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# SEQUENCES & SERIES

**SEQUENCE**

A sequence is a function whose domain is the set N of natural numbers. Since the domain for every sequence is the set N of natural numbers, therefore a sequence is represented by its range. If $f: N \to R$, then $f(n) = t_n$, $n \in N$ is called a sequence and is denoted by $\{f(1), f(2), f(3), ....\}$ $= \{t_1, t_2, t_3, ....\} = \{t_n\}$

**SERIES**

By adding or substracting the terms of a sequence, we get an expression which is called a series. If $a_1, a_2, a_3, ....a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + ..... + a_n$ is a series.

**PROGRESSION**

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression.

**ARITHMETIC PROGRESSION (A.P.) :**

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called Common difference . If 'a' is the first term and 'd' is the common difference, then an AP can be written as

$a + (a + d) + (a + 2d) + (a + 3d) + ......$

**Note :** If a,b,c are in AP $\Leftrightarrow 2b = a + c$

**General Term of an AP:** General term ($n^{th}$ term) of an AP is given by $T_n = a + (n - 1)\,d$

**Note :**

(i) General term is also denoted by $\ell$ (last term)

(ii) n (No. of terms) always belongs to set of natural numbers.

(iii) Common difference can be zero, + ve or – ve.

(iv) $n^{th}$ term from end is given by

$= T_m - (n - 1)\,d$

$= (m - n + 1)^{th}$ term from beginning where m is total no. of terms.

**Sum of n terms of an AP:** The sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}[2a + (n-1)\,d]$ or $S_n = \frac{n}{2}[a + T_n]$

**Some standard results:**

(i) Sum of first n natural numbers $\Rightarrow \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

(ii) Sum of first n odd natural numbers $\Rightarrow \sum_{r=1}^{n}(2r-1) = n^2$

(iii) Sum of first n even natural numbers $\Rightarrow \sum_{r=1}^{n} 2r = n(n+1)$

(iv) Sum of squares of first n natural numbers

$$\Rightarrow \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

(v) Sum of cubes of first n natural numbers

$$\Rightarrow \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(vi) Sum of fourth powers of first n natural numbers $\left(\sum n^4\right)$

$$\sum n^4 = 1^4 + 2^4 + ..... + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

(vii) If $r^{th}$ term of an A.P.

$T_r = Ar^3 + Br^2 + Cr + D$, then sum of n term of AP is

$$S_n = \sum_{r=1}^{n} T_r = A\sum_{r=1}^{n} r^3 + B\sum_{r=1}^{n} r^2 + C\sum_{r=1}^{n} r + D\sum_{r=1}^{n} 1$$

(viii) If for an A.P. $p^{th}$ term is q, $q^{th}$ term is p then $m^{th}$ term is $= p + q - m$.

**Note :**

**(i)** If sum of n terms $S_n$ is given then general term $T_n = S_n - S_{n-1}$ where $S_{n-1}$ is sum of (n –1) terms of A.P.

**(ii)** Common difference of AP is given by $d = S_2 - 2S_1$ where $S_2$ is sum of first two terms and $S_1$ is sum of first term or first term.

**(iii)** Sum of n terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in n, in such case the common difference is twice the coefficient of $n^2$. i.e. 2A

**(iv)** $n^{th}$ term of an A.P. is of the form An + B i.e. a linear expression in n, in such a case the coefficient of n is the common difference of the A.P. i.e. A

**(v)** If for the different A.P.'s

$$\frac{S_n}{S'_n} = \frac{f_n}{\phi_n} \text{ then } \frac{T_n}{T'_n} = \frac{f(2n-1)}{\phi(2n-1)}$$

**(vi)** If for two A.P.'s $\frac{T_n}{T'_n} = \frac{An+B}{Cn+D}$ then $\frac{S_n}{S'_n} = \frac{A\left(\frac{n+1}{2}\right)+B}{C\left(\frac{n+1}{2}\right)+D}$

## ARITHMETIC MEAN (A.M.)

If three or more than three terms are in A.P., then the numbers, lying between first and last term are known as Arithmetic Means between them. i.e.

The A.M. between the two given quantities a and b is A so that a, A, b are in A.P.

i.e. $A - a = b - A \Rightarrow A = \frac{a+b}{2}$

**Note :** A.M. of any n positive numbers $a_1, a_2 .......... a_n$ is

$$A = \frac{a_1 + a_2 + a_3 + .....a_n}{n}$$

**n AM's between two given numbers :**

If in between two numbers 'a' and 'b' we have to insert n AM $A_1, A_2, .....A_n$ then $a_1, A_1, A_2, A_3 ... A_n$, b will be in A.P. The series consist of (n +2) terms and the last term is b and first term is a.

$$\Rightarrow a + (n + 2 - 1)\, d = b \quad \Rightarrow d = \frac{b-a}{n+1}$$

$A_1 = a + d,\ A_2 = a + 2d, ..... A_n = a + nd$ or $A_n = b - d$

**Note :**

**(i)** Sum of n AM's inserted between a and b is equal to n times the single AM between a and b i.e.

$$\sum_{r=1}^{n} A_r = nA, \quad \text{where } A = \frac{a+b}{2}$$

**(ii)** Between two numbers $\frac{\text{sum of m AM's}}{\text{sum of n AM's}} = \frac{m}{n}$

## SUPPOSITION OF TERMS IN A.P.

**(i)** When no. of terms be odd then we take
three terms as : a – d, a, a + d
five terms are a – 2d, a – d, a, a + d, a + 2d
Here we take middle term as 'a' and common difference as 'd'.

**(ii)** When no. of terms be even then we take
4 term as : a – 3d, a – d, a + d, a + 3d
6 term as = a – 5d, a – 3d, a – d, a + d, a + 3d ,a + 5d
Here we take 'a – d, a + d' as middle terms and common difference as '2d'.

**Note :**

**(i)** If no. of terms in any series is odd then only one middle term is exist which $\left(\frac{n+1}{2}\right)^{th}$ term where n is odd.

**(ii)** If no. of terms in any series is even then middle terms are two which are given by $(n/2)^{th}$ and $\left\{\left(\frac{n}{2}\right)+1\right\}^{th}$ term where n is even.

## SOME PROPERTIES OF A.P.

**(i)** If $t_n$ = an + b, then the series so formed is an A.P.

**(ii)** If $S_n = an^2 + bn + c$, then series so formed is an A.P.

**(iii)** If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.

**(iv)** In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.

**(v)** Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$$a_n = \frac{1}{2}(a_{n-k} + a_{n+k}),\ k < n$$

**(vi)** If in a finite AP, the number of terms be odd, then its middle term is the AM between the first and last term and its sum is equal to the product of middle term and no. of terms

**Example 1 :**

Find the sum of all odd numbers of two digits

**Sol.** Required sum $= 11 + 13 + .... + 99 = \frac{1}{2} . 45\,(11 + 99) = 2475$

**Example 2 :**

If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. then which of the following is in A.P. -

(1) a,b,c  (2) $a^2, b^2, c^2$

(3) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  (4) none of these

**Sol. (3).** $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

$$\therefore \frac{b+c-a}{a} + 2, \frac{c+a-b}{b} + 2, \frac{a+b-c}{c} + 2$$

are in A.P. (adding 2 in each term)

or $\frac{a+b+c}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in A.P.

[ dividing by (a+b+c) in each term]

or $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

**Example 3 :**

Find the sum of n term of series 1.3+3.5+5.7+..........

**Sol.** $T_n = [n^{th}$ term of $1.3.5......] \times [n^{th}$ term of $3.5.7....]$

or $T_n = [1 + (n-1) \times 2] \times [3 + (n-1) \times 2]$

or $T_n = (2n-1)(2n+1) = (4n^2 - 1)$

$$S_n = \sum T_n = \sum (4n^2 - 1) = 4.\sum n^2 . \sum 1$$

$$= \frac{4 \times n(n+1)(2n+1)}{6} - n = \frac{2}{3} n\,(n+1)(2n+1) - n$$

**Example 4 :**

If a,b,c in A.P. and $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ then x, y, z are in

(1) AP (2) GP
(3) HP (4) None of these

**Sol.** **(3).** Here a, b, c in A.P, given

Also $x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$, $z = \frac{1}{1-c}$

Now a, b, c in AP

$\Rightarrow 1-a, 1-b, 1-c$ in A.P.

$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ in HP $\Rightarrow$ x, y, z in HP

**Example 5 :**

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the $p^{th}$, $q^{th}$, $r^{th}$ terms respectively of an A.P. then find the value of $ab(p-q) + bc(q-r) + ca(r-p)$

**Sol.** Let x be the first term and y be the c.d. of corresponding A.P.,

$$\frac{1}{a} = x + (p-1)y \quad ......(1)$$

$$\frac{1}{b} = x + (q-1)y \quad ......(2)$$

$$\frac{1}{c} = x + (r-1)y \quad ......(3)$$

Multiplying (1), (2) and (3) respectively by $abc(q-r)$, $abc(r-p)$, $abc(p-q)$ and then adding we get $bc(q-r) + ca(r-p) + ab(p-q) = 0$

**Example 6 :**

If m arithmetic means are inserted between 1 and 31 so that the ratio of the $7^{th}$ and $(m-1)^{th}$ means is 5 : 9, then find the value of m.

**Sol.** Let the means be $x_1, x_2, .....x_m$ so that $1, x_1, x_2, ....x_m, 31$ is an A.P. of (m + 2) terms.

Now, $31 = T_{m+2} = a + (m+1)d = 1 + (m+1)d$

$\therefore d = \frac{30}{m+1}$ Given : $\frac{x_7}{x_{m-1}} = \frac{5}{9}$

$\therefore \frac{T_8}{T_m} = \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$

$\Rightarrow 9a + 63d = 5a + (5m-5)d$

$\Rightarrow 4.1 = (5m-68)\frac{30}{m+1}$

$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$

$\therefore m = \frac{1022}{73} = 14$

## TRY IT YOURSELF-1

**Q.1** Find the sum of all three-digit natural numbers, which are divisible by 7.

**Q.2** The sum of three numbers in A.P. is –3 and their product is 8. Find the numbers.

**Q.3** The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

**Q.4** If eleven A.M.'s are inserted between 28 and 10, then find the number of integral A.M.'s.

**Q.5** Find four numbers in an A.P. whose sum is 20 and sum of their square is 120.

**Q.6** Let $T_r$ be the $r^{th}$ term of an AP, for r = 1, 2, 3..........If for some positive integers m, n we have $T_m = 1/n$ and $T_n = 1/m$, then $T_{mn}$ equals :

(A) 1/mn (B) $\frac{1}{m} + \frac{1}{n}$
(C) 1 (D) 0

**Q.7** If the sum of the first 2n terms of the A. P. 2, 5, 8, ....is equal to the sum of the first n terms of the A.P. 57, 59, 61.., then n=

(A) 10 (B) 12
(C) 11 (D)13

**Q.8** If the sum of first n terms of an A.P. is $cn^2$, then the sum of squares of these n terms is –

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

**Q.9** Let $a_1, a_2, a_3,....., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then $a_2$ .

## ANSWERS

| | | |
|---|---|---|
| **(1)** 70336 | **(2)** –4, –1, 2 or 2, –1, –4 | **(3)** 852 |
| **(4)** 5 | **(5)** 2, 4, 6, 8 or 8, 6, 4, 2. | **(6)** (C) |
| **(7)** (C) | **(8)** (C) | **(9)** 3 or 9 |

### GEOMETRICAL PROGRESSION (G.P.)

Geometric Progression is defined as a series in which ratio between any two consecutive terms is constant throughout the series. This constant ratio is called as a common ratio. If 'a' is the first term and 'r' is the common ratio, then a GP can be written as **:** a, b,c are in G.P. if $\Leftrightarrow b^2 = ac$

**General term of a G.P. :**

General term ($n^{th}$ term) of a G.P. is given by $T_n = ar^{n-1}$

**Note :**

(i) $n^{th}$ term form end is given by $\frac{T_m}{r^{n-1}}$ where m stands for total no. of terms

(ii) If $a_1, a_2, a_3, ......$ are in GP, then $r = \left(\frac{a_k}{a_p}\right)^{\frac{1}{k-p}}$

**Sum of n terms of a G.P. :**

The sum of first n terms of an A.P. is given by

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a - rT_n}{1-r} \quad \text{when } r < 1$$

or $$S_n = \frac{a(r^n-1)}{r-1} = \frac{rT_n - a}{r-1} \quad \text{when } r > 1$$

and $S_n = nr$ when $r = 1$

**Sum of an infinite G.P. :**

The sum of an infinite G.P. with first term a and common ratio r ($-1 < r < 1$ i.e. $|r| < 1$) is $S_\infty = \frac{a}{1-r}$

**Note :** If $r \geq 1$ then $S_\infty \to \infty$

**GEOMETRICAL MEAN (G.M.) :**

If three or more than three terms are in G.P. then all the numbers lying between first and last term are called Geometrical Means between them i.e. The G.M. between two given quantities a and b is G, so that a, G, b, are in G.P.

i.e. $\frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \sqrt{ab}$

**Note :**

(i) G.M. of any n positive numbers $a_1, a_2, a_3, ........ a_n$ is $(a_1, a_2, a_3,.............. a_n)^{1/n}$.

(ii) If a and b are two numbers of opposite signs, then G.M. between them does not exist.

**n GM's between two given numbers:**

If in between two numbers 'a' and 'b', we have to insert n GM $G_1, G_2, .......... G_n$ then $a_1, G_1, G_2, ........G_n$, b will be in G.P. The series consist of (n +2) terms and the last term is b and first term is a.

$$\Rightarrow ar^{n+2-1} = b \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = ar, G_2 = ar^2 ........G_n = ar^n \text{ or } G_n = b/r$$

**Note :** Product of n GM's inserted between 'a' and 'b' is equal to $n^{th}$ power of the single GM between 'a' and 'b' i.e.

$$\prod_{r=1}^{n} G_r = (G)^n \text{ where } G = \sqrt{ab}$$

**SUPPOSITION OF TERMS IN G. P. :**

(i) When no. of terms be odd,

then we take three terms as a/r, a, ar

5 terms as $\frac{a}{r^2}, \frac{a}{r}$, a, ar, $ar^2$

Here we take middle term as 'a' and common ratio as 'r'.

(ii) When no. of terms be even then we take

4 terms as : $\frac{a}{r^3}, \frac{a}{r}$, ar, $ar^3$

6 terms as : $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}$, ar, $ar^3$, $ar^5$

Here we take $\frac{a}{r}$, ar as middle terms and common ratio as $r^2$

.

(iii) In general, if we have to take (2k + 1) terms in G.P. we take them $\frac{a}{r^k}, \frac{a}{r^{k-1}}, ....., \frac{a}{r}, a, ar, ......, ar^k$

**SOME PROPERTIES OF G.P.**

**(i)** If each term of a G.P. be multiplied or divided by the same non zero quantity, then resulting series is also a G.P.

**(ii)** In an G.P. the product of two terms which are at equidistant from the first and the last term, is constant and is equal to product of first and last term

**(iii)** If each term of a G.P. be raised to the same power, then resulting series is also a G.P.

**(iv)** In a G.P. every term (except first) is GM of its two terms which are at equidistant from it.

i.e. $T_r = \sqrt{T_{r-k} T_{r+k}}$ $k < r$

**(v)** In a finite G.P., the number of terms be odd then its middle term is the G.M. of the first and last term.

**(vi)** If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

**(vii)** If $a_1, a_2, a_3..... a_n$ is a G.P. of non zero, non negative terms, then log a1, log $a_2$,......log $a_n$ is an A.P. and vice-versa

**(viii)** If $a_1, a_2, a_3.......$ and $b_1, b_2, b_3$ .... are two G.P.'s then $a_1b_1, a_2b_2, a_3b_3$.... is also in G.P.

**Example 7 :**

The $n^{th}$ term of a GP is 128 and the sum of its n terms is 255. If its common ratio is 2 then find its first term.

**Sol.** Let a be the first term. Then as given

$T_n = 128$ and $S_n = 255$

But $S_n = \frac{rT_n - a}{r-1} \Rightarrow 255 = \frac{2(128) - a}{2-1} \Rightarrow a = 1$

**Example 8 :**

If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, find the sum of the cubes of the terms

**Sol.** Let the GP be a, ar, $ar^2$ ,........, where $0 < r < 1$.
Then, $a + ar + ar^2 + ...... = 3$ and $a^2 + a^2r^2 + a^2r^4 + ..... = 9/2$.

$$\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}$$

$$\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get $a = 2$

Now, the required sum of the cubes is

$$a^3 + a^3r^3 + a^3r^6 + ........... = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$$

**Example 9 :**

If $A_1, A_2$ be two AM's and $G_1, G_2$ be two GM's between two numbers a and b, then find $\frac{A_1 + A_2}{G_1 G_2}$ .

**Sol.** By the property of AP and GP, we have
$A_1 + A_2 = a + b$ ; $G_1 + G_2 = ab$

$$\therefore \frac{A_1 + A_2}{G_1 G_2} = \frac{a+b}{ab}$$

**Example 10 :**

If x,y,z are in G.P. and $a^x = b^y = c^z$ then-

(1) $\log_b a = \log_a c$ (2) $\log_c b = \log_a c$
(3) $\log_b a = \log_c b$ (4) none of these

**Sol. (3).** x,y,z are in G.P. $\Rightarrow y^2 = xz$ ......(i)

We have, $ax = b^y = c^z = \lambda$ (say)

$\Rightarrow$ x log a = y log b = z log c = log $\lambda$

$$\Rightarrow x = \frac{\log \lambda}{\log a}, y = \frac{\log \lambda}{\log b}, z = \frac{\log \lambda}{\log c}$$

Putting x,y,z in (i), we get

$$\left(\frac{\log \lambda}{\log b}\right)^2 = \frac{\log \lambda}{\log a} \cdot \frac{\log \lambda}{\log c}$$

$(\log b)^2 = \log a . \log c$

or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

**Example 11 :**

If a,b,c,d are in G.P., then $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, $(c^3 + d^3)^{-1}$ are in –

(1) A.P. (2) G.P.
(3) H.P. (4) none of these

**Sol. (2).** Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then,

$$\frac{1}{a^3+b^3} = \frac{1}{a^3(1+r^3)}, \frac{1}{b^3+c^3} = \frac{1}{a^3r^3(1+r^3)}$$

$$\text{and } \frac{1}{c^3+d^3} = \frac{1}{a^3r^3(1+r^3)}$$

Clearly, $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$ and $(c^3 + d^3)^{-1}$ are in G.P. with common ratio $1/r^3$.

**SEQUENCES CONVERTIBLE TO G.P.**

**Example 12 :**

Use infinite series to compute the rational number corresponding to $0.4\overline{23}$.

**Sol.** $x = 0.4\overline{23} = 0.4 + 0.023 + 0.00023 + ......$

$$= 0.4 + \frac{23}{10^3} + \frac{23}{10^5} + \frac{23}{10^7} + ......$$

$$= \frac{4}{10} + \frac{23}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + ......\right)$$

$$= \frac{4}{10} + \frac{23}{10^3}\left(\frac{1}{1-1/100}\right)$$

$$x = \frac{4}{10} + \frac{23}{990} = \frac{419}{990}$$

**Example 13 :**

(a) If $9 + 99 + 999 + .... +$ upto 49 terms $= 10\frac{(10^\lambda - 1)}{\mu} - 49$,

where $\lambda, \mu \in N$ then find the value of $\lambda + \mu$

(b) $0.9 + 0.99 + 0.999 + .......$ up to 51 terms

$$= 51 - \frac{1}{p}\left(1 - \frac{1}{10^q}\right) \text{ where p, q } \in N$$

then find the value of p + q.

**Sol.** (a) $S = 9 + 99 + 999 + ...... +$ upto 49 terms

$S = 10 - 1 + 10^2 - 1 + 10^3 - 1 + ...... + 10^{49} - 1$

$= (10 + 10^2 + 10^3 + ...... 10^{49}) - 49$

$$S = 10 \cdot \left(\frac{10^{49} - 1}{9}\right) - 49$$

$\lambda + \mu = 49 + 9 = 58$

(b) $S = 0.9 + 0.99 + 0.999 + .......$up to 51 terms

$$= \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...... \text{ up to 51 terms}$$

$$= 1 - \frac{1}{10} + 1 - \frac{1}{10^2} + 1 - \frac{1}{10^3} + ... 1 - \frac{1}{10^{51}}$$

$$= 51 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ... + \frac{1}{10^{51}}\right)$$

$$= 51 - \frac{\frac{1}{10}\left(1 - \frac{1}{10^{51}}\right)}{1 - \frac{1}{10}} = 51 - \frac{1}{9}\left(1 - \frac{1}{10^{51}}\right)$$

$\therefore$ p + q = 60

**Example 14 :**

Find the sum

$S = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \ldots$ n terms.

**Sol.** It is easy to observe that

$$\frac{x^2-y^2}{x-y} = x+y,\quad \frac{x^3-y^3}{x-y} = x^2+xy+y^2,$$

$$\frac{x^n-y^n}{x-y} = x^{n-1} + x^{n-2}y + \ldots\ldots + xy^{n-2} + y^{n-1}$$

$$S = \frac{1}{x-y}\ [(x^2-y^2) + (x^3-y^3) + \ldots\ldots \text{ n terms}]$$

$$= \frac{1}{x-y}\left[\frac{x^2(1-x^n)}{1-x} - \frac{y^2(1-y^n)}{1-y}\right].$$

**Example 15 :**

Find the sum of series

$$\frac{3}{19} + \frac{33}{19^2} + \frac{333}{19^3} + \frac{3333}{19^4} + \ldots\infty$$

**Sol.** $S = \frac{3}{19} + \frac{33}{19^2} + \frac{333}{19^3} + \frac{3333}{19^4} + \ldots\infty$

$$S = \frac{3}{9}\left[\frac{9}{19} + \frac{99}{19^2} + \frac{999}{19^3} + \ldots\ldots\infty\right]$$

$$= \frac{3}{9}\left[\frac{10-1}{19} + \frac{10^2-1}{19^2} + \frac{10^3-1}{19^3} + \ldots\ldots\infty\right]$$

$$= \frac{3}{9}\left[\left(\left(\frac{10}{19}\right) + \left(\frac{10}{19}\right)^2 + \left(\frac{10}{19}\right)^3 + \ldots\infty\right) - \left(\frac{1}{19} + \frac{1}{19^2} + \ldots\infty\right)\right]$$

$$S = \frac{3}{9}\left[\frac{10/19}{1-10/19} - \left(\frac{1/19}{1-1/19}\right)\right]$$

$$S = \frac{3}{9}\left[\frac{10/19}{9/19} - \frac{1}{18}\right] = \frac{3}{9}\left[\frac{19}{18}\right] = \frac{19}{54}.$$

**ARITHMETICO-GEOMETRICAL PROGRESSION (A.G.P.) :**

If each term of a progression is the product of the corresponding terms of an A.P. and a G.P., then it is called arithmetic-geometric progression (A.G.P.)

e.g. $a, (a+d)r, (a+2d)r^2, \ldots\ldots$

The general term ($n^{th}$ term) of an A.G.P. is

$T_n = [a + (n-1)d]\, r^{n-1}$

To find the sum of n terms of an A.G.P. we suppose its sum S, multiply both sides by the common ratio of the corresponding G.P. and then subtract as in following way and we get a G.P. whose sum can be easily obtained.

$S_n = a + (a+d)r + (a+2d)r^2 + \ldots.. [a+(n-1)d]\, r^{n-1}$

$rS_n = \qquad ar + (a+d)r^2 + \ldots. + [a+(n-1)d]\, r^n$

After subtraction we get

$S_n(1-r) = a + r.d + r^2.d \ldots.. dr^{n-1} - [a+(n-1)d]\, r^n$

After solving

$$S_n = \frac{a}{1-r} + \frac{r.d\left(1-r^{n-1}\right)}{(1-r)^2} \quad \text{and} \quad S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

**Note :** This is not a standard formula. This is only to understand the procedure for finding the sum of an A.G.P. However formula for sum of infinite terms can be used directly.

**Example 16 :**

If $r^{th}$ term of a series is $(2r+1)\, 2^{-r}$, , then find the sum of its infinite terms

**Sol.** Here $T_r = (2r+1)\, 2^{-r}$ $\quad\therefore$ Series is : $\frac{1}{2}\left[3 + \frac{5}{2} + \frac{7}{2^2} + \ldots..\right]$

Obviously the series in the bracket is Arithmetico-Geometrical series. Therefore by the formula

$$S_\infty = \frac{a}{1-r} + \frac{r}{(1-r)^2} \quad ; \quad S_\infty = \frac{1}{2}\left[\frac{3}{1-\frac{1}{2}} + \frac{2\left(\frac{1}{2}\right)}{\left(1-\frac{1}{2}\right)^2}\right] = 5$$

**Example 17 :**

Find the sum of infinite terms of series $3 + 5.\frac{1}{4} + 7.\frac{1}{4^2} + \ldots.$

**Sol.** Given series is an A.G.P. because each term of series is a product of corresponding term of an A.P. 3,5,7.... and a G.P.

$1, \frac{1}{4}, \frac{1}{4^2}$ .... Let $S = 3 + 5.\frac{1}{4} + 7.\frac{1}{4^2} + \ldots.$

$$\frac{1}{4}S = 3.\frac{1}{4} + 5.\frac{1}{4^2} + \ldots.$$

after subtraction we get

$$\frac{3}{4}s = 3 + 2\left[\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots.\right]$$

$$= 3 + 2.\frac{\frac{1}{4}}{1-1/4} = \frac{11}{3}$$

i.e. $S = \frac{11}{3} \times \frac{4}{3} = \frac{44}{9}$

**Alternate :** Using formula $a = 3, d = 2, r = 1/4$

$$S_\infty = \frac{a}{1-r} + \frac{rd}{(1-r)^2} = \frac{3}{1-\frac{1}{4}} + \frac{\frac{1}{4}\times 2}{\left(1-\frac{1}{4}\right)^2} = \frac{44}{9}$$

## TRY IT YOURSELF-2

**Q.1** Fifth term of a G.P. is 2. Find the product of its first nine terms.

**Q.2** If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

**Q.3** If the continued product of three numbers in a G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

**Q.4** Find the product of three geometric means between 4 and 1/4.

**Q.5** Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then :
(A) $a = 7/4, r = 3/7$ (B) $a = 2, r = 3/8$
(C) $a = 3/2, r = 1/2$ (D) $a = 3, r = 1/4$

**Q.6** Let $\alpha, \beta$ be the roots of $x^2 - x + p = 0$ and $\gamma, \delta$ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G. P., then the integral values of p and q respectively, are
(A) –2, –32 (B) –2, 3
(C) –6, 3 (D) –6, –32

**Q.7** Suppose a, b, c are in A.P. $a^2, b^2, c^2$ are in G.P. If $a < b < c$ and $a + b + c = 3/2$, then the value of a is
(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$
(C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

**Q.8** An infinite G.P. has first term 'x' & sum '5', then x belongs to
(A) $x < -10$ (B) $-10 < x < 0$
(C) $0 < x < 10$ (D) $x > 10$

## ANSWERS

**(1)** 512 **(2)** 1/3
**(3)** 18, 6, 2 or 2, 6, 18. **(4)** 1
**(5)** (D) **(6)** (A)
**(7)** (D) **(8)** (B)

### HARMONIC PROGRESSION (H.P.) :

Harmonic progression is defined as a series in which reciprocal of its terms are in A.P.

The standard from of a H.P. is $\frac{1}{a} + \frac{1}{a+d} + \frac{1}{a+2d} + .....$

**Note :** a, b, c are in H.P. $\Leftrightarrow b = \frac{2ac}{a+c}$

**General Term of a H.P. :**

General term ($n^{th}$term) of a H.P. is given by

$$T_n = \frac{1}{a+(n-1)d}$$

**Note :**

(i) If a,b,c are in H.P. then $\frac{a}{c} = \frac{a-b}{b-c}$

(ii) If a, b are first two terms of an H.P. then

$$t_n = \frac{1}{\frac{1}{a} + (n-1)\left(\frac{1}{b} - \frac{1}{a}\right)}$$

### HARMONIC MEAN (H.M.)

If three or more than three terms are in H.P., then all the numbers lying between first and last term are called Harmonic Means between them. i.e;
The H.M. between two given quantities a and b is H so that a, H, b are in H.P.

i.e. $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

$$\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H} \Rightarrow H = \frac{2ab}{a+b}$$

Also $H = \dfrac{n}{\frac{1}{a_1} + \frac{1}{a_2} + .... + \frac{1}{a_n}} = \dfrac{n}{\sum_{j=1}^{n} \frac{1}{a_j}}$

The harmonic mean of n non zero numbers $a_1, a_2, a_3, ..........., a_n$.

**n H.M's between two given numbers :**

To find n HM's between a, and b we first find n AM's between 1/a and 1/b then their reciprocals will be required HM's.
If terms are given in H.P. then the terms could be picked up in the following way

**For three terms** $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$

**For four terms** $\frac{1}{a-3d}, \frac{1}{a-d}, \frac{1}{a+d}, \frac{1}{a+3d}$

**For five terms** $\frac{1}{a-2d}, \frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$

**Note :** In general, If we are to take (2r + 1) terms in H.P. we take them as

$$\frac{1}{a-rd}, \frac{1}{a-(r-1)d}, ............,$$

$$\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}, ............, \frac{1}{a+rd}$$

**Example 18 :**

If the 3rd , 6th and last term of a H.P. are $\frac{1}{3}, \frac{1}{5}, \frac{3}{203}$, find the number of terms.

**Sol.** $T_3 = \frac{1}{3}, T_6 = \frac{1}{5}, T_n = \frac{3}{203}$

then 3rd, 6th and nth term of A.P. series are 3, 5, $\frac{203}{3}$.

$a + 2d = 3 \;;\; a + 5d = 5$

$d = \frac{2}{3}, a = \frac{5}{3}$

$a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1)\frac{2}{3} = \frac{203}{3}$

$(n-1) = 198/2 \;;\; n = 100.$

**Example 19 :**

If a, b, c are in HP, find the value of $\frac{b+a}{b-a} + \frac{b+c}{b-c}$.

**Sol.** a, b, c are in HP, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$S = \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} + \frac{\frac{1}{c}+\frac{1}{b}}{\frac{1}{c}-\frac{1}{b}}$$

Let $\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c} = d$

$$S = \frac{\left(\frac{1}{a}+\frac{1}{b}\right) - \left(\frac{1}{c}+\frac{1}{b}\right)}{d} = \frac{\left(\frac{1}{a}-\frac{1}{c}\right)}{d} = \frac{2d}{d} = 2$$

**Example 20 :**

If between 1 and 1/31 there are n H.M.'s and ratio of 7th and (n –1)th harmonic means is 9 : 5, then find the value of n

**Sol.** Since there are n A.M.'s between 1 and 31 and the ratio of 7th and (n – 1)th A.M.' is 5 : 9

$$\therefore \frac{1 + 7\left(\frac{31-1}{n+1}\right)}{1 + (n-1)\left(\frac{3n-1}{n+1}\right)} = \frac{5}{9} \Rightarrow \frac{n+211}{31n-29} = \frac{5}{9} \Rightarrow n = 14$$

**Example 21 :**

If $H_1, H_2, H_3$ ........$H_n$ be n harmonic means between a and b then find the value of $\frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-b}$

**Sol.** Here $H_1 = \frac{ab(n+1)}{b(n+1)-(b-a)} = \frac{ab(n+1)}{bn+a}$

Similarly $H_n = \frac{ab(n+1)}{an+b}$ (interchange a and b)

Hence $\frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-b}$

$= \frac{(2n+1)b+a}{b-a} + \frac{(2n+1)a+b}{a-b}$

$= \frac{2nb+b+a-2na-a-b}{b-a} = 2n$

**Example 22 :**

If $\frac{a_2a_3}{a_1a_4} = \frac{a_2+a_3}{a_1+a_4} = 3\left(\frac{a_2-a_3}{a_1-a_4}\right)$ then $a_1, a_2, a_3, a_4$ are in

(1) A.P. (2) G.P.
(3) H.P. (4) None of these

**Sol. (1).** $\frac{a_1+a_4}{a_1a_4} = \frac{a_2+a_3}{a_2a_3}$,

So $\frac{1}{a_4} + \frac{1}{a_1} = \frac{1}{a_3} + \frac{1}{a_2}$ or $\frac{1}{a_4} - \frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}$ ........(1)

Also $\frac{3(a_2-a_3)}{a_2a_3} = \frac{a_1-a_4}{a_1a_4}$;

So $3\left(\frac{1}{a_3} - \frac{1}{a_2}\right) = \frac{1}{a_4} - \frac{1}{a_1}$ ........(2)

Clearly, (1) and (2)

$\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3}$;

So $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$ are in A.P.

**RELATION BETWEEN A.M., G.M. & H.M.**

A, G, H are AM, GM and HM respectively between two numbers 'a' and 'b' then

$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$

**(i)** Consider $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0$

So $A \geq G$
In the same way $G \geq H \Rightarrow A \geq G \geq H$

**(ii)** Consider $A.H. = \frac{a+b}{2} \cdot \frac{2ab}{a+b} = ab = G^2$

$\Rightarrow G^2 = A.H.$

If A, G and H are A.M., G.M. and H.M. of two positive numbers a and b, then (a) $G^2 = AH$, (b) $A \ge G \ge H$

**Note :**

**(i)** For given n positive numbers $a_1, a_2, a_3, ........a_n$, A.M. ≥ G.M. ≥ H.M. The equality holds when the numbers are equal

**(ii)** If sum of the given n positive numbers is constant then that their product will be maximum if numbers are equal.

**Example 23 :**

If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then ab + bc + ca is

(1) less than 1 (2) equal to 1

(3) greater than 1 (4) any real number

**Sol.** **(1).** Since a and b are unequal, $\dfrac{a^2 + b^2}{2} > \sqrt{a^2 b^2}$

(A.M. > G.M. for unequal numbers)

$\Rightarrow a^2 + b^2 > 2ab$

Similarly $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$\Rightarrow$ ab + bc + ca < 1

**Example 24 :**

If $x > 0,\ y > 0, z > 0$ then prove that $(x+y)(y+z)(z+x) \ge 8xyz$

**Sol.** $(x+y)(y+z)(z+x)$

$\dfrac{x+y}{2} \ge \sqrt{xy}$ (A.M. ≥ G.M.)

$\dfrac{y+z}{2} \ge \sqrt{yz}$ ; $\dfrac{z+x}{2} \ge \sqrt{zx}$

$\dfrac{(x+y)(y+z)(z+x)}{8} \ge xyz$

$(x+y)(y+z)(z+x) \ge 8xyz$

**Example 25 :**

Prove that a Δ ABC is equilateral if and only if

$\tan A + \tan B + \tan C = 3\sqrt{3}$

**Sol.** $\dfrac{\tan A + \tan B + \tan C}{3} \ge (\tan A \tan B \tan C)^{1/3}$

since $A + B + C = \pi$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\left(\dfrac{\tan A + \tan B + \tan C}{3}\right) \ge (\tan A + \tan B + \tan C)^{1/3}$

$(\tan A + \tan B + \tan C)^3 \ge 27 (\tan A + \tan B + \tan C)$

$(\tan A + \tan B + \tan C)^2 \ge 27$

$\tan A + \tan B + \tan C \ge 3\sqrt{3}$

**Example 26 :**

If a + b + c = 3 and a, b, c are positive then prove that

$a^2b^3c^2 \le \dfrac{3^{10} \cdot 2^4}{7^7}$

**Sol.** $a + b + c = 3$

We can write it as $\dfrac{a}{2} + \dfrac{a}{2} + \dfrac{b}{3} + \dfrac{b}{3} + \dfrac{b}{3} + \dfrac{c}{2} + \dfrac{c}{2} = 3$

Now A.M. ≥ G.M.

$$\frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}}{7} \ge \left(\frac{a^2}{4}\frac{b^3}{27}\frac{c^2}{4}\right)^{1/7}$$

$$\frac{3}{7} \ge \left(\frac{a^2b^3c^2}{2^4 \times 3^3}\right)^{1/7} \quad ; \quad a^2\, b^3\, c^2 \le \frac{3^{10} \cdot 2^4}{7^7}$$

**Example 27 :**

If a, b, c are positive real number then prove that

$$\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a} \ge \frac{5}{4}$$

**Sol.** $\dfrac{a^3}{4b}, \dfrac{b}{8c^2}, \dfrac{1}{2a}, \dfrac{c}{4a}, \dfrac{c}{4a}$

Applying A.M. ≥ G.M.

$$\frac{\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1}{2a} + \frac{c}{4a} + \frac{c}{4a}}{5} \ge \left(\frac{a^3}{4b} \cdot \frac{b}{8c^2} \cdot \frac{1}{2a} \cdot \left(\frac{c}{4a}\right)^2\right)^{1/5}$$

$$\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1}{2a} + \frac{c}{2a} \ge \frac{5}{4}$$

**METHOD OF DIFFERENCE**

Let $T_1, T_2, T_3 ........ T_n$ are the terms of sequence, then

(i) If $(T_2 - T_1), (T_3 - T_2) .... (T_n - T_{n-1})$ are in A.P. then, the sum of the such series may be obtained by using summation formulae in nth term.

(ii) If $(T_2 - T_1), (T_3 - T_2) .... (T_n - T_{n-1})$ are found in G.P. then, the sum of the such series may be obtained by using summation formulae of a G.P.

**Example 28 :**

Find the sum of the series 3 + 7 + 14 + 24 + 37 + ...... 10 terms,

**Sol.** Here the given series is not A.P., G.P., or H.P.

Let $S = 3 + 7 + 14 + 24 + 37 + ..... + T_n$

$S = 3 + 7 + 14 + 24 + ..... + T_n$

after subtracting

$$0 = 3 + \underbrace{4 + 7 + 10 + 13 + ..........}_{\text{A.P.}} - T_n$$

$$\therefore T_n = 3 + \frac{(n-1)}{2}\,[2(4) + (n-2)\,3] = \frac{1}{2}(3n^2 - n + 4)$$

$$\therefore S_n = \frac{1}{2}[3\Sigma n^2 - \Sigma n + 4n]$$

$$= \frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right]$$

Putting n = 10

$$S_{10}=\frac{1}{2}\left[\frac{10\times11\times21}{2}-\frac{10\times11}{2}+40\right]$$

$$=\frac{1}{2}[1155-55+40]=\frac{1140}{2}=570$$

**Splitting the $n^{th}$ term as a difference of two :**

Here is a series in which each term is composed of the reciprocal of the product of r factors in A.P., the first factor of the several terms being in the same A.P.

**Example 29 :**

Find the sum of n terms of the series and also find $S_\infty$ .

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots$$

**Sol.** $\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots+\frac{1}{n(n+1)(n+2)(n+3)}$.

$$S=\frac{1}{3}\left[\frac{4-1}{1\cdot2\cdot3\cdot4}+\frac{5-2}{2\cdot3\cdot4\cdot5}+\ldots+\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}\right]$$

$$T_1=\frac{1}{3}\left(\frac{1}{1\cdot2\cdot3}-\frac{1}{2\cdot3\cdot4}\right)$$

$$T_2=\left(\frac{1}{2\cdot3\cdot4}-\frac{1}{3\cdot4\cdot5}\right)$$

...... ...... ...... ......

...... ...... ...... ......

$$T_n=\frac{1}{3}\left(\frac{1}{n(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S=T_1+T_2+\ldots..+T_n$$

$$=\frac{1}{3}\left[\frac{1}{1\cdot2\cdot3}-\frac{1}{(n+1)(n+2)(n+3)}\right]$$

$$S_n=\frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}\ ;\ S_\infty=\frac{1}{18}$$

**Example 30 :**

Find sum of n terms ($S_n$) for

$$\frac{1}{2\cdot4}+\frac{1\cdot3}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}+\ldots$$

**Sol.** $S_n=\frac{1}{2\cdot4}+\frac{1\cdot3}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}+\ldots+\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\cdot8\ldots(2n+2)}$

$$T_n=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\cdot8\ldots(2n+2)}$$

$$=\frac{1\cdot3\cdot5\ldots(2n-1)\cdot[(2n+2)-(2n+1)]}{2\cdot4\cdot6\cdot8\ldots(2n+2)}$$

$$T_n=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\cdot8\ldots2n}-\frac{1\cdot3\cdot5\ldots(2n+1)}{2\cdot4\cdot6\cdot8\ldots(2n+2)}$$

$$T_1=\frac{1}{2}-\frac{1\cdot3}{2\cdot4}\ ;\quad T_2=\frac{1\cdot3}{2\cdot4}-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}$$

$$T_n=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots2n}-\frac{1\cdot3\cdot5\ldots(2n+1)}{2\cdot4\cdot6\cdot8\ldots(2n+2)}$$

$$S_n=\frac{1}{2}-\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots(2n+2)}$$

**Factor in A.P**

Here is a series in which each terms is composed of r factor in A.P., the first factor of the several terms being in the same A.P.

**Example 31 :**

$1\cdot2\cdot3\cdot4+2\cdot3\cdot4\cdot5+3\cdot4\cdot5\cdot6+$........ up to n terms

**Sol.** $T_n=n(n+1)(n+2)(n+3)$

$$T_n=\frac{1}{5}n(n+1)(n+2)(n+3)[(n+4)-(n-1)]$$

$$T_n=\frac{n(n+1)(n+2)(n+3)(n+4)}{5}-\frac{(n-1)(n)(n+1)(n+2)(n+3)}{5}$$

$$T_1=\frac{1\cdot2\cdot3\cdot4\cdot5}{5}-0$$

$$T_2=\frac{2\cdot3\cdot4\cdot5\cdot6}{5}-\frac{1\cdot2\cdot3\cdot4\cdot5}{5}$$

$$T_3=\frac{3\cdot4\cdot5\cdot6\cdot7}{5}-\frac{2\cdot3\cdot4\cdot5\cdot6}{5}$$

… …… …… …… …… …

… …… …… …… …… …

$$T_n=\frac{n(n+1)(n+2)(n+3)(n+4)}{5}-\frac{(n-1)(n)(n+1)(n+2)(n+3)}{5}$$

$$S_n=T_1+T_2+T_3+\ldots+T_n=\frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$

## SOME IMPORTANT RESULTS

**(i)** If number of terms is an A.P./G.P./H.P. is odd then its mid term is the A.M/G.M./H.M. between the first and last number.

**(ii)** If the number of terms in an A.P./G.P./H.P. is even then A.M./G.M./H.M. of its two middle terms is equal to the A.M./G.M./H.M. between the first and last numbers.

**(iii)** a,b,c are in A.P. and H.P. ⇒ a,b,c are in G.P.

**(iv)** If a,b,c are in A.P. then $\frac{1}{bc},\frac{1}{ac},\frac{1}{ab}$ are in A.P.

(v) If $a^2, b^2, c^2$ are in A.P. then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

(vi) If a,b,c are in G.P. then $a^2, b^2, c^2$ are in G.P.

(vii) If a,b,c,d are in G.P. then a + b, b + c, c + d are in G.P.

(viii) If a,b,c are in H.P. then $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

## TRY IT YOURSELF-3

**Q.1** The $8^{th}$ and $14^{th}$ term of HP are 1/2 and 1/3, respectively. Find its $20^{th}$ term. Also, find its general term.

**Q.2** If first three terms of the sequence 1/16, a, b, 1/6 are in geometric series and last three terms are in harmonic series, then find the values of a and b.

**Q.3** If H is the harmonic mean between P and Q, then find the value of $\frac{H}{P}+\frac{H}{Q}$.

**Q.4** If the A.M. between two numbers exceeds their G.M. by 2 and the G.M. exceeds their H.M. by 8/5, find the numbers.

**Q.5** Find the sum to n terms of the series 3 + 15 + 35 + 63 + ....

**Q.6** If the sum to infinity of the series

$$3+(3+d)\frac{1}{4}+(3+2d)\frac{1}{4^2}+....\infty \text{ is } \frac{44}{9}, \text{ then find d.}$$

**Q.7** If a, b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation

(A) $0 < M \le 1$ (B) $1 \le M \le 2$

(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

**Q.8** The harmonic mean of the roots of the equation

$\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is

(A) 2 (B) 4

(C) 6 (D) 8

**Q.9** Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd and bcd are

(A) Not in A.P./G.P./H.P. (B) in A.P.

(C) in G.P. (D) H.P.

## ANSWERS

**(1)** 1/14, $\frac{6}{n+4}$ **(2)** $b=\frac{2a}{6a+1}$, $(4a+1)(12a-1)=0$

**(3)** 2 **(4)** a = 16 and b = 4

**(5)** $\frac{n}{3}(4n^2+6n-1)$ **(6)** 2 **(7)** (A)

**(8)** (B) **(9)** (D)

# ADDITIONAL EXAMPLES

**Example 1 :**

If $S_n$ denotes the sum of n terms of a G.P. whose first term is a and the common ratio r, then find the sum of $S_1 + S_3 + S_5 + ..... + S_{2n-1}$

**Sol.** We have $S_n = \frac{a(1-r^n)}{1-r}$ $\therefore$ $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

Putting 1, 2, 3,........, n for n is it and summing up we $S_1 + S_3 + S_5 + .......... + S_{2n-1}$

$$=\frac{a}{1-r}[(1+1+....n\text{ term})-(r+r^3+r^5+.....n\text{ term})]$$

$$=\frac{a}{1-r}\left[n-\frac{r\{1-(r^2)^n\}}{1-r^2}\right]=\frac{a}{1-r}\left[n-r.\frac{1-r^{2n}}{1-r^2}\right]$$

**Example 2 :**

Find the maximum sum of the series

$$20+19\frac{1}{3}+18\frac{2}{3}+18+.....$$

**Sol.** The given series is arithmetic whose first term = 20, common difference = – 2/3

As the common difference is negative, the terms will become negative after some stage. So the sum is maximum if only positive terms are added.

Now $t_n = 20 + (n-1)(-2/3) \ge 0$ if $60 - 2(n-1) \ge 0$ or $62 \ge 2n$ or $31 \ge n$

$\therefore$ The first 31 terms are non-negative

$\therefore$ Maximum sum

$$=S_{31}=\frac{31}{2}\left\{2\times 20+(31-1)\left(-\frac{2}{3}\right)\right\}=\frac{31}{2}\{40-20\}=310$$

**Example 3 :**

It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$ then find the value of $\sum_{r=1}^{\infty}\frac{1}{r^2}$

**Sol.** Here $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}.....\infty=\frac{\pi^2}{8}$

Let $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....\infty = x$

Then $x=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....\infty$

$$=\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....\infty\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+.....\infty\right)$$

$$=\frac{\pi^2}{8}+\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....\infty\right)=\frac{\pi^2}{8}+\frac{1}{4}x$$

**Example 4 :**

If $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$ then find the value of a and b.

**Sol.** $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = \sum_{k=1}^{n}\frac{k(k+1)(2k+1)}{6} = \frac{1}{6}\sum_{k=1}^{n}(2k^3+3k^2+k)$

$$= \frac{1}{3}\cdot\left\{\frac{n(n+1)^2}{2}\right\}^2 + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$

$$a = \text{coefficient of } n^4 = \frac{1}{3}\cdot\frac{1}{4},\ b = \text{coefficient of } n^3 = \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{6}$$

**Example 5 :**

$(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ 10 brackets is

(A) 4960 (B) 4860

(C) 5060 (d) none of these

**Sol. (A).** Sum $= (3^3 + 5^3 + 7^3 + \ldots\ldots$ to 10 terms$)$

$- 2(2^3 + 4^3 + 6^3 + \ldots\ldots$ to 10 terms $)$

$= (2^3 + 3^3 + 4^3 + 5^3 + \ldots\ldots$ to 20 terms$)$

$- 2(2^3 + 4^3 + 6^3 + \ldots\ldots$ to 10 terms $)$

$= (1^3 + 2^3 + 3^3 + \ldots\ldots$ to 21 terms$)$

$- 1^3 - 2.2^3(1^3 + 2^3 + 3^3 + \ldots$ to 10 terms $)$

$$= \left\{\frac{21\times(21+1)}{2}\right\}^2 - 1 - 16.\left\{\frac{10(10+1)}{2}\right\}^2$$

$= 231^2 - 220^2 - 1 = (231 + 220)(231 - 220) - 1$

$= 451 \times 11 - 1 = 4961 - 1 = 4960$

**Example 6 :**

If $S_1, S_2$ and $S_3$ denote the sum of first $n_1$, $n_2$ and $n_3$ terms respectively of an A.P., then find

$$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2).$$

**Sol.** We have , $S_1 = \frac{n_1}{2}[2a + (n_1 - 1)d] \Rightarrow \frac{2S_1}{n_1} = 2a + (n_1 - 1)d$

$$S_2 = \frac{n_2}{2}[2a + (n_2 - 1)d] \Rightarrow \frac{2S_2}{n_2} = 2a + (n_2 - 1)d$$

$$S_3 = \frac{n_3}{2}[2a + (n_3 - 1)d] \Rightarrow \frac{2S_3}{n_3} = 2a + (n_3 - 1)d$$

$$\therefore \frac{2S_1}{n_1}(n_2 - n_3) + \frac{2S_2}{n_2}(n_3 - n_1) + \frac{2S_3}{n_3}(n_1 - n_2)$$

$= [2a + (n - 1)d](n_2 - n_3) + [2a + (n_2 - 1)d](n_3 - n_1)$
$+ [2a + (n_3 - 1)d](n_1 - n_2) = 0$

**Example 7 :**

Let a, b, c be positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is –

(A) 0 (B) 4

(C) 8 (D) 3

**Sol. (B)** a, ar, $ar^2$, $0 > 1$ r is integer

$$\frac{a + b + c}{3} = b + 2$$

$a + ar + ar^2 = 3(ar + 2)$

$a + ar + ar^2 = 3ar + 6$

$ar^2 - 2ar + a - 6 = 0$

$a(r - 1)^2 = 6$ ; $a = 6, r = 2$

$$\text{So, } \frac{a^2 + a - 14}{a + 1} = \frac{6^2 + 6 - 14}{6 + 1} = \frac{28}{7} = 4$$

**Example 8 :**

Find the sum up to 16 terms of the series

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots$$

**Sol.** We have $t_n = \frac{1^3 + 2^3 + 3^3 + \ldots\ldots + n^3}{1 + 3 + 5 + \ldots\ldots\text{upto n terms}}$

$$\frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}\{2 + 2(n-1)\}} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4} = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$\therefore S_n = \Sigma t_n = \frac{1}{4}\Sigma n^2 + \frac{1}{2}\Sigma n + \frac{1}{4}\Sigma 1$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2} + \frac{1}{4}n$$

$$S_{16} = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446$$

**Example 9 :**

r & d (d being variable) are $p^{th}$ term and common difference of an A.P. respectively. If the product of $(p - 2)^{th}$ &
$(p + 3)^{th}$ term of the given A.P. is maximum then r/d is equal to –

(A) 3 (B) 4

(C) 2 (D) 8

**Sol. (C).** Let the first term of A.P. is a then

$a + (p - 1)d = r$ ......... (1)

$(p - 2)^{th}$ term $= (r - 2d)$

$(p - 3)^{th}$ term $= (r + 3d)$ then

$\Rightarrow (r - 2d)(r + 3d) \Rightarrow [r^2 + rd - 6d^2]$

$$\Rightarrow\ r^2\left[1+\frac{d}{r}-6\left(\frac{d}{r}\right)^2\right]$$

$$\Rightarrow\ 6r^2\left[-\left(\frac{d}{r}\right)^2+\frac{1}{6}\frac{d}{r}+\frac{1}{6}\right]\Rightarrow r^2\left[\frac{37}{36}-6\left(\frac{d}{r}-\frac{1}{12}\right)^2\right]$$

For max. $\frac{d}{r}-\frac{1}{2}=0$ or $\frac{d}{r}=12$

**Example 10 :**

If a, b, c and d are positive real number, then

$\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}$ belongs to the interval –

(A) $[2,\infty)$ (B) $[3,\infty)$

(C) $[4,\infty)$ (D) $(-\infty,4)$

**Sol.** **(C).** Apply A.M. $\geq$ G.M.

$$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}}{4}\geq\sqrt[4]{\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{d}\cdot\frac{d}{a}}\geq 1$$

$$\therefore\ \frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}\geq 4$$

$$\therefore\ \frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}\in[4,\infty)$$

**Example 11 :**

Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

**Sol.** **9.** $\frac{S_7}{S_{11}}=\frac{6}{11}$ ; $\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]}=\frac{6}{11}$ [Given]

$130<a+6d<140$

$$\frac{7\,(a+3d)}{11\,(a+5d)}=\frac{6}{11}$$

$7a+21d=6a+30d\Rightarrow 130<15d<140$

$a=9d$. Hence, $d=9$, $a=81$

**Example 12 :**

If $\sum_{r=1}^{n} I\,(r)=n\,(2n^2+9n+13)$, then find the sum

$\sum_{r=1}^{n}\sqrt{I\,(r)}$.

**Sol.** $S_n=\sum_{r=1}^{n} I\,(r)=n\,(2n^2+9n+13)$

$\Rightarrow\ I(r)=S_r-S_{r-1}$

$=r(2r^2+9r+13)-(r-1)\,(2\,(r-1)^2+9(r-1)+13)$

$=6r^2+12r+6=6(r+1)^2$

$\Rightarrow\ \sqrt{I\,(r)}=\sqrt{6}(r+1)$

$\Rightarrow\ \sum_{r=1}^{n}\sqrt{I\,(r)}=\sqrt{6}\sum_{r=1}^{n}(r+1)$

$=\sqrt{6}\left(\frac{n^2+3n}{2}\right)=\sqrt{\frac{3}{2}}(n^2+3n)$

**Example 13 :**

If the roots of the equation $x^3-11x^2+36x-36=0$ are in H.P. find the middle root.

**Sol.** $x^3-11x^2+36x-36=0$

If roots are in H.P. then roots of new equation

$\frac{1}{x^3}-\frac{11}{x^2}+\frac{36}{x}-36=0$ are in A.P.

$-36x^3+36x^2-11x+1=0$

$36x^3-36x^2+11x-1=0$

Let the roots be $\alpha$, $\beta$, $\gamma$.

$\alpha+\beta+\gamma=1$

$3\beta=1$ $(2\beta=\alpha+\gamma)$

$\beta=1/3$

So middle root is 3.

# QUESTION BANK

## CHAPTER 6 : SEQUENCES & SERIES

### EXERCISE - 1 [LEVEL-1]

#### PART 1 : ARITHMETIC PROGRESSION

**Q.1** If for an A.P. $T_3 = 18$ and $T_7 = 30$ then $S_{17}$ is equal to-
(A) 612 (B) 622
(C) 306 (D) None of these

**Q.2** The first, second and middle terms of an AP are a, b, c respectively. Their sum is-
(A) $\frac{2(c-a)}{b-a}$ (B) $\frac{2c(c-a)}{b-a}+c$
(C) $\frac{2c(b-a)}{c-a}$ (D) $\frac{2b(c-a)}{b-a}$

**Q.3** The sum of integers in between 1 and 100 which are divisible by 2 or 5 is-
(A) 3100 (B) 3600
(C) 3050 (D) 3500

**Q.4** If $a_1, a_2, a_3.....a_n$ are in AP where $a_i > 0$, then the value of
$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+....+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} =$
(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$
(C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

**Q.5** If $9^{th}$ and $19^{th}$ terms of an AP are 35 and 75 respectively, then $20^{th}$ term is -
(A) 80 (B) 78
(C) 81 (D) 79

**Q.6** If first term of an AP is 5, last term is 45 and the sum of the terms is 400, then the number of terms is-
(A) 8 (B) 10
(C) 16 (D) 20

**Q.7** If the ratio of the sum of n terms of two AP's is $(3n + 1) : (2n + 3)$ then find the ratio of their $11^{th}$ term -
(A) 45 : 64 (B) 3 : 4
(C) 64 : 45 (D) 4 : 3

**Q.8** If 4 AM's are inserted between 1/2 and 3 then 3rd AM is-
(A) – 2 (B) 2
(C) – 1 (D) 1

**Q.9** n AM's are inserted between 2 and 38. If third AM is 14 then n is equal to -
(A) 9 (B) 7
(C) 8 (D) 10

**Q.10** Four numbers are in A.P. If their sum is 20 and the sum of their square is 120, then the middle terms are -
(A) 2,4 (B) 4,6
(C) 6, 8 (D) 8,10

**Q.11** If $(x+1)$, $3x$ and $(4x + 2)$ are first three terms of an AP then its 5th term is-
(A) 14 (B) 19
(C) 24 (D) 28

**Q.12** The sum of first ten terms of a A.P. is four times the sum of its first five terms, then ratio of first term and common difference is-
(A) 2 (B) 1/2
(C) 4 (D) 1/4

**Q.13** If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is -
(A) ± 1 (B) ± 2
(C) ± 3 (D) ± 4

**Q.14** The nos. $\frac{1}{\sqrt{11-4\sqrt{6}}}$, $\frac{1}{\sqrt{6-2\sqrt{3}+2\sqrt{2}-2\sqrt{6}}}$, $\frac{1}{\sqrt{7-4\sqrt{3}}}$ are in-
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.15** If $a^2, b^2, c^2$ are in A.P. then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in-
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.16** Find the maximum sum of the series
$20+19\frac{1}{3}+18\frac{2}{3}+18+..... 20+19\frac{1}{3}+18\frac{2}{3}+18$
(A) 310 (B) 210
(C) 105 (D) 325

**Q.17** The sixth term of an A.P. is equal to 2, the value of the common difference of the A.P. which makes the product $a_1a_4a_5$ least is given by
(A) x = 8/5 (B) x = 5/4
(C) x = 2/3 (D) None of these

**Q.18** The interior angles of a polygon are in A.P. If the smallest angle be $120^o$ and the common difference be $5^o$, then the number of sides is
(A) 8 (B) 10
(C) 9 (D) 6

**Q.19** The ratio of sum of m and n terms of an A.P. is $m^2 : n^2$, then the ratio of $m^{th}$ and $n^{th}$ term will be
(A) $\frac{m-1}{n-1}$ (B) $\frac{n-1}{m-1}$
(C) $\frac{2m-1}{2n-1}$ (D) $\frac{2n-1}{2m-1}$

**Q.20** The number of terms of the A.P. 3,7,11,15...to be taken so that the sum is 406 is –
(A) 5 (B) 10
(C) 12 (D) 14

**Q.21** Four numbers are in arithmetic progression. The sum of first and last term is 8 and the product of both middle terms is 15. The least number of the series is –
(A) 4 (B) 3
(C) 2 (D) 1

**Q.22** If a, b, c are in A.P., then the straight line $ax + by + c = 0$ will always pass through the point –
(A) (–1, –2) (B) (1, –2)
(C) (–1, 2) (D) (1, 2)

**Q.23** If $\frac{S_n}{S_m} = \frac{n^4}{m^4}$ (where $S_k$ is the sum of first k terms of an A.P. $a_1, a_2, ........\infty$ ), then the value of $\frac{a_{m+1}}{a_{n+1}}$ in terms of m and n will be
(A) $\frac{(2m+1)^3}{(2n+1)^3}$ (B) $\frac{(2n+1)^3}{(2m+1)^3}$
(C) $\frac{(2m-1)^3}{(2n-1)^3}$ (D) $\frac{(2m+1)^3}{(2n-1)^3}$

**Q.24** 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is
(A) 15 (B) 20
(C) 25 (D) 30

**Q.25** Given that n A.M.'s are inserted between two sets of numbers a, 2b and 2a, b, where a, b $\in$ R. Suppose further that $m^{th}$ mean between these sets of numbers is same, then the ratio a : b equals
(A) $n - m + 1 : m$ (B) $n - m + 1 : n$
(C) $n : n - m + 1$ (D) $m : n - m + 1$

**Q.26** The number of common terms to the two sequences 17, 21, 25, ....., 417 and 16, 21, 26, ........, 466 is –
(A) 19 (B) 20
(C) 21 (D) 91

**Q.27** The ratio of the sum of n terms of the two A.P.'s be $\frac{7n+1}{4n+27}$ and ratio of 11th term is $\lambda$ then value of $111 \times \lambda$ is –
(A) 138 (B) 128
(C) 122 (D) 148

**Q.28** The $n^{th}$ term of the series 1 + 3 + 7 + 13 + 21 + .......... is 9901. The value of n is –
(A) 100 (B) 90
(C) 900 (D) 99

**Q.29** If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in A.P., then $2a^3 - 9ab =$
(A) 9c (B) 18c
(C) 27c (D) –27c

## PART 2 : GEOMETRIC PROGRESSION

**Q.30** If x , 2x + 2 and 3x + 3 are first three terms of a G.P., then its $4^{th}$ term is-
(A) 27 (B) – 27
(C) – 27/2 (D) 27/2

**Q.31** The sum of 16.2, 5.4, 1.8, ......... to 7 series is-
(A) $\frac{1093}{45}$ (B) $\frac{656}{9}$
(C) $\frac{1039}{41}$ (D) $\frac{566}{9}$

**Q.32** If first, second and eight terms of a G.P. are respectively $n^{-4}$, $n^n$, $n^{52}$, then the value of n is-
(A) 1 (B) 10
(C) 4 (D) None of these

**Q.33** Let a, b and c form a GP of common ratio r, with $0 < r < 1$. If a, 2b and 3c form an AP, then r equals -
(A) 1/2 (B) 1/3
(C) 2/3 (D) None of these

**Q.34** If the sum of first two terms of an infinite GP is 1 and every term is twice the sum of all the successive terms, then its first term is-
(A) 1/3 (B) 2/3
(C) 1/4 (D) 3/4

**Q.35** If 4 GM's be inserted between 160 and 5, then third GM will be -
(A) 8 (B) 118
(C) 20 (D) 40

**Q.36** Three numbers form an increasing GP. If the middle number is doubled, then the new numbers are in AP. The common ratio of the GP is-
(A) $2 - \sqrt{3}$ (B) $2 + \sqrt{3}$
(C) $\sqrt{3} - 2$ (D) $3 + \sqrt{2}$

**Q.37** If product of three terms of a GP is 216, and sum of their products taken in pairs is 156, then greatest term is-
(A) 2 (B) 6
(C) 18 (D) 54

**Q.38** If a,b,c,d are in G.P. then $a^n + b^n$ , $b^n + c^n$, $c^n + d^n$ are in -
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.39** If the sum of first 6 terms of a G.P. is nine times of the sum of its first three terms, then its common ratio is-
(A) 1 (B) 3/2
(C) 2 (D) – 2

**Q.40** If $p^{th}$, $q^{th}$ and $r^{th}$ terms of an A.P. are equal to corresponding terms of a G.P. and these terms are respectively x,y,z, then $x^{y-z} . y^{z-x} . z^{x-y}$ equals-
(A) 0 (B) 1
(C) 2 (D) None of these

**Q.41** If a,b,c,d and p are distinct real numbers such that $(a^2 + b^2 + c^2)\, p^2 - 2p\,(ab + bc + cd) + (b^2 + c^2 + d^2) \le 0$ then a,b,c,d are in -
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.42** If x,y,z are in A.P. and x,y, t are in G.P. then x, x – y, t – z are in
(A) G.P. (B) A.P.
(D) H.P. (D) A.P. and G.P. both

**Q.43** Let $a_n$ be the $n^{th}$ term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio is –

(A) $\alpha/\beta$ (B) $\beta/\alpha$

(C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

**Q.44** The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second terms and 1 is subtracted from the third, the resulting new terms are in arithmetic progression. Then the lowest of the original term is

(A) 1 (B) 2

(C) 4 (D) 8

**Q.45** Let a and b be roots of $x^2 - 3x + p = 0$ and let c and d be the roots of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing G.P. Then the ratio of $(q + p) : (q - p)$ is equal to

(A) 8 : 7 (B) 11 : 10

(C) 17 : 15 (D) None of these

**Q.46** If $\alpha, \beta, \gamma$ are the geometric means between ca, ab; ab, bc; bc, ca respectively where a, b, c are in A.P., then $\alpha^2, \beta^2, \gamma^2$ are in

(A) A.P. (B) H.P.

(C) G.P. (D) None of the above

**Q.47** Two sequences $\{t_n\}$ and $\{s_n\}$ are defined by

$t_n = \log\left(\frac{5^{n+1}}{3^{n-1}}\right), s_n = \left[\log\left(\frac{5}{3}\right)\right]^n$, then

(A) $\{t_n\}$ is an A.P., $\{s_n\}$ is a G.P.

(B) $\{t_n\}$ and $\{s_n\}$ are both G.P.

(C) $\{t_n\}$ and $\{s_n\}$ are both A.P.

(D) $\{s_n\}$ is a G.P., $\{t_n\}$ is neither A.P. nor G.P

**Q.48** If the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then

(A) $c^3a = b^3d$ (B) $ca^3 = bd^3$

(C) $a^3b = c^3d$ (D) $ab^3 = cd^3$

**Q.49** If d, e, f are in G.P. and two quadratic equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root then, $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in-

(A) H.P. (B) G.P.

(C) A.P. (D) None of these

**Q.50** If $x_1, x_2, x_3$ as well as $y_1, y_2, y_3$ are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ & $(x_3, y_3)$

(A) Lie on a straight line (B) Lie on an ellipse

(C) Lie on a circle (D) Are vertices of a triangle

**Q.51** If $(a^2 + b^2 + c^2)\, p^2 - 2\,(ab + bc + cd)\, p + (b^2+c^2 + d^2) \leq 0$, then a, b, c, d are in –

(A) AP (B) GP

(C) HP (D) None of these

**Q.52** Let $x_1, x_2, \ldots\ldots\ldots, x_{10}$ be non-negative real nos. such that $x_1 + x_2 + \ldots.. + x_{10} = 12$ and let $S = x_1x_2 + x_3x_4 + \ldots.+x_9x_{10}$ then

(A) $S \leq 36$ (B) $S > 144$

(C) $S < 18$ (D) None of these

**Q.53** If x, y, z are positive real numbers satisfying $x + y + z = 1$, then maximum value of $\left(1+\frac{1}{x}\right).\left(1+\frac{1}{y}\right).\left(1+\frac{1}{z}\right)$ is –

(A) 8 (B) 16

(C) 64 (D) None of these

**Q.54** Consider an infinite geometric series with first term 'a' and common ratio 'r'. If the sum is 4 and the second term is 3/4, then –

(A) a = 2, r = 3/8 (B) a = 4/7, r = 3/7

(C) a = 3/2, r = 1/2 (D) a = 3, r = 1/4

**Q.55** If the $2^{nd}$ and $5^{th}$ terms of G. P. are 24 and 3 respectively, then the sum of $1^{st}$ six terms is –

(A) 189/2 (B) 189/5

(C) 179/2 (D) 2/189

## PART 3 : HARMONIC PROGRESSION

**Q.56** $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is AM/GM/HM, between a and b if n is equal to respectively-

(A) – 1, – 1/2, 0 (B) 0, 1/2, – 1/2

(C) 0, – 1/2, – 1 (D) None of these

**Q.57** If $a_1, a_2, a_3 \ldots., a_n$ are in HP, then $a_1a_2 + a_2a_3 + \ldots. + a_{n-1}a_n$ is equal to-

(A) $na_1 a_n$ (B) $(n-1)\, a_1a_n$

(C) $(n+1)\, a_1a_n$ (D) None of these

**Q.58** If there are n harmonic means between 1 and $\frac{1}{31}$ and the ratio of $7^{th}$ and $(n - 1)^{th}$ harmonic means is 9 : 5 then the value of n will be

(A) 12 (B) 13

(C) 14 (D) 15

**Q.59** Let $a_1, a_2, \ldots\ldots\ldots a_{10}$ be in A.P. and $h_1, h_2, \ldots\ldots\ldots h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4h_7$ is

(A) 2 (B) 3

(C) 5 (D) 6

**Q.60** Let $a_1, a_2, a_3$ be any positive real numbers, then which of the following statement is not true –

(A) $3a_1a_2a_3 \leq a_1^3 + a_2^3 + a_3^3$

(B) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \geq 3$

(C) $(a_1 + a_2 + a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$

(D) $(a_1 + a_2 + a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \leq 27$

**Q.61** If $p^{th}$ term of a HP be q and $q^{th}$ term be p, then its $(p+q)^{th}$ term is-

(A) $\frac{1}{p+q}$  (B) $\frac{1}{p}+\frac{1}{q}$

(C) $\frac{pq}{p+q}$  (D) $p+q$

**Q.62** If a,b,c are in A.P. and $a^2, b^2, c^2$ are in H.P., then-

(A) $a = b + c$  (B) $b = c + a$

(C) $c = a + b$  (D) $a = b = c$

**Q.63** Five numbers a,b,c,d,e are such that a,b,c, are in AP' b,c,d are in GP and c,d,e, are in HP. If a = 2, e = 18; then values of b,c, d are -

(A) 2, 6, 18  (B) 4, 6, 9

(C) 4, 6, 8  (D) – 2, – 6, 18

**Q.64** a,b,c are first three terms of a GP. If HM of a and b is 12 and that of b and c is 36, then a equals-

(A) 24  (B) 8

(C) 72  (D) 1/3

**Q.65** If x, 1, z are in A.P. x, 2,z are in G.P. then x, 4, z are in-

(A) AP  (B) GP

(C) HP  (D) None of these

**Q.66** If a,b,c in H.P. then value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right) =$

(A) $\frac{2}{bc}-\frac{1}{b^2}$  (B) $\frac{3}{b^2}-\frac{2}{ab}$

(C) $\frac{3}{ac}-\frac{2}{b^2}$  (D) Both (A) and (B)

**Q.67** If a,b,c are in H.P. then $\frac{a}{b+c},\frac{b}{c+a},\frac{c}{a+b}$ will be in-

(A) A.P.  (B) G.P.

(C) H.P.  (D) None of these

**Q.68** If the $(m+1)^{th}$, $(n+1)^{th}$, $(r+1)^{th}$ terms of an A.P. are in G.P. and m,n,r are in H.P. then the ratio of common difference to the first terms in the A.P. is-

(A) n/2  (B) 2/n

(C) – n/2  (D) – 2/n

**Q.69** If a,x, y,z,b are in AP, then x+ y + z = 15 and if a,x,y,z, b are in HP, then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$. Numbers a,b are -

(A) 8, 2  (B) 11, 3

(C) 9, 1  (D) None of these

**Q.70** If H and G are harmonic and geometric mean of positive real nos. a & b such that H : G = 4 : 5 then a : b is –

(A) 5 : 4  (B) 1 : 4

(C) 1 : 5  (D) None of these

## PART 4 : MISCELLANEOUS

**Q.71** The sum of all numbers between 100 and 10,000 which are of the form $n^3$ $(n \in N)$ is equal to -

(A) 55216  (B) 53261

(C) 51261  (D) None of these

**Q.72** $\sum_{r=1}^{n}\frac{1}{\log_{3^r} a}$ is equal to –

(A) $\frac{n(n+1)}{2}\log_3 a$  (B) $\frac{n(n+1)}{2}\log_a 3$

(C) $\frac{(n+1)^2 n^2}{4}\log_3 a$  (D) None of these

**Q.73** The sum to n terms of the series

$$\frac{1}{1-\frac{1}{4}}+\frac{1}{(1+3)-\frac{1}{4}}+\frac{1}{(1+3+5)-\frac{1}{4}}+....... \text{ is}$$

(A) $\frac{2n}{2n+1}$  (B) $\frac{4n}{2n+1}$  (C) $\frac{2}{2n+1}$  (D) $\frac{4n}{2n-1}$

**Q.74** The sum to n terms of the series

$$\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1.3}{3}\left(\frac{1}{2}\right)^4+........ \text{ is}$$

(A) $\frac{1.3.5...(2n-1)}{2^n n}$  (B) $1-\frac{1.3.5...(2n-1)}{2^n n}$

(C) $1-\frac{1.3.5...(2n-1)}{2n^{n-1}n-1}$  (D) $\frac{1.3.5...(2n-1)}{2n^{n-1}n-1}$

**Q.75** If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 +$ ........ is $\frac{35}{16}$, where $|x| < 1$, find 'x'.

(A) 2/5  (B) 1/5

(C) 1/2  (D) 1/4

**Q.76** If $S=\frac{1}{1.2.3}+\frac{2}{3.4.5}+\frac{3}{5.6.7}+......+\infty$, then –

(A) S = 1/4  (B) S = 1/2

(C) S = 2/3  (D) S = 1

**Q.77** $2^{1/4}.\ 2^{2/8}.\ 2^{3/16}.\ 2^{4/32}......\infty$ is equal to-

(A) 1  (B) 2

(C) 3/2  (D) 5/2

**Q.78** Sum of n terms of the series $8 + 88 + 888 + ....$ equals

(A) $\frac{8}{81}[10^{n+1}-9n-10]$  (B) $\frac{8}{81}[10^n-9n-10]$

(C) $\frac{8}{81}[10^{n+1}-9n+10]$  (D) None of these

**Q.79** For all positive integral values of n, the value of $3.1.2+3.2.3+3.3.4+.....+3.n.(n+1)$ is

(A) $n(n+1)(n+2)$  (B) $n(n+1)(2n+1)$

(C) $(n-1)n(n+1)$  (D) $\frac{(n-1)\,n\,(n+1)}{2}$

**Q.80** The sum of the series $\frac{1}{3\times 7}+\frac{1}{7\times 11}+\frac{1}{11\times 25}+....$ is

(A) 1/3 (B) 1/6

(C) 1/9 (D) 1/12

**Q.81** The $9^{th}$ term of the series $27+9+5\frac{2}{5}+3\frac{6}{7}+........$ will be

(A) $1\frac{10}{17}$ (B) $\frac{10}{17}$ (C) $\frac{16}{27}$ (D) $\frac{17}{27}$

**Q.82** A series whose $n^{th}$ term is $\left(\frac{n}{x}\right)+y$, the sum of r terms will

(A) $\left\{\frac{r(r+1)}{2x}\right\}+ry$ (B) $\left\{\frac{r(r-1)}{2x}\right\}$

(C) $\left\{\frac{r(r-1)}{2x}\right\}-ry$ (D) $\left\{\frac{r(r+1)}{2y}\right\}-rx$

**Q.83** The sum of the first five terms of the series

$3+4\frac{1}{2}+6\frac{3}{4}+......$ will be

(A) $39\frac{9}{16}$ (B) $18\frac{3}{16}$ (C) $39\frac{7}{16}$ (D) $13\frac{9}{16}$

**Q.84** Value of $9+99+999+.........$ upto n terms is –

(A) $\frac{10^n-9n-10}{81}$ (B) $\frac{10^{n+1}-9n-10}{9}$

(C) $\frac{10^{n+1}-9n-10}{81}$ (D) $\frac{10^n-9n-10}{9}$

**Q.85** The sum of the series $a-(a+d)+(a+2d)-(a+3d)+...$ upto $(2n+1)$ terms is-

(A) – nd (B) a + 2 nd

(C) a + nd (D) 2nd

**Q.86** The sum to n terms of the series

$1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+....$ is given by-

(A) $n^2$ (B) $n(n+1)$

(C) $n(1+1/n)^2$ (D) None of these

**Q.87** $1+2.2+3.2^2+4.2^3+....+100.2^{99}$ equals-

(A) $99.2^{100}$ (B) $100.2^{100}$

(C) $1+99.2^{100}$ (D) None of these

**Q.88** If $3+\frac{1}{4}(3+d)+\frac{1}{4^2}(3+2d)+....\text{to }\infty=8$, then the value of d is-

(A) 9 (B) 5

(C) 1 (D) None of these

**Q.89** The sum of infinite series $S=1+(1+a)x+(1+a+a^2)x^2+(1+a+a^2+a^3)x^3$ ......... $\infty$ (where $0<a\,;\,x<1$) is –

(A) $\frac{1}{(1-x)(1-a)}$ (B) $\frac{1}{(1-a)(1-ax)}$

(C) $\frac{1}{(1-x)(1-ax)}$ (D) $\frac{1}{(1-x)(1+a)}$

**Q.90** $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}(1)=$

(A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)(2n-1)}{6}$

(C) $\frac{n(n+1)(n+2)}{6}$ (D) None of these

**Q.91** The sum of the first n terms of

$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+.......$ is

(A) $\frac{2n^2-n}{3}$ (B) $\frac{n(n+2)}{3}$ (C) $\frac{2n^2+n}{3}$ (D) $\frac{n^2-2n}{3}$

**Q.92** The sum of the series, $\frac{1}{2\cdot 3}\cdot 2+\frac{2}{3\cdot 4}\cdot 2^2+\frac{3}{4\cdot 5}\cdot 2^3+....$ to n terms is –

(A) $\frac{2^{n+1}}{n+2}+1$ (B) $\frac{2^{n+1}}{n+2}-1$

(C) $\frac{2^{n+1}}{n+2}+2$ (D) $\frac{2^{n+1}}{n+2}-2$

**Q.93** The sum of $1^{st}$ n terms of the series

$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+....$

(A) $\frac{n(n+2)}{3}$ (B) $\frac{n(n-2)}{6}$

(C) $\frac{n+2}{3}$ (D) $\frac{n(n-2)}{3}$

**Q.94** If log 2, log $(2^x - 1)$ and log $(2^x + 3)$ are in A.P. then find the value of x.

(A) $\log_2 5$ (B) $\log_2 3$
(C) $\log_2 8$ (D) $\log_2 6$

**Q.95** If x, y, z are in A.P. and x, y, t are in G.P. then x, x – y, t – z are in

(A) G.P. (B) A.P.
(C) H.P. (D) A.P. and G.P. both

**Q.96** The geometric and harmonic means of two numbers $x_1$ and $x_2$ are 18 and $16\frac{8}{13}$ respectively. The value of $|x_1 - x_2|$ is

(A) 5 (B) 10
(C) 15 (D) 20

**Q.97** If $a_1, a_2, ........, a_{2n+1}$ are in G.P., then

$\dfrac{\sqrt{a_1a_2}+\sqrt{a_3a_4}+.....+\sqrt{a_{2n-1}a_{2n}}}{\sqrt{a_2a_3}+\sqrt{a_4a_5}+.......+\sqrt{a_{2n}a_{2n+1}}}$ is equal to-

(A) $a_1 + a_3 + ........ + a_{2n-1}$ (B) $a_2 + a_4 + .......... + a_{2n}$
(C) $\dfrac{a_2+a_4+......+a_{2n}}{a_1+a_3+......+a_{2n-1}}$ (D) $\dfrac{a_1+a_3+.....+a_{2n-1}}{a_2+a_4+........+a_{2n}}$

**Q.98** The sum of the latter half of the first 1000 terms of any A.P. is equal to one third of the sum of the first n terms of the same A.P. Then n =

(A) 1500 (B) 3000
(C) 2000 (D) 1000

**Q.99** If the $(2p)^{th}$ term of a H.P. is q and the $(2q)^{th}$ term is p, then the $2(p + q)^{th}$ term is-

(A) $\dfrac{pq}{2(p+q)}$ (B) $\dfrac{2pq}{p+q}$
(C) $\dfrac{pq}{p+q}$ (D) $\dfrac{p+q}{pq}$

**Q.100** All terms of a certain A.P are natural numbers. The sum of its nine successive terms begining with the first is larger than 200 and smaller than 220. If the second term is 12, then the common difference is

(A) 2 (B) 3
(C) 4 (D) 6

# EXERCISE - 2 [LEVEL-2]

**ONLY ONE OPTION IS CORRECT**

**Q.1** If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. then which of the following is in A.P. -
(A) a,b,c (B) $a^2, b^2, c^2$
(C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ (D) None of these

**Q.2** If the pth, qth and rth terms of a harmonic progression are a, b, c respectively, then $\frac{q-r}{a}+\frac{r-p}{b}+\frac{p-q}{c}=$
(A) $\frac{pqr}{abc}$ (B) $\frac{p+q+r}{a+b+c}$ (C) $\frac{par}{bqc}$ (D) none of these

**Q.3** If a,b,c,d are in G.P., then $(a^3+b^3)^{-1}, (b^3+c^3)^{-1}, (c^3+d^3)^{-1}$ are in –
(A) A.P. (B) G.P.
(C) H.P. (D) none of these

**Q.4** If $x_i > 0, i = 1, 2, ......, 50$ and $x_1 + x_2 + ........ + x_{50} = 50$, then the minimum value of $\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_{50}}$ equals to –
(A) $(50)^2$ (B) 50
(C) $(50)^3$ (D) $(50)^4$

**Q.5** If 1, $\log_{81}(3^x+48)$ and $\log_9\left(3^x-\frac{8}{3}\right)$ are in A.P., then find x
(A) 1 (B) 2
(C) 9 (D) 3

**Q.6** If x,y,z are in G.P. and $a^x = b^y = c^z$ then-
(A) $\log_b a = \log_a c$ (B) $\log_c b = \log_a c$
(C) $\log_b a = \log_c b$ (D) none of these

**Q.7** a, b, c are first three terms of a G.P. If HM of a and b is 12 and that of b and c is 36, then find the value of a.
(A) 2 (B) 3
(C) 8 (D) 1

**Q.8** The numbers $\frac{1}{\sqrt{11-4\sqrt{6}}}$, $\frac{1}{\sqrt{6-2\sqrt{3}+2\sqrt{2}-2\sqrt{6}}}$, $\frac{1}{\sqrt{7-4\sqrt{3}}}$ are in-
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.9** If $\log_{\frac{x+6}{3}}\left(\log_2\frac{x-1}{x+2}\right) > 0$, then $x \in (a,b) \cup (c,d)$. If a, b, k, c, d are in A.P., then the value of $a^2 + b^2 + k^2 + c^2 + d^2$ is
(A) 115 (B) 125
(C) 118 (D) 130

**Q.10** If $a_1, a_2, a_3, ................a_n, a_{n+1}$ are in A.P. then $\frac{1}{a_1 a_2}+\frac{1}{a_2 a_3}+\frac{1}{a_3 a_4}+ ........+\frac{1}{a_n a_{n+1}}$ is equal to
(A) $\frac{n-1}{a_n a_{n+1}}$ (B) $\frac{1}{a_n a_{n+1}}$
(C) $\frac{n+1}{a_n a_{n+1}}$ (D) $\frac{n}{a_n a_{n+1}}$

**Q.11** If the $(m+1)^{th}$, $(n+1)^{th}$, $(r+1)^{th}$ terms of an A.P. are in G.P. and m,n,r, are in H.P. then find the ratio of common difference to the first terms in the A.P.
(A) n/2 (B) 2/n
(C) – n/2 (D) – 2/n

**Q.12** If n arithmetic means $a_1, a_2, ......, a_n$ are inserted between 50 and 200 and n harmonic means $h_1, h_2, ......, h_n$ are inserted between the same two numbers, then $a_2 h_{n-1}$ is equal to
(A) 500 (B) $\frac{10000}{n}$
(C) 10000 (D) $\frac{250}{n}$

**Q.13** If $a_1, a_2, a_3, a_4, a_5$ are in H.P., then find the value of $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$.
(A) $2a_1a_5$ (B) $8a_1a_5$
(C) $10a_1a_5$ (D) $4a_1a_5$

**Q.14** If positive numbers a, b, c are in H.P. then the value of $e^{\log(a+c)+\log(a-2b+c)}$ is equal to
(A) $\log(a-c)^2$ (B) $(a-c)$
(C) $(a-c)^2$ (D) zero

**Q.15** $\frac{1}{2}\operatorname{cosec}^2\theta, 2\cot\theta, 0 < \theta < \frac{\pi}{2}$, are in G.P. if θ is equal to
(A) π/6 (B) π/4
(C) π/3 (D) None of these

**Q.16** If x, y, z are three real numbers of the same sign then the value of $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$ lies in the interval
(A) $[2, +\infty)$ (B) $[3, +\infty)$
(C) $(3, +\infty)$ (D) $(-\infty, 3)$

**Q.17** If $\frac{a_2a_3}{a_1a_4}=\frac{a_2+a_3}{a_1+a_4}=3\left(\frac{a_2-a_3}{a_1-a_4}\right)$ then $a_1, a_2, a_3, a_4$ are in
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

**Q.18** The sum of the integers lying between 1 and 100 (both inclusive) and divisible by 3 or 5 or 7 is
(A) 2838 (B) 3468
(C) 2738 (D) 3368

**Q.19** The arithmetic mean of two numbers is 3 times their geometric mean and the sum of the squares of the two numbers is 34. The two numbers are

(A) $2\sqrt{3}+\sqrt{5}, 2\sqrt{3}-\sqrt{5}$ (B) $3+2\sqrt{2}, 3-2\sqrt{2}$

(C) $\sqrt{10}+\sqrt{7}, \sqrt{10}-\sqrt{7}$ (D) none of these

**Q.20** In a G.P., if $(2p)^{th}$ term is $q^2$ and $(2q)^{th}$ term is $p^2$ where p and q $\in$ N, then its $(p + q)^{th}$ term is –

(A) pq (B) $p^2q^2$

(C) $\frac{1}{2}p^2q^2$ (D) $\frac{1}{4}p^3q^3$

**Q.21** If $a_1 + a_2 + a_3 + a_4 + a_5 + ..... + a_n = 1$ for all $a_i > 0, i = 1, 2, 3, ....., n$. Then the maximum value of $a_1^2a_2a_3a_4a_5 ........ a_n$ is –

(A) $\frac{2}{(n+1)^n}$ (B) $\frac{4}{(n+1)^{n+1}}$

(C) $\frac{2}{n^n}$ (D) $\frac{4}{n^{n+1}}$

**Q.22** If $\sin\alpha, \sin\beta, \sin\gamma$ are in A.P. $\cos\alpha, \cos\beta, \cos\gamma$ are in G.P.

then $\frac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma} =$

(A) –2 (B) –1

(C) 0 (D) 2

**Q.23** Given $a_{m+n} = A$ ; $a_{m-n} = B$ as the terms of the G.P. $a_1, a_2, a_3$ ,............. then for $A \neq 0$ which of the following holds?

(A) $a_m = \sqrt{AB}$ (B) $a_n = \sqrt[2n]{A^nB^n}$

(C) $a_m = a_1\left(\frac{A}{B}\right)^{\frac{m^2-m-n-mn}{m+n}}$ (D) $a_n = a_1\left(\frac{A}{B}\right)^{\frac{m^2-m-n-n^2}{m+n}}$

**Q.24** If $\log_{(5.2^x+1)} 2$ ; $\log_{(2^{1-x}+1)} 4$ and 1 are in Harmonical Progression then

(A) x is a positive real

(B) x is a negative real

(C) x is rational which is not integral

(D) x is an integer

**Q.25** Consider an A.P. with first term 'a' and the common difference d. Let $S_k$ denote the sum of the first K terms.

Let $\frac{S_{kx}}{S_x}$ is independent of x, then

(A) a = d/2 (B) a = d

(C) a = 2d (D) none

**Q.26** Concentric circles of radii 1, 2, 3......100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equal to

(A) 1000π (B) 5050π

(C) 4950π (D) 5151π

**Q.27** Consider the A.P. $a_1, a_2, ....., a_n, ....$
the G.P. $b_1, b_2, ....., b_n, .....$

such that $a_1 = b_1 = 1$ ; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then

(A) $b_6 = 27$ (B) $b_7 = 27$

(C) $b_8 = 81$ (D) $b_9 = 18$

**Q.28** The point $A(x_1, y_1)$ ; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the parabola $y = 3x^2$. If $x_1, x_2, x_3$ are in A.P. and $y_1, y_2, y_3$ are in G.P. then the common ratio of the G.P. is

(A) $3 + 2\sqrt{2}$ (B) $3 + \sqrt{2}$

(C) $3 - \sqrt{2}$ (D) $3 - 2\sqrt{2}$

**Q.29** A circle of radius r is inscribed in a square. The mid points of sides of the square have been connected by line segment and a new square resulted. The sides of the resulting square were also connected by segments so that a new square was obtained and so on, then the radius of the circle inscribed in the $n^{th}$ square is

(A) $\left[2^{\frac{1-n}{2}}\right]r$ (B) $\left[2^{\frac{3-3n}{2}}\right]r$

(C) $\left[2^{-\frac{n}{2}}\right]r$ (D) $\left[2^{-\frac{5-3n}{2}}\right]r$

**ASSERTION AND REASON QUESTIONS**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement -1 is True, Statement-2 is False.

(D) Statement -1 is False, Statement-2 is True.

(E) Statement -1 is False, Statement-2 is False.

**Q.30** **Statement 1 :** 1, 2, 4, 8, ........... is a G.P., 4, 8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32 ........ is also a G.P.

**Statement 2 :** If $T_k$ denotes $k^{th}$ term of a G.P. of positive common ratio r and $T'_k$ denotes $k^{th}$ term of an other G.P. of common ratio r, then the series whose $k^{th}$ term is $T''_k = T_k + T'_k$ ia also a G..P. with common ratio r.

**Q.31** **Statement–1 :** In the expression $(x + 1)(x + 2) . . . (x + 50)$, coefficient of $x^{49}$ is equal to 1275.

**Statement–2 :** $\sum_{r=i}^{n} r = \frac{n(n+1)}{2}, n \in N$.

**Q.32** Let a, r $\in$ R – {0, 1, – 1} and n be an even number.

**Statement–1 :** $a. ar. ar^2 . . . ar^{n-1} = (a^2 r^{n-1})^{n/2}$.

**Statement–2 :** Product of $k^{th}$ term from the beginning and from the end in a G.P. is independent of k.

**Q.33** **Statement–1 :** Let p, q, r $\in R^+$ and $27pqr^3 (p + q + r)^3$ and $3p + 4q + 5r = 12$, then $p^3 + q^4 + r^5$ is equal to 4.

**Statement–2 :** If A, G, and H are A.M., G.M., and H.M. of positive numbers $a_1, a_2, a_3, . . . , a_n$ then $H \leq G \leq A$.

**Q.34** **Statement 1 :** If a, b, c and d are in harmonic progression then (a + d) > (b + c).
**Statement 2 :** If a, b, c and d are in arithmetic progression, then ab + cd > 2 (ac + bd – bc).

**Q.35** **Statement-1:** If a, b, c are in G.P., $\frac{1}{\log a}, \frac{1}{\log b}, \frac{1}{\log c}$ are in H.P.
**Statement-2:** When we take logarithm of the terms in G.P., they occur in A.P.

**MATCH THE COLUMN TYPE QUESTIONS**

**Q.36** Column II gives sum of n terms of the series given in column I. Match them correctly –

| Column I | Column II |
|---|---|
| (a) $8+88+888+.........$ | (p) $\frac{1}{3}(4^n-1)+n(n+1)$ |
| (b) $3+8+22+72+266+1036+.....$ | (q) $\frac{8}{81}[10^{n+1}-9n+10]$ |
| (c) $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+...$ | (r) $\frac{1}{2}\left(\frac{n(n+1)}{n^2+n+1}\right)$ |
| (d) $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+...$ | (s) $\frac{1}{3}\left[\frac{1}{1.2.3}-\frac{1}{(n+1)(n+2)(n+3)}\right]$ |

Code :
(A) a-q, b-p, c-s, d-r (B) a-s, b-p, c-q, d-r
(C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q

**Q.37** Match the column

| Column I | Column II |
|---|---|
| (a) If $a_k=\int_0^{\pi}\frac{\sin(2k-1)x}{\sin x}dx$ then $a_1, a_2, .......$ form a | (p) constant sequence |
| (b) If x, y, z all greater than '1' are in G.P. then $\frac{1}{1+\log x},\frac{1}{1+\log y},\frac{1}{1+\log z}$ are in | (q) A.P. |
| (c) If a, b, c are in A.P. then $\frac{ab+ac}{bc},\frac{bc+ba}{ca},\frac{ca+bc}{ab}$ are in | (r) H.P. |
| (d) If $x_1, x_2, ...... x_n$ are n non – zero real numbers such that, $(x_1^2+x_2^2+...+x_{n-1}^2)(x_2^2+x_3^2+...+x_n^2) \le (x_1x_2+x_2x_3+...+x_{n-1}x_n)^2$, then $x_1, x_2, ..., x_n$ are in | (s) G.P. |

Code :
(A) a-q, b-p, c-s, d-r (B) a-s, b-p, c-q, d-r
(C) a-r, b-q, c-s, d-p (D) a-p, b-r, c-q, d-s

**PASSAGE BASED QUESTIONS**

**Passage 1- (Q.38-Q.40) :** Four different integers from an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

**Q.38** The smallest number is –
(A) –2 (B) 0 (C) –1 (D) 2

**Q.39** The common difference of A.P. is –
(A) 2 (B) 1 (C) 3 (D) 4

**Q.40** The sum of all the four numbers is –
(A) 10 (B) 8 (C) 2 (D) 6

**Passage 2- (Q.41-Q.43)**
Arithmetic, geometric and harmonic mean of the roots of $x^2+13x+36=0$ and α, β and γ respectively.

**Q.41** $x_1$ and $x_2$ are the roots of $ax^2+bx+c=0$, a, b, c ∈ R and α, β, γ lies between the $x_1$ and $x_2$, $\delta = |x_1-x_2|$, then minimum possible value of δ is –
(A) 1 (B) 3/4
(C) 1/2 (D) 25/26

**Q.42** Set of all values of t if sum of roots of $x^2-(t^2-13t+\alpha+\gamma)x-36=0$ is less than or equal to β, is [ℓ, m] and p = ℓ + m, then p is equal to –
(A) 13 (B) 26
(C) 4 (D) 17

**Q.43** Equation whose roots are 2α, p is (where p obtained from above questions).
(A) $x^2+30x+221=0$ (B) $x^2-39x+338=0$
(C) $x^2+17x+52=0$ (D) $x^2-169=0$

**Passage 3- (Q.44-Q.46)**
Let $A_1, A_2, A_3, ....., A_n$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, ...., G_n$ be geometric means between 1 and 1024. Product of geometric means is $2^{45}$ and sum of arithmetic means is 1025 × 171.

**Q.44** The value of n is –
(A) 7 (B) 9
(C) 11 (D) None of these

**Q.45** The value of m is –
(A) 340 (B) 342 (C) 344 (D) 346

**Q.46** The value of $G_1+G_2+G_3+........+G_n$ is –
(A) 1022 (B) 2044
(C) 512 (D) None of these

**Passage 4- (Q.47-Q.49)**
Let $<a_n>$ and $<b_n>$ be the arithmetic sequences each with common difference 2 such that $a_1 < b_1$ and let

$$c_n=\sum_{k=1}^{n}a_k,\ d_n=\sum_{k=1}^{n}b_k.$$

Suppose that the points $A_n(a_n, c_n)$, $B_n(b_n, d_n)$ are all lying on the parabola C : $y = px^2+qx+r$ where p, q, r are constants.

**Q.47** The value of p equals –
(A) 1/4 (B) 1/3 (C) 1/2 (D) 2

**Q.48** The value of q equals –
(A) 1/4 (B) 1/3 (C) 1/2 (D) 2

**Q.49** If r = 0 then the value of $a_1$ and $b_1$ are –
(A) 1/2 and 1 (B) 1 and 3/2
(C) 0 and 2 (D) 1/2 and 2

## EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

**NOTE : The answer to each question is a NUMERICAL VALUE.**

**Q.1** If the sum $\sum_{k=1}^{\infty} \frac{1}{(k+2)\sqrt{k}+k\sqrt{k+2}} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}}$, where a, b, c $\in$ N and lie in [1, 15], then find the value of a+b+ c.

**Q.2** Numbers are grouped as {1, 1, 1} {3, $3^2$, .......$3^5$} {6, $6^2$, ..... $6^7$} {10, $10^2$, ...., $10^9$}. If sum of numbers in $10^{th}$ bracket is A such that $\left(\frac{54A}{55}+1\right) = 55^B$, then find B.

**Q.3** If $\sum_{n=1}^{49} \frac{1}{\sqrt{n+\sqrt{n^2-1}}} = a + b\sqrt{2}$, then a + b =

**Q.4** If $\tan\left(\frac{\pi}{12}-x\right), \tan\frac{\pi}{12}, \tan\left(\frac{\pi}{12}+x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. The value of k is.

**Q.5** Let $a + ar_1 + ar_1^{\ 2} + ........ \infty$ and $a + ar_2 + ar_2^{\ 2} + ....... \infty$ be two infinite series of positive numbers with the same first term. The sum of the first series is $r_1$ and the sum of the second series is $r_2$. The value of $(r_1 + r_2)$

**Q.6** If the equation $x^4 - (3m+2)x^2 + m^2 = 0$ $(m > 0)$ has four real solutions which are in A.P. then find the value of m.

**Q.7** Consider an A.P. $a_1$ , $a_2$ , $a_3$,...... such that $a_3 + a_5 + a_8 = 11$ and $a_4 + a_2 = -2$, then the value of $a_1 + a_6 + a_7$ is

**Q.8** The sum $\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}$ equal to

**Q.9** Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

**Q.10** Let K is a positive integer such that 36 + K, 300 + K, 596 + K are the squares of three consecutive terms of an arithmetic progression. Find K.

**Q.11** Let $S_k$, k = 1, 2,...., 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is 1/k. Then the value of

$$\frac{100^2}{100!} + \sum_{k=1}^{100} \left| (k^2 - 3k + 1) S_k \right| \text{ is } -$$

**Q.12** Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2_{ak-1} - a_{k-2}$ for k = 3, 4, …, 11. If $\frac{a_1^2 + a_2^2 + .... + a_{11}^2}{11} = 90$, then the value of

$\frac{a_1 + a_2 + .... + a_{11}}{11}$ is equal to :

**Q.13** Let $a_1, a_2, a_3, ....., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then value of $a_2$ greater than 3 is –

**Q.14** Let $a_1, a_2, a_3$, ..... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

**Q.15** A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = ________

**Q.16** The harmonic mean of the roots of the equation

$$\left(5+\sqrt{2}\right)x^2 - \left(4+\sqrt{5}\right)x + 8 + 2\sqrt{5} = 0 \text{ is}$$

**Q.17** The number of solutions of $\log_4(x - 1) = \log_2(x - 3)$ is

# EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

**Q.1** The sum of the series $1^3 - 2^3 + 3^3 - ......+ 9^3 =$ **[AIEEE 2002]**
(A) 300 (B) 125
(C) 425 (D) 0

**Q.2** If the sum of an infinite GP is 20 and sum of their square is 100 then common ratio will be = **[AIEEE 2002]**
(A) 1/2 (B) 1/4
(C) 3/5 (D) 1

**Q.3** If the third term of an A.P. is 7 and its 7th term is 2 more than three times of its 3rd term, then sum of its first 20 terms is- **[AIEEE 2002]**
(A) 228 (B) 74
(C) 740 (D) 1090

**Q.4** If $x_1, x_2, x_3$ and $y_1, y_2, y_3$ are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ **[AIEEE 2003]**
(A) are vertices of a triangle (B) lie on a straight line
(C) lie on an ellipse (D) lie on a circle

**Q.5** If the system of linear equations $x + 2ay + az = 0$; $x + 3by + bz = 0$ ; $x + 4cy + cz = 0$ has a non- zero solution, then a, b, c **[AIEEE 2003]**
(A) satisfy $a + 2b + 3c = 0$ (B) are in A.P.
(C) are in G.P. (D) are in H.P

**Q.6** Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- **[AIEEE 2004]**
(A) $x^2 + 18x + 16 = 0$ (B) $x^2 - 18x + 16 = 0$
(C) $x^2 + 18x - 16 = 0$ (D) $x^2 - 18x - 16 = 0$

**Q.7** Let $T_r$ be the $r^{th}$ term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals- **[AIEEE 2004]**
(A) 0 (B) 1
(C) 1/mn (D) $\frac{1}{m} + \frac{1}{n}$

**Q.8** The sum of the first n terms of the series $1^2 + 2.\,2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + .....$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is- **[AIEEE 2004]**
(A) $\frac{3n(n+1)}{2}$ (B) $\frac{n^2(n+1)}{2}$
(C) $\frac{n(n+1)^2}{4}$ (D) $\left[\frac{n(n+1)}{2}\right]^2$

**Q.9** If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$ then x, y, z are in - **[AIEEE 2005]**
(A) GP (B) AP
(C) AGP (D) HP

**Q.10** If in a $\Delta ABC$, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in **[AIEEE- 2005]**
(A) G.P. (B) A.P.
(C) AGP (D) H.P.

**Q.11** Let $a_1, a_2, a_3$, ..... be terms of an A.P. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals – **[AIEEE- 2006]**
(A) 7/2 (B) 2/7
(C) 11/41 (D) 41/11

**Q.12** If $a_1, a_2$, ..... $a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + .... + a_{n-1}a_n$ is equal to – **[AIEEE- 2006]**
(A) $(n-1)(a_1 - a_n)$ (B) $na_1a_n$
(C) $(n-1)a_1a_n$ (D) $n(a_1 - a_n)$

**Q.13** In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE- 2007]**
(A) $\frac{1}{2}(1-\sqrt{5})$ (B) $\frac{1}{2}\sqrt{5}$
(C) $\frac{1}{2}\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5}-1)$

**Q.14** The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[AIEEE 2008]**
(A) – 12 (B) 12
(C) 4 (D) – 4

**Q.15** Sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + .......$ **[AIEEE 2009]**
(A) 2 (B) 3
(C) 4 (D) 6

**Q.16** A person is to count 4500 currency notes. Let $a_n$ denote the number of notes he counts in the $n^{th}$ minute. If $a_1 = a_2 = ...... = a_{10} = 150$ and $a_{10}, a_{11}$, ...... are in A.P. with common difference –2, then the time taken by him to count all notes is – **[AIEEE 2010]**
(A) 34 minutes (B) 125 minutes
(C) 135 minutes (D) 24 minutes

**Q.17** A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after : **[AIEEE 2011]**
(A) 18 months (B) 19 months
(C) 20 months (D) 21 months

**Q.18** **Statement-1 :** The sum of the series $1+(1+2+4)+(4+6+9)+(9+12+16)+....+(361+380+400)$ is 8000.

**Statement-2 :** $\sum_{k=1}^{n}(k^3-(k-1)^3=n^3$, for any natural number n. **[AIEEE 2012]**

(A) Statement-1 is false, Statement-2 is true.
(B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(C) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1.
(D) Statement-1 is true, statement-2 is false.

**Q.19** If 100 times the $100^{th}$ term of an AP with non zero common difference equals the 50 times its $50^{th}$ term, then the $150^{th}$ term of this AP is : **[AIEEE 2012]**
(A) –150 (B) 150 times its 50th term
(C) 150 (D) zero

**Q.20** The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is – **[JEE MAIN 2013]**
(A) $\frac{7}{81}(179-10^{-20})$ (B) $\frac{7}{9}(99-10^{-20})$
(C) $\frac{7}{81}(179+10^{-20})$ (D) $\frac{7}{9}(99+10^{-20})$

**Q.21** If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then – **[JEE MAIN 2013]**
(A) $x=y=z$ (B) $2x=3y=6z$
(C) $6x=3y=2z$ (D) $6x=4y=3z$

**Q.22** Three positive numbers from an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is – **[JEE MAIN 2014]**
(A) $\sqrt{2}+\sqrt{3}$ (B) $3+\sqrt{2}$
(C) $2-\sqrt{3}$ (D) $2+\sqrt{3}$

**Q.23** If $(10)^9+2(11)^1(10)^8+3(11)^2(10)^7+.....+10(11)^9=k(10)^9$, then k is equal to **[JEE MAIN 2014]**
(A) 121/10 (B) 441/100
(C) 100 (D) 110

**Q.24** The sum of first 9 terms of the series
$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ is **[JEE MAIN 2015]**
(A) 96 (B) 142
(C) 192 (D) 71

**Q.25** If m is the A.M. of two distinct real numbers $l$ and n ($l$, n > 1) and $G_1$, $G_2$ and $G_3$ are three geometric means between $l$ and n, then $G_1^4+2G_2^4+G_3^4$ equals.
(A) $4\,lm^2n$ (B) $4\,lmn^2$ **[JEE MAIN 2015]**
(C) $4\,l^2m^2n^2$ (D) $4\,l^2mn$

**Q.26** If the $2^{nd}$, $5^{th}$ and $9^{th}$ terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is
(A) 4/3 (B) 1 **[JEE MAIN 2016]**
(C) 7/4 (D) 8/5

**Q.27** If the sum of the first ten terms of the series
$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+....$ is $\frac{16}{5}m$, then m is equal to – **[JEE MAIN 2016]**
(A) 101 (B) 100
(C) 99 (D) 102

**Q.28** For any three positive real numbers a, b and c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then : **[JEE MAIN 2017]**
(A) a, b and c are in A.P. (B) a, b and c are in G.P.
(C) b, c and a are in G.P. (D) b, c and a are in A.P.

**Q.29** Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that
$\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9+a_{43}=66$. If $a_1^2+a_2^2+.....+a_{17}^2=140m$, then m equal to
(A) 34 (B) 33 **[JEE MAIN 2018]**
(C) 66 (D) 68

**Q.30** Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+.....$
If $B-2A=100\lambda$, then $\lambda$ is equal to: **[JEE MAIN 2018]**
(A) 464 (B) 496
(C) 232 (D) 248

**Q.31** If a, b and c be three distinct real numbers in G. P. and $a+b+c=xb$, then x cannot be : **[JEE MAIN 2019 (JAN)]**
(A) 4 (B) –3
(C) –2 (D) 2

**Q.32** Let $a_1, a_2, .., a_{30}$ be an A. P., $S=\sum_{i=1}^{30}a_i$ and $T=\sum_{i=1}^{15}a_{(2i-1)}$.
If $a_5=27$ and $S-2T=75$, then $a_{10}=$ **[JEE MAIN 2019 (JAN)]**
(A) 57 (B) 47
(C) 42 (D) 52

**Q.33** The sum of all natural numbers 'n' such that $100<n<200$ and H.C.F. $(91, n)>1$ is : **[JEE MAIN 2019 (APRIL)]**
(A) 3221 (B) 3121
(C) 3203 (D) 3303

**Q.34** The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to- **[JEE MAIN 2019 (APRIL)]**
(A) $2-\frac{3}{2^{17}}$ (B) $2-\frac{11}{2^{19}}$
(C) $1-\frac{11}{2^{20}}$ (D) $2-\frac{21}{2^{20}}$

**Q.35** If three distinct numbers a,b,c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct? **[JEE MAIN 2019 (APRIL)]**

(A) d, e, f are in A.P. (B) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P.
(C) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P. (D) d, e, f are in G.P.

**Q.36** Let the sum of the first n terms of a non-constant A.P., $a_1$, $a_2$, $a_3$, ..... be $50n+\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair $(d, a_{50})$ is equal to **[JEE MAIN 2019 (APRIL)]**
(A) (A, 50 + 46A) (B) (A, 50+45A)
(C) (50, 50+46A) (D) (50, 50+45A)

**Q.37** If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11th term is **[JEE MAIN 2019 (APRIL)]**
(A) –25 (B) 25
(C) –36 (D) –35

**Q.38** The sum of the series $1+2\times3+3\times5+4\times7+....$ upto 11th term is :- **[JEE MAIN 2019 (APRIL)]**
(A) 915 (B) 946
(C) 945 (D) 916

**Q.39** The sum $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+...$ **[JEE MAIN 2019 (APRIL)]**
(A) 660 (B) 620
(C) 680 (D) 600

**Q.40** If $a_1, a_2, a_3, ........., a_n$ are in A.P. and $a_1+a_4+a_7+.........+a_{16}=114$, then $a_1+a_6+a_{11}+a_{16}$ is equal to : **[JEE MAIN 2019 (APRIL)]**
(A) 38 (B) 98
(C) 76 (D) 64

**Q.41** The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...$
$$+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15)$$
**[JEE MAIN 2019 (APRIL)]**
(A) 1240 (B) 1860
(C) 660 (D) 620

**Q.42** Let a, b and c be in G. P. with common ratio r, where $a\neq0$ and $0<r\leq1/2$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is : **[JEE MAIN 2019 (APRIL)]**
(A) (7/3) a (B) a
(C) (2/3) a (D) 5a

**Q.43** If $a_1, a_2, a_3,.....$ are in A.P. such that $a_1+a_7+a_{16}=40$, then the sum of the first 15 terms of this A.P. is : **[JEE MAIN 2019 (APRIL)]**
(A) 200 (B) 280
(C) 120 (D) 150

**Q.44** If $(2^{1-x}+2^{1+x})$, f (x), $(3^x+3^{-x})$ are in A.P. then minimum value of f (x) is **[JEE MAIN 2020 (JAN)]**
(A) 1 (B) 2
(C) 3 (D) 4

**Q.45** Find the sum $\sum_{k=1}^{20}(1+2+3+....+k)$ **[JEE MAIN 2020 (JAN)]**

**Q.46** For an A.P. $T_{10}=1/20$, $T_{20}=1/10$. Find sum of first 200 term. **[JEE MAIN 2020 (JAN)]**
(A) $201\frac{1}{2}$ (B) $101\frac{1}{2}$
(C) $301\frac{1}{2}$ (D) $100\frac{1}{2}$

**Q.47** $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to **[JEE MAIN 2020 (JAN)]**

**Q.48** The product $2^{\frac{1}{4}}.4^{\frac{1}{16}}.8^{\frac{1}{48}}.16^{\frac{1}{128}}...$ to $\infty$ is equal to : **[JEE MAIN 2020 (JAN)]**
(A) $2^{1/2}$ (B) $2^{1/4}$
(C) 2 (D) 1

**Q.49** Let $a_n$ be the $n^{th}$ term of a G.P. of positive terms.
If $\sum_{n=1}^{100}a_{2n+1}=200$ and $\sum_{n=1}^{100}a_{2n}=100$, then $\sum_{n=1}^{200}a_n$ is equal to – **[JEE MAIN 2020 (JAN)]**
(A) 225 (B) 175
(C) 300 (D) 150

**Q.50** The number of terms common to the two A.P.'s 3, 7, 11, ....., 407 and 2, 9, 16, ....., 709 is ______. **[JEE MAIN 2020 (JAN)]**

**Q.51** Let $3+4+8+9+13+14+18+.........40$ terms = S. If S = (102)m then m = **[JEE MAIN 2020 (JAN)]**
(A) 20 (B) 25
(C) 10 (D) 5

**Q.52** $a_1, a_2, a_3 .....a_9$ are in GP where $a_1<0$,
$a_1+a_2=4$, $a_3+a_4=16$, if $\sum_{i=1}^{9}a_i=4\lambda$,
then $\lambda$ is equal to **[JEE MAIN 2020 (JAN)]**
(A) –513 (B) –511/3
(C) –171 (D) 171

# ANSWER KEY

**EXERCISE - 1**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | A | B | C | D | D | C | C | B | C | B | C | B | C | C | A | A | C | C | C | D |
| Q | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| A | D | B | A | C | D | B | D | A | D | C | A | C | B | D | C | B | C | B | C | B |
| Q | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| A | B | A | A | B | C | A | A | A | A | A | B | A | C | D | A | C | B | C | D | D |
| Q | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| A | C | D | B | B | C | D | C | D | C | B | B | B | B | B | B | A | B | A | A | D |
| Q | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| A | A | A | A | B | C | A | C | A | C | C | B | B | A | A | A | C | D | A | C | C |

**EXERCISE - 2**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | C | D | B | B | B | C | C | C | B | D | D | C | D | C | C | B | A | A | B | A | B | A | A | B | A |
| A | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | |
| B | B | B | A | A | C | A | A | D | B | A | A | D | C | B | C | D | A | D | B | B | A | A | C | C | |

**EXERCISE - 3**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 11 | 111 | 8 | 4950 | 1 | 6 | 7 | 8 | 31 | 925 | 3 | 0 | 9 | 25 | 5 | 4 | 1 |

**EXERCISE - 4**

| Q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | C | C | C | B | D | B | A | B | D | B | C | C | D | A | B | A | D | B | D | C | A | D | C | A | A | A | A | D | A | D | D | D |
| Q | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | | | | | | | | | | | | |
| A | B | B | C | A | A | B | A | C | D | B | A | C | 1540 | D | 504 | A | D | 14 | A | C | | | | | | | | | | | | |

# CHAPTER- 6 : SEQUENCES & SERIES

# SOLUTIONS TO TRY IT YOURSELF

## TRY IT YOURSELF-1

**(1)** The smallest and the largest numbers of three digits, which are divisible by 7 are 105 and 994, respectively. So, the sequence of three digit numbers which are divisible by 7 is 105, 112, 119, ... , 994. Clearly, it is an A.P. with first term a = 105 and common difference d = 7. Let there be n terms in this sequence. Then, $a_n = 994$

$\Rightarrow a + (n - 1)d = 994$

$\Rightarrow 105 + (n - 1) \times 7 = 994 \Rightarrow n = 128$

Now, required sum is

$$\frac{n}{2}[2a + (n-1)d] = \frac{128}{2}[2 \times 105 + (128 - 1) \times 7] = 70336$$

**(2)** Let the numbers be (a – d), a, (a + d). Therefore,

$(a - d) + a + (a + d) = -3 \Rightarrow 3a = -3 \Rightarrow a = -1$

and $(a - d)(a)(a + d) = 8$

$\Rightarrow a(a^2 - d^2) = 8 \Rightarrow (-1)(1 - d^2) = 8$  $[\because a = -1]$

$\Rightarrow d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are –4, –1, 2.

If d = –3, the numbers are 2, –1, – 4.

So, the numbers are –4, –1, 2 or 2, –1, –4.

**(3)** Let the digits at ones, tens and hundreds place be (a – d), a and (a + d), respectively. Then the number is

$(a + d) \times 100 + a \times 10 + (a - d) = 111a + 99d$

The number obtained by reversing the digits is

$(a - d) \times 100 + a \times 10 + (a + d) = 111a - 99d$

It is given that $(a - d) + a + (a + d) = 15$ ..... (i)

and $111a - 99d = 111a + 99d - 594$

$\therefore 3a = 15$ and $198d = 594$

$\Rightarrow a = 5$ and $d = 3$

So, the number is $111 \times 5 + 99 \times 3 = 852$.

**(4)** Assume $A_1, A_2, A_3, \ldots, A_{11}$ be the eleven A.M.'s between 28 and 10, so 28, $A_1, A_2, \ldots$, A11, 10 are in A.P.

Let d be the common difference of the A.P.

The number of terms is 13. Now,

$10 = T_{13} = T_1 + 12d = 28 + 12d$

$$\Rightarrow d = \frac{10 - 28}{12} = -\frac{18}{12} = -\frac{3}{2}$$

Hence, the number of integral A.M.'s is 5.

**(5)** Let the four numbers in an A.P. be a – 3d, a – d, a + d, a+3d.

Sum of the terms is , $4a = 20 \Rightarrow a = 5$

Sum of their squares is $4a^2 + 20d^2 = 120$

$\Rightarrow 20d^2 = 120 - 4 \times 25 = 20$

$\Rightarrow d^2 = 1$ or $d = \pm 1$

Hence, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

**(6)** **(C).** $T_m = a + (m - 1)d = 1/n$ and $T_n = a + (n - 1)d = 1/m$

$$\Rightarrow (m - n)d = \frac{1}{n} - \frac{1}{m} = \frac{m - n}{mn} \Rightarrow d = \frac{1}{mn}$$

$$\Rightarrow a = \frac{1}{mn} \therefore T_{mn} = a + (mn - 1)d = \frac{1}{mn} + (mn - 1)\frac{1}{mn}$$

$$= \frac{1}{mn} + 1 - \frac{1}{mn} = 1$$

**(7)** **(C).** 2 + 5 + 8 + ...... 2n terms

= 57 + 59 + 61 + ....... n terms

$$\Rightarrow \frac{2n}{2}[4 + (2n - 1)3] = \frac{n}{2}[114 + (n - 1)2]$$

$\Rightarrow 6n + 1 = n + 56 \Rightarrow 5n = 55 \Rightarrow n = 11$

**(8)** **(C).** $S_n = cn^2$ ; $S_{n-1} = c(n - 1)^2 = cn^2 + c - 2cn$

$T_n = 2cn - c$

$T_n^2 = (2cn - c)^2 = 4c^2 n^2 + c^2 - 4c^2 n$

$$\text{Sum} = \Sigma T_n^2 = \frac{4c^2.n(n+1)(2n+1)}{6} + nc^2 - 2c^2 n(n+1)$$

$$= \frac{2c^2 n(n+1)(2n+1) + 3nc^2 - 6c^2 n(n+1)}{3}$$

$$= \frac{nc^2[4n^2 + 6n + 2 + 3 - 6n - 6]}{3} = \frac{nc^2(4n^2 - 1)}{3}$$

**(9)** **3 or 9.**

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6 + (5n-1)d]}{\frac{n}{2}[6 + (n-1)d]} = \frac{5[(6-d) + 5nd]}{[(6-d) + nd]}$$

d = 6 or d = 0.

Now, if d = 0 then $a_2 = 3$ else $a_2 = 9$

For single choice more appropriate choice is 9, but in principal, seems to have an error.

$\therefore a_2 = 3 + 6 = 9$

## TRY IT YOURSELF-2

**(1)** $t_5 = ar^4 = 2$

Product of its first 9 terms is

$a(ar)(ar^2) \ldots\ldots (ar^8) = a^9 r^{1+2+\ldots+8} = a^9 r^{(8/2)(1+8)}$

$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

**(2)** Let a be the first term and r the common ratio of the G.P.

then, $a_n = 2[a_{n+1} + a_{n+2} + a_{n+3} + \ldots\infty]$, for all $n \in N$ (Given)

$ar^{n-1} = 2[ar^n + ar^{n+1} + \ldots\infty]$

$$\Rightarrow ar^{n-1} = \frac{2ar^n}{1 - r} \Rightarrow 1 = \frac{2a}{1 - r} \Rightarrow r = \frac{1}{3}$$

**(3)** Let the three numbers be a/r, a and ar.

Then, product = 216. Hence, $(a/r) \times a \times ar = 216$

$\Rightarrow a^3 = 216 \Rightarrow a = 6$.

Sum of the products in pairs is 156. Hence,

$$\frac{a}{r}a + a\,ar + \frac{a}{r}ar = 156 \Rightarrow a^2\left(\frac{1}{r} + r + 1\right) = 156$$

$$\Rightarrow 36\left(\frac{1 + r^2 + r}{r}\right) = 156$$

$\Rightarrow 3(r^2 + r + 1) = 13r \Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow$ $(3r-1)(r-3)=0 \Rightarrow r=1/3$ or $r=3$.
Hence, putting the values of a and r, the required numbers are 18, 6, 2 or 2, 6, 18.

**(4)** We have, 4, $g_1$, $g_2$, $g_3$, 1/4 is a G.P.
Here, $a=4$, $g_1=ar=4r$, $g_2=ar^2$, $g_3=ar^3$,
$g_4=ar^4=4r^4=1/4$

$$\Rightarrow \quad r^4=\frac{1}{16}=\left(\frac{1}{2}\right)^4 \Rightarrow r=\frac{1}{2}$$

Now, the product of three G.M.'s

$$g_1g_2g_3=ar\times ar^2\times ar^3=a^3r^6=4^3\times\left(\frac{1}{2}\right)^6=\frac{4^3}{4^3}=1$$

**(5)** **(D).** Sum = 4 and second term = 3/4, it is given that first term is a common ratio r.

$$\frac{a}{1-r}=4 \text{ and } ar=\frac{3}{4}\Rightarrow r=\frac{3}{4a}$$

$$\text{Therefore, } \frac{a}{1-\frac{3}{4a}}=4\Rightarrow\frac{4a^2}{4a-3}=4$$

or $a^2-4a+3\Rightarrow(a-1)(a-3)=0\Rightarrow a=1$ or a 3
When $a=1$, $r=3/4$ and when $a=3$, $r=1/4$

**(6)** **(A).** $\alpha,\beta$ are the roots of $x^2-x+p=0$
$\therefore$ $\alpha+\beta=1$ ......... (1)
$\alpha\beta=p$ ......... (2)
$\gamma,\delta$ are the roots of $x^2-4x+q=0$
$\therefore$ $\gamma+\delta$ are the roots of $x^2-4x+q=0$
$\therefore$ $\gamma+\delta=4$ ......... (3)
$\gamma\delta=q$ ......... (4)
$\alpha,\beta,\gamma,\delta$ are in G.P.
$\therefore$ Let $\alpha=a$, $\beta=ar$, $\gamma=ar^2$, $\delta=ar^3$.
Substituting these values in equations (1), (2), (3) and (4), we get
$a+ar=1$ ......... (5)
$a^2r=p$ ......... (6)
$ar^2+ar^3=4$ ......... (7)
$a^2r^5=q$ ......... (8)
Dividing eq. (7) by eq. (5) we get

$$\frac{ar^2(1+r)}{a(1+r)}=\frac{4}{1}\Rightarrow r^2=4\Rightarrow r=2,-2$$

$$(5)\Rightarrow a=\frac{1}{1+r}=\frac{1}{1+2} \text{ or } \frac{1}{1-2}=\frac{1}{3} \text{ or } -1$$

As p is an integer (given), r is also an integer (2 or –2)

$\therefore$ $(6)\Rightarrow a\neq\frac{1}{3}$. Hence, $a=-1$ and $r=-2$

$\therefore$ $p=(-1)^2\times(-2)=-2p=(-1)^2\times(-2)=-2$
$q=(-1)^2\times(-2)^5=-32$

**(7)** **(D).** Given that a, b, c are in A.P.
$\Rightarrow$ $2b=a+c$
but given $a+b+c=3/2 \Rightarrow 3b=3/2$
$\Rightarrow$ $b=1/2$ and then $a+c=1$
Again, $a^2, b^2, c^2$ are in G.P.

$\Rightarrow$ $b^4=a^2c^2 \Rightarrow b^2=\pm ac$

$\Rightarrow$ $ac=\frac{1}{4}$ or $-\frac{1}{4}$ and $a+c=1$ ........ (1)

Considering $a+c=1$ and $ac=1/4$
$\Rightarrow (a-c)^2=1+1=2 \Rightarrow a-c=\pm\sqrt{2}$

but $a<c \Rightarrow a-c=-\sqrt{2}$ .......... (2)

Solving eq. (1) and (2), we get $a=\frac{1}{2}-\frac{1}{\sqrt{2}}$

**(8)** **(B).** $\frac{x}{1-r}=5\Rightarrow r=1-\frac{x}{5}$
Since G.P. contains infinite terms
$\therefore$ $-1<r<1$

$$\Rightarrow -1<1-\frac{x}{3}<1\Rightarrow 0<-\frac{x}{5}<2\Rightarrow -10<x<0$$

## TRY IT YOURSELF-3

**(1)** Let the H.P. be $\frac{1}{a},\frac{1}{a+d},\frac{1}{a+2d},.......,\frac{1}{a+(n-1)d},...$

Then, $a_8=\frac{1}{2}$ and $a_{14}=\frac{1}{3}$

$$\Rightarrow \frac{1}{a+7d}=\frac{1}{2} \text{ and } \frac{1}{a+13d}=\frac{1}{3}$$

$$\left[\because a_n=\frac{1}{a+(n-1)d}\right]$$

$\Rightarrow$ $a+7d=2$ and $a+13d=3$
$\Rightarrow$ $a=5/6$, $d=1/6$

$$\text{Now, } a_{20}=\frac{1}{a+19d}=\frac{1}{\frac{5}{6}+\frac{19}{6}}=\frac{1}{14}$$

$$\text{and } a_n=\frac{1}{a+(n-1)d}=\frac{1}{\frac{5}{6}+(n-1)\times\frac{1}{6}}=\frac{6}{n+4}$$

**(2)** 1/16, a, b are in G.P. Hence, $a^2=b/16$ or $16a^2=b$ ..... (1)

$$\text{a, b, 1/6 are in H.P. Hence, } b=\frac{2a\frac{1}{6}}{a+\frac{1}{6}}=\frac{2a}{6a+1}$$

From eq. (1) and (2),

$$16a^2=\frac{2a}{6a+1}\Rightarrow 2a\left(8a-\frac{1}{6a+1}\right)=0$$

$\Rightarrow$ $8a(6a+1)-1=0$
$\Rightarrow$ $48a^2+8a-1=0$ $(\because a\neq 0)$
$\Rightarrow$ $(4a+1)(12a-1)=0$

**(3)** $\frac{H}{P}+\frac{H}{Q}=H\left(\frac{1}{P}+\frac{1}{Q}\right)=\frac{2PQ}{P+Q}\frac{P+Q}{PQ}=2$

**(4)** $A-G=2$ ....... (1)

$G-H=8/5$ ....... (2)

$G^2=AH=(G+2)(G-8/5)\Rightarrow G=8$

$\Rightarrow ab=64$ ....... (3)

From eq. (1), $A=10$

$\Rightarrow a+b=20$ ....... (4)

Solving eq. (3) and (4), we get a = 4 and b = 16 or a = 16 and b = 4.

**(5)** The difference between the successive terms are

$15-3=12, 35-15=20, 63-35=28, .....$

Clearly, these differences are in A.P.

Let $T_n$ be the $n^{th}$ term and $S_n$ denote the sum to n terms of the given series. Then,

$S_n=3+15+35+63+......+T_{n-1}+T_n$ ..... (1)

$S_n=\quad 3+15+35+63+......+T_{n-1}+T_n$ ..... (2)

---

$0=3+[12+20+28+......+(n-1)\text{ terms}]-T_n$

[Subtracting (2) from (1)]

$$\Rightarrow T_n=3+\frac{(n-1)}{2}[2\times 12+(n-1-1)\times 8]$$

$=3+(n-1)(12+4n-8)=3+(n-1)(4n+4)$

$=4n^2-1$

$$\Rightarrow S_n=\sum_{k=1}^{n}T_k=\sum_{k=1}^{n}(4k^2-1)=4\sum_{k=1}^{n}k^2-\sum_{k=1}^{n}1$$

$$=4\left\{\frac{n(n+1)(2n+1)}{6}\right\}-n=\frac{n}{3}(4n^2+6n-1)$$

**(6)** $S=3+(3+d)\frac{1}{4}+(3+2d)\frac{1}{4^2}+....\infty$ ....... (1)

$\Rightarrow \frac{1}{4}S=(3)\frac{1}{4}+(3+d)\frac{1}{4^2}+....\infty$ ....... (2)

Subtracting eq. (2) from eq. (1), we have

$$\frac{3}{4}S=3+(d)\frac{1}{4}+(d)\frac{1}{4^2}+....\infty=3+\frac{\frac{d}{4}}{1-\frac{1}{4}}=3+\frac{d}{3}$$

$$\Rightarrow S=4+\frac{4d}{9}, \text{ Given } 4+\frac{4d}{9}=\frac{44}{9}\Rightarrow\frac{4d}{9}=\frac{8}{9}\Rightarrow d=2$$

**(7)** **(A).** Since AM ≥ GM, then

$$\frac{(a+b)+(c+d)}{2}\geq\sqrt{(a+b)(c+d)}\Rightarrow M\leq 1$$

Also, $(a+b)+(c+d)>0$ ($\because$ a, b, c, d > 0)

$\therefore\ 0<M\leq 1$

**(8)** **(B).**

$$\frac{1}{H}=\frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{\alpha+\beta}{2\alpha\beta}=\frac{4+\sqrt{5}}{5+\sqrt{2}}\times\frac{5+\sqrt{2}}{8+2\sqrt{5}}\times\frac{1}{2}=\frac{1}{4}$$

$H=4$

**(9)** **(D).** a, b, c, d are in A.P.

$\therefore$ d, c, b, a are also in A.P.

$\Rightarrow \frac{d}{abcd},\frac{c}{abcd},\frac{b}{abcd},\frac{a}{abcd}$ are also in A.P.

$\Rightarrow \frac{1}{abc},\frac{1}{abd},\frac{1}{acd},\frac{1}{bcd}$ are in A.P.

$\Rightarrow$ abc, abd, acd, bcd are in H.P.

# CHAPTER- 6 : SEQUENCES & SERIES
## EXERCISE-1

**(1)** **(A).** Let first term = a, common difference = d
Then $T_3 = a + 2d = 18$ and $T_7 = a + 6d = 30$
Solving these , a= 12, d= 3

$$\therefore S_{17} = \frac{17}{2}[2a + (17-1)d] = \frac{17}{2}[24 + 16 \times 3] = 612$$

**(2)** **(B).** We have first term = a, second term = b
$\therefore$ d = common difference = b – a
It is given that the middle term is c. This means that there are an odd number of terms in the AP. Let there be (2n+1) terms in the AP. Then $(n+1)^{th}$ term is the middle term.
$\therefore$ middle term = c $\Rightarrow$ a + nd = c

$$\Rightarrow a + n(b-a) = c \Rightarrow n = \frac{c-a}{b-a}$$

$$\therefore \text{Sum} = \frac{2n+1}{2}[2a + (2n+1-1)d]$$

$$= \frac{1}{2}\left\{2\left(\frac{c-a}{b-a}\right)+1\right\}\left[2a + 2\left(\frac{c-a}{b-a}\right)(b-a)\right]$$

$$= \frac{1}{2}\left\{\frac{2(c-a)}{b-a}+1\right\}\{2c\} = \frac{2c(c-a)}{b-a} + c$$

**(3)** **(C).** Required sum = ( sum of integers divisible by 2) + (sum of integers divisible by 5) – (sum of integers divisible by 2 and 5)
$= (2+4+6+....+100) + (5+10+15+....+100) - (10+20+....+100)$

$$= \frac{50}{2}[2\times 2 + (50-1)\times 2] + \frac{20}{2}[2\times 5 + (20-1)\times 10] - \frac{10}{2}[2\times 10 + (10-1)\times 10]$$

$= 50[2+49] + 10[10+95] - 5[20+90]$
$= 51 \times 50 + 105 \times 10 - 110 \times 5 = 3050$

**(4)** **(D).** Let d be the c.d. of the A.P. Now

$$\text{L.H.S.} = \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + .. + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}$$

$$= -\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+.....\sqrt{a_n}}{d}\right)$$

$$= -\frac{(\sqrt{a_1}-\sqrt{a_n})}{d} = \frac{1}{d}\frac{(a_n-a_1)}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{(n-1)d}{d\left[\sqrt{a_n}+\sqrt{a_1}\right]} \quad [\because a_n = a_1 + (n-1)d]$$

$$= \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$$

**(5)** **(D).** If a be the first term and d be the common difference of the AP , then
$T_9 = a + 8d = 35$
$T_{19} = a + 18d = 75$
Subtracting these equations, we get
$-10d = -40 \Rightarrow d = 4, a = 3$
$\therefore T_{20} = 3 + 19 \times 4 = 79$

**(6)** **(C).** Here a = 5, $\ell = 45$ $\quad S_n = 400$

$$S_n = \frac{n}{2}[a+\ell];\ 400 = \frac{n}{2}[5+45] \Rightarrow n = 16$$

**(7)** **(C).** Here $\frac{S_{n_1}}{S_{n_2}} = \frac{3n+1}{2n+3}$

$$\Rightarrow \frac{n/2\left[2a_1+(n-1)d_1\right]}{n/2\left[2a_2+(n-1)d_2\right]} = \frac{3n+1}{2n+3}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{3n+1}{2n+3}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3} \qquad ...(1)$$

$$\therefore \frac{T_{11_1}}{T_{11_2}} = \frac{a_1+10d_1}{a_2+10d_2} \qquad ...(2)$$

$$\frac{n-1}{2} = 10 \Rightarrow n = 21$$

putting the value of n in (1)

$$\frac{a_1+10d_1}{a_2+10d_2} = \frac{3\times 21+1}{2\times 21+3} = \frac{64}{45}$$

**(8)** **(B).** Here $d = \frac{3-\frac{1}{2}}{4+1} = \frac{1}{2} \therefore A_3 = a + 3d \Rightarrow \frac{1}{2} + 3 \times \frac{1}{2} = 2$

**(9)** **(C).** Here $2 + 3d = 14 \Rightarrow d = 4$

$$\therefore 4 = \frac{38-2}{n+1} \Rightarrow 4n + 4 = 36 \Rightarrow n = 8$$

**(10)** **(B).** Let the numbers are a– 3d, a– d, a + d, a + 3d
given a– 3d + a –d + a+ d + a+ 3d = 20
$\Rightarrow 4a = 20 \Rightarrow a = 5$
and $(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$
$\Rightarrow 4a^2 + 20d^2 = 120$
$\Rightarrow 4 \times 5^2 + 20d^2 = 120 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$
Hence numbers are 2, 4, 6, 8

**(11)** **(C).** (x+1), 3x, (4x + 2) in A.P.
$\Rightarrow 3x - (x+1) = (4x+2) - 3x \Rightarrow x = 3$
$\therefore a = 4, d = 9 - 4 = 5$
$\Rightarrow T_5 = 4 + 4(5) = 24$

**(12)** **(B).** Let the A.P. be a + ( a + d) + (a + 2d) + ....

$\because S_{10} = 4S_5 \therefore 2a + 9d = 4a + 8d \Rightarrow \frac{a}{d} = \frac{1}{2}$

**(13)** **(C).** Let roots be $\alpha, \beta, \gamma$ and $\alpha = a - d$, $\beta = a$, $\gamma = a + d$. Then

$\alpha + \beta + \gamma = 3a = -(-12) \Rightarrow a = 4$

$\alpha\beta\gamma = a(a^2 - d^2) = -(-28) \Rightarrow d = \pm 3$

**(14)** **(C).** $\sqrt{11 - 4\sqrt{6}} = 2\sqrt{2} - \sqrt{3}$,

$\sqrt{6 - 2\sqrt{3} + 2\sqrt{2} - 2\sqrt{6}} = 1 + \sqrt{2} - \sqrt{3}$,

$\sqrt{7 - 4\sqrt{3}} = 2 - \sqrt{3}$ and these form an A.P. with common difference $= 1 - \sqrt{2}$.

Hence required numbers are in H.P.

**(15)** **(A).** $\because a^2, b^2, c^2$ are in A.P.

$\therefore a^2 + ab + bc + ca, b^2 + bc + ca + ab, c^2 + ca + ab + bc$ ..... are also in A.P. [adding ab + bc + ca]

or (a+c) (a+ b), (b+ c) (a+b), (c+ a) (b+ c) .. are also in A.P.

or $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

[dividing by (a + b) (b + c) (c + a)]

**(16)** **(A).** The given series is arithmetic whose first term = 20, common difference = – 2/3

As the common difference is negative, the terms will become negative after some stage. So the sum is maximum if only positive terms are added.

Now $t_n = 20 + (n-1)(-2/3) \ge 0$ if $60 - 2(n-1) \ge 0$

or $62 \ge 2n$ or $31 \ge n$

$\therefore$ The first 31 terms are non-negative

$\therefore$ Maximum sum

$$= S_{31} = \frac{31}{2}\left\{2 \times 20 + (31-1)\left(-\frac{2}{3}\right)\right\} = \frac{31}{2}\{40 - 20\} = 310$$

**(17)** **(C).** Let a be the first term and $x$ be the common difference of the A.P. Then $a + 5x = 2 \Rightarrow a = 2 - 5x$

Let $P = a_1 a_4 a_5 = a(a + 3x)(a + 4x)$

$= (2 - 5x)(2 - 2x)(2 - x) = 2(-5x^3 + 17x^2 - 16x + 4)$

Now $\frac{dP}{dx} = 0 \Rightarrow x = \frac{8}{5}, \frac{2}{3}$.

Clearly, $\frac{d^2P}{dx^2} > 0$ for $x = \frac{2}{3}$

Hence $P$ is least for $x = \frac{2}{3}$.

**(18)** **(C).** Let the number of sides of the polygon be n. Then the sum of interior angles of the polygon

$= (2n - 4)\frac{\pi}{2} = (n - 2)\pi$

Since the angles are in A.P. and $a = 120^o, d = 5$,

therefore $\frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$

$\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$

But $n = 16$ gives $T_{16} = a + 15d = 120^o + 15.5^o = 195^o$, which is impossible as interior angle cannot be greater than $180^o$. Hence n = 9.

**(19)** **(C).** Given that $\frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \Rightarrow \frac{a + \frac{1}{2}(m-1)d}{a + \frac{1}{2}(n-1)d} = \frac{m}{n}$

$\Rightarrow an + \frac{1}{2}(m-1)nd = am + \frac{1}{2}(n-1)md$

$\Rightarrow a(n-m) + \frac{d}{2}[mn - n - mn + m] = 0$

$\Rightarrow a(n-m) + \frac{d}{2}(m-n) = 0 \Rightarrow a = \frac{d}{2}$ or $d = 2a$

So, required ratio,

$\frac{T_m}{T_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a}$

$= \frac{1 + 2m - 2}{1 + 2n - 2} = \frac{2m - 1}{2n - 1}$.

**(20)** **(D).** $S = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 406 = \frac{n}{2}[6 + (n-1)4] \Rightarrow 812 = n[6 + 4n - 4]$

$\Rightarrow 812 = 2n + 4n^2 \Rightarrow 406 = 2n^2 + n$

$\Rightarrow 2n^2 + n - 406 = 0$

$\Rightarrow n = \frac{-1 \pm \sqrt{1 + 4.2.406}}{2.2} = \frac{-1 \pm \sqrt{3249}}{4} = \frac{-1 \pm 57}{4}$

Taking (+) sign, $n = \frac{-1 + 57}{4} = 14$.

**(21)** **(D).** Let $A_1, A_2, A_3$ and $A_4$ are four numbers in A.P.

$A_1 + A_4 = 8$ …..(i) and $A_2 . A_3 = 15$ …..(ii)

The sum of terms equidistant from the beginning and end is constant and is equal to sum of first and last terms.

Hence, $A_2 + A_3 = A_1 + A_4 = 8$ …..(iii)

From (ii) and (iii),

$A_2 + \frac{15}{A_2} = 8 \Rightarrow A_2^2 - 8A_2 + 15 = 0$

$A_2 = 3$ or $5$ and $A_3 = 5$ or $3$.

As we know, $A_2 = \frac{A_1 + A_3}{2} \Rightarrow A_1 = 2A_2 - A_3$

$\Rightarrow A_1 = 2 \times 3 - 5 = 1$ and $A_4 = 8 - A_1 = 7$

Hence the series is, 1, 3, 5, 7.

So that least number of series is 1.

**(22)** **(B).** a, b, c are in A.P.

So 2b = a + c, then straight line $ax + by + c = 0$ will pass through $(1, -2)$ because if the line satisfies the condition $a - 2b + c = 0$ or 2b = a + c.

**(23)** **(A).** $\frac{S_n}{S_m} = \frac{n^4}{m^4}$.

Using $S_n = \frac{n}{2}[2a_1 + d(n-1)]$ and $S_m = \frac{m}{2}[2a_1 + d(m-1)]$

$\Rightarrow \frac{a_{m+1}}{a_{n+1}} = \frac{(2m+1)^3}{(2n+1)^3}$ after simplification.

**(24)** **(C).** Let the number of days be n.

Hence a worker can do $\left(\frac{1}{150n}\right)^{th}$ part of the work in a day. Accordingly,

$[150 + 146 + 142 + ....... + \text{upto } (n+8)\text{terms}] \times \frac{1}{150n} = 1$

$\Rightarrow n = 17$

Therefore number of total days in completion

$= 17 + 8 = 25$.

**(25)** **(D).** $m^{th}$ mean between $a, 2b$ is $a + \frac{m(2b-a)}{n+1}$ ……(i)

and $m^{th}$ mean between $2a, b$ is $2a + \frac{m(b-2a)}{n+1}$ ......(ii)

Accordingly, $a + \frac{m(2b-a)}{n+1} = 2a + \frac{m(b-2a)}{n+1}$

$\Rightarrow m(2b-a) = a(n+1) + m(b-2a)$

$\Rightarrow a(n-m+1) = bm$

$\Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$.

**(26)** **(B).** Common terms will be 21, 41, 61, ........

$21 + (n-1)20 \le 417 \Rightarrow n \le 20.8 \Rightarrow n = 20$

**(27)** **(D).** Let first A.P. is $a_1, a_1 + d_1, a_1 + 2d_1$........

$a_1$ (first term), $d_1$ (common difference)

Second A.P. is $a_2, a_2 + d_2, a_2 + 2d_2$........

$a_2$ (first term), $d_2$ (common difference)

given is $\frac{n/2\,[2a_1 + (n-1)d_1]}{n/2\,[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$

$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$

Put $\frac{n-1}{2} = 10$ or $n = 21$ to get

$\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{7 \times 21 + 1}{4 \times 21 + 27} = \frac{148}{111}$

**(28)** **(A).** By the method of differences, $t_n = 1 + (n-1)n$

Given $1 + n(n-1) = 9901 \Rightarrow n(n-1) = 9900$ which is satisfied by n = 100

**(29)** **(D).** $x^3 + ax^2 + bx + c = 0$

Let $\alpha = -1, \beta = 1, \gamma = 3$ and $(x+1)(x-1)(x-3) = 0$

$x^3 - 3x - x + 3 = 0 \Rightarrow a = -3, b = -1$ and $c = 3$

Substitute in options $2a^3 - 9ab = -27c$ satisfies.

**(30)** **(C).** Since x, 2x + 2 and 3x + 3 are in G.P.

$\therefore (2x+2)^2 = x(3x+3)$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+1)(x+4) = 0 \Rightarrow x = -1, -4$

$\Rightarrow x = -4 \qquad (\because x \ne -1)$

$\Rightarrow$ numbers are – 4, – 6, – 9

$\therefore$ First term = – 4 and c.r. = 3/2

Hence $T_4 = (-4)(3/2)^3 = -27/2$

**(31)** **(A).** The terms from a G.P. with common ratio = 1/3

Required form $= 16.2\left(\frac{1-\left(\frac{1}{3}\right)^7}{1-\frac{1}{3}}\right) = 8.1\left(\frac{3^7-1}{3^6}\right)$

$= \frac{2186}{90} = \frac{1093}{45}$

**(32)** **(C).** Here $\left(\frac{T_2}{T_1}\right)^{1/(2-1)} = \left(\frac{T_8}{T_2}\right)^{1/(8-2)} \quad \therefore \frac{n^n}{n^{-4}} = \left(\frac{n^{52}}{n^n}\right)^{1/6}$

or $n^{n+4} = n^{(52-n)/6}$ or $n + 4 = \frac{52-n}{6} \Rightarrow n = 4$

**(33)** **(B).** Let b = ar and c = $ar^2$, where 0 < r < 1. Now, a, 2b and 3c form an AP.

$\therefore 4b = a + 3c \Rightarrow 4ar = a + 3ar^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (3r-1)(r-1) = 0$

$\Rightarrow r = 1/3 \qquad [\because 0 < r < 1]$

(34) (D). As given $a + ar = 1$ ...(1)

$a = 2\left(\frac{ar}{1-r}\right)$ ...(2)

From (2) $1 - r = 2r \quad \therefore r = 1/3$
So from (1) $a = 3/4$

(35) (C). $r = \left(\frac{5}{160}\right)^{\frac{1}{4+1}} = \left(\frac{1}{32}\right)^{\frac{1}{5}} = \frac{1}{2}$

$G_3 = ar^3 \Rightarrow 160 \times \frac{1}{2^3} = 20$

(36) (B). Let the three numbers be a/r, a, ar. As the numbers form an increasing GP. So, r > 1. It is given that a/r, 2a, ar are in A.P.

$\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow r = 2 \pm \sqrt{3} \Rightarrow r = 2 + \sqrt{3} \quad [\because r > 1]$

(37) (C). Let the terms are a/r, a, ar.

then $\frac{a}{r} \times a \times ar = 216 \qquad \Rightarrow a = 6$

and $\frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = 156$

$\Rightarrow a^2\left(\frac{1}{r} + r + 1\right] = 156$

$\Rightarrow 36\,(r^2 + r + 1) = 156r \qquad (\therefore a = 6)$

$3r^2 - 10r + 3 = 0 \Rightarrow (3r-1)(r-3) = 0 \Rightarrow r = 3, \frac{1}{3}$

Terms are 2, 6, 18

(38) (B). Given a,b,c,d in G.P. using property (iii)
$a^n, b^n, c^n, d^n$ are also in G.P.
Let common ratio is k
then $b^n = ka^n \quad c^n = k^2a^n, d^n = k^3 a^n$
Now in $a^n + b^n, b^n + c^n, \quad c^n + d^n$
$\Rightarrow a^n + ka^n, ka^n + k^2a^n, k^2 a^n + k^3a^n$
$\Rightarrow a^n(k+1), ka^n(k+1), k^2a^n(K+1)$
dividing each by $a^n(k+1)$
$\Rightarrow 1, k, k^2$
which are clearly in G.P.

(39) (C). $\frac{a(1-r^6)}{1-r} = 9\,\frac{a(1-r^3)}{1-r}$
$\Rightarrow 1 - r^6 = 9(1 - r^3) \qquad (r \neq 1)$
$\Rightarrow 1 + r^3 = 9$
$\therefore r = 2$

(40) (B). Let first term of an A.P. be a and c.d. be d and first term of a G.P. be A and c.r. be R,then
$a + (p-1)d = AR^{p-1} = x$
$\Rightarrow p - 1 = (x-a)/d$ ...(1)
$a + (q-1)d = AR^{q-1} = y$
$\Rightarrow q - 1 = (y-a)/d$ ...(2)
$a + (r-1)d = AR^{r-1} = z$
$\Rightarrow r - 1 = (z-a)/d$ ...(3)
$\therefore$ Given expression
$= (AR^{p-1})^{y-z}, (AR^{q-1})^{z-x}, (AR^{r-1})^{x-y}$
$= A^0\, R^{(p-1)(y-z)+(q-1)(z-x)+(r-1)+(x-y)}$
$= A^0R^{[(x-a)(y-z)+(y-a)(z-x)+(z-a)(x-y)]/d}$
[By (1), (2) and (3)]
$= A^0R^0 = 1$

(41) (B). Here the given condition
$(a^2 + b^2 + c^2)\, p^2 - 2p\,(ab + bc + ca) + b^2 + c^2 + d^2 \le 0$
$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$
Since the squares can not be negative
$\therefore ap - b = 0, bp - c = 0, cp - d = 0$

$\Rightarrow \frac{1}{p} = \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

$\therefore$ a,b,c,d are in G.P.

(42) (A). x,y,z are in A.P.
$\Rightarrow 2y = x + z$
or $2xy = x^2 + xz$ (multiplying with x)
$\Rightarrow x^2 - 2xy = -xz$ ...(1)
x,y, t are in G.P.
$\Rightarrow y^2 = xt$ ...(2)
or $(x^2 - 2xy + y^2) = -xz + xt$
or $(x-y)^2 = x(t-z)$
x, x–y, t– z are in G.P.

(43) (A). Let the G.P. be $a, ar, ar^2$......., then

$\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots$ upto 100 terms

$= ar + ar^3 + \ldots\ldots$ upto 100 terms

$= ar(1 + r^2 + r^4 + \ldots\ldots r^{198})$

and $\beta = \sum_{n=1}^{100} a_{2n-1} = a + ar^3 + \ldots\ldots$ upto 100 terms

$= a(1 + r^2 + \ldots\ldots + r^{198})$

Obviously $\frac{\alpha}{\beta} = r$.

(44) (B). Let the numbers be $a, ar, ar^2$

$a + ar + ar^2 = 14 \Rightarrow a(1 + r + r^2) = 14$ .....(i)

and $2(ar + 1) = (a + 1) + (ar^2 - 1)$

$a(r^2 - 2r + 1) = 2$ .....(ii)

Put the value of a from (i) to (ii),

$\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow r = 2, \frac{1}{2}$ and $a = 2, 8$

Numbers are 2, 4, 8 or 8, 4, 2. So lowest term in series is 2.

(45) (C). a, b are roots of $x^2 - 3x + p = 0$

$\therefore a + b = 3, ab = p$

$c, d$ are roots of $x^2 - 12x + q = 0$

$\therefore\ c + d = 12,\ cd = q$

$a, b, c, d$ are in GP.

$\therefore\ \frac{b}{a} = \frac{d}{c} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

$\Rightarrow \frac{(a-b)^2}{(a+b)^2} = \frac{(c-d)^2}{(c+d)^2} \Rightarrow 1 - \frac{4ab}{(a+b)^2} = 1 - \frac{4cd}{(c+d)^2}$

$\Rightarrow \frac{ab}{(a+b)^2} = \frac{cd}{(c+d)^2} \Rightarrow \frac{p}{9} = \frac{q}{144}$

$\Rightarrow \frac{p}{1} = \frac{q}{16} \Rightarrow \frac{p}{q} = \frac{1}{16} \Rightarrow \frac{p+q}{q-p} = \frac{17}{15}$.

**(46)** **(A).** By hypothesis, $\alpha^2 = a^2bc, \beta^2 = b^2ca, \gamma^2 = c^2ab$ and $2b = a + c$. Hence $\alpha^2, \beta^2, \gamma^2$ are in A.P.

**(47)** **(A).** $t_n = \log\left(\frac{5^{n+1}}{3^{n-1}}\right)$ ; $s_n = [\log(5/3)]^n$

$t_1 = \log 25$ ; $s_1 = [\log 5/3]^1$

$t_2 = \log\frac{125}{3}$ ; $s_2 = [\log 5/3]^2$

$t_3 = \log\frac{625}{9}$ ; $s_3 = [\log 5/3]^3$

Clearly $t_n$ is an A.P. and $s_n$ is G.P.

**(48)** **(A).** Let $\frac{A}{R}, A, AR$ be the roots of the equation

$ax^3 + bx^2 + cx + d = 0$

then $A^3$ = Product of the roots $= -\frac{d}{a} \Rightarrow A = -\left(\frac{d}{a}\right)^{1/3}$

Since A is a root of the equation.

$\therefore aA^3 + bA^2 + cA + d = 0$

$\Rightarrow a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{2/3} + c\left(-\frac{d}{a}\right)^{1/3} + d = 0$

$\Rightarrow b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3} \Rightarrow b^3\frac{d^2}{a^2} = c^3\frac{d}{a} \Rightarrow b^3d = c^3a$.

**(49)** **(A).** Here $e^2 = df$

Now $dx^2 + 2ex + f = 0$ given

$\Rightarrow dx^2 + 2\sqrt{df}\ x + f = 0 \Rightarrow x = -\sqrt{\frac{f}{d}}$

Putting in $ax^2 + 2bx + c = 0$ we get

$a\frac{f}{d} + c = 2b\sqrt{\frac{f}{d}} \Rightarrow \frac{a}{d} + \frac{c}{f} = \frac{2b}{e}$

$\therefore\ \frac{a}{d}, \frac{b}{e}, \frac{c}{f}$ are in A.P. ; $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in H.P.

**(50)** **(A).** Given $x_2 = rx_1, x_3 = r^2x_1, y_2 = ry_1, y_3 = r^2y_1$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}x_1y_1\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0$$

i.e. lie on a Straight line.

**(51)** **(B).** Given $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$

$\therefore \left.\begin{matrix} ap - b = 0 \\ bp - c = 0 \\ cp - d = 0 \end{matrix}\right\} p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\therefore$ G.P.

**(52)** **(A).**

$\because S = x_1x_2 + x_3x_4 + .... + x_9x_{10} \le (x_1 + x_2 + x_3 + x_5 + x_7 + x_9)$
. $(x_2 + x_4 + x_6 + x_8 + x_{10})$

$\because \left(A \ge G \Rightarrow \frac{a+b}{2} \ge \sqrt{ab} \Rightarrow ab \le \left(\frac{a+b}{2}\right)^2\right)$

$\therefore\ S \le \left[\frac{x_1 + x_2 ...... + x_{10}}{2}\right]^2$ $\therefore S \le 36$

**(53)** **(C).** Using AM $\ge$ GM

$\Rightarrow \frac{x+y+z}{2} \ge (xyz)^{1/3} \Rightarrow \frac{1}{3} \ge (xyz)^{1/3}$ ....... (1)

Also, $\frac{(1+x)+(1+y)+(1+z)}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}$

$\Rightarrow \frac{4}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}$ ....... (2)

Dividing (2) by (1) we get

$4 \ge \left[\frac{(1+x)(1+y)(1+z)}{xyz}\right]^{1/3}$

$\therefore\ \frac{(1+x).(1+y).(1+z)}{xyz} \le 64$

**(54)** **(D).** $4 = \frac{a}{1-r} \Rightarrow 4r = 4 - a$. Check with options.

**(55)** **(A).** $ar = 24$ ; $ar^4 = 3$ $\therefore$ $r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$ and $a = 48$

$S_6 = \frac{a(1-r^6)}{1-r} = 48 \times \left(\frac{1-\frac{1}{64}}{1-\frac{1}{2}}\right) = 2 \times 48 \times \frac{63}{64} = \frac{3 \times 63}{2} = \frac{189}{2}$

**(56)** **(C).** By trial, putting n = 0,

$$\frac{a^{0+1}+b^{0+1}}{a^0+b^0}=\frac{a+b}{2}=\text{A.M.}$$

Putting n = –1/2,

$$\frac{a^{-\frac{1}{2}+1}+b^{-\frac{1}{2}+1}}{a^{-\frac{1}{2}}+b^{-\frac{1}{2}}}=\frac{\sqrt{a}+\sqrt{b}}{\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}}=\sqrt{ab}=\text{G.M.}$$

$$n=-1,\ \frac{a^0+b^0}{a^{-1}+b^{-1}}=\frac{2ab}{a+b}=\text{H.M.}$$

**Alternately :** For AM

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$$

$\Rightarrow\ 2a^{n+1}+2b^{n+1}=a^{n+1}+b^{n+1}+a^nb+ab^n$
$\Rightarrow a^{n+1}-a^nb=-b^{n+1}+ab^n$
$\Rightarrow a^n(a-b)=+b^n(a-b),\ a\neq b$

$\Rightarrow \left(\frac{a}{b}\right)^n=1\Rightarrow n=0$, similarly for GM and HM also.

**(57)** **(B).** Let d be common difference of the corresponding AP. So $\frac{1}{a_2}-\frac{1}{a_1}=d$ $\frac{1}{a_3}-\frac{1}{a_2}=d,..,\ \frac{1}{a_n}-\frac{1}{a_{n-1}}=d$

$\Rightarrow a_1-a_2=d\,(a_1a_2),\ a_2-a_3$
$=d(a_2a_3),\ldots,\ (a_{n-1}-a_n)=d\,(a_{n-1}a_n)$

Adding these relations, we get

$a_1-a_n=d\,(a_1a_2+a_2a_3+\ldots+a_{n-1}a_n)$ ...(1)

Also $\frac{1}{a_n}=T_n=\frac{1}{a_1}+(n-1)d\Rightarrow\frac{1}{a_n}-\frac{1}{a_1}=(n-1)d$

$\Rightarrow\ a_1-a_n=(n-1)\,d\,(a_1a_n)$ ...(2)

From (1) and (2), we have

$(n-1)(a_1a_n)=a_1a_2+a_2a_3+\ldots+a_{n-1}a_n$

**(58)** **(C).** According to the condition

$$\frac{\frac{1}{a+7d}}{\frac{1}{a+(n-1)d}}=\frac{9}{5}\quad\ldots\text{(i)}$$

Also $\frac{1}{a+(n+1)d}=\frac{1}{31}$ …..(ii)

where $a=1$. Hence $d=2,\ n=14$.

**(59)** **(D).** Given $a_1=h_1=2,\ a_{10}=h_{10}=3$

Hence $a_1+9d=3$

For A.P. $2+9d=3$ or $d=\frac{1}{9}$

$\therefore\ a_4=a_1+3d=2+\frac{3}{9}=2+\frac{1}{3}=\frac{7}{3}$

For H.P. $\frac{1}{2}+9d'=\frac{1}{3}$ or $9d'=-\frac{1}{6}$ or $d'=-\frac{1}{54}$

$\therefore\ \frac{1}{h_7}=\frac{1}{h_1}+6d'=\frac{1}{2}-\frac{6}{54}=\frac{1}{2}-\frac{1}{9}=\frac{7}{18}\Rightarrow h_7=\frac{18}{7}$

Hence $a_4h_7=\frac{7}{3}\times\frac{18}{7}=6$.

**(60)** **(D).** $\because$ G.M. $\geq$ H.M.

$$\Rightarrow (a_1.a_2.a_3)^{1/3}\geq\frac{3}{(1/a_1+1/a_2+1/a_3)}$$

$$\Rightarrow (a_1.a_2.a_3)\geq\frac{27}{(1/a_1+1/a_2+1/a_3)^3}$$

$$(a_1.a_2.a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^3\geq 27.$$

**(61)** **(C).** Let a and b be the first term and common difference of the corresponding AP, then its

$$T_p=\frac{1}{q}\text{ and }T_q=\frac{1}{p}$$

$$\Rightarrow a+(p-1)\,d=\frac{1}{q}\text{ and }a+(q-1)\,d=\frac{1}{p}$$

$$\Rightarrow (p-q)\,d=\frac{1}{q}-\frac{1}{p}\Rightarrow d=\frac{1}{pq}$$

Now (p + q)th term of this AP = a + ( p + q – 1) d

$$=[a+(p-1)d]+qd=\frac{1}{q}+q\left(\frac{1}{pq}\right)=\frac{1}{q}+\frac{1}{p}=\frac{p+q}{pq}$$

$$\therefore\ T_{p+q}\text{ of HP}=\frac{pq}{p+q}$$

**(62)** **(D).** a,b, c are in A.P.

$\Rightarrow\ 2b=a+c$ ...(1)

and $a^2, b^2, c^2$ are in H.P. $\Rightarrow b^2=\frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow b^2(a^2+c^2)=2a^2c^2$
$\Rightarrow b^2(4b^2-2ac)=2a^2c^2$ [From (1)]
$\Rightarrow 2b^4-acb^2-a^2c^2=0$
$\Rightarrow (b^2-ac)(2b^2+ac)=0$

$\Rightarrow b^2=ac$ or $b^2=-\frac{1}{2}ac$

If $b^2=ac$, then a,b,c are in G.P. But a,b,c, are also in A.P., therefore a = b = c.

**(63)** **(B).** $b=\frac{2+c}{2}$ ...(1)

$c^2=bd$ ...(2)

$d = \frac{36c}{c+18}$ ...(3)

Eliminate d from (2) and (3) we get $c = \pm 6$
Now from (1) $b = 4, -2$
from (3) $d = 9, -18$
$\therefore b = 4, c = 6, d = 9$

**(64)** **(B).** Let given three terms be br, b, b/r

$\therefore 12 = \frac{2(br)b}{br+b} = \frac{2\,br}{r+1}$ ...(1)

and $36 = \frac{2\,b(b/r)}{b+(b/r)} = \frac{2\,b}{r+1}$ ...(2)

$(1) \div (2) \Rightarrow r = 1/3$
Then from (2) $b = 24$ $\quad\therefore a = br = 8$

**(65)** **(C).** Here $2 = x + z$ ...(1)
$4 = xz$ ...(2)

Now $\frac{2xz}{x+z} = \frac{8}{2} = 4 \quad \therefore x, 4, z$ are H.P.

**(66)** **(D).** Here a,b,c in H.P. $\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Now $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{2}{b}-\frac{1}{b}\right)$

$= \left(\frac{1}{b}+\frac{2}{b}-\frac{1}{a}-\frac{1}{a}\right)\left(\frac{1}{b}\right) = \frac{3}{b^2} - \frac{2}{ab}$

Also $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$

(eliminating 1/a in first factor and $\frac{1}{c}+\frac{1}{a}$ in second)

$= \left(\frac{2}{c}-\frac{1}{b}\right)\left(\frac{1}{b}\right) = \frac{2}{bc} - \frac{1}{b^2}$

**(67)** **(C).** a,b,c are in HP

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in AP

$\Rightarrow 1 + \frac{b+c}{a}, 1 + \frac{c+a}{b}, 1 + \frac{a+b}{c}$ are in AP

$\Rightarrow \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP

$\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP.

**(68)** **(D).** Let the first term of A.P. be a and common difference be d.
Given $(a + md), (a + nd), (a + rd)$ in G.P.
$(a + nd)^2 = (a + md)(a + rd)$

$\Rightarrow \frac{d}{a} = \frac{2n-m-r}{mr-n^2}$

But m, n, r in H.P. $\Rightarrow n = \frac{2mr}{m+r}$

$\therefore \frac{d}{a} = \frac{2n - \frac{2mr}{n}}{mr-n^2} = \frac{2}{n}\left(\frac{n^2-mr}{mr-n^2}\right) = -\frac{2}{n}$

**(69)** **(C).** By property of A.P. $x + z = a + b$ and $y = \frac{1}{2}(a+b)$

$\Rightarrow x + y + z = \frac{3}{2}(a+b) \Rightarrow a + b = 10$ ...(1)

Also $\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ are in AP, so as above

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right) \Rightarrow \frac{1}{a}+\frac{1}{b} = \frac{10}{9} \Rightarrow ab = 9$ ...(2)

From (1) and (2) a,b are 9,1

**(70)** **(B).** If $\frac{A}{G} = \frac{p}{q} \Rightarrow \frac{a}{b} = \frac{p+\sqrt{p^2-q^2}}{p-\sqrt{p^2-q^2}}$

Here, $\frac{H}{G} = \frac{G}{A} = \frac{4}{5} \Rightarrow \frac{A}{G} = \frac{5}{4} \quad \therefore \quad \frac{a}{b} = \frac{5+3}{5-3} = \frac{2}{8} = \frac{1}{4}$

**(71)** **(B).** First number $= 5^3 = 125$. Also since
$20^3 = 8000, 21^3 = 9261, 22^3 = 10648$
so last number is $21^3 = 9261$
$\therefore$ required sum $= 5^3 + 6^3 + 7^3 + .... + 21^3$
$= (1^3 + 2^3 + 3^3 + ... + 21^3) - (1^3 + 2^3 + 3^3 + 4^3)$

$= \sum_{n=1}^{21} n^3 - \sum_{m=1}^{4} m^3 = \left(\frac{21\times 22}{2}\right)^2 - \left(\frac{4\times 5}{2}\right)^2$

$= (231)^2 - (10)^2 = 221 \times 241 = 53261$

**(72)** **(B).** $\sum_{r=1}^{n} \frac{1}{\log_{3^r} a} = \sum_{r=1}^{n} \log_a 3^r = \sum_{r=1}^{n} (r \log_a 3)$

$= \log_a 3 \sum_{r=1}^{n} r = \log_a 3 \frac{n(n+1)}{2} = \frac{n(n+1)}{2} \log_a 3$

**(73)** **(B).** $t_n = \frac{1}{1+3+5+7+........+(2n-1)-\frac{1}{4}} = \frac{1}{n^2 - \frac{1}{4}}$

$= \frac{4}{4n^2-1} = \frac{4}{(2n-1)(2n+1)} = 2\left\{\frac{1}{2n-1} - \frac{1}{2n+1}\right\}$

$\therefore S_n = \Sigma t_n = 2\left\{\frac{1}{1} - \frac{1}{2n+1}\right\} = \frac{4n}{2n+1}.$

**(74)** **(B)** $t_n = \dfrac{1.3.5.....(2n-3)}{n-1.2^{n-1}} - \dfrac{1.3.5.....(2n-3)(2n-1)}{n.2^n}$

$= v_n - v_{n+1}$

$S_n = t_1 + \sum_{2}^{n} t_n = \dfrac{1}{2} + \sum_{2}^{n}(v_n - v_{n+1}) = \dfrac{1}{2} + v_2 - v_{n+1}$

$= \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1.3.5...(2n-1)}{n.2^n} = 1 - \dfrac{1.3.5...(2n-1)}{n.2^n}$

**(75)** **(B).** $S = 1 + 4x + 7x^2 + 10x^3 + ...........$

$x.S = x + 4x^2 + 7x^3 + ...........$

Subtract, $S(1-x) = 1 + 3x + 3x^2 + 3x^3 + ...............$

$S(1-x) = 1 + 3x\left(\dfrac{1}{1-x}\right) \quad |x| < 1 \Rightarrow S = \dfrac{1+2x}{(1-x)^2}$

Given : $\dfrac{1+2x}{(1-x)^2} = \dfrac{35}{16}$

$\Rightarrow 16 + 32x = 35 + 35x^2 - 70x \Rightarrow 35x^2 - 102x + 19 = 0$

$\Rightarrow 35x^2 - 7x - 95x + 19 = 0 \Rightarrow 7x(5x-1) - 19(5x-1) = 0$

$\Rightarrow (5x-1)(7x-19) = 0 \Rightarrow x = \dfrac{1}{5}, \dfrac{19}{7}$ But $|x| < 1$

$\therefore x = 1/5$

**(76)** **(A).** $T_r = \dfrac{r}{(2r-1)\,2r\,(2r+1)} = \dfrac{1}{4}\left(\dfrac{1}{2r-1} - \dfrac{1}{2r+1}\right)$

$S = \dfrac{1}{4}\left\{\left(\dfrac{1}{1} - \dfrac{1}{3}\right) + \left(\dfrac{1}{3} - \dfrac{1}{5}\right) + ..........\right\} = \dfrac{1}{4}$

**(77)** **(B).** The given product $= 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+....} = 2^s$ (say)

Now $S = \dfrac{1}{4} + \dfrac{2}{8} + \dfrac{3}{16} + \dfrac{4}{32} + .....$ ...(1)

$\Rightarrow \dfrac{1}{2}S = \dfrac{1}{8} + \dfrac{2}{16} + \dfrac{3}{32} + ...$ ...(2)

(1) – (2)

$\Rightarrow \dfrac{1}{2}S = \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + ..... = \dfrac{1/4}{1-1/2} = \dfrac{1}{2} \qquad \therefore S = 1$

$\Rightarrow$ Product $= 2^1 = 2$

**(78)** **(A).** Sum $= \dfrac{8}{9}[9 + 99 + 999 + ...n \text{ terms}]$

$= \dfrac{8}{9}[(10–1) + (100–1) + (1000–1) + .... n \text{ terms}]$

$= \dfrac{8}{9}[(10 + 10^2 + 10^3 + ....+ 10^n) - n]$

$= \dfrac{8}{9}\left[\dfrac{10(10^n - 1)}{10-1} - n\right] = \dfrac{8}{81}[10^{n+1} - 9n - 10]$

**(79)** **(A).** Let $T_n$ be the $n^{th}$ term of the series, then $T_n = 3n(n+1)$

If $S_n$ denotes the sum of first n terms

$\therefore S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}(3k^2 + 3k)$

$= 3\sum_{k=1}^{n}(k^2) + 3\sum_{k=1}^{n}(k) = \dfrac{3n(n+1)(2n+1)}{6} + \dfrac{3n(n+1)}{2}$

$= \dfrac{3n(n+1)}{2}\left[\dfrac{2n+1}{3} + 1\right] = \dfrac{3n(n+1)}{2} \cdot \dfrac{2(n+2)}{3}$

$= n(n+1)(n+2)$.

**(80)** **(D).** The series, $\dfrac{1}{3\times7} + \dfrac{1}{7\times11} + \dfrac{1}{11\times15} + ....... \infty$

Let $S = \dfrac{1}{3\times7} + \dfrac{1}{7\times11} + \dfrac{1}{11\times15} + ...... \infty$

$= \dfrac{1}{4}\left[\left\{\dfrac{1}{3} - \dfrac{1}{7}\right\} + \left\{\dfrac{1}{7} - \dfrac{1}{11}\right\} + ......\right]$

$= \dfrac{1}{4}\left[\left\{\dfrac{1}{3} + \dfrac{1}{7} + \dfrac{1}{11} + ....\infty\right\} - \left\{\dfrac{1}{7} + \dfrac{1}{11} + \dfrac{1}{15} + ......\infty\right\}\right]$

$= \dfrac{1}{4}\left[\dfrac{1}{3} + 0\right] = \dfrac{1}{12}$.

**(81)** **(A).** Given series $27 + 9 + 5\,.\,\dfrac{2}{5} + 3\,.\,\dfrac{6}{7} + ......$

$= 27 + \dfrac{27}{3} + \dfrac{27}{5} + \dfrac{27}{7} + ..... + \dfrac{27}{2n-1} + ......$

Hence $n^{th}$ term of given series $T_n = \dfrac{27}{2n-1}$

So, $T_9 = \dfrac{27}{2\times9-1} = \dfrac{27}{17} = 1\dfrac{10}{17}$.

**(82)** **(A).** On putting $n = 1,2,3,.....$

First term of the series $a = \dfrac{1}{x} + y$, Second term $= \dfrac{2}{x} + y$

$\therefore d = \left(\dfrac{2}{x} + y\right) - \left(\dfrac{1}{x} + y\right) = \dfrac{1}{x}$

Sum of $r$ terms of the series

$= \dfrac{r}{2}\left[2\left(\dfrac{1}{x} + y\right) + (r-1)\dfrac{1}{x}\right] = \dfrac{r}{2}\left[\dfrac{2}{x} + 2y + \dfrac{r}{x} - \dfrac{1}{x}\right]$

$= \dfrac{r^2 - r + 2r}{2x} + ry = \left[\dfrac{r(r+1)}{2x} + ry\right]$.

**(83)** **(A).** Given series is

$$3+4\frac{1}{2}+6\frac{3}{4}+........=3+\frac{9}{2}+\frac{27}{4}+.....$$

$$=3+\frac{3^2}{2}+\frac{3^3}{4}+\frac{3^4}{8}+\frac{3^5}{16}+.....\text{ (in G.P.)}$$

Here a = 3, r = 3/2, then sum of the five terms

$$S_5=\frac{a(r^n-1)}{r-1}=\frac{3\left[\left(\frac{3}{2}\right)^5-1\right]}{\frac{3}{2}-1}=\frac{1\left[\frac{3^5}{32}-1\right]}{\frac{1}{2}}$$

$$=6\left[\frac{243-32}{32}\right]=\frac{211\times 3}{16}=\frac{633}{16}=39\frac{9}{16}.$$

**(84)** **(B).** $9+99+999+.........+$ upto n terms

$\Rightarrow (10-1)+(100-1)+(1000-1)+.......+$ upto n terms

$$=\frac{10(10^n-1)}{9}-n=\frac{10^{n+1}-9n-10}{9}$$

**(85)** **(C).** The given series is an A.G.P. with common ratio

$S=a-(a+d)+(a+2d)-(a+3d)+....+(a+2nd)$

$-S=-a+(a+d)-(a+2d)+..+(a+(2n-1)d)-(a+2nd)$

$\therefore 2S=a+\{-d+d-d+d...\text{upto } 2n \text{ terms}\}+(a+2nd)$

$\Rightarrow 2S=2a+2nd \quad S=a+nd$

**(86)** **(A).** Let S be the sum of n terms of the given series and $x=1+1/n$, Then,

$S=1+2x+3x^2+4x^3+....+n\,x^{n-1}$

$\Rightarrow xS=x+2x^2+3x^3+....+(n-1)\,x^{n-1}+nx^n$

$\therefore S-xS=1+[x+x^2+...+x^{n-1}]-nx^n$

$$\Rightarrow S(1-x)=\frac{1-x^n}{1-x}-n\,x^n$$

$\Rightarrow S(-1/n)=-n[1-(1+1/n)^n]-n(1+1/n)^n$

$$\Rightarrow \frac{1}{n}.S=n[1-(1+1/n)^n+(1+1/n)^n]=n$$

$\Rightarrow S=n^2$

**(87)** **(C).** Let $S=1+2.2+3.2^2+4.2^3+...+100.2^{99}$ .....(1)

$\Rightarrow 2S=2+2.2^2+3.2^3+....+99.2^{99}+100.2^{100}$ .....(2)

Subtracting (2) from (1), we get

$-S=(1+2+2^2+2^3+.....+2^{99})-100.2^{100}$

$$\Rightarrow S=100.2^{100}-\frac{2^{100}-1}{2-1}$$

$$=100.2^{100}-2^{100}+1=1+99.2^{100}$$

**(88)** **(A).** The given series is an arithmetico- geometric series. The sum of the series is given by

$$\frac{3}{1-\frac{1}{4}}+\frac{d\times\frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}\quad\left[\text{using : } S=\frac{a}{1-r}+\frac{dr}{(1-r)^2}\right]$$

$$4+\frac{4d}{9}=8\Rightarrow d=9$$

**(89)** **(C).** $\because S=1+(1+a)x+(1+a+a^2)x^2........\infty$ ..... (1)

$\therefore axS= \quad ax+(a+a^2)x^2........\infty$ ..... (2)

Subtracting (2) from (1), we get

$$(1-ax)S=1+x+x^2+x^3.........=\frac{1}{1-x}$$

$$S=\frac{1}{(1-ax)(1-x)}$$

**(90)** **(C).** $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}(1)=\sum_{i=1}^{n}\sum_{j=1}^{i}j$

$$=\sum_{i=1}^{n}\frac{i.(i+1)}{2}=\frac{1}{2}\left[\sum_{i=1}^{n}i^2+\sum_{i=1}^{n}i\right]$$

$$=\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]$$

$$=\frac{n(n+1)}{4}\left[\frac{2n+1}{3}+1\right]=\frac{n(n+1)(2n+4)}{12}$$

**(91)** **(B).** Put n = 2 ; $\frac{1^2}{1}+\frac{1^2+2^2}{1+2}=1+\frac{5}{3}=\frac{8}{3}$

Check with options, option (2) only satisfies.

**(92)** **(B).** Checking with options, putting n = 2

$S_2=\frac{1}{3}+\frac{2}{3}=1$ satisfies only.

**(93)** **(A).** $$t_n=\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\frac{2n+1}{3}$$

$$S_n=\Sigma\, t_n=\frac{2}{3}\Sigma\, n+\frac{1}{3}\Sigma\, 1$$

$$=\frac{2}{3}\times\frac{n(n+1)}{2}+\frac{1}{3}n=\frac{n}{3}(n+2)$$

**(94)** **(A).** $2\log(2^x-1)=\log 2+\log(2^x+3)$

$\Rightarrow (2^x-1)^2=2.(2^x+3) \Rightarrow (2^x)^2-4.2^x-5=0$

$\Rightarrow (2^x-5)(2^x+1)=0$

$\Rightarrow x=\log_2 5$, as $2^x+1\neq 0$

**(95)** **(A).** x, y, z are in A.P. $\Rightarrow 2y=x+z$

or $2xy=x^2+xz$ (multiply with x)

$\Rightarrow x^2-2xy=-xz$ ......(i)

x, y, t are in G.P. $\Rightarrow y^2=xt$ ......(ii)

or $(x^2-2xy+y^2)-xz+xt$

or $(x-y)^2=x(t-z) \Rightarrow x, x-y, t-z$ are in G.P.

**(96)** **(C).** $x_1x_2=18^2=12.27,\ \frac{2x_1x_2}{x_1+x_2}=\frac{216}{13}$ giving

$$x_1+x_2=\frac{26.18^2}{216}=39=27+12,\ |x_1-x_2|=15$$

**(97)** **(D).** If r is the common ratio,

$$\sqrt{a_1a_2} + \sqrt{a_3a_4} + ........ + \sqrt{a_{2n-1}a_{2n}} = \sqrt{r}\ (a_1 + a_3 + ....... + a_{2n-1})$$

$$\sqrt{a_2a_3} + \sqrt{a_4a_5} + ...... + \sqrt{a_{2n}a_{2n+1}} = \sqrt{r}\ (a_2 + a_4 + ...... + a_{2n})$$

**(98)** **(A).** $\frac{1000}{2}\{2a+999d\} - \frac{500}{2}\{2a+499d\}$

$= \frac{1}{3} \times \frac{n}{2}\{2a+(n-1)d\}$

comparing coefficients of a, $\frac{n}{3} = 1000 - 500$

$\Rightarrow n = 1500$

This agrees with the coefficient of d as well

**(99)** **(C).** If a is the first term and d is the common difference of the associated A.P.

$\frac{1}{q} = \frac{1}{a} + (2p-1)d,\ \frac{1}{p} = \frac{1}{a} + (2q-1)d \Rightarrow d = \frac{1}{2pq}$

If h is the $2(p+q)^{th}$ term

$\frac{1}{h} = \frac{1}{a} + (2p+2q-1)d = \frac{1}{q} + \frac{1}{p} = \frac{p+q}{pq},\quad h = \frac{pq}{p+q}$

**(100)** **(C)** $200 < \frac{9}{2}(2a+8d) < 220$ and $a + d = 12$

$\therefore 200 < 9(12+3d) < 220$

$92 < 27d < 112$ ; $3\frac{11}{27} < d < 4\frac{4}{27}$ $\therefore d = 4$

## EXERCISE-2

**(1)** **(C).** $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

$\therefore \frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2$

are in A.P. (adding 2 in each term)

or $\frac{a+b+c}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in A.P.

[ dividing by (a+b+c) in each term]

or $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

**(2)** **(D).** If $\alpha$ is the first derm of the H.P. and d is the common difference of the associated A.P., then

$\frac{1}{\alpha} + (p-1)d = \frac{1}{a},\ \frac{1}{\alpha} + (q-1)d = \frac{1}{b},\ \frac{1}{\alpha} + (r-1)d = \frac{1}{c}$

$\therefore (p-q)d = \frac{1}{a} - \frac{1}{b}$ or $ab(p-q)d = b - a$

By cyclical interchanges $\Sigma ab(p-q) = 0$ or $\Sigma\frac{p-q}{c} = 0$.

**(3)** **(B).** Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then,

$\frac{1}{a^3+b^3} = \frac{1}{a^3(1+r^3)}$ , $\frac{1}{b^3+c^3} = \frac{1}{a^3r^3(1+r^3)}$

and $\frac{1}{c^3+d^3} = \frac{1}{a^3r^3(1+r^3)}$

Clearly, $(a^3+b^3)^{-1}$, $(b^3+c^3)^{-1}$ and $(c^3+d^3)^{-1}$ are in G.P. with common ratio $1/r^3$.

**(4)** **(B).** We have $(x_1+x_2+........+x_{50})\left(\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_{50}}\right)$

$\geq (50)^2$ [since A.M. $\geq$ H.M.]

$\left(\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_{50}}\right) \geq 50$

**(5)** **(B).** The three numbers are $\log_9 9$, $\log_{9^2}(3^x+48)$ and

$\log_9\left(3^x - \frac{8}{3}\right)$,

i.e., $\log_9 9, \frac{1}{2}\log_9(3^x+48), \log_9\left(3^x-\frac{8}{3}\right)$ a e in A.P.

$\Rightarrow \left\{(3^x+48)^{\frac{1}{2}}\right\}^2 = 9\left(3^x - \frac{8}{3}\right) \Rightarrow 8.3^x = 72 \Rightarrow 3^x = 9$

$\Rightarrow x = 2$.

**(6)** **(C).** x, y, z are in G.P. $\Rightarrow y^2 = xz$ ......(1)

We have, $ax = b^y = c^z = \lambda$ (say)

$\Rightarrow x\log a = y\log b = z\log c = \log\lambda \Rightarrow x = \frac{\log\lambda}{\log a}$,

$y = \frac{\log\lambda}{\log b},\quad z = \frac{\log\lambda}{\log c}$

putting x,y,z in (i), we get, $\left(\frac{\log\lambda}{\log b}\right)^2 = \frac{\log\lambda}{\log a}.\frac{\log\lambda}{\log c}$

$(\log b)^2 = \log a.\log c$

or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

**(7)** **(C).** Let given three terms be br, b, b/r

$\therefore 12 = \frac{2(br)b}{br+b} = \frac{2br}{r+1}$ ......(1)

and $36 = \frac{2b\left(\frac{b}{r}\right)}{b+\left(\frac{b}{r}\right)} = \frac{2b}{r+1}$ ......(2)

$(1)/(2) \Rightarrow r = 1/3$ Then from (2) $b = 24$

$\therefore a = br = 8$

**(8)** **(C).** $\sqrt{11-4\sqrt{6}} = 2\sqrt{2}-\sqrt{3}$,

$\sqrt{6-2\sqrt{3}+2\sqrt{2}-2\sqrt{6}} = 1+\sqrt{2}-\sqrt{3}$,

$\sqrt{7-4\sqrt{3}} = 2-\sqrt{3}$ and these form an A.P. with common difference $= 1-\sqrt{2}$.

Hence required numbers are in H.P.

**(9)** **(B).** $\frac{x-y}{\log_2 a} = \frac{y-z}{\log_2 b} = \frac{z-x}{\log_2 c}$

$\Rightarrow \frac{\log_2 a}{x-y} = \frac{\log_2 b}{y-z} = \frac{\log_2 c}{z-x}$

by ratio and proportion the above quantities are equal to

$\frac{\log_2 a + \log_2 b + \log_2 c}{(x-y)+(y-z)+(z-x)} = \frac{\log_2 abc}{0} \Rightarrow abc = 1$

Also equal to

$\frac{z\log_2 a + x\log_2 b + y\log_2 c}{z(x-y)+x(y-z)+y(z-x)} = \frac{\log_e a^z b^x c^y}{0} \Rightarrow a^z b^x c^y = 1$

Similarly above quantities are equal to

$\frac{(x+y)\log_2 a + (y+z)\log_2 b + (z+x)\log_2 c}{(x^2-y^2)+(y^2-z^2)+(z^2-x^2)}$

$= \frac{\log a^{x+y} b^{y+z} c^{z+x}}{0} \Rightarrow a^{x+y} b^{y+z} c^{z+x} = 1$

$\therefore$ Given expression is equal to

$$5^{\left(\frac{a^{x+y}b^{y+z}c^{z+x}}{a^z.b^x.c^y}+\frac{abc}{a^z.b^x.c^y}+\frac{a^{x+y}b^{y+z}c^{z+x}}{abc}\right)} = 5^{(1+1+1)} = 125$$

**(10)** **(D).** $\frac{1}{a_1}-\frac{1}{a_2} = \frac{a_2-a_1}{a_1a_2} = \frac{d}{a_1a_2} \;\therefore\; \frac{1}{a_1a_2} = \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}\right)$.

where d = C.D. of A.P.

$\therefore \frac{1}{a_1a_2}+\frac{1}{a_2a_3}+....+\frac{1}{a_na_{n+1}}$

$= \frac{1}{d}\left\{\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+....+\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)\right\}$

$= \frac{1}{d}\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right) = \frac{1}{d}\left(\frac{a_{n+1}-a_1}{a_1a_{n+1}}\right)$

$= \frac{nd}{da_1a_{n+1}} = \frac{n}{a_1a_{n+1}}$

**(11)** **(D).** Let the first term of A.P. be a and common difference be d.

Given (a + md), (a + nd), (a + rd) in G.P.

$\therefore (a+nd)^2 = (a+md)(a+rd) \Rightarrow \frac{d}{a} = \frac{2n-m-r}{mr-n^2}$

But m, n, r in H.P. $\Rightarrow n = \frac{2mr}{m+n}$

$\therefore \frac{d}{a} = \frac{2n-\frac{2mr}{n}}{mr-n^2} = \frac{2}{n}\left(\frac{n^2-mr}{mr-n^2}\right) = -\frac{2}{n}$

**(12)** **(C).** 50, $a_1, a_2$ ,...................., $a_n$, 200 are in A.P. ........(1)

50, $h_1, h_2$ ,...................., $h_n$, 200 are in H.P.

$\frac{1}{50}, \frac{1}{h_1}, \frac{1}{h_2}$ ,..............., $\frac{1}{h_n}, \frac{1}{200}$ are in A.P.

Reversing, $\frac{1}{200}, \frac{1}{h_n}, \frac{1}{h_{n-1}}$ ,............, $\frac{1}{h_1}, \frac{1}{50}$ are in A.P.

Multiplying by 10000, we get

5 0, $\frac{10000}{h_n}, \frac{10000}{h_{n-1}}$ ,........, $\frac{10000}{h_1}$ , 200 are in A.P.........(2)

(1) and (2) are identical. $\therefore a_2 = \frac{10000}{h_{n-1}}$

$a_2 h_{n-1} = 10000$.

**(13)** **(D).** $a_1, a_2, a_3, a_4, a_5$ are in H.P

$\Rightarrow a_2 = \frac{2a_1a_3}{a_1+a_3} \Rightarrow 2a_1a_3 = a_2a_1 + a_3a_2$

$a_4 = \frac{2a_3a_5}{a_3+a_5} \Rightarrow 2a_3a_5 = a_3a_4 + a_5a_4$

$\Rightarrow a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2a_1a_3 + 2a_3a_5$ ......... (1)

$a_3 = \frac{2(a_1a_5)}{a_1+a_5} \Rightarrow a_1a_3 + a_5a_3 = 2a_1a_5$ ......... (2)

Using eq. (1) and (2)

$a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2(2a_1a_5) = 4a_1a_5$

**(14)** **(C)** log (a + c) + log (a – 2b + c) = log{a + c} (a + c – 2b)

$= \log\left\{(a+c)\left(a+c-\frac{4ac}{a+c}\right)\right\}$ $\left(\text{since } b = -\frac{2ac}{a+c}\right)$

$= \log\{a+c\}^2 - 4ac) = \log(a-c)^2$

The given expression $= (a-c)^2$.

**(15)** **(C).**

$\frac{1}{2}$ $\operatorname{cosec}^2\theta \,.\, \sec\theta = 4\cot^2\theta$ gives $\cos^3\theta = \frac{1}{8}$ and $\theta = \frac{\pi}{3}$.

**(16)** **(B).** $\frac{x}{y}$ etc. are positive A $\geq$ G

$\Rightarrow \frac{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}{3} \geq \sqrt[3]{\frac{x}{y}.\frac{y}{z}.\frac{z}{x}} = 1$

**(17)** **(A).** $\frac{a_1+a_4}{a_1a_4}=\frac{a_2+a_3}{a_2a_3}$,

So $\frac{1}{a_4}+\frac{1}{a_1}=\frac{1}{a_3}+\frac{1}{a_2}$ or $\frac{1}{a_4}-\frac{1}{a_3}=\frac{1}{a_2}-\frac{1}{a_1}$ ........(1)

Also $\frac{3(a_2-a_3)}{a_2a_3}=\frac{a_1-a_4}{a_1a_4}$

So $3\left(\frac{1}{a_3}-\frac{1}{a_2}\right)=\frac{1}{a_4}-\frac{1}{a_1}$ .........(2)

Clearly, (1) and (2)

$\Rightarrow \frac{1}{a_2}-\frac{1}{a_1}=\frac{1}{a_3}-\frac{1}{a_2}=\frac{1}{a_4}-\frac{1}{a_3}$ ; $\frac{1}{a_1},\frac{1}{a_2},\frac{1}{a_3}$ are in A.P.

**(18)** **(A).** The integers divisible by 3 are 33 in number and are 3, 6, ........., 99.
The integers divisible by 5 are 20 in number and are 5, 10, ....., 100.
The integers divisible by 7 are 14 in number and are 7, 14, ....., 98.
The integers divisible by both 3 and 5 are 6 in number and are 15, 30, ......, 90.
The integers divisible by both 3 and 7 are 4 in number and are 21, 42, 63 and 84.
The integers divisible by both 5 and 7 are 2 in number and are 35 and 70.
There are no integers divisible by all three.ss
Hence the sum of the numbers divisible by 3 or 5 or 7 is

$$\frac{33}{2}(3+99)+\frac{20}{2}(5+100)+\frac{14}{2}(7+98)-\frac{6}{2}(15+90)$$

$$-\frac{4}{2}(21+84)-(35+70)=2838.$$

**(19)** **(B).** $a+b=3\,.\,2\sqrt{ab}$ or $\frac{a}{b}-6\sqrt{\frac{a}{b}}+1=0$

$\sqrt{\frac{a}{b}}=3\pm2\sqrt{2}$ or $\frac{a}{b}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}},\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

As $a^2+b^2=34$, the two numbers are

$3+2\sqrt{2}$ and $3-2\sqrt{2}$.

**(20)** **(A).** $q^2=AR^{2p-1}$ and $p^2=AR^{2q-1}$
$T_{p+q}=AR^{p+q-1}=(AR^{2p-1}\,.\,AR^{2q-1})^{1/2}=(p^2q^2)^{1/2}=pq$

**(21)** **(B).** AM $\geq$ GM

$$\frac{\frac{a_1}{2}+\frac{a_1}{2}+a_2+a_3+a_4+.....+a_n}{n+1}\geq\left(\left(\frac{a_1}{2}\right)^2.a_2.a_3.a_4.....a_n\right)^{\frac{1}{n+1}}$$

$$\left(\frac{1}{n+1}\right)^{n+1}\geq\frac{a_1^2a_2a_3a_4.....a_n}{4}$$

$$\Rightarrow \frac{4}{(n+1)^{n+1}}\geq a_1^2a_2a_3a_4.....a_n$$

**(22)** **(A).** From the given conditions we have
$2\sin\beta=\sin\alpha+\sin\gamma$ ......... (1)
$\cos^2\beta=\cos\alpha\cos\gamma$ ......... (2)
Squaring (1), $4\sin^2\beta=\sin^2\alpha+\sin^2\gamma+2\sin\alpha\sin\gamma$
Using (2),
$(1-\cos\alpha\cos\gamma)=1-\cos^2\alpha+1-\cos^2\gamma+2\sin\alpha\sin\gamma$
$\Rightarrow \cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma=2(\sin\alpha\sin\gamma-1)$

$$\Rightarrow \frac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}=-2$$

**(23)** **(A).** $a_1R^{m+n-1}=A$ ....(1)
$a_1R^{m-n-1}=B$ ....(2)
Dividing from (1) and (2) we get
$R^{m+n-1-m+n+1}=A/B$

$$R=\left(\frac{A}{B}\right)^{1/2n};\quad a_1=\frac{A}{R^{m+n-1}}=\frac{A}{\left(\frac{A}{B}\right)^{\frac{m+n-1}{2n}}}$$

$$=A^{\frac{n-m+1}{2n}}.B^{\frac{m+n-1}{2n}}$$

now $a_m=a_1R^{m-1}=A^{\frac{n-m+1}{2n}}.B^{\frac{m+n-1}{2n}}\left(\frac{A}{B}\right)^{\frac{m-1}{2n}}$

$A^{1/2}.B^{1/2}=\sqrt{AB}$

**(24)** **(B).** a , b, c are in H.P. $\Rightarrow b=\frac{2ac}{a+c}$

$$\Rightarrow \frac{\log 4}{\log(2^{1-x}+1)}=\frac{2.\frac{\log 2}{\log(5.2^x+1)}.1}{\frac{\log 2}{\log(5.2^x+1)}+1}\times\frac{2\log 2}{\log(2^{1-x}+1)}$$

$$=\frac{2\log 2}{\frac{\log(5.2^x+1)[\log 2+\log(5.2^x+1)]}{\log(5.2^x+1)}}$$

$10.\,t+2=2/t+1\Rightarrow 10t^2+2t=2+t$ $(2^x=t)$
$10t^2+t-2=0$
$10t^2+5t-4t-2=0$
$5t(2t-1)-2(2t+1)=0\Rightarrow t=2/5$ , $-1/2$ (rejected)
$x\log 2=\log 2/5$ $\Rightarrow 2^x=2/5$
$x\log_2 2=1-\log_2 5$ ; $x=1-\log_2 5$

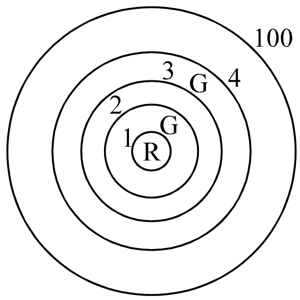
**(25)** **(A).** $\frac{S_{Kx}}{S_x}=\frac{\frac{Kx}{2}[2a+(Kx-1)]}{\frac{K}{2}[2a+(x-1)d]}=K\left[\frac{2a-d+Kxd}{2a-d+xd}\right]$

If $2a-d=0$ then $\frac{S_{Kx}}{S_K}=K\left[\frac{Kxd}{xd}\right]=K^2$

which is possible when $a=d/2$

**(26)** **(B).** $\pi[(r_2^2 - r_1^2)+(r_4^2 - r_3^2)+......+(r_{100}^2 - r_{99}^2)]$

$\therefore\ r_2 - r_1 = r_4 - r_3 = ........ = r_{100} - r_{99} = 1$




$= \pi[r_1 + r_2 + r_3 + r_4 + ...... + r_{100}]$
$= \pi[1 + 2 + 3 + ....... + 100]$
$= 5050\pi$ sq. cm.

**(27)** **(B).** $a_1 = b_1 = 1$ ; $a_9 = 1 + 8d = b_n = 1.\ r^8$

now $\sum_{r=1}^{9} a_r = \frac{9}{2}(1 + a_9)$

$= \frac{9}{2}(1 + r^8) = 369 \Rightarrow r = \sqrt{3}$

$\therefore\ b_7 = b.\ r^6 = 1(\sqrt{3})^6 = 27$

**(28)** **(A).** Let $x_1 = t - a$; $y_1 = 3(t-a)^2$
$x_2 = t$ ; $y_2 = 3t^2$
$x_3 = t + a$; $y_3 = 3(t+a)^2$
since $y_1$, $y_2$ and $y_3$ are in G.P.
however $9t^4 = 9(t-a)^2(t+a)^2$
$t^2 = (t-a)(t+a)$ or $-(t-a)(t+a)$
$t^2 = t^2 - a^2$ rejected as $a \neq 0$
$\therefore\ t^2 = a^2 - t^2$

$2t^2 = a^2 \Rightarrow a = \sqrt{2}\,t$ or $-\sqrt{2}\,t$

$$r = \frac{t^2}{(t-a)^2} = \frac{t^2}{(t-\sqrt{2}\,t)^2} = \frac{1}{(\sqrt{2}-1)^2}$$

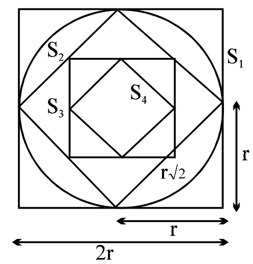
$$= \frac{1}{3-2\sqrt{2}} = 3 + 2\sqrt{2}$$

if $a = -\sqrt{2}\,t$ then $r = 3 - 2\sqrt{2}$

**(29)** **(A).** Side of square $S_1 = 2r$
side of square

$S_2 = r\sqrt{2} = \frac{2r}{2}\left(\frac{1}{\sqrt{2}}\right)^{2-1} = 2r\left[\frac{1}{\sqrt{2}}\right]^{2-1}$

side of square $S_3 = 2r\left(\frac{1}{\sqrt{2}}\right)^{3-1} = 2r\left(\frac{1}{\sqrt{2}}\right)^2$




and so on ,

side of square $S_n = 2r\left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$\therefore$ radius $= r\left(2^{-\frac{1}{2}}\right)^{n-1} = r\left(2^{\frac{1-n}{2}}\right)$ and so on,

side of square $S_n = r(2^{-1/2})^{n-1} = r\left(2^{\frac{1-n}{2}}\right)$

**(30)** **(C).** Let $T_k = 1$, $T'_k = -1$, and $r = 1$, then $T''_k = T_k + T'_k = 0$
$\therefore\ T''_k$ cannot be a term of a G.P.
$\therefore$ statement is false.

**(31)** **(A).** Coefficient of $x^{49}$ is equal $= 1 + 2 + 3 + ...\ 50$

$= \frac{50 \times 51}{2} = 25 \times 51 = 1275.$

**(32)** **(A).** Statement – 1 is true as

a. ar . . . $ar^{n-1} = a^n.\ r^{1+2+...(n-1)} = a^n.\ r^{\frac{n(n-1)}{2}}$

$= (a^2.r^{n-1})^{n/2}$

Statement – 2 is also true as
$(a.\ r^{k-1})(a.\ r^{n-k}) = a^2.\ r^{n-1}$, which is independent of k.
Statement – 2 is the correct reasoning for statement – 1, as in the product of a, ar, $ar^2$,…$ar^{n-1}$, there are n/2 groups of numbers, whose product is $a^2.\ r^{n-1}$.

**(33)** **(D).** $27pqr \geq (p+q+r)^3 \Rightarrow pqr \geq \left(\frac{p+q+r}{3}\right)^3$

$\Rightarrow$ G.M. $\geq$ A.M. but A.M. $\geq$ G.M.
$\therefore$ A.M. = G.M. $\Rightarrow P = Q = r$
Given, $3p + 4q + 5r = 12 \Rightarrow p = q = r = 1$ $\therefore\ p^3 + q^4 + r^5 = 3.$
Hence (D) is the correct answer.

**(34)** **(B).** For statement-1,

$\because$ a, b, c in H.P. $\therefore\ b < \frac{a+c}{2}$ (H.M. < A.M.)

$\because$ b, c, d in H.P. $\therefore\ c < \frac{b+d}{2}$

Adding these two $a + d > b + c$
$\therefore$ statement 1 is true.
for statement 2

$\because$ a, b, c in A.P. $\quad \therefore\ b > \dfrac{2ac}{a+c}$ (A.M. > H.M.)

$\because$ b, c, d in H.P. $\quad \therefore\ c > \dfrac{2bd}{b+d}$ (A.M. > H.M.)

Adding these after simplification

$ab + bc > 2ac$

$bc + cd > 2bd$

---

$ab + cd > 2(ac + bd - bc)$

$\therefore$ statement-2 is true

But statement-2 is not correct explanation of statement-1.

**(35)** **(A).** $a = \log a$

$b = ar \log ar = \log a + \log r$

$c = ar^2 \log ar^2 = \log a + 2 \log r$

So new term are in A.P. and their inverse are in H.P.

**(36)** **(A).** (a) Sum $= \dfrac{8}{9}$ [9 + 99 + 999 +..... n terms ]

$= \dfrac{8}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) +.... n terms]

$= \dfrac{8}{9}\ [(10 + 10^2 + 10^3 + ... + 10^n) - n]$

$= \dfrac{8}{9}\left[\dfrac{10(10^n - 1)}{10-1} - n\right] = \dfrac{8}{81}\ [10^{n+1} - 9n + 10]$

(b) 1st difference 5, 14, 50, 194, 770, ...

2nd difference 9, 36, 144, 576, .....

They are in G.P. whose nth term is $ar^{n-1} = a4^{n-1}$

$\therefore$ $T_n$ of the given series will be of the form

$T_n = a4^{n-1} + bn + c$

$T_1 = a + b + c = 3,\ T_2 = 4a + 2b + c = 8,$

$T_3 = 16a + 3b + c = 22$

Solving we have a = 1, b = 2, c = 0.

$\therefore\ T_n = 4^{n-1} + 2n$

$\therefore\ S_n = \Sigma 4^{n-1} + 2\Sigma n = \dfrac{1}{3}(4^n - 1) + n(n+1)$ .

(c) The rth term of the series is given by

$$t_r = \frac{1}{r(r+1)(r+2)(r+3)} = \frac{1}{3}\left[\frac{(r+3)-r}{r(r+1)(r+2)(r+3)}\right]$$

$$= \frac{1}{3}\left[\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right]$$

$$S_n = \frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{(n+1)(n+2)(n+3)}\right]$$

(d) $\displaystyle\sum_{r=1}^{n}\frac{r}{1+r^2+r^4} = \sum_{r=1}^{n}\frac{r}{(r^2+1-r)(r^2+1+r)}$

$$= \frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{r^2+1-r} - \frac{1}{r^2+1+r}\right)$$

$\therefore\ t_1 = \dfrac{1}{2}\left[\dfrac{1}{1} - \dfrac{1}{3}\right],\ t_2 = \dfrac{1}{2}\left[\dfrac{1}{3} - \dfrac{1}{7}\right],\ t_3 = \dfrac{1}{2}\left[\dfrac{1}{7} - \dfrac{1}{13}\right]$ -----

---- $t_n = \dfrac{1}{2}\left[\dfrac{1}{n^2+1-n} - \dfrac{1}{n^2+n+1}\right]$

Adding we get,

$$t_1 + t_2 + t_3 + .... + t_n = \frac{1}{2}\left[1 - \frac{1}{n^2+n+1}\right] = \frac{1}{2}\left(\frac{n(n+1)}{n^2+n+1}\right)$$

**(37)** **(D).**

(a) $a_k - a_{k-1} = \displaystyle\int_0^{\pi}\frac{\sin(2k-1)x - \sin(2k-3)x}{\sin x}dx$

$$= \int_0^{\pi} 2\cos 2(k-1)x\,dx = \left.\frac{2\sin 2(k-1)x}{2(k-1)}\right]_0^{\pi} = 0$$

for $k = 2, 3, 4 \Rightarrow a_1 = a_2 = a_3 =$ .............

$\Rightarrow$ the sequence is a constant sequence.

(b) Let r is the common ratio of G.P.

$\log y = \log rx = \log r + \log x$

$\log z = \log r^2x = 2\log r + \log x$

Hence, $\dfrac{1}{1+\log x}, \dfrac{1}{1+\log r+\log x}, \dfrac{1}{1+2\log r+\log x}$

are in H.P.

**(38) (C), (39) (B), (40) (C).**

Let four integers be a – d, a, a + d and a + 2d, where a and d are integers and d > 0.

$\because\ a + 2d = (a-d)^2 + a^2 + (a+d)^2$

$\Rightarrow 2d^2 - 2d + 3a^2 - a = 0$ ............. (1)

$\therefore\ d = \dfrac{1}{2}\left[1 \pm \sqrt{1 + 2a - 6a^2}\right]$ ............. (2)

Since d is positive integer

$\therefore\ 1 + 2a - 6a^2 > 0$

$6a^2 - 2a - 1 < 0$

$\Rightarrow \dfrac{1-\sqrt{7}}{6} < a < \dfrac{1+\sqrt{7}}{6}$ $\quad\because$ a is an integer

$\therefore$ a = 0 Put in (2)

$\therefore$ d = 1 or 0 but $\because$ d > 0

$\therefore$ d = 1 $\quad\therefore$ The four numbers are –1, 0, 1, 2

**(41)** **(D).** Roots of $x^2 + 13x + 36 = 0$ are –4, –9

$$\alpha = -\frac{13}{2}, \beta = -6, \gamma = -\frac{72}{13}$$

Minimum distance between roots is $-\dfrac{72}{13} - \left(-\dfrac{13}{2}\right) = \dfrac{25}{26}$

Minimum value of $\delta$ is $\dfrac{25}{26}$

**(42)** **(A).** Sum of roots $\le -6$

$\Rightarrow t^2 - 13t + \alpha + \gamma \le -6$

$\Rightarrow t^2 - 13t + \alpha + \gamma + 6 \le 0$

$\Rightarrow p = \ell + m = 13$

**(43)** **(D).** $2\alpha = -13,\ p = 13$
Equation $x^2 - 169 = 0$

**(44)** **(B).** $G_1 G_2 ......G_n = (\sqrt{1 \times 1024})^n = 2^{5n}$
$\therefore\ 2^{5n} = 2^{45}$ $\therefore\ n = 9$

**(45)** **(B).** $A_1 + A_2 + A_3 + ......A_{m-1} + A_m = 1025 \times 171$
$\therefore\ m\left(\frac{-2+1027}{2}\right) = 1025 \times 171 \ \therefore\ m = 342$

**(46)** **(A).** $\because\ n = 9\ \therefore\ r = (1024)^{\frac{1}{9+1}} = 2\ \therefore\ G_1 = 2, r = 2$

$$G_1 + G_2 + ...... + G_n = \frac{2.(2^9 - 1)}{2-1} = 1024 - 2 = 1022$$

**(47)** **(A).** Given : $c_n = a_1 + a_2 + a_3 + ........ + a_n$
where $a_1, a_2, ........, a_n$ are in A.P. with $d = 2$
and $d_n = b_1 + b_2 + b_3 + ........ + b_n$ are in A.P. with $d = 2$
Also, $(a_n, c_n)$ lies on $y = px^2 + qx + r$
Now, $c_n = pa_n{}^2 + qa_n + r$ .......... (1)
$c_{n-1} = pa^2{}_{n-1} + qa_{n-1} + r$ .......... (2)
$\therefore$ From eq. (1) and (2), we get
$c_n - c_{n-1} = p\,(a^2{}_n - a^2{}_{n-1}) + q\,(a_n - a_{n-1})$
$\therefore\ a_n = p\,(a_n + a_{n-1})\,(a_n - a_{n-1}) + q\,(a_n - a_{n-1})$
$a_n = (a_n - a_{n-1})\,[p\,(a_n + a_{n-1}) + q]$ .......... (3)
$[a_n - a_{n-1} = d]$
On putting n = 2 and 3 in eq. (3), we get
$a_2 = d\,[\,p\,(a_2 + a_1) + q]$ .......... (4)
$a_3 = d\,[\,p\,(a_3 + a_2) + q]$ .......... (5)

Now, (5) – (4), we get

$$\underbrace{a_3 - a_2}_{=d} = dp\,[\underbrace{a_3 - a_1}_{=2d}]$$

$4p = 1 \Rightarrow\ p = 1/4$

**(48)** **(C).** To find q : $c_n = pa^2{}_n + qa_n + r$
On putting n = 1, 2 in above equation, we get
$c_1 = a_1 = pa_1{}^2 + qa_1 + r$ ......... (1)
$c_2 = a_1 + a_2 = pa_2{}^2 + qa_2 + r$ ......... (2)
but $a_2 = a_1 + 2$
$\therefore\ 2a_1 + 2 = p\,(a_1 + 2)^2 + q\,(a_1 + 2) + r = (pa_1{}^2 + qa_1 + r) + 4a_1p + 4p + 2q$ $(4p = 1)$
$2a_1 + 2 = c_1 + a_1 + 1 + 2q$ $(\because\ c_1 = a_1)$

$$2a_1 + 2 = 2a_1 + 1 + 2q\ \Rightarrow\ q = \frac{1}{2}$$

**(49)** **(C).** If r = 0, then $c_1 = pa_1^2 + qa_1$

$a_1 = \frac{1}{4}a_1^2 + \frac{1}{2}a_1$ $(\because\ c_1 = a_1)$

$a_1{}^2 - 2a_1 = 0\ \Rightarrow\ a_1 = 0$ or $a_1 = 2$

Also, $d_1 = \frac{1}{4}b_1^2 + qb_1$ $\left(\because\ q = \frac{1}{2} \text{ and } d_1 = b_1\right)$

$b_1 = \frac{1}{4}b_1^2 + \frac{1}{2}b_1$

$b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0$ or $b_1 = 2$
But $a_1 < b_1 \Rightarrow a_1 = 0$ and $b_1 = 2$

## EXERCISE-3

**(1)** **11.** $t_k = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k\,(k+2)^2 - k^2(k+2)} = \frac{1}{2}\left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+2}}\right)$

as $k \to \infty$

$$\text{Sum} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{1+\sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{1}+\sqrt{2}}{\sqrt{8}} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}}$$

a = 1, b = 2, c = 8 or a = 2, b = 1, c = 8
$\Rightarrow a + b + c = 11$

**(2)** **111.** nth term of 1, 3, 6, 10, ........

$a_n = \frac{1}{2}(n^2 + n)$ at n = 10, $a_n = 55 = k$

So sum of number is $n^{th}$ brackets $= k + k^2 + ....... k^{2n+1}$

$$A = \frac{k\,(k^{2n+1} - 1)}{k-1} \Rightarrow \frac{54A}{55} + 1 = 55^{111} \Rightarrow B = 111$$

**(3)** **8.** $\because$

$$\frac{1}{\sqrt{n+\sqrt{n^2-1}}} = \frac{1}{\sqrt{\left(\sqrt{\frac{n+1}{2}} + \sqrt{\frac{n-1}{2}}\right)^2}}$$

$$= \frac{1}{\sqrt{\frac{n+1}{2}} + \sqrt{\frac{n-1}{2}}} = \frac{\sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}}{\frac{n+1}{2} - \frac{n-1}{2}}$$

$$= \sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}$$

Hence, $a + b\sqrt{2} = \sum_{n=1}^{49}\left(\sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}\right)$

$$\Rightarrow\ a + b\sqrt{2} = \left(\sqrt{\frac{2}{2}} - 0\right) + \left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}\right)$$

$$+ \left(\sqrt{\frac{4}{2}} - \sqrt{\frac{2}{2}}\right) + \left(\sqrt{\frac{5}{2}} - \sqrt{\frac{3}{2}}\right) + ....... + \left(\sqrt{\frac{49+1}{2}} - \sqrt{\frac{49-1}{2}}\right)$$

$$= \sqrt{\frac{49+1}{2}} + \sqrt{\frac{48+1}{2}} - \frac{1}{\sqrt{2}} - 0 = 5 + 3\sqrt{2}$$

$\Rightarrow a = 5, b = 3$ and $a + b = 8$

**(4)** **4950.** $\tan^2\frac{\pi}{12} = \tan\left(\frac{\pi}{12}-x\right)\tan\left(\frac{\pi}{12}+x\right)$

$$\tan^2\frac{\pi}{12} = \frac{\tan\frac{\pi}{12}-\tan x}{1+\tan\frac{\pi}{12}\tan x}\cdot\frac{\tan\frac{\pi}{12}+\tan x}{1-\tan\frac{\pi}{12}\tan x} = \frac{\tan^2\frac{\pi}{12}-\tan^2 x}{1-\tan^2\frac{\pi}{12}\tan^2 x}$$

$$\tan^2\frac{\pi}{12}-\tan^4\frac{\pi}{12}\tan^2 x = \tan^2\frac{\pi}{12}-\tan^2 x$$

$$\Rightarrow \tan^2 x\left(\tan^4\frac{\pi}{12}-1\right) = 0;\ \tan x = 0 \Rightarrow x = k\pi$$

$\therefore \cos 2x = 1 \Rightarrow x = n\pi$

Sum of solutions is $\pi(1+2+3+......+99) = 4950\pi$

$\Rightarrow k = 4950$

**(5)** **1.** $\frac{a}{1-r_1} = r_1$ and $\frac{a}{1-r_2} = r_2$

hence $r_1$ and $r_2$ are the roots of

$$\frac{a}{1-r} = r \Rightarrow r^2 - r + a = 0 \Rightarrow r_1 + r_2 = 1$$

**(6)** **6.** Let the roots are a – 3d, a – d, a + d, a + 3d

sum of roots = 4a = 0 ⇒ a = 0

hence roots are – 3d, – d, d, 3d

product of roots $= 9d^4 = m^2 \Rightarrow d^2 = \frac{m}{3}$ ........ (1)

Again $\Sigma x_1x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2 =$
$-10d^2 = -(3m+2)$ ; $10d^2 = 3m+2$

$$\frac{10m}{3} = 3m+2 = 10m = 9m+6\ ;\ m = 6$$

**(7)** **7.** given : $a_3 + a_5 + a_8 = 11$

$a + 2d + a + 4d + a + 7d = 11$

$3a + 13d = 11$ ....(1)

Given : $a_4 + a_2 = -2$

$a + 3d + a + d = -2$

$a = -1 - 2d$ ....(2)

put (2) in (1)

$3(-1-2d) + 13d = 11 \Rightarrow 7d = 14 \Rightarrow d = 2$ and $a = -5$

Now $a_1 + a_6 + a_7 \Rightarrow a + a+5d + a + 6d$

$\Rightarrow 3a + 11d \Rightarrow 3(-5) + 11(2) = -15 + 22 = 7$

**(8)** **8.** $S = 4\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k = 4\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+.......\right]$

$$= 4\left[\frac{2/3}{1-(2/3)}\right] = 8$$

**(9)** **31.** Let there be 2n + 1 stones ; i.e. n stones on each side of the middle stone. The man will run 20 m, to pick pick up the first stone and return, 40 m. for the second stone and so on. So he runs (n/2) (2 × 20 + (n – 1)20) = 10n(n + 1) meters to pick up the stones on one side, and hence 20 n(n + 1) m, to pick up all the stones.

$\therefore 20n(n+1) = 4800$, or $n = 15$.

∴ there are 2n + 1 = 31 stones

**(10)** **925.** Let the 3 consecutive terms are

$a-d,\ a,\ a+d$ $\qquad d > 0$

hence $a^2 - 2ad + d^2 = 36 + K$ ....(1)

$a^2 = 300 + K$ ....(2)

$a^2 + 2ad + d^2 = 596 + K$ ....(3)

now (2) – (1) gives

$d(2a - d) = 264$ ....(4)

(3) – (2) gives

$d(2a + d) = 296$ ....(5)

(5) – (4) gives

$2d^2 = 32 \Rightarrow d^2 = 16 \Rightarrow d = 4$ (d = – 4 rejected)

Hence from (4)

$4(2a - 4) = 264 \Rightarrow 2a - 4 = 66 \Rightarrow 2a = 70 \Rightarrow a = 35$

$\therefore K = 35^2 - 300 = 1225 - 300 = 925$

**(11)** **3.** $S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$

$$\sum_{k=2}^{100}\left|(k^2-3k+1)\frac{1}{(k-1)!}\right| = \sum_{k=2}^{100}\left|\frac{(k-1)^2-k}{(k-1)!}\right|$$

$$= \sum\left|\frac{k-1}{(k-2)!}-\frac{k}{(k-1)!}\right| = \left|\frac{2}{1!}-\frac{3}{2!}\right|+\left|\frac{3}{2!}-\frac{4}{3!}\right|+.....$$

$$= \frac{2}{1!}-\frac{1}{0!}+\frac{2}{1!}-\frac{3}{2!}+\frac{3}{2!}-\frac{4}{3!}+....+\frac{99}{98!}-\frac{100}{99!} = 3-\frac{100}{99!}$$

**(12)** **0.** $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, ........., a_{11}$ are in A.P.

$$\therefore \frac{a_1^2+a_2^2+.....+a_{11}^2}{11} = \frac{11a^2+35\times 11d^2+10ad}{11} = 90$$

$\Rightarrow 225 + 35d^2 + 150d = 90$

$\Rightarrow 35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$

Given $a_2 < \frac{27}{2}$ ∴ $d = -3$ and $d \neq -9/7$

$$\Rightarrow \frac{a_1+a_2+....+a_{11}}{11} = \frac{11}{2}[30-10\times 3] = 0$$

**(13)** **9.** $\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6+(5n-1)d]}{\frac{n}{2}[6+(n-1)d]} = \frac{5[(6-d)+5nd]}{[(6-d)+nd]}$

d = 6 or d = 0.

Now, if d = 0 then $a_2 = 3$ else $a_2 = 9$

**(14)** **25.** $a_1, a_2, a_3$...........be in H.P

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$.... be in A.P.

in A.P. $T_1 = \frac{1}{a_1} = \frac{1}{5}$ and $T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$

$\Rightarrow T_{20} = T_1 + 19d$

$\frac{1}{25} = \frac{1}{5} + 19d \Rightarrow d = -\frac{4}{19 \times 25}$

$T_n = T_1 + (n-1)d < 0$

$\Rightarrow \frac{1}{5} - \frac{(n-1)\cdot 4}{19 \times 25} < 0 \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25 \times 19}$

$\Rightarrow \frac{5 \times 19}{4} + 1 < n \Rightarrow \frac{99}{4} < n$

$\Rightarrow$ Least positive integer n is 25.

**(15)** **5.** Clearly, $1 + 2 + 3 + ... + n - 2 \le 1224 \le 3 + 4 + ... n$

$\Rightarrow \frac{(n-2)(n-1)}{2} \le 1224 \le \frac{(n-2)}{2}(3+n)$

$\Rightarrow n^2 - 3n - 2446 \le 0$ and $n^2 + n - 2454 \ge 0$

$\Rightarrow 49 < n < 51 \Rightarrow n = 50$

$\therefore \frac{n(n+1)}{2} - (2k+1) = 1224$

$\Rightarrow k = 25 \Rightarrow k - 20 = 5$

**(16)** **4.** $\frac{1}{H} = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{\alpha+\beta}{2\alpha\beta} = \frac{4+\sqrt{5}}{5+\sqrt{2}} \times \frac{5+\sqrt{2}}{8+2\sqrt{5}} \times \frac{1}{2} = \frac{1}{4}$

$H = 4$

**(17)** **1.** $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \frac{\log(x-1)}{\log 4} = \frac{\log(x-2)}{\log 2} \Rightarrow \frac{\log(x-1)}{2\log 2} = \frac{\log(x-2)}{\log 2}$

$\Rightarrow \log(x-1) = 2\log(x-2)$

$\Rightarrow x - 1 = (x-2)^2 \Rightarrow x^2 - 7x + 10 = 0 \Rightarrow x = 5, 2$

Also, $x - 1 > 0$ and $x - 3 > 0$

$\Rightarrow x > 1$ and $x > 3 \Rightarrow x = 5$ is the solution.

## EXERCISE-4

**(1)** **(C).** $1^3 - 2^3 + 3^3 - .......... + 9^3$

$= (1^3 + 2^3 + 3^3 + .......... + 9^3) - 2(2^3 + 4^3 + 6^3 + 8^3)$

$= \left(\frac{9(9+1)}{2}\right)^2 - 2 . 2^3 (1^3 + 2^3 + 3^3 + 4^3)$

$= (9 \times 5)^2 - 16\left(\frac{4 \times (4+1)}{2}\right)^2$

$= (45)^2 - 16 \times (10)^2 = 2025 - 1600 = 425$

$\{\because 1^3 + 2^3 + .......... + n^3 = \left(\frac{n(n+1)}{2}\right)^2\}$

**(2)** **(C).** Let first term of G.P. is 'a' and common ratio is r

$\therefore a + ar + ar^2 + ........ \infty = 20$ (given)

$\frac{a}{1-r} = 20 \Rightarrow a = 20(1-r)$ ......... (1)

and $a^2 + a^2r^2 + a^2r^4 + ........ \infty = 100$ (given)

$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow a^2 = 100(1-r^2)$ ......... (2)

From (1) put value of a in (2)

$\Rightarrow [(20(1-r)]^2 = 100(1-r^2)$

$\Rightarrow 400(1-r)^2 = 100(1-r)(1+r)$

$\Rightarrow 4(1-r) = 1 + r \Rightarrow 5r = 3 \Rightarrow r = 3/5$

**(3)** **(C).** In an A.P. $T_3 = 7$

and $T_7 = 3T_3 + 2$ (according to question)

$= 3 \times 7 + 2 = 23$

$S_{20} = ?$

Let first term of A.P. is a and common difference is d.

$\therefore T_3 = 7$

$\Rightarrow a + 2d = 7 \Rightarrow a = 7 - 2d$ .......... (i)

and $T_7 = 3T_3 + 2 = 23$

$\Rightarrow a + 6d = 23$ .......... (ii)

From (i) put value of a in (ii) we get

$7 - 2d + 6d = 23 \Rightarrow 4d = 16 \Rightarrow d = 4$ & $a = 7 - 2 \times 4 = -1$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{20} = \frac{20}{2}[2 \times (-1) + (20-1) \times 4] = 10 \times (-2 + 76)$

$= 10 \times 74 = 740$

**(4)** **(B).** $\because x_1, x_2, x_3$ are in G.P.

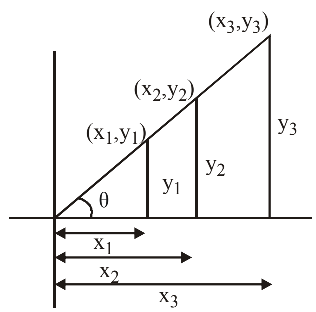
Let common ratio be r

$\therefore \frac{x_2}{x_1} = \frac{x_3}{x_2} = r$ ......... (1)

and $y_1, y_2, y_3$ also are in G.P. with same common ratio as of $x_1, x_2, x_3$

$\therefore \frac{y_2}{y_1} = \frac{y_3}{y_2} = r$ ......... (2)

$\Rightarrow \frac{x_2}{x_1} = \frac{x_3}{x_2} = \frac{y_2}{y_1} = \frac{y_3}{y_2} \Rightarrow \frac{x_2}{y_2} = \frac{x_1}{y_1} = \frac{x_3}{y_3}$ ......... (3)



$\Rightarrow$ point $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on a straight line by graph.

$$\tan\theta = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3}$$

This is possible only when they are in straight line but

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} \text{ (from (3))}$$

**(5)** **(D).** System of linear equation,

$x + 2ay + az = 0;\quad x + 3by + bz = 0; x + 4cy + cz = 0$

has non-zero solution

$$\therefore \text{ For non-zero solution } \Delta = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow 1\,[(3b - 2a)(c - a) - (b - a)(4c - 2a)] = 0$

$\Rightarrow 3bc - 3ab - 2ac + 2a^2 - 4bc + 2ab + 4ac - 2a^2 = 0$

$\Rightarrow - bc - ab + 2ac = 0$

$\Rightarrow 2ac = bc + ab$ {dividing by abc}

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow$ a, b, c are in H.P.

**(6)** **(B).** Quadratic equation A.M. of whose root is A & G.M. is G is $x^2 - 2Ax + G^2 = 0$

$\because$ A.M. of two given no. is

$9 \Rightarrow A = 9$ and G.M. is $4 \Rightarrow G = 4$

$\therefore$ Q.E. with these number as root is

$x^2 - 2Ax + G^2 = 0$

$\Rightarrow x^2 - 2 \times 9x + 4^2 = 0 \Rightarrow x^2 - 18x + 16 = 0$

**(7)** **(A).** $T_m = \frac{1}{n}$ (given)

$a + (m - 1)\,d = \frac{1}{n}$ .......... (1) and $T_n = \frac{1}{m}$ (given)

$a + (n - 1)\,d = \frac{1}{m}$ .......... (2)

Subtracting (2) from (1) we get

$$(m - 1)\,d - (n - 1)\,d = \frac{1}{n} - \frac{1}{m}$$

$\Rightarrow (m - n)\,d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$ .......... (3)

Put the value of d in (2) we get

$$a = \frac{1}{mn} \quad \therefore \quad a - d = \frac{1}{mn} - \frac{1}{mn} = 0$$

**(8)** **(B).** According to question if n is even sequence will be

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots + 2.n^2 = \frac{n(n+1)^2}{2}$$

$\{\because n \to \text{even}\}$

If n is odd sequence will be

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots + n^2$

$\underline{1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots + 2\,(n-1)^2} + n^2$

(n – 1) term (which is even)

$$= \frac{(n-1)(n-1+1)^2}{2} + n^2 = \frac{(n-1)\,n^2}{2} + n^2$$

$$= n^2\left[\frac{n-1}{2} + 1\right] = \frac{n^2(n+1)}{2}$$

$\{\because$ sum of n even terms is $\frac{n(n+1)^2}{2}$

$\therefore$ sum of (n – 1) even term is $(n-1)\,\frac{(n-1+1)^2}{2}\}$

**(9)** **(D).** $\because\ x = \sum_{n=0}^{\infty} a^n = a^0 + a^1 + a^2 + \ldots.. \infty$

$= 1 + a + a^2 + \ldots.. \infty$

$x = \frac{1}{1-a} \Rightarrow \frac{x-1}{x} = a$ ........ (1)

and $y = \sum_{n=0}^{\infty} b^n = b^0 + b^1 + b^2 + \ldots.. \infty$

$y = \frac{1}{1-b} \Rightarrow b = \frac{y-1}{y}$ ......... (2)

and $z = \sum_{n=0}^{\infty} c^n = c^0 + c^1 + c^2 + \ldots.. \infty$

$z = \frac{1}{1-c} \Rightarrow c = \frac{z-1}{z}$ ......... (3) $\because$ a, b, c are in in A.P.

$\therefore \frac{x-1}{x}, \frac{y-1}{y}, \frac{z-1}{z}$ are also in A.P.

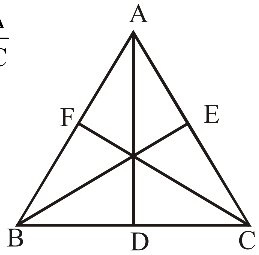
$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are also in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P. (subracting 1from each)

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. (multiplying with – 1 each)

$\Rightarrow$ x, y, z are in H.P.

**(10)** **(B).** $\Delta = \frac{1}{2}BC.AD \Rightarrow AD = \frac{2\Delta}{BC}$




Here, BC = a, AC = b, AB = c

Altitude $AD = \frac{2\Delta}{a}$

Similarly altitude $BE = \frac{2\Delta}{b}$

and $CF = \frac{2\Delta}{c}$

$\because$ AD, BE, CF are in H.P.

$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P.

$\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ sin A, sin B, sin C are in A.P.

$\left\{\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}\right\}$

**(11)** **(C).**

$$\frac{a_1 + a_2 + ......... + a_p}{a_1 + a_2 + ......... + a_q} = \frac{p^2}{q^2} \Rightarrow \frac{p/2\,[2a_1 + (p-1)d]}{q/2\,[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

{$\because$ $a_1$ is first term and d is common difference}

$$\Rightarrow \frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q} \qquad ......... (1)$$

if $\frac{p-1}{2} = 5$ and $\frac{q-1}{2} = 20$

then L.H.S. of (i) represents ratio of $a_6$ and $a_{21}$

$\therefore$ p = 11 and q = 41

Now (i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

**(12)** **(C).** $a_1, a_2, a_3, ........ a_n$ are in H.P

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ..... \frac{1}{a_n}$ are in A.P.

Let common difference of A.P. is d.

$\therefore \frac{1}{a_n} = \frac{1}{a_1} + (n-1)\,d \Rightarrow \frac{1}{a_n} - \frac{1}{a_1} = (n-1)\,d$

$$\Rightarrow \frac{a_1 - a_n}{d} = a_1 a_n (n-1) \qquad ......... (A)$$

$$\therefore \frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow \frac{a_1 - a_2}{d} = a_1 a_2 \qquad ......... (i)$$

$$\frac{1}{a_3} - \frac{1}{a_2} = d \Rightarrow \frac{a_2 - a_3}{d} = a_2 a_3 \qquad ......... (ii)$$

$$\frac{1}{a_4} - \frac{1}{a_3} = d \Rightarrow \frac{a_3 - a_4}{d} = a_3 a_4 \qquad ......... (iii)$$

$\vdots$

$$\frac{1}{a_n} - \frac{1}{a_{n-1}} = d \Rightarrow \frac{a_{n-1} - a_n}{d} = a_n a_{n-1} \qquad ......... (n)$$

Adding all column wise

$a_1 a_2 + a_2 a_3 + a_3 a_4 + ...... + a_n a_{n-1}$

$= \frac{1}{d}[a_1 - a_2 + a_2 - a_3 + a_3 - a_4 + ...... + a_{n-1} - a_n]$

$= \frac{1}{d}[a_1 - a_n]$ ; From (A) $= a_1 a_n (n-1)$

**(13)** **(D).** Let first term of G.P. is a & common ratio is r and then G.P. is a, ar, $ar^2$, $ar^3$, $ar^4$ ......... $ar^{n-1}$

According to question,

$a = ar + ar^2 \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$$r = \frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \sqrt{5}}{2} \quad \{\because\ r > 0\}$$

**(14)** **(A).** Let first term of G.P. is a and common ratio is r

$\therefore$ G.P. is a, ar, $ar^2$, $ar^3$.

According to question, $\quad a + ar = 12 \qquad ....... (1)$

and $ar^2 + ar^3 = 48 \qquad ....... (2)$

Dividing eq. (1) by (2) we get $\frac{1}{r^2} = \frac{1}{4} \Rightarrow r = \pm 2$

$\because$ terms are alternately +ve and –ve $\quad \therefore r = -2$

Put value of r in (1) we get a + a (–2) = 12 ; a = – 12

**(15)** **(B).** $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{10}{3^4} + ......$ $\qquad$ .......... (i)

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...... \qquad .......... (ii)$$

Subtracting (ii) from (i) we get

$$S\frac{2}{3} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + ........$$

$$= \frac{4}{3} + \frac{4}{3^2} + \left(\frac{1}{1 - \frac{1}{3}}\right) = \frac{4}{3} + \frac{4}{3^2} \times \frac{3}{2} = 2 \ ; \ S = 3$$

**(16)** **(A).** Till $10^{th}$ minute number of counted notes = 1500

$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$

$n^2 - 149n + 3000 = 0$

n = 125, 24

n = 125 is not possible.

Total time = 24 + 10 = 34 minutes.

**(17)** **(D).** a = Rs. 200 ; d = Rs. 40
Savings in first two months = Rs. 400
Remained savings = 200 + 240 + 280 + ..... upto n terms
$200n + 20n^2 - 20n = 10640$
$20n^2 + 180n - 10640 = 0$
$n^2 + 9n - 532 = 0$
$(n + 28)(n - 19) = 0$
$n = 19$
$\therefore$ no. of months = 19 + 2 = 21 .

**(18)** **(B).** $T_n = (n-1)^2 + (n-1)n + n^2$

$$= \frac{((n-1)^3 - n^3)}{(n-1) - n} = n^3 - (n-1)^3$$

$T_1 = 1^3 - 0^3$
$T_2 = 2^3 - 1^3$
$\vdots$
$T_{20} = 20^3 - 19^3$
$S_{20} = 20^3 - 0^3 = 8000$

**(19)** **(D).** $100(a + 99d) = 50(a + 49d)$
$2a + 198d = a + 49d$
$a + 149d = 0$
$T_{150} = a + 149d = 0$

**(20)** **(C).** $\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ...... +$ upto 20 terms

$$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + ...... + \text{upto 20 terms}\right]$$

$$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...... + \text{upto 20 terms}\right]$$

$$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + ..... + \text{upto 20 terms}\right]$$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right] = \frac{7}{9}\left[20 - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^{20}\right)\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

**(21)** **(A).** $2y = x + z$
$2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$

$$\tan^{-1}\left(\frac{2y}{1 - y^2}\right) = \tan^{-1}\left(\frac{x + z}{1 - xz}\right)$$

$$\frac{x + z}{1 - y^2} = \frac{x + z}{1 - xz} \Rightarrow y^2 = xz \text{ or } x + z = 0 \Rightarrow x = y = z$$

**(22)** **(D).** Let numbers be a, ar, $ar^2$
Now, $2(2ar) = a + ar^2$ $[a \neq 0]$
$\Rightarrow 4r = 1 + r^2$
$\Rightarrow r^2 - 4r + 1 = 0$
$\Rightarrow r = 2 \pm \sqrt{3} \Rightarrow r = 2 + \sqrt{3}$ (Positive value)

**(23)** **(C).** $S = 10^9 + 2(11)^1(10)^8 + ....... + 10 . 11^9$

$$\frac{11}{10} \cdot S = \quad 11^1 . 10^8 + ....... + 9 . 11^9 + 11^{10}$$

$$-\frac{1}{10} S = 10^9 + 11^1 . 10^8 + 11^2 . 10^7 + ....... 11^9 - 11^{10}$$

$$\Rightarrow -\frac{1}{10} S = 10^9 \left(\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right) - 11^{10}$$

$$\Rightarrow -\frac{1}{10} S = 11^{10} - 10^{10} - 11^{10}$$

$S = 10^{11} = 100 . 10^9 \Rightarrow k = 100$

**(24)** **(A).** $$t_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2} = \frac{(n+1)^2}{4} = \frac{1}{4}[n^2 + 2n + 1]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2(n)(n+1)}{2} + 1\right]$$

$$= \frac{1}{4}\left[\frac{9 \times 10 \times 19}{6} + 9 \times 10 + 9\right] = 96$$

**(25)** **(A).** $\frac{\ell + n}{2} = m$ ; $\ell + n = 2m$ .......... (1)

$$G_1 = \ell\left(\frac{n}{\ell}\right)^{1/4} ; \; G_2 = \ell\left(\frac{n}{\ell}\right)^{2/4} ; \; G_3 = \ell\left(\frac{n}{\ell}\right)^{3/4}$$

Now, $G_1{}^4 + 2G_2{}^4 + G_3{}^4$

$$\ell^4 \cdot \frac{n}{\ell} + 2 \cdot (\ell^2)\left(\frac{n}{\ell}\right)^2 + \ell^4\left(\frac{n}{\ell}\right)^3$$

$= n\ell^3 + 2n^2\ell^2 + n^3\ell = 2n^2\ell^2 + nl(n^2 + \ell^2)$
$= 2n^2\ell^2 + n\ell((n + \ell)^2 - 2n\ell)$
$= n\ell(n + \ell)^2 = n\ell.(2m)^2 = 4n\ell m^2$

**(26)** **(A).** a + d, a + 4d, a + 8d
$(a + 4d)^2 = (a + d)(a + 8d)$
$a^2 + 16d^2 + 8ad = a^2 + 9ad + 8d^2$

$8d^2 = ad$ ; $a = 8d, d \neq 0$ ; $r = \frac{a + 4d}{a + d} = \frac{12d}{9d} = \frac{4}{3}$

**(27)** **(A).** $$S_n = \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2$$

$$S = \frac{16}{25}[2^2 + 3^2 + 4^2 + 5^2 + ..... + 11^2]$$

$$= \frac{16}{25}[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + ..... + 11^2 - 1] = \frac{16}{5} \times 101$$

**(28)** **(D).** $225a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a + c)$
$225a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
$(15a)^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
$15a = 3b = 5c = k$

$$a = \frac{k}{15} = \frac{k}{15},\ b = \frac{k}{3} = \frac{5k}{15},\ c = \frac{k}{5} = \frac{3k}{15}$$

$a + b = 2c$ ; b, c, a are in A.P.

**(29)** **(A).** $a_1 + a_5 + a_9 + \ldots\ldots a_{49} = 416$
$\Rightarrow a + 24d = 32$ ……(i)
$a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ ……(ii)
From (i) and (ii) d = 1 and a = 8
Now, $a_1{}^2 + a_2{}^2 + \ldots\ldots + a^2{}_{17} = 140m$

$$\sum_{r=1}^{17}(8 + (r-1))^2 = 140m \ ; \quad \sum_{r=1}^{17}(7 + r)^2 = 140m$$

$4760 = 140m \Rightarrow m = 34$

**(30)** **(D).** $A = 1^2 + 2.2^2 + 3^2 + 2.4^2 + \ldots\ldots A^2 + 2.20^2$
$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$

$$= \frac{20 \times 21 \times 41}{6} + 4 \times \frac{10 \times 11 \times 21}{6}$$

$= 2870 + 1540 = 4410 = 2870 + 1540 = 4410$

$$B = \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$$

$= 540 \times 41 + 41 \times 280 = 41 \times 820 = 33620$
$33620 - 8820 = 110\lambda ; 100\lambda = 24800 ; \lambda = 248$

**(31)** **(D).** $\frac{b}{r}$, b, br $\to$ G.P. $(|r| \neq 1)$ Given $a + b + c = xb$
$\Rightarrow b/r + b + br = xb \Rightarrow b = 0$ (not possible)

or $1 + r + \frac{1}{r} = x \Rightarrow x - 1 = r + \frac{1}{r}$

$\Rightarrow x - 1 > 2$ or $x - 1 < -2 \Rightarrow x > 3$ or $x < -1$
So x can't be '2'

**(32)** **(D).** $S = a_1 + a_2 + \ldots\ldots + a_{30}$ ; $S = \frac{30}{2}[a_1 + a_{30}]$

$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$
$T = a_1 + a_3 + \ldots\ldots + a_{29}$
$= (a_1) + (a_1 + 2d) \ldots\ldots + (a_1 + 28d)$
$= 15a_1 + 2d(1 + 2 + \ldots\ldots + 14)$
$T = 15a_1 + 210d$. Now use $S - 2T = 75$
$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210d) = 75 \Rightarrow d = 5$
Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$
Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

**(33)** **(B).** $S_A$ = sum of numbers between 100 & 200 which are divisible by 7.
$\Rightarrow S_A = 105 + 112 + \ldots\ldots + 196$

$$S_A = \frac{14}{2}[105 + 96] = 2107$$

SB = Sum of numbers between 100 & 200 which are divisible by 13.

$$S_B = 104 + 117 + \ldots. + 195 = \frac{8}{2}[104 + 195] = 1196$$

$S_C$ = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.
$S_C = 182 \Rightarrow$ H.C.F. $(91, n) > 1 = S_A + S_B - S_C = 3121$

**(34)** **(B).** $S = \sum_{k=1}^{20} \frac{1}{2^k}$

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + \ldots + \frac{20}{2^{20}}$$

$$S \times \frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{19}{2^{20}} + \frac{20}{2^{21}}$$

$$\left(1 - \frac{1}{2}\right)S = \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{20}} - \frac{20}{2^{21}} \Rightarrow S = 2 - \frac{11}{2^{19}}$$

**(35)** **(C).** a, b, c in G.P.
say a, ar, $ar^2$
Satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$
$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d \cdot \frac{c}{a} - \frac{2ce}{b} + f = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

**(36)** **(A).** $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$$

$$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8] = 50 + A(n-4)$$

$d = T_n - T_{n-1} = 50 + A(n-4) - 50 - A(n-5) = A$
$T_{50} = 50 + 46A$
$(d, A_{50}) = (A, 50 + 46A)$

**(37)** **(A).** $a - d + a + a + d = 33 \Rightarrow a = 11$
$a(a^2 - d^2) = 1155$
$121 - d^2 = 105$
$d^2 = 16 \Rightarrow d = \pm 4$
If d = 4 then Ist term = 7
If d = –4 then Ist term = 15
$T_{11} = 7 + 40 = 47$
OR $T_{11} = 15 - 40 = -25$

**(38)** **(B).** $T_r = r(2r - 1)$
$S = \Sigma 2r^2 - \Sigma r$

$$S = \frac{2 \cdot n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S_{11} = \frac{2}{6}(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$$

**(39)** **(A).** $T_n = \frac{(3+(n-1)\times 2\,(1^3+2^3+...+n^3)}{(1^2+2^2+....+n^2)}$

$= \frac{3}{2}n\,(n+1) = \frac{n\,(n+1)\,(n+1)-(n-1)\,n\,(n+1)}{2}$

$\Rightarrow S_n = \frac{n\,(n+1)\,(n+2)}{2} \Rightarrow S_{10} = 660$

**(40)** **(C).** $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}\,(a_1 + a_{16}) = 114. \Rightarrow a_1 + a_{16} = 38$

$a_1 + a_6 + a_{11} + a_{16} = 4 = \frac{4}{2}\,(a_1 + a_{16}) = 2\times 38 = 76$

**(41)** **(D).** $\text{Sum} = \sum_{n=1}^{15} \frac{1^3+2^3+...+n^3}{1+2+....+n} = \frac{1}{2}\cdot\frac{15\cdot16}{2}$

$= \sum_{n=1}^{15}\frac{n\,(n+1)}{2} - 60 = \sum_{n=1}^{15}\frac{n\,(n+1)\,(n+2-(n-1))}{6} - 60$

$= \frac{15\cdot16\cdot17}{6} - 60 = 620$

**(42)** **(B).** $b = ar$

$c = ar^2$

3a, 7b and 15 c are in A.P.

$14b = 3a + 15c$ ; $14\,(ar) = 3a + 15\,ar^2$

$14r = 3 + 15r^2$ ; $15r^2 - 14r + 3 = 0$

$(3r-1)\,(5r-3) = 0$ ; $r = 1/3, 3/5$.

$\in (0, 1/2]$

$\therefore$ c. $d = 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ $4^{th}$ term $= 15\,c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

**(43)** **(A).** $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$3a + 21d = 40$

$a + 7d = \frac{40}{3}$

$S_{15} = \frac{15}{2}\,(2a+14d) = 15\,(a+7d)$; $S_{15} = 15\times\frac{40}{3} = 200$

**(44)** **(C).** $f\,(x) = \frac{2^{1-x}+2^{1+x}+3^x+3^{-x}}{2}$

Using $AM \ge GM$ ; $f\,(x) \ge 3$

**(45)** **1540.** $\sum_{k=1}^{20}\frac{k\,(k+1)}{2} = \frac{1}{2}\sum_{k=1}^{20}k^2 + k$

$= \frac{1}{2}\left[\frac{20\,(21)\,(41)}{6} + \frac{20\,(21)}{2}\right]$

$= \frac{1}{2}\left[\frac{420\times41}{6} + \frac{20\times21}{2}\right] = \frac{1}{2}[2870+210] = 1540$

**(46)** **(D).** $T_{10} = \frac{1}{20} = a + 9d$ .....(i)

$T_{20} = \frac{1}{10} = a + 19d$ .....(ii)

$\Rightarrow a = \frac{1}{200},\ d = \frac{1}{200}$

$\Rightarrow S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2} = 100\frac{1}{2}$

**(47)** **504.** $\frac{1}{4}\left[\sum_{n=1}^{7}(2n^3+3n^2+n)\right]$

$\frac{1}{4}\left[2\left(\frac{7\cdot8}{2}\right)^2 + 3\left(\frac{7\cdot8\cdot15}{6}\right) + \frac{7\cdot8}{2}\right]$

$\frac{1}{4}\,[2\times49\times16 + 28\times15 + 28]$

$\frac{1}{4}\,[1568+420+28] = 504$

**(48)** **(A).** $2^{\frac{1}{4}}\cdot4^{\frac{1}{16}}\cdot8^{\frac{1}{48}}\cdot16^{\frac{1}{128}}...\text{ to }\infty = 2^{\frac{1}{4}}\cdot2^{\frac{2}{16}}\cdot2^{\frac{3}{48}}\cdot2^{\frac{4}{128}}...\infty$

$= 2^{\frac{1}{4}}\cdot2^{\frac{1}{8}}\cdot2^{\frac{1}{16}}\cdot2^{\frac{1}{32}}...\infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}...\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$

**(49)** **(D).** $\sum_{n=1}^{100}a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + .... + a_{201} = 200$

$\Rightarrow ar^2\,\frac{(r^{200}-1)}{(r^2-1)} = 200$

$$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + .... + a_{200} = 100$$

$$\frac{ar\,(r^{200}-1)}{(r^2-1)} = 100$$

On dividing $r = 2$

On adding $a_2 + a_3 + a_4 + a_5 + ... + a_{200} + a_{201} = 300$

$\Rightarrow$ $r\,(a_1 + a_2 + a_3 + .... + a_{200}) = 300$

$\Rightarrow$ $\sum_{n=1}^{200} a_n = 150$

**(50)** **(14)** Common term are : 23, 51, 79, ..... $T_n$

$T_n \leq 407 \Rightarrow 23 + (n-1)\,28 \leq 407$

$\Rightarrow$ $n \leq 14.71$ ; $n = 14$

**(51)** **(A).** $S = \underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19}$....40 terms

$S = 7 + 17 + 27 + 37 + 47 +$.........20 terms

$$S_{40} = \frac{20}{2}[2 \times 7 + (19)\,10] = 10\,[14 + 190]$$

$= 10\,[2040] = (102)\,(20)$

$\Rightarrow$ $m = 20$

**(52)** **(C).** $a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4$ …(i)

$a_3 + a_4 = 16 \Rightarrow a_1 r^2 + a_1 r^3 = 16$ ….(ii)

$$\frac{1}{r^2} + \frac{1}{4} \Rightarrow r^2 = 4 \;;\; r = \pm 2$$

$r = 2,\; a_1\,(1+2) = 4 \Rightarrow a_1 = 4/3$

$r = -2,\; a_1(1-2) = 4 \Rightarrow a_1 = -4$

$$\sum_{i=1}^{a} a_i = \frac{a_1(r^q-1)}{r-1} = \frac{(-4)\,((-2)^9-1)}{-2-1} = \frac{4}{3}(-513) = 4\lambda$$

$\lambda = -171$