# 4

# QUADRATIC EQUATIONS

## INTRODUCTION

Quadratic is another name for a polynomial of the 2nd degree. 2 is the highest exponent.

A quadratic equation is an equation that can be written as $Ax^2 + Bx + C = 0$ where $A \neq 0$. This form is called the standard form.

The Babylonians, as early as 1800 BC (displayed on Old Babylonian clay tablets) could solve a pair of simultaneous equations of the form: $x + y = p$, $xy = q$, which are equivalent to the equation : $x^2 + q = px$

In the Sulba Sutras in India 8th century BC quadratic equations of the form $ax^2 = c$ and $ax^2 + bx = c$ were explored using geometric methods. Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula. Euclid, the Greek mathematician, produced a more abstract geometrical method around 300 BC.

In 628 CE, Brahmagupta gave the first explicit (although still not completely general) solution of the quadratic equation:

To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value. (written in Brahmasphutasiddhanta )

The Bakhshali Manuscript dated to have been written in India in the $7^{th}$ century CE contained an algebraic formula for solving quadratic equations, as well as quadratic indeterminate equations (originally of type $ax/c = y$).

Many times in life we come across problems whose solution we find by hit and trial method with the help of available information. For example (i) Suppose we are to find two consecutive natural numbers whose product is 12, we can easily guess that the numbers are 3 and 4; similarly if the product is 56 again we can find the number as 7 and 8, but if the product is 552, it becomes difficult to answer the problem by a guess. (ii) Similarly suppose we are to find the dimensions of a rectangle with area 168 square metres and the length exceeds breadth by 2 m. It is again difficult to answer the problem by making a guess.

The above type of problems can be solved by a systematic method. For example in the first case if the numbers are $x$ and $x + 1$, we have to solve the equation $x(x + 1) = 552$ or $x^2 + x - 552 = 0$.

Again in 2nd case, when we take breadth as $x$, the length will be $x + 2$ and we will have to solve the equation $x(x + 2) = 168$ or $x^2 + 2x - 168 = 0$. If we put different conditions on the length and breadth, we shall again get an equation of the type $ax^2 + bx + c = 0$ but with different values of a, b and c.

## QUADRATION EQUATIONS

A Polynomial of degree two of the form $ax^2 + bx + c$ $(a \neq 0)$ is called a quadratic expression in x (functioning x).

Ex. $3x^2 - 7x + 5$, $\quad x^2 - 7x + 3$

General form : $f(x) = ax^2 + bx + c$, where $a, b, c \in C$ and $a \neq 0$

A quadratic Polynomial f(x) when equated to zero is called Quadratic Equation.

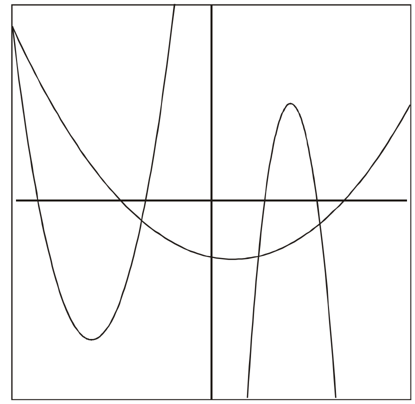
Ex. $3x^2 - 7x + 5 = 0$, $-9x^2 + 7x - 5 = 0$, $-x^2 + 2x = 0$, $2x^2 = 0$

General form $ax^2 + bx + c = 0$

Where $a, b, c \in C$ and $a \neq 0$

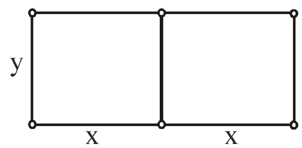
The graph of a quadratic function is a curve called a parabola. Parabolas may open upward or downward and vary in "width" or "steepness", but they all have the same basic "U" shape. The picture below shows three graphs,

and they are all parabolas.
All parabolas are symmetric with respect to a line called the axis of symmetry. A parabola intersects its axis of symmetry at a point called the vertex of the parabola. Quadratic equation are very helpful in solving practical problems.



**Example 1 :**

A rancher has 600 meters of fence to enclose a rectangular corral with another fence dividing it in the middle as in the diagram below.




As indicated in the diagram, the four horizontal sections of fence will each be x meters long and the three vertical sections will each be y meters long.
The rancher's goal is to use all of the fence and enclose the largest possible area.
The two rectangles each have area xy, so we have : total area : A = 2xy.
There is not much we can do with the quantity A while it is expressed as a product of two variables. However, the fact that we have only 1200 meters of fence available leads to an equation that x and y must satisfy.

$3y + 4x = 1200.$

$3y = 1200 – 4x.$

$y = 400 – 4x/3.$

We now have y expressed as a function of x, and we can substitute this expression for y in the formula for total area A.

$A = 2xy = 2x (400 – 4x/3).$

We need to find the value of x that makes A as large as possible. A is a quadratic function of x.

**Example 2 :**

Represent the situation mathematically :
A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

**Sol.** Let the number of toys produced on that day be x.
Therefore, the cost of production (in rupees) of each toy that day = $55 – x$
So, the total cost of production (in rupees) that day = $x (55 – x)$
Therefore, $x (55 – x) = 750$ i.e., $55x – x^2 = 750$ i.e., $– x^2 + 55x – 750 = 0$ i.e., $x^2 – 55x + 750 = 0$
Therefore, the number of toys produced that day satisfies the quadratic equation
$x^2 – 55x + 750 = 0$, which is the required representation of the problem mathematically.

**Example 3 :**

Check whether the following are quadratic equations:
(i) $(x – 2)^2 + 1 = 2x – 3$ (ii) $(x + 2)^3 = x^3 – 4$

**Sol.** (i) LHS = $(x – 2)^2 + 1 = x^2 – 4x + 4 + 1 = x^2 – 4x + 5$
Therefore, $(x – 2)^2 + 1 = 2x – 3$ can be rewritten as $x^2 – 4x + 5 = 2x – 3$ i.e., $x^2 – 6x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.
Therefore, the given equation is a quadratic equation.
(ii) Here, LHS = $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$
Therefore, $(x + 2)^3 = x^3 - 4$ can be rewritten as

$$x^3 + 6x^2 + 12x + 8 = x^3 - 4 \quad \text{i.e.,} \quad 6x^2 + 12x + 12 = 0 \quad \text{or} \quad x^2 + 2x + 2 = 0$$

It is of the form $ax^2 + bx + c = 0$.
So, the given equation is a quadratic equation.

## SELF CHECK

**Q.1** Check whether the following are quadratic equations :
(i) $(x + 1)^2 = 2(x - 3)$ (ii) $(x - 3)(2x + 1) = x(x + 5)$
(iii) $(2x - 1)(x - 3) = (x + 5)(x - 1)$ (iv) $(x + 2)^3 = 2x(x^2 - 1)$

**Q.2** Represent the following situations in the form of quadratic equations :
(i) The area of a rectangular plot is 528 $m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
(ii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
(iii) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

## ANSWERS

**(1)** (i) Yes (ii) Yes (iii) Yes (iv) No
**(2)** (i) $2x^2 + x - 528 = 0$, where x is breadth (in metres) of the plot.
(ii) $x^2 + 32x - 273 = 0$, where x (in years) is the present age of Rohan.
(iii) $u^2 - 2u - 1280 = 0$, where u (in km/h) is the speed of the train.

**Solving quadratic equation:** There are four ways to solve a quadratic equation, namely factorisation, extraction of roots, completing the square and quadratic formula.

**Factoring :** Works well when the quadratic can easily be factored.
You can use the trial and error method of factoring or AC Method of Factoring. The idea behind factoring is to place the equation into standard form, and then factor the left hand side into two factors $(x - a)$ and $(x - b)$. The solutions to the equation are then $x = a$ and $x = b$. The factors will of course vary if $A \neq 1$. Factoring works because there is a rule which says if the product of two factors is zero, then one of the factors must be zero.
Hence steps you can use in solving problems:
**Step 1:** Simplify each side if needed.
This would involve things like removing ( ), removing fractions, adding like terms, etc.
**To remove ( ):** Just use the distributive property.
**To remove fractions:** Since fractions are another way to write division, and the inverse of divide is to multiply, you remove fractions by multiplying both sides by the LCM of all of your fractions.
**Step 2:** Write in standard form, $ax^2 + bx + c = 0$, if needed.
If it is not in standard form, move any term(s) to the appropriate side by using the addition/subtraction property of equality. Also, make sure that the squared term is written first left to right, the x term is second and the constant is third and it is set equal to 0.
**Step 3: Factor.**
**Step 4:** Use the Zero-Product Principle.
If $ab = 0$, then $a = 0$ or $b = 0$.
0 is our magic number because the only way a product can become 0 is if at least one of its factors is 0.

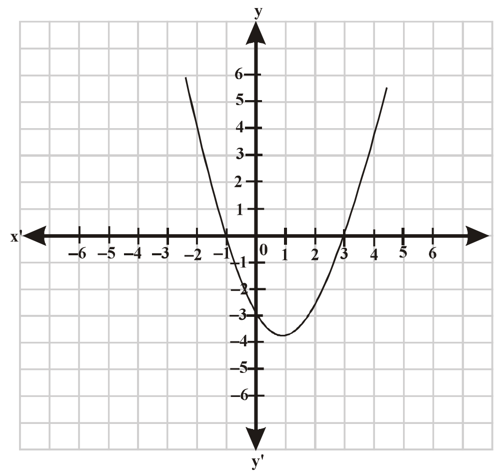
You can not guarantee what the factors would have to be if the product was set equal to any other number. For example if ab = 1, then a = 5 and b = 1/5 or a = 3 and b = 1/3, etc. But with the product set equal to 0, we can guarantee finding the solution by setting each factor equal to 0. That is why it is important to get it in standard form to begin with.

**Step 5:** Solve for the linear equation(s) set up in step 4.

If a quadratic equation factors, it will factor into either one linear factor squared or two distinct linear factors. So, the equations found in step 4 will be linear equations.

**Example 4 :**

$f(x) = x^2 - 2x - 3$. Find the roots of f (x), and sketch the graph of y = f (x).




**Sol.** $x^2 - 2x - 3 = (x + 1)(x - 3)$.

Therefore, the roots are –1 and 3 .

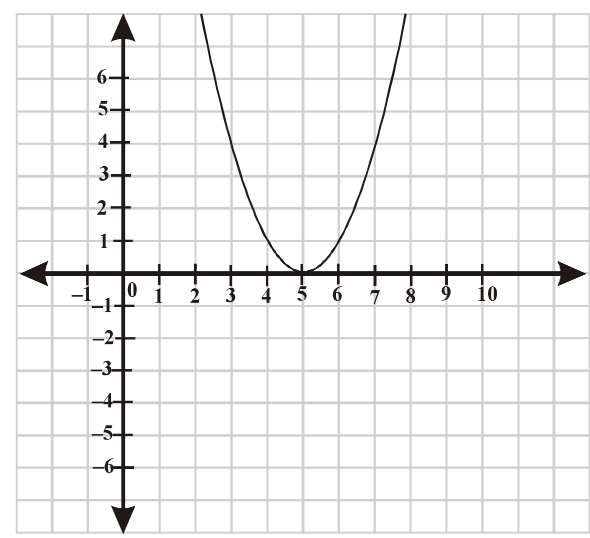
These are the x-intercepts of the graph.

The y-intercept is the constant term, –3.

In every polynomial the y-intercept is the constant term, because the constant term is the value of y when x = 0.

**Example 5 :**

$f(x) = x^2 - 0x + 25$. Find the roots of f (x), and sketch the graph of y = f(x).



**Sol.** $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$.

The "two" roots are 5, 5.

5 is called a double root. At a double root, the graph does not cross the x-axis.

It just touches it.

A double root occurs when the quadratic is a perfect square trinomial: $x^2 \pm 2ax + a^2$; that is, when it is the square of a binomial : $(x \pm a)^2$.

**Example 6 :**

Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

**Sol.** We have, $6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$

$$= 3x(2x + 1) - 2(2x + 1) = (3x - 2)(2x + 1)$$

The roots of $6x^2 - x - 2 = 0$ are the values of x for which $(3x - 2)(2x + 1) = 0$

Therefore, $3x - 2 = 0$ or $2x + 1 = 0$, i.e., $x = \frac{2}{3}$ or $x = -\frac{1}{2}$

Therefore, the roots of $6x^2 - x - 2 = 0$ are $\frac{2}{3}$ and $-\frac{1}{2}$

We verify the roots, by checking that $\frac{2}{3}$ and $-\frac{1}{2}$ satisfy $6x^2 - x - 2 = 0$.

**Example 7 :**

Solve : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x},\ a+b \neq 0$

**Sol.** $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x},\ a+b \neq 0 \Rightarrow \frac{x-(a+b+x)}{a+b+x} = \frac{a+b}{ab} \Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow -ab\ (a+b) = (a+b)\ x + (a+b+x)$

$\Rightarrow (a+b)\ [x\ (a+b+x) + ab] = 0 \Rightarrow x\ (a+b+x) + ab = 0$ $\qquad [\because a+b \neq 0]$

$\Rightarrow x^2 + ax + bx + ab = 0 \Rightarrow x\ (x+a) + b\ (x+b) = 0 \Rightarrow (x+a)\ (x+b) = 0$

$\Rightarrow x+a = 0$ or $x+b = 0 \Rightarrow x = -a$ or $x = -b$

**SELF CHECK**

**Q.1** Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 - x + \frac{1}{8} = 0$

**Q.2** Solve the following quadratic equations by factorization.

(i) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ (ii) $4x^2 + 4bx - (a^2 - b^2) = 0$

(iii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$ (iv) $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

**Q.3.** Find two numbers whose sum is 27 and product is 182.

**Q.4** The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

**ANSWERS**

**(1)** (i) –2, 5 (ii) $\frac{1}{4}, \frac{1}{4}$ **(2)** (i) $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ (ii) $\frac{a-b}{2}, -\left(\frac{a+b}{2}\right)$ (iii) $6, \frac{40}{13}$ (iv) $3, \frac{4}{3}$

**(3)** Numbers are 13 and 14. **(4)** 5 cm. and 12 cm.

**Extraction of Roots :** Works well when there is no linear term, that is, when B = 0.
The extraction of roots is called the square root principle. The goal here is to get the squared variable term by itself on one side and a non-negative constant on the other side.
The square root of both sides is then taken. Remember that the square root of $x^2$ is the absolute value of x.
When you solve an equation involving an absolute value, you will get a plus and minus in the solution. Too often, we bypass the step with the absolute value in it and go straight to the plus/minus phase.

**Example 8 :**

Solve $(2x-1)^2 = 20$

**Sol.** $(2x-1)^2 = 20$

$(2x-1) = \sqrt{20}$ or $(2x-1) = -\sqrt{20}$

If $(2x-1) = \sqrt{20} \Rightarrow (2x-1) = \sqrt{(4)\ (5)} \Rightarrow (2x-1) = 2\sqrt{5} \Rightarrow 2x - 1 + 1 = 2\sqrt{5} + 1 \Rightarrow 2x = 2\sqrt{5} + 1$

$\Rightarrow \frac{2x}{2} = \frac{2\sqrt{5}+1}{2} \Rightarrow x = \frac{2\sqrt{5}+1}{2}$

If $2x-1=-\sqrt{20} \Rightarrow 2x-1=-\sqrt{(4)(5)} \Rightarrow 2x-1=-2\sqrt{5}$

$$\Rightarrow 2x-1+1=-2\sqrt{5}+1 \Rightarrow 2x=-2\sqrt{5}+1$$

$$\Rightarrow \frac{2x}{2}=\frac{-2\sqrt{5}+1}{2} \Rightarrow x=\frac{-2\sqrt{5}+1}{2}$$

There are two solutions to this quadratic equation : $x=\frac{2\sqrt{5}+1}{2}$ and $x=\frac{-2\sqrt{5}+1}{2}$

**Completing the Square :**

Works well when the leading coefficient A is 1 and B is even.

**1.** If A is not 1, then either divide every term by A so that it is 1 or factor an A out of the variable terms only (not out of the constant).

**2.** Move the constant to the right hand side. Be sure and leave space at the end of the left hand side before the equal sign for the constant which will be inserted in there later.

**3.** Take 1/2 of the linear coefficient (B) and call that number "b". On the next line, write $(x+b)^2 =$
We will fill in the right hand side later. Of course, if B is negative, then the expression will look like $(x-b)^2 =$

**4.** Square that value you just found (one-half of B) and write it in that space you left at the end of the left hand side before the equal sign on the previous line. Add the same value to the right hand side, also. It is very important that we add the same thing to both sides. If you chose to factor out the A, rather than dividing through by it, make sure that you add A times that constant to the right hand side.

**5.** Simplify the right hand side

**6.** Continue the process as an extraction of roots problem.

**Example 9 :**

Find the roots of the equation $5x^2-6x-2=0$ by the method of completing the square.

**Sol.** Multiplying the equation throughout by 5, we get

$$25x^2-30x-10=0$$

This is the same as

$$(5x)^2-2\times(5x)\times 3+3^2-3^2-10=0$$

i.e., $(5x-3)^2-9-10=0$

i.e., $(5x-3)^2-19=0$ i.e., $(5x-3)^2=19$

i.e., $5x-3=\pm\sqrt{19}$ i.e., $5x=3\pm\sqrt{19}$ So, $x=\frac{3\pm\sqrt{19}}{5}$

Therefore, the roots are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$. Verify that the roots are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$.

**QUADRATIC FORMULA**

The quadratic formula is a catch-all that can be used to solve any quadratic equation. The equation must first of all be written in standard form, and then the coefficients plugged into the formula. The formula can be derived by completing the square on a generic quadratic equation if $ax^2+bx+c=0$ and $a\neq 0$, then

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

To prove this, we will complete the square. But to do that, the coefficient of $x^2$ must be 1.

Therefore, we will divide both sides of the original equation by a:

$ax^2+bx+c=0$, implies $x^2+\frac{b}{a}x+\frac{c}{a}=0$ (Fortunately, $\frac{0}{a}$ is 0, so that we still have a quadratic equation)

$x^2+\frac{b}{a}x=-\frac{c}{a}$. Now, half of $\frac{b}{a}$ is $\frac{b}{2a}$. Therefore, on adding the square of $\frac{b}{2a}$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2} \quad ; \quad \left(x+\frac{b}{2a}\right)^2=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}=\frac{b^2-4ac}{4a^2}$$

Therefore, $\left(x+\frac{b}{2a}\right)=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\sqrt{\frac{b^2-4ac}{2a}}$ and therefore, $x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{2a}}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

This is the quadratic formula.

## NATURE OF ROOTS

In Quadratic equation $ax^2+bx+c=0$. The term $b^2-4ac$ is called discriminant of the equation , which plays an important role in finding the nature of the roots. It is denoted by $\Delta$ or D

**(A) Suppose a, b, c $\in$ R and a $\neq$ 0 then**

(i) If $D>0 \Rightarrow$ Roots are Real and unequal

(ii) If $D=0 \Rightarrow$ Roots are Real and equal and each equal to $-b/2a$

(iii) If $D<0 \Rightarrow$ Roots are imaginary and unequal or complex conjugate.

**(B) Suppose a, b, c $\in$ Q and a $\neq$ 0 then**

(i) If $D>0$ and D is perfect square $\Rightarrow$ Roots are unequal and Rational

(ii) If $D>0$ and D is not perfect square $\Rightarrow$ Roots are irrational and unequal

## ROOTS UNDER PARTICULAR CASES

For the quadratic equation $ax^2+bx+c=0$

(i) If $b=0 \Rightarrow$ roots are of equal magnitude but of opposite sign

(ii) If $c=0 \Rightarrow$ one root is zero other is $-b/a$

(iii) If $b=c=0 \Rightarrow$ both roots are zero

(iv) If $a=c \Rightarrow$ roots are reciprocal to each other

(v) $\left.\begin{matrix} a>0\ c<0 \\ a<0\ c>0 \end{matrix}\right\} \Rightarrow$ If Roots are of opposite signs

(vi) If $\left.\begin{matrix} a>0,\ b>0,\ c>0 \\ a<0,\ b<0, c<0 \end{matrix}\right\} \Rightarrow$ both roots are negative

(vii) If $\left.\begin{matrix} a>0,\ b<0,\ c>0 \\ a<0,\ b>0, c<0 \end{matrix}\right\} \Rightarrow$ both roots are positive

(viii) If sign of a = sign of b $\neq$ sign of c $\Rightarrow$ Greater root in magnitude is negative

(ix) If sign of b = sign of c $\neq$ sign of a $\Rightarrow$ Greater root in magnitude is positive

(x) If $a+b+c=0 \Rightarrow$ one root is 1 and second root is c/a

(xi) If $a=b=c=0$ then equation will become an identity and will be satisfy by every value of x.

**Sum and product of roots :**
If $\alpha$ and $\beta$ are the roots of quadratic equation $ax^2 + bx + c = 0$, then

(i) Sum of Roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$ (ii) Product of Roots $P = \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Ex. In equation $3x^2 + 4x - 5 = 0$
Sum of Roots $S = -4/3$, Product of roots $P = -5/3$

## RELATION BETWEEN ROOTS AND COEFFICIENTS

If roots of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are $\alpha$ and $\beta$ then:

(i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\frac{\sqrt{b^2-4ac}}{a} = \frac{\pm\sqrt{D}}{a}$ (ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2-2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\frac{b\sqrt{b^2-4ac}}{a^2} = \frac{\pm\sqrt{D}}{a}$

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\frac{b(b^2-3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta) = \sqrt{(\alpha+\beta)^2-4\alpha\beta}\left\{(\alpha+\beta)^2 - \alpha\beta\right\} = \frac{(b^2-ac)\sqrt{b^2-4ac}}{a^3}$

(vi) $\alpha^4 + \beta^4 = \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\frac{b^2-2ac}{a^2}\right)^2 - 2\frac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2-\beta^2)(\alpha^2+\beta^2) = \frac{-b(b^2-2ac)\sqrt{b^2-4ac}}{a^4}$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta$ (ix) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$ (xi) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \frac{\alpha^4+\beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

**Example 10 :**
Write the discriminate of the following quadratic equations :
(i) $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ (ii) $x^2 + px + q = 0$

**Sol.** (i) The given equation is $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Here, $a = \sqrt{3}, b = -2\sqrt{2}$ and $c = -2\sqrt{3}$ $\therefore$ $D = b^2 - 4ac = (-2\sqrt{2})^2 - 4 \times \sqrt{3} \times -2\sqrt{3} = 8 + 24 = 32$
(ii) The equation is $x^2 + px + q = 0$
Here, a = 1, b = p and c = 2q.
$\therefore$ $D = b^2 - 4ac = p^2 - 4 \times 1 \times 2q = p^2 - 8q$

**Example 11 :**

Find the roots of the following quadratic equations, if they exist, using the quadratic formula:

(i) $3x^2 - 5x + 2 = 0$ (ii) $x^2 + 4x + 5 = 0$ (iii) $2x^2 - 2\sqrt{2}\,x + 1 = 0$

**Sol.** (i) $3x^2 - 5x + 2 = 0$.

Here, a = 3, b = – 5, c = 2. So, $b^2 - 4ac = 25 - 24 = 1 > 0$.

Therefore, $x = \frac{5 \pm \sqrt{1}}{6} = \frac{5 \pm 1}{6}$ i.e., x = 1 or $x = \frac{2}{3}$. So, the roots are $x = \frac{2}{3}$ and 1.

(ii) $x^2 + 4x + 5 = 0$. Here, a = 1, b = 4, c = 5. So, $b^2 - 4ac = 16 - 20 = -4 < 0$.

Since the square of a real number cannot be negative, therefore $\sqrt{b^2 - 4ac}$ will not have any real value.

So, there are no real roots for the given equation.

(iii) $2x^2 - 2\sqrt{2}\,x + 1 = 0$. Here, a = 2, $b = -2\sqrt{2}$, c = 1. So, $b^2 - 4ac = 8 - 8 = 0$

Therefore, $x = \frac{2\sqrt{2} \pm \sqrt{0}}{4} = \frac{\sqrt{2}}{2} \pm 0$, i.e., $x = \frac{1}{\sqrt{2}}$. So, the roots are $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

**Example 12 :**

Determine the nature of the roots of the following quadratic equations :

(i) $x^2 - 4x + 4 = 0$ (ii) $2x^2 + 5x + 5 = 0$

**Sol. (i)** The given equation is $x^2 - 4x + 4 = 0$

Here, a = 1, b = – 4 and c = 4.

$\therefore$ $D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4 = 0$. Since D = 0, therefore roots are real and equal.

(ii) The given equation is $2x^2 + 5x + 5 = 0$

Here, a = 2, b = 5 and c = 5. $\therefore$ $D = b^2 - 4ac = (5)^2 - 4 \times 2 \times 5 = 25 - 40 = -15$

Since D < 0, therefore roots are imaginary.

**Example 13 :**

Find two consecutive odd positive integers, sum of whose squares is 290.

**Sol.** Let the smaller of the two consecutive odd positive integers be x. Then, the second integer will be x + 2.

According to the question, $x^2 + (x + 2)^2 = 290$

i.e., $x^2 + x^2 + 4x + 4 = 290$

i.e., $2x^2 + 4x - 286 = 0$ i.e., $x^2 + 2x - 143 = 0$

which is a quadratic equation in x. Using the quadratic formula, we get

$$x = \frac{-2 \pm \sqrt{4 + 572}}{2} = \frac{-2 \pm \sqrt{576}}{2} = \frac{-2 \pm 24}{2} \text{ i.e., } x = 11 \text{ or } x = -13$$

But x is given to be an odd positive integer.

Therefore, $x \neq -13$, x = 11.

Thus, the two consecutive odd integers are 11 and 13.

Check : $11^2 + 13^2 = 121 + 169 = 290$.

**Example 14 :**

Find the values of k for which the given equation has real and equal roots.

(i) $2x^2 - 10x + k = 0$ (ii) $2x^2 + 3x + k = 0$

**Sol.** (i) The given equation is $2x^2 - 10x + k = 0$. Here, a = 2, b = 10, c = k

$\therefore$ $D = b^2 - 4ac = (-10)^2 - 4 \times 2 \times k = 0$

The given equation will have real and equal roots if $D = 0 \Rightarrow 100 - 8k = 0 \Rightarrow k = \frac{100}{8} = \frac{25}{2}$

(ii) The given equation is $2x^2 + 3x + k = 0$

Here, a = 2, b = 3, c = k $\quad\therefore\ D = b^2 - 4ac = 9 - 4 \times 2 \times k = 9 - 8k$

The given equation will have real roots, if $D = 0 \Rightarrow 9 - 8k = 0 \Rightarrow k = \frac{9}{8}$

**Example 15 :**

Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

**Sol.** Here a = 3, b = – 2 and c = 1/3

Therefore, discriminant $b^2 - 4ac = (-2)^2 - 4 \times 3 \times (1/3) = 4 - 4 = 0$.

Hence, the given quadratic equation has two equal real roots.

The roots are, $\frac{-b}{2a}, \frac{-b}{2a}$, i.e., $\frac{2}{6}, \frac{2}{6}$, i.e., $\frac{1}{3}, \frac{1}{3}$

## SELF CHECK

**Q.1** Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$

(iv) $9x^2 + 7x - 2 = 0$ (v) $x^2 + 5x + 5 = 0$

**Q.2** Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

(iii) $2x^2 - kx + 1 = 0$ (iv) $x^2 + k(4x + k - 1) + 2 = 0$

**Q.3** Find the values of k for which the equation $x^2 + 5kx + 16 = 0$ has no real roots.

**Q.4** The number of roots of the quadratic equation $8\sec^2\theta - 6\sec\theta + 1 = 0$ is–

(1) Infinite (2) 1 (3) 2 (4) 0

**Q.5** The roots of the equation $x^2 + 2\sqrt{3}x + 3 = 0$ are–

(1) Real and equal (2) Rational and equal

(3) Irrational and equal (4) Irrational and unequal

**Q.6** If the roots of both the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}\,x + q = 0$ are real, then–

(1) $p = q$, $r \neq 0$ (2) $2q = \pm\sqrt{pq}$ (3) $p/q = q/r$ (4) None of these

**Q.7** The difference between the roots of the equation $x^2 - 7x - 9 = 0$ is–

(1) 7 (2) $\sqrt{85}$ (3) 9 (4) $2\sqrt{85}$

**Q.8** If the sum of the roots of the equation $ax^2 + 4x + c = 0$ is half of their difference, then the value of ac is–

(1) 4 (2) 8 (3) 12 (4) –12

## ANSWERS

**(1)** (i) Real roots do not exist (ii) Equal roots, $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (iii) Distinct roots, $\frac{3 \pm \sqrt{3}}{2}$

(iv) $\frac{2}{9}, 1$ (v) $\frac{-5+\sqrt{5}}{2}, \frac{-5-\sqrt{5}}{2}$ **(2)** (i) $k = \pm 2\sqrt{6}$ (ii) k = 6 (iii) $k = \pm 2\sqrt{2}$ (iv) k = – 1

**(3)** –8/5 < k 8/5 **(4)** 4 **(5)** 3 **(6)** 3 **(7)** 3 **(8)** 4

## FORMATION OF AN EQUATION WITH GIVEN ROOTS

A quadratic equation whose roots are $\alpha$ and $\beta$ is given by

$(x - \alpha)(x - \beta) = 0 \quad \therefore \quad x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\therefore \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 -$ (sum of Roots) x + Product of Roots = 0

$\therefore \quad x^2 - Sx + P = 0$

## EQUATION IN TERMS OF THE ROOTS OF ANOTHER EQUATION

If $\alpha, \beta$ are roots of the equation $ax^2 + bx + c = 0$ then equation whose roots are

(i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$ (Replace x by – x)

(ii) $1/\alpha, 1/\beta \Rightarrow cx^2 + bx + a = 0$ (Replace x by 1/x)

(iii) $\alpha^n, \beta^n; n \in N \Rightarrow a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ (Replace x by $x^{1/n}$)

(iv) $k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2c = 0$ (Replace x by x/k)

(v) $k + \alpha, k + \beta \Rightarrow a(x - k)^2 + b(x - k) + c = 0$ (replace x by (x – k))

(vi) $\frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow k^2 ax^2 + kbx + c = 0$ (Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}; n \in N \Rightarrow a(x^n)^2 + b(x^n) + c = 0$ (replace x by $x^n$)

**Example 16 :**

Find the equation whose roots are 3 and 4.

**Sol.** The quadratic equation is given by

$x^2 -$ (sum of the roots) x + (product of roots) = 0

$\therefore$ The required equation $= x^2 - (3 + 4)x + 3.4 = 0 = x^2 - 7x + 12 = 0$

**Example 17 :**

If $\alpha, \beta$ are root of the equation $x^2 - 5x + 6 = 0$ then find the equation whose roots are $\alpha + 3$ and $\beta + 3$.

**Sol.** Let $\alpha + 3 = x \quad \therefore \alpha = x - 3$ (Replace x by x – 3)

So the required equation is $= (x - 3)^2 - 5(x - 3) + 6 = 0 = x^2 - 6x + 9 - 5x + 15 + 6 = 0 = x^2 - 11x + 30 = 0$

## APPLICATION OF QUADRATIC EQUATION

In this section, we will learn various daily life problems that can be solved with the help of quadratic equation. For this first of all we shall translate the word problem into symbolic language (Mathematical statement). Then solve the quadratic equation thus obtained. Finally retain the meaningful root and reject the root which does not satisfy the given condition. This way the problem will be solved.

**Example 18 :**

The sum of the squares of two consecutive odd positive integers is 290. Find them.

**Sol.** Let the two consecutive odd numbers be 2x – 1 and 2x + 1, where x > 0

Then, according to the given condition, we have

$(2x - 1)^2 + (2x + 1)^2 = 290$

$\Rightarrow \quad 4x^2 - 4x + 1 + 4x^2 + 4x + 1 = 290 \Rightarrow 8x^2 + 2 = 290$

$\Rightarrow \quad 8x^2 = 290 - 2 = 288 \Rightarrow x^2 = \frac{288}{8} = 36$

$\therefore \quad x = +6 \because (x > 0) \Rightarrow 2x - 1 = 2(6) - 1 = 11$ and $2x + 1 = 2(6) + 1 = 13$

Hence the required two consecutive numbers are 11 and 15.

**Example 19 :**

The sides of a right-angled triangle are $2x - 1$, $2x$ and $2x + 1$. Find 'x' and hence the area of the triangle.

**Sol.** Clearly, $2x + 1$ is the hypotenuse of the right-angled triangle.

Hence, using Pythagoras theorem, we have

$(2x + 1)^2 = (2x - 1)^2 + (2x)^2 \quad \therefore \quad 4x^2 + 4x + 1 = 4x^2 - 4x + 1 + 4x^2$

$\therefore \quad -4x^2 + 8x = 0 \quad \therefore \quad -4x(x - 2) = 0 \quad \therefore \quad x = 2$ or $x = 0$

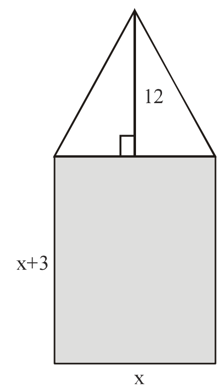
But for $x = 0$, triangle does not exist, hence $x = 2$.

Since $(2x - 1)$ and $2x$ are the sides containing right angle.

$\therefore$ Area $= \frac{1}{2}(2x-1)(2x) = \frac{1}{2}(4-1)(4) = 6$ sq. units $\quad [\because x = 2]$

**Example 20 :**

A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m (Fig). Find its length and breadth.




**Sol.** Let the breadth of the rectangular park be x m.

So, its length $= (x + 3)$ m.

Therefore, the area of the rectangular park $= x(x + 3)\ m^2 = (x^2 + 3x)\ m^2$.

Now, base of the isosceles triangle $= x$ m.

Therefore, its area $= \frac{1}{2} \times x \times 12 = 6x\ m^2$

According to our requirements,,

$x^2 + 3x = 6x + 4 \quad$ i.e., $\quad x^2 - 3x - 4 = 0$

Using the quadratic formula, we get $x = \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2} = 4$ or $-1$

But $x \neq -1 \neq$. Therefore, $x = 4$.

So, the breadth of the park = 4m and its length will be 7m.

**Verification :** Area of rectangular park $= 28\ m^2$, area of triangular park $= 24\ m^2 = (28 - 4)\ m^2$

**Example 21 :**

If a train travelled 5 km/hr faster, it would take one hour less to travel 210 km. Find the speed of the train.

**Sol.** Let the speed of the train be x km/hr. If the speed is x km/hr,

then time to cover 210 km $= \frac{210}{x}$ hours ......... (1)

If the speed is $(x + 5)$ km/hr, then time to cover 210 km $= \frac{210}{x+5}$ hours ........... (2)

Clearly, the time taken in second case is less, since speed is more.

$\Rightarrow \frac{210}{x} - \frac{210}{x+5} = 1 \Rightarrow \frac{210(x+5) - 210x}{x(x+5)} - = 1$

$\Rightarrow 210x + 1050 - 210x = x(x + 5) \Rightarrow 1050 = x^2 + 5x \Rightarrow x^2 + 5x - 1050 = 0$

$\Rightarrow x^2 + 35x - 1050 = 0 \Rightarrow x(x + 35) - 30(x + 35) = 0$

$\Rightarrow (x + 35)(x - 30) = 0 \Rightarrow x = 30$ km/hr.

**Example 22 :**

A lawn 50m long and 34m broad has a pathof uniform width around it. If the area of the path is 540 $m^2$, find its width.

**Sol.** Let the width of the lawn be x/2 m.

Internal dimensions of the lawn are : 50m and 34 m

$\Rightarrow$ External dimensions of the law are : (50 + x) m and (34 + x) m

$\Rightarrow$ External area – Internal area = Area of the path

$\Rightarrow$ $(50 + x)(34 + x) - 50 \times 34 = 540$

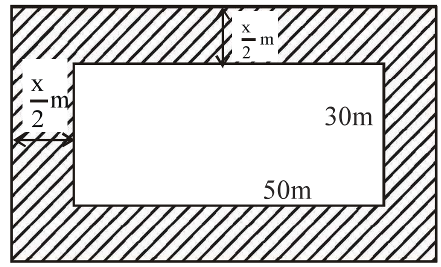
$\Rightarrow$ $50 \times 34 + 50x + 34x - 50 \times 34 + x^2 = 540$

$\Rightarrow$ $x^2 + 84x - 540 = 0 \Rightarrow x^2 + 90x - 6x - 540 = 0$

$\Rightarrow$ $x(x + 90) - 6(x + 90) = 0$

$\Rightarrow$ $(x + 90)(x - 6) = 0 \Rightarrow x = 6m$

$\Rightarrow$ Width of the lawn is x/2 m i.e. 3m.




## SELF CHECK

**Q.1** In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

**Q.2** The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

**Q.3** Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

**Q.4** Sum of the areas of two squares is 468 $m^2$. If the difference of their perimeters is 24 m, find the sides of the two squares.

**Q.5** Find two natural numbers which differ by 3 and the sum of whose squares is 117.

**Q.6** The sides of a right-angled triangle containing the right angle are 5x cm and (3x – 1) cm. If the area of the triangle be 60 $cm^2$, calculate the lengths of the sides of the triangle.

**Q.7** The hypotenuse of a right triangle is 13 cm and the difference between the other two sides is 7 cm.

(i) Taking 'x' as the length of the shorter of the two sides, write an equation in 'x' that represent~ the above statement.

(ii) Solve the equation obtained in (i) above, and hence find the two unknown sides of the triangle.

**Q.8** If the length of a rectangle is increased by 10 and the breadth decreased by 5, the area is unaltered. If the length is decreased by 5 and the breadth is increased by 4, even then the area is unaltered. Find the length and the breadth.

**Q.9** The length of a verandah is 3 m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.

(i) Taking 'x' as the breadth of the verandah, write an equation in 'x' that represents the above statement.

(ii) Solve the equation obtained in (i) above and hence find the dimensions of the verandah.

**Q.10** A journey of 240 km would take half an hour less if the speed were increased by 2 km per hr. Find the usual speed.

**Q.11** If the usual speed is reduced by 5 km per hr. it takes 2 hrs more to cover a distance of 300 km. Find the usual speed.

**Q.12** A plane left an airport 15 min later than the scheduled time and in order to reach its destination 1500 km away in time, had to increase its speed by 200 km/hr. Find its usual speed.

## ANSWERS

**(1)** Marks in mathematics = 12, marks in English = 18, or, Marks in mathematics = 13, marks in English = 17
**(2)** 18, 12 or 18, – 12 **(3)** 15 hours, 25 hours **(4)** 18m, 12m

**EXTRA EDGE**

**Nature of the factors of the quadratic expression :**

The nature of factors of the quadratic expression $ax^2 + bx + c$ is the same as the nature of roots of the corresponding quadratic equation $ax^2 + bx + c = 0$. Thus the factors of the expression are

(i) Real and different, If $b^2 - 4ac > 0$ (ii) Rational and different, if $b^2 - 4ac$ is a prefect square.

(iii) Real and equal, if $b^2 - 4ac = 0$ (iv) Imaginary, if $b^2 - 4ac < 0$

**Ex.** The factors of $x^2 - x + 1$ are

**Sol.** The factors of $x^2 - x + 1$ are imaginary because $b^2 - 4ac = (-1)^2 - 4\,(1)\,(1) = 1 - 4 = -3 < 0$

**Maximum and minimum value of quadratic expression :**

In a Quadratic Expression $ax^2 + bx + c$

**(i)** If $a > 0$ Quadratic expression has least value at $x = -\frac{b}{2a}$. This least value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

**(ii)** If $a < 0$, Quadratic expression has greatest value at $x = -\frac{b}{2a}$. This greatest value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

**SIGN OF THE QUARATIC EXPRESSION**

Let $y = ax^2 + bx + c\ (a \neq 0)$

$$y = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right]$$

Where $D = b^2 - 4ac$ is the Discriminant of the quadratic equation $ax^2 + bx + c = 0$

**Case 1.** $D > 0$ : Suppose the roots of $ax^2 + bx + c = 0$ are $\alpha$ and $\beta$ and $\alpha > \beta$ (say)

$\alpha, \beta$ are real and distinct.

Then $ax^2 + bx + c = a\,(x - \alpha)\,(x - \beta)$

Clearly $(x - \alpha)\,(x - \beta) > 0$ for $x < \beta$ and $x < \alpha$ since both factors are of the same sign and

$(x - \alpha)\,(x - \beta) < 0$ for $\alpha > x > \beta$

For $x = \beta$ or $x = \alpha$, $(x - \alpha)\,(x - \beta) = 0$

$\therefore$ If $a > 0$, then $ax^2 + bx + c > 0$ for all x outside the interval $[\beta, \alpha]$ and is negative for all $x\ (\beta, \alpha)$.

If $a < 0$, then its viceversa.

**Case 2.** $D = 0$ then from (1), $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2$

$\therefore \forall\ x \neq -\frac{b}{2a}$, the quadratic expression takes on values of the same as a ; If $x = -b/2a$ then $ax^2 + bx + c = 0$

If $D = 0$, then

(i) $ax^2 + bx + c > 0$ has a solution any $\left(x \neq -\frac{b}{2a}\right)$ If $a > 0$ and has no solution if $a < 0$

(ii) $ax^2 + bx + c < 0$ has a solution any $\left(x \neq -\frac{b}{2a}\right)$ $a < 0$ and has no solution if $a > 0$

(iii) $ax^2 + bx + c \geq 0$ has any x as a solution if $a > 0$ and the unique solution $x = -\frac{b}{2a}$, if $a < 0$ ;

(iv) $ax^2 + bx + c \leq 0$ has any x as a solution if $a < 0$ and $x = -\frac{b}{2a}$, if $a > 0$ ;

**Case 3.** $D < 0$ from (1)

(i) If $a > 0$, then $ax^2 + bx + c > 0$, for all x ;

(ii) if $a < 0$, then $ax^2 + bx + c < 0$, x for all x.

Ex. The sign of $x^2 + 2x + 3$ is positive for all $x \in R$, because here $b^2 - 4ac = 4 - 12 = -8 < 0$ and $a = 1 > 0$

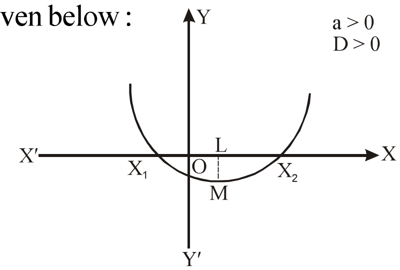
Ex. The sign of $3x^2 + 5x - 8$ is –ve for all $x \in R$ because here $b^2 - 4ac = 25 - 96 = -71 < 0$ and $a = -3 < 0$

## GRAPH OF QUADRATIC EXPRESSION

Consider the expression $y = ax^2 + bx + c$ $(a \neq 0)$ and a, b, c $\in$ R then the graph between x, y is always a parabola if $a > 0$ then the shape of the parabola in concave upward and if $a < 0$ then the shape of the parabola is concave downwards. There is only 6 possible graph of a Quadratic expression as given below :

**Case -I** When $a > 0$

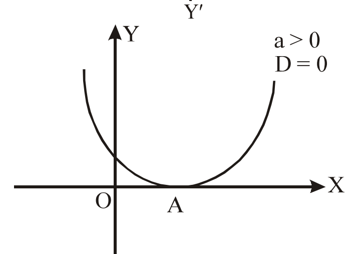
**(i) If D > 0**




Roots are real and different ($x_1$ and $x_2$) Minimum value $LM = \frac{4ac - b^2}{4a}$

at $x = OL = -b/2a$ y is positive for all x out side interval $[x_1, x_2]$ and is negative for all x inside $(x_1, x_2)$

**(ii) If D = 0**

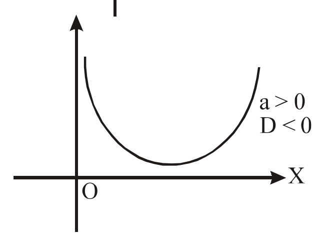



Roots are equal (OA)

Min. value = 0 at $x = OA = -b/2a$

$y > 0$ for all $x \in \left\{R - \frac{-b}{2a}\right\}$

**(iii) If D < 0**

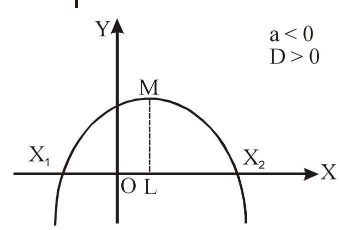



Roots are complex conjugate

y is positive for all $x \in R$

**Case -II** When $a < 0$

**(i) If D > 0**

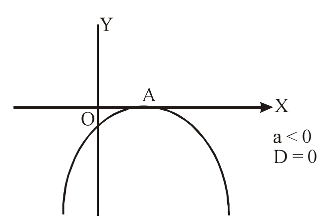



Roots are real and different ($x_1$ and $x_2$)

Max. value = $LM = \frac{4ac - b^2}{4a}$ at $x = OL = -b/2a$

y is positive for all x inside $(x_1, x_2)$ and y is negative for all x outside $[x_1, x_2]$

**(ii) When D = 0**

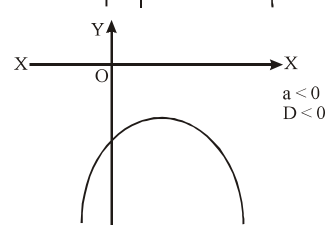



Roots are equal =(OA)

max. value = 0 at $x = OA = -b/2a$

y is negative for all $x \in \left\{R - \frac{-b}{2a}\right\}$

**(iii) When D < 0**




Roots are complex conjugate

y is negative for all $x \in R$

## ADDITIONAL EXAMPLES

**Example 1 :**

If $x_1$ and $x_2$ are non-zero roots of the equation $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ respectively. Prove that $\frac{a}{2}x^2 + bx + c = 0$ has a root between $x_1$ and $x_2$.

**Sol.** If $x_1$ and $x_2$ are roots of $ax^2 + bx + c = 0$ ............. (1)

$-ax^2 + bx + c = 0$ ............. (2)

We have, $-ax_1^2 + bx_1 + c = 0$ ; $-ax_2^2 + bx_2 + c = 0$

Let $f(x) = \frac{a}{2}x^2 + bx + c$ Thus, $f(x_1) = \frac{a}{2}x_1^2 + bx_1 + c = 0$ ............. (3)

$f(x_2) = \frac{a}{2}x_2^2 + bx_2 + c = 0$ ............. (4)

Adding $\frac{1}{2}ax_1^2$ in eq. (3), we get $f(x_1) + \frac{1}{2}ax_1^2 = ax_1^2 + bx_1 + c = 0 \Rightarrow f(x_1) = -\frac{1}{2}ax_1^2$

Subtracting $\frac{3}{2}ax_2^2$ from eq. (4), we get $f(x_2) - \frac{3}{2}ax_2^2 = -ax_2^2 + bx_2 + c = 0 \Rightarrow f(x_1) = -\frac{1}{2}ax_1^2$

Thus, $f(x_1)$ and $f(x_2)$ have opposite signs. Hence $f(x)$ must have a root between x and $x_2$.

**Example 2 :**

Two circles touch externally. The sum of their areas is $130\pi$ sq cm and the distance between their centres is 14 cm. Find the radii of the circles.

**Sol.** Let the radii be $r_1$ and $r_2$ cm.

$\therefore\ r_1 + r_2 = 14 \Rightarrow r_2 = 14 - r_1$.

Sum of the areas = $130\pi$.

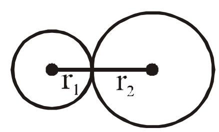
$\therefore\ \pi r_1^2 + \pi r_2^2 = 130\pi \quad \Rightarrow \quad r_1^2 + r_2^2 = 130$

$\Rightarrow r_1^2 + (14 - r_1)^2 = 130 \Rightarrow 2r_1^2 - 28r_1 + 66 = 0$

$\Rightarrow r_1^2 - 14r_1 + 33 = 0 \quad \Rightarrow \quad (r_1 - 11)(r_1 - 3) = 0$

$\Rightarrow r_1 - 11 = 0$ or $r_1 - 3 = 0$

$\Rightarrow r_1 = 11$ or $r_1 = 3$.




$\therefore$ The radii of the two circles are 11 cm and 3 cm.

**Example 3 :**

The speed of a boat in still water is 15 km/h. It can go 30 km upstream and return downstream to the original point in 4 hour 30 min. Find the speed of the stream.

**Sol.** Let the speed of the stream be x km/h.

Then the speed of the boat downstream = (15 + x) and the speed of the boat upstream = (15 – x) km/h

Then, the time taken by the boat to go 30 km upstream = $\frac{\text{distance}}{\text{speed}} = \frac{30}{15 - x}$ h

and the time taken by the boat to return 30 km downstream = $\frac{\text{distance}}{\text{speed}} = \frac{30}{15 + x}$ h

Hence according to the problem, we have $\frac{30}{15-x}+\frac{30}{15+x}$ = 4 hours 30 min.

$\Rightarrow 30\left(\frac{1}{15-x}+\frac{1}{15+x}\right) = 4+\frac{30}{60}$ hours $= 4+\frac{1}{2}=\frac{9}{2}$ hours

$\Rightarrow 30\left[\frac{15+x+15-x}{(15-x)(15+x)}\right]=\frac{9}{2} \Rightarrow 30\left[\frac{30}{(15-x)(15+x)}\right]=\frac{9}{2}$

$\Rightarrow 30\times 30\times\frac{2}{9}=(15-x)(15+x) \quad \Rightarrow 200 = 225 - x^2 \quad \Rightarrow x^2 = 225-200 \quad \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

Now, since the speed of the stream cannot be negative, we reject x = – 5. Hence, we have x = 5.
Thus, the required speed of the stream = 5 km/h.

**Example 4 :**

Determine the value of k such that the quadratic equation $x^2 + 7(3+2k) - 2x(1+3k) = 0$ has equal roots.

**Sol.** The equation is $x^2 - 2x(1+3k) + 7(3+2k) = 0$

Here a = 1, b = – 2(1 + 3k) and c = 7(3 + 2k)

Discriminant $D = b^2 - 4ac = [-2(1+3k]^2 - 4\times 1\times 7(3+2k)$

$= 4(1+9k^2+6k) - 84 - 56k = 3 + 36k^2 + 24k - 84 - 56k$

$= 36k^2 - 32k - 80 = 0.$

For equal roots D = 0

$\therefore\ 36k^2 - 32k - 80 = 0$ or $4(9k^2 - 8k - 20) = 0$ or $9k^2 - 8k - 20 = 0$

On factorization, we get, $9k^2 - 18k + 10k - 2 = 0$

or $9x(k-2) + 10(k-20) = 0$ or $(k-2)(9k+10) = 0$

This implies that, Either $k - 2 = 0$ or $9k + 10 = 0 \Rightarrow k = 2 \quad \Rightarrow k = -10/9$

$\therefore$ The equation has equal roots when k = 2 and –10/9.

**Example 5 :**

If α, β are roots of the equation $x^2 - 3x + 2 = 0$. Construct a quadratic equation whose roots are –α, –β.

**Sol.** The given equation is $x^2 - 3x + 2 = 0$. Here a = 1, b = – 3, c = 2

Discriminant $D = b^2 - 4ac = (-3)^2 - 4\times 1\times 2 = 9 - 8 = 1$

$\therefore$ Roots are $\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{3+\sqrt{1}}{2\times 1} = \frac{3+1}{2} = 2, \qquad \beta = \frac{3-1}{2} = 1.$

Therefore, new equation which we want to find will have roots as –α and –β, i.e. –2 and –1.

Sum of roots = (–2 –1) = –3, product of roots = –2 × –1 = 2

Hence the required quadratic equation is $x^2$ – (sum of roots) x + Products of roots = 0

$x^2 - (-3)x + 2 = 0$ or $x^2 + 3x + 2 = 0.$

**Example 6 :**

Solve the equation : $(x+1)(x+2)(x+3)(x+4) - 8 = 0.$

**Sol.** The given equation is : $(x+1)(x+2)(x+3)(x+4) - 8 = 0$

In such type of equations we combine the factors in such a way that the product of two factors together gives some common polynomial. Rewriting the equation, we have

$(x+1)(x+4)(x+2)(x+3) - 8 = 0$ or $(x^2+5x+4)(x^2+5x+6) - 8 = 0$

Let $x^2 + 5x = y \quad \therefore \quad (y+4)(y+6) - 8 = 0$ or $y^2 + 10y + 24 - 8 = 0$

or $y^2 + 10y + 16 = 0$ or $(y + 8)(y + 2) = 0$

$\therefore$ $y = -8$ or $-2$

But $y = x^2 + 5x$

$\therefore$ $x^2 + 5x = -8$ or $x^2 + 5x = -2$

or $x^2 + 5x + 8 = 0$ or $x^2 + 5x + 2 = 0$

$\therefore x = \dfrac{-5 \pm \sqrt{25-32}}{2}$ $\therefore x = \dfrac{-5 \pm \sqrt{25-8}}{2}$

$= \dfrac{-5 \pm \sqrt{-7}}{2}$ or $x = \dfrac{-5 \pm \sqrt{17}}{2}$ which does not give a real root. Hence $x = \dfrac{-5 \pm \sqrt{17}}{2}$

**Example 7 :**

$\alpha$ and $\beta$ are the roots of quadratic equations $px^2 - qx + r = 0$, from the equation whose roots are $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\alpha}{\beta^2}$

**Sol.** $\alpha$ and $\beta$ are the roots of $px^2 - qx + r = 0$.

Then $\alpha + \beta = \dfrac{q}{p}$ and $\alpha\beta = \dfrac{r}{p}$ We want equation with roots $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\alpha}{\beta^2}$ ,

Sum of the roots 'S' $= \dfrac{\beta}{\alpha^2} + \dfrac{\alpha}{\beta^2} = \dfrac{\beta^3 + \alpha^3}{\alpha^2\beta^2} = \dfrac{(\alpha+\beta)^2 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2}$ as $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$$= \frac{\left(\frac{q}{p}\right)^3 - 3\frac{r}{p}\cdot\frac{q}{p}}{\left(\frac{r}{p}\right)^2} = \frac{\frac{q^3}{p^3} - \frac{3rq}{p^2}}{\frac{r^2}{p^2}} = \frac{q^3 - 3pqr}{p^3} \times \frac{p^2}{r^2} = \frac{q^3 - 3pqr}{r^2 p}$$

Product of roots $P = \dfrac{\beta}{\alpha^2} \cdot \dfrac{\alpha}{\beta^2} = \dfrac{1}{\alpha\beta} = \dfrac{1}{r/p} = \dfrac{p}{r}$

The required equation is $x^2 - (S)\,p + P = 0$ or $x^2 - \left(\dfrac{q^3 - 3pqr}{r^2 p}\right) x + \dfrac{p}{r} = 0$ or $r^2px^2 - (q^3 - 3pqr)x + p^2r = 0$

**Example 8 :**

If one of the roots of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the other root. Also, find the value of p.

**Sol.** The given equation is $2x^2 + px + 4 = 0$.

Here the product of the roots = 4/2 = 2. Since, the one root of the equation = 2.

Therefore the other root = 2/2 = 1 .............(i)

Now the sum of the roots = – p/2. $\because$ One of the roots of the equation is 2

$\therefore$ The other root $= \dfrac{-p}{2} - 2 = \dfrac{-p-4}{2}$ ............(ii)

From (i) and (ii), we have, $\dfrac{-p-4}{2} = 1 \Rightarrow -p - 4 = 2 \Rightarrow -p = 2 + 4 = 6 \Rightarrow p = -6$.

Hence, p = – 6 and the other root = 1.

**Example 9 :**

Solve : $4\left(\frac{7x-1}{x}\right)^2 - 8\left(\frac{7x-1}{x}\right) + 3 = 0.$

**Sol.** Let $\left(\frac{7x-1}{x}\right) = y$ Then $4y^2 - 8y + 3 = 0$

$$\therefore\ y = \frac{-(-8) \pm \sqrt{64-48}}{2 \times 4} = \frac{8 \pm 16}{8} = \frac{8 \pm 4}{8} = \frac{3}{2}, \frac{1}{2}$$

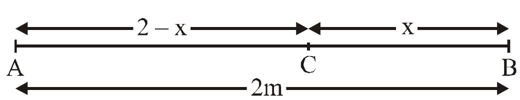
| | |
|---|---|
| $y = \frac{3}{2}$ | $y = \frac{1}{2}$ |
| or $\frac{7x-1}{x} = \frac{3}{2}$ | or $\frac{7x-1}{x} = \frac{1}{2}$ |
| or $14x - 2 = 3x$ | or $14x - 2 = x$ |
| or $11x = 2$ | or $13x = 2$ |
| $\therefore\ x = \frac{2}{11}$ | $\therefore\ x = \frac{2}{13}$ |

$$\therefore\ x = \frac{2}{11}, \frac{2}{13}.$$

**Example 10 :**

A segment AB of 2m length is divided at C into two parts such that $AC^2 = AB \cdot CB$. Find the length of the part CB.

**Sol.** In the figure AB = 2m




Let CB = xm Then AC = (2 – x) m

Now, it is given that

$AC^2 = AB \cdot CB$

$\therefore\ (2-x)^2 = 2x$ or $4 + x^2 - 4x = 2x$ or $x^2 - 6x + 4 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4 \times 1 \times 4}}{2} = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

But $3 + \sqrt{5}$ is not possible as it is more than the total length. Hence CB = $3 - \sqrt{5}$ m.

**Example 11 :**

The product of two successive multiples of 5 is 300. Determine the multiples.

**Sol.** Let the two consecutive multiples of 5 are 5n and 5(n + 1), where $n \in N$

Now by the given condition

$$5n \times 5(n+1) = 300 \Rightarrow 25n(n+1) = 300 \Rightarrow n(n+1) = \frac{300}{25} = 12 \Rightarrow n^2 + n - 12 = 0$$

$\Rightarrow n^2 + 4n - 3n - 12 = 0 \Rightarrow n(n+4) - 3(n+4) = 0 \Rightarrow (n+4)(n-3) = 0$

$\therefore$ Either $n + 4 = 0 \Rightarrow n = -4$ which is rejected as $-4 \notin N$ or $n - 3 = 0 \Rightarrow n = 3$

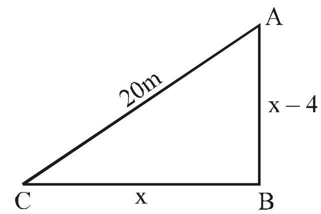
Thus the required numbers are $5 \times 3$ and $5 \times (3 + 1)$ i.e. 15 and 20

**Example 12 :**

The hypotenuse of a right triangle is 20 m. If the difference between the lengths of the other two sides is 4 m, find the other two sides.

**Sol.** In the right triangle ABC, AC = 20 m

Let base = x m . Then altitude = (x – 4) m




Using Pythagoras Theorem, $x^2 + (x-4)^2 = (2)^2$

or $x^2 + x^2 + 16 - 8x = 400$ or $2x^2 - 8x - 384 = 0$

or $x^2 - 4x - 192 = 0$ or $x^2 - 16x + 12x - 192 = 0$

or $x(x-16) + 12(x-16) = 0$ or $(x-16)(x+12) = 0$

Either $x - 16 = 0$ or $x + 12 = 0 \Rightarrow x = 16$ or $x = -12$

This is not permissible. Hence x, i.e. base = 16 m and altitude = 12 cm.

**Example 13 :**

The sides in (cm) of a right triangle are x – 1, x and x + 1. Find the sides of the triangle.

**Sol.** Given three sides of right triangle are x – 1, x and x + 1.

$\because$ Sum of the squares on the sides of a right triangle = square of hypotenuse.

$\therefore (x)^2 + (x-1)^2 = (x+1)^2$ [$\because$ greatest sides being hypotenuse]

$\Rightarrow x^2 + x^2 - 2x + 1 = x^2 + 2x + 1 \Rightarrow x^2 - 2x - 2x + 1 - 1 = 0$

$\Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0$ Either $x - 4 = 0$ or $x = 0$ but $x \neq 0$

$\Rightarrow x = 4$

Thus the sides of a right triangle $x - 1, x, x + 1 \Rightarrow 4 - 1, 4, 4 + 1 \Rightarrow 3, 4, 5$ ; 3 cm. 4 cm. 5 cm.

**Example 14 :**

A farmer wishes to start a 100 sq. m 'rectangular' vegetable garden. Since he has only 30 m barbed wire he fences three sides of the rectangular garden letting his house compound wall act as the fourth side fence. Find the dimensions of his garden.

**Sol.** Let x metres be the breadth and y metres be the length of the rectangular vegetable garden then $x < y$.

Area of garden = xy. But the area of garden = 100 sq. m.

$\Rightarrow xy = 100$ ..........(1)

**Case I.** When the compound wall is parallel to the breadth of the garden.

Length of wire used for fencing three sides = (x + 2y) m.

We are given that $x + 2y = 30 \Rightarrow y = \frac{30-x}{2}$

Setting the value of y in (1), we get $x\left(\frac{30-x}{2}\right) = 100 \Rightarrow 30x - x^2 = 200$

$\Rightarrow x^2 - 30x + 200 = 0 \Rightarrow x^2 - 20x - 10x + 200 = 0$

$\Rightarrow x(x-20) - 10(x-20) = 0 \Rightarrow (x-10)(x-20) = 0$ or $x = 10$ or $x = 20$

When x = 10, from (1), $y = \frac{100}{x} = \frac{100}{10} = 10$. When x = 20, from (1) $y = \frac{100}{20} = 5$

$\therefore x < y$, this case is not possible.

**Case II.** When the compound wall is parallel to the length of the garden.

Length of wire used for fencing three sides = (2x + y)m.

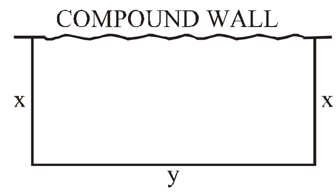
We are given that, $2x + y = 30$ , $y = 30 - 2x$

Setting this value of y in (1), we get $x(30 - 2x) = 100$

$\Rightarrow$ $30x - 2x^2 = 100$ $\Rightarrow$ $x^2 - 15x + 50 = 0$

$\Rightarrow$ $x^2\ 5x - 10x + 5 - 0 = 0$ $\Rightarrow$ $x(x - 5) - 10(x - 5) = 0$

$\Rightarrow$ $(x - 10)(x - 5) = 0$ $\Rightarrow$ $x = 5$ or $10$




When $x = 5$, from (1), $y = \dfrac{100}{x} = \dfrac{100}{5} = 20$ ;

When $x = 10$, from (1) $y = \dfrac{100}{x} = \dfrac{100}{10} = 10$

Since $x < y$, we reject $x = y = 10$. Hence the breadth of vegetable garden = 5 m.
The length of vegetable garden = 20 m. Thus, the dimensions of his garden = 20 m × 5 m.

**Example 15 :**

Solve : $2^{2x} - 3 \,.\, 2^{x+2} + 32 = 0$

**Sol.** $2^{2x} - 3 \,.\, 2^{x+2} + 32 = 0$

$\therefore$ $(2^x)^2 - 3 \,.\, 2^x \,.\, 2^2 + 32 = 0$

Let $2^x = y$ $\therefore$ $y^2 - 12y + 32 = 0$ $\Rightarrow y^2 - 8y - 4y + 32 = 0$ $\Rightarrow y(y - 8) - 4(y - 8) = 0$

$\Rightarrow (y - 4)(y - 8) = 0 \Rightarrow$ $y - 4$ or $y - 8 = 0 \Rightarrow y = 4$ or $y = 8 = 0$

$\Rightarrow$ $2^x = 4 = 2^2$ or $2^x = 8 = 2^3$ $\Rightarrow x = 2$ or $x = 3$

**Example 16 :**

If $\alpha, \beta$ are the roots of the equation $ax^2 + bx + c = 0$, find the values of (a) $\alpha - \beta$ and (b) $\alpha^2 - \beta^2$.

**Sol.** If $\alpha, \beta$ are the roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.

$$\therefore\ \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)} = \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}} = \sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{\sqrt{b^2 - 4ac}}{a}$$

$$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = \left(-\frac{b}{a}\right)\left(\frac{\sqrt{b^2 - 4ac}}{a}\right) = \frac{-b\sqrt{b^2 - 4ac}}{a^2}$$

**Example 17 :**

Solve $\dfrac{x+3}{x+2} = \dfrac{3x-7}{2x-3}$

**Sol.** $(x + 3)(2x - 3) = (x + 2)(3x - 7)$ [By cross multiplication]

$\therefore$ $2x^2 - 3x + 6x - 9 = 3x^2 - 7x + 6x - 14$

$\therefore$ $-x^2 + 4x + 5 = 0$ $\therefore$ $x^2 - 4x - 5 = 0$ $\therefore$ $(x - 5)(x + 1) = 0$ [Factorizing]

$\therefore$ either $x - 5 = 0$ or $x + 1 = 0$ $\therefore$ $x = 5$ or $x = -1$ $\therefore$ Solution set = $\{-1, 5\}$

**Example 18 :**

Solve : $3x^2 - 8x + 2 = 0$. Leave your answer in radical form.

**Sol.** Here, $a = 3$, $b = -8$, $c = 2$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we get $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)} = \dfrac{8 \pm \sqrt{40}}{6} = \dfrac{8 \pm 2\sqrt{10}}{6} = \dfrac{4 \pm \sqrt{10}}{3}$

$\therefore$ Solution set = $\left\{\dfrac{4 + \sqrt{10}}{3}, \dfrac{4 - \sqrt{10}}{3}\right\}$

**Example 19 :**

Solve the equation : $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

**Sol.** Let $5x^2 - 6x = y$. Then, $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1 \Rightarrow \sqrt{y+8} - \sqrt{y-7} = 1$

Squaring both the sides, $(\sqrt{y+8} - \sqrt{y-7})^2 = 1 \Rightarrow y + 8 + y - 7 - 2\sqrt{y^2 + y - 56} = 1$

$\Rightarrow 2y + 1 = 2\sqrt{y^2 + y - 56} + 1 \Rightarrow y = \sqrt{y^2 + y - 56}$

Squaring both the sides, $\Rightarrow y^2 = y^2 + y - 56 \Rightarrow y = 56 \Rightarrow 5x^2 - 6x = 56$ $[\because y = 5x^2 - 6x]$

$\Rightarrow 5x^2 - 6x - 56 = 0 \Rightarrow (5x + 14)(x - 4) = 0 \Rightarrow x = 4, -14/5$

Clearly, both the values satisfy the given equation.

**Example 20 :**

A shopkeeper buys a number of books for Rs 80. If he had bought 4 more books for the same amount, each book would have cost him Rs 1 less. How many books did he buy ?

**Sol.** Suppose the number of books bought = x, Total cost of the books = Rs 80

$\therefore$ Cost of 1 book = Rs 80/x. Number of books bought in 2nd case = x + 4

$\therefore$ Cost of 1 book in this case = Rs $\frac{80}{x+4}$. By the given condition, $\frac{80}{x+4} + 1 = \frac{80}{x}$

$\therefore \frac{80 + x + 4}{x+4} = \frac{80}{x}$ or $84x + x^2 = 80x + 320$

$\therefore x^2 + 4x - 320 = 0$ or $x^2 + 20x - 16x - 320 = 0$

or $x(x + 20) - 16(x + 20) = 0$ or $(x - 16)(x + 20) = 0$

$\therefore$ Either x = 16 or x = – 20 But no. of books can not be negative $\therefore$ No. of books bought = 16.

**Example 21 :**

Solve the equation : $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$

**Sol.** $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$

Dividing both sides of (i) by $x^2$, we get

$$12x^2 - 56x + 89 - \frac{56}{x} + \frac{12}{x^2} = 0 \Rightarrow 12\left(x^2 + \frac{1}{x^2}\right) - 56\left(x + \frac{1}{x}\right) + 89 = 0$$

$$\Rightarrow 12\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 56\left(x + \frac{1}{x}\right) + 89 = 0 \Rightarrow 12\left(x + \frac{1}{x}\right)^2 - 56\left(x + \frac{1}{x}\right) + 65 = 0$$

$\Rightarrow 12y^2 - 56y + 65 = 0$, where $y = x + (1/x) \Rightarrow 12y^2 - 26y - 30y + 65 = 0$

$\Rightarrow (6y - 13)(2y - 5) = 0 \Rightarrow y = 13/6$ or $y = 5/2$

If $y = \frac{13}{6}$, then $x + \frac{1}{x} = \frac{13}{6} \Rightarrow 6x^2 - 13x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0 \Rightarrow x = \frac{2}{3}, \frac{3}{2}$

If $y = \frac{5}{6}$, then $x + \frac{1}{x} = \frac{5}{2} \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow (x - 2)(2x - 1) = 0 \Rightarrow x = 2, \frac{1}{2}$

Hence, the roots of the given equation are 2, 1/2, 2/3, 3/2.

# CONCEPT MAP

## QUADRATIC EQUATIONS

* **Quadratic Polynomials :** If the highest degree of exponent of variable is two in a polynomial it is quadratic polynomial. Number of variables in a quadratic polynomial can be one or more than one.
  The **general form** of a quadratic polynomial in one variable is $ax^2 + bx + c = 0$ where a, b, c are real number and $a \neq 0$.
* **QUADRATIC EQUATION**
  The general form of a quadratic equation in one variable is $ax^2 + bx + c = 0$
  where a, b, c are real number and $a \neq 0$

* **Methods of solving Quadratic equation**

**1.** By factorization
**2.** By using Quadratic formula (completion of square)
The equations, which are unable to solve by factorization are solved by using Quadratic formula.

**Important Results**

**1.** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is the quadratic formula, give two roots or two solutions of a quadratic Equation.

**2.** The expression $b^2 - 4ac$ is called the **discriminant** (D) of the equation, since it determines the nature of roots of the equation.

3 In a Quadratic equation $ax^2 + bx + c = 0$, the roots are
(i) Real and equal if $D = 0$
(ii) Real and Distinct if $D > 0$
(iii) Complex and conjugate if $D < 0$

**4.** If α, β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then

$\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

**5.** If α, β are the roots of a quadratic equation, then the equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

* **To find the roots of a quadratic equation by completing the square**
  (i) First of all make the coefficient of $x^2$ in the quadratic equation 1 by dividing both sides by the coefficient of $x^2$.
  (ii) Write the quadratic equation such that only the terms containing x are on L.H.S. and remaining terms are on R.H.S.
  (iii) Add $\left(\frac{1}{2}\text{coefficient of x}\right)^2$ on both sides.
  (iv) Take the square root of both sides.
  (v) Finally write down the values of x. These values of x will be the requires roots.
* If α is a root of equation $ax^2 + bx + c = 0$, then $a\alpha^2 + b\alpha + c = 0$.
* If α, β are roots of the equation $ax^2 + bx + c = 0$ then equation whose roots are (i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$
  (Replace x by – x)
  (ii) $1/\alpha, 1/\beta \Rightarrow cx^2 + bx + a = 0$
  (Replace x by 1/x)

* **To solve problems based on speed**
  (i) Let the speed be x units.
  (ii) Time taken to cover a distance s will be s/x.
  (iii) Using given conditions form a quadratic equation of the form $ax^2 + bx + c = 0$ and solive it.
* **To solve problems based on age**
  (i) Let the present age of one person be x.
  (ii) Using given conditions form a quadratic equation of the form $ax^2 + bx + c = 0$ and solive it.