A = \sin^2 x + \cos^4 x$

$= \sin^2 x + (1-\sin^2 x)^2 = \sin^4 x - \sin^2 x + 1$

$= \left(\sin^2 x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4} = \dfrac{3}{4} \le A \le 1$

(20) **(B).** $3\sin P + 4\cos Q = 6 \quad$ ...(i)

$4\sin Q + 3\cos P = 1 \quad$ ...(ii)

Squaring and adding (i) & (ii) we get $\sin(P+Q) = 1/2$

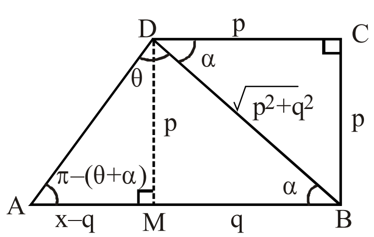
$\Rightarrow \; P + Q = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6} \Rightarrow \; R = \dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$

If $R = \dfrac{5\pi}{6}$ then $0 < P, Q < \dfrac{\pi}{6}$

$\Rightarrow \cos Q < 1$ and $\sin P < 1/2$

$\Rightarrow 3\sin P + 4\cos Q < 11/2$. So, $R = \pi/6$

(21) **(A).**



Let $AB = x$

$\tan(\pi-\theta-\alpha) = \dfrac{p}{x-q} \Rightarrow \tan(\theta+\alpha) = \dfrac{p}{q-x}$

$\Rightarrow \; q - x = p\cot(\theta+\alpha)$

$\Rightarrow \; x = q - p\cot(\theta+\alpha)$

$= q - p\left(\dfrac{\cot\theta\cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$

$= q - p\left(\dfrac{\frac{q}{p}\cot\theta - 1}{\frac{q}{p} + \cot\theta}\right) = q - p\left(\dfrac{q\cot\theta - p}{q + p\cot\theta}\right)$

$= q - p\left(\dfrac{q\cos\theta - p\sin\theta}{q\sin\theta + p\cos\theta}\right)$

$\Rightarrow \; x = \dfrac{q^2\sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$

$\Rightarrow \; AB = \dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

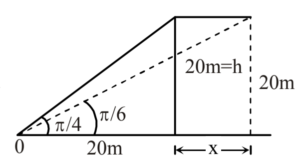
(22) **(B).** Given expression

$= \dfrac{\sin A}{\cos A} \times \dfrac{\sin A}{\sin A - \cos A} + \dfrac{\cos A}{\sin A} \times \dfrac{\cos A}{\cos A - \sin A}$

$= \dfrac{1}{\sin A - \cos A}\left\{\dfrac{\sin^3 A - \cos^3 A}{\cos A \sin A}\right\}$

$= \dfrac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\cos A} = 1 + \sec A \operatorname{cosec} A$

(23) **(D).** $\tan 30° = \dfrac{20}{20+x} = \dfrac{1}{\sqrt{3}}$



$20 + x = 20\sqrt{3}$; $x = 20(\sqrt{3}-1)$

Speed is $20(\sqrt{3}-1)$ m/sec.

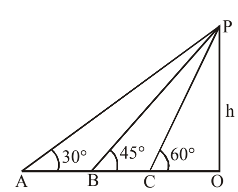
(24) **(D).** $\dfrac{1}{4}(\sin^4 x + \cos^4 x) - \dfrac{1}{6}(\sin^6 x + \cos^6 x)$

$= \dfrac{3(\sin^4 x + \cos^4 x) - 2(\sin^6 x + \cos^6 x)}{12}$

$= \dfrac{3(1 - 2\sin^2 x\cos^2 x) - 2(1 - 3\sin^2 x\cos^2 x)}{12} = \dfrac{1}{12}$

(25) **(D).** $AO = h\cot 30° = h\sqrt{3}$

$BO = h$; $CO = \dfrac{h}{\sqrt{3}}$



$$\therefore \frac{AB}{BC} = \frac{AO - BO}{BO - CO} = \frac{h\sqrt{3} - h}{h - \frac{h}{\sqrt{3}}} = \sqrt{3}$$

(26) **(A).** $2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cdot\cos\frac{x}{2} = 0$

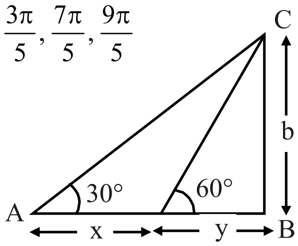
$$2\cos\frac{5x}{2}\left[\cos\frac{3x}{2} + \cos\frac{x}{2}\right] = 0$$

$$2\cos\frac{5x}{2} 2\cos x \cdot \cos\frac{x}{2} = 0$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}; \quad \cos\frac{x}{2} = 0 \Rightarrow x = \pi$$

$$\cos\frac{5x}{2} = 0 \Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

(27) **(C).** Let speed be, v



$$\tan 60° = \frac{b}{y} = \sqrt{3}$$

$$b = \sqrt{3}y$$

$$\frac{1}{\sqrt{3}} = \frac{b}{x+y} \Rightarrow x + y = b\sqrt{3}$$

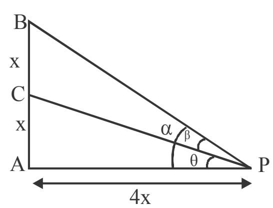
$\Rightarrow 10v + vt_1 = \sqrt{3}y \cdot \sqrt{3}$

$\Rightarrow 10v + vt_1 = 3y \Rightarrow 10v + vt_1 = 3vt_1$

$\Rightarrow 10 + t_1 = 3t_1 \Rightarrow 2t_1 = 10 \Rightarrow t_1 = 5$

(28) **(A).** $\tan\alpha = 1/2, \tan\theta = 1/4$

$\tan\beta = \tan(\alpha - \theta)$



$$\frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{8}} = \frac{\frac{1}{4}}{\frac{9}{8}} = \frac{2}{9}$$

(29) **(B).** $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$$5\left(\frac{\sin^2 x - \cos^4 x}{\cos^2 x}\right) = 2\cos^2 x + 9 \; ; \; \cos^2 x = t$$

$$5\left(\frac{1 - t - t^2}{t}\right) = 2(2t - 1) + 9$$

$5 - 5t - 5t^2 = 4t^2 + 7t \; ; \; 9t^2 + 15t - 3t - 5 = 0$

$3t(3t + 5) - (3t + 5) = 0 \; ; \; t = 1/3$

$$\cos 2x = 2 \times \frac{1}{3} - 1 = \frac{-1}{3}$$

$$\cos 4x = 2 \times \left(\frac{-1}{3}\right)^2 - 1 = \frac{2}{9} - 1 = \frac{-7}{9}$$

(30) **(D).** $8\cos x\left[\left(\cos^2\frac{\pi}{6} - \sin^2 x\right) - \frac{1}{2}\right] = 1$

$$8\cos x\left(\frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x\right) = 1$$

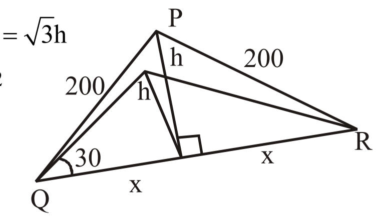
$$\frac{8\cos x}{4} \times (4\cos^2 x - 1 - 2) = 1$$

$\cos 3x = 4\cos^3 x - 3\cos x \; ; \; 2\cos 3x = 1$

$\cos 3x = 1/2 \; ; \; 3x \in [0, 3\pi]$

$$3x = \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}, \; 2\pi + \frac{\pi}{3} \Rightarrow \text{Sum} = \frac{13\pi}{9}$$

(31) **(C).** $\frac{h}{x} = \frac{1}{\sqrt{3}} \; ; \; x = \sqrt{3}h$



$200 = 3h^2 + h^2$

$4h^2 = (200)^2$

$4h^2 = 40000$

$h = 100$

(32) **(A).** We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3 = 13 - 4\cos^6\theta$

(33) **(D).** $0 < \alpha + \beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$

If $\cos(\alpha + \beta) = 3/5, \tan(\alpha + \beta) = 4/3$

and if $\sin(\alpha - \beta) = 5/13, \tan(\alpha - \beta) = 5/12$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{((\alpha + \beta) + (\alpha - \beta)\}$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\cdot\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

(34) **(C).** $a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A \Rightarrow \sin C = 2\sin A \cdot \cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A \Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$$

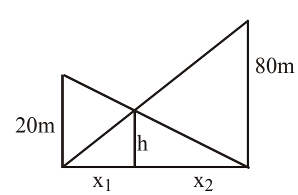
Put $a = b - \lambda, c = b + \lambda, \lambda > 0 \Rightarrow \lambda = \frac{b}{5}$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, \; c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ Required ratio = 4 : 5 : 6

(35) **(C).** By similar triangle

$$\frac{h}{x_1} = \frac{80}{x_1 + x_2} \quad ...(1)$$



$$\frac{h}{x_2} = \frac{20}{x_1 + x_2} \quad ...(2)$$

By (1) and (2)

$$\frac{x_2}{x_1} = 4 \text{ or } x_2 = 4x_1 \; ; \; \frac{h}{x_1} = \frac{80}{5x_1} \text{ or } h = 16\text{m}$$

(36) **(C).** $2(1-\sin^2\theta)+3\sin\theta=0$

$2\sin^2\theta-3\sin\theta-2=0$

$(2\sin\theta+1)(\sin\theta-2)=0$

$\sin\theta=-1/2,\ \sin\theta=2$ (reject)

Roots : $\pi+\frac{\pi}{6},\ 2\pi-\frac{\pi}{6},\ -\frac{\pi}{6},\ -\pi+\frac{\pi}{6}$

Sum of values $=2\pi$

(37) **(B).** $\frac{1}{2}(2\cos^2 10°-2\cos 10°\cos 50°+2\cos^2 50°)$

$\Rightarrow \frac{1}{2}(1+\cos 20°-(\cos 60°+\cos 40°)+1+\cos 100°)$

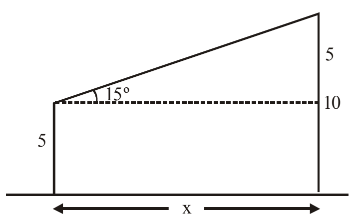
$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20°+2\sin 70°\sin(-30°)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20°-\sin 70°\right)\Rightarrow \frac{3}{4}$

(38) **(D).** (sin 10º sin 30º sin 70º) sin 30º

$\frac{1}{4}(\sin 30°)=\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{16}$

(39) **(C).**



$\tan 15°=\frac{5}{x}$; $2-\sqrt{13}=\frac{5}{x}$; $x=5(2+\sqrt{3})$

(40) **(A).** $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot 4^{-\sin^2 y}\le 1$

$2^{\sqrt{(\sin x-1)^2+4}}\le 2^{2\sin^2 y}$

$\sqrt{(\sin x-1)^2+4}\le 2\sin^2 y\Rightarrow \sin x=1$ and $|\sin y|=1$

(41) **(C).** cot α = & cosec β = $2\sqrt{2}$

So, $\frac{x}{h}=3\sqrt{2}$ ...(i)

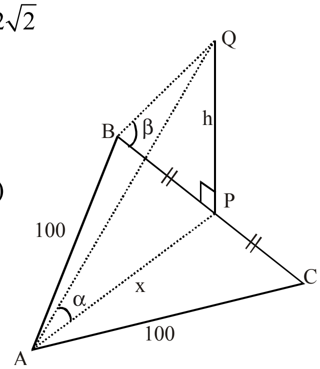
$\frac{h}{\sqrt{10^4-x^2}}=\frac{1}{\sqrt{7}}$ ...(ii)

So, from (i) & (ii)

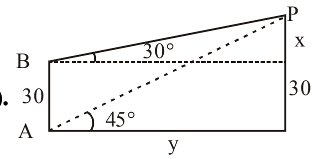
$\Rightarrow \frac{h}{\sqrt{10^4-18h^2}}=\frac{1}{\sqrt{7}}$

$\Rightarrow 25h^2=100\times 100$

$\Rightarrow h=20.$



(42) **(B).**



$\tan 45°=1=\frac{x+30}{y}\Rightarrow x+30=y$ ..... (i)

$\tan 30°=\frac{1}{\sqrt{3}}=\frac{x}{y}\Rightarrow x=\frac{y}{\sqrt{3}}$ ..... (ii)

From (i) and (ii), $y=15(3+\sqrt{3})$

(43) **1.** $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}}=\frac{1}{\sqrt{10}}$

$\tan\alpha=\frac{1}{7}$; $\sin\beta=\frac{1}{\sqrt{10}}$; $\tan\beta=\frac{1}{3}$

$\tan 2\beta=\frac{2\times\frac{1}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{3}{4}$

$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}=\frac{\frac{4+21}{28}}{\frac{25}{28}}=1$

(44) **(C).** $\cos^3\frac{\pi}{8}\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cos\frac{\pi}{8}=\sin\frac{\pi}{8}\cos\frac{\pi}{8}=\frac{1}{2}\sin\frac{\pi}{4}=\frac{1}{2\sqrt{2}}$

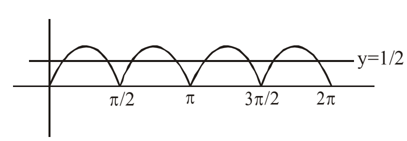
(45) **8.00.**

$\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|;\ x\in[0,2\pi]$

$\Rightarrow \log_{1/2}|\sin x|+\log_{1/2}|\cos x|=2$

$\Rightarrow \log_{1/2}(|\sin x\cos x|)=2$

$\Rightarrow |\sin x\cos x|=\frac{1}{4}\Rightarrow |\sin 2x|=\frac{1}{2}$



$\Rightarrow$ 8 solutions

(46) **(C).** $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta=1-\tan^2\theta+\tan^4\theta+....$

$\Rightarrow x=\cos^2\theta$

$y=\sum_{n=0}^{\infty}\cos^{2n}\theta\Rightarrow y=1+\cos^2\theta+\cos^4\theta+...$

$\Rightarrow y=\frac{1}{\sin^2\theta}\Rightarrow y=\frac{1}{1-x}\Rightarrow y(1-x)=1$