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MOTION IN ONE DIMENSION

INTRODUCTION

Motion is the most fundamental observation about nature at large. It turns out that everything that happens in the world is some type of motion. To describe motion we require terms like time interval, distance, displacement, speed, velocity and acceleration.

To study the motion branch of physics called Mechanics is defined. To simplify study it is further divided into two sections, Kinematics and Dynamics. Kinematic deals with the study of motion of objects without considering the cause of motion, here measurement of time is essential . Dynamics deals with the study of objects taking into consideration and cause of their motion.

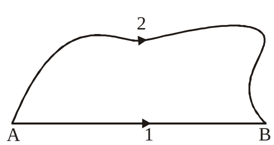
Generally motion we observe in practical life are 2 or 3-dimensional to analyse them we have to break them into single dimension. Hence, we need to study one dimension motion.

We will consider all object as point object for considering one dimensional motion. We will also neglect air resistance if not specified. In analysing any motion consider time as time interval i.e. think initial and final situation according to time interval in which you have to solve the problem.

DISTANCE

The length of the actual path between initial and final positions of a particle in a given interval of time is called distance covered by the particle. Distance is the actual length of the path. It is the characteristic property of any path i.e. path is always associated when we consider distance between two positions.

Distance between A and B while moving through path (1) may or may not be equal to the distance between A and B while moving through path (2).



Characteristics of Distance :

(i) It is a scalar quantity

(ii) It depends on the path

(iii) It never reduces with time.

(iv) Distance covered by a particle is always positive and can never be negative or zero.

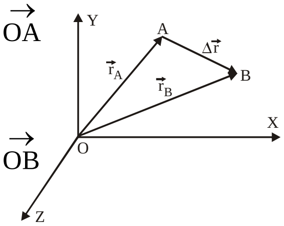
(v) Dimension : $[M^o L^1 T^o]$

(vi) Unit:In CGS centimeter (cm), In S.I. system meter (m).

DISPLACEMENT

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

Displacement of a particle is a position vector of its final position w.r.t, initial position.



Position vector of A w.r.t. O = $\overrightarrow{OA}$

$\Rightarrow \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Position Vector of B w.r.t. O = $\overrightarrow{OB}$

$\Rightarrow \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Displacement = $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

Characteristics of Displacement :

(i) It is a vector quantity.

(ii) The displacement of a particle between any two points is equal to the shortest distance between them.

(iii) The displacement of an object in a given time interval may be +ve, –ve or zero.

(iv) The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement,
i.e. Distance ≥ | Displacement |

(v) Dimension : $[M^o L^1 T^o]$

(vi) Unit: In C.G. S. centimeter (cm), In S.I. system meter (m).

Comparative Study of Displacement & distance

S.No.	Displacement	Distance
1.	It has single value between two points	It may have more than one value between two points
2.	May be +ive, -ive or zero.	Distance > 0
3.	It can decrease with time	It can never decrease with time.
4.	It is a vector quantity	It is a scalar quantity.

Example 1 :

An old person moves on a semi circular track of radius 40m during a morning walk. If he starts at one end of the track and reaches at the other end. Find the displacement of the person.

Sol. Displacement = $2R = 2 \times 40 = 80$ meter.

Example 2 :

An athlete is running on a circular track of radius 50 meter. Calculate the displacement of the athlete after completing 5 rounds of the track.

Sol. Since final and initial positions are same hence displacement of athlete will be $\Delta r = r - r = 0$

Example 3 :

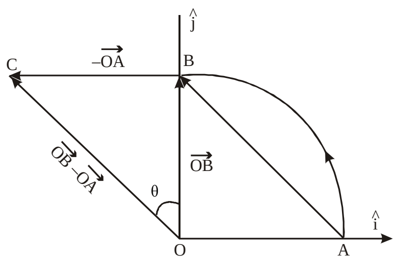
A monkey is moving on circular path of radius 80 m . Calculate the distance covered by the monkey.

Sol. Distance = Circumference of the circle

$$D = 2\pi R \Rightarrow D = 2\pi \times 80 = 160 \times 3.14 = 502.40 \text{ m}$$

Example 4 :

A particle goes along a quadrant from A to B is a circle radius 10m as shown in figure. Find the direction and magnitude of displacement and distance along path AB.



Sol. Displacement $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\hat{j} - 10\hat{i}$

$$|\overrightarrow{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}\text{m}$$

From Δ OBC, $\tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1 \Rightarrow \theta = 45°$

Angle between displacement vector $\overrightarrow{OC}$ and x-axis

$= 90° + 45° = 135°$

Distance of path AB $= \dfrac{1}{4}(\text{circum.}) = \dfrac{1}{4}(2\pi R)$ m $= (5\pi)$ m

SPEED

It is the distance covered by the particle in one second. It is a scalar quantity.

Type of speed :

(i) Instantaneous speed : It is the speed of a particle at particular instant of time or position.

$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

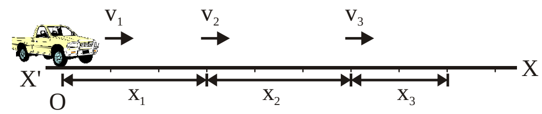
(ii) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

(iii) Uniform speed : If during the entire motion speed of the body remains same, the body is said to have uniform speed.

(iv) Non-Uniform speed : If speed changes, the body is said to have non-uniform speed.

Some important cases related to average speed :

Case : 1



If car covers distances x_1, x_2, and x_3 with speeds v_1, v_2, and v_3 respectively in same direction then average speed of car.

$$\bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} \text{ ; here, } t_1 = \frac{x_1}{v_1},\ t_2 = \frac{x_2}{v_2},\ t_3 = \frac{x_3}{v_3}$$

$$\bar{V} = \frac{x_1 + x_2 + x_3}{\dfrac{x_1}{v_1} + \dfrac{x_2}{v_2} + \dfrac{x_3}{v_3}}$$

If car covers equal distances with different speeds then, $x_1 = x_2 = x_3 = x$

$$\bar{V} = \frac{3x}{\dfrac{x}{v_1} + \dfrac{x}{v_2} + \dfrac{x}{v_3}} = \frac{3}{\dfrac{1}{v_1} + \dfrac{1}{v_2} + \dfrac{1}{v_3}} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Case 2 : If any body travels with speeds v_1, v_2, v_3 during time intervals t_1, t_2, t_3 respectively then the average speed of the body will be.

$$\bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} = \frac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3}$$

If $t_1 = t_2 = t_3 = t = \dfrac{(v_1 + v_2 + v_3) \times t}{3 \times t} = \dfrac{(v_1 + v_2 + v_3)}{3}$

Example 5 :

The distance travelled by a particle in time t is given by $x = 2.5\,t^2$ (m). Find the average speed of the particle during the time 0 to 5 sec.

Sol. Distance covered $x = 2.5\,t^2$; During time 0 to 5 sec.

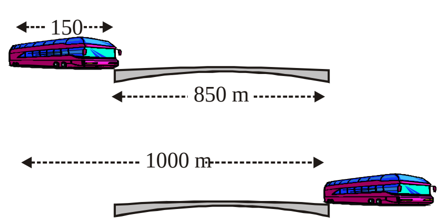
$$x = 2.5 \times (5)^2 = 2.5 \times 25 = 62.5 \text{ m}$$

Average speed, $\bar{V} = \dfrac{x}{t_2 - t_1} = \dfrac{62.5}{5 - 0} = \dfrac{62.5}{5} = 12.5$ m/s

Example 6 :

A train 150 m long is moving with a speed of 90 km/h. In what time shall it cross a bridge 850 m long ?

Sol. Total distance to be covered = 850 + 150 = 1000 m



Speed = 90 km/h = 90 × (5/18) m/s = 25 m/s

Now, time $= \dfrac{1000}{25}$ s = 40 s

Example 7 :

A bicyclist is travelling along a straight road for the first half time with speed v_1 and for second half time with speed v_2. What is the average speed of the bicyclist?

Sol. Let t be the total time taken then distance covered in the

first half time $= v_1\left(\frac{t}{2}\right) = \frac{v_1 t}{2}$

Distance covered in the next half time $= v_2\left(\frac{t}{2}\right) = \frac{v_2 t}{2}$

Average speed $v_{av.} = \frac{\frac{v_1 t}{2}+\frac{v_2 t}{2}}{t} = \frac{v_1+v_2}{2}$

Example 8 :

A person travels along a straight road due east for the first half distance with speed v_1 and the second half distance with speed v_2. What is the average speed of the person?

Sol. Let S be the total distance travelled.

Time taken for the first half distance $= \frac{S/2}{v_1} = \frac{S}{2v_1}$

Time taken for the second half distance $= \frac{S/2}{v_2} = \frac{S}{2v_2}$

Total time taken $= \frac{S}{2v_1}+\frac{S}{2v_2}$

Average speed, $v_{av.} = \frac{S}{\frac{S}{2v_1}+\frac{S}{2v_2}} = \frac{2v_1v_2}{v_1+v_2}$

Example 9 :

A man walks at a speed of 6 km/hr for 1 km and 8 km/hr for the next 1 km. What is his average speed for the walk of 2km.

Sol. $\bar{V} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times6\times8}{6+8} = 7$ km/h.

Example 10 :

The distance travelled by a particle $S = 10t^2$ (m). Find the value of instantaneous speed at t = 2 sec.

Sol. $v = \frac{dx}{dt} = \frac{d}{dt}(10t^2) = 10(2t) = 20\,t$

Put t = 2 sec.

$v = 20\times2 = 40$ m/s.

Calculation of distance by speed :

The distance may be calculated by the speed in the following terms.

(i) Distance by speed-time graph : When the particle moves from time t_1 to t_2 with uniform speed V as shown in the graph:

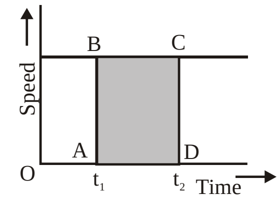
Then distance covered

$S = V(t_2 - t_1)$

$= AB \times AD =$ Area of ABCD

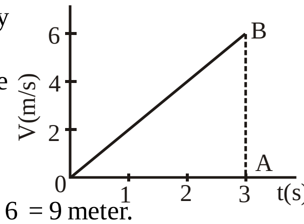
Total distance travelled by particle

= Area of speed-time graph



Example 11 :

Find the distance travelled by the particle during the time t = 0 to t = 3 sec. from the figure.



Sol. Distance S = Area of OAB

$= \frac{1}{2}\times OA\times BA = \frac{1}{2}\times3\times6 = 9$ meter.

(ii) If the speed varies with the time then :

By $v = \frac{ds}{dt} \Rightarrow ds = v\,dt \Rightarrow \int ds = \int v\,dt$ or $s = \int v\,dt$

Example 12 :

If the speed of a particle is $v = 10\,t^2$ m/s. Then find out covered distance from t = 2 sec. to t = 5 sec.

Sol. $s = \int v\,dt = \int_2^5 10\,t^2 dt = 10\int_2^5 t^2 dt = \frac{10}{3}\left(t^3\right)_2^5 = \frac{10}{3}\left(5^3-2^3\right)$

$= 390$ meter.

VELOCITY

The rate of change of displacement of a particle with time is called the velocity of the particle.

i.e. Velocity $= \frac{\text{Displacement}}{\text{Time interval}}$

(i) It is a vector quantity

(ii) The velocity of an object can be positive, zero and negative

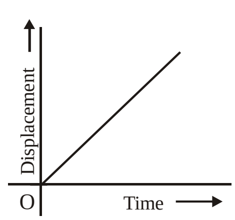
(iii) Unit : C.G.S. : cm/s, S.I. : m/s.

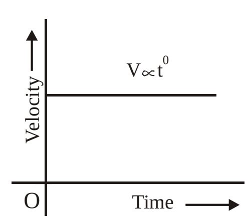
(iv) Dimension : $M^0L^1T^{-1}$

Types of velocity : (a) Uniform Velocity (b) Non-uniform Velocity (c) Average Velocity (d) Instantaneous velocity (e) Relative velocity

1. Uniform Velocity : A body is said to move with uniform velocity, if it covers equal displacements in equal intervals of time, howsoever, small these intervals may be.

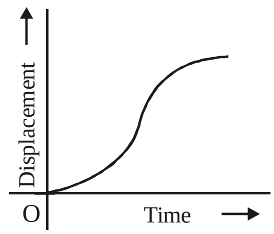
When a body is moving with uniform velocity, then the magnitude and direction of the velocity of the body remains same at all points of its path.



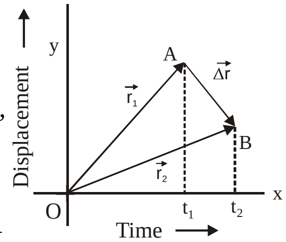


2. Non-uniform Velocity :

The particle is said to have non-uniform motion if it covers unequal displacements in equal intervals of time, howsoever, small these time intervals may be. In this type of motion velocity does not remain constant.



3. Average velocity : The average velocity of an object is equal to the ratio of the displacement, to the time interval for which the motion takes place i.e.,



$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

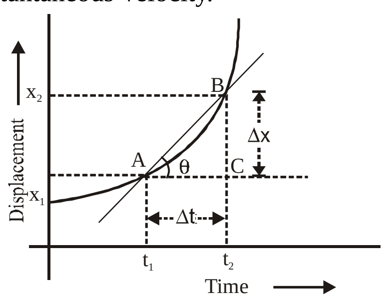
If the initial and final position of a particle are $\vec{r}_1$ and $\vec{r}_2$ at time t_1 and t_2 respectively,

Then Displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

and elapsed time $\Delta t = t_2 - t_1$

$\therefore$ Average velocity $\vec{V}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \dfrac{\Delta\vec{r}}{\Delta t}$

4. Instantaneous velocity : The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity.



Graph (A)

From fig., the average velocity between points A and B is

$$\vec{V}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{\Delta\vec{x}}{\Delta t}$$

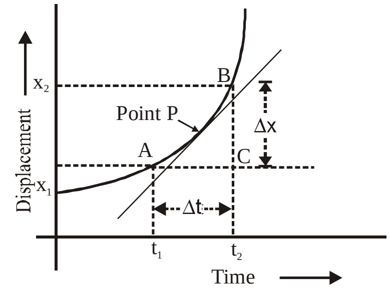
If time interval is small i.e. $t_2 - t_1 = \Delta t$

and $\vec{x}_2 - \vec{x}_1 = \Delta\vec{x}$, then $V_{av} = \dfrac{\Delta x}{\Delta t} = \tan\theta$ from graph (A)

Average velocity is equal to slope of straight line joining two points on displacement time graph. If $\Delta t \to 0$, then average velocity becomes instantaneous velocity

instantaneous velocity, $\vec{V} = \underset{\Delta t \to 0}{\text{Lt}} \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{d\vec{x}}{dt}$

or inst. velocity at point P



Graph (B)

$\vec{V} = \tan\alpha$ (slope of tangent at point P, graph B)

Example 13 :

A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey?

(ii) What is the average velocity?

Sol. (i) Let AB = s, time taken to go from A to B, $t_1 = \dfrac{s}{40}$ h

and time taken to go from B to A, $t_2 = \dfrac{s}{30}$ h

$\therefore$ total time taken $= t_1 + t_2 = \dfrac{s}{40} + \dfrac{s}{30} = \dfrac{(3+4)s}{120} = \dfrac{7s}{120}$ h

Total distance travelled = s + s = 2s

$\therefore$ Average speed

$$= \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{2s}{\frac{7s}{120}} = \frac{120 \times 2}{7} = 34.3 \text{ km/h.}$$

(ii) Total displacement = zero, since the car returns to the original position.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{time taken}} = \frac{0}{2t} = 0$$

Example 14 :

A table clock has its minute hand 4 cm long. Find average velocity of the tip of the minute hand (a) in between 6 a.m. to 6.30 a.m. and (b) 6 a.m. to 6.30 p.m.

Sol. (a) At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m. it is 180° away. Thus the displacement between the initial and final positions of the tip is equal to the diameter of the clock.

Displacement = 2 R = 2 × 4 cm = 8 cm

Time taken from 6 a.m. to 6.30 a.m. is 30 minutes = 1800s.

The average velocity is V_{av}

$$= \frac{\text{Displacement}}{\text{time}} = \frac{8}{1800} = 4.4 \times 10^{-3} \text{ cm/s}$$

(b) Again time taken from 6 am to 6.30 p.m.

= 12 hrs + 30 minutes = 45000 s

$$\therefore V_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8}{45000} = 1.8 \times 10^{-4} \text{ cm/s}$$

Example 15 :

A man walks on a straight road from his home to a market 2.5km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min.

Sol. Time taken by man to go from his home to market,

$$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{2.5}{5} = \frac{1}{2}\ \text{h}$$

Time taken by man to go from market to his home,

$$t_2 = \frac{2.5}{7.5} = \frac{1}{3}\ \text{h}$$

$\therefore$ Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\ \text{h} = 50$ min.

(a) Average velocity $\vec{V}_{ave} = \frac{\text{displacement}}{\text{time}}$

(b) Average speed $V_{ave.} = \frac{\text{distance}}{\text{time}}$

(i) 0 to 30 min

$$\vec{V}_{ave} = \frac{2.5}{1/2} = 5\ \text{km/h towards market}$$

$$V_{ave.} = \frac{2.5}{1/2} = 5\ \text{km/h}$$

(ii) 0 to 50 min

Total distance travelled = 2.5 + 2.5 = 5 km.

Total displacement = zero

$$\vec{V}_{ave.} = 0\ ;\ V_{ave.} = \frac{5}{5/6} = 6\ \text{km/h}$$

(iii) 0 to 40 min

Distance moved in 30 min (from home to market) = 2.5 km.

Distance moved in 10 min (from market to home)

with speed 7.5 km/h $= 7.5 \times \frac{10}{60} = 1.25$ km

So displacement = 2.5 – 1.25 = 1.25 km (towards market)

Distance travelled = 2.5 + 1.25 = 3.75 km

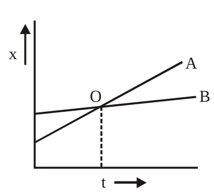
$$\vec{V}_{ave} = \frac{1.25}{40/60}\ ;\ V_{ave} = \frac{3.75}{40/60} = 1.875\ \text{km/h.}$$

= 5.625 km/h.(towards market)

Example 16 :

Give a position-time graph of two objects moving in the same direction with unequal velocities.

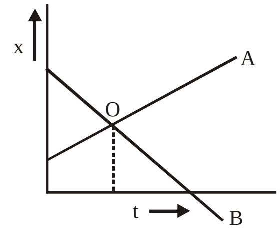
Sol. O is the time of meeting of two bodies A and B.



Example 17 :

Give a position-time graph of two objects moving in the opposite direction with unequal velocities.

Sol.



O is time of meeting of two bodies A and B.

Example 18 :

The position of a particle moving on x-axis is given by $x = At^3 + Bt^2 + Ct + D$. The numerical value of A, B, C, D are 1, 4, –2 and 5 respectively and S.I. units are used. Find velocity of the particle at t = 4 sec.

Sol. $V = \frac{dx}{dt} = \frac{d}{dt}[At^3 + Bt^2 + Ct + D]$

or $V = 3At^2 + 2Bt + C$

at time t = 4 sec.

Considering A = 1, B = 4, C = –2

$V = 3A(4)^2 + 2B(4) + C$

$V = 48(1) + 8(4) + (-2)$

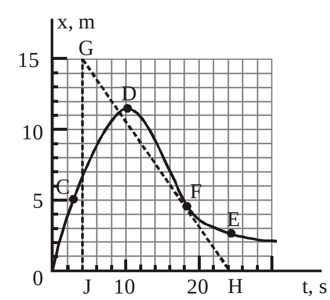
$V = 3A(16) + 8B + C = 78$ m/s

$V = 48A + 8B + C$

Example 19 :

(i) With the help of given fig. find the instantaneous velocity at point F for the object whose motion the curve represents.

(ii) Refer to fig. for the motion of an object along the x-axis. What is the instantaneous velocity of the object (a) at point D? (b) at point C? (c) at point E?



Sol. (i) The tangent at F is the dashed line GH. Taking triangle GHJ, we have

$\Delta t = 24 - 4 = 20$ s

$\Delta x = 0 - 15 = -15$m

Hence slope at F is $v_F = \frac{\Delta x}{\Delta t} = \frac{-15\text{m}}{20\ \text{s}} = -0.75$ m/s

The negative sign tells us that the object is moving in the –x direction.

(ii) (a) Point D is a maximum of the x v/s t curve.

Therefore $v = \frac{dx}{dt} = 0$.

(b) Without the exact equation for x as function of t one cannot get a precise answer. The best we can do is to draw the tangent line at point c and the slope in the same way as in above problem. This yields the answer

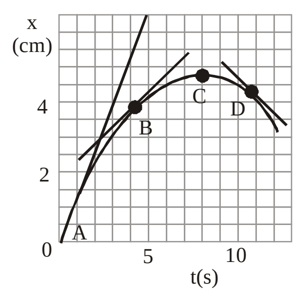
$$v_C = \left.\frac{dx}{dt}\right|_C \approx 1.3 \text{ m/s}$$

(c) We proceed as in part (b), but here the tangent line has a negative slope and the answer should be

$$v_E = \left.\frac{dx}{dt}\right|_E \approx -0.13 \text{ m/s}$$

Example 20 :

The graph of particle's motion along the x-axis is given in fig. Estimate the (a) average velocity for the interval from A to C; instantaneous velocity at (b) D and at (c) A.



Sol. (a) $\vec{v} = \frac{4.8-0}{8.0-0} = 0.60$ cm/s.

From the slope at each point

(b) v = – 0.48 cm/s. and (c) v = 1.3 cm/s.

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let v and v' be the velocity of the object at time t and t' respectively, then acceleration of the body is given by

$$\text{Acceleration } (\vec{a}) = \frac{\text{Change in velocity}}{\text{Time interval}} = \frac{\vec{v}' - \vec{v}}{t' - t}$$

(i) Acceleration is a vector quantity.

(ii) It is positive if the velocity is increasing and is negative if the velocity is decreasing.

(iii) The negative acceleration is also called retardation or deceleration.

(iv) Unit : In S.I. system m/s^2

In C.G.S. system cm/s^2

(v) Dimension : $[M^0L^1T^{-2}]$

Types of Acceleration :

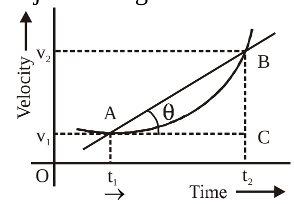
(i) Uniform acceleration : An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

(ii) Variable acceleration : An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time.

(iii) Average Acceleration : When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken i.e.,

Average Acceleration

$$= \frac{\text{total change in velocity}}{\text{total time taken}}$$



Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2. Then, Change in velocity = $\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$

Elapsed time in changing the velocity = $t_2 - t_1 = \Delta t$

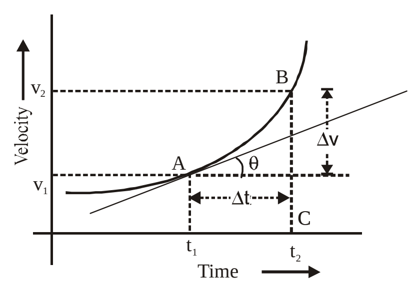
$$\text{Thus, } \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \Rightarrow \vec{a}_{av} = \frac{BC}{AC} = \tan\theta$$

= the slope of chord of v – t graph is average acceleration.

Note : If any body is accelerated with acceleration $\vec{a}_1$ till time t_1 and acceleration $\vec{a}_2$ up to time t_2 then average acceleration will $\vec{a}_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$

(iv) Instantaneous Acceleration :

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.



Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$

& becomes $\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$, then, $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration. Instantaneous acceleration $\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

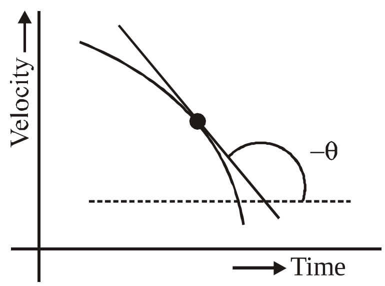
Instantaneous acceleration at a point is equal to slope of tangent at that point on displacement time graph in the graph shown above this point is.

As $\vec{v} = \frac{d\vec{x}}{dt}$, therefore, $\vec{a} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second time derivative of the position of the object at the given instant.

Note :

(i) It is not essential that when velocity is zero acceleration must be zero. e.g. In vertical motion at the top point v = 0 but $a \neq 0$.



(ii) Velocity may vary but $\frac{dv}{dt}$ may be constant.

(iii) The acceleration may vary but v may be constant e.g. In uniform circular motion.

(iv) If velocity decreases w.r.t. time then acceleration is called retardation. Retardation $a = \tan(\pi - \theta) = -\tan\theta$

Example 21 :

An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acc.?

Sol. Here, Initial velocity u = 0,

$_{max}) = 18 \text{ km/h} = 18 \times \frac{5}{18} = 5 \text{ m/s}$,

$t_1 = 0$ sec, $t_2 = 2$ sec.

$a_{av} = \frac{v-u}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$ so $a_{av} = \frac{5.0}{2} = 2.5 \text{ m/s}^2$

Example 22 :

A car starts from rest and acquires velocity equal to 10 m/s after 5 sec. Find the acceleration of the car.

Sol. Here, u = 0 and v = 10 m/s , t = 5 sec

Using , $a = \frac{v-u}{t}$,

we have $a = \frac{(10-0)\text{m/s}}{5\text{ s}} = 2 \text{ m/s}^2$

Example 23 :

The displacement of a particle is proportional to the cube of elapsed time . How does the acceleration of the body depend on time elapsed ?

Sol. Let x be the displacement at time t of an object in motion. Then according to question, $x = kt^3$, where k is a constant.

velocity of object, $v = \frac{dx}{dt} = 3kt^2$ (m/s)

and acceleration of object, $a = \frac{dv}{dt} = 3k \times 2t = 6kt$. (m/s^2)

i.e. $a \propto t$. It means acceleration $\propto$ time.

Example 24 :

The position x of a particle varies with time 't' as $x = at^2 - bt^3$. When will the acceleration of the particle become zero?

Sol. $v = \frac{dx}{dt} = \frac{d}{dt}(at^2 - bt^3) = 2at - 3bt^2$

acc. $= \frac{dv}{dt} = \frac{d}{dt}(2at - 3bt^2) = 2a - 6bt$

According to question acc. = 0

$\therefore$ $2a - 6bt = 0$ hence $t = \frac{a}{3b}$

Example 25 :

The velocity of any particle is related with its displacement As; $x = \sqrt{v+1}$, Calculate acceleration at x = 5 cm.

Sol. $x = \sqrt{v+1}$ $x^2 = v + 1$; $v = (x^2 - 1)$

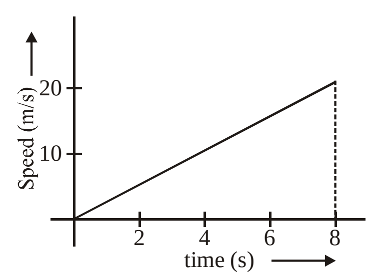
Therefore

$a = \frac{dv}{dt} = \frac{d}{dt}(x^2 - 1) = 2x\frac{dx}{dt} - 0 = 2x\,v = 2x\,(x^2 - 1)$

at x = 5 m, $a = 2 \times 5\,(25 - 1) = 240 \text{ m/s}^2$

TRY IT YOURSELF-1

Q.1 The speed of a car as a function of time as shown in fig. Find the acceleration and distance travelled by the car in 8 seconds.



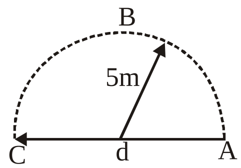
Q.2 If the displacement of a particle is $(2t^2 + t + 5)$ meter then, what will be acc. at t = 5 sec.

Q.3 A car moving with a velocity of 20 ms^{-1} is brought to rest in 5 seconds by applying brakes. Calculate the retardation of the car.

Q.4 A particle moves according to the equation $x = 3 + 4t + 6t^2 + 4t^3$. Find its velocity and acceleration at all times. When does its velocity equal 10 m/s? What is its acceleration at that instant?

Q.5 An object that negatively accelerates slows down.
True or False:

Q.6 A person walks along a circular path of radius 5.00 m. If the person walks around one half of the circle, find (a) the magnitude of the displacement vector and (b) how far the person walked. (c) What is the magnitude of the displacement if the person walks all the way around the circle?



Q.7 A sprinter runs around a 440 meter circular track in 49 seconds.
(a) What is her average speed?
(b) What is her average velocity?

Q.8 A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field.
(a) What distance he has to walk to reach the field?
(b) What is his displacement from his house to the field?

Q.9 Is it possible to have zero velocity but non-zero acceleration at any position in any motion.

Q.10 A particle is moving in east direction with speed 5 m/s after 10 sec it starts moving in north direction with same speed. Find average acceleration.

ANSWERS

(1) (i) 80m (ii) 2.5 m/s^2 **(2)** 4 m/s^2 **(3)** 4 ms^{-2}
(4) $v = 4 + 12t + 12t^2$; $a = 12 + 24t$; $t = 0.37s$; $a = 21\ m/s^2$.
(5) False **(6)** (a) 10.0 m, (b) 15.7 m, (c) 0
(7) (a) 8.98 m/s, (b) 0, **(8)** (a) 110 m, (b) 50m
(9) Yes, **(10)** $\frac{1}{\sqrt{2}}$ m/s^2, 135°

MOTION ANALYSIS

To start solving any motion problem, first analyse whether motion is uniform (velocity constant) or non-uniform (velocity not constant). If motion is uniform use $\vec{v} = \frac{\vec{d}}{t}$, $\vec{v}$ and $\vec{d}$ should be in same direction. If motion is non-uniform check the reason for velocity change. If velocity is changing directionally with constant magnitude then use vector approach. If it is changing magnitudely with fixed direction then use kinematic equation provided acceleration is constant. If acceleration is variable use calculus approach. If velocity is changing magnitudely as well as directionally then use vector approach with calculus.

KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Let $\vec{u}$ = Initial velocity (at t = 0), $\vec{v}$ = Velocity of the particle after time t, $\vec{a}$ = Acceleration (uniform)

$\vec{s}$ = Displacement of the particle during time 't'

(a) Acceleration, $\vec{a} = \frac{\vec{v} - \vec{u}}{t}$

$$\vec{v} = \vec{u} + \vec{a}t \qquad \text{.................. (i)}$$

(b) Displacement $\vec{s}$ = Average velocity x time.

$$\vec{s} = \left(\frac{\vec{u} + \vec{v}}{2}\right) \times t \qquad \text{..................(ii)}$$

[This is very useful equation, when acceleration is not given]

(c) From (i) & (ii) $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ .... (iii)

[$\vec{v} = \vec{u} + \vec{a}t$, $\frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t$

$$\Rightarrow \int d\vec{s} = \int (\vec{u} + \vec{a}t)\, dt \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \text{]}$$

(d) $v^2 = u^2 + 2\vec{a}.\vec{s}$ .... (iv)

[$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$, $\int v\, dv = \int \vec{a}.d\vec{s} \Rightarrow \frac{v^2}{2} = \vec{a}.\vec{s} + c$,

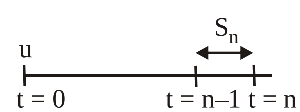
At s = 0, v = u, $\frac{u^2}{2} = c$

$\therefore \frac{v^2}{2} = \vec{a}.\vec{s} + \frac{u^2}{2} \Rightarrow v^2 = u^2 + 2\vec{a}.\vec{s}$]

(e) $\vec{s}_n$ = displacement of particle in nth second

$$\vec{s}_n = \vec{s}_n - \vec{s}_{n-1} = \left\{\vec{u}(n) + \frac{1}{2}an^2\right\} - \left\{\vec{u}(n-1) + \frac{1}{2}\vec{a}(n-1)^2\right\}$$

$$\vec{s}_n = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$$



Equations (i), (iii) and (iv) are called 'equations of motion' and are very useful in solving the problems of motion along a straight line with constant acceleration.

Note : (a) $\vec{v} = \vec{u} + \vec{a}t$ and $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ are vector equation, while $\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$ is a scalar equation.

(b) If the velocity and acceleration are collinear, we conventionally take the direction of motion to be positive, so equation of motion becomes.

$$v = u + at,\ s = ut + \left(\frac{1}{2}\right)at^2,\quad v^2 = u^2 + 2as$$

If the velocity and acceleration are antiparallel then body retards and equation of motion becomes

$v = u - at, \quad s = ut - \frac{1}{2}at^2, \quad v^2 = u^2 - 2as$

(c) In equation $s = ut + \frac{1}{2}at^2$, u is initial speed for time interval t while in $s_{nth} = u + \frac{a}{2}(2n-1)$, u is speed at t = 0.

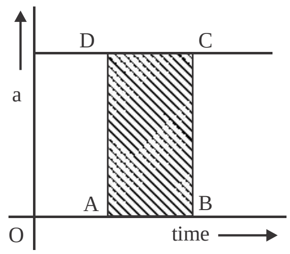
Calculation of speed and distance by acceleration-time graph:

Let a particle be moving with uniform acceleration according to following a – t graph –

$dv = a\,dt$ or $\int_u^v dv = \int_{t_1}^{t_2} a\,dt$

$(v)_u^v = a(t)_{t_1}^{t_2}$

$v - u = a(t_2 - t_1)$



Therefore difference in magnitude of velocity

$(v - u) = AB \times AD$

v – u = Area of rectangle ABCD = area under a – t graph

Example 26 :

A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Find speed after 10s.

Sol. Using $\vec{v} = \vec{u} + \vec{a}t$

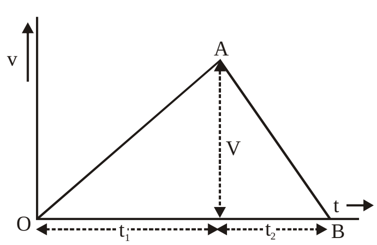
$\Rightarrow \vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10 \Rightarrow \vec{v} = 7\hat{i} + 7\hat{j}$

$\Rightarrow v = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ m/s

Example 27 :

A lift performs the first part of its ascent with uniform acceleration 'a' and the remainder with uniform retardation 2a. Prove that if h the depth of the shaft and t is the time of ascent, then $h = \frac{1}{3}at^2$. Use only the graphical method.

Sol.



$a = \frac{V}{t_1}$ and $2a = \frac{V}{t_2}$ or $t_1 = \frac{V}{a}$ and $t_2 = \frac{V}{2a}$

Total time, $t = t_1 + t_2$;

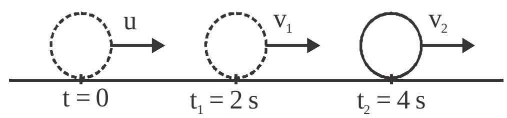
or $t = \frac{V}{a} + \frac{V}{2a} = \frac{3V}{2a}$ or $V = \frac{2}{3}at$

h = area of the $\Delta OAB = \frac{1}{2}tV = \frac{1}{2}t \times \frac{2}{3}at = \frac{1}{3}at^2$

Example 28 :

The velocity acquired by a body moving with uniform acceleration is 20 m/s in first 2 sec and 40 m/s in first 4 sec. Calculate initial velocity.

Sol. $a = \frac{v_2 - v_1}{t_2 - t_1}$; $a = \frac{40-20}{4-2} = \frac{20}{2} = 10$ m/s^2



Now, $v_1 = u + at_1 \Rightarrow 20 = u + 10 \times 2 \Rightarrow u = 0$ m/s

Example 29 :

A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.

Sol. From first eq^n of motion– $v = u + at$

$\Rightarrow 100 = 0 + at$ or $100 = at$ ....(1)

velocity after one second

$v' = 0 + a(t+1) \Rightarrow 150 = a(t+1)$ .....(2)

On subtracting eq^n.(1) from eq^n. (2)

$a = 50$ m/s^2

Example 30 :

A body travels a distance of 2 m in 2 seconds and 2.2 m in next 4 seconds. What will be the velocity of the body at the end of 7th second from the start.

Sol. Here, Case (i) S = 2m, t = 2s

Case (ii) S = 2 + 2.2 = 4.2 m t = 2 + 4 = 6s

Let u and a be the initial velocity and uniform acceleration of the body. $S = ut + \frac{1}{2}at^2$

Case (i), $2 = (u \times 2) + (\frac{1}{2}a \times 2^2)$

or $1 = u + a$ ....(i)

Case (ii), $4.2 = (u \times 6) + (\frac{1}{2}a \times 6^2)$

or $0.7 = u + 3a$ ....(ii)

Subtracting (ii) from (i),

$0.3 = 0 - 2a = -2a$ or $a = -\frac{0.3}{2} = -0.15$ m/s^2

From (i), $u = 1 - a = 1 + 0.15$ or $u = 1.15$ m/s

For the velocity of body at the end of 7th second,

we have $u = 1.15$ m/s ; $a = -0.15$ m/s^2, $v = ?$, $t = 7$s

As, $v = u + at = 1.15 + (-0.15) \times 7 = 0.1$ m/s

Example 31 :

A body travels a distance of 20 m in the 7^{th} second and 24 m in 9^{th} second. How much distance shall it travel in the 15^{th} second ?

Sol. Here , $s_7 = 20$ m ; $s_9 = 24$ m, $s_{15} = ?$

Let u = initial velocity and a = uniform acc. of the body.

Distance travelled in n^{th} second $s_n = u + \frac{a}{2}(2n - 1)$

Distance travelled in 7^{th} second $s_7 = u + \frac{a}{2}(2 \times 7 - 1)$

or $20 = u + \frac{13a}{2}$ ...(i)

Distance travelled in 9^{th} second $s_9 = u + \frac{a}{2}(2 \times 9 - 1)$

or $24 = u + \frac{17}{2}a$ ...(ii)

Subtracting (ii) from (i), $4 = 2a$ or $a = 2\ m/s^2$

Putting this value of a in eq^n (i)

$20 = u + \frac{13}{2} \times 2$ or $20 = u + 13$ or $u = 20 - 13 = 7$ m/s

distance travelled in 15^{th} second

$s_{15} = u + \frac{a}{2}(2 \times 15 - 1) = 7 + \frac{2}{2} \times 29 = 36$ m

Example 32 :

A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping ?

Sol. Here, $u = 43.2$ km/h $= 43.2 \times \frac{5}{18}$ m/s

Deceleration ; $a = 6\ m/s^2$ $v = 0$ $s = ?$

$0 = (12)^2 - 2 \times 6\,s$ [using $v^2 = u^2 - 2as$]

or $144 = 2 \times 6s$ or $s = \frac{144}{12} = 12$ m

Example 33 :

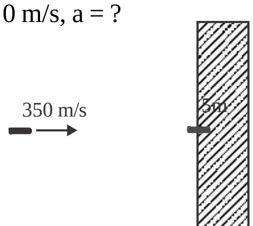
A bullet going with speed 350 m/s enters in a concrete wall and penetrates a distance of 5 cm before coming to rest. Find deceleration.

Sol. Here, u = 350 m/s, s = 5 cm, v = 0 m/s, a = ?

By using $v^2 = u^2 + 2as$

we get $0 = u^2 + 2as$

or $u^2 = -2as$ or $a = -\frac{u^2}{2s}$



or $a = -\frac{350 \times 350}{2 \times .05} = -12.25 \times 10^5\ m/sec^2$

Negative answer represents retardation.

Example 34 :

A driver takes 0.20 s to apply the brakes after he see a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after he see the need to put the brakes on.

Sol. $u = 54$ km/h $= 54 \times \frac{5}{18}$ m/s $= 15$ m/s

before applying brakes by driver, distance covered by the car $s_1 = ut = 15 \times 0.2 = 3.0$ m

After applying the brakes

$v = 0$, $u = 15$ m/s, $a = 6\ m/s^2$, $s_2 = ?$

Using $v^2 = u^2 - 2as$ or $0 = (15)^2 - 2 \times 6 \times s_2$

$12\,s_2 = -225 \Rightarrow s_2 = \frac{225}{12} = 18.75$ m

Distance travelled by the car after driver see the need for it $s = s_1 + s_2 = 3 + 18.75 = 21.75$ m

Example 35 :

Two cars travelling towards each other on a straight road at velocity 10m/s and 12 m/s respectively when they are 150 meter apart, both drivers apply their brakes and each car decelerates at 2 m/s^2 until it stops. (a) How far apart will the cars be after stopping. (b) Will the car collide to another?

Sol. Here $u_1 = 10$ m/s, $u_2 = 12$ m/s , $v_1 = 0$ m/s , $v_2 = 0$ m/s, $a = -2\ m/s^2$, D = 150 m

For first car $v^2 = u^2 - 2as \Rightarrow 0 = u_1^2 - 2as_1$ or $s_1 = \frac{u_1^2}{2a}$

For second car $v^2 = u^2 - 2as$

$\Rightarrow$ $0 = u_2^2 - 2as_2$ $2as_2 = u_2^2$ or $s_2 = \frac{u_2^2}{2a}$

distance travelled by both cars

$= s_1 + s_2 = \frac{u_1^2}{2a} + \frac{u_2^2}{2a} = \frac{u_1^2 + u_2^2}{2a}$

Now, substituting the values of u_1, u_2 and a we get

$s = \frac{10^2 + 12^2}{2 \times 2} = \frac{100 + 144}{4} = \frac{244}{4} = 61$ m

Thus, distance between cars after stopping

$\Delta s = D - s = 150 - 61 = 89$ m

(b) Because D > s hence there will be no collision

Example 36 :

A particle starts moving from the position of rest under a constant acc. If it covers a distance x in t sec, what distance will it travel in next t sec?

Sol. As acc. is constant so from $s = ut + \frac{1}{2}at^2$ we have

$x = \frac{1}{2}at^2$ [u = 0] ....(1)

Now if it travels a distance y in next t sec.

in 2t sec total distance travelled

$x + y = \frac{1}{2}a(2t)^2$ ....(2) (t + t = 2t)

Dividing eq^n. (2) by eq^n (1)

$\frac{x + y}{x} = 4$ or $y = 3x$

Example 37 :

At an instant as the traffic light turns green a car starts with a constant acc. 2 m/s^2. At the same instant a truck, travelling with a constant speed of 10 m/s, overtakes and passes the car. (a) How far beyond the starting point will the car overtake the truck? (b) How fast will the car be travelling at that instant? (c) Draw s/t curves for each vehicle.

Sol. Let the two vehicles meet after time t.

Then from 2nd eq^n of motion

The distance travelled by car

$$s_C = \frac{1}{2} \times 2t^2 \quad [\text{as } u = 0] \quad ...(1)$$

And distance travelled by truck

$$s_T = 10 \times t \quad [\text{as } a = 0]$$

According to given problem

$s_C = s_T$, i.e. $t^2 = 10\,t$ or $t = 10$ sec.

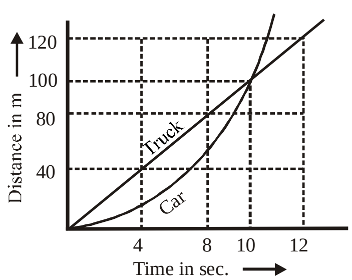
(a) The distance travelled by the car in overtaking the truck, $s_C = 10^2 = 100$ m

(b) The speed of car at t = 10 sec.

from eq^n $v = u + at$, or $v = 0 + 2 \times 10 = 20$ m/s

(c) s/t curves for car and truck,

i.e., Eq^n. (1) and Eq^n. (2), are plotted in figure



Example 38 :

A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus.

Sol. Let the passenger catch the bus after time t.

The distance travelled by the bus,

$$s_1 = 0 + \frac{1}{2}at^2 \quad ....(1)$$

and the distance travelled by the passenger

$$s_2 = ut + 0 \quad ....(2)$$

Now the passenger will catch the bus if

$$d + s_1 = s_2 \quad ....(3)$$

Substituting the values of s_1 and s_2 from eq^n. (1) and eq^n. (2) in (3)

$$d + \frac{1}{2}at^2 = ut \text{ i.e. } \frac{1}{2}at^2 - ut + d = 0 \text{ or } t = \frac{[u \pm \sqrt{u^2 - 2ad}\,]}{a}$$

So the passenger will catch the bus if t is real, i.e.,

$$u^2 \geq 2\,ad \quad \text{or} \quad u \geq \sqrt{2\,ad}$$

So the minimum speed of passenger for catching the bus is $\sqrt{2\,ad}$.

TRY IT YOURSELF-2

Q.1 A particle starts with initial velocity 2.5 m/s along the x direction and accelerates uniformly at the rate 50 cm/s^2. Find time taken to increase the velocity to 7.5 m/s.

Q.2 A truck starts from rest with an acceleration of 1.5 m/s^2 while a car 150 meter behind starts from rest with an acceleration of 2 m/s^2. How long will it take before both the truck and car are side by side.

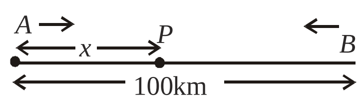
Q.3 A car is moving at a speed 50 km/h. Two seconds there after it is moving at 60 km/h. Calculate the acceleration of the car.

Q.4 A bullet moving with 10 m/s hits the wooden plank the bullet is stopped when it penetrates the plank 20 cm. deep calculate retardation of the bullet.

Q.5 A particle starts from rest and travel a distance x with uniform acceleration, then moves uniformly a distance 2x and finally comes to rest after moving further 5x with uniform retardation. Find the ratio of maximum speed to average speed.

Q.6 A particle starts from rest with constant acceleration = 2m/s^2. Find displacement in 5th sec.

Q.7 Two trains A and B, 100 km. apart, are travelling towards each other with starting speeds of 50 km/hr. for both. The train A is accelerating at 18 km/hr^2 and B is decelerating at 18 km/hr^2. Find the distance from the initial position of A of the point when the engines cross each other.



Q.8 A particle moving with uniform acceleration along a straight line passes three successive points A, B and C where the distances AB : BC is 3 : 5 & the time taken from A to B is 40 sec. If the velocities at A & C are 5 m/s & 15 m/s respectively. Find (a) the velocity of the particle at B. (b) acceleration of the particle

Q.9 A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively. If C is the mid point between A and B then determine the velocity of the particle at C.

Q.10 A train travelling along a straight line with constant acceleration is observed to travel consecutive distances of 1 km in times of 30s and 60s respectively. Find the initial velocity of the train.

Q.11 A particle is moving in a straight line with initial velocity u and uniform acceleration f. If the sum of the distances travelled in tth and (t + 1)th seconds is 100 cm, then find its velocity after t seconds, in cm/s.

ANSWERS

(1) 10 sec	**(2)** 24.5 sec	**(3)** 1.39 m/s^2
(4) 2500 m/s^2	**(5)** 4/7	**(6)** 9 m
(7) 59 km.	**(8)** 10 m/s, (1/8) m/s^2	
(9) $\sqrt{\frac{v_1^2 + v_2^2}{2}}$	**(10)** $\frac{350}{9}$ m/s	**(11)** 50 cm/s

MOTION UNDER GRAVITY

The most important example of motion in a straight line with constant acceleration is motion under gravity. In case of motion under gravity.

(1) The acceleration is constant, i.e.
$a = g = 9.8\ m/s^2$ and directed vertically downwards.

(2) The motion is in vacuum, i.e., viscous force or thrust of the medium has no effect on the motion.

1. Body falling freely under gravity :

Taking initial position as origin and downward direction of motion as positive, we have

$u = 0$ [as body starts from rest]

$a = +g$ [as acc. is in the direction of motion]

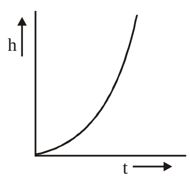
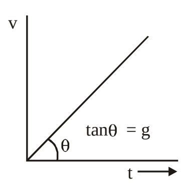
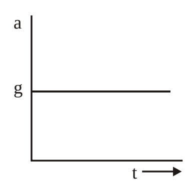
So if the body acquires velocity v after falling a distance h in time t, equations of motion, viz.

$$v = u + at\ ;\ \ s = ut + \frac{1}{2}at^2 \quad \text{and} \quad v^2 = u^2 + 2as$$

reduces to $v = gt$ ....(1), $h = \frac{1}{2}gt^2$ ....(2) and

$v^2 = 2gh$ ....(3)

These equations can be used to solve most of the problems of freely falling bodies as if.

If t is given use eq^n. (1) and eq^n (2)	If h is given use eq^n. (2) and eq^n (3)	If v is given use eq^n (3) and eq^n (1)
$v = gt$	$t = \sqrt{\frac{2h}{g}}$	$t = \frac{v}{g}$
and $h = \frac{1}{2}gt^2$	$v = \sqrt{2gh}$	$h = \frac{v^2}{2g}$
		

(i) If the body is dropped from a height H, as in time t is has fallen a distance h from its initial position, the height of the body from the ground will be

$$h' = H - h \text{ with } h = \frac{1}{2}gt^2$$

(ii) As $h = \frac{1}{2}gt^2$, i.e., $h \propto t^2$,

distance fallen in time t, 2t, 3t etc., will be in the ratio of
$1^2 : 2^2 : 3^2$, i.e., square of integers.

(iii) The distance fallen in the n^{th} sec

$$= h_{(n)} - h_{(n-1)} = \frac{1}{2}g(n)^2 - \frac{1}{2}g(n-1)^2 = \frac{1}{2}g(2n-1)$$

So distances fallen in I^{st}, 2^{nd}, 3^{rd} sec etc. will be in the ratio of 1 : 3 : 5 i.e., odd integers only.

2. Body projected vertically up :

Taking initial position as origin and direction of motion (i.e., vertically up) as positive,

here we have $v = 0$ [at highest point velocity = 0]

$a = -g$ [as acc. is downwards while motion upwards]

If the body is projected with velocity u and reaches the highest point at a distance h above the ground in time t, the equations of motion viz.,

$$v = u + at,\ \ s = ut + \frac{1}{2}at^2 \text{ and } \quad v^2 = u^2 + 2as$$

reduces to $0 = u - gt$, $h = ut - \frac{1}{2}gt^2$ and $0 = u^2 - 2gh$

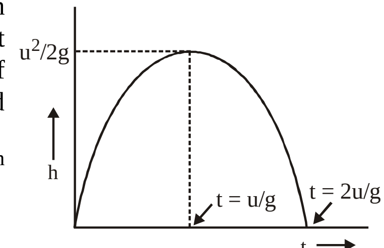
Substituting the value of u from first equation in second and rearranging these,

$u = gt$ ....(1)

$h = \frac{1}{2}gt^2$ ....(2)

and $u^2 = 2gh$ ....(3)

These equations can be used to solve most of the problems of bodies projected vertically up as.



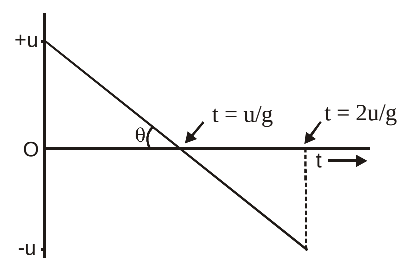
(A)

If t is given, use eq^n (1) and eq^n (2)

$u = gt$ and $h = \frac{1}{2}gt^2$

if h is given, use eq^n (2) and eq^n (3)

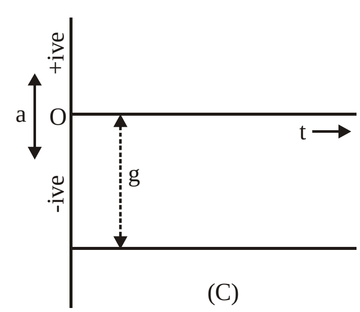
$$t = \sqrt{\frac{2h}{g}}\ ;\quad v = \sqrt{2g\,h}$$



(B)

if u is given, use eq^n (3) and eq^n (1)

$$t = \frac{u}{g}\ ;\quad h = \frac{u^2}{2g}$$



(C)

IMPORTANT POINTS

1. In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

2. The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. This is why a heavy and lighter body when released from the same height, reach the ground simultaneously and with same velocity. i.e. $t = \sqrt{\frac{2h}{g}}$ and $v = \sqrt{2gh}$

However, momentum, kinetic energy or potential energy depend on the mass of the body (all $\propto$ mass)

3. As from eq^n.(2) time taken to reach a height h,

$$t_U = \sqrt{\frac{2h}{g}}$$

Similarly, time taken to fall down through a distance h,

$$t_D = \sqrt{\frac{2h}{g}} \quad \text{so} \quad t_U = t_D = \sqrt{\frac{2h}{g}}$$

So in case of motion under gravity time taken to go up a height h is equal to the time taken to fall down through the same height h.

4. If a body is projected vertically up and it reaches a height h, then $u = \sqrt{2gh}$

and if a body falls freely through a height h, then

$$v = \sqrt{2gh} = u$$

So in case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Example 39 :

A ball is dropped from height 'h' in the last second it travels $\frac{9h}{25}$. Find h.

Sol. **Method I :** Let us say ball take 't' sec to fall height h as it falls $\frac{9h}{25}$ in last sec., it travel $h - \frac{9h}{25} = \frac{16h}{25}$ in (t – 1) sec

$$\therefore \; h = \frac{1}{2}gt^2 \;\; ......(1) \qquad \frac{16h}{25} = \frac{1}{2}g(t-1)^2 \;\; ....(2)$$

Divide (2) by (1), $\frac{16}{25} = \frac{(t-1)^2}{t^2} \Rightarrow h = \frac{1}{2}g(5)^2 = \frac{25g}{2}$ m

Method II : Let us say ball take n sec to fall height h last sec will be nth sec. (student usually think it wrongly as n – 1) (Remember in nth sec. formula u is speed at t = 0)

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

$$\frac{9h}{25} = \frac{g}{2}(2n-1) \qquad ....(1)$$

$$h = \frac{1}{2}gn^2 \qquad ....(2)$$

Divide (1) by (2), $\frac{9}{25} = \frac{2n-1}{n^2} \Rightarrow n = 5$ sec

Put in (2), $h = \frac{1}{2}g(5)^2 = \frac{25g}{2}$ m.

Example 40 :

A ball is thrown upwards from the ground with an initial speed of u. The ball is at a height of 80m at two times, the time interval being 6s. Find u. Take g = 10 m/s^2.

Sol. Here, u = u m/s, a = g = – 10 m/s^2 and s = 80m.

Substituting the values in

$s = ut + \frac{1}{2}at^2$, we have, $80 = ut - 5t^2$

or $5t^2 - ut + 80 = 0$ or $t = \frac{u + \sqrt{u^2 - 1600}}{10}$

and $\frac{u - \sqrt{u^2 - 1600}}{10}$

It is given that, $\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6$

or $\frac{\sqrt{u^2 - 1600}}{5} = 6$ or $\sqrt{u^2 - 1600} = 30$

or $u^2 - 1600 = 900$

$\therefore$ $u^2 = 2500$ or $u = \pm 50$ m/s

Ignoring the negative sign, we have, u = 50 m/s

Example 41 :

A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped. [Take g = 10 m/s^2]

Sol. When 6th ball is being dropped, the positions of the other (previously fallen) balls can be calculated by using the time of falling of each ball till this instant.

For 5th ball, it was dropped just one second before. Thus it has fallen a distance = $\frac{1}{2}gt^2 = 5$m.

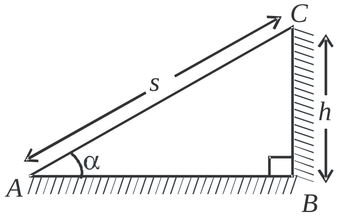
For 4th ball, it was dropped two second before this instant. It has fallen a distance = $\frac{1}{2}(10)\,2^2 = 20$m.

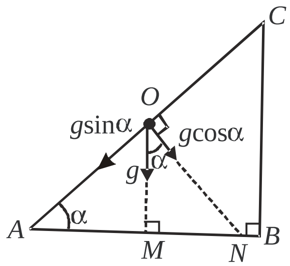
For 3rd ball, it was dropped two second before this instant. It has fallen a distance = $\frac{1}{2}(10)3^2 = 45$m.

MOTION ALONG SMOOTH INCLINED PLANE

Acceleration due to gravity being a vector quantity can be resolved, along and perpendicular to the inclined plane. The component of g along the plane is g sin α and perpendicular to the plane is g cos α.

The component g cos α, being perpendicular to the direction of motion (AC), does not contribute towards accelerating the object. Thus the effective acceleration on the body is g sin α along CA.





In applying kinematic equation , $v^2 = u^2 + 2as$
where v, u, a, s should be same direction hence, use
$a = g \sin \alpha$ along inclined plane.
Let a particle, sliding down C to A, along the inclined plane CA, acquire a final velocity v_1, covering a distance s.

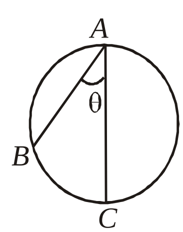
Now for the sliding particle, $u = 0$, $a = g \sin \alpha$, $v = v_1$.
[Taking the direction C to A as positive]
Using, $v^2 = u^2 + 2as$

$$v_1^2 = 2g \sin\alpha . s = 2g\left[\frac{h}{s}\right]s = 2gh$$ [If α be the angle of inclination then, $\sin\alpha = \frac{h}{s}$]

$$\therefore v_1 = \sqrt{2gh}$$

Example 42 :

Show that time to slide along AB and AC (diameter) of circle is same.



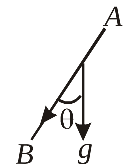
Sol. For motion along AC,

$$2R = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{4R}{g}} \quad ....(1)$$

For motion along AB,

$$AB = \frac{1}{2}g\cos\theta t^2 \; ; \; 2R\cos\theta = \frac{1}{2}g\cos\theta t^2$$

$$t = \sqrt{\frac{4R}{g}} \quad ....(2)$$



From (1) and (2) we can conclude the result.

MOTION UNDER GRAVITY IN PRESENCE OF AIR RESISTANCE

An object is thrown with speed u in upward direction during its motion its experiences constant air resistance R in the direction opposite to its motion.

(a) **Motion in upward direction :** Total force during upward motion mg + R

Hence, total acceleration, $a_1 = g + \frac{R}{m}$

Time in upward motion, $t_1 = \frac{u}{g + \frac{R}{m}}$

Maximum height, $h = \frac{u^2}{2\left(g + \frac{R}{m}\right)}$

(b) **Motion in downward direction :** Total force during downward motion mg – R

Hence, total acceleration, $a_2 = g - \frac{R}{m}$

Time in downward motion (from IInd kinematic equation)

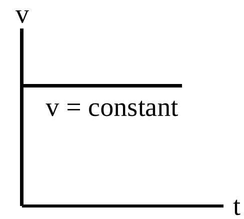
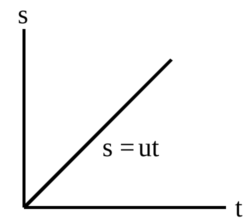
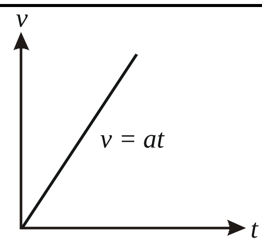
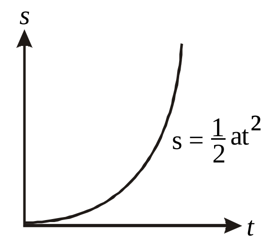
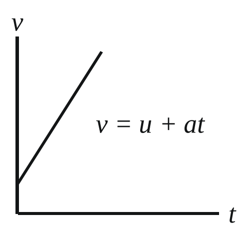
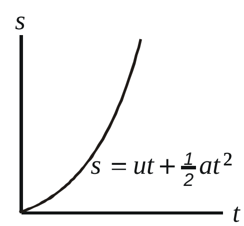
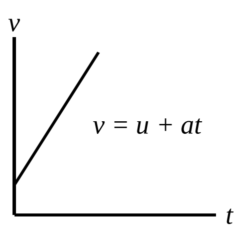
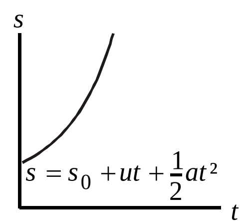
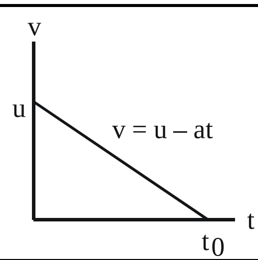
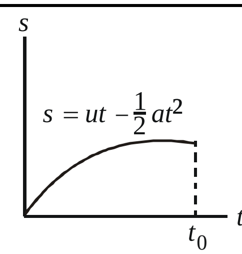
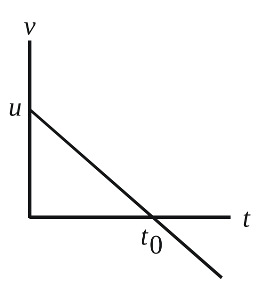
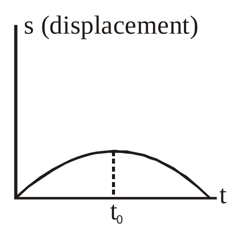
$$h = 0 + \frac{1}{2}\left(g - \frac{R}{m}\right)t_2^2 \Rightarrow \frac{u^2}{2\left(g + \frac{R}{m}\right)} = \frac{1}{2}\left(g - \frac{R}{m}\right)t_2^2$$

$$\Rightarrow t_2 = \frac{u}{\sqrt{g^2 - \left(\frac{R}{m}\right)^2}}$$

hence, $\frac{t_1}{t_2} = \sqrt{\frac{g - \frac{R}{m}}{g + \frac{R}{m}}} < 1$

SOME IMPORTANT GRAPHS RELATED TO MOTION

All the following graphs are drawn for one-dimensional motion with uniform velocity or with constant acceleration.

Different Case	v-t graph	s-t graph	Important Points
1. Uniform motion			(i) Slope of s-t graph = v = constant (ii) In s-t graph s = 0 at t = 0
2. Uniformly accelerated motion with u = 0 and s = 0 at t = 0			(i) u = 0, i.e., v = 0 at t = 0 (ii) u = 0, i.e., slope of s-t graph at t = 0, should be zero (iii) a or slope of v-t graph is constant
3. Uniformly accelerated motion with $u \neq 0$ but s = 0 at t = 0			(i) $u \neq 0$, i.e., v or slope of s – t graph at t = 0 is not zero (ii) v or slope of s-t graph gradually goes on increasing
4. Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) $s = s_0$ at t =0
5. Uniformly retarded motion till velocity becomes zero			(i) Slope of s-t graph at t = 0 gives u (ii) Slope of s-t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero
6. Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

TRY IT YOURSELF-3

Q.1 From the foot of a tower 90m high, a stone is thrown up so as to reach the top of the tower. Two second later another stone is dropped from the top of the tower. Find when and where two stones meet.

Q.2 A stone is dropped from a height h. Simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time after which two stones cross each other.

Q.3 A falling stone takes 0.2 seconds to fall past a window which is 1m high. From how far above the top of the window was the stone dropped ?

Q.4 You are throwing a ball straight up in the air. At the highest point, the ball's
(A) velocity and acceleration are zero.
(B) velocity is nonzero but its acceleration is zero.
(C) acceleration is nonzero, but its velocity is zero.
(D) velocity and acceleration are both nonzero.

Q.5 A person standing at the edge of a cliff throws one ball straight up and another ball straight down, each at the same initial speed. Neglecting air resistance, which ball hits the ground below the cliff with the greater speed:
(A) ball initially thrown upward;
(B) ball initially thrown downward;
(C) neither; they both hit at the same speed.

Q.6 Two buildings stand side by side. The taller is 20 meters higher than the shorter. Rocks are dropped from rest from both roofs at the same time. When the rock from the taller building passes the top of the shorter building, the rock from the shorter building will be
(A) 20 meters below its start point
(B) less than 20 meters below its start point
(C) farther than 20 meters below its start point.

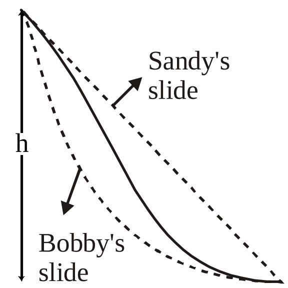
Q.7 A bag of sand dropped by a would be assassin from the roof of a building just misses Tough Tony, a gangster 2m tall. The missile traverses the height of Tough Tony in 0.20s, landing with a thud at his feet. How high was the building? Ignore friction.

Q.8 A person throws a ball vertically upward with an initial velocity of 15 m/s. Calculate (i) how high it goes and (ii) how long the ball is in air before it comes to his hand.

Q.9 With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m ?

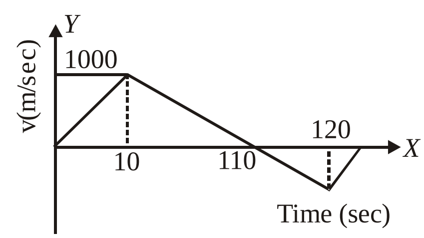
Q.10 A 1 kg mass is found to be moving 18 m/s up a 30^{o} incline. How fast is the mass moving 3 seconds later? Take g to be $10\ m/s^2$.
(A) 2 m/s. (B) 3 m/s.
(C) 6 m/s. (D) None of the above.

Q.11 Two children on the playground, Bobby and Sandy, travel down slides of identical height h but different shapes as shown. The slides are frictionless. Assuming they start down the slides at the same time with zero initial velocity, which of the following statements is true?



(A) Bobby reaches the bottom first with the same average velocity as Sandy.
(B) Bobby reaches the bottom first with a larger average acceleration than Sandy.
(C) Bobby reaches the bottom first with the same average acceleration as Sandy.
(D) They reach the bottom at the same time with the same average acceleration.

Q.12 Adjacent graph shows the variation of velocity of a rocket with time. Find the time of burning of fuel from the graph-



(A) 10 sec (B) 110 sec (C) 120 sec
(D) Cannot be estimated from the graph

ANSWERS

(1) 3.11 s, 83.82m.	**(2)** $\sqrt{\frac{h}{8g}}$	**(3)** (4/5) m
(4) (C)	**(5)** (C)	**(6)** (A)
(7) 6.1m	**(8)** 11.5m, 3.06s	**(9)** 31 m/s
(10) (B)	**(11)** (B)	**(12)** (A)

RELATIVE VELOCITY

Relative velocity of an object A with respect to another object B, when both are in motion is the time rate at which object A changes its position with respect to object B.

Position of object A and B are given as

$$x_A = x_{OA} + v_A t \quad \text{and} \quad x_B = x_{OB} + v_B t$$

$$x_B - x_A = (x_{OB} - x_{OA}) + (v_B - v_A)\,t$$

or $x = x_O + (v_B - v_A)\,t$

or $\frac{x - x_O}{t} = v_B - v_A$

If $\vec{v}_A$ and $\vec{v}_B$ be the respective velocities of object A and B then relative velocity of A w.r.t. B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

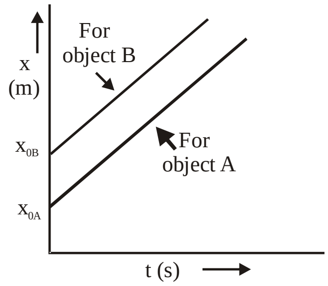
Similarly, relative velocity of B w.r.t. A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

SPECIAL CASES

(1) When the two objects move with equal velocities :

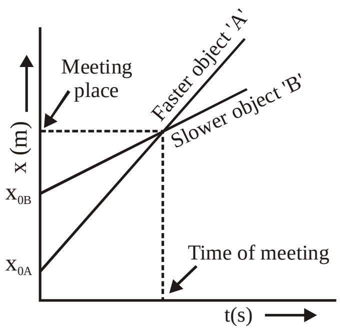
i.e. $v_A = v_B$ or $v_B - v_A = 0$

It means the two objects stay at constant distance apart during the whole journey. In this case, the position-time graphs of two objects are parallel straight lines as shown in figure.

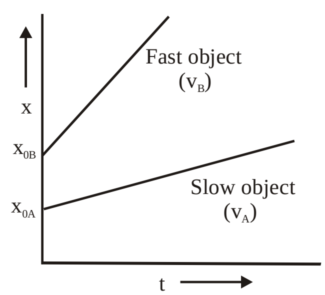


(2) When the two objects move with unequal velocities :

(i) When $v_A > v_B$, then $v_B - v_A$ is negative. This shows that the separation between two moving objects will go on decreasing with time. After some time, the two moving objects will meet and then the relative distance between the objects will increase with time as shown in figure.



(ii) When $v_B > v_A$, then $v_B - v_A$ is positive. This shows that the separation between two moving objects will go on increasing with time as shown in figure.



(3) When two trains A and B move with same velocity v but in opposite in direction :

The relative velocity of train A w.r.t. train B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v(\hat{i}) - v(-\hat{i}) = 2v(\hat{i})$$

Relative velocity of train B w.r.t. A

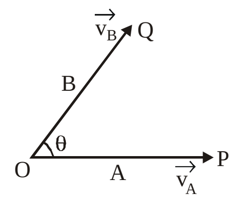
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = v(-\hat{i}) - v(\hat{i}) = 2v(-\hat{i})$$

Thus, when two trains cross each other in opposite directions, then each train appears to move very fast (i.e. double the actual speed) relative to the other.

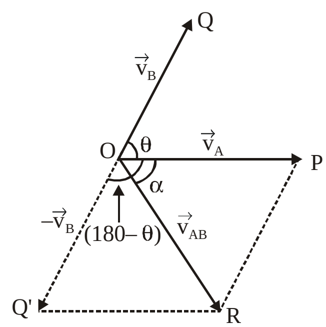
(4) The bodies moving in directions inclined to each other :

Relative velocity of A w.r.t B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$



The relative velocity of A with respect to B is given by the diagonal OR of the parallelogram OPRQ' as shown in fig.



The magnitude of the relative velocity v_{AB} is given by

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180 - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

Let α be the angle made by v_{AB} with v_A, then

$$\tan\alpha = \frac{v_B \sin(180 - \theta)}{v_A + v_B \cos(180 - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

$$\text{or } \alpha = \tan^{-1}\left(\frac{v_B \sin\theta}{v_A - v_B \cos\theta}\right)$$

∠α gives the direction of the relative velocity with $\vec{v}_A$.

(i) When both the bodies are moving along parallel straight lines in the same direction :

Then the angle between them is θ = 0º

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 0}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B} \quad [\because \cos 0^\circ = 1]$$

$$= \sqrt{(v_A - v_B)^2} = (v_A - v_B)$$

Thus magnitude of relative velocity of A with respect to B is equal the difference between the magnitude of individual velocities.

(ii) When two bodies are moving along parallel straight lines in the opposite direction i.e. θ = 180º :

$\therefore \; v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 180^o}$

$= \sqrt{v_A^2 + v_B^2 + 2v_A v_B}$ $\quad [\because \cos 180^o = -1]$

$= \sqrt{(v_A + v_B)^2} = (v_A + v_B)$

Thus magnitude of relative velocity of body A w.r.t. body B is equal to the sum of the magnitudes of individual velocities.

Note : When two bodies move in opposite directions, the magnitude of relative velocity of one with respect to the other is equal to the sum of the magnitudes of two velocities.

Example 43 :

Two trains are moving east ward with velocities 10 ms^{-1} and 15 ms^{-1} on parallel tracks. Calculate the relative velocity of slow train w.r.t. the fast train.

Sol. $v_1 = 10$ ms^{-1} , $v_2 = 15$ ms^{-1}

Relative velocity of slow train w.r.t. the fast train

$= v_1 - v_2 = 10 - 15 = -5$ ms^{-1}

–ve sign shows that slow train appears to move westward w.r.t. fast train with velocity of 5 ms^{-1}.

Example 44 :

A police van moving on a highway with a speed of 30 km/h fires a bullet at thief's car speeding away in the same direction with a speed of 192 km/hr. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car.

Sol. Speed of police van = 30 km/h = $\frac{30 \times 1000\,\text{m}}{3600\ \text{s}} = \frac{25}{3}$ m/s

Speed of thief's car = 192 km/h = $\frac{160}{3}$ m/s

$\therefore$ Relative speed of theif's car w.r.t. police van

$= \frac{160}{3} - \frac{25}{3} = 45$ m/s

Speed of bullet w.r.t. van = 150 m/s

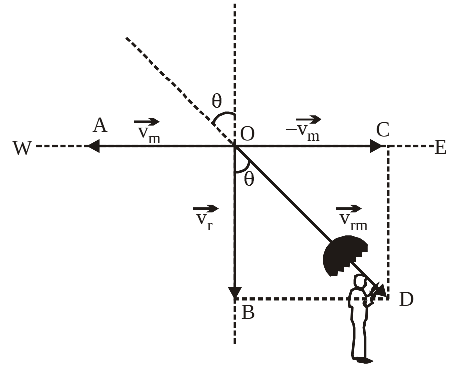
Speed with which bullet hits the car = 150 – 45 = 105 m/s

RAIN BASED PROBLEMS

Relative velocity of rain w.r.t. the moving Man :

A man walking west with velocity $\vec{v}_m$, represented by $\vec{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\vec{OB}$ as shown in fig.

The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$, will be represented by diagonal $\vec{OD}$ of rectangle OBDC.



$\therefore \; v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^o} = \sqrt{v_r^2 + v_m^2}$

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then $\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r}$ or $\theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$

Here angle θ is from vertical towards west and is written as θ, west of vertical.

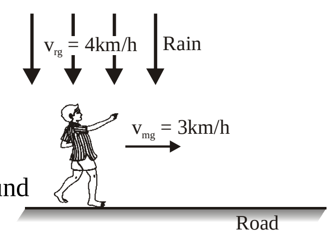
Note : In the above case if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain w.r.t. man i.e. the umbrella should be hold making an angle $\theta \left(= \tan^{-1}\frac{v_m}{v_r}\right)$ west of vertical.

Example 45 :

A man is walking on a level road at a speed of 3 km/h. Raindrops fall vertically with a speed of 4 km/h. Find the velocity of raindrops with respect to the men.

Sol. If we consider velocity of rain with respect to the man is V km/h.

Relative velocity of man w.r.t. ground



$\vec{v}_{mg} = \vec{v}_m - \vec{v}_g$ ....(1)

velocity of rain w.r.t. ground

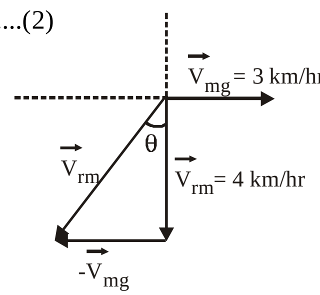
$\vec{v}_{rg} = \vec{v}_r - \vec{v}_g$ ....(2)

Velocity of rain w.r.t. man

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

On subtracting eq^n. 1 from eq^n. 2

$\vec{v}_{rm} = \vec{v}_{rg} - \vec{v}_{mg}$

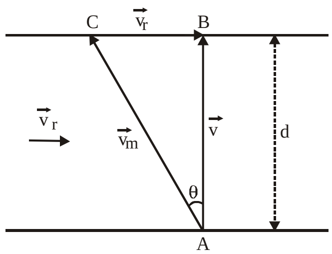


$|v_{rm}| = \sqrt{{v_{rg}}^2 + {v_{mg}}^2} = \sqrt{4^2 + 3^2} = 5$ km / hr

Direction : $\tan\theta = \frac{3}{4}$ or $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

RIVER PROBLEMS

1. Minimum distance approach :



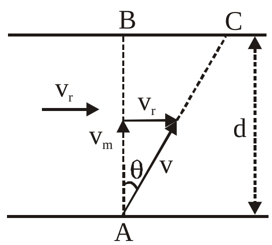
d = width of river, v_r = velocity of river,
v_m = velocity of swimmer

The swimmer should swim in a direction such that resultant $\vec{v}$ of $\vec{v}_m$ and $\vec{v}_r$ is along AB which is the shortest path

$$\sin\theta = \frac{v_r}{v_m} \;;\quad v = \sqrt{v_m^2 - v_r^2} \;;\quad t = \frac{d}{\sqrt{v_m^2 - v_r^2}}$$

2. Minimum time of approach :

Time of crossing the river $t = \dfrac{d}{v_m \cos\theta}$



$t = t_{min}$ when $\cos\theta = +1 = \cos 0°$ i.e. $\theta = 0°$

$$t_{min} = \frac{d}{v_m}$$

To cross the river in shortest time man should swim perpendicular to direction of flow.

Man will reach C instead of B

It BC = x then $\tan\theta = \dfrac{v_r}{v_m} = \dfrac{x}{d}$ so $x = \dfrac{v_r}{v_m}.d$

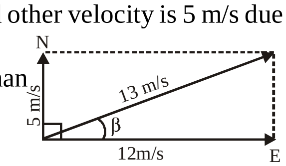
Example 46 :

A ship is steaming towards east at a speed of 12 ms^{-1}. A woman runs across the deck at a speed of 5 ms^{-1} in the direction at right angles to the direction of motion of the ship i.e. towards north. What is the velocity of the woman relative to sea.

Sol. The woman has two velocities simultaneously while running on the deck, one velocity is equal to the velocity of ship i.e. 12 m/s due east and other velocity is 5 m/s due north.

The resultant velocity of woman

$= \sqrt{(12)^2 + (5)^2} = 13$ m/s



Let β be the angle made by the resultant velocity with the direction of motion of the ship (i.e. East).

$$\therefore\ \tan\beta = \frac{5\sin 90°}{12 + 5\cos 90} = \frac{5}{12} = 0.4167$$

$\therefore\quad \beta = 22°37'$ north of east.

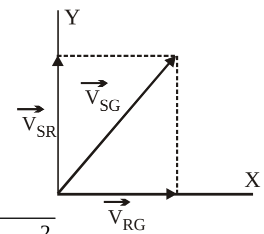
Thus, the direction of the velocity of the woman is 22°37' north of east.

Example 47 :

A swimmer can swim in still water at a rate 4 km/h. If he swims in a river flowing at 3 km/h and keeps his direction (w.r.t. water) perpendicular to the current. Find his velocity w.r.t. the ground.

Sol. The velocity of the swimmer w.r.t. water $\vec{v}_{SR} = 4.0$ km/h in the direction perpendicular to the river. The velocity of river w.r.t. the ground is $\vec{v}_{RG} = 3.0$ km/h along the length of river.

The velocity of the swimmer w.r.t. the ground is $\vec{v}_{SG}$ where



$$\vec{V}_{SG} = \vec{V}_{SR} + \vec{V}_{RG}$$

$$V_{SG} = \sqrt{V_{SR}^2 + V_{RG}^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ km/hr}$$

The angle θ made with the direction of flow is

$$\theta = \tan^{-1}\left[\frac{V_{SR}}{V_{RG}}\right] = \tan^{-1}\left(\frac{4}{3}\right)$$

TRY IT YOURSELF-4

Q.1 Balls A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively ($g = 10\ m/s^2$). Find separation between them after one second.

Q.2 A man standing on a road has to hold his umbrella at 30º with the vertical to keep the rain away. He throws the umbrella and starts running at 10km/hr. He finds that rain drop are hitting his head vertically. Find the speed of raindrops with respect to (a) road (b) the moving man.

Q.3 To a man walking at the rate of 3 km/hr the rain appears to fall vertically. When he increases his speed to 6 km/hr it appears to meet him at an angle of 45º with vertical. Find the speed of rain.

Q.4 A man swims at an angle θ = 120º to the direction of water flow with a speed v_{mw} = 5 km/hr relative to water. If the speed of water v_w = 3 km/hr, find the speed of the man.

Q.5 A man crosses the river in shortest time at an angle θ = 60º to the direction of flow of water. If the speed of water is v_w = 5km/hr, find the speed of the man.

Q.6 Two points P and Q move in same plane such that the relative acceleration of P with respect to Q is zero. They are moving such that the distance between them is decreasing. Pick the correct statement for P and Q to collide

(A) The line joining P and Q should not rotate.

(B) The line joining P and Q should rotate with constant angular speed

(C) The line joining P and Q should rotate with variable angular speed.

(D) All the above statements are correct.

Q.7 An eagle flies at constant velocity horizontally across the sky, carrying a turtle in its talons. The eagle releases the turtle while in flight. From the eagle's perspective, the turtle falls vertically with speed v_1. From an observer on the ground's perspective, at a particular instant the turtle falls at an angle with speed v_2. What is the speed of the eagle with respect to an observer on the ground?

(A) $v_1 + v_2$ (B) $v_1 - v_2$

(C) $\sqrt{v_1^2 - v_2^2}$ (D) $\sqrt{v_2^2 - v_1^2}$

Q.8 A man who is wearing a hat of extended length of 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops do not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be:

(A) (15/2) m/s (B) (40/3) m/s

(C) 10 m/s (D) zero

Q.9 A train is moving with velocity $\vec{v}_{TG} = 3\hat{i} + 4\hat{j}$ relative to the ground. A bullet is fired in the train with velocity $\vec{v}_{BT} = 15\hat{i} - 6\hat{j}$ relative to the train. What is the bullets' velocity $\vec{v}_{BG}$ relative to the ground?

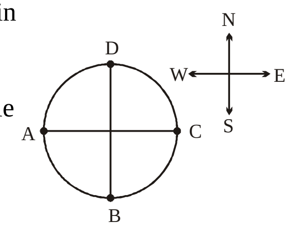
Q.10 Two aeroplanes fly from their respective positions A and B starting at the same time and reach the point C simultaneously when wind was not blowing. On a windy day they head towards C but both reach the point D simultaneously in the same time which they took to reach C. Then the wind is blowing in

(A) North-East direction

(B) North-West direction

(C) Direction making an angle $0 < \theta < 90$ with North towards West.

(D) North direction



ANSWERS

(1) 5m	**(2)** (a) 20kph, (b) $10\sqrt{3}$ kph	
(3) $3\sqrt{2}\ \frac{km}{hr}$	**(4)** $\sqrt{19}$ m/sec.	**(5)** 8 km/hr
(6) (A)	**(7)** (D)	**(8)** (A)
(9) $18\hat{i} - 2\hat{j}$	**(10)** (B)	

ADDITIONAL EXAMPLES

Example 1 :

A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is $10 m/s^2$.

(a) How long does it take to reach the highest point?

(b) How high does it rise above the point of projection?

(c) How long will it take for the ball to reach a point 10 m above the point of projection?

Sol. As here motion is vertically upwards,

$a = -g$ and $v = 0$

(a) From 1st equation of motion, i.e., $v = u + at$,

$0 = 20 - 10t$ i.e. $t = 2$ sec.

(b) Using $v^2 = u^2 + 2ax$

$0 = (20)^2 - 2 \times 10 \times h$ i.e. $h = 20$ m

(c) Using $s = ut + \frac{1}{2}at^2$, $10 = 20t - \frac{1}{2} \times 10 \times t^2$

i.e. $t^2 - 4t + 2 = 0$ or $t = 2 \pm \sqrt{2}$,

i.e. $t = 0.59$ sec. or 3.41 sec.

i.e., there are two times, at which the ball passes through h = 10 m, once while going up and then coming down.

Example 2 :

A ball is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2 s. If acc due to gravity is 9.8 m/s^2 (a) What is the height of the bridge? (b) With which velocity does the ball strike the water?

Sol. Taking the point of projection as origin and downward direction as positive,

(a) Using $s = ut + \frac{1}{2}at^2$ we have

$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times 2^2 = 9.8$ m

(u is taken to be negative as it is upwards.)

(b) Using $v = u + at$

$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

Example 3 :

A rocket is fired vertically up from the ground with a resultant vertical acc. of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached ?

(b) After how much time from then will the maximum height be reached? (Take g = 10 m/s^2).

Sol. (a) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acc. is vertically upwards and 10 m/s^2 will be

$h_1 = 0 \times 60 + \frac{1}{2} \times 10 \times 60^2 = 18000$ m ....(1)

Velocity acquired by it is

$v = 0 + 10 \times 60 = 600$ m/s ....(2)

After one minute the rocket moves vertically up with initial velocity of 600 m/s and continues till height h_2 till its velocity becomes zero.

$0 = (600)^2 - 2gh_2$

or $h_2 = 18000$ m ....(3) [as g = 10 m/s^2]

From eq^n. (1) and (3) the maximum height reached by the rocket from the ground is

$H = h_1 + h_2 = 18 + 18 = 36$ km

(b) The time to reach maximum height after burning of fuel is $0 = 600 - gt$ or $t = 60$ s

After finishing fuel the rocket goes up for 60 s.

Example 4 :

A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies after 2 sec the release of the second body, if $g = 9.8\ m/s^2$.

Sol. The 2nd body falls for 2s, so $h_2 = \frac{1}{2}g(2)^2$ ....(1)

while 1st has fallen for 2 + 1 = 3 sec so

$$h_1 = \frac{1}{2}g(3)^2 \quad ....(2)$$

$\therefore$ Separation between two bodies after 2 sec the release of 2nd body, $d = h_1 - h_2 = \frac{1}{2}g(3^2 - 2^2) = 4.9 \times 5 = 24.5$ m

Example 5 :

If a body travels half its total path in the last second of its fall from rest, find : (a) The time and (b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. ($g = 9.8\ m/s^2$)

Sol. In time t, the body falls a height $h = \frac{1}{2}gt^2$

[u = 0 as the body starts from rest] ....(1)

Now, as the distance covered in (t – 1) s is

$$h' = \frac{1}{2}g(t-1)^2 \quad ....(2)$$

from eq^{n}s (1) and (2) distance travelled in the last sec.

$$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$$

i.e., $h - h' = \frac{1}{2}g(2t-1)$

But according to given problem as $(h - h') = \frac{h}{2}$

i.e., $\frac{1}{2}h = \frac{1}{2}g(2t-1)$

or $\frac{1}{2}gt^2 = g(2t-1)$ [as from eq^n. (1) $h = \frac{1}{2}gt^2$]

or $t^2 - 4t + 2 = 0$

or $t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 2}}{2} = 2 \pm \sqrt{2}$

hence t = 0.59 s or t = 3.41 sec.

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1 s.

so $t = 3.41$ s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Example 6 :

A stone is dropped into a well and the sound of impact of stone on the water is heard after 2.056 sec. of the release of stone from the top. If acc. due to gravity is 980 cm/sec^2 and velocity of sound in air is 350 m/s, calculate the depth of the well.

Sol. If the depth of well is h and time taken by stone to reach the bottom is t_1, then $h = \frac{1}{2}gt_1^2$ ....(1)

time taken by sound to reach surface

$$t_2 = \frac{h}{350} \quad ....(2)$$

But $t_1 + t_2 = 2.056$ ....(3)

Now as negative time is not physically acceptable, so $t_1 = 2$ sec

the depth of well $h = \frac{1}{2} \times 9.8 \times 2^2 = 19.6$ m

Example 7 :

Two railway tracks are parallel to North-South direction. Train A is moving with a speed of 40 ms^{-1} from North to South along one track, while train B is moving with a speed of 30 ms^{-1} from South to North. Calculate (i) relative velocity of B w.r.t. A and (ii) relative velocity of ground w.r.t. A.

Sol. Consider the direction from North to South as positive.

$\therefore$ $v_A = +40$ ms^{-1} and $v_B = -30$ ms^{-1}

(i) Relative velocity of B w.r.t.

A $= v_B - v_A = -30 - 40 = -70$ ms^{-1}

Thus train B appears to move from South to North with speed of 70 m s^{-1} for an observer in A.

(ii) Velocity of ground, $v_g = 0$

$\therefore$ Relative velocity of ground w.r.t.

A $= v_g - v_A = 0 - 40 = -40$ ms^{-1}

Thus, the ground will appear to move from south to north with speed of 40 ms^{-1} w.r.t. A.

Example 8 :

The velocity of the bullet with respect to gun is 60 m/s. The gun is mounted on a tank moving with a speed 20 m/s with respect to the ground. If the bullet is fired in the direction of tank's motion than calculate velocity of bullet with respect to the ground.

Sol. Velocity of tank w.r.t. ground $\vec{v}_{Tg} = \vec{v}_T - \vec{v}_g = 20$ .....(1)

Velocity of bullet w.r.t. tank $\vec{v}_{BT} = \vec{v}_B - \vec{v}_T = 60$ .....(2)

Velocity of bullet w.r.t. ground $\vec{v}_{Bg} = \vec{v}_B - \vec{v}_g = ?$

On adding eq^n (1) and eq^n (2)

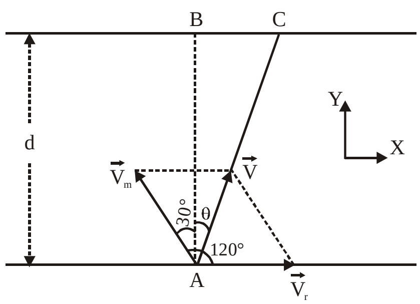
$\therefore$ $\vec{v}_{Tg} + \vec{v}_{BT} = \vec{v}_B - \vec{v}_g = \vec{v}_{Bg}$

$\vec{v}_{Bg} = 20 + 60 = 80$ m / sec

Example 9 :

A man can swim in still water at a speed of 3 km/h. He wants to, cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming. (a) Find the time he takes to cross the river. (b) At what point on the opposite bank will he arrive.

Sol. Width of river AB = d = 500 m = 1/2 km.



$V_m = 3$ km/hr velocity of man in still water

$V_r = 2$ km/hr velocity of river

V = resultant velocity of man in flowing river

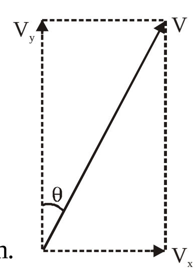
$$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j}$$

Now, $V_x = V_r - V_m \sin 30° = 2 - 3 \times \frac{1}{2} = \frac{1}{2}$ km / hr

$$V_y = V_m \cos 30° = \frac{3\sqrt{3}}{2}\text{ km / hr}$$

Displacement along Y-axis, $d = V_y \times t$

or $t = \frac{d}{V_y} = \frac{\frac{1}{2}}{\frac{3\sqrt{3}}{2}}$ $\therefore$ $t = \frac{1}{3\sqrt{3}}$ hr.



Displacement along X-axis,

$$BC = V_x \times t = \frac{1}{2} \times \frac{1}{3\sqrt{3}} = \frac{1}{6\sqrt{3}}\text{ km.}$$

Example 10 :

A girl standing on a road has to hold her umbrella at 30° with the vertical to keep the rain away. She throws the umbrella and starts running at 10 km/h. She finds that raindrops are hitting her head vertically. Find the speed of raindrops with respect to (a) the road (b) the moving girl.

Sol. Suppose the velocity of rain with respect to girl = V_{RG}

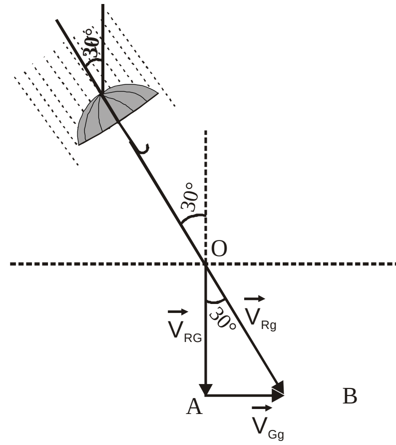
The velocity of rain with respect to the ground = V_{Rg}

The velocity of girl with respect to ground = V_{Gg} = 10 km/h

$$\vec{v}_{RG} = \vec{v}_R - \vec{v}_G \quad ....(1)$$

$$\vec{v}_{Rg} = \vec{v}_R - \vec{v}_g \quad ....(2)$$

$$\vec{v}_{Gg} = \vec{v}_G - \vec{v}_g \quad ....(3)$$



On adding eq^n. (1) and eq^n. (3)

$$\vec{v}_{RG} + \vec{v}_{Gg} = \vec{v}_R - \vec{v}_g = \vec{v}_{Rg}$$

(a) By triangle AOB, $\sin 30° = \frac{AB}{OB} = \frac{10}{V_{Rg}}$

$$V_{Rg} = \frac{10}{\sin 30°} = \frac{10}{1/2} = 20 \text{ km / hr}$$

(b) Now, taking $\frac{V_{RG}}{V_{Gg}} = \cot 30°$

$$\frac{V_{RG}}{10} = \sqrt{3} \quad \text{or} \quad V_{RG} = 10\sqrt{3}\text{ km / h}$$

Example 11 :

Two boats A and B move in perpendicular direction to a buoy anchored at some point O on a river. They travel with velocity 1.2 v, where v is the stream velocity. Boat A moves along the river, whereas boat B moves perpendicular to it. After traversing an equal distance from O the two boats return. Find the ratio of the time taken by the two boats.

Sol. Let ℓ = distance covered by the boat A along the river as well as by the boat B across the river in one direction.

Resultant velocity of boat A with respect to the ground when boat goes along the river = 1.2 v + v.

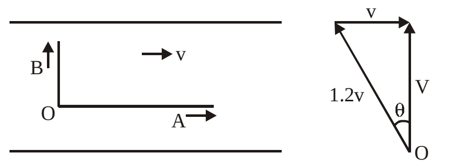
Resultant velocity of boat A with respect to the ground when the boat goes against the stream = 1.2 v – v.

$\therefore$ Time taken by the boat A to cover the whole journey is

$$t_A = \frac{\ell}{1.2v + v} + \frac{\ell}{1.2v - v} = \frac{\ell(1.2v + v + 1.2v - v)}{(1.2v)^2 - v^2}$$

$$= \frac{2.4v\ell}{0.44v^2} = \frac{5.45\ell}{v} \quad .....(1)$$

For boat B to move from O perpendicular to the direction of flow of stream, its velocity must be at an angle θ to the direction of the stream velocity so that the resultant velocity is directed perpendicular to the flow of stream



(a) (b)

$\therefore$ Resultant speed of boat is given by

$$V = \sqrt{(1.2v)^2 - v^2} = v\sqrt{0.44} = 0.66\,v$$

$\therefore$ Time taken by the boat B to cover the whole journey is

$$t_B = \frac{2\ell}{V} = \frac{2\ell}{0.66v} = \frac{\ell}{0.33v} \qquad ....(2)$$

From (1) and (2), we have

$$\frac{t_A}{t_B} = \frac{5.45\ell}{v} \times \frac{0.33v}{\ell} = 1.80$$

Example 12 :

The velocity of a particle moving in the positive direction of x-axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0, find (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Sol. (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \quad \therefore \frac{dx}{\sqrt{x}} = \alpha\, dt \quad \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

$$2\sqrt{x} = \alpha\, t \Rightarrow x = (\alpha^2 t^2 / 4)$$

Velocity, $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres

$$s = \frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}\,\alpha$$

Example 13:

A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the particle at the origin of coordinates.

Sol. $$\frac{dy}{dt} = p\frac{dx}{dt} - q.2x\frac{dx}{dt}$$

and $$\frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$$= (p - 2qx)\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$\because \frac{d^2x}{dt^2} = 0$ (no acceleration along x-axis) and $\frac{d^2y}{dt^2} = -a$

$$\therefore v_x^2 = \frac{a}{2q} \Rightarrow v_x = \sqrt{\frac{a}{2q}}.$$

Further, $$\left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt} \Rightarrow v_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

$$v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)} \Rightarrow v = \sqrt{\left[\frac{a\,(p^2+1)}{2q}\right]}$$

Example 14 :

A particle start with initial velocity v_0 and acceleration a = kt, where k is constant. Find velocity and displacement after time t.

Sol. Given : $$a = kt \Rightarrow \frac{dv}{dt} = kt \Rightarrow dv = kt\,dt \Rightarrow \int_{v_0}^{v} dv = k\int_0^t t\,dt$$

$$\Rightarrow v - v_0 = \frac{kt^2}{2} \Rightarrow v = v_0 + \frac{k}{2}t^2$$

(ii) $$v = \frac{ds}{dt} = v_0 + \frac{k}{2}t^2 \Rightarrow ds = (v_0 + \frac{k}{2}t^2)\,dt$$

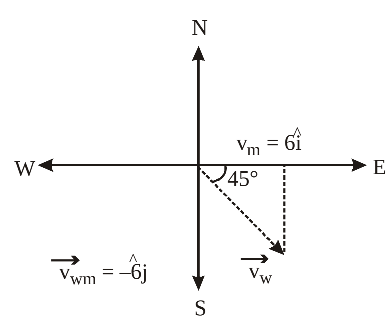
$$\Rightarrow \int_0^s ds = \int_0^t v_0 dt + \frac{k}{2}\int_0^t t^2 dt$$

$$\Rightarrow s = v_0 t + \frac{k}{2}\frac{t^3}{3} \Rightarrow s = v_0 t + \frac{k}{6}t^3$$

Example 15 :

A person moves due east at speed 6 m/s and feels the wind is blowing to south at speed 6 m/s. (a) Find the actual velocity of wind blow. (b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

Sol. (i)

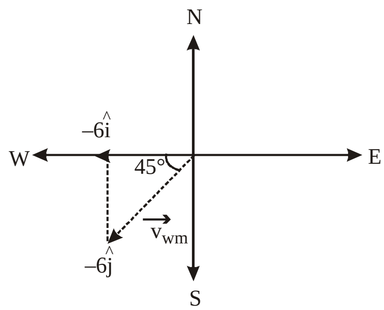


$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6\hat{j} + 6\hat{i}$; $\vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}$ m/s and it blowing to S–E

(ii) Person doubles its velocity then $\vec{v}_m = 12\hat{i}$



but actual wind velocity remain unchanged.

$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i} = -6\hat{i} - 6\hat{j}$

Now relative velocity of wind is $6\sqrt{2}$ m/s to S-W.

Example 16 :

A particle moves along x-axis with acc. $a = a_0(1 - t/T)$ where a_0 and T are constant if velocity at t = 0 is zero then find the average velocity from t = 0 to the time when a = 0.

Sol. (a) $\frac{dv}{dt} = a_0\left(1 - \frac{t}{T}\right)$

$$\Rightarrow \int_0^v dv = \int_0^t a_0\left(1 - \frac{t}{T}\right) dt \Rightarrow v = a_0\left(t - \frac{t^2}{2T}\right)$$

$$\because \frac{dx}{dt} = v \text{ so, } \int dx = \int v\, dt \Rightarrow x = \int_0^t a_0\left(t - \frac{t^2}{2T}\right) dt$$

$$\Rightarrow x = a_0\left(\frac{t^2}{2} - \frac{t^3}{6T}\right) \quad \because a = 0 \Rightarrow t = T$$

$$\text{Av. velocity} = \frac{\text{displacement}}{\text{time}} = \frac{a_0\left(\frac{T^2}{2} - \frac{T^3}{6T}\right)}{T} = \frac{a_0 T}{3}$$

Example 17 :

A particle moves along a straight line path such that its magnitude of velocity is given by $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O then determine the magnitude of particle's average velocity and average speed in time interval from t = 0 to t = 4s.

Sol. Av. velocity $= \dfrac{\int v\,dt}{\int dt} = \dfrac{\int_0^4 (3t^2 - 6t)\,dt}{\int_0^4 dt} = \dfrac{(t^3 - 3t^2)_0^4}{(t)_0^4} = 4$ m/s

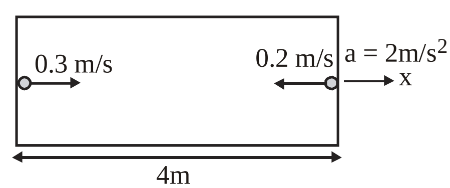
Average speed

$$= \frac{\int |v|\,dt}{\int dt} = \frac{\int_0^4 |3t^2 - 6t|\,dt}{\int_0^4 dt} = \frac{\int_0^2 (6t - 3t^2)\,dt + \int_2^4 (3t^2 - 6t)\,dt}{\int_0^4 dt}$$

$$= \frac{(3t^2 - t^3)_0^2 + (t^3 - 3t^2)_2^4}{(t)_0^4} = \frac{24}{4} = 6 \text{ m/s}$$

Example 18 :

A rocket is moving in a gravity free space with a constant acceleration of 2 m/s^2 along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is –



Sol. **8.** $S_1 = 0.2t + \frac{1}{2} \times 2 \times t^2$

$S_2 = 0.3t - \frac{1}{2} \times 2 \times t^2$

$S_1 + S_2 = 4$; $0.5t = 4$; $t = 8$

QUESTION BANK

CHAPTER 3 : MOTION IN ONE DIMENSION

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : POSITION, PATH LENGTH AND DISPLACEMENT

Q.1 The numerical ratio of distance to displacement is
(A) always equal to one
(B) always less than one
(C) always greater than one
(D) equal to or more than one

Q.2 An athlete is running on a circular track of radius 50 meter. Calculate the displacement (in m) of the athlete after completing 5 rounds of the track.
(A) 0 (B) 50
(C) 100 (D) 75

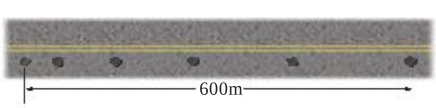
Q.3 A monkey is moving on circular path of radius 80m. Calculate the distance covered by the monkey in one round.
(A) 160.00 m (B) 542.40m
(C) 502.40m (D) 602.40m

Q.4 Which of the following statements is incorrect?
(A) Displacement is independent of the choice of origin of the axis.
(B) Displacement may or may not be equal to the distance travelled.
(C) When a particle returns to its starting point, its displacement is not zero.
(D) Displacement does not tell the nature of the actual motion of a particle between the points.

Q.5 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Determine how long the drunkard takes to fall in a pit 13 m away from the start.
(A) 29s (B) 32s
(C) 37s (D) 24s

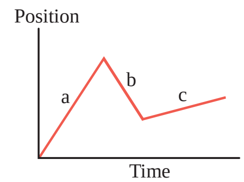
PART - 2 : AVERAGE VELOCITY AND AVERAGE SPEED

Q.6 One drop of oil falls straight down onto the road from the engine of a moving car every 5 s. Figure shows the pattern of the drops left behind on the pavement. What is the average speed of the car over this section of its motion?



(A) 20 m/s (B) 24 m/s
(C) 30 m/s (D) 100 m/s

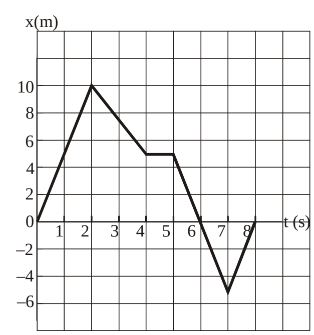
Q.7 The graph accompanying this problem shows a three-part motion. For each of the three parts, a, b, and c, identify the direction of the motion. A positive velocity denotes motion to the right.



(A) a right, b left, c right (B) a right, b right, c left
(C) a right, b left, c left (D) a left, b right, c left

Q.8 A jogger runs along a straight and level road for a distance of 8.0 km and then runs back to her starting point. The time for this round-trip is 2.0h. Which one of the following statements is true?
(A) Her average speed is 8.0 km/h, but there is not enough information to determine her average velocity.
(B) Her average speed is 8.0 km/h, and her average velocity is 8.0 km/h.
(C) Her average speed is 8.0 km/h, and her average velocity is 0 km/h.
(D) None of these

For Q.9-Q.13
The position versus time for a certain particle moving along the x axis is shown in Figure.



Q.9 Find the average velocity in the time intervals 0 to 2 s.
(A) 3 m/s (B) 4 m/s
(C) 5 m/s (D) 2 m/s

Q.10 Find the average velocity in the time intervals 0 to 4s.
(A) 1.2 m/s (B) 3.2 m/s
(C) 4.2 m/s (D) 5.2 m/s

Q.11 Find the average velocity in the time intervals 2 s to 4 s.
(A) – 0.5 m/s (B) – 1.5 m/s
(C) – 2.5 m/s (D) – 3.5 m/s

Q.12 Find the average velocity in the time intervals 4 s to 7 s
(A) – 1.3 m/s (B) – 2.5 m/s
(C) – 6.1 m/s (D) – 3.3 m/s

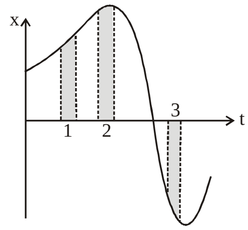
Q.13 Find the average velocity in the time intervals 0 to 8 s.
(A) 1 m/s (B) 0 m/s
(C) 5 m/s (D) 2 m/s

Q.14 A bicyclist is travelling along a straight road for the first half time with speed v_1 and for second half time with speed v_2. What is the average speed of the bicyclist?

(A) $\frac{v_1 + v_2}{2}$ (B) $\frac{v_1 - v_2}{2}$

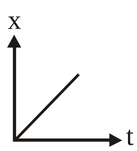
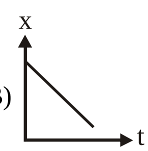
(C) $\frac{2v_1v_2}{v_1 + v_2}$ (D) None of these

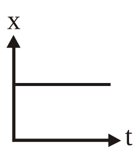
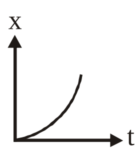
Q.15 Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown.



Choose the correct statement –
(A) Average speed is greatest in interval 3.
(B) Average speed is least in interval 2.
(C) Average speed is greatest in interval 1.
(D) Both (A) and (B)

Q.16 Which of the following graphs represents the position time graph of a particle moving with negative velocity?

(A)  (B) 

(C)  (D) 

Q.17 The area under velocity-time graph for a particle in a given interval of time represents
(A) velocity (B) acceleration
(C) work done (D) displacement

Q.18 A table clock has its minute hand 4 cm long. Choose the correct statement
(A) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 a.m. is 4.4×10^{-3} cm/s
(B) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 p.m is 1.8×10^{-4} cm/s
(C) Average velocity of the tip of the minute hand in between 6 a.m. to 6.30 p.m is 4.4×10^{-4} cm/s
(D) Both (A) and (B)

Q.19 Which of the following changes when a particle is moving with uniform velocity?
(A) Position (B) Speed
(C) Velocity (D) Acceleration

For Q.20-Q.21

The position of an object moving along x-axis is given by $x = a + bt^2$, where a = 8.5 m and b = 2.5 m s^{-2} and t is measured in seconds.

Q.20 The average velocity of the object between t = 2 s and t = 4 s is
(A) 5 m s^{-1} (B) 10 m s^{-1}
(C) 15 m s^{-1} (D) 20 m s^{-1}

Q.21 The velocity of the object at t = 2s is
(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s

Q.22 A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is

(A) $\frac{v_1 + v_2}{2}$ (B) $\frac{2v_1 + v_2}{v_1 + v_2}$

(C) $\frac{2v_1v_2}{v_1 + v_2}$ (D) $\frac{L(v_1 + v_2)}{v_1v_2}$

For Q.23-Q.24

A particle moves according to the equation $x = 10t^2$ where x is in meters and t is in seconds.

Q.23 Find the average velocity for the time interval from 2.00s to 3.00 s.
(A) 50.0 m/s (B) 31.0 m/s
(C) 41.0 m/s (D) 20.0 m/s

Q.24 Find the average velocity for the time interval from 2.00 to 2.10 s.
(A) 50.0 m/s (B) 31.0 m/s
(C) 41.0 m/s (D) 20.0 m/s

Q.25 A cyclist moving on a circular track of radius 40m completes half a revolution in 40 s. His average velocity is
(A) zero (B) 4π m/s
(C) 2 m/s (D) 8π m/s

For Q.26-Q.27

A person walks first at a constant speed of 5m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s.

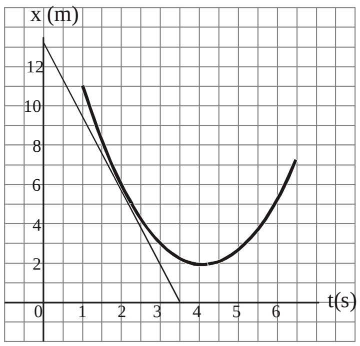
Q.26 What is her average speed over the entire trip?
(A) 1.25 m/s (B) 3.75 m/s
(C) 4.15 m/s (D) 5.75 m/s

Q.27 What is her average velocity over the entire trip?
(A) 0 m/s (B) 1 m/s
(C) 2 m/s (D) 3 m/s

PART - 3 : INSTANTANEOUS VELOCITY AND SPEED

For Q.28-Q.30

A position-time graph for a particle moving along the x axis is shown in figure.



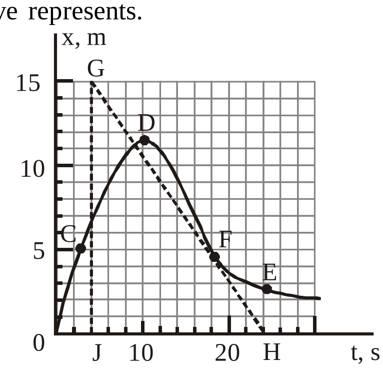
Q.28 Find the average velocity in the time interval t = 1.50 s to t = 4.00 s.
(A) – 1.2 m/s (B) – 2.4 m/s
(C) – 3.8 m/s (D) – 4.2 m/s

Q.29 Determine the instantaneous velocity at t=2.00s by measuring the slope of the tangent line shown in the graph.
(A) – 1.2 m/s (B) – 2.4 m/s
(C) – 3.8 m/s (D) – 4.2 m/s

Q.30 At what value of t is the velocity zero?
(A) 4s (B) 2s
(C) 6s (D) 8s

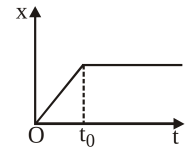
Q.31 A particle moves with uniform velocity. Which of the following statements about the motion of the particle is true?
(A) Its speed is zero.
(B) Its acceleration is zero.
(C) Its acceleration is opposite to the velocity.
(D) Its speed may be variable.

Q.32 With the help of given fig. find the instantaneous velocity at point F for the object whose motion the curve represents.



(A) – 0.25 m/s (B) – 0.5 m/s
(C) – 0.75 m/s (D) – 0.1 m/s

Q.33 Figure shows the displacement (x)-time (t) graph of the particle moving on the x-axis.



(A) The particle is at rest.
(B) The particle is continuously going along x-direction.
(C) The velocity of the particle increases upto time t_0 and then becomes constant.
(D) The particle moves at a constant velocity up to a time t_0 and then stops.

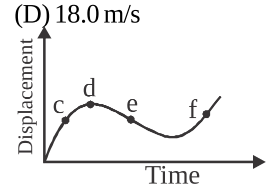
For Q.34-Q.35

The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds.

Q.34 Evaluate its position at t = 3 s
(A) 7.0 m (B) 17.0 m
(C) 27.0 m (D) 20.0 m

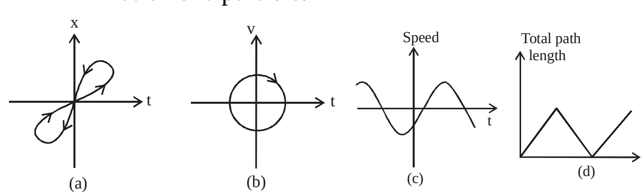
Q.35 Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero, to find the velocity at t = 3 s.
(A) 7.0 m/s (B) 20.0 m/s
(C) 27.0 m/s (D) 18.0 m/s

Q.36 The displacement-time graph of a moving particle is as shown in the figure. The instantaneous velocity of the particle is negative at the point



(A) c (B) e
(C) d (D) f

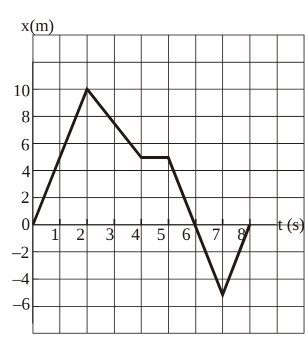
Q.37 Look at the graphs (a) to (d) (Fig.) carefully which of these cannot possibly represent one-dimensional motion of a particle.



(A) a, b (B) c, d
(C) a, b, c (D) all of these

For Q.38-Q.40

Find the instantaneous velocity of the particle described in figure at the following times:



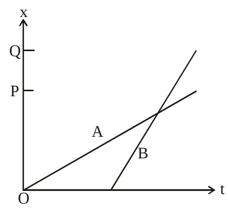
Q.38 t = 1.0 s,
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.39 t = 3.0 s,
(A) –1.5 m/s (B) –3.5 m/s
(C) –2.5 m/s (D) –4.5 m/s

Q.40 t = 4.5 s
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.41 t = 7.5 s.
(A) 5 m/s (B) 3 m/s
(C) 0 m/s (D) 4 m/s

Q.42 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig.



Choose the INCORRECT statement –
(A) A lives closer to the school than B.
(B) A starts from the school earlier than B.
(C) A walks faster than B.
(D) A and B reach home at the same time.

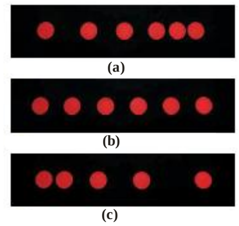
PART - 4 : ACCELERATION

Q.43 The velocity of a train is 80.0 km/h, due west. One and a half hours later its velocity is 65.0 km/h, due west. What is the train's average acceleration?
(A) 10.0 km/h^2, due west (B) 43.3 km/h^2, due west
(C) 10.0 km/h^2, due east (D) 43.3 km/h^2, due east

Q.44 When the pilot reverses the propeller in a boat moving north, the boat moves with an acceleration directed south. Assume the acceleration of the boat remains constant in magnitude and direction. What happens to the boat?
(A) It eventually stops and remains stopped.
(B) It eventually stops and then speeds up in the forward direction.
(C) It eventually stops and then speeds up in the reverse direction.
(D) It never stops but loses speed more and more slowly forever.

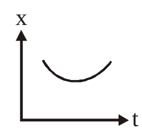
Q.45 As an object moves along the x axis, many measurements are made of its position, enough to generate a smooth, accurate graph of x versus t. Which of the following quantities for the object cannot be obtained from this graph alone?
(A) the velocity at any instant.
(B) the acceleration at any instant.
(C) the displacement during some time interval
(D) the average velocity during some time interval

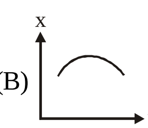
Q.46 Each of the strobe photographs (a), (b), and (c) in Figure was taken of a single disk moving toward the right, which we take as the positive direction. Within each photograph, the time interval between images is constant.

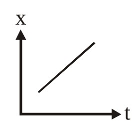


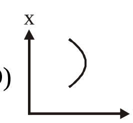
Choose the correct option –
(A)Photograph (b) shows motion with zero acceleration.
(B)Photograph(c) shows motion with positive acceleration.
(C) Photograph (a) shows motion with negative acceleration.
(D) All of these

Q.47 Position-time graph for motion with zero acceleration is

(A) 

(B) 

(C) 

(D) 

Q.48 An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acceleration ?
(A) 1.5 m/s^2 (B) 2.5 m/s^2
(C) 3.5 m/s^2 (D) 0.5 m/s^2

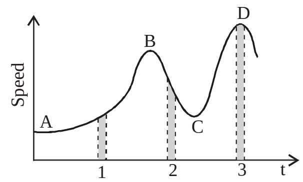
Q.49 A car starts from rest and acquires velocity equal to 10 m/s after 5 sec. Find the acceleration of the car.
(A) 1.5 m/s^2 (B) 2.5 m/s^2
(C) 3.5 m/s^2 (D) 2.0 m/s^2

Q.50 The position x of a particle varies with time 't' as $x = at^2 – bt^3$. When will the acceleration of the particle become zero?
(A) t = a/3b (B) t = a/2b
(C) t = a/b (D) t = 2a/b

Q.51 A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval?
(Note: 1 ms = $10^{–3}$ s.)
(A) 0.34×10^4 m/s^2 (B) 1.34×10^6 m/s^2
(C) 2.17×10^4 m/s^2 (D) 1.34×10^4 m/s^2

Q.52 The area under acceleration-time graph represents the
(A) initial velocity (B) final velocity
(C) change in velocity (D) distance travelled

Q.53 Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown.

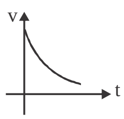


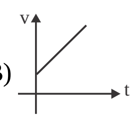
Choose the correct statement –
(a) Average acceleration is greatest in interval 2
(b) Average speed is greatest in interval 2
(c) Velocity is positive only in interval 3
(d) Acceleration is positive in intervals 1 and 3 and negative in interval 2

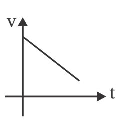
(A) a, b (B) c, d
(C) b, c (D) a, d

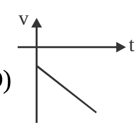
Q.54 The slope of the tangent drawn on velocity-time graph at any instant of time is equal to the instantaneous
(A) acceleration (B) velocity
(C) impulse (D) momentum

Q.55 Given below are four curves describing variation of velocity with time of a particle. Which one of these describe the motion of a particle initially in positive direction with constant negative acceleration?

(A) 

(B) 

(C) 

(D) 

PART - 5 : KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Q.56 In which one of the following situations can the equations of kinematics not be used?
(A) When the velocity changes from moment to moment.
(B) When the velocity remains constant.
(C) When the acceleration changes from moment to moment.
(D) When the acceleration remains constant.

Q.57 In a race two horses, Silver Bullet and Shotgun, start from rest and each maintains a constant acceleration. In the same elapsed time Silver Bullet runs 1.20 times farther than Shotgun. According to the equations of kinematics, which one of the following is true concerning the accelerations of the horses?
(A) $a_{\text{Silver Bullet}} = 1.44\, a_{\text{Shotgun}}$
(B) $a_{\text{Silver Bullet}} = a_{\text{Shotgun}}$
(C) $a_{\text{Silver Bullet}} = 2.40\, a_{\text{Shotgun}}$
(D) $a_{\text{Silver Bullet}} = 1.20\, a_{\text{Shotgun}}$

Q.58 A skateboarder starts from rest and moves down a hill with constant acceleration in a straight line, traveling for 6 s. In a second trial, he starts from rest and moves along the same straight line with the same acceleration for only 2 s. How does his displacement from his starting point in this second trial compare with that from the first trial?
(A) one-third as large (B) three times larger
(C) one-ninth as large (D) nine times larger

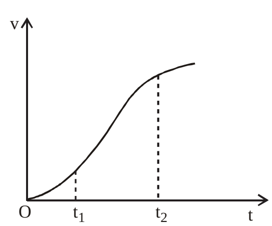
Q.59 A racing car starts from rest at t = 0 and reaches a final speed v at time t. If the acceleration of the car is constant during this time, which of the following statements are true?
(a) The car travels a distance vt.
(b) The average speed of the car is v/2.
(c) The magnitude of the acceleration of the car is v/t.
(d) The velocity of the car remains constant.
(A) a, b (B) b, c
(C) a, d (D) c, d

Q.60 The velocity of a particle (moving with uniform acceleration) at an instant is 10m/s. After 3s its velocity will becomes 16 m/s. The velocity at 2s, before the given instant will be
(A) 6 m/s (B) 4 m/s
(C) 2 m/s (D) 1 m/s

Q.61 A particle starts moving from the position of rest under a constant acc. If it covers a distance x in t sec, what distance will it travel in next t sec?
(A) y = 3x (B) y = x
(C) y = 2x (D) y = 4x

Q.62 Which of the following statements is not correct?
(A) The zero velocity of a body at any instant does not necessarily imply zero acceleration at that instant.
(B) The kinematic equation of motions are true only for motion in which the magnitude and the direction of acceleration are constants during the course of motion.
(C) The sign of acceleration tells us whether the particle's speed is increasing or decreasing.
(D) All of these

Q.63 The velocity-time graph of a particle in one-dimensional motion is shown in figure :



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2:
(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (½)a(t_2 - t_1)^2$
(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$
(c) $v_{average} = (x(t_2) - x(t_1))/(t_2 - t_1)$
(d) $a_{average} = (v(t_2) - v(t_1))/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + (1/2)\, a_{average}(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the v-t curve bounded by the t-axis and the dotted line shown.

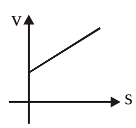
(A) (c), (d) and (f) (B) (a), (b) and (e)
(C) (b), (c) and (d) (D) (d), (e) and (f)

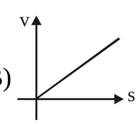
Q.64 Stopping distance of a moving vehicle is directly proportional to (Assume uniform retardation)
(A) square of the initial velocity
(B) square of the initial acceleration
(C) the initial velocity
(D) the initial acceleration

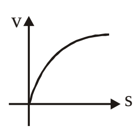
Q.65 A car moving along a straight road with speed 144 km h^{-1} is brought to a stop within a distance 200m. How long does it take for the car to stop?
(A) 5s (B) 10s
(C) 15s (D) 20s

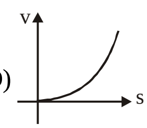
Q.66 Which of the following equations does not represent the kinematic equations of motion?
(A) $v = u + at$ (B) $S = ut + ½ at^2$
(C) $S = vt + ½ at^2$ (D) $v^2 - u^2 = 2aS$
where, u = initial velocity of a body
v = final velocity of the body
a = uniform acceleration of the body
S = distance travelled by the body in time t

Q.67 A body starting from rest moves along a straight line with a constant acceleration. The variation of speed (v) with distance (s) is given by

(A) 

(B) 

(C) 

(D) 

Q.68 A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.
(A) 50 m/s^2 (B) 15 m/s^2
(C) 30 m/s^2 (D) 40 m/s^2

Q.69 A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping ?
(A) 12m (B) 18m
(C) 16m (D) 24m

For Q.70-Q.72
A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2.

Q.70 Find the distance travelled by it in the first two seconds.
(A) 2.0 m (B) 4.0 m
(C) 6.0 m (D) 8.0 m

Q.71 How much time does it take to reach the velocity 7.5 m/s?
(A) 2s (B) 5s
(C) 7s (D) 10s

Q.72 How much distance will it cover in reaching the velocity 7.5 m/s?
(A) 40 m (B) 50 m
(C) 30 m (D) 20 m

Q.73 A particle starts from rest with constant acceleration = 2m/s^2. Find displacement in 5^{th} sec.
(A) 9 m (B) 18 m
(C) 25 m (D) 20 m

PART - 6 : MOTION UNDER GRAVITY

Q.74 A juggler throws a bowling pin straight up in the air. After the pin leaves his hand and while it is in the air, which statement is true?
(A) The velocity of the pin is always in the same direction as its acceleration.
(B) The velocity of the pin is never in the same direction as its acceleration.
(C) The acceleration of the pin is zero.
(D) The velocity of the pin is opposite its acceleration on the way up.

Q.75 A rocket is sitting on the launch pad. The engines ignite, and the rocket begins to rise straight upward, picking up speed as it goes. At about 1000 m above the ground the engines shut down, but the rocket continues straight upward, losing speed as it goes. It reaches the top of its flight path and then falls back to earth. Ignoring air resistance, decide which one of the following statements is true.
(A) All of the rocket's motion, from the moment the engines ignite until just before the rocket lands, is free-fall.
(B) Only part of the rocket's motion, from just after the engines shut down until just before it lands, is free-fall.
(C) Only the rocket's motion while the engines are firing is free-fall.
(D) Only the rocket's motion from the top of its flight path until just before landing is free-fall.

Q.76 The top of a cliff is located a distance H above the ground. At a distance H/2 there is a branch that juts out from the side of the cliff, and on this branch a bird's nest is located. Two children throw stones at the nest with the same initial speed, one stone straight downward from the top of the cliff and the other stone straight upward from the ground. In the absence of air resistance, which stone hits the nest in the least amount of time?
(A) There is insufficient information for an answer.
(B) Both stones hit the nest in the same amount of time.
(C) The stone thrown from the ground.
(D) The stone thrown from the top of the cliff.

Q.77 A rock is thrown downward from the top of a 40.0-m-tall tower with an initial speed of 12m/s. Assuming negligible air resistance, what is the speed of the rock just before hitting the ground?
(A) 28 m/s (B) 30 m/s
(C) 56 m/s (D) 784 m/s

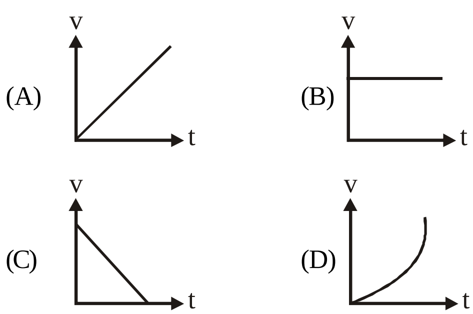
Q.78 On another planet, a marble is released from rest at the top of a high cliff. It falls 4.00 m in the first 1 s of its motion. Through what additional distance does it fall in the next 1 s?
(A) 4.00 m (B) 8.00 m
(C) 12.0 m (D) 16.0 m

Q.79 A pebble is dropped from rest from the top of a tall cliff and falls 4.9 m after 1.0 s has elapsed. How much farther does it drop in the next 2.0 s?
(A) 9.8 m (B) 19.6 m
(C) 39 m (D) 44 m

Q.80 A cannon shell is fired straight up from the ground at an initial speed of 225 m/s. After how much time is the shell at a height of 6.20×10^2 m above the ground and moving downward?
(A) 2.96 s (B) 17.3 s
(C) 25.4 s (D) 43.0 s

Q.81 A player throws a ball vertically upwards with velocity u. At highest point,
(A) both the velocity and acceleration of the ball are zero.
(B) the velocity of the ball is u but its acceleration zero.
(C) the velocity of the ball is zero but its acceleration g.
(D) the velocity of the ball is u but its acceleration g.

Q.82 Which of the following graphs represents the velocity-time variation of an object falls freely under gravity?



Q.83 A girl standing on a stationary lift (open from above) throws a ball upwards with initial speed 50 m/s.The time taken by the ball to return to her hands is
(Take $g = 10\ m\ s^{-2}$)
(A) 5 s (B) 10 s
(C) 15 s (D) 20 s

Q.84 A body falling freely under gravity passes two points 30 m apart in 1 s. From what point above the upper point it began to fall?
(A) 32.1 m (B) 16.0 m
(C) 8.6 m (D) 4.0 m

Q.85 Free fall of an object in vacuum is a case of motion with
(A) uniform velocity (B) uniform acceleration
(C) variable acceleration (D) uniform speed

Q.86 The distances traversed during equal intervals of time by a body falling from rest stand to one another in the same ratio as the odd numbers beginning with unity that is, 1 : 3 : 5 : 7 : . This law was established
(A) Galileo Galilei (B) Isaac Newton
(C) Johannes Kepler (D) Albert Einstein

Q.87 A player throws a ball upwards with an initial speed of 30 m/s. How long does the ball take to return to the player's hands? (Take $g = 10\ m\ s^{-2}$)
(A) 3s (B) 6s
(C) 9s (D) 12s

Q.88 A ball is dropped from height 'h' in the last second it travels $\frac{9h}{25}$. Find h.
(A) (25/2) g (B) (15/2) g
(C) (5/2) g (D) (35/2) g

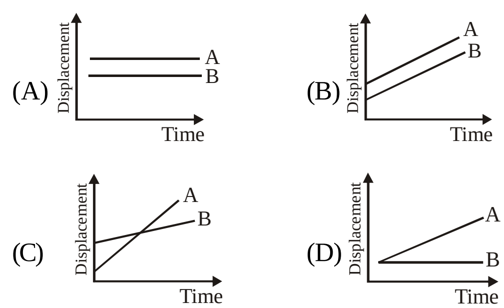
Q.89 A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of –
(A) 3s (B) 5s
(C) 7s (D) 9s

Q.90 Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant
(A) 2.50 m (B) 3.75 m
(C) 4.00 m (D) 1.25 m

Q.91 A stone is shot straight upward with a speed of 20m/sec from a tower 200 m high. The speed with which it strikes the ground is approximately–
(A) 60 m/sec (B) 65 m/sec
(C) 70 m/sec (D) 75 m/sec

PART - 7 : RELATIVE VELOCITY

Q.92 Which one of the following represents displacement time graph of two objects A and B moving with zero relative velocity?



For Q.93-Q.94
Two cars A and B are running at velocities of $60\ km\ h^{-1}$ and $45\ km\ h^{-1}$.

Q.93 What is the relative velocity of car A with respect to car B, if both are moving eastward?
(A) $15\ km\ h^{-1}$ (B) $45\ km\ h^{-1}$
(C) $60\ km\ h^{-1}$ (D) $105\ km\ h^{-1}$

Q.94 What is the relative velocity of a car A with respect to car B, if car A is moving eastward and car B is moving westward?
(A) 15 km h^{-1} (B) 45 km h^{-1}
(C) 60 km h^{-1} (D) 105 km h^{-1}

Q.95 A jet airplane travelling at the speed of 500km/h ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the combustion with respect to an observer on the ground ?
(A) – 500 km h^{-1}. (B) – 1000 km h^{-1}.
(C) – 1500 km h^{-1}. (D) – 2000 km h^{-1}.

For Q.96-Q.98
Two parallel rail tracks run north-south. On one track train A moves north with a speed of 54 km/h and on the other track train B moves south with a speed of 90km/h.

Q.96 The velocity of train A with respect to train B is
(A) 10 m/s (B) 15 m/s
(C) 25 m/s (D) 40 m/s

Q.97 What is the velocity of a monkey running on the roof of the train A against its motion with a velocity of 18 km/h with respect to the train A as observed by a man standing on the ground?
(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s

Q.98 On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?
(A) 1 ms^{-2} (B) 2 ms^{-2}
(C) 3 ms^{-2} (D) 4 ms^{-2}

EXERCISE - 2 [LEVEL-2]

Choose one correct response for each question.

Q.1 The average velocity of a particle moving with constant acceleration a and initial velocity u in a straight line in first t seconds is
(A) $u+\frac{1}{2}at$ (B) $\frac{u}{2}$
(C) u + at (D) $\frac{u+at}{2}$

Q.2 The velocity of any particle is related with its displacement As; $x=\sqrt{v+1}$, calculate acceleration at x = 5 cm.
(A) 140 m/s^2 (B) 240 m/s^2
(C) 40 m/s^2 (D) 340 m/s^2

Q.3 The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is
(A) increasing with time (B) decreasing with time
(C) constant but not zero (D) zero

Q.4 The velocity of the particle at any time t is given by v = 2t (3 – t) m/s. At what time is its velocity maximum?
(A) 2 s (B) 3 s
(C) (2/3) s (D) (3/2) s

Q.5 Which of the following statements is not correct regarding the motion of a particle in a straight line?
(A) x-t graph is a parabola, if motion is uniformly accelerated.
(B) v-t is a straight line inclined to the time axis, if motion is uniformly accelerated.
(C) x-t graph is a straight line inclined to the time axis if motion is uniform and acceleration is zero.
(D) v-t graph is a parabola if motion is uniform and acceleration is zero.

Q.6 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20km/h in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min. in the opposite direction. What is the time period T of the bus service. Assume buses ply on the road with constant speed.
(A) 5 mins (B) 9 mins
(C) 18 mins (D) 27 mins

Q.7 A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has speed of 27 km/h while the other has the speed of 18 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h when the two cars were separated by 36 km. The total distance covered by the bird is –
(A) 28.8 km (B) 38.8 km
(C) 48.8 km (D) 58.8 km

Q.8 It is a common observation that rain clouds can be at about 1 km altitude above the ground. If a rain drop falls from such a height freely under gravity, then what will be its speed in km h^{-1}?
(Take g = 10 m s^{-2})
(A) 510 (B) 610
(C) 710 (D) 910

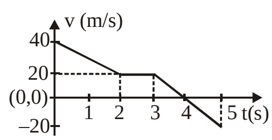
Q.9 In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.
(A) The displacement in time T must always take non-negative values.
(B) The displacement x in time T satisfies $-v_0T < x < v_0T$.
(C) The acceleration is always a non-negative number.
(D) The motion has no turning points.

Q.10 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car ? (Obtain that speed which is relevant for damaging the thief's car).
(A) 125 m/s (B) 160 m/s
(C) 95 m/s (D) 105 m/s

Q.11 A boy walks on a straight road from his home to a market 2.5 km with a speed of 5 km h^{-1}. Finding the market closed he instantly turns and walks back with a speed of 7.5 km h^{-1}. What is the average speed and average velocity of the boy between t = 0 to t = 50 min?
(A) 0, 0 (B) 6 km h^{-1}, 0
(C) 0, 6 km h^{-1} (D) 6 km h^{-1}, 6 km/h

Q.12 A particle moving with uniform acceleration has average velocities v_1, v_2 and v_3 over the successive intervals of time t_1, t_2 and t_3 respectively. The value of $\frac{v_1 - v_2}{v_2 - v_3}$ will be –
(A) $\frac{t_1 - t_2}{t_2 - t_3}$ (B) $\frac{t_1 - t_2}{t_2 + t_3}$
(C) $\frac{t_1 + t_2}{t_2 - t_3}$ (D) $\frac{t_1 + t_2}{t_2 + t_3}$

Q.13 An auto travelling along a straight road increases its speed from 30.0 m/s to 50.0 m/s in a distance of 180 m. If the acceleration is constant, how much time elapses while the auto moves this distance?
(A) 6.0 s (B) 4.5 s
(C) 3.6 s (B) 7.0 s

Q.14 In the given v-t graph the distance travelled by the body in 5 sec. will be


(A) 100 m (B) 80 m
(C) 40 m (D) 20 m

Q.15 Which of the following statements may be correct?
(i) Average velocity is path length divided by time interval.
(ii) In general, speed is greater than the magnitude of the velocity.
(iii) A particle moving in a given direction with a nonzero velocity can have zero speed.
(iv) The magnitude of average velocity is the average speed.
(A) (ii) and (iii) (B) (ii) and (iv)
(C) (i), (iii) and (iv) (D) (iv)

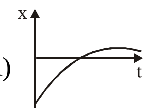
Q.16 For the one-dimensional motion, described by $x = t - \sin t$
(A) x (t) > 0 for all t > 0 (B) v (t) > 0 for all t > 0
(C) a (t) > 0 for all t > 0 (D) all of these

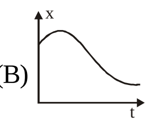
Q.17 A body A starts from rest with an acceleration a_1. After 2 seconds, another body B starts from rest with an acceleration a_2. If they travel equal distances in the 5^{th} second, after the start of A, then the ratio $a_1 : a_2$ is equal to
(A) 5 : 9 (B) 5 : 7
(C) 9 : 5 (D) 9 : 7

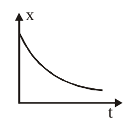
Q.18 A bus is moving with a speed of 10 m/s on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist with what speed should the scooterist chase the bus?
(A) 40 m/s (B) 25 m/s
(C) 10 m/s (D) 20 m/s

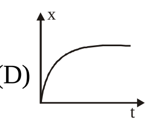
Q.19 A ball A is thrown vertically upwards with speed u. At the same instant another ball B is released from rest at height h. At time t, the speed of A relative to B is
(A) u (B) u – 2gt
(C) $\sqrt{u^2 - 2gh}$ (D) u – gt

Q.20 Among the four graphs, there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?

(A)


(B)


(C)


(D)


Q.21 At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be
(A) $\frac{t_1 + t_2}{2}$ (B) $\frac{t_1 t_2}{t_2 - t_1}$
(C) $\frac{t_1 t_2}{t_2 + t_1}$ (D) $t_1 - t_2$

For Q.22-Q.23
A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building of 25 m high. (Take g = 10 m s^{-2})

Q.22 How high will the ball rise?
(A) 10 m (B) 15 m
(C) 20 m (D) 25 m

Q.23 Time taken by the ball to reach the ground is
(A) 2s (B) 3s
(C) 5s (D) 4s

Q.24 A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 second is
(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$
(C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(n+1)}{n}$

Q.25 An object falling through a fluid is observed to have acceleration given by a = g – bv where g = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. The value of constant speed is
(A) g/b (B) b/g
(C) bg (D) b

Q.26 A body covers a distance of 4 m in 3^{rd} second and 12m in 5^{th} second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds?
(A) 10 m (B) 30 m
(C) 40 m (D) 60 m

Q.27 A ball A is dropped from a building of height 45m. Simultaneously another identical ball B is thrown up with a speed 50 m/s. The relative speed of ball B w.r.t ball A at any instant of time is (Take $g = 10\ m/s^2$)
(A) 0 m/s (B) 10 m/s
(C) 25 m/s (D) 50 m/s

Q.28 Two cars A and B are travelling in the same direction with velocities v_1 and v_2 ($v_1 > v_2$). When the car A is at a distance d ahead of the car B, the driver of the car A applied the brake producing a uniform retardation a. There will be no collision when

(A) $d < \frac{(v_1 - v_2)^2}{2a}$ (B) $d < \frac{v_1^2 - v_2^2}{2a}$

(C) $d > \frac{(v_1 - v_2)^2}{2a}$ (D) $d > \frac{v_1^2 - v_2^2}{2a}$

Q.29 The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in metres and t in sec. The displacement, when velocity is zero, is
(A) 24 metres (B) 12 metres
(C) 5 metres (D) Zero

Q.30 If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will
(A) Zero (B) 8 m/s^2
(C) – 8 m/s^2 (D) 4 m/s^2

Q.31 If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is

(A) $\frac{1}{2}\sqrt{v_1 v_2}$ (B) $\frac{v_1 + v_2}{2}$

(C) $\frac{2v_1 v_2}{v_1 + v_2}$ (D) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

Q.32 A ball is projected upwards from a height h above the surface of the earth with velocity v. The time at which the ball strikes the ground is

(A) $\frac{v}{g} + \frac{2hg}{\sqrt{2}}$ (B) $\frac{v}{g}\left[1 - \sqrt{1 + \frac{2h}{g}}\right]$

(C) $\frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$ (D) $\frac{v}{g}\left[1 + \sqrt{v^2 + \frac{2g}{h}}\right]$

Q.33 A man throws ball with the same speed vertically upwards one after the other at an interval of 2seconds. What should be the speed of the throw so that more than two balls are in the sky at any time?
(Given $g = 9.8\ m/s^2$)
(A) More than 19.6 m/s
(B) At least 9.8 m/s
(C) Any speed less than 19.6 m/s
(D) Only with speed 19.6 m/s

Q.34 A particle moves in a straight line with a constant acceleration. It changes its velocity from 10m/s to 20 m/s while passing through a distance 135m in t second. The value of t is –
(A) 12 (B) 9
(C) 10 (D) 1.8

Q.35 Balls A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively ($g = 10 m/s^2$). Find separation between them after one second.
(A) 2m (B) 3m
(C) 5m (D) 6m

Q.36 An electron starting from rest has a velocity that increases linearly with the time that is v = kt, where $k = 2 m/sec^2$. The distance travelled in the first 3seconds will be
(A) 9m (B) 16 m
(C) 27m (D) 36m

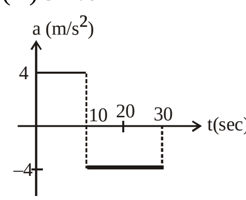
Q.37 The initial velocity of a body moving along a straight line is 7m/s. It has a uniform acceleration of 4 m/s^2. The distance covered by the body in the 5^{th} second of its motion is –
(A) 25 m (B) 35 m
(C) 50 m (D) 85 m

Q.38 If a body starts from rest and travels 120 cm in the 6^{th} second, then what is the acceleration
(A) 0.20 m/s^2 (B) 0.027 m/s^2
(C) 0.218 m/s^2 (D) 0.03 m/s^2

Q.39 A particle starts from rest, accelerates at 2 m/s^2 for 10s and then goes for constant speed for 30s and then decelerates at 4 m/s^2 till it stops. What is the distance travelled by it
(A) 750 m (B) 800 m
(C) 700 m (D) 850 m

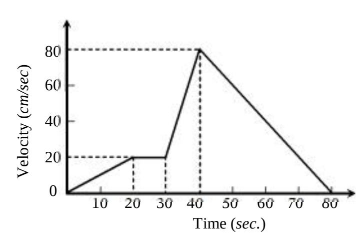
Q.40 A particle moves along X-axis in such a way that its coordinate X varies with time t according to the equation $x = (2 - 5t + 6t^2)$ m. The initial velocity of the particle is
(A) – 5m/s (B) 6 m/s
(C) – 3m/s (D) 3 m/s

Q.41 The acceleration versus time graph for a particle moving along a straight line is shown in the figure. If the particle starts from rest at t = 0, then its speed at t = 30 sec. will be–



(A) 20m/sec (B) 0 m/sec
(C) – 40 m/sec. (D) 40 m/sec.

Q.42 The v – t graph of a moving object is given in figure. The maximum acceleration is –



(A) 1 cm/sec^2 (B) 2 cm/sec^2
(C) 3 cm/sec^2 (D) 6 cm/sec^2

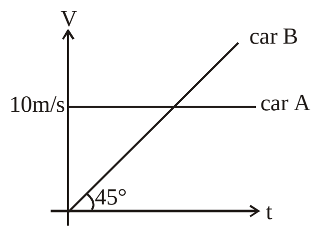
EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Snow is falling vertically at a constant speed of 8.0 m/s. At π/A angle from the vertical the snowflakes appear to tbe falling as viewed by the driver of a car traveling on a straight, level road with a speed of 50 km/h. Find the value of A.

Q.2 A body moves with speed 10 m/s for 10 sec, then with a speed of 20 m/s for distance 300m. Find its average speed (in m/sec).

Q.3 Initially car A is 10.5 m ahead of car B. Both start moving at time t = 0 in the same direction along a straight line. The velocity time graph of two cars is shown in figure. Find the time (in sec) when the car B will catch the car A



Q.4 A particle starts from the origin at t = 0 and moves in the x-y plane with constant acceleration 'a' in the y direction. Its equation of motion is $y = bx^2$. The x-component of its velocity is $\sqrt{\frac{a}{Ab}}$. Find the value of A.

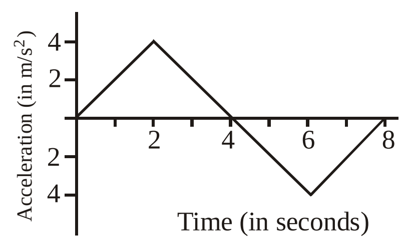
Q.5 A particle is moving on a straight line with a constant retardation of 1 m/s^2. Find the average speed (in m/sec) of particle in the last two meters before it stops.

Q.6 A boat travels upstream in a river and at t = 0 a wooden cork is thrown over the side with zero initial velocity. After 7.5 minutes the boat turns and starts moving downstream catches the cork when it has drifted 1 km downstream. Find the velocity (km/hr) of water current.

Q.7 A particle starts moving rectilinearly at time t = 0 such that its velocity v changes with time t according to the equation
$v = t^2 - t$ where t is in seconds and v is in m/s. The time interval for which the particle retards is $\frac{1}{A} < t < 1$. Find the vlaue of A.

Q.8 The velocity of a particle moving in the direction of x-axis varies as $v = \alpha.x$, where α is a constant. At the moment t=0, the particle was located at x = 0, then find the value of α if the magnitude of average velocity and average acceleration over the above internal is same.

Q.9 A car starts from rest and accelerates as shown in the accompanying diagram.



At what time (in sec.) would the car be moving with the greatest velocity

Q.10 In above question, at what time (in sec.) would the car be farthest from its original starting position.

Q.11 A river is flowing with a velocity of 2m/s. A boat is moving downstream along the river. Velocity of the boat in still water is 3m/s. A person standing on the boat throws a ball (w.r.t. himself) in a plane perpendicular to the direction of motion of the boat with 10m/s at 60° with the horizontal. When the ball reaches highest point of its path. The speed of ball w.r.t. man standing on boat is A m/s

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 From a builiding two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then- **[AIEEE-2002]**
(A) $v_B > v_B$
(B) $v_A = v_B$
(C) $v_A > v_B$
(D) their velocities depends on their masses

Q.2 A body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest- **[AIEEE-2002]**
(A) 1 cm (B) 2 cm
(C) 3 cm (D) 4 cm

Q.3 A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively **[AIEEE-2002]**
(A) g, g (B) g – a, g – a
(C) g – a, g (D) a, g

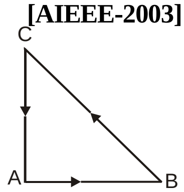
Q.4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is - **[AIEEE-2002]**
(A) 1 : 1 (B) 1 : 4
(C) 1 : 8 (D) 1 : 16

Q.5 A car, moving with a speed of 50 km/hr. can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is- **[AIEEE-2003]**
(A) 24 m (B) 6 m
(C) 12 m (D) 18 m

Q.6 The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by **[AIEEE-2003]**
(A) $3t\sqrt{\alpha^2+\beta^2}$ (B) $3t^2\sqrt{\alpha^2+\beta^2}$
(C) $t^2\sqrt{\alpha^2+\beta^2}$ (D) $\sqrt{\alpha^2+\beta^2}$

Q.7 A body is moved along a straight line by machine delivering a constant power. The distance moved by the body in time t is proportional to - **[AIEEE-2003]**
(A) $t^{3/4}$ (B) $t^{3/2}$
(C) $t^{1/4}$ (D) $t^{1/2}$

Q.8 Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity **[AIEEE-2003]**



(A) less than $\vec{v}$
(B) greater than $\vec{v}$
(C) $|\vec{v}|$ in the direction of largest force BC
(D) $\vec{v}$, remaining unchanged

Q.9 A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[AIEEE-2004]**
(A) x^2 (B) e^x
(C) x (D) $\log_e x$

Q.10 A ball is released from the topof a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T/3 second ? **[AIEEE-2004]**
(A) h/9 meter from the ground
(B) 7h/9 meter from the ground
(C) 8h/9 meter from the ground
(D) 17h/18 meter from the ground

Q.11 An automobile travelling with a speed of 60km/h, can brake to stop within a distance of 20m. If the car is going twice as fast, i.e. 120 km/h, the stopping distance will be **[AIEEE-2004]**
(A) 20 m (B) 40 m
(C) 60 m (D) 80 m

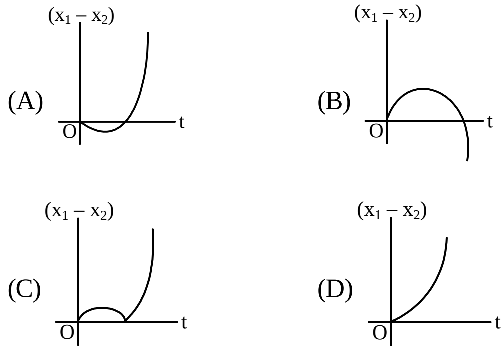
Q.12 The relation between time t and distance x is $t=ax^2 + bx$ where a and b are constants. The acceleration is **[AIEEE-2005]**
(A) $-2av^3$ (B) $2av^2$
(C) $-2av^2$ (D) $2bv^3$

Q.13 A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate f/2 to come to rest. If the total distance traversed is 15 S, then **[AIEEE-2005]**
(A) $S = \frac{1}{72}ft^2$ (B) $S = \frac{1}{4}ft^2$
(C) S = ft (D) $S = \frac{1}{6}ft^2$

Q.14 A particle is moving eastwards with a velocity of 5m/s. In 10s the velocity changes to 5m/s northwards. The average acceleration in this time is **[AIEEE-2005]**
(A) zero
(B) $1/\sqrt{2}$ ms^{-2} towards north-west
(C) $1/\sqrt{2}$ ms^{-2} towards north-east
(D) 1/2 ms^{-2} towards north

Q.15 A particle located at x = 0 at time t = 0, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[AIEEE-2006]**
(A) $t^{1/2}$ (B) t^3
(C) t^2 (D) t

Q.16 A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by x_1 (t) after time t and that of second body by x_2 (t) after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t **[AIEEE-2008]**

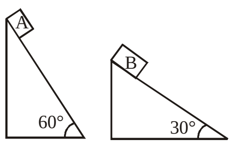


Q.17 A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10s is **[AIEEE-2009]**

(A) 8.5 units (B) 10 units

(C) $7\sqrt{2}$ units (D) 7 units

Q.18 Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are shown in the figure. Two block A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B **[AIEEE 2010]**



(A) 4.9 ms^{-2} in horizontal direction

(B) 9.8 ms^{-2} in vertical direction

(C) zero

(D) 4.9 ms^{-2} in vertical direction

Q.19 An object moving with a speed of 6.25 m/s, is decelerated at a rate given by : $\frac{dv}{dt} = -2.5\sqrt{v}$,

where v is the instananeous speed. The time taken by the object, to come to rest, would be – **[AIEEE 2011]**

(A) 1s (B) 2s

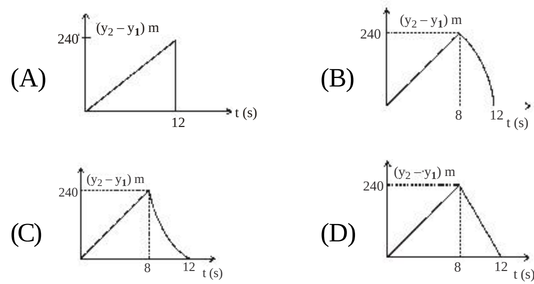
(C) 4s (D) 8s

Q.20 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is – **[JEE MAIN 2014]**

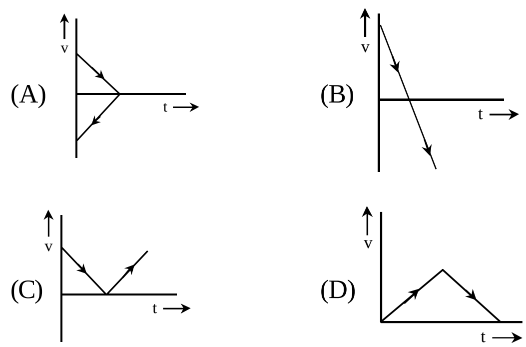
(A) $2gH = nu^2 (n-2)$ (B) $gH = (n-2)\,u^2$

(C) $2gH = n^2u^2$ (D) $gH = (n-2)^2u^2$

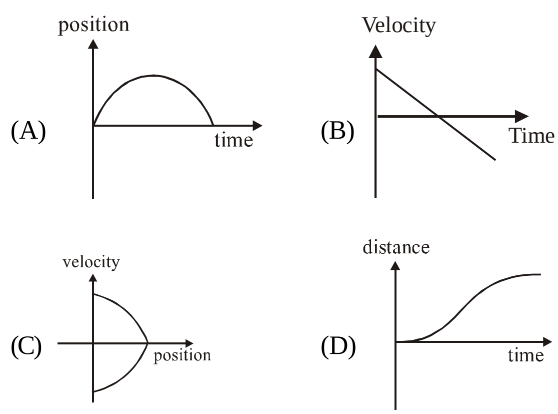
Q.21 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s & 40m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$) (The figures are schematic and not drawn to scale) **[JEE MAIN 2015]**



Q.22 A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE MAIN 2017]**



Q.23 All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE MAIN 2018]**



Q.24 A particle is moving with a velocity $\vec{v} = K\,(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[JEE MAIN 2019 (JAN)]**

(A) xy = constant (B) $y^2 = x^2$ + constant

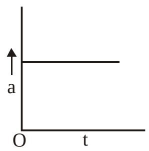
(C) $y = x^2$ + constant (D) y = x + constant

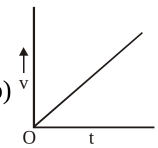
Q.25 Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[JEE MAIN 2019 (APRIL)]**

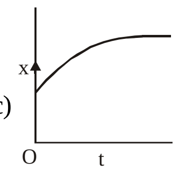
(A) 4.2 hrs. (B) 2.2 hrs.

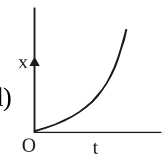
(C) 3.2 hrs. (D) 2.6 hrs.

Q.26 A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively.
(a = acceleration, v = velocity, x = displacement, t = time)
[JEE MAIN 2019 (APRIL)]

(a) 

(b) 

(c) 

(d) 

(A) (a), (b), (c)   (B) (a)
(C) (a), (b), (d)   (D) (b), (c)

Q.27 A particle is moving along the x-axis with its coordinate with the time 't' given be x (t) = 10 + 8t – $3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by y (t) = 5 – $8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v is __________.
[JEE MAIN 2020 (JAN)]

Q.28 A ball is dropped from the top of a 100 m high tower on a planet. In the last (1/2) s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _____
[JEE MAIN 2020 (JAN)]

Q.29 A particle starts from the origin at t = 0 with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m / s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is :
[JEE MAIN 2020 (JAN)]

(A) 50   (B) 32
(C) 60   (D) 40

EXERCISE - 5 (PREVIOUS YEARS AIPMT / NEET EXAM QUESTIONS)

Choose one correct response for each question.

Q.1 A ball is dropped from a high platform at t = 0 starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t = 18 s. What is the value of v? (Take $g = 10\ m/s^2$) **[AIPMT (PRE) 2010]**
(A) 75 m/s (B) 55 m/s
(C) 40 m/s (D) 60 m/s

Q.2 A particle has initial velocity $(3\hat{i}+4\hat{j})$ and has acceleration $(0.4\hat{i}+0.3\hat{j})$. Its speed after 10s is – **[AIPMT (PRE) 2010]**
(A) 7 units (B) $7\sqrt{2}$ units
(C) 8.5 units (D) 10 units

Q.3 A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to **[AIPMT (PRE) 2010]**
(A) $(\text{velocity})^{3/2}$ (B) $(\text{velocity})^2$
(C) $(\text{velocity})^{-2}$ (D) $(\text{velocity})^{2/3}$

Q.4 A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g = 10\ ms^{-2}$, the velocity with which it hits the ground is – **[AIPMT (PRE) 2011]**
(A) 5.0 m/s (B) 10.0 m/s
(C) 20.0 m/s (D) 40.0 m/s

Q.5 A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The magnitude of average acceleration of the body is – **[AIPMT (PRE) 2011]**
(A) $5\ m/s^2$ (B) $1\ m/s^2$
(C) $7\ m/s^2$ (D) $8\ m/s^2$

Q.6 A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is : **[AIPMT (MAINS) 2011]**
(A) $\frac{v_1v_2}{v_1+v_2}$ (B) $\frac{2v_1v_2}{v_1+v_2}$
(C) $\frac{2v_1^2v_2^2}{v_1^2+v_2^2}$ (D) $\frac{v_1+v_2}{2}$

Q.7 A particle has initial velocity $(2\hat{i}+3\hat{j})$ and acceleration $(0.3\hat{i}+0.2\hat{j})$. The magnitude of velocity after 10 seconds will be : **[AIPMT (MAINS) 2011]**
(A) $9\sqrt{2}$ units (B) $5\sqrt{2}$ units
(C) 5 units (D) 9 units

Q.8 The motion of a particle along a straight line is described by equation : $x = 8 + 12t - t^3$, where x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is : **[AIPMT (PRE) 2012]**
(A) $24\ ms^{-2}$ (B) zero
(C) $6\ ms^{-2}$ (D) $12\ ms^{-2}$

Q.9 A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is – **[NEET 2013]**
(A) $h_1 = h_2 = h_3$ (B) $h_1 = 2h_2 = 3h_3$
(C) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ (D) $h_2 = 3h_1$ & $h_3 = 3h_2$

Q.10 A particle is moving such that its position coordinates (x, y) are (2m, 3m) at time t = 0, (6m, 7m) at time t = 2 s and (13m, 14m) at time t = 5 s. Average velocity vector $(\vec{V}_{av})$ from t = 0 to t = 5 s is – **[AIPMT 2014]**
(A) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (B) $\frac{7}{3}(\hat{i}+\hat{j})$
(C) $2(\hat{i}+\hat{j})$ (D) $\frac{11}{5}(\hat{i}+\hat{j})$

Q.11 A ship A is moving Westwards with a speed of 10 km/h and a ship B 100 km South of A, is moving Northwards with a speed of 10 km/h. The time after which the distance between them becomes shortest, is: **[AIPMT 2015]**
(A) 5 h (B) $5\sqrt{2}$ h
(C) $10\sqrt{2}$ h (D) 0 h

Q.12 A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to : $v(x) = b\,x^{-2n}$, where b and n are constants and x is the position of the particle. The acceleration of the particle as function of x, is given by : **[AIPMT 2015]**
(A) $-2nb^2x^{-4n-1}$ (B) $-2b^2x^{-2n+1}$
(C) $-2nb^2e^{-4n+1}$ (D) $-2nb^2x^{-2n-1}$

Q.13 If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is **[NEET 2016 PHASE 1]**
(A) $\frac{3}{2}A+4B$ (B) $3A+7B$
(C) $\frac{3}{2}A+\frac{7}{3}B$ (D) $\frac{A}{2}+\frac{B}{3}$

Q.14 Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity? **[NEET 2016 PHASE 2]**
(A) $\frac{a-f}{1+b}$ (B) $\frac{a+f}{2(b-1)}$ (C) $\frac{a+f}{2(1+b)}$ (D) $\frac{f-a}{2(1+b)}$

Q.15 Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be – **[NEET 2017]**
(A) $\frac{t_1t_2}{t_2-t_1}$ (B) $\frac{t_1t_2}{t_2+t_1}$ (C) t_1-t_2 (D) $\frac{t_1+t_2}{2}$

Q.16 A toy car with charge q moves on a frictionless horizontal plane surface under the influence of a uniform electric field $\vec{E}$. Due to the force $q\vec{E}$, its velocity increases from 0 to 6 m/s in one second duration. At that instant the direction of the field is reversed. The car continues to move for two more seconds under the influence of this field. The average velocity and the average speed of the toy car between 0 to 3 seconds are respectively **[NEET 2018]**

(A) 1 m/s, 3.5 m/s (B) 1 m/s, 3 m/s
(C) 2 m/s, 4 m/s (D) 1.5 m/s, 3 m/s

Q.17 When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be : **[NEET 2019]**

(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$
(C) $1:\sqrt{3}$ (D) $1:2\sqrt{3}$

Q.18 The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path the angle at which he should make his strokes w.r.t. north is given by : **[NEET 2019]**

(A) 30° west (B) 0°
(C) 60° west (D) 45° west

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	D	A	C	C	C	B	A	C	C	A	C	D	B	A	D	B	D	D	A	C	B	C	A	C	C
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	B	A	B	C	A	B	C	D	C	D	B	D	A	C	C	A	C	C	C	B	D	C	B	D	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	D	C	D	A	C	C	D	C	B	A	A	C	A	A	B	C	C	A	A	C	D	B	A	D	B
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98		
A	D	B	C	C	D	C	A	D	A	B	A	B	A	B	B	B	B	A	D	B	D	B	A		

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	B	A	D	D	B	A	A	B	D	B	D	B	A	D	A	A	D	A	B	C	C	C	A	A
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42								
A	D	D	C	D	C	D	C	A	B	C	A	A	C	A	A	C	D								

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11
A	3	16	21	2	1	4	2	1	4	8	5

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	A	C	D	A	B	B	D	A	C	D	A	A	B	C	A	C	D	B	A
Q	21	22	23	24	25	26	27	28	29											
A	B	B	D	B	D	C	580	8	C											

EXERCISE - 5

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	A	B	A	C	A	B	B	D	C	D	A	A	C	D	B	B	C	A

MOTION IN ONE DIMENSION
TRY IT YOURSELF-1

(1) (i) Distance travelled = Area under speed - time graph

$$= \frac{1}{2} \times 20 \times 8 = 80 \text{ m}$$

(ii) Acc $= \frac{\Delta v}{\Delta t} = \frac{20}{8} = \frac{5}{2} = 2.5 \text{ m/s}^2$

(2) $v = \frac{dx}{dt} = \frac{d}{dt}(2t^2 + t + 5) = 4t + 1$ m/s

and $a = \frac{dv}{dt} = \frac{d}{dt}(4t+1)$; $a = 4$ m/s^2

(3) Here, $u = 20$ ms^{-1}, $v = 0$, $t = 5$ sec. Using $a = \frac{v-u}{t}$,

we have $a = \frac{(0-20)}{5} = -4$ m/s^2

–ve acceleration is known as retardation. Thus, retardation of the car = 4 ms^{-2}.

(4) Here we assume that t is given in seconds and x in meters, so that v is m/s and a is m/s^2.

$$v = \frac{dx}{dt} = 4 + 12t + 12t^2 \,;\quad a = \frac{dv}{dt} = 12 + 24t$$

For a given v we have

$$12t^2 + 12t + 4 - v = 0 \Rightarrow t^2 + t + \frac{4-v}{12} = 0$$

So the quadratic formula gives

$$t = \frac{-1 \pm \sqrt{1-(4-v)/3}}{2}$$

and for v = 10 we have

$$t = \frac{-1+\sqrt{1-(4-10)/3}}{2} = 0.37\text{s}$$

where we take the positive sign as usual. The acceleration at this time is $a = 21$ m/s^2.

(5) The direction of an acceleration actually identifies for you the direction of the change of velocity of an object. The meaning of this is not intuitively obvious, at least as far as most people are concerned. The easiest way to get a handle on it is to notice that acceleration and net force are directly proportional to one another. The idea of a negative force isn't mysterious. If an object is moving in the negative direction and a force (hence acceleration) in the negative direction is applied to it, the body will speed up in the negative direction. By the same token, if an object is moving in the negative direction and a force (hence acceleration) in the positive direction is applied to it, the body will slow down. The rule of thumb is: if the net force (hence acceleration) is in the same direction as the velocity vector (i.e., they have the same sign), the body will speed up. If the net force (hence acceleration) is in the opposite direction of the velocity vector (i.e., they have different signs), the body will slow down. In short, a negative acceleration does NOT necessarily mean slowing down.

(6) See the sketch

(a) $|d| = |-10.0\,\mathbf{i}| = 10.0$ m

Since the displacement is a straight line from point A to point C.

(b) The actual distance walked is not equal to the straight-line displacement. The distance follows the curved path of the semicircle (ABC).

$$s = \frac{1}{2}(2\pi r) = 5.00\pi \text{ m} = 15.7\text{m}$$

(c) If the circle is complete, **d** begins and ends at point A. Hence, $|\mathbf{d}| = 0$.

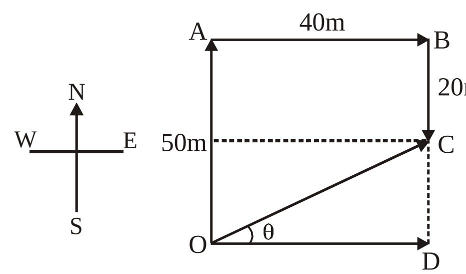
(7) (a) The total distance traversed (versus the net displacement) divided by the elapsed time. That scalar is: s = dist/time = (440 m)/(49 sec) = 8.98 m/s.

(b) The magnitude of the average velocity is the net displacement divided by the elapsed time. That is:

v = (net disp)/time = (0 m)/(49 sec) = 0 m/s.

Making sense of this: The woman finished where she started, so her net displacement is zero. The average velocity tells us the constant velocity she would have to travel to effect that displacement in 49 seconds. That velocity is zero.

(8) Let origin be O then :



(a) Distance covered $= OA + AB + BC = 50 + 40 + 20 = 110$m

(b) First method : Displacement OC $= \sqrt{OD^2 + CD^2}$

$$= \sqrt{40^2 + 30^2} = 50 \text{ m}$$

Second method : Displacement

$$\vec{d} = 50\hat{j} + 40\hat{i} - 20\hat{j} = 30\hat{j} + 40\hat{i}$$

$$|\vec{d}| = \sqrt{40^2 + 30^2} = 50\text{m}$$

(9) Yes, at turning point of motion. If ball is thrown upward then at highest point velocity will be zero but acceleration is not zero (= g = acceleration due to gravity).

(10) $|\vec{v}_f| = |\vec{v}_i|| = 5$ m/s

Acceleration $\neq 0$ (due to change in direction of velocity

Av. acceleration, $\Delta\vec{v} = \vec{v}_v - \vec{v}_i = \vec{v}_f + (-\vec{v}_i)$

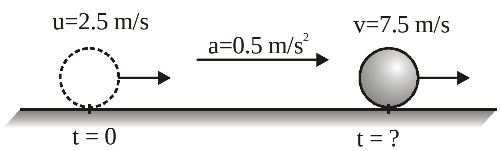
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{5\hat{j} - 5\hat{i}}{10} \Rightarrow \vec{a} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$$|\vec{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$$\tan\theta = \frac{1/2}{-1/2} = -1, \theta = 135^\circ$$

TRY IT YOURSELF-2

(1) v = u + at



$7.5 = 2.5 + 0.5\,t$

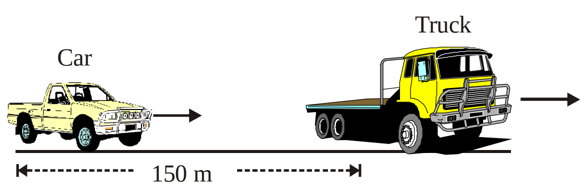
$5.0 = 0.5t$

$$t = \frac{50}{5} = 10 \text{ sec}$$

(2) Let car overtakes after t second

In time t distance travelled by truck

$$s_T = \frac{1}{2}at^2 \qquad \text{or} \qquad s_T = \frac{1}{2}(1.5)\,t^2 \qquad ....(1)$$



Distance covered by car when car overtakes the truck

$$s_c = \frac{1}{2}(2)\,t^2 \qquad [\text{acc. of car} = 2 \text{ m/s}^2]$$

$$\text{or} \quad (s_T + 150) = \frac{1}{2}(2)\,t^2 \qquad ....(2)$$

divide eq^n. (2) by eq^n. (1)

$$\frac{s_T + 150}{s_T} = \frac{2}{1.5} \quad \text{or} \quad 1 + \frac{150}{s_T} = \frac{20}{15} = \frac{4}{3}$$

$$\text{or} \quad \frac{150}{s_T} = \frac{4}{3} - 1 = \frac{1}{3} \quad \text{or} \quad s_T = 450$$

distance travelled by car = 450 + 150 = 600 meter

Now by eq^n (1)

$$s_T = \frac{1}{2}at^2 \text{ or } \qquad 450 = \frac{1}{2} \times 1.5 \times t^2$$

$$t^2 = \frac{450 \times 2}{1.5} \quad \Rightarrow \quad t = \sqrt{300 \times 2} = 24.5 \text{ sec.}$$

Therefore car will over take the truck after 24.5 sec.

(3) Here, $v_0 = 50$ km/h $= 50 \times \frac{5}{18}$ m/s $= \frac{250}{18}$ m/s

and $v = 60$ km/h $= 60 \times \frac{5}{18} = \frac{300}{18}$ m/s

$$\text{Since} \quad a = \frac{v - v_0}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{\frac{50}{18}}{2} = \frac{50}{36} = 1.39 \text{ m/s}^2$$

(4) Here, $v_0 = 10$ m/s, $v = 0$ & s = 20 cm. $= \frac{2}{100} = 0.02$m

Using $v^2 - v_0{}^2 = 2ax$

$$0 - (10)^2 = 2a\,(0.2) \Rightarrow \frac{-100}{2 \times 0.02} = a$$

or $a = -2500$ m/s^2

Retardation = 2500 m/s^2

(5) Using, $x = \left(\frac{u+v}{2}\right)t$

$$x = \frac{1}{2}vt_1 \;;\; 2x = vt_2 \;;\; 5x = \frac{1}{2}vt_3$$

Average speed

$$= \frac{x + 2x + 5x}{t_1 + t_2 + t_3} = \frac{8x}{\frac{2x}{v} + \frac{2x}{v} + \frac{10x}{v}} = \frac{8x}{14x}v = \frac{4}{7}v$$

(6) Method I :

Using $S_{nth} = u + \frac{2}{2}(2n - 1) = 0 + \frac{2}{2}(2 \times 5 - 1) = 9$m

(In S_nth formula, u is speed at t = 0)

Method II : $S = u' \times 1 + \frac{1}{2}a\,(1)^2;$ $u' = 0 + 2 \times 4 = 8$ m/s

(5^{th} sec $\rightarrow$ time interval = 1 sec., u' initial speed for 5^{th} sec)

$$S = 8 \times 1 + \frac{1}{2} \times 2\,(1)^2 = 9 \text{ m}$$

(7) Let P be the point, where the two engines cross each other. If t hr be the time to occur this event, then total distance covered by the two trains should be equal to 100 km.(fig.) i.e., AP + BP = 100

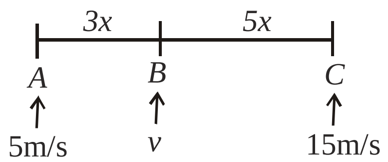
$$\Rightarrow \quad 50t + \frac{1}{2} \times 18t^2 + 50t - \frac{1}{2} \times 18t^2 = 100$$

$\Rightarrow \ 100t = 100 \Rightarrow t = 1$ hr.

$\therefore \ x = AP = 50\,(1) + \frac{1}{2} \times 18(1) \Rightarrow x = 50 + 9 = 59$ km.

(8) $t_{AB} = 40$ sec.,

$$15^2 = 5^2 + 2a\,(8x) \Rightarrow ax = \frac{15^2 - 5^2}{16} = \frac{200}{16}$$



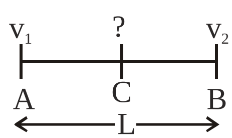
$v^2 = 5^2 + 2a\,(3x) = 25 + 6 \times (200/16) = 100 \Rightarrow v = 10$ m/s

As a = constant using, $v = u + at$

$$10 = 5 + a \times 50 \Rightarrow a = \frac{5}{40} = \frac{1}{8}\,\text{m/s}^2$$

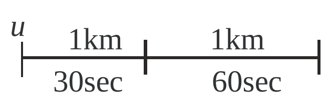
(9) $v_2^2 = v_1^2 + 2aL \Rightarrow aL = \frac{v_2^2 - v_1^2}{2}$

$$v'^2 = v_1^2 + 2a \times \frac{L}{2} = v_1^2 + \frac{v_2^2 - v_1^2}{2}$$



$$v'^2 = \frac{v_2^2 + v_1^2}{2} \Rightarrow v' = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

(10) $1000 = u \times 30 + \frac{1}{2}a(30)^2$ .... (1)



$2000 = u \times 90 + \frac{1}{2}a(90)^2$ .... (2)

Multiply eq. (1) by a both side & sub. (2)

$$7000 = 180\,u \Rightarrow u = \frac{700}{18}\,\text{m/s} = \frac{350}{9}\,\text{m/s}$$

(11) Let distance travelled in t^{th} second = s_1 and in $(t + 1)^{th}$ seconds = s_2 then

$$S_1 = u + \frac{f}{2}(2t - 1) \text{ and } S_2 = u + \frac{f}{2}[2\,(t+1) - 1]$$

$S_1 + S_2 = 100$

$$2u + \frac{f}{2}(2t - 1 + 2t + 2 - 1) = 100 \Rightarrow 2u + 2ft = 100$$

$\Rightarrow u + ft = 50 \Rightarrow v = u + ft = 50$ cm/s

TRY IT YOURSELF-3

(1) First stone is thrown so as to reach the top of the tower, so its initial velocity is

$$u = \sqrt{2gH} = \sqrt{2 \times 10 \times 90} = 42.5 \text{ m/s}$$

Let us take the time $t = t_0$, when the two stones meet at a height h above the foot of the tower. The first stone travelled a height h in the duration t_0 and the second stone has fallen a distance $(90 - h)$ in time $(t_0 - 2)$.

For first stone, $h = 42.5\,t_0 - \frac{1}{2}(10)\,t_0^2$;

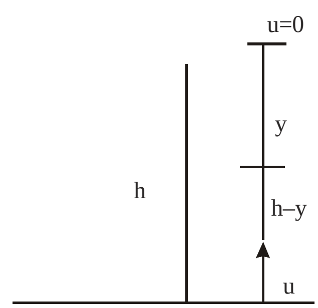
For second stone, $90 - h = \frac{1}{2}(10)\,(t_0 - 2)^2$

Adding above two equation, $22.5\,t_0 = 70$ or $t_0 = 3.11$ s

Thus height h is given as,

$$h = 42.5\,(3.11) - \frac{1}{2}(10)\,(3.11)^2 = 83.82\text{m}.$$

(2) Max. height, $4h = \frac{u^2}{2g} \Rightarrow u = \sqrt{8gh}$

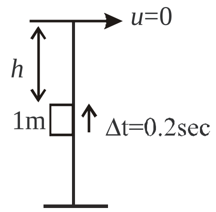


$$y = \frac{1}{2}gt^2, \quad h - y = ut - \frac{1}{2}gt^2$$

$$h = ut \Rightarrow t = \frac{h}{\sqrt{8gh}} = \sqrt{\frac{h}{8g}}$$

(3) Using, $h = ut + \frac{1}{2}gt^2$; $h = \frac{1}{2}gt^2$; $h + 1 = \frac{1}{2}g(t + 0.2)^2$

$$\frac{1}{2}gt^2 + 1 = \frac{1}{2}gt^2 + \frac{1}{2}g(0.2)^2 + \frac{1}{2}g \times 2 \times 0.2t$$



$$1=\frac{1}{5}+0.2gt \quad ; \quad \frac{4}{5}=2t \Rightarrow t=\frac{2}{5} \quad ; \quad h=\frac{1}{2}g\frac{4}{25}=\frac{4}{5}\text{m}$$

(4) **(C).** The ball reaches its highest point when its velocity is zero; the acceleration of gravity is never zero (it is always 9.8 m/s^2 downward).

(5) **(C).** (Coordinate system: positive x-axis upwards.)

Upon its descent, the velocity of an object thrown straight up with an initial x-component of velocity

$v_{x,0} > 0$ has velocity $v_x = -v_{x,0} < 0$ when it passes the point at which it was first released. This is exactly the same x-component of velocity as the ball that was thrown downward, so both balls will hit the ground with the same x-component of velocity. Let t_f denote the time interval that the ball thrown downwards takes to hit the ground, then the x-component of the velocity of both balls when they hit the ground is given by $v_s(t_f) = v_{x,0} - gt_f$.

(6) **(A).** Both objects are falling with the same acceleration (gravity), and as both are accelerating without friction and with the same initial velocity, the two ought to stay the same distance apart throughout the motion.

(7) Down is positive; over his height $s = ut + \frac{1}{2}at^2$

$$2\text{m} = u\,(0.20\text{s}) + \frac{1}{2}\,(9.81\ \text{m/s}^2)\,(0.20\text{s})^2\ ;$$

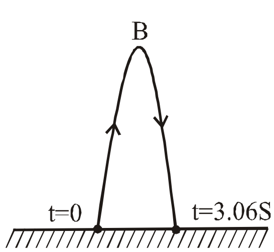
$u = 9.02$ m/s + $(9.81\ \text{m/s}^2)(0.20\text{s}) = 10.98$ m/s; for total fall,

$$v^2 = u^2 + 2as_B\ ;$$

$$(10.98\ \text{m/s})^2 = 0 + 2\,(9.81\ \text{m/s}^2)\,s_B\ ;\ s_B = 6.1\text{m}$$

(8) We select earth as the origin so that $g = -9.8\ \text{ms}^{-2}$

(i) At the highest point, velocity is zero



$$v^2 - v_0^2 = 2gh\,,$$

Here $v = 0,\ v_0 = +15$ m/s, $g = -9.8\ \text{ms}^{-2}$

$\therefore\ (0)^2 - (15)^2 = 2 \times (-9.8)\,h$

$\therefore$ max. height, $h = \dfrac{-(+15)^2}{2\times(-9.8)} = 11.5\ \text{m}\ ;\ h = v_0t + \dfrac{1}{2}gt^2,$

Here $h = 0,\ v_0 = \pm 15$ m/s, $g = -9.8\ \text{ms}^{-2}$

$$0 = 15t - \frac{1}{2}\times 9.8\,t^2\ ;\ 0 = 15t - 4.9\,t^2$$

or $\quad 0 = t\,(15 - 4.9\,t)$

$$t = 0 \text{ and } 15 - 4.9\,t = 0 \quad \text{or} \quad t = \frac{15}{4.9} = 3.06\ \text{s}$$

That there are two solution for t, 0 S and 3.06 S. The first solution corresponds to initial point A and second solution corresponds to return point C. Therefore, the ball is in the air for 3.06 S.

(9) At the highest point the velocity of the ball is instantaneously zero. Take the y-axis to be upward, set v = 0 in

$v^2 = v_0^2 - 2gy$, and solve for v_0 : $v_0 = \sqrt{2gy}$.

Substitute $g = 9.8\ \text{m/s}^2$ and y = 50m to get

$$v_0 = \sqrt{2\,(9.8\text{m/s}^2)\,(50\text{m})} = 31\ \text{m/s}$$

(10) **(B).** $v = u - at$

$v = 18 - 10\sin 30°\,t = 18 - 15 = 3$ m/s

(11) **(B).** Both children begin with gravitational potential energy mgh at the top of the slide, which is completely transferred to kinetic energy at the end of the slide. Bobby's potential energy is transferred more quickly, however, therefore he attains a higher average velocity and beats Sandy to the end of the slide. Average acceleration is the change in velocity divided by the time interval. Each child has the same change in velocity, but Bobby observes this change over a shorter period of time, resulting in a larger average acceleration.

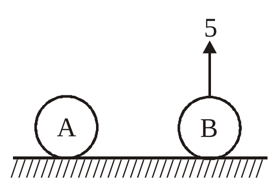
(12) (A)

TRY IT YOURSELF-4

(1) Relative acceleration,

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$$

Also, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s



As relative acceleration is zero we can use

$$\vec{s}_{BA}\ (\text{in 1 sec}) = \vec{v}_{BA}\times t = 5\times 1 = 5\text{m}$$

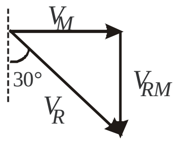
$\therefore$ Distance between A and B after 1 sec = 5m

(2) Given that the velocity of rain drops with respect to road is making an angle 30° with the vertical, and the velocity of the man is 10kph, also the velocity of rain drops with respect to main is vertical. We have

$$v_{RM} = v_R - v_M$$

hence $v_R = v_{RM} - v_M$

The situation is shown in velocity triangle in figure.



It shows clearly that, $v_R = V_M \operatorname{cosec}\theta = 10 \times 2 = 20$kph

and $V_{RM} = V_M \cos\theta = 10 \times \sqrt{3} = 10\sqrt{3}$ kph.

(3) Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain be $\vec{v}_r = a\hat{i} + b\hat{j}$ .... (i)

Then speed of rain will be $|\vec{v}| = \sqrt{a^2 + b^2}$ .... (ii)

In the first case $\vec{v}_m$ = velocity of man = $3\hat{i}$

$\therefore \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\,\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence, a – 3 = 0 or a = 3

In the second case $\vec{v}_m = 6\hat{i}$

$\therefore \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with vertical. Hence, | b | = 3

Therefore, from eq. (i) speed of rain is

$$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}\ \frac{\text{km}}{\text{hr}}$$

(4) Using relative velocity concept :

$\vec{v}_{mw} = \vec{v}_m - \vec{v}_w$

$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$

$\Rightarrow \quad v_m = |\vec{v}_{mw} + \vec{v}_w| = \sqrt{v_{mw}^2 + v_w^2 + 2v_{mw}.v_w \cos\theta}$

$\Rightarrow v_m = \sqrt{5^2 + 3^2 + 2(5)(3)\cos 120^\circ}$

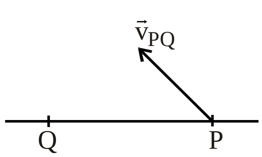
$\Rightarrow v_m = \sqrt{25+9-15} = \sqrt{19}$ m/sec.

(5) For minimum time of crossing the man should head perpendicular to the shore

$\vec{v}_{mw} \perp \vec{v}_w$

$\cos\theta = \dfrac{v_w}{v_m} \quad \Rightarrow \quad \cos 60^\circ = \dfrac{4}{v_m} \Rightarrow v_m = 8$ km/hr

(6) **(A).**



Q measures acceleration of P to be zero.

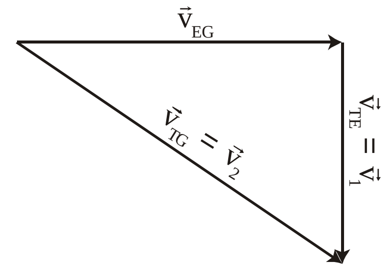
$\therefore$ Q measures velocity of P, i.e. $\vec{v}_{PQ}$ to be constant. Hence Q observes P to move along straight line.

$\therefore$ For P and Q to collide Q should observe P to move along line PQ.

Hence, PQ should not rotate.

(7) **(D).** Call the velocity of the turtle with respect to the eagle v_{TE}, also known as v_1.

Call the velocity of the turtle with respect to the ground v_{TG}, also known as v_2.



You are asked to find the velocity of the eagle with respect to the ground, v_{EG}.

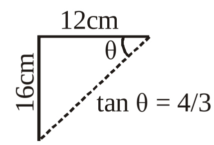
Analyzing the right triangle, you can use the Pythagorean Theorem to solve for the magnitude of v_{EG}

$$\vec{v}_{TG} = \vec{v}_{EG} + \vec{v}_{TE}$$

$$\vec{v}_{TG} = \vec{v}_{EG} + \vec{v}_{TE} \xrightarrow[\vec{v}_{TE}=\vec{v}_1]{\vec{v}_{TG}=\vec{v}_2} \vec{v}_2 = \vec{v}_{BG} + \vec{v}_1$$

$$\Rightarrow \quad v_2^2 = v_{EG}^2 + v_1^2 \Rightarrow v_{EG} = \sqrt{v_2^2 - v_1^2}$$

(8) **(A).** $V_{R/G(x)} = 0$, $V_{R/G(y)} = 10$ m/s



Let, velocity of man = v

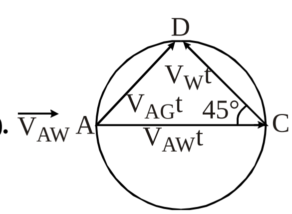
then, $v_{R/man}$ = v (opposite to man)

For the required condition :

$$\tan\theta = \frac{V_{R/M(y)}}{V_{R/M(x)}} = \frac{10}{v} = \frac{4}{3} \Rightarrow V = \frac{10 \times 3}{4} = 7.5$$

(9) $\vec{v}_{BG} = \vec{v}_{BT} + \vec{v}_{TG} = 18\hat{i} - 2\hat{j}$

(10) **(B).**



In absence of wind A reaches to C and in presence of wind it reaches to D in same time so wind must deflect from C to D so wind blow in the direction of CD.

$$\vec{V}_{AG} = \vec{V}_{AW} + \vec{V}_{WG}$$

$$\Rightarrow \quad \vec{V}_{AG}t = \vec{V}_{AW}t + \vec{V}_{WG}t$$

$$AC = \vec{V}_{AW}t \ ; \ CD = \vec{V}_{WG}t$$

CHAPTER-3 :
MOTION IN ONE DIMENSION
EXERCISE-1

(1) **(D).** Distance $\geq$ [Displacement]

(2) **(A).** Since final and initial positions are same hence displacement of athlete will be
$\Delta r = r - r = 0$

(3) **(C).** Distance = Circumference of the circle
$D = 2\pi R \Rightarrow D = 2\pi \times 80 = 160 \times 3.14 = 502.40m$

(4) **(C).** When a particle returns to its starting point its displacement is zero.

(5) **(C).** Distance covered with 1 step = 1 m
Time taken = 1 s
Time taken to move first 5 m forward = 5 s
Time taken to move 3 m backward = 3 s
Net distance covered = 5 – 3 = 2 m
Net time taken to cover 2 m = 8 s
Drunkard covers 2 m in 8 s.
Drunkard covered 4 m in 16 s.
Drunkard covered 6 m in 24 s.
Drunkard covered 8 m in 32 s.
In the next 5 s, the drunkard will cover a distance of 5m and a total distance of 13m and falls into the pit.
Net time taken by the drunkard to cover 13 m
= 32 + 5 = 37s

(6) **(B).** Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at $5 \times 5\ s = 25\ s$.
The average speed is 600 m/25 s = 24 m/s.

(7) **(A).** The slope of the line in a position versus time graph gives the velocity of the motion. The slope for part a is positive. For part b the slope is negative. For part c the slope is positive.

(8) **(C).** The average speed is the distance of 16.0km divided by the elapsed time of 2.0 h. The average velocity is the displacement of 0km divided by the elapsed time. The displacement is 0 km, because the jogger begins and ends at the same place.

(9) **(C).** $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10m}{2s} = 5\ m/s$

(10) **(A).** $\bar{v} = \frac{5m}{4s} = 1.2\ m/s$

(11) **(C).** $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5m - 10m}{4s - 2s} = -2.5\ m/s$

(12) **(D).** $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5m - 5m}{7s - 4s} = -3.3\ m/s$

(13) **(B).** $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = 0\ m/s$

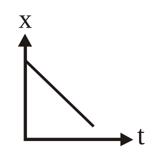
(14) **(A).** Let t be the total time taken then distance covered in the first half time $= \frac{v_1 t}{2}$

Distance covered in the next half time $= \frac{v_2 t}{2}$

Average speed $v_{av.} = \frac{\frac{v_1 t}{2} + \frac{v_2 t}{2}}{t} = \frac{v_1 + v_2}{2}$

(15) **(D).** Interval 3 (Greatest), Interval 2 (Least)
Positive (Intervals 1 & 2),
Negative (Interval 3)
The average speed of a particle shown in the x-t graph is obtained from the slope of the graph in a particular interval of time.
It is clear from the graph that the slope is maximum and minimum in intervals 3 and 2 respectively. Therefore, the average speed of the particle is the greatest in interval 3 and is the least in interval 2. The sign of average velocity is positive in both intervals 1 and 2 as the slope is positive in these intervals. However, it is negative in interval 3 because the slope is negative in this interval.

(16) **(B).** The position-time graph of a particle moving with negative velocity is as shown in the figure.



(17) **(D).** The area under the velocity-time graph represents the displacement over a given time interval.

(18) **(D).** At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m. it is 180º away. Thus the displacement between the initial and final positions of the tip is equal to the diameter of the clock.
Displacement = 2 R = 2 × 4 cm = 8 cm
Time taken from 6 a.m. to 6.30 a.m. is 30 minutes
= 1800s.
The average velocity is V_{av}

$$= \frac{\text{Displacement}}{\text{time}} = \frac{8}{1800} = 4.4 \times 10^{-3}\ cm/s$$

Again time taken from 6 am to 6.30 p.m.
= 12 hrs + 30 minutes = 45000 s

$$\therefore\ V_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8}{45000} = 1.8 \times 10^{-4}\ cm/s$$

(19) **(A).** Both speed and velocity are constant in the case of a particle moving with uniform velocity.
A particle moving with uniform velocity has zero acceleration.

(20) **(C).** Average velocity, $\bar{v} = \frac{(x)_{t=4} - (x)_{t=2}}{4 - 2}$

$$\bar{v} = \frac{(a + b(4)^2) - (a + b(2)^2)}{4 - 2}$$

$$= \frac{(a + 16b) - (a + 4b)}{4 - 2} = 6b = 6\ (2.5)\ m/s = 15\ m/s$$

(21) **(B).** Here, $x = a + bt^2$
Where, $a = 8.5$ m and $b = 2.5$ m/s^2

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt$

At t = 2 s, $v = 2\,(2.5\text{ m s}^{-2})\,(2s) = 10$ m/s

(22) **(C).** Time taken to travel first half distance,

$$t_1 = \frac{L/2}{v_1} = \frac{L}{2v_1}$$

Time taken to travel second half distance,

$$t_2 = \frac{L/2}{v_2} = \frac{L}{2v_2}$$

Total time taken $= t_1 + t_2 = \frac{L}{2v_1} + \frac{L}{2v_2}$

Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{L}{\frac{L}{2v_1} + \frac{L}{2v_2}} = \frac{1}{\frac{1}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} = \frac{2v_1v_2}{v_2 + v_1}$$

(23) **(A).** $x = 10\,t^2$:

t (s) =	2.0	2.1	3.0
x (m) =	40	44.1	90

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50\text{m}}{1.0\text{s}} = 50.0 \text{ m/s}$$

(24) **(C).** $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1\text{m}}{0.1\text{s}} = 41.0 \text{ m/s}$

(25) **(C).** Average velocity $= \frac{\text{Displacement}}{\text{Time taken}}$

$$= \frac{2R}{t} = \frac{2 \times 40}{40} = 2 \text{ m/s}$$

(26) **(B).** Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in 5.00 m/s = d/t_1.
Let t_2 represent the longer time for the return trip in –3.00 = – d/t_2.
Then the times are

$$t_1 = \frac{d}{(5.00\text{ m/s})} \text{ and } t_2 = \frac{d}{(3.00\text{ m/s})}.$$

The average speed is

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{d + d}{\frac{d}{(5.00\text{ m/s})} + \frac{d}{(3.00\text{ m/s})}} = \frac{2d}{\frac{(8.00\text{ m/s})\,d}{(15.0\text{ m}^2/\text{s}^2)}}$$

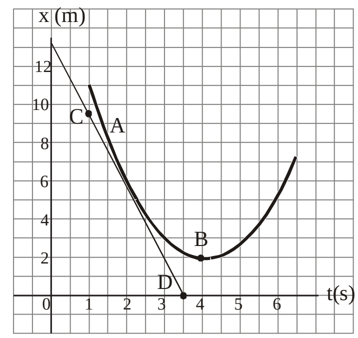
$$\bar{v} = \frac{2\,(15.0\text{ m}^2/\text{s}^2)}{8.00\text{ m/s}} = 3.75 \text{ m/s}$$

(27) **(A).** She starts and finishes at the same point A. With total displacement = 0,
Average velocity = 0 .

(28) **(B).** At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)
At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\text{ m}}{(4 - 1.5)\text{ s}} = -\frac{6.0\text{m}}{2.5\text{s}} = -2.4 \text{ m/s}$$

(29) **(C).** The slope of the tangent line is found from points C and D.
$(t_C = 1.0\text{ s}, x_C = 9.5\text{ m})$ and $(t_D = 3.5\text{ s}, x_D = 0)$,
$v \approx -3.8$ m/s



(30) **(A).** The velocity is zero when x is a minimum.
This is at $t \approx 4$ s .

(31) **(B).** A particle moving with uniform velocity has zero acceleration.

(32) **(C).** The tangent at F is the dashed line GH. Taking triangle GHJ, we have
$\Delta t = 24 - 4 = 20$ s
$\Delta x = 0 - 15 = -15$m

Hence slope at F is $v_F = \frac{\Delta x}{\Delta t} = \frac{-15\text{m}}{20\text{ s}} = -0.75$ m/s

The negative sign tells us that the object is moving in the –x direction.

(33) **(D).** The displacement-time graph is a straight line inclined to time axis upto time t_0 indicates a uniform velocity. After time t_0, the displacement-time graph is a straight line parallel to time axis indicates particle at rest.

(34) **(C).** At any time, t, the position is given by
$x = (3.00\text{ m/s}^2)\,t^2$. Thus, at $t_i = 3.00$ s
$x_i = (3.00\text{ m/s}^2)\,(3.00\text{ s})^2 = 27.0$ m

(35) **(D).** $v = \frac{dx}{dt} = 6t$. At t = 3 ; v = 18 m/s

(36) **(B).** The slope of the tangent at any point on the displacement-time graph gives instantaneous velocity at that instant. In the given graph, the slope is negative at point e.

(37) **(D).**
(a) The given x-t graph, shown in (a), does not represent one-dimensional motion of the particle. This is because a particle cannot have two positions at the same instant of time.
(b) The given v-t graph, shown in (b), does not represent one-dimensional motion of the particle. This is because a particle can never have two values of velocity at the same instant of time.

(c) The given v-t graph, shown in (c), does not represent one-dimensional motion of the particle. This is because speed being a scalar quantity cannot be negative.

(d) The given v-t graph, shown in (d), does not represent one-dimensional motion of the particle. This is because the total path length travelled by the particle cannot decrease with time.

(38) **(A).** $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = 5\text{ m/s}$

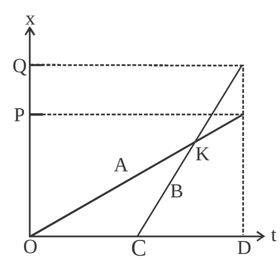
(39) **(C).** $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = -2.5\text{ m/s}$

(40) **(C).** $v = \dfrac{(5-5)\text{ m}}{(5-4)\text{ s}} = 0$

(41) **(A).** $v = \dfrac{0-(-5\text{m})}{(8-7)\text{ s}} = +5\text{m/s}$

(42) **(C).**

(A) It is clear from the graph that OQ > OP. So, A lives closer to the school than B.



(B) The position-time graph of A starts from the origin (t=0) while the position-time graph of B starts from C which indicates that B started later than A after a time interval OC. So, A started earlier than B.

(C) The speed is represented by the steepness (or slope) of the position-time graph. Since the position-time graph of B is steeper than the position-time of graph A, therefore, we conclude that B is faster than A.

(D) Corresponding to both P and Q, the time interval is the same, i.e., OD. This indicates that both A and B reach their homes at the same time.

(43) **(C).** The average acceleration is the change in velocity (final velocity minus initial velocity) divided by the elapsed time. The change in velocity has a magnitude of 15.0 km/h. Since the change in velocity points due east, the direction of the average acceleration is also due east.

(44) **(C).** The object has an initial positive (northward) velocity and a negative (southward) acceleration; so, a graph of velocity versus time slopes down steadily from an original positive velocity. Eventually, the graph cuts through zero and goes through increasing-magnitude- negative values.

(45) **(B).** In a position vs. time graph, the velocity of the object at any point in time is the slope of the line tangent to the graph at that instant in time. The speed of the particle at this point in time is simply the magnitude (or absolute value) of the velocity at this instant in time. The displacement occurring during a time interval is equal to the difference in x coordinates at the final and initial times of the interval, $\Delta x = x_f - x_i$. The average velocity during a time interval is the slope of the straight line connecting the points on the curve corresponding to the initial and final times of the interval, $\bar{v} = \dfrac{\Delta x}{\Delta t}$

Thus, we see how the quantities in choices (A), (C), and (D) can all be obtained from the graph. Only the acceleration, choice (B), cannot be obtained from the position vs. time graph.

(46) **(D).** (i) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (ii) (c) shows positive acceleration throughout. (iii) (a) shows negative (leftward) acceleration in the first four images.

(47) **(C).** For zero acceleration, the position-time graph is a straight line.

(48) **(B).** Here, Initial velocity u = 0,

$v = (v_{max}) = 18\text{ km/h} = 18 \times \dfrac{5}{18} = 5\text{ m/s}$; $t_1 = 0$ sec, $t_2 = 2$ sec.

$$a_{av} = \frac{v-u}{t_2 - t_1} = \frac{\Delta v}{\Delta t}, \text{ so } a_{av} = \frac{5.0}{2} = 2.5\text{ m/s}^2$$

(49) **(D).** Here, u = 0 and v = 10 m/s, t = 5 sec

Using, $a = \dfrac{v-u}{t}$, we have $a = \dfrac{(10-0)\text{m/s}}{5\text{ s}} = 2\text{ m/s}^2$

(50) **(A).** $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(at^2 - bt^3) = 2at - 3bt^2$

$$\text{acc.} = \frac{dv}{dt} = \frac{d}{dt}(2at - 3bt^2) = 2a - 6bt$$

According to question acc. = 0

$\therefore$ $2a - 6bt = 0$ hence $t = \dfrac{a}{3b}$

(51) **(D).** Choose the positive direction to be the outward direction, perpendicular to the wall.

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0\text{ m/s} - (-25.0\text{m/s})}{3.50 \times 10^{-3}\text{s}} = 1.34 \times 10^4\text{ m/s}^2$$

(52) **(C).** The area under acceleration-time graph represents the change in velocity.

(53) **(D).**

(54) **(A).** The slope of the tangent drawn on velocity-time graph at any instant of time is equal to the instantaneous acceleration.

(55) **(C).** Slope of velocity-time graph shows acceleration.

(56) **(C).** The equations of kinematics can be used only when the acceleration remains constant and cannot be used when it changes from moment to moment.

(57) **(D).** According to one of the equation of kinematics

$x = v_0t + \frac{1}{2}at^2$, with $v_0 = 0$ m/s, the displacement is proportional to the acceleration.

(58) **(C).** With original velocity zero, displacement is proportional to the square of time in $(1/2)\,at^2$. Making the time one-third as large makes the displacement one-ninth as large.

(59) **(B).** The initial velocity of the car is $v_0 = 0$ and the velocity at time t is v. The constant acceleration is therefore

given by $a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} = \frac{v - 0}{t} = \frac{v}{t}$

and the average velocity of the car is

$\bar{v} = \frac{v + v_0}{2} = \frac{v + 0}{2} = \frac{v}{2}$

The distance traveled in time t is $\Delta x = \bar{v}\,t = \frac{vt}{2}$.

In the special case where a= 0 (and hence $v = v_0 = 0$), we see that statements (a), (b), (c), and (d) are all correct. However, in the general case ($a \neq 0$) and hence ($v \neq 0$). Only statement (b) and (c) are true.

(60) **(A).** Here, u = 10 m/s, t = 3 s, v = 16 m/s

$a = \frac{v - u}{t} = \frac{16 - 10}{3} = 2\text{ m/s}^2$

Now velocity at 2s, before the given instant

$10 = u + 2 \times 2$ $(\because v = u + at)$

$\therefore$ u = 6 m/s

(61) **(A).** As acc. is constant so from $s = ut + \frac{1}{2}at^2$ we have

$x = \frac{1}{2}at^2$ [u = 0] ....(1)

Now if it travels a distance y in next t sec. in 2t sec total distance travelled

$x + y = \frac{1}{2}a(2t)^2$ ....(2) (t + t = 2t)

Dividing eq^n. (2) by eq^n (1)

$\frac{x + y}{x} = 4$ or $y = 3x$

(62) **(C).** The sign of acceleration does not tell us whether the particle's speed is increasing or decreasing. The sign of acceleration depends on the choice of the positive direction of the axis. For example, if the vertically upward direction is chosen to be positive direction of the axis, the acceleration due to gravity is negative. If a particle is falling under gravity, this acceleration though negative results in increase in speed.

(63) **(A).** The velocity time graph is not a straight line, the acceleration is not uniform. Hence relation (a), (b) and (e) are not correct, but relation, (c), (d) and (f) are correct.

(64) **(A).** Let d_s is the distance travelled by the vehicle before it stops. Here, final velocity v = 0,
initial velocity = u, S = d,
Using equation of motion
$v^2 = u^2 + 2aS$ $\therefore (0)^2 = u^2 + 2ad_s$

$d_s = -\frac{u^2}{2a}$; $d_s \propto u^2$

(65) **(B).** Here, $u = 144\text{ km/h} = 144 \times \frac{5}{18}\text{ m/s} = 40\text{ m/s}$

v = 0, S = 200m
As $v^2 - u^2 = 2aS$
$(0)^2 - (40)^2 = 2 \times a \times (200)$

$a = -\frac{(40)^2}{2 \times 200} = -4\text{ m/s}^2$

As v = u + at
$\therefore$ $0 = 40 - (4)(t) \Rightarrow t = 40/4 = 10s$

(66) **(C).** $S = vt + \frac{1}{2}at^2$

It is not a kinematic equation of motion.
All others are three kinematic equations of motion.

(67) **(C).** Here, u = 0 $\therefore v^2 = 2as$
It is a parabola of the type $y^2 = 4ax$.
Hence, option (C) represents the correct graph.

(68) **(A).** From first eq^n of motion– v = u + at
$\Rightarrow$ 100 = 0 + at or 100 = at ....(1)
velocity after one second
v' = 0 + a(t + 1)
$\Rightarrow$ 150 = a(t + 1) .....(2)
On subtracting eq^n.(1) from eq^n. (2)
$a = 50\text{ m/s}^2$

(69) **(A).** $u = 43.2\text{ km/h} = 43.2 \times \frac{5}{18}\text{ m/s} = 12\text{ m/s}$

Deceleration ; $a = 6\text{ m/s}^2$ v = 0, s = ?
$0 = (12)^2 - 2 \times 6\,s$ [using $v^2 = u^2 - 2as$]

or 144 = 2 × 6s or $s = \frac{144}{12} = 12\text{ m}$

(70) **(C).** We have, $x = ut + \frac{1}{2}at^2$

$= (2.5\text{ m/s})(2s) + \frac{1}{2}(0.50\text{ m/s}^2)(2s)^2$

= 5.0 m + 1.0 m = 6.0 m

(71) **(D).** We have, v = u + at
or $7.5\text{ m/s} = 2.5\text{ m/s} + (0.50\text{ m/s}^2)\,t$

or $t = \frac{7.5\text{m/s} - 2.5\text{m/s}}{0.50\text{m/s}^2} = 10s$

(72) **(B).** We have, $v^2 = u^2 + 2ax$
or $(7.5\text{ m/s})^2 = (2.5\text{ m/s})^2 + 2\,(0.50\text{ m/s}^2)\,x$

or $x = \frac{(7.5\text{m/s})^2 - (2.5\text{m/s})^2}{2 \times 0.50\text{m/s}} = 50\text{m}$

(73) **(A).** Using $S_{nth} = u + \frac{2}{2}(2n-1) = 0 + \frac{2}{2}(2\times5-1) = 9m$

(In S_nth formula, u is speed at t = 0)

(74) **(D).** The bowling pin has a constant downward acceleration while in flight. The velocity of the pin is directed upward on the ascending part of its flight and is directed downward on the descending part of its flight. Thus, only (D) is a true statement.

(75) **(B).** Free-fall is the motion that occurs while the acceleration is solely the acceleration due to gravity. While the rocket is picking up speed in the upward direction, the acceleration is not just due to gravity, but is due to the combined effect of gravity and the engines. In fact, the effect of the engines is greater than the effect of gravity. Only when the engines shut down does the free-fall motion begin.

(76) **(D).** The acceleration due to gravity points downward, in the same direction as the initial velocity of the stone thrown from the top of the cliff. Therefore, this stone picks up speed as it approaches the nest. In contrast, the acceleration due to gravity points opposite to the initial velocity of the stone thrown from the ground, so that this stone loses speed as it approaches the nest. The result is that, on average, the stone thrown from the top of the cliff travels faster than the stone thrown from the ground and hits the nest first.

(77) **(B).** Using $v_f^2 = v_i^2 + 2a\Delta y$, with $v_i = -12$m/s and $\Delta y = -40$ m:

$v_f^2 = v_i^2 + 2a\Delta y$,

$v^2 = (-12\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-40\text{ m})$

$v = -30$ m/s

(78) **(C).** We take downward as the positive direction with y = 0 and t = 0 at the top of the cliff. The freely falling marble then has $v_0 = 0$ and its displacement at t = 1.00 s is $\Delta y = 4.00$ m. To find its acceleration, we use $y = y_0 + v_0 t + at^2$

$$y - y_0 = \Delta y = \frac{1}{2}at^2 ; \quad a = \frac{2\Delta y}{t^2}$$

$$a = \frac{2(4.00\text{ m})}{(1.00\text{ s})^2} = 8.00\text{ m/s}^2$$

The displacement of the marble (from its initial position) at t = 2.00 s is found from

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}(8.00\text{ m/s}^2\})(2.00\text{ s})^2 = 16.0\text{ m.}$$

The distance the marble has fallen in the 1.00 s interval from t = 1.00 s to t = 2.00 s is then

$\Delta y = 16.0\text{ m} - 4.0\text{ m} = 12.0$ m.

(79) **(C).** We take downward as the positive direction with y = 0 and t = 0 at the top of the cliff. The freely falling pebble then has $v_0 = 0$ and $a = g = +9.8$ m/ s^2.

The displacement of the pebble at t = 1.0 s is given: $y_1 = 4.9$m. The displacement of the pebble at t = 3.0 s is found from

$$y_3 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.8\text{ m/s}^2)(3.0\text{ s})^2 = 44\text{ m}$$

The distance fallen in the 2.0-s interval from t = 1.0 s to t = 3.0 s is then

$\Delta y = y_3 - y_1 = 44\text{ m} - 4.9\text{ m} = 39$ m.

(80) **(D).** The maximum height (where v = 0) reached by a freely falling object shot upward with an initial velocity $v_0 = +225$m/s is found from

$v_f^2 = v_i^2 + 2a(y_f - y_i) = v_i^2 + 2a\Delta y$,

where we replace a with –g, the downward acceleration due to gravity.

Solving for Δy then gives

$$\Delta y = \frac{v_f^2 - v_i^2}{2a} = \frac{-v_0^2}{2(-g)} = \frac{-(225\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 2.58\times10^3\text{ m}$$

Thus, the projectile will be at the $\Delta y = 6.20 \times 10^2$ m level twice, once on the way upward and once coming back down.

The elapsed time when it passes this level coming downward can be found by using

$v_f^2 = v_i^2 + 2a\,\Delta y$ again by substituting a = –g and solving for the velocity of the object at height (displacement from original position)

$\Delta y = +6.20 \times 10^2$ m

$v_f^2 = v_i^2 + 2a\,\Delta y$

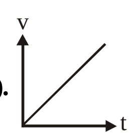
$v^2 = (225\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(6.20\times10^2\text{m})$

$v = \pm 196$ m/s

The velocity coming down is –196m/s. Using $v_f = v_i + at$, we can solve for the time the velocity takes to change from + 225 m/s to –196 m/ s:

$$t = \frac{v_f - v_i}{a} = \frac{(-196-225)\text{ m/s}}{-9.80\text{ m/s}^2} = 43.0\text{s}$$

(81) **(C).** At the highest point velocity of the ball becomes zero, but its acceleration is equal to g.

(82) **(A).** 

(83) **(D).** $t = \frac{2u}{g} = \frac{2\times50}{g} = 10\text{s}$

(84) **(A).** Suppose the body passes the upper point at t second and lower point at (t + 1) s, then

$$S_2 - S_1 = \frac{1}{2}g(t+1)^2 - \frac{1}{2}gt^2 = \frac{1}{2}g(2t+1)$$

or $30\text{m} = \frac{1}{2}\times9.8(2t+1) \quad \therefore \; t = 2.56$ s

$$S_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times9.8\times(2.56)^2 = 32.1\text{ m}$$

(85) **(B).** Free fall of an object in vacuum is a case of motion with uniform acceleration.

(86) **(A).** The given law is known as Galileo's law of odd numbers. This law was established by Galileo Galilei who was the first to make quantitative studies of free fall.

(87) **(B).** $t = \frac{2u}{g} = \frac{2 \times 30}{10} = 6\ \text{sec}.$

(88) **(A).** Let us say ball take 't' sec to fall height h as it falls (9h/25) in last sec., it travel

$$h - \frac{9h}{25} = \frac{16h}{25} \text{ in } (t-1) \text{ sec} \quad \therefore \quad h = \frac{1}{2} gt^2 \ ..... (1)$$

$$\frac{16h}{25} = \frac{1}{2} g (t-1)^2 \qquad ... (2)$$

Divide (2) by (1),

$$\frac{16}{25} = \frac{(t-1)^2}{t^2} \Rightarrow h = \frac{1}{2} g (5)^2 = \frac{25g}{2} \text{ m}$$

(89) **(B).** $\frac{1}{2} g(3)^2 = \frac{g}{2}(2n-1) \Rightarrow n = 5\ s$

(90) **(B).** Time taken by first drop to reach the ground

$$t = \sqrt{\frac{2h}{g}} \Rightarrow t = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ sec.}$$

As the water drops fall at regular intervals from a tap therefore time difference between any two drops = 1/2 sec

In this given time, distance of second drop from the tap $= \frac{1}{2} g \left(\frac{1}{2}\right)^2 = \frac{5}{5} = 1.25\,\text{m}$

Its distance from the ground = 5 – 1.25 = 3.75m

(91) **(B).** Speed of stone in a vertically upward direction is 20m/s. So for vertical downward motion we will consider u = – 20m/s

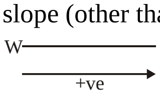
$$v^2 = u^2 + 2gh = (-20)^2 + 2 \times 9.8 \times 200 = 4320 \text{ m/s}$$

$\therefore\ v \simeq 65\,\text{m/s}.$

(92) **(B).** When velocity of A = velocity of B, then, relative velocity is zero.

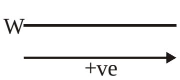
∴ Displacement-time graphs of A and B must have same slope (other than zero)

(93) **(A).**



Velocity of car A w.r.t. ground, $v_{AG} = 60$ km/h
Velocity of car B w.r.t. ground, $v_{BG} = 45$ km/h
Relative velocity of car A w.r.t. B
$v_{AB} = v_{AG} + v_{GB} = v_{AG} - v_{BG}$ $(\because v_{GB} = -v_{BG})$
= 60 km/h – 45 km/h = 15 km/h

(94) **(D).**



Velocity of car A w.r.t. ground, $v_{AG} = 60$ km/h
Velocity of car B w.r.t. ground, $v_{BG} = -45$ km/h
Relative velocity of car A w.r.t. B
$v_{AB} = v_{AG} + v_{GB} = v_{AG} - v_{BG}$ $(\because v_{GB} = -v_{BG})$
= 60 km/h – (– 45 km/h) = 105 km/h

(95) **(B).** Speed of combustion products w.r.t. observer on the ground = ?
Velocity of jet air plane w.r.t. observer on ground = 500 kmh^{-1}

If $\vec{v}_j$ and $\vec{v}_0$ represent the velocities of jet and observer respectively, then $v_j - v_0 = 500\ \text{kmh}^{-1}$

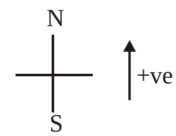
Similarly, if $\vec{v}_c$ represents the velocity of the combustion products w.r.t. jet plane, then

$v_c - v_j = -1500\ \text{km h}^{-1}$

The negative sign indicates that the combustion products move in a direction opposite to that of jet. Speed of combustion products w.r.t. observer

$$= v_c - v_0 = (v_c - v_j) + (v_j - v_0) = (-1500 + 500)\ \text{km h}^{-1}$$
$$= -1000\ \text{km h}^{-1}.$$

(96) **(D).** Let the positive direction of motion be from south to north.



Velocity of train A with respect to ground

$$v_{AG} = +54\ \text{km/h} = +54 \times \frac{5}{18}\ \text{m/s} = +15\ \text{m/s}$$

Velocity of train B with respect to ground

$$v_{BG} = -90\ \text{km/h} = -90 \times \frac{5}{18}\ \text{m/s} = -25\ \text{m/s}$$

Relative velocity of train A with respect to train B is
$v_{AB} = v_{AG} + v_{GB} = v_{AG} - v_{BG}$ $(\because v_{GB} = -v_{BG})$
= +15 m/s – (–25 m/s) = 40 m/s

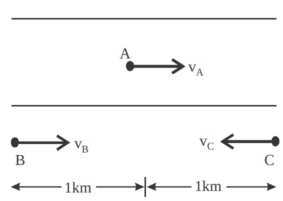
(97) **(B).** Let the velocity of the monkey with respect ground be v_{MG}.
Relative velocity of the monkey with respect to train A, $v_{MA} = -18\ \text{km/h} = -18 \times \frac{5}{18}\ \text{m/s} = -5\ \text{m/s}$

$v_{MG} = v_{MA} + v_{AG} = -5\ \text{m/s} + 15\ \text{m/s} = 10\ \text{m/s}$

(98) **(A).** $v_A = 36\ \text{km h}^{-1} = 36 \times \frac{5}{18}\ \text{ms}^{-1} = 10\text{ms}^{-1}$

$v_B = 54\ \text{km h}^{-1} = 54 \times \frac{5}{18}\ \text{ms}^{-1} = 15\text{ms}^{-1}$

$v_C = -15$ m/s



Relative velocity of B w.r.t. A,
$v_{BA} = v_B - v_A = 15 - 10 = 5\ \text{ms}^{-1}$
Relative velocity of C w.r.t. A,
$v_{CA} = v_C - v_A = -15 - 10 = -25\ \text{ms}^{-1}$

Time taken by C to cover distance AC $= \frac{1000\text{m}}{25\text{ms}^{-1}} = 40\text{s}$

In order to avoid an accident, the car B accelerates such that it overtakes car A in less than 40sec. Let the minimum required acceleration be a. Now, for B,

$$1000 = 5 \times 40 + \frac{1}{2} a \times 40 \times 40$$

On simplification, $a = 1\ ms^{-2}$.

EXERCISE-2

(1) **(A).** The average velocity, $v_{av} = \dfrac{\text{Total displacement}}{\text{Total time}}$

Total displacement, $s = ut + \frac{1}{2}at^2$

and total time = t $\therefore$ $v_{av} = \dfrac{ut + \frac{1}{2}at^2}{t} = u + \frac{1}{2}at$

(2) **(B).** $x = \sqrt{v+1}$; $x^2 = v + 1$; $v = (x^2 - 1)$

$$a = \frac{dv}{dt} = \frac{d}{dt}(x^2 - 1) = 2x\frac{dx}{dt} - 0$$

$= 2x\,v = 2x\,(x^2 - 1)$

At x = 5 m, $a = 2 \times 5\,(25 - 1) = 240\ m/s^2$

(3) **(A).** As $x \propto t^3$

Velocity, $v \propto 3t^2$

Acceleration, $a \propto 6t$

(4) **(D).** Given: $v = 2t\,(3 - t)$ or $v = 6t - 2t^2$

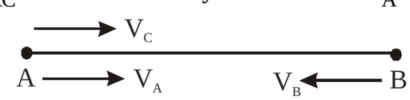
$\dfrac{dv}{dt} = 6 - 4t$. At maximum velocity,

$\dfrac{dv}{dt} = 0$ $\therefore$ $6 - 4t = 0$ or $t = (3/2)$ s

(5) **(D).** For uniform motion with zero acceleration, v-t graph is a straight line parallel to the time axis.

(6) **(B).** Let Bus A leaves town A and bus B leaves town B at regular intervals. Let C represents the cyclist and V_A, V_B and V_C are velocities of bus A, bus B and the cyclist respectively.

V_{AC} = Relative velocity of A w.r.t. C = $V_A - V_C$



Similarly, $V_{BC} = V_B - V_C$

Let T = Time interval at which buses are leaving from town A and B.

The distance between two buses plying in the same direction at the same constant speed will remain the same whether measured by an observer moving at some constant speed or by a standing observer.

The distance between two consecutive buses A for an observer standing on ground = $V_A T$ ....(1)

This distance as measured by the cyclist

= V_{AC} T', where, T' = Time interval between two consecutive buses for the cyclist = 18 minutes

$\therefore$ Distance between two consecutive A-buses for the cyclist = 18 V_{AC} = 18 $(V_A - V_C)$ .... (2)

$\therefore$ $V_A T = 18\,(V_A - V_C)$ .... (3)

Similarly, $V_B T = 6\,(V_B + V_C)$ .... (4)

[$V_{BC} = |V_B| + |V_C|$, because B and C are moving in opposite directions]

Given, $|V_A| = |V_B| = V$, say and $|V_C| = 20$ km/hr

$\therefore$ Equation (3) and (4) become

V.T = 18 (V – 20) .... (5)

V.T = 6 (V + 20) .... (6)

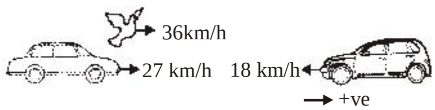
$\therefore$ 18 (V – 20) = 6 (V + 20)

18V – 360 = 6V + 120

12V= 480 $\Rightarrow$ V = 40 km/hr

Putting it in eq. (5) we get, T = 9 mins.

(7) **(A).**



Velocity of car A, v_A = + 27 km/h

Velocity of car B, v_B = –18 km/h

Relative velocity of car A with respect to car B

= $v_A - v_B$ = + 27 km/h – (–18 km/h) = 45 km/h

Time taken by the two cars to meet

$$= \frac{36\ km}{45\ km/h} = 0.8\ h$$

Thus, distance covered by the bird

= 36 km/h × 0.8 h = 28.8 km

(8) **(A).** Here, u = 0, g = 10 m/s^2, h=1km=1000 m

As $v^2 - u^2 = 2gh$ $\therefore$ $v^2 = 2gh$

or $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000} = 100\sqrt{2}\ m/s$

$$= 100\sqrt{2} \times \frac{18}{5}\ km/h$$

$$= 360\sqrt{2}\ km/h = 510\ km/h$$

(9) **(B).** The maximum distance covered in time T = v_0T. Therefore, for the object having one dimensional motion the displacement x in time T satisfies $-v_0T < x < v_0T$.

(10) **(D).** Speed of police van, $v_p = 30\ km\ h^{-1}$

$$= \frac{30 \times 1000}{3600} ms^{-1} = \frac{25}{3} ms^{-1}$$

Speed of thief's car, $v_t = 192\ km\ h^{-1}$

$$= \frac{192 \times 1000}{3600} ms^{-1} = \frac{160}{3} ms^{-1}$$

Speed of bullet, v_b = speed of police van + speed with which bullet is actually fired

$$\therefore\ v_b = \left(\frac{25}{3} + 150\right) ms^{-1} = \frac{475}{3} ms^{-1}$$

Relative velocity of bullet w.r.t. thief's car,

$$v_{bt} = v_b - v_t = \left(\frac{475}{3} - \frac{160}{3}\right) ms^{-1} = 105\ ms^{-1}$$

(11) **(B).** Time taken by the boy to go from his home to the market, $t_1 = \dfrac{2.5\text{ km}}{5\text{ km h}^{-1}} = \dfrac{1}{2}\text{h}$

Time taken by the boy to return back from the market to his home, $t_2 = \dfrac{2.5\text{ km}}{7.5\text{ km h}^{-1}} = \dfrac{1}{3}\text{h}$

$\therefore$ Total time taken = $t_1 + t_2$

$\dfrac{1}{2}\text{h} + \dfrac{1}{3}\text{h} = \dfrac{5}{6}\text{h} = 50\text{ min}$

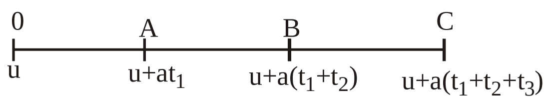
In t = 0 to 50 min, Total distance travelled
$= 2.5\text{ km} + 2.5\text{ km} = 5\text{ km}$
Displacement = 0
(As the boy returns back home)

$\therefore$ Average speed

$$= \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{5\text{ km}}{(5/6)\text{ h}} = 6\text{ km/h}$$

Average velocity $= \dfrac{\text{Displacement}}{\text{Time taken}} = 0$

(12) **(D).** Let u be initial velocity and a be uniform acceleration.



Average velocities in the intervals from 0 to t_1, t_1 to t_2 and t_2 to t_3 are

$$v_1 = \frac{u + u + at_1}{2} = u + \frac{a}{2}t_1 \quad ..... (1)$$

$$v_2 = \frac{u + at_1 + u + a(t_1 + t_2)}{2} = u + at_1 + \frac{a}{2}t_2 \quad ..... (2)$$

$$v_3 = \frac{u + a(t_1 + t_2) + u + a(t_1 + t_2 + t_3)}{2}$$

$$= u + at_1 + at_2 + \frac{a}{2}t_3 \quad ..... (3)$$

Subtract (1) from (2), we get

$$v_2 - v_1 = \frac{a}{2}(t_1 + t_2) \quad ..... (4)$$

Subtract (2) from (3), we get

$$v_3 - v_2 = \frac{a}{2}(t_2 + t_3) \quad ..... (5)$$

Divide (4) by (5), we get

$$\frac{v_2 - v_1}{v_3 - v_2} = \frac{t_1 + t_2}{t_2 + t_3} \text{ or } \frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 + t_2}{t_2 + t_3}$$

(13) **(B).** Let a be constant acceleration of auto.
Here, u = 30 m/s, v = 50 m/s, S = 180 m
As $v^2 - u^2 = 2aS$
$(50)^2 - (30)^2 = 2 \times a \times 180$
$(2500) - (900) = 2 \times a \times 180$

$a = \dfrac{1600}{2 \times 180} = \dfrac{40}{9}\text{ m/s}^2$. As $S = ut + \dfrac{1}{2}at^2$

$$180 = 30 \times t + \frac{1}{2} \times \frac{40}{9} \times t^2$$

$$180 = 30t + \frac{20}{9}t^2 \;;\; 18 = 3t + \frac{2}{9}t^2$$

$$\frac{2}{9}t^2 + 3t - 18 = 0 \;;\; 2t^2 + 27t - 162 = 0$$

Solving this quadratic equation by quadratic formula,

we get $t = \dfrac{-27 \pm \sqrt{(27)^2 - 4(2)(-162)}}{4} = 4.5, -18$

t can't be negative $\quad \therefore t = 4.5$ s

(14) **(A).** The distance is equal to total area under v-t graph

$$= \frac{20 \times 2}{2} + 20 \times 2 + 20 \times 1 + \frac{20 \times 1}{2} + \frac{20 \times 1}{2}$$

$= 20 + 40 + 20 + 10 + 10 = 100$ m

(15) **(D).** Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}}$

A particle moving in a given direction with non-zero velocity cannot have zero speed.
In general, average speed is not equal to magnitude of average velocity. However it can be so if the motion is along a straight line without change in direction.

(16) **(A).** x = t – sin t

$$v = \frac{dx}{dt} = 1 - \cos t \;;\; a = \frac{dv}{dt} = \sin t$$

$\therefore$ x (t) > 0 for all values of t > 0 and v (t) can be zero for one value of t. a (t) can zero for one value of t.

(17) **(A).** Time taken by body A, $t_1 = 5$ s
Acceleration of body A = a_1
Time taken by body B, $t_2 = 5 - 2 = 3$ s
Acceleration of body B = a_2
Distance covered by first body in 5th second after its start,

$$S_5 = u + \frac{a_1}{2}(2t_1 - 1) = 0 + \frac{a_1}{2}(2 \times 5 - 1) = \frac{9}{2}a_1$$

Distance covered by the second body in the 3rd second after its start,

$$S_3 = u + \frac{a_2}{2}(2t_2 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1) = \frac{5}{2}a_2$$

Since $S_5 = S_3 \therefore \dfrac{9}{2}a_1 = \dfrac{5}{2}a_2$ or $a_1 : a_2 = 5 : 9$

(18) **(D).** Let v_s be the velocity of scooter. The distance between the scooter and the bus $= 1\text{ km} = 1000\text{ m}$.
The velocity of bus, $v_b = 10$ m/s
Time taken to overtake the bus, t = 100 s.
Relative velocity of the scooter w.r.t. the bus
$= (v_s - 10)$

$$\therefore \quad t = \frac{1000}{v_s - 10} = 100 \text{ or } v = 20\text{ m/s}$$

(19) **(A).** Taking upwards motion of ball A for time t, velocity is $v_A = u - gt$.

Taking downwards motion of ball B for time t, its velocity is $v_B = gt$.

$\therefore$ Relative velocity of A w.r.t. B

$= v_{AB} = v_A - (-v_B) = (u - gt) - (-gt) = u$

(20) **(B).** In the graph (B), for one value of displacemet, there are two timings. As a result of it, for one time, the average velocity is positive and for other time is equal but negative. Due to it the average velocity for timings (equal to time period) can vanish.

(21) **(C).** Let L be the length of escalator.

Velocity of girl w.r.t. escalator, $v_{ge} = L/t_1$

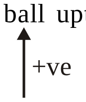
Velocity of escalator, $v_e = L/t_2$

Velocity of girl w.r.t. ground would be

$$v_g = v_{ge} + v_e = L\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

The desired time is $t = \dfrac{L}{v_g} = \dfrac{L}{L\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)} = \dfrac{t_1 t_2}{t_1 + t_2}$

(22) **(C).** Taking vertical upward motion of the ball upto highest point.



Here, $u = 20\ m\ s^{-1}$

$v = 0$ (At highest point velocity is zero)

$a = -g = -10\ ms^{-2}$

As $v^2 = u^2 + 2aS$; $0 = (20)^2 + 2(-10)(S)$

$$S = \frac{20 \times 20}{20} = 20\ m$$

(23) **(C).** Let t_1 be the time taken by the ball to reach the highest point.

$v = 0$, $u = 20$ m/s, $a = -g = -10\ m/s^2$,

$t = t_1$ As $v = u + at$

$\therefore$ $0 = 20 + (-10)\,t_1$ or $t_1 = 2s$

Taking vertical downward motion of the ball from the highest point to ground.

Here, $u = 0$, $a = +g = 10\ m\ s^{-2}$,

$S = 20\ m + 25\ m = 45m$, $t = t_2$

As $S = ut + \frac{1}{2}at^2$ $\therefore$ $45 = 0 + \frac{1}{2}(10)\,t_2^2$

$$t_2^2 = \frac{45 \times 2}{10} = \frac{90}{10} = 9 \text{ or } t_2 = 3s$$

Total time taken by the ball to reach the ground

$= t_1 + t_2 = 2s + 3s = 5s$

(24) **(A).** Here, $a = \dfrac{v-u}{t} = \dfrac{v-0}{n} = \dfrac{v}{n}$

Displacement in last 2 sec.

$$S_n - S_{n-2} = \frac{1}{2}an^2 - \frac{1}{2}a\,(n-2)^2$$

$$= 2a\,(n-1) = 2\frac{v}{n}(n-1) = \frac{2v\,(n-1)}{n}$$

(25) **(A).** Here, $a = g - bv$

When an object falls with constant speed v_c, in acceleration becomes zero.

$\therefore$ $g - bv_c = 0$ or $v_c = g/b$

(26) **(D).** $S_3 = u + \frac{a}{2}(2 \times 3 - 1) = 4$ or $u + \frac{5}{2}a = 4$

$$S_5 = u + \frac{a}{2}(2 \times 5 - 1) = 12 \text{ or } u + \frac{9}{2}a = 12$$

On solving, $u = -6$ m/s, $a = 4\ m/s^2$

Distance travelled in next 3 seconds

$$= S_8 - S_5 = [-6 \times 8 + \frac{1}{2} \times 4 \times (8)^2]$$

$$-[-6 \times 5 + \frac{1}{2} \times 4 \times (5)^2] = 80 - 20 = 60m$$

(27) **(D).** Here, $u_A = 0$, $u_B = +50$ m/s

$a_A = -g$, $a_B = -g$

$u_{BA} = u_B - u_A = 50\ m/s - 0\ m/s = 50\ m/s$

$a_{BA} = a_B - a_A = -g - (-g) = 0$

$v_{BA} = u_{BA} + a_{BA}t$ (As $a_{BA} = 0$)

$\therefore$ $v_{BA} = u_{BA}$

As there is no acceleration of ball B w.r.t to ball A, therefore the relative speed of ball B w.r.t ball A at any instant of time remains constant (= 50 m/s).

(28) **(C).** Initial relative velocity $= v_1 - v_2$,

Final relative velocity = 0

From $v^2 = u^2 - 2as$

$$\Rightarrow 0 = (v_1 - v_2)^2 - 2 \times a \times s \Rightarrow s = \frac{(v_1 - v_2)^2}{2a}$$

If the distance between two cars is 's' then collision will take place. To avoid collision $d > s$

$\therefore$ $d > \dfrac{(v_1 - v_2)^2}{2a}$, where d = actual initial distance between two cars.

(29) **(D).** $3t = \sqrt{3x} + 6 \Rightarrow 3x = (3t - 6)^2 \Rightarrow x = 3t^2 - 12t + 12$

$v = \dfrac{dx}{dt} = 6t - 12$, for $v = 0$, $t = 2$ sec.

$x = 3(2)^2 - 12 \times 2 + 12 = 0$

(30) **(C).** $v^2 = 180 - 16x$

Compare with $v^2 = u^2 - 2ax$; $a = -8\ m/s^2$

(31) **(D).** Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

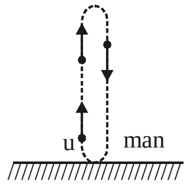
$$= \frac{x}{\dfrac{2x/5}{v_1} + \dfrac{3x/5}{v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2}$$

(32) **(C).** Since direction of v is opposite to the direction of g and h so from equation of motion

$$h = -vt + \frac{1}{2}gt^2 \Rightarrow gt^2 - 2vt - 2h = 0$$

$$\Rightarrow t = \frac{2v \pm \sqrt{4v^2 + 8gh}}{2g} \Rightarrow t = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$$

(33) **(A).** In this case time of flight of a ball ≥ 2 × 2 = 4 sec.



∵ Time of flight = $\frac{2u}{g} \geq 4$

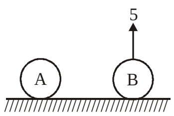
$\Rightarrow$ $u \geq 2g$

$\Rightarrow$ $u \geq 19.6$ m/s (∵ $g = 9.8$ m/s^2)

(34) **(B).** ∵ Average velocity × time = distance

∴ $\left(\frac{10+20}{2}\right)(t) = 135 \Rightarrow t = 9s$

(35) **(C).**



Relative acceleration,

$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$

Also, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s

As relative acceleration is zero we can use

$\vec{s}_{BA}$ (in 1 sec) = $\vec{v}_{BA} \times t = 5 \times 1 = 5m$

∴ Distance between A & B after 1 sec = 5m

(36) **(A).** $S = \int_0^3 v\,dt = \int_0^3 kt\,dt = \left[\frac{1}{2}kt^2\right]_0^3 = \frac{1}{2} \times 2 \times 9 = 9m$

(37) **(A).** $S_n = u + \frac{a}{2}[2n-1]$

$S_{5^{th}} = 7 + \frac{4}{2}[2 \times 5 - 1] = 7 + 18 = 25m$.

(38) **(C).** $S_n = u + \frac{a}{2}(2n-1) \Rightarrow \quad 1.2 = 0 + \frac{a}{2}(2 \times 6 - 1)$

$\Rightarrow \quad a = \frac{1.2 \times 2}{11} = 0.218\ m/s^2$

(39) **(A).** Velocity acquired by body in 10sec

$v = 0 + 2 \times 10 = 20m/s$

and distance travelled by it in 10 sec

$S_1 = \frac{1}{2} \times 2 \times (10)^2 = 100$ m then

it moves with constant velocity (20 m/s) for 30 sec

$S_2 = 20 \times 30 = 600m$

After that due to retardation (4m/s^2) it stops

$S_3 = \frac{v^2}{2a} = \frac{(20)^2}{2 \times 4} = 50m$

Total distance travelled $S_1 + S_2 + S_3 = 750m$

(40) **(A).** The velocity of the particle is

$\frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2) = (0 - 5 + 12t)$

For initial velocity t = 0, hence v = – 5m/s.

(41) **(C).** For a–t curve, area under give change in velocity at t = 10 sec, v = 40 m/s

For 10 – 30 sec, Δv = – 80,

$v_{30sec} - 40 = -80$

Speed at 30 sec = – 40 m/s

(42) **(D).** Maximum acceleration means maximum change in velocity in minimum time interval.

In time interval t = 30 to t = 40 sec

$a = \frac{\Delta v}{\Delta t} = \frac{80-20}{40-30} = \frac{60}{10} = 6\ cm/sec^2$

EXERCISE-3

(1) **3.** Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is given by $\tan\theta = \frac{v_h}{v_0} = \frac{13.9\ m/s}{8.0\ m/s} = 1.74$. The angle is 60°.

(2) **16.** Average speed = $\frac{100+300}{10+\frac{300}{20}} = \frac{400}{25} = 16\ m/s$

(3) **21.** $x_A = x_B$; $10.5 + 10t = \frac{1}{2}at^2$ Þ $a = \tan 45° = 1$

$t^2 - 20t - 21 = 0$ Þ $t = \frac{20 \pm \sqrt{400+84}}{2}$ Þ $t = 21$ sec.

(4) **2.** $y = bx^2$

$\frac{dy}{dt} = 2bx.\frac{dx}{dt} \Rightarrow \frac{d^2y}{dx^2} = 2b\left(\frac{dx}{dt}\right)^2 + 2bx\frac{d^2x}{dt^2}$

$a = 2bv^2 + 0 \Rightarrow v = \sqrt{\frac{a}{2b}}$

(5) **1.** $v_{av} = \frac{\Delta s}{\Delta t}$; u → Initial velocity for last 2 metre

$0 = u^2 - 2 \times 1 \times 2$; u = 2 m/s

$2 = \left(\frac{2+0}{2}\right)t \Rightarrow t = 2$ sec ; $v_{av} = \frac{2}{2} = 1\ m/s$

(6) **4.** For downstream relative distance travelled by cork $x_1 = v_r t$ and for upstream relative distance travelled by cork $x_2 = v_r t$

$1\ km = 2v_r \times \frac{7.5}{60} \Rightarrow v_r = 4\ km/hr$

(7) **2.** Acceleration of the particle a = 2t – 1

The particle retards when acceleration is opposite to velocity.

$\Rightarrow a\,.\,v < 0 \Rightarrow (2t-1)(t^2 - t) < 0 \Rightarrow t(2t-1)(t-1) < 0$

Now t is always positive ∴ $(2t-1)(t-1) < 0$

or $2t - 1 < 0$ and $t - 1 > 0 \Rightarrow t < 1/2$ and $t > 1$.

This is not possible

or $2t - 1 > 0$ & $t - 1 < 0 \Rightarrow 1/2 < t < 1$

(8) Given $u = \alpha x$ .... (1)

$$\frac{dx}{dt} = \alpha.x \quad \text{or} \quad \int \frac{dx}{x} = \int \alpha\, dt$$

$\ln x = \alpha t + C$

as $t = 0,\ x = 0 \Rightarrow C = 0$

$\therefore\ x = e^{\alpha t}$ .... (2)

Again diff. eq. (1) with respect to t, we get

$$a = \frac{dv}{dt} = \alpha.1\frac{dv}{dt} = \alpha.v = \alpha^2 e^{\alpha t} \quad \text{......... (3)}$$

If T time taken to travel distnace S, then

$$S = e^{\alpha t} \text{ or } T = \frac{\ln s}{\alpha}$$

$$\text{Again, } v_{avg} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{T}\int_0^T \alpha\, e^{\alpha t} dt = \frac{e^{\alpha T}}{T} = \frac{\alpha s}{\ln s}$$

$$a_{avg} = \frac{1}{T}\int_0^T a\,dt = \frac{1}{T}\int_0^T \alpha^2 e^{\alpha t} dt = \frac{\alpha e^{\alpha T}}{T}$$

$$a_{avg} = \frac{\alpha^2 s}{\ln s} \quad ; \quad v_{avg} = a_{avg} \quad ;$$

$$\frac{\alpha s}{\ln s} = \frac{\alpha^2 S}{\ln s} \Rightarrow \alpha = 1$$

(9) (4), (10) (8).

a is maximum when v change its sign.

Area of at-graph = 0

(11) **5.** $10 \cos 60° = 5$ m/s

EXERCISE-4

(1) **(B).** Both will reach with same speed.

(2) **(A).** $v^2 = u^2 + 2as$

$$\frac{u^2}{4} = u^2 + 2a(3) \ ;\ 6a = \frac{-3u^2}{4} \Rightarrow a = \frac{-u^2}{8}$$

and $v^2 = u^2 + 2as$

$$0 = u^2 + 2\left(-\frac{u^2}{8}\right)s' \ ;\ s' = 4\text{cm}$$

Distance travelled further = 4 – 3 = 1 cm.

(3) **(C).** $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g - a$

(4) **(D).** $v^2 = u^2 + 2as$; $0 = u^2 + 2(-a)s$

$$s = \frac{u^2}{2a} \ ;\ s \propto u^2$$

(5) **(A).** $v^2 = u^2 + 2as$; $0 = u^2 + 2(-a)s$

$$s = \frac{u^2}{2a} \ ; s \propto u^2$$

So, distance travelled before coming to rest = 24m.

(6) **(B).** $x = \alpha t^3,\ y = \beta t^3$

$$\frac{dx}{dt} = 3\alpha t^2,\ \frac{dy}{dt} = 3\beta t^2$$

So resultant velocity $v = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2}$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

(7) **(B).** $P = Fv$; $mav = P$

$$\int mv\,dv = \int P dt \ ;\ \frac{mv^2}{2} = Pt \ ;\ v \propto \sqrt{t}$$

$$\int dx = \int k\sqrt{t}\,dt \ ;\ x \propto t^{3/2}$$

(8) **(D).** Net force = 0 $\quad\therefore\ \vec{v}$ = const.

(9) **(A).** According to problem, $a = -kx$

$$v\frac{dv}{dx} = -kx\ ;\ \int v dv = -k\int x dx\ ;\ \frac{v^2}{2} = -k\frac{x^2}{2}$$

So, kinetic energy $\propto x^2$

(10) **(C).** Initially $s = ut + \frac{1}{2}at^2$; $h = 0 + \frac{1}{2}gT^2$

at time = T/3

$$h' = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{9}\left(\frac{1}{2}gT^2\right) \ ;\ h' = \frac{h}{9}$$

So height above ground = $h - \frac{h}{9} = \frac{8h}{9}$

(11) **(D).** $v^2 = u^2 + 2as$; $0 = u^2 + 2(-a)s$

$$s = \frac{u^2}{2a} \Rightarrow s \propto u^2$$

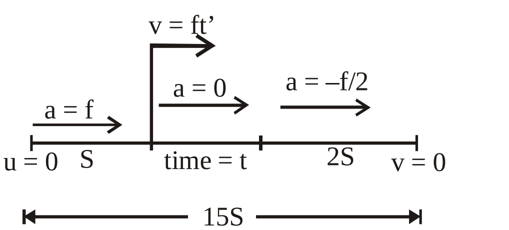
(12) **(A).** $t = ax^2 + bx$. So, $\frac{dt}{dx} = 2ax + b$

So velocity $v = \frac{1}{2ax + b}$ .....(1)

and $a = \frac{dv}{dt} = -\frac{(2a)}{(2ax+b)^2}\frac{dx}{dt}$; $a = -\frac{2a}{(2ax+b)^2}v$

From equation (1), $a = -2av^3$

(13) **(A).**



If time taken in first part is t', then

$$S = 0 + \frac{1}{2}ft^2 \quad \text{.......(i)}$$

then distance traveled in last part would be = 2S

So the distance up to which particle move with constant velocity = 15S – 3S = 12S

So $12S = (ft')\,t$

$$12\left(\frac{1}{2}ft'^2\right) = f\,tt'\;;\;6t' = t$$

From equation (i), $S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$

(14) **(B).** Initial velocity $\vec{v}_i = 5\hat{i}$; Final velocity $\vec{v}_f = 5\hat{j}$

Average acceleration $a = \frac{\vec{v}_f - \vec{v}_i}{t}$

$$\vec{a} = \frac{5\hat{i} - 5\hat{j}}{10} = \frac{1}{2}(\hat{j} - \hat{i})\;;\;|\vec{a}| = \frac{1}{\sqrt{2}}, \text{ direction} = N - W$$

(15) **(C).** $v = \alpha\sqrt{x}$

$$\frac{dx}{\sqrt{x}} = \alpha\,dt\;;\;\int_{x=0}^{x} x^{-1/2}dx = \int_{t=0}^{t}\alpha\,dt$$

$$\frac{x^{1/2}}{1/2} = \alpha t\;;\;x \propto t^2$$

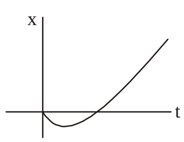
(16) **(A).** $x_1 = 0 + \frac{1}{2}at^2$; $x_2 = ut$

$$x = x_1 - x_2 = \frac{1}{2}at^2 - ut$$

at t = 0, x = 0 and $\frac{1}{2}at^2 - ut = 0$

$x = 0,\; t = \frac{2u}{a}$. Slope $\frac{dx}{dt} = at - u$

So the graph of x and t



(17) **(C).** $V_x = 3 + (0.4)(10) = 7$ units
$V_y = 4 + (0.3)(10) = 7$ units
Speed = $7\sqrt{2}$ units

(18) **(D).** $mg\sin\theta = ma \quad \therefore a = g\sin\theta$
where a is along the inclined plane
$\therefore$ vertical component of acceleration is $g\sin^2\theta$
$\therefore$ relative vertical acceleration of A with respect to B is

$$g[\sin^2 60 \sin^2 30] = \frac{g}{2} = 4.9\text{ m/s}^2 \text{ in vertical direction}$$

(19) **(B).** $\int_{6.25}^{0}\frac{dv}{\sqrt{v}} = -2.5\int_{0}^{t}dt\;;\;\left|2\sqrt{v}\right|_{6.25}^{0} = -2.5t$

$2\sqrt{6.25} = 2.5t$; t = 2 sec.

(20) **(A).** Time to reach the maximum height, $t_1 = \frac{u}{g}$

If t_2 be the time taken to hit the ground

$$-H = ut_2 - \frac{1}{2}gt_2^2$$

But $t_2 = nt_1$ (given) $\Rightarrow -H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$

$\Rightarrow\; 2gH = nu^2(n-2)$

(21) **(B).** Till both are in air (From t = 0 to t = 8 sec)
$\Delta x = x_2 - x_1 = 30t \Rightarrow \Delta x \propto t$. When second stone hits ground and first stone is in air Δx decreases.

(22) **(B).** During the whole journey acceleration remains constant $(a = -g) \Rightarrow V = V_0 - gt$

(23) **(D).** The (A), (B) and (C) graphs can represent the motion of a ball that is thrown in vertically upward direction. Initially speed decreases, becomes zero and then on the return trip, speed increases. Slope of graph in option (D) does not explain it.

(24) **(B).** $\frac{dx}{dt} = ky,\;\; \frac{dy}{dt} = kx$;

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$$

Integrating both side, $y^2 = x^2 + c$

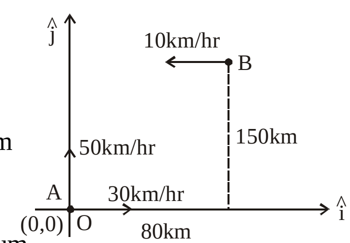
(25) **(D).** If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as

$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km/hr

$\vec{v}_B = -10\hat{i}$ km/hr

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = (80\hat{i} + 150\hat{j})$ km



Time after which distance between them will be minimum

$$t = -\frac{\vec{r}_{BA}\cdot\vec{v}_{BA}}{|\vec{v}_{BA}|^2}, \text{ where, } \vec{r}_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$\vec{v}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j})$

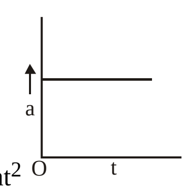
$(-40\hat{i} - 50\hat{j})$ km/hr

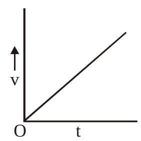
$$\therefore\quad t = -\frac{(80\hat{i} + 150\hat{j})\cdot(-40\hat{i} - 50\hat{j})}{|-40\hat{i} - 50\hat{j}|^2}$$

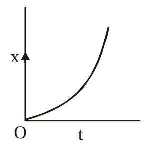
$$= \frac{3200 + 7500}{4100}\text{hr} = \frac{10700}{4100}\text{hr} = 2.6 \text{ hrs}$$

(26) **(C).** Given initial velocity u = 0
and acceleration is constant
At time t, $v = 0 + a\,t \Rightarrow v = at$

Also $x = 0\,(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$







Graph (a) ; (b) and (d) are correct.

(27) **580.00**

$x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$; $(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$; $(v_y)_{t=1} = -24\hat{j}$

$v' = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$; $v = 580$

(28) **8.** Time to travel 81 m is t sec.

Time to travel 100 m is $t + \frac{1}{2}$ sec.

$$81 = \frac{1}{2} \times a \times t^2 \Rightarrow t = 9\sqrt{\frac{2}{a}}$$

$$100 = \frac{1}{2} \times a \times \left(t + \frac{1}{2}\right)^2 \Rightarrow t + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$$

$$9\sqrt{\frac{2}{a}} + \frac{1}{2} = 10\sqrt{\frac{2}{a}} \;;\; \frac{1}{2} = \sqrt{\frac{2}{a}} \;;\; a = 8\ m/s^2$$

(29) **(C).** $x = u_x t + \frac{1}{2} a_x t^2$; $y = u_y t + \frac{1}{2} a_y t^2$

$$32 = 0 \times t + \frac{1}{2}(4)\,(t)^2 \;;\; t^2 = 16 \;;\; t = 4\text{ sec}$$

$$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 12 + 48 = 60\text{m}$$

EXERCISE-5

(1) **(A).** Let the two balls meet after t s at distance x from the platform. For the first ball, u = 0, t = 18s, $g = 10\ m/s^2$

Using $h = ut + \frac{1}{2}gt^2$ $\therefore$ $x = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (18)^2$ ....(i)

For the second ball, u = u, t = 12s, $g = 10\ m/s^2$

$x = 12u + (1/2) \times 10 \times 12^2$ ....(ii)

From equations (i) and (ii), we get

$\frac{1}{2} \times 10 \times 18^2 = 12u + \frac{1}{2} \times 10 \times (12)^2$

or $12u = (1/2) \times 10 \times [(18)^2 - (12)^2]$

$= (1/2) \times 10 \times [(18 + 12)\,(18 - 12)]$

$12u = (1/2) \times 10 \times 30 \times 6$

or $u = \frac{1 \times 10 \times 30 \times 6}{2 \times 12} = 75\text{m/s}$

(2) **(B).** Here, Initial velocity, $\vec{u} = 3\hat{i} + 4\hat{j}$

Acceleration, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, Time, t = 10s

Let $\vec{v}$ be velocity of a particle after 10s.

$\vec{v} = \vec{u} + \vec{a}t$ $\therefore$ $\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})\,(10)$

$3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

Speed of the particle after 10s

$= |\vec{v}| = \sqrt{(7)^2 + (7)^2} = 7\sqrt{2}$ units

(3) **(A).** Distance, $x = (t + 5)^{-1}$ ...(i)

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t + 5)^{-1} = -(t + 5)^{-2}$ ...(ii)

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[-(t + 5)^{-2}] = 2\,(t + 5)^{-3}$ ...(iii)

From equation (ii), we get

$v^{3/2} = -(t + 5)^{-3}$ ...(iv)

Substituting this in equation (iii) we get

Acceleration, $a = -2v^{3/2}$

or $a \propto (\text{velocity})^{3/2}$ ...(v)

(4) **(C).** $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20\text{m/s}$

(5) **(A).** $a = \frac{|\vec{v}_f - \vec{v}_i|}{t} = \frac{\sqrt{30^2 + 40^2}}{10} = 5\ m/s^2$

(6) **(B).** $V_{av} = \frac{S + S}{\frac{S}{v_1} + \frac{S}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$

(7) **(B).** $\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j}) \times 10 = 5\hat{i} + 5\hat{j}$

$|\vec{v}| = 5\sqrt{2}$

(8) **(D).** $X = 8 + 12t - t^3$; $V = 0 + 12 - 3t^2 = 0$

$3t^2 = 12$; t = 2sec ; a = dv/dt = 0 – 6t

a [t = 2] = –12 m/s^2 ; Retardation = 12 m/s^2

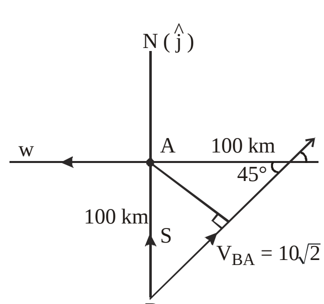
(9) **(C).** $h_1 = \frac{1}{2}g\,(5)^2$, $h_2 = \frac{1}{2}g\,(10)^2$

and $h_3 = \frac{1}{2}g\,(15)^2 \Rightarrow h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

(10) **(D).** $\vec{V}_{av} = \frac{(x_2 - x_1)\,\hat{i} + (y_2 - y_2)\,\hat{j}}{t_2 - t_1}$

$= \frac{(13 - 2)\,\hat{i} + (14 - 3)\,\hat{j}}{5 - 0} = \frac{11\hat{i} + 11\hat{j}}{5} = \frac{11}{5}(\hat{i} + \hat{j})$

(11) **(A).**



$V_A = 10(-\hat{i}),\ V_B = 10(\hat{j})\ ;\ V_{BA} = 10\hat{j} + 10\hat{i}$

Time for shortest distance $= \frac{100/\sqrt{2}}{10\sqrt{2}} = 5$

(12) **(A).** $V(x) = bx^{-2n}$

$a = v\frac{dv}{dx} = bx^{-2n}\{b(-2n)x^{-2n-1}\} = -2b^2nx^{-4n-1}$

(13) **(C).** $v = At + Bt^2$

$\Rightarrow\ dx/dt = At + Bt^2 \Rightarrow dx = (At + Bt^2)\,dt$

$\Rightarrow\ x = \left[\frac{At^2}{2} + \frac{Bt^3}{3}\right]_1^2 = \frac{A}{2}(4-1) + \frac{B}{3}(8-1) = \frac{3}{2}A + \frac{7}{3}B$

(14) **(D).** $v_P = \frac{dx_P}{dt} = a + 2bt\ ;\ v_Q = \frac{dx_Q}{dt} = f - 2t$

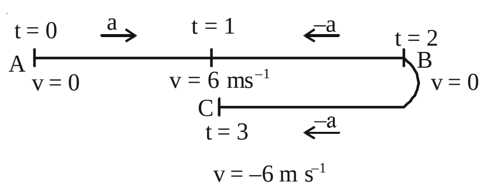
$v_P = v_Q \Rightarrow a + 2bt = f - 2t$

$2t + 2bt = f - a \Rightarrow t = \frac{f-a}{2(b+1)}$

(15) **(B).** $V_1 \to$ velocity of Preeti ; $V_2 \to$ velocity of escalator

$\ell \to$ distance ; $t = \frac{\ell}{V_1 + V_2} = \frac{\ell}{\frac{\ell}{t_1} + \frac{\ell}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

(16) **(B).**



Acceleration, $a = \frac{6-0}{1} = 6\ ms^{-2}$

For t = 0 to t = 1 s, $S_1 = \frac{1}{2} \times 6\,(1)^2 = 3m$ ...(i)

For t = 1 s to t = 2 s, $S_2 = 6 \times 1 - \frac{1}{2}6\,(1)^2 = 3m$ ...(ii)

For t = 2 s to t = 3 s, $S_3 = 0 - \frac{1}{2} \times 6\,(1)^2 = -3m$ ...(iii)

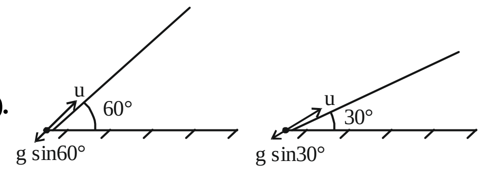
Total displacement $S = S_1 + S_2 + S_3 = 3$ m

Average velocity= 3/3 = 1m/s,

Total distance travelled =9m

Average speed = 9/3 = 3 m/s

(17) **(C).**



(Stopping distance) $x_1 = \frac{u^2}{2g\sin 60°}$

(Stopping distance) $x_2 = \frac{u^2}{2g\sin 30°}$

$\frac{x_1}{x_2} = \frac{\sin 30°}{\sin 60°} = \frac{1 \times 2}{2 \times \sqrt{3}} = 1 : \sqrt{3}$

(18) **(A).** $V_{SR} = 20$ m/s, $V_{RG} = 10$ m/s

$\vec{V}_{SG} = \vec{V}_{SR} + \vec{V}_{RG}$

$\sin\theta = \left|\frac{\vec{V}_{RG}}{\vec{V}_{SR}}\right| = \frac{10}{20} = \frac{1}{2}$

θ = 30° west