



# **MATHEMATICAL INDUCTION**

#### **THEOREM-I**

If  $P(n)$  is a statement depending upon n, then to prove it by induction, we proceed as follows :

(i) Verify the valdity of P (n) for  $n = 1$ 

(ii) Assume that P (n) is true for some positive integer m and then using it establish the validity of P (n) for  $n=m+1$ . Then P (n) is true for each  $n \in N$ .

#### **THEOREM-II**

If P (n) is a statement depending upon n, but beginning  $\mathbf S$ with some positive integer k, then to prove P (n), we proceed as follows :

(i) Verify the valdity of P (n) for  $n = k$ 

(ii) Assume that the statement is true for  $n = m \ge k$ . Then using it establish the validity of P (n) for  $n = m + 1$ . Then P (n) is true for each  $n \ge k$ .

#### **SUMS USEFUL RESULT BASED ON PRINCIPLE OF MATHEMEMATICAL INDUCTION**

For any natural number n

(1) 
$$
1+2+3+4+......+n=\sum_{n=1}^{\infty} \frac{n(n+1)}{2}
$$

(2) 
$$
1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}
$$

(3) 
$$
1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \sum n^3 = (\sum n)^2 = \left\{ \frac{n(n+1)}{2} \right\}^2
$$

(4) 
$$
1^4 + 2^4 + 3^4 + 4^4 + \dots + n^4 = \frac{n (n+1) (2n+1) (3n^2 + 3n - 1)}{30}
$$
  
Assume P(m+1)

(5) 
$$
1^5 + 2^5 + 3^5 + 4^5 + \dots + n^5 = \frac{n^2(n+1)^2(2n^2 + 2n - 1)}{12}
$$

(6) 
$$
2+4+6+......+2n = \sum 2n = n (n + 1)
$$
  
\n(7)  $1+3+5+.....+ (2n - 1) = \sum (2n - 1) = n^2$   
\n(8)  $x^n - y^n = (x - y) (x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ..... + xy^{n-2} + y^{n-1})$   
\n(9)  $x^n + y^n = (x + y) (x^{n-1} - x^{n-2}y + x^{n-3}y^2 + ..... - xy^{n-2} + y^{n-1})$   
\nwhen n is odd positive integer.

#### **NOTE :**

- (i) Product of r consecutive integers is divisible by r!
- (ii) For  $x \neq y$ ,  $x^n y^n$  is divisible by
	- (a)  $x + y$  if n is even (b)  $x y$  if n is even or odd
- (iii) For solving objective question related to natural numbers we find out the correct alternative by negative examination of this principle. If the given statement is  $P(n)$ , then by

putting  $n = 1, 2, 3, \dots$  in P(n) we decide the correct answer. We also use the above formulae established by this principle to find the sum of n terms of a given series. For this we first express  $T_n$  as a polynomial in n and then for finding  $S_n$ , we put  $\Sigma$  before each term of this polynomial and then use above resulsts of  $\Sigma$  n,  $\Sigma$ n<sup>2</sup>,  $\Sigma$ n<sup>3</sup> etc.

#### **ADDITIONAL EXAMPLES**

#### **Example 1 :**

Prove that  $n < 2^n$  for all positive integers n.

**ATHEMATICAL INDUCTION**  
\n
$$
\begin{array}{ll}\n\text{perading upon n, then to prove it by} & \text{putting n = 1, 2, 3, ....... in P (n) we decide the correct answer. } \n\text{is follows: } \n\text{in follows: } \n\text{by (n) for n = 1} & \text{for if } n \text{ is not lower. } \n\text{is follows: } \n\text{by (n) for n = 1} & \text{for } n \text{ is not lower. } \n\text{by (n) for n = 2} & \text{for } n \text{ is not lower. } \n\text{by (n) for n = 3} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 4} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 5} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 6} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 7} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 8} & \text{for } n \text{ is not even. } \n\text{by (n) for n = 1} & \n\text{by (n) for n = 2} & \n\text{by (n) for n = 4} & \n\text{by (n) for n = 5} & \n\text{by (n) for n = 6} & \n\text{by (n) for n = 7} & \n\text{by (n) for n = 8} & \n\text{by (n) for n = 6} & \n\text{by (n) for n = 6} & \n\text{by (n) for n = 7} & \n\text{by (n) for n = 8} & \n\text{by (n) for n = 1} & \n\text{by (n)
$$

 products of every pair of squares of the first n natural Prove by mathematical induction that the sum of the

numbers is 
$$
\frac{1}{360}
$$
 n (n<sup>2</sup> - 1) (5n + 6).

**Sol.** Let 
$$
p(n) = 1^2 \cdot 2^2 + 1^2 \cdot 3^2 + \dots + 1^2 \cdot n^2 + 2^2 \cdot 3^2 + 2^2 + 4^2
$$

6  
+........+2<sup>2</sup>. n<sup>2</sup>+........+ (n-1)<sup>2</sup>. n<sup>2</sup> = 
$$
\frac{1}{360}
$$
 n (n<sup>2</sup>-1)(5n+6)

$$
\left[ \frac{2}{2} \right]^2 = 1^2 \cdot 2^2 = \frac{2 \times 3 \times 15 \times 16}{360}
$$
 is true

Assume P(m+1) = 
$$
\frac{1}{360}
$$
 m (m<sup>2</sup> – 1) (4m<sup>2</sup> – 1) (5m+6)

$$
+\frac{(m+1)^2 m(m+1)(2m+1)}{6}
$$

$$
x \sin \alpha \tan \alpha \tan \alpha
$$
\n
$$
= \sum n^2 = \frac{n(n+1)(2n+1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+2)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{12}
$$
\n
$$
= \frac{n^2(n+1)^2(2n^2+2n-1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{12}
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= \frac{n(n+1)(2n+1)}{12}
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= \frac{n^2(n+1)^2(2n^2+2n-1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{12}
$$
\n
$$
= \frac{n(n+1)(2n+1)}{360}
$$
\n
$$
= \frac{n(n+1)(2m+1)}{360}
$$
\n<math display="</math>

$$
= \frac{m(m+1)(2m+1)}{360} (10m^3 + 57m^2 + 107m + 66)
$$
  
 
$$
m(m+1)(2m+1)
$$

$$
=\frac{m(m+1)(2m+1)}{360} (m+2)(2m+3)(5m+11)
$$

 $= (m+1)(m^2+2m)(4(m+1)^2-1(5m+11))$  $=\{(m+1)(m+1)^2-1\}$   $\{4(m+1)^2-1\}$   $\{m+1\}+6$ This being of the same form the RHS of P (m),  $P(m + 1)$  is true. Hence,  $P(n)$  is true by mathematical induction.



#### **Example 3 :**

Prove by mathematical induction or that the sum or otherwise that  $3^{2n+2}$ ,  $5^{2n}$  -  $3^{3n+2}$ ,  $2^{2n}$  is divisible by 1053 for  $n \leq 1$ .

**Sol.** Let 
$$
f(n) = 3^{2n+2} (5^{2n} - 3n, 2^{2n}) = 9^{n+1} (25^n - 12^n)
$$
  
\n $= 81.9^{n-1} (25^n - 12^n)$   
\nNow,  $a^n - b^n$  is divisible by  $a - b$   
\n $\therefore 25^n - 12^n$  is divisible by  $25 - 12 = 13$   
\n $\therefore f(n) = 81.9^{n-1} \cdot 13 \times k = 1053.9^{n-1} \cdot k$   
\nHence  $f(n)$  is divisible by 1053.

#### **Example 4 :**

Find  $P_{k+1}$  for the following.

(a) 
$$
P_k
$$
:  $S_k = \frac{k^2 (k+1)^2}{4}$   
\n(b)  $P_k$ :  $S_k = 1 + 5 + 9 + ... + [4 (k-1) - 3] + (4k-3)$   
\n(c)  $P_k$ :  $S_k \ge 2k+1$ 

**Sol.** (a) 
$$
P_{k+1}: S_{k+1} = \frac{(k+1)^2 (k+1+1)^2}{4}
$$
 [Replace k by k+1]

4 (b) P<sup>k</sup> : S<sup>k</sup> = 1 + 5 + 9 + . . . + {[4 (k + 1)–1] – 3}+ [4 (k+3) – 3] = 1 + 5 + 9 + . . . + (4k – 3) + (4k + 1) (c) Pk+1 : 3k+1 2 (k + 1) + 1 3k+1 2k + 3. n 1 2 2 7 (7 1) 49 (64) <sup>784</sup> 4 4 

**Example 5 :**

Find 
$$
\sum_{n=1}^{7} n^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3
$$

**Sol.** Using the formula for the sum of the cubes of the first n positive integers, you obtain the following.

(b) 
$$
P_k: S_k = 1 + 5 + 9 + ... + \{[4(k+1)-1]-3\} + [4(k+3)-3]
$$
  
\t $= 1 + 5 + 9 + ... + (4k-3) + (4k+1)$   
(c)  $P_{k+1}: 3^{k+1} \ge 2(k+1)+1$   
\t $3^{k+1} \ge 2k+3$ .  
\t\t\t\t**Sol.** E  
\t\t\t\t $\sum_{n=1}^{7} n^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$   
\t\t\t\t $\sum_{n=1}^{7} n^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$   
\t\t\t\t $\sum_{n=1}^{7} n^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 =$   
\t\t\t\t $\sum_{n=1}^{7} n^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 =$   
\t\t\t\t $\frac{7^2(7+1)^2}{4} = \frac{49(64)}{4} = 784$   
\t\t\t\t $\sum_{n=1}^{7} n^3 = 784$   
\t\t\t\t $\sum_{n=1}^{7} n^3 = 784$   
\t\t\t\t $\sum_{n=1}^{7} (7+1)^2 = 49(64) = 784$   
\t\t\t\t $\sum_{n=1}^{7} (7+1)^2 = 784$   
\t\t\t\t $\sum_{n=1}^{7$ 

Check this sum by adding the numbers 1, 8, 27, 64, 125, 216 and 343.

#### **Example 6 :**

Use mathematical induction to prove the following formula.  $S_n = I + 3 + 5 + 7 + ... + (2n - 1) = n^2$ . .

**Sol.** Mathematical induction consists of two distinct parts. First, you must show that the formula is true when  $n = 1$ .

(1) When  $n = 1$ , the formula is valid, because  $S_1 = 1 = 1^2$ . The second part of mathematical induction has two steps. The first step is to assume that the formula is valid for some integer k. The second step is to use this assumption to prove that the formula is valid for the next integer,  $k + 1$ . (2) Assuming that the formula

$$
S_k = 1 + 3 + 5 + 7 + ... + (2k - 1) = k^2
$$

is true, you must show that the formula  $S_{k+1} = (k + 1)^2$  is true.  $S_{k+1} = 1 + 3 + 5 + 7 + ... + (2k-1) + [2 (k+1) - 1]$  $= [1 + 3 + 5 + 7 + ... + (2k-1)] + (2k+2-1)$  $= S_k + (2k +$  $+(2k+1)$  [Group terms to form  $S_k$ ]  $=k^2+2k+$  $+ 2k + 1$  [Replace S<sub>k</sub> by k<sup>2</sup>]  $=(k+1)^2$ 

Combining the results of parts (1) and (2), you can conclude by mathematical induction that the formula is valid for all positive integer values of n.

**STUDY MATER**<br> **EXECUTE THE EXECUTE OF THE CONDUMATER**<br> **EXECUTE TO ALL THE CONDUMATER**<br> **EXECUTE 2** 2 2 2 is divisible by 1053 for<br>  $32^{2n+2}$ ,  $52^{2n}$ ,  $3^{2n+2}$ ,  $2^{2n}$  is divisible by 1053 for<br>  $= 2 \times 2 \times 12^{2n}$ <br> **Example 3:**<br>
We by mathematical induction or that the sum or other<br>
wise that  $3^{2n+2}$ ,  $5^{2n} - 3^{3n+2}$ ,  $2^{2n}$  is divisible by 1063 for<br>
wise that  $3^{2n+2}$ ,  $5^{2n} - 3^{3n+2}$ ,  $2^{2n}$  is divisible by 1063 for<br>
Sol. L **STUDY MATERIAL: MAT**<br>
cal induction or that the sum or other<br>
true. Space the time signal time is the formula  $S_{k+1}$ <br>  $-3^{3n+2}$ ,  $2^{2n}$  is divisible by 1053 for<br>  $S_{k+1} = [1+3+5+7+...+(2k-1)+[2(1+3+2+1)]$ <br>  $S_{k+1} = [1+3+5+7$ <sup>=</sup> 2 2 (k 1) (k 2) cal induction or that the sum or other-<br>  $x^3 - 3^{2n+2}$ ,  $x^2 - 3^{2n+2}$  is divisible by 1053 for<br>  $x^2 - 12^n$ <br>  $x^3 - 3^{2n-2} = 9^{n+1} (25^n - 12^n)$ <br>  $x^4 + 1 + 1^2 (k + 1) - 1 - 3 + [4(k+3) - 3]$ <br>  $x + 1 + 1 + (4(k-1) - 1) - 3 + [4(k+3) - 3]$ <br>  $x^$ It occasionally happens that a statement involving natural numbers is not true for the first  $k - 1$  positive integers but is true for all values of  $n \geq k$ . In these instances, you use a slight variation of the Principle of Mathematical Induction in which you verify  $P_k$  rather than  $P_1$ . This variation is called the extended principle of mathematical induction. To see the validity of this variation, note from Figure that all but the first  $k - 1$  dominoes can be knocked down by knocking over the kth domino. This suggests that you can prove a statement  $P_n$  to be true for  $n \geq k$  by showing that  $P_k$  is true and that  $P_k$  implies  $P_{k+1}$ . =  $k^2 + 2k + 1$  [Replace S<sub>k</sub> by  $k^2$ ]<br>
=  $(k + 1)^2$ <br>
combining the results of parts (1) and (2), you can conclude<br>
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Combining the results of parts (1) and (2), you can conclude<br>
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positive integer values of n.<br>
It occasionally happens that a statement in = S<sub>k</sub> + (2k + 1) [Group terms to form S<sub>k</sub>]<br>
= k + 2k + 1 [Replace S<sub>k</sub> by k<sup>2</sup>]<br>
= (k + 1)<sup>2</sup><br>
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in.<br>
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and a statement involving natural<br>  $\geq$  K. In these instances, you use a<br>  $\geq$  K. In these instances, you use a<br>
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after than P<sub>1</sub>. This va is true for all values of  $n \ge k$ . In these instances, you use a<br>sight variation of the Principle of Mathematical Induction<br>sight variation of the Principle of Mathematical Induction<br>in which you verify  $P_k$  rather than  $P$ variation of the Principle of Mathematical Induction<br>
inch you verify  $P_k$  rather than  $P_1$ . This variation is<br>
the extended principle of mathematical induction. To<br>
te validity of this variation, note from Figure that a nbers is not true for the first k - 1 positive integers but<br>ue for all values of  $n \ge k$ . In these instances, you use a<br>ht variation of the Principle of Mathematical Induction<br>which you verify  $P_k$  rather than  $P_1$ . This nt variation of the Principle of Mathematical induction<br>
which you verify P<sub>K</sub> rather than P<sub>1</sub>. This variation is<br>
ed the extended principle of mathematical induction. To<br>
the validity of this variation, note from Figure

#### **Example 7 :**

Find a formula for the following finite sum :

$$
\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{n(n+1)}
$$

**Sol.** Begin by writing out the first few sums.

in which you verify 
$$
P_k
$$
 rather than  $P_1$ . This variation is  
\ncalled the extended principle of mathematical induction. To  
\nsee the validity of this variation, note from Figure that all  
\nbut the first  $k - 1$  domains, can be knocked down by  
\nknowledge over the kth domino. This suggests that you can  
\nprove a statement  $P_n$  to be true for  $n \ge k$  by showing that  
\n $P_k$  is true and that  $P_k$  implies  $P_{k+1}$ .  
\n $\text{while } 7$ :  
\nFind a formula for the following finite sum :  
\n $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{n (n + 1)}$   
\nBegin by writing out the first few sums.  
\n $S_1 = \frac{1}{1.2} = \frac{1}{2} = \frac{1}{1+1}$ ;  $S_2 = \frac{1}{1.2} + \frac{1}{2.3} = \frac{4}{6} = \frac{2}{3} = \frac{2}{2+1}$   
\n $S_3 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} = \frac{9}{60} = \frac{3}{5} = \frac{3}{4+1}$   
\n $S_4 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} = \frac{48}{60} = \frac{4}{5} = \frac{3}{4+1}$   
\nFrom this sequence, it appears that the formula for the kth  
\nsum is  
\n $S_k = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$   
\nTo prove the validity of this hypothesis, use mathematical  
\ninduction, as follows. Note that you have already verified  
\nthe formula for  $n = 1$ , so you can begin by assuming that  
\nthe formula is valid for  $n = k$  and trying to show that it is  
\nvalid for  $n = k + 1$ .  
\n $S_{k+1} = \left[ \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{k(k+1)} \right] + \frac{1}{(k+1)($ 

From this sequence, it appears that the formula for the kth sum is

$$
S_k = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}
$$

To prove the validity of this hypothesis, use mathematical induction, as follows. Note that you have already verified the formula for  $n = 1$ , so you can begin by assuming that the formula is valid for  $n = k$  and trying to show that it is valid for  $n = k + 1$ .

Begin by writing out the first few sums.  
\n
$$
S_1 = \frac{1}{1.2} = \frac{1}{2} = \frac{1}{1+1}
$$
;  $S_2 = \frac{1}{1.2} + \frac{1}{2.3} = \frac{4}{6} = \frac{2}{3} = \frac{2}{2+1}$   
\n $S_3 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} = \frac{9}{12} = \frac{3}{4} = \frac{3}{3+1}$   
\n $S_4 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} = \frac{48}{60} = \frac{4}{5} = \frac{3}{4+1}$   
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\n $S_k = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$   
\nTo prove the validity of this hypothesis, use mathematical induction, as follows. Note that you have already verified the formula for n = 1, so you can begin by assuming that the formula is valid for n = k and trying to show that it is valid for n = k + 1.  
\n $S_{k+1} = \left[\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{k(k+1)}\right] + \frac{1}{(k+1)(k+2)}$   
\n $= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)}$   
\n $= \frac{k^2 + 2k + 1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$ .  
\nSo, the hypothesis is valid.

So, the hypothesis is valid.



## **QUESTION BANK**

### **EXERCISE**

- **Q.1**  $2^n > n^2$  when  $n \in N$  such that (A)  $n > 2$  (B)  $n > 3$ (C) n < 5 (D) n  $\ge$  5
- **Q.2** If  $n \in N$  and n is odd, then  $n(n^2 1)$  is divisible by  $(A) 24$  (B) 16  $(C)$  32 (D) 8
- **Q.3** If  $49^n + 16n + \lambda$  is divisible by 64 for all  $n \in N$ , then the least negative internal value of  $\lambda$  is –  $(A) -2$  (B) –1 **EXERCISE**<br>  $(2)$ <br>  $(1)$  (2)  $10^{-5}$ <br>  $(3)$  (2)  $10^{-5}$ <br>  $(4)$  (2)  $10^{-5}$ <br>  $(5)$  (2)  $(6)$   $(7)$ <br>  $(8)$  (3)  $(9)$ <br>  $(10)$ <br> **CULESTION BANK**<br>
EXERCISE<br>
S when n ∈ N such that<br>
S (B) n ≥ 3<br>
S and n is odd, then n (n<sup>2</sup> - 1) is divisible by -<br>
(B) 16<br>
(D) 8<br>
(D) a - 4<sup>n</sup> - 1<br>
(D) -4<br>
(D) -4<br>
(B) -1<br>
(D) -4<br>
(B) -1<br>
(D) -4<br>
(D) -4<br>
(D) -4<br>
(D) -(C) n<sup>-2</sup> (n<sup>-2</sup>)<br>
(A) n<sup>-2</sup> (h) n<sup>-2</sup> (h) n<sup>-2</sup> (h) n<sup>-2</sup> (h) n<sup>-2</sup> (h) n<sup>-2</sup> (h) 2<br>
(C) n<sup>-2</sup> (n) n (B) n>3<br>
and n is odd, then n (n<sup>2</sup> = 1) is divisible by –<br>
(B) 16<br>
(D) 16<br>
(D) 8<br>
(D) 8<br>
(D) 8<br>
(D) -4<br>
(D) Equal to the formula<br>
Lead negative internal value of  $\lambda$  is divisible by 64 for all n  $\in$  N, then the<br>
Lead negative internal value of  $\lambda$  is  $(S)$ <br>
(C)-3 (B)-1<br>
C(-)-3 (B)-1<br>
C(-)-3 (C)--4 (C) S(E)--1 (C) S(E)--1 (C)
	- $(C)$  3 (D) 4
- **Q.4** For  $n \in N$ ,  $x^{n+1} + (x+1)^{2n-1}$  is divisible by (A) x (B)  $x + 1$  $(C) x^2 + x + 1$  $+x+1$  (D)  $x^2-x+1$
- **Q.5** The sum of the terms in the n<sup>th</sup> bracket of the series  $(1) + (2 + 3 + 4) + (5 + 6 + 7 + 8 + 9) + \dots$  is –  $(A) (n-1)<sup>3</sup> + n<sup>3</sup>$  $(B) (n+1)<sup>3</sup> + 8n<sup>2</sup>$

(C) 
$$
\frac{(n+1)(n+2)}{6n}
$$
 (D)  $(n+1)^3 + n^3$ 

**Q.6** If  $n \in N$  and  $n > 1$ , then –

(A) 
$$
n! > \left(\frac{n+1}{2}\right)^n
$$
 (B)  $n! \ge \left(\frac{n+1}{2}\right)^n$ 

$$
1! < \left(\frac{n+1}{2}\right)^n \tag{D) None of these}
$$

**Q.7** If  $\omega$  is an imaginary cube root of unity then value of expression 1.  $(2-\omega)$ .  $(2-\omega^2)$  + 2.  $(3-\omega)$ .  $(3-\omega^2)$  + ........  $+(n-1)(n-\omega)$ .  $(n-\omega^2)$  is -

(A) 
$$
\frac{1}{4}n^2(n+1)^2
$$
 (B)  $\frac{1}{4}n^2(n+1)^2 - n$  (A)  $\left(\frac{n(n+1)}{2}\right)^2$ 

(C) 
$$
\frac{1}{6}n^2(n+1)^2 - 1
$$
 (D) None of these

**Q.8**  $\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \dots$  upto n terms, is equal to–

(A) 
$$
\frac{6n}{n+1}
$$
 (B)  $\frac{9n}{n+1}$  (C)  $\frac{12n}{n+1}$  (D)  $\frac{5n}{n+1}$  (A)

**Q.9** If 
$$
A = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}
$$
, then for some  $n \in N$ ,  $A^n$  is equal to –

(A) 
$$
\begin{pmatrix} n & k \\ 0 & n \end{pmatrix}
$$
 (B)  $\begin{pmatrix} 1 & k^n \\ 0 & 1 \end{pmatrix}$  (C)  $\begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix}$  (D)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

**Q.10** 
$$
\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \dots \dots \dots \dots \text{to n terms} =
$$

$$
\frac{\sqrt{3}}{4} \sqrt{4NK}
$$
\n
$$
\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \dots \dots \text{ to } n \text{ terms } =
$$
\n(A)  $n - \frac{4^n}{3} - \frac{1}{3}$  (B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$   
\n(C)  $n + \frac{4^n}{3} - \frac{1}{3}$  (D)  $n - \frac{4^n}{3} + \frac{1}{3}$   
\nLet S(K) = 1 + 3 + 5 + \dots + (2K - 1) = 3 + K<sup>2</sup>. Then which

$$
\text{(D) } n + \frac{4^n}{3} - \frac{1}{3} \qquad \qquad \text{(D) } n - \frac{4^n}{3} + \frac{1}{3}
$$

- 3 3 (B) <sup>n</sup> 4 1 MORTHOWARE DIEARNING<br>
MORTHOWARE DIEARNING<br>
(A)  $n - \frac{4^n}{3} - \frac{1}{3}$ <br>
(B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$ <br>
(C)  $n + \frac{4^n}{3} - \frac{1}{3}$ <br>
(D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>
Let S(K) = 1 + 3 + 5 + ........ + (2K – 1) = 3 + K<sup>2</sup>. Then which<br>
of t 3 3 (D) <sup>n</sup> 4 1 **Q.11** Let  $S(K) = 1 + 3 + 5 + \dots + (2K - 1) = 3 + K^2$ . Then which of the following is true? (A) S (1) is correct (B)  $S(K) \Rightarrow S(K+1)$ A)  $n - \frac{4^n}{3} - \frac{1}{3}$  (B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$ <br>
(B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$ <br>
(D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>
(D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>
et S(K) = 1 + 3 + 5 + ........ + (2K – 1) = 3 + K<sup>2</sup>. Then which<br>
f the following is true  $\frac{3}{4} + \dots$  to n terms =<br>  $-\frac{1}{3}$  (B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$ <br>
(D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>  $+ 3 + 5 + \dots$   $+ (2K - 1) = 3 + K^2$ . Then which<br>
wing is true?<br>
(B)  $S(K) \Rightarrow S(K + 1)$ <br>  $> S(K + 1)$ <br>
le of mathematical induction can be used t  $+\frac{15}{16} + \frac{63}{64} + \dots$  to n terms =<br>
(B) n +  $\frac{4^{n}}{3} - \frac{1}{3}$ <br>
(B) n +  $\frac{4^{n}}{3} - \frac{1}{3}$ <br>
(D) n  $-\frac{4^{n}}{3} + \frac{1}{3}$ <br>
(D) n  $-\frac{4^{n}}{3} + \frac{1}{3}$ <br>
xt S(K) = 1 + 3 + 5 + ........ + (2K – 1) = 3 + K<sup>2</sup>. Then which<br>  $\frac{15}{16} + \frac{63}{64} + \dots \dots \text{ to } n \text{ terms} = \frac{1}{16} - \frac{4^n}{3} - \frac{1}{3}$ (B)  $n + \frac{4^{n}}{3} - \frac{1}{3}$ <br>  $n + \frac{4^n}{3} - \frac{1}{3}$ (D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>  $\frac{4^n}{3} - \frac{1}{3}$ (D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>  $\frac{5}{(K)} = 1 + 3 + 5 + \dots + (2K - 1) = 3 + K^2$ 
	-
	- (C)  $S(K) \Rightarrow S(K+1)$

(D) Principle of mathematical induction can be used to prove the formula

**Q.12 Statement-1:** For every natural number  $n \ge 2$ 

$$
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}
$$

**Statement -2:** For every natural number  $n \ge 2$ 

$$
\sqrt{n(n+1)} < n+1
$$

5<br>
S and n is odd, then  $(n^2-1)$  is divisible by -<br>
(B) 16<br>
(B) 4<br>
(B) 16<br>
(B) 11<br>
2.11<br>
1.1 Let S(K) = 1+3 + 5 + ...........  $\overline{2}$  (A) Statement-1 is true, Statement-2 is true; Statement-2 Q.10  $\frac{4}{4} + \frac{4}{16} + \frac{4}{64} + \dots$ ......to n terms =<br>
is divisible by –<br>
(A)  $n - \frac{4^n}{3} - \frac{1}{3}$  (B)  $n + \frac{4^{-n}}{3} - \frac{1}{3}$ <br>
all  $n \in \mathbb{N}$ , then the<br>
(C)  $n + \frac{4^n}{3} - \frac{1}{3}$  (D)  $n - \frac{4^n}{3} + \frac{1}{3}$ <br>
Q.11 Let S(K)=1+ For all n ∈ N, then the<br>  $\frac{4}{3} - \frac{1}{3}$  (B) n +  $\frac{4^{-n}}{3} - \frac{1}{3}$ <br>
or all n ∈ N, then the<br>  $\frac{1}{2}$ <br>
or all n ∈ N, then the<br>  $\frac{1}{2}$ <br>
O.11 Let S(K) = 1+3 + 5 + .......+(2K-1) = 3 + K<sup>2</sup>. Then which<br>
sible by<br>
o (B) 8<br>
(B) 8<br>
(B) 8<br>
(B) 8<br>
(B) -4<br>
(B) -2<br>
(B) -2<br>
(B) -2<br>
(B) -2<br>
(B) -1<br>
(B) -1<br>
(B) -1<br> (D) 8<br>  $x + 1$ <br>
EN,  $x^{n+1} + (x + 1)^{2n-1}$  is divisible by  $\begin{array}{ll}\n\text{(B)}-1 & \text{(C)} n + \frac{4^n}{3} - \frac{1}{3} & \text{(D)}-4 \\
\text{(B)}-1 & \text{(B)}-1 & \text{(B)}-1 & \text{(C)}-4 & \text{(D)}-4 & \text{(D)}-4 \\
\text{(D)}-4 & \text{(D)}-4 & \text{(D)}-1 & \text{(D)}-4 & \text{(D)}-1 & \text{(D)}-4 \\
\text{(D)}-4 & \text{(D)}-2 & \text{($ +  $\frac{15}{16}$  +  $\frac{63}{64}$  + ........to n terms =<br>
(b) n +  $\frac{4^{-n}}{3}$  -  $\frac{1}{3}$ <br>
(c) n +  $\frac{4^{-n}}{3}$  -  $\frac{1}{3}$ <br>
(d) n +  $\frac{4^{-n}}{3}$  -  $\frac{1}{3}$ <br>
(d) n +  $\frac{4^{-n}}{3}$  -  $\frac{1}{3}$ <br>
(d) n +  $\frac{4^{-n}}{3}$  +  $\frac{1}{3}$ is a correct explanation for Statement-1 (B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1 (C) Statement-1 is true, Statement -2 is false (D) Statement-1 is false, Statement-2 is true nder n ≥ 2<br>
mber n ≥ 2<br>
2 is true; Statement-2<br>
2 is true; Statement-2<br>
nt-1<br>
2 is true; Statement-2<br>
ement-1<br>
2 is false<br>
2 is true<br>  $\frac{n(n+1)}{2}$ <br>  $\frac{n(n-1)}{3}$  S (K)  $\Rightarrow$  S(K+1)<br>
duction can be used to<br>
umber n  $\ge$  2<br>
umber n  $\ge$  2<br>
-2 is true; Statement-2<br>
eent-1<br>
-2 is true; Statement-2<br>
atement-1<br>
-2 is false<br>
tt-2 is true<br>
( $\frac{n(n+1)}{2}$ )<sup>3</sup><br>
( $\frac{n(n-1)}{3}$ )<sup>2</sup> mber n  $\ge$  2<br>
2 is true; Statement-2<br>
2 is true; Statement-2<br>
nt-1<br>
2 is false<br>
2 is false<br>
2 is true<br>  $\left(\frac{n(n+1)}{2}\right)^3$ <br>  $\left(\frac{n(n-1)}{3}\right)^2$ umber n ≥ 2<br>
umber n ≥ 2<br>
-2 is true; Statement-2<br>
eent-1<br>
-2 is true; Statement-2<br>
tement-1<br>
-2 is false<br>
t-2 is true<br>  $\left(\frac{n(n+1)}{2}\right)^3$ <br>  $\left(\frac{n(n-1)}{3}\right)^2$ umber n ≥ 2<br>
-2 is true; Statement-2<br>
eent-1<br>
-2 is true; Statement-2<br>
tement-1<br>
-2 is false<br>
t-2 is true<br>  $\left(\frac{n(n+1)}{2}\right)^3$ <br>  $\left(\frac{n(n-1)}{3}\right)^2$ ment -2 is true; Statement-2<br>
atement-1<br>
ment -2 is true; Statement-2<br>
r Statement-1<br>
ment -2 is false<br>
ment-2 is true<br>
(B)  $\left(\frac{n(n+1)}{2}\right)^3$ <br>
(D)  $\left(\frac{n(n-1)}{3}\right)^2$ <br>
(B) 13<br>
(D) 14<br>
2=<br>
(B)  $\frac{n(2n-1)(2n+1)}{3}$ <br>
(D)  $\frac{n(2$ rue; Statement-2<br>
rue; Statement-2<br>
tt-1<br>
dalse<br>
rue<br>  $\left(-1\right)^3$ <br>  $\left(-1\right)^2$ <br>  $\left(-1\right)(2n+1)$ <br>
3<br>  $\left(-1\right)(n+1)$ <br>
3 rue; Statement-2<br>
t-1<br>
alse<br>
rue<br>  $\left(-1\right)^3$ <br>  $\left(-1\right)^2$ <br>  $\left(-1\right)(2n+1)$ <br>
3<br>  $\left(-1\right)(n+1)$ <br>
3

<sup>1</sup> 2 2 n (n 1) <sup>4</sup> <sup>1</sup> 2 2 n (n 1) n <sup>1</sup> 2 2 n (n 1) 1 (D) None of these **Q.8** 2 2 2 2 2 2 3 5 7 ..... 1 1 2 1 2 3 n 1 n 1 n 1 n 1 1 k 0 1 , then for some n N, A<sup>n</sup> n k 0 n (B) <sup>n</sup> 1 k 0 1 (C) 1 nk 0 1 (D) 1 0 0 1 **Q.13** 3 3 3 3 1 2 3 ...... n <sup>=</sup> (A) <sup>2</sup> n(n 1) 2 (B) 2 (C) <sup>3</sup> n(n 1) 2 (D) 3 (A) n (n 1) (2n 1) (C) n (n 1) (n 1) (D) n (2n 1) (n 1)

**Q.14** 
$$
10^{2n-1} + 1
$$
 is divisible by –  
(A) 12 (B) 13  
(C) 11 (D) 14

$$
Q.15 \quad 1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 =
$$

(A) 
$$
\frac{n (n-1) (2n+1)}{3}
$$
 (B)  $\frac{n (2n-1) (2n+1)}{3}$ 

(C) 
$$
\frac{n (n-1) (n+1)}{3}
$$
 (D)  $\frac{n (2n-1) (n+1)}{3}$ 

**Q.16** 
$$
3^{2n+2} - 8^n - 9
$$
 is divisible by –  
\n(A) 2 (B) 3  
\n(C) 4 (D) 8





(C) 
$$
\frac{2n}{(n-1)}
$$
 (D)  $\frac{2n}{(n+1)}$ 

Method	STUDV MATERIAL: MATFENAL: MATFEMATICS				
$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} =$	$Q.22 \left(1 + \frac{3}{1}\right)\left(1 + \frac{2}{4}\right)\left(1 + \frac{2}{4}\right)\left(1 + \frac{(2n+1)}{n^2}\right) =$				
(A) $\frac{n}{(2n+3)}$	(B) $\frac{n}{(2n-3)}$	(A) $2(n+1)^2$	(B) $(n+1)^2$		
(C) $\frac{n}{3(2n+3)}$	(D) $\frac{2n}{(2n-3)}$	(A) $(1 + 1)$	(B) $(n+1)^2$		
(C) $\frac{n}{3}$	(D) $\frac{2n}{(2n-3)}$	(A) $(n+1)$	(B) $(n+1)^2$		
(A) $\frac{(2n+1)^{2n+1}+3}{4}$	(B) $\frac{(n-1)^{2n+1}+3}{4}$	(C) $\frac{(2n-1)^{2n+1}+3}{4}$	(D) $\frac{(2n-1)^{2n+1}+3}{2}$	(A) $\frac{(3^{2n}-1)}{2}$	(B) $\frac{(3^{n}-1)}{2}$
(C) $\frac{(2n-1)3^{n+1}+2}{4}$	(D) $\frac{(2n-1)3^{n+1}+3}{2}$	(A) $\frac{(3^{n}+1)}{2}$	(B) $\frac{(3^{n}-1)}{3}$		
(C) $\frac{(3^{n}+1)}{3}$	(D) $\frac{(3^{n}+$				







(7) **(B)**. Sum = 
$$
\sum_{n=2}^{n} (n-1) (n - \omega) (n - \omega^2)
$$
  
\n
$$
= \sum_{n=1}^{n} (n-1) [(n^2 - n (\omega + \omega^2) + \omega^3]
$$
\n[ $\because$  when  $n = 1$ , sum = 0]  
\n
$$
= \sum (n-1) (n^2 + n + 1)
$$
\n
$$
= \sum (n^3 - 1) = \sum n^3 - \sum 1 = \frac{1}{4} n^2 (n + 1)^2 - n
$$
\n(8) **(A).**  
\n
$$
T_n = \frac{2n + 1}{1^2 + 2^2 + .... + n^2} = \frac{(2n + 1)6}{n(n + 1)(2n + 1)}
$$
\n
$$
= \frac{6}{n(n + 1)} = 6 \left[ \frac{1}{n} - \frac{1}{n + 1} \right]
$$
\n
$$
\Rightarrow S_n = 6 \left[ \left( 1 - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + .... + \left( \frac{1}{n} - \frac{1}{n + 1} \right) \right]
$$
\n
$$
= 6 \left[ 1 - \frac{1}{n + 1} \right] = \frac{6n}{n + 1}
$$
\n(9) **(C)**. We find that  
\n
$$
A^2 = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}
$$
\n
$$
A^3 = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}
$$
\nSimilarly,  $A^4 = \begin{pmatrix} 1 & 4k \\ 0 & 1 \end{pmatrix}$ ,  $A^5 = \begin{pmatrix} 1 & 5k \\ 0 & 1 \end{pmatrix}$  etc.  
\nSo,  $A^n = \begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix}$   
\n(10) **(B)**. For  $n = 1$ , we have  
\n<math display="</p>

le by 24 is  
\nbe true.  
\n
$$
\Rightarrow S_n = 6 \left[ \left( 1 - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + ... + \left( \frac{1}{n} - \frac{1}{n+1} \right) \right]
$$
\n
$$
= -1
$$
\n
$$
= 6 \left[ 1 - \frac{1}{n+1} \right] = \frac{6n}{n+1}
$$
\n(fλ = -1  
\n(fλ = -1  
\n(fλ = -1  
\n(fλ = 1  
\n(9) (C). We find that  
\n
$$
A^2 = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}
$$
\n
$$
A^3 = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}
$$
\n
$$
+x + 1,
$$
\nSimilarly,  $A^4 = \begin{pmatrix} 1 & 4k \\ 0 & 1 \end{pmatrix}, A^5 = \begin{pmatrix} 1 & 5k \\ 0 & 1 \end{pmatrix}$  etc.  
\nSo,  $A^n = \begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix}$   
\n $3 + 4 = 9$   
\n(10) (B). For n = 1, we have  
\n $n - \frac{4^n}{3} - \frac{1}{3} = 1 - \frac{4}{3} - \frac{1}{3} = -\frac{2}{3}$   
\n $n + \frac{4^{-n}}{3} - \frac{1}{3} = 1 + \frac{4^{-1}}{3} - \frac{1}{3} = \frac{3}{4}$   
\n $\times \left( \frac{n+1}{2} \right)^n$   
\n $n + \frac{4^n}{3} - \frac{1}{3} = 1 + \frac{4}{3} - \frac{1}{3} = \frac{5}{4}$   
\nAlso, for n = 2, we have  
\n $n + \frac{4^{-n}}{3} - \frac{1}{3} = 2 + \frac{1}{48} - \frac{1}{3} = \frac{27}{1$ 

**(10) (B).** For 
$$
n = 1
$$
, we have

A<sup>3</sup> = 
$$
\begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}
$$
  
\nSimilarly, A<sup>4</sup> =  $\begin{pmatrix} 1 & 4k \\ 0 & 1 \end{pmatrix}$ , A<sup>5</sup> =  $\begin{pmatrix} 1 & 5k \\ 0 & 1 \end{pmatrix}$  etc.  
\nSo, A<sup>n</sup> =  $\begin{pmatrix} 1 & nk \\ 0 & 1 \end{pmatrix}$   
\n= 9 (10) (B). For n = 1, we have  
\n $n - \frac{4^n}{3} - \frac{1}{3} = 1 - \frac{4}{3} - \frac{1}{3} = -\frac{2}{3}$   
\n $n + \frac{4^{-n}}{3} - \frac{1}{3} = 1 + \frac{4^{-1}}{3} - \frac{1}{3} = \frac{3}{4}$   
\n $\frac{+1}{2}$   
\n $n + \frac{4^n}{3} - \frac{1}{3} = 1 + \frac{4}{3} - \frac{1}{3} = 2$   
\n $\frac{+1}{2}$   
\n $n - \frac{4^{-n}}{3} + \frac{1}{3} = 1 - \frac{4^{-1}}{3} + \frac{1}{3} = \frac{5}{4}$   
\nAlso, for n = 2, we have  
\n $n + \frac{4^{-n}}{3} - \frac{1}{3} = 2 + \frac{1}{48} - \frac{1}{3} = \frac{27}{16}$  and  $\frac{3}{4} + \frac{15}{16} = \frac{27}{16}$   
\nHence, option (B) is correct.

Similarly, 
$$
A^4 = \begin{pmatrix} 1 & 4k \ 0 & 1 \end{pmatrix}
$$
,  $A^5 = \begin{pmatrix} 1 & 5k \ 0 & 1 \end{pmatrix}$  etc.  
\nSo,  $A^n = \begin{pmatrix} 1 & nk \ 0 & 1 \end{pmatrix}$   
\n**(B).** For  $n = 1$ , we have  
\n
$$
n - \frac{4^n}{3} - \frac{1}{3} = 1 - \frac{4}{3} - \frac{1}{3} = -\frac{2}{3}
$$
\n
$$
n + \frac{4^{-n}}{3} - \frac{1}{3} = 1 + \frac{4^{-1}}{3} - \frac{1}{3} = \frac{3}{4}
$$
\n
$$
n + \frac{4^n}{3} - \frac{1}{3} = 1 + \frac{4}{3} - \frac{1}{3} = 2
$$
\n
$$
n - \frac{4^{-n}}{3} + \frac{1}{3} = 1 - \frac{4^{-1}}{3} + \frac{1}{3} = \frac{5}{4}
$$
\nAlso, for  $n = 2$ , we have  
\n
$$
n + \frac{4^{-n}}{3} - \frac{1}{3} = 2 + \frac{1}{48} - \frac{1}{3} = \frac{27}{16}
$$
\nHence, option (B) is correct.



**(11) (C).**  $S(K) = 1 + 3 + 5 + 7 + \dots + (2K - 1) = 3 + K^2$ For  $K = 1$  $L.H.S. = 1$  and  $R.H.S. = 4$ Option (A) cancel out  $\therefore$  we know  $1 + 3 + 5 + 7 + \dots + (2K - 1) = K^2$ but in question S (K) =  $3 + K^2$  $\therefore$  by principle of mathematical induction **(O.B. SOLUTIONS** STUDY MAT<br>
CO. S(K) = 1 + 3 + 5 + 7 + ...... + (2K - 1) = 3 + K<sup>2</sup><br>
Eor K = 1 and R.H.S. = 4<br>
E.H.S. = 1 and R.H.S. = 4<br>
Deption (A) cancel out<br>  $\therefore$  we know 1 + 3 + 5 + 7 + ..... + (2K - 1) = K<sup>2</sup><br>
Let  $\{ \because S(K) \text{ is not true for } K = 1, 2, 3, \dots \}$ **(12) (B).** Let p (n) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$  $p(2) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$ 1 1 <sup>2</sup> **10.8 SOLUTIONS** STUDY MATE:<br>  $\begin{aligned}\n &\text{1,1} & \text{1,2} & \text{1,3} & \text{1,3} & \text{1,4} & \text{1,5} & \text{1,6} \\
 &\text{1,6} & \text{1,7} & \text{1,8} & \text{1,9} \\
 &\text{1,1} & \text{1,1} & \text{1,1} & \text{1,1} \\
 &\text{1,1} & \text{1,1} & \text{1,1} \\
 &\text$ **(0.8. SOLUTIONS** STUDY MATE<br>  $=1+3+5+7+......+(2K-1)=3+K^2$ <br>  $\Rightarrow$  by 6. Thus, P(1) is true for n = i<br>
c., P(k) = 1 (1+1)(1+2) = 1 × 2<br>
cancel out<br>
c. P(k) = b(k) = the first interval of  $S(K) = 3 + K^2$ <br>  $\Rightarrow$  be the  $k(k+1)(k+2) = 6\lambda$ **EXERCISE ANTIFALL: MATHEMAL: MATHEMATICS**<br>
CC, S(K)=1+3+5+7+......+(2K-1)=3+K<sup>2</sup><br>
For K=1<br>
Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ...$ ,  $\frac{1}{\sqrt{8}} \times \sqrt{k}$  is true.<br>
Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ...$ ,  $\frac{1}{\sqrt$ 1 2 k For K = 1<br>
LHS. = 1 and RHS. = 4<br>
LOP (A) example of the state and HS.<br>
Cyricol (A) example 1<br>
option (A) example of the state and HS.<br>  $\therefore$  we know 1+3+5+7+.... + (2K - 1) = K2<br>  $\therefore$  thet,  $(k+1)(k+2)$  is divisible by 6 CH.S. = 4<br>
Step 2: For m = k, Let P(k) be true,<br>  $3 + 5 + 7 + ...... + (2K - 1) = K^2$ <br>  $3 + 5 + 7 + ...... + (2K - 1) = K^2$ <br>
Step 3: For m = k +1, we have to show that P(k + 1) is true,<br>
of mathematical induction<br>
of  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + .... + \frac{1$ +5+7+.......+(2K-1)=3+K<sup>2</sup><br>
H.S.-4<br>
H.S.-2 For n=k, Left P(k) be true,<br>  $\text{Step 2: For } n = k, \text{ Let } P(n)$  (k +  $\therefore p(k+1) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ has to be true.  $\therefore$  L.H.S. >  $\sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k+1} + \sqrt{k+1}}{\sqrt{k+1}}$ 1 = 3 + K<sup>2</sup><br>
athematical induction<br>
ie. P(k + 1) is this blue by 6.<br>
athematical induction<br>  $P(k+1) = (k + 1)(k + 1 + 2)$ <br>
= (k + 1) (k + 2) + 3 (k + 1) (k + 2)<br>
= 6x + 3 (k + 1) is the + 2)<br>
= 6x + 3 (k + 1) is the + 2)<br>
= 6 and hematical induction<br>  $P(k+1) = (k+1)(k+1+2)$ <br>  $P(k+1) = (k+1)(k+1+2)$ <br>  $= k(k+1)(k+2)$ <br>  $= k(k+1)(k+2)$ <br>  $= 6k+3(k+1)(k+2)$ <br>  $= 6k+3(k+1)(k+2)$ <br>  $= 6k+6+6+6(k+1)$ <br>  $= 6k+6+6+6(k+1)$ <br>
Thus, P (k i) is to these (b, + 0) is tive.<br>
Thus, P ( 3+5+7+......+(2K-1)=K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
S(K)=3+K<sup>2</sup><br>
(sep 3: For n = k + 1, we have to show the<br>
i.e. P(k+1) is divisible by 6.<br>
P(k+1) is divisible by 6.<br> = 3 + K<sup>2</sup><br>
= 3 + K<sup>2</sup><br>
= 3 + K<sup>2</sup><br>
= 3 + K<sup>2</sup><br>
= 8 + K<sup>2</sup><br>
= (k + 1) s divisible b 6.<br>
= (k + 1) (k + 2) + 3 (k + 1) + 1) (k + 2) + 3 (k + 2)<br>
= (k + 1) (k + 2) + 3 (k + 1) (k + 2)<br>
= 6λ + 3 (k + 1) is divisible by 6.<br> SINTER (SET)<br>
(SET)<br>
(SET)<br>
(SET)<br>
(SET)<br>
(B). Let p (n) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>
(B). Let p (n) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>
(B). Let p (n) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>
(Pa) is not use C  $\therefore \frac{\sqrt{k(k+1)}+1}{\sqrt{k+1}} > \frac{k+1}{\sqrt{k+1}} = \sqrt{k+1}$ (k + 1) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>
= 6. + 6t = 6(x+ 1)<br>
Thus, P (k + 1) is true.<br>
Thus, P (k + 1) is true.<br>
∴ P (k is true.<br>
∴ P (k is true.<br>
Hence, by principle of mathematical induction, 1<br>
sassume that  $\$ Thus, P (k + 1) is divisible by 6.<br>
Therefore, P (k + 1) is divisible by 6.<br>
Therefore, P (k + 1) is true.<br>
Hence, by principle of mathematical in<br>
sume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
Hen s not true for K = 1, 2, 3,........)<br>  $= \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>  $\Rightarrow$   $= 6\lambda + 3(k + 1)(k + 2) + 3k$ <br>  $+ \frac{1}{\sqrt{2}} > \sqrt{2}$ <br>  $\therefore$  P (k) is virue  $\Rightarrow$  P (k + 1) is virue  $\Rightarrow$  P (k + 1) is virue  $\Rightarrow$  P (k + 1) is virue n) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ <br>
= 6). + 6(- A+1)<br>
Thus, P(k+1) is true.<br>
me that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
me that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
me intervention is t p(2) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$ <br>
Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
∴ p(k+1) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\$  $\therefore$  Statement (1) is true. Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ....\frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ....\frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>
(19) (A). Let P(n) be the statement given by<br>  $P(0) = 1.3 + 2.3^2 + 3.3^3 + .......n3^{2n} = \$ Let us assume that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{k}} > \sqrt{k}$  is true.<br>  $\therefore p(k+1) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ <br>
Also to be true.<br>  $\therefore$  L.H.S.>  $\sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k(k+1)} + 1}{\sqrt{k+1}}$ <br>
S .. p (k+1) =  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ...., \frac{1}{\sqrt{k+1}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ <br>
has to be true.<br>
has to be true.<br>
<br>  $\therefore$  L.H.S.>  $\sqrt{k} + \frac{1}{k+1} = \frac{\sqrt{k(k+1)} + 1}{\sqrt{k+1}}$ <br>  $\therefore$  Thus, P (1) is true<br>
Since,  $\sqrt{k(k+1)} > k$  ( $\sqrt$ ∴  $p(k+1) = \frac{1}{\sqrt{1 + \frac{1}{\sqrt{2}}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k + 1}} > \sqrt{k + 1}$ <br>
Step 1: For n = 1, we have<br>
Ans to be true.<br>
∴ L.H.S.>  $\sqrt{k} + \frac{1}{\sqrt{k + 1}} = \frac{\sqrt{k(k+1)} + 1}{\sqrt{k + 1}}$ <br>
Step 2: For n = k, assume that P (k) is true.<br>
Since,  $\$ Since  $(k + 1) < k + 2$ ∴ I.H.S. >  $\sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k(k+1)} + 1}{\sqrt{k+1}}$ <br>
Since,  $\sqrt{k(k+1)} > k$  ( $\forall k \ge 0$ )<br>
Since,  $\sqrt{k(k+1)} > k + 1$ <br>
Since,  $\sqrt{k(k+1)} + \sqrt{k+1} = \sqrt{k+1}$ <br>
LH.S.  $\sqrt{k+1}$ <br>
LH.S.  $\sqrt{k+1} + \sqrt{k+1} = \sqrt{k+1}$ <br>
LH.S.  $\sqrt{k+1} + \sqrt{k+1} = \sqrt{k+1}$ <br>
LH.S.

Hence statement 2 is also true but not correct explanation of statement (1).

- **(13) (A).**
- **(14) (A).**
- **(15) (B).**
- **(16) (D).**
- **(17) (C).**
- **(18) (C).** Let P (n) be the statement " $n(n+1)(n+2)$  is divisible by 6", i.e.  $P(n) = n(n + 1)(n + 2)$  is divisible by 6. Step 1 : For  $n = 1$ , we have

**(O.B. SOLUTIONS** STUDY MATERIAL: MATHE<br>  $+5+7+......+ (2K-1)=3+K^2$   $P(1)=1 (1+1) (1+2)=1 \times 2 \times 3=6$ , which is<br>
by 6. Thus, P (1) is true for n = 1.<br>
H.S. =4<br>  $+5+7+......+ (2K-1)=K^2$  i.e.,  $P(k)=k(k+1) (k+2) = 6\lambda$ , for some  $\lambda \in \mathbb{N}$ .<br> **(O.B. SOLUTIONS** STUDY MATERIAL: MATHEN<br>  $+5+7+......+ (2K-1)=3+K^2$   $P(1)=1 (1+1) (1+2)=1 \times 2 \times 3=6$ , which is<br>  $+5+7+......+ (2K-1) = K^2$ <br>  $1 \text{ out }$ <br>  $+5+7+......+ (2K-1) = K^2$ <br>  $(K)=3+K^2$   $\text{Step 3: For } k, l+1 (k+2) \text{ is divisible by } 6.$ <br>  $\text{Let, } k (k+1) (k+2) = 6\lambda$ **(0.B. SOLUTIONS**) STUDY MATERIAL: MATI<br>  $5+7+......+ (2K-1)=3+K^2$ <br>  $P(1)=1 (1+1) (1+2)=1 \times 2 \times 3=6$ , which<br>
by 6. Thus, P(1) is true for n = 1.<br>
S. =4<br>
Step 2 : For n = k, Let P(k) be true,<br>
i.e., P(k)= k(k + 1) (k + 2) is divisib **(Q.B. SOLUTIONS** ) STUDY MATERIAL: MATHEMATIE<br>  $\therefore$   $\begin{aligned}\n &\text{SUDY MATERIAL: MATHEMATHEMATIF  
\n by 6. Thus, P (1) is true for n = 1.\n \end{aligned}$ <br>  $\begin{aligned}\n &\text{Step 2: For } n = k, \text{ Let } P (k) \text{ be true, } \\
 &\text{for } p = k, \text{ Let } P (k) \text{ be true, } \\
 &\text{for } p = k, \text{ Let } P (k) \text{ be true, } \\
 &\text{for } p = k, \text{ Let } P (k) \$ 2K-1)=K<sup>2</sup><br>
Let, k(k+1)(k+2)=6), for some  $\lambda \in \mathbb{N}$ .<br>
Step 3 : Forn = k+1, we have to show that P(k+1) is<br>
duction<br>
i.e. P(k+1) is divisible by 6.<br>
P(k+1)=(k+1)(k+1+1)(k+1+2)<br>
=(k+1)(k+2)+3 (k+3)<br>
=(k+1)(k+2)+3 (k+3)<br>  $P(1) = 1 (1 + 1) (1 + 2) = 1 \times 2 \times 3 = 6$ , which is divisible by 6. Thus, P (1) is true for  $n = 1$ . Step 2 : For  $n = k$ , Let P (k) be true, i.e.,  $P(k) = k (k + 1) (k + 2)$  is divisible by 6. Let,  $k(k+1)(k+2) = 6\lambda$ , for some  $\lambda \in N$ . ......... (1) Step 3 : For  $n = k + 1$ , we have to show that P  $(k + 1)$  is true, i.e.  $P(k+1)$  is divisible by 6.  $P (k+1) = (k+1) (k+1+1) (k+1+2)$  $=(k + 1) (k + 2) (k + 3)$  $=k (k + 1) (k + 2) + 3 (k + 1) (k + 2)$  $= 6\lambda + 3 (k + 1) (k + 2)$  [From (1)  $= 6\lambda + 6t = 6 (\lambda + t)$ Thus,  $P(k + 1)$  is divisible by 6. Therefore,  $P(k+1)$  is true.  $\therefore$  P (k) is true  $\Rightarrow$  P (k + 1) is true. Hence, by principle of mathematical induction, P (n) is true for all natural number n. <sup>=</sup> n 1 (2n 1) 3 3 **SITHEMATICS**<br>hich is divisible<br>5.<br> $P(k+1)$  is true,<br>(2)<br>(2)<br>(3)<br>(3)<br>action, P (n) is<br> $\frac{-1}{3} \frac{3^{n+1} + 3}{4}$ c(k+1) (k+2) is divisible by 6.<br>
(k+2) = 6λ, for some λ ∈ N. ..........(1)<br>
is divisible by 6.<br>
is divisible by 6.<br>
is divisible by 6.<br>
is divisible by 6.<br>
k+1) (k+1) where to show that P (k+1) is true,<br>
k+1) (k+2) (k+3) sible by 6.<br>
ne  $\lambda \in N$ . ..........(1)<br>
(1) (k+2)<br>
[From(1)<br>
[From(1)<br>
[From(1)<br>
wen by<br>  $n = \frac{(2n-1)3^{n+1} + 3}{4}$ <br>  $3 = \frac{9+3}{4}$  or 3 = 3<br>
(k) is true.<br>  $\frac{(2k-1)3^{k+1} + 3}{4}$  ...(1) + 1) (k + 2)<br>
[From (1)<br>
e.<br>
e.<br>
e.<br>
e.<br>
e.<br>
e.<br>
e.<br>
e.<br>
e.<br>
given by<br>  $3^n = \frac{(2n-1)3^{n+1} + 3}{4}$ <br>  $3 = \frac{9+3}{4}$  or  $3 = 3$ <br>
(k) is true.<br>  $= \frac{(2k-1)3^{k+1} + 3}{4}$  ... (1)<br>
o show that<br>  $k + 1$ ) $3^{k+1}$ <br>  $\frac{2(k+1) - 1] . 3^{(k+1$  ... (1) 1 induction, P (n) is<br>  $\frac{(2n-1)3^{n+1}+3}{4}$ <br>  $\frac{9+3}{4}$  or 3 = 3<br>
is true.<br>  $\frac{2k-13^{k+1}+3}{4}$  ... (1)<br>
ow that<br>  $\frac{13^{k+1}}{4}$ <br>  $+1) - 11 \cdot 3^{(k+1)+1} + 3$ <br>
4<br>
..k.3<sup>k</sup> + (k + 1)3<sup>k+1</sup><br>
[Using (1)]

$$
(19) \quad (A). Let P(n) be the statement given by
$$

$$
P(n) = 1.3 + 2.3^{2} + 3.3^{3} + \dots + n.3^{n} = \frac{(2n-1)3^{n+1} + 3}{4}
$$

Step 1 : For 
$$
n = 1
$$
, we have

P(1): 1: 
$$
3^1 = \frac{(2-1)3^{1+1} + 3}{4}
$$
 or  $3 = \frac{9+3}{4}$  or  $3 = 3$ 

Thus,  $P(1)$  is true Step 2 : For  $n = k$ , assume that P (k) is true.

Then, 
$$
1.3 + 2.3^2 + 3.3^3 + \dots k.3^k = \frac{(2k-1)3^{k+1} + 3}{4} \dots (1)
$$

Step 3 : For  $n = k + 1$ , we have to show that  $1.3 + 2.3^2 + 3.3^3 + \dots k.3^k + (k+1)3^{k+1}$ 

Therefore, P (k+1) is true.  
\n
$$
\therefore
$$
 P (k) is true  $\Rightarrow$  P (k+1) is true.  
\nHence, by principle of mathematical induction, P (n) is true for all natural number n.  
\n(A). Let P (n) be the statement given by  
\nP (n) = 1.3 + 2.3<sup>2</sup> + 3.3<sup>3</sup> + .........n.3<sup>n</sup> =  $\frac{(2n-1)3^{n+1}+3}{4}$   
\nStep 1: For n = 1, we have  
\nP (1) : 1 : 3<sup>1</sup> =  $\frac{(2-1)3^{1+1}+3}{4}$  or 3 =  $\frac{9+3}{4}$  or 3 = 3  
\nThus, P (1) is true  
\nStep 2: For n = k, assume that P (k) is true.  
\nThen, 1.3 + 2.3<sup>2</sup> + 3.3<sup>3</sup> + .........k.3<sup>k</sup> =  $\frac{(2k-1)3^{k+1}+3}{4}$  ...(1)  
\nStep 3: For n = k + 1, we have to show that  
\n1.3 + 2.3<sup>2</sup> + 3.3<sup>3</sup> + .........k.3<sup>k</sup> + (k + 1)3<sup>k+1</sup>  
\n=  $\frac{[2(k+1)-1]3^{(k+1)+1}+3}{4}$   
\nNow, L.H.S. = 1.3 + 2.3<sup>2</sup> + 3.3<sup>3</sup> + .........k.3<sup>k</sup> + (k + 1)3<sup>k+1</sup>  
\n=  $\frac{(2k-1) .3^{k+1} + 3}{4}$  + (k + 1)3<sup>k+1</sup>  
\n=  $\frac{(2k-1) .3^{k+1} + 3 + 4 (k+1)3^{k+1}}{4}$   
\n=  $\frac{(2k-1+4k+4).3^{k+1} + 3}{4} = \frac{(6k+3)3^{k+1} + 3}{4}$   
\n=  $\frac{3 (2k+1) .3^{k+1} + 3}{4} = \frac{(2k+1)3^{k+2} + 3}{4}$   
\n=  $\frac{12 (k+1) - 1! .3^{(k+1)-1} + 3}{4}$   
\nTherefore, P (k + 1) is true. Thus, P

$$
= \frac{(2k-1) \cdot 3^{k}}{4} + (k+1)3^{k+1}
$$
 [Using (1)]

$$
=\frac{(2k-1) \cdot 3^{k+1} + 3 + 4 (k+1) 3^{k+1}}{4}
$$

$$
=\frac{(2k-1+4k+4) \cdot 3^{k+1} + 3}{4} = \frac{(6k+3) 3^{k+1} + 3}{4}
$$

$$
=\frac{3(2k+1) \cdot 3^{k+1} + 3}{4} = \frac{(2k+1)3^{k+2} + 3}{4}
$$

$$
=\frac{[2 (k+1) - 1] \cdot 3^{(k+1)-1} + 3}{4}
$$

$$
f_{\rm{max}}
$$

Therefore, P (k + 1) is true. Thus, P (k) is true  $\Rightarrow$  P (k + 1) is true.

4

Hence, by principle of mathematical induction P (n) is true for all  $n \in N$ .

**(20) (A).**

**(21) (D).**

#### **MATHEMATICAL INDUCTION Q.B. SOLUTIONS**



**(22) (B).** Let P (n) be the statement given by :

**HEMATICAL INDUCTION**  
\n**(B).** Let P (n) be the statement given by :  
\n
$$
P(n): \left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2
$$
\nStep 1 : For n = 1, we have  
\n
$$
P(1) = \left(1 + \frac{2 \times 1 + 1}{1^2}\right) = (1+1)^2 \text{ or } \left(1 + \frac{3}{1}\right) = 2^2 \text{ or } 4 = 4
$$
\nStep 2 : For n = k, assume that P (k) is true.  
\n**(b)** If  $P(n) = \left(1 + \frac{2 \times 1 + 1}{1^2}\right) = (1+1)^2 \text{ or } \left(1 + \frac{3}{1}\right) = 2^2 \text{ or } 4 = 4$   
\n
$$
P(1): 3^{1-1} = \frac{3^1 - 1}{2} \Rightarrow 1 = 1
$$
\nThus, P (1) is true.  
\n**(b)** If  $P(n) = \left(1 + \frac{2 \times 1 + 1}{1^2}\right) = (1+1)^2 \text{ or } \left(1 + \frac{3}{1}\right) = 2^2 \text{ or } 4 = 4$   
\n
$$
P(1): 3^{1-1} = \frac{3^1 - 1}{2} \Rightarrow 1 = 1
$$
\nThus, P (1) is true.  
\n**(c)** If  $P(n) = \frac{3^n - 1}{2}$   
\n
$$
P(n): 1 + 3 + 3^2 + \dots + 3^{k-1} = \frac{3^k - 1}{2}
$$
\nThus, P (2): For n = k, assume that P (k) is true.  
\n**(d)** If  $P(n) = \frac{3^n - 1}{2}$   
\n
$$
P(n) = \frac{3^n - 1}{2} \Rightarrow 1 = 1
$$
\nThus, P (3): For n = k, assume that P (4): for n = k, assume that P (5): for n = k, assume that P (6): for n = k, assume that P (6): for n = k + 1, we have to show that  $\text{Step 3}: For n = k + 1, we have to show that  $\text{Step 3}: For n = k +$$ 

Step 1 : For  $n = 1$ , we have

$$
P(1) = \left(1 + \frac{2 \times 1 + 1}{1^2}\right) = (1 + 1)^2 \text{ or } \left(1 + \frac{3}{1}\right) = 2^2 \text{ or } 4 = 4
$$
 
$$
P(1): 3^{1-1} = \frac{3}{4}
$$

Thus,  $P(1)$  is true.

**Step 2 :** For  $n = k$ , assume that P (k) is true. Then,

$$
\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\dots\dots\left(1+\frac{2k+1}{k^2}\right) = (k+1)^2 \dots (1)
$$

Step 3 : For  $n = k + 1$ , we have to show that

**ALTHEMATICAL INDUCTIONS**  
\n**(B).** Let P (n) be the statement given by : **(25)** (A). Let P(n) be the statement given by  
\nP (n): 
$$
\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)... \left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2
$$
  
\nStep 1: For n = 1, we have  
\n
$$
P(1) = \left(1+\frac{2 \times 1+1}{1^2}\right) = (1+1)^2 \text{ or } \left(1+\frac{3}{1}\right) = 2^2 \text{ or } 4=4
$$
\n
$$
P(1): 3^{1-1} = \frac{3^1-1}{2} \Rightarrow 1 = 1
$$
\nThus, P (1) is true.  
\nStep 2: For n = k, assume that P (k) is true. Then,  
\n
$$
\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)...... \left(1+\frac{2k+1}{k^2}\right) = (k+1)^2 ....(1)
$$
\n
$$
P(2): P(3) = 12 \text{ or } 12 \text{ or }
$$

Therefore,  $P(k+1)$  is true.

Thus P (k) is true  $\Rightarrow$  P (k + 1) is true

Hence, by principle of mathematical induction P (n) is true for all  $n \in N$ 

- **(23) (A).**
- **(24) (B).**

**(25) (A).** Let P(n) be the statement given by

**Q.B. SOLUTIONS**  
\n**o** by : **(25) (A).** Let **P**(n) be the statement given by  
\n
$$
+\frac{(2n+1)}{n^2} = (n+1)^2
$$
\n
$$
P(n): 1+3+3^2+....+3^{n-1} = \frac{(3^n-1)}{2}
$$
\nStep 1: For n = 1, we have  
\n
$$
\left(1+\frac{3}{1}\right) = 2^2
$$
 or 4=4  
\n
$$
P(1): 3^{1-1} = \frac{3^1-1}{2} \Rightarrow 1 = 1
$$
\nThus, **P**(1) is true.  
\nStep 2: For n = k, assume that **P**(k) is true.  
\n
$$
\frac{1}{2} = (k+1)^2
$$
\nNow that  
\nStep 3: For n = k + 1, we have to show that  
\n
$$
\frac{1}{1} \left(1+\frac{2(k+1)+1}{1+3+3^2+....+3^{k-1}}+\frac{3^{k-1}}{1+3^{k+1-1}}+\frac{3^{k+1}-1}{2}\right)
$$

Step 1 : For  $n = 1$ , we have

$$
P(1): 3^{1-1} = \frac{3^1 - 1}{2} \Rightarrow 1 = 1
$$

Thus,  $P(1)$  is true.

Step 2 : For  $n = k$ , assume that P (k) is true.

$$
2 \dots (1) \qquad \text{Then } 1+3+3^2+\dots+3^{k-1} = \frac{3^k-1}{2}
$$

Step 3 : For  $n = k + 1$ , we have to show that

(a) Let P(n) be the statement given by  
\n(a) i. Let P(n) be the statement given by  
\n(a) i. 
$$
1+3+3^2+....+3^{n-1} = \frac{(3^n-1)}{2}
$$
  
\n(b) i.  $3^{1-1} = \frac{3^1-1}{2} \Rightarrow 1 = 1$   
\nthus, P (1) is true.  
\nthen  $1+3+3^2+....+3^{k-1} = \frac{3^k-1}{2}$   
\nthen  $1+3+3^2+....+3^{k-1} = \frac{3^k-1}{2}$   
\nthen  $1+3+3^2+....+3^{k-1} + 3^{k+1-1} = \frac{3^{k+1}-1}{2}$   
\nNow, L.H.S. =  $1+3+3^2+....+3^{k-1}+3^{k+1-1}$ 

30. **Q.B. SOLUTIONS**  
\n10. Let 
$$
f(n)
$$
 be the statement given by:  
\n
$$
\frac{7}{9} \int ... \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2
$$
\n
$$
= (n+1)^2 + 3 + 3^2 + .... + 3^{k-1} = \frac{3^k - 1}{2}
$$
\n
$$
= \frac{3^{k-1} - 1}{2}
$$
\n
$$
= \frac{3^{k+1} - 1}{2}
$$
\n
$$
= \frac{3^{k-1} - 1}{2}
$$
\n
$$
= \frac{3^{k-1} - 1}{2}
$$
\n
$$
= \frac{3^k - 1}{2} + 3^{k+1-1}
$$
\n
$$
= \frac{3^k - 1}{2} + 3^{k+1-1}
$$
\n
$$
= \frac{3^k - 1}{2} + 3^{k+1-1}
$$
\n
$$
= \frac{3^k - 1}{2} + 3^k = \frac{3^k - 1 + 2 \cdot 3^k}{2} = \frac{3^k (1 + 2) - 1}{2}
$$
\n
$$
= \frac{3 \cdot 4!}{2} + 4k + 4 = (k +
$$

There P (k + 1) is true. Thus, p (k) is true  $\Rightarrow$  P (k + 1) is true for all  $n \in N$ .

- **(26) (D).**
- **(27) (A).**