4

QUADRATIC EQUATION AND INEQUALITIES

QUADRATIC EXPRESSION

A Polynomial of degree two of the form $ax^2 + bx + c\ (a \neq 0)$ is called a quadratic expression in x.

Ex $3x^2 - 7x + 5,\ x^2 - 7x + 3$

General form : $f(x) = ax^2 + bx + c$, where $a, b, c \in C\ \&\ a \neq 0$

QUADRATIC EQUATION

A quadratic Polynomial f (x) when equated to zero is called Quadratic Equation.

Ex. $3x^2 - 7x + 5 = 0, -9x^2 + 7x - 5 = 0, -x^2 + 2x = 0, 2x^2 = 0$

General form $ax^2 + bx + c = 0$

where $a, b, c \in C$ and $a \neq 0$

Roots of a Quadratic Equation : The values of variable x which satisfy the quadratic equation is called as Roots (also called solutions or zeros) of a Quadratic Equation.

SOLUTION OF QUADRATIC EQUATION

Factorization Method: Let $ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$

Then $x = \alpha$ and $x = \beta$ will satisfy the given equation

Hence factorize the equation and equating each to zero gives roots of equation.

Ex. $3x^2 - 2x - 1 = 0, (x - 1)(3x + 1) = 0;\quad x = 1, -1/3$

Hindu Method (Shri Dharacharya Method) :

Quadratic equation $ax^2 + bx + c = 0\ (a \neq 0)$ has two roots,

given by $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Note : If one roots is k times the other root of quadratic equation $ax^2 + bx + c = 0$ then $\dfrac{(k+1)^2}{k} = \dfrac{b^2}{ac}$

SUM AND PRODUCT OF ROOTS

If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then

(i) Sum of Roots $S = \alpha + \beta = -\dfrac{b}{a} = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

(ii) Product of Roots $P = \alpha\beta = \dfrac{c}{a} = \dfrac{\text{cons tan t term}}{\text{coefficient of } x^2}$

Ex. In equation $3x^2 + 4x - 5 = 0$

Sum of Roots $S = -\dfrac{4}{3}$, Product of roots $P = -\dfrac{5}{3}$

NATURE OF ROOTS

In Quadratic equation $ax^2 + bx + c = 0$. The term $b^2 - 4ac$ is called discriminant of the equation , which plays an important role in finding the nature of the roots. It is denoted by Δ or D

(A) Suppose $a, b, c \in R$ and $a \neq 0$ then

(i) If $D > 0 \Rightarrow$ Roots are Real and unequal

(ii) If $D = 0 \Rightarrow$ Roots are Real and equal and each equal to $-b/2a$

(iii) If $D < 0 \Rightarrow$ Roots are imaginary and unequal or complex conjugate.

(B) Suppose $a, b, c \in Q$ and $a \neq 0$ then

(i) If $D > 0$ and D is perfect square $\Rightarrow$ Roots are unequal and Rational.

(ii) If $D > 0$ and D is not perfect square $\Rightarrow$ Roots are irrational and unequal.

Conjugate Roots : The irrational and complex roots of a quadratic equation with rational coefficient are always occurs in pairs. Therefore

If One Root then	Other Root
$\alpha + i\beta$	$\alpha - i\beta$
$\alpha + \sqrt{\beta}$	$\alpha - \sqrt{\beta}$

Example 1 :

Find two consecutive odd positive integers, sum of whose squares is 290.

Sol. Let the smaller of the two consecutive odd positive integers be x. Then, the second integer will be x + 2.

According to the question,

$x^2 + (x + 2)^2 = 290$

i.e., $x^2 + x^2 + 4x + 4 = 290$

i.e., $2x^2 + 4x - 286 = 0$ i.e., $x^2 + 2x - 143 = 0$

which is a quadratic equation in x. Using the quadratic formula, we get

$$x = \frac{-2 \pm \sqrt{4 + 572}}{2} = \frac{-2 \pm \sqrt{576}}{2} = \frac{-2 \pm 24}{2}$$

i.e., $x = 11$ or $x = -13$

But x is given to be an odd positive integer.

Therefore, $x \neq -13, x = 11$.

Thus, the two consecutive odd integers are 11 and 13.

RELATION BETWEEN ROOTS AND COEFFICIENTS

If roots of quadratic equation $ax^2 + bx + c = 0\ (a \neq 0)$ are α and β then:

(i) $(\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \pm\dfrac{\sqrt{b^2 - 4ac}}{a} = \dfrac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dfrac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\dfrac{b\sqrt{b^2-4ac}}{a^2} = \dfrac{\pm\sqrt{D}}{a}$

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta\,(\alpha+\beta) = -\dfrac{b(b^2-3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta\,(\alpha-\beta)$

$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\,\left\{(\alpha+\beta)^2 - \alpha\beta\right\} = \dfrac{(b^2-ac)\sqrt{b^2-4ac}}{a^3}$

(vi) $\alpha^4 + \beta^4 = \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\dfrac{b^2-2ac}{a^2}\right)^2 - 2\dfrac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2-\beta^2)(\alpha^2+\beta^2) = \dfrac{-b(b^2-2ac)\sqrt{b^2-4ac}}{a^4}$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta$

(ix) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta\,(\alpha+\beta)$

(xi) $\left(\dfrac{\alpha}{\beta}\right)^2 + \left(\dfrac{\beta}{\alpha}\right)^2 = \dfrac{\alpha^4+\beta^4}{\alpha^2\beta^2} = \dfrac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

FORMATION OF AN EQUATION WITH GIVEN ROOTS

A quadratic equation whose roots are α and β is given by

$(x-\alpha)(x-\beta) = 0$

$\therefore\ x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\therefore\ x^2 - (\alpha+\beta)x + \alpha\beta = 0$

i.e. x^2 – (sum of Roots) x + Product of Roots = 0

$\therefore\ x^2 - Sx + P = 0$

Equation in terms of the Roots of another Equation :

If α, β are roots of the equation $ax^2 + bx + c = 0$ then equation whose roots are

(i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$ ¼Replace x by – x)

(ii) $1/\alpha, 1/\beta \Rightarrow cx^2 + bx + a = 0$ ¼Replace x by 1/x)

(iii) α^n, β^n ; $n \in N \Rightarrow a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ ¼Replace x by $x^{1/n}$)

(iv) $k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2c = 0$ ¼Replace x by x/k)

(v) $k+\alpha, k+\beta \Rightarrow a(x-k)^2 + b(x-k) + c = 0$ ¼replace x by (x – k))

(vi) $\dfrac{\alpha}{k}, \dfrac{\beta}{k} \Rightarrow k^2ax^2 + kbx + c = 0$ ¼Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}$; $n \in N \Rightarrow a(x^n)^2 + b(x^n) + c = 0$ ¼replace x by x^n)

Symmetric Expressions :

The symmetric expressions of the roots α, β of an equation are those expression in α and β, which do not change by interchanging α and β. To find the value of such an expression, we generally express that in terms of $\alpha + \beta$ and $\alpha\beta$.

Some example of symmetric expression are– If α, β are the roots of equation $ax^2 + bx + c = 0$, then the equation whose roots are –

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 + \alpha\beta + \beta^2$ (iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ (iv) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

(v) $\alpha^2\beta + \beta^2\alpha$ (vi) $\left(\dfrac{\alpha}{\beta}\right)^2 + \left(\dfrac{\beta}{\alpha}\right)^2$ (vii) $\alpha^3 + \beta^3$

(viii) $\alpha^4 + \beta^4$

Example 2 :

Form the quadratic equation whose roots are α and β such that $\alpha\beta = 4$ and $\dfrac{\alpha}{\alpha-1}$ 9+ $\dfrac{\beta}{\beta-1} = \dfrac{a^2-7}{a^2-4}$

Sol. $\alpha\beta = 4$ and $\dfrac{\alpha}{\alpha-1} + \dfrac{\beta}{\beta-1} = \dfrac{a^2-7}{a^2-4}$

i.e., $\dfrac{2\alpha\beta - (\alpha+\beta)}{\alpha\beta + 1 - (\alpha+\beta)} = \dfrac{a^2-7}{a^2-4}$

i.e., $\dfrac{(\alpha+\beta) - 8}{(\alpha+\beta) - 5} = \dfrac{a^2-7}{a^2-4}$

or $\dfrac{(\alpha+\beta-1) - 7}{(\alpha+\beta-1) - 4} = \dfrac{a^2-7}{a^2-4}$

Comparison gives $\alpha+\beta - 1 = a^2$ or $\alpha+\beta = a^2 + 1$

Hence the required equation is $x^2 - x(\alpha+\beta) + \alpha\beta = 0$

i.e. $x^2 - x(a^2+1) + 4 = 0$

ROOTS UNDER PARTICULAR CASES

For the quadratic equation $ax^2 + bx + c = 0$

(i) If $b = 0 \Rightarrow$ roots are of equal magnitude but of opposite sign

(ii) If $c = 0 \Rightarrow$ one root is zero other is $-b/a$

(iii) If $b = c = 0 \Rightarrow$ both roots are zero

(iv) If $a = c \Rightarrow$ roots are reciprocal to each other

(v) $\left.\begin{array}{l} a > 0\ c < 0 \\ a < 0\ c > 0 \end{array}\right\} \Rightarrow$ If Roots are of opposite signs

(vi) If $\left.\begin{array}{l} a > 0,\ b > 0,\ c > 0 \\ a < 0,\ b < 0,\ c < 0 \end{array}\right\} \Rightarrow$ both roots are negative

(vii) If $\left.\begin{array}{l} a > 0,\ b < 0,\ c > 0 \\ a < 0,\ b > 0,\ c < 0 \end{array}\right\} \Rightarrow$ both roots are positive

(viii) If sign of a = sign of b $\neq$ sign of c $\Rightarrow$ Greater root in magnitude is negative

(ix) If sign of b = sign of c $\neq$ sign of a $\Rightarrow$ Greater root in magnitude is positive

(x) If $a + b + c = 0 \Rightarrow$ one root is 1 and second root is c/a

(xi) If $a = b = c = 0$ then equation will become an identity and will be satisfy by every value of x.

Example 3 :

If both roots of equation $x^2-(p-4)x+2e^{2\ln p}-4=0$ are negative then find the value of p.

Sol. Both roots are negative

$\Rightarrow$ Sum of the roots < 0 and product of the roots > 0

$\Rightarrow p-4<0$ and $e^{2\ln p}-4>0$

$\Rightarrow p<4$ and $p^2>2 \Rightarrow p\in(-\sqrt{2},4)$

CONDITION FOR COMMON ROOTS

Only one root common : Let α be the common root of quadratic equations

$a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ then

$\therefore\ a_1\alpha^2+b_1\alpha+c_1=0$; $a_2\alpha^2+b_2\alpha+c_2=0$

By Cramer's rule : $$\frac{\alpha^2}{\begin{vmatrix}-c_1 & b_1\\ -c_2 & b_2\end{vmatrix}}=\frac{\alpha}{\begin{vmatrix}a_1 & -c_1\\ a_2 & -c_2\end{vmatrix}}=\frac{1}{\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$$

or $$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-a_2b_1}$$

$$\therefore\ \alpha=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}=\frac{b_1c_2-b_2c_1}{a_2c_1-a_1c_2},\ \alpha\neq 0$$

$\therefore$ The condition for only one Root common is

$(c_1a_2-c_2a_1)^2=(b_1c_2-b_2c_1)(a_1b_2-a_2b_1)$

Both roots are common :

Then required conditions is $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$.

Two different quadratic equations with rational coefficients cannot have single common root which is complex or irrational, as imaginary and surd roots always occur in pair.

Example 4 :

If the equation $2x^2+x+k=0$ and $x^2+x/2-1=0$ have 2 common roots then find the value of k.

Sol. Since the given equation have two roots in common so from the condition

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow\frac{2}{1}=\frac{1}{1/2}=\frac{k}{-1}\quad\therefore\ k=-2$$

Example 5 :

If the quadratic equation $x^2+bx+c=0$ and $x^2+cx+b=0$ $(b\neq c)$ have a common root then prove that their uncommon roots are the roots of the equation $x^2+x+bc=0$

Sol. $\alpha^2+b\alpha+c=0$; $\alpha^2+c\alpha+b=0$

$$\therefore\ \frac{\alpha^2}{b^2-c^2}=\frac{\alpha}{c-b}=\frac{1}{c-b}$$

Hence $\alpha=1$ or $\alpha=-(b+c)$

if $\alpha=1$ then $\alpha\beta_1=c \Rightarrow \beta_1=c$

and $\alpha\beta_2=b \Rightarrow \beta_2=b$

where β_1 and β_2 are the common root

$\therefore$ required equation $x^2-(b+c)x+bc=0$

but $-(b+c)=1$ $\quad\therefore x^2+x+bc=0$

NATURE OF THE FACTORS

The nature of factors of the quadratic expression ax^2+bx+c is the same as the nature of roots of the corresponding quadratic equation $ax^2+bx+c=0$. Thus the factors of the expression are :

(i) Real and different, If $b^2-4ac>0$

(ii) Rational and different, if b^2-4ac is a prefect square.

(iii) Real and equal, if $b^2-4ac=0$

(iv) Imaginary, if $b^2-4ac<0$

Ex. The factors of x^2-x+1 are

Sol. The factors of x^2-x+1 are imaginary because $b^2-4ac=(-1)^2-4(1)(1)=1-4=-3<0$

MAXIMUM & MINIMUM VALUE OF QE

In a Quadratic Expression ax^2+bx+c

(i) If $a>0$ Quadratic expression has least value at $x=-\dfrac{b}{2a}$.

This least value is given by $\dfrac{4ac-b^2}{4a}=-\dfrac{D}{4a}$

(ii) If $a<0$, Quadratic expression has greatest value at $x=-\dfrac{b}{2a}$.

This greatest value is given by $\dfrac{4ac-b^2}{4a}=-\dfrac{D}{4a}$

Example 6 :

If expression $\left(mx-1+\dfrac{1}{x}\right)$ is non-negative for all real positive values of x then find the minimum value of m.

Sol. We know that $ax^2+bx+c\geq 0$ if $a>0$ and $b^2-4ac\leq 0$

So, $mx-1+\dfrac{1}{x}\geq 0\Rightarrow\dfrac{mx^2-x+1}{x}\geq 0\Rightarrow mx^2-x+1\geq 0$

$\because x>0$. Now, $mx^2-x+1\geq 0$ if $m>0$

and $1-4m\leq 0$ or if $m>0$ and $m\geq\dfrac{1}{4}$

Hence, minimum value of m is 1/4

SIGN OF THE QUADRATIC EXPRESSION

Let $y=ax^2+bx+c\ (a\neq 0)$

$$y=a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right]$$

$$=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a^2}\right]$$

where $D = b^2 - 4ac$ is the Discriminant of the quadratic equation $ax^2 + bx + c = 0$

Case 1 : $D > 0$: Suppose the roots of $ax^2 + bx + c = 0$ are α and β and $\alpha > \beta$ (say) $\alpha]$ β are real and distinct.

Then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

Clearly $(x - \alpha)(x - \beta) > 0$ for $x < \beta$ and $x < \alpha$ since both factors are of the same sign and

$(x - \alpha)(x - \beta) < 0$ for $\alpha > x > \beta$

For $x = \beta$ or $x = \alpha$, $(x - \alpha)(x - \beta) = 0$

$\therefore$ If $a > 0$, then $ax^2 + bx + c > 0$ for all x outside the interval $[\beta, \alpha]$ and is negative for all x (β, α).

If $a < 0$, then its vice-versa.

Case 2 : $D = 0$ then from (1), $ax^2 + bx + c = a\left(x + \dfrac{b}{2a}\right)^2$

$\therefore \forall\ x \neq -\dfrac{b}{2a}$, the quadratic expression takes on values of the same as a ; If $x = -b/2a$ then $ax^2 + bx + c = 0$

If $D = 0$, then

(i) $ax^2 + bx + c > 0$ has a solution any $\left(x \neq -\dfrac{b}{2a}\right)$

If $a > 0$ and has no solution if $a < 0$.

(ii) $ax^2 + bx + c < 0$ has a solution any $\left(x \neq -\dfrac{b}{2a}\right)$ $a < 0$ and has no solution if $a > 0$

(iii) $ax^2 + bx + c \geq 0$ has any x as a solution if $a > 0$ and the unique solution $x = -\dfrac{b}{2a}$, if $a < 0$;

(iv) $ax^2 + bx + c \leq 0$ has any x as a solution if $a < 0$ and

$x = -\dfrac{b}{2a}$, if $a > 0$

Case 3 : $D < 0$ from (1)

(i) If $a > 0$, then $ax^2 + bx + c > 0$, for all x ;

(ii) if $a < 0$, then $ax^2 + bx + c < 0$, x for all x.

Ex. The sign of $x^2 + 2x + 3$ is positive for all $x \in R$, because here $b^2 - 4ac = 4 - 12 = -8 < 0$ and $a = 1 > 0$

Ex. The sign of $3x^2 + 5x - 8$ is negative for all $x \in R$ because here $b^2 - 4ac = 25 - 96 = -71 < 0$ and $a = -3 < 0$

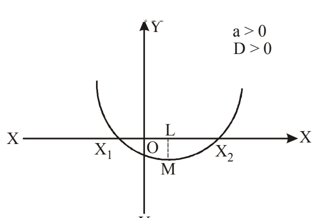
Graph of Quadratic Expression :

Consider the expression $y = ax^2 + bx + c$ $(a \neq 0)$ & $a, b, c \in R$ then the graph between x, y is always a parabola if $a > 0$ then the shape of the parabola in concave upward and if $a < 0$ then the shape of the parabola is concave downwards.

There is only 6 possible graph of a Quadratic expression :

<u>Case -I</u> When $a > 0$

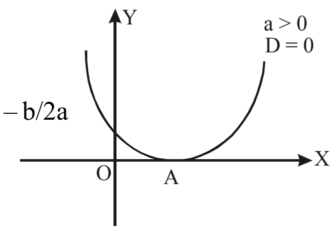
(i) If D > 0



Roots are real and different (x_1 and x_2)

Minimum value $LM = \dfrac{4ac - b^2}{4a}$ at $x = OL = -b/2a$ y is positive for all x out side interval $[x_1, x_2]$ and is negative for all x inside (x_1, x_2)

(ii) If D = 0

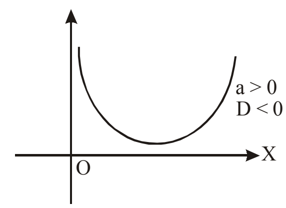


Roots are equal (OA)

Min. value = 0 at x = OA = – b/2a

$y > 0$ for all $x \in \left\{R - \dfrac{-b}{2a}\right\}$

(iii) If D < 0

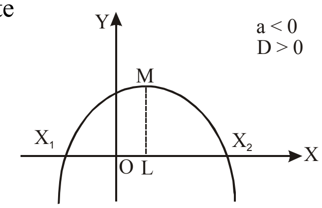


Roots are complex conjugate

y is positive for all $x \in R$

<u>Case -II</u> When $a < 0$

(i) If D > 0

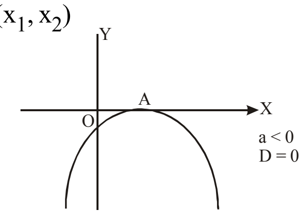


Roots are real and different (x_1 and x_2)

Max. value $= LM = \dfrac{4ac - b^2}{4a}$ at $x = OL = -b/2a$

y is positive for all x inside (x_1, x_2) and y is negative for all x outside $[x_1, x_2]$

(ii) When D = 0

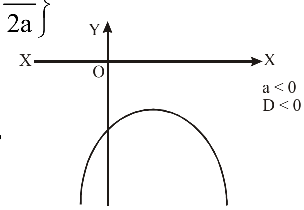


Roots are equal =(OA)

max. value = 0 at x = OA = – b/2a

y is negative for all $x \in \left\{R - \dfrac{-b}{2a}\right\}$

(iii) When D < 0



Roots are complex conjugate,

y is negative for all $x \in R$

Example 7 :

Find all the values of the parameter 'd' for which both roots of the equation

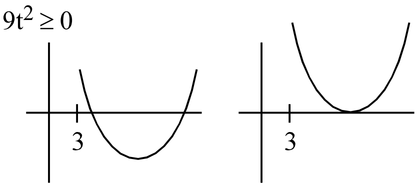
$x^2 - 6dx + (2 - 2d + 9d^2) = 0$ exceed the number 3.

Sol. (i) $D \geq 0$

$9t^2 - 2 + 2t - 9t^2 \geq 0$

$t - 1 \geq 0$

$t \geq 1$



(ii) $-\dfrac{b}{2a} > 3$; $3t > 3$; $t > 1$

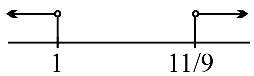
(iii) $f(3) > 0$

$9 - 18t + 2 - 2t + 9t^2 > 0$

$9t^2 - 20t + 11 > 0$

$9t^2 - 9t - 11t + 11 > 0$

$(t - 1)(9t - 11) > 0$



$\therefore$ $t \in (-\infty, 1) \cup (11/9, \infty)$

$\therefore$ Intersection of (i), (ii) and (iii) is $t > 11/9$

QUADRATIC EXPRESSION IN TWO VARIABLES

The general form of a quadratic expression in two variables x & y is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

The condition that this expression may be resolved into two linear rational factors is

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and $h^2 - ab > 0$

This expression is called discriminant of the above quadratic expression.

Example 8 :

Find the value of m for which expression $y^2 + 2xy + 2x + my - 3$ can be resolved in two rational factors.

Sol. Here, $a = 0, b = 1, c = -3$

$h = 1, g = 1, f = m/2$

$$\therefore \quad \Delta = 0 \Rightarrow \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & m/2 \\ 1 & m/2 & -3 \end{vmatrix} = 0$$

$\Rightarrow (m/2 + 3) + (m/2 - 1) = 0 \Rightarrow m + 2 = 0 \Rightarrow m = -2$

THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3 ...... \alpha_n$ are the roots of the equation

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... a_{n-1} x + a_n = 0$

Where $a_0, a_1, a_2 .... a_n$ are all real, $a_0 \neq 0$

then $\Sigma \alpha_1 = -\dfrac{a_1}{a_0}$, $\Sigma \alpha_1 \alpha_2 = \dfrac{a_2}{a_0}$,

$$\Sigma \alpha_1 \alpha_2 \alpha_3 = -\frac{a_3}{a_0}, ...$$

$$\Sigma \alpha_1 \alpha_2 \alpha_3 \,.............. \alpha_n = (-1)^n \frac{a_n}{a_0}$$

Ex. If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$

then $\alpha + \beta + \gamma = -b/a$, $\beta\gamma + \gamma\alpha + \alpha\beta = c/a$

$\alpha\beta\gamma = -d/a$

Note :

(i) Every equation of n^{th} degree ($n \geq 1$) has exactly n roots and if the equation has more than n roots, it is an identity.

(ii) If α is a root of the equation $f(x) = 0$ then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$.

LINEAR INEQUALITIES

Let us first recall the basic definitions and solution methods.

Inequalities of the form $ax + b > px + q$ or $ax + b \geq px + q$. where a, b, p, q are certain numbers, are termed linear. Both sides of a linear inequality are linear functions.

It is obvious that the inequality $ax + b > px + q$ is equivalent to the inequality $(a - p)\,x > q - b$, as also to this inequality $(p - a)\,x < p - q$.

Thus, the study of linear inequalities is reduced to studying inequalities of the form $ax > b$ and $ax < b$, where a and b are some numbers. It is obvious that

(i) If $a > 0$, then, $ax > b \Leftrightarrow x > \dfrac{b}{a}$, $ax < b \Leftrightarrow x < \dfrac{b}{a}$, i.e., the set of solutions of the inequality $ax > b$ is the infinite interval $\left(\dfrac{b}{a}, +\infty\right)$, and that of inequality $ax < b$ the infinite interval $\left(-\infty, \dfrac{b}{a}\right)$.

(ii) If $a < 0$, then, $ax > b \Leftrightarrow x < \dfrac{b}{a}$, $ax < b \Leftrightarrow x > \dfrac{b}{a}$, i.e, the set of solution of the inequality $ax > b$ is the interval $\left(-\infty, \dfrac{b}{a}\right)$ and that of the inequality $ax < b$ the interval $\left(\dfrac{b}{a}, +\infty\right)$.

The case $a = 0$, that is, inequalities of the form $0.x > b$ and $0.x < b$ should be given special consideration. Indeed, if $b > 0$, then the inequality $0.x > b$ has no solution, where as the inequality $0.x < b$ is satisfied by any real number.

If $b < 0$, then the inequality $0.x < b$ has no solution, and the inequality $0.x > b$ is satisfied by any real number. If $b = 0$, then the inequalities $0.x > b$ and $0.x < b$ have no solution.

PROPERTIES OF INEQUALITIES

(A) **(i)** If $a < b$, then $a + c < b + c$, for any real c

(ii) If $a \leq b$, then $a + c \leq b + c$

(B) **(i)** If $a < b$, then $ac < bc$, if $c > 0$ then $ac > bc$, if $c < 0$

(ii) If $a \leq b$, then $ac \leq bc$, if $c \geq 0$ and $ac \geq bc$, if $c \leq 0$

(C) **(i)** If $a < b < c$, then $a^2 < b^2 < c^2$ if $a, b, c, \in [0, \infty)$

(ii) If $a < b < c$, then $a^2 > b^2 > c^2$ if $a, b, c, \in (-\infty, 0)$

(iii) If $a \le b \le c$, then $a^2 \le b^2 \le c^2$ if a, b, c, $\in [0, \infty)$

(iv) If $a \le b \le c$, then $a^2 \ge b^2 \ge c^2$ if a, b, c, $\in (-\infty, 0)$

(D) **(i)** If $a < b < c$, then $\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$ if a, b, c, $\in (0, \infty)$

or a, b, c, $\in (-\infty, 0)$

(ii) If $a \le b \le c$, then $\frac{1}{a} \ge \frac{1}{b} \ge \frac{1}{c}$ if a, b, c, $\in (0, \infty)$

or a, b, c, $\in (-\infty, 0)$

(D) **(i)** If $a < b$ and $c < d$, then $a + c < b + d$

(ii) If $a \le b$ and $c \le d$, then $a + c \le b + d$

(iii) If $a < b$ and $c \le d$, then $a + c < b + d$

(F) If $a < b$ and $c < d$, then $ac < bd$ if a, b, c, d $\in (0, \infty)$

(G) If $a \le b$ and $c \le d$, then $ac \le bd$ if a, b, c, d $\in [0, \infty)$

Note : Never subtract or divide two inequalities.

Example 9 :

Solve the inequation $12 + 1\frac{5}{6}x \le 5 + 3x,\ x \in R$. Represent the solution set on a number line.

Sol. We have $12 + 1\frac{5}{6}x \le 5 + 3x \Rightarrow 12 + \frac{11}{6}x \le 5 + 3x$

$\Rightarrow 72 + 11x \le 30 + 18$ [Multiplying both sides by 6]

$\Rightarrow 11x \le 18x - 42$ [Adding –72 on both sides]

$\Rightarrow -7x \le -42$ [Adding –18x on both sides]

$\Rightarrow x \ge 6$ [Dividing both sides by –7]

$\therefore$ Solution set $= \{x : x \ge 6, x \in R\}$

This set can be represented on the number line, as shown

Example 10 :

Solve the inequality $2x + |x| < 1$.

Sol. The given inequality is not a linear one, but its solution is reduced to solving linear inequalities. Indeed, if we consider only $x \ge 0$, then $2x + |x| = 3x$ and consequently, the given inequality takes the form $3x < 1$. Its nonegative solutions are all the numbers from the interval [0, 1/3).

Let now $x < 0$, then $2x + |x| = 2x - x = x$ and the given inequality takes the form $x < 1$.

Its negative solutions are all numbers $x < 0$.

Bringing together the nonnegative and negative solutions of the given inequality we find that any number $x < 1/3$ is a solution and there are no other solutions.

RATIONAL INEQUALITIES

Inequalities of the form $\frac{P_1(x)}{Q_1(x)} = \frac{P_2(x)}{Q_2(x)}$,

where $P_1(x)$, $Q_1(x)$, $P_2(x)$ and $Q_2(x)$ are certain polynomials are called rational. Linear and quadratic inequalities are the simplest examples of rational inequalities.

We shall illustrate the methods of solving rational inequalities by several examples.

Example 11 :

Solve the inequality $\frac{x+1}{x-2} > 1$.

Sol. The given inequality is equivalent to the inequality

$$\frac{x+1}{x-2} - 1 > 0,\ \text{i.e.}\ \frac{3}{x-2} > 0,$$

and therefore any number $x > 2$ is a solution and there are no other solutions.

SYSTEM OF INEQUALITIES

The number a is called the solution of a system of inequalities in one unknown if it is a solution of each inequality of the system. For instance, the number 1 is a solution of the system of two inequalities $\begin{cases} x^2 + x - 1 > 0, \\ x + 2 > 0, \end{cases}$

while the number –2 is not a solution of this system since it does not satisfy the second inequality, though it is a solution of the first inequality.

To solve a system of inequalities means to find the set of all the solutions of the system.

Two system of inequalities are called equivalent if they have the same solution sets.

We shall consider the methods of solving systems of inequalities by demonstrating concrete examples.

Example 12 :

Solve the system of inequalities $\begin{cases} 2x + 1 < x + 2, \\ x - 1 > 2x \end{cases}$

Sol. The given system of inequalities is equivalent to the system $\begin{cases} x < 1, \\ -1 > x \end{cases}$

since either inequality of the system is replaced by an equivalent inequality. The obtained system has no solution, consequently, the given system has no solution either.

Ans. : ϕ

THE GRAPH OF AN INEQUALITY

The following statements are inequalities in two variables :

$3x - 2y < 6$ and $2x^2 + 3y^2 \ge 6$.

An ordered pair (a, b) is a solution of an inequality in x and y if the inequality is true when a and b are substituted for x and y, respectively. The graph of an inequality is the collection of all solutions of the inequality. To sketch the graph of an inequality, begin by sketching the graph of the corresponding equation. The graph of the equation will normally separate the plane into two or more regions. In each such region, one of the following must be true.

(a) All points in the region are solutions of the inequality.

(b) No point in the region is a solution of the inequality.

So, you can determine whether the points in an entire region satisfy the inequality by simply testing one point in the region.

Systems of Inequalities : Many practical problems in business, science, and engineering involve systems of linear inequalities. A solution of a system of inequalities in x and y is a point (x, y) that satisfies each inequality in the system.

To sketch the graph of a system of inequalities in two variables, first sketch the graph of each individual inequality (on the same coordinate system) and then find the region that is common to every graph in the system. For systems of linear inequalities, it is helpful to find the vertices of the solution region.

Example 13 :

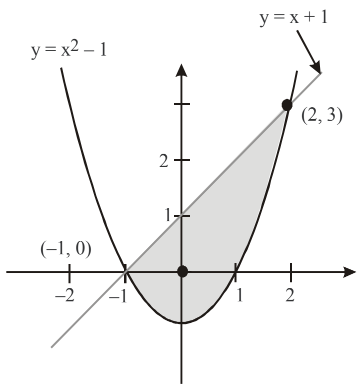
Sketch the region containing all points that satisfy the following system. $x^2 - y \le 1$; $-x + y \le 1$

Sol. As shown in Figure, the points that satisfy the inequality $x^2 - y \le 1$ are the points lying above (or on) the parabola given by $y = x^2 - 1$. [Parabola]

The points satisfying the inequality $-x + y \le 1$ are the points lying on or below the line given by $y = x + 1$ [Line]

To find the points of intersection of the parabola and the line, solve the system of corresponding equations.

$$x^2 - y = 1$$
$$-x + y = 1$$



Using the method of substitution, you can find the solutions to be (– 1,0) and (2,3), as shown in figure.

When solving a system of inequalities, you should be aware that the system might have no solution. For instance, the system $x + y > 3$
$x + y < -1$
has no solution points, because the quantity (x + y) cannot be both less than –1 and greater than 3, as shown in figure.

Another possibility is that the solution set of a system of inequalities can be unbounded. For instance, the solution set of $x + y < 3$
$x + 2y > 3$
forms an infinite wedge as shown in figure.

SOLUTION OF QUADRATIC INEQUALITIES

The values of x satisfying the inequality $ax^2 + bx + c > 0 \ (a \ne 0)$ are :

(i) If D > 0, i.e. the equation $ax^2 + bx + c = 0$ has two different roots $\alpha < \beta$

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\beta, \infty)$

$a < 0 \Rightarrow x \in (\alpha, \beta)$

(ii) If D = 0, i.e. roots are equal i.e. $\alpha = \beta$

then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$

$a < 0 \Rightarrow x \in \phi$

(iii) If D < 0 i.e., the equation $ax^2 + bx + c = 0$ has no real root

Then $a > 0 \Rightarrow x \in R$

$a < 0 \Rightarrow x \in \phi$

(iv) Inequalities of the form $\dfrac{P(x)Q(x)R(x)....}{A(x)B(x)C(x).........} \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ can be quickly solved using the method of intervals, where A, B, C, ..., P, Q, R..........x are linear functions of x.

Example 14 :

If $-3 < \dfrac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$ for all $x \in R$, then find $[\lambda]$, where [.] denotes the greatest integer function

Sol. Given : $-3 < \dfrac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$

$\Rightarrow -3x^2 - 3x - 3 < x^2 - \lambda x - 2 < 2x^2 + 2x + 2$
(i) (ii)

$(\because x^2 + x + 1 > 0 \text{ when } x \in R)$

From inequality (i)

$4x^2 - (\lambda - 3)x + 1 > 0 \Rightarrow (\lambda - 3)^2 - 4 \cdot 1 \cdot 4 \cdot 1 < 0$

$\Rightarrow -4 < \lambda + 2 < 4 \Rightarrow -1 < \lambda < 7$

From inequality (ii), $x^2 + (\lambda + 2)x + 4 > 0$

$\Rightarrow (\lambda + 2)^2 - 4 \cdot 1 \cdot 4 \cdot 1 < 0$

$\Rightarrow -4 < \lambda + 2 < 4 \Rightarrow -6 < \lambda < 2$

From (i) & (ii), $-1 < \lambda < 2$

$\therefore [\lambda] = -1, 0, 1$

TRY IT YOURSELF

Q.1 The roots of the quadratic equation $2x^2 - 7x + 4 = 0$ are –
(A) Rational and different (B) Rational and equation
(C) Irrational and different (D) Imaginary and different

Q.2 The quadratic equation with rational coefficients whose one root is $2 + \sqrt{3}$ is –
(A) $x^2 - 4x + 1 = 0$ (B) $x^2 + 4x + 1 = 0$
(C) $x^2 + 4x - 1 = 0$ (D) $x^2 + 2x + 1 = 0$

Q.3 Find range of k for which graph of $y = x^2 - 3x + k$ lies completely above x-axis.

Q.4 If $x = 3 + \sqrt{5}$ find the value of $x^4 - 12x^3 + 44x^2 - 48x + 17$.

Q.5 Solve : $(x + 1)(x + 3)(x - 2)^2 \ge 0$

Q.6 Solve : $\dfrac{x+1}{x-1} \ge \dfrac{x+5}{x+1}$

Q.7 Solve : $1 < \frac{3x^2 - 7x + 8}{x^2 + 1} \le 2$

Q.8 If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are α + 3 and β + 3 is –
(A) $x^2 - 11x + 30 = 0$ (B) $(x-3)^2 - 5(x-3) + 6 = 0$
(C) Both (A) and (B) (D) None

Q.9 Find the value of k for which the equation $3x^2 + 4kx + 2 = 0$ and $2x^2 + 3x - 2 = 0$ have a common root.

Q.10 If $Q_1(x) = x^2 + (k-29)x - k$ and $Q_2(x) = 2x^2 + (2k-43)x + k$ both are factors of a cubic polynomial P (x), then the largest value of k is –
(A) 0 (B) 33
(C) 23 (D) 30

Q.11 Let $P(x) = ax^2 + bx + 8$ is a quadratic polynomial. If the minimum value of P (x) is 6 when x = 2, find the values of a and b.

Q.12 For x ≥ 0, what is the smallest possible value of the expression $\log_{10}(x^3 - 4x^2 + x + 26) - \log_{10}(x + 2)$?

Q.13 If x is real then find the range of the function

$$y = \frac{x+2}{x^2 + 3x + 6}$$

Q.14 If x is real then find the range of the function

$$y = \frac{x^2 - 3x + 2}{x^2 + x - 6}$$

Q.15 Find the set of values of m for which exactly one root of the equation $x^2 + mx + (m^2 + 6m) = 0$ lie in (–2, 0).

Q.16 Solve the system of inequalities : $\begin{cases} 3x < x + 2, \\ x + 1 < \frac{1}{2}x + 2 \end{cases}$

ANSWERS

(1) (C) **(2)** (A) **(3)** (9/4, ∞)
(4) 1 **(5)** $x \in (-\infty, -1] \cup \{2\} \cup [3, \infty)$
(6) $x \in (-\infty, -1) \cup (1, 3]$ **(7)** $x \in [1, 6]$
(8) (C) **(9)** 11/8 **(10)** (D)
(11) b = –2, a = 1/2 **(12)** $\log_{10} 4$
(13) $y \in [-1/5, 1/3]$ **(14)** $R - \left\{\frac{1}{5}, 1\right\}$
(15) $m \in (-6, -2) \cup (-2, 0)$ **(16)** $(-\infty, 1)$

ADDITIONAL EXAMPLES

Example 1:
Find the values of 'a' for which both roots of the equation $(a + 1)x^2 - 3ax + 4a = 0$ are greater than unity.
Sol. Since the roots are real.

$D \ge 0$ & $\frac{3a}{a+1} > 2$ and $(a + 1)(2a + 1) > 0$

$\Rightarrow \frac{-16}{7} \le a \le 0$ and $a < -1$, $a > 2$ and

$a < -1, a > -\frac{1}{2} \Rightarrow a \in \left[\frac{-16}{7}, -1\right)$

Example 2 :
If a < b < c < d, then find the nature of roots of $(x - a)(x - c) + 2(x - b)(x - d) = 0$.
Sol. Here, $3x^2 - (a + c + 2b + 2d)x + (ac + 2bd) = 0$
∴ Discriminant $= (a + c + 2b + 2d)^2 - 12(ac + 2bd)$
$= [(a + 2d) - (c + 2b)]^2 + 4(a + 2d)(c + 2b) - 12(ac + 2bd)$
$= [(a + 2d) - (c + 2b)]^2 + 8(c - b)(d - a) > 0$.
Hence roots are real and unequal

Example 3 :
If $x^2 - 2px + q = 0$ has real roots then prove that the equation $(1 + y)x^2 - 2(p + y) + (q + y) = 0$ will have its roots real and distinct if and only if y is negative and p is not unity.
Sol. Here $4p^2 - 4q = 0 \Rightarrow p^2 = q$
Also $D = 4(p + y)^2 - 4(1 + y)(q + y)$
$= 4[p^2 + 2py + y^2 - q - qy - y - y^2]$
$= +4y(2p - q - 1)$
$= 4y(2p - p^2 - 1) = -4y(p - 1)^2$
Here D > 0 if y is negative and p is not one.

Example 4 :
For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then find the sum of roots.
Sol. $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$; $\frac{b-a}{x^2 + (b+a)x + ab} = \frac{1}{x+c}$
or $x^2 + (a + b)x + ab = (b - a)x + (b - a)c$
or $x^2 + 2ax + ab + ca - bc = 0$
Since product of the roots = 0
$ab + ca - bc = 0$

$a = \frac{bc}{b+c}$. Thus sum of roots $= -2a = \frac{-2bc}{b+c}$

Example 5 :
If p and q are roots of the equation $x^2 - 2x + A = 0$ and r and s be roots of the equation $x^2 - 18x + B = 0$ if p < q < r < s be in A.P., then find the value of A and B.
Sol. Here p, q are roots of $x^2 - 2x + A = 0$
∴ p + q = 2 .....(1)
Also r, s are roots of $x^2 - 18x + B = 0$
∴ r + s = 18 .....(2)

Now since p, q, r, s in A.P. say with common difference d.

$\therefore \quad q = p + d, r = p + 2d, s = p + 3d$

From (1) and (2)

$$\left.\begin{array}{l} 2p + d = 2 \\ 2p + 5d = 18 \end{array}\right\} \Rightarrow 4d = 16 \Rightarrow d = 4$$

$\therefore 2p + 4 = 2 \Rightarrow p = -1$

Hence $p = -1, q = -1 + 4 = 3$

$r = -1 + 8 = 7, s = -1 + 12 = 11$

$A = pq = -3, B = rs = 77$

Example 6 :

If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$ and λ_1 and λ_2 are two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then find the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}$.

Sol. Here $\alpha + \beta = \frac{\lambda - 1}{\lambda}$, $\alpha\beta = \frac{5}{\lambda}$

Given $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \Rightarrow 5[\alpha^2 + \beta^2] = 4\alpha\beta$

$$\Rightarrow 5\left[\frac{\lambda - 1}{\lambda}\right]^2 = 14 \times \frac{5}{\lambda} \Rightarrow \lambda^2 - 16\lambda + 1 = 0$$

Now $\lambda_1 + \lambda_2 = 16, \lambda_1\lambda_2 = 1$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1\lambda_2} = \frac{(-16)^2 - 2}{1} = 254$$

Example 7 :

If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then prove that $aS_{n+1} + cS_{n-1} = -bS_n$

Sol. Here $\alpha + \beta$ are roots

$\therefore \quad a\alpha^2 + b\alpha + c = 0$ ...... (1)

$a\beta^2 + b\beta + c = 0$ ....(2)

Now let us consider

$a\,S_{n+1} + b\,S_n + C\,S_{n-1}$

$= a[\alpha^{n+1} + \beta^{n+1}] + b[\alpha^n + \beta^n] + c[\alpha^{n-1} + \beta^{n-1}]$

$= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$

$= \alpha^{n-1}[a\alpha^2 + b\alpha + c] \mid \beta^{n-1}[a\beta^2 + b\beta + c]$

$= 0 + 0 = 0$

Hence $aS_{n+1} + cS_{n-1} = -bS_n$

Example 8 :

If the equation $x^2 + 2(k+1)x + 9k - 5 = 0$ has only negative roots, then find the value of k.

Sol. Let $f(x) = x^2 + 2(k+1)x + 9k - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots, if (i) Disc. ≥ 0 (ii) $\alpha < 0, \beta < 0$, i.e. $(\alpha + \beta) < 0$ and (iii) $f(0) > 0$

Now, Discriminant $\geq 0 \Rightarrow 4(k+1)^2 - 36k + 20 \geq 0$

$\Rightarrow k^2 - 7k + 6 \geq 0 \Rightarrow (k-1)(k-6) \geq 0$

$\Rightarrow k \leq 1$ or $k \geq 6$ ......(i)

$(\alpha + \beta) < 0 \Rightarrow -2(k+1) < 0$

$\Rightarrow k + 1 > 0 \Rightarrow k > -1$ ......(ii)

and, $f(0) > 0 \Rightarrow 9k - 5 > 0 \Rightarrow k < \frac{5}{9}$ ......(iii)

From (i), (ii), (iii), we get $k \geq 6$

Example 9 :

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Sol. Let x litres of 30% acid solution is required to be added. Then

Total mixture = (x + 600) litres

Therefore 30% x + 12% of 600 > 15% of (x + 600)

and 30% x + 12% of 600 < 18% of (x + 600)

or $\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$

and $\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$ i.e., $120 < x < 300$.

Thus the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

Example 10 :

If α and β are the real roots of the equation $x^2 - (k-2)x + (k^2 + 3k + 5) = 0$ $(k \in R)$. Find the maximum and minimum values of $(\alpha^2 + \beta^2)$.

Sol. For real roots $D \geq 0$

$(k-2)^2 - 4(k^2 + 3k + 5) \geq 0$

$(k^2 + 4 - 4k) - 4k^2 - 12k - 20 \geq 0$

$-3k^2 - 16k - 16 \geq 0$; $3k^2 + 16k + 16 \leq 0$

$$\left(k + \frac{4}{3}\right)(k + 4) \leq 0$$

Now, $E = \alpha^2 + \beta^2$

$E = (\alpha + \beta)^2 - 2\alpha\beta$

$E = (k-2)^2 - 2(k^2 + 3k + 5) = -k^2 - 10k - 6$

$E = (k^2 + 10k + 6) = -[(k+5)^2 - 19] = 19 - (k+5)^2$

$\therefore \quad E_{min}$ occurs when $k = -4/3$

$$\therefore \quad E_{min} = 19 - \frac{121}{9} = \frac{171 - 121}{9} = \frac{50}{9}$$

E_{max} occurs when $k = -4$

$E_{max} = 19 - 1 = 18$

Example 11 :

If $ax^2 + 2bx + c = 0$ and $y = x + \frac{1}{x}$, prove that

$acy^2 + 2b(a + c)y + (a - c)^2 + 4b^2 = 0.$

Sol. $ax^2 + 2bx + c = 0$ .... (1)

$a + \frac{2b}{x} + \frac{c}{x^2} = 0$ .... (2)

eq. (1) × c + eq. (2) × a gives

$$ac\left(x^2 + \frac{1}{x^2}\right) + 2b\left(cx + \frac{a}{x}\right) + c^2 + a^2 = 0$$

$$\text{Now } cx + \frac{a}{x} = c\left(x + \frac{1}{x}\right) + a\left(x + \frac{1}{x}\right) - \left(\frac{c}{x} + ax\right)$$

$$= c\left(x + \frac{1}{x}\right) + a\left(x + \frac{1}{x}\right) + 2b, \text{ using (1)}$$

Substituting $x + \frac{1}{x} = y$ in equation (3), we get

$ac(y^2 - 2) + 2b\{cy + ay + 2b\} + (c^2 + a^2) = 0$

i.e.) $acy^2 + 2b(a + c)y + (a - c)^2 + 4b^2 = 0.$

Example 12 :

Let α and β be the roots of the quadratic equation $ax^2 + 2bx + c = 0$ and $\alpha + \gamma$ and $\beta + \gamma$ be the roots of the quadratic equation $Ax^2 + 2Bx + C = 0$. Prove that $A^2(b^2 - ac) = a^2(B^2 - AC)$.

Sol. From the given equation

$$\alpha + \beta = \frac{-2b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\text{and } \alpha + \beta + 2\gamma = -\frac{2B}{A}, \quad (\alpha + \gamma)(\beta + \gamma) = \frac{C}{A}$$

$$\Rightarrow 2\gamma = \frac{-2B}{A} + \frac{2b}{a}, \quad \alpha\beta + \gamma(\alpha + \beta) + \gamma^2 = \frac{C}{A}$$

$$\Rightarrow \gamma = \frac{-B}{A} + \frac{b}{a}, \alpha\beta + \gamma(\alpha + \beta + \gamma) = \frac{C}{A}$$

Eliminating α, β and γ we get

$$\frac{c}{a} + \left(-\frac{B}{A} + \frac{b}{a}\right)\left[-\frac{2b}{a} - \frac{B}{A} + \frac{b}{a}\right] = \frac{C}{A} \Rightarrow \frac{B^2}{A^2} - \frac{C}{A} = \frac{b^2}{a^2} - \frac{c}{a}$$

$$\Rightarrow a^2(B^2 - AC) = A^2(b^2 - ac)$$

Example 13 :

The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Sol. **2.** (i) $x^2 - 8kx + 16(k^2 - k + 1) = 0$

$D = 64(k^2 - (k^2 - k + 1)) = 64(k - 1) > 0$

$k > 1$

(ii) $-\frac{b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4 \Rightarrow k > 1$

(iii) $f(4) \geq 0$

$16 - 32k + 16(k^2 - k + 1) \geq 0$

$k^2 - 3k + 2 \geq 0$; $(k - 2)(k - 1) \geq 0$; $k \leq 1$ or $k \geq 2$

Hence $k = 2$

Example 14 :

Let p & q be real numbers such that $p \neq 0$, $p^3 \neq q$ & $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is –

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Sol. **(B).** $\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = q \Rightarrow -p^3 + 3p\alpha\beta = q$

$$\Rightarrow \alpha\beta = \frac{q + p^3}{3p}\ ; \quad x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}.\frac{\beta}{\alpha} = 0$$

$$x^2 - \frac{(\alpha^2 + \beta^2)}{\alpha\beta}x + 1 = 0$$

$$\Rightarrow x^2 - \left(\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$$

$$\Rightarrow x^2 - \frac{p^2 - 2\left(\frac{p^3 + q}{3p}\right)}{\frac{p^3 + q}{3p}}x + 1 = 0$$

$\Rightarrow (p^3 + q)x^2 - (3p^3 - p^3 - 2q)x + (p^3 + q) = 0$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

QUESTION BANK

CHAPTER 4 : QUADRATIC EQUATION AND INEQUALITIES

EXERCISE - 1 [LEVEL-1]

PART - 1 : SOLUTION OF QUADRATIC EQUATIONS AND NATURE OF ROOTS

Q.1 The roots of the equation $x^2 - 4x + 1 = 0$ are –
(A) $2 \pm \sqrt{3}$ (B) 2, 4
(C) $-2 \pm \sqrt{3}$ (D) $\sqrt{3} \pm 2$

Q.2 The roots of the quadratic equation $2x^2 - 7x + 4 = 0$ are -
(A) Rational and different (B) Rational and equal
(C) Irrational and different (D) Imaginary and different

Q.3 The roots of the quadratic equation $x^2 - 2(a+b)x + 2(a^2+b^2) = 0$ are -
(A) Rational and different (B) Rational and equal
(C) Irrational and different (D) Imaginary and different

Q.4 The roots of the equation $x^2 - 2\sqrt{2}\,x + 1 = 0$ are –
(A) Real and different (B) Imaginary and different
(C) Real and equal (D) Rational and different

Q.5 The roots of the equation $x^2 - 3x - 4 = 0$ are–
(A) Opposite and greater root in magnitude is positive
(B) Opposite and greater root in magnitude is negative
(C) Reciprocal to each other
(D) None of these

Q.6 The roots of the equation $2x^2 - 3x + 2 = 0$ are -
(A) Negative of each other (B) Reciprocal to each other
(C) Both roots are zero (D) None of these

Q.7 If the roots of the equation $x^2 + 2x + P = 0$ are real then the value of P is -
(A) $P \le 2$ (B) $P \le 1$
(C) $P \le 3$ (D) None of these

PART - 2 : RELATION BETWEEN ROOTS AND COEFFICIENTS

Q.8 If equation $\frac{x^2 - bx}{ax - c} = \frac{k-1}{k+1}$ has equal and opposite roots then the value of k is -
(A) $\frac{a+b}{a-b}$ (B) $\frac{a-b}{a+b}$
(C) $\frac{a}{b}+1$ (D) $\frac{a}{b}-1$

Q.9 If the product of the roots of the quadratic equation $mx^2 - 2x + (2m-1) = 0$ is 3 then the value of m is -
(A) 1 (B) 2
(C) –1 (D) 3

Q.10 If α and β are roots of the equation $x^2 - 5x + 6 = 0$ then the value of $\alpha^3 + \beta^3$ is-
(A) 35 (B) 40
(C) 45 (D) None of these

Q.11 The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is-
(A) 0, 2 (B) 0,–2
(C) 2,–2 (D) None of these

Q.12 The equation whose roots are 3 and 4 will be-
(A) $x^2 + 7x + 12 = 0$ (B) $x^2 - 7x + 12 = 0$
(C) $x^2 - x + 12 = 0$ (D) $x^2 + 7x - 12 = 0$

Q.13 The quadratic equation with rational coefficients whose one root is $2 + \sqrt{3}$ is –
(A) $x^2 - 4x + 1 = 0$ (B) $x^2 + 4x + 1 = 0$
(C) $x^2 + 4x - 1 = 0$ (D) $x^2 + 2x + 1 = 0$

Q.14 If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are α + 3 and β + 3 is-
(A) $x^2 - 11x + 30 = 0$ (B) $(x-3)^2 - 5(x-3) + 6 = 0$
(C) Both (A) and (B) (D) None

Q.15 The quadratic equation whose one root is $\frac{1}{2+\sqrt{5}}$ will be-
(A) $x^2 + 4x - 1 = 0$ (B) $x^2 - 4x - 1 = 0$
(C) $x^2 + 4x + 1 = 0$ (D) None of these

Q.16 If the roots of the equations $x^2 + 3x + 2 = 0$ and $x^2 - x + \lambda = 0$ are in the same ratio then the value of λ is given by-
(A) 2/7 (B) 2/9
(C) 9/2 (D) 7/2

Q.17 If α and β are the roots of $2x^2 + 7x + c = 0$ and $|\alpha^2 - \beta^2| = \frac{7}{4}$ then c is equal to
(A) 2 (B) 3
(C) 6 (D) 5

Q.18 If α and β are the roots of $x^2 - ax + b^2 = 0$, then $\alpha^2 + \beta^2$ is equal to –
(A) $a^2 - 2b^2$ (B) $2a^2 - b^2$
(C) $a^2 - b^2$ (D) $a^2 + b^2$

Q.19 If $4ax^2 - 4ax + b = 0$ is a quadratic equation having real and distinct roots then –
(A) $a > b \le 0$
(B) Roots are of opposite sign
(C) Both roots are greater than unity if a > b
(D) Both roots are positive if a and b have same sign

Q.20 If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is
(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$
(C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

Q.21 Set of all values of k for which one root of $x^2 + 2(k-3)x + 111 = 0$ is smaller than –2 and the other is greater than 2, is –
(A) ϕ (B) (57, 61)
(C) (2, 3) (D) (–37, 40)

PART - 3 : COMMON ROOTS

Q.22 If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, γ respectively-
(A) a,b,c (B) b,c,a
(C) c,a,b (D) None of these

Q.23 If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to-
(A) –2 (B) –1
(C) 0 (D) 1

Q.24 If one root of the equations $x^2 + 2x + 3k = 0$ and $2x^2 + 3x + 5k = 0$ is common then the values of k is -
(A) 1, 2 (B) 0, –1
(C) 1, 3 (D) None of these

Q.25 If the equations $2x^2 + x + k = 0$ and $x^2 + x/2 - 1 = 0$ have 2 common roots then the value of k is-
(A) 1 (B) 3
(C) –1 (D) –2

Q.26 If $x^2 + x - 1 = 0$ and $2x^2 - x + \lambda = 0$ have a common root then–
(A) $\lambda^2 - 7\lambda + 1 = 0$ (B) $\lambda^2 + 7\lambda - 1 = 0$
(C) $\lambda^2 + 7\lambda + 1 = 0$ (D) $\lambda^2 - 7\lambda - 1 = 0$

Q.27 If $\alpha, \beta \in R$ and the quadratic equations $x^2 + 2x + 7 = 0$ and $4x^2 + \alpha x + \beta = 0$ have atleast one common roots, then the value of $\alpha + \beta$ is –
(A) 36 (B) – 4
(C) 24 (D) 20

Q.28 If a, p, q are non-zero real numbers, the two equations $2a^2x^2 - 2ab + b^2 = 0$ and $p^2x^2 + 3pqx + q^2 = 0$ have
(A) No common root
(B) One common root if $2a^2 + b^2 = p^2 + q^2$
(C) Two common roots if $3pq = 2ab$
(D) Two common roots if $3qb = 2ap$

PART - 4 : QUADRATIC EXPRESSIONS

Q.29 The value of the expression $x^2 + 2bx + c$ will be positive if-
(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$
(C) $c^2 < b$ (D) $b^2 < c$

Q.30 If $a + b + c > \frac{9c}{4}$ and equation $ax^2 + 2bx - 5c = 0$ has non-real roots then :-
(A) $a > 0, c > 0$ (B) $a > 0, c < 0$
(C) $a < 0, c < 0$ (D) $a < 0, c > 0$

Q.31 If the roots of equation $3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0$ are opposite in sign then interval of k lies –
(A) $(-\infty, 0)$ (B) $(-\infty, -1)$
(C) $(1, 2)$ (D) $(3/2, 2)$

Q.32 Set of value of k for which roots of equation $x^2 - (2k - 1)x + k(k - 1) = 0$ are :-
(A) both less then 2 if $k \in (2, \infty)$
(B) of opposite sign if $k \in (-\infty\ 0) \cup (1, \infty)$
(C) of same sign if $k \in (-\infty, 0) \cup (1, \infty)$
(D) both greater than 2 if $k \in (2, \infty)$

Q.33 If p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$ then minimum value of $p^2 + q^2 =$
(A) 2 (B) 3
(C) 6 (D) 5

Q.34 Let $f(x) = ax^2 + bx + c$; $a, b, c \in R, a \neq 0$. If $f(1) + f(2) = 0$, the equation $f(x) = 0$ has –
(A) no real root (B) 1 and 2 as real roots
(C) two equal roots (D) two distinct real roots

Q.35 The minimum value of the expression $4x^2 + 2x + 1$ is –
(A) 1/4 (B) 1/2
(C) 3/4 (D) 1

Q.36 The maximum value of $5 + 20x - 4x^2$ for all real value of x is-
(A) 10 (B) 20
(C) 25 (D) 30

Q.37 If x is real, then the minimum value of $x^2 - 8x + 17$ is –
(A) 1 (B) 2
(C) 3 (D) 4

Q.38 Let $a, b, c \in R$ and $ax^2 + bx + c = 0$ has two negative roots, then –
(A) a, b, c are of same sign
(B) a, –b, c are of same sign
(C) a, b, –c are of same sign
(D) a, – c are of same sign

Q.39 The equation $\pi^x = -2x^2 + 6x - 9$ has –
(A) one solution (B) two solutions
(C) infinite solutions (D) no solution

Q.40 The quadratic expression $21 + 12x - 4x^2$ are
(A) The least value 5 (B) The highest value 30
(C) The highest value 21 (D) None of these

PART - 5 : THEORY OF EQUATIONS AND INEQUALITIES

Q.41 For real values of x, $2x^2 + 5x - 3 > 0$, if-
(A) $x < -2$ (B) $x > 0$
(C) $x > 1$ (D) None of these

Q.42 If x is real then the value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between –
(A) –3 and 3 (B) – 4 and 5
(C) – 4 and 4 (D) –5 and 4

Q.43 If roots of the equation $x^3 - 12x^2 + kx - 28 = 0$ are in A.P. then k is –
(A) 29 (B) 39
(C) 34 (D) None of these

Q.44 The range of the values of $\frac{x}{x^2 + 4}$ for all real value of x is-
(A) $\frac{-1}{4} \leq y \leq \frac{1}{4}$ (B) $\frac{-1}{2} \leq y \leq \frac{1}{2}$
(C) $\frac{-1}{6} \leq y \leq \frac{1}{6}$ (D) None of these

Q.45 If a, –a, b are the roots of $x^3 - 5x^2 - x + 5 = 0$, then b is a

root of –
(A) $x^2 + 3x - 20 = 0$ (B) $x^2 - 5x + 10 = 0$
(C) $x^2 - 3x - 10 = 0$ (D) $x^2 + 5x - 30 = 0$

Q.46 If $x - 1$ is a factor of $x^5 - 4x^3 + 2x^2 - 3x + k = 0$, then k is –
(A) 3 (B) 4
(C) –4 (D) 2

Q.47 If α, β, γ are the roots of the equation $x^3 + 4x + 2 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$
(A) – 6 (B) 2
(C) 6 (D) –2

Q.48 The solution set of the inequation $\frac{x^2 + 6x - 7}{|x+4|} < 0$ is –
(A) (–7, 1) (B) (– 7, –4)
(C) $(-7, -4) \cup (-4, 1)$ (D) $(-7, -4) \cup (-4, 1)$

Q.49 If $\frac{(x-1)\sqrt{x}}{(x+1)(x-3)^2} < 0$ then –
(A) $x \in (1, 3)$ (B) $x \in (3, \infty)$
(C) $x \in (0, 1)$ (D) None of these

Q.50 Solution of inequality $\log_{10}(x^2 - 12x + 36) < 2$ is –
(A) (–4, 16) (B) (–4, 6)
(C) (6, 16) (D) $(-4, 6) \cup (6, 16)$

PART - 6 : MISCELLANEOUS

Q.51 The number of solutions of the equation $2x^2 + 9|x| - 5 = 0$ is
(A) 4 (B) 2
(C) 1 (D) 0

Q.52 The quadratic equation, whose roots are A.M. and H.M. between the roots of the equation $ax^2 + bx + c = 0$, is –
(A) $abx^2 + (b^2 + ac)\,x + bc = 0$
(B) $2abx^2 + (b^2 + 4ac)\,x + 2bc = 0$
(C) $2abx^2 + (b^2 + ac)\,x + bc = 0$
(D) None of these

Q.53 The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$, is -
(A) 0 (B) 8
(C) 4 (D) none of these

Q.54 Solution set of the equation $3^{2x^2} - 2.3^{x^2+x+6} + 3^{2(x+6)} = 0$ is –
(A) {–3, 2} (B) {6, –1}
(C) {–2, 3} (D) {1, –6}

Q.55 The number of solutions of the equation $4x(x - 3) - 5|2x - 3| + 13 = 0$ is
(A) 1 (B) 2
(C) 3 (D) 4

Q.56 If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then, the common factor is –
(A) (x – 3) (B) (x – 6)
(C) (x – 8) (D) none of these

Q.57 The equation $\log_5 x + \left(\log_{(x^2+3)} 25\right)^{-1} = \log_{25} 10$ has
(A) No solution (B) One solution
(C) Two solutions (D) Four solutions

Q.58 The roots of $x^2 - 8|x| + 12 = 0$
(A) Do not form a progression
(B) Form an A..P. with Zero sum
(C) Form an A.P. with non-zero sum
(D) Form a G.P.

Q.59 If $a > b > 0$ are two real numbers, the value of $\sqrt{ab + (a-b)\sqrt{ab + (a-b)\sqrt{ab + (a-b)\sqrt{ab + ...}}}}$ is
(A) Independent of b
(B) Independent of a
(C) Independent of both A and B
(D) Dependent on both A and B

Q.60 If A and H are the arithmetic and harmonic means of the roots of the equation $ax^2 + bx + c = 0$, $a, b \neq 0$, then the equation whose roots are AH and A–H are
(A) $ax^2 + (b^2 - 4ac)\,x + c = 0$
(B) $ab\,x^2 + (b^2 - 4ac)\,x + bc = 0$
(C) $2abx^2 + (b^2 - 4ac)\,x + 2bc = 0$
(D) None of these

Q.61 $\frac{8x^2 + 16x - 51}{(2x-3)\,(x+4)} > 3$, if x satisfies
(A) $x < -4$ (B) $-3 < x < 3/2$
(C) $x > 5/2$ (D) all the above

Q.62 If a, b, c are real and the difference between the two roots of the quadratic equation $ax^2 + bx + c = 0$ is less than 2, then Δ, the discriminant satisfies the relation-
(A) $0 \leq \Delta < 4a^2$ (B) $4a^2 < \Delta$
(C) $\Delta = 4a^2$ (D) None of these

Q.63 Find the largest integral value of m for which the inequality $\frac{x^2 - mx - 2}{x^2 - 3x + 4} > -1$ satisfied for all $x \in R$.
(A) (–3, 1) (B) (–4, 2)
(C) (–7, 1) (D) (–3, 1)

Q.64 The mid point of the interval in which $3x + 8\sqrt{x} - 3 \leq 0$ is satisfied is
(A) $\frac{19}{18}$ (B) $\frac{1}{18}$
(C) –4/3 (D) 1/6

Q.65 If α, β be the roots of $ax^2 + bx + c = 0$ and γ, δ those of $\ell x^2 + mx + n = 0$, then the equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$ is
(A) $a^2\ell^2x^2 - ab\ell mx + (b^2 - 4ac)n\ell + m^2ac = 0$
(B) $a^2\ell^2x^2 + ab\ell mx - (b^2 - 4ac)n\ell = 0$
(C) $a^2\ell^2x^2 - ab\ell mx - (b^2 - 4ac)n\ell = 0$
(D) none of these

Q.66 The minimum value of $f(x) = x^2 - 2bx + 2c^2$ is more than the maximum value of $g(x) = -x^2 - 2cx + b^2$, x being real, for–
(A) $|c| < |b|\sqrt{2}$ (B) $0 < c < b\sqrt{2}$
(C) $|c| > |b|\sqrt{2}$ (D) $b\sqrt{2} < c < 0$

Q.67 The solutions of the equation $x^2 - |2x - 3| - 3x + 3 = 0$ form-

(A) An A.P. (B) A G.P.
(C) A H.P. (D) A set of numbers with zero sum

Q.68 The roots of the equation $(x - a)(x - b) = a^2 - ab^2$ are real and distinct for all $a > 0$, provided

(A) $-a \le b < \frac{5}{7}a$ (B) $-a < b < \frac{5}{7}a$
(C) $-a < b \le \frac{5}{7}a$ (D) $-2a < b < \frac{-7}{5}a$

Q.69 The equation $x^2 + (1 + 2\sin\theta)x + \sin 2\theta(\sin\theta - \cos\theta) = 0$ has roots of equal magnitude but opposite signs for

(A) Only one value of θ
(B) Only two values of θ
(C) Infinitely many values of θ
(D) No value of θ

Q.70 The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is

(A) 0, 2 (B) 0, – 2
(C) 2, – 2 (D) none of these

Q.71 If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is –

(A) 0 (B) 2
(C) 3 (D) 1

Q.72 The quadratic equation with real coefficients one of whose complex roots has the real part 12 & modulus 13 is

(A) $x^2 - 12x + 13 = 0$ (B) $x^2 - 24x + 13 = 0$
(C) $x^2 - 24x + 169 = 0$ (D) $x^2 - 24x - 13 = 0$

Q.73 If $x^2 - 2px + q = 0$ has real roots then the equation $(1 + y)x^2 - 2(p + y) + (q + y) = 0$ will have its roots real and distinct if and only if –

(A) y is negative
(B) p is not unity
(C) y is negative and p is not unity
(D) none of these

Q.74 The number of solutions of the equation $\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3$ is

(A) 1 (B) 2
(C) 3 (D) 4

Q.75 If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is

(A) $x^2 + 10x - 11 = 0$ (B) $x^2 + 10x + 11 = 0$
(C) $11x^2 + 10x + 1 = 0$ (D) $11x^2 - 10x + 1 = 0$

Q.76 Set of all values of t if sum of roots of $x^2 - (t^2 - 13t + \alpha + \gamma)x - 36 = 0$ is less than or equal to β, is $[\ell, m]$ and $p = \ell + m$, then p is equal to –

(A) 13 (B) 26
(C) 4 (D) 17

Q.77 If α, β are root of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is

(A) $x^2 - 11x + 30 = 0$ (B) $(x-3)^2 - 5(x-3) + 6 = 0$
(C) Both (A) and (B) (D) none

Q.78 The value of the expression $x^4 - 8x^3 - 8x + 2$ when $x = 2 + \sqrt{3}$ is –

(A) 0 (B) 1
(C) 2 (D) 3

Q.79 If each root of the equation $x^3 - px - 19 = 0$ is one less than corresponding root of the equation $x^3 - Ax^2 + Bx - C = 0$ where A, B, C, p are constants, then the value of C – B is equal to –

(A) 18 (B) 17
(C) 19 (D) 20

For Q.80-85

Find the set of values of 'a' for which the equation $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$ has

Q.80 All four real and distinct roots.

(A) $a \in (-\infty, -4]$ (B) $a \in (65/4, \infty)$
(C) $a \in (-4, 65/4)$ (D) $a \in \phi$

Q.81 Two real roots which are distinct.

(A) $a \in (-\infty, -4]$ (B) $a \in (65/4, \infty)$
(C) $a \in (-4, 65/4)$ (D) $a \in \phi$

Q.82 All four roots are imaginary.

(A) $a \in (-\infty, -4]$ (B) $a \in (65/4, \infty)$
(C) $a \in (-4, 65/4)$ (D) $a \in \phi$

Q.83 Four real roots in which two are equal.

(A) $a \in (-\infty, -4]$ (B) $a \in (65/4, \infty)$
(C) $a \in (-4, 65/4)$ (D) $a \in \phi$

Q.84 All four real roots which are equal.

(A) $a \in \phi$ (B) $a = 65/4$
(C) $a \in (-4, 65/4)$ (D) $a \in (-\infty, -4]$

Q.85 Two real roots which are equal.

(A) $a \in \phi$ (B) $a = 65/4$

(C) $a \in (-4, 65/4)$ (D) $a \in (-\infty, -4]$

Q.86 Find the range of values of k if the roots of the quadratic equation $(2k-5)x^2 - 2(k-1)x + 3 = 0$ are equal.

(A) k = 4 (B) k = 13

(C) k = ±10 (D) [0, 1/2)

Q.87 Find the range of values of k if the equation $(k-12)x^2 + 2(k-12)x + 2 = 0$ possess no real roots.

(A) k = 4 (B) k = 13

(C) k = ±10 (D) [0, 1/2)

Q.88 Find the range of values of k if the curve $y = x^2 + kx + 25$ touches the x-axis.

(A) k = 4 (B) k = 13

(C) k = ±10 (D) [0, 1/2)

Q.89 Find the range of values of k if the inequality $kx^2 + 2kx + 0.5 > 0$ is satisfied $\forall\, x \in R$.

(A) k = 4 (B) k = 13

(C) k = ±10 (D) [0, 1/2)

Q.90 Find the range of values of k if the quadratic trinomial $(k-2)x^2 + 8x + k + 4$ is positive for all values of x.

(A) $(-\infty, -4) \cup (2, \infty)$ (B) $(-\infty, -1) \cup (2, \infty)$

(C) $(-\infty, -3) \cup (5, \infty)$ (D) $(-\infty, -6) \cup (4, \infty)$

Q.91 Find the largest integral value of x satisfying the inequality $(x+1)(x-3)^2(x-5)(x-4)^2(x-2) < 0$.

(A) –2 (B) –6

(C) –3 (D) –4

Q.92 Find the smallest integral value of x satisfying the in- .... $\dfrac{x-5}{x^2+5x-14} > 0$.

(A) –2 (B) –6

(C) –3 (D) –4

For Q.93-99

Solve the following logrithmic equalities. Wherever base is not given take it as 10.

Q.93 $\dfrac{1}{1+\log x} + \dfrac{1}{1-\log x} > 2$

(A) $(0.1, 1) \cup (1, 10)$ (B) $(0.1, 2) \cup (1, 5)$

(C) $(0.2, 1) \cup (2, 8)$ (D) $(0.4, 1) \cup (3, 9)$

Q.94 $\dfrac{x-1}{\log_3(9-3^x)-3} \le 1$.

(A) [log 0.8, 1)] (B) [log 1.9, 2)]

(C) [log 0.7, 2)] (D) [log 0.5, 3)]

Q.95 $\log_{1/3}(2^{x+2} - 4^x) \ge -2$.

(A) $(-\infty, 1)$ (B) $(-2, 3)$

(C) $(-\infty, 2)$ (D) $(-1, 2)$

Q.96 $\log_{0.5}\left(\log_6 \dfrac{x^2+x}{x+4}\right) < 0$.

(A) $(-1, -3) \cup (4, \infty)$ (B) $(-5, -2) \cup (2, \infty)$

(C) $(-1, -4) \cup (7, \infty)$ (D) $(-4, -3) \cup (8, \infty)$

Q.97 $\log_x(\log_9(3^x - 9)) < 1$.

(A) $x \in (\log_3 10, 1)$ (B) $x \in (\log_3 10, \infty)$

(C) $x \in (\log_3 10, 2)$ (D) $x \in (\log_3 10, 5)$

Q.98 $\dfrac{(x-0.5)(3-x)}{\log_2|x-1|} > 0$

(A) $(0, 1) \cup (2, 3)$ (B) $(0, 1/2) \cup (3, 3)$

(C) $(0, 1/2) \cup (4, 5)$ (D) $(0, 1/2) \cup (2, 3)$

Q.99 $\log_{\log_2(0.5)x}(x^2 - 10x + 22) > 0$.

(A) $(3, 5-\sqrt{3}) \cup (7, \infty)$ (B) $(2, 5-\sqrt{3}) \cup (3, \infty)$

(C) $(1, 7-\sqrt{3}) \cup (2, \infty)$ (D) $(1, 3-\sqrt{2}) \cup (1, \infty)$

EXERCISE - 2 [LEVEL-2]

Q.1 When $y^2 + my + 2$ is divided by $(y - 1)$ then the quotient is $f(y)$ and the remainder is R_1. When $y^2 + my + 2$ is divided by $(y + 1)$ then quotient is $g(y)$ and the remainder is R_2. If $R_1 = R_2$ then the value of m is –
(A) –1 (B) 0
(C) 1 (D) 2

Q.2 The largest interval in which $x^{10} - x^7 + x^4 - x + 1 > 0$ is-
(A) $[0, \infty)$ (B) $(-\infty, 0]$
(C) $(-\infty, \infty)$ (D) None of these

Q.3 The sum of real roots of the equation
$x^2 - 2^{2008}x + |x - 2^{2007}| + 2(2^{4013} - 1) = 0$ is-
(A) 2^{2007} (B) 2^{2006}
(C) 2^{2008} (D) None of these

Q.4 If α, β are roots of the equation $x^2 - x - 1 = 0$ and $A_n = \alpha^n + \beta^n$, then $A_{n+2} + A_{n-2} =$
(A) $3A_{n+1}$ (B) $3A_n$
(C) $3A_{n-1}$ (D) None of these

Q.5 If exactly one root of the equation $x^2 - 2ax + a^2 - 1 = 0$ lies between 2 and 4, then-
(A) $a \in (-\infty, 1)$ (B) $a \in [5, \infty)$
(C) $a \in (1, 5)$ (D) None of these

Q.6 If x and y are two positive numbers such that x + y = a, then the minimum value of $\sqrt{\left(x+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)}$ is
(A) $1 + 2a^{-1}$ (B) $1 + a^{-1}$
(C) $1 - 2a^{-1}$ (D) $a - a^{-1}$

Q.7 Let α_1, α_2 be the roots of $x^2 - 2x + p = 0$ and α_3, α_4 be the roots of $x^2 - 8x + q = 0$. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in$ G.P. and p and q are integers, then the quadratic equation whose roots are p and q is-
(A) $x^2 - 136x + 1024 = 0$ (B) $x^2 + 136x + 1024 = 0$
(C) $x^2 - 136x - 1024 = 0$ (D) $x^2 + 136x - 1024 = 0$

Q.8 If $f(x) = x^2 + bx + c$ and $f(2 + t) = f(2 - t)$ for all real numbers t, then which of the following is true ?
(A) $f(1) < f(2) < f(4)$ (B) $f(2) < f(1) < f(4)$
(C) $f(2) < f(4) < f(1)$ (D) $f(4) < f(2) < f(1)$

Q.9 Number of real roots of the equation $x|x| + a|x| - 1 = 0$ (where $a \in R$) is-
(A) 0 (B) 2
(C) 3 (D) 4

Q.10 If a, b, c are sides of a triangle, where $a \neq b \neq c$ and each quadratic equation $ax^2 + bx + \frac{c}{4} = 0$, $bx^2 + cx + \frac{a}{4} = 0$ and $cx^2 + ax + \frac{b}{4} = 0$ are real roots. If $p = \frac{a^2 + b^2 + c^2}{ab + bc + ca}$, then-
(A) $p > 1$ (B) $p < 2$
(C) $1 \le p < 2$ (D) $p \le 1$

Q.11 Given two equations $x^2 + x + k - k^2 = 0$, $x^2 - (k + 2)x + 2k = 0$. If exactly one root of any equation lies between roots of other, then set of values of k is $(a, b) \cup (c, \infty)$. Find the least integral value of a.
(A) 3 (B) 4
(C) 8 (D) 6

Q.12 Set of all real value of a such that
$f(x) = \frac{(2a-1)x^2 + 2(a+1)x + (2a-1)}{x^2 - 2x + 40}$ always negative is –
(A) $(-\infty, 0)$ (B) $(0, \infty)$
(C) $(-\infty, 1/2)$ (D) $(2, \infty)$

Q.13 The ordered pair (p,q) giving the least and the greatest positive solution of $\sqrt{-x^2 + 8x - 12} > 10 - 2x$ is equal to
(A) (4, 5) (B) (4, 28/5)
(C) (4, 6) (D) (2, 6)

Q.14 The set of values of a such that $x^2 - 2ax + a^2 - 6a < 0$ in [1, 2] is
(A) $\left[4-\sqrt{15}, 4+\sqrt{15}\right]$ (B) $\left[5-\sqrt{21}, 4+\sqrt{15}\right]$
(C) $\left[5-\sqrt{21}, 5+\sqrt{21}\right]$ (D) $\left[4-\sqrt{15}, 5+\sqrt{21}\right]$

Q.15 If a > 1 and the equation
$\log_{ax} a + 2\log_{a^2x} a + 3\log_{a^3x} a = 3$ is solved for x, then the equation has –
(A) exactly one real root
(B) two real roots, both greater than 1
(C) three real roots less than or equal to 1
(D) no real root

Q.16 Find the value(s) of 'a' for which the inequality $\tan^2 x + (a + 1)\tan x - (a - 3) < 0$, is true for at least one $x \in \left(0, \frac{\pi}{2}\right)$
(A) $a \in (-\infty, -(2\sqrt{5} + 3)) \cup (3, \infty)$
(B) $a \in (-\infty, -(2\sqrt{5} + 3)) \cup (4, \infty)$
(C) $a \in (-\infty, -(2\sqrt{5} + 3)) \cup (3, \infty)$.
(D) $a \in (-\infty, -(2\sqrt{5} + 2)) \cup (3, \infty)$

Q.17 If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, λ respectively -
(A) a, b, c (B) b, c, a
(C) c, a, b (D) none of these

Q.18 If $k \notin [0, 8]$, find the value of x for which the inequality $\frac{x^2 + k^2}{k(6 + x)} \ge 1$ is satisfied.
(A) $-1 < x < 1$ (B) $-1 < x < 2$
(C) $-2 < x < 1$ (D) $-3 < x < 1$

Q.19 If the equation $\left(\frac{1}{4}\right)^{x} + \left(\frac{1}{2}\right)^{x-1} + b = 0$ has a positive solution, then the real number b lies in the interval –

(A) $(-\infty, 1)$ (B) $(-\infty, -2)$
(C) $(-3, 1)$ (D) $(-3, 0)$

Q.20 If the function $f(x) = x^2 + bx + 3$ is not injective for values of x in the interval $0 \le x \le 1$ then b lies in –

(A) $(-\infty, \infty)$ (B) $(-2, \infty)$
(C) $(-2, 0)$ (D) $(-\infty, 2)$

Q.21 Consider the following statements :
S_1 : Number of integrals values of 'a' for which the roots of the equation $x^2 + ax + 7 = 0$ are imaginary with positive real parts is 5.
$_2$: Let α, β are roots $x^2 - (a + 3)x + 5 = 0$ and α, a, β are in A.P. then roots are 2 and 5/2
S_3 : Solution set of $\log_x (2 + x) \le \log_x (6 - x)$ is (1, 2]
State, in order, whether S_1, S_2, S_3 are true or false.
(A) FFT (B) TFT
(C) TFF (D) TTT

Q.22 The set of values of m for which both roots of the equation $x^2 - (m + 1)x + m + 4 = 0$ are real and negative consists of all m such that –

(A) $-4 < m \le -3$ (B) $-3 < m \le -1$
(C) $-3 \le m \le 5$ (D) $m \le -3$ or $m \ge 5$

Q.23 Number of integers satisfying either $\log_3 |x| < 2$ or $|\log_3 x| < 2$ are –
(A) 18 (B) 16
(C) 20 (D) 23

Q.24 The solution set of the inequality

$\log_{\cos(\pi/4)} (2x^2 - 5x + 3) \ge 2$ is –

(A) $\left[\frac{5-\sqrt{5}}{4}, 1\right) \cup \left(\frac{3}{2}, \frac{5+\sqrt{5}}{4}\right]$ (B) $\left(\frac{1}{2}, 2\right)$

(C) $\left(\frac{1}{2}, 3\right) \cup \left(3, \frac{5}{2}\right)$ (D) $\left(\frac{1}{2}, 1\right) \cup \left(2, \frac{9}{2}\right)$

Q.25 Number of integral values of parameter 'c' for which the inequality $1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) > \log_2 (cx^2 + c)$, hold good $\forall\ x \in R$, is –
(A) 0 (B) 2
(C) 7 (D) Infinite

Q.26 Set of solution for $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$, $\frac{7x-1}{3} - \frac{7x+2}{6} > x$

(A) $x \in (4, 9)$ (B) $x \in (2, 9)$
(C) $x \in (2, 3)$ (D) $x \in (2, 4)$

Q.27 A solution of the equation $4^x + 4 \cdot 6^x = 5.9^x$, is
(A) – 1 (B) 1
(C) 2 (D) 0

Q.28 If α, β are the roots of the quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma)(\alpha - \delta)$ is equal to
(A) $q + r$ (B) $q - r$
(C) $-(q + r)$ (D) $-(p + q + r)$

Q.29 The roots of the equation $x^2 + 6x + a = 0$ are real and distinct and they differ by atmost 4, then the range of values of a, is
(A) (5, 9] (B) [5, 9)
(C) [4, 8) (D) [3, 9)

Q.30 The values of k for which the quadratic equation $(1 - 2k)x^2 - 6kx - 1 = 0$ and $kx^2 - x + 1 = 0$ have atleast one root in common are
(A) {1/2} (B) {1/3, 2/9}
(C) {2/9} (D) {1/2, 2/9}

Q.31 If x, y, z are real such that $x + y + z = 4$, $x^2 + y^2 + z^2 = 6$, then the range x is
(A) (–1, 1) (B) [0, 2]
(C) [2, 3] (D) [2/3, 2]

Q.32 If tan θ and cot θ are the roots of the equation $x^2 + 2x + 1 = 0$, then the least value of $x^2 + \tan\theta\, x + \cot\theta = 0$, is
(A) 3/4 (B) 5/4
(C) –3/4 (D) –5/4

Q.33 Let α, β, γ are roots of the equation $x^3 + qx + q = 0$ then find the value of $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$.
(A) 0 (B) – 1
(C) 1 (D) none

Q.34 If all values of x obtained from the equation $4^x + (k - 3)\,2^x + k = 4$ are non-positive, then the largest integral value of k is
(A) 1 (B) 2
(C) 3 (D) 4

Q.35 Let m(b) be the minimum value of $f(x) = (2 + b + b^2)\,x^2 - 2\sqrt{2}\,(2b + 1)\,x + 8$, where $b \in [-3, 10]$. The maximum value of m (b) is
(A) 2 (B) 4
(C) 6 (D) 8

For Q.36 to Q.38
Consider the quadratic equation $(1 + k)x^2 - 2(1 + 2k)x + (3 + k) = 0$, where $k \in R - \{-1\}$.

Q.36 The number of integral values of k such that the given quadratic equation has imaginary roots are
(A) 0 (B) 1
(C) 2 (D) 3

Q.37 The set of values of k such that the given quadratic has both the roots positive is
(A) $k \in R$
(B) $k \in (-\infty, -3) \cup [\sqrt{2/3}, \infty)$
(C) $k \in (-\infty, -3) \cup (-1, \infty)$
(D) $k \in (-\infty, -\sqrt{2/3}\,] \cup [\sqrt{2/3}, \infty)$

Q.38 The number of real values of k such that the given quadratic equation has roots in the ratio 1 : 2 is
(A) 0 (B) 1
(C) 2 (D) ∞

For Q.39 to Q.41

Consider a rational function $f(x) = \frac{x^2 - 3x - 4}{x^2 - 3x + 4}$ and a quadratic function $g(x) = x^2 - (b + 1)x + b - 1$, where b is a parameter.

Q.39 The sum of integers in the range of $f(x)$, is
(A) – 5 (B) – 6
(C) – 9 (D) – 10

Q.40 If both roots of the equation $g(x) = 0$ are greater than – 1, then b lies in the interval
(A) $(-\infty, -2)$ (B) $(-\infty, -1/4)$
(C) $(-2, \infty)$ (D) $(-1/2, \infty)$

Q.41 The largest natural number b satisfying $g(x) > -2\ \forall\ x \in R$, is
(A) 1 (B) 2
(C) 3 (D) 4

ASSERTION AND REASON QUESTIONS

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.
(E) Statement -1 is False, Statement-2 is False.

Q.42 **Statement 1 :** All the real roots of the equation $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ lie in the interval [0, 3].
Statement 2 : The equation reduces to a quadratic equation in the variable $t = x + \frac{1}{x}$.

Q.43 **Statement 1 :** $(|x| + 1)^2 = 4|x| + 9$ has only two real solutions.
Statement 2 : $\frac{x-8}{n-10} = \frac{n}{x}$ has no solutions for some (more than one) values of $n \in N$.

Q.44 **Statement 1 :** There exists no solution to $(\sin x + \cos x + 2)^4 = 128 \sin 2x$.
Statement 2 : Let a, b, c, d be positive numbers, then $\frac{a+b+c+d}{4} \ge \sqrt[4]{abcd}$.
Further $\frac{a+b+c+d}{4} = \sqrt[4]{abcd}$ only if a = b = c = d.

Q.45 Consider $f(x) = (x^2 + x + 1)a^2 - (x^2 + 2)a - 3(2x^2 + 3x + 1) = 0$
Statement–1 : Number of values of 'a' for which f(x) = 0 will be an identity in x is 1.
Statement–2 : a = 3 the only value for which f(x) = 0 will represent an identity.

Q.46 **Statement 1 :** $|x - 2| + |x - 7| = 6$ has no solution.
Statement 2 : $|x - a| + |x - b| = c$ has no solution, where $0 < c < b - a$.

Q.47 Let a, b, c be real such that $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root
Statement–1 : a = b = c
Statement–2 : Two quadratic equations with real coefficients can not have only one imaginary root common.

Q.48 **Statement 1 :** Sum of the roots of the equation $x^2 + 3x + 2 = 0$ is negative and its discriminant is positive
Statement 2 : If sum of the roots of the equation $ax^2 + bx + c = 0$ is negative where $a > 0$, $b, c \in R$, then discriminant of $ax^2 + bx + c = 0$ must be positive.

Q.49 **Statement 1 :** If a, b, c are non real complex and α, β are the roots of the equation $ax^2 + bx + c = 0$ then $\text{Im}(\alpha\beta) \neq 0$.
because
Statement 2 : A quadratic equation with non real complex coefficient do not have root which are conjugate of each other.

Q.50 Let $f(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial with real coefficients and all real roots. Also $|f(i)| = 1$ where $i = \sqrt{-1}$.
Statement 1 : All 3 roots of f (x) = 0 are zero.
because
Statement 2 : a + b + c = 0

Q.51 **Statement–1 :** If $0 < a < \pi/4$, then the equation $(x - \sin a)(x - \cos a) - 2 = 0$ has both roots in $(\sin a, \cos a)$
Statement–2 : If f (a) and f(b) possess opposite signs then there exist at least one solution of the equation f(x) = 0 in open interval (a, b).

Q.52 **Statement 1 :** If $f(x) = 3(x-2)(x-6) + 4(x-3)(x-7)$, then f (x) = 0 has two different and real roots.
Statement 2 : If $f(x) = 3(x-a)(x-c) + 4(x-b)(x-d)$ and $0 < a < b < c < d$, then f (x) = 0 has two different and real roots.

Q.53 **Statement–1** : If the roots of $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in G.P. and sum of their reciprocal is 10, then $|S| = 32$
Statement–2 : $x_1 . x_2 . x_3 . x_4 . x_5 = S$, where x_1, x_2, x_3, x_4, x_5 are the roots of given equation.

Q.54 **Statement-1:** If all real values of x obtained from the equation $4^x - (a - 3)2^x + (a - 4) = 0$ are non-positive, then $a \in (4, 5]$
Statement-2: If $ax^2 + bx + c$ is non-positive for all real values of x, then $b^2 - 4ac$ must be –ve or zero and 'a' must be –ve.

MATCH THE COLUMN TYPE QUESTIONS

Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

Q.55 Match the column –

Column I

(a) Set of all values of x satisfying the inequation $\log_2\left(\frac{x-2}{4}\right) < 4$ is

(b) Set of all values of x satisfying the inequation $5^{x^2-10x} < 5^{11x-54}$ is

(c) Set of all values of x satisfying the inequation $\frac{(x-4)^2(x-6)}{(x-2)^2x^4(x-8)^2} > 0$ is

(d) Set of all values of x satisfying the inequation $||x|+2| \le 1$ is

Column II

(p) ϕ (q) $(6, \infty)$

(r) $(6, 8) \cup (8, \infty)$ (s) $(3, 18)$

Code :

(A) a-s, b-s, c-r, d-p (B) a-s, b-q, c-s, d-r

(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q

Q.56 Match the column –

Column I

(a) If $4^x - 2^{x+2} + 5 + |b-1|-3| = |\sin y|$, x, y, b $\in$ R, then possible values of b is/are

(b) Let $f(x) = \min\{x^2, 2\}$, then possible integral values of k for which $f(x) < k$ for atleast one real x is/are

(c) Let $f(x) = \begin{cases} |2x-1| & -2 \le x \le 1 \\ x^2-4 & 1 < x \le 8 \end{cases}$, then possible integersin the domain of $y = f(f(x))$ is/are

(d) Let $f(x) = x^3 + px^2 + qx + 6$, where p, q $\in$ R and $f'(x) < 0$ in largest possible interval $(-5/3, -1)$ then $(p + q)$ is greater than

Column II

(p) – 2 (q) 2

(r) 4 (s) 6

Code :

(A) a-r, b- rs, c-p, d-pqr

(B) a-pr, b- qrs, c-q, d-prs

(C) a-pqr, b- s, c-pq, d-pqrs

(D) a-pr, b- qrs, c-pq, d-pqrs

Q.57 For the quadratic equation $x^2 - (k-3)x + k = 0$ match the items in column I with items in column II.

Column I

(a) If both roots are positive, then k exhaustively belongs to interval

(b) If both roots are negative, then k exhaustively belongs to interval

(c) If both roots are equal, then k exhaustively belongs to interval

(d) If no root lies between (–1,1), then k exhaustively belongs to interval

Column II

(p) $\{1\} \cup [9, \infty)$ (q) $(0, 1)$

(r) $\{1, 9\}$ (s) $(9, \infty)$

Code :

(A) a-p, b-s, c-r, d-p (B) a-p, b-q, c-r, d-s

(C) a-s, b-q, c-r, d-p (D) a-r, b-s, c-p, d-q

Q.58 Match the column –

Column I

(a) The equation $x^3 - 6x^2 + 9x + \lambda = 0$ have exactly one root in (1, 3) then $[\lambda + 1]$ is (where [.] denotes the greatest integer function)

(b) If $-3 < \frac{x^2-\lambda x-2}{x^2+x+1} < 2$ for all x $\in$ R, then $[\lambda]$ is can be where [.] denotes the greatest integer function

(c) If $x^2 + \lambda x + 1 = 0$ and $(b-c)x^2 + (c-a)x + (a-b) = 0$ have both the roots common, then $[\lambda - 1]$ is (where [.] denotes the greatest integer function).

(d) If N be the number of solutions of the equation $|x - |4-x|| - 2x = 4$, then the value of –N is

Column II

(p) – 3 (q) – 2

(r) – 1 (s) 0

Code :

(A) a-pqrs, b- rs, c-p, d-p

(B) a-pr, b- qrs, c-q, d-prs

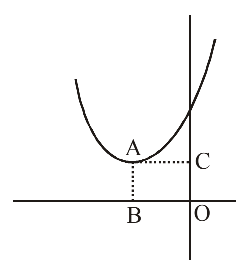
(C) a-pqr, b- s, c-pq, d-pqrs

(D) a-pr, b- qrs, c-pq, d-pqrs

PASSAGE BASED QUESTIONS

Passage 1- (Q.59-Q.61)

Graph $f(x) = ax^2 + bx + c$ of is shown adjacently, for which $\ell(AB) = 2$, $\ell(AC) = 3$ and $b^2 - 4ac = -4$



Q.59 The value of a + b + c is equal to –

(A) 7 (B) 8

(C) 9 (D) 10

Q.60 The quadratic equation with rational coefficients whose one of the roots is $b + \sqrt{a+c}$

(A) $x^2 - 6x + 2 = 0$ (B) $x^2 - 6x - 1 = 0$

(C) $x^2 + 6x + 2 = 0$ (D) $x^2 + 6x - 1 = 0$

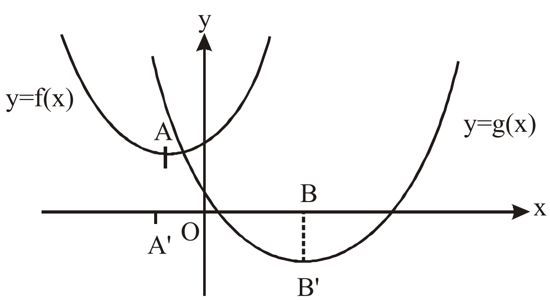
Q.61 Range of $g(x) = \left(a + \frac{1}{2}\right)x^2 + (b+2)x - \left(c - \frac{1}{2}\right)$ when $x \in [-4, 0]$ is

(A) $[-10, -6]$ (B) $\left[\frac{-49}{4}, -10\right]$

(C) $\left[\frac{-49}{4}, -6\right]$ (D) $\left[\frac{-49}{4}, \infty\right)$

Passage 2- (Q.62-Q.64)

Let $f(x) = x^2 + 2ax + b$, $g(x) = cx^2 + 2dx + 1 = 0$ be quadratic expressions whose graph is shown in the figure.



Here it is given that $|AA'| = |BB'|$ and $|OA'| = |OB|$

Q.62 Which of the following statement is correct –

(A) $bc + c + ad = d^2$ (B) $a + d = b + c$
(C) $a^2 + d^2 = c + b$ (D) $bc + c = ac^2 + d^2$

Q.63 If $|OA'| = |AA'| = 2$, then sum of real roots of equation $f(x)\,g(x) = 0$ is –

(A) 1 (B) 2
(C) 3 (D) 4

Q.64 If $|OA'| = |AA'| = 2$, then the range of $g(x)$ is –

(A) $[-2, \infty)$ (B) $[-1, \infty)$
(C) $[2, \infty)$ (D) $[1, \infty)$

Passage 3- (Q.65-Q.67)

Let $f(x) = x^3 - ax^2 + bx - 1$, $g(x) = x^3 - bx^2 + ax - 1$ be polynomials with coefficients real or complex numbers and α, β be roots of $f(x) = x^3$.

Q.65 If roots of $g(x) = 0$ are in A.P. then –

(A) $a = b$ (B) $2a^3 + 27 = 9ab$
(C) $ab^3 + 27 = 9ab$ (D) $9ab = 2b^3 + 27$

Q.66 If a, b are roots of $x^2 + x + 2 = 0$, then $f(x)\,g(x)$

(A) $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$
(B) $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$
(C) $-x^6 + x^5 - x^4 + x^3 - x^2 + x - 1$
(D) $x^6 + 2x^5 - x^4 + 2x^3 - x^2 + 2x + 1$

Q.67 If a, b are roots of $x^2 + x + 2 = 0$ then which of the following is correct –

(A) $f(x) + g(x) = 0$ has one real root and two imaginary roots
(B) $x = 1$ is a root of $f(x) = 0$
(C) $x = 1$ is a root of $g(x) = 0$
(D) $x = 1$ is not a root of $f(x) + g(x) = 0$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 Sum of real solutions of the equation

$5^{2x^2} - 2.5^{x^2+x+1} - 3.5^{2x+3} = 0$ is

Q.2 Let x be real number and $k = \dfrac{x^2 - 2x + 9}{x^2 + 2x + 9}$. If the set of all real values of k is [a, b], then find the value of 2 (a + b).

Q.3 Inequality $\dfrac{ax^2 + 3x + 4}{x^2 + 2x + 2} < 5$ is satisfied for all real values of x then, find out greatest integral value of a.

Q.4 If set of all positive values of x satisfying

$\sqrt{\log_2\left(\dfrac{x-4}{1-x^2}\right)} > 1$ is (a, b), then 2 (a + b) =

Q.5 If $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ have a common root and a, b, c ∈ N, then find the minimum value of (a + b + c).

Q.6 If set of all real values of x satisfying $\dfrac{|x-2|}{x^2 - 3x + 2} \le 1$ is

$(-\infty, a] \cup (b, c) \cup (d, \infty)$, then a + b + c + d = ?

Q.7 If every pair of equation among the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root then the sum of the three common roots is

Q.8 Net N = αααααα be a 6 digit number (all digit repeated) and N is divisible by 924 and let α, β be the roots of the equation $x^2 - 11x + \lambda = 0$, then product of all possible values of λ is ....

Q.9 Suppose a, b, c ∈ I such that greatest common divisor of $x^2 + ax + b$ and $x^2 + bx + c$ is (x + 1) and the least common multiple of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x^3 - 4x^2 + x + 6)$. The value of | a + b + c | is equal to –

Q.10 If the sum of the roots of the equation

$2^{333x-2} + 2^{111x+1} = 2^{222x+2} + 1$ is expressed in the form S_1/S_2 find $S_1 + S_2$, where S_1/S_2 is in its lowest form.

Q.11 If the equation $\dfrac{ax^2 - 24x + b}{x^2 - 1} = x$, has exactly two distinct real solutions and their sum is 12 then find the value of (a – b).

Q.12 Let $f(x) = 2^{kx} + 9$ where k is a real number. If $3f(3) = f(6)$, then the value of $f(9) - f(3)$ is equal to N, where N is a natural number. Find all the composite divisors of N.

Q.13 For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to

Q.14 The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Q.15 Let α and β be the roots of $x^2 - 6x - 2 = 0$, with α > β. If $a_n = \alpha^n - \beta^n$ for n ≥ 1, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then- **[AIEEE-2002]**
(A) p = 1 and q = 56 (B) p = 1 and q = – 56
(C) p = – 1 and q = 56 (D) p = – 1 and q = – 56

Q.2 If α and β be the roots of the equation $(x - a)(x - b) = c$ and $c \neq 0$, then roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are - **[AIEEE-2002]**
(A) a and c (B) b and c
(C) a and b (D) a+ b and b + c

Q.3 If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is **[AIEEE-2002]**
(A) 19/3 (B) 25/3
(C) – 19/3 (D) None of these

Q.4 If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in- **[AIEEE-2003]**
(A) Arithmetic Geometric Progression
(B) Arithmetic Progression
(C) Geometric Progression
(D) Harmonic Progression

Q.5 The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is- **[AIEEE-2003]**
(A) – 1/3 (B) 2/3
(C) – 2/3 (D) 1/3

Q.6 The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[AIEEE-2003]**
(A) 3 (B) 2
(C) 4 (D) 1

Q.7 If (1– p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are- **[AIEEE-2004]**
(A) 0,1 (B) – 1, 1
(C) 0, – 1 (D) – 1, 2

Q.8 If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is- **[AIEEE-2004]**
(A) 49/4 (B) 12
(C) 3 (D) 4

Q.9 The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is- **[AIEEE-2005]**
(A) 1 (B) 0
(C) 3 (D) 2

Q.10 If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals - **[AIEEE-2005]**
(A) –2 (B) 3
(C) 2 (D) 1

Q.11 In a triangle PQR, $\angle R = \frac{\pi}{2}$, If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then - **[AIEEE-2005]**
(A) a = b + c (B) c = a + b
(C) b = c (D) b = a + c

Q.12 If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[AIEEE-2005]**
(A) (5, 6] (B) $(6, \infty)$
(C) $(-\infty, 4)$ (D) [4, 5]

Q.13 If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan15°, respectively then the value of 2+q – p is – **[AIEEE-2006]**
(A) 3 (B) 0
(C) 1 (D) 2

Q.14 All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval– **[AIEEE-2006]**
(A) m > 3 (B) – 1 < m < 3
(C) 1 < m < 4 (D) – 2 < m < 0

Q.15 If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is – **[AIEEE-2006]**
(A) 41 (B) 1
(C) 17/7 (D) 1/4

Q.16 If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is- **[AIEEE-2007]**
(A) (–3, 3) (B) $(-3, \infty)$
(C) $(3, \infty)$ (D) $(-\infty, -3)$

Q.17 The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE-2008]**
(A) 4 (B) 3
(C) 2 (D) 1

Q.18 How many real solution does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ? **[AIEEE-2008]**
(A) 1 (B) 3
(C) 5 (D) 7

Q.19 If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is - **[AIEEE-2009]**
(A) Greater than 4 ab (B) Less than 4ab
(C) Greater than – 4ab (D) Less than – 4ab

Q.20 If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE 2010]**
(A) –1 (B) 1
(C) 2 (D) –2

Q.21 The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[AIEEE 2012]**
(A) infinite number of real roots (B) no real roots
(C) exactly one real root (D) exactly four real roots

Q.22 If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then a : b : c is **[JEE MAIN 2013]**
(A) 1 : 2 : 3 (B) 3 : 2 : 1
(C) 1 : 3 : 2 (D) 3 : 1 : 2

Q.23 If $a \in R$ and the equation $-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where [x] denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval **[JEE MAIN 2014]**
(A) $(-1, 0) \cup (0, 1)$ (B) $(1, 2)$
(C) $(-2, -1)$ (D) $(-\infty, -2) \cup (2, \infty)$

Q.24 Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \ne 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is – **[JEE MAIN 2014]**
(A) $\frac{\sqrt{61}}{9}$ (B) $\frac{2\sqrt{17}}{9}$ (C) $\frac{\sqrt{34}}{9}$ (D) $\frac{2\sqrt{13}}{9}$

Q.25 Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \ge 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9} =$
(A) –6 (B) 3 **[JEE MAIN 2015]**
(C) –3 (D) 6

Q.26 The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is – **[JEE MAIN 2016]**
(A) – 4 (B) 6
(C) 5 (D) 3

Q.27 If, for a positive integer n, the quadratic equation,
$x(x+1) + (x-1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$
has two consecutive integral solutions , then n is equal to **[JEE MAIN 2017]**
(A) 10 (B) 11
(C) 12 (D) 9

Q.28 Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to : **[JEE MAIN 2019 (JAN)]**
(A) 512 (B) –512
(C) –256 (D) 256

Q.29 If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $(\alpha/\beta)^n = 1$ is : **[JEE MAIN 2019 (APRIL)]**
(A) 2 (B) 3
(C) 4 (D) 5

Q.30 The sum of the solutions of the equation
$|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to : **[JEE MAIN 2019 (APRIL)]**
(A) 4 (B) 9
(C) 10 (D) 12

Q.31 The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is **[JEE MAIN 2019 (APRIL)]**
(A) infinitely many (B) 2
(C) 3 (D) 1

Q.32 If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :- **[JEE MAIN 2019 (APRIL)]**
(A) $8\sqrt{3}$ (B) $4\sqrt{3}$
(C) $10\sqrt{5}$ (D) $10\sqrt{5}$

Q.33 The least positive value of 'a' for which the equation
$2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is **[JEE MAIN 2020 (JAN)]**

Q.34 The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is **[JEE MAIN 2020 (JAN)]**
(A) 4 (B) 2
(C) 3 (D) 1

Q.35 Let $a, b \in R$, $a \ne 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : **[JEE MAIN 2020 (JAN)]**
(A) 26 (B) 25
(C) 28 (D) 24

Q.36 Let α and β are the roots of $x^2 - x - 1 = 0$ such that $P_k = \alpha^k + \beta^k$, $k \ge 1$ then which one is incorrect? **[JEE MAIN 2020 (JAN)]**
(A) $P_5 = P_2 \times P_3$
(B) $P_1 + P_2 + P_3 + P_4 + P_5 = 26$
(C) $P_3 = P_5 - P_4$
(D) $P_4 = 11$

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	A	C	D	A	A	B	B	B	C	A	B	B	A	C	A	B	C	A	D	B	A	C	C	B	D
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	C	A	A	D	B	C	C	D	D	C	D	A	A	D	B	C	D	B	A	C	B	A	C	C	A
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	B	B	C	C	D	C	B	B	A	C	D	A	C	B	A	C	A	B	D	B	D	C	C	A	C
Q	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	
A	A	C	B	B	A	B	C	D	A	B	A	B	C	D	D	A	B	A	B	C	D	B	D	A	

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	C	B	C	A	B	B	B	C	A	A	C	B	C	A	C	A	D	C	B	A	B	A	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A	A	D	C	B	C	D	D	C	C	D	B	B	C	B	D	B	D	B	A	D	D	A	C	D	B
Q	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67								
A	D	A	C	B	A	D	C	A	D	A	C	A	D	A	D	B	A								

EXERCISE - 3

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	5	2	5	9	5	0	672	6	113	5854	11	3	2	3

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
A	D	C	A	D	B	C	C	A	A	D	B	C	A	B	A	A	C	A	C	B	B	A	A	D	B	D	B	C
Q	29	30	31	32	33	34	35	36																				
A	C	C	A	D	8	D	B	A																				

CHAPTER- 4 : QUADRATIC EQUATION AND INEQUALITIES
SOLUTIONS TO TRY IT YOURSELF

(1) **(C).** $b^2 - 4ac = 49 - 32 = 17 > 0$ (not a perfect square)

$\therefore$ Its roots are irrational and different.

(2) **(A).** The required equation is

$$x^2 - \{(2+\sqrt{3}) + (2-\sqrt{3})\}\,x + (2+\sqrt{3})(2-\sqrt{3}) = 0$$

or $x^2 - 4x + 1 = 0$

(3) $D < 0$

$9 - 4k < 0$; $k > 9/4$; $(9/4, \infty)$

(4) $x = 3 + \sqrt{5} \Rightarrow x - 3 = \sqrt{5}$

$\Rightarrow (x-3)^2 = 5 \Rightarrow x^2 - 6x + 4 = 0$

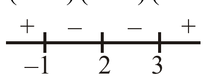
$x^4 - 12x^3 + 44x^2 - 48x + 17 = (x^2 - 6x + 4)(x^2 - 6x + 4) + 1$

We know that, divided = (divisor) (quotient) + R

But $x^2 - 6x + 4 = 0$

$\Rightarrow x^4 - 12x^3 + 44x^2 - 48x + 17 = 1$

(5) $(x+1)(x+3)(x-2)^2 \ge 0$



$x \in (-\infty, -1] \cup \{2\} \cup [3, \infty)$

(6) $\dfrac{x+1}{x-1} \ge \dfrac{x+5}{x+1} \Rightarrow \dfrac{x+1}{x-1} - \dfrac{x+5}{x+1} \ge 0$

$\Rightarrow \dfrac{(x+1)^2 - (x-1)(x+5)}{(x-1)(x+1)} \ge 0$

$\Rightarrow \dfrac{-2x+6}{(x-1)(x+1)} \ge 0 \Rightarrow \dfrac{x-3}{(x-1)(x+1)} \le 0$

$\Rightarrow x \in (-\infty, -1) \cup (1, 3]$

(7) $1 < \dfrac{3x^2 - 7x + 8}{x^2 + 1} \le 2$

Here, we need to make sure

$\Rightarrow \dfrac{3x^2 - 7x + 8}{x^2 + 1} > 1$ and $\dfrac{3x^2 - 7x + 8}{x^2 + 1} \le 2$

$\Rightarrow 3x^2 - 7x + 7 > x^2 + 1$ and $3x^2 - 7x + 8 \le 2x^2 + 2$

$\Rightarrow 2x^2 - 7x + 7 > 0$ and $x^2 - 7x + 6 \le 0$

Here, $a > 0$ and $D < 0$ and $(x-1)(x-6) \le 0$

$\Rightarrow x \in R$ and $x \in [1, 6]$

Taking intersection of both, we get $x \in [1, 6]$

(8) **(C).** Let $\alpha + 3 = x$

$\therefore \alpha = x - 3$ (Replace x by $x - 3$)

So, the required equation is

$(x-3)^2 - 5(x-3) + 6 = 0$

$x^2 - 6x + 9 - 5x + 6 = 0$

$x^2 - 11x + 30 = 0$

(9) $2x^2 + 3x - 2 \Rightarrow x = -2$ or $1/2$

when $x = -2$ is common root

$\Rightarrow 12 - 8k + 2 = 0 \Rightarrow 8k = 14 \Rightarrow k = 7/4$

when $x = 1/2$ is common root

$\Rightarrow \dfrac{3}{4} + 2k + 2 = 0 \Rightarrow 2k = -11/4 \Rightarrow k = 11/8$

(10) **(D).** Two quadratic polynomials can be a factor of cubic polynomial only when they have atleast one root common.

$\Rightarrow \quad x^2 + (b - 2a)x - k = 0$ ...... (1)

and $\quad 2x^2 + (2k - 43)x + k = 0$ ...... (2)

Must have a common roots

Multiple eq. (1) by 2 and subtracting, we get

$15x + 3k = 0 \Rightarrow x = -k/5$ is the common root

and it must satisfy equation (1)

$\Rightarrow \dfrac{k^2}{25} + (k - 29)\left(-\dfrac{k}{5}\right) - k = 0$

$\Rightarrow \left(-\dfrac{k}{5}\right)\left[-\dfrac{k}{5} + k - 29 + 5\right] = 0 \Rightarrow k = 0$ or $k = 30$

(11) $\dfrac{-b}{2a} = 2 \Rightarrow 4a = -b$ .... (1)

$P(2) = 4a + 2b + 8 = 6$

$4a + 2b = -2$ .... (2)

Using eq. (1) & eq. (2), $b = -2$, $a = 1/2$

(12) $y = \log_{10}(x^3 - 4x^2 + x + 26) - \log_{10}(x + 2)$

$x^3 - 4x^2 + x + 26 > 0$, $x + 2 > 0$

$(x+2)(x^2 - 6x + 13) > 0$

$x + 2 > 0$, $x^2 - 6x + 13 > 0$

Since, $x^2 - 6x + 13$ is always positive therefore $x > -2$

$$y = \log \dfrac{(x+2)(x^2 - 6x + 13)}{(x+2)} = \log_{10}(x^2 - 6x + 13)$$

$y = \log_{10}[(x-3)^2 + 4]$

$y_{min} = \log_{10} 4$

(13) $y = \dfrac{x+2}{x^2+3x+6}$

$\Rightarrow x^2y + 3xy + 6y = x + 2$

$\Rightarrow x^2y + x(3y-1) + 6y - 2 = 0$

$\because$ x is real $\quad \therefore D \ge 0$

$\Rightarrow (3y-1)^2 - 4y(6y-2) \ge 0$

$\Rightarrow 9y^2 - 6y + 1 - 24y^2 + 8y \ge 0$

$\Rightarrow -15y^2 + 2y + 1 \ge 0$

$\Rightarrow 15y^2 - 2y - 1 \le 0$

$\Rightarrow (5y+1)(3y-1) \le 0$

$\Rightarrow y \in [-1/5, 1/3]$

(14) $y = \dfrac{x^2-3x+2}{x^2+x-6} \Rightarrow \dfrac{(x-1)(x-2)}{(x+3)(x-2)},\ x \ne 2$

$\Rightarrow \quad y = \dfrac{x-1}{x+3}$ .... (1)

$\Rightarrow xy + 3y = x - 1 \Rightarrow x(1-y) = 3y + 1$

$\Rightarrow \quad x = \dfrac{3y+1}{1-y},\ y \ne 1$

y is not defined at $x = 2$

$\therefore$ on putting $x = 2$ in eq. (1)

$y = \dfrac{2-1}{2+3} = \dfrac{1}{5}$

Hence, range of the function is $R - \left\{\dfrac{1}{5}, 1\right\}$

(15) $x^2 + mx + (m^2 + 6m) = 0$

If exactly one root lies in $(-2, 0)$ then $f(-2)\, f(0) < 0$

$(m^2 + 4m + 4)(m^2 + 6m) < 0$

$m \in (-6, -2) \cup (-2, 0)$

We have to find out the conditions when one of the root is -2, or 0.

Case I : If one root is -2 then $f(-2) = 0$

$m = -2$

$x^2 - 2x - 8 = 0$

$x = 4, -2$, no root lie in $(-2, 0)$ for $m = -2$.

Case II : If one root is zero, then $m = 0$, or -6

If $m = 0$, $x^2 = 0$ both the roots are zero and no root lies in $(-2, 0)$ If $m = -6$, $x = 0, 6$, no roots lie in $(2, 0)$

Hence, $m \in (-6, -2) \cup (-2, 0)$

(16) The given system is equivalent to the system : $\begin{cases} x < 1, \\ x < 2 \end{cases}$

from which it follows that any number from the interval $(-\infty, 1)$ will be a solution and there are no other solutions.

CHAPTER-4 : QUADRATIC EQUATION AND INEQUALITIES

EXERCISE-1

(1) **(A).** Here a = 1, b = 4, c = 1

$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

(2) **(C).** $b^2 - 4ac = 49 - 32 = 17 > 0$ (not a perfect square)

$\therefore$ Its roots are irrational and different.

(3) **(D).** 1, B = –2 (a+b), C = 2 $(a^2 + b^2)$

$B^2 - 4AC = 1[2(a+b)]^2 - 4(1)(2a^2 + 2b^2)$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 - 4b^2 + 8ab = -4(a-b)^2 < 0$

So roots are imaginary and different.

(4) **(A).** The discriminant of the equation

$(-2\sqrt{2})^2 - 4(1)(1) = 8 - 4 = 4 > 0$ and a perfect square

so roots are real and different but we can't say that roots are rational because coefficients are not rational therefore.

$\alpha, \beta = \frac{2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4}}{2} = \frac{2\sqrt{2} \pm 2}{2} = \sqrt{2} \pm 1$

$\therefore$ This is irrational. The roots are real and different.

(5) **(A).** The roots of the equation $x^2 - 3x - 4 = 0$ are of opposite sign and greater root is positive

($\because a > 0, b < 0, c < 0$)

(6) **(B).** The roots of the equations $2x^2 - 3x + 2 = 0$ are reciprocal to each other because here a = c.

(7) **(B).** Here a = 1, b = 2, c = P

$\Rightarrow$ discriminant $= (2)^2 - 4(1)(P) \geq 0$ (Since roots are real)

$\Rightarrow 4 - 4P \geq 0 \Rightarrow 4 \geq 4P \Rightarrow P \leq 1$

(8) **(B).** Let the roots are α & $-\alpha$.

Given equation is $(x^2 - bx)(k+1) = (k-1)(ax - c)$

$\Rightarrow x^2(k+1) - bx(k+1) = ax(k-1) - c(k-1)$

$\Rightarrow x^2(k+1) - bx(k-1) - ax(k-1) + c(k-1) = 0$

Now sum of roots = 0 ($\because \alpha - \alpha = 0$)

$\therefore b(k+1) + a(k-1) = 0 \Rightarrow k = \frac{a-b}{a+b}$

(9) **(C).** Product of the roots $\frac{c}{a} = 3 = \frac{2m-1}{m}$

$\therefore 3m - 2m = -1 \Rightarrow m = -1$

(10) **(A).** Here $\alpha + \beta = 5$, $\alpha\beta = 6$

Now $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (5)^3 - 3.6(5) = 125 - 90 = 35$

(11) **(B).** Let α be the root of $x^2 - x + m = 0$ and 2α be the root of $x^2 - 3x + 2m = 0$. Then,

$\alpha^2 - \alpha + m = 0$ and $4\alpha^2 - 6\alpha + 2m = 0$

$\Rightarrow \frac{\alpha^2}{-4m} = \frac{\alpha}{-2m} = \frac{1}{2}$

$\Rightarrow m^2 = -2m \Rightarrow m = 0, m = -2$

(12) **(B).** The quadratic equation is given by

x^2 – (sum of the roots) x + (product of roots) = 0

$\therefore$ The required equation $= x^2 - (3+4)x + 3.4 = 0$

$= x^2 - 7x + 12 = 0$

(13) **(A).** The required equation is

$x^2 - \{(2+\sqrt{3}) + (2-\sqrt{3})\}x + (2+\sqrt{3})(2-\sqrt{3}) = 0$

or $x^2 - 4x + 1 = 0$

(14) **(C).** Let $\alpha + 3 = x$ $\therefore$ $\alpha = x - 3$ (Replace x by x – 3)

So the required equation is

$(x-3)^2 - 5(x-3) + 6 = 0$ ...(1)

$\Rightarrow x^2 - 6x + 9 - 5x + 15 + 6 = 0$

$\Rightarrow x^2 - 11x + 30 = 0$ ...(2)

(15) **(A).** Given root $= \frac{1}{2+\sqrt{5}} = \sqrt{5} - 2$

So the other root $= -\sqrt{5} - 2$.

Then sum of the roots = – 4, product of the roots = – 1

Hence the equation is $x^2 + 4x - 1 = 0$

(16) **(B).** If roots are in same ratio then

$\frac{3^2}{(-1)^2} = \frac{(1).(2)}{(1).(\lambda)} \Rightarrow 9 = \frac{(2)}{(\lambda)} \Rightarrow \lambda = 2/9$

(17) **(C).** $|\alpha^2 - \beta^2| = |(\alpha+\beta)|\,|\alpha - \beta|$

$= |\alpha+\beta|\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \frac{7}{4}\sqrt{49-8c}$

$= \frac{7}{4} \Rightarrow 49 - 8c = 1 \Rightarrow c = \frac{48}{8} = 6$

(18) **(A).** $\alpha + \beta = a$, $\alpha\beta = b^2$

$\alpha^2 + \beta^2 = (\alpha+\beta) - 2\alpha\beta = a^2 - 2b^2$

(19) **(D).** Sum of roots = 1 and product of roots $= \frac{b}{4a}$

For real roots D > 0 $\Rightarrow 16a^2 - 16ab > 0 \Rightarrow 16a(a-b) > 0$

(A) Not true because if a < 0, the a < b.

(B) If a and b both have the same sign, then sum as well as product of roots are positive.

$\therefore$ Both the roots are positive.

(C) sum of roots = 1 $\therefore$ (C) is not true

(D) If a and b are of the same signs, then the both the roots are positive.

(20) **(B).** Since α, β are the roots of equation $x^2 - 3x + 5 = 0$

So, $\alpha^2 - 3\alpha + 5 = 0$; $\beta^2 - 3\beta + 5 = 0$

$\therefore \alpha^2 - 3\alpha = -5$; $\beta^2 - 3\beta = -5$

Putting in $(\alpha^2 - 3\alpha + 7)$ & $(\beta^2 - 3\beta + 7)$ ......(1)

$-5 + 7, -5 + 7$ $\therefore$ 2 and 2 are the roots

$\therefore$ The required equation is $x^2 - 4x + 4 = 0$

(21) **(A).** One root less than – 2 and other greater than 2

$\Rightarrow$ product of roots < 0, which is not so

$\therefore$ there is no value of k.

(22) **(C).** From the given two equation

$\alpha + \beta = -b$ ...(1)

$\alpha\beta = ac$ ...(2)

$\alpha + \gamma = -a$ ...(3)

$\alpha\gamma = bc$ ...(4)

$(1) - (3) \Rightarrow \beta - \gamma = a - b$ ...(5)

(2) / (4) $\Rightarrow \beta/\gamma = a/b$

$\beta = \dfrac{a\gamma}{b}$ ..(6)

putting the value of β in (5)

$\dfrac{a\gamma}{b} - \gamma = a - b \Rightarrow \gamma\dfrac{(a-b)}{b} = (a-b)$

$\therefore \gamma = b \quad \therefore \beta = a \quad \& \alpha = c$

(23) **(C).** Let α be the common root of the given equations.
Then $a\alpha^2 + 2c\alpha + b = 0$
and $a\alpha^2 + 2b\alpha + c = 0$
$\Rightarrow 2\alpha(c - b) + (b - c) = 0$
$\Rightarrow \alpha = 1/2$ $[\because b \neq c]$
Putting $\alpha = 1/2$ in $a\alpha^2 + 2c\alpha + b = 0$,
we get $a + 4b + 4c = 0$.

(24) **(B).** Since one root is common, let the root is α.

$\dfrac{\alpha^2}{10k - 9k} = \dfrac{\alpha}{6k - 5k} = \dfrac{1}{3-4}$

$\alpha^2 = -k$ ...(1) $\alpha = -k$ ...(2)
$\therefore \alpha^2 = k^2 \Rightarrow k^2 = -k \Rightarrow k^2 + k = 0$
$\Rightarrow k(k+1) = 0 \Rightarrow k = 0$ and $k = -1$

(25) **(D).** Since the given equation have two roots in common so from the condition

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} \Rightarrow \dfrac{2}{1} = \dfrac{1}{1/2} = \dfrac{k}{-1} \quad \therefore k = -2$

(26) **(C).** Let the common root is α then
$\alpha^2 + \alpha - 1 = 0$
$2\alpha^2 - \alpha + \lambda = 0$

By cross multiplication, $\dfrac{\alpha^2}{\lambda - 1} = \dfrac{\alpha}{-2-\lambda} = \dfrac{1}{-1-2}$

$\alpha^2 = \dfrac{\lambda - 1}{-3} = \dfrac{1-\lambda}{3}$, $\alpha = \dfrac{2+\lambda}{3}$

$\left(\dfrac{2+\lambda}{3}\right)^2 = \dfrac{1-\lambda}{3} \Rightarrow \lambda^2 + 7\lambda + 1 = 0$

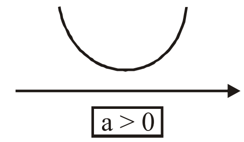
(27) **(A).** $x^2 + 2x + 7 = 0$ has imaginary roots.
$\Rightarrow$ Both roots of $4x^2 + \alpha x + \beta = 0$ and $x^2 + 2x + 7 = 0$ are common.

$\dfrac{4}{1} = \dfrac{\alpha}{2} = \dfrac{\beta}{7} \Rightarrow \alpha = 8 ; \beta = 28$

(28) **(A).** Discriminant of the equation $2a^2x^2 - 2abx + b^2 = 0$ is $-4a^2b^2 < 0$ and that of the equation
$p^2x^2 + 3pqx + q^2 = 0$ is $5p^2q^2 > 0$.
There cannot be any common root.

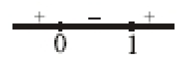
(29) **(D).** Expression $= (x + b)^2 - b^2 + c = (x + b)^2 + (c - b^2)$
$\therefore$ expression will be positive if $c - b^2 > 0$
$\Rightarrow b^2 < c$

(30) **(B).** $a + b + c > \dfrac{9c}{4}$
$4a + 4b - 5c > 0$
$f(x) = ax^2 + 2bx - 5c = 0$
$f(2) = 4a + 4b - 5c > 0$
$f(0) > 0 \Rightarrow -5c > 0$
$c < 0$



(31) **(C).** $\alpha\beta < 0$
$k^2 - 3k + 2 < 0$; $(k-1)(k-2) < 0 \; k \in (1, 2)$

(32) **(C).** Roots are same sign if $\alpha\beta > 0$
$k(k-1) > 0$
$k \in (-\infty, 0) \cup (1, \infty)$



(33) **(D).** $p^2 + q^2 = (p+q)^2 - 2pq = \alpha^2 - 2\alpha + 6$
$p^2 + q^2 = (\alpha - 1)^2 + 5 \geq 5$

(34) **(D).** $\because f(1) + f(2) = 0$
$\therefore a + b + c + 4a + 2b + c = 0$

$\Rightarrow 5a + 3b + 2c = 0 \Rightarrow b = -\dfrac{1}{3}(5a + 2c)$

Putting this value of b in $D = b^2 - 4ac$, we get $D > 0$.
Hence, two distinct real roots.

(35) **(C).** Since $a = 4 > 0$ therefore its minimum value is

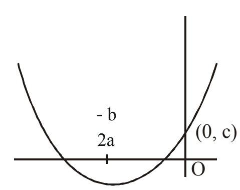
$= \dfrac{4(4)(1) - (2)^2}{4(4)} = \dfrac{16-4}{16} = \dfrac{12}{16} = \dfrac{3}{4}$

(36) **(D).** Since $a = -4 < 0$ therefore its maximum value is -

$= \dfrac{4(-4)(5) - (20)^2}{4(-4)} = \dfrac{-80-400}{-16} = \dfrac{-480}{-16} = 30$

(37) **(A).** $x^2 - 8x + 17 = (x-4)^2 + 1 \geq 1$

(38) **(A).** Let $a > 0,\ c > 0,\ \dfrac{-b}{2a} < 0 \Rightarrow -b < 0$
$\Rightarrow b > 0 \Rightarrow a > 0, b > 0, c > 0$
Similarly, if $a < 0$, we obtain $b < 0, c < 0$.



$\therefore$ a, b, c are of same sign

(39) **(D).** $-2x^2 + 6x - 9 = -(2x^2 - 6x + 9) < 0 \ \{\because D = 36 - 72 < 0\}$
thus L.H.S. $> 0 \ \forall$ x and R.H.S. < 0 $\therefore$ no solution

(40) **(B).** $21 + 12x - 4x^2 = 21 - 4\left(x - \dfrac{3}{2}\right) + 9$

$= 30 - 4\left(x - \dfrac{3}{2}\right)^2 \leq 30.$

(41) **(C).** Discriminant $b^2 - 4ac = 25 + 24 = 49 > 0$
$\Rightarrow$ Roots are real.
$\Rightarrow$ The given expression is positive for those real values of x for which $x \notin (-3, 1/2)$, because $a = 2 > 0$.
$\Rightarrow x > 1$ is true.

(42) **(D).** Let $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 3} = y$
$\Rightarrow x^2 (1 - y) + 2x (7 - y) + 3 (3 - y) = 0$
Hence $4 (7 - y)^2 - 12 (1 - y) (3 - y) \ge 0$
gives $-2y^2 - 2y + 40 \ge 0$
$\Rightarrow y^2 + y - 20 \le 0 \Rightarrow (y + 5) (y - 4) \le 0 \Rightarrow -5 \le y \le 4$

(43) **(B).** Let roots are $\alpha - d, \alpha, \alpha + d$
Using theory of equation
$\alpha - d + \alpha + \alpha + d = 12 \Rightarrow \alpha = 4$ .... (1)
$\alpha (\alpha - d) + \alpha (\alpha + d) + (\alpha - d) (\alpha + d) = k$ .... (2)
$\alpha (\alpha - d) (\alpha + d) = 28$ .... (3)
From eq. (1) and eq. (3)
$d^2 = 9$ i.e. $d = \pm 3$ using in eq. (2) and get k.

(44) **(A).** Let $\dfrac{x}{x^2 + 4} = y \Rightarrow x^2 y - x + 4y = 0$
Now, $x \in R \Rightarrow B^2 - 4AC \ge 0 \Rightarrow 1 - 4y . 4y \ge 0$
$\Rightarrow (4y - 1) (4y + 1) \le 0 \quad \therefore \dfrac{-1}{4} \le y \le \dfrac{1}{4}$

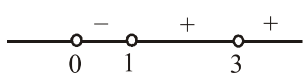
(45) **(C).** $a + (-a) + b = 5 \Rightarrow b = 5$, which is a root of $x^2 - 3x - 10 = 0$

(46) **(B).** Since $(x - 1)$ is a factor, $f(1) = 0$
$\therefore 1 - 4 + 2 - 3 + k = 0 ; k = 4$

(47) **(A).** $x^3 + 0x^2 + 4x + 2 = 0.$ Since $\alpha + \beta + \gamma = 0$
$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma = 3 (-2) = -6$

(48) **(C).** As $|x + 4| > 0$ $x^2 + 6x - 7 < 0$ but $x \ne -4$
x should lie between the roots of $x^2 + 6x - 7 = 0$ i.e. -7 and 1 $\therefore x \in (-7, 1) - \{-4\}$ i.e. $x \in (-7, -4) \cup (-4, 1)$

(49) **(C).** $\dfrac{(x - 1)\sqrt{x}}{(x + 1) (x - 3)^2} < 0$



$x \in (0, 1)$

(50) **(A).** $\log_{10} (x - 6)^2 < 2$ and $x - 6 \ne 0$
$(x - 6)^2 < 100$
$(x - 6 - 10) (x - 6 - 10) < 0$
$(x + 14) (x - 16) < 0 \Rightarrow x \in (-4, 16)$ and $x \ne 6$

(51) **(B).** $2x^2 + 9 |x| - 5 \equiv 2 |x|^2 + 9 |x| - 5 \equiv (2 |x| - 1) (|x| + 5) = 0$
$|x| + 5 \ne 0$ so that $x = \pm \dfrac{1}{2}$.

(52) **(B).** $\alpha + \beta = -\dfrac{b}{a}, \quad \alpha\beta = \dfrac{c}{a}$

$$x^2 - x \left(\frac{\alpha + \beta}{2} + \frac{2\alpha\beta}{\alpha + \beta} \right) + \alpha\beta = 0$$

$$2abx^2 + (b^2 + 4ac) x + 2bc = 0$$

$$x^2 - x \left(-\frac{b}{26} + \frac{2c / a}{-b / a} \right) + \frac{c}{a} = 0 \quad ;$$

$$x^2 + x \left(\frac{b^2 + 4ac}{2ab} \right) + \frac{c}{a} = 0$$

i.e. $2ab\, x^2 + (b^2 + 4ac) x + 2bc = 0$

(53) **(C). Case I** $x - 2 > 0$, Putting $x - 2 = y, \; y > 0$
$x > 2$
$\therefore Y^2 + Y - 2 = 0 \Rightarrow Y = -2, 1 \Rightarrow x = 0 , 3$
But $0 < 2$, Hence $x = 3$ is the real root.
Case II $x - 2 < 0 \Rightarrow x < 2, y < 0$
$y^2 - y - 2 = 0 \Rightarrow y = 2, -1 \Rightarrow x = 4 , x = 1$
Since $4 \nless 2$, only $x = 1$ is the real root
Hence the sum of the real roots = 3 + 1 = 4

(54) **(C).** $3^{2x^2} - 2.3^{x^2 + x + 6} + 3^{2(x + 6)} = 0$

$$(3^{x^2})^2 - 2 (3^{x^2}).3^{x + 6} + (3^{x + 6})^2 = 0$$

$$(3^{x^2} - 3^{x + 6})^2 = 0 \Rightarrow 3^{x^2} = 3^{x + 6} \Rightarrow x^2 - x - 6 = 0$$

$\Rightarrow (x - 3) (x + 2) = 0 \Rightarrow x = -2, 3$

(55) **(D).** $4x (x - 3) - 5|2x - 3| + 13 = (2x - 3)^2 - 5|2x - 3| + 4 = 0$
As $(2x - 3)^2 \equiv |2x - 3|^2$, this gives $|2x - 3| = 1$ or 4 giving 4 solutions.

(56) **(C).** Here Let $x - \alpha$ is the common factor then $x = \alpha$ is root of the corresponding equation
$\therefore \alpha^2 - 11\alpha + a = 0 \; ; \; \alpha^2 - 14\alpha + 2a = 0$
Subtracting $3 \alpha - a = 0 \Rightarrow \alpha = a/3$

Hence $\dfrac{a^2}{9} - 11 \dfrac{a}{3} + a = 0$, $a = 0$ or $a = 24$

since $a \ne 0, a = 24$ $\therefore$ the common factor of $\begin{cases} x^2 - 11x + 24 \\ x^2 - 14x + 48 \end{cases}$

is clearly $x - 8$

(57) **(B).** The equation can be written as

$$\log x + \frac{1}{2} \log_5 (x^2 + 3) = \frac{1}{2} \log_5 10$$

leading to $x \sqrt{x^2 + 3} = \sqrt{10}$ i.e., $(x^2 + 5) (x^2 - 2) = 0$

Of the two values $x = \pm \sqrt{2}$ log x exists only when $x = \sqrt{2}$.

(58) **(B).** The equation is
$x^2 - 8 |x| + 12 = (|x| - 6) (|x| - 2) = 0$
or $x = -6, -2, 2, 6$ which form an A.P. with zero sum

(59) **(A).** If the radical is equal to x, then x, then
$x = \sqrt{ab + (a - b)x}$ which gives $(x - a) (x + b) = 0.$
Thus x = a, independent of b.

(60) **(C).** If α, β are the roots of

$ax^2 + bx + c = 0$, then $\alpha + \beta = \dfrac{-b}{a}, \; \alpha \beta = \dfrac{c}{a}$

$A = \dfrac{\alpha + \beta}{2} = -\dfrac{b}{2a}, H = \dfrac{2\alpha\beta}{\alpha + \beta} = -\dfrac{2c}{b}, AH = \dfrac{c}{a},$

$A - H = \frac{-b^2 + 4ac}{2ab}$

The required equation is $2abx^2 + (b^2 - 4ac)x + 2bc = 0$.

(61) **(D).** Consider $\frac{8x^2 + 16x - 51}{(2x-3)(x+4)} - 3 > 0$

$\Rightarrow \frac{2x^2 + x - 15}{2x^2 + 5x - 12} > 0 \Rightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} > 0$

Hence both Nr and Dr are positive if $x < -4$ or $x > 5/2$ and both negative if $-3 < x < 3/2$

Hence all the statements are true.

(62) **(A).** a, b, c being real, the roots can be complex conjugates. But their difference cannot be real.

Thus the roots α, β are real and

$|\alpha - \beta| < 2$ giving $(\alpha + \beta)^2 - 4\alpha\beta = (\alpha - \beta)^2 < 4$.

$b^2 - 4ac = 0$ and $\frac{b^2}{a^2} - 4\frac{c}{a} < 4$ i.e. $0 \le b^2 - 4ac < 4a^2$.

(63) **(C).** For all $n \in R$

$\frac{x^2 - mx - 2}{x^2 - 3x + 4} + 1 > 0$

$\Rightarrow \frac{x^2 - mx - 2 + x^2 - 3x + 4}{x^2 - 3x + 4} > 0$

$\Rightarrow \frac{2x^2 - x(m+3) + 2}{x^2 - 3x + 4} > 0$

$\Rightarrow 2x^2 - (m+3)x + 2 > 0$

$[x^2 - 3x + 4 > 0$ for all $x \in R]$

$\Rightarrow$ Discriminent should be < 0

$\Rightarrow (m+3)^2 - 4 \times 2 \times 2 < 0$

$\Rightarrow (m+3+4)(m+3-4) < 0$

$\Rightarrow (m+7)(m-1) < 0 \Rightarrow -7 < m < 1$

(64) **(B).** $3x + 8\sqrt{x} - 3 = 0$ is a quadratic in $\sqrt{x}$

$(3\sqrt{x} - 1)(\sqrt{x} + 3) = 0 \quad \therefore 3x + 8\sqrt{x} - 3 \le 0$

$\Rightarrow (3\sqrt{x} - 1)(\sqrt{x} + 3) \le 0 \Rightarrow 3\sqrt{x} - 1 \le 0,$

since $(\sqrt{x} + 3)$ is positive for all $x \ge 0$]

$\Rightarrow \sqrt{x} \le \frac{1}{3}$ or $x \le \frac{1}{9}$. The term $\sqrt{x}$ implies $x \ge 0$.

Hence $x \in \left[0, \frac{1}{9}\right]$ and the interval $= \frac{1}{18}$.

(65) **(A).** Here $S = (\alpha\gamma + \beta\delta) + (\alpha\delta + \beta\gamma)$

$= \alpha(\gamma + \delta) + \beta(\gamma + \delta) = (\alpha + \beta)(\gamma + \delta) = \frac{bm}{a\ell}$

Also $P = (\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma)$

$= (\alpha^2 + \beta^2)\gamma\delta + \alpha\beta(\gamma^2 + \delta^2)$ ... (note)

$= \frac{b^2 n\ell + m^2 ac - 4acn\ell}{a^2\ell^2}$.

Use $x^2 - Sx + P = 0$ to obtain the equatuion.

(66) **(C).** Minimum vlaue of $f(x)$ is $\frac{4.1.2c^2 - 4b^2}{4.1} = 2c^2 - b^2$

Maximum value of $g(x)$ is $\frac{4.(-1).b^2 - 4c^2}{4.(-1)} = b^2 + c^2$

$2c^2 - b^2 > b^2 + c^2 \Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}\,|b|$

(67) **(A).** $x^2 - 3x - |2x-3| + 3 = \frac{1}{4}(2x-3)^2 - \frac{9}{4} - |2x-3| + 3$

$\Rightarrow |2x-3|^2 - 4|2x-3| + 3 = 0$

i.e., $(|2x-3| - 3)(|2x-3| - 1) = 0$

$\Rightarrow |2x-3| = 1$ or 3 and $x = 1, 2, 0, 3$ which are in A.P.

(68) **(B).** The equation is $x^2 - (a+b)x + ab - a^2 + 2b^2 = 0$

Discriminant $= (a+b)^2 - 4(ab - a^2 + 2b^2)$

$= 5a^2 - 2ab - 7b^2 = (5a - 7b)(a + b) > 0$

i.e., $\left(b - \frac{5}{7}a\right)(b + a) < 0$ giving $-a < b < \frac{5}{7}a$

(69) **(D).** The given conditions imply

$1 + 2\sin 2\theta(\sin\theta - \cos\theta) < 0.$

$1 + 2\sin\theta = 0$ given $\sin\theta = -\frac{1}{2}$ and $\cos\theta = \pm\frac{\sqrt{3}}{2}$

leading to $\sin 20(\sin\theta - \cos\theta) > 0$

(70) **(B).** Let α be the root of $x^2 - x + m = 0$ and 2α be the root of $x^2 - 3x + 2m = 0$. Then,

$\alpha^2 - \alpha + m = 0$ and $4\alpha^2 - 6\alpha + 2m = 0$

$\Rightarrow \frac{\alpha^2}{-m} = \frac{\alpha}{-m} = \frac{1}{2} \Rightarrow m^2 = -2m \Rightarrow m = 0, m = -2$

(71) **(D).** Let roots be $n, n+1, n \in I$

$b = n + n + 1, c = n(n+1)$

$b^2 - 4c = (2n+1)^2 - 4n(n+1) = 4n^2 + 4n + 1 - 4n^2$

$4n = 1$

(72) **(C).** The coefficients beings real, the complex roots occur in conjugate pairs. If $\alpha + i\beta$ is a root then $\alpha - i\beta$ is the other root.

Sum of the roots $- 2\alpha = 24$ and the product $= [\alpha + i\beta]^2 = 169$.

(73) **(C).** Here $4p^2 - 4q = 0 \Rightarrow p^2 = q$

Also $D = 4(p + y)^2 - 4(1 + y)(q + y)$

$= 4[p^2 + 2py + y^2 - q - qy - y - y^2]$

$= +4y(2p - q - 1) = 4y(2p - p^2 - 1) = -4y(p-1)^2$

Here $D > 0$ if y is negative and p is not one

(74) **(A).** The given equation can be written as

$x^3 - 3x^2 - 4x + 8 = (x-3)^3 \equiv x^3 - 9x^2 + 27x - 27$

i.e., $6x^2 - 31x + 35 \equiv (3x-5)(2x-7) = 0$

$x = 7/2$ is the only solution as the base $x - 3$ is –ve for $x = 5/3$.

(75) **(C).** α, β are the roots of $2x^2 + 4x - 5 = 0$ so that

$\alpha + \beta = -2, \alpha\beta = \frac{5}{2}$.

$2\alpha-3+2\beta-3=-10,\ (2\alpha-3)(2\beta-3)$
$=4\alpha\beta-6(\alpha+\beta)+9=11$
$2\alpha-3$ and $2\beta-3$ are the roots of $x^2+10x+11=0$ and their reciprocals are the roots of $11x^2+10x+1=0$.

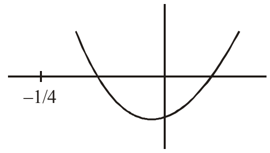
(76) **(A).** Sum of roots ≤ -6
$\Rightarrow t^2-13t+\alpha+\gamma\le -6$
$\Rightarrow t^2-13t+\alpha+\gamma+6\le 0$
$\Rightarrow p=\ell+m=13$

(77) **(C).** Let $\alpha+3=x$
$\therefore\ \alpha=x-3$ (Replace x by $x-3$)
So the required equation is
$=(x-3)^2-5(x-3)+6=0\quad =x^2-6x+9-5x+15+6=0$
$=x^2-11x+30=0$

(78) **(B).** $x=2+\sqrt{3}$
$(x-2)^2=3$ i.e., $x^2-4x+1=0$ .... (1)
$x^4-8x^3+24x^2-32x+16=9$
$x^4-8x^3+18x^2-8x+2+6(x^2-4x+1)-1=0$
From (1), $x^4-8x^3+18x^2-8x+2=1$

(79) **(B).** Let α,β,δ be roots of $x^2-Ax^2+Bx-C=0$
$\alpha+\beta+\delta=A,\ \alpha\beta+\alpha\delta+\delta\beta=B,\ \alpha\beta\delta=C$
$\alpha-1,\beta-1,\delta-1$ are roots of $x^3+px-19=0$
$\Rightarrow \alpha-1+\beta-1+\delta-1=0\Rightarrow \alpha+\beta+\delta$
$\Rightarrow A=3$ and product $(\alpha-1)(\beta-1)(\delta-1)=19$
$\alpha\beta\delta-(\alpha\beta+\beta\delta+\delta\alpha)+(\alpha+\beta+\delta)-1=19$
$C-B=19+1-3=17$

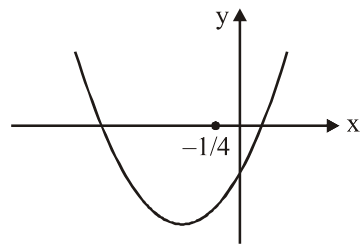
For Q.80-85
Let $x^2+x=k$ ...(1)
then $(x^2+x)^2+a(x^2+x)+4=0$
$\Rightarrow\ t^2+at+4=0$ ...(2)
If x is real then $D>0$ for equation (1)
$1+4\in>0$
or $t>-1/4$
and if t is real different then $D>0$ for equation (2)



$a^2-16>0$
$\therefore\ a<-4$ or $a>4$ ...(3)

(80) **(A).** If all for real & distinct roots then
$t>-1/4$
or both roots of equation (2) must be greater than $-1/4$
$\therefore\ f(t)=t^2+at+4$
$\therefore\ f(-1/4)>0$ and $-\frac{a}{2}>-\frac{1}{4}$
$\Rightarrow \frac{1}{16}-\frac{a}{4}+4>0$
or $a<65/4$ and $a<1/2$ ...(4)
$\therefore$ By (4) & (3)
$(-\infty,-4)$ **Ans.**

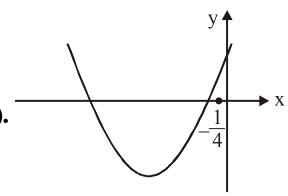
(81) **(B).** $f(t)=t^2+at+4=0$
For two real roots which is distinct imaginary & distinct $\Rightarrow$ one root (2) $>-1/4$
Other root of (2) $<-1/4$



$\therefore\ \left(-\frac{1}{4}<0\right)$

$\frac{1}{16}-\frac{a}{4}+4<0\ \Rightarrow a>\frac{65}{4}$. Hence $a\in\left(\frac{65}{4},\infty\right)$.

(82) **(C).**



All four roots are imaginary
Case-I : $D<0$ for eq. (2)
$\Rightarrow a\in(-4,4)$
Case-II : If $D\ge 0$ for eq.(2)
$a\in(-\infty,-4]\cup[4,\infty)$ ......(1)
Both root must be less then $-1/4$ & vertex (x-coordinate) $<-1/4$
$=\left(\frac{1}{4}>0\right)>0$ and $-\frac{a}{2}<-\frac{1}{4}$
$\frac{1}{16}-\frac{a}{4}+4>0$ and $a>\frac{1}{4}$
$a<\frac{65}{4}$ and $a>\frac{1}{4}$ ......(2)
From (1) and (2), $a\in\left[4,\frac{65}{4}\right)$
From Case-I and Case-II : $a\in\left(-4,\frac{65}{4}\right)$

(83) **(D).** Four real roots in which two are equal.
$t^2+ax+4=0$
two roots are equal
$t=x^2+x=\alpha$
This equation be perfect square
$\alpha=-\frac{1}{4}\ \Rightarrow\ f\left(-\frac{1}{4}\right)=0\ \Rightarrow a=\frac{65}{4}$ ......(1)
And other roots are unequal & real
So other roots of eq.(2) $>-1/4$
So $f\left(-\frac{1}{4}\right)>0\ \Rightarrow a<\frac{65}{4}$ ......(2)
From (1) and (2), $a\in\phi$

(84) **(A).** All four real roots which are equal $t^2 + ax + 4 = 0$

This equation should be perfect square

$\therefore$ $D = 0 \Rightarrow a = \pm 4$

But when we put $t = -2$ in $x^2 + x = -2$ then this gives no solution for a.

$\therefore$ $a \in \phi$

(85) **(B).** Two real roots which are equal.

$a = 65/4$ at $\alpha = -1/4$

Product of root $\alpha\beta = 4$. Hence $\beta = -16$

Now $x^2 + x = -16 \Rightarrow$ This equation has no solution

Hence value of a for which two real roots which are equal $a = 65/4$

(86) **(A).** $(2k - 5)x^2 - 2(k - 1)x + 3 = 0$

If roots of the equation are equal then

Discriminant should be zero.

$\therefore$ Discriminant $= (-2(k-1))^2 - 4(2k-5)(3)$

$\Rightarrow 4(k-1)^2 - 4(2k-5)(3) = 0$

$\Rightarrow (k^2 - 2k + 1) - (6k - 15) = 0$

$\Rightarrow k^2 - 8k + 16 = 0 \Rightarrow (k-4)^2 = 0 \quad \therefore k = 4$

(87) **(B).** Equation $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ posses no real roots then discriminant should be negative.

$\therefore$ $4(k-12)^2 - 4(k-12)(2) < 0$

$\Rightarrow (k-12)^2 - (k-12)^2 < 0$

$\Rightarrow (k-12)[k - 12 - 2] < 0 \Rightarrow (k-12)(k-14) < 0$

$\Rightarrow 12 < k < 14 \Rightarrow k = 13$ (Integral value)

(88) **(C).** If the curve $y = x^2 + kx + 25$ touches the x axis then discriminant should be zero.

$\therefore$ $k^2 - (4)(25) = 0 \Rightarrow k = \pm 10$

(89) **(D).** $kx^2 + 2kx + 1/2 > 0$ is satisfied for $x \in R$ then discriminant should be negative and $k > 0$

$\Rightarrow 4k^2 - 4(k)(1/2) < 0$ and $k > 0$

$\Rightarrow 2k^2 - k < 0 \Rightarrow k(2k - 1) < 0 \Rightarrow 0 < k < 1/2$

For $k = 0$ is also satisfied the inequality.

$k \in (0, 1/2]$

(90) **(D).** $(k - 2)x^2 + 8x + k + 4 > 0$ for all values of x then discriminant < 0 and $(k - 2) > 0$

$\Rightarrow 64 - 4(k-2)(k+4) < 0$ and $(k-2) > 0$

$\Rightarrow 16 - (k^2 + 2k - 8) < 0 \Rightarrow k^2 + 2k - 24 > 0$

$\Rightarrow (k+6)(k-4) > 0 \Rightarrow k \in (-\infty, -6) \cup (4, \infty)$

$\Rightarrow k \in (-\infty, -6) \cup (4, \infty)$ at $k > 2$

$\therefore$ $k \in (-\infty, -6) \cup (4, \infty)$.

(91) **(A).** $(x + 1)(x - 3)^2(x - 5)(x - 4)^2(x - 2) < 0$

Arrange the critical values on number line and mark appropriate sign. $x \in (-\infty, -1) \cup (2, 5) - \{3, 4\}$

$\therefore$ $x = -2$ is largest integral.

⊖ ⊕ ⊖ ⊖ ⊖ ⊕
–5 2 3 4 5

(92) **(B).** $\dfrac{(x-5)}{x^2 + 5x - 14} > 0 \Rightarrow \dfrac{(x-5)}{(x+7)(x-2)} > 0$

$\Rightarrow$ ⊖ ⊕ ⊖ ⊕
–7 3 5

$\Rightarrow x \in (-7, 2) \cup (5, \infty)$

$\Rightarrow x = -6$ is smallest integral value.

(93) **(A).** $\dfrac{1}{1 + \log x} + \dfrac{1}{1 - \log x} > 2$

$\Rightarrow \dfrac{1 - \log x + 1 + \log x - \alpha(1 - \log^2 x)}{(1 - \log^2 x)} > 0 \Rightarrow \dfrac{2\log^2 x}{1 - \log^2 x} > 0$

$\Rightarrow 1 - \log^2 x > 0$ & $\log^2 x > 0$

$\Rightarrow (\log^2 x - 1) < 0$, $\log^2 x > 0$

$-1 < \log x < 1$ $\quad x \neq 1$ & $n > 0$

$\dfrac{1}{10} < x < 10 \Rightarrow x \in \left(\dfrac{1}{10}, 10\right) - \{1\}$

(94) **(B).** $\dfrac{x - 1}{\log_3(9 - 3^x) - 3} \le 1$

$\Rightarrow 9 - 3^x > 0 \Rightarrow 3^x < 9 \Rightarrow 3^x < 3^2 \Rightarrow x < 2$

Since $x < 2$ & $3^x > 0 \Rightarrow 9 - 3^x < 9$

Hence $\log_3(9 - 3^x) - 3 < 0$

$x - 1 \ge \log_3(9 - 3^x) - 3 \Rightarrow x - 4 \ge \log_3(9 - 3^x)$

$\Rightarrow 3^{x-4} \ge 9 - 3^x$

$\Rightarrow 3^x + \dfrac{3^x}{81} \ge 9 \Rightarrow 3^x\left(\dfrac{82}{81}\right) \ge 9$

$\Rightarrow 3^x \ge \dfrac{9^3}{82} \Rightarrow \log_3\left(\dfrac{9^3}{82}\right) \le x < 2$

(95) **(C).** $2^{x+2} - 4^x > 0$

$4 \cdot 2^x - 2^{2x} > 0$

$2^x(4 - 2^x) > 0$; $2^x < 4$; $x < 2$

$2^{x+2} - 4^x \le \left(\dfrac{1}{3}\right)^{-2}$; $4 \cdot 2^x - 2^{2x} \le 9$

$2^{2x} - 4 \cdot 2^x + 9 \ge 0$; Put $2^x = t$; $t^2 - 4t + 9 \ge 0$

$x \in (-\infty, 2)$

(96) **(D).** $\log_6\left(\dfrac{x^2 + x}{x + 4}\right) > 1 \Rightarrow \dfrac{x^2 + x}{x + 4} > 6$

$\Rightarrow \dfrac{x^2 + x - 6x - 24}{x + 4} > 0 \Rightarrow \dfrac{x^2 - 5x - 24}{x + 4} > 0$

$\Rightarrow \dfrac{(x-8)(x+3)}{(x+4)} > 0$

⊖ ⊕ ⊖ ⊕
–4 –3 8

$x \in (-4, -3) \cup (8, \infty)$

(97) **(B).** $\log_x(\underbrace{\log_9(3^x - 9)}) < 1$

This log to define $3^x - 9 > 0 \Rightarrow x > 2$ ……(1)

$\log_x\underbrace{\left(\log_9(3^x - 9)\right)} < 1$

This log to define $\log_9(3^x - 9) > 0 \Rightarrow 3^x - 9 > 1$

$\Rightarrow x > \log_3 10$

$\Rightarrow$ $x > \log_3 10$ ......(2)

Now solving of question

$\log_x(\log_3(3^x - 9)) < 1$

$\Rightarrow$ $\log_9(3^x - 9) < x$

$\Rightarrow$ $3^x - 9 < 9^x \Rightarrow 3^{2x} - 3^x + 9 > 0$

$D < 0 \quad \forall x \in R$ ......(3)

From (1), (2) and (3) we get

$x \in (\log_3 10, \infty)$.

(98) **(D).** Case-I : $x \neq 1,\ x \in (0, 2) - \{1\}$ ...(i)

$\Rightarrow$ $(x - 0.5)(3 - x) < 0$

$\Rightarrow$ $(x - 0.5)(x - 3) > 0$

$x \in (-\infty, 0.5) \cup (3, \infty)$ ...(ii)

$x \in (0, 0.5)$

Case-II : $x \in (-\infty, 0) \cup (2, \infty)$ ...(iii)

$(x - 0.5)(3 - x) > 0$

$\Rightarrow$ $(x - 0.5)(x - 3) < 0$

$\Rightarrow$ $x \in (0.5, 3)$ ...(iv)

Using (iii) & (iv), $x \in (2, 3)$

hence $x \in (0, 0.5) \cup (2, 3)$.

(99) **(A).** $\log_2(0.5)x > 0$; $0.5x > 1$; $x > 2$

$x^2 - 10x + 22 > 0$

$x \in (-\infty, 5 - \sqrt{3}) \cup (5 + \sqrt{3}, \infty)$

Case-I : $2 < x < 4$	Case-II : $4x < x\infty$
$x^2 - 10x + 22 < 1$	$x^2 - 10x + 22 > 1$
$x^2 - 10x + 21 < 0$	$(x - 3)(x - 7) > 0$
$(x - 3)(x - 7) < 0$	$x < 3$ or $x > 7$
$x \in (3, 7)$	hence $x \in (3, 5 - \sqrt{3}) \cup (7, \infty)$.

EXERCISE-2

(1) **(B).** Let $F(y) = y^2 + my + 2$

$(y - 1) f(y) + R_1 = y^2 + my + 2$ .....(1)

$(y + 1) g(y) + R_2 = y^2 + my + 2$ .....(2)

$$\left.\begin{array}{l} R_1 = F(1) = m + 3 \\ R_2 = F(-1) = 3 - m \end{array}\right] = R_1 = R_2$$

$\Rightarrow m + 3 = 3 - m \Rightarrow m = 0$

Theorem : When a polynomial $f(x)$ is divided by $(x - a)$ then the remainder is $f(a)$.

(2) **(C).** Let

$f(x) = x^{10} - x^7 + x^4 - x + 1$

$= (x^{10} + x^4) - (x^7 + x) + 1$

$= x^4(x^6 + 1) - x(x^6 + 1) + 1$

$= (x^6 + 1)(x^4 - x) + 1$

$= x(x^6 + 1)(x^3 - 1) + 1 > 0 \ \forall\ x \leq 0$

$\therefore x \in (-\infty, 0]$ .... (i)

Again $f(x) = x^{10} - x^7 + x^4 - x + 1$

$= x^7(x^3 - 1) + x(x^3 - 1) + 1$

$= x(x^6 + 1)(x^3 - 1) + 1 > 0 \ \forall\ x \geq 0$

$\therefore \quad x \in [1, \infty]$ .... (ii)

Also $f(x) = x^{10} - x^7 + x^4 - x + 1$

$= x^{10} + x^4(1 - x^3) + (1 - x) > 0 \ \forall\ 0 < x < 1$

$\therefore x \in (0, 1)$ .... (iii)

Now from (i), (ii) and (iii) we have

$x \in (-\infty, 0] \cup (0, 1) \cup [1, \infty) \Rightarrow x \in (-\infty, \infty)$

(3) **(C).**

$x^2 - 2^{2008}.x + |x - 2^{2007}| + 2(2^{4013} - 1) = 0$

$\Rightarrow x^2 - x.2^{2008} + 2.2^{4013} + |x - 2^{2007}| - 2 = 0$

$\Rightarrow x^2 - 2.x.2^{2007} + 2^{4014} + |x - 2^{2007}| - 2 = 0$

$\Rightarrow (x - 2^{2007})^2 + |x - 2^{2007}| - 2 = 0$

$\Rightarrow t^2 + t - 2 = 0$ putting $t = |x - 2^{2007}|$

$$\Rightarrow \left(t + \frac{1}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow t + \frac{1}{2} = \pm\frac{3}{2} \Rightarrow t = -\frac{1}{2} \pm \frac{3}{2}$$

$\Rightarrow t = 1$ & $t = -2$ which is not possible

$\Rightarrow |x - 2^{2007}| = 1$

$\therefore x - 2^{2007} = \pm 1$

$\therefore x = 2^{2007} \pm 1$

$\therefore x = 2^{2007} + 1,\ x = 2^{2007} - 1$

$\therefore$ sum of real roots is $2^{2007} + 1 + 2^{2007} - 1 = 2^{2008}$

(4) **(B).** As α, β are roots of $x^2 - x - 1 = 0$

$\therefore \alpha^2 = \alpha + 1$ & $\beta^2 = \beta + 1$ and $A_n = \alpha^n + \beta^n$

$\therefore A_{n+2} + A_{n-2} = \alpha^{n+2} + \beta^{n+2} + \alpha^{n-2} + \beta^{n-2}$

$= \alpha^{n-2}(\alpha^4 + 1) + \beta^{n-2}(\beta^4 + 1)$

$= \alpha^{n-2}[(\alpha^2 + 1)^2 - 2\alpha^2] + \beta^{n-2}[(\beta^2 + 1)^2 - 2\beta^2]$

$= \alpha^{n-2}[(\alpha + 2)^2 - 2\alpha^2] + \beta^{n-2}[(\beta + 2)^2 - 2\beta^2]$

$= \alpha^{n-2}(3\alpha^2) + \beta^{n-2}(3\beta^2)$

$= 3(\alpha^n + \beta^n) = 3A_n$

(5) **(C).** Let $f(x) = x^2 - 2ax + a^2 - 1$.

Let α, β be the roots of $f(x) = 0$, now between 2 & 4 either α lies or β lies

$\therefore$ we have the two conditions only

(i) $D \geq 0$ (ii) $f(2) f(4) < 0$

Now $D = 4a^2 - 4(1)(a^2 - 1) = 4 > 0$ which is true $\forall\ a \in R$.

$\therefore a \in (-\infty, \infty)$ .... (i)

Again $f(2) f(4) < 0$

$\Rightarrow (4 - 4a + a^2 - 1)(16 - 8a + a^2 - 1) < 0$

$\Rightarrow (a^2 - 4a + 3)(a^2 - 8a + 15) < 0$

$\Rightarrow (a - 1)(a - 3)(a - 3)(a - 5) < 0$

$\Rightarrow (a - 3)^2 (a - 1)(a - 5) < 0$

$\Rightarrow a \in (1, 5)$.

(6) **(A).** $\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right) = 1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = 1 + \frac{a + 1}{xy}$.

But $xy = x(a - x) = \frac{a^2}{4} - \left(x - \frac{a}{2}\right)^2$

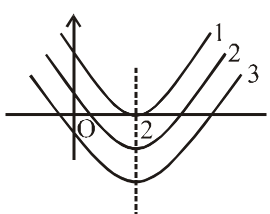
so that $xy \leq \frac{a^2}{4}$ and $\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right) \geq 1 + \frac{4(a + 1)}{a^2}$

$$= \left(\frac{a + 2}{a}\right)^2$$

$$\therefore \sqrt{\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right)} \geq 1 + 2a^{-1}$$

(7) **(B).**
As α_1, α_2 are roots of $x^2 - 2x + p = 0$
$\therefore \quad \alpha_1 + \alpha_2 = 2$ and $\alpha_1 \alpha_2 = p$ .... (i)
Again α_3, α_4 are roots of $x^2 - 8x + q = 0$
$\therefore \quad \alpha_3 + \alpha_4 = 8$ and $\alpha_3 \alpha_4 = q$ .... (ii)
As $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in$ G.P., let R be the common ratio
$\therefore \quad \alpha_2 = \alpha_1 R, \alpha_3 = \alpha_1 R^2, \alpha_4 = \alpha_1 R^3$ .... (*)
Now from (i) and (ii) by using (*) we get
$\alpha_1 + \alpha_2 = \alpha_1(1 + R)$
$\Rightarrow \alpha_1(1 + R) = 2$ .... (iii)
and $\alpha_3 + \alpha_4 = 8 = \alpha_1 R^2 (1 + R)$
$\alpha_1(R^2(1 + R)) = 8$ .... (iv)
On dividing (iv) by (iii) we get $R = \pm 2$.
* when $R = 2 \Rightarrow \alpha_1 = 2/3$
$\therefore \alpha_2 = 4/3 \qquad \therefore \alpha_1\alpha_2 = 8/9 = p$
but p is an integer, so rejecting the case.
* when $R = -2 \Rightarrow \alpha_1 = 2$
$\therefore \alpha_2 = \alpha_1 R = 4 \qquad \therefore \alpha_1 \alpha_2 = -8 = p$
Now $q = \alpha_3\alpha_4 = \alpha_1 R^2.\alpha_1 R^3 = \alpha_1{}^2 R_5 = 4(-2)^5 = -128$
Now quadratic equation whose roots are
$p = -8, q = -128$ is given by
$x^2 - (p + q)x + pq = 0$
$\Rightarrow x^2 - (-136)x + (-8)(-128) = 0$
$\Rightarrow x^2 + 136x + 1024 = 0$

(8) **(B).** Since $f(2 + t) = f(2 - t)$
$\Rightarrow$ function is symmetric about the line $x = 2$
Also $x^2 + bx + c = 0$ is symmetric about $x = -b/2$
$$\therefore \; -\frac{b}{2} = 2 \Rightarrow b = -4$$
$\Rightarrow f(x) = x^2 - 4x + c$
Now 3 graphs are possible. In (1) and (2) 'c' is positive and in (3) 'c' is negative.
$f(0) = c$
Let c is positive
Now $f(1) = c - 3, f(2) = c - 4, f(4) = c$ say $c = 3$



then $f(1) = 0; f(2) = -1; f(3) = 3$
$\Rightarrow f(2) < f(1) < f(3)$
again c is negative
Let $c = -3$
$f(1) = -6; f(2) = -7; f(4) = -3$
$\therefore f(2) < f(1) < f(4)$ ..... (B)
Also if $c = 0$ the statement 'B' is true.

(9) **(B).** $x|x| + a|x| - 1 = 0$
Case (I) : When $x > 0$,
$$\Rightarrow x^2 + ax - 1 = 0 \quad \Rightarrow x = \frac{-a \pm \sqrt{a^2+4}}{2}$$
So two value of x are :
$$\frac{-a + \sqrt{a^2+4}}{2} \text{ which is positive,}$$
$$\& \; \frac{-a - \sqrt{a^2+4}}{2} \text{ which is negative}$$
So when $x > 0$ then real root is only one.
Case (II) :
When $x < 0, -x^2 - ax - 1 = 0$
$$\Rightarrow x^2 + ax + 1 = 0 \Rightarrow x = \frac{-a \pm \sqrt{a^2-4}}{2}$$
For real roots so $a^2 - 4 \geq 0 \Rightarrow a \leq -2$ or $a \geq 2$
$$\text{Roots of equation are } \frac{-a - \sqrt{a^2-4}}{2} \text{ and } \frac{-a + \sqrt{a^2+4}}{2}$$
$$\text{When } a < -2, \text{ so } \frac{-a - \sqrt{a^2-4}}{2} > 0 \text{ and } \frac{-a + \sqrt{a^2+4}}{2} > 0$$
But when $a > 2$,
$$\frac{-a - \sqrt{a^2-4}}{2} < 0 \text{ and } \frac{-a + \sqrt{a^2+4}}{2} > 0$$
$$\text{So only two roots are } \frac{-a + \sqrt{a^2+4}}{2}, \frac{-a - \sqrt{a^2-4}}{2}$$

(10) **(C)** a, b, c are sides of a triangle and $a \neq b \neq c$
$\therefore |a - b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$
Similarly, we have
$b^2 + c^2 - 2bc < a^2 \;\&\; c^2 + a^2 - 2ca < b^2$
On adding, we get
$a^2 + b^2 + c^2 < 2(ab + bc + ca)$
$$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} < 2 \qquad \text{.......(I)}$$
and roots are real of quadratic equation, so
$b^2 \geq ac, c^2 \geq ab$ and $a^2 \geq bc$.
On adding, we get
$a^2 + b^2 + c^2 \geq ab + bc + ca$
$$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} \geq 1 \qquad \text{........(II)}$$
So, from (I) and (II)
$1 \leq p < 2$

(11) **(A).** $\left(x + \frac{1}{x}\right)^2 + 2(a-1)\left(x + \frac{1}{x}\right) + 1 = 0$ has two distinct negative roots.
If $t^2 + 2(a - 1)t + 1 = 0$ has at least one root < -2
$\therefore \quad 4(a - 1)^2 - 4 \geq 0 \qquad$ i.e., $\quad (a - 1) \geq 1$
i.e., $a^2 - 2a \geq 0 \qquad$ i.e., $\quad a \leq 0$ or $a \geq 2$ .... (1)
The roots are
$$t = \frac{-2(a-1) \pm \sqrt{4a^2 - 8a}}{2} = 1 - a \pm \sqrt{a^2 - 2a}$$

$\therefore\ 1-a\sqrt{a^2-2a} < -2$ i.e., $3-a<\sqrt{a^2-2a}$

which is value if either $a>3$ or $a\le 3$

and $9+a^2-6a<a^2-2a$

i.e., $9<4a$ i.e., $a>\frac{9}{4}$ $\therefore\ a>\frac{9}{4}$

$\therefore$ least the integral value of a is 3.

(12) **(A).** $\frac{(2a-1)x^2+2(a+1)x+(2a-1)}{x^2-2x+40}<0$, for all $x\in R$

$\Rightarrow (2a-1)x^2+2(a+1)x+(2a-1)<0$, for all $x\in R$

$\Leftrightarrow 2a-1<0$ and $4(a+1)^2-4(2a-1)^2<0$

$\Leftrightarrow a<\frac{1}{2}$ and $3a(-a+2)<0 \Leftrightarrow a<\frac{1}{2}$ and $a(a-2)>0$

$\Leftrightarrow a<\frac{1}{2}$ and $\{a<0$ or $a>2\}$. Thus $a<0$

(13) **(C)** For $\sqrt{-x^2+8x-12}$ to be real, x^2+12 should be negative.

$(x-2)(x-6)<0$ (i.e.) $2<6$ is the initial condition ....(1)

Now, if $10-2x>0$ (i.e.) $x<5$. ....(2)

Then $\sqrt{-x^2+8x-12}>10-2x$

$\Rightarrow x^2+8x-12>(10-2x)^2$

$5x^2-48x+112<0$ (i.e.) $(5x-28)(x-4)<0$

(i.e.) $4<x<\frac{28}{5}$ ....(3)

From (2) and (3), we get $4<x<5$ ....(4)

When it $10-2x\le 0$, RHS is negative and

$\sqrt{-x^2+8x-12}>a$

Negative number, is true $\therefore\ 5\le x<6$ ....(5)

Combining (4) and (5), the solution is $x\in(4,6)$

(14) **(B).** $y=(x-a)^2-6a$ is a parabola with its vertex at $(a,-6a)$. It intersects the x-axis at the points

$x=a+\sqrt{6a}$, $y<0$ in $[1,2]$ provided that

$a>0$ and $a-\sqrt{6a}\le 1$ and $2\le a+\sqrt{6a}$. If $a\ ^3\ 1$, the second inequality is satisfied and the first given

$a^2-8a+1\le 0$ leading to $4-\sqrt{15}\le a\le 4+\sqrt{15}$.

Since $4-\sqrt{15}<1$,

We need to take $1\le a\le 4+\sqrt{15}$.

If $0<a<1$, the first inequality is satisfied and to satisfy the second,

we need to choose a such that $a^2-10a+4\le 0$ which gives $5-\sqrt{21}\le a\le 5+\sqrt{21}$. As $5+\sqrt{21}>1$,

we need to take $5-\sqrt{21}\le a<1$.

It follows $y<0$ in $[1,2]$ for all $a\in[5-\sqrt{21}, 4+\sqrt{15}]$.

(15) **(C).** $\frac{1}{1+\log_a x}+\frac{2}{2+\log_a x}+\frac{3}{3+\log_a x}-3=0$

$\frac{-\log_a x}{1+\log_a x}+\frac{-\log_a x}{2+\log_a x}+\frac{-\log_a x}{3+\log_a x}=0$

$\therefore\ x=1$ is a solution

further $3(\log_a x)^2+12\log_a x+11=0$

i.e. $\log_a x=\frac{-12\pm\sqrt{12}}{6}$

$\therefore\ x<1$ for both the values of $\log_a x$

Hence all the three real roots are less than or equal to 1.

(16) **(A).** The required condition will be satisfied if

(i) The quadratic expression (quadratic in tanx)

$f(x)=\tan^2x+(a+1)\tan x-(a-3)$ has positive discriminant, and (ii) At least one root of $f(x)=0$ is positive,

as $\tan x>0,\ \forall x\in(0,\pi/2$

For (i) Discriminant $>0 \Rightarrow (a+1)^2+4(a-3)>0$

$\Rightarrow a>2\sqrt{5}-3$ or $a<-(2\sqrt{5}+3)$

For (ii), we first find the condition, that both the roots of $t^2+(a+1)t-(a-3)=0$

($t=\tan x$) are non-positive for which

Sum of roots <0 product of roots $^3\ 0$

$\Rightarrow -(a+1)<0$ and $-(a-3)\ ^3\ 0 \Rightarrow -1<a\le 3$

Condition (ii) will be fulfilled if $a\le -1$ or $a>3$ ... (2)

Required values of a is given by intersection of (1) and (2). Hence $a\in(-\infty,-(2\sqrt{5}+3))\cup(3,\infty)$.

(17) **(C).** From the given two equation

$\alpha+\beta=-b$ ..... (1)

$\alpha\beta=ac$ ...... (2)

$\alpha+\gamma=-a$ ..... (3)

$\alpha\gamma=bc$ ...... (4)

$(1)-(3)\Rightarrow \beta-\gamma=a-b$ ...... (5)

$(2)/(4)\Rightarrow \beta/\gamma=a/b$

$\beta=\frac{a\gamma}{b}$ ...... (6)

putting the value of β in (5)

$\frac{a\gamma}{b}-\gamma=a-b\Rightarrow \gamma\frac{(a-b)}{b}=(a-b)$

$\therefore\ \gamma=b$ $\therefore\ \beta=a\ \&\ \alpha=c$

(18) **(A).** $\frac{x^2+k^2}{k(6+x)}\ge 1 \Rightarrow \frac{x^2-kx+k^2-6k}{k(6+x)}\ge 0$ ....(1)

Now the discriminant of the numerator is

$24k-3k^2=3k(8-k)$ is negative for all $k<0$ and for all $k>8$. For these values of k, the numerator is positive.

(i) For $k<0$, inequality (1) is true only if $x<-6$.

But $x\in(-1,1)$ ....(2)

Hence for $k<0$, the inequality is not valid.

(ii) For $k>8$, inequality (1) is true only if $x>-6$ ....(3)

and $x \in (-1, 1)$ and hence the inequality is valid for all $k > 8$.
For $k = 0$, the inequality is indeterminate.

(19) (D). $\left(\frac{1}{2}\right)^{2x} + 2\left(\frac{1}{2}\right)^{x} + b = 0$

Let $\left(\frac{1}{2}\right)^{x} = y$ hence, $y^2 + 2y + b = 0$

$\therefore\ y = \frac{-2 \pm \sqrt{4-4b}}{2} = -1 \pm \sqrt{1-b}$; $y = -1 - \sqrt{1-b}$

or $y = -1 + \sqrt{1-b}$

$\therefore$ the equation must have a +ve solution hence

$-1 - \sqrt{1-b}$ is not possible.

here $y = -1 + \sqrt{1-b}$; $\left(\frac{1}{2}\right)^{x} = \sqrt{1-b} - 1$

$-x \log_2 2 = \log_2(\sqrt{1-b} - 1)$

$x = -\log_2(\sqrt{1-b} - 1)$

for x to be positive $\log_2(\sqrt{1-b} - 1) < 0$

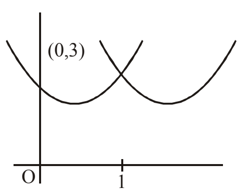
hence $0 < \sqrt{1-b} - 1 < 1$

$1 < \sqrt{1-b} < 2$; $1 < 1 - b < 4$; $0 < -b < 3$

$-3 < b < 0$. Also $1 - b > 0$, $b < 1$ which is true, hence

$b \in (-3, 0)$

(20) (C).



For many one in [0, 1]

$0 < -\frac{b}{2} < 1$; $0 < -b < 2$; $-2 < b < 0$

Note : If f is injective then $b \in R - (-2, 0)$

(21) (B). S_1 : if $x^2 + ax + 7 = 0$ are imaginary roots with positive real parts then
D < 0 and sum of roots > 0
$\Rightarrow\ a^2 - 28 < 0$ and $-a > 0$

$\Rightarrow\ -\sqrt{28} < a < \sqrt{28}$ and $a < 0 \Rightarrow a = -1, -2, -3, -4, -5$

S_2 : $x^2 - (a+3)x + 5 = 0$ has roots α, a, β
If α, a, β are in AP. then $2a = \alpha + \beta \Rightarrow 2a = a + 3 \Rightarrow a = 3$
The equation becomes $x^2 - 6x + 5 = 0$ which has roots 1 and 5.
S_3 : Case- I : If $0 < x < 1$, then $2 + x \ge 6 - x > 0$
$\Rightarrow 2x \ge 4$ and $x < 6 \Rightarrow x \ge 2$ and $x < 6 \Rightarrow x \in [2, 6)$
$\therefore x \in (0, 1) \cap [2, 6) = \phi$ $\therefore x \in \phi$
Case II : If $x > 1$, then $0 < 2 + x \le 6 - x \Rightarrow x > -2$ and $x \le 2$ $\therefore x \in (1, 2]$

(22) (A). $f(x) = x^2 - (m+1)x + m + 4 = 0$

$f(0) > 0$, $\frac{(m+1)}{2} < 0$ and $(m+1)^2 - 4(m+4) \ge 0$

$m + 4 > 0$, $m < -1$ and $m^2 + 2m + 1 - 4m - 16 \ge 0$
$m^2 - 2m - 15 \ge 0$
$m > -4$, $m < -1$ and $m^2 + 3m - 5m - 15 \ge 0$
$m > -4$, $m < -1$ and $(m+3)(m-5) \ge 0$
$\therefore -4 < m \le -3$

(23) (B). $\log_3 |x| < 2 \Rightarrow |x| < 3^2 = 9$, $x \ne 0$
$\therefore -9 < x < 9, x \ne 0$
$\therefore$ set of integral values of
$x = \{-8, -7, ...-1, 1, 2, 3, ...8\}$
$\Rightarrow -2 < \log_3 x < 2$, $x \ne 0$ $\Rightarrow 3^{-2} < \log x < 3^2$, $x \ne 0$
$\Rightarrow 1/9 < \log x < 9$, $x \ne 0$
$\therefore$ set of integral values of $x = \{1, 2, 3, ..., 8\}$
$\Rightarrow$ set of integral values of x satisfying either
$\log_3 |x| < 2$ or $|\log_3 x| < 2$ is
$\{-8, -7, ...-1, 1, 2, ..., 8\}$
$\therefore$ number of values of x is 16.

(24) (A). $2x^2 - 5x + 3 > 0 \Rightarrow (2x-3)(x-1) > 0$

$\Rightarrow\ x \in (-\infty, 1) \cup \left(\frac{3}{2}, \infty\right)$ .... (1)

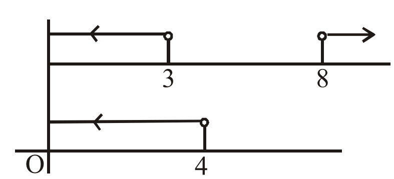
Also, $2x^2 - 5x + 3 \le \left(\frac{1}{\sqrt{2}}\right)^2$; $4x^2 - 10x + 6 \le 1$

$\Rightarrow\ x \in \left[\frac{5-\sqrt{5}}{4}, \frac{5+\sqrt{5}}{4}\right]$ ...... (2)

$x \in \left[\frac{5-\sqrt{5}}{4}, 1\right) \cup \left(\frac{3}{2}, \frac{5+\sqrt{5}}{4}\right]$

(25) (B). $\log_2 2 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) > \log\left[(x^2+1)c\right]$

$\log(4x^2 + 4x + 7) > \log[c(x^2+1)]$
$4x^2 + 4x + 7 > cx^2 + c$ $(c > 0)$
$(4-c)x^2 + 4x + (7-c) > 0$
Hence $4 > c$ and $16 - 4(4-c)(7-c) < 0$
$0 < c < 4$ and $4c(28 - 11c + c^2)$
$c^2 - 11c + 24 > 0$
$(c-8)(c-3) > 0$



$\Rightarrow$ common solution $c \in (0, 3)$
$\therefore$ no. of integral values of c = 2

(26) **(A).** The given system of linear inequalities is

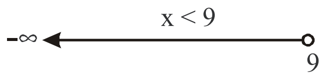
$$\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4} \quad .... (1)$$

$$\frac{7x-1}{3}-\frac{7x+2}{6}>x \quad .... (2)$$

From inequality (1), we have

$$\frac{16x-27}{12}<\frac{4x+3}{4} \Rightarrow 16x-27<12x+9$$

$\Rightarrow 4x<36 \;\Rightarrow\; x<9$



Thus solution of inequality (1) is given by $x<9$.......... (3)

$\Rightarrow \; x \in (-\infty, 9)$

From inequality (2), we get

$$\frac{14x-2}{6}-\frac{7x+2}{6}>x \Rightarrow 14x-2-7x-2>6x$$

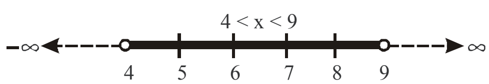
$\Rightarrow 7x-4>6x \;\Rightarrow x>4 \Rightarrow\; x \in (4,\infty)$

Thus solution of inequality (2) is given by

$x>4$ .... (4)

$\Rightarrow \; x \in (4,\infty)$

The solution set of inequality (1) and (2) are represented graphically on real line :



Clearly the common values of x satisfying (3) and (4), lie between 4 and 9. Hence the solution of the given system is given by $4<x<9 \;\Rightarrow\; x \in (4,9)$

(27) **(D).** $4^x+4.6^x=5.9^x$

$$\Rightarrow \left(\frac{4}{6}\right)^x+4=5.\left(\frac{4}{6}\right)^x \Rightarrow \left(\frac{2}{3}\right)^x+4=5.\left(\frac{3}{2}\right)^x$$

Let $\left(\frac{2}{3}\right)^x = t$ then

$$t+4=5\frac{1}{t} \Rightarrow t^2+4t-5=0 \;\Rightarrow\; (t+5)(t-1)=0$$

$t \neq -5$ (as t can't be negative)

$t=1$

$$\left(\frac{2}{3}\right)^x=1 \Rightarrow x=0 \text{ solution.}$$

Option D is correct.

(28) **(C).** If roots of equation $x^2+px+q=0$ are α and β then

$\alpha+\beta=-p$ and $\gamma\delta=q$

and if roots of equation $x^2+px-r=0$ are g, d then

$r+d=-p,\ rd=-r$

then $(\alpha-\gamma)(\alpha-\delta)$

$\Rightarrow \; \alpha^2-(\gamma+\delta)\alpha+\gamma\delta$

$\Rightarrow \; \alpha^2+p\alpha-\gamma$

$\Rightarrow \; -(q+r) \qquad \{\alpha^2+p\alpha+q=0 \Rightarrow a^2+p\alpha=-q\}.$

(29) **(B).** If roots are real and distinct then $\Delta>0$

$\therefore$ For equaton $x^2+6x+a=0$

then $36-4a>0$

or $a<9$ .. (1)

$\alpha-\beta \le 4$

$\Rightarrow \; (\alpha-\beta)^2 \le 16 \Rightarrow (\alpha-\beta)^2-4\alpha\beta \le 16 \Rightarrow 4a \ge 20$

$a \ge 5$

(30) **(C).** Let common root be a $(1-2k)\alpha^2-6k\alpha-1=0$

$k\alpha^2-\alpha+1=0$

$$\frac{\alpha^2}{-6k-1}=\frac{\alpha}{-k-(1-2k)}=\frac{1}{-(1-2k)+6k^2}$$

$$\frac{\alpha^2}{-(6k+1)}=\frac{\alpha}{k-1}=\frac{1}{6k^2+2k-1}$$

$$\alpha^2=\frac{-(6k+1)}{6k^2+2k-1},\ \alpha=\frac{k-1}{6k^2+2k-1}$$

$(k-1)^2=-(6k+1)(6k^2+2k-1)$

$-k^2+2k-1=36k^3+12k^2-6k+6k^2+2k-1$

$36k^3+19k^2-6k=0 \;\Rightarrow\; k(36k^2+19k-6)=0$

$k \neq 0$ then $36k^2+19k-6=0$

$36k^2+27k-8k-6=0;\ 9k(4k+3)-2(4k+3)=0$

$$(4k+3)(9k-2)=0;\ k=\frac{-3}{4}\ k=\frac{2}{9}$$

(31) **(D).** $x+y+z=4$ and $x^2+y^2+z^2=6$

then $xy+yz+zx=5$

$\therefore \; y+z=4-x$ ... (1)

$yz=5-x(4-x)$ ... (2)

then if y and z are roots of any quadratic then

$f(t)=t^2-(4-x)t+5-x(4-x)$

If t is real then $D \ge 0$

$\Rightarrow \; (4-x)^2-4(5-x(4-x)) \ge 0$

$\Rightarrow \; 16+x^2-8x-20+4x(4-x) \ge 0$

$\Rightarrow \; 16+x^2-8x-20+16x-4x^2 \ge 0$

$\Rightarrow \; -3x^2+8x-4 \ge 0 \Rightarrow -3x^2-8x+4 \le 0$

$x \in [2/3, 2]$

(32) **(D).** If $\tan\theta+\cot\theta$ are roots of the equation $x^2+2x+1=0$

then $\tan\theta+\cot\theta=-2$ then $\tan\theta=-1$ and $\cot\theta=-1$

then the least value of $x^2+x\tan\theta+\cot\theta$

$$\Rightarrow x^2-x-1 \Rightarrow \left(x^2-x+\frac{1}{4}\right)-\frac{5}{4}$$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+\left(-\frac{5}{4}\right)$$

$\therefore$ least value of function is –5/4.

(33) **(C).** If α, β, γ are roots of the equation $x^3+qx+q=0$ then

$\Rightarrow \; (\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1}$

$$\Rightarrow -\frac{1}{\gamma}-\frac{1}{\alpha}-\frac{1}{\beta} \quad \Rightarrow -\left(\frac{\alpha\beta+\beta\gamma+\gamma\delta}{\alpha\beta\gamma}\right)$$

$\{\alpha\beta+\beta\gamma+\gamma\alpha=\theta,\ \alpha\beta\gamma=-q\} \Rightarrow 1.$

(34) **(C).** Let $2^x = t$ then $t^2 + (k-3)t + (k-4) = 0$

$$\Rightarrow \ t = \frac{-(k-3) \pm \sqrt{(k-3)^2 - 4(k-4)}}{2}$$

$$\Rightarrow \ t = \frac{-(k-3) \pm (k-5)}{2} = \frac{-2k+8}{2} \Rightarrow -k+4$$

$\therefore$ x is non positive then $x \le 0$

$\therefore 0 < 2^x \le 1 \ \Rightarrow 0 < -k+4 \le 1$; $-3 \le k < 4$

$\therefore$ largest integral value of k is 3

(35) **(D).** $f(x) = (2+b+b^2)x^2 = 2\sqrt{2}\,(2b+1)x + 8$

Min value of

$$f(x) = \frac{-D}{4a} = \frac{-[8(2b+1)^2 - 32(2+b+b^2)]}{4(b^2+b+2)}$$

$$m(b) = \frac{56}{4(b^2+b+2)}$$

Maximum value of m (b) is obtain when minimum value of b^2+b+2 is obtain minimum value

$$b^2+b+2 = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 2 = \frac{1}{4} - \frac{1}{2} + 2 = \frac{7}{8}$$

$$\text{Maximum value of } m(b) = \frac{56}{4 \times \frac{7}{4}} = 8$$

(36) **(B).** $(1+k)x^2 - 2(1+2k)x + (3+k) = 0, \forall k \in R - \{-1\}$

$\therefore$ If the roots are imaginary them $D < 0$

$\Rightarrow 4(1+2k)^2 - 4(1+k)(3+k) < 0$

$\Rightarrow (1+2k)^2 - (1+k)(3+k) < 0$

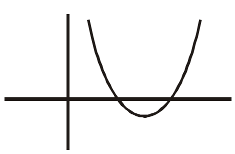
$\Rightarrow 4k^2 + 4k + 1 - (k^2 + 4k + 3) < 0 \Rightarrow 3k^2 - 2 < 0$

$$\Rightarrow -\sqrt{\frac{2}{3}} < k < \sqrt{\frac{2}{3}}$$

$\therefore$ $k = 0$ (Only one integral values)

(37) **(B).** If both the roots are positive

(1) $D \ge 0$ (2) $-\dfrac{b}{2a} > 0$ (3) $4f(0) > 0$



(I) $k \ge \sqrt{\frac{2}{3}}$ or $k \le -\sqrt{\frac{2}{3}}$

(II) $\dfrac{2(1+2k)}{2(1+k)} \ge 0$, $k > -\dfrac{1}{2}$ or $k < -1$

(III) $(1+k)(3+k) > 0$

$k > -1$ or $k < -3$

By (1) & (2) & (3)

$$k \in (-\infty, -3) \cup \left[\sqrt{\frac{2}{3}}, \infty\right)$$

(38) **(C).** Let the roots are α & 2α

then sum $3\alpha = 2(1+2k)/(1+k)$

product $2a^2 = (3+k)/(1+k)$

Eliminate the α

$$2\left(\frac{2}{3}\left(\frac{1+2k}{1+k}\right)\right)^2 = \frac{(3+k)}{(1+k)}$$

$\Rightarrow 8(1+2k)^2 = 9(3+k)(1+k)$

$\Rightarrow (4k^2+4k-1)8 = 9(k^2+4k+3)$

$\Rightarrow 32k^2 + 32k + 8 - (9k^2 + 36k + 27) = 0$

$\Rightarrow 23k^2 - 4k - 19 = 0$

$\therefore D > 0 \quad \therefore$ Two real values of k.

(39) **(B).** $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 3x + 4}$

Sum of integer in range f(x)

$$f(x) = \frac{x^2 - 3x - 4 + 4 - 4}{x^2 - 3x + 4}$$

$$\Rightarrow f(x) = 1 - \frac{8}{(x^2 - 3x + 4)}$$

$$\text{Now } f(x)_{min} = 1 - \frac{8}{(x^2 - 3x + 4)_{min}}$$

$$(x^2 - 3x + 4)_{min} = \frac{-D}{4a} = \frac{-(9-16)}{4} = \frac{7}{4}$$

$$f(x)_{min} = 1 - \frac{8}{7/4} \Rightarrow (f(x))_{min} = \frac{-25}{7}$$

$$f(x)_{max} = \text{at } (x \to \infty) \Rightarrow f(x)_{max} \to 1 - \frac{1}{\infty}$$

$f(x)_{max} \to 1$

So range of f(x); $f(x) \in \left[\dfrac{-25}{7}, 1\right)$

Integers in rage $f(x) = -3, -2, -1, 0$

So answer = Sum of integers.

$\Rightarrow -3 - 2 - 1 + 0 = -6$

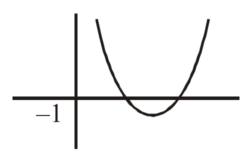
(40) **(D).** $g(x) = x^2 - (b+1)x + b - 1$

If both root of the equation $g(x) = 0$ are greater than -1 then

(1) $D \ge 0$

(2) $-\dfrac{b}{2a} > -1$

(3) $4f(-1) > -1$



(i) $(b+1)^2 - 4(b-1) \ge 0$

$\Rightarrow b^2 + 2b + 1 - 4b + 4 \ge 0$

$\Rightarrow b^2 - 2b + 5 \ge 0$

$\therefore D < 0$

$\therefore b \in R$ ...(1)

(ii) $\dfrac{b+1}{2} > -1 \Rightarrow b + 1 > -2$

$\Rightarrow b > -3$ ...(2)

(iii) $1 + (b + 1) + (b - 1) > 0$
$2b + 1 > 0$
$b > -1/2$ …(3)

$\therefore\ b \in \left(-\frac{1}{2}, \infty\right)$

(41) **(B).** $g(x) > -2 \ \forall \ x \in R$ then
$\Rightarrow x^2 - (b + 1)x + (b - 1) + 2 > 0$
$\Rightarrow x^2 - (b + 1)x + (b + 1) > 0$
$\therefore$ x is real then $D < 0$
$\Rightarrow (b + 1)^2 - 4(b + 1) < 0$
$\Rightarrow (b + 1)(b + 1 - 4) < 0$
$\Rightarrow (b + 1)(b - 3) < 0$
$-1 < b < 3$
$\Rightarrow b = 2$ is largest natural number.

(42) **(D)** Dividing by x^2, we get

$$x^2 - 3x - 2 - \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 3\left(x + \frac{1}{x}\right) - 2 = 0$$

$\Rightarrow t^2 - 3t - 4 = 0 \quad \Rightarrow t = -1, 4$

$x + \frac{1}{x} = -1 \quad \Rightarrow x^2 + x + 1 = 0$

$\therefore x = \omega, \omega^2$, the complex cube roots of unity

$x + \frac{1}{x} = 4 \quad \Rightarrow x = 2 \pm \sqrt{3}$

$\therefore$ One root is outside [0, 3].

(43) **(B)**
$|x|^2 + 2|x| + 1 = 4|x| + 9$
$\Rightarrow |x|^2 - 2|x| - 8 = 0$
$\Rightarrow (|x| - 4)(|x| + 2) = 0$
$\Rightarrow |x| = 4, -2.$
But $|x| \neq -2 \Rightarrow |x| = 4 \Rightarrow x = \pm 4$
i.e. has only two real solutions.
$\therefore$ S-1 is correct.
In S-2, for n = 10, it has no solution.
$x^2 - 8x = n(n - 10)$

$$x = \frac{8 \pm \sqrt{64 + 4n^2 - 40n}}{2} = 4 \pm \sqrt{16 + n^2 - 10n}$$

$n^2 - 10n + 16 = (n - 8)(n - 2)$
$n^2 - 10n + 16 < 0$ for $2 < n < 8$
i.e., for n = 3, 4, 5, 6, 7 it has no solutions.
$\therefore$ S-2 is correct.
But S-2 is not the correct explanation of S-1.

(44) **(A).** $\frac{\sin x + \cos x + 1 + 1}{4} \geq (\sin x \cos x)^{1/4}$
$(\sin x + \cos x + 2)^4 \geq 128 \sin 2x.$
$\Rightarrow \sin x = \cos x = 1$ (not possible)
Statement-1 is correct and statement-2 is correct explanation to statement-1.

(45) **(D).** $f(x) = 0$ represents an identity if
$a^2 - a - 6 = 0 \Rightarrow a = 3, -2$
$a^2 - a - 6 = 0 \Rightarrow a = 3, -2$
$a^2 - a = 0 \Rightarrow a = 3, -3$
$a^2 - 2a - 3 = 0 \Rightarrow a = 3, -1 \Rightarrow a = 3$ is the only values.

(46) **(D).** Statement 1 : Least value of $|x - 2| + |x - 7|$ is 5
$\therefore\ |x - 2| + |x - 7| = 6$ has two solutions
$\therefore$ Statement is false.
Statement 2 : Least value of $|x - a| + |x - b|$ is $b - a$
Since $c < b - a$
$\therefore$ no solution
$\therefore$ Statement is true

(47) **(A).** $x^2 + x + 1 = 0$
$D = -3 < 0$
$\therefore x^2 + x + 1 = 0$ and $ax^2 + bx + c = 0$ have both the roots common $\Rightarrow a = b = c$.

(48) **(C).** Statement 1 : Sum of the roots = –3 and discriminant $D = 1 > 0$
$\therefore$ Statement is true

(49) **(D).** $ix^2 + (1 + i)x + i = 0$
$\Rightarrow \alpha\beta = 1$
$\Rightarrow \text{Im}(\alpha\beta) = 0$

(50) **(B).** Let $x_1, x_2, x_3 \in R$ be the roots of $f(x) = 0$
$\therefore\ f(x) = (x - x_1)(x - x_2)(x - x_3)$
$f(i) = (i - x_1)(i - x_2)(i - x_3)$
$|f(i)| = |x_1 - i||x_2 - i||x_3 - i| = 1$
$\therefore\ \sqrt{x_1^2 + 1}\sqrt{x_2^2 + 1}\sqrt{x_3^2 + 1} = 1$
This is possible only if $x_1 = x_2 = x_3 = 0$
$\Rightarrow f(x) = x^3 \Rightarrow a = 0 = b = c \Rightarrow a + b + c = 0 \not\Rightarrow$ all roots are zero.

(51) **(D).** Let, $f(x) = (x - \sin\alpha)(x - \cos\alpha) - 2$
then, $f(\sin\alpha) = -2 < 0$; $f(\cos\alpha) = -2 < 0$
Also as $0 < \alpha < \pi/4$;
$\therefore\ \sin\alpha < \cos\alpha$
Therefore equation $f(x) = 0$
has one root in $(-\infty, \sin\alpha)$
and other in $(\cos\alpha, \infty)$

(52) **(A).** Since $f(a) f(b) < 0$ and $f(c) . f(d) < 0$
$\therefore$ The equation has one root between a and b and the other root between c and d.
$\therefore$ Statement-2 is true and explains statement-1
for a = 2, b = 3, c = 6, d = 7

(53) **(C).** Roots of the equation $x^5 - 40x^4 + px^3 + qx^2 + rx + s = 0$ are in G.P., let roots be a, ar, ar^2, ar^3, ar^4
$\therefore a + ar + ar^2 + ar^3 + ar^4 = 40$ . . . (i)

and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4} = 10$ . . . (ii)

from (i) and (ii); $ar^2 = \pm 2$ . . . (iii)
Now, - S = product of roots $= a^5 r^{10} = (ar^2)^5 = \pm 32.$
$\therefore |s| = 32$

(54) **(B).** equation can be written as
$(2^x)^2 - (a - 4)2^x - (a - 4) = 0$
$\Rightarrow 2^x = 1 \ \& \ 2^x = a - 4$
Since $x \leq 0$ and $2^x = a - 4$ [$\because$ x is non positive]
$\therefore 0 < a - 4 \leq 1 \Rightarrow 4 < a \leq 5$ i.e., $a \in (4, 5]$

(55) **(A).**

(a) $-2 < \log_2 \frac{x-2}{4} < 2 \;\Rightarrow\; \frac{1}{4} < \frac{x-2}{4} < 4$

$\Rightarrow\; 1 < x-2 < 16 \;\;\Rightarrow\; 3 < x < 18$

(b) $x^2 - 10x < 11x - 54$

$\Rightarrow\; x^2 - 21x + 54 < 0 \Rightarrow\; 3 < x < 18$

(c) $\frac{(x-4)^2(x-6)}{(x-2)^2 x^4 (x-8)^2} > 0 \;\Rightarrow\; x \in (6,8) \cup (8,\infty)$

(d) $||x|+2| \le 1 \Leftrightarrow\; |x|+2 \le 1 \qquad \therefore\; x \in \phi$

(56) **(D).**

(a) $(2^{2x} - 4 \cdot 2^x + 4) + 1 + ||b-1|-3| = |\sin y|$

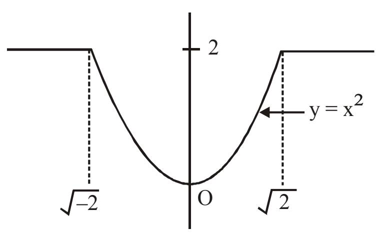
$(2^x - 2)^2 + 1 + ||b-1|-3| = |\sin y|$

LHS ≥ 1 & RHS ≤ 1

$\therefore\; 2x = 2,\; |b-1|-3 = 0 \Rightarrow (b-1) = \pm 3$

$x = 1,\; b = 4, -2 \Rightarrow$ p, r.

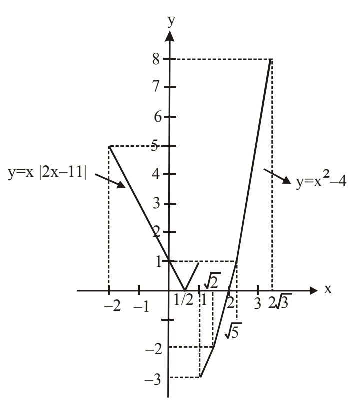
(b) $f(x) = \begin{cases} x^2 & x \in [-\sqrt{2}, \sqrt{2}] \\ 2 & x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty) \end{cases}$



$k \in (0, \infty)$

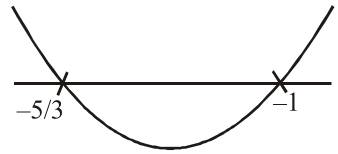
hence, q, r, s

(c) $D_{fof(x)} = [-2, 1] \cup [\sqrt{2}, 2\sqrt{3}]$



hence, p, q.

(d) $f'(x) = 3x^2 + 2px + q < 0, \qquad x \in \left(-\frac{5}{3}, -1\right)$



$\Rightarrow 3x^2 + 2px + q = (3x+5)(x+1)$

$\Rightarrow 2p = 8\,;\, p = 4$ & $q = 5 \Rightarrow p + q = 9 \Rightarrow$ p, q, r, s.

(57) **(C).**

$D > 0 \Rightarrow (k-3)^2 - 4k > 0$

Sum of roots $> 0 \Rightarrow k - 3 > 0$;

Product of roots $> 0 \Rightarrow k > 0$

(b) $D > 0 \;\Rightarrow (k-3)^2 - 4k > 0$

Sum of roots $< 0 \Rightarrow k - 3 < 0$;

Products of roots $> 0 \Rightarrow k > 0$

(c) $D = 0$

(d) $f(x) = x^2 - (k-3)x + k$

Now $D \ge 0 \Rightarrow k \in (-\infty, 1] \cup [9, \infty)$ .... (i) and

because $f(1) = 4$ is positive, it means $f(-1)$ is also positive.

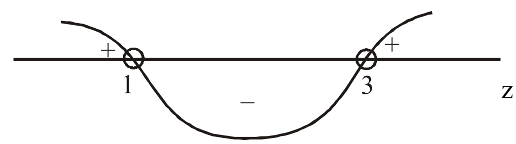
$\Rightarrow k \ge 1$ .... (ii)

From (i) and (ii) $\Rightarrow k \in \{1\} \cup [9, \infty)$

(58) **(A).**

Let $f(x) = x^3 - 6x^2 + 9x + \lambda$

$\therefore\; f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$



$\therefore\; f'(x) = < 0$ in $(1, 3)$

But $f(1) = 4 + \lambda$ & $f(3) = \lambda$

$f(x) = 0$ to have exactly one root in $(1, 3)$, $f(1)$ and $f(3)$ should have opposite signs

$\therefore\; f(1)\,f(3) < 0$

$\Rightarrow \lambda(\lambda + 4) < 0$

$\Rightarrow\; -3 < \lambda + 1 < 0$

$\therefore\; -3 < \lambda + 1 < 1$

$\Rightarrow\; [\lambda + 1] = -3, -2, -1, 0$

(b) $-3 < \frac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$

$\Rightarrow -3x^2 - 3x - 3 < x^2 - \lambda x - 2 < 2x^2 + 2x + 2$

(i) (ii)

($\because\; x^2 + x + 1 > 0$ when $x \in R$)

From inequality (i),

$4x^2 - (\lambda - 3)x + 1 > 0$

$\Rightarrow\; (\lambda - 3)^2 - 4 \,.\, 1 \,.\, 4 \,.\, 1 < 0 \Rightarrow\; -4 < \lambda + 2 < 4$

$\Rightarrow\; -1 < \lambda < 7$

From inequality (ii),

$x^2 + (\lambda + 2)x + 4 > 0$

$\Rightarrow\; (\lambda + 2)^2 - 4 \,.\, 1 \,.\, 4 \,.\, 1 < 0 \Rightarrow\; -4 < \lambda + 2 < 4$

$\Rightarrow\; -6 < \lambda < 2$

From (i) & (ii), $\; -1 < \lambda < 2 \;\therefore\; [\lambda] = -1, 0, 1$

(c) $\because\; x = 1$ satisfies

$(b-c)x^2 + (c-a)x + (a-b) = 0$

$\therefore\; x = 1$ satisfies $x^2 + \lambda x + 1 = 0$

then $1+\lambda+1=0 \Rightarrow [\lambda-1]=[-2-1]=-3$

(d) $\because |x^2-x-6|=x+2$

$\Rightarrow |(x-3)(x+2)|=x+2 \Rightarrow |x-3||x+2|=x+2$

$$\Rightarrow \begin{cases}(x-3)(x+2)=x+2, & x<-2\\ -(x-3)(x+2)=x+2, & -2\le x<3\\ (x-3)(x+2)=x+2, & x>3\end{cases}$$

$$\Rightarrow \begin{cases}x=4, & x<-2\\ x=-2,2 & -2\le x<3\\ x=4, & x>3\end{cases}$$

So, $x=-2, 2, 4$ $\quad\therefore N=3$

(59) (D), (60) (A), (61) (C).

$$AB=\frac{-D}{4a}=2\Rightarrow a=\frac{1}{2}$$

$$AC=3\Rightarrow\frac{-b}{2a}=-3\Rightarrow b=6a\Rightarrow b=3$$

$$b^2-4ac=-4\Rightarrow c=\frac{13}{2}\ ;\ b+\sqrt{a+c}=3+\sqrt{\frac{1}{2}+\frac{13}{2}}=3+\sqrt{7}$$

(62) **(A).**

AA' = –BB'

$$\Rightarrow b-a^2=\frac{d^2-c}{c}$$

$bc-a^2c=d^2-c$

$bc+c=a^2c+d^2$

$bc+c=ac.a+d^2$ .... (1)

$$-OA'=OB=a=-\frac{d}{c}$$

$ac=-d$ .... (2)

From (1) and (2), we get

$ad+bc+c=d^2$

(63) **(D).** $a=2$, sum of roots $=-\dfrac{2d}{c}=2a=4$

f (x) has no real root

$\therefore$ sum of real roots of g (x) f (x) = 0 is 4.

(64) **(A).** Least value of g (x) is BB' = – AA' = –2

Range of g (x) is $[-2,\infty)$

(65) **(D).** Let $A-d,\ A,\ A+d$ be roots

$\Rightarrow A-d+A+A+d=b$

$\Rightarrow A=\dfrac{b}{3}$ must satisfy $g(x)=0 \Rightarrow \dfrac{b^3}{27}-\dfrac{b^3}{9}+\dfrac{ab}{3}-1=0$

$\Rightarrow 9ab=2b^3+27$

(66) **(B).** $a+b=-1,\ ab=2,\ a^2+b^2+2ab=1$

$a^2+b^2=-3$

$f(x).\,g(x)=x^6-(a+b)x^5+(a+b+ab)x^4$

$-(a^2+b^2+2)x^3+(a+b+ab)x^2-(a+b)x+1$

$=x^6+x^5+x^4+x^3+x^2+x+1$

(67) **(A).** $a+b=-1,\ ab=2$

putting x = 1 in $f(x)\Rightarrow f(1)=1-a+b-1=b-a\ne 0$

x = 1 is not a root of f (x) = 0

putting x = 1 in g (x)

$\Rightarrow g(1)=1-b+a-1=a-b\ne 0$

x = 1 is not a root of g (x) = 0

putting x = 1 in f (x) + g (x)

$\Rightarrow f(1)+g(1)=1-a+b-1+1-b+a-1=0$

$\therefore$ x = 1 is a root of f (x) + g (x) = 0

$f(x)+g(x)=0$

$\Rightarrow 2x^3-(a+b)x^2+(a+b)x-2=0$

$\because a+b=-1$

$\Rightarrow 2x^3+x^2-x-2=0$

$\Rightarrow (x-1)(2x^2+3x+2)=0$

$x=1$ and $2x^2+3x+2=0$

$D=9-4.2.2<0\ \therefore$ one real root and two imaginary roots.

EXERCISE-3

(1) **1.** $5^{2x^2}-2.5^{x^2+x+1}-3.5^{2x+3}=0$

$5^{2x^2}-10.5^{x^2-x}-3.5^3=0$

$t^2-10t-375=0$

$(t-25)(t+15)=0\Rightarrow t=25,-15$

$\therefore\ 5^{x^2-x}=5^2\Rightarrow x^2-x=2$

$\Rightarrow x^2-x-2=0\Rightarrow(x-2)(x+1)=0$

$\Rightarrow x=2,-1 \quad\therefore$ sum = 1

(2) **5.** $k=\dfrac{x^2-2x+9}{x^2+2x+9}\Rightarrow kx^2+2kx+9k-x^2+2x-9=0$

$\Rightarrow (k-1)x^2+2(k+1)x+9(k-1)=0$

Case I : If k = 1, then x = 0

$\therefore$ k can take the value 1

Case II : If $k\ne 1$, then $4(k+1)^2-36(k-1)^2\ge 0$

i.e. $(k+1)^2-(3k-3)^2\ge 0$

i.e. $(4k-2)(-2k+4)\ge 0$

i.e. $(2k-1)(k-2)\le 0$

$\therefore \dfrac{1}{2}\le k\le 2,\ k\ne 1 \qquad \therefore \dfrac{1}{2}\le k\le 2$

$\therefore a=\dfrac{1}{2},\ b=2 \qquad \therefore 2(a+b)=5$

(3) **2.** $\dfrac{ax^2+3x+4}{x^2+2x+2}<5$

$ax^2+3x+4<5x^2+10x+10\quad(\because x^2+2x+2>0,\ \forall\, x\in R)$

$x^2(a-5)-7x-6<0\quad\forall\, x\in R$

$a-5<0$ and $(-7)^2-4(a-5)(-6)<0$

i.e. $24a-71<0$

$a<5$ and $a<\dfrac{71}{24}\quad\therefore$ greatest integral value of a is 2.

(4) **5.** $\sqrt{\log_2\left(\dfrac{x-4}{1-x^2}\right)} > 1$ i.e. $\log_2\left(\dfrac{x-4}{1-x^2}\right) > 1$

i.e. $\dfrac{x-4}{1-x^2} > 2$

i.e. $\dfrac{x-4}{1-x^2} - 2 > 0$ i.e. $\dfrac{(x+2)(2x-3)}{(x-1)(x+1)} < 0$

$\therefore\ x \in (-2,-1) \cup (1, 3/2)$

$\therefore\ a = 1, b = 3/2 \qquad \therefore\ 2(a+b) = 5$

(5) **9.** Since roots of $x^2 + 3x + 5 = 0$ are imagainary.
$\therefore$ both the roots of $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ are common. $\therefore$ minimum value of $a + b + c$ is 9.

(6) **5.** Case I : If $x < 2$, then $\dfrac{-(x-2)}{(x-1)(x-2)} \le 1$

$\therefore\ x \in (-\infty, 0] \cup (1, 2)$ .... (1)

Case II : If $x > 2$, then $\dfrac{x-2}{(x-1)(x-2)} - 1 \le 0$

$\therefore\ x \in (2, \infty)$ .... (2)

From (1) and (2)
$\therefore\ x \in (-\infty, 0] \cup (1, 2) \cup (2, \infty)$
$\therefore\ a = 0, b = 1, c = 2, d = 2$
$\therefore\ a + b + c + d = 5$

(7) **0.** $x^2 + px + qr = 0$ .... (i)
$x^2 + qx + rp = 0$ .... (ii)
$x^2 + rx + pq = 0$ .... (iii)
Every pair has a common root.
Let the roots are α, β for (i), β, γ for (ii); γ, α for (iii).
$\alpha + \beta = -p$ .... (iv)
$\alpha\beta = qr$ .... (v)
$\beta + \gamma = -q$ .... (vi)
$\beta\gamma = rp$ .... (vii)
Common roots are α, β, γ
By (i) and (ii), $\beta^2 + p\beta + qr = 0,\ \beta^2 + p\beta + rp = 0$
Subtracting, $(p - q)\beta + r(q - p) = 0$, or $\beta = r$
Put this in (vii), $r\gamma = rp$ or $\gamma = p$.
Put $\beta = r$ in (v), $\alpha = p$.
$\therefore\ \alpha + \beta + \gamma = q + r + p$ .... (I)
But $\alpha + \beta = -p$; $\beta + \gamma = -q$, $\gamma + \alpha = -r$

$\Rightarrow\ \alpha + \beta + \gamma = -\dfrac{1}{2}(p + q + r)$ .... (II)

By (I) and (II) $\Rightarrow p + q + r = 0 \Rightarrow \alpha + \beta + \gamma = 0$

(8) **672.** $N = \alpha\,(111111)$ is divisible by $7 \times 11 \times 3$
Hence for N to be a divisible by 924, $\alpha = 4$ or 8
and α and β are roots of $x^2 - 11x + \lambda = 0$.
$\Rightarrow \alpha + \beta = 11 \Rightarrow (\alpha, \beta) \equiv (4, 7), (8, 3)$
$\Rightarrow$ Possible value of $\lambda = 28, 24$
$\Rightarrow$ Product of $\lambda = 672$

(9) **6.** $x^2 + ax + b \equiv (x + 1)(x + b) \Rightarrow b + 1 = a$ ....(1)
also $x^2 + bx + c \equiv (x + 1)(x + c)$
$\Rightarrow c + 1 = b$ or $b + 1 = c + 2$ ....(2)
hence $b + 1 = a = c + 2$
also $(x + 1)(x + b)(x + c) \equiv x^3 - 4x^2 + x + 6$
$x^3 + (1 + b + c)x^2 + (b + bc + c)x + bc \equiv x^3 - 4x^2 + x + 6$
$1 + b + c = -4$
$2c + 2 = -4 \Rightarrow c = -3$; $b = -2$ and $a = -1$
$\Rightarrow a + b + c = -6$

(10) **113.** Let $2^{111x} = y$

so that $\log_2 y = 111x \Rightarrow x = \dfrac{\log_2 y}{111}$

equation becomes $\dfrac{y^3}{4} + 2y = 4y^2 + 1$

$y^3 - 16y^2 + 8y - 4 = 0$
sum of the roots of the given equation is

$x_1 + x_2 + x_3 = \dfrac{\log_2 y_1 + \log_2 y_2 + \log_2 y_3}{111}$

$= \dfrac{\log_2(y_1 y_2 y_3)}{111} = \dfrac{\log_2 4}{111} = \dfrac{2}{111} \Rightarrow S_1 + S_2 = 113$

(11) **5854.** Cross multiplication and rearranging gives the cubic.

$x^3 - ax^2 + 23x - b = 0$ with roots α, α, β

$2\alpha + \beta = a$ ....(1)
$\alpha^2 + 2\alpha\beta = 23$ ....(2)
and $\alpha^2\beta = \beta$ ....(3)
Also given $\alpha + \beta = 12$ ....(4)
from (2) and (4)
$\alpha^2 + 2\alpha(12 - \alpha) = 23$
$\alpha^2 + 24\alpha - 2\alpha^2 = 23$
$\alpha^2 - 24\alpha + 23 = 0$
$\alpha = 1$ (rejected) since $x \ne \pm 1$
$\therefore\ \alpha = 23 \quad \therefore\ \beta = -11$
$\therefore\ a = 35$ from (4)
and $b = \alpha^2\beta = 529 \times -11$
$\Rightarrow b = -5819 \Rightarrow a - b = 35 - (-5819) = 5854$

(12) **11.** $\dfrac{f(3)}{f(6)} = \dfrac{2^{3k} + 9}{2^{6k} + 9} = \dfrac{1}{3}$

$f(9) - f(3) = (2^{9k} + 9) - (2^{3k} + 9) = 2^{9k} - 2^{3k}$ ....(1)
$3(2^{3k} + 9) = 2^{6k} + 9$
$\Rightarrow 2^{6k} - 3(2^{3k}) - 18 = 0$
$2^{3k} = y$
$y^2 - 3y - 18 = 0$
$(y - 6)(y + 3) = 0$
$y = 6;\quad y = -3$ (rejected)
$2^{3k} = 6$
now $f(9) - f(3) = 2^{9k} - 2^{3k}$ { from (1) }
$= (2^{3k})^3 - 2^{3k} = 6^3 - 6 = 210$
hence $N = 210 = 2 \cdot 3 \cdot 5 \cdot 7$
Total number of divisor $= 2 \cdot 2 \cdot 2 \cdot 2 = 16$
number of divisors which are composite
$= 16 - (1, 2, 3, 5, 7) = 11$

(13) **3.** Let α, α^2 be the roots of $3x^2 + px + 3 = 0$

Now $\alpha + \alpha^2 = -p/3,\ p = \alpha^3 = 1$

$\Rightarrow \alpha = 1, \omega, \omega^2,$

$\alpha + \alpha^2 = -p/3 \Rightarrow \omega + \omega^2 = -p/3$

$\Rightarrow -1 = -p/3 \Rightarrow p = 3$

(14) **2.** (i) $x^2 - 8kx + 16(k^2 - k + 1) = 0$

$D = 64(k^2 - (k^2 - k + 1)) = 64(k-1) > 0$

$k > 1$

(ii) $-\frac{b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4 \Rightarrow k > 1$

(iii) $f(4) \geq 0$

$16 - 32k + 16(k^2 - k + 1) \geq 0$; $k^2 - 3k + 2 \geq 0$

$(k-2)(k-1) \geq 0$; $k \leq 1$ or $k \geq 2$. Hence $k = 2$

(15) **3.** $x^2 - 6x - 2 = 0$ having roots α and β

$\Rightarrow \alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

$\Rightarrow \alpha^{10} - 2\alpha^8 = 6\alpha^9$ .... (1)

Similarly, $\beta^{10} - 2\beta^8 = 6\beta^9$ .... (2)

By eq. (1) and eq. (2)

$(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9)$

$\Rightarrow a_{10} - 2a_8 = 6a_9 \Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$

Aliter : $\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$= \frac{\alpha^{10} - \beta^{10} + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$= \frac{\alpha^9(\alpha+\beta) - \beta^9(\alpha+\beta)}{2(\alpha^9 - \beta^9)} = \frac{\alpha+\beta}{2} = \frac{6}{2} = 3$

EXERCISE-4

(1) **(D).** α, β are roots of equation $x^2 - 5x + 16 = 0$

$\Rightarrow \alpha + \beta = 5$ and $\alpha\beta = 16$ .... (1)

and $(\alpha^2 + \beta^2), \frac{\alpha\beta}{2}$ are roots of equation $x^2 + px + q = 0$

$\Rightarrow \alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p$ and $(\alpha^2 + \beta^2)\ \frac{\alpha\beta}{2} = q$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 5^2 - 2 \times 16 = -7$

and $\frac{\alpha\beta}{2} = \frac{16}{2} = 8$

again, $\alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p \Rightarrow -7 + 8 = -p \Rightarrow p = -1$

$(\alpha^2 + \beta^2)\ \frac{\alpha\beta}{2} = q \Rightarrow (-7) \times 8 = q \Rightarrow q = -56$

(2) **(C).** $\alpha\ \beta$ are the roots of equation

$(x-a)(x-b) = c \quad [c \neq 0]$

$\Rightarrow (x-a)(x-b) - c = 0$ .... (1)

We know that if α and β are roots of any equation then the equation will be

$(x-\alpha)(x-\beta) = 0$ .... (2)

(1) and (2) are same

$\therefore (x-a)(x-b) - c = 0 = (x-\alpha)(x-\beta)$

$\Rightarrow (x-\alpha)(x-\beta) + c = 0 = (x-a)(x-b)$ .... (3)

$\therefore$ Root of equation $(x-\alpha)(x-\beta) + c = 0$ are a and b.

(3) **(A).** $\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0$ .... (1)

$\beta^2 = 5\beta - 3 \Rightarrow \beta^2 - 5\beta + 3 = 0$ .... (2)

From (1) and (2) we see that α and β satisfies equation $x^2 - 5x + 3 = 0$

$\therefore \alpha, \beta$ are roots of equation $x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5$ 1 ; $\alpha\beta = 3$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 2 \times 3}{3} = \frac{19}{3}$

(4) **(D).** Let the roots of equation $ax^2 + bx + c = 0$ are α and β

$\therefore \alpha + \beta = -\frac{b}{a}$; $\alpha\beta = \frac{c}{a}$

According to question,

$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} \Rightarrow \alpha + \beta = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$\Rightarrow \alpha + \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \Rightarrow \frac{-b}{a} = \frac{b^2/a^2 - 2c/a}{c^2/a^2}$

$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{c^2} \Rightarrow -bc^2 = ab^2 - 2a^2c$

$\Rightarrow 2a^2c = ab^2 + bc^2$

dividing by abc

$\Rightarrow \frac{2a}{b} = \frac{b}{c} + \frac{c}{a} \Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P. $\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

(5) **(B).** Let roots of quadratic equation $(a^2 - 5a + 3)x^2 + (3a-1)x + 2 = 0$ are α and 2α

$\therefore$ sum of roots $\alpha + 2\alpha = -\left(\frac{3a-1}{a^2 - 5a + 3}\right)$

$\Rightarrow 3\alpha = -\left(\frac{3a-1}{a^2 - 5a + 3}\right) \Rightarrow \alpha = -\frac{1}{3}\left(\frac{3a-1}{a^2 - 5a + 3}\right)$ ...... (1)

and product of roots

$\alpha(2\alpha) = \frac{2}{a^2 - 5a + 3} \Rightarrow 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$ .... (2)

From (1) put value of α in (2)

$\Rightarrow 2\left[-\frac{1}{3}\left(\frac{3a-1}{a^2 - 5a + 3}\right)\right]^2 = \frac{2}{a^2 - 5a + 3}$

$\Rightarrow \frac{2}{9}\frac{(3a-1)^2}{(a^2-5a+3)^2} = \frac{2}{a^2-5a+3}$

$\Rightarrow (3a-1)^2 = 9(a^2-5a+3)$

$\Rightarrow 9a^2+1-6a = 9a^2-45a+27$

$\Rightarrow 39a = 26 \Rightarrow a = 2/3$

(6) **(C).** Equation, $x^2 - 3|x| + 2 = 0$

Case (i), If $x \geq 0$ then $|x| = x$

Now equation, $x^2 - 3x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2$ (two real solution)

Case (ii), if $x < 0$, then $|x| = -x$

Now equation, $x^2 + 3x + 2 = 0$

$\Rightarrow (x+1)(x+2) = 0$

$\Rightarrow x = -1, -2$ (two real solution)

$\therefore$ equation have four real solution.

(7) **(C).** $x^2 + px + (1-p) = 0$

$\therefore$ $1-p$ is the root of quadratic equation

$\therefore$ it will satisfy the Q.E.

$\Rightarrow (1-p)^2 + p(1-p) + (1-p) = 0$

$\Rightarrow (1-p)[1-p+p+1] = 0$

$\Rightarrow 2(1-p) = 0 \Rightarrow p = 1$

Put the value of p in equation, then equation becomes

$\Rightarrow x^2 + x + 0 = 0 \Rightarrow x(x+1) = 0 \Rightarrow x = 0$ or -1

(8) **(A).** $\because$ One root of equation $x^2 + px + 12 = 0$ is 4.

$\therefore$ it will satisfy the equation

$\Rightarrow 4^2 + 4p + 12 = 0$

$\Rightarrow 4p = -28 \quad \Rightarrow p = -7$ .... (i)

equation $x^2 + px + q = 0$ has equal roots

putting value of p from (i) equation becomes

$x^2 - 7x + q = 0$

$\therefore$ equation has equal roots let roots are α, α

$\therefore 2\alpha = 7 \Rightarrow \alpha = 7/2$ (sum of roots 2α)

and $\alpha^2 = q \Rightarrow 49/4 = q$ (product of roots α^2)

$\Rightarrow q = 49/4$

(9) **(A).** Let roots of equation $x^2 - (a-2)x - a - 1 = 0$ are α and β.

$\therefore \alpha + \beta = a - 2$ .... (1) $\alpha\beta = -a-1$ .... (2)

again $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (a-2)^2 - 2(-a-1) = a^2 + 4 - 4a + 2a + 2$

$= a^2 - 2a + 6 = a^2 - 2a + 1 + 5$

$\alpha^2 + \beta^2 = (a-1)^2 + 5$

$\alpha^2 + \beta^2$ will be minimum if $a - 1 = 0 \Rightarrow a = 1$

(10) **(D).** Roots of equation $x^2 - bx + c = 0$ are two consecutive integer. Let roots are α and $\alpha + 1$

$\therefore$ sum of roots $= \alpha + \alpha + 1 = b$

$\Rightarrow 2\alpha + 1 = b \Rightarrow \alpha = \frac{b-1}{2}$ .... (1)

and product of roots

$\alpha(\alpha+1) = c \Rightarrow \alpha^2 + \alpha = c$ .... (2)

on putting value of α from (1) in (2)

$\Rightarrow \frac{(b-1)^2}{4} + \frac{(b-1)}{2} = c \Rightarrow \frac{b-1}{2}\left[\frac{b-1}{2}+1\right] = c$

$\Rightarrow \frac{b-1}{2}\left(\frac{b+1}{2}\right) = c \Rightarrow \frac{b^2-1}{4} = c \Rightarrow b^2 - 1 = 4c$

$\Rightarrow b^2 - 4c = 1$

(11) **(B).** In a triangle PQR, $\angle R = \frac{\pi}{2}$

$\therefore \angle P + \angle Q + \angle R = \pi$

$\Rightarrow \angle P + \angle Q = \pi/2 \quad \{\because \angle R = \pi/2\}$

$\Rightarrow \frac{\angle P}{2} + \frac{\angle Q}{2} = \frac{\pi}{4} \Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$ $[\angle P = P, \angle Q = Q]$

$\Rightarrow \tan(P/2 + Q/2) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan(P/2) + \tan(Q/2)}{1 - \tan(P/2).\tan(Q/2)} = 1$ .... (1)

Now roots of equation $ax^2 + bx + c = 0$ are $\tan(P/2)$ and $\tan(Q/2)$ $\therefore$ $\tan(P/2) + \tan(Q/2) = -b/a$

and $\tan(P/2)\tan(Q/2) = c/a$

Put these values in (1), we get

$\frac{-b/a}{1-c/a} = 1 \Rightarrow \frac{-b}{a} = 1 - \frac{c}{a}$

$\Rightarrow -b = a - c \Rightarrow a + b = c$

(12) **(C).** Equation $x^2 - 2kx + k^2 + k - 5 = 0$ has two roots (according to question)

If we consider the expression

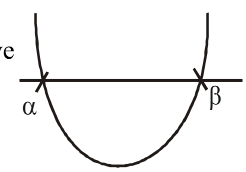
$f(x) = x^2 - 2kx + k^2 + k - 5 = 0$

Coeff. of x^2 is 1 which is positive

$\therefore$ graph of f(x) will

let α, β are roots of equation

$\because$ both roots are less than 5



$\therefore f(5) > 0$

$\Rightarrow 5^2 - 2k(5) + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0$

$\Rightarrow (k-5)(k-4) > 0 \Rightarrow k < 4$ or $k > 5$ .... (1)

but equation has two roots

$\therefore D > 0$

$\Rightarrow 4k^2 - 4.1(k^2 + k - 5) > 0 \Rightarrow -4k + 20 > 0 \Rightarrow k < 5$ .... (2)

From eq. (1) and (2), $k \in (-\infty, 4)$

(13) **(A).** $x^2 + px + q = 0$ roots are $\tan 30°$ and $\tan 15°$ (given)

$\Rightarrow \tan 30° + \tan 15° = -p$ .... (1)

and $\tan 30° \tan 15° = q$ .... (2)

$\therefore 45° = 30° + 15°$

$\Rightarrow \tan 45° = \tan(30° + 15°)$

$\Rightarrow 1 = \frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} \Rightarrow 1 = \frac{-p}{1-q}$

$\Rightarrow 1 = q - p \Rightarrow 1 + 2 = 2 + q - p \Rightarrow 2 + q - p = 3$

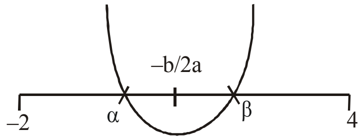
(14) **(B).** Given equation $x^2 - 2mx + m^2 - 1 = 0$ has two roots (as given in question)

$\therefore D > 0$

$\Rightarrow 4m^2 - 4.1(m^2 - 1) > 0$

$\Rightarrow 4 > 0 \Rightarrow m \in R$ .... (i)

Now if we consider $f(x) = x^2 - 2mx + m^2 - 1 = 0$
Coefficient of x^2 is 1 which is +ve
$\Rightarrow$ Curve of $f(x)$ will be



$\Rightarrow f(-2) > 0 \Rightarrow (-2)^2 - 2(m)(-2) + m^2 - 1 > 0$
$\Rightarrow 4 + 4m + m^2 - 1 > 0 \Rightarrow m^2 + 4m + 3 > 0$
$\Rightarrow (m+1)(m+3) > 0 \Rightarrow m < -3$ or $m > -1$ .... (ii)
and $f(4) > 0 \Rightarrow 4^2 - 2m(4) + m^2 - 1 > 0$
$\Rightarrow 16 - 8m + m^2 - 1 > 0$
$\Rightarrow m^2 - 8m + 15 > 0 \Rightarrow (m-3)(m-5) > 0$
$\Rightarrow m < 3$ or $m > 5$ .... (iii)
and for expression
$f(x) = ax^2 + bx + c, \alpha < -b/2a < \beta$

$\therefore -2 < \frac{2m}{2.1} < 4 \Rightarrow -2 < m < 4$ .... (iv)

$$\left\{\begin{matrix} \therefore 2 < \alpha\beta < 4 \\ \therefore -2 < \frac{-b}{2a} < 4 \end{matrix}\right\}$$

from (i), (ii), (iii) & (iv), $m \in (-1, 3)$

(15) **(A).** Let $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$
$\Rightarrow 3x^2y + 9xy + 7y = 3x^2 + 9x + 17$
$\Rightarrow 3x^2(y-1) + 9x(y-1) + 7y - 17 = 0$

$\because$ x is real $\left\{\begin{matrix} \because y - 1 \neq 0 \\ \therefore y \neq 1 \end{matrix}\right.$ .... (i) $\therefore D \geq 0$

$[9(y-1)]^2 - 4.3(y-1)(7y-17) \geq 0$
$\Rightarrow 81(y-1)^2 - 12(y-1)(7y-17) \geq 0$
$\Rightarrow 27(y-1)^2 - 4[(y-1)(7y-17)] \geq 0$
$\Rightarrow -y^2 + 42y - 41 \geq 0 \Rightarrow y^2 - 42y + 41 \leq 0$
$\Rightarrow (y-1)(y-41) \leq 0 \Rightarrow 1 \leq y \leq 41$
$\Rightarrow 1 < y \leq 41$ (from (i)) $\therefore y_{max} = 41$

(16) **(A).** Let roots of equation $x^2 + ax + 1 = 0$ are α and β.
$\therefore \alpha + \beta = -a; \ \alpha\beta = 1$
Now, $|\alpha - \beta| < \sqrt{5} \Rightarrow (\alpha-\beta)^2 < 5 \Rightarrow (\alpha+\beta)^2 - 4\alpha\beta < 5$
$\Rightarrow a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$

(17) **(C).** Let the root of equation $x^2 - 6x + a = 0$ are α, β.
$\Rightarrow \alpha + \beta = 6$ .... (1)
$\Rightarrow \alpha\ \beta = a$ .... (2)
and root of equation $x^2 - cx + 6 = 0$ are α, γ
$\Rightarrow \alpha + \gamma = c$ .... (3) { $\because$ one root is common}
$\Rightarrow \alpha\ \gamma = 6$ .... (4)

According to quesiton, $\frac{\beta}{\gamma} = \frac{4}{3}$ (given)

(2) divided by (3), $\frac{\alpha\beta}{\alpha\gamma} = \frac{a}{6} \Rightarrow \frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$

Put value of a in equation $x^2 - 6x + 8 = 0$
Equation becomes $x^2 - 6x + a = 0$
$\Rightarrow (x-4)(x-2) = 0 \Rightarrow x = 2, 4$
$\because$ If we take $\alpha = 4$ then $\alpha\gamma = 6 \Rightarrow \gamma = 6/4$ (not integer)
If we take $\alpha = 2$ then $\alpha\gamma = 6 \Rightarrow \gamma = 3$ (integer)
$\therefore$ common root is 2

(18) **(A).** Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$
$f(x) = 7x^6 + 70x^4 + 48x^2 + 30 \ \because f(x) > 0 \ \forall x \in R$
$\therefore$ f (x) is always increasing function
$\therefore$ curve of f (x) cut x axis at only and only one point
$\therefore$ f (x) = 0 has only one real solution.

(19) **(C).** Given $c^2 < 4ab$
$3b^2x^2 + 6bcx + 2c^2 = 3(bx+c)^2 - c^2$
Now, $3(bx+c)^2 - c^2 \geq -c^2 > -4ab$

(20) **(B).** $x^2 - x + 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1-4}}{2}$

$x = \frac{1 \pm \sqrt{3}i}{2}$; $\alpha = \frac{1}{2} + i\frac{\sqrt{3}}{2}$, $\beta = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

$\alpha = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$; $\beta = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$\alpha^{2009} + \beta^{2009} = 2\cos 2009\left(\frac{\pi}{3}\right)$

$= 2\cos\left[668\pi + \pi + \frac{2\pi}{3}\right] = 2\cos\left(\pi + \frac{2\pi}{3}\right)$

$= -2\cos\frac{2\pi}{3} = -2\left(-\frac{1}{2}\right) = 1$

(21) **(B).** Let $e^{\sin x} = t \Rightarrow t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} \Rightarrow t = e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}, \ e^{\sin x} = 2 + \sqrt{5}$

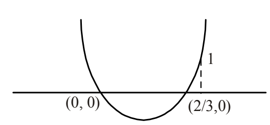
$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0 \Rightarrow \sin x = \ln(2 + \sqrt{5}) > 1$
so rejected so rejected
Hence, no solution.

(22) **(A).** $x^2 + 2x + 3 = 0$ ...(1)
$ax^2 + bx + c = 0$ ... (2)
Since equation (1) has imaginary roots
So equation (2) will also have both roots same as (1).

Thus, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} \Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$

Hence, 1 : 2 : 3

(23) **(A).** $a^2 = 3t^2 - 2t$



For non-integral solution $0 < a^2 < 1$
$a \in (-1, 0) \cup (0, 1)$
Note : It is assumed that a real solution of given equation exists.

(24) **(D).** $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ **;** $2q = p + r$; $-2(\alpha + \beta) = 1 + \alpha\beta$

$$-2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}+1 \Rightarrow \frac{1}{\alpha\beta}=-9$$

Equation having roots α, β is $9x^2+4x-1=0$

$$\alpha,\beta=\frac{-4\pm\sqrt{16+36}}{2\times 9}\ ;\ |\alpha-\beta|=\frac{2\sqrt{13}}{9}$$

(25) **(B).** From equation, $\alpha+\beta=6\ ;\ \alpha\beta=-2$

The value of $\frac{a_{10}-2a_8}{2a_9}=\frac{\alpha^{10}-\beta^{10}-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}$

$$=\frac{\alpha^{10}-\beta^{10}+\alpha\beta(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}$$

$$=\frac{\alpha^9(\alpha+\beta)-\beta^9(\alpha+\beta)}{2(\alpha^9-\beta^9)}=\frac{\alpha+\beta}{2}=\frac{6}{2}=3$$

(26) **(D).** $(x^2-5x+5)^{x^2+4x-60}=1$

Case – I : $x^2+4x-60=0$; $x=-10$; $x=6$

Case – II : $x^2-5x+5=1$

$x^2-5x+4=0$; $x=1$; $x=4$

Case – III : $x^2-5x+5=-1$

$x^2-5x+6=0$; $x=2$ or 3

For $x=2$, $x^2+4x-60=-48$

For $x=3$, $x^2+4x-60=-39$ $\quad\therefore x=2$

Sum of all real value = 3

(27) **(B).** $\sum_{r=1}^{n}(x+(r-1))(x+r)=10n$

$\sum x^2+(2r-1)x+r(r-1)=10n$

$nx^2+x.n^2+\frac{n(n^2-31)}{3}=0$

$3x^2+3nx+(n^2-31)=0$

$\frac{\sqrt{D}}{|\alpha|}=\frac{\sqrt{9n^2-12n^2+372}}{3}=1$

$372-3n^2=9$; $3n^2=372-9=363$

$n^2=121$; $n=11$

(28) **(C).** We have, $(x+1)^2+1=0$

$\Rightarrow (x+1)^2-(i)^2=0\Rightarrow(x+1+i)(x+1-i)=0$

$\therefore\ x=-\underbrace{(1+i)}_{\alpha\text{ (let)}}-\underbrace{(1-i)}_{\beta\text{ (let)}}$

So, $\alpha^{15}+\beta^{15}=(\alpha^2)^7\alpha+(\beta^2)^7\beta$

$=-128(-i+1+i+1)=-256$

(29) **(C).** $(x-1)^2+1=0\Rightarrow x=1+i,1-i$

$\therefore\ (\alpha/\beta)^n=1\Rightarrow(\pm 1)^n=1$

$\therefore$ n (least natural number) = 4

(30) **(C).** $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0$

$|\sqrt{x}-2|+(\sqrt{x})^2-4\sqrt{x}+2=0$

$|\sqrt{x}-2|^2+|\sqrt{x}-2|-2=0$

$|\sqrt{x}-2|=-2$ (not possible) or $\quad |\sqrt{x}-2|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}=3,1$; $x=9,1$; Sum = 10

(31) **(A).** $D<0$

$4(1+3m)^2-4(1+m^2)(1+8m)<0$

$\Rightarrow m(2m-1)^2>0\Rightarrow m>0$

(32) **(D).** $SOR=\frac{3}{m^2+1}\Rightarrow(SOR)_{max}=3$

When $m=0$

$x^2-3x+1=0\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$

$\alpha+\beta=3$; $\alpha\beta=1$

$|\alpha^3-\beta^3|=||\alpha-\beta|(\alpha^2+\beta^2+\alpha\beta)|$

$=|\sqrt{(\alpha-\beta)^2-\alpha\beta}\,((\alpha+\beta)^2-\alpha\beta)|$

$=|\sqrt{9-4}\,(9-1)|=\sqrt{5}\times 8$

(33) **8.00** $D\ge 0$

$(a-10)^2-4(2)\left(\frac{33}{2}-2a\right)\ge 0$

$(a-10)^2-4(33-4a)\ge 0$

$a^2-4a-32\ge 0\Rightarrow a\in(-\infty,-4]\cup[8,\infty)$

(34) **(D).** $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$

Divide by e^{2x}

$e^{2x}+e^x-4+\frac{1}{e^x}+\frac{1}{e^{2x}}=0$

$\left(e^{2x}+\frac{1}{e^{2x}}\right)+\left(e^x+\frac{1}{e^x}\right)-4=0$

$\left(e^x+\frac{1}{e^x}\right)^2-2+\left(e^x+\frac{1}{e^x}\right)-4=0$

Let $e^x+\frac{1}{e^x}=t\Rightarrow(e^x-1)^2=0\Rightarrow x=0$

$\therefore$ Number of real roots = 1

(35) **(B).** $ax^2-2bx+5=0 \begin{smallmatrix}\alpha\\ \alpha\end{smallmatrix}$

$\Rightarrow\ \alpha=\frac{5}{6},\alpha^2=\frac{5}{a}\Rightarrow b^2=5a$

$x^2-2bx-10=0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

$\Rightarrow\ \alpha^2-2b\alpha-10=0$

$\Rightarrow\ a=1/4\Rightarrow\alpha^2=20;\ \alpha\beta=-10\Rightarrow\beta^2=5$

$\Rightarrow\ \alpha^2+\beta^2=25$

(36) **(A).** $\alpha^5=5\alpha+3$

$\beta^5=5\beta+3$

$P_5=5(\alpha+\beta)+6=5(1)+6$

$P_5=11$ and $P_5=\alpha^2+\beta^2=\alpha+1+\beta+1$

$P_2=3$ and $P_3=\alpha^3+\beta^3=2\alpha+1+2\beta+1$

$=2(1)+2=4$

$P_2\times P_3=12$ and $P_5=11\Rightarrow P_5\ne P_2\times P_3$