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SEQUENCES & SERIES

SEQUENCE

A sequence is a function whose domain is the set N of natural numbers. Since the domain for every sequence is the set N of natural numbers, therefore a sequence is represented by its range. If $f : N \to R$, then $f(n) = t_n$, $n \in N$ is called a sequence and is denoted by $\{f(1), f(2), f(3),\ldots\}$ $= \{t_1, t_2, t_3, \ldots\} = \{t_n\}$

SERIES

By adding or substracting the terms of a sequence, we get an expression which is called a series. If $a_1, a_2, a_3, \dots a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + \dots + a_n$ is a series.

PROGRESSION

When the terms of a sequence or series are arranged under a definite rule then they are said to be in a Progression.

ARITHMETIC PROGRESSION (A.P.) :

Arithmetic Progression is defined as a series in which difference between any two consecutive terms is constant throughout the series. This constant difference is called Common difference . If 'a' is the first term and 'd' is the common difference, then an AP can be written as

 $a + (a + d) + (a + 2d) + (a + 3d) + \dots$ **Note :** If a,b,c are in $AP \Leftrightarrow 2b = a + c$

General Term of an AP: General term (nth term) of an AP is given by $T_n = a + (n-1) d$ **Note :**

- (i) General term is also denoted by ℓ (last term)
- (ii) n (No. of terms) always belongs to set of natural numbers.
- (iii) Common difference can be zero, $+$ ve or $-$ ve.
- (iv) nth term from end is given by
	- $T_m (n-1)d$

 $=$ (m – n + 1)th term from beginning where m is total no. of terms.

Sum of n terms of an AP: The sum of first n terms of an A.P.

is given by
$$
S_n = \frac{n}{2} [2a + (n-1) d]
$$
 or $S_n = \frac{n}{2} [a + T_n]$
 $\frac{S_n}{R_n} = \frac{f_n}{L_n}$ then

Some standard results:

(i) Sum of first n natural numbers
$$
\Rightarrow \sum_{r=1}^{n} r = \frac{n(n+1)}{2}
$$
 (ii) 16.

(ii) Sum of first n odd natural numbers
$$
\Rightarrow \sum_{r=1}^{n} (2r-1) = n^2
$$

- (iii) Sum of first n even natural numbers $\Rightarrow \sum_{r=1}^{n} 2r = n(n+1)$ **ES**
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of first n even natural numbers $\Rightarrow \sum_{r=1}^{n} 2r = n(n+1)$

of squares of first n natural numbers
 $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

of cubes of first n natural numbers
 $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$
- (iv) Sum of squares of first n natural numbers

$$
\Rightarrow \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}
$$

(v) Sum of cubes of first n natural numbers

$$
\Rightarrow \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2} \right]^2
$$

(vi) Sum of fourth powers of first n natural numbers $\left(\sum n^4\right)$

RIES
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\nSum of fourth powers of first n natural numbers $(\sum n^4)$
\n $\sum n^4 = 1^4 + 2^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30}$
\nIf r^{th} term of an A.P.
\n $\Gamma_r = Ar^3 + Br^2 + Cr + D$, then sum of n term of AP is

(vii) If rth term of an A.P. T^r = Ar³ + Br²

RIES
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$$
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\nIf r^{th} term of an A.P.
\n $T_r = Ar^3 + Br^2 + Cr + D$, then sum of n term of AP is
\n $S_n = \sum_{r=1}^{n} T_r = A \sum_{r=1}^{n} r^3 + B \sum_{r=1}^{n} r^2 + C \sum_{r=1}^{n} r + D \sum_{r=1}^{n} 1$
\nIf for an A.P. pth term is q, qth term is p then mth term is
\n= p + q - m.
\nNote:
\n $T_x = S_x - S_x$, where S_x, is sum of (n - 1) terms of A.P.

(viii) If for an A.P. pth term is q, qth term is p then mth term is $= p + q - m$.

Note :

- (i) If sum of n terms S_n is given then general term
- $T_n = S_n S_{n-1}$ where S_{n-1} is sum of $(n-1)$ terms of A.P. (ii) Common difference of AP is given by $d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term or first term. $= p + q - m$.
 (i) If sum of n terms S_n is given then general term
 $T_n = S_n - S_{n-1}$ where S_{n-1} is sum of $(n-1)$ terms of A.P.
 (ii) Common difference of AP is given by $d = S_2 - 2S_1$ where
 S_2 is sum of first two t
- (iii) Sum of n terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in n, in such case the common difference is twice the coefficient of n^2 . i.e. 2A
- **(iv)** n^{th} term of an A.P. is of the form $An + B$ i.e. a linear expression in n, in such a case the coefficient of n is the common difference of the A.P. i.e. A

$$
\frac{S_n}{S'_n} = \frac{f_n}{\phi_n} \text{ then } \frac{T_n}{T'_n} = \frac{f(2n-1)}{\phi(2n-1)}
$$

r 1 ⁼ n (n 1) 2 r 1 (2r 1) = n² n n S f ⁿ ^T f (2n 1) T (2n 1) **(vi)** If for two A.P.'s ⁿ ⁿ ^T An B T Cn D then ⁿ ⁿ n 1 A B ^S ² ^S n 1 C D 2

ARITHMETIC MEAN (A. M.)

If three or more than three terms are in A.P., then the numbers, lying between first and last term are known as Arithmetic Means between them. i.e. **EXECUTE AN (A.M.)**

SOME PROPERTIES OF A.P.

IETICMEAN(A.M.) SOME PROPERTIES OF A.P.

then the series so formed

and last term are known as **STUDY MATERI**

SOME PROPERTIES OF A.P.

more than three terms are in A.P., then the (i) If $r_n = an + b$, then the series so form

imp between first and last term are known as (ii) If $r_n = an + b$, then series so form

Means betw

The A.M. between the two given quantities a and b is A so that a, A, b are in A.P.

i.e.
$$
A - a = b - A \Rightarrow A = \frac{a + b}{2}
$$

Note : A.M. of any n positive numbers a_1, a_2 a_n is $\qquad \mathbf{v}$

$$
A = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}
$$

nAM's between two given numbers :

If in between two numbers 'a' and 'b' we have to insert n AM A_1 , A_2 ,..... A_n then a_1 , A_1 , A_2 , A_3 ... A_n , b will be in A.P. The series consist of $(n+2)$ terms and the last term is b and first term is a. **n AM's between two given numbers :**

If in between two numbers 'a' and 'b' we have to insert n

AMA₁, A₂,A_n then a₁, A₁, A₂, A₃...A_n, b will be in A.P.

The series consist of (n +2) terms and the la

$$
\Rightarrow a + (n+2-1) d = b \Rightarrow d = \frac{b-a}{n+1}
$$

A₁ = a + d, A₂ = a + 2d, A_n = a + nd or A_n = b - d
Sol. Required su

Note :

(i) Sum of n AM's inserted between a and b is equal to n times the single AM between a and b i.e.

$$
\sum_{r=1}^{n} A_r = nA, \text{ where } A = \frac{a+b}{2} \quad \text{the}
$$

(ii) Between two numbers
$$
\frac{\text{sum of m AM's}}{\text{sum of n AM's}} = \frac{\text{m}}{\text{n}}
$$

SUPPOSITION OFTERMS INA.P.

- **(i)** When no. of terms be odd then we take three terms as : $a-d$, a , $a+d$ five terms are $a - 2d$, $a - d$, a , $a + d$, $a + 2d$ Here we take middle term as 'a' and common difference as 'd'.
- **(ii)** When no. of terms be even then we take 4 term as : $a - 3d$, $a - d$, $a + d$, $a + 3d$ 6 term as $= a - 5d$, $a - 3d$, $a - d$, $a + d$, $a + 3d$, $a + 5d$ Here we take 'a – d, a + d' as middle terms and common difference as '2d'.

Note :

(i) If no. of terms in any series is odd then only one middle

term is exist which
$$
\left(\frac{n+1}{2}\right)^{th}
$$
 term where n is odd.

(ii) If no. of terms in any series is even then middle terms are

two which are given by
$$
(n/2)
$$
th and $\left\{ \left(\frac{n}{2} \right) + 1 \right\}^{th}$ term where $= \frac{4 \times}{}$

n is even.

SOME PROPERTIES OFA.P.

(i) If $t_n = an + b$, then the series so formed is an A.P.

(ii) If $S_n = an^2 + bn + c$, then series so formed is an A.P.

- **SOME PROPERTIES OFA.P.**

SOME PROPERTIES OFA.P.

first and last term are known as (ii) If $t_n = an + b$, then the series so formed is

first and last term are known as (ii) If $S_n = an^2 + bn + c$, then series so formed

en them. i.e **(iii)** If each term of a given A.P. be increased, decreased, multiplied or divided by some non zero constant number then resulting series thus obtained will also be in A.P.
	- 2 $\frac{1}{1}$ $+ b$ (iv) $\ln a_1$ **(iv)** In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last term.
		- **(v)** Any term of an AP (except the first term) is equal to the half of the sum of terms equidistant from the term i.e.

$$
a_n = \frac{1}{2} (a_{n-k} + a_{n+k}), k \le n
$$

SOME PROPERTIES OF A.P.

SOME PROPERTIES OF A.P.

and last term are known as

(ii) If $Y_n = an + b$, then the series so formed is an A.P.

m. i.e.

and it is can term of a given A.P. be increased, de-

multiplied or divided erms are in A.P., then the **(i)** If $\mathbf{k}_0 = \mathbf{a} + \mathbf{b} + \mathbf{c}$. (ii) If $\mathbf{k}_0 = \mathbf{a} + \mathbf{b} + \mathbf{c}$, then the eriss of formed is an A.P.

and last term are known as (ii) If $\mathbf{s}_n = \mathbf{a} + \mathbf{b} + \mathbf{c}$, then series **(vi)** If in a finite AP, the number of terms be odd, then its middle term is the AM between the first and last term and its sum is equal to the product of middle term and no. of terms

Example 1 :

Find the sum of all odd numbers of two digits

Sol. Required sum =
$$
11 + 13 + \dots + 99 = \frac{1}{2}
$$
. 45 $(11 + 99) = 2475$

Example 2 :

The A.M. between the two given quantities a and b is A so
\nthe A.M. between the two given quantities a and b is A so
\nthe A.M. of any no positive numbers.
\ni.e.
$$
A - a = b - A \Rightarrow A = \frac{a+b}{2}
$$

\nNote: A.M. of any no positive numbers.
\nNote: A.M. of any non-
\nA. If $a + b = 2$, $a + 3$, $a + b = 4$
\n $A = \frac{a_1 + a_2 + a_3 +a_n}{n}$
\n $A = \frac{a_1 + a_2 + a_3 +a_n}{n}$
\n $A = \frac{a_1 + a_2 + a_3 +a_n}{n}$
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\n $A = \frac{a_1 + b_1}{n}$
\n $A = \frac{a_1 + a_2 + a_3 +a_n}{n}$
\n $A = \frac{a_1 + b_1}{n}$
\n $A = \frac{a_1$

Sol. (3).
$$
\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}
$$
 are in A.P.

$$
\therefore \frac{b+c-a}{a} + 2, \frac{c+a-b}{b} + 2, \frac{a+b-c}{c} + 2
$$

are in A.P. (adding 2 in each term)

or
$$
\frac{a+b+c}{a}
$$
, $\frac{c+a+b}{b}$, $\frac{a+b+c}{c}$ are in A.P.
\n[dividing by (a+b+c) in each term]

or
$$
\frac{1}{a}, \frac{1}{b}, \frac{1}{c}
$$
 are in A.P.

Example 3 :

S

s $\frac{1}{\text{sum of n AM}'}$ = $\frac{m}{n}$

(3) $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c}$

(4) none

s $\frac{1}{\text{sum of n AM}'}$ = $\frac{m}{n}$

(4) none

s $\frac{1}{\text{sum of n AM}'}$ = $\frac{m}{n}$

Sol. (3). $\frac{b+c-a}{a} \cdot \frac{c+a-b}{b} \cdot \frac{a+b-c}{c}$ are in

s odd then we tak re A = $\frac{a+b}{2}$

the following is in A.P.

(1) a,b,e

ers $\frac{\text{sum of m AM}^{\prime}}{\text{sum of n AM}^{\prime}} = \frac{m}{n}$

(3) $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c}$

(4) none

for AM's = $\frac{m}{m}$

(3) $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c}$

(4) none

for AM's = $\frac{$ For $\sinh A$
 $\sinh A$
 the state of t $\left\{\frac{n}{2}\right\}+1$ term where $= \frac{4 \times n(n+1)(2n+1)}{6} - n = \frac{2}{3}n(n+1)(2n+1) - n$ M's n

Sol. (3). $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in A.P.

take
 $\frac{b+c-a}{a} + 2$, $\frac{c+a-b}{b} + 2$, $\frac{a+b-c}{c} + 2$

are in A.P. (adding 2 in each term)

te take

or $\frac{a+b+c}{a}$, $\frac{a+b+c}{b}$, $\frac{a+b+c}{c}$ are in A.P.
 Sol. (3). $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in A.P.

take
 $a + 2d$
 $\therefore \frac{b+c-a}{a} + 2$, $\frac{c+a-b}{b} + 2$, $\frac{a+b-c}{c} + 2$

are in A.P. (adding 2 in each term)

e take

or $\frac{a+b+c}{a}$, $\frac{c+a+b}{b}$, $\frac{a+b+c}{c}$ are in A Find the sum of n term of series $1.3+3.5+5.7+$ **Sol.** $T_n = [n^{th} \text{ term of } 1.3.5 \dots] \times [n^{th} \text{ term of } 3.5.7 \dots]$ or $T_n = [1 + (n-1) \times 2] \times [3 + (n-1) \times 2]$ or $T_n = (2n-1)(2n+1) = (4n^2-1)$ $S_n = \sum T_n = \sum (4n^2 - 1) = 4.\sum n^2.\sum 1$ ⁼ ² T (4n 1) ⁿ ⁼ ² 4. n . 1 $\frac{a}{a}$, $\frac{b+c-a}{b}$, $\frac{c}{c}$ are tanced.
 $\frac{a+b+c}{a}$ + 2, $\frac{a+b-c}{b}$ + 2, $\frac{a+b-c}{c}$ + 2
 $\frac{a+b+c}{a}$, $\frac{c+a+b}{b}$, $\frac{a+b+c}{c}$ are in A.P.

[dividing by (a+b+c) in each term]
 $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.
 $\frac{-a}{b}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in A.P.
 $\frac{a}{b}$ + 2, $\frac{c+a-b}{b}$ + 2, $\frac{a+b-c}{c}$ + 2

((adding 2 in each term)
 $\frac{+c}{b}$, $\frac{c+a+b}{c}$ are in A.P.

[dividing by (a+b+c) in each term]
 $\frac{1}{c}$ are in A.P.
 $(n+1)(2n+1) - n$

Example 4 :

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\n <p>If a, b, c in A.P. and x = $\sum_{n=0}^{\infty} a^n$, y = $\sum_{n=0}^{\infty} b^n$, z = $\sum_{n=0}^{\infty} c^n$ then\n \na_n by a function of a function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n.\n\n</p>	
\n <p>If a, b, c in A.P. and x = $\sum_{n=0}^{\infty} a^n$, y = $\sum_{n=0}^{\infty} b^n$, z = $\sum_{n=0}^{\infty} c^n$ then the sum of a function a_n is a function of the function a_n. \n (1) AP\n \na_n by a function of the function a_n and the function a_n is a function of the function a_n.\na_n by a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n.\n\n</p>	
\n <p>If a, b, c in A.P. and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n is a function of the function a_n and the function a_n and the function a_n is a function of the function a_n and the</p>	

 $(3) HP$ (4) None of these **Sol. (3).** Here a, b, c in A.P, given

QUENCES & SERIES)	Figure 4:	
If a, b, c in A.P. are in a $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ then	Q.1 Find the sum of all three-digit natural of 1.2. The sum of three numbers in A.P. is	
<i>x, y, z are in</i>	(2) GP	Q.3 The digits of a positive integer, having is 8. Find the numbers.
(1) AP	(2) GP	Q.3 The digits of a positive integer, having a. The sum of the number of integers in A.P. is
(3) HP	(4) None of these reversing the digits is 594 less than t	
<i>A</i> Is $x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$, $z = \frac{1}{1-c}$	Q.4 If eleven A.M.'s are inserted between from the square is 120.	
<i>A</i> Is $x = 1 - a$, $1 - b$, $1 - c$ in A.P.	Q.5 Find four numbers in an A.P. whose there is 120.	
$\Rightarrow \frac{1}{1-a}$, $\frac{1}{1-b}$, $\frac{1}{1-c}$ in HP $\Rightarrow x, y, z$ in HP	Q.6 Let T _r be the r th term of an AP, for r = some positive integers m, n we have the result of a b (p-q) + b c (q-r) + ca (r-p)	Q.7 If the sum of the first 2n terms of the A.
<i>A</i> Is $x = 1 - a$, $1 - b$, $1 - c$ in the the sum of the first 2n terms of the A.		
<i>A</i> Is $x = 1 - a$, $a = 1 - b$, $a = 1 - b$, $a = 1 - b$, $a = 1$		

Example 5 :

- If $\frac{1}{\epsilon}$, $\frac{1}{\epsilon}$, $\frac{1}{\epsilon}$ are the pth, qth, rth terms respectively of an A.P.
- then find the value of $ab(p q) + bc (q r) + ca (r p)$ **Sol.** Let x be the first term and y be the c.d. of corresponding A.P.,

$$
\frac{1}{a} = x + (p-1)y
$$
(1)

$$
\frac{1}{b} = x + (q-1)y
$$
(2)

$$
\frac{1}{c} = x + (r-1)y
$$
(3)

Multiplying (1) , (2) and (3) respectively by abc $(q - r)$, abc $(r - p)$, abc $(p - q)$ and then adding we get $Q.9$ bc $(q - r)$ + ca $(r - p)$ + ab $(p - q) = 0$

Example 6 :

If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7th and $(m-1)$ th means is 5 : 9, then find the value of m.

Sol. Let the means be x_1, x_2, \dots, x_m so that $1, x_1, x_2, \dots, x_m$, 31 is an A.P. of $(m+2)$ terms.

Now,
$$
31 = T_{m+2} = a + (m+1)d = 1 + (m+1)d
$$

$$
\frac{1}{a} = x + (p-1)y
$$
(1)
\n
$$
\frac{1}{b} = x + (q-1)y
$$
(2)
\n
$$
\frac{1}{b} = x + (q-1)y
$$
(3)
\nMultiplying (1), (2) and (3) respectively by
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(3)
\nMultiplying (1), (2) and (3) respectively by
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(3)
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(3)
\n
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\frac{1}{c} = x + (r-1)y
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\frac{1}{c} = x + (r-1)y
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\frac{1}{c} = x + (r-1)y
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\frac{1}{c} = x + (r-1)y
$$
(3)
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(4)
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(5)
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(7)
\n
$$
\frac{1}{c} = x + (r-1)y
$$
(8)
\n
$$
\frac{1}{c} = x + r(1) + r(2) + r(3) + r(4) + r(5) + r(6) + r(7) + r(8) + r(8) + r(9) + r(9) + r
$$

$$
m+1
$$

\n
$$
\Rightarrow 2m+2=75m-1020 \Rightarrow 73m=1022
$$

$$
\therefore \quad m = \frac{1022}{73} = 14
$$

TRY IT YOURSELF-1

- $\sum_{z=0}^{\infty} b^{n} z = \sum_{z=0}^{\infty} c^{n}$ then divisible by 7. **Q.1** Find the sum of all three-digit natural numbers, which are divisible by 7.
	- is 8. Find the numbers.
- **TRYITYOURSEI**
 $\sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ then
 O.1 Find the sum of all three-digit nature-

divisible by 7.
 O.2 The sum of three numbers in A.P.

is 8. Find the numbers.
 O.3 The digits of a positive in **EDIMADIATELY**
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 CONTADIATELY ERYITYOURSELF-1
 $= \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ then
 $= \sum_{n=0}^{\infty} b^n$ (1) Find the sum of all three-digit natural numbers, we divisible by 7.

2) GP

(1) The sum of three numbers in A.P. is -3 and their

is 8. **Q.3** The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. **E-1**
 IODMADVANCED LEARNING

IS -3 and their product

ving three digits, are in

number obtained by

in the original number.

ween 28 and 10, then

sum is 20 and sum of
 $r = 1, 2, 3$ **E-1**

ral numbers, which are

is -3 and their product

ving three digits, are in

number obtained by

in the original number.

ween 28 and 10, then

s.

see sum is 20 and sum of

r = 1, 2, 3..............................
	- **Q.4** If eleven A.M.'s are inserted between 28 and 10, then find the number of integral A.M.'s.
		- **Q.5** Find four numbers in an A.P. whose sum is 20 and sum of their square is 120.
		- **Q.6** Let T^r be the rth term of an AP, for r = 1, 2, 3...........If for some positive integers m, n we have $T_m = 1/n$ and $T_n = 1/m$, then T_{mn} equals :

(A) $1/mn$ (B) $\frac{1}{m} + \frac{1}{n}$ $+\frac{1}{2}$

$$
(C) 1 \t\t (D) 0
$$

Q.7 If the sum of the first 2n terms of the A. P. 2, 5, 8,is equal to the sum of the first n terms of the A.P. 57, 59, 61.., then n=

(A) 10 (B) 12 (C) 11 (D)13

Q.8 If the sum of first n terms of an A.P. is cn^2 , then the sum of squares of these n terms is –

$$
\frac{1}{1-c}
$$

\nFind the number.
\nQ.4 If eleven A.M.'s are inserted between 28 and 10, then
\n
$$
\frac{1}{1-c}
$$

\nQ.5 Find the number of integral A.M.'s.
\nQ.5 Find four numbers in an A.P. whose sum is 20 and sum of
\ntheir square is 120.
\nQ.6 Let T₁ be the rth term of an A.P. whose sum is 20 and sum of
\ntheir square is 120.
\nQ.6 Let T₁ be the rth term of an A.P. (or
\nsome positive integers m, n we have T_m = 1/n and
\n
$$
T_n = 1/m
$$
, then T_{mn} equals :
\n(A) 1/mm (B) $\frac{1}{m} + \frac{1}{n}$
\n $+ bc$ (q-r) + ca (r-p) (C) 1 (D) 13
\n $+ bc$ (q-r) + ca (r-p) (A) 10 (B) 12
\n $...$ (1) (B) 12
\n $...$ (1) (B) 12
\n $...$ (2) (1) (1) (13) (15) 13
\n(A) 10 (B) 12
\n $...$ (3) (C) 1 (D) 13
\n(A) 10 (B) 12
\n $...$ (1) (2) 1 (3) 13
\n(A) 10 (B) 12
\n $...$ (1) (2) 1 (3) 13
\n(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$
\n $...$ (3)
\n $-(q)$ and then adding we get Q.9 Let a₁, a₂, a₃,..., a₁₀₀ be an arithmetic progression with
\na₁ = 3 and S_p = $\sum_{i=1}^{p} a_i$, 1 ≤ p ≤ 100. For any integer n with
\n $a_1 = 3$ and S_p = $\sum_{i=1}^{p} a_i$, 1 ≤ p ≤ 100. For any integer n with
\nthe
\no that 1, x

Q.9 Let $a_1, a_2, a_3, \dots, a_{100}$ be an arithmetic progression with

x, y, z in HP
\n
$$
x, y, z in HP
$$
\n
$$
T_n = 1/m, then T_{mn}
$$
\n
$$
T_n = 1/m, then T_{mn}
$$
\n
$$
T_n = 1/m
$$
\n
$$
T_n = 1/m, then T_{mn}
$$
\nequals :
\n(A) 1/mn
\n(B) $\frac{1}{m} + \frac{1}{n}$
\n
$$
F_n = 1/m
$$
 and
\n(B) 0
\n
$$
F_n = 1/m
$$
\n
$$
F_{mn} = 1/m
$$
\n<

 $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then

 a_2 .

ANSWERS

GEOMETRICAL PROGRESSION (G. P.)

Geometric Progression is defined as a series in which ratio between any two consecutive terms is constant throughout the series. This constant ratio is called as a common ratio. If 'a' is the first term and 'r' is the common ratio, then a GP can be written as **:** a, b,c are in G.P. if \Leftrightarrow $b^2 = ac$

STUDY MATERIAL : MATHEMATICS

General term of a G.P.:

General term (nth term) of a G.P. is given by $T_n = ar^{n-1}$ **Note :**

(i) nth term form end is given by $\frac{T_m}{n-1}$ where m stands for 5 terms as $\frac{a}{2}, \frac{a}{r}$ $\frac{m}{r^{n-1}}$ where m stands for $\frac{5}{r^{n}}$ total no. of terms

(ii) If
$$
a_1, a_2, a_3, \dots
$$
 are in GP, then $r = \left(\frac{a_k}{a_p}\right)^{\frac{1}{k-p}}$ 4 term

Sum of n terms of a G.P.:

The sum of first n terms of an A.P. is given by

Sⁿ ⁼ ⁿ ⁿ a (1 r) a r T 1 r 1 r when r < 1 or Sⁿ ⁼ ⁿ ⁿ a(r 1) rT a r 1 r 1 when r > 1 and Sⁿ = nr when r = 1 G b a G

Sum of an infinite G.P.:

The sum of an infinite G.P. with first term a and common

ratio
$$
r(-1 < r < 1
$$
 i.e. $|r| < 1$) is $S_{\infty} = \frac{a}{1-r}$
\n**Note :** If $r \ge 1$ then $S_{\infty} \to \infty$
\n**Note :** If $r \ge 1$ then $S_{\infty} \to \infty$
\n**(i)** If each non zero
\n**(ii)** In an G

GEOMETRICAL MEAN (G. M.) :

If three or more than three terms are in G.P. then all the numbers lying between first and last term are called Geometrical Means between them i.e. The G.M. between two given quantities a and b is G, so that a, G, b, are in G.P.

i.e.
$$
\frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \sqrt{ab}
$$
 i.e. $T_r =$
(v) In a fini

Note :

- (i) G.M. of any n positive numbers $a_1, a_2, a_3, \dots, a_n$ is $(a_1, a_2, a_3, \dots, a_n)^{1/n}.$
- (ii) If a and b are two numbers of opposite signs, then G.M. between them does not exist.

n GM's between two given numbers:

If in between two numbers 'a' and 'b', we have to insert n GMG_1, G_2, \dots, G_n then $a_1, G_1, G_2, \dots, G_n$, b will be in G.P. The series consist of $(n+2)$ terms and the last term is b and first term is a. b is c, so unat a, c, o, are in cr.

which are at equidistant from

i.e. $T_r = \sqrt{T_{r-k}T_{r+k}}$ k < r

(v) In a finite G.P., the number of

term is the G.M. of the first an

expected between a label and b is

time in the new s **M.):**
 IVENTUALS:

IF and OHER IDOM to the first and the last term, is constant

if each case of the stand and term and case of the stand and term

is constant in its. The GM. between the since of a GP, be raised to th **M.):**

from the first and the last term, is constant

from the first and the last term, is constant

from for first and last term are called

(iii) If each term of a GP, be raised to the same

resulting series is also a **Al's between two given numbers:**

Let between two numbers 'a' and 'b', we have to insert n
 G_1, G_2, \dots, G_n then $a_1, G_1, G_2, \dots, G_n$, b will be in GP.

Series consist of $(n+2)$ terms and the last term is b and

term is a

$$
\Rightarrow ar^{n+2-1} = b \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}
$$

$$
G_1 = ar, G_2 = ar^2 \dots G_n = ar^n \text{ or } G_n = b/r
$$

Note : Product of n GM's inserted between 'a' and 'b' is equal to nth power of the single GM between 'a' and 'b' i.e.

the sq
\n
$$
\prod_{r=1}^{n} G_r = (G)^n
$$
 where $G = \sqrt{ab}$ the ter

SUPPOSITION OFTERMS IN G. P. :

STUDY MATE

supposition of terms IN G P.

is given by T_n = arⁿ⁻¹ (i) When no. of terms be odd,

then we take three terms as a/r, a
 $\frac{\Gamma_m}{n-1}$ where m stands for

5 terms as $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar²

H (i) When no. of terms be odd, then we take three terms as a/r, a, ar

5 terms as
$$
\frac{a}{r^2}
$$
, $\frac{a}{r}$, a, ar, ar²

1 (ii) When no. of terms be even then we take **STUDY MATERIAL: M**

SUPPOSITION OF TERMS ING P.:

(i) When no. of terms be odd,

then we take three terms as a/r , a, ar

there m stands for

5 terms as $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar²

Here we take middle term as ' **STUDY MATERIAL: MATHEMAT**

given by T_n = arⁿ⁻¹ (i) When no. of terms be odd,

then we take three terms as a/r, a, ar
 $\frac{a_k}{a_p}$ $\frac{1}{k-p}$ (ii) When no. of terms as a², a, ar, ar²

Here we take middle term as ' **STUDY MATERIAL: MATHEMAT**

given by T_n = arⁿ⁻¹ (i) When no. of terms be odd,

then we take three terms as a/r, a, ar

where m stands for 5 terms as $\frac{a}{r^2}$, a, a, ar, ar²

Here we take middle term as 'a' and **STUDY MATERIAL: MATHEMATICS**
 OF TERMS IN G P. :

f terms be odd,

e three terms as a/r , a, ar
 $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar²

e middle term as 'a' and common ratio as 'r'.

f terms be even then we take
 $\frac{a}{$ Here we take middle term as 'a' and common ratio as 'r'. **STUDY MATERIAL: MATHEMATICS**
 DFTERMS IN G P.:

terms be odd,

three terms as a/r, a, ar
 $\frac{1}{2}$, $\frac{a}{r}$, a, ar, ar²

middle term as 'a' and common ratio as 'r'.

terms be even then we take
 $\frac{a}{r^3}$, $\frac{a}{$

$$
\left(\frac{a}{a_p}\right)
$$
 4 terms as: $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

6 terms as:
$$
\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}
$$
, ar, ar³, ar⁵

STUDY MATERIAL: MATHEMATICS
 POSITION OF TERMS IN G P.:

When no. of terms be odd,

then we take three terms as a/r, a, ar

5 terms as $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar²

Here we take middle term as 'a' and common r **STUDY MATERIAL: MATHEMATICS**
 DETERMS ING P.:

terms be odd,

three terms as a/r, a, ar
 $\frac{a}{2}, \frac{a}{r}$, a, ar, ar²

e middle term as 'a' and common ratio as 'r'.

ferms be even then we take
 $\frac{a}{r^3}, \frac{a}{r}$, ar **STUDY MATERIAL: MATHEMATICS**
 OF TERMS IN G P.:

f terms be odd,

e three terms as a/r, a, ar
 $\frac{a}{2}, \frac{a}{r}$, a, ar, ar²

e middle term as 'a' and common ratio as 'r'.

f terms be even then we take
 $\frac{a}{r^3}, \frac{a$ Here we take $\frac{a}{a}$, ar as middle terms and common ra $\frac{a}{r}$, ar as middle terms and common ratio as r² as $\frac{1}{r^2}$, $\frac{1}{r}$, a, ar, ar-

e take middle term as 'a' and common ratio as 'r'.

no. of terms be even then we take

as : $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar³

as : $\frac{a}{r^5}$, $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar³, ar **ION OF TERMS IN G P.:**

no. of terms be odd,

e take three terms as a/r, a, ar

sas $\frac{a}{r^2}$, a, ar, ar²

e take middle term as 'a' and common ratio as 'r'.

no. of terms be even then we take

sas: $\frac{a}{r^3}$, a, a **NOFTERMS INGP:**

..., of terms be odd,

ake three terms as a/r, a, ar
 $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar²

take middle term as 'a' and common ratio as 'r'.

... of terms be even then we take
 $s: \frac{a}{r^3}, \frac{a}{r}$, ar, ^r r r

(iii) In general, if we have to take $(2k + 1)$ terms in G.P. we take

them
$$
\frac{a}{r^k}
$$
, $\frac{a}{r^{k-1}}$, ..., $\frac{a}{r}$, a, ar, ..., ar^k

SOME PROPERTIES OF G.P.

.

- **(i)** If each term of a G.P. be multiplied or divided by the same non zero quantity, then resulting series is also a G.P.
- where m stands for
 $\frac{1}{1-r}$

Here we take middle term as 'a' and common ratio as 'r'.

Here we take middle term as 'a' and common ratio as 'r'.
 $\frac{a_k}{a_p}$ $\Big)$ $\frac{1}{k-p}$

(ii) When no. of terms be even then we take **(ii)** In an G.P. the product of two terms which are at equidistant from the first and the last term, is constant and is equal to product of first and last term we take $\frac{a}{r}$, ar as middle terms and common ratio as r²

teral, if we have to take (2k + 1) terms in G.P. we take
 $\frac{a}{r^k}$, $\frac{a}{r^{k-1}}$,, $\frac{a}{r}$, a, ar,, ar^k
 OPERTIES OF G.P.

term of a G
	- **(iii)** If each term of a G.P. be raised to the same power, then resulting series is also a G.P.
	- **(iv)** In a G.P. every term (except first) is GM of its two terms which are at equidistant from it.

i.e.
$$
T_r = \sqrt{T_{r-k} T_{r+k}}
$$
 $k < r$

- **(v)** In a finite G.P., the number of terms be odd then its middle term is the G.M. of the first and last term.
- **(vi)** If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.
- (vii) If $a_1, a_2, a_3, \dots, a_n$ is a G.P. of non zero, non negative terms, then $\log a_1$, $\log a_2$,...... $\log a_n$ is an A.P. and vice-versa
- (viii) If a_1, a_2, a_3 and b_1, b_2, b_3 are two G.P.'s then a_1b_1 , a_2b_2 , a_3b_3 is also in G.P.

Example 7 :

The nth term of a GP is 128 and the sum of its n terms is 255. If its common ratio is 2 then find its first term.

Sol. Let a be the first term. Then as given $T_n = 128$ and $S_n = 255$

But
$$
S_n = \frac{rT_n - a}{r - 1} \Rightarrow 255 = \frac{2(128) - a}{2 - 1} \Rightarrow a = 1
$$

Example 8 :

From a 20.1. be tasted to the same power, then

series is also a GP.

every term (except first) is GM of its two terms

at equidistant from it.
 $T_{r-k}T_{r+k}$ k < r

GP, the number of terms be odd then its middle

GM. of t very term (except first) is GM of its two terms
tequidistant from it.
tequidistant from it.
 $\frac{\Gamma_{r-k} \Gamma_{r+k}}{\Gamma_{r+k}}$ k < r
i.h., the number of terms be odd then its middle
GM. of the first and last term.
of a given GP. are Gr. to transact to the same power, then
also a GP.
m (except first) is GM of its two terms
listant from it.
 $\frac{1}{k}$ k < r
the first and last term.
iven GP are chosen at regular intervals,
ence is also a GP.
s a GP. of n rst) is GM of its two terms

t.

t.

terms be odd then its middle

last term.

chosen at regular intervals,

a GP.

n zero, non negative terms,

an A.P. and vice-versa

.... are two G.P.'s then

P.

the sum of its n terms If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, find the sum of the cubes of the terms

SEQUENCES & SERIES

Sol. Let the GP be a, ar, ar²,........, where $0 < r < 1$. Then, $a + ar + ar^2 + \dots = 3$ and $a^2 + a^2r^2 + a^2r^4 + \dots = 9/2$. NCES & SERIES

he GP be a, ar, ar²,......., where $0 < r < 1$.

h, a + ar + ar² + = 3 and a² + a² r² + a²r⁴ + = 9/2.

a
 $\frac{a}{1-r} = 3$ and $\frac{a^2}{1-r^2} = \frac{9}{2}$
 $\frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} =$

$$
\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}
$$
 SEQ
Exa

$$
\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}
$$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get a = 2 Now, the required sum of the cubes is

$$
a^3 + a^3r^3 + a^3r^6 + \dotsb = \frac{a^3}{1 - r^3} = \frac{8}{1 - (1/27)} = \frac{108}{13} = \frac{4}{10} + \frac{23}{10}
$$

Example 9 :

If A_1 , A_2 be two AM's and G_1 , G_2 be two GM's between two $A_1 + A_2$.

numbers a and b, then find
$$
\frac{P_1 + P_2}{G_1 G_2}
$$

Sol. By the property of AP and GP, we have $A_1 + A_2 = a + b$; $G_1 + G_2 = ab$ $\therefore \frac{1}{C C}$ ling $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get $a = 2$
 $a^3r^3 + a^3r^6 + \dots = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$
 $a^3r^3 + a^3r^6 + \dots = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$

9

9

5
 $\frac{a^3r^3 + a^3r^6 + \dots}{a^3r^3 + a^3r^6 + \dots} = \frac{a^3}{1-($ If $I = \frac{1}{3}$ in $\frac{1}{1-r} = 3$, we get $a = 2$

the required sum of the cubes is $3r^3 + a^3r^6 + \dots + \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$
 \therefore
 $\frac{A_2}{2}$ be two AM's and G_1, G_2 be two GM's between two

ers a and b, th $\frac{+A_2}{ } = \frac{a+b}{ }$

Example 10 :

If x, y, z are in G.P. and $a^x = b^y = c^z$ then-(1) $\log_b a = \log_a c$ $(2) \log_c b = \log_a c$ (3) $log_b a = log_c b$ (4) none of these **Sol.** (3). x, y, z are in $GP \Rightarrow y^2 = xz$ $\dots(i)$ We have, $ax = b^y = c^z = \lambda$ (say) \Rightarrow x log a = y log b = z log c = log λ \Rightarrow $x = \frac{\log \lambda}{\lambda}, y = \frac{\log \lambda}{\lambda}, z = \frac{\log \lambda}{\log \lambda}$ be two AM's and G₁, G₂ be two GM's between two

is a and b, then find $\frac{A_1 + A_2}{G_1 G_2}$
 $x = \frac{4}{10} + \frac{23}{990} = \frac{419}{990}$

roperty of AP and GP, we have
 $x = a + b$; G₁ + G₂ = ab
 $\frac{A_2}{2} = \frac{a + b}{ab}$

(a) If Is and G₁, G₂ be two GM's between two

len find $\frac{A_1 + A_2}{G_1 G_2}$.
 $\frac{A_1 + A_2}{G_1 G_2}$

AP and GP, we have
 $\frac{A_1 + A_2}{G_1 G_2}$.

AP and GP, we have
 $\frac{A_1 + A_2}{G_1 G_2}$.
 $\frac{A_1 + B_2}{G_1 G_2}$.
 $\frac{A_1 + B_2}{G_1$ $\log \lambda$ and the set of t $\lambda = (10$ Putting x,y,z in (i), we get $\left(\frac{\log \lambda}{\log b}\right)^2 = \frac{\log \lambda}{\log a} \cdot \frac{\log \lambda}{\log c}$ ers a and b, then find $\frac{1}{G_1 G_2}$.
 $\frac{1}{G_2}$ is property of AP and GP, we have
 $\frac{1}{G_2}$ is $\frac{1}{G_2}$ is e property of AP and GP, we have
 $\frac{1}{2}$ = $\frac{a+b}{ab}$
 $\frac{1}{2}$ = $\frac{a+b}{ab}$
 $\frac{1}{2}$ = $\frac{a+b}{ab}$
 $\frac{1}{2}$ = $\frac{a+b}{ab}$
 $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{1}{2}$
 $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{1}{2}$
 $\frac{1}{2}$ = $\frac{1}{2}$ bers a and b, then find $\frac{A_1 + A_2}{G_1 G_2}$.
 $x = \frac{4}{10} + \frac{23}{990} = \frac{419}{990}$

he property of AP and GP, we have
 $A_2 = a + b$; G₁+G₂ = ab
 $\frac{13}{13}G_2$
 $10 - \frac{1}{31}G_2$
 $11 - \frac{1}{31}G_2$
 $12 - \frac{1}{31}G_2$
 13 bers a and b, then find $\frac{A_1 + A_2}{G_1 G_2}$

the property of AP and GR, we have
 $A_2 = a + b$; $G_1 + G_2 = ab$
 $G_1 G_2$
 $G_2 = ab$
 $G_3 G_3$
 $G_1 G_2$
 $G_2 = ab$
 $G_3 G_3$
 $G_3 G_2$
 G_4
 G_5
 G_6
 G_7
 G_8
 G_8 G_9
 $(\log b)^2 = \log a$. log c or $\log_a b = \log_b c \implies \log_b a = \log_c b$ **Example 11 :** If a,b,c,d are in G.P., then $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, $(c^3 + d^3)^{-1}$ are in – (1) A.P. (2) GP. (3) H.P. (4) none of these **Sol.** (2). Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then, titing x, y,z in (i), we get
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \frac{\log \lambda}{\log a} \cdot \frac{\log \lambda}{\log c}$
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \log a \cdot \log c$
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \log a \cdot \log c$
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \log a \cdot \log c$
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \log a \cdot \log c$
 $\left(\frac{\log \lambda}{\log b}\right)^2 = \log a \cdot \log c$
 are in GP. \Rightarrow is $\cos \theta = 0.9 + 0.99 + 0.999 + ...$ in the same in GP. \Rightarrow is $\theta = 4x^3$. Then, $\theta = 4x^3$. Then, $\theta = 4x^3 - 4x^3$ is $\theta = 4x^3 - 4x^3$ is $\theta = 4x^3 - 4x^3$ is $\theta = 4x^3 - 4x^3 \frac{1}{25}$, $\frac{1}{25}$, \frac 2are in GR. \Rightarrow if $\sec \theta$ and $\sec \theta$ and $\sec \theta$ by $\sec \theta$ and $\sec \theta$ by $\sec \theta$ b $\frac{\log x}{\log b}$ = $\frac{\log x}{\log a}$ $\log c$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{c} b$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{c} b$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{c} b$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{c} b$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{c} b$
 $\log_{a} b = \log_{b} c \Rightarrow \log_{b} a = \log_{$ 1. $y = \frac{\log \lambda}{\log b}$, $z = \frac{\log \lambda}{\log c}$
 $= \log \frac{\log \lambda}{\log a}$. $\log \frac{\log \lambda}{\log c}$
 $= \frac{1}{\log a} \cdot \log \frac{\log \lambda}{\log c}$
 $= \log \frac{\log \lambda}{\log a}$. $\frac{\lambda + \mu}{\log a} = \log \frac{\lambda}{\log c}$
 $= \log \frac{\lambda}{\log a}$
 $= \log \frac{\lambda}{\log a}$
 $= \log \frac{\lambda + \mu}{\log a} = \log \frac{\lambda}{\log a}$
 $= \log \frac{\lambda +$ g a $y = \log x$, $y = \log x$

g a x, yz in (i), we get
 $\log x$, yz in (i), we get
 $\log x$ b $\log x$
 $= \log x$ complex b $\log x$
 $= \log x$ c $\frac{1}{2}$, $y = \frac{\log \lambda}{\log b}$, $z = \frac{\log \lambda}{\log c}$
 $y = \frac{\log \lambda}{\log b}$, $z = \frac{\log \lambda}{\log c}$
 $= (10 + 10^2 + 10^3 + \dots 10^{49}) - 49$
 $= \frac{1}{\log a} \cdot \frac{\log \lambda}{\log c}$
 $= \log a \cdot \log c$
 $= \log b$
 $= \frac{9}{1$

Clearly,
$$
(a^3 + b^3)^{-1}
$$
, $(b^3 + c^3)^{-1}$ and $(c^3 + d^3)^{-1}$ are in G.P.
with common ratio $1/r^3$.

SEQUENCES CONVERTIBLE TO GP. Example 12 :

S

²,...,..., where $0 < r < 1$.

...= 3 and $a^2 + a^2r^2 + a^2r^4 + ... = 9/2$.

 $\frac{a^2}{1+r} = \frac{9}{2}$
 SEQUENCES CONVERTIBLE TO GP.
 EXAMPLE 12:
 EXAMPLE 12:
 EXAMPLE 12:
 EXAMPLE 12:
 EXAMPLE 12:
 EXAMPLE 12:
 IES

If 2^{n^2} , where $0 < r < 1$.

If 2^{n^2} with common ratio $1/r^3$.
 $\frac{a^2}{1-r^2} = \frac{9}{2}$
 $\frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$
 $\frac{a}{1+r} = 3$, we get $a = 2$

BEQUENCES CONVERTIBLE TO GP.

Example 12:

Use infinite ser 9(1 r) 9 1 r 1 1 SERIES

a, ar, ar²,........, where $0 < r < 1$.

ar² + = 3 and $a^2 + a^2r^2 + a^2r^4 + ... = 9/2$.

ard $\frac{a^2}{1 - r^2} = \frac{9}{2}$
 $\Rightarrow \frac{1 - r}{1 + r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$
 $\Rightarrow \frac{a}{1 - r} = 3$, we get a = 2

Foother the cubes is
 \Rightarrow Use infinite series to compute the rational number corresponding to 0.423 .

Sol.
$$
x = 0.4\overline{23} = 0.4 + 0.023 + 0.00023 + \dots
$$

1 r 2 1 r 2 3 1 r a 8 108 1 r 1 (1/ 27) 13 numbers a and b, then find 1 2 1 2 A A G G A A a b G G ab = 0.4 + 3 5 7 23 23 23 10 10 10 3 2 4 4 23 1 1 ¹ ¹⁰ 10 10 10 3 4 23 1 10 1 1/100 ¹⁰ 4 23 419 x 10 990 990 (a) If 9 + 99 + 999 + + upto 49 terms = 10 (10 1) 1 1 51 1 where p, q ^N

Example 13 :

– 49,

where
$$
\lambda
$$
, $\mu \in N$ then find the value of $\lambda + \mu$
(b) 0.9 + 0.99 + 0.999 + up to 51 terms

$$
= 51 - \frac{1}{p} \left(1 - \frac{1}{10^q} \right)
$$
 where p, q \in N

then find the value of $p + q$.

$$
\frac{4}{10} + \frac{23}{10^3} \left(\frac{1}{1-1/100} \right)
$$
\n
$$
x = \frac{4}{10} + \frac{23}{10^3} = \frac{419}{990}
$$
\nwe have
\nhence
\n
$$
x = \frac{4}{10} + \frac{23}{990} = \frac{419}{990}
$$
\nwe have
\nExample 13:
\n(a) If 9+99+999+....+upto 49 terms = $10 \frac{(10^{\lambda} - 1)}{\mu} - 49$,
\nwhere $\lambda, \mu \in \mathbb{N}$ then find the value of $\lambda + \mu$
\n(b) 0.9+0.99+0.999+.....+upto 51 terms
\n
$$
x^2
$$
 then.
\n
$$
x^2
$$
 then.
\n
$$
x = \frac{51 - \frac{1}{p} \left(1 - \frac{1}{10^4} \right) \text{ where } p, q \in \mathbb{N}
$$

\n
$$
x = \frac{51 - \frac{1}{p} \left(1 - \frac{1}{10^4} \right) \text{ where } p, q \in \mathbb{N}
$$

\n
$$
x = \frac{51 - \frac{1}{p} \left(1 - \frac{1}{10^4} \right) \text{ where } p, q \in \mathbb{N}
$$

\n
$$
x = \frac{100 + 10^2 - 1 + 10^3 - 1 + \dots + 10^{49} - 1}{5! \text{ where } p, q \in \mathbb{N}
$$

\n
$$
x = \frac{100 + 10^2 + 10^3 + \dots + 10^{49} - 49}{5! \text{ terms}} = 10 \cdot \frac{10^{49} - 1}{9} - 49
$$
\n
$$
\lambda + \mu = 49 + 9 = 58
$$
\n
$$
x = \frac{9}{10} + \frac{99}{100} + \dots + \frac{999}{100} + \dots + \mu
$$
 to 51 terms
\n
$$
x = \frac{9}{10} + \frac{99}{100} + \dots + \mu
$$
 to 51 terms
\

where
$$
\lambda
$$
, $\mu \in N$ then find the value of $\lambda + \mu$
\n $0.9 + 0.99 + 0.999 + \dots$ up to 51 terms
\n $= 51 - \frac{1}{p} \left(1 - \frac{1}{10^4} \right)$ where p, q $\in N$
\nthen find the value of p + q.
\n $S = 9 + 99 + 999 + \dots +$ upto 49 terms
\n $S = 10 - 1 + 10^2 - 1 + 10^3 - 1 + \dots + 10^{49} - 1$
\n $= (10 + 10^2 + 10^3 + \dots + 10^{49}) - 49$
\n $S = 10 \cdot \left(\frac{10^{49} - 1}{9} \right) - 49$
\n $\lambda + \mu = 49 + 9 = 58$
\n $S = 0.9 + 0.99 + 0.999 + \dots +$ up to 51 terms
\n $= \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots +$ up to 51 terms
\n $= 1 - \frac{1}{10} + 1 - \frac{1}{10^2} + 1 - \frac{1}{10^3} + \dots + \frac{1}{10^{51}}$
\n $= 51 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots + \frac{1}{10^{51}} \right)$
\n $= 51 - \frac{\frac{1}{10} \left(1 - \frac{1}{10^{51}} \right)}{1 - \frac{1}{10}} = 51 - \frac{1}{9} \left(1 - \frac{1}{10^{51}} \right)$
\n $p + q = 60$

 \therefore p + q = 60

5

Example 14 :

Find the sum $S = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$ n terms. **Sol.** It is easy to observe that

EXAMPLE 14. EXAMPLE 14. EXAMPLE 14. EXAMPLE 15. Example 15:
$$
= \frac{1}{x-y} \left[\frac{x^2(1-x^n)}{1-x} - \frac{y^2(1-y^n)}{1-y} \right].
$$
 Example 15: Example 16: Find the sum of series
$$
\frac{3}{19} + \frac{33}{19^2} + \frac{333}{19^3} + \frac{3333}{19^3} + \frac{3333}{19^3} + \frac{3333}{19^3} + \frac{333}{19^3} + \frac{333}{19^3} + \frac{333}{19^3} + \frac{333}{19^3} + \frac{31}{19^3} + \frac{1}{19^3} + \frac{1}{19^
$$

Find the sum of series

$$
\frac{x-y}{x-y} = x+y, \quad \frac{x'-y'}{x-y} = x^2 + xy + y^2,
$$
\n
$$
\frac{x-y}{x-y} = x^2 + xy + y^2,
$$
\n
$$
\frac{x-y}{x-y} = x^2 - 1 + x^2 - 2y + + xy^2 - 2 + y^2 - 1
$$
\n
$$
\frac{1}{x-y} \left[(x^2 - y^2) + (x^2 - y^2) + + (x^2 - y^2) \right]
$$
\n
$$
S = \frac{1}{x-y} \left[(x^2 - y^2) + (x^2 - y^2) + + 2x^2 \right]
$$
\nExample 15:
\n
$$
= \frac{1}{x-y} \left[\frac{x^2(1-x^n)}{1-x} - \frac{y^2(1-y^n)}{1-y} \right].
$$
\nExample 16:
\nExample 15:
\nFind the sum of series
\n
$$
\frac{3}{19} + \frac{33}{19^2} + \frac{333}{19^2} + \frac{3333}{19^4} + ...
$$
\n
$$
S = \frac{3}{9} \left[\frac{10}{19} + \frac{10}{19^2} + \frac{10^3}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left[\frac{10-1}{19} + \frac{10^2-1}{19^2} + \frac{10^3}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left[\frac{10-1}{19} + \frac{10^2-1}{19^2} + \frac{10^3-1}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left[\frac{10-1}{19} + \frac{10^2-1}{19^2} + \frac{10^3-1}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left[\frac{10 \cdot 19}{19} + \frac{11}{19^2} + \frac{10^3-1}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left[\frac{10 \cdot 19}{19} + \frac{11}{19^3} + ... \right]
$$
\n
$$
= \frac{3}{9} \left
$$

ARITHMETICO-GEOMETRICAL PROGRESSION (A.G.P.) :

If each term of a progression is the product of the corresponding terms of an A.P. and a G.P., then it is called arithmetic-geometric progression (A.G.P.)

e.g. $a, (a+d)r, (a+2d) r^2, \dots$

The general term $(n^{th}$ term) of an A.G.P. is

 $T_n = [a + (n-1)d] r^{n-1}$

To find the sum of n terms of an A.G.P. we suppose its sum S, multiply both sides by the common ratio of the corresponding G.P. and then subtract as in following way and we get a G.P. whose sum can be easily obtained.

$$
S_n = a + (a + d) r + (a + 2d) r^2 + \dots [a + (n - 1) d] r^{n-1}
$$

rS_n = $ar + (a + d) r^2 + \dots + [a + (n - 1)d] r^n$

After subtraction we get

 $S_n(1-r) = a + r.d + r^2.d \dots dr^{n-1} - [a + (n-1)d] r^n$ After solving

STUDY MATERIAL: MATHEMATICS
\nAfter subtraction we get
\n
$$
S_n (1-r) = a + r.d + r^2.d..... \text{dr}^{n-1} - [a + (n-1)d] r^n
$$
\nAfter solving
\n
$$
S_n = \frac{a}{1-r} + \frac{r.d(1-r^{n-1})}{(1-r)^2} \text{ and } S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}
$$
\nNote: This is not a standard formula. This is only to understand the procedure for finding the sum of an A.G.P.

STUDY MATERIAL: MATHEMATICS
subtraction we get
 $\frac{n}{(1-r)} = a + r \cdot d + r^2 \cdot d \dots dr^{n-1} - [a + (n-1)d] r^n$
solving
 $\frac{a}{1-r} + \frac{r \cdot d(1-r^{n-1})}{(1-r)^2}$ and $S_{\infty} = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$
: This is not a standard formula. This is only to
s **STUDY MATERIAL: MATHEMATICS**
tion we get
= $a + r \cdot d + r^2 \cdot d$ $dr^{n-1} - [a + (n-1)d] r^n$
 $\frac{r \cdot d(1-r^{n-1})}{(1-r)^2}$ and $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$
is not a standard formula. This is only to
e procedure for finding the sum of **STUDY MATERIAL: MATHEMATICS**

n we get
 $a + r \cdot d + r^2 \cdot d \dots d r^{n-1} - [a + (n-1)d] r^n$
 $\frac{(1 - r^{n-1})}{(1 - r)^2}$ and $S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2}$

not a standard formula. This is only to

procedure for finding the sum of an A.G.P.
 : **MATHEMATICS**
 $[a+(n-1)d] r^n$
 $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

la. This is only to

ne sum of an A.G.P.

terms can be used [EMATICS]
1)d] r^n
 $\frac{dr}{(1-r)^2}$
is only to
f an A.G.P.
an be used **Note :** This is not a standard formula. This is only to understand the procedure for finding the sum of an A.G.P. However formula for sum of infinite terms can be used directly. **AL: MATHEMATICS**

¹-[a + (n - 1)d] rⁿ
 $= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

mula. This is only to

g the sum of an A.G.P.

te terms can be used

hen find the sum of its
 $\frac{1}{2} \left[3 + \frac{5}{2} + \frac{7}{2^2} + \right]$

ket is Arithmetico-AL: MATHEMATICS
 $1-[a+(n-1)d]r^{n}$
 $= \frac{a}{1-r} + \frac{dr}{(1-r)^{2}}$

mula. This is only to

g the sum of an A.G.P.

te terms can be used

hen find the sum of its
 $\frac{1}{2} \left[3+\frac{5}{2}+\frac{7}{2^{2}}+....\right]$

ket is Arithmetico-

formula
 L: **MATHEMATICS**
 $-[a + (n-1)d] r^n$
 $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

ala. This is only to

the sum of an A.G.P.

terms can be used

en find the sum of its
 $\left[3 + \frac{5}{2} + \frac{7}{2^2} + \dots \right]$

et is Arithmetico-

formula L: **MATHEMATICS**
 $-[a + (n-1)d]r^n$
 $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

alla. This is only to

the sum of an A.G.P.

terms can be used

en find the sum of its
 $\left[3 + \frac{5}{2} + \frac{7}{2^2} +\right]$

et is Arithmetico-

formula
 $2\left(\frac{1}{2}\right)$

Example 16 :

If rth term of a series is $(2r + 1) 2^{-r}$, then find the sum of its infinite terms

Sol. Here
$$
T_r = (2r + 1) 2^{-r}
$$
 \therefore Series is $\frac{1}{2} \left[3 + \frac{5}{2} + \frac{7}{2^2} + \dots \right]$

Obviously the series in the bracket is Arithmetico-Geometrical series. Therefore by the formula

After subtraction we get
\n
$$
S_n (1-r) = a + r \cdot d + r^2 \cdot d \dots dr^{n-1} - [a + (n-1)d]r^n
$$
\nAfter solving
\n
$$
S_n = \frac{a}{1-r} + \frac{r!}{(1-r)^2}
$$
 and $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$
\nNote: This is not a standard formula. This is only to
\nunderstand the procedure for finding the sum of an A.G.P.
\nHowever formula for sum of infinite terms can be used
\ndirectly.
\n
$$
P = 16
$$
:
\n
$$
P = 1
$$

Example 17 :

Find the sum of infinite terms of series $3 + 5 \cdot \frac{1}{4} + 7 \cdot \frac{1}{4^2} + \dots$

Sol. Given series is an A.G.P. because each term of series is a product of corresponding term of an A.P. 3,5,7.... and a G.P.

$$
\int_{-\infty}^{\infty} \frac{1}{1-r} \left(1-r\right)^2 \quad , \quad S_{\infty} = \frac{1}{2} \left[\frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{1}{2}} \right] = 5
$$
\n**Example 17 :**

\nFind the sum of infinite terms of series 3 + 5. $\frac{1}{4} + 7 \cdot \frac{1}{4^2} + \dots$

\n**Sol.** Given series is an A.G.P. because each term of series is a product of corresponding term of an A.P. 3,5,7... and a G.P.

\n
$$
\frac{1}{2} + \dots \infty \left[\frac{1}{2} + \frac{1}{4^2} + \dots \right]
$$
\n
$$
\frac{1}{4}S = 3 \cdot \frac{1}{4} + 5 \cdot \frac{1}{4} + \dots
$$
\nafter subtraction we get

\n
$$
\frac{3}{4}s = 3 + 2\left[\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots \right]
$$
\n**N(A.G.P)::**

\nuct of the

\n
$$
= 3 + 2 \cdot \frac{\frac{1}{4}}{1 - 1/4} = \frac{11}{3}
$$
\ni.e. $S = \frac{11}{3} \times \frac{4}{3} = \frac{44}{9}$

\nAlternate: Using formula a = 3, d = 2, r = 1/4

\nuse its sum to the sum of the two of the two ways way

\nUsing way

\n
$$
S_{\infty} = \frac{a}{1-r} + \frac{rd}{(1-r)^2} = \frac{3}{1-\frac{1}{4}} + \frac{\frac{1}{4} \times 2}{\left(1-\frac{1}{4}\right)^2} = \frac{44}{9}
$$
\nFind the sum of the two ways.

\n**Solution**

\n
$$
S_{\infty} = \frac{1}{1-r} + \frac{1}{(1-r)^2} = \frac{3}{1-\frac{1}{4}} + \frac{\frac{1}{4} \times 2}{\left(1-\frac{1}{4}\right)^2} = \frac{44}{9}
$$
\nSince the sum of the two ways.

after subtraction we get

$$
\frac{3}{4}s = 3 + 2\left[\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots\right]
$$

$$
= 3 + 2 \cdot \frac{\frac{1}{4}}{1 - 1/4} = \frac{11}{3}
$$

i.e. $S = \frac{11}{3} \times \frac{4}{3} = \frac{44}{9}$

Alternate : Using formula $a = 3$, $d = 2$, $r = 1/4$

Given series is an A.C.F. because each term of series is a
product of corresponding term of an A.P. 3,5,7.... and a G.P.

\n
$$
\frac{1}{4}, \frac{1}{4}, \frac{1}{4^2} \text{ Let } S = 3 + 5. \frac{1}{4} + 7. \frac{1}{4^2} +
$$
\n
$$
\frac{1}{4}S = 3. \frac{1}{4} + 5. \frac{1}{4^2} +
$$
\nfter subtraction we get

\n
$$
\frac{3}{4}s = 3 + 2\left[\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} +\right]
$$
\n
$$
= 3 + 2. \frac{\frac{1}{4}}{1 - 1/4} = \frac{11}{3}
$$
\ne. $S = \frac{11}{3} \times \frac{4}{3} = \frac{44}{9}$

\nAlternate: Using formula $a = 3$, $d = 2$, $r = 1/4$

\n
$$
S_{\infty} = \frac{a}{1 - r} + \frac{rd}{(1 - r)^2} = \frac{3}{1 - \frac{1}{4}} + \frac{\frac{1}{4} \times 2}{(1 - \frac{1}{4})^2} = \frac{44}{9}
$$

SEQUENCES & SERIES

TRY IT YOURSELF-2

- **Q.1** Fifth term of a G.P. is 2. Find the product of its first nine terms.
- **Q.2** If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.
- **Q.3** If the continued product of three numbers in a G.P. is 216 and the sum of their products in pairs is 156, find the numbers.
- **Q.4** Find the product of three geometric means between 4 and 1/4.
- **Q.5** Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then :

(A)
$$
a = 7/4
$$
, $r = 3/7$
\n(B) $a = 2$, $r = 3/8$
\n(C) $a = 3/2$, $r = 1/2$
\n(B) $a = 2$, $r = 3/8$
\n(D) $a = 3$, $r = 1/4$

(SEQUENCES & SERIES)
 CALLACT COURSELF-2
 CALLACT COURSELF-2
 CALLACT COURSELE-1
 CALLACT COURSELE-1
 CALLACT COURSELE-1
 CALLACT COURS (I) Fra, b, c are in H.P. the

following it, then find the common ratio roots of $x^2 - 4x + q = 0$. If α , β , γ , δ are in G. P., then the integral values of p and q respectively, are $(A) -2, -32$ (B) –2, 3 (C) –6, 3 (D) –6, –32 the sum of their products in pairs is 156, find the

lotes.

identic product of three geometric means between 4

1/4.

identify the product of three geometric series with first term 'a' **HARMONICMEAN(H.M.)**

identify and

Q.7 Suppose a, b, c are in A.P. a^2 , b^2 , c^2 are in G.P. If $a < b < c$ and $a + b + c = 3/2$, then the value of a is

Find the product of three geometric means between 4
and 1/4.
Consider an infinite geometric series with first term 'a' **HARMONICMEAN(H.M.)**
and common ratio r. If the sum is 4 and the second term is
3/4, then:
(A) a = 7/4, r = 3/7
(B) a = 2, r = 3/8
(C) a = 3/2, r = 1/2
(D) a = 3, r = 1/4
and r, b are in H.P.
Let
$$
\alpha
$$
, β be the roots of $x^2 - x + p = 0$ and γ , δ be the
roots of $x^2 - 4x + q = 0$. If α , β , γ , δ are in G.P., then the
integral values of p and q respectively, are
(A) -2, -32
(B) -2, 3
(C) -6, 3
Suppose a, b, c are in A.P. a², b², c² are in G.P. If a < b < c
and a + b + c = 3/2, then the value of a is
(A) $\frac{1}{2\sqrt{2}}$
(B) $\frac{1}{2\sqrt{3}}$
(C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$
(D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
(E) $\frac{1}{2\sqrt{2}}$
Therefore, the value of a is
in $\frac{1}{a_1}$, a₂, a₃,........a, a_n.
(C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$
(D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$
Therefore, the function of a₁, a₂, a₃,........a, a_n.
(A) x < -10
(B) -10 < x < 0
(C) 0 < x < 10
(D) x > 10
Therefore, the terms

Q.8 An infinite G.P. has first term 'x' & sum '5', then x belongs to

ANSWERS

HARMONIC PROGRESSION (H.P.) :

Harmonic progression is defined as a series in which reciprocal of its terms are in A.P.

The standard from of a H.P. is $-+$ $+$ $+$ $+$ $-$

Note : a, b, c are in H.P. \Leftrightarrow b = $\frac{2ac}{2ac}$ a - rd

General Term of a H.P. :

General term $(nth$ term) of a H.P. is given by

$$
T_n = \frac{1}{a + (n-1)d}
$$

Note :

(i) If a,b,c are in H.P. then
$$
\frac{a}{c} = \frac{a-b}{b-c}
$$

(ii) If a, b are first two terms of an H.P. then

$$
t_n = \frac{1}{\frac{1}{a} + (n-1)\left(\frac{1}{b} - \frac{1}{a}\right)}
$$

HARMONIC MEAN (H.M.)

9. Examples and H.P. then $\frac{a}{c} = \frac{a-b}{b-c}$

b are first two terms of an H.P. then
 $= \frac{1}{\frac{1}{a} + (n-1)(\frac{1}{b} - \frac{1}{a})}$

CMEAN(H.M.)

or more than three terms are in H.P., then all the

is lying between them. i.e.

i. FORM ADVANCED LEARNING

FORM ADVANCED LEARNING

The first two terms of an H.P. then
 $\frac{1}{(n-1)(\frac{1}{b}-\frac{1}{a})}$

EAN (H.M.)

more than three terms are in H.P., then all the

ring between first and last term are called If three or more than three terms are in H.P., then all the numbers lying between first and last term are called Harmonic Means between them. i.e; $=\frac{a-b}{b-c}$
of an H.P. then
st and last term are called
em. i.e;
quantities a and b is H so that
 $\frac{2ab}{a+b}$
 $=\frac{n}{\sum_{j=1}^{n} \frac{1}{a_j}}$
zero numbers LAN (H.M.)

more than three terms are in H.P., then all the

ing between first and last term are called

fleans between them. i.e;

tween two given quantities a and b is H so that

H.P.

are in A.P.
 $=\frac{1}{b} - \frac{1}{H} \Rightarrow H = \$ Solution and last term are called

i.e;

i.e;

antities a and b is H so that

b

b
 $\frac{n}{b}$
 $\sum_{j=1}^{n} \frac{1}{a_j}$

co numbers
 $\sum_{j=1}^{n} \frac{1}{a_j}$ P. then $\frac{1}{c} = \frac{1}{b-c}$
wo terms of an H.P. then
 $\frac{1}{b} = \frac{1}{a}$
M.)
M.
N.
tween first and last term are called
etween them. i.e;
two given quantities a and b is H so that
A.P.
 $\Rightarrow H = \frac{2ab}{a+b}$
 $+ \dots + \frac{1}{a_n} = \frac{n$ In H.P. then $\frac{1}{c} = \frac{1}{b-c}$

first two terms of an H.P. then
 $\frac{1}{(n-1)\left(\frac{1}{b}-\frac{1}{a}\right)}$

N(H.M.)

ore than three terms are in H.P., then all the

g between first and last term are called

ans between them. i.e;

o

The H.M. between two given quantities a and b is H so that a, H, b are in H.P.

EXERIES)
\n**TRY IT YOLIRSEL F-2**
\nof a G.P. is 2. Find the product of its first nine
\nin of an infinite G.P. is twice the sum of the terms
\nin the common ratio of the G.P.
\nSince the sum of the terms of the G.P. is 216
\nand the common ratio of the G.P. is 156, find the
\nproduct of three geometric means between 4
\nto right to the G.P. is 156, find the
\nproduct of three geometric series with first term 'a'
\nand the second term is
\n
$$
r = 3/7
$$

\nIn infinite geometric series with first term 'a'
\n $r = 1/2$
\n $(B)a = 2, r = 3/8$
\n $(C)a = 2, r = 3/8$
\n $(D)a = 3, r = 1/4$
\n $(D)a = 3, r = 1/4$
\n $(D)a = 3, r = 1/4$
\n $(E)a = 2, r = 3/8$
\n $(E)a = 2, F = 3$

are in G.P. If
$$
a < b < c
$$

f a is

$$
\frac{1}{a_1 + \frac{1}{a_2} + \dots + \frac{1}{a_n}} = \frac{\frac{n}{n}}{\sum_{j=1}^{n} \frac{1}{a_j}}
$$

The harmonic mean of n non zero numbers

 $a_1, a_2, a_3, \dots, a_n.$

$\frac{1}{2} - \frac{1}{\sqrt{2}}$
n H.M's between two given numbers :

To find n HM's between a, and b we first find n AM's between 1/a and 1/b then their reciprocals will be required HM's.

If terms are given in H.P. then the terms could be picked up in the following way

b be the roots of
$$
x^2 - x + p = 0
$$
 and γ , δ be the
\n $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the
\nvalues of p and q respectively, are
\n $\frac{1}{10}, -\frac{1}{6}, -\frac{1}{32}$
\n $\frac{1}{(10)}, -\frac{1}{(10)}, -\frac{1}{$

Note : In general, If we are to take $(2r + 1)$ terms in H.P. we take them as

$$
\frac{1}{-rd}, \frac{1}{a - (r-1)d}, \dots, \frac{1}{a - d}, \frac{1}{a}, \frac{1}{a + d}, \dots, \frac{1}{a + rd}
$$

Example 18 :

If the 3rd, 6th and last term of a H.P. are $\frac{1}{3}, \frac{1}{5}, \frac{3}{203}$, find the number of terms.

Sol.
$$
T_3 = \frac{1}{3}, T_6 = \frac{1}{5}, T_n = \frac{3}{203}
$$

then 3rd, 6th and nth termof A.P. series are 3, 5, $\frac{203}{2}$.

$$
a + 2d = 3; a + 5d = 5
$$

\n
$$
d = \frac{2}{3}, a = \frac{5}{3}
$$

\n
$$
a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}
$$

\n
$$
(n-1) = 198/2; n = 100.
$$

\nExample 22:

Example 19 :

If a, b, c are in HP, find the value of $\frac{0.1a}{1} + \frac{0.1c}{1}$. (3) H.P.

Sol. a, b, c are in HP, then $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are in A.P.

$$
S = \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} + \frac{\frac{1}{c} + \frac{1}{b}}{\frac{1}{c} - \frac{1}{b}}
$$

Let
$$
\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c} = d
$$

$$
S = \frac{\left(\frac{1}{a} + \frac{1}{b}\right) - \left(\frac{1}{c} + \frac{1}{b}\right)}{d} = \frac{\left(\frac{1}{a} - \frac{1}{c}\right)}{d} = \frac{2d}{d} = 2
$$

Example 20 :

If between 1 and $1/31$ there are n H.M.'s and ratio of $7th$ and $(n-1)$ th harmonic means is 9 : 5, then find the value of n

Sol. Since there are n A.M.'s between 1 and 31 and the ratio of $7th$ and $(n-1)th$ A.M.' is 5 : 9

The 20:
\nIf between 1 and 1/31 there are n H.M.'s and ratio of 7th and
\n(n-1)th harmonic means is 9 : 5, then find the value of n
\nSince there are n A.M.'s between 1 and 31 and the ratio of
\n
$$
7^{th}
$$
 and $(n-1)^{th}$ A.M.' is 5 : 9
\n $1+\frac{7(\frac{31-1}{n+1})}{1+(n-1)(\frac{3n-1}{n+1})} = \frac{5}{9} \Rightarrow \frac{n+211}{31n-29} = \frac{5}{9} \Rightarrow n=14$
\n**1** The number's a' and b' then
\n $A = \frac{a+b}{2}$, $G = \sqrt{ab}$, $H = \frac{2ab}{a+b}$
\n $1+f(\frac{1}{n+1}) = \frac{5}{9} \Rightarrow \frac{n+211}{31n-29} = \frac{5}{9} \Rightarrow n=14$
\n**1** (i) Consider A – G = $\frac{a+b}{2}$ – \sqrt{ab} =
\n $50 A \ge G$
\nIn the same way G ≥ H ⇒ A ≥
\nthen find the value of $\frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b}$
\n(ii) Consider A. H. = $\frac{a+b}{2} \cdot \frac{2ab}{a+b} = a$
\n $\Rightarrow G^2 = A.H.$
\n(b) Consider A. H. = $\frac{a+b}{2} \cdot \frac{2ab}{a+b} = a$
\n $\Rightarrow G^2 = A.H.$

Example 21 :

If H_1 , H_2 , H_3 H_n be n harmonic means between a and b

then find the value of $\frac{11 + a}{H_1 - a} + \frac{11}{H_1 - b}$

STUDY MATERIAL: MATHEMATICS
\n
$$
\text{m of a H.P. are } \frac{1}{3}, \frac{1}{5}, \frac{3}{203}, \text{ find the}
$$
\n
$$
\text{Sol. Here } H_1 = \frac{ab(n+1)}{b(n+1) - (b-a)} = \frac{ab(n+1)}{b+n}
$$
\n
$$
\text{Similarly } H_n = \frac{ab(n+1)}{an+b} \text{ (interchange a and b)}
$$
\n
$$
\text{Hence } \frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b}
$$
\n
$$
= 5
$$
\n
$$
\text{m of A.P. series are 3, 5, } \frac{203}{3}.
$$
\n
$$
= \frac{(2n+1)b+a}{b-a} + \frac{(2n+1)a+b}{a-b}
$$
\n
$$
= \frac{2nb+b+a-2na-a-b}{b-a} = 2n
$$
\n
$$
\Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}
$$
\n**Example 22:**\n
$$
\text{If } \frac{a_2a_3}{a_1a_4} = \frac{a_2+a_3}{a_1+a_4} = 3\left(\frac{a_2-a_3}{a_1-a_4}\right) \text{ then } a_1, a_2, a_3, a_4 \text{ are in}
$$
\n
$$
\text{the value of } \frac{b+a}{b-a} + \frac{b+c}{b-c}.
$$
\n
$$
\text{If } \frac{1}{a_1a_4} = \frac{a_2+a_3}{a_1+a_4} = 3\left(\frac{a_2-a_3}{a_1-a_4}\right) \text{ then } a_1, a_2, a_3, a_4 \text{ are in}
$$
\n
$$
\text{or } \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}
$$
\n
$$
\text{So } \frac{1}{a_1} + \frac{1}{a_1} = \frac{1}{a_2a_3}, \text{So } \frac{1}{a_4} + \frac{1}{a_1} = \frac{1}{a_3} + \frac{1}{a_2} \text{ or } \frac{1}{a_4} - \frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1} \text{ (1
$$

$$
\frac{(2n+1)b+a}{b-a} + \frac{(2n+1)a+b}{a-b}
$$

$$
= \frac{2nb+b+a-2na-a-b}{b-a} = 2n
$$

If
$$
\frac{a_2 a_3}{a_1 a_4} = \frac{a_2 + a_3}{a_1 + a_4} = 3 \left(\frac{a_2 - a_3}{a_1 - a_4} \right)
$$
 then a_1, a_2, a_3, a_4 are in
(1) A.P. (2) G.P.
(3) H.P. (4) None of these

If the 3¹² (b¹⁰ and last term of a H.P. are
$$
\frac{1}{3}, \frac{1}{3}, \frac{1}{203}
$$
, find the
\nnumber of terms.
\n $15 = \frac{1}{3}, 1\frac{1}{6} = \frac{1}{5}, 1\frac{1}{6} = \frac{3}{5}, 1\frac{1}{203}$
\n $a - 2d = 3$; $a + 5d = 5$
\n $d = \frac{2}{3}, a = \frac{5}{3}$
\n $a - 2d = 3$; $a + 5d = 5$
\n $d = \frac{2}{3}, a = \frac{5}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) \frac{2}{3} = \frac{203}{3}$
\n $a + (n-1)d = \frac{1}{3} \Rightarrow \frac{1}{6} \Rightarrow (n-1) \frac{2}{3} \Rightarrow (n-1) \$

So
$$
\overline{a_1}
$$
, $\overline{a_2}$, $\overline{a_3}$ are in A.P.

RELATION BETWEEN A.M., G.M. & H.M.

A, G, H are AM, GM and HM respectively between two numbers 'a' and 'b' then

$$
= \frac{3}{9} \Rightarrow n = 14
$$

A = $\frac{a+b}{2}$, G = \sqrt{ab} , H = $\frac{2ab}{a+b}$

(i) Consider A – G =
$$
\frac{a+b}{2}
$$
 – \sqrt{ab} = $\frac{(\sqrt{a} - \sqrt{b})^2}{2} \ge 0$

So $A \ge G$ In the same way $G \geq H \Rightarrow A \geq G \geq H$

(ii) Consider A.H. =
$$
\frac{a+b}{2} \cdot \frac{2ab}{a+b} = ab = G^2
$$

\n $\Rightarrow G^2 = A.H.$

SEQUENCES & SERIES

If A, G and H are A.M., G.M. and H.M. of two positive S numbers a and b, then (a) $G^2 = AH$, \ldots \geq G \geq H **Note :**

- (i) For given n positive numbers $a_1, a_2, a_3, \dots, a_n$, A.M. \ge G.M. \ge H.M. The equality holds when the numbers are equal
- **(ii)** If sum of the given n positive numbers is constant then that their product will be maximum if numbers are equal.

Example 23 :

If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is (1) less than 1 (2) equal to 1 (3) greater than 1 (4) any real number

Sol. (1). Since a and b are unequal,
$$
\frac{a^2 + b^2}{2} > \sqrt{a^2 b^2}
$$

 $(A.M. > G.M.$ for unequal numbers) \Rightarrow $a^2 + b^2 > 2ab$ Similarly $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$ Hence $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$ \Rightarrow ab + bc + ca < 1

Example 24 :

If $x > 0$, $y > 0$, $z > 0$ then prove that $(x + y)(y + z)(z + x) \ge 8xyz$ **Sol.** $(x+y)(y+z)(z+x)$

that their product will be maximum if numbers are equal.
\n
$$
\frac{2}{7} \ge \left(\frac{a}{2}, \frac{b}{3}, \frac{c}{2}\right)^{1/7}
$$
\n
$$
\frac{1}{2} \ge 3 \Rightarrow \frac{3}{7} \ge \frac{1}{2} \left(\frac{a}{4}, \frac{b}{27}, \frac{d}{4}\right)
$$
\n
$$
\frac{1}{2} \ge 1 \Rightarrow b^2 \ge 2 \Rightarrow 3 \Rightarrow 2 \ge \frac{1}{2} \left(\frac{a}{4}, \frac{b}{27}, \frac{d}{4}\right)
$$
\n
$$
\frac{1}{2} \ge 1 \Rightarrow b^2 \ge 2 \Rightarrow 1 \Rightarrow 2 \ge 2 \Rightarrow 3 \Rightarrow 2 \ge 2 \Rightarrow 2 \Rightarrow 3 \Rightarrow 2 \ge 2 \Rightarrow 2 \Rightarrow 3 \Rightarrow 2 \ge 2 \Rightarrow 2 \Rightarrow 3 \
$$

Example 25 :

Prove that a \triangle ABC is equilateral if and only if

Sol.
$$
\frac{\tan A + \tan B + \tan C}{3} \ge (\tan A \tan B \tan C)^{1/3}
$$

since A + B + C = π

 $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$$
\tan A + \tan B + \tan C = 3\sqrt{3}
$$

\n
$$
\frac{\tan A + \tan B + \tan C}{3} \ge (\tan A \tan B \tan C)^{1/3}
$$

\nsince A + B + C = π
\n
$$
\tan A + \tan B + \tan C = \tan A \tan B \tan C
$$

\n
$$
\left(\frac{\tan A + \tan B + \tan C}{3}\right) \ge (\tan A + \tan B + \tan C)^{1/3}
$$

\n
$$
\left(\tan A + \tan B + \tan C\right)^{3} \ge 27 (\tan A + \tan B + \tan C)
$$

\n
$$
\left(\tan A + \tan B + \tan C\right)^{2} \ge 27
$$

\n
$$
\tan A + \tan B + \tan C \ge 3\sqrt{3}
$$

\n
$$
\therefore \tan A + \tan B + \tan C \ge 3\sqrt{3}
$$

\n
$$
\therefore \tan B + \tan C = 3 \text{ and a, b, c are positive, then prove that}
$$

\n
$$
\therefore S_n = \frac{1}{2} [3\Sigma r]
$$

\n
$$
a^2b^3c^2 \le \frac{3^{10} \cdot 2^4}{7^7}
$$

\n
$$
= \frac{1}{2} \left[3 \frac{n}{3} \right]
$$

Example 26 :

If $a + b + c = 3$ and a, b, c are positive then prove that

$$
a^2b^3c^2 \le \frac{3^{10} \cdot 2^4}{7^7}
$$

$$
Sol. a+b+c=3
$$

$$
\frac{\sqrt{3}}{\sqrt{2}}
$$
\n
$$
\frac{10}{3} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3
$$
\n
$$
\frac{10.2^4}{7^7}
$$
\n
$$
\frac{10.2^4}{7^7}
$$
\nthen prove that

and H.M. of two positive **Sol.**
$$
a + b + c = 3
$$

\nAH, $\dots \ge G \ge H$
\nWe can write it as $\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} = 3$
\nWe can write it as $\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} = 3$
\nby holds when the numbers
\nnumbers is constant then
\n $\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} \ge \left(\frac{a^2 b^3 c^2}{4 \cdot 27 \cdot 4}\right)^{1/7}$
\n= real numbers and
\n $\frac{3}{2} \ge \left(\frac{a^2 b^3 c^2}{2^4 \cdot 3^3}\right)^{1/7}$; $a^2 b^3 c^2 \le \frac{3^{10} \cdot 2^4}{7^7}$
\n(2) equal to 1
\n(4) any real number
\n $\frac{a^2 + b^2}{2} > \sqrt{a^2 b^2}$
\nIf a, b, c are positive real number then prove that
\n $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1 + c}{2a} \ge \frac{5}{4}$
\n $c^2 + a^2 > 2ca$
\n $ab + bc + ca$
\n**Sol.** $\frac{a^3}{4b} + \frac{b}{8c^2}, \frac{1}{2a}, \frac{c}{4a}, \frac{c}{4a}$
\nApplying A.M. ≥ G.M.
\n $t(x+v)(v+z)(z+x) \ge 8xyz$
\n $a^3, b \ne 1, c \ne 2$

$$
Now A.M. \ge G.M.
$$

$$
\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}
$$
\n
$$
\frac{a}{4} + \frac{b}{27} + \frac{c}{4} + \frac{c}{27} + \frac{c}{4}
$$

$$
\frac{3}{7} \ge \left(\frac{a^2 b^3 c^2}{2^4 \times 3^3}\right)^{1/7} \quad ; \quad a^2 b^3 c^2 \le \frac{3^{10} \cdot 2^4}{7^7}
$$

Example 27 :

 $\frac{1+b^2}{2}$ > $\sqrt{a^2 b^2}$ If a, b, c are positive real number then prove that

$$
\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a} \ge \frac{5}{4}
$$

Sol.
$$
\frac{a^3}{4b}, \frac{b}{8c^2}, \frac{1}{2a}, \frac{c}{4a}, \frac{c}{4a}
$$

$$
Applying A.M. \geq G.M.
$$

a + b + c = 3
\nWe can write it as
$$
\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} = 3
$$

\nNow A.M. ≥ G.M.
\n $\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}$
\n $\left(\frac{a^2b^3c^2}{4 \times 3^3}\right)^{1/7}$; a² b³ c² ≤ $\frac{3^{10} \cdot 2^4}{7^7}$
\n**mple 27:**
\nIf a, b, c are positive real number then prove that
\n $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a} \ge \frac{5}{4}$
\n $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a} \ge \frac{5}{4}$
\nApplying A.M. ≥ G.M.
\n $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1}{2a} + \frac{c}{4a} + \frac{c}{4a} \ge \left(\frac{a^3}{4b} \cdot \frac{b}{8c^2} \cdot \frac{1}{2a} \cdot \left(\frac{c}{4a}\right)^2\right)^{1/5}$
\n $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1}{2a} + \frac{c}{2a} \ge \frac{5}{4}$
\n**THODOF DIFFERENCE**
\nLet T₁, T₂, T₃,...... T_n are the terms of sequence, then
\nsum of P₁, T₂, T₃,...... T_n = T_n =

METHOD OF DIFFERENCE

Let T_1 , T_2 , T_3 T_n are the terms of sequence, then

- (i) If $(T_2 T_1)$, $(T_3 T_2)$ $(T_n T_{n-1})$ are in A.P. then, the sum of the such series may be obtained by using summation formulae in nth term.
- (ii) If $(T_2 T_1)$, $(T_3 T_2)$ $(T_n T_{n-1})$ are found in G.P. then, the sum of the such series may be obtained by using summation formulae of a GP.

Example 28 :

Find the sum of the series $3 + 7 + 14 + 24 + 37 + ...$ 10 terms, **Sol.** Here the given series is not A.P., G.P., or H.P.

Let $S = 3 + 7 + 14 + 24 + 37 + \dots + T_n$
 $S = 3 + 7 + 14 + 24 + \dots + T_n$

after subtracting

$$
(x+y)(y+z)(z+x)
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{xy}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
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\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x+y}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x+x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x}{2} \ge \sqrt{yz}
$$
\n
$$
\frac{x}{2} \ge \sqrt{zx}
$$
\n
$$
\frac{x}{2} \ge \frac{x}{2}
$$
\n

Putting $n = 10$ $=\frac{1}{2}[1155 - 55 + 40] = \frac{1140}{2} = 570$

Splitting the nth term as a difference of two :

Here is a series in which each term is composed of the reciprocal of the product of r factors in A.P., the first factor of the several terms being in the same A.P.

Example 29 :

Find the sum of n terms of the series and also find S_{∞} .

... 1·2·3·4 2·3·4·5 3·4·5·6 **Sol.** 1 1 1 1 ... 1·2·3·4 2·3·4·5 3·4·5·6 n(n 1)(n 2)(n 3) . 1 4 1 5 2 (n 3) n S ... 3 1·2 ·3 ·4 2 ·3 ·4 ·5 n(n 1)(n 2)(n 3) 1 1 1 1 ^T 3 1·2 ·3 2 ·3 ·4 2 1 1 ^T 2 ·3 ·4 3 ·4 ·5 ⁿ 1 1 1 ^T S = T¹ + T² + + Tⁿ 1 1 1 3 1·2 ·3 (n 1)(n 2)(n 3) ⁿ 1 1 ^S 18 3 (n 1) (n 2) (n 3) ; S⁼ ¹ 18 1 1·3 1·3·5 ... 2·4 2·4·6 2·4·6·8 **Sol.** ⁿ 1 1·3 1·3·5 1·3·5 (2n 1) ^S ⁿ 1·3·5 (2n 1) ^T 2·4·6·8 (2n 2) 1·3·5 (2n 1)·[(2n 2) (2n 1)] 2·4·6·8 (2n 2)

Example 30 :

Find sum of n terms (S_n) for

$$
\frac{1}{2\cdot 4} + \frac{13}{2\cdot 4\cdot 6} + \frac{13\cdot 5}{2\cdot 4\cdot 6\cdot 8} + \dots
$$

ol. $S_n = \frac{1}{2\cdot 4} + \frac{13}{2\cdot 4\cdot 6} + \frac{13\cdot 5}{2\cdot 4\cdot 6\cdot 8} + \dots + \frac{13\cdot 5 \dots (2n-1)}{2\cdot 4\cdot 6\cdot 8 \dots (2n+2)}$

$$
T_n = \frac{133... (2n-1)}{2 \cdot 4 \cdot 6 \cdot 8... (2n+2)}
$$

=
$$
\frac{135...(2n-1) \cdot [(2n+2)-(2n+1)]}{2 \cdot 4 \cdot 6 \cdot 8...(2n+2)}
$$

IDENTIFY	STUDY MATERIAL: MATHEMATICS
\n <p>Putting $n = 10$</p> \n <p>\n$S_{10} = \frac{1}{2} \left[\frac{10 \times 11 \times 21}{2} - \frac{10 \times 11}{2} + 40 \right]$\n</p> \n <p>\n$= \frac{1}{2} \left[1155 - 55 + 40 \right] = \frac{1140}{2} = 570$\n</p> \n <p>\n$T_1 = \frac{1}{2} - \frac{13}{24}$; \n $T_2 = \frac{13}{24} - \frac{135}{246}$\n</p> \n <p>\n$T_1 = \frac{1}{2} - \frac{13}{24}$; \n $T_2 = \frac{13}{24} - \frac{135}{246}$\n</p> \n <p>\n$T_1 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \dots (2n + 2)}$\n</p> \n <p>\n$T_1 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \dots (2n + 2)}$\n</p> \n <p>\n$T_1 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \dots (2n + 2)}$\n</p> \n <p>\n$T_1 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \dots (2n + 2)}$\n</p> \n <p>\n$T_2 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \cdot (2n + 2)}$\n</p> \n <p>\n$T_3 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \cdot (2n + 2)}$\n</p> \n <p>\n$T_4 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \cdot (2n + 2)}$\n</p> \n <p>\n$T_5 = \frac{13 \cdot 5 \dots (2n + 1)}{2 \cdot 4 \cdot 6 \cdot (2n + 2)}$\n</p> \n <p>\n$T_6 = \frac{13 \cdot 5 \dots (2n + 1)}{$</p>	

Factor in A.P

Here is a series in which each terms is composed of r factor in A.P., the first factor of the several terms being in the same A.P.

Example 31 :

$$
\frac{1}{2} \frac{1}{1155-55+40} = \frac{1140}{2} = 570
$$
\n
$$
\frac{1}{2} \frac{1}{21155-55+40} = \frac{1140}{2} = 570
$$
\n
$$
\frac{1}{2} \frac{1}{3} \left(\frac{1}{11235-56} + \frac{1}{11235-56}
$$

SOME IMPORTANT RESULTS

- **(i)** If number of terms is an A.P./G.P./H.P. is odd then its mid term is the A.M/G.M./H.M. between the first and last number.
- **(ii)** If the number of terms in an A.P./G.P./H.P. is even then A.M./ G.M./H.M. of its two middle terms is equal to the A.M./ G.M./H.M. between the first and last numbers.
- (iii) a,b,c are in A.P. and H.P. \Rightarrow a,b,c are in G.P.

(iv) If a,b,c are in A.P. then
$$
\frac{1}{bc}
$$
, $\frac{1}{ac}$, $\frac{1}{ab}$ are in A.P.

SEQUENCES & SERIES

(v) If
$$
a^2
$$
, b^2 , c^2 are in A.P. then $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ are in A.P.
\n(vi) If a,b,c are in G.P. then a^2 , b^2 , c^2 are in G.P.

(vii) If a, b, c, d are in G.P. then $a + b$, $b + c$, $c + d$ are in G.P.

(viii) If a,b,c are in H.P. then
$$
\frac{b+c}{a}
$$
, $\frac{c+a}{b}$, $\frac{a+b}{c}$ are in A.P.

TRY IT YOURSELF-3

- **Q.1** The 8th and 14th term of HP are 1/2 and 1/3, respectively. Find its 20th term. Also, find its general term.
- **Q.2** If first three terms of the sequence 1/16, a, b, 1/6 are in geometric series and last three terms are in harmonic series, then find the values of a and b. **EXERIES**

are in A.P. then $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ are in A.P.

n G.P. then a^2 , b^2 , c^2 are in G.P.

in G.P. then a^2 , b^2 , c^2 are in G.P.

in H.P. then $\frac{b+c}{a}$, $\frac{c+a}{b}$, $\frac{a+b}{c}$ are in **EXERIES**

are in A.P. then $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ are in A.P.

n G.P. then a^2 , b^2 , c^2 are in G.P.

in G.P. then $a + b$, $b + c$, $c + d$ are in G.P.

in G.P. then $a + b$, $b + c$, $c + d$ are in G.P.

n H.P. t
- **Q.3** If H is the harmonic mean between P and Q, then find the

value of
$$
\frac{H}{P} + \frac{H}{Q}
$$
.

- **Q.4** If the A.M. between two numbers exceeds their G.M. by 2 and the G.M. exceeds their H.M. by 8/5, find the numbers.
- **Q.5** Find the sum to n terms of the series $3 + 15 + 35 + 63 + ...$

Q.6 If the sum to infinity of the series

$$
3 + (3 + d)\frac{1}{4} + (3 + 2d)\frac{1}{4^2} + \dots \infty
$$
 is $\frac{44}{9}$, then find d.

1 1 3 (3 d) (3 2d) ⁴ ⁴ b,e are in GP: then a^2 , b², e² are in GP:

b,e, are in GP: then $a + b$, $b + c$, $c + d$ are in GP:

b,e are in H.P. then $\frac{b+c}{a}$, $\frac{c}{b}$, $\frac{a}{c}$ are in A.P.

b,e are in H.P. then $\frac{b+c}{a}$, $\frac{c}{b}$, $\frac{a}{c$ **Q.7** If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b) (c + d)$ satisfies the relation **The d³⁸** and 14° term of HP are 1/2 and 13, respectively.

The fit is 20³ term of HP are 1/2 and 13, respectively.

First three term Also, find is general term.

If first three terms of the sequence 1/16, a, b, The 8^{10} and 14^{10} term of HP are 1/2 and 1/3, respectively.

The 8^{10} and 14^{10} term of HP are 1/2 and 1/3, respectively.

Find its 20th term Also, find its general term

Eind its 20th term Also, find its and the control of the square of the square 1/16, a, b, 1/6 are in
 π if first three terms of the square 1/16, a, b, 1/6 are in
 π first three terms of the square 1/16, a, b, 1/6 are in
 π if first three terms of

$$
(C) 2 \le M \le 3
$$

$$
(D) 3 \le M \le 4
$$

Q.8 The harmonic mean of the roots of the equation

$$
(5 + \sqrt{2})x^{2} - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0
$$
 is
(A)2
(B)4
(C)6
(D)8

$$
= S_{31} =
$$

 (C) in G.P. (D) H.P.

Q.9 Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd and bcd are (A) Not in A.P./G.P./H.P. (B) in A.P.

ANSWERS

3 + (3 + 0)
$$
\frac{1}{4}
$$
 + (3 + 20) $\frac{1}{4^2}$ +²⁰ is 9, then find d.
\nIf a, b, c, d are positive real numbers such that
\nrelation
\nrelation
\n(a) 0 < M \le 1
\n(b) 1 \le M \le 2
\n(c) 2 \le M \le 3
\n(d) 2
\n(e) 3
\n(f) 3x + 8 + 2\sqrt{5} = 0 is
\n(e) 3x + 8 + 2\sqrt{5} = 0 is
\n(e) 0
\n(f) 4
\n3h) 2
\n3i = 1
\n3j = 2
\n**Answer** and the equation
\n
$$
5 + \sqrt{2} x^2 - (4 + \sqrt{5}) x + 8 + 2\sqrt{5} = 0 is\n2j = 2\nAnswer and the equation\n
$$
5 + \sqrt{2} x^2 - (4 + \sqrt{5}) x + 8 + 2\sqrt{5} = 0 is
$$
\n
$$
5x = 3
$$
\n
$$
5x = 4
$$
\n
$$
5x = 1
$$
$$

$$
(8) (B) \qquad \qquad (9) (D)
$$

ADDITIONAL EXAMPLES

Example 1 :

ADDITIONAL EXAMPLES
 $\frac{1}{+c} \cdot \frac{1}{c+a} \cdot \frac{1}{a+b}$ are in A.P.
 2 are in G.P.
 $\frac{1}{c} \cdot \frac{1}{ac} \cdot \frac{1}{a+b}$ are in A.P.
 $\frac{1}{c} \cdot \frac{1}{c} \cdot \frac{1}{a+b}$ are in A.P.
 $\frac{1}{c} \cdot \frac{1}{c} \cdot \frac{1}{c}$ are in A.P.
 $\frac{1}{s_1 + s_3 + s$ **SURVENUES**
 $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.
 $\frac{1}{c^2 \text{ are in G.P.}}$
 $\frac{1}{c^2 \text{ are in G.P.}}$
 $\frac{1}{c^2 \text{ are in G.P.}}$
 $\frac{1}{b}$, $\frac{1}{c} + \frac{1}{c}$ are in G.P.
 $\frac{1}{c^2 \text{ are in G.P.}}$
 $\frac{1}{c^2 \text{ are in G.P.}}$
 $\frac{1}{c^2 \text{ are in A.P.}}$
 SODITIONAL EXAMPLES

ten $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.
 $2, b^2, c^2$ are in G.P.
 $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.
 Example 1:
 $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in G.P.
 Sol. We have $S_n = \frac{a(1-r^n)}{1-r}$ $\frac{1}{b+c} \cdot \frac{1}{c+a} \cdot \frac{1}{a+b}$ are in A.P.
 $\frac{1}{b+c} \cdot \frac{1}{c+a} \cdot \frac{1}{a+b}$ are in A.P.
 $\frac{1}{b^2,c^2}$ are in G.P.
 $\frac{1}{b^2,c^2}$ are in G.P.
 $\frac{1}{b^2,c^2}$ are in G.P.
 $\frac{1}{b^2,c^2}$ are in G.P.
 $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac$ **ADDITIONAL EXAMPLES**
 $\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$ are in A.P.
 b^2 , c^2 are in G.P.
 $\frac{1}{b^2}$, c^2 are in G.P.
 $\frac{1}{b^2}$, c^2 are in G.P.
 $\frac{1}{b^2}$, $\frac{1}{c}$ are in G.P.
 $\frac{1}{a}$, $\frac{1}{c}$, $\$ If S_n denotes the sum of n terms of a G.P. whose first term is a and the common ratio r, then find the sum of $S_1 + S_3 + S_5 + \dots + S_{2n-1}$ **SOMADULES**

s the sum of n terms of a G.P. whose first term is

mmon ratio r, then find the sum of
 y^+ +S_{2n-1}

= $\frac{a(1-r^n)}{1-r}$:... S_{2n-1} = $\frac{a}{1-r}[1-r^{2n-1}]$

3, 3,......., n for n is it and summing up we
 SOMAL EXAMPLES

sum of n terms of a G.P. whose first term is

n ratio r, then find the sum of
 $+S_{2n-1}$
 \therefore $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

..., n for n is it and summing up we

...... $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

...., **SPARAMPLES**

terms of a G.P. whose first term is

; then find the sum of
 \therefore S_{2n-1}= $\frac{a}{1-r}[1-r^{2n-1}]$

r n is it and summing up we

n-1
 $)-(r+r^3+r^5+......n$ term)] **ADDITIONAL EXAMPLES**
 ADDITIONAL EXAMPLES
 ADDITIONAL EXAMPLES
 Ending the common ratio r, then find the sum of
 $+ S_3 + S_5 + ... + S_{2n-1}$
 e have $S_n = \frac{a(1 - r^n)}{1 - r}$ \therefore $S_{2n-1} = \frac{a}{1 - r} [1 - r^{2n-1}]$

thing 1, 2, **DINAL EXAMPLES**

m of n terms of a G.P. whose first term is
 S_{2n-1}
 $\frac{r^n}{r}$ \therefore $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$
 \therefore $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$
 \therefore n for n is it and summing up we
 $\therefore + S_{2n-1}$
 $\text{nterm} - (\text{r} + \$ **ADDITIONAL EXAMPLES**
 EDDITIONAL EXAMPLES

1:

a denotes the sum of n terms of a G.P. whose first term is

ad the common ratio r, then find the sum of
 $S_3 + S_5 + \dots + S_{2n-1}$

have $S_n = \frac{a(1-r^n)}{1-r}$ \therefore $S_{2n-1} = \frac{a}{$ **ADDITIONAL EXAMPLES**
 ADDITIONAL EXAMPLES

De 1:

S_n denotes the sum of n terms of a G.P. whose first term is

and the common ratio r, then find the sum of
 $1 + S_3 + S_5 + ... + S_{2n-1}$
 e have $S_n = \frac{a(1 - r^n)}{1 - r}$ \therefore **DDITIONAL EXAMPLES**
 DDITIONAL EXAMPLES
 EXAMPLES
 EXAMPLES DDITIONAL EXAMPLES
 EDDITIONAL EXAMPLES

PROMADVANCED LEARNING

PROMADVANCED LEARNING

PROMADVANCED LEARNING

The common ratio r, then find the sum of
 $S_1 + S_2 + \dots + S_{2n-1}$
 \vee $S_n = \frac{a(1 - r^n)}{1 - r}$ \therefore $S_{2n-1} = \$ **DDITIONAL EXAMPLES**
 EXAMPLES

PODITIONAL EXAMPLES

PODITIONAL EXAMPLES

PODITIONAL EXAMPLES
 $x^3 + 5x^4 + ... +52n-1$
 $x^4 + 5x^3 + ... +52n-1$
 $x^5 + 5x^2 + ... +52n-1$
 $x^4 + 5x^3 + ...$ is it and summing up we
 $x^4 + 5x^4 + ...$ is the **EXAMPLES**

so f a GP. whose first term is
 $S_{2n-1} = \frac{a}{1-r} [1-r^{2n-1}]$

it and summing up we
 $+r^3 + r^5 + ...$ therm]
 $\frac{a}{-r} \left[n - r \cdot \frac{1-r^{2n}}{1-r^2} \right]$

series **AMPLES**

a GP. whose first term is

d the sum of
 $h^{-1} = \frac{a}{1-r} [1-r^{2n-1}]$

d summing up we
 h^{-1} f h^{-1} n term)
 h^{-1} f $\frac{1-r^{2n}}{1-r^2}$ **EXAMPLES**

TOD MADVANCED LEARNING

IS ON A G.P. whose first term is

en find the sum of
 $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

s it and summing up we
 $(r+r^3 + r^5 + \dots n \text{ term})]$
 $\frac{a}{1-r}\left[n-r, \frac{1-r^{2n}}{1-r^2}\right]$

e series **SPON ADVANCED LEARNING**
 AMPLES

Fa G.P. whose first term is

d the sum of
 $2n-1 = \frac{a}{1-r}[1-r^{2n-1}]$

and summing up we
 $+r^5 + \dots$ therm)]
 $\left[n-r, \frac{1-r^{2n}}{1-r^2}\right]$ **SOMADVANCED LEARNING**
 AMPLES

Fa GP. whose first term is

d the sum of
 $2n-1 = \frac{a}{1-r}[1-r^{2n-1}]$

and summing up we
 $+r^5 + \dots$ term)]
 $\left[n-r, \frac{1-r^{2n}}{1-r^2}\right]$ EXAMPLES

so f a GP. whose first term is
 $S_{2n-1} = \frac{a}{1-r} [1-r^{2n-1}]$

it and summing up we
 $+r^3 + r^5 + ...$ therm)]
 $\frac{a}{1-r} \left[n - r \cdot \frac{1-r^{2n}}{1-r^2} \right]$

series **ADDITIONAL EXAMPLES**

1:

n denotes the sum of n terms of a G.P. whose first term is
 $+ S_3 + S_5 + + S_{2n-1}$

have $S_n = \frac{a(1-r^n)}{1-r}$ \therefore $S_{2n-1} = \frac{a}{1-r} [1-r^{2n-1}]$

ting 1, 2, 3,......., n for n is it and summing up we

Sol. We have
$$
S_n = \frac{a(1 - r^n)}{1 - r}
$$
 : $S_{2n-1} = \frac{a}{1 - r} [1 - r^{2n-1}]$

Putting 1, 2, 3,........, n for n is it and summing up we S¹ + S³ + S⁵ +..........+ S2n–1

$$
= \frac{a}{1-r} [(1+1+.... n \text{ term}) - (r + r^3 + r^5 + n \text{ term})]
$$

$$
= \frac{a}{1-r} \left[n - \frac{r \left\{ 1 - (r^2)^n \right\}}{1 - r^2} \right] = \frac{a}{1-r} \left[n - r \cdot \frac{1 - r^{2n}}{1 - r^2} \right]
$$

Example 2 :

Find the maximum sum of the series

$$
20 + 19\frac{1}{3} + 18\frac{2}{3} + 18 + \dots
$$

44 μ **Sol.** The given series is arithmetic whose first term = 20, common 9 , then find a. difference $=-2/3$

phe 1 :

If S_n denotes the sum of n terms of a GP. whose first term is

and the common ratio r, then find the sum of
 $S_1 + S_3 + S_5 ++S_{2n-1}$

We have $S_n = \frac{a(1-r^n)}{1-r}$ \therefore $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

Putting 1, 2, 3 notes the sum of n terms of a G.P. whose first term is

e common ratio r, then find the sum of
 $+S_5 ++S_{2n-1}$
 $S_n = \frac{a(1-r^n)}{1-r}$ \therefore $S_{2n-1} = \frac{a}{1-r}[1-r^{2n-1}]$

1, 2, 3,........, n for n is it and summing up we
 $+S_5$ As the common difference is negative, the terms will become negative after some stage. So the sum is maximum if only positive terms are added. $\left[\text{min} \left\{ \text{ln} - \frac{r \left\{1 - (r^2)^n\right\}}{1 - r^2} \right\} \right] = \frac{a}{1 - r} \left[\text{ln} - r \cdot \frac{1 - r^{2n}}{1 - r^2} \right]$
 sple 2 :

Find the maximum sum of the series
 $20 + 19\frac{1}{3} + 18\frac{2}{3} + 18 + \dots$

The given series is arithmetic whose firs +1 +n term) - $(r+r^3 + r^5 + ... n \text{ term})$
 $\left[\frac{r(1-(r^2)^n)}{1-r^2}\right] = \frac{a}{1-r} \left[n - r \cdot \frac{1-r^{2n}}{1-r^2}\right]$

aximum sum of the series
 $+18\frac{2}{3} + 18 + ...$

series is arithmetic whose first term = 20, common
 $=-2/3$

mon difference i arithmetic whose first term = 20, common

fference is negative, the terms will become

ne stage. So the sum is maximum if only

added.

1) $(-2/3) \ge 0$ if $60 - 2(n - 1) \ge 0$ or $62 \ge 2n$

erms are non-negative

1
 $+(31-1)\left(-\$ sum of the series

18 +.....

arithmetic whose first term = 20, common

ference is negative, the terms will become

e stage. So the sum is maximum if only
 $)(-2/3) \ge 0$ if $60 - 2$ (n - 1) ≥ 0 or $62 \ge 2n$

rms are non-As the common difference is negative, the terms will become
eigative after some stage. So the sum is maximum if only
oositive terms are added.
Now $t_n = 20 + (n-1) (-2/3) \ge 0$ if $60 - 2 (n-1) \ge 0$ or $62 \ge 2n$
r31 $\ge n$
 \therefore T wen series is arithmetic whose first term = 20, common

noce = - 2/3

common difference is negative, the terms will become

ve after some stage. So the sum is maximum if only

ve terms are added.
 $1 = 20 + (n-1)(-2/3) \ge 0$ i ence = -2/3

common difference is negative, the terms will become

vive after some stage. So the sum is maximum if only

ve terms are added.
 $\frac{1}{2}$ =20+(n-1)(-2/3) ≥ 0 if 60 - 2 (n-1) ≥ 0 or 62 ≥ 2n
 \geq n

he first 3 s strategive that the term = 20, common
neries is arithmetic whose first term = 20, common
mmon difference is negative, the terms will become
after some stage. So the sum is maximum if only
terms are added.
20+(n-1)(-2/3) common difference is negative, the terms will become

e after some stage. So the sum is maximum if only
 $t = \tan x$ are added.
 $= 20 + (n-1)(-2/3) \ge 0$ if $60 - 2 (n-1) \ge 0$ or $62 \ge 2n$

n

or first 31 terms are non-negative

x

Now $t_n = 20 + (n-1)(-2/3) \ge 0$ if $60 - 2(n-1) \ge 0$ or $62 \ge 2n$ or $31 \ge n$

- \therefore The first 31 terms are non-negative
- \therefore Maximum sum

negative after some stage. So the sum is maximum if only
positive terms are added.
Now t_n = 20 + (n-1) (-2/3) ≥ 0 if 60 – 2 (n-1) ≥ 0 or 62 ≥ 2n
r 31 ≥ n
∴ The first 31 terms are non-negative
∴ Maximum sum

$$
S_{31} = \frac{31}{2} \left\{ 2 \times 20 + (31-1) \left(-\frac{2}{3} \right) \right\} = \frac{31}{2} \left\{ 40 - 20 \right\} = 310
$$

ple 3:
it is known that
$$
\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}
$$
 then find the value of

$$
\sum_{r=1}^{\infty} \frac{1}{r^2}
$$

Here
$$
\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty = \frac{\pi^2}{8}
$$

Let
$$
\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... \infty = x
$$

Then
$$
x = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... \infty
$$

$$
= \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty \right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ... \infty \right)
$$

Example 3 :

It is known that \sum 2 $2\frac{1}{8}$ then find the value of $\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$ then find the value of

$$
\sum_{r=1}^{\infty} \frac{1}{r^2}
$$

$$
\therefore \text{ Maximum sum}
$$
\n
$$
= S_{31} = \frac{31}{2} \left\{ 2 \times 20 + (31 - 1) \left(-\frac{2}{3} \right) \right\} = \frac{31}{2} \left\{ 40 - 20 \right\} = 310
$$
\n**Example 3:**\nIt is known that
$$
\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}
$$
 then find the value of

\n
$$
\sum_{r=1}^{\infty} \frac{1}{r^2}
$$
\n**Sol.** Here
$$
\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty = \frac{\pi^2}{8}
$$
\n
$$
\text{Let } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty = x
$$
\n
$$
\text{Then } x = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty
$$
\n
$$
\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \dots \infty \right) = \frac{1}{1^2} + \frac{1}{1^2} + \frac{1}{1^2} + \dots \infty
$$

Then
$$
x = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty
$$

Now t_n = 20 + (n-1) (-2/3) ≥ 0 if 60 – 2 (n-1) ≥ 0 or 62 ≥ 2n
\nor 31 ≥ n
\n
$$
\therefore
$$
 The first 31 terms are non-negative
\n \therefore Maximum sum
\n= S₃₁ = $\frac{31}{2}$ {2 × 20 + (31-1) $\left(-\frac{2}{3}\right)$ } = $\frac{31}{2}$ {40 – 20} = 310
\n**mple 3:**
\nIt is known that $\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$ then find the value of
\n $\sum_{r=1}^{\infty} \frac{1}{r^2}$
\nHere $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$ $\infty = \frac{\pi^2}{8}$
\nLet $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ $\infty = x$
\nThen $x = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ ∞
\n $= \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$ $\infty\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ...$ $\infty\right)$
\n $= \frac{\pi^2}{8} + \frac{1}{4} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ $\infty\right) = \frac{\pi^2}{8} + \frac{1}{4}x$

Example 4 :

**EXAMPLE A RANING
\n**CPV MATI**
\n**Prove and B**
\n**Example 7:**
\nLet a, b, c be positive integers su
\n
$$
\left(\sum_{k=1}^{n} \left(\sum_{m=1}^{k} m^{2}\right) = an^{4} + bn^{3} + cn^{2} + dn + e
$$
 then find the
\nvalue of a and b.
\n
$$
\sum_{k=1}^{n} \left(\sum_{m=1}^{k} m^{2}\right) = \sum_{k=1}^{n} \frac{k(k+1)(2k+1)}{6} = \frac{1}{6} \sum_{k=1}^{n} (2k^{3} + 3k^{2} + k)
$$
\n(A) 0
\n(C) 8
\n(D) a, ar, ar², 0 > 1 r is integer**

value of a and b.

Sol. n k n 2 k 1 m 1 k 1 6 1 ⁼ ² a = coefficient of n⁴ , b = coefficient of n³

Example 5 :

36.6. (a)
$$
\frac{1}{2} \left(\frac{1}{10-1}\right)^2
$$

\n $= \frac{1}{3} \cdot \left(\frac{n(n+1)^2}{2}\right)^2 + \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \frac{n(n+1)}{2}$
\n $= \csc{n\pi}(\arctan{6r}n^4 = \frac{1}{3} \cdot \frac{1}{4}, b = \csc{n\pi}(\arctan{6r}n^3 = \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{2} \arctan{6r}n^3 = \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{2} \arctan{6r}n^2 = \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \arctan{6r}n^3 = \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \arctan{6r}n^2 = 6$
\n66. (A) Sum = (3³ + 5³ + 7³ +........ (b) brackets is
\n(A) 4960
\n56. (A) Sum = (3³ + 5³ + 7³ +........ (c) 10 terms)
\n $= (2^3 + 3^3 + 4^3 + 5^3 +........ (a) 20 terms)$
\n $= (2^3 + 3^3 + 4^3 + 5^3 +........ (b) 10 terms)$
\n $= (1^3 + 2^3 + 3^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^2 terms)$
\n $= (1^3 + 2^3 + 3^3 + + 10^$

$$
=451 \times 11 - 1 = 4961 - 1 = 4960
$$

Example 6 :

If S_1 , S_2 and S_3 denote the sum of first n_1 , n_2 and n_3 terms respectively of an A.P., then find

$$
\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2).
$$

 $\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(2a + (n_1 - 1)d)$ $\frac{S_2}{n_1} = \frac{16.17.33}{24} + \frac{16.1733}{24}$

Sol. We have , $S_1 = \frac{n_1}{2}[2a + (n_2 - 1)d] \Rightarrow \frac{2S_2}{2} = 2a + (n_2 - 1)d$ $\frac{n_2}{2}(S_2 + 2(n-1))$ $\frac{S_1}{2}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac$

$$
S_2 = {n_2 \over 2} [2a + (n_2 - 1)d] \Rightarrow {2S_2 \over n_2} = 2a + (n_2 - 1)d
$$

$$
S_3 = \frac{n_3}{2} [2a + (n_3 - 1)d] \Rightarrow \frac{2S_3}{n_3} = 2a + (n_3 - 1)d
$$

nple 6:	\n $1 \text{ f } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms}$ \n	\n $\therefore S_n = \Sigma t_n = \frac{1}{4} \Sigma n^2 + \frac{1}{2} \Sigma n + \frac{1}{4} \Sigma 1$ \n
respectively of an A.P., then find	\n $\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2)$ \n	\n $S_16 = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446$ \n
\n We have, $S_1 = \frac{n_1}{2}[2a + (n_1 - 1)d] \Rightarrow \frac{2S_1}{n_2} = 2a + (n_2 - 1)d$ \n	\n Example 9:	
\n $S_2 = \frac{n_2}{2}[2a + (n_2 - 1)d] \Rightarrow \frac{2S_2}{n_2} = 2a + (n_2 - 1)d$ \n	\n Example 9:	
\n $S_3 = \frac{n_3}{2}[2a + (n_3 - 1)d] \Rightarrow \frac{2S_3}{n_3} = 2a + (n_3 - 1)d$ \n	\n (p + 3) th term of the given A.P. is a equal to –\n	
\n $\therefore \frac{2S_1}{n_1}(n_2 - n_3) + \frac{2S_2}{n_2}(n_3 - n_1) + \frac{2S_3}{n_3}(n_1 - n_2)$ \n	\n (c) Let the first term of A.P. is a th \n $= [2a + (n - 1)d] (n_2 - n_3) + [2a + (n_2 - 1)d] (n_3 - n_1)$ \n	\n (d) Using variable) are

Example 7 :

Let a, b, c be positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean

SALB
\n
$$
\int_{\text{Re1}}^{\text{MSE}} \int_{\text{Re2}}^{\text{NES}} \int_{\text{Re3}}^{\text{NES}} \int_{\text{Re4}}^{\text{NES}} \int_{\text{Re5}}^{\text{NES}} \int_{\text{Re6}}^{\text{NES}} \int_{\text{Re7}}^{\text{NES}} \int_{\text{Re8}}^{\text{NES}} \int_{\text{Re9}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re2}}^{\text{NES}} \int_{\text{Re3}}^{\text{NES}} \int_{\text{Re4}}^{\text{NES}} \int_{\text{Re5}}^{\text{NES}} \int_{\text{Re9}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re2}}^{\text{NES}} \int_{\text{Re3}}^{\text{NES}} \int_{\text{Re4}}^{\text{NES}} \int_{\text{Re5}}^{\text{NES}} \int_{\text{Re9}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re2}}^{\text{NES}} \int_{\text{Re3}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{NES}} \int_{\text{Re1}}^{\text{N
$$

So,
$$
\frac{a^2 + a - 14}{a + 1} = \frac{6^2 + 6 - 14}{6 + 1} = \frac{28}{7} = 4
$$

Example 8 :

Find the sum up to 16 terms of the series

$$
\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots
$$

Sol. We have $t_n = \frac{1+2+3+3+3+3+3}{1+2+5+3+3+3}$

$$
\left(\frac{1}{2}\right)^2 + \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \frac{n(n+1)}{2}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ brackets}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ brackets}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ brackets}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ brackets}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ brackets}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ 834860}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + (7^2 - 6^2) + = 10 \text{ 0 terms}
$$
\n
$$
\left(\frac{3}{2} - 4\right) + 3\left(\frac{3}{2} + 4\right) + 5\left(\frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + = 10 \text{ 0 terms}\right)
$$
\n
$$
\left(\frac{3
$$

$$
\begin{aligned}\n&= \left\{ \frac{21 \times (21+1)}{2} \right\}^2 - 1 - 16 \cdot \left\{ \frac{10(10+1)}{2} \right\}^2 & \frac{\left\{ n(n+1) \right\}^2}{2} \\
&= 231^2 - 220^2 - 1 = (231 + 220)(231 - 220) - 1 \\
&= 451 \times 11 - 1 = 4961 - 1 = 4960\n\end{aligned}
$$
\n
$$
\begin{aligned}\n&\text{mple 6: } \\
&\text{If } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms} \\
&\text{If } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms} \\
&\text{If } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms} \\
&\text{If } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms} \\
&\text{If } S_1, S_2 \text{ and } S_3 \text{ denote the sum of first } n_1, n_2 \text{ and } n_3 \text{ terms} \\
&\text{If } S_1 = \frac{n}{2} (2n + \frac{1}{2} \Sigma n + \frac{1}{4} \Sigma 1 \\
&= \frac{1}{4} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2} + \frac{1}{4} \frac{n(n+1)}{4} + \frac{1}{4} \frac{n(n+1)}{2} + \frac{1}{4} \frac{n(n+
$$

Example 9 :

r & d (d being variable) are pth term and common difference of an A.P. respectively. If the product of $(p (2)$ th &

 $(p + 3)$ th term of the given A.P. is maximum then r/d is equal to –

(A) 3 (B) 4
\n(C) 2 (D) 8
\n**Sol.** (C). Let the first term of A.P. is a then
\n
$$
a + (p-1) d = r
$$
(1)

$$
(p-2)th term = (r-2d)
$$

$$
(p-3)th term = (r+3d) then
$$

\n
$$
\Rightarrow (r-2d)(r+3d) \Rightarrow [r^2 + rd - 6d^2]
$$

QUENCES & SERIES	Example 12:
\n $\Rightarrow r^{2} \left[1 + \frac{d}{r} - 6 \left(\frac{d}{r} \right)^{2} \right]$ \n	\n Example 12:\n $If \sum_{r=1}^{n} I(r) = n (2n^{2} + 9n + 13), \text{ then find } \sum_{r=1}^{n} I(r) = n (2n^{2} + 9n + 13)$ \n
\n $\Rightarrow 6r^{2} \left[-\left(\frac{d}{r} \right)^{2} + \frac{1}{6} \frac{d}{r} + \frac{1}{6} \right] \Rightarrow r^{2} \left[\frac{37}{36} - 6 \left(\frac{d}{r} - \frac{1}{12} \right)^{2} \right]$ \n	\n $\sum_{r=1}^{n} \sqrt{I(r)}$ \n
\n $For max. \frac{d}{r} - \frac{1}{2} = 0 or \frac{d}{r} = 12\n$	\n $Sol = S = \sum_{r=1}^{n} I(r) = n (2n^{2} + 9n + 13)$ \n

Example 10 :

If a, b, c and d are positive real number, then

$$
\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}
$$
 belongs to the interval –
(A) [2, ∞)
(B) [3, ∞)
(C) [4, ∞)
(D) (-∞, 4)

Sol. (C). Apply $A.M. \ge G.M.$

$$
\frac{\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}}{4} \ge \sqrt[4]{\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{d} \cdot \frac{d}{a}} \ge 1
$$

$$
\therefore \quad \frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} \ge 4
$$

$$
\therefore \quad \frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} \in [4, \infty)
$$

Example 11 :

Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

$$
\frac{b+c+\frac{b+c}{d+\frac{b+c}{d+\frac{c}{d+\frac{d}{d+\frac{c}{d+\frac{d}{d+\frac{c}{d+\frac{d}{d+\frac{c}{d+\frac{d}{d+\frac{d}{d+\frac{c}{d+\frac{d}{d+\frac{
$$

Example 12 :

EXECUTE: **Q. S EXERCISE 3**
\n
$$
\Rightarrow t^2 \left[1 + \frac{d}{r} - 6\left(\frac{d}{r}\right)^2\right]
$$
\n
$$
\Rightarrow 6t^2 \left[-\left(\frac{d}{r}\right)^2 + \frac{1}{6} + \frac{1}{6}\right] \Rightarrow t^2 \left[\frac{37}{36} - 6\left(\frac{d}{r} - \frac{1}{12}\right)^2\right]
$$
\n
$$
\Rightarrow 6t^2 \left[-\left(\frac{d}{r}\right)^2 + \frac{1}{6} + \frac{1}{6}\right] \Rightarrow t^2 \left[\frac{37}{36} - 6\left(\frac{d}{r} - \frac{1}{12}\right)^2\right]
$$
\n
$$
\Rightarrow 6t = \sum_{r=1}^{n} 1(r) = n(2n^2 + 9n + 13), \text{ then find the sum}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = S_r - S_{r-1}
$$
\n
$$
\Rightarrow 1(r) = \sqrt{6}(r + 1)^2
$$
\n
$$
\Rightarrow \frac{1}{6}r^2 + 12r + 6 = 6(r + 1)^2
$$
\n
$$
\Rightarrow \frac{1}{6}r^2 + 12r + 6 = 6(r + 1)^2
$$
\n
$$
\Rightarrow \frac{1}{16}(r) = \sqrt{6}(r + 1)
$$
\n
$$
\Rightarrow \frac{1}{\sqrt{1}}\sqrt{1(r)} = \sqrt{6}(r + 1)
$$
\n
$$
\Rightarrow \frac{1}{\sqrt{1}}\sqrt{1(r)} = \sqrt{6}(r + 1)
$$
\n
$$
\Rightarrow \frac{1}{\sqrt{1}}\sqrt{1(r)} = \sqrt{6}(r + 1)
$$
\n
$$
\Rightarrow \frac{1}{\sqrt{1}}\sqrt{1(r)} = \sqrt{6}(r + 1)
$$
\n
$$
\Rightarrow \frac{1}{
$$

Example 13 :

If the roots of the equation $x^3 - 11x^2 + 36x - 36 = 0$ are in H.P. find the middle root.

Sol. $x^3 - 11x^2 + 36x - 36 = 0$

If roots are in H.P. then roots of new equation

$$
\frac{1}{a} \ge \sqrt[4]{\frac{a \cdot b \cdot c \cdot d}{c \cdot d}} \ge 1
$$
\n
$$
= \sqrt{6} \left(\frac{n^2 + 3n}{2} \right) = \sqrt{\frac{3}{2}} (n^2 + 3n)
$$
\nExample 13:
\n
$$
\frac{1}{a} \in [4, \infty)
$$
\nExample 13:
\nIf the roots of the equation $x^3 - 11x^2 + 36x - 36 =$
\nH.P. find the middle root.
\n**Sol.** $x^3 - 11x^2 + 36x - 36 = 0$
\nIf roots are in H.P. then roots of new equation
\nnumbers. If the ratio of the sum of the
\no the sum of the first eleven terms is 6
\nsoth term lies in between 130 and 140,
\ndifference of this A.P. is
\n
$$
\frac{7}{2}[2a + 6d]
$$
\n
$$
= \frac{7}{2}[2a + 10d] = \frac{6}{11}
$$
\n[Given]
\n
$$
\frac{1}{2} = \frac{1}{2} \left[\frac{1}{2}a + 10d \right] = \frac{6}{11}
$$
\n
$$
\frac{11}{2} \left[2a + 10d \right]
$$
\n
$$
= \frac{6}{11}
$$
\n
$$
\frac{1}{11}
$$
\n
$$
\frac{1}{1
$$

So middle root is 3.

SEQUENCES & SERIES QUESTION BANK

Q.43 Let a_n be the nth term of the G.P. of positive numbers. Let **Q** 100 **OUESTION BANK**

Let a_n be the nth term of the G.P. of positive numbers. Let **Q.52** L
 $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the the common ratio is –

(A) α/β (B) β/α (D 100

 $_{2n}$ = α and $\sum a_{2n-1}$ – β , such the

common ratio is –

 $(A) \alpha / \beta$ (B) β / α

(C)
$$
\sqrt{\frac{\alpha}{\beta}}
$$
 (D) $\sqrt{\frac{\beta}{\alpha}}$

- **Q.44** The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second terms and 1 is subtracted from the third, the resulting new terms are in arithmetic progression. Then the lowest of the original term is
	- $(A) 1$ (B) 2 (C) 4 (D) 8
- **Q.45** Let a and b be roots of $x^2 3x + p = 0$ and let c and d be the Q.55 roots of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing G.P. Then the ratio of $(q + p)$: $(q - p)$ is equal to $(A) 8 : 7$ (B) 11 : 10 (C) 17:15 (D) None of these ca, ab, bc; ca respectively where a, b, care in A.P.

(A) Note (C) $\sqrt{\frac{\alpha}{\beta}}$ (B) $\frac{\sqrt{\alpha}}{\alpha}$ (D) $\sqrt{\frac{\beta}{\alpha}}$ then maximum value of $\left(1 + \frac{1}{x}\right)$.

The sum of three consecutive terms in a geometric (A) 8

respectiv n=1

(a) $\sqrt{\frac{\alpha}{\beta}}$

(b) β/α

(c) $\sqrt{\frac{\alpha}{\beta}}$

(c) $\$ inal term is

(B) 2

(D) 8

be roots of $x^2 - 3x + p = 0$ and let c and d be the
 $-12x + q = 0$, where a, b, c, d form an increasing

then the sum

then the (A) a = 2, r = 3/8

(B) 2 (A) a = 2, r = 3/8

et a and b b roots of $x^2 - 3x + p = 0$ and let c and b e the $Q.55$ If the 2nd and 5^{11}

costs of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing then the sum of

i.P.
- **Q.46** If α , β , γ are the geometric means between

 (A) A.P. (B) H.P. (C) GP. (D) None of the above **Q.47** Two sequences $\{t_n\}$ and $\{s_n\}$ are defined by

$$
t_n = \log\left(\frac{5^{n+1}}{3^{n-1}}\right), s_n = \left[\log\left(\frac{5}{3}\right)\right]^n
$$
, then

- (A) $\{t_n\}$ is an A.P., $\{s_n\}$ is a G.P.
- (B) $\{t_n\}$ and $\{s_n\}$ are both G.P.
- (C) $\{t_n\}$ and $\{s_n\}$ are both A.P.
- (D) $\{s_n\}$ is a G.P., $\{t_n\}$ is neither A.P. nor G.P
- **Q.48** If the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then

(C)
$$
a^3b = c^3d
$$
 (D) $ab^3 = cc$

Q.49 If d, e, f are in G.P. and two quadratic equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root

Q.50 If x_1 , x_2 , x_3 as well as y_1 , y_2 , y_3 are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$ (A) Lie on a straight line (B) Lie on an ellipse (C) Lie on a circle (D)Are vertices of a triangle **Q.51** If $(a^2 + b^2 + c^2) p^2 - 2(ab + bc + cd) p + (b^2 + c^2 + d^2) \le 0$,

EXECUTE ARNING

Let a_n be the nth term of the G.P. of positive numbers. Let **Q.52** Let x_1, x_2, \dots, x_{10} be
 $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the then

(A) S
is dependen **QUESTION BANK** STUDY

of the G.P. of positive numbers. Let Q.52 Let x_1, x_2, \dots, x_{10} b
 $x_1 + x_2 + \dots + x_{10} = 12$ is
 $x_1 + x_2 + \dots + x_{10} = 12$ is
 $(x_1, x_2, \dots, x_{10}) = 12$ is
 $x_1 + x_2 + \dots + x_{10} = 12$ is

then (A) S ≤ 36

(**QUESTION BANK**

serm of the G.P. of positive numbers. Let **Q.52** Let $x_1, x_2,$..., ..., ..., $x_1 + x_2 + ... + x_{10}$
 $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the then

(A) S \leq 36

(B) β/α
 Q.53 If x, y, **QUESTION BANK**

sTUDY MATERIAL: MATHEMATICS

erm of the G.P. of positive numbers. Let Q.52 Let x_1, x_2, \dots, x_{10} be non-negative real nos. such that
 $x_1 + x_2 + \dots + x_{10} = 12$ and let $S = x_1x_2 + x_3x_4 + \dots + x_{9}x_{10}$
 $\sum_{n=$ **Q.52** Let x_1, x_2, \dots, x_{10} be non-negative real nos. such that $x_1 + x_2 + \dots + x_{10} = 12$ and let $S = x_1x_2 + x_3x_4 + \dots + x_9x_{10}$ then $(A) S \le 36$ (B) S > 144 $(C) S < 18$ (D) None of these

Q.53 If x, y, z are positive real numbers satisfying $x + y + z = 1$,

(D) $\sqrt{\frac{\beta}{\alpha}}$ then maximum value then maximum value of $\left(1 + -\right)$ **MATERIAL: MATHEMATICS**

non-negative real nos. such that

l let $S = x_1x_2 + x_3x_4 + ...+x_9x_{10}$

(B) $S > 144$

(D) None of these

umbers satisfying $x + y + z = 1$,
 $1 + \frac{1}{x}$, $\left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is –

(B) 16

(D **ERIAL: MATHEMATICS**

negative real nos. such that

S = $x_1x_2 + x_3x_4 + ... + x_9x_{10}$

(B) S > 144

(D) None of these

bers satisfying $x + y + z = 1$,
 $\frac{1}{x}$, $\left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is –

(B) 16

(D) None of these
 MATERIAL: MATHEMATICS
non-negative real nos. such that
d let $S = x_1x_2 + x_3x_4 ++x_9x_{10}$
(B) $S > 144$
(D) None of these
numbers satisfying $x + y + z = 1$,
 $\left(1 + \frac{1}{x}\right) \cdot \left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is –
(B) 16
(D) No **MATERIAL: MATHEMATICS**

non-negative real nos. such that

d let $S = x_1x_2 + x_3x_4 + ... + x_9x_{10}$

(B) $S > 144$

(D) None of these

numbers satisfying $x + y + z = 1$,
 $\left(1 + \frac{1}{x}\right) \cdot \left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is –

(B) 16 $(A) 8$ (B) 16 (C) 64 (D) None of these **Q.54** Consider an infinite geometric series with first term 'a' en maximum value of $\left(1+\frac{1}{x}\right)$. $\left(1+\frac{1}{y}\right)$. $\left(1+\frac{1}{z}\right)$ is $-$

1) 8 (B) 16

1) 96 (B) 10

1) 96 (B) 10

consider an infinite geometric series with first term 'a'

and common ratio 'r'. If the sum is 4 and t A) \ge s 350

(D) None of these

f x, y, z are positive real numbers satisfying $x + y + z = 1$,

hen maximum value of $\left(1 + \frac{1}{x}\right) \cdot \left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is -

A) 8 (B) 16

C)64 (D) None of these

Consider an in x, y, z are positive real numbers satisfying $x + y + z = 1$,

en maximum value of $\left(1 + \frac{1}{x}\right) \cdot \left(1 + \frac{1}{y}\right) \cdot \left(1 + \frac{1}{z}\right)$ is $-$

18 (B) 16 (D) None of these

msider an infinite geometric series with first term 'a'

and common ratio 'r'. If the sum is 4 and the second term is $3/4$, then –

(A)
$$
a = 2
$$
, $r = 3/8$
\n(B) $a = 4/7$, $r = 3/7$
\n(C) $a = 3/2$, $r = 1/2$
\n(D) $a = 3$, $r = 1/4$
\nQ.55 If the 2nd and 5th terms of G. P. are 24 and 3 respectively,

then the sum of $1st$ six terms is – (A) 189/2 (B) 189/5 (C) 179/2 (D) 2/189

PART 3 : HARMONIC PROGRESSION

Q.56 $\frac{1}{a^n + b^n}$ is AM/GM/HM, between a and b if n is

equal to respectively-

Q.58 If there are n harmonic means between 1 and $\frac{1}{31}$ and the

ratio of $7th$ and $(n-1)th$ harmonic means is 9 : 5 then the value of n will be

- in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then a_4h_7 is $(A) 2$ (B) 3 $(C) 5$ (D) 6 (A) na₁ a_n (B) (n-1) a₁ a_n

(C) (n+1) a₁ a_n (D) None of these

If there are n harmonic means between 1 and $\frac{1}{31}$ and the

ratio of 7th and (n - 1)th harmonic means is 9 : 5 then the

value of n will b
- **Q.60** Let a_1 , a_2 , a_3 be any positive real numbers, then which of the following statement is not true **–**

(c) 0, -1/2, -1 (D) Note of these
\nIf a₁, a₂, a₃,..., a_n are in HP, thena₁a₂ + a₂a₃ +....+ a_{n-1}
\na_n is equal to-
\n(A) na₁ a_n (B) (n-1) a₁a_n
\n(C) (n+1) a₁a_n (D) None of these
\nIf there are n harmonic means between 1 and
$$
\frac{1}{31}
$$
 and the
\nratio of 7th and (n-1)th harmonic means is 9 : 5 then the
\nvalue of n will be
\n(A) 12 (B) 13
\n(C) 14 (D) 15
\nLet a₁, a₂,......a₁₀ be in A.P. and h₁, h₂,......h₁₀ be
\nin H.P. If a₁ = h₁ = 2 and a₁₀ = h₁₀ = 3, then a₄h₇ is
\n(A) 2 (B) 3
\n(C) 5 (D) 6
\nLet a₁, a₂, a₃ be any positive real numbers, then which of
\nthe following statement is not true –
\n(A) 3a₁a₂a₃ \le a₁³ + a₂³ + a₃³
\n(B) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \ge 3$
\n(C) (a₁ + a₂ + a₃) $(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3})^3 \le 27$
\n(D) (a₁ + a₂ + a₃) $(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3})^3 \le 27$

Q.61 If pth term of a HP be q and q^{th} term be p, then its $\overline{(p+q)^{th}}$ term is-

JENCES & SERIES	QUESTION BANK	
\n $\text{If } p^{\text{th}}$ term of a HP be q and q^{\text{th}} term be p, then its $(p+q)^{\text{th}}$ \n $(A) \frac{1}{p+q}$ \n	\n $(B) \frac{1}{p} + \frac{1}{q}$ \n	\n $(A) \frac{n(n+1)}{2} \log_{3} a$ \n
\n $(C) \frac{pq}{p+q}$ \n	\n $(D) p+q$ \n	\n $(A) \frac{n(n+1)}{2} \log_{3} a$ \n
\n $(A) a = b + c$ \n	\n $(B) b = c + a$ \n	\n $(C) \frac{(n+1)^{2}n^{2}}{4} \log_{3} a$ \n
\n $(A) a = b + c$ \n	\n $(B) b = c + a$ \n	\n $(C) \frac{(n+1)^{2}n^{2}}{4} \log_{3} a$ \n
\n $(A) a = b + c$ \n	\n $(B) b = c + a$ \n	\n $(C) a = b + c$ \n
\n $(C) c = a + b$ \n	\n $(D) a = b = c$ \n	

(C)
$$
\frac{pq}{p+q}
$$
 (D) $p+q$

- CES & SERIES
 $\frac{1}{p+q}$ (DUESTION BANK)
 $\frac{1}{p+q}$ (B) $\frac{1}{p} + \frac{1}{q}$
 $\frac{pq}{p+q}$ (B) $\frac{1}{p+q}$ (A) $\frac{n(n+1)}{2} \log_3 a$
 $\frac{pq}{p+q}$ (D) $p+q$
 $\log_3 a$ **Q.62** If a,b,c are in A.P. and a^2 , b^2 , c^2 are in H.P., then-(A) $a = b + c$ (B) $b = c + a$ (C) $c = a + b$ (D) $a = b = c$
- **Q.63** Five numbers a,b,c,d,e are such that a,b,c, are in AP' b,c,d are in GP and c,d,e, are in HP. If $a = 2$, $e = 18$; then values of b,c, d are - $(A) 2, 6, 18$ (B) 4, 6, 9
	- (C) 4, 6, 8 (D) 2, 6, 18
- **Q.64** a,b,c are first three terms of a GP. If HM of a and b is 12 and that of b and c is 36, then a equals- $(A) 24$ (B) 8
	- $(C) 72$ $(D) 1/3$
- **Q.65** If x, 1, z are in A.P. x, 2, z are in G.P. then x, 4, z are in- $(A)AP$ (B) GP (C) HP (D) None of these
- **Q.66** If a,b,c in H.P. then value of $\left(\frac{1}{b} + \frac{1}{c} \frac{1}{a}\right) \left(\frac{1}{c} + \frac{1}{a} \frac{1}{b}\right) =$

(A)
$$
\frac{2}{bc} - \frac{1}{b^2}
$$
 (B) $\frac{3}{b^2} - \frac{2}{ab}$ (C) $\frac{3}{ac} - \frac{2}{b^2}$ (D) Both (A) and (B)

- **Q.67** If a,b,c are in H.P. then $\frac{1}{1}$, $\frac{1}{1}$, $\frac{1}{1}$ will be in- (A) A.P. (B) GP. (C) H.P. (D) None of these
- **Q.68** If the $(m+1)$ th, $(n+1)$ th, $(r+1)$ th terms of an A.P. are in G.P. and m,n,r are in H.P. then the ratio of common difference to the first terms in the A.P. is- (A) $n/2$ (B) $2/n$ $(C) - n/2$ (D) – 2/n P. then value of $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) =$

(A) $\frac{2^n}{2^n}$ (B) $1-\frac{1 \cdot 3 \cdot 5 \cdot \cdot (2n-1)}{2n^{n-1}n-1}$ (D) $\frac{1 \cdot 3 \cdot 5}{2n^n}$

(B) $\frac{3}{b^2} - \frac{2}{ab}$

(D) Both (A) and (B) is $\frac{3$ P. then value of $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) =$

(C) $1 - \frac{1.3.5...(2n-1)}{2n^{n-1}n-1}$ (D) $\frac{1.3.5}{2n^n}$

(B) $\frac{3}{b^2} - \frac{2}{ab}$

(D) Both (A) and (B)
 $\frac{3}{16}$, where $|x| < 1$, find 'x'.

(D) B
- **Q.69** If a,x, y,z,b are in AP, then $x+y+z=15$ and if a,x, y,z, b are
- in HP, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$. Numbers a,b are - $(A) 8, 2$ (B) 11, 3 (C) 9, 1 (D) None of these **Q.70** If H and G are harmonic and geometric mean of positive
- real nos. a & b such that $H : G = 4 : 5$ then a : b is $(A) 5 : 4$ (B) 1 : 4 $(C) 1:5$ (D) None of these

PART 4 : MISCELLANEOUS

Q.71 The sum of all numbers between 100 and 10,000 which are of the form n^3 ($n \in N$) is equal to -(A) 55216 (B) 53261 (C) 51261 (D) None of these

OUESTION BANK

mbe p, then its $(p+q)^{th}$
 $Q.72 \sum_{r=1}^{n} \frac{1}{\log_{3}r}$ a is equal to –
 $(A) \frac{n(n+1)}{2} \log_{3} a$ (B) $\frac{n(n+1)}{2} \log_{a} 3$
 $p+q$

re in H.P., then.
 $Q = 0.78$ F.P. 2002 F.P. **QUESTION BANK**

mbe p, then its $(p+q)^{th}$
 $\frac{1}{p} + \frac{1}{q}$
 $(1 + \frac{1}{p})^{th}$
 $(2.72 + \sum_{r=1}^{n} \frac{1}{\log_{3}r}a)$ is equal to –
 $(2.73 + \sum_{r=1}^{n} \frac{1}{\log_{3}r}a)$
 $(3.74 - \sum_{r=1}^{n} \frac{1}{\log_{3}r}a)$
 $(4.74 - \sum_{r=1}^{n} \frac{1}{\log_{$ $+\frac{1}{2}$ **Q.72** $\sum_{r=1}^{n} \frac{1}{\log_{3} r}$ is equal to – B

B
 $\frac{\sum_{r=1}^{n} \frac{1}{\log_{3} r} a}$ is equal to –

(A) $\frac{n (n+1)}{2} \log_3 a$ (B) $\frac{n (n+1)}{2} \log_a 3$
 $(n+1)^2 n^2$ $\frac{1}{\log_{3} \pi a}$ is equal to -
 $\frac{n (n+1)}{2} \log_3 a$ (B) $\frac{n (n+1)}{2} \log_a 3$
 $\frac{(n+1)^2 n^2}{4} \log_3 a$ (D) None of these **SPONTAD VANSED LEARNING**
 $\frac{1}{\log_{3} a}$ is equal to –
 $\frac{n (n+1)}{2} \log_{3} a$ (B) $\frac{n (n+1)}{2} \log_{a} 3$

(D) None of these

sum to n terms of the series

1 **EXECUTE ARNING**

SOM ADVANCED LEARNING

is equal to –
 $\frac{11}{\log_3 a}$ (B) $\frac{n (n+1)}{2} \log_a 3$
 $\frac{2n^2}{2} \log_3 a$ (D) None of these

on terms of the series
 $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

(A)
$$
\frac{n (n+1)}{2} \log_3 a
$$
 \t\t (B) $\frac{n (n+1)}{2} \log_a 3$

(C)
$$
\frac{(n+1)^2 n^2}{4}
$$
 log₃ a (D) None of these

Q.73 The sum to n terms of the series

A
\n
$$
\sum_{r=1}^{n} \frac{1}{\log_{3} r} \text{ a is equal to } -
$$
\n(A) $\frac{n (n+1)}{2} \log_{3} a$ (B) $\frac{n (n+1)}{2} \log_{a} 3$
\n(C) $\frac{(n+1)^{2} n^{2}}{4} \log_{3} a$ (D) None of these
\nThe sum to n terms of the series
\n $\frac{1}{1-\frac{1}{4}} + \frac{1}{(1+3)-\frac{1}{4}} + \frac{1}{(1+3+5)-\frac{1}{4}} + \dots$ is

A
\n
$$
\sum_{r=1}^{n} \frac{1}{\log_{3} r} a
$$
 is equal to –
\n(A) $\frac{n (n+1)}{2} \log_{3} a$ (B) $\frac{n (n+1)}{2} \log_{a} 3$
\n(C) $\frac{(n+1)^{2} n^{2}}{4} \log_{3} a$ (D) None of these
\nThe sum to n terms of the series
\n
$$
\frac{1}{1-\frac{1}{4}} + \frac{1}{(1+3)-\frac{1}{4}} + \frac{1}{(1+3+5)-\frac{1}{4}} + \dots
$$
 is
\n(A) $\frac{2n}{2n+1}$ (B) $\frac{4n}{2n+1}$ (C) $\frac{2}{2n+1}$ (D) $\frac{4n}{2n-1}$
\nThe sum to n terms of the series
\n
$$
\frac{1}{1-\frac{1}{2} + \frac{1}{2n+1}} + \frac{1}{2n+1} + \dots
$$

Q.74 The sum to n terms of the series

1 1 1 b c a 1 1 1 c a b ⁼ 2 1 3 2 3 2 a b c , , b c c a a b . Numbers a,b are - 2n 1 2n 1 2n 1 1 2 + 1 2 ² 1 2 + 1.3 3 ⁴ 1 2 + is (A) ⁿ 1.3.5...(2n 1) 2 n (B) 1– ⁿ 1.3.5...(2n 1) 2 n (C) 1– n 1 1.3.5...(2n 1) 2n n 1 (D) n 1 1.3.5...(2n 1) 2n n 1 1 2 3 S 1.2.3 3.4.5 5.6.7 , then –

 $\frac{b^2}{b^2} - \frac{2}{ab}$ **Q.75** If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 +$

........ is
$$
\frac{35}{16}
$$
, where |x| < 1, find 'x'.
(A) 2/5 (B) 1/5
(C) 1/2 (D) 1/4

- **Q.76** If $S = \frac{1}{1.2.3} + \frac{2}{3.4.5} + \frac{3}{5.6.7} + \dots + \infty$, then -(A) $S = 1/4$ (B) $S = 1/2$
- $(C) S = 2/3$ (D) $S = 1$ **Q.77** $2^{1/4}$. $2^{2/8}$. $2^{3/16}$. $2^{4/32}$ ∞ is equal to- $(A) 1$ (B) 2 (C) 3/2 (D) 5/2
- **Q.78** Sum of n terms of the series $8 + 88 + 888 + \dots$ equals

(A)
$$
\frac{8}{81}
$$
 [10ⁿ⁺¹ - 9n - 10] (B) $\frac{8}{81}$ [10ⁿ - 9n - 10]

(C)
$$
\frac{8}{81}
$$
 [10ⁿ⁺¹ – 9n + 10] (D) None of these

2.
\n3.
$$
\frac{35}{16}
$$
, where $|x| < 1$, find 'x'.
\n(A) 2/5 (B) 1/5
\n(C) 1/2 (D) 1/4
\nQ.76 If $S = \frac{1}{1.2.3} + \frac{2}{3.4.5} + \frac{3}{5.6.7} + \dots + \infty$, then –
\n(A) $S = 1/4$ (B) $S = 1/2$
\n(C) $S = 2/3$ (D) $S = 1$
\nQ.77 $2^{1/4}$, $2^{2/8}$, $2^{3/16}$, $2^{4/32}$, ..., ∞ is equal to-
\n(A) 1 (B) 2
\n(C) 3/2 (D) 5/2
\nQ.78 Sum of n terms of the series
\n8 + 88 + 888 + equals
\n(A) $\frac{8}{81}$ [10ⁿ⁺¹ – 9n – 10] (B) $\frac{8}{81}$ [10ⁿ – 9n – 10]
\n(C) $\frac{8}{81}$ [10ⁿ⁺¹ – 9n + 10] (D) None of these
\nQ.79 For all positive integral values of n, the value of
\n3.1.2 + 3.2.3 + 3.3.4 + + 3.n.(n + 1) is
\n(A) n (n + 1) (n + 2) (B) n (n + 1) (2n + 1)
\n(C) (n – 1) n (n + 1) (D) $\frac{(n-1) n (n + 1)}{2}$

- **Q.80** The sum of the series $\frac{1}{2 \times 7} + \frac{1}{2 \times 11} + \frac{1}{11 \times 25} + \dots$ is Q $(A) 1/3$ (B) 1/6 **Q.80** The sum of the series $\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 25} + ...$ is **Q.88** If $3 + \frac{1}{4}(3 + 4) + \frac{1}{4^2}(3 + 24) + ...$ to $\infty = 8$, then the value (A) 1/3 (B) 1/6 of dis-
 Q.81 The 9th term of the series $27 + 9 + 5\frac{$
	- (C) 1/9 (D) 1/12

(A)
$$
1\frac{10}{17}
$$
 \t(B) $\frac{10}{17}$ \t(C) $\frac{16}{27}$ \t(D) $\frac{17}{27}$

Q.82 A series whose nth term is $\left(\frac{n}{x}\right)$ + y, the sum of r terms will

COMRE 5 LVALUATE		STUDY MA		
The sum of the series $\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 25} + \dots$ is	Q.88 If $3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d)$			
(A) 1/3	(B) 1/6	(C) 1/9		
The 9 th term of the series $27 + 9 + 5\frac{2}{5} + 3\frac{6}{7} + \dots$ will be	Q.89 The sum of infinite series $S = +(1 + a + a^2 + a^3)x^3 \dots \dots$			
(A) $1\frac{10}{17}$	(B) $\frac{10}{17}$	(C) $\frac{16}{27}$	(D) $\frac{17}{27}$	(A) $\frac{1}{(1-x)(1-a)}$
A series whose n th term is $(\frac{n}{x}) + y$, the sum of r terms will	(C) $\frac{1}{(1-x)(1-ax)}$			
(A) $\left\{\frac{r(r+1)}{2x}\right\} + ry$	(B) $\left\{\frac{r(r-1)}{2x}\right\}$	Q.90 $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} (1) =$		
(C) $\left\{\frac{r(r-1)}{2x}\right\} - ry$	(D) $\left\{\frac{r(r+1)}{2y}\right\} - rx$	Q.90 $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} (1) =$		
The sum of the first five terms of the series	(A) $\frac{n(n+1)(2n+1)}{6$			

Q.83 The sum of the first five terms of the series

(A) 1/3 (B) 1/6 (C) 1/9 (D) 1/12 (A) 9 (B
\nThe 9th term of the series 27 + 9 + 5
$$
\frac{2}{5}
$$
 + 3 $\frac{4}{7}$ + will be
\n(A) 1 $\frac{10}{17}$ (B) $\frac{10}{17}$ (C) $\frac{16}{27}$ (D) $\frac{17}{27}$ (A) $\frac{1}{(1-x)(1-a)}$ (B)
\nA series whose nth term is $(\frac{a}{x})$ + y, the sum of r terms will
\n(A) $\left{\frac{r(r+1)}{2x}\right}$ + ry (B) $\left{\frac{r(r-1)}{2x}\right}$
\n(C) $\left{\frac{r(r+1)}{2x}\right}$ - ry (D) $\left{\frac{r(r+1)}{2y}\right}$ - rx (E) $\left{\frac{n(n+1)(2n+1)}{6}\right}$ (E)
\n3 + 4 $\frac{1}{2}$ + 6 $\frac{3}{4}$ + will be
\n(A) 39 $\frac{9}{16}$ (B) 18 $\frac{3}{16}$ (C) 39 $\frac{7}{16}$ (D) 13 $\frac{9}{16}$ (E) 11 $\frac{1}{12}$ (E) 13 $\frac{1}{12}$ (E) 14 $\frac{10^{n-9n-10}}{81}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{3}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{3}$ (E) $\frac{10^{n+1}-9n-10}{9}$ (E) $\frac{10^{n+1}-9n-10}{3}$ (E) $\frac{10^{n+1}-9n-10}{3}$

Q.84 Value of $9 + 99 + 999 + \dots$ upto n terms is –

(A)
$$
\frac{10^{n} - 9n - 10}{81}
$$

\n(B)
$$
\frac{10^{n+1} - 9n - 10}{9}
$$

\n(C)
$$
\frac{10^{n+1} - 9n - 10}{81}
$$

\n(D)
$$
\frac{10^{n} - 9n - 10}{9}
$$

Q.85 The sum of the seriesa – $(a+d) + (a+2d) - (a+3d) + ...$ **Q.92** The sum of the series, upto $(2n + 1)$ terms is-

 $(A) - nd$ (B) $a + 2 nd$ (C) a + nd (D) 2nd

Q.86 The sum to n terms of the series

3+4
$$
\frac{1}{2}
$$
+6 $\frac{3}{4}$ +...... will be
\n(A) 39 $\frac{9}{16}$ (B) 18 $\frac{3}{16}$ (C) 39 $\frac{7}{16}$ (D) 13 $\frac{9}{16}$ (D) 13 $\frac{9}{16}$ (E) 142
\nQ.84 Value of 9 + 99 + 999 +
\n(A) $\frac{10^{n}-9n-10}{81}$ (B) $\frac{10^{n+1}-9n-10}{9}$ (C) $\frac{10^{n+1}-9n-10}{9}$ (D) $\frac{10^{n}-9n-10}{9}$ (E) $\frac{10^{n-1}-9n-10}{9}$ (E) $\frac{10^{n-1}-9n-10}{9}$ (E) $\frac{10^{n-1}-9n-10}{9}$ (E) $\frac{10^{n-1}-9n-10}{9}$ (E) $\frac{10^{n-1}-9n-10}{3}$ (E) $\frac{10^{n-1}-9n-10}{3}$ (E) $\frac{10^{n-1}-10}{3}$ (E) $\frac{10^{n-1}-10}{3}$ (E) $\frac{10^{n-1}-10}{3}$ (E) $\frac{10^{n-1}-10}{3}$
\nE) $\frac{10^{n-1}+10}{3}$ (E) $\frac{10^{n-1}-10}{3}$ (E) $\frac{10$

QUESTION BANK	STUDY MATERIAL: MATHEMATICS		
$\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 25} + \dots$ is	Q.88 If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + \dots$ to $\infty = 8$, then the value of d is-		
(B) 1/6	(A) 9	(B) 5	
(C) 1	(D) None of these		
$28 \times 27 + 9 + 5\frac{2}{5} + 3\frac{6}{7} + \dots$ will be	Q.89 The sum of infinite series $S = 1 + (1 + a) \times + (1 + a + a^2) \times 2 + (1 + a + a^2 + a^3) \times 3 + \dots$ so (where $0 < a; x < 1$) is		
(C) $\frac{16}{27}$	(D) $\frac{17}{27}$	(A) $\frac{1}{(1 - x)(1 - a)}$	(B) $\frac{1}{(1 - a)(1 - ax)}$
$118 \left(\frac{n}{x}\right) + y$, the sum of r terms will	(C) $\frac{1}{(1 - x)(1 - ax)}$	(D) $\frac{1}{(1 - x)(1 + a)}$	
(B) $\left\{\frac{r(r-1)}{2x}\right\}$	Q.90 $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} (1) =$		
(A) $\frac{n(n+1)(2n+1)}{6}$	(B) $\frac{n(n+1)(2n-1)}{6}$		
$118 \left(\frac{n(n+1)(2n-1)}{6}\right)$	Q.90		

(A) 9 (B) 5 (C) 1 (D) None of these

Q.89 The sum of infinite series $S = 1 + (1 + a)x + (1 + a + a^2)x^2$ $+(1+a+a^2+a^3)x^3$ ∞ (where $0 \le a$; $x \le 1$) is –

QUESTION BANK	STUDY MATERIAL: MATHEMATICS				
$7^+ \frac{1}{7 \times 11} + \frac{1}{11 \times 25} + \dots$ is	Q.88 If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + \dots$ to $\infty = 8$, then the value of d is-				
(B) 1/6	(A) 9	(B) 5			
$7 + 9 + 5\frac{2}{5} + 3\frac{6}{7} + \dots$ will be	Q.89 The sum of infinite series $S = 1 + (1 + a)x + (1 + a + a^2)x^2 + (1 + a + a^2 + a^3)x^3 - \dots$ so (where $0 < a; x < 1$) is				
(C) $\frac{16}{27}$	(D) $\frac{17}{27}$	(A) $\frac{1}{(1 - x)(1 - a)}$	(B) $\frac{1}{(1 - a)(1 - ax)}$		
(C) $\frac{1}{(x + a)(1 - a)(1 - a)}$	(D) $\frac{1}{(1 - a)(1 - a)}$				
(D) $\left\{\frac{r(r+1)}{2x}\right\}$	Q.90 $\sum_{i=1}^{n} \sum_{j=1}^{i} (1) =$				
(D) $\left\{\frac{r(r+1)}{2y}\right\} - rx$	Q.90 $\sum_{i=1}^{n} \sum_{j=1}^{i} k = 1$	Q.11 $\frac{n(n+1)(2n+1)}{6}$	Q.22 $\frac{n(n+1)(n+2)}{6}$	Q.33 $\frac{7}{16}$	Q.34 The sum of the first n terms of the series

$$
\left\{\frac{r(r+1)}{2y}\right\} - rx
$$
 Q.90
$$
\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} (1) =
$$

$$
\frac{n(n+1)(2n+1)}{6}
$$
 (B)
$$
\frac{n(n+1)(2n-1)}{6}
$$

$$
\text{(C)} \frac{n(n+1)(n+2)}{6} \qquad \qquad \text{(D) None of these}
$$

 $\frac{39}{16}$ (D) $\frac{13}{16}$ **Q.91** The sum of the first n terms of

e
\n
$$
\begin{vmatrix}\n1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0\n\end{vmatrix} = \frac{1}{27} \qquad \frac
$$

to n terms is – 3
 $2+\frac{2}{3\cdot4}\cdot2^2+\frac{3}{4\cdot5}\cdot2^3+....$

(B) $\frac{2^{n+1}}{n+2}-1$

(D) $\frac{2^{n+1}}{n+2}-2$

series

....

(B) $\frac{n (n-2)}{6}$

(D) $\frac{n (n-2)}{3}$

(A) 1 (B) 1 (C) 2 (D) 2 n 2

Q.93 The sum of 1st n terms of the series

$$
\frac{1^2}{1} + \frac{1^2 + 2^2}{1 + 2} + \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3} + \dots
$$

$$
\dots \quad n (n+2) \qquad \dots \quad n (n-2)
$$

(A)
$$
\frac{n(n+2)}{3}
$$
 (B) $\frac{n(n-2)}{6}$

(C)
$$
\frac{n+2}{3}
$$
 (D) $\frac{n(n-2)}{3}$

SEQUENCES & SERIES QUESTION BANK

Q.94 If $\log 2$, $\log (2^x - 1)$ and $\log (2^x + 3)$ are in A.P. then find Q.98 the value of x.

Q.95 If x, y, z are in A.P. and x, y, t are in G.P. then x, $x - y$, $t - z$ are in

> (A) G.P. (B) A.P. (C) H.P. (D) A.P. and G.P. both

Q.96 The geometric and harmonic means of two numbers x_1

and x_2 are 18 and 16 $\frac{8}{13}$ respectively. The value of

 $|x_1 - x_2|$ is

- $(A) 5$ (B) 10 $(C) 15$ (D) 20
- **Q.97** If $a_1, a_2, \dots, a_{2n+1}$ are in G.P., then

JENCESS & SERIES	QUESTION BANK	EXAMPLES	
If log 2, log (2 ^x - 1) and log (2 ^x + 3) are in A.P. then find Q.98 The sum of the latter half of the first 1000 ter the value of x.	(A) log ₂ 5	(B) log ₂ 3	(C) log ₂ 6
(A) log ₂ 5	(D) log ₂ 6	(A) 1500	
(A) G.P. (B) A.P. (C) H.P. (D) A.P. and G.P. both the geometric and harmonic means of two numbers x ₁	(A) $\frac{pq}{2(p+q)}$	(B) $\frac{2pq}{p+q}$	
(A) 5 (D) 1000	(D) 11000		
(B) 6 (D) 11000	(E) 11000		
(C) 15 (D) 15 (D) 20 (E) 15 (D) 20 (E) 15 (E) 25 (E) 26 (E) 27 (E) 27 (E) 27 (E) 28 (E) 28 (E) 28 (E) 28 (E) 29 (E) 29 (E) 20 (E) 29 (E) 20 (E) 20 (E) 29 (E) 21 (E) 22 (E) 23 (E) 24 (E) 25 (E) 26 (E) 27 (E) 27 (E) 27 (E) 28 (E) 28 (E) 29 (E) 29 (E) 29 (E) 29 (E) 20 (E) 20 (E) 20 (E) 20 (E) 21 (E)			

Q.98 The sum of the latter half of the first 1000 terms of any A.P. is equal to one third of the sum of the first n terms of the same A.P. Then $n =$ **EXERCISE AN INTERENT CONSTANT SURFAINING**

SUM of the latter half of the first 1000 terms of any

is equal to one third of the sum of the first n terms of

ame A.P. Then n =

500 (B) 3000

(D) 1000

(D) 1000

(D) $\frac{2pq}{$ **SPONDIVANCED LEARNING**

First 1000 terms of any

in of the first n terms of

1000

(A) 1500 (B) 3000
(C) 2000 (D) 1000 $(C) 2000$

Q.99 If the $(2p)^{th}$ term of a H.P. is q and the $(2q)^{th}$ term is p, then the 2 $(p+q)$ th term is-

(A)
$$
\frac{pq}{2(p+q)}
$$
 (B) $\frac{2pq}{p+q}$

(C)
$$
\frac{pq}{p+q}
$$
 (D) $\frac{p+q}{pq}$

s of two numbers x_1

(A) $2(p+q)$

(B) $p+q$

(D) $\frac{p+q}{pq}$

(D) y. The value of

(C) $\frac{pq}{p+q}$ (D) $\frac{p+q}{pq}$

Q.100 All terms of a certain A.P are natural numbers.

of its nine successive terms begining with the larger than 200 and smaller than 220. If the secon

12, then the comm iP. then x, x – y, t – zare

(C) 2000

(D) 1000

(D) 1000

(D) 1000

(D) 1000

(P) 11 the (2p^{yh} term of a H.P. is q and the (2q^{yh} term is p,

then the 2 (p + q^{1th} term is-

shen is of two numbers x₁

(A) $\frac{pq}{2(p+$ **Q.99** If the $(2p)^{1h}$ term of a H.P. is q and the $(2q)^{1h}$ term is p,

then the $2 (p+q)^{1h}$ term is-

and GP. both

ans of two numbers x_1
 (x) $\frac{pq}{2(p+q)}$
 (x) $\frac{2pq}{p+q}$
 (x) $\frac{p+q}{pq}$
 (x) $\frac{p+q}{pq}$
 2.100 All terms of a certain A.P are aduced then x, x - y, t - zare

2.99 If the (2pth term of a H.P. is q and the (2qth term is p,

2.99 If the (2pth term is -

2. and G.P. both

of two numbers x₁

(A) $\frac{pq}{2(p+q$ **EDIMADUMNEED LEARNING**

SUM of the latter half of the first 1000 terms of any

is equal to one third of the sum of the first n terms of

same A.P. Then n =

(B) 3000

(D) 1000

(D) 1000

(D) 1000

(D) 1000

(D) 1000

(D) **EDENTADVANCED LEARNING**
irst 1000 terms of any
n of the first n terms of
000
000
d the $(2q)^{th}$ term is p,
 $\frac{2pq}{p+q}$
 $\frac{p+q}{pq}$
ral numbers. The sum
ining with the first is
0. If the second term is **Q.100** All terms of a certain A.P are natural numbers. The sum of its nine successive terms begining with the first is larger than 200 and smaller than 220. If the second term is 12, then the common difference is

EXERCISE - 2 [LEVEL-2]

ONLY ONE OPTION IS CORRECT

- **Q.1** If $\frac{1}{\sqrt{1-\lambda}}$, $\frac{1}{\sqrt{1-\lambda}}$ are in A.P. then which of b c a c a b a b c EXERCISE - 2 [LEVEL-2]

DPTIONISCORRECT
 $c-a$, $c+a-b$, $a+b-c$

lowing is in A.P.

(B) a^2 , b^2 , c^2

(B) a^2 , b^2 , c^2

(D) None of these

(D) None of these

(D) Accuration (D) $\frac{n+1}{a_1 a_{n+1}}$

(D) Accuration the following is in A.P. - (A) a,b,c (B) a^2 , b^2 , c^2 (C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ **EXERCISE - 2 [LEVEL-2]**
 EXERCISE - 2 [LEVEL-2]
 EXERCISE - 2 [LEVEL-2]
 EXERCISE - 2 [LEVEL-2]
 COPTION IS CORRECT
 $+ c - a$, $c + a - b$, $a + b - c$ are in A.P. then which of $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \frac{1$ **EXERCISE - 2 [LEVEL-2]**
 OUDESTION IS CORRECT
 OUDESTION BANK
 EXERCISE - 2 [LEVEL-2]
 OUDESTION BANK
 OUDE EXERCISE - 2 [LEVEL-2]
 EXERCISE - 2 (10 If a₁, a₂, a₃,........................ **EXERCISE - 2 [LEVEL-2]**
 EXERCISE - 2 [LEVEL-2]
 EXERCISE - 2 [LEVEL-2]
 EXERCISE - 2 [LEVEL-2]
 OUPION ISCORRECT
 EXERCISE - 2 [LEVEL-2]
 OUPION ISCORRECT
 OUPION ISCORRECT
 OUPION ISCORRECT
 OUPION ISC EXERCISE - 2 [LEVEL-2]
 CORRECT
 CORRECT
- **Q.2** If the pth, qth and rth terms of a harmonic progression

are a, b, c respectively, then
$$
\frac{q-r}{a} + \frac{r-p}{b} + \frac{p-q}{c} =
$$

- (A) $\frac{pqr}{r}$ (B) $\frac{p+q+r}{r}$ (C) $\frac{par}{pqc}$ par **particular** $\frac{1}{\text{bqc}}$ (D) none of these
- **Q.3** If a,b,c,d are in G.P., then $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, $(c^3 + a^2)^{-1}$ $(d^3)^{-1}$ are in – (A) A.P. (B) G.P. (C) H.P. (D) none of these
- **Q.4** If $x_i > 0$, i = 1, 2, ..., 50 and $x_1 + x_2 +$..., $x_5 = 50$, then the minimum value of $\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{50}}$ equals to – $(A) (50)^2$ $(B) 50$
	- $(C) (50)^3$ $(D)(50)^4$
- **Q.5** If 1, $\log_{81} (3^{x} + 48)$ and $\log_{9} \left(3^{x} \frac{8}{3}\right)$ are in A.P., then $\qquad \begin{array}{c} +a_2a_3 + a_4 \\ +a_2a_3 + a_3 \\ (A) 2a_1a_5 \end{array}$ find x
	- $(A) 1$ (B) 2
	- $(C) 9$ (D) 3
- **Q.6** If x, y, z are in G.P. and $a^x = b^y = c^z$ then-(A) $\log_b a = \log_a c$ $(B) \log_c b = \log_a c$ (C) $log_b a = log_c b$ (D) none of these **Q.7** a, b, c are first three terms of a G.P. If HM of a and b is 12
- and that of b and c is 36, then find the value of a. $(A) 2$ (B) 3 $(C) 8$ (D) 1
- **Q.8** The numbers $\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{n}}}}}, \frac{1}{\sqrt{1-\frac{1}{n}}}, \frac{1}{$

(A) (30)³ (B) 30 (C) 10000
\n(C) (50)³ (D) (50)⁴ (D) 10000
\n(D) (50)⁴ (D) 10000
\n40.5 If 1, log₈₁ (3^x + 48) and log₉ (3^x -
$$
\frac{8}{3}
$$
) are in A.P., then
\n $\frac{1}{2}$ and that
\n(A) log₈ = log₈ (B) log₈ = log₈ (C) log₈ = log₈ (D) none of these
\n(A) log₈ = log₈ (D) none of these
\n(A) log₈ = log₈ (D) none of these
\n(A) 2² = 2² (E) 2² (E) 2² (E) log₈ = log₈ (E) log₈ = log₈ (E) log₈ = log₈ (E) log₈ = log₈ (E) 10000
\n5. If x,y,z are in G.P. and a^x = b^y = c² then
\n(A) log₈ = log₈ b (D) none of these
\n5. If A, A₂, A₃, A₃, A₄, A₅ are im-
\n(A) log₈ = log₈ (A) log₈ = 2² (B) log₈ = log₈ (C) 14
\nD. 14 If positive numbers a, b
\n⁸ = log₈ (A) 2⁸ = 2⁸ (B) log₈ = 2⁸ (C) 10⁸ (D) 10⁸ (E) 10⁸ (E) 10⁸ (E) 10⁸ (

- If $\log_{\frac{x+6}{3}}\left(\log_2 \frac{x-1}{x+2}\right) > 0$, then $x \in (a, b) \cup (c, d)$. If a,
	- b, k, c, d are in A.P., then the value of $a^2 + b^2 + k^2 + c^2 + d^2$ is (A) 115 (B) 125 (C) 118 (D) 130

Q.10 If
$$
a_1, a_2, a_3, \dots, a_n, a_{n+1}
$$
 are in A.P. then

a b c ⁼ 1 1 1 x x x equals to – 1 2 2 3 3 4 1 1 1 a a a a a a n n 1 1 a a is equal to (A) n n 1 n 1 a a (B) n n 1 1 a a n n 1 n 1 a a n n 1 a a

(C)
$$
\frac{n+1}{a_n a_{n+1}}
$$
 (D) $\frac{n}{a_n a_{n+1}}$

Q.11 If the $(m+1)$ th, $(n+1)$ th, $(r+1)$ th terms of an A.P. are in G.P. and m,n,r, are in H.P. then find the ratio of common difference to the first terms in the A.P.

(A) n/2 (B) 2/n (C) – n/2 (D) – 2/n

 \overline{Q} **Q.12** If n arithmetic means a_1, a_2, \dots, a_n are inserted between rms of a harmonic progression

len $\frac{q-r}{a} + \frac{r-p}{b} + \frac{p-q}{c} =$
 0.11 If the $(m+1)^{th}$, $(n+1)^{th}$, $(r+1)^{th}$ terms

and m, n, r, are in H.P. then find the

difference to the first terms in the A

difference to the fir 50 and 200 and n harmonic means h_1 , h_2 ,....., h_n are inserted between the same two numbers, then a_2 h_{n-1} is equal to

(A) 500
 (B)
$$
\frac{10000}{n}
$$

 (C) 10000

$$
(D) \frac{250}{n}
$$

- B) and a memoin progression

and n_{n+1} (B) $\frac{1}{a_{n}}a_{n+1}$
 $\frac{1-r}{a} + \frac{r-p}{b} + \frac{p-q}{c} =$

C. 11 If the (m+1)th, (n+1)th, (r+1)th (r+1)th (r+1)th (r+1)th (r+1)th (r+1)th (r+1)th (r+1)th (r+1)th e of $\frac{1}{x_1} + \frac{1}{x_2} + + \frac{1}{x_{50}}$ equals to -

(B) 50

(D) (50)⁴

(D) (50)⁴

(D) (50)⁴

(D) (50)⁴

(D) (50)⁴

(D) (50)⁴

(D) 3

(B) and log₉ $\left(3^x - \frac{8}{3}\right)$ are in A.P., then

(A) 2a₁a₃ 4a₃ 4 2⁺ + $\frac{1}{x_{50}}$ equals to -

(A) 500

(B) $\frac{10000}{n}$

(50)⁴
 $\frac{1}{x_{50}}$ equals to -

(A) 500

(B) $\frac{250}{n}$

(B) **Q.13** If a_1 , a_2 , a_3 , a_4 , a_5 are in H.P., then find the value of a_1a_2 2 $+a_2a_3+a_3a_4+a_4a_5$. (A) $2a_1a_5$ $(B) 8a_1a_5$ 5 (C) $10a_1a_5$ 5 (D) 4a₁a₅ 5 (A) 500 (B) $\frac{100000}{n}$

(C) 10000 (D) $\frac{250}{n}$

If a_1, a_2, a_3, a_4, a_5 are in H.P., then find the value of a_1a_2
 $+ a_2a_3 + a_3a_4 + a_4a_5$.

(A) $2a_1a_5$ (B) $8a_1a_5$

(C) $10a_1a_5$ (D) $4a_1a_3$

(C) $10a_1$ (D) $\frac{250}{n}$
 a_3 , a_4 , a_5 are in H.P., then find the value of a_1a_2
 $\frac{1}{3}3a_4 + a_4a_5$. (B) $8a_1a_5$
 $\frac{1}{5}$ (D) $4a_1a_5$
 $\frac{1}{2}$ e numbers a, b, c are in H.P. then the value of
 $\frac{1}{9}(8a-2b+c)$ (B) $\frac{10000}{n}$

(D) $\frac{250}{n}$

(D) $\frac{250}{n}$

(B) $8a_1a_5$

(B) $8a_1a_5$

(D) $4a_1a_5$

(D) $2a_1a_5$

(B) $(a-c)$

(B) $(a-c)$

(D) zero

2
	- **Q.14** If positive numbers a, b, c are in H.P. then the value of $e^{\log(a + c) + \log(a-2b + c)}$ is equal to

(A) log (a – c)² (B) (a – c) (C) (a – c)² (D) zero

- **Q.15** $\frac{1}{2}$ cosec² θ , 2cot θ , 0 < θ < $\frac{\pi}{2}$, are in G.P. $\frac{1}{2}$ cosec² θ , 2cot θ , $0 < \theta < \frac{\pi}{2}$, are in G.P. if θ is equal to $\frac{\pi}{2}$, are in G.P. if θ is equal to (A) $\pi/6$ (B) $\pi/4$ $(C) \pi/3$ (D) None of these
- **Q.16** If x, y, z are three real numbers of the same sign then the

value of
$$
\frac{x}{y} + \frac{y}{z} + \frac{z}{x}
$$
 lies in the interval

$$
(A) [2, +\infty) \qquad (B) [3, +\infty)
$$

$$
(C) (3, +\infty) \qquad (D) (-\infty, 3)
$$

- d x

dx

dx

(C) $10a_1a_5$ (B) a_3a_6

(C) $10a_1a_5$

dthat of b and c is 36, then the these

degra $+ b = e$ (B)2

are in GP. and a^x = b^y = c² then-
 v_6 a = log₄c
 v_7 a = log₄c
 v_8 a = log₄c

(A) log (a - c)²

(B (3^x + 48) and log₉ $\begin{pmatrix} 3^x - \frac{8}{3} \end{pmatrix}$ are in A.P., then

(A) 2₄₁₄ and a^x = b² = c² and

(C) 10a₁₄₅ (D) 3

(C) 10a₁₄₅ (D) 4a₁

(C) 10a₁₄₅ (D) 4a₁

(C) 10a₁₄₅ (D) 4a₁

(C) 10a₁₄₅ (D) 4a₁ (3³+48) and log₉ $\begin{pmatrix} 3 & -\frac{1}{3} \end{pmatrix}$ are in A.P., then

(A) $2a_1a_3$ (B) $8a_1$

(B) 2

in G.P. and $a^x = b^y = c^x$ then

(B) $a_2b^x = 0$ (B) $a_3b^x = 0$

(B) $a_4c^x = 0$ (B) $a_5c^y = 0$

(B) $a_6c^y = 0$

(B) $a_$ (b) -3 and $a^x = b^y = c^x$ (h) a_1

(h) a_2

in G.P. and $a^x = b^y = c^x$ (h) a_3

(h) a_4

(h) a_5

in G.P. and $a^x = b^y = c^x$ (h) a_6b

(h) a_7b
 a_8b , cere and the value of a.
 a_8b , cere and a_8b

(h) $s_9\begin{bmatrix} 3^5 - \frac{1}{3} \end{bmatrix}$ are in A.P., then $(2)3a_1a_5$ (B) $8a_1a_5$ (D) $4a_1a_5$ (D) $4a_1a_5$ (D) $4a_1a_5$ (D) $4a_1a_5$ (D) $4a_1a_5$ (D) $4a_1a_5$ (D) s_1s_2 (d) $2a_1a_5$ (d) $2a_1a_5$ (d) $2a_1a_5$ (d) $2a$ **(A)** $\log(a - c)^2$ (B) $(a - c)$
 Q.15 $\frac{1}{2} \csc^2\theta$, $2 \cot\theta$, $0 \le \theta < \frac{\pi}{2}$, are in GP. if θ is equal to
 Q.15 $\frac{1}{2} \csc^2\theta$, $2 \cot\theta$, $0 \le \theta < \frac{\pi}{2}$, are in GP. if θ is equal to
 Q.16 If x, y, z are three (B) $(a-c)^2$

(B) $(a-c)$

(D) zero

osec² θ , 2cot θ , $0 < \theta < \frac{\pi}{2}$, are in G.P. if θ is equal to
 $\frac{\pi}{3}$

(B) $\pi/4$

(B) $\pi/4$

(D) None of these

y, z are three real numbers of the same sign then the

e of $\$ 2^{2a} , 3^{2a} , 3^{2a 2a₁a₅

(B) 8a₁a₅

(D) 4a₁a₅

(D) 4a₁a₅

(D) 4a₁a₅

(a + c) + log(a-2)² is equal to

log (a - c)²

(B) (a - c)

(a - c)²

(D) zero

cosec²θ, 2cotθ, $0 < \theta < \frac{\pi}{2}$, are in G.P. if θ is equal to
 a₃, a₄, a₅ are in H.P., then find the value of a₁a₂

a₃a₄ + a₄a₅. (B) 8a₁a₅

(B) 8a₁a₅

(D) 4a₁a₅

(D) 4a₁a₅

(D) 2cro

(B) (a-c)

(D) zero

2-0, 2cotθ, $0 < \theta < \frac{\pi}{2}$, are in G.P. if θ i , a_2 , a_3 , a_4 are in (A) A.P. (B) GP. (C) H.P. (D) None of these **Q.18** The sum of the integers lying between 1 and 100 (both
	- inclusive) and divisible by 3 or 5 or 7 is (A) 2838 (B) 3468 (C) 2738 (D) 3368

3

(SEQUENCES & SERIES)\n

CEQUENCES & SERIES	QUESTION BANK	EXAMPLE A
Q.19 The arithmetic mean of two numbers is 3 times their one.	Q.27 Consider the A.P. $a_1, a_2, \ldots, a_n, \ldots$ the GP. $b_1, b_2, \ldots, b_n, \ldots$ the OP. $b_1, b_2, \ldots, b_n, \ldots$	

(C)
$$
\frac{1}{2}p^2q^2
$$
 (D) $\frac{1}{4}p^3q^3$

Q.21 If $a_1 + a_2 + a_3 + a_4 + a_5 + \dots + a_n = 1$ for all $a_i > 0$, $i = 1, 2$, 3,, n. Then the maximum value of $a_1^2a_2a_3a_4a_5$ a_n Q.29 $is -$

(A)
$$
\frac{2}{(n+1)^n}
$$

\n(B) $\frac{4}{(n+1)^{n+1}}$
\n(B) $\frac{4}{(n+1)^{n+1}}$
\n(B) $\frac{4}{(n+1)^{n+1}}$
\n(B) $\frac{4}{(n+1)^{n+1}}$
\n(C) $\frac{2}{n^n}$
\n(D) $\frac{4}{n^{n+1}}$
\n5. $\frac{1-n}{n}$

Q.22 If sin α , sin β , sin γ are in A.P. cos α , cos β , cos γ are in G.P.

then
$$
\frac{\cos^2 \alpha + \cos^2 \gamma - 4 \cos \alpha \cos \gamma}{1 - \sin \alpha \sin \gamma} =
$$

(A) -2 (B) -1
(C) 0 (D) 2

Q.23 Given $a_{m+n} = A$; $a_{m-n} = B$ as the terms of the G.P. a_1 , a_2 , **ASS** a_3 ,............. then for $A \neq 0$ which of the following holds?

(C) ² m m n mn m n m 1 A a a ^B (D) (5.2 1) log 2

Q.24 If $\log_{(5.2^X+1)} 2$; $\log_{(2^{1-x}+1)} 4$ and 1 are in
and 1 +4 2 +

Harmonical Progression then

- (A) x is a positive real
- (B) x is a negative real
- (C) x is rational which is not integral
- (D) x is an integer
- **Q.25** Consider an A.P. with first term 'a' and the common difference d. Let S_k denote the sum of the first K terms.

Let
$$
\frac{S_{kx}}{S_x}
$$
 is independent of x, then
(A) a = d/2 (B) a = d

$$
(C) a = 2d
$$
 (D) none

Q.26 Concentric circles of radii 1, 2, 3......100 cms are drawn. The interior of the smallest circle is coloured red and the 0.33 angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equal to (A) 1000π (B) 5050π (C) 4950π (D) 5151π

Q.27 Consider the A.P.
$$
a_1, a_2, \dots, a_n, \dots
$$

the GP. $b_1, b_2, \dots, b_n, \dots$

such that
$$
a_1 = b_1 = 1
$$
; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then

(A)
$$
b_6 = 27
$$

\n(B) $b_7 = 27$
\n(C) $b_8 = 81$
\n(B) $b_7 = 27$
\n(D) $b_9 = 18$

Q.28 The point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the parabola $y = 3x^2$. If x_1, x_2, x_3 are in A.P. and y_1, y_2, y_3 are in G.P. then the common ratio of the G.P. is

$$
\frac{1}{4}p^3q^3
$$
\n(A) $3 + 2\sqrt{2}$
\n1 for all $a_i > 0$, $i = 1, 2$,
\n(C) $3 - \sqrt{2}$
\n(D) $3 - 2\sqrt{2}$

ers are

(B) $3+2\sqrt{2}$, $3-2\sqrt{2}$

(D) none of these

(A) $b_6 = 27$

(B) $b_7 =$

are m is $-$
 $(20)^{\text{th}}$ term is p^2 where p
 $(30)^2 q^2$
 $a_n = 1$ for all $a_i > 0$, $i = 1, 2$,
 $a_1 = 1$ for all $a_i > 0$, $i = 1, 2$,
 a 4
resulting square were also connected by segments so **COUESTION BANK**

Then since the interval of the A.P. a₁, a₂,, a_n, ...

the squares of the two

the GiP. b₁, b₂,, b_n, ...

are

are
 $3+2\sqrt{2}$, $3-2\sqrt{2}$

such that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{$ (D) none of these (A) $b_6 = 27$ (B)

erm is -

(C) $b_8 = 81$ (D)

erm is -

(B) p^2q^2 parabola $y = 3x^2$. If x_1, x_2, x_3 are

(D) $\frac{1}{4}p^3q^3$ (A) $3 + 2\sqrt{2}$ (B)
 $a_n = 1$ for all $a_i > 0, i = 1, 2,$ (C) $3 - \sqrt{2}$ (D $\frac{1}{4}p^3q^3$ (A) $3 + 2\sqrt{2}$ (B) $3 + \sqrt{2}$

1 for all $a_i > 0, i = 1, 2,$ (C) $3 - \sqrt{2}$ (D) $3 - 2\sqrt{2}$

of $a_1^2a_2a_3a_4a_5......a_n$ Q.29 A circle of radius r is inseribled in a square. The mid points

of sides of the squar I for all a_i > 0, i = 1, 2, (C) 3 - $\sqrt{2}$ (D) 3 - 2/2

of a_i²a₂a₃a₄a₅........ a_n **Q.29** A circle of radius r is inscribed in a square. The mid points

of sides of the square have been connected by line Consider the A.P. a_1 , a_2 ,...., a_n ,....

the G.P. b_1 , b_2 ,...., b_n ,....

such that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then

(A) $b_6 = 27$ (B) $b_7 = 27$

(C) $b_8 = 81$ (D) $b_9 = 18$

The point A Consider the A.P. a_1 , a_2 ,....., a_n ,....

the G.P. b_1 , b_2 ,....., b_n ,....

such that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $(A) b_6 = 27$
 $(B) b_7 = 27$
 $(B) b_7 = 27$
 $(C) b_8 = 81$

The point $A(x_1,$ **Q.29** A circle of radius r is inscribed in a square. The mid points of sides of the square have been connected by line segment and a new square resulted. The sides of the that a new square was obtained and so on, then the radius of the circle inscribed in the nth square is = 27 (B) b₇ = 27

= 81 (D) b₉ = 18

oint A(x₁, y₁); B(x₂, y₂) and C(x₃, y₃) lie on the

lay = 3x². If x₁, x₂, x₃ are in A.P. and y₁, y₂, y₃ are

then the common ratio of the GP. is
 $2\sqrt{2}$ the G.P. b₁, b₂,....., b_n⁵,.....

that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $\frac{5}{5} = 27$ (B) $b_7 = 27$
 $\frac{1}{7} = 81$ (D) $b_9 = 18$

point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the

col **EDENTIFY**

Sider the A.P. $a_1, a_2, ..., a_n, ...$

the G.P. $b_1, b_2, ..., b_n, ...$

that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $b_6 = 27$
 $b_6 = 27$
 $b_8 = 81$
 $b_9 = 18$
 b_9 sider the A.P. a_1 , a_2 ,...., a_n ,....

the G.P. b_1 , b_2 ,...., a_n ,....

that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $b_6 = 27$ (B) $b_7 = 27$
 $b_8 = 81$ (D) $b_9 = 18$

point A(x_1 , y_1); B($x_$ the GP. b₁, b₂,...., b_n,.....

that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $b_6 = 27$ (B) $b_7 = 27$
 $b_8 = 81$ (D) $b_9 = 18$

point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the

bolay = 3x². I =1

=1

=27

=18

C(x₃, y₃) lie on the

i.P. and y₁, y₂, y₃ are

GP. is
 $\sqrt{2}$

-2 $\sqrt{2}$

-2 $\sqrt{2}$

iuare. The mid points

connected by line

d. The sides of the

ted by segments so

o on, then the radius

a 2 $\left[\frac{9}{2}a_r = 369$ then
 $7 = 27$
 $9 = 18$

d C(x₃, y₃) lie on the

A.P. and y₁, y₂, y₃ are

e G.P. is
 $+\sqrt{2}$
 $-2\sqrt{2}$

square. The mid points

n connected by line

ed. The sides of the

cted by segments so **EDMADVANCED LEARNING**

1,....
 $\frac{9}{r=1}$
 $b_7 = 27$
 $b_9 = 18$

and $C(x_3, y_3)$ lie on the

m A.P. and y_1, y_2, y_3 are

are G.P. is
 $3 + \sqrt{2}$
 $3 - 2\sqrt{2}$

square. The mid points

en connected by line

lited. The si $\frac{9}{15}$
 $\frac{1}{15}$
 $\frac{1}{2}$
 3.....

do $\sum_{r=1}^{9} a_r = 369$ then

b₇ = 27

b₉ = 18

nd C(x₃, y₃) lie on the

n A.P. and y₁, y₂, y₃ are

e G.P. is

3 + $\sqrt{2}$

3 - 2 $\sqrt{2}$

square. The mid points

en connected by line

lted. The sides 6=27 (B) b_7 =27

g=81 (D) b_9 =18

g=81 (D) b_9 =18

point A(x_1 , y_1); B(x_2 , y_2) and C(x_3 , y_3) lie on the

point $A(x_1, y_1)$; B(x_2 , y_3 are in A. P. and y_1 , y_2 , y_3 are

2. then the commo 1 that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
 $b_6 = 27$ (B) $b_7 = 27$
 $b_8 = 81$ (D) $b_9 = 18$
 $b_9 = 18$
 $b_9 = 18$
 $b_9 = 1001$ A(x_1, y_1); $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the
 b_0 and y_1, y_2, y_3 1 that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{n} a_r = 369$ then
 $b_6 = 27$ (B) $b_7 = 27$
 $b_8 = 81$ (D) $b_9 = 18$

point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the

bololay = 3x². If x_1, x_2, x_3 are in A.P. and $b_6 = 27$ (B) $b_7 = 27$
 $b_8 = 81$ (D) $b_9 = 18$

point A(x₁, y₁); B(x₂, y₂) and C(x₃, y₃) lie on the

bolola y = 3x². If x₁, x₂, x₃ are in A.P. and y₁, y₂, y₃ are

1.P. then the common ratio of P. and y_1 , y_2 , y_3 are

i.P. is
 $\sqrt{2}$
 $2\sqrt{2}$

aare. The mid points

connected by line

. The sides of the

d by segments so

on, then the radius

re is
 $\left[\frac{-3n}{2}\right]_1^{\frac{3n}{2}}$
 $\left[\frac{5-3n}{2}\right]_1^{\frac{5-3n}{2$ $7 = 27$
 $9 = 18$

d C(x₃, y₃) lie on the

A.P. and y₁, y₂, y₃ are

e G.P. is
 $+\sqrt{2}$
 $-2\sqrt{2}$

quare. The mid points

n connected by line

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cted by segments so

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ua d $\sum_{r=1}^{9} a_r = 369$ then
 $27 = 27$
 $27 = 18$

od $C(x_3, y_3)$ lie on the

n A.P. and y_1, y_2, y_3 are

ee G.P. is
 $3 + \sqrt{2}$
 $3 - 2\sqrt{2}$

square. The mid points

en connected by line

ted. The sides of the

ected by se d $\sum_{r=1}^{n} a_r = 369$ then
 $27 = 27$
 $29 = 18$

and $C(x_3, y_3)$ lie on the

and $C(x_3, y_3)$ lie on the

and $C(x_3, y_3)$ are

according to the
 $3 + \sqrt{2}$
 $3 - 2\sqrt{2}$

square. The mid points

enconnected by line

ted. The $p_7 = 27$
 $p_9 = 18$

and $C(x_3, y_3)$ lie on the

nA.P. and y_1, y_2, y_3 are

e G.P. is
 $3 + \sqrt{2}$
 $3 - 2\sqrt{2}$

square. The mid points

en connected by line

ted. The sides of the

ected by segments so

d so on, then th

n 2 2 cos cos 4cos cos 1 sin sin (A) a AB ^m (B) 2n n n 2 2 m m n n m n n 1 a a ^B (A) (B) (C) ⁿ (D)

, **ASSERTION AND REASON QUESTIONS**

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
- (C) Statement -1 is True, Statement-2 is False.
- \overline{A} $\overline{m+n}$ (D) Statement -1 is False, Statement-2 is True.
	- (E) Statement -1 is False, Statement-2 is False.
- **Q.30** Statement 1: 1, 2, 4, 8, is a G.P., 4, 8, 16, 32 is a G.P. and $1 + 4$, $2 + 8$, $4 + 16$, $8 + 32$ is also a G.P. **Statement 2 :** If T_k denotes kth term of a G.P. of positive common ratio r and T_k denotes k^{th} term of an other G.P. of common ratio r, then the series whose kth term is $T''_k = T_k + T'_k$ ia also a G..P. with common ratio r. r is True, statement-2 is True, statement-
correct explanation for Statement-1.
1 is True, Statement-2 is False.
1 is False, Statement-2 is True.
1 is False, Statement-2 is True.
2 is False.
2, 4, 8, is a GP, 4 (D) $\left[2^{-\frac{5-3n}{2}}\right]r$

(QUESTIONS

e, Statement-2 is True; Statement-

nation for Statement-1.

e, Statement-2 is True; Statement-

explanation for Statement-1.

ue, Statement-2 is False.

lse, Statement-2 is False.
 (D) $\begin{bmatrix} 2 & 2 \ 2 & 2 \end{bmatrix}$ $\begin{bmatrix} \text{r} \\ \text{r} \end{bmatrix}$
 SON QUESTIONS

True, Statement-2 is True; Statement-

xplanation for Statement-1.

True, Statement-2 is True; Statement-

rect explanation for Statement-1.

S Tru CC $\left[2\frac{n}{2}\right]$ r (D) $\left[2\frac{5-3n}{2}\right]$ r (D) $\left[2\frac{5-3n}{2}\right]$ r (D) $\left[2\frac{3-3n}{2}\right]$ r (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1. (B) Statement-1 is True,
	- **Q.31 Statement–1**: In the expression $(x + 1)(x + 2) \dots (x + 50)$, coefficient of x^{49} is equal to 1275.

Statement-2:
$$
\sum_{r=i}^{n} r = \frac{n(n+1)}{2}
$$
, $n \in N$.

- **Statement–1**: a. ar. ar^2 ... $ar^{n-1} = (a^2 r^{n-1})^{n/2}$. **Statement–2 :** Product of kth term from the beginning and from the end in a G.P. is independent of k.
- **Q.33 Statement–1**: Let p, q, $r \in R^+$ and 27 pqr³ (p + q + r)³ and $3p + 4q + 5r = 12$, then $p^3 + q^4 + r^5$ is equal to 4. **Statement–2 :** If A, G, and H are A.M., G.M., and H.M. of positive numbers $a_1, a_2, a_3, \ldots, a_n$ then $H \le G \le A$.

- **Q.34 Statement 1 :** If a, b, c and d are in harmonic progression then $(a + d) > (b + c)$. **Statement 2 :** If a, b, c and d are in arithmetic progression, then $ab + cd > 2$ (ac + bd – bc). **QUESTION BANK**
 QUESTION BANK
 Q.34 Statement 1: If a, b, c and d are in harmonic progression

then (a + d) > (b + c).

Statement 2: If a, b, c and d are in arithmetic progression,

then ab + cd > 2 (a + bd – bc).

- **Q.35** Statement-1: If a, b, c are in G.P., $\frac{1}{\log a}$, $\frac{1}{\log b}$, $\frac{1}{\log c}$ are

in H.P.

Statement-2: When we take logarithm of the terms in G.P., they occur in A.P.

MATCH THE COLUMN TYPE QUESTIONS

Q.36 Column II gives sum of n terms of the series given in column I. Match them correctly –

Column I Column II

(a)
$$
8 + 88 + 888 + \dots
$$
 (p) $\frac{1}{3}(4^n - 1) + n(n+1)$ (A) 1
(C) 1/2

(b) $3 + 8 + 22 + 72 + 266 + 1036 + \dots$

(q)
$$
\frac{8}{81}
$$
 [10ⁿ⁺¹ - 9n + 10]

Hence (4 + 0) × (0 + 0).
\n**Sistement-2:** If a, b, c and d are in arithmetic progression.
\n**Sistement-2:** If a, b, c are in G.P.,
$$
\frac{1}{\log a}, \frac{1}{\log b}, \frac{1}{\log c}
$$

\n**Statement-2:** When we take logarithm of the terms in
\n $\frac{0.38}{0.39}$ The smallest number is –
\n $\frac{0.39}{0.39}$ The smallest number is –
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
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\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
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\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{0.30}{0.39}$ The sum of the terms of the terms in
\n $\frac{$

(s)
$$
\frac{1}{3} \left[\frac{1}{1.2.3} - \frac{1}{(n+1)(n+2)(n+3)} \right]
$$

Code :

- (A) a-q, b-p, c-s, d-r (B) a-s, b-p, c-q, d-r (C) a-r, b-q, c-s, d-p (D) a-r, b-s, c-p, d-q
- **Q.37** Match the column **Column I Column II**
	-
	- (a) If $a_k = \int \frac{1}{\sin x} dx$ $=\int_{0}^{\pi} \frac{\sin(2k-1)x}{\sin x} dx$ (p) constant sequence Q.46 then $a_1, a_2, \dots,$ form a
	- (b) If x, y, z all greater than '1' are (q) A.P.

in G.P. then
$$
\frac{1}{1 + \log x}
$$
, $\frac{1}{1 + \log y}$, $\frac{1}{1 + \log z}$ are in

(c) If
$$
a, b, c
$$
 are in A.P. then $(r) H.P.$

$$
\frac{ab+ac}{bc}, \frac{bc+ba}{ca}, \frac{ca+bc}{ab}
$$
 are in

ab ac bc ba ca bc (s) $\frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{(n+1)(n+2)(n+3)}\right]$

Let A₁, A₂, A₃,, A₃ be arithmetic manuscription

b-p, c.-s, d.-r

b-p, c.-s, d.-r

the column (B) a.-r, b-s, c-p, d-r

the column II

column II

Column II

Co (d) If x_1, x_2, \dots, x_n are n non-zero (s) G.P. real numbers such that, $(x^{2}_{1} + x^{2}_{2} + ... + x^{2}_{n-1}) (x^{2}_{2} + x^{2}_{3} + ... + x^{2}_{n})$ $\leq (x_1x_2 + x_2x_3 + ... + x_{n-1}x_n)^2$, then $x_1, x_2, ..., x_n$ are in are in Code : (A) a-q, b-p, c-s, d-r (B) a-s, b-p, c-q, d-r (C) a-r, b-q, c-s, d-p (D) a-p, b-r, c-q, d-s

PASSAGE BASED QUESTIONS

- **EXECTION BANK**
 EXECTION MATERIAL: MATHEMATICS
 EXECTION BANK
 EXECTIONS
 EXECTIONS Passage 1- (Q.38-Q.40) : Four different integers from an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then
	- **Q.38** The smallest number is $(A) -2$ (B) 0 (C) –1 (D) 2 **Q.39** The common difference of A.P. is –
	- (A) 2 (B) 1 (C) 3 (D) 4
	- **Q.40** The sum of all the four numbers is (A) 10 (B) 8 (C) 2 (D) 6
	- **Passage 2- (Q.41-Q.43)**

Arithmetic, geometric and harmonic mean of the roots of $x^2 + 13x + 36 = 0$ and α , β and γ respectively.

- **OUESTION BANK** STUDY MATERIAL: MATHEMATICS
 PASSAGE BASED OUESTIONS
 PASSAGE ALSED Q.41 x_1 and x_2 are the roots of $ax^2 + bx + c = 0$, a, b, $c \in R$ and α , β , γ lies between the x_1 and x_2 , $\delta = |x_1 - x_2|$, then minimum possible value of δ is – (A) 1 (B) $3/4$
	- (C) $1/2$ (D) $25/26$
- **COUESTION BANK**
 COUESTION BANK
 COUESTION BANK
 COUESTION MATERIAL:MATHEMATICS
 COUESTION BANK
 COUEST 8 $x^2 - (t^2 - 13t + \alpha + \gamma)x - 36 = 0$ is less than or equal to β , metic progression, creasing A.P. One of these numbers is equal to the sum

of the squares of the of ending the other three numbers. Then
 $\frac{1}{10}$, $\frac{1}{10}$ are
 $\frac{1}{10}$ are
 $\frac{1}{10}$ are
 $\frac{1}{10}$ are
 $\frac{1}{1$ **1 0.48** For sydens of the content line functions. Then

1² $\frac{1}{\log b}$, $\frac{1}{\log c}$ are **(A)** -2 (B) 0 (C)-1 (D) 2
 1 or of the terms in **Q.40** The sum of all the four numbers is -

(A) 2 (B) 1 (C) 3 (D) 4
 1 P Passage 1- (Q.38-Q.40): Four different integers from an in-

titic progression,

or disease A.P. One of these unubers is equal to the sum

of the squares of the other three numbers. Then

or different manns of the squar tic progression,

a reasing A.P. One of these numbers is equal to the sum

of the squares of the other three numbers is equal to the sum

of the squares of the other three numbers. Then
 $\frac{1}{(b)g}$, $\frac{1}{\log g}$ are
 \frac If a, b, c are in GP., $\frac{1}{\log a}$, $\frac{1}{\log b}$, $\frac{1}{\log c}$, $\frac{1}{\log a}$, $\frac{1}{\log b}$, $\frac{1}{\log c}$, $\frac{1}{\log a}$, $\frac{1}{\log b}$, $\frac{1}{\log c}$ (b) $\frac{1}{\log a}$. (f) $\frac{1}{\log a}$ (h) $\frac{1}{\log a}$ (h) $\frac{1}{\log a}$ (h) $\frac{1}{\log a}$ (e take logarithm of the terms in 0.40 The sum of all the four numbers is $(0, 1, 1)$
 Passage 2. Q.41-Q.43)
 Column II EXERIONS

Passage 2-(0,41-0,43)
 $\frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{(n+1)(n+2)(n+3)}\right]$

All $\frac{1}{2}$
 $\frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{(n+1)(n+2)(n+3)}\right]$
 $\left[\frac{1}{3(4^n-1)} + \frac{1}{2(4^n+1)(n+3)}\right]$
 $\left[\frac{1}{3(4^n-1)} + \frac{1}{2(4^n+1)(n+2)}\right]$
 $\left[\frac$ take logarithm of the terms in **Q.40** The sum of all the four numbers is $(0, 13)$
 DIESTIONS
 Passage 2 (**Q.410.44)**
 CALES COMENTIONS
 Passage 3 (**Q.410.44)**
 CALES COMENTIONS
 Passage 3 (**Q.410.44)**
 CALES take logarithm of the terms in Q.40 The sum of all the four numbers is—

(A) 10 e sum of all the four numbers is—
 α (A) 10 (B) 8
 PASSEQ (Q.41-Q.43)
 PASSEQ (Q.41-Q.43)
 PASSEQ (Q.41-Q.43)
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 take logarithm of the terms in Q.40 The sum of all the four numbers is -
 Passage 2-(Q.41-Q.43)
 Passage 2-(Q.41-Q.43)
 Passage 2-(Q.41-Q.43)
 Passage 2-(Q.41-Q.43)
 C.41-Q.43
 C.41-Q.43
 C.41-Q.43
 C.41-Q. Q.42 Set of all values of t if sum of roots of is $[\ell, m]$ and $p = \ell + m$, then p is equal to – $(A) 13$ (B) 26 (C) 4 (D) 17 x² + 1 xx² - 30 = 0 and α , p and γ respectively.
 x_1 and x_2 are the roots of $ax^2 + bx + c = 0$, a, b, c e R and
 α , β ; γ less between the x_1 and x_2 , $\delta = |x_1 - x_2|$, then

minimum possible value o
	- **Q.43** Equation whose roots are 2α , p is (where p obtained from above questions).
		- $(A) x² + 30x + 221 = 0$ $+30x+221=0$ (B) $x^2-39x+338=0$ $(C) x^2 + 17x + 52 = 0$ $+17x+52=0$ (D) $x^2-169=0$

Passage 3- (Q.44-Q.46)

 $\frac{1}{3} + \frac{1}{3.4.5.6} + ...$ (r) $\frac{1}{2} + \frac{3}{1+3+3+3+} + ...$
 $\frac{2^2 + 2^4}{1+3^2 + 3^4} + \frac{3^2}{1+3^2 + 3^4} + ...$
 $\frac{1}{3} \left[\frac{1}{1 \cdot 2 \cdot 3} - \frac{1}{(n+1)(n+2)(n+3)} \right]$
 DAS Equation whose roots are 2*a*, p is (where p obtained from Let A_1 , A_2 , A_3 ,, A_n be arithmetic means between -2 and 1027 and G_1 , G_2 , G_3 ,, G_n be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025×171 . , A₂, A₃,, A_n be arithmetic means between -2

227 and G₁, G₂, G₃,, G_n be geometric means

in 1 and 1024. Product of geometric means is 2⁴⁵

m of arithmetic means is 1025 × 171.

llue of n is -
 bow-

download (usations).

(A) $x^2 + 30x + 221 = 0$ (B) $x^2 - 39x + 338 = 0$

(C) $x^2 + 17x + 52 = 0$ (D) $x^2 - 169 = 0$
 $x^3 - 169 = 0$
 $x^2 + 17x + 52 = 0$ (D) $x^2 - 169 = 0$
 $x^3 - 169 = 0$
 $x^2 - 169 = 0$
 $x^4 - 169 = 0$
 $x^3 - 16$

- 2 + 1⁴ 1 + 2² + 2⁴ ¹ 1 + 3² + 3⁴ (C) x² + 17x + 52= 0 (D) x² 16

(s) $\frac{1}{3} \left[\frac{1}{1.2.3} \frac{1}{(n+1)(n+2)(n+3)} \right]$

Bassage **Q.44** The value of n is – $(A) 7$ (B) 9 (C) 11 (D) None of these **Q.45** The value of m is – nd 1027² and G_1 , G_2 , G_3 ,, G_n be geometric means
etween 1 and 1024. Product of geometric means is 2⁴⁵
etween 1 and 1024. Product of geometric means is 2⁴⁵
he value of n is –
(B) 9
(B) 0
(B) 100 000 00 bend 1 and 1024. Product of geometric means is 245

en 1 and 1024. Product of geometric means is 245

um of arithmetic means is 1025 × 171.

alue of n is –

(B) (D) None of these

alue of n is –

(B) (D) None of these

(B
	- (A) 340 (B) 342 (C) 344 (D) 346 **Q.46** The value of $G_1 + G_2 + G_3 + \dots + G_n$ is – (A) 1022 (B) 2044 (C) 512 (D) None of these

Passage 4- (Q.47-Q.49)

Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be the arithmetic sequences each with common difference 2 such that $a_1 < b_1$ and le

$$
{n} = \sum{k=1}^{n} a_{k}, d_{n} = \sum_{k=1}^{n} b_{k}.
$$

 (a_n, c_n) , B_n (b_n, d_n) are all lying on the parabola C : $y = px^2 + qx + r$ where p, q, r are constants.

- **Q.47** The value of p equals (A) $1/4$ (B) $1/3$ (C) $1/2$ (D) 2
- **Q.48** The value of q equals (A) $1/4$ (B) $1/3$ (C) $1/2$ (D) 2 **Q.49** If $r = 0$ then the value of a_1 and b_1 are $-$ (A) $1/2$ and 1 (B) 1 and $3/2$
	- (C) 0 and 2 (D) $1/2$ and 2

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

EXECUTES & SERIES **EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**\nNOTE: The answer to each question is a NUMBERICAL VALUE. Q.11 Let
$$
S_k
$$
, $k = 1, 2, \ldots, 100$, denote the sum of the is Q .11 If the sum $\sum_{k=1}^{\infty} \frac{1}{(k+2)\sqrt{k} + k\sqrt{k+2}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, where $a, b, c \in N$ and lie in [1, 15], then find the value of $a+b+c$.\n\nQ.2 Numbers are grouped as $\{1, 1, 1\}$ and $\{3, 3^2, \ldots, 3^5\}$.\n\nQ.3 If $\sum_{n=1}^{49} \frac{1}{\sqrt{n} + \sqrt{n^2 - 1}} = a + b\sqrt{2}$, then $a + b = 25^B$, then find B.\n\nQ.4 If $\tan\left(\frac{\pi}{12} - x\right)$, $\tan\frac{\pi}{12}$, $\tan\left(\frac{\pi}{12} + x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. The value of $x = 1, 2, \ldots, 100$, and $x = 2k + 1$. The value of $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 100$. If $\tan\left(\frac{\pi}{12} - x\right)$, $\tan\frac{\pi}{12}$, $\tan\left(\frac{\pi}{12} + x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. The value of $x = 1, 2, \ldots, \infty$ and $x = 2 + 1$. The value of $2x + 1$ and $2x + 1$. The value

a, b, $c \in N$ and lie in [1, 15], then find the value of $a+b+c$. **Q.2** Numbers are grouped as $\{1, 1, 1\}$ $\{3, 3^2, \ldots, 3^5\}$

$$
\{6, 6^2, \dots, 6^7\} \{10, 10^2, \dots, 10^9\}.
$$
 If sum of numbers in 10th

bracket is A such that $\left(\frac{54A}{2.5} + 1\right) = 55^B$, then find B. $\frac{100!}{100!} + \sum_{k=1}^{B} |k|^{-1}$. 55)

Q.3 If
$$
\sum_{n=1}^{49} \frac{1}{\sqrt{n + \sqrt{n^2 - 1}}} = a + b\sqrt{2}
$$
, then $a + b =$

Q.4 If
$$
\tan\left(\frac{\pi}{12} - x\right)
$$
, $\tan\frac{\pi}{12}$, $\tan\left(\frac{\pi}{12} + x\right)$ in order are three

consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. The value of k is.

- **EXERCISE 3 (NUMERICAL VALUE BASED QUESTIONS)**
 EXERCISE 3 (NUMERICAL VALUE BASED QUESTIONS)
 SEVENCES 3 (NUMERICAL VALUE BASED QUESTIONS)
 SEVENCES 3 (NUMERICAL VALUE Q.11 Let S_k , $k = 1, 2,..., 100$, denot **Q.5** Let $a + ar_1 + ar_1^2 + \dots + \infty$ and $a + ar_2 + ar_2^2 + \dots + \infty$ be two infinite series of positive numbers with the same first term. The sum of the first series is r_1 and the sum of the second series is r_2 . The value of $(r_1 + r_2)$
- **Q.6** If the equation $x^4 (3m + 2)x^2 + m^2 = 0$ (m > 0) has four real solutions which are in A.P. then find the value of m.
- **Q.7** Consider an A.P. a_1 , a_2 , a_3 ,...... such that $a_3 + a_5 + a_8 = 11$ and $a_4 + a_2 = -2$, then the value of $a_1 + a_6 + a_7$ is

Q.8 The sum
$$
\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}
$$
 equal to

- **Q.9** Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is
- **Q.10** Let K is a positive integer such that $36 + K$, $300 + K$, $596 + K$ are the squares of three consecutive terms of an arithmetic progression. Find K.

Q.11 Let S_k , $k = 1, 2, \dots, 100$, denote the sum of the infinite

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

and lie in [1, 15], then find the value of a+b+ c.

The geometric series whose first term is $\frac{k-1}{k!}$ a

and lie in [1, 15 **EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

For to each question is a NUMERICAL VALUE. Q.11 Let S_k , $k = 1, 2, ..., 100$, denote the sum of the infinite
 $\sum_{k=1}^{\infty} \frac{$ $c \qquad \qquad$ geometric se **QUESTION BANK**
 E. -3 (NUMERICAL VALUE BASED QUESTIONS)
 E. -3 (NUMERICAL VALUE. Q.11 Let S_k , $k = 1, 2,..., 100$, denote the sum of the in
 $\frac{1}{k+2} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, where geometric series whose first term is $\$ **ERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**
 ERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)
 ERCISE - 3 (NUMERICAL VALUE. Q.11 Let S_k , $k = 1, 2, ..., 100$, denote the sum of the
 $\frac{1}{k + k\sqrt{k + 2}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, w **ERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**
 ERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)
 ESSED ANUMERICAL VALUE. Q.11 Let S_k , $k = 1, 2, ..., 100$, denote the sum of
 $\frac{1}{k + k\sqrt{k + 2}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, where geom **EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)**
 EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)
 I $\sqrt{k} + k\sqrt{k+2} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, where geometric series whose first term is
 $1, 113$, then find the value of $a^2 +$ **EXERCISE - 3 (NUMERICALVALUE BASED QUESTIONS)**
 EXERCISE - 3 (NUMERICALVALUE BASED QUESTIONS)
 EXERCISE - 3 (NUMERICALVALUE BASED QUESTIONS)
 $\lim_{k \to 1} \sum_{k=1}^{\infty} \frac{1}{(k+2)\sqrt{k} + k\sqrt{k+2}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$, where
 $\$ **EXERCISE -3 (NUMERICAL VALUE BASED QUESTIONS)**
 $\lim_{k \to 1} \frac{8}{(k+2)\sqrt{k} + k$ geometric series whose first term is $\frac{k-1}{k!}$ and the WANCED LEARNING

of the infinite
 $\frac{k-1}{k!}$ and the **ED QUESTIONS)**
 $k = 1, 2, ..., 100$, denote the sum of the infinite

ric series whose first term is $\frac{k-1}{k!}$ and the

ratio is 1/k. Then the value of
 $\sum_{k=1}^{100} |(k^2 - 3k + 1) S_k|$ is -
 $2, a_3, ..., a_{11}$ be real numbers sa **E BASED QUESTIONS)**
 EXECTIONS
 EXE EXECTIONS
 EXECTIONS

Let S_k , $k = 1, 2, ..., 100$, denote the sum of the infinite

recometric series whose first term is $\frac{k-1}{k!}$ and the

remmon ratio is 1/k. Then the value of
 $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k$ **BASED QUESTIONS)**

Et S_k, k = 1, 2,...., 100, denote the sum of the infinite

cometric series whose first term is $\frac{k-1}{k!}$ and the

mmon ratio is 1/k. Then the value of
 $\frac{00^2}{00!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ i **E BASED QUESTIONS)**

Let S_k, k = 1, 2,...., 100, denote the sum of the infinite

geometric series whose first term is $\frac{k-1}{k!}$ and the

common ratio is 1/k. Then the value of
 $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k +$ **EXED QUESTIONS)**

S_k, k = 1, 2,...., 100, denote the sum of the infinite

metric series whose first term is $\frac{k-1}{k!}$ and the

mon ratio is 1/k. Then the value of
 $\frac{y^2}{0!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ is –
 a_1 **EXECTIONS**

SubMADYANGED **EXECTIONS**

S_k, k = 1, 2,..., 100, denote the sum of the infinite

metric series whose first term is $\frac{k-1}{k!}$ and the

mmon ratio is 1/k. Then the value of
 $\frac{0^2}{0!} + \sum_{k=1}^{100} |(k^2 - 3$

common ratio is 1/k. Then the value of

$$
\frac{00^2}{100!} + \sum_{k=1}^{100} | (k^2 - 3k + 1) S_k | is -
$$

 $\sum \frac{1}{\sqrt{2}} = a + b\sqrt{2}$, then $a + b = 27 - 2a_2 > 0$ and $a_k = 2_{ak-1} - a_{k-2}$ for $k = 3, 4, ..., 11$. If **Q.12** Let $a_1, a_2, a_3, ..., a_{11}$ be real numbers satisfying $a_1 = 15$,

$$
\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90
$$
, then the value of

$$
\frac{a_1 + a_2 + \dots + a_{11}}{11}
$$
 is equal to :

EBASED QUESTIONS)

Let S_k, k = 1, 2,...., 100, denote the sum of the infinite

geometric series whose first term is $\frac{k-1}{k!}$ and the

common ratio is 1/k. Then the value of
 $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1$ **Q.13** Let $a_1, a_2, a_3, \dots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_{i}$, $1 \le p \le 100$. For any integer $-3k + 1$) S_k is $-$
 a_{11} be real numbers satisfying $a_1 = 15$,
 $a_k = 2_{ak-1} - a_{k-2}$ for $k = 3, 4, ..., 11$. If
 $\frac{21}{11} = 90$, then the value of
 $\frac{11}{11}$ is equal to :
 a_{100} be an arithmetic progression with
 whose first term is $\frac{k-1}{k!}$ and the

c. Then the value of
 $3k + 1$) S_k is $-$
 $s = 2$ is $k-1 - a_{k-2}$ for $k = 3, 4, ..., 11$. If
 $s = 90$, then the value of

is equal to :
 $s = 100$ be an arithmetic progression with
 a, 100, denote the sum of the infinite

is whose first term is $\frac{k-1}{k!}$ and the

1/k. Then the value of
 $-3k+1$ S_k is $-$
 a_{11} be real numbers satisfying $a_1 = 15$,
 $a_k = 2_{ak-1} - a_{k-2}$ for $k = 3, 4, ..., 11$. If $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then $\frac{a_1 + a_2 + + a_{11}}{11}$ is equal to :

Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with
 $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_{i,1} \le p \le 100$. For any integer n with
 $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does no

value of a_2 greater than 3 is –

- **Q.8** The sum $\sum_{n=1}^{\infty} \frac{1}{2^{k}}$ equal to and $a_{20} = 25$. The least positive integer n for which **Q.14** Let a_1, a_2, a_3, \dots be in harmonic progression with $a_1 = 5$ $a_n < 0$ is
	- **Q.15** A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$
	- **Q.16** The harmonic mean of the roots of the equation

$$
(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0
$$
 is

Q.17 The number of solutions of $log_4(x-1) = log_2(x-3)$ is

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

 (C) AGP

- **Q.6** Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- **[AIEEE 2004]** $(A) x² + 18x + 16 = 0$ (B) $x² - 18x +$ (B) $x^2 - 18x + 16 = 0$ $(C) x^2 + 18 x - 16 = 0$ (D) $x^2 - 18 x -$ (D) $x^2 - 18x - 16 = 0$
- **Q.7** Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m,

n, m \neq n, T_m = $\frac{1}{n}$ and T_n = $\frac{1}{m}$, then a – d equals- Q.15 Sur **[AIEEE 2004]** $(A) 0$ (B) 1

$$
(\overline{}) \cdot
$$

$$
(D) \frac{-}{m} + \frac{1}{m}
$$

Q.8 The sum of the first n terms of the series

$$
1^2 + 2
$$
, $2^2 + 3^2 + 2$, $4^2 + 5^2 + 2$, $6^2 + \dots$ is $\frac{n (n + 1)^2}{2}$ when

n is even. When n is odd the sum is- **[AIEEE 2004]**

mean 4. Then these numbers are the roots of the
\nquadratic equation.
\n(A)
$$
x^2 + 18x + 16 = 0
$$
 (B) $x^2 - 18x - 16 = 0$ (C) $x^2 + 18x - 16 = 0$ (D) $x^2 - 18x - 16 = 0$ (E) $x^2 - 18x - 16 = 0$ (D) $x^2 - 18x - 16 = 0$ (E) x^2

Q.9 If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in -

- **Q.10** If in a $\triangle ABC$, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in (A) G.P. (B) A.P. **[AIEEE- 2005]** $(C) \text{AGP}$ (D) H.P.
- **Q.11** Let a_1, a_2, a_3, \dots be terms of an A.P. If

SK STUDY MATERIAL: MATHEMATICS
\n**(A)** GP **(B)** AP **[AIEEE 2005]**
\n**(C)**AGP **(D)** HP
\n**1** If in a
$$
\triangle ABC
$$
, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in (A) G.P.
\n**(C)** AGP **(D)** H.P.
\n**(A)** G.P. **(B)** A.P. **[AIEEE-2005]**
\n**(C)** AGP **(D)** H.P.
\nLet a_1 , a_2 , a_3 , be terms of an A.P. If
\n
$$
\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}
$$
, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals –
\n**(A)** 7/2 **(B)** 2/7 **[AIEEE-2006]**
\n**(C)** 11/41 **(D)** 41/11
\n**2** If a_1 , a_2 , a_n are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to –
\n**(A)** $(n-1)(a_1 - a_n)$ **(B)** na_1a_n

- **Q.12** If a_1, a_2, \dots, a_n are in H.P., then the expression $a_1 a_2 + a_2 a_3$ 3 $+ ... + a_{n-1}a_n$ is equal to – [AIEEE- 2006] (A) $(n-1) (a_1 - a_n)$ (B) $na_1 a_n$ (D) $n(a_1 - a_n)$ (C) (n – 1) a_1a_n **Q.13** In a geometric progression consisting of positive terms,
- each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE- 2007]**

(A)
$$
\frac{1}{2}
$$
 (1 - $\sqrt{5}$)
\n(B) $\frac{1}{2}\sqrt{5}$
\n(C) $\frac{1}{2}\sqrt{5}$
\n(D) $\frac{1}{2}(\sqrt{5}-1)$

- **Q.14** The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[AIEEE 2008]** $(A) - 12$ (B) 12 (C) 4 (D) – 4
- $\frac{1}{\text{m}}$, then a d equals-
Q.15 Sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$ 3^3 3^4 10 3^2 3^3 3^4 6 $10 \t14$ 3^{2} 3^{3} 3^{4} $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$ (A) 2 (B) 3 **[AIEEE 2009]**
	- (C) 4 (D) 6
- assuance of the meanus the sum of the next two terms.

an acircle

na circle

non-zero solution,

(A) $\frac{1}{2}$ (1 $\sqrt{5}$) (B) $\frac{1}{2}$ $\sqrt{5}$

1.41 EEE 2003]

1.8.18 EEE 2004]

1.8.18 EEE 2004]

1.8.16 EE 2004]

1.8 and and good and term equals the sum of the next two terms. Then the

non-zero solution,
 $\frac{1}{2}$ (A) $\frac{1}{2}$ (1 - $\sqrt{5}$) (B) $\frac{1}{2}$ $\sqrt{5}$
 $\frac{1}{2}$ (A) $\frac{1}{2}$ (A) $\frac{1}{2}$ (D) $\frac{1}{2}$ ($\sqrt{5}$ -1)

no a $t + \frac{1}{2}$ the number of notes he counts in the nth minute. If [AILEEE 2003]

1. P.

A.P.

A.P. IS IS with the fourth diamed the fourth terms is 48. If 2 (C) 135 minutes (D) 24 minutes (A) 34 minutes when when ne roos of the COM and the Counter of netation and up to
 μ (AIEEE 2004] (2.14 The first two terms of a geometric progression add up to
 μ (2.18x + 16 = 0

tems of the direct progression are alternately positive

an -16 x-16 -0

and negative, then the first term is [AIEEE 2008]

2 positive integers m,

(C) 4 (B) 12

m a -d equals-

(L) -4

(A) -12 (B) 12

(B) 12

m a -d equals-

(C) 4 (B) 3

(D) -4

(D) -4

(D) -4

(D) -4

(B) 3

(B) $x^2-18x+16=0$
 $x^2-18x-16=0$
 $x^2-18x-16=0$

Lems of the geometric progression are alternately positive

and negative, then the first term is and

(A)-12

me positive integers m,

(C)4

(C) 4

(C) (B) 3

(LAIEEE 2008]
 $x^2-18x-16=0$ terms of the geometric progression are alternately positive

nose first term is a and
 $(A) - 12$

nose first term is a and
 $(A) - 12$

nose institute in the first term is and
 $(A) - 12$
 $(B) - 4$
 $(C) + 4$
 $(D) -$ (D) $\frac{1}{m} + \frac{1}{n}$

the number of notes he counts
 $a_1 = a_2 = = a_{10} = 150$ and a_{10}
 $a_1 = a_2 = = a_{10} = 150$ and a_{10}
 $a_2 = 150$ and a_{11}
 $a_3 = 150$ and a_{11}
 $a_4 = a_2 = = a_{10} = 150$ and a_{11}

count all n **Q.16** A person is to count 4500 currency notes. Let a_n denote $a_1 = a_2 = \dots = a_{10} = 150$ and a_{10} , a_{11} , are in A.P. with common difference –2, then the time taken by him to count all notes is – **[AIEEE 2010]** (B) 125 minutes
	- 2 previous month. His total saving from the start of service $\frac{1}{10}$ increases by Rs. 40 more than the saving of immediately (C) 20 months **Q.17** A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving will be Rs. 11040 after : **[AIEEE 2011]** (A) 18 months (B) 19 months (D) 21 months

SEQUENCES & SERIES QUESTION BANK

- **Q.18 Statement-1 :** The sum of the series
	- $1+(1+2+4)+(4+6+9)+(9+12+16)$ $+ ... + (361 + 380 + 400)$ is 8000.

Statement-2:
$$
\sum_{k=1}^{n} (k^3 - (k-1)^3 = n^3
$$
, for any natural

number n. **[AIEEE 2012]**

(A) Statement-1 is false, Statement-2 is true.

(B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1. (C) Statement-1 is true, statement-2 is true; statement-2

is not a correct explanation for Statement-1. (D) Statement-1 is true, statement-2 is false.

- **Q.19** If 100 times the 100th term of an AP with non zero common difference equals the 50 times its $50th$ term, then the $150th$ term of this AP is : **[AIEEE 2012]** (A) –150 (B) 150 times its 50th term (C) 150 (D) zero From 1.1 is the statement-2 is the statement-2 (A) 130 if the statement-2 (C) 74

From 1.1 (C) 74

Statement-1 is false, Statement-2 is true; statement-2 c quality of $(1\frac{3}{5})^2 + (2\frac{2}{5})^2 + (3\frac{1}{5})^2 + 4^2 + (4\frac{4}{5})^2$ ement-2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3 = n^3)$, for any natural 0.27 The sum of the interst of

ber n. [AIEEE 2012] $\left(\frac{1^3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2$

Statement-1 is false, Statement-2 is true; st
- **Q.20** The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is – **[JEE MAIN 2013]**

(A)
$$
\frac{7}{81}(179-10^{-20})
$$
 (B) $\frac{7}{9}(99-10^{-20})$
(C) $\frac{7}{81}(179+10^{-20})$ (D) $\frac{7}{9}(99+10^{-20})$

- **Q.21** If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then – **[JEE MAIN 2013]** (A) $x = y = z$ (B) $2x = 3y = 6z$ (C) $6x = 3y = 2z$ (D) $6x = 4y = 3z$
- **Q.22** Three positive numbers from an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is –

[JEE MAIN 2014]

- **Q.23** If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9 = k$ $(10)^9$, then k is equal to **[JEE MAIN 2014]** (A) 121/10 (B) 441/100 $(C) 100$ (D) 110
- **Q.24** The sum of first 9 terms of the series

(C)
$$
\frac{7}{81}(179+10^{-20})
$$
 (D) $\frac{7}{9}(99+10^{-20})$ the first 40 terms of the series
\n11x, y, z are in A.P. and tan⁻¹x, tan⁻¹y and tan⁻¹z are also
\nin A.P., then -
\n(D) $\frac{7}{9}(99+10^{-20})$ (B) 2x=2y=6z
\n(A) x = y = z
\n(B) 2x = 3y = 6z
\n(C) 6x = 3y = 2z
\n(D) 6x = 4y = 3z
\n(D) 6x = 4y = 3z
\n311 f, the
\n120.311 f, the
\n211 f, the
\n222 h, the
\n233 f, the
\n234 f, the
\n245 h, the
\n246 f, the
\n247 g, the
\n248 g, the
\n249 h, the
\n240 h, the
\n241 h, the
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\n245 h, the
\n245 h, the
\n246 h, the
\n247 h, the
\n249 h, the
\n25 h, the
\n261 h, the
\n27 h, the
\n28 h, the
\n29 h, the

Q.25 If m is the A.M. of two distinct real numbers *l* and $n(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between *l* and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. (A) 4 lm^2 n (B) ⁴ lmn^2 [JEE MAIN 2015] (C) 4 $l^2m^2n^2$ (D) 4 l^2 mn

- **QUESTION BANK**

m of the series
 $+9)+(9+12+16)$
 $+...+(361+380+400)$ is 8000.

(A) 4/3
 $(k-1)^3 = n^3$, for any natural
 Q.27 If the sum of the first ten terms of the first ten terms of the first ten terms of the first ten **RIES**

he sum of the series
 $(4+6+9)+(9+12+16)$
 $+....+(361+380+400)$ is 8000.
 C.26 If the 2nd, 5th and

G.P., then the com
 $+....+(361+380+400)$ is 8000.

(A) 4/3

(C) 7/4
 $\sum_{k=1}^{n} (k^3 - (k-1)^3 = n^3)$, for any natural
 S

Sum of the series
 $(6+9)+(9+12+16)$
 $+...+(361+380+400)$ is 8000.
 $(k^3-(k-1)^3 = n^3$, for any natural
 $(k^3-(k-1)^3 = n^3)$ **EXECUTE:**

He sum of the series

(4+6+9)+(9+12+16)
 $+...+ (361+380+400)$ is 8000.
 $\sum_{k=1}^{n} (k^3 - (k-1)^3 = n^3)$, for any natural
 $\begin{array}{ll} \textbf{(A + B + B)} & \textbf{(A + B)} & \textbf{(B + B)} & \textbf{(C + B)} \\ & \textbf{(D + B)} & \textbf{(E + B)} \\ & \textbf{(E + B)} & \textbf{(E + B)} \\ & \textbf{(D + B$ **Q.26** If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is (A) 4/3 (B) 1 **[JEE MAIN 2016]** (C) 7/4 (D) 8/5 **Q.27** If the sum of the first ten terms of the series **EDENTE ADVANCED LEARNING**

F the 2nd, 5th and 9th terms of a non-constant A.P. are in

2P., then the common ratio of this G.P. is

A) 4/3 (B) 1 [JEE MAIN 2016]

2D 7/4 (D) 8/5

f the sum of the first ten terms of t **SOMADVANCED LEARNING**
the 2nd, 5th and 9th terms of a non-constant A.P. are in
P., then the common ratio of this G.P. is
 (19)
 $1/3$
 (19)
 $1/4$
the sum of the first ten terms of the series
 $\left(\frac{3}{5}\right)^2 + \left(2\frac{2$ If the 2nd, 5th and 9th terms of a non-constant A.P. are in

GP., then the common ratio of this G.P. is

(A) 4/3 (B) 1 **JJEE MAIN 2016**

(C) 7/4 (D) 8/5

If the sum of the first ten terms of the series
 $\left(1\frac{3}{5}\$
	-

$$
\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots
$$
 is $\frac{16}{5}$ m, then m is

- equal to **[JEE MAIN 2016]** (A) 101 (B) 100 (C) 99 (D) 102
- **Q.28** For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **[JEE MAIN 2017]**
	- (A) a, b and c are in A.P. (B) a, b and c are in G.P. (C) b, c and a are in G.P. (D) b, c and a are in A.P.
- **Q.29** Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that

80+400) is 8000. (A) 4/3 (B) 1 [JEE MANN 2016]
\nfor any natural
\n
$$
Q.27
$$
 If the sum of the first ten terms of the series
\n[**AIEEE 2012**]
\n
$$
= \left(\frac{13}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 +
$$
\nis $\frac{16}{5}$ m, then m is
\ntrue.
\ntrue; statement-2
\n(0, 101 (B) 100
\n(10, 101 (C) 99 (D) 102
\n(10, 102 (D) 102
\n(10, 102 (E) 102
\n(10, 102

- 21 = 22 = 15b (3a + 400) is 8000.

(A) 43

(B) 1 [DEE MAIN 2016]

(A) 43

(B) 1 [DEE MAIN 2016]

(B) 1 [DEE MAIN 2016]

(A) 40 a = n³, for any natural **0.27** The sum of the intertains of the series
 -2 is true.
 $+10^{-20}$)
 $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+....$ difference equals to $\frac{7}{81}(179+10^{-20})$

(B) 150 times its 50th term, then the 150th cand a arc in GP.

(A) -150

(A) 150 times its 50th term

(A) 150 times its 50th term

(A) 150 times its 50th term

(A) $\frac{7}{81}(1$ (A) 150 (B) 150 lines its 50th term

(C) 150

(C) 150

The sum of first 20 terms of the sequence 0.7, 0.77,

(C) 2π

(D) 2π

(A) $\frac{7}{81}(179-10^{-20})$

(B) $\frac{7}{9}(99-10^{-20})$

(B) $\frac{7}{9}(99-10^{-20})$

(C) $\frac{7}{81}(1$ 179-10) (B) $\frac{7}{9}$ (D) $\frac{7}{9}$ (99-10)

179+10⁻²⁰) (B) $\frac{7}{9}$ (99-10⁻²⁰)

a the first 40 terms of the sum of the first 20 terms

are in A.P. and tan⁻¹x, tan⁻¹y and tan⁻¹z are also

a the first 40 terms 81

81
 $\frac{7}{81}(179+10^{-20})$
 $\frac{7}{8}(99+10^{-20})$
 $\frac{12}{12}+2.2^2+3^2+2.4^2+5^2+2.6^2+....$
 $\frac{1}{12}+2.2^2+3^2+2.4^2+5^2+2.6^2+....$
 $\frac{1}{12}+2.2^2+3^2+2.4^2+5^2+2.6^2+....$
 $\frac{1}{12}+2.2^2+3^2+2.4^2+5^2+2.6^2+....$
 $\frac{1}{12}+$ 179+10⁻²⁰) (D) $\frac{7}{9}(99+10^{-20})$

are in A.P. and tan⁻¹x, tan⁻¹y and tan⁻¹z are also

are in A.P. and tan⁻¹x, tan⁻¹y and tan⁻¹z are also

IFB-224 -3²+22²+2²+2²+2² -10², then is step is

(A)464
 Q.30 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series If $B - 2A = 100\lambda$, then λ is equal to: **[JEE MAIN 2018]** (A) 464 (B) 496 (C) 232 (D) 248 then m equal to

(3) 33 [JEE MAIN 2018]

(3) 68

erms and B be the sum of
 $+$

1to: [JEE MAIN 2018]

(3) 496

(3) 248

al numbers in G. P. and

JEE MAIN 2019 (JAN)]

(3) -3

(3) 2

(3) 2

(4) a

(4) =

(3) EC MAIN 2 IN 2018]
e sum of
N 2018]
P. and
P. (JAN)]
(2i-1). I c are in G.P.

1 a are in A.P.

t

ual to

IE **MAIN 2018**

B be the sum of

E **MAIN 2018**

s in G. P. and

IN 2019 (JAN)]

T = $\sum_{i=1}^{15} a_{(2i-1)}$.

IN 2019 (JAN)]
	- **Q.31** If a, b and c be three distinct real numbers in G. P. and $a + b + c = xb$, then x cannot be :**[JEE MAIN 2019 (JAN)]** (A) 4 (B) –3 $(C) -2$ (D) 2 (D) 248

	(D) 248

	= xb, then x cannot be :[JEE MAIN 2019 (JAN)]

	(B) -3

	(D) 2

	...,a₃₀ be an A. P., S = $\sum_{i=1}^{30} a_i$ and T = $\sum_{i=1}^{15} a_{(2i-1)}$.

	7 and S - 2T = 75, then a_{10} =

	[JEE MAIN 2019 (JAN)]

	(B) 47

	- **Q.32** Let $a_1, a_2, ..., a_{30}$ be an A. P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.

If $a_5 = 27$ and $S - 2T = 75$, then $a_{10} =$ **[JEE MAIN 2019 (JAN)]**

(A) 57 (B) 47 (C) 42 (D) 52

Q.33 The sum of all natural numbers 'n' such that $100 \le n \le 200$ and H.C.F. $(91, n) > 1$ is:

[JEE MAIN 2019 (APRIL)]

(A) 3221 (B) 3121 (C) 3203 (D) 3303

Q.34 The sum $\sum k \frac{1}{k}$ is eq. 20 $\overline{1}$ k is equal to [order which $\left(\text{m} \right)$ $\sum_{k=1}^{20}$ k $\frac{1}{2^k}$ is equal to-**[JEE MAIN 2019 (APRIL)]**

(A)
$$
2 - \frac{3}{2^{17}}
$$
 (B) $2 - \frac{11}{2^{19}}$

(C)
$$
1 - \frac{11}{2^{20}}
$$
 (D) $2 - \frac{21}{2^{20}}$

Q.35 If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **[JEE MAIN 2019 (APRIL)]** (A) d, e, f are in A.P. (B) $\frac{d}{\rho}$, $\frac{e}{b}$, $\frac{f}{c}$ are in G.P. **EXECUTE ANTIFICATE (SUESTION BANK**

1GP. and the equations
 O.43 If a_1 , a_2 , a_3 ,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$,

then the sum of the first 15 terms of this A.P. is :
 MAIN 2019 (APRIL)
 MAIN COUBSTION BANK STUDY MATERIAL: MATHEMATICS

1GP. and the equations Q.43 If a_1 , a_2 , a_3 ,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$,
 $= 0$ have a common root, then the sum of the first 15 terms of this A.P. (C) $\frac{d}{d}$, $\frac{e}{b}$, $\frac{f}{c}$ are in A.P. **CUESTION BANK** STUDY MATERIAL: M.

STUDY MATERIAL: M.

The struct numbers a,b,c are in G.P. and the equations Q.43 If a_1 , a_2 , a_3 ,..... are in A.P. such that $a_1 + 2bx + c = 0$ and $\alpha^2 + 2ex + f = 0$ have a common root, **EXERIBING**

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THE MAIN COUBSTION BANK THE SUBSERVERTED SUBSTION BANK

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(A) d) and the first is correc **Q.36** Let the sum of the first n terms of a non-constant A.P., a_1 , a_2, a_3, \dots be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If Q.45 Find the sum **EXERCISE TON BANK**

SIUDY MATERIAL: MATHEMATICS

edistinct numbers a,b,c are in GP. and the equations **Q.43** If a_1 , a_2 , a_3 ,..... are in A.P. such that $a_1 + a_2 + a_16 = 40$,

then the sum of the first 15 terms of t d is the common difference of this A.P., then the ordered pair (d, a₅₀) is equal to **[JEE MAIN 2019 (APRIL)]**
(A) (A, 50+46A) (B) (A, 50+45A) $(A) (A, 50 + 46A)$ $(C)(50, 50+46A)$ $(D)(50, 50+45A)$ **Q.37** If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11th term is **[JEE MAIN 2019 (APRIL)]** (A) –25 (B) 25 (D) –36 (D) –35 **Q.38** The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \dots$ upto 11th term is :- **[JEE MAIN 2019 (APRIL)]** (A) 915 (B) 946 (C) 945 **Q.39** The sum $\frac{3 \times 1^3}{1^2} + \frac{5 \times (1^3 + 2^3)}{1^2 + 2^2} + \frac{7 \times (1^3 + 2^3 + 3^3)}{1^2 + 2^2 + 3^2} + ...$ e in A.P. (D) d, e, f are in G.P.

the first n terms of a non-constant A.P., a₁,

(D) 4

the first n terms of a non-constant A.P., a₁,

on + $\frac{n(n-7)}{2}$ A, where A is a constant. If **Q.45** Find the sum $\sum_{k=1}^{20} (1$ of the first n terms of a non-constant A.P., a₁,
 $20 + 3 + 2 = 2$
 $20 + 2 = 2$ 10 one to the COMERA SOMERTIAL (A) 200

are in A.P. (B) $\frac{d}{dt} \cdot \frac{8}{b} \cdot \frac{5}{c}$ are in GP.

(B) $\frac{d}{dt} \cdot \frac{8}{b} \cdot \frac{c}{c}$ are in GP.

(D) $\frac{d}{dt} \cdot \frac{8}{b} \cdot \frac{c}{c}$ are in GP.

(D) $\frac{1}{a} \cdot \frac{c}{b} \cdot \frac{c}{c}$ are i e in A.P.

(B) $\frac{d}{a} = \frac{c}{b}$ are in GP.

(B) $\frac{d}{a} = \frac{c}{b}$ are in GP.

(D) d,e, fare in GP.

(D) d,e, fare in GP.

(D) d,e, fare in GP.

(C) a (B) and the first temms of the CHAIN 2020 (JAN)

(C) a (B) d) 4

of are in A.P. (b) distants as content (A) 2000

one of the following statematics is content (A) 2000

(D) 2000

in A.P. (B) $\frac{d}{dt} \times \frac{d}{dt} = \frac{d}{dt} \times \frac{d}{dt}$ are in AP. (C) 120

one of the find statematic is content?

(and $x^2 + 2x + 1 = 0$ have common root,

(B) $\frac{d}{dx} = \frac{6}{b} = \frac{6}{c}$ are in GP.

(B) $\frac{d}{dx} = \frac{6}{b} = \frac{6}{c}$ are in GP.

(D) d, e, f are in GP.

(D following statements is orcered?

(B) $\frac{d}{dt} \sum_{i=1}^{n} \sum_{j=1}^{n} \arcsin(2R/L)$

(B) $\frac{d}{dt} \sum_{i=1}^{n} \frac{f}{t_i}$ are in GP.

(B) $\frac{d}{dt} \sum_{i=1}^{n} \frac{f}{t_i}$ are in GP.

(D) d, e, f.are in GP.

(D) d, e, f.are in GP.

(D) d, **[JEE MAIN 2019 (APRIL)]** $(A) 660$ (B) 620 (C) 680 (D) 600 **Q.40** If $a_1, a_2, a_3, \dots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \dots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to : **[JEE MAIN 2019 (APRIL)]** (A) 38 (B) 98 (C) 76 (D) 64 **Q.41** The sum $1 + \frac{1^3 + 2^3}{1 \cdot 2} + \frac{1^3 + 2^3 + 3^3}{1 \cdot 2 \cdot 3} + ...$ squal to **FREMANY 2019 (APRIL)** Q.46 For an A.P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of first 200

(AN) (6) (6), 50-45A) the model of the fact there teen in an A.P. are

spectrively, then a value of its 11th term is

(A ... 1 2 1 2 3 EVERTAIN 2020 (JAN)

(B) (DE MAIN 2020 (JAN)

(B) (A, S0+45A)

(B) (A, S0+45A)

(D) (A, S0+45A)

(D) (A, S0+45A)

(D) (A, S0+45A)

(C) 301 $\frac{1}{2}$

(P) -13

(C) 301 $\frac{1}{2}$

(D) -3

(C) -3020 $\frac{1}{2}$

(D) -3

(D) -3 difference of this AP, then the ordered

(a) [B(A, 50+43A) [B(A) + 43₁₆ = 1/20, T₂₀ = 1/10. Find sum of first 2010

(a) (B)(A, 50+45A)

(B)(A) = (B)(A) + (B) = (B)(B) + (B) = (B)(B) = (B)

(B) = (B) = (B) = (B) = (B) 3 3 $\frac{(3)}{(2)}$... 15 $\frac{(3) \times 10^{-15}}{(10 \times 20)}$... 15 ... 15 ... 16 ... 15 ... 16 ... 15 ... 16 ... 16 ... 15 ... 16 ... 16 ... 16 ... 16 ... 16 ... 16 ... 16 ... 16 ... 15 ... 16 ... 15 ... 15 ... 15 ... 15 ... 17 ... 1 (a) (b)(A,50+45A)

(a) (i)(S0,50+45A)

(a) (i)(S0,50+45A)

(a) (d)(S0,50+45A)

(a) (d)(S0,50+45A)

(a) (d) (d) 35

(a) (d) -35

(b) -35

(d) -35

(e) -35

(d) -35

(e) +6A) (D)(60,50+45A)

(D)(60,50+45A)

(D)(60,50+45A)

(D)(60,50+45A)

(D)(60,50+45A)

(A)201 ¹₂

(B) 101 ¹₂

(B) 101 ¹₂

(B) 101 ¹₂

(B) 101 ¹₂

(B) 102

(B) 35

(B) 35

(B) 26

(D)(60,50+45A)

(C) 30 oduct of the first three term in an A.P. are
 (B) 1014 $\frac{1}{2}$

(B) 101 $\frac{1}{2}$

(B) 1013 $\frac{1}{2}$

(B) 103 $\frac{1}{2}$

(B) 103 $\frac{1}{2}$

(B) 25

(B) 25

(B) 25

(B) 25

(B) 25

(B) 27

(B) 27

(B) 27

(B) 27

(B) 27
 [JEE MAIN 2019 (APRIL)] (A) 1240 (B)1860 (C) 660 (D) 620 **Q.42** Let a, b and c be in G. P. with common ratio r, where $a \neq 0$ and $0 < r \le 1/2$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is : **[JEE MAIN 2019 (APRIL)] Q.43** If a_1 , a_2 , a_3 ,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : **[JEE MAIN 2019 (APRIL)]** (A) 200 (B) 280 (C) 120 (D) 150 **Q.44** If $(2^{1-x} + 2^{1+x})$, $f(x)$, $(3^{x} + 3^{-x})$ are in A.P. then minimum value of $f(x)$ is **[JEE MAIN 2020 (JAN)**] $(A) 1$ (B) 2 (C) 3 (D) 4 **Q.45** Find the sum $\sum (1 + 2 + 3 + 3)$ 20 **STUDY MATERIAL: MATHEMATICS**

.. are in A.P. such that $a_1 + a_7 + a_{16} = 40$,

f the first 15 terms of this A.P. is :
 [JEE MAIN 2019 (APRIL)]

(B) 280

(D) 150

5
 [JEE MAIN 2020 (JAN)]

(B) 2

(D) 4

20

20

20

20
 STUDY MATERIAL: MATHEMATICS

.. are in A.P. such that $a_1 + a_7 + a_{16} = 40$,

f the first 15 terms of this A.P. is :

[JEE MAIN 2019 (APRIL)]

(B) 280

(D) 150

), f (x), (3^x + 3^{-x}) are in A.P. then minimum
 IJEE MAI [JEE MAIN 2020 (JAN)] Q.46 For an A.P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of first 200 [**JEE MAIN 2020 (JAN)**] $(A) 201\frac{1}{2}$ $\frac{1}{2}$ (B) $101\frac{1}{2}$ 2 $\sqrt{2}$ 1 2 (C) 301 $\frac{1}{2}$ $\frac{1}{2}$ (D) 100 $\frac{1}{2}$ 2 $(D) \frac{100}{2}$ 1 2 a set of \sim 3 a set of \sim **Q.47** $\sum_{ }^{\mathbf{m}}$ $\frac{7}{2}$ n (n + 1) (2n + 1) CO 120

If (2^{1-x} + 2^{1+x}), f(x), (3^x + 3^{-x}) are in A.P. then minimum

value of f(x) is

(A) 1

(A) 1

(B) 2

(C) 3

(C) 3

Find the sum $\sum_{k=1}^{20} (1+2+3+....+k)$

For an A.P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of STUDY MATERIAL: MATHEMATICS

1, a₂, a₃,..... are in A.P. such that a₁ + a₇ + a₁₆ = 40,

the sum of the first 15 terms of this A.P. is:

[JEE MAIN 2019 (APRIL)]

200 (B) 280

212

201 (B) 280

201 (B) 150

201 (B 4 **STUDY MATERIAL: MATHEMATICS**

If a_1 , a_2 , a_3 ,...., are in A.P. such that $a_1 + a_7 + a_{16} = 40$,

then the sum of the first 15 terms of this A.P. is:

[JEE MAIN 2019 (APRIL)]

(D) 150

(C) 2120

(C) 220

(D) 150

(D **Q.48** The product $\frac{1}{2^4}$, $\frac{1}{4^{16}}$, $\frac{1}{8^{48}}$, $\frac{1}{16^{128}}$, to ∞ is equal to : , f(x), (3^x + 3^{-x}) are in A.P. then minimum

[JEE MAIN 2020 (JAN)]

(B) 2

(D) 4

20

20

20

20

21 (1+ 2 + 3 + ... + k)

[JEE MAIN 2020 (JAN)]

= 1 /20, T₂₀ = 1/10. Find sum of first 200

[JEE MAIN 2020 (JAN)]

(B ... are in A.P. such that $a_1 + a_7 + a_16 = 40$,

of the first 15 terms of this A.P. is :

[JEE MAIN 2019 (APRIL)]

(B) 280

(D) 150

S, f(x), (3x + 3^{-x})</sup> are in A.P. then minimum

(B) 2

(D) 4

(B) 2

(D) 4

(D) 4

(D) 4
 [JEE MAIN 2020 (JAN)] (B) 2^{1/4} $(C) 2$ (D) 1 **Q.49** Let a_n be the nth term of a G.P. of positive terms. If 2^{a_2} x=1

x=1

100 100 1 100 1 100 1

m.

100 1 2

100 1

100 1
 A.P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of first 200

[JEE MAIN 2020 (JAN)]
 $\frac{1}{2}$ (B) 101 $\frac{1}{2}$

(B) 101 $\frac{1}{2}$

(D) 100 $\frac{1}{2}$

(D) 10 r an A.P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of first 200

m.

1 **JEE MAIN 2020 (JAN)**

1 201 $\frac{1}{2}$

(B) 101 $\frac{1}{2}$

(D) 100 $\frac{1}{2}$

1 (D) 100 \frac (D) 4

(D) 4

the sum $\sum_{k=1}^{20} (1+2+3+....+k)$

[JEE MAIN 2020 (JAN)]

14. P. T₁₀ = 1/20, T₂₀ = 1/10. Find sum of first 200

[JEE MAIN 2020 (JAN)]

11

12

(B) 101 $\frac{1}{2}$

(D) 100 $\frac{1}{2}$

(D) 100 $\frac{1}{2}$

(D) **STUDY MATERIAL: MATHEMAITCS**

1, a₂, a₃,..... are in A.P. such that a₁ + a₇ + a₁₆ = 40,

in the sum of the first 15 terms of this A.P. is:
 IFEE MAIN2019 (APRIL)

2200 (B) 280

(D) 150

122¹ + z³+ 3^{-x} + 200 $\sum_{n=1}^{\infty} a_n$ is equal um of first 200
 N 2020 (JAN)
 S equal to :
 N 2020 (JAN)

terms.
 $\sum_{n=1}^{200} a_n$ is equal
 N 2020 (JAN)
 A.P.'s
 $\sum_{n=1}^{200}$ (**JAN)** to – **[JEE MAIN 2020 (JAN)]** (A) 225 (B) 175 (C) 300 (D) 150 **Q.50** The number of terms common to the two A.P.'s 3, 7, 11,, 407 and 2, 9, 16,, 709 is **[JEE MAIN 2020 (JAN)] Q.51** Let $3 + 4 + 8 + 9 + 13 + 14 + 18 + \dots$ 40 terms = S. If $S = (102)$ m then m = **[JEE MAIN 2020 (JAN)]** $(A) 20$ (B) 25 $(C) 10$ (D) 5 **Q.52** $a_1, a_2, a_3, \ldots a_9$ are in GP where $a_1 < 0$, $a_1 + a_2 = 4$, $a_3 + a_4 = 16$, if $\sum a_i$ 9 $i = 4\lambda,$ i 1 to [JEE MAIN 2020 (JAN)]
 $16^{\frac{1}{128}}$... to ∞ is equal to :

[JEE MAIN 2020 (JAN)]

(B) 2^{1/4}

(D) 1

a.P. of positive terms.

n = 100_, then $\sum_{n=1}^{200} a_n$ is equal

[JEE MAIN 2020 (JAN)]

(B) 175

(D) 150

no then λ is equal to **[JEE MAIN 2020 (JAN)**] (A) –513 (B)–511/3

 (C) –171 (D) 171

ANSWER KEY

CHAPTER- 6 : SEQUENCES & SERIES SOLUTIONS TO TRY IT YOURSELF TRY IT YOURSELF-1

(1) The smallest and the largest numbers of three digits, which are divisible by 7 are 105 and 994, respectively. So, the sequence of three digit numbers which are divisible by 7 is 105, 112, 119, ... , 994. Clearly, it is an A.P. with first term $a = 105$ and common difference $d = 7$. Let there be n terms in this sequence. Then, $a_n = 994$ \implies a + (n - 1)d = 994 \Rightarrow 105 + (n - 1) × 7 = 994 \Rightarrow n = 128 Now, required sum is

$$
\frac{n}{2}[2a + (n-1)d] = \frac{128}{2}[2 \times 105 + (128 - 1) \times 7] = 70336
$$

- **(2)** Let the numbers be $(a-d)$, a, $(a+d)$. Therefore, $(a-d) + a + (a+d) = -3 \Rightarrow 3a = -3 \Rightarrow a = -1$ and $(a-d)(a)(a+d) = 8$ \implies a (a²-d²) = 8 \implies (-1) (1-d²) = 8 $[\because a = -1]$ (9) $\Rightarrow d^2 = 9 \Rightarrow d = \pm 3$ If $d = 3$, the numbers are $-4, -1, 2$. If $d = -3$, the numbers are $2, -1, -4$. So, the numbers are -4 , -1 , 2 or 2, -1 , -4 .
- **(3)** Let the digits at ones, tens and hundreds place be $(a-d)$, a and $(a + d)$, respectively. Then the number is $(a+d) \times 100 + a \times 10 + (a-d) = 111a + 99d$ The number obtained by reversing the digits is $(a-d) \times 100 + a \times 10 + (a+d) = 111a - 99d$ It is given that $(a-d) + a + (a+d) = 15$ (i) and $111a - 99d = 111a + 99d - 594$ \therefore 3a = 15 and 198d = 594 \Rightarrow a = 5 and d = 3 So, the number is $111 \times 5 + 99 \times 3 = 852$. 9 \Rightarrow d = + 3

the numbers are -4,-1, 2.

the numbers are -4,-1, 2

the numbers are $-4, -1, 2$

tunkers are $-4, -1, 2$

tunk numbers are -4, -1, 2.

holes are -2, -1, -4.

libs at ones, tens and hundreds place be $(a-d)$,

d), respectively. Then the number is

d), respectively. Then the number is
 $0 + a \times 10 + (a + d) = 11a + 99d$
 $-99d = 111a + 99d - 594$ 8 ⇒ (-1) (1 - d²) = 8 [∴ a = -1] (9) 3 or 9.

sum c= 4, -1, 2

ones are -4, -1, 2

ones are -4, -1, -4.

and hundreds place be (a - d),

mes, tens and hundreds place be (a - d),

pectively. Then the number is
 $\times 10 + ($
- (4) Assume A_1 , A_2 , A_3 , ..., A_{11} be the eleven A.M.'s between 28 and 10, so $28, A_1, A_2, \dots, A_{11}$, 10 are in A.P. Let d be the common difference of the A.P. The number of terms is 13. Now, $10 = T_{13} = T_1 + 12d = 28 + 12d$

$$
\Rightarrow d = \frac{10 - 28}{12} = -\frac{18}{12} = -\frac{3}{2}
$$

Hence, the number of integral A.M.'s is 5. **(5)** Let the four numbers in an A.P. be $a - 3d$, $a - d$, $a + d$, a+3d. Sum of the terms is , $4a = 20 \Rightarrow a = 5$ Sum of their squares is $4a^2 + 20d^2 = 120$ \implies 20d² = 120 - 4 × 25 = 20 \Rightarrow d² = 1 or d = ± 1 Hence, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2. **(6) (C).** $T_m = a + (m-1) d = 1/n$ and $T_n = a + (n-1) d = 1/m$

$$
\Rightarrow (m-n) d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}
$$

$$
\Rightarrow a = \frac{1}{mn} \therefore T_{mn} = a + (mn-1) d = \frac{1}{mn} + (mn-1) \frac{1}{mn} \Rightarrow 36(\frac{1+n}{2}) = 3(n^2+n)
$$

STUDY MATERIAL: MATHEMATICS
\n
$$
= \frac{1}{mn} + 1 - \frac{1}{mn} = 1
$$
\n5 + 8 + 2n terms
\n+ 59 + 61 + n terms
\n
$$
\frac{2n}{n} [4 + (2n - 1) 3] = \frac{n}{n} [114 + (n - 1) 2]
$$

$$
= 57 + 59 + 61 + \dots \dots \text{ n terms}
$$

 $(0, 2)$

ITIONS STUDY MATERIAL: MATHEMATICS
\n
$$
= \frac{1}{mn} + 1 - \frac{1}{mn} = 1
$$
\n(7) **(C).** 2 + 5 + 8 + 2n terms
\n
$$
= 57 + 59 + 61 + ... \text{ in terms}
$$
\n
$$
\Rightarrow \frac{2n}{2} [4 + (2n - 1) 3] = \frac{n}{2} [114 + (n - 1) 2]
$$
\n
$$
\Rightarrow 6n + 1 = n + 56 \Rightarrow 5n = 55 \Rightarrow n = 11
$$
\n(8) **(C).** S_n = cn² ; S_{n-1} = c (n - 1)² = cn² + c - 2cn
\n
$$
T_n = 2cn - c
$$
\n
$$
T_n^2 = (2cn - c)^2 = 4c^2 n^2 + c^2 - 4c^2 n
$$
\n
$$
Sum = \sum T_n^2 = \frac{4c^2.n(n+1)(2n+1)}{6} + nc^2 - 2c^2 n(n+1)
$$
\n
$$
= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2 n(n+1)}{3}
$$
\n
$$
= \frac{nc^2[4n^2 + 6n + 2 + 3 - 6n - 6]}{3} = \frac{nc^2(4n^2 - 1)}{3}
$$
\n(9) **3 or 9.**
\n
$$
\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6 + (5n - 1) d]}{2[6 + (n - 1) d]} = \frac{5[(6 - d) + 5nd]}{[(6 - d) + nd]}
$$
\n
$$
d = 6 \text{ or } d = 0.
$$
\nNow, if d = 0 then a₂ = 3 else a₂ = 9
\nFor single choice more appropriate choice is 9, but in principal, seems to have an error.

(9) 3 or 9.

$$
\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6 + (5n - 1) d]}{\frac{n}{2}[6 + (n - 1) d]} = \frac{5[(6 - d) + 5nd]}{[(6 - d) + nd]}
$$

 $d = 6$ or $d = 0$. Now, if $d = 0$ then $a_2 = 3$ else $a_2 = 9$ For single choice more appropriate choice is 9, but in principal, seems to have an error. \therefore a₂ = 3 + 6 = 9 $\frac{1}{2}[6 + (n-1) d]$
 $\frac{1}{2}$
 $\frac{1}{2} = 0$
 $\frac{1}{2} = 0$ then $a_2 = 3$ else $a_2 = 9$

le choice more appropriate choice is 9, but in

1, seems to have an error.
 $+6 = 9$
 RY IT YOURSELF-2

sfirst 9 terms is
 $= a^9 r^{36}$ = 3 else a₂ = 9

re appropriate choice is 9, but in

ave an error.
 URSELF-2

si is

= $a^9r^{1+2+...+8} = a^9r^{8/2}$ (1+8)
 $a^9r^{36} = (ar^4)^9 = 2^9 = 512$

d r the common ratio of the G.P.

-2 + a_{n+3} +∞], for all n ∈

TRY IT YOURSELF-2

(1) $t_5 = a r^4 = 2$ Product of its first 9 terms is a (ar) (ar^2) $(ar^8) = a^9r^{1+2+...+8} = a^9r^{(8/2)(1+8)}$ $= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

(2) Let a be the first term and r the common ratio of the G.P. then, $a_n = 2 [a_{n+1} + a_{n+2} + a_{n+3} + \dots \infty]$, for all $n \in N$ (Given) $ar^{n-1} = 2 [ar^{n} + ar^{n+1} + ... \infty]$

$$
\Rightarrow \quad \text{ar}^{\text{n}-1} = \frac{2\text{ar}^{\text{n}}}{1-\text{r}} \quad \Rightarrow \quad 1 = \frac{2\text{a}}{1-\text{r}} \quad \Rightarrow \quad \text{r} = \frac{1}{3}
$$

(3) Let the three numbers be a/r, a and ar. Then, product = 216. Hence, $(a/r) \times a \times ar = 216$ \Rightarrow $a^3 = 216$ \Rightarrow $a = 6$.

Sum of the products in pairs is 156. Hence,

1 1 m n 1 ^d n m mn mn 1 mn a a a a ar ar 156 r r ² 1 a r 1 156 r ² 1 r r 36 156 r 3 (r² + r + 1) = 13r 3r² – 10r + 3 = 0

28

$$
\Rightarrow (3r-1)(r-3) = 0 \Rightarrow r = 1/3 \text{ or } r = 3.
$$

Hence, putting the values of a and r, the required numbers
are 18, 6, 2 or 2, 6, 18.

(4) We have, 4 , g_1 , g_2 , g_3 , $1/4$ is a G.P. Here, $a = 4$, $g_1 = ar = 4r$, $g_2 = ar^2$, $g_3 = ar^3$, $g_4 = ar^4 = 4r^4 = 1/4$ $4 \t1 \t(1)^4$ 1

$$
\Rightarrow r^4 = \frac{1}{16} = \left(\frac{1}{2}\right) \Rightarrow r = \frac{1}{2}
$$

Now, the product of three G.M.'s

$$
g_1g_2g_3 = ar \times ar^2 \times ar^3 = a^3r^6 = 4^3 \times \left(\frac{1}{2}\right)^6 = \frac{4^3}{4^3} = 1
$$
 (8) (B). $\frac{x}{1-r} = 5 \implies x = \frac{1}{2}$

(5) (D). Sum = 4 and second term = $3/4$, it is given that first term is a common ratio r.

^a ⁴ 1 r and 3 ar ⁴ 3 r 4a Therefore, 4a or a² – 4a + 3 (a – 1) (a – 3) = 0 a = 1 or a 3 When a = 1, r = 3/4 and when a = 3, r = 1/4 **(6) (A).** , are the roots of x² – x + p = 0 + = 1 (1) = p (2) , are the roots of x² – 4x + q = 0 + are the roots of x² – 4x + q = 0 + = 4 (3) = q (4) ,,, are in G.P. Let = a, = ar, = ar² , = ar³ . Substituting these values in equations (1), (2), (3) and (4), we get a + ar = 1 (5) a 2 r = p (6) ar² + ar³ = 4 (7) a 2 r⁵ = q (8) Dividing eq. (7) by eq. (5) we get ² ar (1 r) 4 a (1 r) 1 r² = 4 r = 2, – 2 (5) 1 1 a 1 r 1 2 or 1 1 1 2 3 or – 1 As p is an integer (given), r is also an integer (2 or –2) (6) 1 a 3 . Hence, a = – 1 and r = – 2 p = (–1)² × (–2) = – 2p = (–1)² × (–2) = – 2 q = (–1)² × (–2)⁵ = – 32

(7) (D). Given that a, b, c are in A.P. \implies 2b = a + c but given $a + b + c = 3/2 \implies 3b = 3/2$ \Rightarrow b = 1/2 and then a + c = 1 Again, a^2 , b^2 , c^2 are in G.P.

EXECUTE: Q SERIIES
\n
$$
(3r-1)(r-3)=0 \Rightarrow r=1/3 \text{ or } r=3.
$$
\n
$$
8, 6, 2 \text{ or } 2, 6, 18.
$$
\n
$$
8, 6, 2 \text{ or } 2, 6, 18.
$$
\n
$$
8, 4, 8_1, 8_2, 8_3, 1/4 \text{ is a G.P.}
$$
\n
$$
a^4 = 4q^4 = 4q^4 = 1
$$
\n
$$
a^4 =
$$

Solving eq. (1) and (2), we get $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$

$$
\left(\frac{1}{2}\right) = \frac{4^2}{4^3} = 1
$$
 (8) (B). $\frac{x}{1-r} = 5 \Rightarrow r = 1 - \frac{x}{5}$
given that first

$$
\therefore \quad -1 < r < 1
$$

$$
\Rightarrow -1 < 1 - \frac{4}{3} < 1 \Rightarrow 0 < -\frac{4}{5} < 2 \Rightarrow -10 < x < 0
$$

TRY IT YOURSELF-3

(1) Let the H.P. be
$$
\frac{1}{a}
$$
, $\frac{1}{a+d}$, $\frac{1}{a+2d}$,, $\frac{1}{a+(n-1)d}$,...

Considering a + c = 1 and ac = 1/4
\n⇒ (a-c)² = 1 + 1 = 2 ⇒ a-c = ±√2
\nbut a < c ⇒ a-c = -√2
\nSolving eq. (1) and (2), we get
$$
a = \frac{1}{2} - \frac{1}{\sqrt{2}}
$$

\n(B). $\frac{x}{1-r} = 5 \Rightarrow r = 1 - \frac{x}{5}$
\nSince G.P. contains infinite terms
\n∴ -1 < c < 1
\n $\Rightarrow -1 < 1 - \frac{x}{3} < 1 \Rightarrow 0 < -\frac{x}{5} < 2 \Rightarrow -10 < x < 0$
\n**TRYITYOUBSELF-3**
\nLet the H.P. be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots, \frac{1}{a+(n-1)d}, \dots$
\nThen, $a_8 = \frac{1}{2}$ and $a_{14} = \frac{1}{3}$
\n $\Rightarrow \frac{1}{a+7d} = \frac{1}{2}$ and $\frac{1}{a+13d} = \frac{1}{3}$
\n $\therefore a_n = \frac{1}{a+(n-1)d}$
\n $\Rightarrow a = 5/6, d = 1/6$
\nNow, $a_{20} = \frac{1}{a+19d} = \frac{1}{\frac{5}{6} + \frac{19}{6}} = \frac{1}{14}$
\nand $a_n = \frac{1}{a+(n-1)d} = \frac{1}{\frac{5}{6} + (n-1) \times \frac{1}{6}} = \frac{6}{n+4}$
\n $1/16, a, b are in G.P. Hence, a2 = b/16 or 16a2 = b (1)$
\na, b, 1/6 are in H.P. Hence, $b = \frac{2a\frac{1}{6}}{a+1} = \frac{2a}{6a+1}$
\nFrom eq. (1) and (2),
\n $16a^2 = \frac{2a}{6a+1} \Rightarrow 2a (8a - \frac{1}{6a+1}) = 0$
\n $\Rightarrow 8a(6a+1)-1=0$
\n $\Rightarrow 48a^2 + 8a - 1 = 0$ (∴ $a \ne 0$)
\n $($

and
$$
a_n = \frac{1}{a + (n-1) d} = \frac{1}{\frac{5}{6} + (n-1) \times \frac{1}{6}} = \frac{6}{n+4}
$$

(2)
$$
1/16
$$
, a, b are in G.P. Hence, $a^2 = b/16$ or $16a^2 = b$ (1)

a, b, 1/6 are in H.P. Hence,
$$
b = \frac{2a\frac{1}{6}}{a + \frac{1}{6}} = \frac{2a}{6a + 1}
$$

From eq. (1) and (2),

$$
a + (n-1) a
$$

\n⇒ a + 7d = 2 and a + 13d = 3
\n⇒ a = 5/6, d = 1/6
\nNow, a₂₀ = $\frac{1}{a + 19d} = \frac{1}{\frac{5}{6} + \frac{19}{6}} = \frac{1}{14}$
\nand a_n = $\frac{1}{a + (n-1)d} = \frac{1}{\frac{5}{6} + (n-1) \times \frac{1}{6}} = \frac{6}{n+4}$
\n1/16, a, b are in G.P. Hence, a² = b/16 or 16a² = b(1)
\na, b, 1/6 are in H.P. Hence, b = $\frac{2a\frac{1}{6}}{a + \frac{1}{6}} = \frac{2a}{6a + 1}$
\nFrom eq. (1) and (2),
\n $16a^2 = \frac{2a}{6a + 1} \Rightarrow 2a\left(8a - \frac{1}{6a + 1}\right) = 0$
\n⇒ 8a (6a + 1) - 1 = 0
\n⇒ 48a² + 8a - 1 = 0 (∴ a ≠ 0)
\n⇒ (4a + 1)(12a - 1) = 0

30
$$
\frac{H}{P} + \frac{H}{Q} = H(\frac{1}{P} + \frac{1}{Q}) = \frac{2PQ}{P+Q} \frac{P+Q}{PQ} = 2
$$

\n**(4)** $A-G=2$ (1)
\n $G-H=8/5$ (2)
\n $G^2=AH = (G+2) (G-8/5) \Rightarrow G=8$

H H 1 1 2PQ P Q H 2 **EXECUTIONS**
 EXECUTIONS (4) $A - G = 2$ (1) $G - H = 8/5$ (2) $G^2 = AH = (G + 2) (G - 8/5) \Rightarrow G = 8$ \Rightarrow ab = 64 (3) From eq. (1) , $A = 10$ \Rightarrow a + b = 20 (4)

Solving eq. (3) and (4), we get $a = 4$ and $b = 16$ or $a = 16$ and $b = 4$.

- **(5)** The difference between the successive terms are $15 - 3 = 12, 35 - 15 = 20, 63 - 35 = 28, ...$
	- Clearly, these differences are in A.P.

Let T_n be the nth term and S_n denote the sum to n terms of the given series. Then,

$$
S_n = 3 + 15 + 35 + 63 + \dots + T_{n-1} + T_n \qquad \dots (1)
$$

\n
$$
S_n = 3 + 15 + 35 + 63 + \dots + T_{n-1} + T_n \qquad \dots (2)
$$

 $0 = 3 + [12 + 20 + 28 + \dots + (n-1) \text{ terms}] - T_n$ [Subtracting (2) from (1)]

$$
x^2 + 3x + 3 = 10
$$

\n⇒ a + b = 20(4)
\nSolving eq. (3) and (4), we get a = 4 and b = 16 or a = 16
\nand b = 4.
\nThe difference between the successive terms are
\n15-3=12,35-15=20,63-35=28,......
\n15-3=12,35-15=20,63-35=28,......
\nClearly, these differences are in A.P.
\nLet T_n be the nth term and S_n denote the sum to n terms of
\nthe given series. Then,
\nS_n = 3+15+35+63+......+T_{n-1}+T_n (1)
\nS_n = 3+15+35+63+......+T_{n-1}+T_n (2)
\n
$$
\overline{0=3+[12+20+28+......+(n-1) terms]-Tn}
$$
\n(Subtracting (2) from (1)]
\n⇒ T_n = 3+ $\frac{(n-1)}{2}$ [2 × 12+ (n-1-1) × 8]
\n= 3+(n-1) (12+4n-8)=3+(n-1) (4n+4)
\n= 4n²-1
\nS_n = $\frac{n}{2}$ (12 × 12+ (n-1-1) × 8]
\n $\frac{1}{n} = \frac{1}{2}(\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4}{5}$
\n $\frac{1}{\alpha}$, b, a are also in A.P.
\n= 4n²-1
\nS_n = $\frac{n}{2}$ (12² + 12+20+28+....+ (n-1) (4n+4)
\n= 4n²-1
\n $\Rightarrow \frac{d}{abcd}$, $\frac{c}{abcd}$, $\frac{b}{abcd}$, $\frac{a}{abcd}$

 n n n n n k k 1 k 1 k 1 k 1 S T (4k 1) 4 k 1 n (n 1) (2n 1) 4 n 6 n ²

STUDY MATERIAL: MATHEMATICS
\n(6)
$$
S=3+(3+d)\frac{1}{4}+(3+2d)\frac{1}{4^2}+...\infty
$$
(1)
\n $\Rightarrow \frac{1}{4}S = (3)\frac{1}{4}+(3+d)\frac{1}{4^2}+...\infty$ (2)
\nSubtracting eq. (2) from eq. (1), we have
\n $\frac{d}{d}$

$$
\Rightarrow \frac{1}{4}S = (3)\frac{1}{4} + (3+d)\frac{1}{4^2} + \dots \infty
$$
 (2)

Subtracting eq. (2) from eq. (1) , we have

STUDY MATERIAL: MATHEMATICS
\nS = 3 + (3 + d)
$$
\frac{1}{4}
$$
 + (3 + 2d) $\frac{1}{4^2}$ + ...∞(1)
\n $\frac{1}{4}S = (3)\frac{1}{4}$ + (3 + d) $\frac{1}{4^2}$ + ...∞(2)
\nracting eq. (2) from eq. (1), we have
\n $\frac{3}{4}S = 3$ + (d) $\frac{1}{4}$ + (d) $\frac{1}{4^2}$ + ...∞ = 3 + $\frac{\frac{d}{4}}{1-\frac{1}{4}} = 3 + \frac{\frac{d}{3}}{3}$
\nS = 4 + $\frac{4d}{9}$, Given 4 + $\frac{4d}{9} = \frac{44}{9} \Rightarrow \frac{4d}{9} = \frac{8}{9} \Rightarrow d = 2$
\nSince AM ≥ GM, then
\n $\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)(c+d)} \Rightarrow M \le 1$
\nAlso, (a + b) + (c + d) > 0 (∴ a, b, c, d > 0)

$$
\Rightarrow S = 4 + \frac{4d}{9}, \text{Given } 4 + \frac{4d}{9} = \frac{44}{9} \Rightarrow \frac{4d}{9} = \frac{8}{9} \Rightarrow d = 2
$$

(7) (A). Since
$$
AM \ge GM
$$
, then

S **STUDY MATERIAL: MATHEMATICS**
\n
$$
S = 3 + (3 + d)\frac{1}{4} + (3 + 2d)\frac{1}{4^2} + \dots \infty
$$
(1)
\n $\Rightarrow \frac{1}{4}S = (3)\frac{1}{4} + (3 + d)\frac{1}{4^2} + \dots \infty$ (2)
\nSubtracting eq. (2) from eq. (1), we have
\n $\frac{3}{4}S = 3 + (d)\frac{1}{4} + (d)\frac{1}{4^2} + \dots \infty = 3 + \frac{d}{1 - \frac{1}{4}} = 3 + \frac{d}{3}$
\n $\Rightarrow S = 4 + \frac{4d}{9}$, Given $4 + \frac{4d}{9} = \frac{44}{9} \Rightarrow \frac{4d}{9} = \frac{8}{9} \Rightarrow d = 2$
\n(A). Since AM \ge GM, then
\n $\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)(c+d)} \Rightarrow M \le 1$
\nAlso, $(a+b)+(c+d) > 0$ (\because a, b, c, d > 0)
\n $\therefore 0 < M \le 1$
\n(B).
\n $\frac{1}{H} = \frac{1}{2}(\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{8 + 2\sqrt{5}} \times \frac{1}{2} = \frac{1}{4}$
\nH = 4
\n(D). a, b, c, d are in A.P.
\n \therefore d, c, b, a are also in A.P.
\n \therefore d = c b

_______________________________________ **(8) (B).**

$$
\frac{1}{4}S = (3)\frac{1}{4} + (3+d)\frac{1}{4^2} + \dots \infty \qquad \qquad \dots \dots (2)
$$

tracting eq. (2) from eq. (1), we have

$$
\frac{3}{4}S = 3 + (d)\frac{1}{4} + (d)\frac{1}{4^2} + \dots \infty = 3 + \frac{4}{1 - \frac{1}{4}} = 3 + \frac{d}{3}
$$

$$
S = 4 + \frac{4d}{9}, \text{ Given } 4 + \frac{4d}{9} = \frac{44}{9} \Rightarrow \frac{4d}{9} = \frac{8}{9} \Rightarrow d = 2
$$

$$
\therefore \text{ Since AM} \ge GM, \text{ then}
$$

$$
\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)(c+d)} \Rightarrow M \le 1
$$
Also, $(a+b)+(c+d) > 0$ (\because a, b, c, d > 0)
 $0 < M \le 1$
$$
\frac{1}{H} = \frac{1}{2}(\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{8 + 2\sqrt{5}} \times \frac{1}{2} = \frac{1}{4}
$$

$$
H = 4
$$

$$
\therefore \text{ a, b, c, d are in A.P.}
$$

$$
\therefore \text{ d, c, b, a are also in A.P.}
$$

$$
\frac{d}{d} = \frac{c}{d} = \frac{b}{d} = \frac{a}{d} = \text{ or also in A.P.}
$$

(9) (D). a, b, c, d are in A.P.

e get a = 4 and b = 16 or a = 16
\nne successive terms are
\na-3-35 = 28,
\n(a + b) + (c + d) = 9
\n(a + b) + (c + d) = 16
\n(a + b) + (c + d) = 16
\n(a + b) + (c + d) = 16
\n(b + c) + (c + d) = 16
\n(c + d) + (c + d) = 16
\n(d + b) + (c + d) = 16
\n(b + c) + (c + d) = 16
\n
\n
$$
= 16
$$

 \Rightarrow abc, abd, acd, bcd are in H.P.

CHAPTER- 6 : SEQUENCES & SERIES EXERCISE-1

(1) (A). Let first term = a, common difference = d Then $T_3 = a + 2d = 18$ and $T_7 = a + 6d = 30$ Solving these, $a=12$, $d=3$

$$
\therefore S_{17} = \frac{17}{2} [2a + (17 - 1)d] = \frac{17}{2} [24 + 16 \times 3] = 612
$$
 (6) $\therefore T_{20}$

(2) (B). We have first term $= a$, second term $= b$ \therefore d = common difference = b – a It is given that the middle term is c. This means that there are an odd number of terms in the AP. Let there be $(2n+1)$ terms in the AP. Then $(n+1)$ th term is the middle term. \therefore middle term = c \Rightarrow a + nd = c

UENCES & SERIES	Q.B.-SOLUTIONS	Supers
HAPTER 6: SEQUENCES & SERIES	(5)	(D). If ab the first term and do be the common difference of the AP, then
Chap: A. Let first term = a, common difference = d	$T_9 = a + 8d = 35$	
Chap: A. Let first term = a, common difference = d	$T_9 = a + 8d = 75$	
Solving these , $a = 12, d = 3$	$T_{19} = a + 18d = 75$	
Stolving these , $a = 12, d = 3$	$T_{19} = a + 18d = 75$	
Obving these , $a = 12, d = 3$	$T_{10} = 40 \Rightarrow d = 4, a = 3$	
Obving these , $a = 12, d = 3$	$T_{12} = 14$	
Obving these , $a = 12, d = 3$	$T_{13} = a + 18d = 75$	
Obving these , $a = 12, d = 3$	$T_{14} = 40$	
Obving these , $a = 12, d = 3$	$T_{15} = 12$	
Obving these , $a = 12, d = 3$	$T_{16} = 12$	
Obving these , $a = 12, d = 3$	$T_{17} = 4 + 18d = 75$	
Obving these , $a = 12, d = 3$		

(3) (C). Required sum = (sum of integers divisible by 2) + (sum of integers divisible by 5) – (sum of integers divisible by 2 and 5)

$$
= (2+4+6+....+100) + (5+10+15+....+100)
$$

- (10+20+....+100)

$$
= \frac{50}{2} [2 \times 2 + (50 - 1) \times 2] + \frac{20}{2} [2 \times 5 + (20 - 1) \times 10]
$$

\n
$$
- \frac{10}{2} [2 \times 10 + (10 - 1) \times 10]
$$

\n
$$
\frac{a_1 + a_2}{a_2 + a_1}
$$

 $= 50$ $[2 + 49] + 10$ $[10 + 95] - 5$ $[20 + 90]$ $= 51 \times 50 + 105 \times 10 - 110 \times 5 = 3050$ **(4) (D).** Let d be the c.d. of the A.P. Now

$$
\begin{aligned}\n&= \frac{1}{2} \left\{ 2\left(\frac{c-a}{b-a}\right) + 1 \right\} \left[2a + 2\left(\frac{c-a}{b-a}\right) (b-a) \right] \\
&= \frac{1}{2} \left\{ \frac{2(c-a)}{b-a} + 1 \right\} \{ 2c\} = \frac{2c(c-a)}{b-a} + c \\
&= \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3} \qquad ...(1) \\
&= \frac{3n}{2} \left(\frac{2 \times (3n-1)}{b-a} \right) \left(\frac{c-a}{b-a} \right) (c-a) \left(\frac{c-a}{b-a} \right) (d-a) \\
&= \frac{3n}{2} \left[2 \times 2 + (50-1) \times 2 \right] + \frac{20}{2} \left[2 \times 5 + (20-1) \times 10 \right] \\
&= \frac{50}{2} \left[2 \times 2 + (50-1) \times 2 \right] + \frac{20}{2} \left[2 \times 5 + (20-1) \times 10 \right] \\
&= \frac{50}{2} \left[2 \times 10 + (10-1) \times 10 \right] \\
&= \frac{50}{2} \left[2 \times 10 + (10-1) \times 10 \right] \\
&= \frac{50}{2} \left[2 \times 10 + (10-1) \times 10 \right] \\
&= \frac{50}{4} \left[2 \times 10 + (10-1) \times 10 \right] \\
&= \frac{10}{2} \left[2 \times 10 + (10-1) \times 10 \right] \\
&= \frac{10}{4} \left[10 \frac{3 \times 21+1}{4} = \frac{64}{45} \\
&= 50 \left[2 + 49 \right] + 10 \left[10 + 95 \right] - 5 \left[20 + 90 \right] \\
&= 51 \times 50 + 105 \times 10 - 110 \times 5 = 3050 \\
&= 3050 \\
&= 1.11 \text{S.} = \frac{\sqrt{a_1} - \sqrt{a_2} + \sqrt{a_2} - \sqrt{a_3} + \dots + \
$$

CES & SERIES)\n**CHAPTER 6: SEQUENCES & SERIES**\n**EXERCISE-1**\n**EXERCISE-1**\n**1**
$$
T_3 = a + 2d = 1
$$
 3
\n $T_4 = 2d = 1$ 4
\n $T_5 = a + 2d = 1$ 5
\n**2** $T_6 = 4 \times 10^2$ 6
\n**3** $T_7 = a + 6d = 30$
\n $T_8 = 42 - 18$ 7
\n $T_9 = a + 18 d = 75$
\n $T_1 = a + 18 d = 75$
\n $T_1 = 2a + (17 - 1)dI = \frac{17}{2}[24 + 16 \times 3] = 612$
\n $T_1 = \frac{17}{2}[2a + (17 - 1)dI] = \frac{17}{2}[24 + 16 \times 3] = 612$
\n $T_2 = 3 + 19 \times 4 = 79$
\n $T_3 = 3 + 19 \times 4 = 79$
\n $T_4 = 3$
\n $T_5 = 3 + 19 \times 4 = 79$
\n $T_6 = 16 \text{ (m-th)}$
\n $T_7 = 2 \text{ (m-th)}$
\n $T_8 = \frac{n}{2}[a + \ell]; 400 = \frac{n}{2}[5 + 45] \Rightarrow n = 16$
\nWe have first term = a, second term = b
\n $T_7 = 3 + 19 \times 4 = 79$
\n $T_8 = \frac{n}{2}[a + \ell]; 400 = \frac{n}{2}[5 + 45] \Rightarrow n = 16$
\n $T_9 = 3 + 19 \times 4 = 79$
\n $T_1 = 3 + 19 \times 4 = 79$
\n $T_2 = 3 + 19 \times 4 = 79$
\n $T_3 = 3 + 19 \times 4 = 79$
\n $T_4 = 16 \text{ (m-th)}$
\n $T_5 = \frac{n}{2}[a + \ell]; 400 = \frac{n}{$

2 ^[2] $\left(6\right)$ **(6) (C)**. Here a= 5, $\ell = 45$ S_n = 400 $S_n = 400$

$$
S_n = \frac{n}{2} [a + \ell]; \ 400 = \frac{n}{2} [5 + 45] \Rightarrow n = 16
$$

(7) **(C).** Here
$$
\frac{S_{n_1}}{S_{n_2}} = \frac{3n+1}{2n+3}
$$

= a, common difference = d
\n18 and T₇ = a + 6d = 30
\n12, d=3
\n17-1) d] =
$$
\frac{17}{2}
$$
 [24+16×3] = 612
\n∴ T₉₉ = 3+18d = 75
\n∴ T₉₉ = 3+19x4=79
\n∴ T₉₀ = 3+1
\n∴ T₉₀ = 2+1
\n∴ T₉₀ =

$$
\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3} \qquad ...(1)
$$

$$
\therefore \frac{T_{11_1}}{T_{11_2}} = \frac{a_1 + 10d_1}{a_2 + 10d_2} \qquad ...(2)
$$

$$
\frac{n-1}{2} = 10 \Rightarrow n = 21
$$

putting the value of n in (1)

$$
\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{3 \times 21 + 1}{2 \times 21 + 3} = \frac{64}{45}
$$

$$
\frac{1}{2} \frac{1}{2a_2 + (n-1)d_2} = \frac{3n+1}{2n+3}
$$
\n
\n⇒ $\frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3}$...(1)
\n
\n $\therefore \frac{T_{11_1}}{T_{11_2}} = \frac{a_1 + 10d_1}{a_2 + 10d_2}$...(2)
\n
\n100)
\n+20+....+100) $\frac{n-1}{2} = 10 \Rightarrow n = 21$
\n $\therefore 10 - 1 \times 10$ putting the value of n in (1)
\n $\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{3 \times 21 + 1}{2 \times 21 + 3} = \frac{64}{45}$
\n(8) (B). Here $d = \frac{3 - \frac{1}{2}}{4 + 1} = \frac{1}{2}$ ∴ A₃ = a + 3d ⇒ $\frac{1}{2} + 3 \times \frac{1}{2} = 2$
\n $\frac{a_{n-1} - \sqrt{a_n}}{a_{n-1} - a_n}$ (9) (C). Here 2 + 3d = 14 ⇒ d = 4
\n∴ 4 = $\frac{38-2}{n+1} \Rightarrow 4n + 4 = 36 \Rightarrow n = 8$
\n(10) (B). Let the numbers are a-3d, a-4, a + d, a + 3d given a-3d + a-d + a+d + a+3d = 20

$$
\frac{-1 - \sqrt{a_n}}{-1 - a_n}
$$
 (9) (C). Here $2 + 3d = 14 \Rightarrow d = 4$

$$
\therefore 4 = \frac{38 - 2}{n + 1} \Rightarrow 4n + 4 = 36 \Rightarrow n = 8
$$

- $\frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3}$ (a)

integers divis
 $\therefore \frac{T_1 I_1}{T_{112}} = \frac{a_1 + 10d_1}{a_2 + 10d_2}$...(2)
 $\therefore \frac{100}{112} = 10 \Rightarrow n = 21$
 $0-1) \times 10$ putting the value of n in (1)
 $+(10-1) \times 10$ $\$ $\frac{2}{\frac{(n-1)}{2}d_2} = \frac{3n+1}{2n+3}$...(1)
 $\frac{1}{2} = \frac{a_1 + 10d_1}{a_2 + 10d_2}$...(2)
 $= 10 \Rightarrow n = 21$

g the value of n in (1)
 $\frac{0d_1}{0d_2} = \frac{3 \times 21 + 1}{2 \times 21 + 3} = \frac{64}{45}$

re d = $\frac{3 - \frac{1}{2}}{4 + 1} = \frac{1}{2}$ ∴ A₃ $\frac{(n-1)}{2}d_2$ $= \frac{a_1 + 10d_1}{a_2 + 10d_2}$ $= 10 \Rightarrow n = 21$ g the value of n in (1)
 $\frac{d_1}{d_2} = \frac{3 \times 21 + 1}{2 \times 21 + 3} = \frac{64}{45}$ $e d = \frac{3 - \frac{1}{2}}{4 + 1} = \frac{1}{2}$ ∴ A₃ = a + 3d ⇒ $\frac{1}{2} + 3 \times \frac{1}{2} = 2$
 $e 2 + 3d = 14$ **(10) (B).** Let the numbers are $a-3d$, $a-d$, $a+d$, $a+3d$ given $a-3d + a-d + a+d + a+3d = 20$ \Rightarrow 4a = 20 \Rightarrow a = 5 and $(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$ \Rightarrow 4a² + 20 d² = 120 \Rightarrow 4 x 5² + 20 d² = 120 \Rightarrow d² = 1 \Rightarrow d = \pm 1 Hence numbers are 2, 4, 6, 8
	- **(11) (C).** $(x+1)$, $3x$, $(4x+2)$ in A.P. \Rightarrow 3x-(x+1)=(4x+2)-3x \Rightarrow x=3 \therefore a = 4, d = 9 - 4 = 5 \Rightarrow T₅ = 4 + 4 (5) = 24

**EXAMPLERLA. MATERAL. MATHEMATI (12) (B). Let the A.P. be a + (a+d) + (a+2d) +
\n
$$
\therefore S_{10} = 4S_5 \therefore 2a + 9d = 4a + 8d \Rightarrow \frac{a}{d} = \frac{1}{2}
$$
\n
$$
\therefore S_{10} = 4S_5 \therefore 2a + 9d = 4a + 8d \Rightarrow \frac{a}{d} = \frac{1}{2}
$$
\n
$$
\therefore S_{10} = 6S_6 \therefore S_{11} \text{ and } \alpha = a - d, \beta = a,
$$
\n
$$
\therefore S_{12} = (a^2 - d^2) \Rightarrow a = 4
$$
\n
$$
\therefore S_{13} = (a^2 - d^2) \Rightarrow a = 4
$$
\n
$$
\therefore S_{14} = 120^\circ + 15.5^\circ = 19
$$
\n
$$
\therefore S_{15} = 120^\circ + 15.5^\circ = 19
$$
\n
$$
\therefore S_{16} = 2\sqrt{3} + 2\sqrt{2} = 2\sqrt{5} = 14
$$
\n
$$
\therefore S_{17} = 4\sqrt{3} = 2 - \sqrt{3} = 4
$$
\n
$$
\therefore S_{18} = 2 - \sqrt{3} = 4
$$
\n
$$
\therefore S_{19} = 4\sqrt{3} = 2\sqrt{2} - \sqrt{3} = 4
$$
\n
$$
\therefore S_{10} = 2\sqrt{3} + 2\sqrt{2} = 2\sqrt{6} = 1 + \sqrt{2} - \sqrt{3}.
$$
\n
$$
\therefore S_{10} = 2\sqrt{3} + 2\sqrt{2} = 2\sqrt{6} = 1 + \sqrt{2} - \sqrt{3}.
$$
\n
$$
\therefore S_{12} = 2\sqrt{3} \text{ and these form an A.P. with common difference } = 1 - \sqrt{2}.
$$
\nHence required numbers are in H.P.
\n
$$
\therefore S_{16} = 2\sqrt{3} + 2\sqrt{2} = 2\sqrt{6} = 1 + \sqrt{2} - \sqrt{3}.
$$
\n
$$
\therefore S_{18} = 2\sqrt{3} \text{ and the 5 for } n \text{ an A
$$**

Hence *P* is least for $x = \frac{2}{3}$.

(18) (C). Let the number of sides of the polygon be n. Then the sum of interior angles of the polygon

$$
=(2n-4)\frac{\pi}{2}=(n-2)\pi
$$

Since the angles are in A.P. and $a = 120^\circ$, $d = 5$,

(O.B.-SOLUTIONS) STUDY MATERIAL: MATHEMATICS
 $\frac{a}{d} = \frac{1}{2}$ therefore $\frac{n}{2} [2 \times 120 + (n-1)5] = (n-2)180$
 $\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$

But $n = 16$ gives $T_{16} = a + 15d = 120^\circ + 15.5^\circ = 195^\circ$,

which **(O.B.-SOLUTIONS)**

STUDY MATERIAL: MATHEMATICS
 $\frac{a}{d} = \frac{1}{2}$
 $\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$

Bra,

But $n = 16$ gives $T_{16} = a + 15d = 120^\circ + 15.5^\circ = 195^\circ$,

which is impossible as interior angle canno therefore $\frac{n}{2} [2 \times 120 + (n-1)5] = (n-2)180$ \Rightarrow $n^2 - 25n + 144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$ But $n = 16$ gives $T_{16} = a + 15d = 120^\circ + 15.5^\circ = 195^\circ$, which is impossible as interior angle cannot be greater than 180° . Hence $n = 9$.

(19) (C). Given that
$$
\frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}
$$

$$
\Rightarrow \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \Rightarrow \frac{\frac{a + \frac{1}{2}(m-1)d}{2}}{a + \frac{1}{2}(n-1)d} = \frac{m}{n}
$$

$$
\Rightarrow \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \Rightarrow \frac{2}{a + \frac{1}{2}(n-1)d} = \frac{m}{n}
$$

$$
\Rightarrow an + \frac{1}{2}(m-1)nd = am + \frac{1}{2}(n-1)md
$$

$$
\Rightarrow a(n-m) + \frac{d}{2}[mn - n - mn + m] = 0
$$

$$
\Rightarrow a(n-m)+\frac{d}{2}[mn-n-mn+m]=0
$$

$$
\Rightarrow a(n-m) + \frac{d}{2}(m-n) = 0 \Rightarrow a = \frac{d}{2} \text{ or } d = 2a
$$

So, required ratio,

$$
\frac{T_m}{T_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a}
$$

$$
= \frac{1 + 2m - 2}{1 + 2n - 2} = \frac{2m - 1}{2n - 1}.
$$

$$
= 310 \quad (20) \quad (D). \ S = \frac{n}{2} [2a + (n-1)d]
$$

$$
\Rightarrow 406 = \frac{n}{2} [6 + (n-1)4] \Rightarrow 812 = n[6 + 4n - 4]
$$

\n
$$
\Rightarrow 812 = 2n + 4n^2 \Rightarrow 406 = 2n^2 + n
$$

\n
$$
\Rightarrow 2n^2 + n - 406 = 0
$$

\n
$$
\Rightarrow n = \frac{-1 \pm \sqrt{1 + 4.2.406}}{2.2} = \frac{-1 \pm \sqrt{3249}}{4} = \frac{-1 \pm 57}{4}
$$

\nTaking (+) sign, $n = \frac{-1 + 57}{4} = 14$.

(21) **(D).** Let
$$
A_1, A_2, A_3
$$
 and A_4 are four numbers in A.P.
\n $A_1 + A_4 = 8$ (i) and $A_2, A_3 = 15$ (ii)
\nThe sum of terms equidistant from the beginning and end
\nis constant and is equal to sum of first and last terms.

Hence, $A_2 + A_3 = A_1 + A_4 = 8$ (iii) From (ii) and (iii),

$$
A_2 + \frac{15}{A_2} = 8 \implies A_2^2 - 8A_2 + 15 = 0
$$

\n
$$
A_2 = 3 \text{ or } 5 \text{ and } A_3 = 5 \text{ or } 3.
$$

\nAs we know, $A_2 = \frac{A_1 + A_3}{2} \implies A_1 = 2A_2 - A_3$
\n
$$
\implies A_1 = 2 \times 3 - 5 = 1 \text{ and } A_4 = 8 - A_1 = 7
$$

\nHence the series is, 1, 3, 5, 7.
\nSo that least number of series is 1.
\n(B) a, b, c are in A.P.
\nSo $2b = a + c$, then straight line ax + by + c = 0 will pass
\n
$$
a - 2b + c = 0
$$
 or $2b = a + c$.
\nHence, the series is 1.
\n
$$
B = \frac{a_1 + 10a_1}{2} = 10 \text{ or } n = \frac{a_1 + 10a_1}{2} = \frac{7 \times 21}{4 \times 214}
$$

(28) (A).
$$
\frac{S_n}{S_m} = \frac{n^4}{m^4}.
$$

 (22)

Using
$$
S_n = \frac{n}{2} [2a_1 + d(n-1)]
$$
 and $S_m = \frac{m}{2} [2a_1 + d(m-1)]$ (29) (D). $x^3 + ax^2 + bx + c = 0$
Let $\alpha = -1, \beta = 1, \gamma = 3$ and $(x + 1)^2$

$$
\Rightarrow \frac{a_{m+1}}{a_{n+1}} = \frac{(2m+1)^3}{(2n+1)^3} \text{ after simplification.}
$$
 (30) (C).S

(24) (C). Let the number of days be n.

Hence a worker can do $\left(\frac{1}{150n}\right)^n$ part of the work in a $\left(\frac{1}{150 n}\right)^{m}$ part of the work in a $\begin{pmatrix} 1 \end{pmatrix}^m$ and $\begin{pmatrix} 1 \end{pmatrix}$ $150n$ μ μ α α β α $\frac{1}{10}$ part of the work in a day. Accordingly,

[150 + 146 + 142 + + upto (n + 8) terms] ×
$$
\frac{1}{150n}
$$
 = 1 (31) (A). The terms from a G.

 \Rightarrow $n=17$ Therefore number of total days in completion

$$
= 17 + 8 = 25.
$$

(25) **(D).**
$$
m^{th}
$$
 mean between a , $2b$ is $a + \frac{m(2b-a)}{n+1}$ (i)

and $_m$ th mean between 2*a*, *b* is $2a + \frac{m(b-2a)}{n+1}$ (ii) $a + \frac{m(b-2a)}{n+1}$ (ii) $a = \frac{1}{\sqrt{90}}$

Accordingly,
$$
a + \frac{m(2b - a)}{n + 1} = 2a + \frac{m(b - 2a)}{n + 1}
$$

\n $\Rightarrow m(2b - a) = a(n + 1) + m(b - 2a)$
\n $\Rightarrow a(n - m + 1) = bm$ or $n^{n + 4} = \frac{a}{b} = \frac{m}{n - m + 1}$. (33) (B). Let b

(26) **(B).** Common terms will be 21, 41, 61,
21 + (n-1) 20
$$
\le
$$
 417 \Rightarrow n \le 20.8 \Rightarrow n = 20

(27) (D). Let first A.P. is a_1 , $a_1 + d_1$, $a_1 + 2d_1$ a_1 (first term), d_1 (common difference)

Second A.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$ a_2 (first term), d_2 (common difference)

$$
\frac{1}{2} \frac{1
$$

$$
\frac{8}{\text{SDMADVARLE}} \text{Second A.P. is a}_{2}, \quad a_{2} + d_{2}, \quad a_{2} + 2d_{2} \dots \dots
$$
\n
$$
a_{2} \text{ (first term), } d_{2} \text{ (common difference)}
$$
\ngiven is
$$
\frac{n/2[2a_{1} + (n-1)d_{1}]}{n/2[2a_{2} + (n-1)d_{2}]} = \frac{7n+1}{4n+27}
$$
\n
$$
\Rightarrow \frac{a_{1} + \left(\frac{n-1}{2}\right)d_{1}}{a_{2} + \left(\frac{n-1}{2}\right)d_{2}} = \frac{7n+1}{4n+27}
$$
\n
$$
\text{Put } \frac{n-1}{2} = 10 \text{ or } n = 21 \text{ to get}
$$
\n
$$
\frac{a_{1} + 10d_{1}}{a_{2} + 10d_{2}} = \frac{7 \times 21 + 1}{4 \times 21 + 27} = \frac{148}{111}
$$
\n
$$
\text{(A). By the method of differences, } t_{n} = 1 + (n-1) \text{ in}
$$
\n
$$
\text{Given } 1 + n(n-1) = 9901 \Rightarrow n(n-1) = 9900 \text{ which is}
$$

Put
$$
\frac{n-1}{2} = 10
$$
 or $n = 21$ to get

$$
\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{7 \times 21 + 1}{4 \times 21 + 27} = \frac{148}{111}
$$

EDMANDRANGED LEARNING

and A.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$

Then its term), d_2 (common difference)

m is $\frac{n/2[2a_1 + (n-1)d_1]}{n/2[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$
 $\frac{a_1 + (\frac{n-1}{2})d_1}{a_2 + (\frac{n-1}{2})d_2} = \frac{7$ ond A.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$

first term), d_2 (common difference)
 $\ln \frac{\ln/2[2a_1 + (n-1)d_1]}{n/2[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$
 $\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$
 $\frac{n-1}{2} = 1$ cond A.P. is a₂, a₂ + d₂, a₂ + 2d₂.......

(first term), d₂ (common difference)

en is $\frac{n/2[2a_1 + (n-1)d_1]}{n/2[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$
 $\frac{a_1 + (\frac{n-1}{2})d_1}{a_2 + (\frac{n-1}{2})d_2} = \frac{7n+1}{4n+27}$
 $\frac{t \frac{n-1}{2$ cond A.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$

(first term), d_2 (common difference)

ven is $\frac{n/2[2a_1 + (n-1)d_1]}{n/2[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$
 $\frac{a_1 + (\frac{n-1}{2})d_1}{a_2 + (\frac{n-1}{2})d_2} = \frac{7n+1}{4n+27}$
 $\frac{1}{a_2 + (\$ MA.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$

st term), d_2 (common difference)

is $\frac{n/2 \left[2a_1 + (n-1)d_1\right]}{n/2 \left[2a_2 + (n-1)d_2\right]} = \frac{7n+1}{4n+27}$
 $+\left(\frac{n-1}{2}\right) d_1 + \left(\frac{n-1}{2}\right) d_2 = \frac{7n+1}{4n+27}$
 $+\left(\frac{n-1}{2}\right) d_2 = \frac$ Sa a_2 , $a_2 + d_2$, $a_2 + 2d_2$
 a_2 (common difference)
 $2[2a_1 + (n-1)d_1] = \frac{7n+1}{4n+27}$
 $\frac{1}{1}$
 $\frac{1}{1}$
 $\frac{1}{1}$
 $\frac{1}{1}$
 $\frac{1}{2}$
 $\frac{7n+1}{4n+27}$
 $\frac{1}{1}$
 $\frac{1}{1}$
 $\frac{1}{4}$
 $\frac{7n+1}{4n+2$ MA.P. is a_2 , $a_2 + d_2$, $a_2 + 2d_2$

st term), d_2 (common difference)

is $\frac{n/2 \left[2a_1 + (n-1)d_1\right]}{n/2 \left[2a_2 + (n-1)d_2\right]} = \frac{7n+1}{4n+27}$
 $+\left(\frac{n-1}{2}\right) d_1$
 $+\left(\frac{n-1}{2}\right) d_2$
 $\frac{n-1}{2} = \frac{7n+1}{4n+27}$
 $+\left(\frac$ **(28) (A).** By the method of differences, $t_n = 1 + (n-1)n$ Given $1 + n(n-1) = 9901 \Rightarrow n(n-1) = 9900$ which is satisfied by $n = 100$

$$
\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{7 \times 21 + 1}{4 \times 21 + 27} = \frac{148}{111}
$$
\n(28) (A). By the method of differences, t_n = 1 + (n-1) n
\nGiven 1 + n (n-1) = 9901 ⇒ n (n-1) = 9900 which is
\nsatisfied by n = 100
\n(29) (D). x³ + ax² + bx + c = 0
\nLet α = -1, β = 1, γ = 3 and (x + 1) (x - 1) (x - 3) = 0
\nx³ - 3x - x + 3 = 0 ⇒ a = -3, b = -1 and c = 3
\nSubstitute in options 2a³ - 9ab = -27c satisfies.
\n(30) (C). Since x, 2x + 2 and 3x + 3 are in G.P.
\n∴ (2x+2)² = x (3x+3)
\n⇒ x² + 5x + 4 = 0
\n⇒ (x+1) (x+4) = 0 ⇒ x = -1, -4
\n⇒ x = -4 (∴ x ≠ -1)
\n⇒ numbers are -4, -6, -9
\n∴ First term = -4 and c.r. = 3/2
\nHence T₄ = (-4) (3/2)³ = -27/2
\n(31) (A). The terms from a G.P. with common ratio = 1/3
\nRequired form = 16.2
$$
\begin{pmatrix} 1 - (\frac{1}{3})^7 \ 1 - \frac{1}{3} \end{pmatrix} = 8.1 (\frac{3^7 - 1}{3^6})
$$
\nRequired form = 16.2
$$
\begin{pmatrix} 1 - (\frac{1}{3})^7 \ 1 - \frac{1}{3} \end{pmatrix} = 8.1 (\frac{3^7 - 1}{3^6})
$$

Put
$$
\frac{1}{2} = 10
$$
 or $n = 21$ to get
\n
$$
\frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{7 \times 21 + 1}{4 \times 21 + 27} = \frac{148}{111}
$$
\n(28) (A). By the method of differences, $t_n = 1 + (n-1)n$
\nGiven 1 + n (n-1) = 9901 $\Rightarrow n (n-1) = 9900$ which is satisfied by $n = 100$
\n(29) (D). $x^3 + ax^2 + bx + c = 0$
\nLet $\alpha = -1$, $\beta = 1$, $\gamma = 3$ and $(x + 1) (x - 1) (x - 3) = 0$
\n $x^3 - 3x - x + 3 = 0 \Rightarrow a = -3$, $b = -1$ and $c = 3$
\nS. Substitute in options 2a³ - 9ab = -27c satisfies.
\n(30) (C). Since x, 2x + 2 and 3x + 3 are in G.P.
\n \therefore (2x+2)² = x (3x + 3)
\n $\Rightarrow x^2 + 5x + 4 = 0$
\n \Rightarrow (x+1) (x+4) = 0 $\Rightarrow x = -1$, -4
\n $\Rightarrow x = -4$ ($\therefore x \ne -1$)
\n \Rightarrow numbers are -4, -6, -9
\n \therefore First term = -4 and c.r. = 3/2
\nHence $T_4 = (-4)(3/2)^3 = -27/2$
\n(31) (A). The terms from a G.P. with common ratio = 1/3
\nRequired form = 16.2 $\begin{pmatrix} 1 - (\frac{1}{3})^7 \\ 1 - \frac{1}{3} \end{pmatrix} = 8.1 (\frac{3^7 - 1}{3^6})$
\nRequired form = 16.2 $\begin{pmatrix} 1 - (\frac{1}{3})^7 \\ 1 - \frac{1}{3} \end{pmatrix} = 8.1 (\frac{3^7 - 1}{3^6})$
\n(32) (C). Here $(\frac{T_2}{T_1})$

 $\frac{1}{150 n} = 1$ (31) (A). The terms from a 0 **(31) (A).** The terms from a G.P. with common ratio $= 1/3$

$$
\begin{array}{ll}\n\text{ation} \\
5. & \text{Required form} = 16.2 \left(\frac{1 - \left(\frac{1}{3} \right)^7}{1 - \frac{1}{3}} \right) = 8.1 \left(\frac{3^7 - 1}{3^6} \right) \\
\frac{-a}{a} & \text{and} \n\end{array}
$$

$$
=\frac{2186}{90}=\frac{1093}{45}
$$

\n (equivalently,
$$
\tan \theta = 2\pi
$$
), $\tan \theta = 2\pi$.\n

\n\n (30)
\n (C). Since *x*, 2*x* + 2 and 3*x* + 3 are in G.P.\n

\n\n ∴ $(2x+2)^2 = x(3x+3)$ \n $\Rightarrow x^2 + 5x + 4 = 0$ \n $\Rightarrow (x+1)(x+4) = 0 \Rightarrow x = -1, -4$ \n

\n\n part of the work in a\n $\Rightarrow x = -4 \quad (\because x \neq -1)$ \n \Rightarrow numbers are -4, -6, -9\n \therefore First term = -4 and c.r. = 3/2\n

\n\n 8) terms\n $|x| = 1$ \n

\n\n (31)
\n (A). The terms from a G.P. with common ratio = 1/3\n

\n\n (32)
\n $\tan(\theta - 2a)$ \n $\frac{m(b-2a)}{n+1}$ \n $\frac{m(b-2a)}{n+1}$ \n

\n\n (33)
\n (B). Let $b = \arctan(\theta - \sinh(\theta - \sinh(\theta))$ \n $\frac{2186}{1-\frac{1}{3}} = \left(\frac{1-\left(\frac{1}{3}\right)^7}{1-\frac{1}{3}}\right) = 8.1 \left(\frac{3^7-1}{3^6}\right)$ \n

\n\n (32)
\n (C). Here $\left(\frac{T_2}{T_1}\right)^{1/(2-1)} = \left(\frac{T_8}{T_2}\right)^{1/(8-2)}$ \n $\therefore \frac{\mathbf{n}^{\mathbf{n}}}{\mathbf{n}^{-4}} = \left(\frac{\mathbf{n}^{52}}{\mathbf{n}}\right)^{1/6}$ \n

\n\n (33)
\n (34)
\n (35)
\n (36) Let $b = \arctan(c = \ar^2$, where $0 < r < 1$. Now, a, 2b, and 3c form an AP.\n

\n\n (37)
\n (38)
\n (39). Let $b = \arctan$

or
$$
n^{n+4} = n^{(52-n)/6}
$$
 or $n + 4 = \frac{52-n}{6} \Rightarrow n = 4$

ommon ratio = 1/3

= 8.1 $\left(\frac{3^7 - 1}{3^6}\right)$
 $\therefore \frac{n^n}{n^{-4}} = \left(\frac{n^{52}}{n^n}\right)^{1/6}$
 $\frac{52 - n}{6} \Rightarrow n = 4$

0< r < 1. Now, a, 2b and **(33) (B).** Let $b = ar$ and $c = ar^2$, where $0 < r < 1$. Now, a, 2b and 3c form an AP. \therefore 4b = a + 3c \Rightarrow 4 ar = a + 3ar² \Rightarrow 3r² - 4r + 1 = 0 \Rightarrow (3r – 1) (r – 1) = 0 \Rightarrow r = 1/3 [\because 0 < r < 1]

(34) **(D).** As given a + ar = 1 ...(1)

$$
a = 2 \left(\frac{ar}{1-r} \right) \qquad ...(2)
$$

From (2)
$$
1 - r = 2r
$$
 $\therefore r = 1/3$
So from (1) $a = 3/4$

(35) (C).
$$
r = \left(\frac{5}{160}\right)^{\frac{1}{4+1}} = \left(\frac{1}{32}\right)^{\frac{1}{5}} = \frac{1}{2}
$$

$$
G_3 = ar^3 \Rightarrow 160 \times \frac{1}{2^3} = 20
$$

(36) (B). Let the three numbers be a/r, a, ar. As the numbers form an increasing GP. So, $r > 1$. It is given that a/r , 2a, ar are in A.P.

$$
\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r^2 - 4r + 1 = 0
$$

$$
\Rightarrow r = 2 \pm \sqrt{3} \Rightarrow r = 2 + \sqrt{3} \quad [\because r > 1]
$$

(37) (C). Let the terms are a/r, a, ar.

then
$$
\frac{a}{r} \times a \times ar = 216
$$
 $\Rightarrow a = 6$
\nand $\frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = 156$ (4)
\n $\Rightarrow a^2 \left(\frac{1}{r} + r + 1\right) = 156$
\n $\Rightarrow 36 (r^2 + r + 1) = 156r$ ($\therefore a = 6$)
\n $3r^2 - 10r + 3 = 0 \Rightarrow (3r - 1)(r - 3) = 0 \Rightarrow r = 3, \frac{1}{3}$

Terms are 2, 6, 18

(38) (B). Given a,b,c,d in G.P. using property (iii) a^n , b^n , c^n , d^n are also in G.P. Let common ratio is k then $b^n = ka^n$ $= ka^{n}$ $c^{n} = k^{2}a^{n}$, $d^{n} = k^{3} a^{n}$ Now in $a^{n} + b^{n}$, $b^{n} + c^{n}$, $c^{n} + d^{n}$ \Rightarrow aⁿ + kaⁿ, kaⁿ + k²aⁿ, k² aⁿ + k³aⁿ \Rightarrow aⁿ (k+1), kaⁿ (k+1), k² aⁿ (K+1) dividing each by a^n ($k+1$) \Rightarrow 1, k, k² which are clearly in G.P. F × a + a × ar+ ar × F = 156

(43) (A). Let the G.P. be *a, ar, ar*

($\frac{1}{x}$ + r+1) = 156

(a) (x + r+1) = 156

(a) $(x + 1) = 156$

(a) $x = \sum_{n=1}^{100} a_n = a_2 + a_4 +$
 $= ar + ar^3 + ...$

sare 2, 6, 18

diven a,b,c,d in G.P. usi $\left[\begin{array}{c} 1-r+1 \ 1-r+1 \end{array}\right] = 156$
 $\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots$
 $\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots$
 $\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots$
 $\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots$
 $\alpha = \sum_{n=1}^{100} a_{2n-1} = a + ar$
 $\alpha = ar(1 + r^2 - 1)$

(39) (C)
$$
\frac{a(1-r^{6})}{1-r} = 9 \frac{a(1-r^{3})}{1-r}
$$
 and $2(ar$
\n $\Rightarrow 1-r^{6} = 9(1-r^{3})$ (r $\neq 1$)
\n $\Rightarrow 1+r^{3} = 9$
\n $\therefore r=2$ Put the va

 $a + (r-1) d = AR^{r-1} = z$

(40) (B). Let first term of an A.P. be a and c.d. be d and first term of a G.P. be A and c.r. be R,then $a + (p-1) d = AR^{p-1} = x$ \Rightarrow p-1 = (x-a) /d ...(1) $a + (q - 1) d = AR^{q-1} = y$ \Rightarrow q - 1 = (y-a)/d ...(2)

1 r 1 1 5 1 1 4 1 5 160 32 2 r 1 r – 1 = (z–a) / d ...(3) Given expression = (ARp–1)y–z, (ARq–1) z–x, (ARr–1)x–y = A⁰ R(p-1)(y-z)+(q-1)(z-x)+(r-1)+(x-y) = A0R [(x–a)(y-z)+(y-a)(z-x)+(z-a)(x-y)]/d [By (1), (2) and (3)] = A0R ⁰ = 1 **(41) (B).** Here the given condition (a² + b² + c²) p² – 2p (ab + bc + ca) + b² + c² + d² 0 (ap – b)² + (bp – c)² + (cp– d)² 0 Since the squares can not be negative ap – b = 0, bp – c = 0, cp – d = 0 1 a b c p b c d a,b,c,d are in G.P. **(42) (A).** x,y,z are in A.P. 2y = x + z or 2xy = x² + xz (multiplying with x) x 2 – 2xy = – xz ...(1) x,y, t are in G.P. y 2 = xt ...(2) or (x² – 2xy + y²) = – xz + xt or (x–y)² = x (t– z) x, x–y, t– z are in G.P. **(43) (A).** Let the G.P. be , ,, ² *a ar ar* then

$$
43 \text{ (A). Let the G.P. be } a, ar, ar^2 \text{......, then}
$$
\n
$$
56 \text{ (} \text{:. } a=6 \text{)}
$$
\n
$$
\Rightarrow (3r-1)(r-3)=0 \Rightarrow r=3, \frac{1}{3}
$$
\n
$$
a = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \text{...... } \text{ upto } 100 \text{ terms}
$$
\n
$$
= ar + ar^{3} + \text{...... } \text{ upto } 100 \text{ terms}
$$
\n
$$
= ar(1 + r^{2} + r^{4} + \text{...... } r^{198})
$$
\n
$$
1 \text{ G.P. using property (iii)}
$$
\n
$$
a \text{ and } \beta = \sum_{n=1}^{100} a_{2n-1} = a + ar^{3} + \text{...... } \text{ upto } 100 \text{ terms}
$$
\n
$$
= ar(1 + r^{2} + r^{4} + \text{...... } r^{198})
$$
\n
$$
= ar(1 + r^{2} + r^{4} + \text{...... } r^{198})
$$
\n
$$
a \text{ and } \beta = \sum_{n=1}^{100} a_{2n-1} = a + ar^{3} + \text{...... } \text{ upto } 100 \text{ terms}
$$
\n
$$
= a(1 + r^{2} + \text{...... } + r^{198})
$$
\n
$$
a \text{ and } \beta = \sum_{n=1}^{100} a_{2n-1} = a + ar^{3} + \text{...... } \text{ upto } 100 \text{ terms}
$$
\n
$$
= ar(1 + r^{2} + r^{4} + \text{...... } r^{198})
$$
\n
$$
= a(1 + r^{2} + \text{...... } + r^{198})
$$
\n
$$
= a(1 + r^{2} + \text{...... } + r^{198})
$$
\n
$$
= a(1 + r^{2} + \text{...... } + r^{198})
$$
\n
$$
= a(1 + r^{2} + \text{...... } + r^{198})
$$
\n
$$
= a(1 + r^{2} + r^{4} + \text{...... } r^{198})
$$
\n
$$
= a(1 + r^{2} + r^{4
$$

Obviously $\frac{\alpha}{\beta} = r$. $\frac{\alpha}{\beta} = r$.

(44) (B). Let the numbers be a, ar, ar^2

$$
a + ar + ar2 = 14 \implies a(1 + r + r2) = 14
$$
(i)

and
$$
2(ar+1) = (a+1) + (ar^2 - 1)
$$

$$
a(r^2 - 2r + 1) = 2
$$
 (ii)

Put the value of a from (i) to (ii),

$$
\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow r = 2, \frac{1}{2} \text{ and } a = 2,8
$$

Numbers are 2, 4, 8 or 8, 4, 2. So lowest term in series is 2.

(45) (C). a, b are roots of
$$
x^2 - 3x + p = 0
$$

 $\therefore a + b = 3, ab = p$

c, d are roots of
$$
x^2 - 12x + q = 0
$$

\n $\therefore c+d = 12, cd = q$
\n a, b, c, d are in G.P.
\n(a, b, c, d are in G.P.
\n(b, c, d) are in G.P.
\n $\therefore \frac{b}{a} = \frac{d}{c} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$
\n $\Rightarrow \frac{(a-b)^2}{(a+b)^2} = \frac{(c-d)^2}{(c+d)^2} \Rightarrow 1 - \frac{4ab}{(a+b)^2} = 1 - \frac{4cd}{(c+d)^2}$
\n $\Rightarrow \frac{ab}{(a+b)^2} = \frac{cd}{(c+d)^2} \Rightarrow \frac{p}{9} = \frac{q}{144}$
\n $\Rightarrow \frac{p}{1} = \frac{q}{16} \Rightarrow \frac{p}{q} = \frac{1}{16} \Rightarrow \frac{p+q}{q-p} = \frac{17}{15}$
\n(d6) (A). By hypothesis, $a^2 = a^2bc$, $\beta^2 = b^2ca$, $\gamma^2 = c^2ab$ and
\n $2b = a + c$. Hence a^2 , β^2 , γ^2 are in A.P.
\n(d7) (A). $t_n = \log \left(\frac{5^{n+1}}{3^{n-1}}\right)$; $s_n = [\log(5/3)]^n$
\n $t_1 = \log 25$; $s_1 = [\log 5/3]^1$
\n $t_2 = \log \frac{125}{3}$; $s_2 = [\log 5/3]^2$
\n $t_3 = \log \frac{625}{9}$; $s_3 = [\log 5/3]^3$
\n $t_4 = \log 25$
\n $\therefore s = \frac{x_1 + x_2 + ... + x_n}{2}$
\n $t_5 = \log \frac{x_1 + x_2 ...}{2}$
\n $x_6 = \log \frac{x_1 + x_2 ...}{2}$
\n $x_7 = \log \frac{4b}{b}$
\n $t_8 = \log \frac{a+b}{b}$
\n $t_9 = \log \frac{125}{3}$; $s_1 = [\$

then A³ = Product of the roots $= -\frac{d}{a} \Rightarrow A = -\left(\frac{d}{a}\right)^{1/3}$ 4 > 1(1)

0

Since A is a root of the equation.

$$
ax^{3} + bx^{2} + cx + d = 0
$$

\nthen A³ = Product of the roots = $-\frac{d}{a} \Rightarrow A = -\left(\frac{d}{a}\right)^{1/3}$
\nSince A is a root of the equation.
\n $\therefore aA^{3} + bA^{2} + cA + d = 0$
\n $\Rightarrow a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{2/3} + c\left(-\frac{d}{a}\right)^{1/3} + d = 0$
\n $\Rightarrow b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3} \Rightarrow b^{3}\frac{d^{2}}{a^{2}} = c^{3}\frac{d}{a} \Rightarrow b^{3}d = c^{3}a$
\n(A). Here e² = df
\nNow dx² + 2x + f = 0 given
\n $\Rightarrow dx^{2} + 2\sqrt{df} x + f = 0 \Rightarrow x = -\sqrt{\frac{f}{d}}$
\nPutting in ax² + 2bx + c = 0 we get
\n $a\frac{f}{d} + c = 2b\sqrt{\frac{f}{d}} \Rightarrow \frac{a}{d} + \frac{c}{f} = \frac{2b}{e}$
\n(b) $A = \frac{a}{1 - r} \Rightarrow A = 4 - a$. Check with opt

Now $dx^2 + 2ex + f = 0$ given

 (49)

$$
\Rightarrow dx^2 + 2\sqrt{df} \ x + f = 0 \Rightarrow x = -\sqrt{\frac{f}{d}}
$$

Putting in $ax^2 + 2bx + c = 0$ we get

$$
a\frac{f}{d} + c = 2b\sqrt{\frac{f}{d}} \Rightarrow \frac{a}{d} + \frac{c}{f} = \frac{2b}{e}
$$

$$
\therefore \frac{a}{d}, \frac{b}{e}, \frac{c}{f} \text{ are in A.P.}; \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in H.P.}
$$

(50) (A). Given
$$
x_2 = rx_1
$$
, $x_3 = r^2x_1$, $y_2 = ry_1$, $y_3 = r^2y_1$

Q.B.-SOLUTIONS
\n
$$
\frac{a}{d} \cdot \frac{b}{e} \cdot \frac{c}{f} \text{ are in A.P.}; \quad \frac{d}{a} \cdot \frac{e}{b} \cdot \frac{f}{c} \text{ are in H.P.}
$$
\n(50) (A). Given $x_2 = rx_1$, $x_3 = r^2 x_1$, $y_2 = ry_1$, $y_3 = r^2 y_1$
\nArea of $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & py_1 & 1 \\ r^2 x_1 & r^2 y_1 & 1 \\ r^2 x_1 & r^2 y_1 & 1 \end{vmatrix}$

$$
= \frac{1}{2} x_1 y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0
$$

i.e. lie on a Straight line.

(51) (B). Given
$$
(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0
$$

$$
\begin{array}{c|c}\n\text{ap} - \text{b} = 0 \\
\text{bp} - \text{c} = 0 \\
\text{cp} - \text{d} = 0\n\end{array} p = \frac{\text{b}}{\text{a}} = \frac{\text{c}}{\text{b}} = \frac{\text{d}}{\text{c}} \qquad \therefore \text{ G.P.}
$$

UITONS
\n
$$
\frac{a}{d} \cdot \frac{b}{e} \cdot \frac{c}{f} \text{ are in A.P.}; \quad \frac{d}{a} \cdot \frac{e}{b} \cdot \frac{f}{c} \text{ are in H.P.}
$$
\n(50) (A). Given $x_2 = rx_1$, $x_3 = r^2x_1$, $y_2 = ry_1$, $y_3 = r^2y_1$
\nArea of $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0$
\ni.e. lie on a Straight line.
\n(51) (B). Given $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$
\n $\therefore \begin{cases} ap - b = 0 \\ cp - d = 0 \end{cases}$ $p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ \therefore G.P.
\n(52) (A).
\n $\therefore S = x_1x_2 + x_3x_4 + ... + x_9x_{10} \le (x_1 + x_2 + x_3 + x_5 + x_7 + x_9)$
\n $\therefore \begin{pmatrix} A \ge G \Rightarrow \frac{a+b}{2} \ge \sqrt{ab} \Rightarrow ab \le (\frac{a+b}{2})^2 \\ 2 \end{pmatrix}$
\n $\therefore S \le \left[\frac{x_1 + x_2 + ... + x_{10}}{2} \right]^2 \therefore S \le 36$
\n(53) (C). Using $AM \ge GM$
\n $\Rightarrow \frac{x + y + z}{2} \ge (xyz)^{1/3} \Rightarrow \frac{1}{3} \ge (xyz)^{1/3}$ (1)
\nAlso, $\frac{(1+x) + (1+y) + (1+z)}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}$
\n $\Rightarrow \frac{4}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}$
\n $\Rightarrow \frac{4}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}$
\n $\$

$$
\therefore S \leq \left\lfloor \frac{x_1 + x_2 + \dots + x_{10}}{2} \right\rfloor \quad \therefore S \leq 36
$$

$$
(53) \quad (C). Using AM \ge GM
$$

$$
\therefore \left(A \ge G \Rightarrow \frac{4}{2} \ge \sqrt{ab} \Rightarrow ab \le \left(\frac{4}{2} \right) \right)
$$

\n
$$
\therefore S \le \left[\frac{x_1 + x_2 + ... + x_{10}}{2} \right]^2 \therefore S \le 36
$$

\n(C). Using AM ≥ GM
\n
$$
\Rightarrow \frac{x + y + z}{2} \ge (xyz)^{1/3} \Rightarrow \frac{1}{3} \ge (xyz)^{1/3} \qquad (1)
$$

\nAlso, $\frac{(1 + x) + (1 + y) + (1 + z)}{3} \ge [(1 + x)(1 + y)(1 + z)]^{1/3}$
\n
$$
\Rightarrow \frac{4}{3} \ge [(1 + x)(1 + y)(1 + z)]^{1/3} \qquad (2)
$$

\nDividing (2) by (1) we get
\n
$$
4 \ge \left[\frac{(1 + x)(1 + y)(1 + z)}{xyz} \right]^{1/3}
$$

\n
$$
\therefore \frac{(1 + x)(1 + y) \cdot (1 + z)}{xyz} \le 64
$$

\n(D). $4 = \frac{a}{1 - r} \Rightarrow 4r = 4 - a$. Check with options.
\n(A) . ar = 24 ; ar⁴ = 3 :. r³ = $\frac{1}{8} \Rightarrow r = \frac{1}{2}$ and a = 48
\nS₆ = $\frac{a(1 - r^6)}{1 - r} = 48 \times \left(\frac{1 - \frac{1}{64}}{1 - \frac{1}{2}} \right) = 2 \times 48 \times \frac{63}{64} = \frac{3 \times 63}{2} = \frac{189}{2}$

$$
A = -\left(\frac{d}{a}\right)^{1/3}
$$

\n
$$
\Rightarrow \frac{4}{3} \ge [(1+x)(1+y)(1+z)]^{1/3}
$$
 (2)
\nDividing (2) by (1) we get

$$
4 \ge \left[\frac{\left(1+x\right)\left(1+y\right)\left(1+z\right)}{xyz} \right]^{1/3}
$$

$$
\therefore \frac{\left(1+x\right)\left(1+y\right)\left(1+z\right)}{xyz} \le 64
$$

(54) **(D).**
$$
4 = \frac{a}{1-r} \Rightarrow 4r = 4-a
$$
. Check with options.

$$
-\sqrt{\frac{f}{d}}
$$
 (55) (A) $ar = 24$; $ar^4 = 3$: $r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$ and $a = 48$

$$
\Rightarrow \frac{1}{2} \geq (xyz)^{1/3} \Rightarrow \frac{1}{3} \geq (xyz)^{1/3} \quad \dots (1)
$$

Also, $\frac{(1+x) + (1+y) + (1+z)}{3} \geq [(1+x)(1+y)(1+z)]^{1/3}$

$$
\Rightarrow \frac{4}{3} \geq [(1+x)(1+y)(1+z)]^{1/3} \quad \dots (2)
$$

Dividing (2) by (1) we get

$$
4 \geq \left[\frac{(1+x)(1+y)(1+z)}{xyz} \right]^{1/3}
$$

$$
\therefore \frac{(1+x)(1+y)(1+z)}{xyz} \leq 64
$$

(D). $4 = \frac{a}{1-r} \Rightarrow 4r = 4 - a$. Check with options.
(A). $ar = 24$; $ar^4 = 3$ $\therefore r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$ and $a = 48$

$$
S_6 = \frac{a(1-r^6)}{1-r} = 48 \times \left(\frac{1-\frac{1}{64}}{1-\frac{1}{2}} \right) = 2 \times 48 \times \frac{63}{64} = \frac{3 \times 63}{2} = \frac{189}{2}
$$

 (60)

(56) (C). By trial, putting $n = 0$,

$$
\frac{a^{0+1} + b^{0+1}}{a^0 + b^0} = \frac{a+b}{2} = A.M.
$$

Putting $n = -1/2$,

Q.B.-SOLUTIONS
\n(a) By trial, putting n = 0,
\n
$$
\frac{a^{0+1} + b^{0+1}}{a^0 + b^0} = \frac{a+b}{2} = A.M.
$$
\n
$$
\frac{a^{0+1} + b^{0+1}}{a^0 + b^0} = \frac{a+b}{2} = A.M.
$$
\nFor H.P. $\frac{1}{2} + 1$
\n
$$
\frac{1}{a^2 + b^2} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a^2 + b^2} = \frac{1}{\sqrt{a} + \sqrt{b}} = \sqrt{ab} = GM.
$$
\nHence $a_4 h_7 = a^0 + b^0 = 2ab$ (60) (D). $\because GM \ge 0$

 $n = -1$, $\frac{a + b}{a^{-1} + b^{-1}} = \frac{2ab}{a + b} = H.M.$

Alternately : For AM

n 1 n 1 a b ² 2an+1 + 2bn+1 = an+1 + bn+1+ anb + abⁿ a n+1 – aⁿ b = – bn+1 + abⁿ a n (a – b) = + bⁿ (a – b) , a b ⁿ ^a b =1 n = 0, similarly for GM and HM also.

(57) (B). Let d be common difference of the corresponding

$$
\frac{a^{-\frac{1}{2}+1} + b^{-\frac{1}{2}+1}}{a^{-\frac{1}{2}+b^{-\frac{1}{2}}}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \frac{1}{\sqrt{b}}} = \sqrt{ab} = GM.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = -1, \frac{a^{0} + b^{0}}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
n = (a_{1}.a_{2}.a_{3})^{1/3} \ge \frac{7}{(1/a_{1} + 1/a_{2} + 1/a_{3})} = \frac{27}{(1/a_{1} + 1/a_{3})
$$

$$
\frac{\frac{1}{a+7d}}{\frac{1}{a+(n-1)d}} = \frac{9}{5}
$$
(i)

Also
$$
\frac{1}{a + (n+1)d} = \frac{1}{31}
$$
 (ii)

where $a = 1$. Hence $d = 2, n = 14$.

(59) **(D).** Given
$$
a_1 = h_1 = 2
$$
, $a_{10} = h_{10} = 3$ \Rightarrow b²
Hence $a_1 + 9d = 3$

For A.P.
$$
2 + 9d = 3
$$
 or $d = \frac{1}{9}$

Q.B. SOLUTIONS **STUDY MATERIAL: MATHEMATICS**
\nC). By trial, putting n = 0,
\n
$$
\frac{a^{0+1} + b^{0+1}}{a^0 + b^0} = \frac{a+b}{2} = A.M.
$$
\n
$$
\frac{a^{\frac{1}{2}+1} + b^{-\frac{1}{2}+1}}{a^{-\frac{1}{2}+b^{-\frac{1}{2}}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \sqrt{ab} = GM.
$$
\n
$$
a = a_1 + 3d = 2 + \frac{3}{9} = 2 + \frac{1}{3} = \frac{7}{3}
$$
\nFor H.P. $\frac{1}{2} + 9d' = \frac{1}{3}$ or $9d' = -\frac{1}{6}$ or $d' = -\frac{1}{54}$
\n
$$
\frac{1}{a^{-\frac{1}{2}+b^{-\frac{1}{2}+1}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \sqrt{ab} = GM.
$$
\n
$$
a = -1, \quad \frac{a^0 + b^0}{a^{-1} + b^{-1}} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
a = -1, \quad \frac{a^0 + b^0}{a^1 + b^1} = \frac{2ab}{a+b} = H.M.
$$
\n
$$
a = \frac{a+b}{a^2 + b^2} = \frac{2ab}{a^2 + b^2} =
$$

$$
\Rightarrow (a_1 \cdot a_2 \cdot a_3) \ge \frac{27}{\left(1/a_1 + 1/a_2 + 1/a_3\right)^3}
$$

$$
(a_1 \cdot a_2 \cdot a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \ge 27.
$$

(61) (C). Let a and b be the first term and common difference of the corresponding AP, then its

$$
\frac{1}{a} + \sqrt{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \sqrt{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \frac{1}{b} = \sqrt{ab} = GM.
$$
\n
$$
\frac{1}{a} + \frac{1}{a} = \frac{1}{b} = \frac{1}{b} = \frac{1}{c} = \frac{1
$$

(63) **(B).**
$$
b = \frac{2+c}{2}
$$
 ...(1)
 $c^2 = bd$...(2)

$$
\frac{36c}{c+18}
$$
...(3)

 $d = \frac{1}{c+18}$ Eliminate d from (2) and (3) we get $c = \pm 6$ Now from (1) b = 4, - 2 from (3) $d = 9, -18$ \therefore b = 4, c = 6, d = 9

...(3)

(64) (B). Let given three terms be br, b, b/r

$$
\therefore 12 = \frac{2(br)b}{br+b} = \frac{2 br}{r+1}
$$
...(1)

and
$$
36 = \frac{2 b(b/r)}{b+(b/r)} = \frac{2 b}{r+1}
$$
 ...(2)
\n(1) ÷ (2) ⇒ r = 1/3
\nThen from (2) b = 24
\n(65) (C). Here $2 = x + z$...(1)

$$
4 = xz
$$
...(2)
Now $\frac{2x z}{x + z} = \frac{8}{2} = 4$ \therefore x, 4, z are H.P.

∴
$$
12 = \frac{2(br)}{br + b} = \frac{2 br}{r + 1}
$$
 ...(1)
\nand $36 = \frac{2 b(b/r)}{b + (b/r)} = \frac{2 b}{r + 1}$...(2)
\nand $36 = \frac{2 b(b/r)}{b + (b/r)} = \frac{2 b}{r + 1}$...(2)
\n $(1) \div (2) \Rightarrow r = 1/3$
\n $(1) \div (2) \Rightarrow r = 1/3$
\n $(2) \div (3) \Rightarrow 24$...(2)
\n $(4) \div (5) \Rightarrow 24$...(2)
\n $(4) \div (2) \Rightarrow r = 1/3$
\n $(5) \div (69) \text{ (C). By property of A.P. x + z = a + b and y = 10\n $(65) \text{ (C). Here } 2 \le x + z$...(1)
\n $4 \times z = \frac{8}{2} = 4$ ∴ x, 4, z are H.P.
\nNow $\frac{2xz}{x+z} = \frac{8}{2} = 4$ ∴ x, 4, z are H.P.
\nNow $(1) \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{2} (\frac{1}{a} + \frac{1}{b}) \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{10}{9} \Rightarrow ab$
\n $10 \text{ (a) } \frac{1}{b} + \frac{1}{c} - \frac{1}{a} \frac{1}{b} (\frac{2}{b} - \frac{1}{b})$
\n $= (\frac{1}{b} + \frac{2}{b} - \frac{1}{a} - \frac{1}{a})(\frac{1}{b}) = \frac{3}{b^2} - \frac{2}{ab}$
\n $= (\frac{1}{b} + \frac{2}{b} - \frac{1}{a})(\frac{1}{b} + \frac{1}{b} - \frac{1}{b})$
\n $= (\frac{2}{c} - \frac{1}{b})(\frac{1}{b}) = \frac{3}{bc} - \frac{2}{ab}$
\n $= (\frac{2}{c} - \frac{1}{b})(\frac{1}{b}) = \frac{3}{bc} - \frac{2}{ab}$
\n $= (\frac{2}{c} - \frac{$$

(eliminating $1/a$ in first factor and $-+-$ in second)

$$
= \left(\frac{2}{c} - \frac{1}{b}\right)\left(\frac{1}{b}\right) = \frac{2}{bc} - \frac{1}{b^2}
$$

(67) (C). a,b,c are in HP

$$
\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in AP}
$$
\n
$$
\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c} \text{ are in AP}
$$
\n
$$
\Rightarrow 1 + \frac{b+c}{a}, 1 + \frac{c+a}{b}, 1 + \frac{a+b}{c} \text{ are in AP}
$$
\n
$$
\Rightarrow \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c} \text{ are in AP}
$$
\n
$$
\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \text{ are in AP}
$$
\n
$$
\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \text{ are in HP.}
$$
\n
$$
\Rightarrow \frac{4}{a^2 - 16} = \frac{4}{a^2 - 16}
$$

(68) (D). Let the first term of A.P. be a and common difference be d. Given $(a + md)$, $(a + nd)$, $(a + rd)$ in G.P. $(a + nd)² = (a + md) (a + rd)$

NCES & SERIES)\n

Q.B. SOLUTIONS	Q.B. SOLUTIONS	Logarithms
$d = \frac{36c}{c + 18}$...3	$\Rightarrow \frac{d}{a} = \frac{2n - m - r}{mr - a^2}$
$\lim_{\text{min and } b} \frac{1}{a} = \frac{1}{b + 1} = \frac{1}{b + 1}$	36 = $\frac{2 \text{ (b)}(r)}{b + (b + r)} = \frac{2r}{r + 1}$...40
$36 = \frac{2 \text{ b}(\text{b)}(r)}{b + (b + r)} = \frac{2r}{r + 1}$...41	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...42	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...43	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...42	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...43	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...45	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...42	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b + r)} = \frac{2r}{r + 1}$...45	
$36 = \frac{2 \text{ b}(\text{b})}{b + (b$		

From (1) and (2) a,b are $9,1$

(70) **(B).** If
$$
\frac{A}{G} = \frac{p}{q} \Rightarrow \frac{a}{b} = \frac{p + \sqrt{p^2 - q^2}}{p - \sqrt{p^2 - q^2}}
$$

Here,
$$
\frac{H}{G} = \frac{G}{A} = \frac{4}{5} \Rightarrow \frac{A}{G} = \frac{5}{4}
$$
 $\therefore \frac{a}{b} = \frac{5+3}{5-3} = \frac{2}{8} = \frac{1}{4}$

12 =
$$
\frac{2(br)}{br + b} = \frac{2br}{r+1}
$$
 ...(1)
\n $36 = \frac{2b(b/r)}{b + (b/r)} = \frac{2b}{r+1}$...(2)
\n $36 = \frac{b + (b/r)}{b + (b/r)} = \frac{2b}{r+1}$...(3)
\n $42 = \frac{2b + (b/r)}{b + (b/r)} = \frac{2b}{r+1}$...(4)
\n $43 = \frac{b + (b/r)}{c + 1}$...(5)
\n $43 = \frac{b + 2b}{c + 1}$...(6)
\n $43 = \frac{b + 2b}{c + 1}$...(7)
\n $43 = \frac{b + 2b}{c + 1}$...(8)
\n $43 = \frac{b + 2b}{c + 1}$...(9)
\n $43 = \frac{1}{a} \times \frac{1}{b} = \frac{1}{b} = \frac{1}{a} \times \frac{1}{b} = \frac{1}{b} \times$

$$
= \frac{1}{4n^2 - 1} = \frac{1}{(2n - 1)(2n + 1)} = 2 \left\{ \frac{1}{2n - 1} - \frac{1}{2n + 1} \right\}
$$

$$
\therefore S_n = \Sigma t_n = 2 \left\{ \frac{1}{1} - \frac{1}{2n + 1} \right\} = \frac{4n}{2n + 1}.
$$

(74) (B) tⁿ ⁼ n 1 1.3.5..... 2n 3 n 1.2 = vⁿ – vn + 1 Sⁿ = t¹ ⁺ ⁿ n 2 ^t ⁼ ¹ 2 + n n n 1 2 + v² – vn + 1 ⁼ ¹ 2 + 1 2 – n.2 = 1 – 1 2x 35 (1 x) ¹⁶

(75) **(B).** $S = 1 + 4x + 7x^2 + 10x^3 + \dots$ $x.S = x + 4x^2 + 7x^3 + \dots$ Subtract, $S(1-x) = 1 + 3x + 3x^2 + 3x^3 + \dots$

$$
S(1-x) = 1 + 3x \left(\frac{1}{1-x}\right) \quad |x| < 1 \quad \Rightarrow \quad S = \frac{1+2x}{(1-x)^2}
$$

Given:
$$
\frac{1+2x}{(1-x)^2} = \frac{35}{16}
$$

\n
$$
\Rightarrow 16+32x = 35+35x^2 - 70x \Rightarrow 35x^2 - 102x + 19 = 0
$$

\n
$$
\Rightarrow 35x^2 - 7x - 95x + 19 = 0 \Rightarrow 7x(5x - 1) - 19(5x - 1) = 0
$$

\n
$$
\Rightarrow (5x-1)(7x-19) = 0 \Rightarrow x = \frac{1}{5}, \frac{19}{7} \text{ But } |x| < 1
$$

\n $\therefore x = 1/5$

(76) **(A).**
$$
T_r = \frac{r}{(2r-1) 2r (2r+1)} = \frac{1}{4} \left(\frac{1}{2r-1} - \frac{1}{2r+1} \right)
$$

$$
S = \frac{1}{4} \left\{ \left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \dots \right\} = \frac{1}{4}
$$

Given:
$$
\frac{1+2x}{(1-x)^2} = \frac{35}{16}
$$

\n⇒ 16+32x=35x²−70x ⇒ 35x²−102x+19=0
\n⇒ 35x²−7x-95x+19=0 ⇒ 7x(5x-1)-19(5x-1)=0
\n⇒ (5x-1)(7x-19)=0 ⇒ x = $\frac{1}{5}, \frac{19}{7}$ But $|x| < 1$
\n∴ x=1/5
\n(76) (A). T_t = $\frac{r}{(2r-1) 2r (2r+1)} = \frac{1}{4} (\frac{1}{2r-1} - \frac{1}{2r+1})$
\n $\Rightarrow \frac{1}{4} [(\frac{1}{3} + \frac{1}{7} + \frac{1}{11} + \frac{1}{11}$

 $-\frac{1.3.5....(2n-3)(2n-1)}{2^n}$ (79) (A). Let T_n be the nth term of the series, then $T_n = 3n(n+1)$ **(O.B.-SOLUTIONS** STUDY MATERIAL: MATHEMATICS
 $\frac{2n-3(2n-1)}{2n}$ (79) (A). Let T_n be the nth term of the series, then $T_n = 3n(n+1)$

If S_n denotes the sum of first n terms
 $\therefore S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n (3k^2 + 3k)$ If S_n denotes the sum of first n terms

Q.B.-SOLUTIONS
\nSTUDY MATERIAL: MATHEMATICS
\n
$$
i\mathbf{r}_n = \frac{1.3.5....(2n-3)}{n-1.2^{n-1}} - \frac{1.3.5....(2n-3)(2n-1)}{n.2^n}
$$
\n
$$
= v_n - v_{n+1}
$$
\n
$$
= t_1 + \sum_{n=1}^{n} t_n = \frac{1}{2} + \sum_{n=1}^{n} (v_n - v_{n+1}) = \frac{1}{2} + v_2 - v_{n+1}
$$
\n
$$
= t_1 + \sum_{n=1}^{n} t_n = \frac{1}{2} + \sum_{n=1}^{n} (v_n - v_{n+1}) = \frac{1}{2} + v_2 - v_{n+1}
$$
\n
$$
\therefore S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (3k^2 + 3k)
$$
\n
$$
\frac{1}{2} + \frac{1}{2} - \frac{1.3.5...(2n-1)}{n.2^n} = 1 - \frac{1.3.5...(2n-1)}{n.2^n}
$$
\n
$$
= 3 \sum_{k=1}^{n} (k^2) + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$
\n
$$
= 3 \sum_{k=1}^{n} (k^2) + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$
\n
$$
= 3 \sum_{k=1}^{n} (k^2) + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$
\n
$$
= 3 \sum_{k=1}^{n} (k^2) + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$
\n
$$
= 3 \sum_{k=1}^{n} (k^2) + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$
\n
$$
= 3 \sum_{
$$

$$
\Rightarrow S = \frac{1+2x}{(1-x)^2}
$$
 (80) (D). The series, $\frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 15} + \dots$

$$
= v_{n} - v_{n+1}
$$

\n
$$
S_{n} = t_{1} + \sum_{2}^{n} t_{n} = \frac{1}{2} + \sum_{2}^{n} (v_{n} - v_{n+1}) = \frac{1}{2} + v_{2} - v_{n+1}
$$

\n
$$
= \frac{1}{2} + \frac{1}{2} - \frac{1.3.5...(2n-1)}{n.2^{n}} = 1 - \frac{1.3.5...(2n-1)}{n.2^{n}}
$$

\n
$$
= 3 \sum_{k=1}^{n} (k^{2} + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$

\n
$$
= 3 \sum_{k=1}^{n} (k^{2} + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$

\n
$$
= 3 \sum_{k=1}^{n} (k^{2} + 3 \sum_{k=1}^{n} (k) = \frac{3n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}
$$

\n
$$
= n(n+1)(n+2).
$$

\n
$$
S(1-x) = 1 + 3x \left(\frac{1}{1-x}\right) - |x| < 1 \Rightarrow S = \frac{1+2x}{(1-x)^{2}}
$$

\n
$$
= n(n+1)(n+2).
$$

\n
$$
S(1-x) = 1 + 3x \left(\frac{1}{1-x}\right) - |x| < 1 \Rightarrow S = \frac{1+2x}{(1-x)^{2}}
$$

\n
$$
= n(n+1)(n+2).
$$

\n
$$
S(1-x) = 1 + 3x \left(\frac{1}{1-x}\right) - |x| < 1 \Rightarrow S = \frac{1+2x}{(1-x)^{2}}
$$

\n
$$
= n(n+1)(n+2).
$$

\n
$$
S(1-x) = 1 + 3x \left(\frac{1}{1-x}\right) - |x| < 1 \Rightarrow S = \frac{1+2x}{(1-x)^{2}}
$$

\n
$$
= n(n+1)(n+2).
$$

\n
$$
S
$$

(81) (A). Given series
$$
27 + 9 + 5 \cdot \frac{2}{5} + 3 \cdot \frac{6}{7} + \dots
$$

$$
= 27 + \frac{27}{3} + \frac{27}{5} + \frac{27}{7} + \dots + \frac{27}{2n - 1} + \dots
$$

Hence n^{th} term of given series $T_n = \frac{27}{2n-1}$ -1

So,
$$
T_9 = \frac{27}{2 \times 9 - 1} = \frac{27}{17} = 1 \frac{10}{17}
$$
.

(82) (A). On putting $n = 1, 2, 3, \dots$

First term of the series $a = \frac{1}{x} + y$, Second term $= \frac{2}{x} + y$

$$
\therefore d = \left(\frac{2}{x} + y\right) - \left(\frac{1}{x} + y\right) = \frac{1}{x}
$$

Sum of *r*terms of the series

$$
= \frac{r}{2}\left[2\left(\frac{1}{x} + y\right) + (r - 1)\frac{1}{x}\right] = \frac{r}{2}\left[\frac{2}{x} + 2y + \frac{r}{x} - \frac{1}{x}\right]
$$

$$
= \frac{r^2 - r + 2r}{2x} + ry = \left[\frac{r(r+1)}{2x} + ry\right].
$$

(83) (A). Given series is

$$
3 + 4\frac{1}{2} + 6\frac{3}{4} + \dots \dots = 3 + \frac{9}{2} + \frac{27}{4} + \dots
$$
Subtra

$$
= 3 + \frac{3^2}{2} + \frac{3^3}{4} + \frac{3^4}{8} + \frac{3^5}{16} + \dots (in G.P.)
$$

Here $a = 3$, $r = 3/2$, then sum of the five terms

$$
S_5 = \frac{a(r^n - 1)}{r - 1} = \frac{3\left[\left(\frac{3}{2}\right)^5 - 1\right]}{\frac{3}{2} - 1} = \frac{1\left[\frac{3^5}{32} - 1\right]}{\frac{1}{2}}
$$
(90) (C)

$$
= 6\left[\frac{243 - 32}{32}\right] = \frac{211 \times 3}{16} = \frac{633}{16} = 39\frac{9}{16}.
$$

(84) **(B).**
$$
9+99+999+...
$$
 + upto n terms
\n $\Rightarrow (10-1)+(100-1)+(1000-1)+...$ + upto n terms

$$
=\frac{10(10^{n}-1)}{9}-n=\frac{10^{n+1}-9n-10}{9}
$$

- **(85) (C).** The given series is an A.G.P. with common ratio $S = a-(a+d)+(a+2d)-(a+3d)+....+(a+2nd)$ $- S = -a + (a + d) - (a + 2d) + ... + (a + 2nd) - (a + 2nd)$ (9 \therefore 2S = a + {-d + d -d + d...upto 2n terms} + (a+2nd) \Rightarrow 2S = 2a + 2nd S = a + nd
- **(86) (A).** Let S be the sum of n terms of the given series and $x = 1 + 1/n$, Then, $S = 1 + 2x + 3x^2 + 4x^3 + \dots + n x^{n-1}$ \Rightarrow xS = x + 2x²+3x³ +....+ (n–1) x^{n–1} + nxⁿ \therefore S – xS = 1 + [x + x² + ...+ xⁿ⁻¹] –nxⁿ

$$
\Rightarrow S(1-x) = \frac{1-x}{1-x} - n x^n
$$

\n
$$
\Rightarrow S(-1/n) = -n[1-(1+1/n)^n] - n(1+1/n)^n
$$

\n
$$
\Rightarrow \frac{1}{n} \cdot S = n[1-(1+1/n)^n + (1+1/n)^n] = n
$$

\n
$$
\Rightarrow S = n^2
$$

(87) (C). Let $S = 1 + 2.2 + 3.2^2 + 4.2^3 + ... + 100.2^{99}$ (1) \Rightarrow 2S = 2+2.2² +3.2³ +....+99.2⁹⁹ +100.2¹⁰⁰(2) Subtracting (2) from (1) , we get $-S = (1+2+2^2+2^3+....+2^{99})-100.2^{100}$

$$
\Rightarrow S = 100.2^{100} - \frac{2^{100} - 1}{2 - 1}
$$
(95)
= 100.2¹⁰⁰ - 2¹⁰⁰ + 1 = 1 + 99.2¹⁰⁰
(A) The given series is an arithmetic.

(88) (A). The given series is an arithmetico- geometric series. The sum of the series is given by

$$
\frac{3}{1-\frac{1}{4}} + \frac{d \times \frac{1}{4}}{\left(1-\frac{1}{4}\right)^2} \left[u \sin g : S = \frac{a}{1-r} + \frac{dr}{\left(1-r\right)^2} \right]
$$

4 + $\frac{4d}{9} = 8 \Rightarrow d = 9$

27 \sim **(89) (C).** \because S = 1 + (1 + a) x + (1 + a + a²) x² ... ∞ ... (1) \therefore axS = $ax + (a + a^2) x^2 \dots x^2$ ∞ (2) Subtracting (2) from (1) , we get

IDENTIFY (A). Given series is
\n(A). Given series is
\n(A) Given series
\n(A) Given series
\n(A) Given
$$
\frac{1}{2} + 6\frac{3}{4} + \dots = 3 + \frac{9}{2} + \frac{27}{4} + \dots
$$

\n \therefore $\frac{1}{2} + 3\frac{3}{2} + \frac{3^3}{4} + \frac{3^5}{16} + \dots$ (in G.P.)
\n(B) $\frac{1}{2} + \frac{3^3}{4} + \frac{3^4}{8} + \frac{3^5}{16} + \dots$ (in G.P.)
\n(B) $\frac{1}{2} + \frac{3}{4} + \frac{3^5}{8} + \frac{3^4}{16} + \dots$ (in G.P.)
\n $S_5 = \frac{a(p^8 - 1)}{r - 1} = \frac{3}{2-1} = \frac{3}{2-1}$
\n $= 3 + \frac{3^2}{2} + \frac{3^3}{4} + \frac{3^4}{8} + \frac{3^5}{16} + \dots$ (in G.P.)
\n(A) $\frac{1}{2} + \frac{1}{2} + \$

Check with options, option (2) only satisfies. **(92) (B).** Checking with options, putting $n = 2$

$$
S_2 = \frac{1}{3} + \frac{2}{3} = 1
$$
 satisfies only.

1 x – ¹⁰⁰ 2 1 2 1 a dr u sin g : S 1 r 1 r 2 n n 2 1 S t n 1 3 3 2 n (n 1) 1 n n (n 2) 3 2 3 3 **(94) (A).** 2 log (2^x – 1) = log 2 + log (2^x + 3) (2^x – 1)² = 2. (2^x

⇒
$$
(2^{x}-1)^{2} = 2.(2^{x}+3)
$$
 ⇒ $(2^{x})^{2} - 4.2^{x} - 5 = 0$
\n⇒ $(2^{x}-5)(2^{x}+1) = 0$
\n⇒ $x = \log_{2} 5$, as $2^{x}+1 \neq 0$

(93) (A).
$$
t_n = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}
$$

\n $S_n = \sum t_n = \frac{2}{3} \sum n + \frac{1}{3} \sum 1$
\n $= \frac{2}{3} \times \frac{n(n+1)}{2} + \frac{1}{3}n = \frac{n}{3}(n+2)$
\n(94) (A). 2 log (2^x - 1) = log 2 + log (2^x + 3)
\n⇒ (2^x - 1)² = 2. (2^x + 3) ⇒ (2^x)² - 4.2^x - 5 = 0
\n⇒ (2^x - 5) (2^x + 1) = 0
\n⇒ x = log₂5, as 2^x + 1 ≠ 0
\n(95) (A). x, y, z are in A.P. ⇒ 2y = x + z
\nor 2xy = x² + xz (multiply with x)
\n⇒ x² - 2xy = -xz (multiply with x)
\n⇒ x² - 2xy = -xz (multiply with x)
\nor (x² - 2xy + y²) - xz + xt
\nor (x² - 2xy + y²) - xz + xt
\nor (x - y)² = x (t - z) ⇒ x, x - y, t – z are in G.P.
\n(96) (C). x₁x₂ = 18² = 12.27, $\frac{2x_1x_2}{x_1 + x_2} = \frac{216}{13}$ giving
\n $x_1 + x_2 = \frac{26.18^2}{216} = 39 = 27 + 12$, $|x_1 - x_2| = 15$

(96) (C).
$$
x_1x_2 = 18^2 = 12.27
$$
, $\frac{2x_1x_2}{x_1 + x_2} = \frac{216}{13}$ giving

$$
x_1 + x_2 = \frac{26.18^2}{216} = 39 = 27 + 12
$$
, $|x_1 - x_2| = 15$

(97) (D). If r is the common ratio,

Q.B.-SOLUTIONS	STUDY MATERIAL: MATHEM	
(97)	(D). If r is the common ratio,	By cyclical interchanges $\Sigma ab (p - q) = 0$ or $\Sigma \frac{p - q}{c}$
$= \sqrt{r} (a_1 + a_3 + \dots + a_{2n-1})$	(3)	(B). Let $b = ar, c = ar^2$ and $d = ar^3$. Then,
$\sqrt{a_2 a_3} + \sqrt{a_4 a_5} + \dots + \sqrt{a_{2n} a_{2n+1}}$	(3)	(C). Let $b = ar, c = ar^2$ and $d = ar^3$. Then,
$\sqrt{a_2 a_3} + \sqrt{a_4 a_5} + \dots + \sqrt{a_{2n} a_{2n+1}}$	(3)	(D). Let $b = ar, c = ar^2$ and $d = ar^3$. Then,
$\sqrt{a_2 a_3} + \sqrt{a_4 a_5} + \dots + \sqrt{a_{2n} a_{2n+1}}$	(3)	(4)
$\sqrt{a_2 a_3} + \sqrt{a_4 a_5} + \dots + \sqrt{a_{2n} a_{2n+1}}$	(5)	
$= \sqrt{r} (a_2 + a_4 + \dots + a_{2n})$	(6)	$\frac{1}{a^3 + b^3} = \frac{1}{a^3 (1 + r^3)}$
$= \frac{1}{3} \times \frac{n}{2} \{2a + (n-1)d\}$	$\frac{500}{2} \{2a + 499d\}$	$\frac{1}{c^3 + d^3} = \frac{1}{a^3 r^3 (1 + r^3)}$
$= \frac{1}{3} \times \frac{n}{2} \{2a + (n-1)d\}$	$\frac{4}{n}$	

comparing coefficients of a, $\frac{n}{2} = 1000 - 500$ $\frac{1}{3}$ = 1000 – 500 \Rightarrow n = 1500

This agrees with the coefficient of d as well

(99) (C). If a is the first term and d is the common difference of the associated A P the associated A.P.

$$
\frac{1}{q} = \frac{1}{a} + (2p - 1)d, \frac{1}{p} = \frac{1}{a} + (2q - 1)d \Rightarrow d = \frac{1}{2pq}
$$
lo; If h is the 2(p + q)th term

$$
\frac{1}{h} = \frac{1}{a} + (2p + 2q - 1)d = \frac{1}{q} + \frac{1}{p} = \frac{p+q}{pq}, \quad h = \frac{pq}{p+q}
$$
 i.e., log

(100) (C) 200
$$
\leq \frac{9}{2}
$$
 (2a + 8d) < 220 and a + d = 12
\n∴ 200 < 9(12+3d) < 220
\n
\n92 < 27 d < 112 ; 3 $\frac{11}{27}$ < d < 4 $\frac{4}{27}$ ∴ d = 4
\n
\n6) (C). x, y,

EXERCISE-2

(1) (C).
$$
\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c} \text{ are in A.P.}
$$

$$
\therefore \frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2
$$

are in A.P. (adding 2 in each term)

or
$$
\frac{a+b+c}{a}
$$
, $\frac{c+a+b}{b}$, $\frac{a+b+c}{c}$ are in A.P.
\n[dividing by (a+b+c) in each term] (7)

or
$$
\frac{1}{a}, \frac{1}{b}, \frac{1}{c}
$$
 are in A.P

(2) (D). If α is the first derm of the H.P. and d is the common difference of the associated A.P., then

$$
\frac{1}{\alpha} + (p-1)d = \frac{1}{a}, \frac{1}{\alpha} + (q-1)d = \frac{1}{b}, \frac{1}{\alpha} + (r-1)d = \frac{1}{c}
$$
\n
$$
\therefore (p-q)d = \frac{1}{a} - \frac{1}{b} \text{ or } ab (p-q)d = b-a
$$
\n
$$
\therefore 1/(2) \Rightarrow r = \frac{1}{a} \Rightarrow r = 8
$$
\n
$$
\therefore a = br = 8
$$

 $\frac{-q}{c} = 0.$ **TERIAL: MATHEMATICS**

(p-q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

= ar³. Then,
 $\frac{1}{+c^3} = \frac{1}{a^3r^3(1+r^3)}$ **TERIAL: MATHEMATICS**

(p-q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

= ar³. Then,
 $\frac{1}{+c^3} = \frac{1}{a^3r^3(1+r^3)}$
 $\frac{1}{a^3r^3(1+r^3)}$

(3) **(B).** Let
$$
b = ar, c = ar^2
$$
 and $d = ar^3$. Then,

S **STUDY MATERIAL: MATHEMATICS**
\nBy cyclical interchanges
$$
\Sigma ab (p - q) = 0
$$
 or $\Sigma \frac{p - q}{c} = 0$.
\n**(B).** Let $b = ar, c = ar^2$ and $d = ar^3$. Then,
\n
$$
\frac{1}{a^3 + b^3} = \frac{1}{a^3(1 + r^3)}, \frac{1}{b^3 + c^3} = \frac{1}{a^3r^3(1 + r^3)}
$$
\nand
$$
\frac{1}{c^3 + d^3} = \frac{1}{a^3r^3(1 + r^3)}
$$
\nClearly, $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$ and $(c^3 + d^3)^{-1}$ are in G.P.
\nwith common ratio $1/r^3$.
\n**(B).** We have $(x_1 + x_2 + + x_{50}) \left(\frac{1}{x} + \frac{1}{x} + + \frac{1}{x} \right)$

and
$$
\frac{1}{c^3 + d^3} = \frac{1}{a^3 r^3 (1 + r^3)}
$$

IATERIAL: MATHEMATICS

Cab (p-q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

d = ar³. Then,
 $\frac{1}{b^3 + c^3} = \frac{1}{a^3r^3(1+r^3)}$
 $\frac{3}{b^3}$
 $\frac{1}{b^3} = \frac{1}{(a^3 + d^3)^{-1}}$ are in G.P. Clearly, $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$ and $(c^3 + d^3)^{-1}$ are in G.P. with common ratio $1/r^3$.

STUDY MATERIAL: MATHEMATICS

lical interchanges Σ ab (p-q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

t b = ar, c = ar² and d = ar³. Then,
 $\frac{1}{b^3} = \frac{1}{a^3(1+r^3)}$, $\frac{1}{b^3+c^3} = \frac{1}{a^3r^3(1+r^3)}$
 $\frac{1}{c^3+d^3} = \frac{1}{a^3r^3(1+r^$ **(4) (B).** We have $(x_1 + x_2 + \dots + x_{50}) \Big| \frac{1}{x}$ = 0 or $\Sigma \frac{p-q}{c} = 0$.
hen,
 $\frac{1}{a^3r^3(1+r^3)}$
 $c^3 + d^3$)⁻¹ are in G.P.
 $\frac{1}{1} + \frac{1}{x_2} + \dots + \frac{1}{x_{50}}$ L: MATHEMATICS
 $) = 0$ or $\Sigma \frac{p-q}{c} = 0$.

Then,
 $\frac{1}{a^3 r^3 (1+r^3)}$
 $(c^3 + d^3)^{-1}$ are in G.P.
 $\frac{1}{x_1} + \frac{1}{x_2} + + \frac{1}{x_{50}}$ **L: MATHEMATICS**

(1) = 0 or $\Sigma \frac{p-q}{c} = 0$.

Then,
 $= \frac{1}{a^3 r^3 (1+r^3)}$
 $(c^3 + d^3)^{-1}$ are in G.P.
 $\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{50}}$
 $\log_{2} 3(3^x + 48)$ and IAL: MATHEMATICS

q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

Then,
 $= \frac{1}{a^3r^3(1+r^3)}$

d $(c^3 + d^3)^{-1}$ are in G.P.
 $\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{50}}\right)$ IAL: MATHEMATICS

q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

Then,
 $= \frac{1}{a^3 r^3 (1+r^3)}$

d $(c^3 + d^3)^{-1}$ are in G.P.
 $\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_{50}}\right)$ \geq (50)² [since A.M. \geq H.M.] $rac{1}{1^{3} + b^{3}} = \frac{1}{a^{3}(1 + r^{3})}$, $rac{1}{b^{3} + c^{3}} = \frac{1}{a^{3}r^{3}(1 + r^{3})}$
 $\frac{1}{c^{3} + d^{3}} = \frac{1}{a^{3}r^{3}(1 + r^{3})}$
 $\frac{1}{c^{3} + d^{3}} = \frac{1}{a^{3}r^{3}(1 + r^{3})}$
 $\frac{1}{c^{3} + d^{3}} = \frac{1}{(b^{3} + b^{3})^{-1}}$, $(b^{3} + c^{3})^{-1}$ and $(c^{3} + d^{3})^{-1}$ STUDY MATERIAL: MATHEMATICS

v cyclical interchanges $\Sigma ab (p - q) = 0$ or $\Sigma \frac{p - q}{c} = 0$.

). Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then,
 $\frac{1}{a^3 + b^3} = \frac{1}{a^3(1 + r^3)}$, $\frac{1}{b^3 + c^3} = \frac{1}{a^3r^3(1 + r^3)}$
 $\frac{1}{c^3 + d^3} = \frac{1}{a$ STUDY MATERIAL: MATHEMATICS

y cyclical interchanges $\Sigma ab (p - q) = 0$ or $\Sigma \frac{p - q}{c} = 0$.

S). Let $b = ar, c = ar^2$ and $d = ar^3$. Then,
 $\frac{1}{a^3 + b^3} = \frac{1}{a^3(1+r^3)}$, $\frac{1}{b^3 + c^3} = \frac{1}{a^3r^3(1+r^3)}$
 $\frac{1}{a^3 + d^3} = \frac{1}{a^3r^$ By cyclical interchanges $\Sigma ab (p - q) = 0$ or $\Sigma \frac{p - q}{c} = 0$.
 (B). Let $b = ar$, $c = ar^2$ and $d = ar^3$. Then,
 $\frac{1}{a^3 + b^3} = \frac{1}{a^3(1 + r^3)}$, $\frac{1}{b^3 + c^3} = \frac{1}{a^3r^3(1 + r^3)}$

and $\frac{1}{c^3 + d^3} = \frac{1}{a^3r^3(1 + r^3)}$

Clear S STUDY MATERIAL: MATHEMATICS

By cyclical interchanges Σab (p-q) = 0 or $\Sigma \frac{p-q}{c} = 0$.

(B). Let b = ar, c = ar² and d = ar³. Then,
 $\frac{1}{a^3 + b^3} = \frac{1}{a^3(1+r^3)}$, $\frac{1}{b^3+c^3} = \frac{1}{a^3r^3(1+r^3)}$

and $\frac{1}{c^3$

(5) (B). The three numbers are
$$
\log_9 9
$$
, $\log_{9^2} (3^{x} + 48)$ and

$$
\frac{1}{42} + \sqrt{a_3 a_4} + \dots + \sqrt{a_{2n+1} a_n}
$$
\nBy cyclical interchanges 2ab (p - q) = 0 or $\sum \frac{p-q}{c} = 0$.
\n
$$
= \sqrt{r} (a_1 + a_3 + \dots + a_{2n-1})
$$
\n
$$
= \sqrt{r} (a_2 + a_3 + \dots + a_{2n-1})
$$
\n
$$
= \sqrt{r} (a_2 + a_4 + \dots + a_{2n-1})
$$
\n
$$
= \sqrt{r} (a_2 + a_4 + \dots + a_{2n})
$$
\n
$$
\frac{1}{a^2 + b^2} = \frac{1}{a^2(1 + r^2)}, \frac{1}{b^2 + c^2} = \frac{1}{a^2r^2(1 + r^2)}
$$
\n
$$
= \sqrt{r} (a_2 + a_4 + \dots + a_{2n})
$$
\n
$$
\frac{1}{3} \times \frac{n}{2} [2a + (n-1)d]
$$
\n
$$
\frac{1}{3} \times \frac{n}{2} [2a + (n-1)d]
$$
\n
$$
= \frac{1500}{3} (2a + 999d) - \frac{500}{2} (2a + 499d)
$$
\n
$$
= \frac{n}{3} [2a + (n-1)d]
$$
\n
$$
= \frac{1}{3} \times \frac{n}{2} [2a + (n-1)d]
$$
\n
$$
= \frac{1}{3} \times \frac{n}{2} [2a + (n-1)d]
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$
\n
$$
= \frac{1}{3} \times (2n + (n-1)d)
$$

$$
y = \frac{\log \lambda}{\log b}, \quad z = \frac{\log \lambda}{\log c}
$$

putting x,y,z in (i), we get, $|$

 $(\log b)^2 = \log a$. $\log c$ or $\log_a b = \log_b c \implies \log_b a = \log_c b$ **(7) (C).** Let given three terms be br, b, b/r

$$
\therefore 12 = \frac{2(br)b}{br+b} = \frac{2br}{r+1}
$$
(1)

 $\log \lambda$ ² $\log \lambda$ $\log \lambda$

$$
\frac{1}{\alpha} + (r-1) d = \frac{1}{c}
$$
 and $36 = \frac{2b(\frac{b}{r})}{b+(\frac{b}{r})} = \frac{2b}{r+1}$ (2)

 $(1) / (2) \Rightarrow r = 1/3$ Then from (2) b = 24 \therefore a = br = 8

 $+n$

 $-n^2$

 \therefore $(a + nd)^2 = (a + md) (a + rd) \Rightarrow \frac{d}{a} = \frac{2n - nd - 1}{mr}$

But m, n, r in H.P. \Rightarrow n = $\frac{2mr}{\sqrt{m}}$

**EXECUTE:
$$
\vec{A} \times \vec{B}
$$**
\n**EXECUTE: $\vec{B} \times \vec{B}$**
\n**EXECUTE:**
\n**EXEC**

EXECUTES A SERIES)
\n(C)
$$
\sqrt{11-4\sqrt{6}} = 2\sqrt{2} - \sqrt{3}
$$

\n $\sqrt{6-2\sqrt{3}+2\sqrt{2}-2\sqrt{6}} = 1 + \sqrt{2} - \sqrt{3}$
\n $\sqrt{6-2\sqrt{3}+2\sqrt{2}-2\sqrt{6}} = 1 + \sqrt{2} - \sqrt{3}$
\n $\sqrt{7-4\sqrt{3}} = 2 - \sqrt{3}$ and these form an A.P. with common
\ndifference $= 1 - \sqrt{2}$.
\nHence required numbers **are in H.P.**
\n**EXECUTE:**
\n**EXECUTE**

$$
\Rightarrow \frac{\frac{x}{y} + \frac{y}{z} + \frac{z}{x}}{3} \ge \sqrt[3]{\frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{x}} = 1
$$

$$
\therefore \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_{n+1}}
$$
\n
$$
= \frac{1}{d} \left\{ \left(\frac{1}{a_1} - \frac{1}{a_2} \right) + \left(\frac{1}{a_2} - \frac{1}{a_3} \right) + \dots + \left(\frac{1}{a_n} - \frac{1}{a_{n+1}} \right) \right\}
$$
\n
$$
= \frac{1}{d} \left(\frac{1}{a_n} - \frac{1}{a_{n+1}} \right) = \frac{1}{d} \left(\frac{a_{n+1} - a_1}{a_1 a_{n+1}} \right)
$$
\n(15) (15)

nd n

(11) (D). Let the first term of A.P. be a and common difference be d. Given $(a + md)$, $(a + nd)$, $(a + rd)$ in G.P.

EXAMPLE ARENAL: MATHEMATICS
\n(17) (A)
$$
\frac{a_1 + a_4}{a_1 a_4} = \frac{a_2 + a_3}{a_2 a_3}
$$
,
\nSo $\frac{1}{a_4} + \frac{1}{a_1} = \frac{1}{a_3} + \frac{1}{a_2}$ or $\frac{1}{a_4} - \frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}$(1)
\nAlso $\frac{3(a_2 - a_3)}{a_2 a_3} = \frac{a_1 - a_4}{a_1 a_4}$
\nSo $\frac{1}{a_4} + \frac{1}{a_1} = \frac{1}{a_3} + \frac{1}{a_2}$ or $\frac{1}{a_4} - \frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}$(1)
\nAlso $\frac{3(a_2 - a_3)}{a_2 a_3} = \frac{a_1 - a_4}{a_1 a_4}$
\nSo $3(\frac{1}{a_3} - \frac{1}{a_2}) = \frac{1}{a_4} - \frac{1}{a_1}$
\nSo $3(\frac{1}{a_3} - \frac{1}{a_2}) = \frac{1}{a_4} - \frac{1}{a_1}$
\nClearly, (1) and (2)
\n $\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3}$; $\frac{1}{a_1} \frac{1}{a_2} \frac{1}{a_3}$
\n(23) (A) . a₁Rm⁺ⁿ⁻¹ = A(1)
\n $\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3}$; $\frac{1}{a_1} \frac{1}{a_2} \frac{1}{a_3}$
\n(23) (A) . a₁Rm⁺ⁿ⁻¹ = B(2)
\nDividing (2) we get
\n (18) (A) . The integers divisible by 3 are 33 in

Clearly, (1) and (2)

$$
\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3}; \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} \text{ are in A.P.}
$$

(18) (A). The integers divisible by 3 are 33 in number and are 3, 6,, 99.

> The integers divisible by 5 are 20 in number and are 5, 10,, 100.

> The integers divisible by 7 are 14 in number and are 7, 14,, 98.

The integers divisible by both 3 and 5 are 6 in number and are 15, 30,, 90.

The integers divisible by both 3 and 7 are 4 in number and are 21, 42, 63 and 84.

The integers divisible by both 5 and 7 are 2 in number and are 35 and 70.

There are no integers divisible by all three.ss

Hence the sum of the numbers divisible by 3 or 5 or 7 is

$$
\frac{33}{2}(3+99) + \frac{20}{2}(5+100) + \frac{14}{2}(7+98) - \frac{6}{2}(15+90)
$$
\n
$$
-\frac{4}{2}(21+84) - (35+70) = 2838.
$$
\n
$$
\Rightarrow \frac{\log 4}{\log(2^{1-x}+1)}
$$

$$
2^{(3+3)/3} 2^{(3+16)/3} 2^{(3+3)/3} 2^{(3+3)/3} 2^{(3+3)/3}
$$
\n
$$
-\frac{4}{2}(21+84)-(35+70)=2838.
$$
\n
$$
\frac{1}{\log(2^{1-x}+1)} = \frac{1}{\frac{\log 4}{\log(2^{1-x}+1)}} = \frac{2 \cdot \frac{1}{10}}{\frac{1}{10}}
$$
\n
$$
\sqrt{\frac{a}{b}} = 3 \pm 2\sqrt{2} \text{ or } \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}, \frac{3-2\sqrt{2}}{3+2\sqrt{2}}
$$
\n
$$
\sqrt{a} = 3 \pm 2\sqrt{2} \text{ or } \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}, \frac{3-2\sqrt{2}}{3+2\sqrt{2}}
$$
\n
$$
\sqrt{a} = 3 \pm 2\sqrt{2} \text{ or } \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}, \frac{3-2\sqrt{2}}{3+2\sqrt{2}}
$$
\n
$$
10. t+2 = 2/t+1 \Rightarrow 10t
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
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\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + t - 2 = 0
$$
\n
$$
10t^2 + 5t - 4t - 2 = 0
$$
\n
$$
10t^2 + 5t - 4t - 2 = 0
$$
\n
$$
10t^2 - 2(2t+1) = 0 = 0
$$
\n
$$
10t^2 - 2(2t+1) = 0 = 0
$$
\n
$$
10t^2
$$

$$
3 + 2\sqrt{2} \text{ and } 3 - 2\sqrt{2}.
$$

(20) (A). q² = AR^{2p-1} and p² = AR^{2q-1}
T_{p+q} = AR^{p+q-1} = (AR^{2p-1}. AR^{2q-1})^{1/2} = (p²q²)^{1/2} = pq
(21) (B). AM ≥ GM

$$
\sqrt{\frac{a}{b}} = 3 \pm 2\sqrt{2} \text{ or } \frac{a}{b} = \frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}, \frac{3 - 2\sqrt{2}}{3 + 2\sqrt{2}} = \frac{1}{2}
$$
\nAs $a^2 + b^2 = 34$, the two numbers are
\n $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$.
\n(A). $q^2 = AR^{2p-1}$ and $p^2 = AR^{2q-1}$
\n $T_{p+q} = AR^{p+q-1} = (AR^{2p-1} \cdot AR^{2q-1})^{1/2} = (p^2q^2)^{1/2} = pq$
\n(B). $AM \ge GM$
\n $\frac{a_1}{2} + \frac{a_1}{2} + a_2 + a_3 + a_4 + \dots + a_n}{n+1} \ge \left(\left(\frac{a_1}{2} \right)^2 \cdot a_2 \cdot a_3 \cdot a_4 \dots a_n \right)^{\frac{1}{n+1}}$
\n
$$
\left(\frac{1}{n+1} \right)^{n+1} \ge \frac{a_1^2 a_2 a_3 a_4 \dots a_n}{4}
$$
\nIf $2a -$ which

$$
\Rightarrow \frac{4}{(n+1)^{n+1}} \ge a_1^2 a_2 a_3 a_4 \dots a_n
$$

 $\frac{4}{3}$ = $\frac{1}{a_2} - \frac{1}{a_1}$ (22) (A). From the given conditions we have
 $\frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}$ (22) (A). From the given conditions we have
 $\cos^2 \beta = \cos \alpha \cos \gamma$

Squaring (1), 4sin²β = sin²α + sin²γ
 (O.B.-SOLUTIONS STUDY MATERIAL: MATHEMATICS
 $\Rightarrow \frac{4}{(n+1)^{n+1}} \ge a_1^2 a_2 a_3 a_4a_n$
 $\frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}(1)$ (22) (A). From the given conditions we have
 $2 \sin \beta = \sin \alpha + \sin \gamma$ (1)
 $\cos^2 \beta = \cos \alpha \cos \gamma$... **(O.B.- SOLUTIONS** STUDY MATERIAL: MATHEMATICS
 $\Rightarrow \frac{4}{(n+1)^{n+1}} \ge a_1^2 a_2 a_3 a_4a_n$
 $\frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}(1)$ (22) (A). From the given conditions we have
 \therefore and $\frac{1}{a_3} = \frac{1}{a_2} - \frac{1}{a_1}(1)$ (22) 1 2 3 4 n n 1 a a a aa (n 1) **(22) (A).** From the given conditions we have $2 \sin \beta = \sin \alpha + \sin \gamma$ (1) $\cos^2\beta = \cos \alpha \cos \gamma$ (2) Squaring (1), $4\sin^2\beta = \sin^2\alpha + \sin^2\gamma + 2\sin\alpha \sin\gamma$ Using (2) , $(1 - \cos \alpha \cos \gamma) = 1 - \cos^2 \alpha + 1 - \cos^2 \gamma + 2 \sin \alpha \sin \gamma$ \Rightarrow cos² α + cos² γ – 4cos α cos γ = 2 (sin α sin γ – 1) 2 2 cos cos 4cos cos ² 1 sin sin

$$
\Rightarrow \frac{\cos^2 \alpha + \cos^2 \gamma - 4 \cos \alpha \cos \gamma}{1 - \sin \alpha \sin \gamma} = -2
$$

(23) (A). $a_1 R^{m+n-1} = A$ (1) $a_1R^{m-n-1} = B$ (2) Dividing from (1) and (2) we get $R^{m+n-1-m+n+1} = A/B$

$$
R = \left(\frac{A}{B}\right)^{1/2n} \; ; \; a_1 = \frac{A}{R^{m+n-1}} = \frac{A}{\left(\frac{A}{B}\right)^{\frac{m+n-1}{2n}}}
$$

$$
= A^{\frac{n-m+1}{2n}} . B^{\frac{m+n-1}{2n}}
$$

now
$$
a_m = a_1 R^{m-1} = A^{\frac{n-m+1}{2n}} B^{\frac{m+n-1}{2n}} \left(\frac{A}{B}\right)^{\frac{m-1}{2n}}
$$

 $A^{1/2} B^{1/2} = \sqrt{AB}$

6 (24) (B). a, b, c are in H.P.
$$
\Rightarrow
$$
 b = $\frac{2ac}{a+c}$

⁼ 3 2 2 3 2 2 , 3 2 2 3 2 2 2 3 4 n n 1 2 3 4 n a a a a a a ^a 2 2 .a .a .aa n 1 2 1 2 3 4 n ¹ a a a aa n 1 4 log(2 1) log 4 ¹ ^x = 1 log(5.2 1) log 2 .1 log(5.2 1) log 2 2. x x × log2 1 2log 2 ¹ ^x ⁼ log(5.2 1) log(5.2 1)[log 2 log(5.2 1) 2log 2 x x x 10. t +2 = 2/t + 1 10 t² + 2t = 2 + t (2^x = t) 10 t² + t – 2 = 0 10t² + 5t – 4t – 2 = 0 5t (2t–1) – 2(2t+1) = 0 t = 2/5 , –1/2 (rejected) x log2 = log 2/5 2^x = 2/5 x log22 = 1 – log25 ; x = 1 – log25

(25) **(A).**
$$
\frac{S_{Kx}}{S_x} = \frac{\frac{Kx}{2}[2a + (Kx - 1)]}{\frac{K}{2}[2a + (x - 1)d]} = K \left[\frac{2a - d + Kxd}{2a - d + xd} \right]
$$

If $2a - d = 0$ then $\frac{S_{Kx}}{S_K} = K \left[\frac{Kxd}{xd} \right] = K^2$
which is possible when $a = d/2$

SEQUENCES & SERIES Q.B. - SOLUTIONS

(29) (A). Side of square $S_1 = 2r$ side of square

$$
S_2 = r \sqrt{2} = \frac{2r}{2} \left(\frac{1}{\sqrt{2}}\right)^{2-1} = 2r \left[\frac{1}{\sqrt{2}}\right]^{2-1}
$$
 : a, b,
side of square $S_3 = 2r \left(\frac{1}{\sqrt{2}}\right)^{3-1} = 2r \left(\frac{1}{\sqrt{2}}\right)^2$: b, c,
Adding

and so on,
\nside of square S_n =
$$
2r\left(\frac{1}{\sqrt{2}}\right)^{n-1}
$$

\n∴ radius = $r\left(2^{-\frac{1}{2}}\right)^{n-1} = r\left(2^{\frac{1-n}{2}}\right)$ and so on,
\nside of square S_n = $r\left(2^{-1/2}\right)^{n-1} = r\left(2^{\frac{1-n}{2}}\right)$
\n(C). Let T_k = 1, T'_k = -1, and r = 1, then T''_k = T_k + T'_k = 0
\n∴ T''_k cannot be a term of a G.P.
\n∴ statement is false.
\n(A). Coefficient of x⁴⁹ is equal = 1 + 2 + 3 + ... 50
\n $= \frac{50 \times 51}{2} = 25 \times 51 = 1275$.
\n(A). Statement – 1 is true as
\na. ar ... arⁿ⁻¹ = aⁿ. r<sup>1+2+... (n-1) = aⁿ. $\frac{n(n-1)}{r^2}$
\n= $\left(a^2 \cdot r^{n-1}\right)^{n/2}$</sup>

 $\sum_{r=1}^{5} a_r = \frac{9}{2} (1 + a_9)$ side of square $S_n = r \left(2^{-1/2} \right)^{n-1} = r \left(2^{-2} \right)$ \int $\sqrt{2}$ $\left(\begin{array}{c}\n1-n \\
1-n\n\end{array}\right)$ $-n$) 2 | \blacksquare $r \mid 2^{2}$

 $\frac{9}{2}(1+r^8) = 369 \implies r = \sqrt{3}$
 $\therefore T'_k$ cannot be a term of a G.P. **(30) (C).** Let $T_k = 1$, $T'_k = -1$, and $r = 1$, then $T''_k = T_k + T'_k = 0$ \therefore T''_k cannot be a term of a G.P. \therefore statement is false. o on,
 $\Gamma^{n}_{k} = T_{k} + T_{k} = 0$
 $3 + ... 50$
 $= 25 \times 51 = 1275.$

n. $\frac{n(n-1)}{r}$ on,
 $\begin{aligned}\n\mathbf{n}_k &= \mathbf{T}_k + \mathbf{T}_k = 0 \\
3 + \dots 50 \\
&= 25 \times 51 = 1275.\n\end{aligned}$
 $\frac{\mathbf{n}(\mathbf{n}-\mathbf{l})}{\mathbf{r}^2}$

(31) (A). Coefficient of x^{49} is equal = $1 + 2 + 3 + ... 50$

$$
=\frac{50\times51}{2}=25\times51=1275.
$$

(32) (A). Statement – 1 is true as

a. ar ... arⁿ⁻¹ = aⁿ. r¹⁺²⁺... (n-1) = aⁿ.
$$
\frac{n(n-1)}{r^2} = (a^2 \cdot r^{n-1})^{n/2}
$$

Statement -2 is also true as

 $(\sqrt{2}-1)^2$ of numbers, whose product is a². rⁿ⁻¹. and so on,
 $= r\left(2^{\frac{1-n}{2}}\right)$

1, then T''_k = T_k + T'_k = 0

1 + 2 + 3 + . . . 50
 $\frac{2(0 \times 51)}{2} = 25 \times 51 = 1275$.
 -1) = aⁿ. $\frac{n(n-1)}{2}$

a². r^{n-1})^{n/2}

aich is independent of k.

oning for statemen $(a. r^{k-1})$ $(a. r^{n-k}) = a^2 \cdot r^{n-1}$, which is independent of k. Statement – 2 is the correct reasoning for statement – 1, as in the product of a, ar, ar^2 ,... ar^{n-1} , there are n/2 groups r $\left(2^{\frac{1-n}{2}}\right)$

then T''_k = T_k + T'_k = 0

+ 2 + 3 + . . . 50
 \times 51

= 25 × 51 = 1275.

1) = aⁿ. $\frac{n(n-1)}{r}$

2. rⁿ⁻¹)^{n/2}

ch is independent of k.

ning for statement - 1,

-1, there are n/2 groups

r

(33) (D).
$$
27pqr \ge (p+q+r)^3 \Rightarrow pqr \ge \left(\frac{p+q+r}{3}\right)^3
$$

en T"_k = T_k + T'_k = 0

2 + 3 + ... 50
 $\frac{(51)}{2} = 25 \times 51 = 1275.$
 $= a^{n} \cdot \frac{n(n-1)}{r^2}$

rⁿ⁻¹)^{n/2}

i is independent of k.

ng for statement - 1,

there are n/2 groups

-1

1-1

2 + q¹ + r⁵ = 3. \Rightarrow G.M. \geq A.M. but A.M. \geq G.M. \therefore A.M. = G.M. \Rightarrow P = Q = r Given, $3p + 4q + 5r = 12 \implies p = q = r = 1$: $p^3 + q^4 + r^5 = 3$. Hence (D) is the correct answer.

(34) (B). For statement-1,

(32) (A). Statement – 1 is true as
\n
$$
1 + a)^2
$$
\n
$$
= (1 - a)(1 + a)
$$
\n
$$
= 0
$$
\n
$$
= \frac{1}{(1\sqrt{2})^{2-1}}
$$
\n(33) (B). For statement – 1,
\n
$$
1 = 2r \left(\frac{1}{\sqrt{2}}\right)^{2-1}
$$
\n
$$
= 2r \left(\frac{1}{\sqrt{2}}\right)^{2-1}
$$
\n
$$
= 2r \left(\frac{1}{\sqrt{2}}\right)^{2}
$$
\n

 t_1

(37) (D).

Q.B. - SOLUTIONS STUDY MATERIAL : MATHEMATICS

1. a, b, c, in A.P. \n2. b, c, in A.P. \n3. b, c, in A.P. \n4. c 120
\n
$$
x, b, c, d
$$
 in H P. $\therefore c > \frac{2bd}{b+d}(A.M. > H.M.)$ \n
$$
x, t, t = \frac{1}{2}[\frac{1}{1 \cdot 3}, \frac{1}{3}, t = \frac{1}{2}[\frac{1}{3 \cdot 7}, 1, t = \frac{1}{2}[\frac{1}{7}, \frac{1}{1 \cdot 3}] ...]
$$
\nAdding these after simplification
\n
$$
x + b, c, d
$$
 in H P. $\therefore c > \frac{2bd}{b+d}(A.M. > H.M.)$ \n
$$
x + t, t = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t}, t + ... + t] = \frac{1}{2}[\frac{1}{1 \cdot 2 + t},
$$

a c b d 1 1 1 1 t 2 1 3 , ² 1 1 1 t 2 3 7 , ³ 1 1 1 t 2 7 13 ----- ---- ⁿ 2 2 1 1 1 t ² n 1 n n n 1 Adding we get, + t² + t³ + + tⁿ ² 1 1 ¹ ² n n 1 ² 1 n(n 1) ² n n 1 (a) k k 1 0 sin (2k 1)x sin(2k 3)x a a dx sin x **=** 0 0 2sin 2 (k 1) x 2cos 2 (k 1)x dx 0 2 (k 1) 1 1 1 , , 1 log x 1 log r log x 1 2log r log x

for $k = 2, 3, 4 \implies a_1 = a_2 = a_3 = \dots$ \Rightarrow the sequence is a constant sequence. (b) Let r is the common ratio of G.P. $\log y = \log rx = \log r + \log x$ $\log z = \log r^2x = 2 \log r + \log x$

Hence,
$$
\frac{1}{1 + \log x}
$$
, $\frac{1}{1 + \log r + \log x}$, $\frac{1}{1 + 2\log r + \log x}$
are in H.P.

(38) (C), (39) (B), (40) (C).

Let four integers be $a-d$, a , $a+d$ and $a+2d$, where a and d are integers and $d > 0$. \therefore a + 2d = (a - d)² + a² + (a + d)² 2d² – 2d + 3a² – a = 0 (1) 2 2
 $\left(\frac{1}{2} \cos 2(k-1)x\right) dx = \frac{2 \sin 2(k-1)x}{2(k-1)} \Big|_0^{\pi} = 0$
 $k=2, 3, 4 \Rightarrow a_1 = a_2 = a_3 = \dots$

the sequence is a constant sequence.

Let r is the common ratio of GP.
 $y = \log rx = \log r + \log x$
 $z = \log r^2x = 2 \log r + \log x$
 $z = \log r^2x = 2 \log r + \log x$
 $k = a_{k-1} = \int_{0}^{\pi} \frac{\sin (2k-1)x - \sin (2k-3)x}{\sin x} dx$
 $\cos 2 (k-1)x dx = \frac{2 \sin 2 (k-1) x}{2 (k-1)} \Big|_{0}^{\pi} = 0$
 $k = 2, 3, 4 \Rightarrow a_1 = a_2 = a_3 = \dots$

exequence is a constant sequence.

et r is the common ratio of GP.
 $= \log x = \log r + \log x$
 $= \log r^2 x = 2$ Since d is positive integer \therefore 1 + 2a – 6a² > 0 $6a^2 - 2a - 1 < 0$ Are $2x, 3x \rightarrow 4a_1$ to 2 as mattemation
the sequence is a constant sequence.
Let r is the common ratio of GP.
 $y = log rx = log r + log x$
 $z = log r²x = 2 log r + log x$
 $z = log r²x = 2 log r + log x$
 $sin H.P$.
 $x = 2 log r + log x$
 $sin H.P$.
 $x = 2 log r + log x$
 $sin H.P$.
 $z = 2 cos x$ fr is the common ratio of GP.
 $\left[\log rx = \log r + \log x\right]$
 $\left[\log r^2x = 2 \log r + \log x\right]$
 $\left[\frac{1}{\sqrt{1 + \log x}}, \frac{1}{1 + \log r + \log x}, \frac{1}{1 + 2 \log r + \log x}\right]$
 $\left[\frac{1}{\log 2}\right]$, (40) (C).
 $\left[\frac{1}{\log 2}\right] = \frac{1}{2} \left[\frac{1 + \sqrt{1 + 2a - 6a^2}}{1 + \sqrt{1 + 2a - 6a^2$ k = 2, 3, 4 \Rightarrow a₁ = a₂ = a₃ =

e sequence is a constant sequence.

et r is the common ratio of GP.

= log r^x = 2 log r + log x
 \therefore i₁ + log x³ 1 + log x³ 1 + 2 log r + log x
 \therefore i₁ 3, $4 \Rightarrow a_1 = a_2 = a_3 = \dots$

uence is a constant sequence.

the common ratio of G.P.

rx = $\log r + \log x$
 $\frac{1}{\log x}, \frac{1}{1 + \log r + \log x}, \frac{1}{1 + 2\log r + \log x}$
 $\frac{1}{\log x}, \frac{1}{1 + \log r + \log x}, \frac{1}{1 + 2\log r + \log x}$

(40) (C).

(eggrs be a - d, a, a + ence, $\frac{1 + \log x}{1 + \log x}$, $\frac{1 + \log x}{1 + \log x}$, $\frac{1 + 2 \log x + \log x}{1 + 2 \log x + \log x}$

in H.P.

(39) (B), (40) (C).

(40) (C),

a + 2d = (a - d)² + a² + (a + d)²

a + 2d = (a - d)² + a² + (a + d)²

care integers and $d >$ 72 13 25 13 2 26

$$
\Rightarrow \frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6} \qquad \therefore \text{ a is an integer}
$$

$$
\therefore a = 0 \text{ Put in (2)}
$$

$$
\therefore d = 1 \text{ or } 0 \text{ but } \therefore d > 0
$$

$$
\therefore d = 1 \qquad \therefore \text{ The four numbers are } -1, 0, 1, 2
$$

(41) **(D).** Roots of
$$
x^2 + 13x + 36 = 0
$$
 are -4, -9

$$
\alpha = -\frac{13}{2}, \beta = -6, \gamma = -\frac{72}{13}
$$

Minimum distance between roots is $-\frac{72}{13} - \left(-\frac{13}{2}\right) = \frac{25}{26}$

Minimum value of δ is $\frac{25}{26}$ 25 and 26 an 26

(42) (A). Sum of roots
$$
\leq -6
$$

\n $\Rightarrow t^2 - 13t + \alpha + \gamma \leq -6$
\n $\Rightarrow t^2 - 13t + \alpha + \gamma + 6 \leq 0$
\n $\Rightarrow p = \ell + m = 13$

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SEQUENCES & SERIES Q.B. - SOLUTIONS

(43) (D). 2 = –13, p = 13 Equation x² – 169 = 0 **(44) (B).** n 5n G GG (1 1024) 2 1 2 n 2 5n = 2⁴⁵ n = 9 **(45) (B).** A A AA A 1025 171 1 2 3 m 1 m 2 1027 m 1025 171

(45) **(B).** A₁ + A₂ + A₃ +A_{m-1} + A_m = 1025 × 171
\n
$$
m\left(\frac{-2 + 1027}{2}\right) = 1025 \times 171 \text{ m} \quad m = 342
$$

$$
\therefore m\left(\frac{2+1627}{2}\right) = 1025 \times 171 \quad \therefore m = 342
$$

(46) (A). :
$$
n=9
$$
 : $r = (1024)^{9+1} = 2$: $G_1 = 2, r = 2$

$$
G_1 + G_2 + \dots + G_n = \frac{2.(2^9 - 1)}{2 - 1} = 1024 - 2 = 1022
$$
 Sum =

SEQUENCES & SERIES	Q.B. SOLUTIONS	Resimorphism
(43) (a) $2(2x-13, p=13$	$b_1 - \frac{1}{4}b_1^2 + \frac{1}{2}b_1$	
(44) (B) $G_1 G_2.....G_n = (\sqrt{x}1024)^n = 2^{5n}$	$b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2$	
(45) (B) $A_1 + A_2 + A_3 +A_{m-1} + A_m = 1025 \times 171$	$But. a_1 < b_1 > a_1 = 0$ and $b_1 = 2$	
(46) (A) $\because n = 9$ $\therefore r = (1024)^{\frac{1}{9} + 1} = 2$ $\therefore G_1 = 2, r = 2$		
(47) (A) Given: $c_n = a_1 + a_2 + a_3 + + a_n$	(1) 11. $t_k = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k(k+2)^2 - k^2(k+2)} = \frac{1}{2}(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{8}})$	
(49) (A) $\therefore n = 9$ $\therefore r = (1024)^{\frac{1}{9} + 1} = 2$ $\therefore G_1 = 2, r = 2$		
(A) \therefore Given: $c_n = a_1 + a_2 + a_3 + + a_n$	(1) 11. $t_k = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k(k+2)^2 - k^2(k+2)} = \frac{1}{2}(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{8}})$	
(47) (A) Given: $c_n = a_1 + a_2 + a_3 + + a_n$	(2) 111. <math< td=""></math<>	

Now,
$$
(5) - (4)
$$
, we get

$$
\frac{a_3 - a_2}{-d} = dp \left[\frac{a_3 - a_1}{-2d}\right]
$$
\n
$$
4p = 1 \Rightarrow p = 1/4
$$
\n
$$
4p = 1 \Rightarrow p = 1/4
$$
\n(48) (C). To find q : c_n = pa_n² + qa_n + r
\nOn putting n = 1, 2 in above equation, we get
\nc₁ = a₁ = pa₁² + qa₁ + r
\nc₂ = a₁ + a₂ = pa₂² + qa₂ + r
\n∴ 2a₁ + 2 = p(a₁ + 2)² + q(a₁ + 2) + r = (pa₁² + qa₁ + r
\nr) + 4a₁p + 4p + 2q (4p = 1)
\n2a₁ + 2 = c₁ + a₁ + 1 + 2q (∴ c₁ = a₁)
\n2a₁ + 2 = 2a₁ + 1 + 2q ⇒ q = \frac{1}{2}
\n(49) (C). If r = 0, then c₁ = pa₁² + qa₁ ⇒ a + b\sqrt{2} = a₁ + 2a₁ ⇒ a + b\sqrt{2} = a₁ + 2a₁ + 2a₁ ⇒ a₁ = 0 ⇒ a₁ = 0 or a₁ = 2
\na₁ = \frac{1}{4}a₁² + \frac{1}{2}a₁ (∴ c₁ = a₁)
\na₁² - 2a₁ = 0 ⇒ a₁ = 0 or a₁ = 2
\nAlso, d₁ = \frac{1}{4}b₁

(49) (C). If
$$
r = 0
$$
, then $c_1 = pa_1^2 + qa_1$

$$
a_1 = \frac{1}{4}a_1^2 + \frac{1}{2}a_1 \quad (\because c_1 = a_1)
$$

\n
$$
a_1^2 - 2a_1 = 0 \implies a_1 = 0 \text{ or } a_1 = 2
$$

\nAlso, $d_1 = \frac{1}{4}b_1^2 + qb_1 \qquad (\because q = \frac{1}{2} \text{ and } d_1 = b_1)$

EXERCISE3
\n
$$
\frac{x=-13, p=13}{x^2-169=0}
$$
\n
$$
\frac{b_1}{b_1} = \frac{1}{4}b_1^2 + \frac{1}{2}b_1
$$
\n
$$
\frac{b_1}{b_1} = \frac{1}{2}b_1^2 + \frac{1}{2}b_1
$$
\n
$$
\frac{b_1}{b_1} = \frac{1}{2}b_1^2 + \frac{1}{2}b_1
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n
$$
\frac{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}{b_1^2 - 2b_1 = 0 \Rightarrow b_1 = 0 \text{ or } b_1 = 2}
$$
\n**EXERCISE3**
\n**11.** $t_k = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k(k+2)^2 - k^2(k+2)} = \frac{1}{2}(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+2}})$
\n
$$
\therefore n = 9 \therefore r = (1024)^{\frac
$$

EXERCISE-3

If IONS

\n
$$
b_{1} = \frac{1}{4}b_{1}^{2} + \frac{1}{2}b_{1}
$$
\n
$$
b_{1}^{2} - 2b_{1} = 0 \Rightarrow b_{1} = 0 \text{ or } b_{1} = 2
$$
\nBut $a_{1} < b_{1} \Rightarrow a_{1} = 0$ and $b_{1} = 2$

\n**EXERCISE-3**

\n(1)

\n
$$
11. \quad t_{k} = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k(k+2)^{2} - k^{2}(k+2)} = \frac{1}{2} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+2}} \right)
$$
\n
$$
as k \to \infty
$$
\n
$$
Sum = \frac{1}{2} \left(1 + \frac{1}{\sqrt{2}} \right) = \frac{1 + \sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{1} + \sqrt{2}}{\sqrt{8}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}
$$

as k $\rightarrow \infty$

EXECUTE:
\n
$$
\frac{1}{2} \int_{0}^{2} \frac{1}{16} 2 \dots 6n = (\sqrt{1 \times 1024})^n = 2^{5n}
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = (\sqrt{1 \times 1024})^n = 2^{5n}
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 2^{45} \quad \therefore n = 9
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 2^{45} \quad \therefore n = 9
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 2^{45} \quad \therefore n = 9
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 2^{45} \quad \therefore n = 9
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 342
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 342
$$
\n
$$
\frac{1}{2} \int_{0}^{2} 2 \dots 6n = 2^{35} \quad \text{(1)} \quad \text{11. } t_k = \frac{(k+2)\sqrt{k} - k\sqrt{k+2}}{k(k+2)^2 - k^2(k+2)} = \frac{1}{2} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+2}} \right)
$$
\n
$$
\therefore n = 9 \quad \therefore r = (1024)^{9+1} = 2 \quad \therefore G_1 = 2, r = 2
$$
\n
$$
G_2 = 1024 - 2 = 1022
$$
\n
$$
G_3 = 1, 2, \dots, a_1
$$
\n
$$
G_4 = 2, G_5 = 2, G_6 = 2, G_7 = 2
$$
\n
$$
G_5 = 2, G_7 = 2, G_8 =
$$

$$
a_n = \frac{1}{2} (n^2 + n)
$$
 at $n = 10$, $a_n = 55 = k$

So sum of number is nth brackets = $k + k^2 + ...$ k^{2n+1}

$$
\begin{pmatrix}\n\frac{1}{2} & 2 & 5 \\
\frac{1}{2} & 2 & 5 \\
\frac{1}{2} & 2 & 5\n\end{pmatrix}
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix}
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 0 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 0 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 0 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 0 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 0 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 10 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 10 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = 10 \text{ or } b_1 = 2
$$
\n
$$
\begin{pmatrix}\n\frac{1}{2} & 1 \\
\frac{1}{2} & 2\n\end{pmatrix} = \frac{1}{2} \text{ or } \frac{1}{2
$$

 \Rightarrow a = 5, b = 3 and a + b = 8

45

(4) 4950. ² tan tan x tan x 12 12 12 2 tan tan x tan tan x tan tan x 12 12 12 tan . ¹² 1 tan tan x 1 tan tan x 1 tan tan x 12 12 12 2 4 2 2 2 tan tan tan x tan tan x 12 12 12 2 4 tan x tan 1 12 = 0 ; tan x = 0 x = k cos 2x = 1 x = n Sum of solutions is (1 + 2 + 3 + + 99) = 4950 k = 4950 **(5) 1.** ¹ 1 a r 1 r ⁼ and ² 2 a r 1 r ⁼ hence r¹ and r² are the roots of 1 r ⁼

$$
\frac{a}{1-r} = r \implies r^2 - r + a = 0 \implies r_1 + r_2 = 1
$$

(6) 6. Let the roots are $a - 3d$, $a - d$, $a + d$, $a + 3d$ sum of roots = $4a = 0 \implies a = 0$ hence roots are $-3d$, $-d$, d, 3d

product of roots =
$$
9d^4 = m^2 \Rightarrow d^2 = \frac{m}{3}
$$
(1)
\nAgain $\sum x_1x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2 =$
\n $-10d^2 = -(3m+2)$; $10d^2 = 3m+2$
\n $\frac{10m}{3} = 3m+2 = 10m = 9m+6$; $m = 6$
\n $= \frac{2}{11} - \frac{1}{21} + \frac{1}{3}$

smin of roots = -a - 0 − a
\nhence roots are - 3d, -d, d, 3d
\nproduct of roots = 9d⁴ = m² ⇒ d² =
$$
\frac{m}{3}
$$
(1)
\nAgain $\sum x_1x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2 =$
\n $-10d^2 = -(3m+2)$; $10d^2 = 3m+2$
\n $\frac{10m}{3} = 3m + 2 = 10m = 9m + 6$; m = 6
\n(7) 7, given: a₃ + a₅ + a₈ = 11
\na + 2d + a + 4d + a + 7d = 11
\na + 2d + a + 4d + a + 7d = 11
\nGiven: a₄ + a₂ = -2
\na + 3d = 11
\na + 13d = 11
\nGiven: a₄ + a₂ = -2
\na + 3d = 11
\na + 2d + a + 4d + a + 7d = 11
\na = -1 - 2d
\na = -1 - 2d
\nPut (2) in (1)
\n $3(-1-2d) + 13d = 11 \Rightarrow 7d = 14 \Rightarrow d = 2$ and a = -5
\nNow $a_1 + a_6 + a_7 \Rightarrow a + a + 5d + a + 6d$
\n $\Rightarrow 3a + 11d \Rightarrow 3a + 3d + 5d + a + 6d$
\n $\Rightarrow 3a + 11d = 3(-3a + 13d) = 11 \Rightarrow 7d = 14 \Rightarrow d = 2$ and a = -5
\nNow $a_1 + a_6 + a_7 \Rightarrow a + a + 5d + a + 6d$
\n $\Rightarrow 3a + 11d = 3(-3a + 11) \Rightarrow 7d = 14 \Rightarrow d = 2$ and a = -5
\n $\Rightarrow 3a + 11d = 3(-3a + 11) \Rightarrow 7d = 14 \Rightarrow d = 2$ and a = -5
\n $\Rightarrow 3a + 11d = 3(-3a +$

(8) 8.
$$
S = 4 \sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k = 4 \left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots \right]
$$

$$
= 4 \left[\frac{2/3}{1 - (2/3)}\right] = 8
$$

(9) 31. Let there be
$$
2n + 1
$$
 stones; i.e. n stones on each side of the middle stone. The man will run 20 m , to pick pick up the first stone and return, 40 m . for the second stone and

so on. So he runs $(n/2)$ $(2 \times 20 + (n-1)20) = 10n(n + 1)$ meters to pick up the stones on one side, and hence 20 $n(n + 1)$ m, to pick up all the stones.

$$
\therefore 20n(n+1) = 4800, or n = 15.
$$

\n
$$
\therefore there are 2n+1 = 31 stones
$$

(0.8. - SOOLUTIONS) STUDY MATERIAL : MATHEMATICS
\nso on. So he runs (n/2) (2 × 20 + (n – 1)20) = 10n(n + 1)
\nmeters to pick up the stones on one side, and hence 20
\n∴ 20n (n + 1) = 4800, or n = 15.
\n∴ 20n (n + 1) = 4800, or n = 15.
\n∴ 20n (n + 1) = 4800, or n = 15.
\n(10) 925. Let the 3 consecutive terms are
\n
$$
a - d
$$
, a, a + d d > 0
\nhence
\n $a^2 - 2ad + d^2 = 36 + K$ (1)
\n $a^2 + 2ad + d^2 = 596 + K$ (2)
\n $a^2 + 2ad + d^2 = 596 + K$ (3)
\nnow (2) – (1) gives
\n $d(2a - d) = 264$ (4)
\n(3) - (2) gives
\n $d(2a - d) = 264$ (4)
\n $d(3-4) = 296$ (5)
\n $e^{3/2} = 32$ ⇒ $d^2 = 16$ ⇒ $d = 4(d = -4$ rejected)
\nHence from (4)
\n $4(2a - 4) = 264$ ⇒ $2a - 4 = 66$ ⇒ $2a = 70$ ⇒ $a = 35$
\n∴ K = 35² - 300 = 1225 - 300 = 925
\n $\frac{k-1}{k-1}$
\n(11) 3. $S_k = \frac{k!}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$
\n $\frac{100}{k-2} | (k^2 - 3k + 1) \frac{1}{(k-1)!} | = \frac{100}{k-2} | \frac{(k-1)^2 - k}{(k-1)!} |$
\n $\frac{1}{2} | \frac{k-1}{(k-1)!} - \frac{k}{(k-1)!} | = \frac{1}{1!} - \frac{3}{2!} | + \frac{3}{2!} - \frac{4}{3!} + ...$
\n $=$

$$
\therefore K = 35^2 - 300 = 1225 - 300 = 925
$$

(11) 3.
$$
S_k = \frac{\frac{k-1}{k!}}{1 - \frac{1}{k}} = \frac{1}{(k-1)!}
$$

$$
(\frac{1}{2}) - \frac{1}{28} = 4950\pi
$$
\n
$$
(\frac{1}{2}) - \frac{1}{28} = 4050\pi
$$
\n
$$
(\frac{1}{2}) - \frac{
$$

$$
= \frac{2}{1!} - \frac{1}{2!} + \frac{2}{1!} - \frac{3}{2!} + \frac{3}{2!} - \frac{4}{2!} + \dots + \frac{99}{20!} - \frac{100}{20!} = 3 - \frac{100}{20!}
$$

(12) 0.
$$
a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \dots, a_{11}
$$
 are in A.P.

$$
\sum_{k=2}^{\infty} \left| \frac{k-1}{(k-1)!} - \frac{k}{(k-1)!} \right| = \sum_{k=2}^{\infty} \left| \frac{k-1}{(k-1)!} \right|
$$
\n
$$
= \sum \left| \frac{k-1}{(k-2)!} - \frac{k}{(k-1)!} \right| = \left| \frac{2}{1!} - \frac{3}{2!} \right| + \left| \frac{3}{2!} - \frac{4}{3!} \right| + \dots
$$
\n
$$
= \frac{2}{1!} - \frac{1}{0!} + \frac{2}{1!} - \frac{3}{2!} + \frac{3}{2!} - \frac{4}{3!} + \dots + \frac{99}{98!} - \frac{100}{99!} = 3 - \frac{100}{99!}
$$
\n**0.** $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \dots, a_{11}$ are in A.P.
\n
$$
\therefore \frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90
$$
\n
$$
\Rightarrow 225 + 35d^2 + 150d = 90
$$
\n
$$
\Rightarrow 35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7
$$
\nGiven $a_2 < \frac{27}{2}$ \therefore $d = -3$ and $d \neq -9/7$ \n
$$
\Rightarrow \frac{a_1 + a_2 + \dots + a_{11}}{11} = \frac{11}{2}[30 - 10 \times 3] = 0
$$
\n**9.**
$$
\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6 + (5n - 1) d]}{\frac{n}{2}[6 + (n - 1) d]} = \frac{5[(6 - d) + 5nd]}{[(6 - d) + nd]}
$$
\n
$$
d = 6 \text{ or } d = 0.
$$
\nNow, if $d = 0$ then $a_2 = 3$ else $a_2 = 9$

$$
\Rightarrow 35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7
$$

Given
$$
a_2 < \frac{27}{2}
$$
 : $d = -3$ and $d \neq -9/7$

$$
\Rightarrow \frac{a_1 + a_2 + \dots + a_{11}}{11} = \frac{11}{2} [30 - 10 \times 3] = 0
$$

(13) **9.**
$$
\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2} [6 + (5n - 1) d]}{\frac{n}{2} [6 + (n - 1) d]} = \frac{5 [(6 - d) + 5nd]}{[(6 - d) + nd]}
$$

 $d = 6$ or $d = 0$. Now, if $d = 0$ then $a_2 = 3$ else $a_2 = 9$

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 θ | y_1 y_2

 $\frac{x_2}{x_3}$

 (x_2,y_2)
(x₁,y₁)

 $\overline{x_1}$

(14) 25. a_1, a_2, a_3be in H.P \Rightarrow $\frac{1}{\cdot}$, $\frac{1}{\cdot}$, $\frac{1}{\cdot}$ be in A.P. **EXERIES**

1₃...........be in H.P
 (2) (C). Let first term
 \therefore a + ar + ar² +
 $\frac{1}{1-r}$, $\frac{1}{a_2}$, $\frac{1}{a_3}$ be in A.P.
 $T_1 = \frac{1}{a_1} = \frac{1}{5}$ and $T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$
 $\Rightarrow \frac{1}{1-r^2} = 100 =$ **& SERIES**

a₃..........be in H.P
 (2) (C). Let first term of G.P. is 'a' an
 \therefore a + ar + ar² + $\infty = 20$ (g
 $\frac{1}{4_1}, \frac{1}{4_2}, \frac{1}{4_3}...$ be in A.P.

2. T₁ = $\frac{1}{4_1} = \frac{1}{5}$ and T₂₀ = $\frac{1}{$ **& SERIES**

, a₃...........be in H.P

(2) (C). Let first term of G.P. is 'a' and
 \therefore a + ar + ar² + $\infty = 20$ (giv
 $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$... be in A.P.

P. T₁ = $\frac{1}{a_1} = \frac{1}{5}$ and T₂₀ = $\$ CES & SERIES
 a_1, a_2, a_3be in H.P
 $\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$ be in A.P.
 $\text{in A.P. } T_1 = \frac{1}{a_1} = \frac{1}{5} \text{ and } T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$
 $\Rightarrow T_{20} = T_1 + 19d$
 $\frac{1}{25} = \frac{1}{5} + 19d \Rightarrow d = -\frac{4}{19 \times 2$ ES

be in H.P

(2) (C). Let first term of G.P. is 'a' and
 \therefore a + ar + ar² + $\infty = 20$ (giv
 $\frac{1}{1-\frac{1}{1-\frac{1}{5}}}$ and T₂₀ = $\frac{1}{a_{20}} = \frac{1}{25}$
 \Rightarrow $\frac{a^2}{1-r^2} = 100 \Rightarrow a^2 = 100(1-r^2)$
 $19d \Rightarrow d = -\frac{4}{1$ \Rightarrow T₂₀ = T₁ + 19d $\times 25$ \rightarrow $(20(1-1))$ $T_n = T_1 + (n-1) d < 0$ $\Rightarrow \frac{1}{5} - \frac{(n-1)\cdot 4}{19 \times 25} < 0 \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25 \times 19}$ $\Rightarrow \frac{5 \times 19}{4} + 1 < n \Rightarrow \frac{99}{4} < n$ **EXERIES**

3............be in A.P.
 $T_1 = \frac{1}{a_1} = \frac{1}{3}$ and $T_20 = \frac{1}{a_{20}} = \frac{1}{25}$
 $a_0 = T_1 + 19d \Rightarrow d = -\frac{4}{19 \times 25}$
 $T_1 = T_1 + (n-1) d < 0$
 $T_2 = \frac{1}{3} + 19d \Rightarrow d = -\frac{1}{19 \times 25}$
 $T_3 = \frac{1}{3} + 19d \Rightarrow d = -\frac{4}{19 \times 25}$ \Rightarrow Least positive integer n is 25. **(15) 5.** Clearly, $1+2+3+...+n-2 \le 1224 \le 3+4+...$ n $\Rightarrow \frac{(n-2)(n-1)}{2} \le 1224 \le \frac{(n-2)}{2}(3+n)$ \implies n² – 3n – 2446 \leq 0 and n² + n – 2454 \geq 0 \Rightarrow 49 < n < 51 \Rightarrow n = 50 $\therefore \frac{24+1}{2} - (2k+1) = 1224$ a
 $\frac{a_1}{2s} = \frac{1}{5} + 194 \Rightarrow d = -\frac{4}{19 \times 25}$
 $\frac{1}{2s} = \frac{1}{5} + 194 \Rightarrow d = -\frac{4}{19 \times 25}$
 $\frac{1}{2s} = \frac{1}{5} + 194 \Rightarrow d = -\frac{4}{19 \times 25}$
 $\frac{1}{2s} = \frac{1}{5} + 194 \Rightarrow d = -\frac{4}{19 \times 25}$
 $\frac{1}{2s} = \frac{(20(1 - r)^2 + 100(1 - r)}{(1 - r)^2 - 100($ $+1)$ (21 1) 1221 $\Rightarrow k = 25 \Rightarrow k - 20 = 5$ **(16) 4.** $\frac{1}{H} = \frac{1}{2} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{2 \cdot 2\beta} \times \frac{5 + \sqrt{2}}{2 \cdot 2\beta} \times \frac{1}{2} = \frac{1}{4}$ **(4) (B).** \because **x**₁, **x**₂, **x**₃ is 25 3
 $\frac{1}{2}$
 $\frac{1}{3} - \frac{(n-1)\cdot 4}{3} < 0 \Rightarrow \frac{1}{3} < \frac{4(n-1)}{2 \times 5} < 0 \Rightarrow \frac{1}{3} < \$ T_n = T₁ + (n - 1) d < 0

⇒ $\frac{1}{3} - \frac{(n-1)\cdot 4}{19 \times 25} < 0 \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25 \times 19}$

⇒ $\frac{5 \times 19}{4} + 1 <$ n ⇒ $\frac{99}{4} <$ n

⇒ Least positive integer n is 25.

⇒ Least positive integer n is 25.

⇒ Least positive $rac{1}{25} = \frac{1}{5} + 194 \Rightarrow d = -\frac{4}{19 \times 25}$
 $T_n = T_1 + (n-1) d < 0$
 $T_n = T_1 + (n-1) d < 0$
 $T_2 = 19 \times 25$
 $T_3 = 19 \times 25$
 $T_4 = 14 \text{ N} \Rightarrow \frac{99}{4} < 1$
 $T_5 = 1224 \text{ N} \Rightarrow \frac{1}{4} < 1$
 $T_6 = 2 \text{ N} \Rightarrow 1244 \text{ N} \Rightarrow T_6 = 2 \text{ N} \Rightarrow T_7 = 2 \text$ $H = 4$ **(17) 1.** $\log_4(x-1) = \log_2(x-3)$ $\Rightarrow \frac{\log (x-1)}{\log x} = \frac{\log (x-2)}{\log 2} \Rightarrow \frac{\log (x-1)}{2\log 2} = \frac{\log (x-2)}{\log 2}$ of x_1, x_2, x_3 $\frac{5 \times 19}{4} + 1 <$ n $\frac{9}{4} <$ n
 \Rightarrow Lead tristic integer n is 25.

Clearly, 1+2+3+...+n -2 ≤ 1224 ≤ 3+4+... π
 \Rightarrow Clearly, 1+2+3+...+n -2 ≤ 1224 ≤ 3+4+... π
 \Rightarrow a + 2d- 7 \Rightarrow a + 2d- 7 \Rightarrow a + 2d- 7
 \Rightarrow and 5 19×25 3 25×19
 5×19
 5×19

Let first term of A.P. is a and common

Learly, $1 + 2 + 3 + ... + n - 2 \le 1224 \le 3 + 4 + ...$

Let first term of A.P. is a and common

Learly, $1 + 2 + 3 + ... + n - 2 \le 1224 \le 3 + 4 + ...$
 \Rightarrow 49×1 $\Rightarrow \log(x-1) = 2 \log(x-2)$ \Rightarrow $x-1=(x-2)^2 \Rightarrow x^2-7x+10=0 \Rightarrow x=5,2$ Also, $x - 1 > 0$ and $x - 3 > 0$ \Rightarrow x > 1 and x > 3 \Rightarrow x = 5 is the solution. ⇒ $k = 25 \Rightarrow k = 20 = 5$
 $\frac{1}{2} = \frac{1}{2} (\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{5 + 2\sqrt{5}} \times \frac{1}{2} = \frac{1}{4}$
 $= \frac{1}{2} (\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{8 + 2\sqrt{5}} \times \frac{1}{2$:. $\frac{n(n+1)}{2} - (2k+1) = 1224$

⇒ $k = 25 \Rightarrow k - 20 = 5$

⇒ $S_{20} = \frac{20}{2} [2 \times (-1) + (20 - 1)]$

⇒ $S_{20} = \frac{20}{2} [2 \times (-1) + (20 - 1)]$
 $\frac{1}{2} = \frac{1}{2} (\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{8 + 2\sqrt{5}} \times \frac{1}{2} =$ $\frac{1}{2}$
 $\Rightarrow k = 25 \Rightarrow k - 20 = 5$
 $\frac{1}{2} = \frac{1}{2} (\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2}}{8 + 2\sqrt{5}} \times \frac{1}{2} = \frac{1}{4}$
 $\frac{1}{2} = \frac{1}{2} (\frac{1}{\alpha} + \frac{1}{\beta}) = \frac{\alpha + \beta}{2\alpha\beta} = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \times \frac{5 + \sqrt{2$

EXERCISE-4

(1)
$$
(C) \t1^{3} - 2^{3} + 3^{3} - \dots + 9^{3}
$$

\n
$$
= (1^{3} + 2^{3} + 3^{3} + \dots + 9^{3}) - 2(2^{3} + 4^{3} + 6^{3} + 8^{3})
$$

\n
$$
= \left(\frac{9(9+1)}{2}\right)^{2} - 2 \cdot 2^{3} (1^{3} + 2^{3} + 3^{3} + 4^{3})
$$

\n
$$
= (9 \times 5)^{2} - 16 \left(\frac{4 \times (4+1)}{2}\right)^{2}
$$

\n
$$
= (45)^{2} - 16 \times (10)^{2} = 2025 - 1600 = 425
$$

\n
$$
\{ \because 1^{3} + 2^{3} + \dots + n^{3} = \left(\frac{n(n+1)}{2}\right)^{2} \}
$$

1 1 1 1 T and T a 5 a 25 1 1 4 19d d 25 5 19 25 1 (n 1) 4 1 4 (n 1) ⁰ 5 19 25 5 25 19 5 19 99 1 n n (n 2) (n 1) (n 2) 1224 (3 n) 2 2 4 5 5 2 1 1 5 2 8 2 5 2 4 log (x 1) log (x 2) 2log 2 log 2 ² 4 (4 1) ⁼ ² n (n 1) } **(2) (C).** Let first term of G.P. is 'a' and common ratio is r a + ar + ar² + = 20 (given) ^a ²⁰ 1 r a = 20 (1 – r) (1) and a² + a² r² + a² r⁴ + = 100 (given) 2 2 ^a ¹⁰⁰ 1 r a 2 = 100 (1 – r²) (2) From (1) put value of a in (2) [(20 (1 – r)]² = 100 (1 – r²) 400 (1 – r)² = 100 (1 – r) (1 + r) 4 (1 – r) = 1 + r 5r = 3 r = 3/5 **(3) (C).** In an A.P. T³ = 7 and T⁷ = 3T³ + 2 (according to question) = 3 × 7 + 2 = 23 S20 = ? Let first term of A.P. is a and common difference is d. T³ = 7 a + 2d = 7 a = 7 – 2d (i) and T⁷ = 3T³ + 2 = 23 a + 6d = 23 (ii) From (i) put value of a in (ii) we get 7 – 2d + 6d = 23 4d = 16 d = 4 & a = 7 – 2 × 4 = – 1 Now, Sⁿ ⁼ ⁿ 2 [2a + (n – 1) d] S20⁼ ²⁰ 2 [2 × (–1) + (20 – 1) × 4] = 10 × (–2 + 76) = 10 × 74 = 740 , x² , x³ are in G.P. Let common ratio be r 2 3 1 2 x x r x x (1) and y¹ , y² , y³ also are in G.P. with same common ratio as of x¹ , x² , x³ 2 3 1 2 y y r y y (2) 2 2 3 3 1 2 1 2 x y x x y y 2 1 ³ 2 1 3 x x ^x y y y (3) (x ,y) 2 2 (x ,y) 3 3

y³

 \Rightarrow point (x₁, y₁), (x₂, y₂), (x₃, y₃) lie on a straight line by (8) graph.

 $\tan \theta = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3}$

This is possible only when they are in straight line but

CO.B.-SOLUTIONS
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3}$

this is possible only when they are in straight line but
 $\frac{1}{2} = \frac{y_2}{x_1} = \frac{y_3}{x_2}$
 $\frac{y_1}{x_3} = \frac{y_2}{x_3}$
 $\frac{y_2}{x_1} = \frac{y_3}{x_2$ **(5) (D).** System of linear equation, $x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$ has non-zero solution

$$
\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} \text{ (from (3))}
$$
\n(5) (b) System of linear equation,
\n $x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$
\nhas non-zero solution
\n $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 4c & c \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 4c & c \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 4c & c \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 4c & c \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & a \\ 1 & 4c & c \\ 1 & 4c & c \end{vmatrix} = 0$ \n $\begin{vmatrix} 1 & 2a & 1 \\ 1 & 4c &$

Put the value of d in (2) we get

$$
a = \frac{1}{mn} \quad \therefore \quad a - d = \frac{1}{mn} - \frac{1}{mn} = 0
$$

(8) (B). According to question if n is even sequence will be

1 2 ³ 1 2 3 y y ^y x x x 1 2 3 y y ^y x x x (from (3)) 1 2a a 1 3b b 0 1 4c c 1 2a a 0 3b 2a b a 0 0 4c 2a c a 1 1 1 , , a b c are in A.P. m 1 1 n m m n mn 1 1 a d 0 mn mn 1 2 + 2.2² + 3² + 2.4² + 5² + 2.6² + + 2.n² ⁼ ² n(n 1) 2 {n even} If n is odd sequence will be 1 2 + 2.2² + 3² + 2.4² + 5² + 2.6² + + n² 1 2 + 2.2² + 3² + 2.4² + 5² + 2.6² + + 2 (n – 1)² + n² (n – 1) term (which is even) ⁼ ² 2 (n 1) (n 1 1) ⁿ 2 ⁼ ² 2 (n 1) n n 2 ⁼ ² ² n 1 n (n 1) n 1 2 2 {sum of n even terms is ² n(n 1) 2 sum of (n – 1) even term is (n – 1) 2 (n 1 1) 2 } **(9) (D).** ⁿ n 0 x a = a⁰ + a¹ + a² + = 1 + a + a² + 1 x 1 x a 1 a x (1) and ⁿ n 0 y b = b⁰ + b¹ + b² + 1 y 1 y b 1 b y (2) and ⁿ n 0 z c = c⁰ + c¹ + c² + 1 z 1 z c 1 c z (3)a, b, c are in in A.P. x 1 y 1 z 1 , , x y z are also in A.P. 1 1 1 1 ,1 ,1 x y z are also in A.P. 1 1 1 , , x y z are in A.P. (subracting 1from each) 1 1 1 , , x y z are in A.P. (multiplying with – 1 each) x, y, z are in H.P.

(3) (B.2-SOLUTIONS

\n(10) (B).
$$
\Delta = \frac{1}{2}BC
$$
AD $\Rightarrow AD = \frac{2\Delta}{BC}$

\nAlitude $AB = \frac{2\Delta}{a}$

\nSimilarly altitude BE = $\frac{2\Delta}{b}$

\nSimilarly altitude BE = $\frac{2\Delta}{b}$

\nSimilarly, a It is the $AB = c$

\nand $CF = \frac{2\Delta}{c}$

\nThus, $\Delta = \frac{2\Delta}{a}$

\nThus, $\Delta = \frac{2\Delta}{b}$

\nThus, $\Delta = \frac{2\Delta}{b}$

\nThus, $\Delta = \frac{2\Delta}{b}$

\nThus, $\Delta = \frac{2\Delta}{b}$

\nThus, $\Delta = \frac{2\Delta}{a}$

\nThus, $\Delta = \frac{1}{b}$

\nThus, $\$

 $\{\cdot : a_1$ is first term and d is common difference}

$$
\frac{a_1 + (\frac{p-1}{2})d}{a_1 + (\frac{q-1}{2})d} = \frac{p}{q}
$$

\n
$$
\frac{p-1}{2} = 5 \text{ and } \frac{q-1}{2} = 20
$$

\nthen L.H.S. of (i) represents ratio of a₆ and a₂₁
\n∴ p = 11 and q = 41
\nNow (i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_21} = \frac{11}{41}$
\n
$$
\frac{a_1 + 5d}{a_1 a_2 a_3, \dots, a_n \text{ are in H.P}}
$$

\n
$$
\frac{1}{a_1} + \frac{1}{a_2} \cdot \frac{1}{a_3}, \dots, \frac{1}{a_n} \text{ are in A.P.}
$$

\nLet common difference of A.P. is d.
\n
$$
\frac{a_1 - a_n}{a_1} = a_1 a_n (n - 1)
$$

\n
$$
\frac{a_1 - a_n}{a_2 - a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = \frac{1}{a_1} \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n
$$
\therefore \frac{1}{a_2 - a_1} = 4 \Rightarrow \frac{a_1 - a_2}{a_1} = a_1 a_2
$$

\n<

if
$$
\frac{p-1}{2} = 5
$$
 and $\frac{q-1}{2} = 20$ (13)

then L.H.S. of (i) represents ratio of a_6 and a_{21} \therefore p = 11 and q = 41

Now (i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_1} = \frac{11}{41}$ $+20d$ 41 a_{21} 41 (15) (B). $S = \frac{1}{2}$
 $\frac{1}{2} = 5$ and $\frac{q-1}{2} = 20$

L.H.S. of (i) represents ratio of a_6 and a_{21}
 $\frac{S}{3} = \frac{1}{3} + \frac{1}{3}$

(i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$
 $a_1, a_2, a_3,$ $\frac{1}{2}$ = 5 and $\frac{q-1}{2}$ = 20

L.H.S. of (i) represents ratio of a₆ and a₂₁

(i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_21} = \frac{11}{41}$

(i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_21} = \frac{11$ (i) becomes, $\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$
 $a_1, a_2, a_3, \dots, a_n$ are in H.P
 $\frac{1}{2}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in A.P.

formmon difference of A.P. is d.
 $\frac{1}{a} = \frac{1}{a_1} + (n-1) d \Rightarrow \frac{1}{$

(12) (C). $a_1, a_2, a_3, \dots, a_n$ are in H.P

$$
\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}
$$
 are in A.P.

Let common difference of A.P. is d.

$$
\therefore \frac{1}{a_n} = \frac{1}{a_1} + (n-1) d \Rightarrow \frac{1}{a_n} - \frac{1}{a_1} = (n-1) d
$$

$$
\Rightarrow \frac{a_1 - a_n}{d} = a_1 a_n (n-1) \qquad \qquad \dots \dots \dots (A)
$$

(C).
$$
a_1, a_2, a_3, \dots, a_n
$$
 are in H.P
\n $\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in A.P.
\nLet common difference of A.P. is d.
\n $\therefore \frac{1}{a_n} = \frac{1}{a_1} + (n-1) d \Rightarrow \frac{1}{a_n} - \frac{1}{a_1} = (n-1) d$ (16) (A). 1
\n $\Rightarrow \frac{a_1 - a_n}{d} = a_1 a_n (n-1)$ (A)
\n $\therefore \frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow \frac{a_1 - a_2}{d} = a_1 a_2$ (i)
\n $\frac{1}{a_3} - \frac{1}{a_2} = d \Rightarrow \frac{a_2 - a_3}{d} = a_2 a_3$ (ii)
\n $\frac{49}{a_3}$

$$
\frac{1}{a_3} - \frac{1}{a_2} = d \Rightarrow \frac{a_2 - a_3}{d} = a_2 a_3
$$
(ii)

$$
\frac{1}{a_4} - \frac{1}{a_3} = d \Rightarrow \frac{a_3 - a_4}{d} = a_3 a_4
$$
\n
$$
\frac{1}{a_n} - \frac{1}{a_{n-1}} = d \Rightarrow \frac{a_{n-1} - a_n}{d} = a_n a_{n-1}
$$
\n
$$
\frac{1}{a_4 a_2} + a_2 a_3 + a_3 a_4 + \dots + a_n a_{n-1}
$$
\n
$$
= \frac{1}{d} [a_1 - a_2 + a_2 - a_3 + a_3 - a_4 + \dots + a_{n-1} - a_n]
$$

$$
\frac{1}{a_n} - \frac{1}{a_{n-1}} = d \Rightarrow \frac{a_{n-1} - a_n}{d} = a_n a_{n-1}
$$
 (n)

Adding all column wise

MS
\n
$$
\frac{1}{a_4} - \frac{1}{a_3} = d \Rightarrow \frac{a_3 - a_4}{d} = a_3 a_4
$$
\n........(iii)
\n
$$
\frac{1}{a_n} - \frac{1}{a_{n-1}} = d \Rightarrow \frac{a_{n-1} - a_n}{d} = a_n a_{n-1}
$$
\n............ (n)
\nAdding all column wise
\n
$$
a_1 a_2 + a_2 a_3 + a_3 a_4 + + a_n a_{n-1}
$$
\n
$$
= \frac{1}{d} [a_1 - a_2 + a_2 - a_3 + a_3 - a_4 + + a_{n-1} - a_n]
$$
\n
$$
= \frac{1}{d} [a_1 - a_n]; \text{ From (A)} = a_1 a_n (n - 1)
$$
\n(D). Let first term of G.P. is a & common ratio is r and then G.P. is a, ar, ar², ar³, ar⁴...... arⁿ⁻¹
\nAccording to question,
\n
$$
a = ar + ar2 \Rightarrow 1 = r + r2 \Rightarrow r2 + r - 1 = 0
$$
\n
$$
= \frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \sqrt{5}}{2} \{:: r > 0\}
$$
\n(A). Let first term of G.P. is a and common ratio is r
\n \therefore G.P. is a, ar, ar², ar³.
\nAccording to question, $a + ar = 12$ (1)
\nand ar² + ar³ = 48

(13) (D). Let first term of G.P. is a & common ratio is r and then G.P. is a, ar, ar^2 , ar^3 , ar^4 ar^{n-1} According to question, $a = ar + ar^2 \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$ (iii)

......... (n)

+ a_{n-1} - a_n]

()

n ratio is r and then

= 0

<u>1+ $\sqrt{5}$ </u> {: r > 0}

2 (1)

....... (2) 1 1 r 2 [a₁ - a_n]; From (A) = a₁a_n (n - 1)

Let first term of GP. is a & common ratio is r and then

is a, ar, ar², ar³, ar⁴ ar¹⁻¹

ording to question,

ar + ar² ⇒ 1 = r + r² ⇒ r² + r - 1 = 0
 $\$ 1; From (A) = a₁a_n (n - 1)

term of G.P. is a & common ratio is r and then

ar², ar³, ar⁴ arⁿ⁻¹

o question,
 $\frac{2}{3}$ = 1 = r + r² ⇒ r² + r - 1 = 0
 $\frac{1}{(n-1)} = \frac{-1 \pm \sqrt{5}}{2}$ \Rightarrow r = $\frac{-1 + \$ 1₁ - a₂ + a₂ - a₃ + a₃ - a₄ + + a_{n-1} - a_n]

1₁ - a_n]; From (A) = a₁a_n (n - 1)

21 first term of GP, is a & common ratio is r and then

a, ar, ar², ar³, ar⁴ arⁿ⁻¹

1ing to **D).** Let first term of GP. is a & common ratio is r and then

GP. is a, ar, ar², ar³, ar⁴, arⁿ⁻¹

According to question,
 $a = ar + ar^2 \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$
 $\frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \$ 1 a₁ - a_n 1; From (A) = a₁ a_n (n - 1)

1. Let first term of GP. is a & common ratio is r and then

2. is a, ar, ar², ar³, ar⁴ arⁿ⁻¹

cording to question,
 $\frac{\pm \sqrt{1-4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac$

$$
r = \frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \sqrt{5}}{2} \ \{ \because \ r > 0 \}
$$

(14) (A). Let first term of G.P. is a and common ratio is r $\frac{1 + (\mathbf{p} - 1)\mathbf{d}}{1} = \frac{\mathbf{p}^2}{2}$ \therefore GP. is a, ar, ar², ar³. $2 \quad \text{According to question,} \quad a + ar = 12 \quad \dots (1)$ $1 + (q-1)q$ q^2 Acc A.P.

According to question,
 $a = ar + ar^2 \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$
 $r = \frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \sqrt{5}}{2}$ {: r > 0
 $\frac{p}{2} \frac{p}{2} \left(\frac{2[2a_1 + (p-1)d]}{2} \right) = \frac{p^2}{q^2}$

(14) (A). Let first term of GP. is and $ar^2 + ar^3 = 48$ \ldots (2) According to question,
 $a = ar + ar^2 \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$
 $r = \frac{-1 \pm \sqrt{1 - 4.1(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{-1 + \sqrt{5}}{2} \{:: r > 0\}$
 (14) (A). Let first term of G.P. is a and common ratio is r
 \therefore G.P. is a, ar, ar², ar³ 4.1(-1) = $\frac{-1 \pm \sqrt{5}}{2}$ ⇒ $r = \frac{-1 + \sqrt{5}}{2}$ { : r > 0}

rst term of G.P. is a and common ratio is r

, ar, ar², ar³.

g to question, a + ar = 12(1)

ar³ = 48(2)

eq. (1) by (2) we get $\frac{1}{r^2}$ 3.P. is a, ar, ar², ar³, ar⁴ arⁿ⁻¹
According to question,
a = ar + ar² \Rightarrow 1 = r + r² \Rightarrow r² + r - 1 = 0
According to question,
 2
A. Let first term of G.P. is a and common ratio is r
 \therefore G.P.

Dividing eq. (1) by (2) we get $\frac{1}{r^2} = \frac{1}{4} \Rightarrow r = \pm 2$ \therefore terms are alternately +ve and –ve \therefore r = – 2

Put value of r in (1) we get $a + a(-2) = 12$; $a = -12$

(15) (B).
$$
S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{10}{3^4} + \dots
$$
 (i)

$$
\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots
$$
 (ii)

Subtracting (ii) from (i) we get

3. a
$$
\frac{1}{4}
$$
, $\frac{1}{4}$ and $\frac{1}{4}$ and $\frac{1}{2}$ and $\frac{1}{$

(16) (A). Till 10^{th} minute number of counted notes = 1500

d
$$
\frac{A}{2} = 20
$$

\n(i) represents ratio of a₆ and a₂₁
\n $1 = 41$
\n $= 8, \frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_21} = \frac{11}{41}$
\n $= 8, \frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_21} = \frac{11}{41}$
\n $= 8, \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots$
\n $= 8, \frac{1}{3} + \frac{1}{3^2} + \frac{4}{3^3} + \dots$
\n $= \frac{4}{3} + \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$
\n $= \frac{4}{3} + \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$
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\n $= \frac{4}{3} + \frac{4}{3} + \frac{4}{3} + \dots$
\n<

(17) (D). a = Rs. 200 ; d = Rs. 40 Savings in first two months = Rs. 400 Remained savings = 200 + 240 + 280 + upto n terms 200n + 20n² – 20n = 10640 20n² + 180n – 10640 = 0 n 2 + 9n – 532 = 0 (n + 28) (n – 19) = 0 n = 19 no. of months = 19 + 2 = 21 . **(18) (B).** Tⁿ = (n – 1)² + (n – 1) n + n² ⁼ 3 3 (n 1) n T¹ = 1³ – 0³ T² = 2³ – 1³ ^T20 = 20³ – 19³ ^S20 = 20³ – 0³ = 8000 **(19) (D).** 100 (a + 99d) = 50 (a + 49d) 2a + 198 d = a + 49d a + 149d = 0 T150 = a + 149d = 0 **(20) (C).** ³ 7 77 777 upto 20 terms 10 100 ¹⁰ 3 2 3 ²⁰ 1 1 ¹ 10 10 ⁷ 20 9 1 1 10 **(21) (A).** 2y = x + z 2 tan–1 y = tan–1 x + tan–1 (z) 1 1 2 tan tan 1 y 1 xz 2 x z x z 1 y 1 xz y² = xz or x + z = 0 x = y = z **(22) (D).** Let numbers be a, ar, ar² Now, 2 (2ar) = a + ar² [a 0] 4r = 1 + r²

3 3 ((n 1) n) n (n 1) 1 11 111 7 upto 20 terms 10 100 ¹⁰ 7 9 99 999 upto 20 terms 9 10 100 1000 7 1 1 1 1 1 1 upto 20 terms 9 10 10 10 ²⁰ 7 1 1 20 1 9 9 10 ²⁰ 7 179 1 1 7 ²⁰ [179 (10)] 9 9 9 10 81 2y x z r² – 4r + 1 = 0 r = 2 3 r = 2 3 (Positive value) **(23) (C).** S = 10⁹ + 2 (11)¹ (10)⁸ + + 10 . 11⁹ ¹¹ ^S 10 11¹ . 10⁸ + + 9 . 11⁹ + 11¹⁰ ¹ ^S ¹⁰ = 10⁹ + 11¹ . 10⁸ + 11² . 10⁷ + 11⁹ – 11¹⁰ ¹ ^S ¹⁰ 10 9 10 ¹¹ ¹ ¹⁰ 10 11 ¹¹ ¹ 10 ¹ ^S ¹⁰ ¹¹10 – 1010 – 11¹⁰ S = 10¹¹ = 100 . 109k = 100 **(24) (A).** 2 2 2 n 2 n (n 1) ² (n 1) 1 t [n 2n 1] ⁿ 4 4 1 n (n 1) (2n 1) 2 (n) (n 1) ¹ 4 6 2 1 9 10 19 9 10 9 96 4 6 **(25) (A).** n m 2 **;** n 2m (1) 1/4 1 ⁿ ^G ; 2/4 2 ⁿ ^G ; 3/4 3 ⁿ ^G Now, G¹ 4 + 2G² 4 + G³ 4 2 3 4 2 4 n n n 2 () = n 3 + 2n2 2 + n3= 2n2 2 + nl (n² + 2)= 2n² 2 + n((n +)² – 2n) = n(n +)² = n .(2m)² = 4 nm² **(26) (A).** a + d, a + 4d, a + 8d (a + 4d)² = (a + d) (a + 8d) a 2 + 16d² + 8ad = a² + 9ad + 8d² 8d² = ad ; a = 8d, d 0 ; a 4d 12d 4 r a d 9d 3 **(27) (A).** 2 2 2 2 ⁿ 8 12 16 20 ^S 5 5 5 5 ¹⁶ 2 2 2 2 2 S [2 3 4 5 11] ²⁵ ¹⁶ 2 2 2 2 2 2 [1 2 3 4 5 11 1] ²⁵ ¹⁶ ¹⁰¹ ⁵

SEQUENCES & SERIES Q.B. - SOLUTIONS

(28) (D). 225 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$ $225 a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$ $(15a)^{2} + (3b)^{2} + (5c)^{2} = 45ab + 75ac + 15bc$ $15a = 3b = 5c = k$ **& SERIES**
 $x^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$
 $x^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
 $x^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $x = 3b = 5c = k$
 $x = \frac{k}{15} = \frac{k}{15}$, $b = \frac{k}{3} = \frac{5k}{15}$, $c = \frac{k}{5} = \frac{3k}{15}$
 $= 2c$; b, c, a are in A. $a = \frac{R}{1.5} = \frac{R}{1.5}$, $b = \frac{R}{2} = \frac{5R}{1.5}$, $c = \frac{R}{5} = \frac{5R}{1.5}$ **EXAMPLE SOLUTIONS**

15 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a + c)$
 $5a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a + c)$
 $5a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
 $a = 3b = 5c = k$
 $a = 3b = 5c = k$
 $\frac{k}{15} = \frac{k}{15}$, $b = \frac{k}{3} = \frac{5k}{15}$, $c = \frac{k}{5} = \frac{3k}{1$ $a + b = 2c$; b, c, a are in A.P. **(29) (A).** $a_1 + a_5 + a_9 + \dots + a_{49} = 416$ \Rightarrow a + 24d = 32 ……(i) $a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ (ii) From (i) and (ii) $d = 1$ and $a = 8$ Now, $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$ $\sum_{r=1}^{17} (8 + (r-1)^2 - 140m) \sum_{r=1}^{17} (7 + r)^2 - 140m$ 225 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$

225 $a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
 $(15a)^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $15a = 3b = 5c = k$
 $a = \frac{k}{15} = \frac{k}{15}$, $b = \frac{k}{3} = \frac{5k}{15}$, $c = \frac{k}{5} = \frac{3k}{15}$
 $a+b=2c$; b, c, a are in **S & SERIES**

(**O.B.- SOLUTIONS**
 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$
 $a^2 + 9b^2 + 25c^2 - 75ac + 45ab + 15bc$
 $a^2 + 9b^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $a^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $a^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $b = 2c$; b, e, **CES & SERIES**
 O.B. SOLUTIONS

225 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a + c)$
 $S_C = 5 \text{mm of numbers between } 225 a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
 $\text{divisible by both } 7 \& 13.$
 $15a = 3b = 5c = k$
 $a = \frac{k}{15} = \frac{k}{15}$, $b = \frac{k}{3} = \frac{5k}{15}$, $c = \frac{k}{5} = \frac{$ $4760 = 140 \text{ m} \Rightarrow \text{m} = 34$ **(30) (D).** A = $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + \dots$ A $^2 + 2 \cdot 20^2$ $=(1^2+2^2+3^2+4^2+...+20^2)+(2^2+4^2+...+20^2)$ Extraces

Extraces
 $225 \text{ a}^2 + 9b^2 + 25c^2 - 75ac = 15b (3a+c)$
 $25a^2 + 9b^2 + 25c^2 - 75ac = 15b (3a+c)$
 $25a^2 + 9b^2 + 25c^2 - 75ac = 15b (3a+c)$
 $25a^2 + 9b^2 + 25c^2 - 75ac = 15b (3a+c)$
 $3b_1 + 8b_1 = 8c$
 $5a = 3b = 5c = k$
 $k = k$
 $k = k$
 $= 2870 + 1540 = 4410 = 2870 + 1540 = 4410$ Are 3b = 5c = k
 $x_0 = 3$ = $\frac{1}{15}$ = $\frac{1}{15}$, b = $\frac{1}{8}$ = $\frac{5}{15}$, c = $\frac{1}{8}$ = $\frac{3}{15}$ + c = $\frac{1}{8}$ = $\frac{3}{15}$ + c = $\frac{1}{8}$ = $\frac{5}{15}$ + c = $\frac{1}{8}$ = $\frac{5}{15}$ + c = $\frac{1}{8}$ = $\frac{1}{2$ $= 540 \times 41 + 41 \times 280 = 41 \times 820 = 33620$ $33620 - 8820 = 110\lambda$; $100\lambda = 24800$; $\lambda = 248$ **(31) (D).** $\frac{b}{r}$, b, br \rightarrow G.P. **(|r|** \neq 1) Given a + b $r^{(1)}$ \Rightarrow b/r + b + br = xb \Rightarrow b = 0 (not possible) or $1 + r + \frac{1}{r} = x \implies x - 1 = r + \frac{1}{r}$ $\frac{1}{r} = x \implies x - 1 = r + \frac{1}{r}$ $1 \qquad \qquad \blacksquare$ r and the state of the stat \Rightarrow x-1>2 or x-1 < -2 \Rightarrow x>3 or x < -1 So x can't be '2' **(32) (D).** $S = a_1 + a_2 + \dots$ S = 15 (a₁ + a₃₀) = 15 (a₁ + a₁ + 29d) $T = a_1 + a_3 + \dots + a_{29}$ $=(a_1)+(a_1+2d) \dots + (a_1+28d)$ $= 15a_1 + 2d(1 + 2 + \dots + 14)$ $T = 15a_1 + 210$ d. Now use $S - 2T = 75$ \Rightarrow 15 (2a₁ + 29d) – 2 (15a₁ + 210 d) = 75 \Rightarrow d = 5 Given $a_5 = 27 = a_1 + 4d \implies a_1 = 7$ Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$ **(33) (B).** S_A = sum of numbers between 100 & 200 which are divisible by 7. \Rightarrow S_A = 105 + 112 + + 196 $S_A = \frac{14}{2} [105 + 96] = 2107$ $SB = Sum of numbers between 100 \& 200 which are$ divisible by 13.

> $S_B = 104 + 117 + ... + 195 = \frac{8}{2} [104 + 195] = 1196$ $\frac{1}{2}$ [104 + 195] = 1196

Q.B. - SOLUTIONS

Sc² - 75ac = 15b (3a+c)

Sc² = 75ac + 45ab + 15bc

Sc² = 75ac + 45ab + 15bc

(sc)² = 45ab + 75ac + 15bc

sc₂ = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

sc₂ = $\frac{1}{3} = \frac{5k}{$ **(Q.B. - SOLUTIONS**

Sb (3a+c)

5 15 (3a+c)

15 15 (3a+c)

5 15 (34)
 $S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$
 $S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$
 $S = \$ = 15b (3a+ c)

+45ab + 15bc

+45ab + 15bc

divisible by both 7 & 13.

c = $\frac{k}{5}$ = $\frac{3k}{15}$

(34) (B). S = $\sum_{k=1}^{20} \frac{1}{2^k}$

P.

(34) (B). S = $\sum_{k=1}^{20} \frac{1}{2^k}$

P.

(35) (C). a, b, c in GP.

 $k =$ **(O.B. SOLUTIONS**

Sb (3a + c)

Sb (3a + c)

Sc = Sum of numbers between 100 & 200 which are

divisible by both 7 & 13.

Sc = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\frac{1}{5} = \frac{3k}{15}$

(34) **(B).** S = **(O.B.-SOLUTIONS**
 EDENTADE FRAME SERVE AND SET UP:
 $c = \frac{k}{5} = \frac{3k}{15}$
 $c = \frac{k}{5} = \frac{3k}{15}$

(34) **(B)**. $S = \frac{20}{k+1} = \frac{1}{2^k}$
 $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 \cdots .(i)
 $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{$ 5 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$
 $a^2 + 9b^2 + 25c^2 - 75ac = 45b(3a+c)$
 $a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a+c)$
 $a^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$
 $a = 3b = 5c = k$
 $\frac{1}{15} = \frac{1}{15}$, $b = \frac{k}{3} = \frac{5k}{15}$, $c = \frac{k}{5} = \frac{3k}{15}$
 EXERIES
 EXERCISE
 EXERCISE 1902-2502-75ac = 15b (3a+ 5b) **6**
 EXERCISE 1904-2502-75ac = 15b (3a+ 5b)
 EXECUTIONS
 $x^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$
 $x^2 + 3b^2 - 5ca + 45ab + 75ac + 15bc$
 $x^2 - 8b + 75ac + 15bc$
 $x^2 - 8b$ La = 30 x = $\frac{1}{2}$ = $\frac{1}{15}$ + $\frac{1}{15}$ + (3b)²+(5c)²=45ab + 75ac + 15bc

= 5c = k

= $\frac{k}{15}$, b = $\frac{5}{3} = \frac{5k}{15}$, c = $\frac{k}{5} = \frac{3k}{15}$

(34) (B). S = $\frac{20}{k-1} = \frac{1}{2^{k}}$
 $\frac{k}{15}$, b = $\frac{5}{3} = \frac{5k}{15}$, c = $\frac{k}{5} = \frac{3k}{15}$
 $\frac{34}{15}$
 $\$ $\frac{1}{244-32}$ = 66 ⇒ a + 25d = 33
 $\frac{1}{24}$ + a₄₃ = 66 ⇒ a + 25d = 33

om (i) and (ii) d = 1 and a = 8
 $\frac{1}{24}$ + a₄₃ = 66 ⇒ a + 25d = 33
 $\frac{1}{24}$ + a₄₃ = 6 ⇒ a + 25d = 33
 $\frac{1}{24}$ + a₄₂ + a + a + a² $S = \frac{1}{2} + \frac{2}{2} + \frac{3}{3} + ... + \frac{20}{20}$
 $a_9 + ... + a_{21} = 140$
 $a_1 + a_2^2 + ... + a_{21}^2 = 140m$
 $a_2 + a_3 + a_2^2 + ... + a_{21}^2 = 140m$
 $a_3 + a_3^2 + ... + a_{21}^2 = 140m$
 $a_4 + a_2^2 + ... + a_{21}^2 = 140m$
 $a_5 + a_6^2 + ... + a_{21}^2 = 140m$
 $a_7 + a_8^2$ + + a³⁰ ; 1 30 200² + (220²
 $\frac{2(3-2) + (2^2 + 4^2 + ... + 20^2)}{8 \text{ satisfies } 24^2 + 20 \times 4 - 2}$
 $= -\frac{11 \times 21}{6}$
 $= \frac{11 \times 21 \times 41}{6$ (35) (C).a, b, c in GP.
 $x^2 + 220^2$
 $x = -r$ statisties $ax^2 + 2bx + c = 0 \Rightarrow x = -r$
 $\times 21$
 $\times 21$
 $\times 3620$
 $\times 3620$
 $\times 3620$

(36) (A). $S_n = 50n + \frac{n(n-7)}{b}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$

(36) (A). $S_n = 50n + \frac{n(n-7)}{2}A$
 S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13. $S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$ **(34) (B).** $S = \sum_{i=1}^{20} \frac{1}{i}$ $20₁$ k^k SUPERMOUNTED LEARNING

SUBRADYANCED LEARNING

SIDE by both 7 & 13.

= 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\frac{20}{k=1} \frac{1}{2^k}$
 $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^$ $=\sum_{k=1}^{\infty} \frac{1}{2^k}$ **EXECUTE ARNING**

1 of numbers between 100 & 200 which are

by both 7 & 13.
 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\frac{2}{2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{1}{2} + \frac{2}{2^3} + ... + \frac{19}{2^{20}} + \frac{20}{2^{21}}$
 $= \frac{1}{2}$ S_C = Sum of numbers between 100 & 200 which are

sites by both 7 & 13.

S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

S = $\frac{20}{k=1} \frac{1}{2^k}$

S = $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$

S $\times \frac$ SUM ANXINEED LEARNING

SUM of numbers between 100 & 200 which are

sible by both 7 & 13.
 $22 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$
 $\frac{20}{2} = \frac{1}{2^k}$
 $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3}$ SUPERBURBANNING

SUPERBANNING

SUPERBAN COM ADVANCED LEARNING

ODM ADVANCED LEARNING

by both 7 & 13.
 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\frac{2}{2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{1}{2} + \frac{2}{2^3} + ... + \frac{19}{2^{20}} + \frac{20}{2^{21}}$
 $= \frac{1}{2} + \frac{1}{2^2} + ... + \$ **1**

S_C = Sum of numbers between 100 & 200 which are

divisible by both 7 & 13.

S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

S = $\frac{20}{k} \frac{1}{2^k}$

S = $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 EXECUTE 2
 SODM ADVANCED LEARNING

SODM ADVANCED LEARNING

visible by both 7 & 13.
 $z = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$
 $= \sum_{k=1}^{20} \frac{1}{2^k}$
 $= \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\times \frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + ... + \frac{1$ **EDIMADVANCED LEARNING**

DOM ADVANCED LEARNING

7 & 13.
 $\therefore (91, n) > 1 = S_A + S_B - S_C = 3121$
 $\therefore + \frac{20}{2^{20}}$
 $\therefore + \frac{19}{2^{20}} + \frac{20}{2^{21}}$
 $\therefore + \frac{1}{2^{20}} - \frac{20}{2^{21}} \Rightarrow S = 2 - \frac{11}{2^{19}}$
 $x + c = 0 \Rightarrow x = -r$

mon root, satisfies Sommably Settled DIERRINING

Sommably South 7 & 13.

Signification 1 20 U which are
 $15\left(2\pi\right) = 182 \Rightarrow \text{H.C.F. } (91, \text{n}) > 1 = \text{S}_A + \text{S}_B - \text{S}_C = 3121$
 $\Rightarrow \sum_{k=1}^{20} \frac{1}{2^k}$
 $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{2$ Sum of numbers between 100 & 200 which are

sum of numbers between 100 & 200 which are

sible by both 7 & 13.
 $182 \Rightarrow \text{H.C.F.} (91, \text{n}) > 1 = S_A + S_B - S_C = 3121$
 $\frac{20}{2} \frac{1}{2}$
 $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 \frac S_C = Sum of numbers between 100 & 200 which are

divisible by both 7 & 13.

S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

S = $\frac{20}{k+1} \frac{1}{2^k}$

S = $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$

S NGEDILEARNING
 $S_C = 3121$
 $S = 2 - \frac{11}{2^{19}}$ EDILEARNING

Which are
 $C = 3121$
 $= 2 - \frac{11}{2^{19}}$ **(35) (C).** a, b, c in G.P. say a, ar, ar^2 Satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$ $x = -r$ is the common root, satisfies second equation d $(-r)^2 + 2e(-r) + f = 0$ $\Rightarrow d \cdot \frac{c}{a} - \frac{2ce}{b} + f = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$ So = Sum of numbers between 100 & 200 which are

divisible by both 7 & 13.

So = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

S = $\sum_{k=1}^{20} \frac{1}{2^k}$

S = $\frac{1}{2^2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$

S = = Sum of numbers between 100 & 200 which are

is bile by both 7 & 13.

= 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\sum_{k=1}^{20} \frac{1}{2^k}$
 $\frac{1}{2^k} \frac{1}{2^k}$
 $\frac{1}{2^k} \frac{1}{2^k} \frac{2}{3^2} + \frac{3}{3^2} + ... + \frac{2$ Solution of numbers between 100 & 200 which are

visible by both 7 & 13.
 $z = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121$
 $\frac{2}{k-1} \frac{1}{2^k}$
 $= \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\times \frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + ... + \frac{19}{$ EDIMADVANCED LEARNING

dDIMADVANCED LEARNING

3.

n) > 1 = S_A + S_B - S_C = 3121

20
 $\frac{20}{2^{20}}$
 $+ \frac{1}{2^{20}} - \frac{20}{2^{21}}$
 $+ \frac{1}{2^{20}} - \frac{20}{2^{21}}$ \Rightarrow S = 2 - $\frac{11}{2^{19}}$

= 0 \Rightarrow x = -r

coot, satisfies se EDIMENTARINE

etween 100 & 200 which are

3.

n) > 1 = S_A + S_B - S_C = 3121
 $\frac{20}{2^{20}}$
 $\frac{19}{2^{20}} + \frac{20}{2^{21}}$
 $+ \frac{1}{2^{20}} - \frac{20}{2^{21}} \Rightarrow S = 2 - \frac{11}{2^{19}}$
 $= 0 \Rightarrow x = -r$

oot, satisfies second

r) + f = 0
 $\frac{$ EDENTIFICATION ANCEDE EXERUNG

Version 100 & 200 which are
 $y>1=S_A+S_B-S_C=3121$
 $\frac{1}{0}$
 $\frac{20}{0}$
 $\frac{1}{2^{20}}-\frac{20}{2^{21}} \Rightarrow S=2-\frac{11}{2^{19}}$
 $0 \Rightarrow x=-r$

ot, satisfies second
 $+ f=0$
 $+\frac{f}{c}=\frac{2e}{b}$ Sc Servent to the servent to x 200 which are

divisible by both 7 & 13.

S_C = 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121

(34) (B). $S = \sum_{k=1}^{20} \frac{1}{2^k}$
 $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{(1-7)}{2}A$ = Sum of numbers between 100 & 200 which are

sisible by both 7 & 13.

= 182 \Rightarrow H.C.F. (91, n) > 1 = S_A + S_B - S_C = 3121
 $\sum_{k=1}^{20} \frac{1}{2^k}$
 $\frac{1}{2^k} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$
 $\frac{1}{2} = \frac{1}{2^2} + \$ $T_n = S_n - S_{n-1}$ $\sum_{k=1}^{\infty} \frac{1}{2^{k}}$ $= \frac{1}{2} + \frac{2}{2^{2}} + \frac{3}{3^{2}} + ... + \frac{20}{2^{20}}$ $\times \frac{1}{2} = \frac{1}{2^{2}} + \frac{2}{2^{3}} + ... + \frac{19}{2^{20}} + \frac{20}{2^{21}}$ $- \frac{1}{2}$ $\sum_{k=1}^{\infty} S = \frac{1}{2} + \frac{1}{2^{2}} + ... + \frac{1}{2^{20}} - \frac{20}{2^{21}} \Rightarrow S = 2 - \frac{11}{2^{19}}$ b, c i $=50n+\frac{n (n-7)}{2}A-50 (n-1)-\frac{(n-1) (n-8)}{2}A$ S = $\frac{20}{\sum \frac{1}{2}} \frac{1}{2^{k}}$

S = $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$

S $\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^2} + ... + \frac{19}{2^{20}} + \frac{20}{2^{21}}$
 $\left(1-\frac{1}{2}\right)$ S = $\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{20}} - \frac{20}{2^{21}} \Rightarrow S = 2 - \frac{11}{2^{19}}$ $\frac{20}{(20)}$
 $\frac{19}{(220)} + \frac{20}{(220)} = \frac{20}{(220)}$
 $+ \frac{1}{220} - \frac{20}{(220)} = 10$
 $= 0 \Rightarrow x = -1$

oot, satisfies second
 $x + f = 0$
 $\frac{d}{dx} + \frac{f}{c} = \frac{2e}{b}$

50 (n -1) - $\frac{(n-1)(n-8)}{2}$ A
 $+ 9n - 8$] = 50 + A (n -4)

(n $S = 2 - \frac{11}{2^{19}}$
 $\Rightarrow S = 2 - \frac{11}{2^{19}}$
 \Rightarrow coord
 $\Rightarrow \frac{-1 (n-8)}{2} A$
 $+ A (n-4)$
 $A (n-5) = A$ $\frac{20}{2^{20}}$
 $-\frac{19}{2^{20}} + \frac{20}{2^{21}}$

... + $\frac{1}{2^{20}} - \frac{20}{2^{21}}$ ⇒ S = 2 - $\frac{11}{2^{19}}$

 $z = 0$ ⇒ x = -r

root, satisfies second

(-r) + f = 0

 $\frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

A

A

A

2 + 9n - 8] = 50 + $= 50 + {A \over 2}[n^2 - 7n - n^2 + 9n - 8] = 50 + A(n-4)$ $\frac{\text{A}}{2}$ [n² – 7n – n² + 9n – 8] = 50 + A (n – 4) $d = T_n - T_{n-1}$ = 50 + A (n – 4) – 50 – A (n – 5) = A $T_{50} = 50 + 46A$ $(d, A_{50}) = (A, 50 + 46A)$ **(37) (A).** $a-d+a+a+d=33 \implies a=11$ $a (a^2-d^2) = 1155$ $121 - d^2 = 105$ $d^2 = 16 \Rightarrow d = \pm 4$ If $d = 4$ then Ist term = 7 If $d = -4$ then Ist term = 15 $T_{11} = 7 + 40 = 47$ OR $T_{11} = 15 - 40 = -25$ **(38) (B).** $T_r = r(2r - 1)$ $S = \Sigma 2r^2 - \Sigma r$ S_n = 50n + $\frac{n(n-7)}{2}$ A

T_n = S_n - S_{n-1}

= 50n + $\frac{n(n-7)}{2}$ A - 50 (n - 1) - $\frac{(n-1)(n-8)}{2}$ A

= 50 + $\frac{A}{2}$ [n² - 7n - n² + 9n - 8] = 50 + A (n - 4)

d = T_n - T_{n-1} = 50 + A (n - 4) - 50 - A (n - $\frac{(n-7)}{2}A$
 $-7n-n^2+9n-8]=50+A(n-4)$
 $=50+A(n-4)-50-A(n-5)=A$
 $-33 \Rightarrow a=11$
 155
 $5 + 46A$
 $-1 = 33 \Rightarrow a = 11$
 155
 $5 = 44$
 $st term = 7$
 $Ist term = 15$
 $Ist term = 15$
 $-40 = -25$

r
 $-40 = -25$
 $t = 44$
 $-40 = -25$
 $t = 24$
 -2 $\frac{2}{b} + f = 0 \Rightarrow \frac{a}{a} + \frac{1}{c} = \frac{2b}{b}$
 $\frac{5}{b} + f = 0 \Rightarrow \frac{a}{a} + \frac{1}{c} = \frac{2b}{b}$
 $\frac{5}{b} - S_{n-1}$
 $\frac{1}{a} + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$
 $\frac{A}{2}[n^2 - 7n - n^2 + 9n - 8] = 50 + A(n-4)$
 $\frac{-1}{n-1} = 50 + A(n-4) - 50 - A(n-5) = A$ a b $h^{-1} = 0 \rightarrow a^{-1}e^{-b}$
 $h = 50n + \frac{n(n-7)}{2}A$
 $= S_n - S_{n-1}$
 $50n + \frac{n(n-7)}{2}A - 50 (n-1) - \frac{(n-1)(n-8)}{2}A$
 $50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8] = 50 + A(n-4)$
 $= T_{n-1}T_{n-1} = 50 + A(n-4) - 50 - A(n-5) = A$
 $\frac{1}{60} = 50 + 46A$
 $A_{50} = (A, 50$ $\Gamma_n = S_{n-1}$
 $\Gamma_n = S_{n-1}$
 $= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$
 $= 50 + \frac{\lambda}{2} [n^2 - 7n - n^2 + 9n - 8] = 50 + A(n-4)$
 $d = T_{n} - T_{n-1} = 50 + A(n-4) - 50 - A(n-5) = A$
 $T_{S0} = 50 + 46A$
 $(d, A_{S0}) = (A, 50 + 46A)$
 $= 4 - 4 + a + d = 33 \Rightarrow a = 1$

$$
S_{11} = \frac{2}{6}(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946
$$

Using $AM \ge GM$; $f(x) \ge 3$

(39) (A). 3 3 3 ⁿ 2 2 2 (3 (n 1) 2 (1 2 ... n) ^T (1 2 n) 3 n (n 1) (n 1) (n 1) n (n 1) n (n 1) 2 2 ⁿ n (n 1) (n 2) ^S 2 S10 = 660 **(40) (C).** a 1 + a⁴ + a⁷ + a10 + a13 + a16 = 114 6 2 (a¹ +a16) = 114. a 1 + a16 = 38 a 1 + a⁶ + a¹¹ + a16 = 4 = 4 2 (a¹ + a16) = 2 × 38 = 76 **(41) (D).** ¹⁵ 3 3 3 n 1 1 2 ... n 1 15 16 Sum 1 2 n 2 2 15 n 1 n (n 1) ⁶⁰ 2 15 n 1 6 15 16 17 60 620 6 **(42) (B).** b = ar c = ar² 3a, 7b and 15 c are in A.P. 14b = 3a + 15c ; 14 (ar) = 3a + 15 ar² 14r = 3 + 15r2 ; 15r² – 14r + 3 = 0 (3r – 1) (5r – 3) = 0 ; r = 1/3, 3/5. O n l y a c c e p t a b l e v a l u e i s r = 1 / 3 , b e c a u s e r (0, 1/2] c. d = 7b – 3a = 7ar – 3a 4 th term = 15 c – **(43) (A).** a 1 + a⁷ + a16 = 40 a + a + 6d + a + 15d = 40 3a + 21d = 40 ⁴⁰ a 7d 3 ^S15⁼ ¹⁵ 2 (2a + 14d) = 15 (a + 7d); 15 **(44) (C). (45) 1540.** 2

n (n 1) (n 2 (n 1)) ⁶⁰ 7 2 a 3a a 3 3 2 15 2 a a a a 3 9 3 ⁴⁰ S 15 200 ³ 20 20 k 1 k 1 k (k 1) 1 k k 2 2 1 20 (21) (41) 20 (21) 2 6 2 1 420 41 20 21 2 6 2 1 [2870 210] 1540 2 **(46) (D).** ^T10⁼ ¹ ²⁰ = a + 9d …..(i) ^T20⁼ ¹ ¹⁰ = a + 19d …..(ii) 1 1 a , d 200 200 200 200 2 199 201 1 S 100 2 200 200 2 2 **(47) 504.** 7 3 2 n 1 1 (2n 3n n) ⁴ ² 1 7 8 7 8 15 7 8 2 3 4 2 6 2 1 4 [2 × 49 × 16 + 28 × 15 + 28] 1 4 [1568 + 420 + 28] = 504 **(48) (A).** ¹ 1 1 1 ⁴ 16 48 128 2 4 8 16 ... to ¹ 2 3 4 ⁴ 16 48 128 2 2 2 2 ... ¹ 1 1 1 ⁴ 8 16 32 2 2 2 2 ... 1 1 1 1 1/4 ... 4 8 16 32 1 1/2 1/2 2 (2) 2 **(49) (D).** 100 2n 1 n 1 a 200 ^a 3 + a⁵ + a⁷ + + a201 = 200 2 200 2 (r 1) ar 200 (r 1)

EXECUTES 8 SERIES
\n(a)
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\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + + a_{200} = 100
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\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + + a_{200} = 100
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\sum_{n=1}^{100} a_{2n} = \frac{20}{2} [2 \times 7 + (19) 10] = 10 [14 + 190]
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T_n \le 407 \Rightarrow 23 + (n-1) \ 28 \le 407
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\Rightarrow n \le 14.71 ; n = 14
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5 & SERIES
\na_{2n} = 100
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\Rightarrow a_2 + a_4 + a_6 + + a_{200} = 100
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