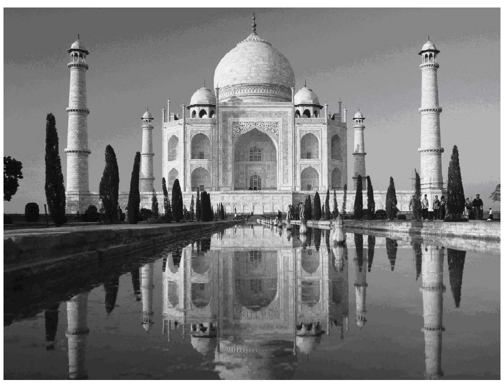
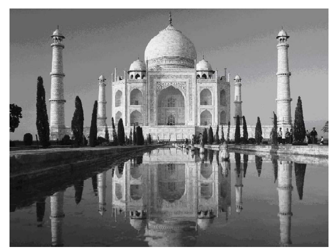
6

TRIANGLES

SIMILAR FIGURES

There are so many examples of similar figures in our daily life. For example, two different sizes of photographs of a person have prepared from the same negative then the shape of figure is same but size is different.
Similarly, when we compare the figures of models of Taj Mahal, Hawa Mahal, and Qutub meenar to their buildings then we observe that the shape of their figures are same but sizes are different.





In this chapter, we shall study the linear figures and specially triangles understand the concept of similarlity we study the following geometrical figures.

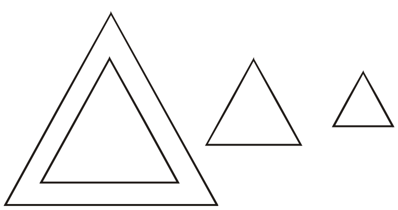


Figure (i)

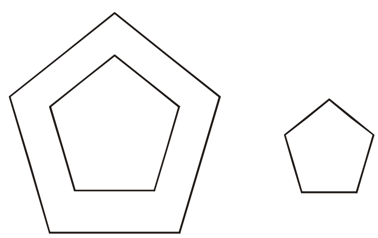


Figure (ii)

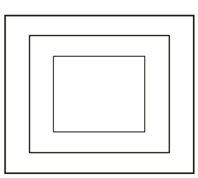


Figure (iii)

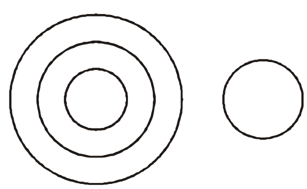
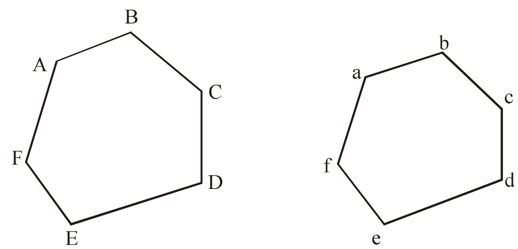


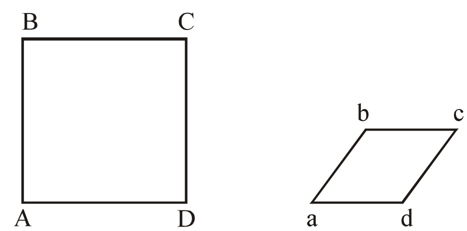
Figure (iv)

In figure (i) there are four equilateral triangles in figure (ii) three regular pentagons, in figure (iii) four squares and in figure (iv) four circles. We observe that their respective shapes are same and sizes are different and if two figures are similar then we can put the smaller one inside the bigger one in such a way that their corresponding sides are parallel. Hence we can say that regular polygons with same number of sides (equilateral triangle, square, regular pentagon) are similar and all circles are also similar figures.

If to change (to increase or to decrease) all sizes of a plane figure in the same ratio (ratio of similarity), then an old and a new figures are called similar ones. For example, a picture and its photograph are similar figures.
In two similar figures any corresponding angles are equal, that is, if points A, B, C, D of one figure correspond to points a, b, c, d of another figure, then $\angle ABC = \angle abc$, $\angle BCD = \angle bcd$ and so on. Two polygons (ABCDEF and abcdef) are similar, if their angles are equal: $\angle A = \angle a$, $\angle B = \angle b$, ..., $\angle F = \angle f$, and sides are proportional : $\frac{AB}{ab} = \frac{BC}{bc} = \frac{CD}{cd} = ... = \frac{FA}{fa}$



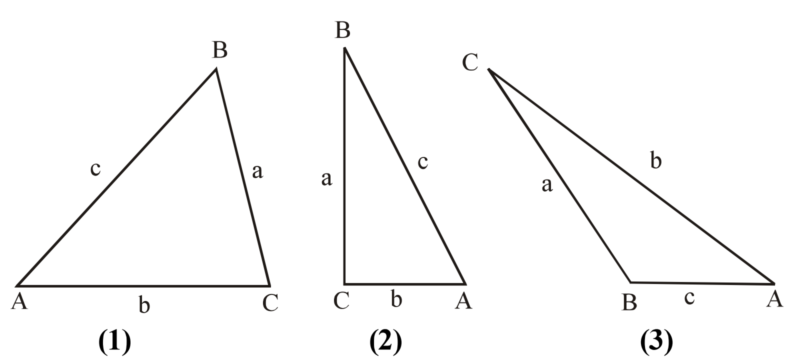
Only proportionality of sides is not enough for similarity of polygons. For example, the square ABCD and the rhombus abcd have proportional sides: each side of the square is twice more than of the rhombus, but the diagonals have not changed proportionally.



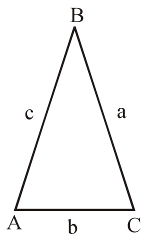
But, for similarity of triangles proportionality of its sides is enough.
Areas of similar figures are proportional to squares of their resembling lines (for instance, sides). So, areas of circles are proportional to ratio of squares of diameters (or radii).

Example 1 :
A round metallic disc by diameter 20 cm weighs 6.4 kg. What is the weight of a round metallic disc by diameter 10 cm ?
Sol. Because the material and the thick of a new disc are the same, the weights of the discs are proportional to their areas, and a ratio of an area of the small disc to an area of the big disc is equal to $(10/20)^2 = 0.25$.
Hence, the weight of the small disc is $6.4 \times 0.25 = 1.6$ kg.

SIMILARITY OF TRIANGLES
Let us first review what you have learnt about triangle in earlier classes.
Triangle is a polygon with three sides (or three angles). Sides of triangle are signed often by small letters, corresponding to designations of opposite vertices, signed by capital letters.



If all the three angles are acute (Fig. 1), then this triangle is an acute-angled triangle; if one of the angles is right (C, Fig 2), then this triangle is a right-angled triangle; sides a, b, forming a right angle, are called legs; side c, opposite to a right angle, called a hypotenuse; if one of the angles is obtuse (B, Fig. 3), then this triangle is an obtuse-angled triangle.



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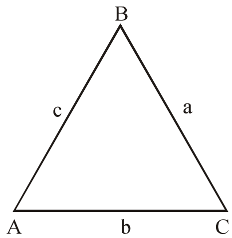

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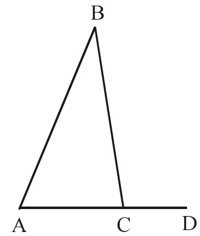


(6)

A triangle ABC is an isosceles triangle (Fig.4), if the two of its sides are equal (a = c); these equal sides are called lateral sides, the third side is called a base of triangle. A triangle ABC is an equilateral triangle (Fig.5), if all of its sides are equal (a=b=c). In general case (a b c) we have a scalene triangle.

Main properties of triangles. In any triangle:

1. An angle, lying opposite the greatest side, is also the greatest angle, and inversely.
2. Angles, lying opposite the equal sides, are also equal, and inversely. In particular, all angles in an equilateral triangle are also equal.
3. A sum of triangle angles is equal to 180 deg.
   From the two last properties it follows, that each angle in an equilateral triangle is equal to 60 deg.
4. Continuing one of the triangle sides (AC , Fig. 6), we receive an exterior angle BCD.
   An exterior angle of a triangle is equal to a sum of interior angles, not supplementary with it: $\angle BCD = \angle A + \angle B$.
5. Any side of a triangle is less than a sum of two other sides and more than their difference
   ($a < b + c$, $a > b - c$; $b < a + c$, $b > a - c$; $c < a + b$, $c > a - b$).

Theorems about congruence of triangles.

Two triangles are congruent, if they have accordingly equal:

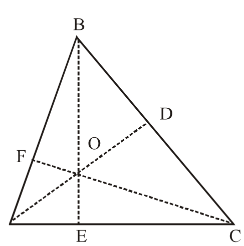
(a) two sides and an angle between them; (b) two angles and a side, adjacent to them; (c) three sides.

Theorems about congruence of right-angled triangles.

Two right-angled triangles are congruent, if one of the following conditions is valid:

1. Their legs are equal;
2. A leg and a hypotenuse of one of triangles are equal to a leg and a hypotenuse of another;
3. A hypotenuse and an acute angle of one of triangles are equal to a hypotenuse and an acute angle of another;
4. A leg and an adjacent acute angle of one of triangles are equal to a leg and an adjacent acute angle of another
5. A leg and an opposite acute angle of one of triangles are equal to a leg and an opposite acute angle of another.

REMARKABLE LINES AND POINTS OF TRIANGLE



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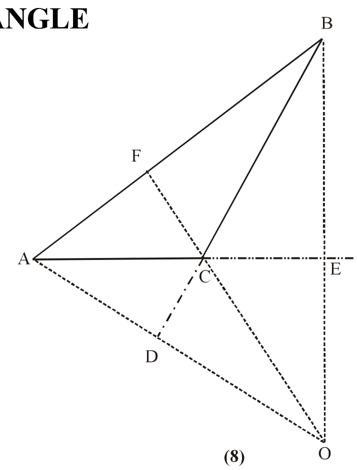


(8)

Altitude (height) of a triangle is a perpendicular, dropped from any vertex to an opposite side (or to its continuation). This side is called a base of triangle in this case. Three heights of triangle always intersect in one point, called an orthocenter of a triangle. An orthocenter of an acute-angled triangle (point O, Fig.7) is placed inside of the triangle; and an orthocenter of an obtuse-angled triangle (point O, Fig.8) – outside of the triangle; an orthocenter of a right-angled triangle coincides with a vertex of the right angle.

Median is a segment, joining any vertex of triangle and a midpoint of the opposite side. Three medians of triangle (AD, BE, CF, Fig.9) intersect in one point O (always lied inside of a triangle), which is a center of gravity of this triangle. This point divides each median by ratio 2 : 1, considering from a vertex.

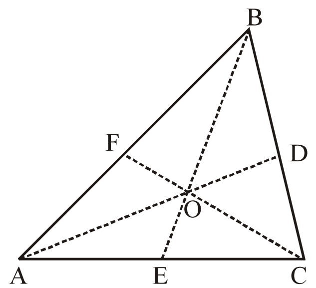


Fig. (9)

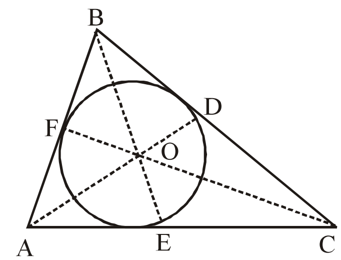


Fig. (10)

Bisector is a segment of the angle bisector, from a vertex to a point of intersection with an opposite side. Three bisectors of a triangle (AD, BE, CF, Fig.10) intersect in the one point (always lied inside of triangle), which is a center of an inscribed circle (see the section "Inscribed and circumscribed polygons").

A bisector divides an opposite side into two parts, proportional to the adjacent sides; for instance, on Fig.10 AE : CE = AB : BC.

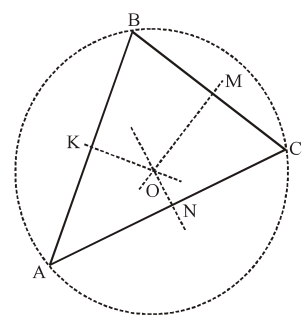


Fig. (11)

Mid-perpendicular is a perpendicular, drawn from a middle point of a segment (side).Three midperpendiculars of a triangle (ABC, Fig.11), each drawn through the middle of its side (points K, M, N, Fig.11), intersect in one point O, which is a center of circle, circumscribed around the triangle (circumcircle).

In an acute-angled triangle this point lies inside of the triangle; in an obtuse-angled triangle - outside of the triangle; in a right-angled triangle - in the middle of the hypotenuse. An orthocenter, a center of gravity, a center of an inscribed circle and a center of a circumcircle coincide only in an equilateral triangle.

SIMILAR TRIANGLES

The mathematical definition for similar triangles is that they both have corresponding angles that are equal, while the lengths of the corresponding sides are in proportion. Similarity criteria of triangles.

Two triangles are similar, if: (1) all their corresponding angles are equal;

(2) all their sides are proportional;

(3) two sides of one triangle are proportional to two sides of another and the angles concluded between these sides are equal.

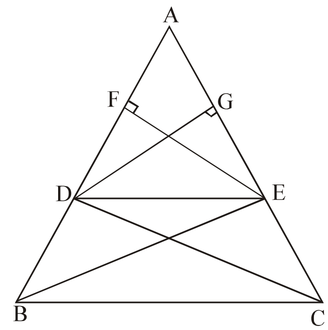
Two right-angled triangles are similar, if

(1) their legs are proportional;

(2) a leg and a hypotenuse of one triangle are proportional to a leg and a hypotenuse of another;

(3) two angles of one triangle are equal to two angles of another.

Theorem 1 : (Basic proportionality theorem)



Statement : In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.

Given : In Δ ABC, DE is drawn parallel to BC and it intercepts AB and AC at D

and E respectively. To prove $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw EF $\perp$ AB and DG $\perp$ AC

Proof : Δ DBE and Δ CDE are on the same base DE and between the same parallels DE and BC.

$\therefore$ Area (Δ BDE) = Area (Δ CDE) .... (1)

Now Δ ADE and Δ BDE have the same vertex D an their bases AD and DB are on the same straight line AB, then the height of both triangles are EF.

$$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} \qquad \text{.......... (2)}$$

Similarly for Δ ADE and Δ CDE

$$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \qquad \text{.......... (3)}$$

Hence from (1), (2) and (3), we get $\frac{AD}{DB} = \frac{AE}{EC}$

Corollary : In Δ ABC, DE is parallel to BC and intersects AB and AC at D and E respectively, then

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ and (ii) $\frac{AB}{AD} = \frac{AC}{AE}$

Proof : (i) By proportionality Theorm $\frac{AD}{DB} = \frac{AE}{EC}$

On adding 1 to both sides $\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1 \Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

(ii) $\frac{AD}{DB} = \frac{AE}{EC}$ (By basic proportionality Theorem)

Taking inverse and then adding 1 to both sides

$$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} \qquad \text{or} \qquad \frac{AD + DB}{AD} = \frac{AE + AC}{AE} \qquad \text{or} \qquad \frac{AB}{AD} = \frac{AC}{AE}$$

Theorem 2 : (Converse of Basic proportionality theorem) :

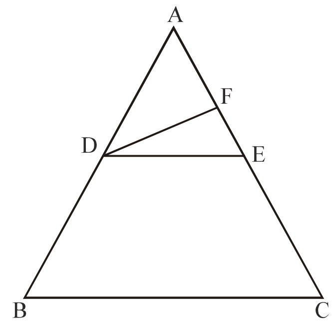
If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third line.

Given : A triangle ABC and line DE intersecting AB in D and AC in E such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : DE || BC

Construction : Draw another line DF through D.

Proof : Let us suppose that DE is not parallel to BC.
Then, through D there must
be some other line DF parallel to BC.
Since DF || BC, By basic proportionality theorem, we get



$\frac{AD}{DB} = \frac{AF}{FC}$ .... (1) But, $\frac{AD}{DB} = \frac{AE}{EC}$ (given) .... (2)

From (1) and (2) $\frac{AF}{FC} = \frac{AE}{EC}$,

On adding 1 to both sides

$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$ or $\frac{AF + FC}{FC} = \frac{AE + EC}{EC}$ or $\frac{AC}{FC} = \frac{AC}{EC}$. Hence, FC = EC

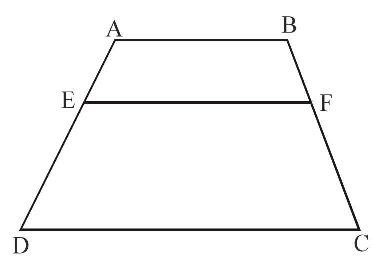
But this is impossible unless the points F and E coincide. i.e., DF and DE are coincident lines.
Hence, DE || BC

SOME IMPORTANT RESULTS AND THEOREMS

1. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
2. In a triangle ABC, if D is a point on BC such that D divides BC in the ratio AB:AC, then AD is the bisector of $\angle$A.
3. The external bisector of an angle of a triangle divides the opposite sides externally in the ratio of the sides containing the angle.
4. The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.
5. The line Joining the mid-points of two sides of a triangle is parallel to the third side.
6. The diagonals of a trapezium divide each other proportionally.
7. If the diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.
8. Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.
9. If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.

Example 2 :

ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB (Fig.).



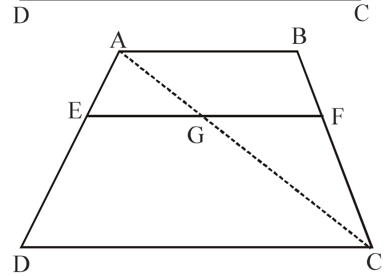
Show that $\frac{AE}{ED} = \frac{BF}{FC}$

Sol. Let us join AC to intersect EF at G (Fig.).

AB || DC and EF || AB (Given)

So, EF || DC (Lines parallel to the same line are parallel to each other)

Now, in ΔADC, EG || DC (As EF || DC)



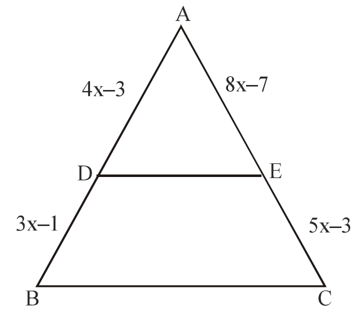
So, $\frac{AE}{ED} = \frac{AG}{GC}$ .... (1)

Similarly, from ΔCAB,

$\frac{CG}{AG} = \frac{CF}{BF}$ i.e., $\frac{AG}{GC} = \frac{BF}{FC}$ .... (2) Therefore, (1) and (2), $\frac{AE}{ED} = \frac{BF}{FC}$

Example 3 :

In a ΔABC. D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, find the value of x.



Sol. In ΔABC, we have DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By basic proportionality Theorem)

$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3} \Rightarrow 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$\Rightarrow 20x^2 - 27x + 9 = 24x^2 - 29x + 7 \Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0 \Rightarrow (2x + 1)(x - 1) = 0 \Rightarrow x = 1$ or $x = -1/2$

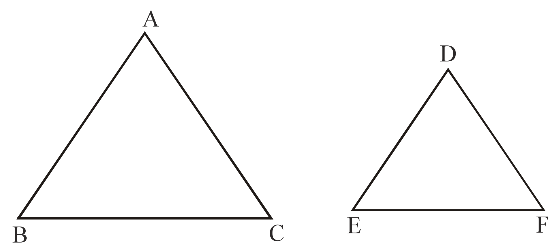
So, the required value of x is 1. [x = –1/2 is neglected as length cannot be negative]

PROPERTIES OF SIMILAR TRIANGLES

Theorem : (Angle-Angle-Angle Similarity)

In two triangles, if the corresponding angles are equal then the triangles are similar.

OR Two equiangular triangles are similar.



Given : ΔABC and ΔDEF are equiangular.

Hence $\angle A = \angle D$, $\angle B = \angle F$ and $\angle C = \angle F$

To prove : $\Delta ABC \sim \Delta DEF$

Proof : Here, ΔABC and ΔDEF are equiangular, then

$\angle A = \angle D$, $\angle B = \angle F$ and $\angle C = \angle F$ .... (1)

Three cases arises for sides AB of ΔABC and DE of ΔDEF :

(i) AB = DE (ii) AB > DE (iii) AB < DE

Case (1) : When AB = DE in ΔABC and ΔDEF

$\angle A = \angle D$ (Given)

AB = DE (Given)

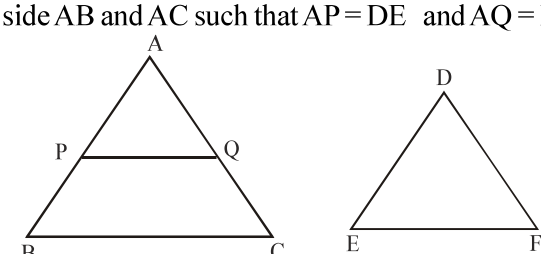
$\angle B = \angle E$ (Given)

Then by ASA rule of congruence, $\Delta ABC \cong \Delta DEF$

Therefore BC = EF, AC = DF, AB = DE $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \Delta ABC \sim \Delta DEF$

Case (2) : When AB > DE

Construction : As in figure, taking the point P and Q on side AB and AC such that AP = DE and AQ = DF.



Proof : In ΔAPQ and ΔDEF

AP = DE (By Construction)

AQ = DF (By Construction)

$\angle A = \angle D$ (Given)

Therefore by Side-Angle-Side Rule for congruency

$\Delta APQ \cong \Delta DEF$

So, $\angle APQ = \angle E$ .... (1)

But $\angle B = \angle E$ (Given) .... (2)

$\Rightarrow \angle APQ = \angle B$, which is corresponding angle

Consequently, PQ || BC

Hence $\frac{AP}{AB} = \frac{AQ}{AC}$ (By basic proportionality theorem .... (3)

Also, $\frac{AP}{DE} = \frac{AQ}{DF}$ (By Construction) .... (4)

From (3) and (4), $\frac{AB}{DE} = \frac{AC}{DF}$ .... (5) Similarly, $\frac{AB}{DE} = \frac{BC}{EF}$ .... (6)

From (5) amd (6), we get, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$. Hence, $\Delta ABC \sim \Delta DEF$

Case (3) : When AB < DE. Proof is the same as for case (2).
Taking points P and Q on the side DE and DF respectively one can prove $\Delta ABC \sim \Delta DEF$

Corollary : (AA similarity) :
If two angles of one triangle are equal to two angle of another triangle, then the triangles are similiar.

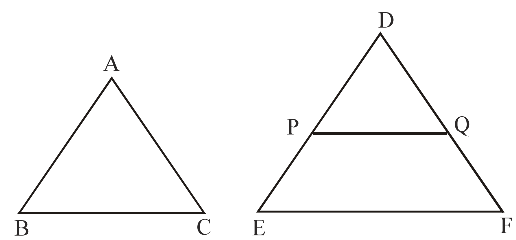
Theorem : (Side-Side-Side Similarity)
If the corresponding sides of two triangles are proportional, then they are similar.

Given : ΔABC and ΔDEF

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on DF such that DP = AB and DQ = AC then join PQ.



Proof : In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ .... (1)

and $\frac{AB}{DP} = \frac{AC}{DQ}$ (By Construction) .... (2)

From (1) and (2), $\frac{DP}{DE} = \frac{DQ}{DF}$

Therefore, by basic Proportionality theorem, $PQ \parallel EF$

So $\angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$ (corresponding angles)

Hence by AA similarity, $\Delta DPQ \sim \Delta DEF$ .... (3)

Hence the corresponding sides of similar triangles ΔDPQ and ΔDEF are proportional.

i.e., $\frac{DP}{DE} = \frac{PQ}{EF} \Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ .... (4)

From (1) and (4), $\frac{PQ}{EF} = \frac{BC}{EF} \Rightarrow PQ = BC$ .... (5)

Now, in ΔABC and ΔDPQ

AB = DQ (By Construction)
AC = DQ (By Construction)
BC = PQ [by (5)]

So by SSS congruence rule

$\Delta ABC \cong \Delta DPQ$ .... (6)

From (3) and (6)

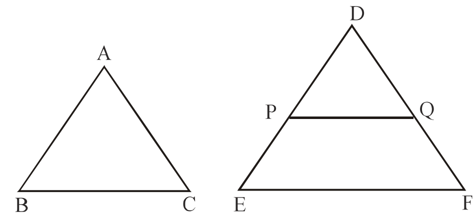
$\Delta ABC \sim \Delta DPQ \sim \Delta DEF \Rightarrow \Delta ABC \sim \Delta DEF$

Theorem : (Side-Angle-Side Similarity)

If one angle of one triangle is equal to an angle of other triangle and if the sides including the angles are proportional, then the two triangles are similar.

Given : Δ ABC and Δ DEF

$\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$



To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on sides DE and DF respectively such that AB = DP and AC = DQ, join PQ.

Proof : In Δ ABC and Δ DPQ

AB = DP (By Construction)

AC = DQ (By Construction)

$\angle A = \angle D$ (Given)

By SAS rule of congruence

$\Delta ABC \cong \Delta DPQ$ .... (1) $\frac{AB}{DE} = \frac{AC}{DF}$ .... (2)

and $\frac{AB}{DP} = \frac{AC}{DQ}$ (By Construction) .... (3)

From (2) and (3), $\frac{DP}{DE} = \frac{PQ}{DF}$

By converse of basic Proportionality theorem, PQ || EF

So $\angle DPQ = \angle E$ and $\angle DQP = \angle F$ (corresponding angles)

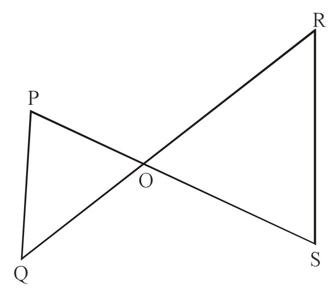
Consequently, by AA similarity, $\Delta DPQ \sim \Delta DEF$ .... (4)

From (1) and (4), we get, $\Delta ABC \sim \Delta DPQ \sim \Delta DEF$

$\Rightarrow \Delta ABC \sim \Delta DEF$

Example 4 :

In Fig., if PQ || RS, prove that $\Delta POQ \sim \Delta SOR$



Sol. PQ || RS (Given)

So, $\angle P = \angle S$ (Alternate angles)

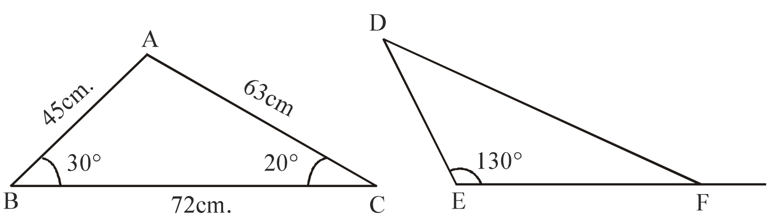
and $\angle Q = \angle R$

Also, $\angle POQ = \angle SOR$ (Vertically opposite angles)

Therefore, $\Delta POQ \sim \Delta SOR$ (AAA similarity criterion)

Example 5 :

In figure, prove that ΔABC and ΔDEF are similar. If EF : DE = 5 : 7, then find DF.



Sol. In Δ ABC, AB = 45 cm, BC = 72 cm, AC = 63 cm and $\angle A = 180° - (20° + 30°) = 130°$ and in Δ DEF, $\angle E = 130°$ (Given)

$\frac{AB}{AC} = \frac{45}{63} = \frac{5}{7}$ and $\frac{EF}{DE} = \frac{5}{7}$ .... (1)

Now for ΔABC and ΔDEF, $\angle A = \angle E = 130°$ and $\frac{AB}{AC} = \frac{EF}{DE}$ [by eq. (1)]

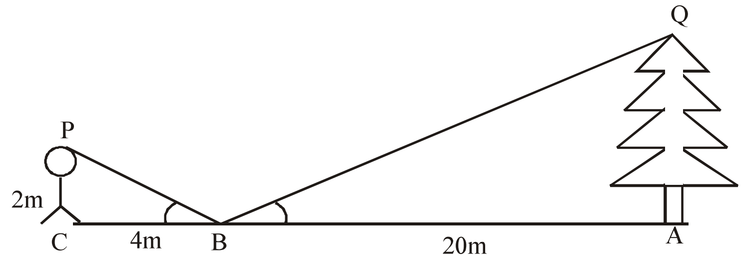
By SAS rule of congruency, $\Delta ABC \sim \Delta EFD \Rightarrow \angle B = \angle F = 30°$ and $\angle D = \angle C$

and $\frac{AB}{EF} = \frac{BC}{DF} \Rightarrow DF = \frac{BC \times EF}{AB} = \frac{72 \times 5}{45} = 8cm$

Example 6 :

A clever outdoorsman whose eye-level is 2 meters above the ground, wishes to find the height of a tree. He places a mirror horizontally on the ground 20 meters from the tree, and finds that if he stands at a point C which is 4 meters from the mirror B, he can see the reflection of the top of the tree. How high is the tree?

Sol.



We make the assumption that the man and the tree are both standing up straight and that the ground is flat.

So $\angle PBC = \angle QBA$ also, the triangles ΔPCB and ΔQAB are similar. Thus,

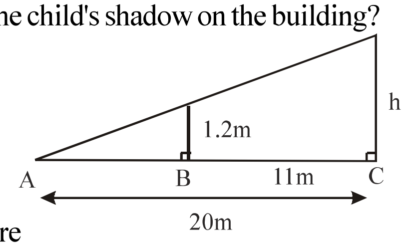
$$\frac{|QA|}{|PC|} = \frac{|AB|}{|CB|} \text{ or } \frac{|QA|}{2} = \frac{20}{4} \text{ or } |QA| = 10$$

Therefore, the height of the tree is 10 meters.

Example 7 :

A child 1.2 meters tall is standing 11 meters away from a tall building. A spotlight on the ground is located 20 meters away from the building and shines on the wall. How tall is the child's shadow on the building?

Sol. Let h be the height of the shadow on the building. Then draw a diagram assuming the ground to be flat, as in the diagram. There are two triangles: one formed by the spotlight and the child, and one formed by the spotlight and the height of the shadow, h. These two triangles share a common angle A at the spotlight. If we assume that the child and the wall of the building are perpendicular to the ground, then the angle formed by the child and the ground (angle C) are both right angles. So the triangles have another pair of equal angles. Therefore, the trianlges are similar.



Now we must look at the lenghts of the corresponding sides. We know that the child must be 9 meters from the spotlight (i.e. 20 m- 11 m). This length in the smaller triangle corresponds to the distance from the spotlight to the building in the larger triangle (i.e. 20 m). The height of the child in the smaller triangle (1.2 m) corresponds to the height of the shadow in the larger triangle (h). Since the triangles are similar, these lengths are in proportion.

Therefore: $\frac{9}{20} = \frac{1.2}{h}$; $9h = 20 (1.2)$

$h = 24/9 = 8/3 = 2.67$ meters

The height of the shadow is 8/3 meters (approx. 2.67 meters).

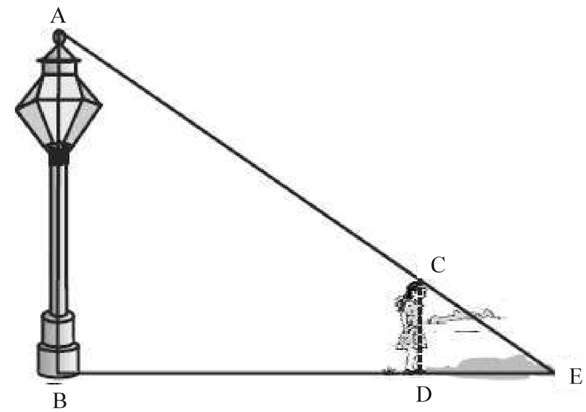
Example 8 :

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Sol. Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post.

From the figure, you can see that DE is the shadow of the girl. Let DE be x metres.

Now, BD = 1.2m × 4 = 4.8m



Note that in ΔABE and $\angle$ CDE,

$\angle B = \angle D$

(Each is of 90° because lamp-post as well as the girl are standing vertical to the ground)

and $\angle E = \angle E$ (same angle)

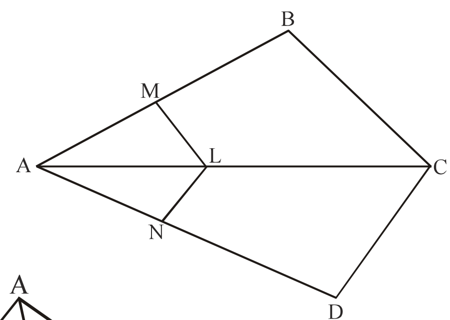
So, $\Delta ABE \sim \Delta CDE$

Therefore, $\dfrac{BE}{DE} = \dfrac{AB}{CD}$

i.e., $\dfrac{4.8+x}{x} = \dfrac{3.6}{0.9}$ $\left(90cm = \dfrac{90}{100}m = 0.9m\right)$

i.e., 4.8 + x = 4x i.e., 3x = 4.8 i.e., x = 1.6

So, the shadow of the girl after walking for 4 seconds is 1.6m long.



SELF CHECK

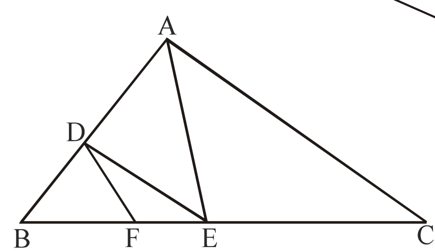
Q.1 E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

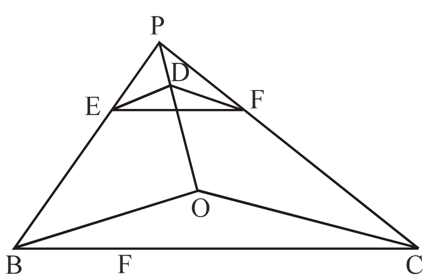
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Q.2 In figure, DE || AC and DF || AE. Prove that $\dfrac{BF}{FE} = \dfrac{BE}{EC}$

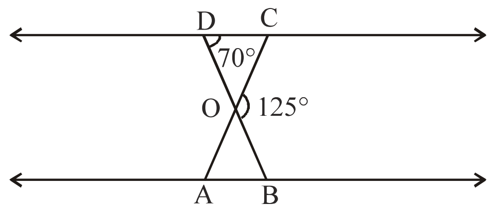


Q.3 In Fig., A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



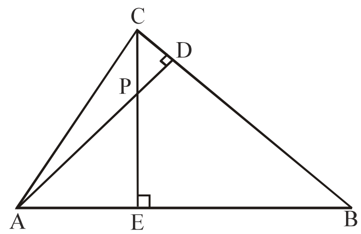
Q.4 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

Q.5 In figure, Δ ODC ~ Δ OBA, $\angle$ BOC = 125° and $\angle$ CDO = 70°. Find $\angle$ DOC, $\angle$ DCO AND $\angle$ OAB.



Q.6 S and T are points on sides PR and QR of Δ PQR such that $\angle$ P = $\angle$ RTS. Show that Δ RPQ ~ Δ RTS.

Q.7 In figure, altitudes AD and CE of Δ ABC intersect each other at the point. Show that :



(i) Δ AEP ~ Δ CDP (ii) Δ ABD ~ Δ CBE (iii) Δ AEP ~ Δ ADB (iv) Δ PDC ~ Δ BEC

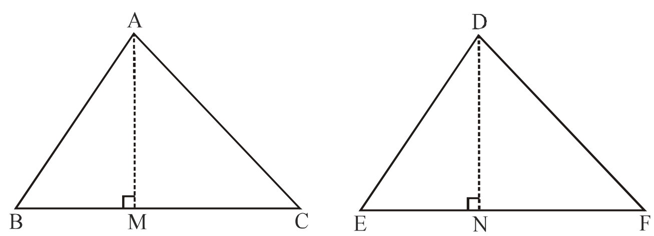
Q.8 D is a point the side BC of a triangle ABC such that $\angle$ ADC = $\angle$ BAC. Show that CA^2 = CB.CD.

ANSWERS

(1) (i) No (ii) Yes (iii) Yes (5) 55°, 55°, 55°

AREA OF SIMILAR TRIANGLE

Theorem : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.



Given : Δ ABC ~ Δ DEF .... (1)

To prove : $\frac{\Delta ABC}{\Delta DEF} = \frac{BC^2}{EF^2} = \frac{AC^2}{DE^2} = \frac{AC^2}{EF^2}$

Construction : Draw AM $\perp$ BC and DN $\perp$ EF

Proof : In Δ AMB and Δ DNE,

$\angle B = \angle E$ [Given]

$\angle M = \angle N = 90°$ [Construction]

$\Rightarrow$ $\Delta AMB \sim \Delta DNE$ [A.A.A.]

$\frac{AM}{DN} = \frac{AB}{DE} = \frac{BC}{EF}$ [From (1) Δ ABC ~ Δ DEF]

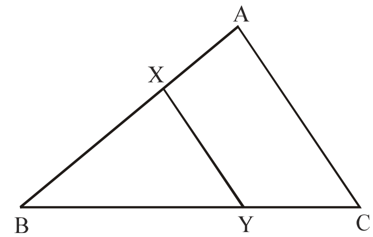
$\frac{\Delta ABC}{\Delta DEF} = \frac{1/2.BC.AM}{1/2.EF.DN} = \frac{BC}{EF}\left(\frac{BC}{EF}\right)$ [Area of a $\Delta = \frac{1}{2}\text{base} \times \text{ht.}$]

$\therefore$ $\frac{\Delta ABC}{\Delta DEF} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$ [$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF}$, given]

Thus, in the above similar triangles ABC and DEF : $\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Example 9 :

In figure, the line segment XY is parallel to side AC of Δ ABC and it divides the triangle into two parts of eqal areas. Find the ratio $\frac{AX}{AB}$.



Sol. We have, XY || AC (Given)

So, $\angle BXY = \angle A$ and $\angle BYX = \angle C$ (Corresponding angles)

Therefore, $\Delta ABC \sim \Delta XBY$ (AA similarity criterion)

So, $\frac{ar(ABC)}{ar(XBY)} = \left(\frac{AB}{XB}\right)^2$ .... (1)

Also, ar (ABC) = 2 ar (XBY) (Given)

So, $\frac{ar(ABC)}{ar(XBY)} = \frac{2}{1}$ .... (2)

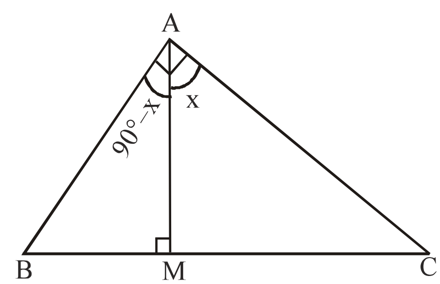
Therefore, from (1) and (2),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}, \text{ i.e., } \frac{AB}{XB} = \frac{\sqrt{2}}{1} \quad \text{or} \quad \frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$\text{or } 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} \quad \text{i.e.,} \quad \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

Example 10 :

From the diagram, prove that $\Delta ABM \sim \Delta AMC \sim \Delta ABC$.



Sol. Let $\angle B = x$

$\angle BAM = 90 - x$ $[\angle x + \angle BAM = 90°]$

$\Rightarrow \angle MAC = x$ $[\angle BAM + \angle MAC = 90°]$

In ΔABM and ΔAMC :

$\angle B = \angle MAC = x$

$\angle M = \angle M = 90°$ [Given]

$\Rightarrow \Delta MBA \sim \Delta MAC$ [A.A.A.]

$$\Rightarrow \frac{\Delta ABM}{\Delta AMC} = \frac{AB^2}{AC^2}$$

In ΔAMB and ΔABC : $\angle B = \angle B$ (Common)

$\angle AMB = \angle BAC = 90°$ [Given]

$\Rightarrow \Delta MBA \sim \Delta ABC$ [A.A.A.]

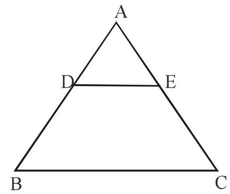
$$\Rightarrow \frac{\Delta AMB}{\Delta ABC} = \frac{AM^2}{AC^2}$$

SELF CHECK

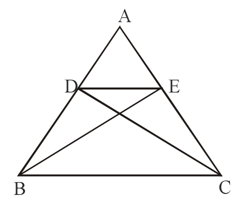
Q.1 If the areas of two similar triangles are equal, prove that they are congruent.

Q.2 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

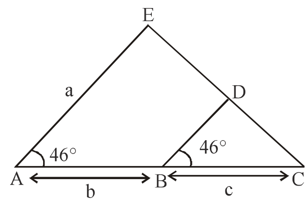
Q.3 In figure, DE || BC and AD : DB = 2 : 3 then find the ratio of the areas of Δ ADE and Δ ABC.



Q.4 In figure, DE || BC and AD : DB = 5 : 4, then find the ratio of the areas of Δ ADE and Δ ABC.



Q.5 In figure, express x in terms of a, b, c.



ANSWERS

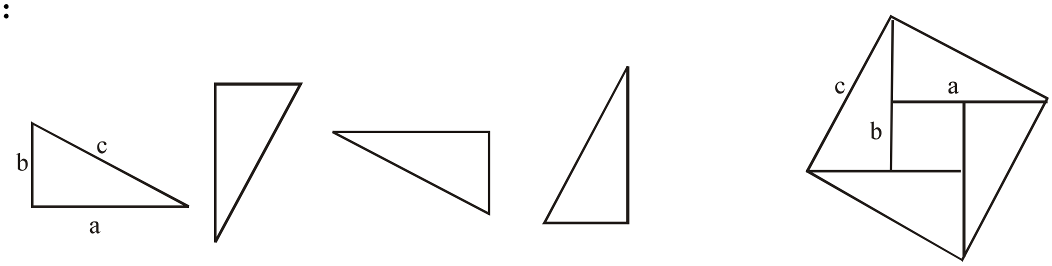
(3) 4 : 25 **(4)** 25 : 81 **(5)** $x = \frac{ac}{b+c}$

THE PYTHAGOREAN THEOREM

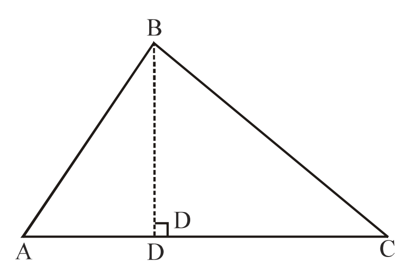
The Pythagorean theorem is about right triangles, that is, triangles, one of whose angles is a 90° angle. The right angle be labeled C and the hypotenuse c, while A and B denote the other two angles, and a and b the sides opposite them, respectively, often called the legs of a right triangle.

The Pythagorean theorem states that the square of the hypotenuse is the sum of the squares of the other two sides, that is, $c^2 = a^2 + b^2$

Proof :



start with four copies of the same triangle. Three of these have been rotated 90°, 180°, and 270°, respectively. Each has area ab/2. Let's put them together without additional rotations so that they form a square with side c. The square has a square hole with the side (a – b). Summing up its area $(a-b)^2$ and 2ab, the area of the four triangles (4 × ab/2), we get , $c^2 = (a-b)^2 + 2ab = a^2 - 2ab + b^2 + 2ab = a^2 + b^2$



Proof on basis of similar triangle :

We are given a right triangle ABC right angled at B.

We need to prove that $AC^2 = AB^2 + BC^2$

Let us draw $BD \perp AC$ (figure)

Now, $\Delta ADB \sim \Delta ABC$

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (sides are proportional)

or $AD.AC = AB^2$ .... (1)

Also, Δ BDC ~ Δ ABC

So, $\frac{CD}{BC} = \frac{BC}{AC}$ or $CD.AC = BC^2$ .... (2)

Adding (1) and (2)

$AD. AC + CD. AC = AB^2 + BC^2$ or $AC (AD + CD) = AB^2 + BC^2$

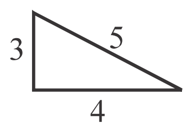
or $AC.AC = AB^2 + BC^2$ or $AC^2 = AB^2 + BC^2$

The above theorem was earlir given by an ancient Indian mathematician Baudhayan (about 800 B.C.) in the following form :

The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e., length and breadth).

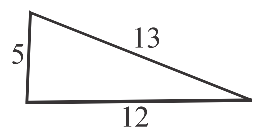
For this reason, this theorem is sometimes also referred to as the Baudhayan theorem.

Examples:



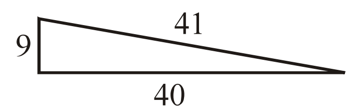
3, 4, 5 Triangle

$3^2 + 4^2 = 5^2$



5, 12, 13 triangle

$5^2 + 12^2 = 13^2$



9, 40, 41 Triangle

$9^2 + 40^2 = 41^2$

Converse of pythagoreous theorem :

Staetment : In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

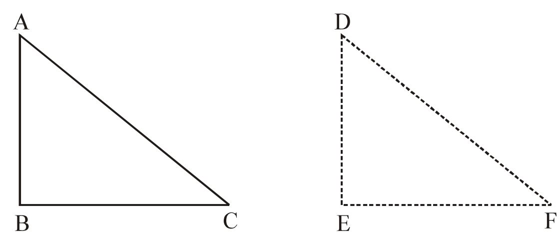
Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$

Construction : Construct a triangle DEF such that DE = AB, EF = BC and $\angle E = 90°$

Proof : In order to prove that $\angle B = 90°$, it is sufficient to show Δ ABC ~ Δ DEF. For this we proceed as follows.

Since Δ DEF is a right-angled triangle with right angle at E.

Therefore, by Pythagoras theorem, we have :



$DF^2 = DE^2 + EF^2$

$\Rightarrow DF^2 = AB^2 + BC^2$ [$\because$ DE = AB and EF = BC (By construction)]

$\Rightarrow DF^2 = AC^2$ [$\because AB^2 + BC^2 = AC^2$ (Given)]

$\Rightarrow DF = AC$ .... (1)

Thus, in Δ ABC and Δ DEF, we have

AB = DE, BC = EF [By construction]

and AC = DF [From eq. (1)]

$\therefore$ $\Delta ABC \cong \Delta DEF$, [By SSS criteria of congruency]

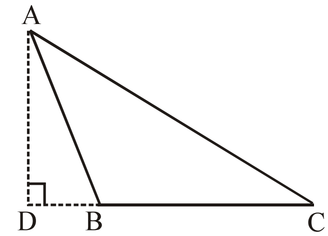
$\Rightarrow \angle B = \angle E = 90°$

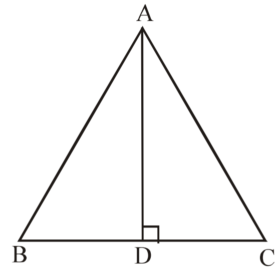
Hence, Δ ABC is a right triangle, right angled at B

Some results deduced from phythagoreous theorem

(1) In the given ABC is an obtuse angled at B. If AD ⊥ CB, then

$AC^2 = AB^2 + BC^2 + 2BC.BD$





(2) In the given figure, if ∠B of Δ ABC is an acute angle and AD ⊥ BC, then

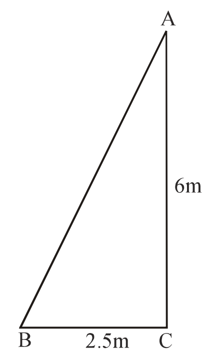
$AC^2 = AB^2 + BC^2 - 2BC.BD$

(3) In any triangle, the sum of the squares of any two sides is equal to twice the square of the third side together with twice the square of the median which bisects the third side.

(4) Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Example 11 :

A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.



Sol. Let AB be the ladder and CA be the wall with the window at A (Fig.).

Also, BC = 2.5 m and CA = 6 m

From Pythagoras Theorem, we have:

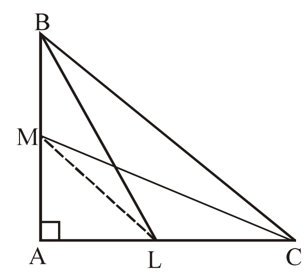
$AB^2 = BC^2 + CA^2 = (2.5)^2 + (6)^2 = 42.25$

So, AB = 6.5

Thus, length of the ladder is 6.5 m.

Example 12 :

BL and CM are medians of Δ ABC right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



Sol. In Δ BAL, $BL^2 = AL^2 + AB^2$ (using Pythagoreous theorem) .... (1)

and, in Δ CAM

$CM^2 = AM^2 + AC^2$ (using Pythagoreous theorem) .... (2)

Adding (1) and (2) and then multiplying by 4, we get

$4(BL^2 + CM^2) = 4(AL^2 + AB^2 + AM^2 + AC^2)$

$= 4\{AL^2 + AM^2 + (AB^2 + AC^2)\}$ [∴ Δ ABC is a right triangle]

$= 4(AL^2 + AM^2 + BC^2)$

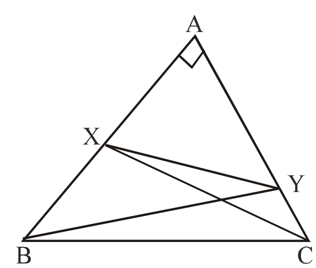
$= 4(ML^2 + BC^2)$ [∴ Δ LAM is a right triangle]

$= 4ML^2 + 4BC^2$ (A line joining mid-points of two parallel to third side and is equal to

$= BC^2 + 4BC^2 = 5BC^2$ half of it, ML = BC/2)

Example 13 :

From the adjoining figure, diagram that $BC^2 + YX^2 = BY^2 + C^2$.



Sol. In Δ ABC : $BC^2 = AB^2 + AC^2$ [∠A = 90°] .... (1)

In Δ AXY : $XY^2 = AX^2 + AY^2$ [∠A = 90°] .... (2)

∴ $BC^2 + XY^2 = AB^2 + AC^2 + AX^2 + AY^2$ [Adding (1) and (2)]

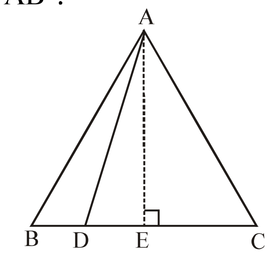
$= (AB^2 + AY^2) + (AC^2 + AX^2)$ [By grouping]

$= BY^2 + CX^2$ [In Δ ABY & Δ ACX]

∴ $BC^2 + YX^2 = BY^2 + CX^2$

Example 14 :

In an equilateral traingle ABC, the side BC is trisected at D. Prove that $9AD^2 = 7AB^2$.



Sol. ABC be an euilateral triangle and D be point on BC such that

$BD = \frac{1}{3}BC$ (Given)

Draw $AE \perp BC$, Join AD

BE = EC (Altitude drown from any vertex of an equilateral triangle bisects the opposite side)

So, $BE = EC = \frac{BC}{2}$

In Δ ABC, $AB^2 = AE^2 + EB^2$ .... (1) ; $AD^2 = AE^2 + ED^2$ .... (2)

From (1) and (2), $AB^2 = AD^2 - ED^2 + EB^2$

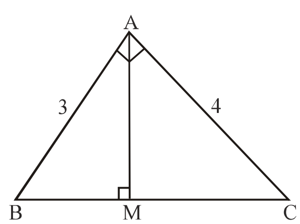
$$AB^2 = AD^2 - \frac{BC^2}{36} + \frac{BC^2}{4} \quad (\because BD + DE = \frac{BC}{2} \Rightarrow \frac{BC}{3} + DE = \frac{BC}{2} \Rightarrow DE = \frac{BC}{6})$$

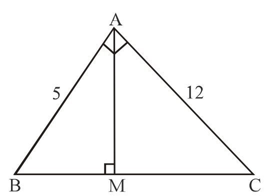
$$AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AD^2 \quad \text{or} \quad \frac{36AB^2 + AB^2 - 9AB^2}{36} = AD^2$$

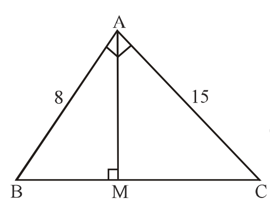
$$\text{or} \quad \frac{28AB^2}{36} = AD^2 \quad \text{or} \quad 7AB^2 = 9AD^2$$

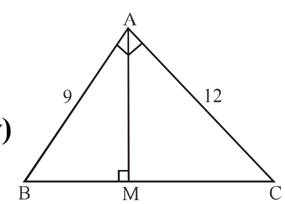
SELF CHECK

Q.1 Calculate the length of the side BC and AM in the followight right –

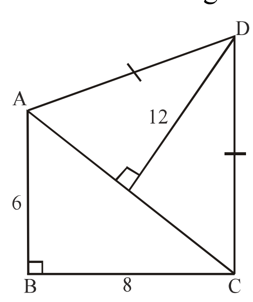
(i) 

(ii) 

(iii) 

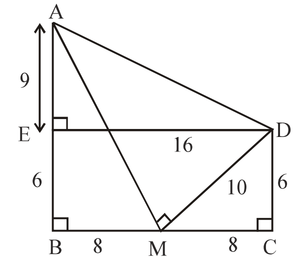
(iv) 

Q.2 From the adjoining figure, calculate the perimeter of the triangle ADC.

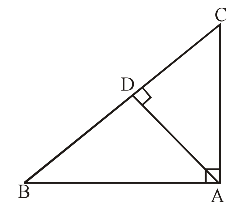


Q.3 From the adjoining figure, calculate

(i) MD (ii) AM (iii) AD



Q.4 In figure $\angle BAC = 90°$, $AD \perp BC$. Prove that $AB^2 = BD^2 + AC^2$.



Q.5 ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Q.6 ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Q.7 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut ?

Q.8 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Q.9 In an equilateral triangle ABC, D is a point on side BC such that BD = (1/3) BC. Prove that $9\,AD^2 = 7\,AB^2$.

ANSWERS

(1) (i) 5, 2.4 **(ii)** 13, $4\frac{8}{13}$ **(iii)** $17, \frac{120}{17}$ **(iv)** 15, 7.2

(2) 36 **(3) (i)** 10 **(ii)** 17 **(iii)** 18.36

(6) $a\sqrt{3}$ **(7)** $6\sqrt{7}$m **(8)** 13m

ADDITIONAL EXAMPLES

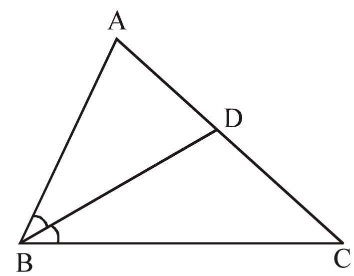
Example 1 :

In ΔABC, $\angle B = 2\angle C$ and the bisector of $\angle B$ intersects AC at D. Prove that $\frac{BD}{DA} = \frac{BC}{BA}$.

Sol. In ΔABC, $\angle ABC = 2\angle ACB$ and $\angle ABD = \angle DBC$

In ΔABC, BD is angle bisector

$\therefore \frac{BA}{BC} = \frac{AD}{DC}$



Now in ΔBDC,

$\angle ABC = 2\angle ACB$

$\Rightarrow \frac{1}{2}\angle ABC = \angle ACB \Rightarrow \angle DBC = \angle DCB \Rightarrow$ BD = DC

Substituting the value of DC = BD in (1), we get $\frac{BA}{BC} = \frac{AD}{BD} \Rightarrow \frac{BD}{AD} = \frac{BC}{BA}$

Example 2 :

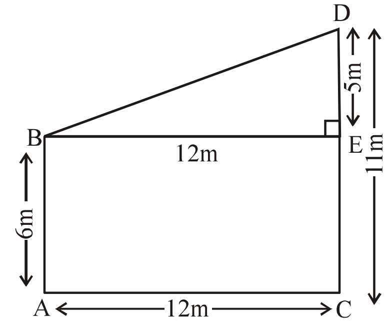
Two poles of heights 6m and 11m stand vertically on the ground. If the distance between their feet is 12m, find the distance between their tops.

Sol. Let AB and CD represent the poles and AC is the distance between their feet.

Let $BE \perp CD$

$\therefore$ BE = AC = 12 m

DE = 11 – 6 = 5m



In rt. ΔBED

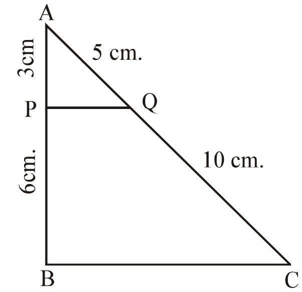
$BD^2 = BE^2 + DE^2$ [Pythagoras' theorem]

$= 12^2 + 5^2 = 144 + 25 = 169$

$\therefore BD = \sqrt{169} = 13$ $\therefore$ Distance between the tops of the poles = 13m

Example 3 :

P and Q are points on side AB and AC respectively of Δ ABC. If AP = 3 cm, PB = 6 cm, AQ = 5 cm. and QC = 10cm, show that BC = 3PQ.



Sol. In Δ ABC, P and Q are the points on AB and AC.

It is given that, AP = 3 cm, PB = 6 cm, AQ = 5 cm and QC = 10 cm.

Now, $\frac{AP}{PB} = \frac{AQ}{QC} \Rightarrow \frac{3}{6} = \frac{5}{10} \Rightarrow \frac{1}{2}$ Hence PQ || BC

In Δ APQ and Δ ABC

$\angle P = \angle B$ [Corresponding angles]

$\angle Q = \angle C$ [Corresponding angles]

$\angle A = \angle A$ [Common angle]

$\Rightarrow$ Δ APQ ~ Δ ABC [AAA Similarity]

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} \Rightarrow \frac{3}{9} = \frac{5}{15} = \frac{PQ}{BC}$ [$\because$ AB = 3 + 6 = 9 cm. , AC = 5 + 10 = 15 cm]

$\Rightarrow \frac{1}{3} = \frac{PQ}{BC} \Rightarrow BC = 3PQ$

Example 4 :

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Prove that $AB^2 = 2AC^2$, if Δ ABC is an isosceles triangle right angled at C.

Sol. Given Δ ABC is an isosceles triangle, right angled at C.

To prove : $AB^2 = 2AC^2$

Proof : $\because$ Δ ABC is an isosceles right angled triangle.

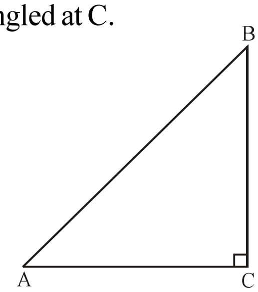
$\therefore$ AC = BC

Using Pythagoras' theorem, we have

$AB^2 = AC^2 + BC^2$

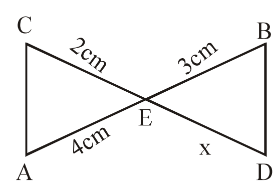
$AB^2 = AC^2 + AC^2$ .... (1) [$\because$ AC = BC (Given)]

$AB^2 = 2AC^2$



Example 5:

Two line segments AB and CD intersect at the point E such that Δ ACE ~ Δ DBE. If AE = 4 cm., BE = 3 cm, CE = 2cm and DE = x, find x.



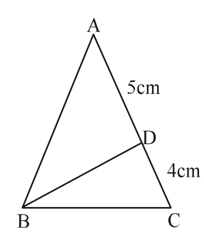
Sol. Δ ACE ~ Δ BDE (Given)

$\therefore \frac{AE}{DE} = \frac{CE}{BE} \quad \therefore \frac{4}{x} = \frac{2}{3}$

$\therefore 2x = 12 \Rightarrow x = 6cm.$

Example 6 :

In Δ ABC, AB = AC and BC = 6 cm. D is a point on side AC such that AD = 5 cm and CD = 4 cm. Show that Δ BCD – Δ ACB and hence find BD.



Sol. Consider Δ ABC and Δ BCD.

It is given that AB = AC, BC = 6 cm, AD = 5 cm and CD = 4 cm.

Then, $\frac{BC}{AC} = \frac{6}{5+4} = \frac{6}{9} = \frac{2}{3}$ and $\frac{CD}{AB} = \frac{4}{6} = \frac{2}{3}$ $\therefore$ $\frac{BC}{AC} = \frac{CD}{CB}$

Also, $\angle BCD = \angle ACB$ (common)

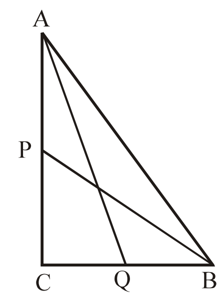
$\therefore$ $\Delta BCD \sim \Delta ACB$ (SAS similarly)

$\therefore$ $\frac{BD}{AB} = \frac{CD}{CB} = \frac{2}{3}$ $\therefore$ $\frac{BD}{AC} = \frac{2}{3}$ ($\because AB = AC$)

$\therefore$ $BD = \frac{2}{3}AC = \frac{2}{3}(5+4) = \frac{2}{3} \times 9 = 6cm.$

Example 7 :

P and Q are the midpoints of the sides CA and CB respectively of a ΔABC in which C is a right angle. Prove that (i) $4\,AQ^2 = 4\,AC^2 + BC^2$ and (ii) $4\,(AQ^2 + BP^2) = 5AB^2$



Sol. Given $\angle C = 90°$, P is the midpoint of AC, Q is the midpoint of BC.

Proof : $AQ^2 = AC^2 + CQ^2$ (Pythagoras's throerem)

$$= AC^2 + \left(\frac{1}{2}BC\right)^2 = AC^2 + \frac{1}{4}BC^2$$

$\therefore$ $4\,AQ^2 = 4\,AC^2 + BC^2$ .... (1)

Similarly, $4\,BP^2 = 4\,BC^2 + AC^2$ .... (2)

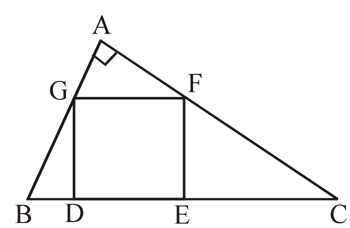
Adding (1) and (2),

$$4\,AQ^2 + 4\,BP^2 = (4AC^2 + BC^2) + (4\,BC^2 + AC^2) = 5\,AC^2 + 5\,BC^2 = 5\,(AC^2 + BC^2)$$

$\Rightarrow$ $4\,(AQ^2 + BP^2) = 5\,AB^2$ (Phythagoras's theorem)

Example 8 :

ΔABC is right-angled at A. DEFG is a square as shown in the figure. Prove that $DE^2 = BD \times EC$.



Sol. Given ΔABC is right-angled at . DEFG is a square

To prove $DE^2 = BD \times EC$.

Proof : In ΔAGF and ΔDBG

$\angle GAF = \angle BDG = 90°$

$\angle AGF = \angle DBG$ (corrsp. angles)

$\therefore$ $\Delta AGF \sim \Delta DBG$ .... (i) (AA similarlity)

In ΔAGF and ΔEFC,

$\angle GAF = \angle CEF = 90°$

$\angle AFG = \angle FCE$ (corrsp. angles

$\therefore$ $\Delta AFG \sim \Delta EFC$ .... (ii) (AA similarity)

From (i) and (ii), $\Delta DBG \sim \Delta EFC$

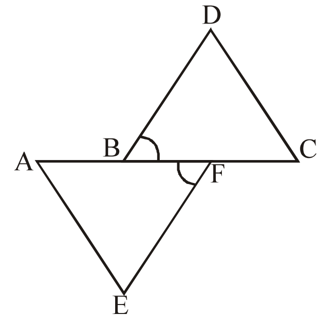
$\therefore$ $\frac{DB}{EF} = \frac{DG}{EC}$ But $EF = DG = DE$ (sides of a square)

$\therefore$ $\frac{DB}{DE} = \frac{DE}{EC}$ $\therefore DE^2 = DB \times EC$

Example 9 :

In the given figure, AB = CF, EF = D, ∠ AFE = ∠ DBC.

Prove that Δ AFE ≅ Δ CBD.



Sol. ∴ AB = CF .... (1)

∴ AB + BF = BF + FC

⇒ AF = CB

In Δ AFE and Δ CBD

AF = CB [From (1)]

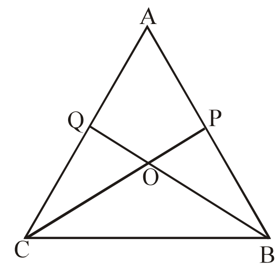
EF = BD [given]

∠ AFE = ∠ DBC [given]

∴ Δ AFE ≅ Δ CBD [By SSA congruence rule]

Example 10 :

P and Q are two points on equal sides AB and AC of an isosceles Δ ABC such that AP = AQ. Prove that PC = QB.



Sol. ∵ AP = AQ and AB = AC

∴ AB – AP = AC – AQ ⇒ PB = QC .... (1)

In Δ s PBC and QBC, we have

PB = QC [by (1)]

BC = BC [Common]

∠ PBC = ∠ PCB [∵ AB = AC]

∴ Δ PBC ≅ Δ QBC [By SAS congruence rule]

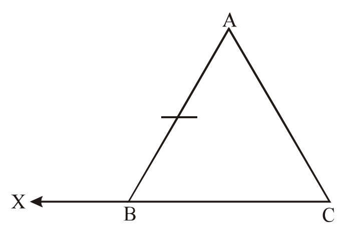
PC = QB

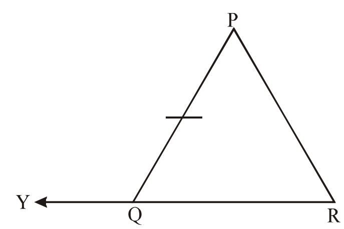
Example 11 :

In ∠ ABC and Δ PQR, AB = PQ, BC = QR, CB and RQ are extended to X and Y respectively.

∠ ABX = ∠ PQY.

Prove that Δ ABC ≅ Δ PQR





Sol. ∵ ∠ ABX = ∠ PQY

180° – ∠ ABC =180° – ∠ PQR

[∵ ∠ ABX + ∠ ABC = 180°,

∠ PQY + ∠ PQR = 180° (linear pair)]

∠ ABC = ∠ PQR .... (1)

In Δ ABC and ∠ PQR

AB = PQ (given)

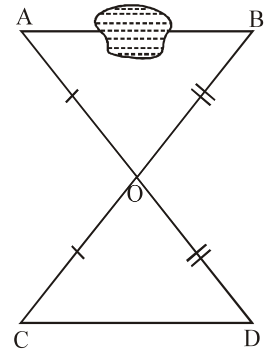
∠ ABC = ∠ PQR [From (1)]

BC = QR (given)

Δ ABC ≅ Δ PQR [By SAS congruence rule]

Example 12 :

Hari wishes to determine the distance between two objects A and B, but there is an obstacle between these two objects which prevent him from making a direct measurement. He devises an ingenious way to overcome this difficulty. First he fixes a pole at a convenient point O so that from O, both A and B are visible. Then he fixes another pole at the point D on the line AO (produced) such that AO = DO. In a similar way he fixes a third pole at the point C on the line BO (produced) such that BO = CO. Then he measures CD which is equal to 170 cm. Prove that the distance between the objects A and B is also 170 cm.



Sol. In Δ s AOB and Δ COD

$OA = OD$ (given)

$\angle AOB = \angle COD$ (vertically opposite angles)

$OB = OC$ (given)

$\therefore \Delta AOB \cong \Delta COD$

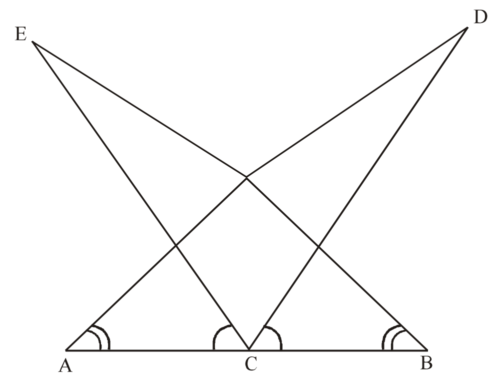
$\Rightarrow AB = CD$ (c.p.c.t)

$\Rightarrow AB = 170$ cm. [$\because CD = 170$ cm]

Example 13 :

In the figure, C is the mid point of AB, $\angle BAD = \angle CBE$, $\angle ECA = \angle DCB$. Prove that

(a) $\Delta DAC \cong \Delta EBC$ (b) $DA = EB$



Sol. In Δ DAC and Δ EBC

$AC = CB$ [$\because$ C is mid point of AB]

$\angle CAD = \angle CBE$ [given]

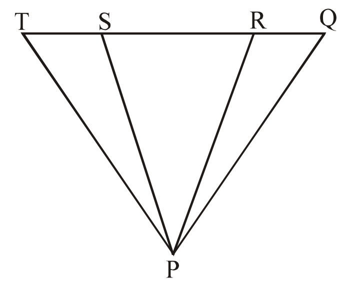
$\angle ACD = \angle BCE$ [$\because \angle ACD = 180° - \angle DCB$, $\angle BCE = 180° - \angle ECA$ and $\angle DCB = \angle ECA$]

$\Delta DAC \cong \Delta EBC$ [By AAS rule]

$DA = AB$ [c.p.c.t.]

Example 14 :

In figure, $PS = PR$, $\angle TPS = \angle QPR$. Prove that $PT = PQ$



Sol. In Δ PRS, $PS = PR$

$\Rightarrow \angle PRS = \angle PSR$ [$\because$ Angle opposite to equal sides are equal]

$\Rightarrow 180° - \angle PRS = 180° - \angle PSR$

$\Rightarrow \angle PRQ = \angle PST$ .... (1)

In Δ PST and Δ PRQ

$\angle TPS = \angle QPR$ (given)

$PS = PR$ (given)

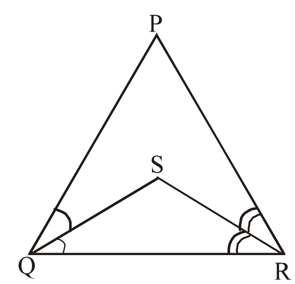
$\angle PST = \angle PRQ$ [From (1)]

$\Delta PST \cong \Delta PRQ$ [By SAS rule]

$\Rightarrow PT = PQ$

Example 15 :

In figure $PQ > PR$. QS and RS are the bisectors of $\angle Q$ and $\angle R$ respectively. Prove that $SQ > SR$.



Sol. In Δ PQR, $PQ > PR$ [Given]

$\Rightarrow \angle PRQ > \angle PQR$ [Angle opposite to greater side of a Δ is greater]

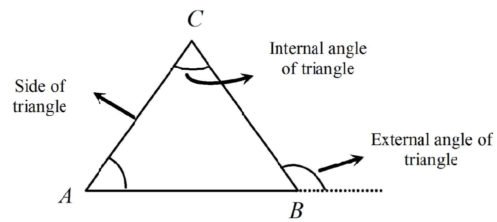
$\Rightarrow \frac{1}{2} \angle PRQ > \frac{1}{2} \angle PQR$

$\Rightarrow \angle SRQ > \angle SQR$ [$\because$ RS and QS are bisector of $\angle PRQ$ and $\angle PQR$ respectively]

$\Rightarrow SQ > SR$ [Side opposite to greater angle is greater]

CONCEPT MAP

* A three side closed figure made by straight lines is a **triangle** or a polygon of three sides is called **triangle.**
Triangle ABC has six elements namely angle ABC (or ∠B), angle ACB (or ∠C) , angle BAC (∠A)and three sides AB, BC and CA.



* **Types of Triangle:**

(A) On the Basis of sides:

(i) A triangle with no sides equal is called **scalene triangle**.
(ii) A triangle with two sides equal is called an **isosceles triangle**.
(iii) A triangle with all sides equal is called an **equilateral triangle**.

(B) On the basis of Angles:

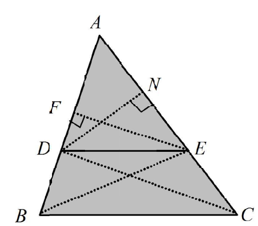
(i) A triangle with all angles acute is called an **acute angled triangle**.
(ii) A triangle with one angle a right angle is called a **right angled triangle**.
(iii) A triangle with one angle an obtuse angle is called an **obtuse angled triangle**.

* **Similarity of triangles**
- Two figures are **similar**, if they are of the **same shape** but not necessarily of the same size**.**
- Where as two congruent figures have the 'same shape' and the 'same size'. Hence two congruent figures are similar but the converse is not necessarily true.
- All regular polygons of same number of sides such as equilateral triangles, squares, hexagons etc. are similar. In particular, all circles are also similar.

TRIANGLES

Two triangles are said to be similar, if
(i) Their corresponding angles are equal
(ii) Their corresponding sides are in the same ratio

* **Basic Proportionality Theorem (Thales Theorem)**: If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

* **Criteria for similarity of two triangles**

1. **AAA Similarity rule:** If in two triangles, the corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar.
Corollary: If two angles of a triangle are equal to two angles of another triangle, then the two triangles are similar. This is referred to as the AA similarity criterion for two triangles.
2. **SSS Similarity rule**: If the corresponding sides of two triangles are proportional their corresponding angles are equal and hence the triangles are similar.